

Homework 3:

Buffon's needle

1.

$$\begin{aligned} P &= \int_0^{t/2} \int_{\arcsin(t/l)}^{\pi/2} \frac{4}{t\pi} (\pi/2 - \alpha) d\alpha dt \\ &= \int_0^{t/2} \frac{4}{t\pi} \left(\frac{t}{2} \arcsin(t/l) + \sqrt{l^2 - t^2} \right) dt \\ &= 1 + \frac{2l}{t\pi} - \frac{2}{\pi} \arcsin(t/l) - \frac{2}{\pi} \sqrt{1 - (t/l)^2} \\ &= 1 + \frac{2}{\pi} \frac{l}{t} - \frac{2}{\pi} \arcsin\left(\frac{1}{l/t}\right) - \frac{2}{\pi} \sqrt{1 - \frac{1}{(l/t)^2}} \end{aligned}$$

2.

General Code for $l > t$ and $l < t$ two different cases:

```
import numpy as np
import matplotlib.pyplot as plt
```

```

class BuffonNeedle():
    def
__init__(self,lLength,tLength,times=100):
    self.l = lLength
    self.t = tLength
    self.N = int(times)

    def run(self):
        ratio = self.l/self.t
        x =
np.random.random(self.N)*self.t/2
        theta =
(np.random.random(self.N))*90
        count = 0
        for i in range(0,self.N):
            test = 0.5*self.l *
np.sin(math.radians(theta[i]))
            if x[i] < test :
                count +=1
        Pi =
(2*self.l/self.t/(count/self.N))
        return Pi

```

When $l=2$, $t=3$, $N = 10E5$, the result is 3.1372549019607843

Part 3:

```
import numpy as np
import matplotlib.pyplot as plt

class BuffonNeedle():
    def
__init__(self, lLength, tLength, times=100):
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```

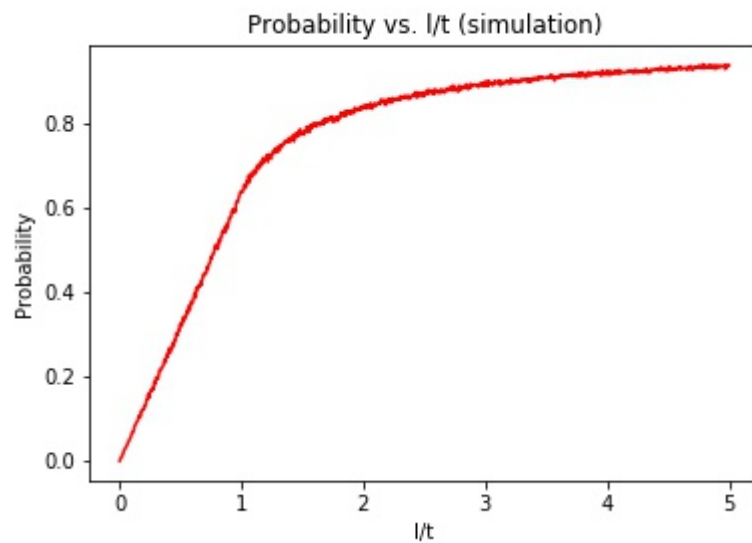
```

Pi = ( 2*ratio-
2*np.arcsin(1/ratio)-2*np.sqrt(ratio**2-
1))/(count/self.N-1)
return Pi

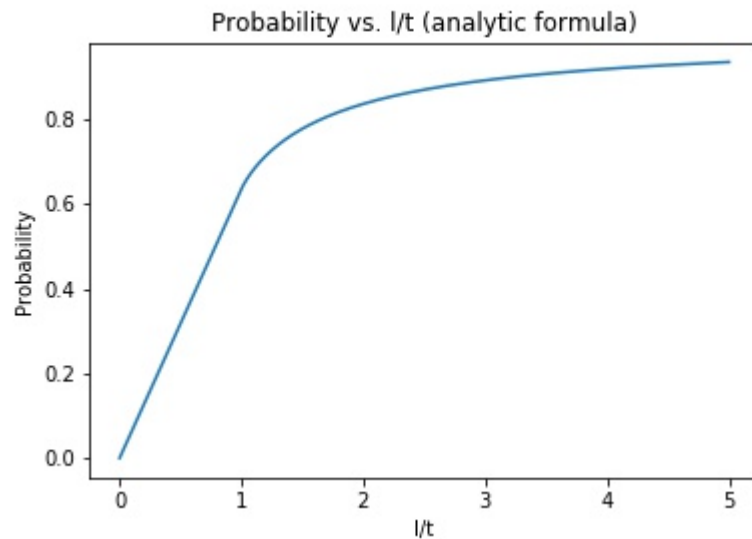
```

when $l=3$, $t=2$ $N = 10E5$, result is 3.13848721387351

Part 4:



Part 5:



We can find that our simulation is pretty good: they are almost the same. When I try to plot them in one pad, I find I couldn't distinguish them because they are almost coincide.

So we can say that the for this problem, MC simulation perform well enough to solve it.