Homework 3:

Buffon's needle

1.

$$P = \int_0^{t/2} \int_{arcsin(}^{\pi/2}$$

$$= \int_0^{t/2} \frac{4}{t\pi}(\pi/2 - a$$

$$= 1 - \frac{4}{t\pi}(\frac{t}{2}arcsin(t/l) + \sqrt{}$$

$$= 1 + \frac{2l}{t\pi} - \frac{2}{\pi}arcsin(t/l) -$$

$$= 1 + \frac{2}{\pi}\frac{l}{t} - \frac{2}{\pi}arcsin(\frac{1}{l/}$$

2.

General Code for l>t and l<t two different cases:

```
import numpy as np
import matplotlib.pyplot as plt
```

```
class BuffonNeedle():
    def
__init__(self,lLength,tLength,times=100):
        self.l = lLength
        self.t = tLength
        self.N = int(times)

    def run(self):
        ratio = self.l/self.t
        x =
np.random.random(self.N)*self.t/2
        theta =
(np.random.random(self.N))*90
        count = 0
        for i in range(0,self.N):
            test = 0.5*self.l *
np.sin(math.radians(theta[i]))
            if x[i] < test :
                count +=1
        Pi =
(2*self.l/self.t/(count/self.N))
        return Pi
```

When l=2, t=3 ,N = 10E5, the result is 3.1372549019607843

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```
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            if x[i] < test :
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```
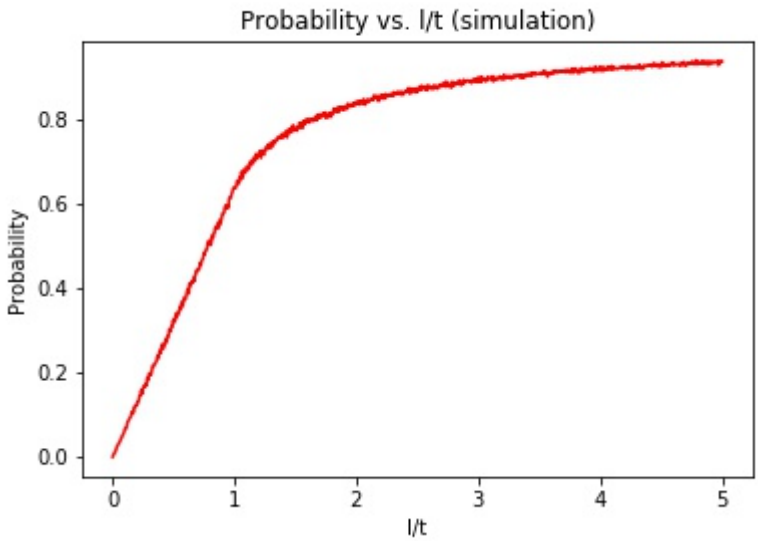
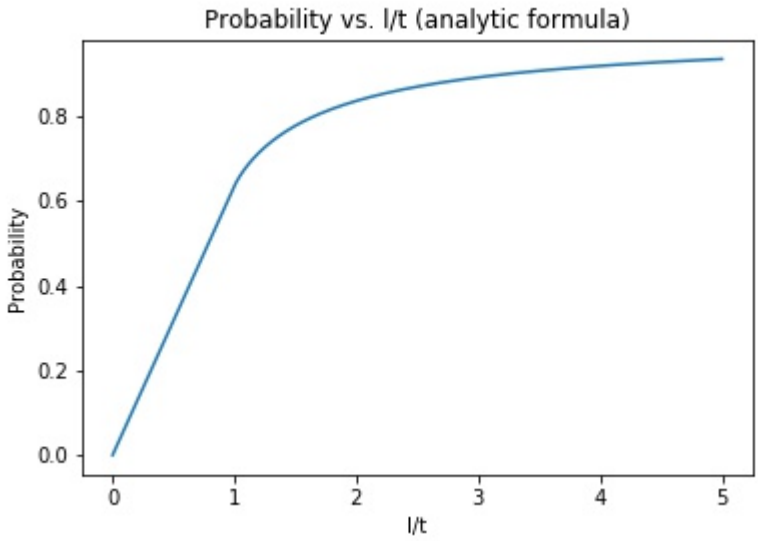

```
            Pi = ( 2*ratio-
2*np.arcsin(1/ratio)-2*np.sqrt(ratio**2-
1))/(count/self.N-1)
            return Pi
```

when l=3, t=2 N = 10E5, result is 3.13848721387351

Part 4:



Part 5:



We can find that our simulation is pretty good: they are almost the same. When I try to plot them in one pad, I find I couldn't distinguish them because they are almost coincide.

So we can say that the for this problem, MC simulation perform well enough to solve it.