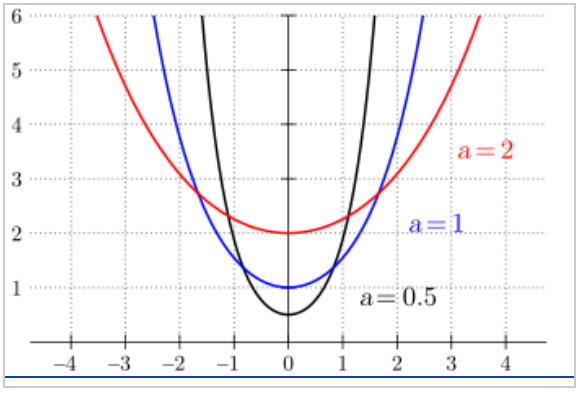
Catenary Function

Problem:
To investigate a Maclaurin series approximation to the Catenary function, imagine that an architecture company wishes to code a real-time simulation walk-through that includes a Catenary Arch (as shown below). To do this they are prepared to implement the Catenary function by a fast but approximate Maclaurin series instead of slower but more accurate math library functions. This program investigates the accuracy of the series approximation to this function.

A hanging chain forms a catenary.

The silk on a spider's web forming multiple (approximate) catenaries.

$$y = a\cosh\left(\frac{x}{a}\right) = \frac{a\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)}{2}$$



The graph at right shows f(x) = y as the vertical axis and x as the horizontal axis. a is a scale factor. Increasing the value of a is equivalent to magnifying the curve, but not changing its shape. You can think of a as a parameter to increase the length of the Catenary or increase the separation of its ends.

This C program is a a Maclaurin series expansion of the Catenary function that goes up to and including the term in x^{12}. The purpose of the series approximation is to speed up its execution so math library function (such as exp() or pow() or any other) are not used in the evaluation of the terms of the series and terms such as x^3 are written explicitly in your code as x*x*x.

Algorithm: While the user wishes to continue, the integer value of the highest power of x in the series is selected (you should reject invalid integers). Then the user selects a range of x, somewhere between -10.0 and +10.0 over which the series is evaluated from 0 in ten equal increments (reject values outside this range). Then the user chooses the value of the scale factor a.

For each value of x the Maclaurin series approximation to the Catenary function is output together with the exact value calculated from the math library using the exp() function.
The error that results from using the series approximation is calculated in two different ways.

1. From comparison with the exact value calculated using the math library cosh() function:
   Exact % Error = 100*(exact value – series value)/exact value

2. From the first truncated term. This gives you an idea of how well the first truncated term approximates to the error.
   Truncation % Error = 100*first truncated term/series value.

Example Output

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**********************************

       Catenary Series
1. Evaluate the function
2. Quit
**********************************
1

       EVALUATING THE CATENARY SERIES APPROXIMATION

Please enter the highest power of x in the catenary series (0, 2, 4, 6, 8, or 10): 8

       CHOOSE THE RANGE OF EVALUATION - low x to high x
Please enter low x - in the range -10.0 to 0.0: -2
Please enter high x - in the range 0.0 to +10.0: 2

Please enter the scale factor the range 0.0 to +10.0: 1

CATENARY SERIES TO x^8 from x = -2.000000 to x = 2.000000
  x          Series         Exact         Exact % Error    Trunc. % Error
-2.000e+00   3.76190e+00   3.76220e+00      7.73296e-03      7.50117e-03
-1.600e+00   2.57743e+00   2.57746e+00      1.19868e-03      1.17557e-03
-1.200e+00   1.81065e+00   1.81066e+00      9.52715e-05      9.42354e-05
-8.000e-01   1.33743e+00   1.33743e+00      2.22317e-06      2.21240e-06
-4.000e-01   1.08107e+00   1.08107e+00      2.67613e-09      2.67290e-09
+0.000e+00   1.00000e+00   1.00000e+00      0.00000e+00      0.00000e+00
+4.000e-01   1.08107e+00   1.08107e+00      2.67615e-09      2.67290e-09
+8.000e-01   1.33743e+00   1.33743e+00      2.22317e-06      2.21240e-06
+1.200e+00   1.81065e+00   1.81066e+00      9.52715e-05      9.42354e-05
+1.600e+00   2.57743e+00   2.57746e+00      1.19868e-03      1.17557e-03
+2.000e+00   3.76190e+00   3.76220e+00      7.73296e-03      7.50117e-03

**********************************
       Catenary Series
1. Evaluate the function
2. Quit
**********************************
1

       EVALUATING THE CATENARY SERIES APPROXIMATION

Please enter the highest power of x in the catenary series (0, 2, 4, 6, 8, or 10): 10

       CHOOSE THE RANGE OF EVALUATION - low x to high x
Please enter low x - in the range -10.0 to 0.0: -4
Please enter high x - in the range 0.0 to +10.0: 2

Please enter the scale factor the range 0.0 to +10.0: 2

CATENARY SERIES TO x^10 from x = -4.000000 to x = 2.000000
  x          Series         Exact         Exact % Error    Trunc. % Error
-4.000e+00   7.52437e+00   7.52439e+00      2.32370e-04      2.27291e-04
-3.400e+00   5.65663e+00   5.65663e+00      4.36965e-05      4.30053e-05
-2.800e+00   4.30180e+00   4.30180e+00      5.56249e-06      5.50275e-06
-2.200e+00   3.33704e+00   3.33704e+00      3.95309e-07      3.92685e-07
-1.600e+00   2.67487e+00   2.67487e+00      1.07646e-08      1.07268e-08
-1.000e+00   2.25525e+00   2.25525e+00      4.52507e-11      4.51916e-11
-4.000e-01   2.04013e+00   2.04013e+00      0.00000e+00      0.00000e+00
+2.000e-01   2.01001e+00   2.01001e+00      0.00000e+00      0.00000e+00
+8.000e-01   2.16214e+00   2.16214e+00      3.22467e-12      3.24521e-12
+1.400e+00   2.51034e+00   2.51034e+00      2.30839e-09      2.30218e-09
+2.000e+00   3.08616e+00   3.08616e+00      1.36039e-07      1.35293e-07

**********************************
       Catenary Series
1. Evaluate the function
2. Quit
**********************************
1

       EVALUATING THE CATENARY SERIES APPROXIMATION

Please enter the highest power of x in the catenary series (0, 2, 4, 6, 8, or 10): 2

       CHOOSE THE RANGE OF EVALUATION - low x to high x
Please enter low x - in the range -10.0 to 0.0: -10
Please enter high x - in the range 0.0 to +10.0: 2

Please enter the scale factor the range 0.0 to +10.0: 10

CATENARY SERIES TO x^2 from x = -10.000000 to x = 2.000000
  x          Series         Exact         Exact % Error    Trunc. % Error
-1.000e+01   1.50000e+01   1.54308e+01      2.79186e+00      2.77778e+00
-8.800e+00   1.38720e+01   1.41284e+01      1.81488e+00      1.80128e+00
-7.600e+00   1.28880e+01   1.30297e+01      1.08762e+00      1.07859e+00
-6.400e+00   1.20480e+01   1.21189e+01      5.84762e-01      5.80221e-01
-5.200e+00   1.13520e+01   1.13827e+01      2.70067e-01      2.68367e-01
-4.000e+00   1.08000e+01   1.08107e+01      9.91952e-02      9.87654e-02
-2.800e+00   1.03920e+01   1.03946e+01      2.47030e-02      2.46446e-02
-1.600e+00   1.01280e+01   1.01283e+01      2.69838e-03      2.69616e-03
-4.000e-01   1.00080e+01   1.00080e+01      1.06587e-05      1.06581e-05
+8.000e-01   1.00320e+01   1.00320e+01      1.70158e-04      1.70122e-04
+2.000e+00   1.02000e+01   1.02007e+01      6.54424e-03      6.53595e-03
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