Part 1:

$$M = \begin{pmatrix} 1 & -4 & 2 \\ -4 & 1 & -2 \\ 2 & -2 & -2 \end{pmatrix}$$

$$\det(M) = 1 \times (-2-4) - (-4) \times (8+4) + 2 \times (8-2)$$
$$= -6 + 48 + 12 = 54 \neq 0 \Rightarrow \text{inverse exist}$$

$$M^T = \begin{pmatrix} 1 & -4 & 2 \\ -4 & 1 & -2 \\ 2 & -2 & -2 \end{pmatrix}$$

$$M^{-1} \cdot M = I \qquad \Rightarrow M^{-1} = \begin{pmatrix} \frac{-1}{9}, & -\frac{2}{9}, & \frac{1}{9} \\ -\frac{2}{9}, & -\frac{1}{9}, & -\frac{1}{9} \\ \frac{1}{9}, & \frac{-1}{9}, & \frac{5}{18} \end{pmatrix}$$

Part 2: $M \cdot X = \lambda X$

$$\Rightarrow \begin{vmatrix} 1-\lambda & -4 & 2 \\ -4 & 1-\lambda & -2 \\ 2 & -2 & -2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)((\lambda-1)(2+\lambda)-4) + 4(8+4\lambda+4) + 2(8-2+\lambda) = 0$$

$$\Rightarrow \lambda = -3, 6$$

$$\Rightarrow x_1 = \begin{pmatrix} 1 \\ -1 \\ 1/2 \end{pmatrix} \quad x_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad x_3 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

Part 3:

$$\nabla_A f(A) = \begin{pmatrix} 2x_{11}x_{22}x_{23} + x_{12}x_{13}x_{31} & x_{11}x_{13}x_{31} & x_{11}x_{12}x_{31} \\ -x_{11}^2 x_{22} & x_{11}^2 x_{22} & x^2 x \end{pmatrix}$$

$$\nabla_A f(A) = \begin{pmatrix} 2x_{11}x_{32}x_{23} + x_{12}x_{33}x_{31} & x_{11}x_{13}x_{31} & x_{11}x_{12}x_{31} \\ -x_{33}^2 x_{32} & x_{11}^2 x_{23} & x_{11}^2 x_{22} \\ x_{11}x_{12}x_{13} & -x_{33}^2 x_{21} & -2x_{13}x_{32}x_{21} \end{pmatrix}$$

Part 4:

$$\frac{dg}{dx} = 3x^2y + yz\cos(x) + y^2z^5 \qquad \frac{dg}{dy} = x^3 + z\sin(x) + 2xyz^5$$

$$\frac{dg}{dz} = y\sin(x) + 5xy^2z^4$$

$$H(g) = \begin{pmatrix} 6xy - yz\sin(x) & 3x^2 + z\cos(x) + 2yz^5 & y\cos(x) + 5y^2z^4 \\ 3x^2 + z\cos(x) + 2yz^5 & 2xz^5 & \sin(x) + 10xyz^4 \\ y\cos(x) + 5y^2z^4 & \sin(x) + 10xyz^4 & 20xy^2z^3 \end{pmatrix}$$