Computer Assignment 3

In this computer assignment, we wrote a general program for computing the maximum likelihood values of the parameters and applied it to 20 sample data points given in the table by considering 4 cases in the univariate normal mixture as follows.

$$p(\mathbf{x}|\boldsymbol{\theta}) = \frac{P(\omega_1)}{\sqrt{2\pi}\sigma_1}\exp\left[-\frac{1}{2}\left(\frac{x-\mu_1}{\sigma_1}\right)^2\right] + \frac{1-P(\omega_1)}{\sqrt{2\pi}\sigma_2}\exp\left[-\frac{1}{2}\left(\frac{x-\mu_2}{\sigma_2}\right)^2\right].$$

We have defined four functions for four cases: a, b, c, and d. We know the P_w1 and sigma values for the first case, which are 1 for two Gaussians. So, we have initialized the two mean values randomly. We have used the following equation to obtain the necessary conditions on the maximum likelihood estimate for both means.

$$\hat{\boldsymbol{\mu}}_i = \frac{\sum_{k=1}^{n} P(\omega_i|\mathbf{x}_k, \hat{\boldsymbol{\mu}})\mathbf{x}_k}{\sum_{k=1}^{n} P(\omega_i|\mathbf{x}_k, \hat{\boldsymbol{\mu}})}. \qquad P(\omega_i|\mathbf{x}_k, \hat{\boldsymbol{\mu}}) = \frac{p(\mathbf{x}_k|\omega_i, \hat{\boldsymbol{\mu}}_i)P(\omega_i)}{\sum_{j=1}^{c} p(\mathbf{x}_k|\omega_j, \hat{\boldsymbol{\mu}}_j)P(\omega_j)} \qquad \hat{\boldsymbol{\mu}}_i(j+1) = \frac{\sum_{k=1}^{n} P(\omega_i|\mathbf{x}_k, \hat{\boldsymbol{\mu}}(j))\mathbf{x}_k}{\sum_{k=1}^{n} P(\omega_i|\mathbf{x}_k, \hat{\boldsymbol{\mu}}(j))}$$

The mixture model assumes two Gaussian distributions, and we used the Expectation-Maximization (EM) algorithm to iteratively estimate the unknown parameters. First, we compute the probability of each data belonging to each Gaussian component and update the required parameters of the model based on the probability value.

In the case of b, we only know that the P_w1 and variances are equal and unknown, and the mean values are unknown. We estimate the means and variances. For Case of c, both the means and variance of the two Gaussians are unknown and estimate the means and variance of it. All parameters are unknown in the case of d. So, we estimated by updating the means, variance, and mixing coefficient at each iteration.

The estimated parameters after convergence are shown below.

```
{eng-tamale1:~/Third Computer Assignment} python3 RANA-CA3.py
Sample Data X1:[-7.82 -6.68  4.36  6.72 -8.64 -6.87  4.47  6.73 -7.71 -6.91  6.18  6.
72
 -6.25 -6.94  8.09  6.81 -5.19 -6.38  4.08  6.27]
Question A: Mean 1 = 6.042999999999999, Mean 2 = -6.939
Question B: Mean 1 = 6.042999999999999, Mean 2 = -6.939, sigma = 0.24279878500519728
Question C: Mean 1 = 6.042999999999999, Mean 2 = -6.939, sigma 1 = 1.240871065018441,
 sigma 2 = 0.9045932787722889
Question D: Mean 1 = -6.939, Mean 2 = 6.042999999999999, sigma 1 = 0.9045932787722889
, sigma 2 = 1.240871065018441, P(ω1) = 0.5
```

In all cases, the convergence criteria were based on the change in parameter values between consecutive iterations, which were considered converged when the change was smaller than a predefined threshold (1e-6). In this way, we estimate the parameters of a univariate normal mixture model under different assumptions.