

# Epipolar Geometry

12 questions

1  
point

1.

Consider two images  $x_1, x_2$  of the same point  $p$  from two camera positions with relative pose  $(R, T) \in SE(3)$ , where  $R \in SO(3)$  is the relative orientation and  $T \in \mathbb{R}^3$  is the relative position. Then,  $x_1, x_2$  always satisfy

☐  $x_2^T \widehat{T} R x_1 = 0$

☐  $x_2^T \widehat{T} x_1 = 0$

☐  $x_2^T R x_1 = 0$

☐  $x_2^T x_1 = 0$

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2.

Let  $\wedge : \mathbb{R}^3 \rightarrow \mathbb{R}^{3 \times 3}$  defined by

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \mapsto \widehat{u} = \begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix}$$

If  $R$  is a rotation matrix, which of the following properties hold ?

☐  $\widehat{u} u = 0$

☐  $\widehat{u}^T = -\widehat{u}$

☐  $u^T \hat{u} = 0$

☐  $R^T \hat{u} R = \widehat{R^T u}$

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3.

Let two cameras with poses  $g_1 = (R_1, T_1) \in SE(3)$  and  $g_2 = (R_2, T_2) \in SE(3)$ . Note that the poses are such that a point  $X_w$  in the world frame is transformed to the camera frame as  $X_c = R^T (X_w - T)$ . Which of the following matrices are valid essential matrices, that is they satisfy  $x_1^T E x_2 = 0$  for all point correspondences  $x_1 \leftrightarrow x_2$ ? For convenience let  $T_{ij} \doteq T_j - T_i$  and  $R_{ij} \doteq R_i^T R_j$ .

☐  $E = R_1^T \widehat{T_{12}} R_2$

☐  $E = R_1^T \widehat{T_{12}} R_{12}$

☐  $E = R_1^T \widehat{T_{21}} R_{12}$

☐  $E = R_1^T \widehat{T_{21}} R_2$

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4.

The relative pose between two views is  $(R, T) \in SE(3)$  where  $R = I$  and  $T$  corresponds to a translation of 1m in the direction of the  $z$ -axis, which of the following is a valid essential matrix? Hint: use the fact that  $E = \widehat{T} R$ .

☐  $E = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

☐  $E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

☐  $E = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

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5.

The relative pose between two views is  $(R, T) \in SE(3)$  where  $R = I$  and  $T$  corresponds to a translation of 1m in the direction of the  $x$ -axis, which of the following is a valid essential matrix? Hint: use the fact that  $E = \widehat{T} R$ .

☐  $E = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

☐  $E = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

☒  $E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

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6.

A nonzero matrix  $E \in \mathbb{R}^{3 \times 3}$  is an essential matrix if and only if  $E$  has a singular value decomposition (SVD)  $E = U \Sigma V^T$  with

☒  $\Sigma = \text{diag}(\sigma, \sigma, 0)$  for some  $\sigma > 0$  and  $U, V \in SO(3)$ .

☐  $\Sigma = \text{diag}(\sigma, 0, 0)$  for some  $\sigma > 0$  and  $U, V \in SO(3)$ .

☐  $\Sigma = \text{diag}(\sigma, \sigma, 0)$  for some  $\sigma < 0$  and  $U, V \in SO(3)$ .

☐  $\Sigma = \text{diag}(\sigma, 0, 0)$  for some  $\sigma < 0$  and  $U, V \in SO(3)$ .

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point

7.

Given a real matrix  $F \in \mathbb{R}^{3 \times 3}$  with SVD  $F = U \text{diag}(\lambda_1, \lambda_2, \lambda_3) V^T$  with  $U, V \in SO(3)$ ,  $\lambda_1 \geq \lambda_2 \geq \lambda_3$ , then the essential matrix that minimizes the error  $\|E - F\|_F^2$  is given by

- ☐  $E = U \text{diag}(\sigma, 0, 0) V^T$  with  $\sigma = (\lambda_1 + \lambda_2 + \lambda_3)/2$ .
- ☒  $E = U \text{diag}(\sigma, \sigma, 0) V^T$  with  $\sigma = (\lambda_1 + \lambda_2)/2$ .
- ☐  $E = U \text{diag}(\sigma, 0, 0) V^T$  with  $\sigma = (\lambda_1 + \lambda_2)/2$ .
- ☐  $E = U \text{diag}(\sigma, \sigma, 0) V^T$  with  $\sigma = (\lambda_1 + \lambda_2 + \lambda_3)/2$ .

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8.

How many point correspondences are required to obtain an essential matrix using the linear algorithm?

- ☐ 4
- ☐ 5
- ☐ 6
- ☒ 8

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9.

Which of the following are valid essential matrices?

☐  $E = \begin{bmatrix} 0 & 0 & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix}$

☐  $E = \begin{bmatrix} 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix}$



$$E = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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10.

Suppose we know the camera motion always moves on a plane, say the  $XY$ - plane. The essential matrix  $E = \widehat{T} R$  has the special form

☐  $E = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & c & d \end{bmatrix}, \quad a, b, c, d \in \mathbb{R}$

☐  $E = \begin{bmatrix} 0 & a & 0 \\ b & 0 & 0 \\ c & 0 & d \end{bmatrix}, \quad a, b, c, d \in \mathbb{R}$

☐  $E = \begin{bmatrix} 0 & 0 & a \\ 0 & 0 & b \\ c & d & 0 \end{bmatrix}, \quad a, b, c, d \in \mathbb{R}$

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11.

Now, assuming the same scenario as in the previous question, which of the following solutions for  $(R, T)$  in terms of  $a, b, c, d$  are valid? Assume that  $a^2 + b^2 = 1$  and  $c^2 + d^2 = 1$ .

☐  $T = \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix}, \quad R = \begin{bmatrix} -bd - ac & -ad + bc & 0 \\ ad - bc & -bd - ac & 0 \\ 0 & 0 & 1 \end{bmatrix}$

☐  $T = \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix}, \quad R = \begin{bmatrix} bd - ac & -ad - bc & 0 \\ ad + bc & bd - ac & 0 \\ 0 & 0 & 1 \end{bmatrix}$



$$T = \begin{bmatrix} -b \\ a \\ 0 \end{bmatrix}, \quad R = \begin{bmatrix} -bd + ac & -ad + bc & 0 \\ ad + bc & -bd - ac & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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point

12.

In general, given a normalized essential matrix, we get  $m$  distinct poses  $(R, T)$  and by enforcing the positive depth constraint, we end up with  $n$  valid poses. Which of the following is true?



$(m, n) = (4, 2)$



$(m, n) = (4, 1)$



$(m, n) = (2, 1)$



$(m, n) = (8, 2)$

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8 questions unanswered

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