π Day [Mrs. Armstrong]

Rancho San Joaquin Middle School ASB

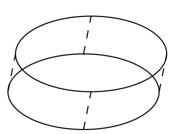
3.14.23

Instructions

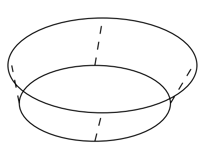
All problems below have answers in the set $\{\, n \in \mathbb{N} \cup \{0\} \mid 0 \leq n \leq 100 \,\}$, and are arranged in random order (but all classes still have the same problems). Between Friday, March 10th, and Tuesday, March 14th, you may submit your solutions at this link. For both seventh and eighth grade, the advisement class who submits the most correct answers will receive a pie party. Good luck and happy π Day!

Problems

1. Mrs. Armstrong makes pie. She bakes P pies. If $\pi^P < P^\pi$, how many pies did she bake?
2. Mrs. Armstrong makes pie. She has 3 pumpkin pies and 19 pineapple pies, and pies of the same type are indistinguishable. If she randomly arranges the pie in a line, what is the expected number of distinct locations where a fourth pumpkin pie could be placed? Placing a pumpkin pie in front of a pumpkin pie is not distinct from placing it after.
3. Mrs. Armstrong makes pie. When selling the pie, the total price includes tax and tip (both multiplicative). Tip is 3.75 times the percentage of tax (e.g., if tax were 10%, then tip would be 37.5%). If only tax is added to the subtotal, the price would be $\$16$, but if only tip were added, the price would be $\$18.75$. What is the price, in dollars, when both tax and tip are added?
4. Mrs. Armstrong makes pie. To optimize the process, she first creates a huge cone of pie, and then makes 24 cuts perpendicular to the axis, for equally thick pies. How many of those pies are actually pie-shaped (a ratio of greater to lesser base areas between 100% and 144%, inclusive)?



100%



144%

5. Mrs. Armstrong makes pie. She bakes one per day. If she starts baking on Thursday, $\sqrt{5}$ Day (February 24th), how many will she have after Tuesday, π Day, March 14th? The year is not necessarily 2023.
6. Mrs. Armstrong makes pie. Her pies are so tasty that she even participates in live pie cook-offs. Each of her 15 pies is judged against each of her opponents 15 pies. A pie is considered *delicious* if it wins against at least 10 opponents. What is the maximum possible number of *delicious* pies?
7. Mrs. Armstrong makes pie. You have a finite number of nickels, while infinite pennies—but no other denominations of coin. If you can buy a $\$5$ pie with a minimum of 128 coins, how many coins are necessary to buy a $\$10$ pie? Express your answer as the sum of the digits of the number of coins.
8. Mrs. Armstrong makes pie. To prevent cooling, she places them under a heat lamp, which will heat any pie sufficiently if completely within a distance of 6.5 units. However, the lamp is placed 6 units above the ground. How many full pies (disks of one unit in diameter) on the ground can be heated at one time?

9. Mrs. Armstrong makes pie. She currently has 36 apple pies, 48 pineapple pies, 20 strawberry pies, 16 cherry pie, and 24 pumpkin pies. If she chooses 80 pies to serve at a feast, with each pie being equally likely, how many are expected to be apple?

10. Mrs. Armstrong makes pie. A pie can contain apples or no apples (two equally likely options), and cinammon or no cinammon (also both equally likely). However, a pie with cinammon can not have no apples (she first determines whether to add cinammon, and based on that result, determines whethere there are apples). If adding apples contributes 4 grams of sugar, adding cinammon contributes 8 grams of sugar, and everything else is already 6 grams of sugar, what is the expected amount of sugar in grams in a pie?

11. Mrs. Armstrong makes pie. A pie contains 10 congruent slices, each a 36° sector of a circle, arranged in that circle. If someone randomly eats a subset of these pies, what is the probability that no remaining slice shares an edge with any other remaining slice? If your answer can be expressed as $\frac{a}{b}$, where a and b are integers with no common factor other than 1, find the sum of the digits of $a+b$.

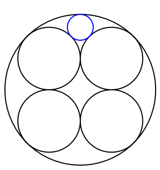
12. Mrs. Armstrong makes pie. People enjoy her pie so much, that you, the last person in line, see 2023 people before you, all waiting to buy pie. Pie can contain apples or no apples, and each person in line has a 50% chance of buying a pie with apples. However, Mrs. Armstrong only has enough apple pies to sell 1012 of them. If you really want to buy apple pie, and will do so if they are not already sold out, what is $a+b$, where $\frac{a}{b}$ is the probability that you will buy apple pie, and $a, b \in \mathbb{N}, \gcd(a,b) = 1$?

13. Mrs. Armstrong makes pie. A pie is a cylinder, with a volume equal to three-fifths the surface area. If the radius is two units, and the surface area may be expressed as $a\sqrt[b]{c}\pi$, where $a, b, c \in \mathbb{N}$ find the minimum possible value of $a+b+c$.

14. Mrs. Armstrong makes pie. Three types of pie, pineapple, pear, and papaya pie, are put in a bag. Pies are then randomly removed, without replacement. For all three pies, the probability that the pie will be the second type of pie completely removed, is equal. If there are 10 pineapple pies, what is the sum of all possible number of total pies in the bag?

15. Mrs. Armstrong makes pie. She lays out all her pies in rows. If she groups them 20 to a row, she will have n left over. If she groups them n to a row, she will have 8 left over. If she groups them 8 to a row, she will have $\frac{n}{2}$ left over. Finally, if she groups them $\frac{n}{2}$ to a row, there will not be any left over. Find the smallest possible number of pies she has.

16. Mrs. Armstrong makes pie. A pie is a disk. Mrs. Armstrong only has a single pie with a diameter of 12 inches, but wants to create more pies by cutting, to make it look like she did more work. She first creates four pies, each internally tangent to the large pie, and externally tangent to two others, all of equal radii. She then cuts out a sixth pie, externally tangent to the first pie and two of the second through fifth pies that themselves are tangent to each other. If the diameter of the sixth pie is 12 inches, and given that the diameter of the sixth pie may be expressed in the form $a + b\sqrt{p}$, where a and b are integers, and p is a prime, what is the value of $a+b+p$?



17. Mrs. Armstrong makes pie. If she starts the day with 60 pies, but then sells three-quarters of them, how many more did she sell than keep?

18. Mrs. Armstrong makes pie. Two pies of radius 1 are externally tangent to each other, while a third pie is also externally tangent to both. If the sum of the areas of the pies is a hundred times the area of the triangle connecting the three circle centers, how many solutions are there for the radius of the third pie?

19. Mrs. Armstrong makes pie. The amount of pie baked each day may be modelled by a cubic polynomial. If she bakes one on Monday, two on Tuesday, four on Wednesday, and eight on Thursday, how many are baked on Friday?

20. Mrs. Armstrong makes pie. She makes 2 cylindrical pies. One pie has a diameter of 3 feet and a height of $\frac{1}{2}$ foot, while the other has a radius of 4 inches and a height of 2 inches. What is the maximum number of small pies she can fit in the large pie, if she can reshape pies?

21. Mrs. Armstrong makes pie. One of her pies has a radius of 10 and an area A. When she expresses A in scientific notation with a base of b, she gets $a \times b^n$, where $n \in \mathbb{Z}$ and $1 \le a < b$. There is a number $a_{\text{max}} \in \mathbb{R}$ such that for all a, where $b \in \mathbb{Z}$ and $b < A$, the inequality $a < a_{\text{max}}$ holds. If a_{max} can be expressed as $a\pi^{\frac{1}{b}}$, where a is an integer, find $a+b$.

22. Mrs. Armstrong makes pie. She also delivers pie; today, Mrs. Armstrong begins at the origin and makes deliveries to the points $(1, 2)$ and $(2, 1)$, in any order, before returning back to the origin. Mrs. Armstrong can only travel vertically or horizontally, one unit at a time, and can only move toward her next stop. How many paths are possible?

23. Mrs. Armstrong makes pie. However, her oven is special, and she can only increase or decrease the temperature by 1 degree Fahrenheit or double the temperature in degrees Fahrenheit. Given that the current temperature is 70 degrees Fahrenheit, there is a temperature between 360 and 555 degrees Fahrenheit that requires the fewest number of operations to reach. Find the sum of the digits of that temperature in degrees Fahrenheit.

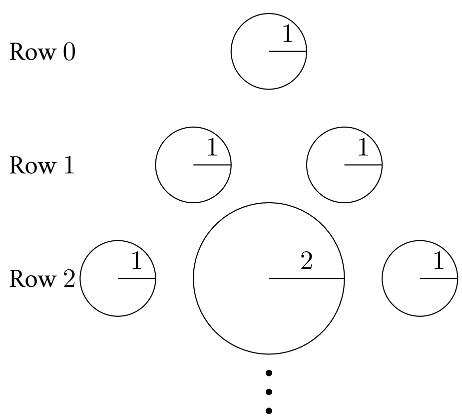
24. Mrs. Armstrong makes pie. Her pies (two-dimensional disk) each contain one white and one black chocolate chip (points distributed uniformly at random within the pie). In each pie, denote the diameter of the pie through the black chocolate chip by d. Let p be the probability that the distance from the white chip to d is less than the distance to the edge of the pie. Find the sum of the numerator and denominator of p when expressed in simplest form.

25. Mrs. Armstrong makes pie. Each apple pie weighs the same as four apples, but by weight, only 12.5% of a pie is actually apple. How many pies can be made with 10 apples and sufficient other ingredients?

26. Mrs. Armstrong makes pie. The number of pies of certain, not necessarily distinct types of her pies, forms the set $\{n+1, n+2, 8, n+4, n+5\}$, where n is an integer. Given that she has no other pies, and that the median number of each type of pie is 7, find the sum of all possible values of n.

27. Mrs. Armstrong makes pie. Three types of pie cost different amounts, in an arithmetic progression. Once tip is applied (a constant proportional increase), the cost of the pies now form a geometric progression. However, once tax is applied after tip (another constant proportional increase), the costs are again in an arithmetic progression. How many distinct pairs of tax and tip exist?

28. Mrs. Armstrong makes pie. She arranges them in Pie-scal's Triangle, which has the structure of Pascal's Triangle, except each value n of the triangle is replaced with a circular pie of radius n units, instead. What is the total perimeter of the pies in Row Five of Pie-scal's Triangle? Given that the answer is of the form $a\pi$, where a is an integer, find a.



29. Mrs. Armstrong makes pie. A person is considered fed when they receive a nonzero portion of pie. If she is to feed 280 total people, and makes four planar cuts per solid right circular cylindrical pie, how many pies are necessary?

30. Mrs. Armstrong makes pie. We also happen to be playing chess. I receive a pie for every win, of probability $\frac{2}{3}$, but lose a pie otherwise. If we play until one person reaches 40 wins, how many pies am I expected to have? Round your answer to the nearest whole number of pies.

31. Mrs. Armstrong makes pie. She makes two types of pies: blueberry, cherry, and watermelon. She sells blueberry pies for $\$7$, cherry pies for $\$4$, and watermelon also for $\$4$. If she sold 28 pies in total and made $\$178$, how many blueberry pies did she sell?

32. Mrs. Armstrong makes pie. A pie is composed of both a plate, which has a constant mass, and the edible portion, which can decrease. Currently, no slices have been eaten. If she were to eat five sixth of the edible portion of the pie, or a thousand grams, the mass of the entire pie would decrease by a factor of 5. Furthermore, eating all the remaining slices would again decrease the mass by a factor of 5. Now with just the pan left, subtracting the mass of three more slices, would increase the mass by a factor of -5 (the mass would go negative). How many slices are there?

33. Mrs. Armstrong makes pie. A pie is made with two ounces of flour. If she has five pounds of flour, and sufficient other ingredients, how many pies can she make?

34. Mrs. Armstrong makes pie. She now has so many apple and pumpkin pies, that she puts several of both in a bag. She then pulls out pies, until either the last apple or the last pumpkin pie is chosen. If the current probability for the last pie chosen being apple is $\frac{3}{5}$, but after adding 10 pumpkin pies, the new probability is $\frac{2}{5}$. How many pumpkin pies need to be added to move the probability from $\frac{2}{5}$ to $\frac{1}{5}$?

35. Mrs. Armstrong makes pie. A pie costs $\$10$ per pound. I have $\$800$, and want to buy as many pies as possible. However, I feel scammed, and only want to pay for the 80% that is actually edible. How many more pies could I get this way?

36. Mrs. Armstrong makes pie. The pie is the region bounded between two horizontal planes of an inverted right-circular cone. Two cross-sections are created, one vertical and one horizontal, both passing through the point which is the intersection of the vertical center and the axis of the cone. The ratio of the vertical cross-section area to the horizontal cross-section area is $1 : 20\pi$. What is the ratio of the height of the pie to the average horizontal radius of the pie?

37. Mrs. Armstrong makes pie. When approximating the pie to be a polyhedron, it turns out that the pie has 182 faces and 200 edges. How many vertices does it have?

38. Mrs. Armstrong makes pie. She has numbered each of the thousand slices of the circular pie from 1 to 12, clockwise, in that order. She first eats the slice numbered 12, so that there are now 11 slices and a gap. At each step, the gap can be filled by an adjacent slice (each of the two adjacent slices have an equal likelihood of being that slice), but a new gap appears. For example, in the first step, with probability $\frac{1}{2}$, the slice numbered 11 could move to occupy the gap, thus moving the gap counterclockwise. When the gap first reaches the slice numbered 6, what is the expected value of the sum of the number of times that the slices numbered 5 or 7 were reached? Round your answer to the nearest integer.

39. Mrs. Armstrong makes pie. Her first pie has a volume of π. However, with each successive pie, the volume decreases by a factor of 10. What is the total volume of all her pies? Round your answer to the nearest integer.

40. Mrs. Armstrong makes pie. A pie is circular, and slices are formed by creating lines through the center. When Mrs. Armstrong serves eight people, four cuts are necessary, and each will be chosen by randomly selecting a point on the edge, uniformly at random, and drawing the diameter through that point. Given that the probability that no single person eats more than a fourth of the pie can be expressed in lowest terms as $\frac{a}{b}$, find $a + b$.