initial utility : $U_0 = 0$ for all states.

The states for this problem represents the number of fish in lake from 0 to 10.

Given growth rates and their probabilities:

$P(R=1) = 0.2$

$P(R=1.25) = 0.3$

$P(R=1.5) = 0.3$

$P(R=1.75) = 0.2$

For state $s=0$

$$U_{i+1}(s) = R(s) + \gamma \sum_{s'} P(s'|s, \pi_{i}(s)) . U_i(s')$$

$U_1(s=0) = 0$ and $U_2(s=0) = 0$ since there is no fish.

First iteration

$$U_{i+1}(s) = R(s) + \gamma \sum_{s'} P(s'|s, \pi_i(s)) . U_i(s')$$

1. $s=1$, there is 1 fish in lake

$\pi_0(s=1) = 0$, 0 fish are caught

$R(s)$ is the immediate reward, which is the number of fish caught

$R(s=1) = 0$

Remaining fish $= 1$, so next year's population:

For $R=1 \rightarrow 1$

For $R=1.25 \rightarrow 1$

For $R=1.5 \rightarrow 2$

For $R=1.75 \rightarrow 2$

$$\sum_{s'} P(s'|s=1, \pi_0(s=1)) . U_0(s') = 1 \times 0.2 \times 0 + 1 \times 0.3 \times 0 + 2 \times 0.3 \times 0 + 2 \times 0.2 \times 0 = 0$$

$$U_1 = R(s=1) + 0.9 \sum_{s'} P(s'|s=1, \pi_0(s=1)) . U_0(s') = 0$$

* $\pi_0(s=1) = 1$, 1 fish is caught

$R(s=1) = 1$

No fish left, so next year's population is 0.

$$\sum_{s'} P(s' | s=1, \pi_0(s=1)) \cdot U_0(s') = 0$$

$$U_1 = R(s=1) + 0.9 \sum_{s'} P(s' | s=1, \pi_0(s=1)) \cdot U_0(s') = 1$$

(2) s=2 , there is 2 fish in lake

* $\pi_0(s=2) = 0$, 0 fish are caught

$R(s=2) = 0$

$$U_1 = R(s=2) + 0.9 \sum_{s'} P(s' | s=2, \pi_0(s=2)) \cdot U_0(s') = 0$$

* $\pi_0(s=2) = 1$, 1 fish is caught

$R(s=2) = 1$

$$U_1 = R(s=2) + 0.9 \sum_{s'} P(s' | s=2, \pi_0(s=2)) \cdot U_0(s') = 1$$

* $\pi_0(s=2) = 2$, 2 fish is caught

$R(s=2) = 2$

$$U_1 = R(s=2) + 0.9 \sum_{s'} P(s' | s=2, \pi_0(s=2)) \cdot U_0(s') = 2$$

As we can see, for the first iteration, utilities are equal to the immediate reward which is the number of fish that were caught because initial utility is 0.

So, utilities from first iteration are (we take the maximum utility):

- $U_1(s=0) = 0$
- $U_1(s=1) = 1$
- $U_1(s=2) = 2$
- $U_1(s=3) = 3$
- $U_1(s=4) = 4$
- $U_1(s=5) = 5$
- $U_1(s=6) = 6$
- $U_1(s=7) = 7$
- $U_1(s=8) = 8$
- $U_1(s=9) = 9$
- $U_1(s=10) = 10$

Second Iteration

$U_{i+1}(s) = R(s) + \gamma \sum_{s'} P(s'|s, \pi_i(s)) . U_i(s')$

① $s=1$, there is 1 fish in lake.

$\pi_1(s=1) = 0$, 0 fish are caught

$R(s=1) = 0$

Remaining fish $=1$, so next year's population:

For $R=1 \rightarrow 1$

For $R=1.25 \rightarrow 1$

For $R=1.5 \rightarrow 2$

For $R=1.75 \rightarrow 2$

$\sum_{s'} P(s'|s=1, \pi_1(s=1)) . U_1(s') = 0.2 \times 1 + 0.3 \times 1 + 0.3 \times 2 + 0.2 \times 2 = 1.5$

$U_2 = R(s=1) + 0.9 \times 1.5 = 1.35$

$\pi_1(s=1) = 1$, 1 fish are caught

$R(s=1) = 1$

Remaining fish $=0$, so next year's population will be 0.

$\sum_{s'} P(s'|s=1, \pi_1(s=1)) . U_1(s') = 0.2 \times 0 + 0.3 \times 0 + 0.3 \times 0 + 0.2 \times 0 = 0$

$U_2 = R(s=1) + 0.9 \times 0 = 1$

② $s=2$, there is 2 fish in lake.

* $\pi_1(s=2) = 0$, 0 fish are caught.

$R(s=2) = 0$

Remaining fish $= 2$, so next year's population:

For $R=1 \longrightarrow 2$

For $R=1.25 \longrightarrow 3$

For $R=1.5 \longrightarrow 3$

For $R=1.75 \longrightarrow 4$

$\sum_{s'} P(s'|s=2, \pi_1(s=2)) U_1(s') = 0.2 \times 2 + 0.3 \times 3 + 0.3 \times 3 + 0.2 \times 4 = 3$

$U_2 = 0 + 0.9 \times 3 = 2.7$

* $\pi_1(s=2) = 1$, 1 fish are caught.

$R(s=2) = 1$

Remaining fish = 1 , so next year's population:

For $R=1 \longrightarrow 1$

For $R=1.25 \longrightarrow 1$

For $R=1.5 \longrightarrow 2$

For $R=1.75 \longrightarrow 2$

$$\sum_{s'} P(s' \mid s=2, \pi_1(s=2))\, U_1(s') = 0.2\times1 + 0.3\times1 + 0.3\times2 + 0.2\times2 = 1.5$$

$$U_2 = 1 + 0.9 \times 1.5 = 2.35$$

* $\pi_1(s=2) = 2$, 2 fish are caught.

$R(s=2) = 2$

Remaining fish = 2 , so next year's population will be 0.

$$\sum_{s'} P(s' \mid s=2, \pi_1(s=2))\, U_1(s') = 0.2\times0 + 0.3\times0 + 0.3\times0 + 0.2\times0 = 0.$$

$$U_2 = 2 + 0.9 \times 0 = 2$$

③ $s=3$, there is 3 fish in lake.

* $\pi_1(s=3) = 0$, 0 fish are caught.

$R(s=3) = 0$

Remaining fish = 3 , so next year's population:

For $R=1 \longrightarrow 3$

For $R=1.25 \longrightarrow 4$

For $R=1.5 \longrightarrow 5$

For $R=1.75 \longrightarrow 5$

$$\sum_{s'} P(s' \mid s=3, \pi_1(s=3))\, U_1(s') = 0.2\times3 + 0.3\times4 + 0.3\times5 + 0.2\times5 = 4.3$$

$$U_2 = 0 + 0.9 \times 4.3 = 3.87$$

* $\pi_1(s=3) = 1$, 1 fish are caught.

$R(s=3) = 1$

Remaining fish = 2 , so next year's population :

For $R=1 \longrightarrow 2$

For $R=1.25 \longrightarrow 3$

For $R=1.5 \longrightarrow 3$

For $R=1.75 \longrightarrow 4$

$$\sum_{s'} P(s'|s=3, \pi_1(s=3))\, U_1(s') = 0.2\times 2 + 0.3\times 3 + 0.3\times 3 + 0.2\times 4 = 3$$

$$U_2 = 1 + 0.9\times 3 = 3.7$$

* $\pi_1(s=3) = 2$, 2 fish are caught.

$R(s=3) = 2$

Remaining fish = 1 , so next year's population :

For $R=1 \longrightarrow 1$

For $R=1.25 \longrightarrow 1$

For $R=1.5 \longrightarrow 2$

For $R=1.75 \longrightarrow 2$

$$\sum_{s'} P(s'|s=3, \pi_1(s=3))\, U_1(s') = 0.2\times 1 + 0.3\times 1 + 0.3\times 2 + 0.2\times 2 = 1.5$$

$$U_2 = 2 + 0.9\times 1.5 = 3.35$$

* $\pi_1(s=3) = 3$, 3 fish are caught.

$R(s=3) = 3$

Remaining fish = 0 , so next year's population will be 0.

$$\sum_{s'} P(s'|s=3, \pi_1(s=3))\, U_1(s') = 0.2\times 0 + 0.3\times 0 + 0.3\times 0 + 0.2\times 0 = 0$$

$$U_2 = 3 + 0.9\times 0 = 3$$

4) $s = 4$, there is 4 fish in lake.

* $\pi_1(s=4) = 0$, 0 fish are caught.

$R(s=4) = 0$

Remaining fish = 4 , so next year's population:

For $R=1 \longrightarrow 4$

For $R=1.25 \longrightarrow 5$

For $R=1.5 \longrightarrow 6$

For $R=1.75 \longrightarrow 7$

$\sum_{s'} P(s'|s=4, \pi_1(s=4))\, U_1(s') = 0.2 \times 4 + 0.3 \times 5 + 0.3 \times 6 + 0.2 \times 7 = 5.5$

$U_2 = 0 + 0.9 \times 5.5 = 4,95$

* $\pi_1(s=4) = 1$, 1 fish are caught.

$R(s=4) = 1$

Remaining fish = 3 , so next year's population:

For $R=1 \longrightarrow 3$

For $R=1.25 \longrightarrow 4$

For $R=1.5 \longrightarrow 5$

For $R=1.75 \longrightarrow 5$

$\sum_{s'} P(s'|s=4, \pi_1(s=4))\, U_1(s') = 0.2 \times 3 + 0.3 \times 4 + 0.3 \times 5 + 0.2 \times 5 = 4.3$

$U_2 = 1 + 0.9 \times 4.3 = 4.87$

* $\pi_1(s=4) = 2$, 2 fish are caught.

$R(s=4) = 2$

Remaining fish = 2 , so next year's population:

For $R=1 \longrightarrow 2$

For $R=1.25 \longrightarrow 3$

For $R=1.5 \longrightarrow 3$

For $R=1.75 \longrightarrow 4$

$\sum_{s'} P(s'|s=4, \pi_1(s=4))\, U_1(s') = 0.2 \times 2 + 0.3 \times 3 + 0.3 \times 3 + 0.2 \times 4 = 3$

$U_2 = 2 + 0.9 \times 3 = 4.7$

* $\pi_1(s=4) = 3$, 3 fish are caught.

$R(s=4) = 3$

Remaining fish = 1 , so next year's population:

For $R=1 \longrightarrow 1$

For $R=1.25 \longrightarrow 1$

For $R=1.5 \longrightarrow 2$

For $R=1.75 \longrightarrow 2$

$$\sum_{s'} P(s' \mid s=4, \pi_1(s=4))\, U_1(s') = 0.2 \times 1 + 0.3 \times 1 + 0.3 \times 2 + 0.2 \times 2 = 1.5$$

$$U_2 = 3 + 0.9 \times 1.5 = 4.35$$

* $\pi_1(s=4) = 4$, 4 fish are caught.

$R(s=4) = 4$

Remaining fish = 0 , so next year's population will be 0.

$$\sum_{s'} P(s' \mid s=4, \pi_1(s=4))\, U_1(s') = 0.2 \times 0 + 0.3 \times 0 + 0.3 \times 0 + 0.2 \times 0 = 0$$

$$U_2 = 4 + 0.9 \times 0 = 4$$

5 $s=5$, there is 5 fish in lake.

* $\pi_1(s=5) = 0$, 0 fish are caught.

$R(s=5) = 0$

Remaining fish = 5 , so next year's population:

For $R=1 \longrightarrow 5$

For $R=1.25 \longrightarrow 6$

For $R=1.5 \longrightarrow 8$

For $R=1.75 \longrightarrow 9$

$$\sum_{s'} P(s' \mid s=5, \pi_1(s=5))\, U_1(s') = 0.2 \times 5 + 0.3 \times 6 + 0.3 \times 8 + 0.2 \times 9 = 7$$

$$U_2 = 0 + 0.9 \times 7 = 6.3$$

* $\pi_1(s=5) = 1$, 1 fish are caught.

$R(s=5) = 1$

Remaining fish = 4 , so next year's population:

For $R=1 \longrightarrow 4$

For $R=1.25 \longrightarrow 5$

For $R=1.5 \longrightarrow 6$

For $R=1.75 \rightarrow 7$

$\sum_{s'} P(s'|s=5, \pi_1(s=5))\, U_1(s') = 0.2 \times 4 + 0.3\times5 + 0.3\times 6 + 0.2 \times 7 = 5.5$

$U_2 = 1 + 0.9 \times 5.5 = 5{,}95$

* $\pi_1(s=5) = 2$, 2 fish are caught.

$R(s=5) = 2$

Remaining fish = 3 , so next year's population:

For $R=1 \longrightarrow 3$

For $R=1.25 \longrightarrow 4$

For $R=1.5 \longrightarrow 5$

For $R=1.75 \rightarrow 5$

$\sum_{s'} P(s'|s=5, \pi_1(s=5))\, U_1(s') = 0.2 \times 3 + 0.3\times4 + 0.3\times 5 + 0.2 \times 5 = 4.3$

$U_2 = 2 + 0.9 \times 4.3 = 5.87$

* $\pi_1(s=5) = 3$, 3 fish are caught.

$R(s=5) = 3$

Remaining fish = 2 , so next year's population:

For $R=1 \longrightarrow 2$

For $R=1.25 \longrightarrow 3$

For $R=1.5 \longrightarrow 3$

For $R=1.75 \rightarrow 4$

$\sum_{s'} P(s'|s=5, \pi_1(s=5))\, U_1(s') = 0.2 \times 2 + 0.3\times3 + 0.3\times 3 + 0.2 \times 4 = 3$

$U_2 = 3 + 0.9 \times 3 = 5.7$

* $\pi_1(s=5) = 4$, 4 fish are caught.

$R(s=5) = 4$

Remaining fish = 1 , so next year's population:

For $R=1 \longrightarrow 1$

For $R=1.25 \longrightarrow 1$

For $R=1.5 \longrightarrow 2$

For $R=1.75 \longrightarrow 2$

$\sum_{s'} P(s' | s=5, \pi_1(s=5))\, U_1(s') = 0.2 \times 1 + 0.3 \times 1 + 0.3 \times 2 + 0.2 \times 2 = 1.5$

$U_2 = 4 + 0.9 \times 1.5 = 5.35$

* $\pi_1(s=5) = 5$, 5 fish are caught.

$R(s=5) = 5$

Remaining fish = 0 , so next year's population will be 0.

$\sum_{s'} P(s' | s=5, \pi_1(s=5))\, U_1(s') = 0.2 \times 0 + 0.3 \times 0 + 0.3 \times 0 + 0.2 \times 0 = 0$

$U_2 = 5 + 0.9 \times 0 = 5$

6. $s = 6$, there is 6 fish in lake.

* $\pi_1(s=6) = 0$, 0 fish are caught.

$R(s=6) = 0$

Remaining fish = 6 , so next year's population:

For $R=1 \longrightarrow 6$

For $R=1.25 \longrightarrow 8$

For $R=1.5 \longrightarrow 9$

For $R=1.75 \longrightarrow 10$ (found 11 but capacity is 10)

$\sum_{s'} P(s' | s=6, \pi_1(s=6))\, U_1(s') = 0.2 \times 6 + 0.3 \times 8 + 0.3 \times 9 + 0.2 \times 10 = 8.3$

$U_2 = 0 + 0.9 \times 8.3 = 7.47$

* $\pi_1(s=6) = 1$, 1 fish are caught.

$R(s=6) = 1$

Remaining fish = 5 , so next year's population:

For $R=1 \longrightarrow 5$

For $R=1.25 \longrightarrow 6$

For $R=1.5 \longrightarrow 8$

For $R=1.75 \rightarrow 9$

$$\sum_{s'} P(s' \mid s=6, \pi_1(s=6))\, U_1(s') = 0.2\times5 + 0.3\times6 + 0.3\times8 + 0.2\times9 = 7$$

$$U_2 = 1 + 0.9 \times 7 = 7.3$$

* $\pi_1(s=6) = 2$, 2 fish are caught.

$R(s=6) = 2$

Remaining fish = 4 , so next year's population:

For $R=1 \longrightarrow 4$

For $R=1.25 \longrightarrow 5$

For $R=1.5 \longrightarrow 6$

For $R=1.75 \rightarrow 7$

$$\sum_{s'} P(s' \mid s=6, \pi_1(s=6))\, U_1(s') = 0.2\times4 + 0.3\times5 + 0.3\times6 + 0.2\times7 = 5.5$$

$$U_2 = 2 + 0.9 \times 5.5 = 6,95$$

* $\pi_1(s=6) = 3$, 3 fish are caught.

$R(s=6) = 3$

Remaining fish = 3 , so next year's population:

For $R=1 \longrightarrow 3$

For $R=1.25 \longrightarrow 4$

For $R=1.5 \longrightarrow 5$

For $R=1.75 \rightarrow 5$

$$\sum_{s'} P(s' \mid s=6, \pi_1(s=6))\, U_1(s') = 0.2\times3 + 0.3\times4 + 0.3\times5 + 0.2\times5 = 4.3$$

$$U_2 = 3 + 0.9 \times 4.3 = 6.87$$

* $\pi_1(s=6) = 4$, 4 fish are caught.

$R(s=6) = 4$

Remaining fish = 2 , so next year's population:

For $R=1 \longrightarrow 2$

For $R=1.25 \longrightarrow 3$

For $R=1.5 \longrightarrow 3$

For $R=1.75 \longrightarrow 4$

$$\sum_{s'} P(s'|s=6, \pi_1(s=6))\, U_1(s') = 0.2\times 2 + 0.3\times 3 + 0.3\times 3 + 0.2\times 4 = 3$$

$$U_2 = 4 + 0.9 \times 3 = 6.7$$

* $\pi_1(s=6) = 5$, 5 fish are caught.

$R(s=6) = 5$

Remaining fish = 1 , so next year's population:

For $R=1 \longrightarrow 1$

For $R=1.25 \longrightarrow 1$

For $R=1.5 \longrightarrow 2$

For $R=1.75 \longrightarrow 2$

$$\sum_{s'} P(s'|s=6, \pi_1(s=6))\, U_1(s') = 0.2\times 1 + 0.3\times 1 + 0.3\times 2 + 0.2\times 2 = 1.5$$

$$U_2 = 5 + 0.9 \times 1.5 = 6.35$$

* $\pi_1(s=6) = 6$, 6 fish are caught.

$R(s=6) = 6$

Remaining fish = 0 , so next year's population will be 0.

$$\sum_{s'} P(s'|s=6, \pi_1(s=6))\, U_1(s') = 0.2\times 0 + 0.3\times 0 + 0.3\times 0 + 0.2\times 0 = 0$$

$$U_2 = 6 + 0.9 \times 0 = 6$$

7 $s=7$, there is 7 fish in lake.

* $\pi_1(s=7)=0$, 0 fish are caught.

$R(s=7)=0$

Remaining fish = 7 , so next year's population :

For $R=1 \longrightarrow 7$

For $R=1.25 \longrightarrow 9$

For $R=1.5 \longrightarrow 10$ (found 11 but capacity is 10)

For $R=1.75 \longrightarrow 10$ (found 12 but capacity is 10)

$\sum_{s'} P(s'|s=7, \pi_1(s=7))\, U_1(s') = 0.2\times 7 + 0.3\times 9 + 0.3\times 10 + 0.2\times 10 = 9.1$

$U_2 = 0 + 0.9 \times 9.1 = 8.19$

* $\pi_1(s=7)=1$, 1 fish are caught.

$R(s=7)=1$

Remaining fish = 6 , so next year's population :

For $R=1 \longrightarrow 6$

For $R=1.25 \longrightarrow 8$

For $R=1.5 \longrightarrow 9$

For $R=1.75 \longrightarrow 10$ (found 11 but capacity is 10)

$\sum_{s'} P(s'|s=7, \pi_1(s=7))\, U_1(s') = 0.2\times 6 + 0.3\times 8 + 0.3\times 9 + 0.2\times 10 = 8.3$

$U_2 = 1 + 0.9 \times 8.3 = 8.47$

* $\pi_1(s=7)=2$, 2 fish are caught.

$R(s=7)=2$

Remaining fish = 5 , so next year's population :

For $R=1 \longrightarrow 5$

For $R=1.25 \longrightarrow 6$

For $R=1.5 \longrightarrow 8$

For $R=1.75 \longrightarrow 9$

$\sum_{s'} P(s'|s=7, \pi_1(s=7))\, U_1(s') = 0.2\times 5 + 0.3\times 6 + 0.3\times 8 + 0.2\times 9 = 7$

$U_2 = 2 + 0.9 \times 7 = 8.3$

* $\pi_1(s=7) = 3$, 3 fish are caught.

$R(s=7) = 3$

Remaining fish = 4 , so next year's population:

For $R=1 \longrightarrow 4$

For $R=1.25 \longrightarrow 5$

For $R=1.5 \longrightarrow 6$

For $R=1.75 \longrightarrow 7$

$\sum_{s'} P(s'|s=7, \pi_1(s=7))\, U_1(s') = 0.2 \times 4 + 0.3 \times 5 + 0.3 \times 6 + 0.2 \times 7 = 5.5$

$U_2 = 3 + 0.9 \times 5.5 = 7{,}95$

* $\pi_1(s=7) = 4$, 4 fish are caught.

$R(s=7) = 4$

Remaining fish = 3 , so next year's population:

For $R=1 \longrightarrow 3$

For $R=1.25 \longrightarrow 4$

For $R=1.5 \longrightarrow 5$

For $R=1.75 \longrightarrow 5$

$\sum_{s'} P(s'|s=7, \pi_1(s=7))\, U_1(s') = 0.2 \times 3 + 0.3 \times 4 + 0.3 \times 5 + 0.2 \times 5 = 4.3$

$U_2 = 4 + 0.9 \times 4.3 = 7.87$

* $\pi_1(s=7) = 5$, 5 fish are caught.

$R(s=7) = 5$

Remaining fish = 2 , so next year's population:

For $R=1 \longrightarrow 2$

For $R=1.25 \longrightarrow 3$

For $R=1.5 \longrightarrow 3$

For $R=1.75 \longrightarrow 4$

$\sum_{s'} P(s'|s=7, \pi_1(s=7))\, U_1(s') = 0.2 \times 2 + 0.3 \times 3 + 0.3 \times 3 + 0.2 \times 4 = 3$

$U_2 = 5 + 0.9 \times 3 = 7.7$

* $\pi_1(s=7) = 6$, 6 fish are caught.

$R(s=7) = 6$

Remaining fish = 1 , so next year's population:

For R = 1 $\longrightarrow$ 1

For R = 1.25 $\longrightarrow$ 1

For R = 1.5 $\longrightarrow$ 2

For R = 1.75 $\longrightarrow$ 2

$$\sum_{s'} P(s' \mid s=7, \pi_1(s=7))\, U_1(s') = 0.2 \times 1 + 0.3 \times 1 + 0.3 \times 2 + 0.2 \times 2 = 1.5$$

$$U_2 = 6 + 0.9 \times 1.5 = 7.35$$

* $\pi_1(s=7) = 7$, 7 fish are caught.

$R(s=7) = 7$

Remaining fish = 0 , so next year's population will be 0.

$$\sum_{s'} P(s' \mid s=7, \pi_1(s=7))\, U_1(s') = 0.2 \times 0 + 0.3 \times 0 + 0.3 \times 0 + 0.2 \times 0 = 0$$

$$U_2 = 7 + 0.9 \times 0 = 7$$

8) s = 8 , there is 8 fish in lake.

* $\pi_1(s=8) = 0$, 0 fish are caught.

$R(s=8) = 0$

Remaining fish = 8 , so next year's population:

For R = 1 $\longrightarrow$ 8

For R = 1.25 $\longrightarrow$ 10

For R = 1.5 $\longrightarrow$ 10 (found 12 but capacity is 10)

For R = 1.75 $\longrightarrow$ 10 (found 14 but capacity is 10)

$$\sum_{s'} P(s' \mid s=8, \pi_1(s=8))\, U_1(s') = 0.2 \times 8 + 0.3 \times 10 + 0.3 \times 10 + 0.2 \times 10 = 9.6$$

$$U_2 = 0 + 0.9 \times 9.6 = 8.64$$

* $\pi_1(s=8) = 1$, 1 fish are caught.

$R(s=8) = 1$

Remaining fish = 7 , so next year's population :

For $R=1 \longrightarrow 7$

For $R=1.25 \longrightarrow 9$

For $R=1.5 \longrightarrow 10$ (found 11 but capacity is 10)

For $R=1.75 \rightarrow 10$ (found 12 but capacity is 10)

$$\sum_{s'} P(s'|s=8, \pi_1(s=8))\, U_1(s') = 0.2 \times 7 + 0.3 \times 9 + 0.3 \times 10 + 0.2 \times 10 = 9.1$$

$$U_2 = 1 + 0.9 \times 9.1 = 9.19$$

* $\pi_1(s=8) = 2$, 2 fish are caught.

$R(s=8) = 2$

Remaining fish = 6 , so next year's population :

For $R=1 \longrightarrow 6$

For $R=1.25 \longrightarrow 8$

For $R=1.5 \longrightarrow 9$

For $R=1.75 \rightarrow 10$ (found 11 but capacity is 10)

$$\sum_{s'} P(s'|s=8, \pi_1(s=8))\, U_1(s') = 0.2 \times 6 + 0.3 \times 8 + 0.3 \times 9 + 0.2 \times 10 = 8.3$$

$$U_2 = 2 + 0.9 \times 8.3 = 9.47$$

* $\pi_1(s=7) = 3$, 3 fish are caught.

$R(s=7) = 3$

Remaining fish = 5 , so next year's population :

For $R=1 \longrightarrow 5$

For $R=1.25 \longrightarrow 6$

For $R=1.5 \longrightarrow 8$

For $R=1.75 \rightarrow 9$

$$\sum_{s'} P(s'|s=8, \pi_1(s=8))\, U_1(s') = 0.2 \times 5 + 0.3 \times 6 + 0.3 \times 8 + 0.2 \times 9 = 7$$

$$U_2 = 3 + 0.9 \times 7 = 9.3$$

* $\pi_1(s=8) = 4$, 4 fish are caught.

$R(s=8) = 4$

Remaining fish = 4 , so next year's population:

For $R=1 \longrightarrow 4$

For $R=1.25 \longrightarrow 5$

For $R=1.5 \longrightarrow 6$

For $R=1.75 \longrightarrow 7$

$\sum_{s'} P(s'|s=8, \pi_1(s=8))\, U_1(s') = 0.2 \times 4 + 0.3 \times 5 + 0.3 \times 6 + 0.2 \times 7 = 5.5$

$U_2 = 4 + 0.9 \times 5.5 = 8{,}95$

* $\pi_1(s=8) = 5$, 5 fish are caught.

$R(s=8) = 5$

Remaining fish = 3 , so next year's population:

For $R=1 \longrightarrow 3$

For $R=1.25 \longrightarrow 4$

For $R=1.5 \longrightarrow 5$

For $R=1.75 \longrightarrow 5$

$\sum_{s'} P(s'|s=8, \pi_1(s=8))\, U_1(s') = 0.2 \times 3 + 0.3 \times 4 + 0.3 \times 5 + 0.2 \times 5 = 4.3$

$U_2 = 5 + 0.9 \times 4.3 = 8.87$

* $\pi_1(s=8) = 6$, 6 fish are caught.

$R(s=8) = 6$

Remaining fish = 2 , so next year's population:

For $R=1 \longrightarrow 2$

For $R=1.25 \longrightarrow 3$

For $R=1.5 \longrightarrow 3$

For $R=1.75 \longrightarrow 4$

$\sum_{s'} P(s'|s=8, \pi_1(s=8))\, U_1(s') = 0.2 \times 2 + 0.3 \times 3 + 0.3 \times 3 + 0.2 \times 4 = 3$

$U_2 = 6 + 0.9 \times 3 = 8.7$

* $\pi_1(s=8) = 7$, 7 fish are caught.

$R(s=8) = 7$

Remaining fish = 1 , so next year's population:

For $R=1 \longrightarrow 1$

For $R=1.25 \longrightarrow 1$

For $R=1.5 \longrightarrow 2$

For $R=1.75 \rightarrow 2$

$$\sum_{s'} P(s' \mid s=8, \pi_1(s=8))\, U_1(s') = 0.2 \times 1 + 0.3 \times 1 + 0.3 \times 2 + 0.2 \times 2 = 1.5$$

$$U_2 = 7 + 0.9 \times 1.5 = 6.35$$

* $\pi_1(s=8) = 8$, 8 fish are caught.

$R(s=8) = 8$

Remaining fish = 0 , so next year's population will be 0.

$$\sum_{s'} P(s' \mid s=8, \pi_1(s=8))\, U_1(s') = 0.2 \times 0 + 0.3 \times 0 + 0.3 \times 0 + 0.2 \times 0 = 0$$

$$U_2 = 8 + 0.9 \times 0 = 8$$

9 $s = 9$, there is 9 fish in lake.

* $\pi_1(s=9) = 0$, 0 fish are caught.

$R(s=9) = 0$

Remaining fish = 9 , so next year's population:

For $R=1 \longrightarrow 9$

For $R=1.25 \longrightarrow 10$ (found 11 but capacity is 10)

For $R=1.5 \longrightarrow 10$ (found 14 but capacity is 10)

For $R=1.75 \rightarrow 10$ (found 16 but capacity is 10)

$$\sum_{s'} P(s' \mid s=9, \pi_1(s=9))\, U_1(s') = 0.2 \times 9 + 0.3 \times 10 + 0.3 \times 10 + 0.2 \times 10 = 9.8$$

$$U_2 = 0 + 0.9 \times 9.8 = 8.82$$

* $\pi_1(s=9) = 1$, 1 fish are caught.

$R(s=9) = 1$

Remaining fish = 8 , so next year's population:

For $R=1 \longrightarrow 8$

For $R=1.25 \longrightarrow 10$

For $R=1.5 \longrightarrow 10$ (found 12 but capacity is 10)

For $R=1.75 \rightarrow 10$ (found 14 but capacity is 10)

$\sum_{s'} P(s'|s=9, \pi_1(s=9))\, U_1(s') = 0.2 \times 8 + 0.3 \times 10 + 0.3 \times 10 + 0.2 \times 10 = 9.6$

$U_2 = 1 + 0.9 \times 9.6 = 9.64$

* $\pi_1(s=9) = 2$, 2 fish are caught.

$R(s=9) = 2$

Remaining fish = 7 , so next year's population:

For $R=1 \longrightarrow 7$

For $R=1.25 \longrightarrow 9$

For $R=1.5 \longrightarrow 10$ (found 11 but capacity is 10)

For $R=1.75 \rightarrow 10$ (found 12 but capacity is 10)

$\sum_{s'} P(s'|s=9, \pi_1(s=9))\, U_1(s') = 0.2 \times 7 + 0.3 \times 9 + 0.3 \times 10 + 0.2 \times 10 = 9.1$

$U_2 = 2 + 0.9 \times 9.1 = 10.19$

* $\pi_1(s=9) = 3$, 3 fish are caught.

$R(s=9) = 3$

Remaining fish = 6 , so next year's population:

For $R=1 \longrightarrow 6$

For $R=1.25 \longrightarrow 8$

For $R=1.5 \longrightarrow 9$

For $R=1.75 \rightarrow 10$ (found 11 but capacity is 10)

$\sum_{s'} P(s'|s=9, \pi_1(s=9))\, U_1(s') = 0.2 \times 6 + 0.3 \times 8 + 0.3 \times 9 + 0.2 \times 10 = 8.3$

$U_2 = 3 + 0.9 \times 8.3 = 10.47$

* $\pi_1(s=9) = 4$, 4 fish are caught.

$R(s=9) = 4$

Remaining fish = 5 , so next year's population :

For $R=1 \longrightarrow 5$

For $R=1.25 \longrightarrow 6$

For $R=1.5 \longrightarrow 8$

For $R=1.75 \rightarrow 9$

$\sum_{s'} P(s'|s=9, \pi_1(s=9))\, U_1(s') = 0.2 \times 5 + 0.3 \times 6 + 0.3 \times 8 + 0.2 \times 9 = 7$

$U_2 = 4 + 0.9 \times 7 = 10.3$

* $\pi_1(s=9) = 6$, 6 fish are caught.

$R(s=9) = 6$

Remaining fish = 3 , so next year's population :

For $R=1 \longrightarrow 3$

For $R=1.25 \longrightarrow 4$

For $R=1.5 \longrightarrow 5$

For $R=1.75 \rightarrow 5$

$\sum_{s'} P(s'|s=9, \pi_1(s=9))\, U_1(s') = 0.2 \times 3 + 0.3 \times 4 + 0.3 \times 5 + 0.2 \times 5 = 4.3$

$U_2 = 6 + 0.9 \times 4.3 = 9.87$

* $\pi_1(s=9) = 7$, 7 fish are caught.

$R(s=9) = 7$

Remaining fish = 2 , so next year's population :

For $R=1 \longrightarrow 2$

For $R=1.25 \longrightarrow 3$

For $R=1.5 \longrightarrow 3$

For $R=1.75 \rightarrow 4$

$\sum_{s'} P(s'|s=9, \pi_1(s=9))\, U_1(s') = 0.2 \times 2 + 0.3 \times 3 + 0.3 \times 3 + 0.2 \times 4 = 3$

$U_2 = 7 + 0.9 \times 3 = 9.7$

* $\pi_1(s=9) = 8$, 8 fish are caught.

$R(s=7) = 8$

Remaining fish = 1 , so next year's population:

For $R=1 \longrightarrow 1$

For $R=1.25 \longrightarrow 1$

For $R=1.5 \longrightarrow 2$

For $R=1.75 \longrightarrow 2$

$$\sum_{s'} P(s' \mid s=9, \pi_1(s=9))\, U_1(s') = 0.2\times1 + 0.3\times1 + 0.3\times2 + 0.2\times2 = 1.5$$

$U_2 = 8 + 0.9 \times 1.5 = 7.35$

* $\pi_1(s=9) = 9$, 9 fish are caught.

$R(s=9) = 9$

Remaining fish = 0 , so next year's population will be 0.

$$\sum_{s'} P(s' \mid s=9, \pi_1(s=9))\, U_1(s') = 0.2\times0 + 0.3\times0 + 0.3\times0 + 0.2\times0 = 0$$

$U_2 = 9 + 0.9 \times 0 = 9$

10 $s=10$, there is 10 fish in lake.

* $\pi_1(s=10) = 0$, 0 fish are caught.

$R(s=10) = 0$

Remaining fish = 10 , so next year's population:

For $R=1 \longrightarrow 10$

For $R=1.25 \longrightarrow 10$ (found 13 but capacity is 10)

For $R=1.5 \longrightarrow 10$ (found 15 but capacity is 10)

For $R=1.75 \longrightarrow 10$ (found 18 but capacity is 10)

$$\sum_{s'} P(s' \mid s=10, \pi_1(s=10))\, U_1(s') = 0.2\times10 + 0.3\times10 + 0.3\times10 + 0.2\times10 = 10$$

$U_2 = 0 + 0.9 \times 10 = 9$

* $\pi_1(s=10) = 1$, 1 fish are caught.

$R(s=10) = 1$

Remaining fish = 9 , so next year's population:

For $R=1 \longrightarrow 9$

For $R=1.25 \longrightarrow 10$ (found 11 but capacity is 10)

For $R=1.5 \longrightarrow 10$ (found 14 but capacity is 10)

For $R=1.75 \rightarrow 10$ (found 16 but capacity is 10)

$\sum_{s'} P(s' \mid s=9, \pi_1(s=9)) \, U_1(s') = 0.2 \times 9 + 0.3 \times 10 + 0.3 \times 10 + 0.2 \times 10 = 9.8$

$U_2 = 1 + 0.9 \times 9.8 = 9.82$

* $\pi_1(s=10) = 2$, 2 fish are caught.

$R(s=10) = 2$

Remaining fish = 8 , so next year's population:

For $R=1 \longrightarrow 8$

For $R=1.25 \longrightarrow 10$

For $R=1.5 \longrightarrow 10$ (found 12 but capacity is 10)

For $R=1.75 \rightarrow 10$ (found 14 but capacity is 10)

$\sum_{s'} P(s' \mid s=10, \pi_1(s=10)) \, U_1(s') = 0.2 \times 8 + 0.3 \times 10 + 0.3 \times 10 + 0.2 \times 10 = 9.6$

$U_2 = 2 + 0.9 \times 9.6 = 10.64$

* $\pi_1(s=10) = 3$, 3 fish are caught.

$R(s=10) = 3$

Remaining fish = 7 , so next year's population:

For $R=1 \longrightarrow 7$

For $R=1.25 \longrightarrow 9$

For $R=1.5 \longrightarrow 10$ (found 11 but capacity is 10)

For $R=1.75 \rightarrow 10$ (found 12 but capacity is 10)

$\sum_{s'} P(s' \mid s=10, \pi_1(s=10)) \, U_1(s') = 0.2 \times 7 + 0.3 \times 9 + 0.3 \times 10 + 0.2 \times 10 = 9.1$

$U_2 = 3 + 0.9 \times 9.1 = 11.19$

* $\pi_1(s=10) = 4$, 4 fish are caught.

$R(s=10) = 4$

Remaining fish = 6 , so next year's population:

For $R=1 \longrightarrow 6$

For $R=1.25 \longrightarrow 8$

For $R=1.5 \longrightarrow 9$

For $R=1.75 \longrightarrow 10$ (found 11 but capacity is 10)

$\sum_{s'} P(s' \mid s=10, \pi_1(s=10))\, U_1(s') = 0.2 \times 6 + 0.3 \times 8 + 0.3 \times 9 + 0.2 \times 10 = 8.3$

$U_2 = 4 + 0.9 \times 8.3 = 11.47$

* $\pi_1(s=10) = 5$, 5 fish are caught.

$R(s=10) = 5$

Remaining fish = 5 , so next year's population:

For $R=1 \longrightarrow 5$

For $R=1.25 \longrightarrow 6$

For $R=1.5 \longrightarrow 8$

For $R=1.75 \longrightarrow 9$

$\sum_{s'} P(s' \mid s=10, \pi_1(s=10))\, U_1(s') = 0.2 \times 5 + 0.3 \times 6 + 0.3 \times 8 + 0.2 \times 9 = 7$

$U_2 = 5 + 0.9 \times 7 = 11.3$

* $\pi_1(s=10) = 7$, 7 fish are caught.

$R(s=10) = 7$

Remaining fish = 3 , so next year's population:

For $R=1 \longrightarrow 3$

For $R=1.25 \longrightarrow 4$

For $R=1.5 \longrightarrow 5$

For $R=1.75 \longrightarrow 5$

$\sum_{s'} P(s' \mid s=10, \pi_1(s=10))\, U_1(s') = 0.2 \times 3 + 0.3 \times 4 + 0.3 \times 5 + 0.2 \times 5 = 4.3$

$U_2 = 7 + 0.9 \times 4.3 = 10.87$

* $\pi_1(s=10) = 8$, 8 fish are caught.

$R(s=10) = 8$

Remaining fish = 2 , so next year's population:

For $R=1 \longrightarrow 2$

For $R=1.25 \longrightarrow 3$

For $R=1.5 \longrightarrow 3$

For $R=1.75 \rightarrow 4$

$\sum_{s'} P(s'|s=10, \pi_1(s=10))\, U_1(s') = 0.2 \times 2 + 0.3 \times 3 + 0.3 \times 3 + 0.2 \times 4 = 3$

$U_2 = 8 + 0.9 \times 3 = 10.7$

* $\pi_1(s=10) = 9$, 9 fish are caught.

$R(s=10) = 9$

Remaining fish = 1 , so next year's population:

For $R=1 \longrightarrow 1$

For $R=1.25 \longrightarrow 1$

For $R=1.5 \longrightarrow 2$

For $R=1.75 \rightarrow 2$

$\sum_{s'} P(s'|s=10, \pi_1(s=10))\, U_1(s') = 0.2 \times 1 + 0.3 \times 1 + 0.3 \times 2 + 0.2 \times 2 = 1.5$

$U_2 = 9 + 0.9 \times 1.5 = 8.35$

* $\pi_1(s=10) = 10$, 10 fish are caught.

$R(s=10) = 10$

Remaining fish = 0 , so next year's population will be 0.

$\sum_{s'} P(s'|s=10, \pi_1(s=10))\, U_1(s') = 0.2 \times 0 + 0.3 \times 0 + 0.3 \times 0 + 0.2 \times 0 = 0$

$U_2 = 10 + 0.9 \times 0 = 10$

Utilities after second iteration and optimal policies

- $U_2(s=0) = 0$, there is no fish to catch.
- $U_2(s=1) = 1.35$, $\pi_1(s=1) = 0$, 0 fish should be caught.
- $U_2(s=2) = 2.7$, $\pi_1(s=2) = 0$, 0 fish should be caught.
- $U_2(s=3) = 3.87$, $\pi_1(s=3) = 0$, 0 fish should be caught.
- $U_2(s=4) = 4.95$, $\pi_1(s=4) = 0$, 0 fish should be caught
- $U_2(s=5) = 6.3$, $\pi_1(s=5) = 0$, 0 fish should be caught.
- $U_2(s=6) = 7.47$, $\pi_1(s=6) = 0$, 0 fish should be caught.
- $U_2(s=7) = 8.47$, $\pi_1(s=7) = 1$, 1 fish should be caught.
- $U_2(s=8) = 9.47$, $\pi_1(s=8) = 2$, 2 fish should be caught.
- $U_2(s=9) = 10.47$, $\pi_1(s=9) = 3$, 3 fish should be caught.
- $U_2(s=10) = 11.47$, $\pi_1(s=10) = 4$, 4 fish should be caught.