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Analytical Approximation to the Nonlinear Power Spectrum of Gravitational Clustering

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ABSTRACT

This Letter presents an analytical formula that closely approximates the fully nonlinear power spectrum of matter fluctuations for redshift $z\approx5$ to 0 over a wide range of cosmologically interesting flat models with a varying matter density Ω_m and neutrino fraction Ω_ν. The functional form is motivated by analytical solutions in asymptotic regimes, but in order to obtain accurate approximations, the coefficients are calculated from fits to the nonlinear power spectra that are computed from numerical simulations of four cosmological models. The transformation from the linear to nonlinear power spectrum depends on Ω_m, Ω_ν, and time. A simple scaling rule is introduced, which greatly simplifies the construction of the functional form and allows the formula to depend directly on the rms linear mass fluctuation σ_8 instead of on an effective spectral index, as in earlier work.

Subject headings: cosmology: theory—dark matter—elementary particles—large-scale structure of universe—methods: analytical

§1. INTRODUCTION

The power spectrum of matter fluctuations $P(k)$ provides the most fundamental statistical measure of gravitational clustering. When the amplitude of the density fluctuations is small, the power spectrum can be calculated easily from the linear perturbation theory of gravitational collapse. In the nonlinear regime, however, determination of the fully evolved power spectrum for a given cosmological model requires numerical simulations. Since nearly all observable astronomical systems have experienced some nonlinear collapse, it would provide much physical insight and practical convenience to devise a general analytical approximation (based on simulation results) for the nonlinear power spectrum for a wide range of cosmologically interesting models.

By fitting to *N*-body results, Hamilton et al. (1991) studied scale-free models with a power-law spectrum and presented a simple analytical formula that relates the spatially averaged nonlinear and linear two-point correlation function, $\bar{\xi}_{\rm nl}(r)$ and $\bar{\xi}_l(r_0)$, where r is related to its precollapsed linear scale r_0 by $r_0=r(1+\bar{\xi}_{\rm nl})^{1/3}$. This transformation then appeared to be magically independent of the spectral index n that was assumed in the model. Further tests against numerical simulations, however, found significant errors when the Hamilton et al. function was applied to models with n<-1 (Jain, Mo, & White 1995; Padmanabhan 1996). Jain et al. (1995) instead proposed n-dependent formulas to relate $\bar{\xi}$ and $P(k)$ in the linear and nonlinear regimes in both scale-free models and the standard cold dark matter (CDM) model. For the more realistic CDM model, for which the spectral index changes from the primordial value $n\approx1$ on large scales to nearly -3 on small scales, they used an effective index given by $n_{\rm eff}=d\ln P(k)/d\ln k|_{k_c}$, where k_c is the scale at which the rms mass fluctuation σ is unity. The index $n_{\rm eff}$ therefore reflects the slope of the power spectrum at the length scale where nonlinearity becomes important. Peacock & Dodds (1996) extended this work to allow for a low matter density Ω_m and a nonzero cosmological constant.

No previous work has investigated in detail the subject of linear-to-nonlinear mapping in cold + hot dark matter (C + HDM) models that assume massive neutrinos are a component of the dark matter. This is perhaps true for the following reason: the physics in C + HDM models is generally more complicated than in pure CDM models or CDM models with a cosmological constant (LCDM) because of the additional length scale associated with the free-streaming of the neutrinos (Ma 1996). Nevertheless, massive neutrinos remain a prime dark matter candidate, and the recent evidence for neutrino masses from the Super-Kamiokande experiment has made this possibility particularly intriguing (Fukuda et al. 1998). Although neither Jain et al. nor Peacock & Dodds has tested these models, one may surmise that their formulas can be naturally extended to C + HDM models as long as the spectral index in the formula is calculated from the C + HDM power spectrum. This unfortunately does not work. Both fitting functions underestimate the nonlinear density variance [1] $\Delta_{\rm nl}=4\pi k^3 P_{\rm nl}(k)$, at $k\gtrsim2$ h Mpc^{-1} in C + HDM models, and the errors in Peacock & Dodds, for example, reach $\sim$50% at $k\sim10$ h Mpc^{-1}. The linear-to-nonlinear transformation is therefore regulated by more than simply $n_{\rm eff}$. (Smith et al. 1998 recently reported an agreement between the Peacock-Dodds formula and the results from two C + HDM simulations. Their simulation resolution of $\sim$0.3 h^{-1} Mpc, however, limited their test to only the mildly nonlinear regime and could not probe the nonlinear regime where the large discrepancies reside.)

This Letter differs from previous work in two ways. First, the simple analytical formula presented here closely approximates the fully nonlinear power spectrum of mass fluctuations at $z\lesssim5$ in the previously unexplored C + HDM models as well as the LCDM models with varying Ω_m. Numerical simulations of four *COBE*-normalized flat C + HDM and flat LCDM models are performed in order to calibrate the coefficients in the analytical formula. Second, the formula introduced here depends directly on σ_8 (the rms linear mass fluctuation on 8 h^{-1} Mpc scale) instead of a spectral index, as in previous work. This is achieved by recognizing a scaling rule (see § 3), which also greatly simplifies the construction of the analytical formula. This work also extends into the nonlinear regime a previous investigation of the effects of the free-streaming of the neutrinos on the linear C + HDM power spectrum (Ma 1996).

FOOTNOTES

[1] The notations Δ and $P(k)$ here are the same as in Jain et al. (1995) and are equivalent to Δ^2 and $P(k)/(2\pi)^3$ in Peacock & Dodds (1996).

§2. INPUT LINEAR POWER SPECTRUM

For a wide range of CDM and LCDM models that assume neutrinos are massless, a good approximation to the linear power spectrum is given by

$$P(k,\ a,\ \Omega_\nu = 0) = \frac{Ak^n[D(a)/D_0]^2[\ln(1+\alpha_1 q)/\alpha_1 q]^2}{[1+\alpha_2 q + (\alpha_3 q)^2 + (\alpha_4 q)^3 + (\alpha_5 q)^4]^{1/2}}, \quad (1)$$

where k is the wavenumber in units of Mpc^{-1}, $q=k/\Gamma h$ (Γ is a shape parameter), and $\alpha_1=2.34$, $\alpha_2=3.89$, $\alpha_3=16.1$, $\alpha_4=5.46$, and $\alpha_5=6.71$ (Bardeen et al. 1986). The shape parameter Γ characterizes the dependence on cosmological parameters and is well approximated by $\Gamma=\Omega_m h/\exp[\Omega_b(1+1/\Omega_m)]$ (Efstathiou, Bond, & White 1992; Sugiyama 1995; see also Bunn & White 1997). The function $D(a)$ is the linear growth factor, whose present value is $D_0=D(a{=}1)$, and it can be expressed as $D(a)=ag$, where the relative growth factor g is well approximated by $g[\Omega_m(a), \Omega_\Lambda(a)] = 2.5\Omega_m(a) \times \{\Omega_m(a)^{4/7} - \Omega_\Lambda(a) + [1 + \Omega_m(a)/2](1 + \Omega_\Lambda(a)/70]\}^{-1}$ (Lahav et al. 1991; Carroll, Press, & Turner 1992). In LCDM models, $g\approx 1$ until the universe becomes Λ-dominated at $1+z \approx \Omega_m^{-1/3}$; the value of g then decreases with increasing a. The normalization factor A can be chosen by fixing the value of σ_8; if instead the *COBE* normalization is desired, it is $A=\delta_H^2(c/H_0)^{n+3}/(4\pi)$, where (for flat models) $\delta_H = 1.94\times 10^{-5}\,\Omega_m^{-0.785-0.05\ln\Omega_m}\exp(-0.95\tilde{n}-0.169\tilde{n}^2)$, with $\tilde{n}=n-1$ (Bunn & White 1997).

The linear power spectra for the C + HDM models require additional treatment since the effect of massive neutrinos on the shape of the power spectrum is both time and scale dependent. It is found that by introducing a second shape parameter, $\Gamma_\nu=a^{1/2}\Omega_\nu h^2$, to characterize the neutrino free-streaming distance, one can obtain a good approximation to the linear power spectra (density averaged over the cold and hot components) in flat C + HDM models at $z\lesssim 5$ when neutrinos are adequately nonrelativistic (Ma 1996):

$$P(k,\ a,\ \Omega_\nu) = P(k,\ a,\ \Omega_\nu = 0)\left(\frac{1 + d_1 x^{d_4/2} + d_2 x^{d_4}}{1 + d_3 x_0^{d_4}}\right)^{\Omega_\nu^{1.05}}, \quad (2)$$

where $x=k/\Gamma_\nu$, $x_0=x(a{=}1)$, $P(k, a, \Omega_\nu{=}0)$ for the pure CDM model is given by equation (1), and $d_1=0.004321$, $d_2=2.217\times 10^{-6}$, $d_3=11.63$, and $d_4=3.317$ for k in units of Mpc^{-1}. (The scale factor in Γ_ν assumes *COBE* normalization at a=1. If another normalization is used, replace a by σ_8/σ_8^{cobe}, where σ_8 is the rms mass fluctuation at the epoch of interest.)

§3. NONLINEAR POWER SPECTRUM

Numerical simulations of the structure formation in two flat C + HDM models with neutrino fractions of $\Omega_\nu=0.1$ and 0.2 and in two flat LCDM models with matter densities of $\Omega_m=0.3$ and 0.5 are performed in order to obtain the nonlinear power spectra of matter fluctuations. All four simulations are performed in a (100 Mpc)^3 comoving box. The gravitational forces are computed with a particle-particle particle-mesh (P^{3}M) algorithm

(Bertschinger & Gelb 1991; Ma et al. 1997) with a comoving Plummer force softening length of 50 kpc. An identical set of random phases is used in the initial conditions for all four runs. The primordial power spectrum has an index of n=1, with density fluctuations drawn from a random Gaussian field. A total of 128^3 simulation particles are used to represent the cold dark matter. For the C + HDM models, 128^3 and 10×128^3 particles are used to represent the hot component in the Ω_ν=0.1 and 0.2 models, respectively. Although the larger particle number of 10×128^3 is needed to sample finely the velocity phase space (Ma & Bertschinger 1994), tests performed for the Ω_ν=0.2 model show that the power spectrum itself is little affected when 128^3 hot particles are used. Since structure forms too late in flat C + HDM models with Ω_ν>0.2 (Ma et al. 1997 and references therein), only the Ω_ν=0.1 and 0.2 models are studied here. Both models assume Ω_b=0.05 and h=0.5, while $\Omega_{\rm cdm}$=0.85 and 0.75 for the two models, respectively. The two LCDM models chosen for the simulations have (Ω_m, Ω_Λ, h)=(0.3, 0.7, 0.75) and (0.5, 0.5, 0.7) and Ω_b=0. All four models are normalized to the 4-yr *COBE* results (Bennett et al. 1996; Gorski et al. 1996).

Figure 1 contrasts the linear (*dotted*) and nonlinear (*solid*) power spectra at various redshifts for the four simulated models. The hierarchical nature of gravitational collapse in these models is evident: the high-k modes have become strongly nonlinear, whereas the low-k modes still follow the linear power spectrum. The dashed curves are from the analytical approximation described below. Figure 2 illustrates the dependence of the linear-to-nonlinear transformation on both time and cosmological parameters by plotting, at various redshifts, the ratio of the nonlinear and linear density variance, $\Delta_{\rm nl}(k)/\Delta_l(k_0)$, against the linear $\Delta_l(k_0)$ (where $\Delta=4\pi k^3 P$). Note that $\Delta_{\rm nl}$ and Δ_l are evaluated at different wavenumbers, where $k_0=k(1+\Delta_{\rm nl})^{-1/3}$ corresponds to the precollapsed scale of k, as introduced in Hamilton et al. (1991). However, the dependence of $\Delta_{\rm nl}(k)/\Delta_l(k_0)$ on Ω_m, Ω_ν, and time demonstrates that the universality seen by Hamilton et al. is limited to the power-law models with n>-1 studied there. The linear-to-nonlinear mapping is clearly more complicated in more realistic cosmological models (such as LCDM and C + HDM) whose spectral indices change continuously with length scale.

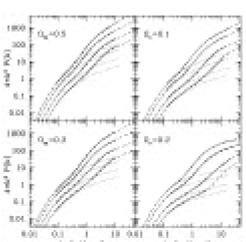
Fig. 1

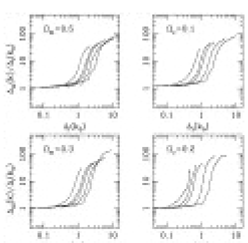
Fig. 2

Despite the apparent complication from the time dependence in Figure 2, I find the scaling rule given below very helpful in simplifying the construction of an analytical approximation. As Figure 3 shows, the time dependence is largely removed when $\Delta_{\rm nl}/\Delta_l$ is plotted against the scaled quantity $\tilde{\Delta}_l$,

$$\tilde{\Delta}_l = \frac{\Delta_l}{\sigma_8^\beta}, \quad \beta = 0.7 + 10\Omega_\nu^2, \qquad (3)$$

instead of Δ_l. For example, β=0.7 for the CDM and LCDM models, while β=0.8 and 1.1 for the Ω_ν=0.1 and 0.2

C + HDM models. The deviations in the two LCDM models at late times ($a \gtrsim 0.7$) result from the retardation in the relative growth factor g in $\Omega_m < 1$ models. As shown in equation (4), they can be easily accounted for by including a factor of g^3 in the highly nonlinear regime in the fitting formula.

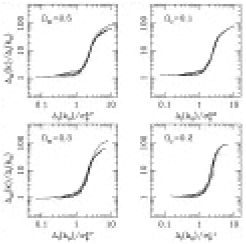
Fig. 3

The scaling behavior in equation (3) can be understood in terms of the time dependence of the local slope of the linear power spectrum near the scale where nonlinear effects become important. An example is the effective spectral index $n_{\rm eff}+3 = d\ln\Delta_l/d\ln k|_{k_c}$ considered by Jain et al. (1995), where k_c is the scale at which the rms linear mass fluctuation σ equals unity. Figure 4 illustrates the dependence of $n_{\rm eff}$ on time and cosmological parameters. The value of $n_{\rm eff}$ generally increases with time because, as σ grows, the wavenumber k_c at which $\sigma = 1$ decreases and the spectral index at k_c (i.e., $n_{\rm eff}$) becomes larger since the slope of the power spectrum for the models studied here always increases with decreasing k. In Figure 4, $n_{\rm eff}$ exhibits the fastest growth in the $\Omega_\nu = 0.2$ C + HDM model at $\sigma_8 \gtrsim 0.3$ because the neutrino free-streaming effect is more prominent in higher Ω_ν models; this effect acts to suppress the structure growth below the free-streaming scale, causing Δ_l to bend more at $0.1 < k < 1$ h Mpc^{-1} (see the dotted curves in Fig. 1). At $\sigma_8 \lesssim 0.3$ in the $\Omega_\nu = 0.2$ model, on the other hand, $n_{\rm eff}$ stays nearly constant because it is probing the nearly flat, $k > 5$ h Mpc^{-1} part of Δ_l. Despite this interesting behavior, Figure 4 shows that for all models, the dependence of $n_{\rm eff}+3$ on σ_8 is well approximated by a power law at $\sigma_8 \gtrsim 0.3$: $d\ln(n_{\rm eff}+3)/d\ln\sigma_8 \propto \beta$, where $\beta = 0.7 + 10\Omega_\nu^2$, as given in equation (3). This therefore explains why replacing the factor $n_{\rm eff}+3$ in the earlier work with σ_8^β works well here.

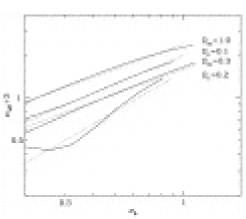
Fig. 4

The simple scaling behavior introduced by equation (3) allows one to approximate the evolution of the nonlinear power spectrum directly in terms of σ_8 instead of $n_{\rm eff}$. Combining these factors, I find that a close approximation for the nonlinear power spectrum is given by

$$\frac{\Delta_{\rm nl}(k)}{\Delta_l(k_0)} = G\left(\frac{\Delta_l}{g_0^{1.5}\sigma_8^{\beta}}\right),$$

$$G(x) = [1 + \ln(1 + 0.5x)]\,\frac{1 + 0.02x^4 + c_1 x^8/g^3}{1 + c_2 x^{7.5}}, \qquad (4)$$

where σ_8 is the rms linear mass fluctuation at the epoch of interest, β is given by equation (3), $k_0 = k(1+\Delta_{\rm nl})^{-1/3}$, and $g_0 = g(\Omega_m, \Omega_\Lambda)$ and $g = g[\Omega_m(a), \Omega_\Lambda(a)]$ are, respectively, the relative growth factor [2] at the present (i.e., a=1) and at a discussed in § 2. The time dependence is in factors of σ_8^{β} and g. For CDM and LCDM, a good fit is given by $c_1 = 1.08\times10^{-4}$ and $c_2 = 2.10\times10^{-5}$. For C + HDM, a good fit is given by $c_1 = 3.16\times10^{-3}$ and $c_2 = 3.49\times10^{-4}$ for $\Omega_\nu = 0.1$ and by $c_1 = 6.96\times10^{-3}$ and $c_2 = 4.39\times10^{-4}$ for $\Omega_\nu = 0.2$. The dependence of c_1 and c_2 on Ω_ν can, in principle, be cast in a functional form (see Ma 1998), but since the allowed range of Ω_ν is narrow, separate coefficients are given here in order to obtain the highest possible fitting accuracy. The accuracy of equation (4) is illustrated in Figure 1 (*dashed curves*), where the rms error for each model ranges from 3% to 10% for $k \lesssim 10\ h$ Mpc^{-1} at all times except $z \gtrsim 4$, when the errors are about 15%.

The functional form of $G(x)$ in equation (4) is chosen to give the appropriate asymptotic behavior $\Delta_{\rm nl} \rightarrow \Delta_l$ in the linear regime ($x \ll 1$) and $\Delta_{\rm nl} \propto \Delta_l^{3/2}$ in the stable clustering regime ($x \gg 1$). In the mildly nonlinear regime, $0.1 < \Delta_l < 1$, the prefactor $[1+\ln(1+0.5x)]$ is introduced to approximate the nonnegligible positive slope of $\Delta_{\rm nl}/\Delta_l$. This factor is needed because $\Delta_{\rm nl}$ and Δ_l are evaluated at different wavenumbers k and k_0, where the precollapsed k_0 is always smaller than k. Due to the steep positive slope of Δ_l in this region, Δ_l at k_0 is noticeably smaller than at k, and $\Delta_{\rm nl}(k)/\Delta_l(k_0)$ is thus significantly above unity. Without the logarithmic prefactor in equation (4) to account for this elevation, the approximation to $\Delta_{\rm nl}$ can be underestimated by up to 30% at $0.1 < \Delta_l < 1$.

FOOTNOTES

[2] Even for C + HDM models, g in eq. (4) is taken to be the familiar $g[\Omega_m(a), \Omega_\Lambda(a)]$ for $\Omega_\nu = 0$ models. The true relative growth factor for C + HDM models is in fact given by eq. (2), but due to its complicated scale dependence, attempts thus far to incorporate this factor directly into eq. (4) have not led to approximations with high accuracies (see Ma 1998).

§4. SUMMARY AND DISCUSSION

This Letter presents a single formula, equation (4), that accurately approximates the fully nonlinear power spectrum of matter fluctuations for redshift $z \lesssim 5$ for flat CDM, LCDM, and C + HDM models with varying matter density Ω_m and neutrino fraction Ω_ν. Equations (1), (2), and (4) together offer a complete description of the shape and time evolution of the matter power spectrum in both linear and nonlinear regimes for a wide range of cosmologically interesting models. Figure 1 summarizes the analytical and simulation results for the four models that are used to calibrate the coefficients in equation (4). Depending on the models and epochs, the rms errors are between 3% and 10% for $k\lesssim 10\ h\ \mathrm{Mpc}^{-1}$ at $z\lesssim 4$ in equation (4). In comparison, the Peacock & Dodds formula has a rms error between 6% and 17% for the two LCDM simulations studied here, and the error reaches 50% at $k\sim 10\ h\ \mathrm{Mpc}^{-1}$ for the two C + HDM models. It should be remembered, however, that equation (4) has not been tested beyond the range of models explored in this Letter.

In contrast to the scale-free models studied in Hamilton et al. (1991), the relation between the linear and the nonlinear power spectrum is far from universal. The different panels of Figure 2 illustrate the dependence on cosmological parameters Ω_m and Ω_ν. Moreover, within a given model, the fact that the curves for different times in Figure 2 do not overlap is important because it implies that the ratio $\Delta_{\mathrm{nl}}(k)/\Delta_l(k_0)$ depends not only on $\Delta_l(k_0)$ but also on time or, equivalently, the overall amplitude of $\Delta_l(k_0)$. This amplitude (or time) dependence is present, albeit in a somewhat subtle form, in earlier work. The effective spectral index used by Jain et al. (1995) is clearly time-varying, as shown in Figure 4 of this Letter. The local spectral index $n=d\ln P/d\ln k|_{k_0}$ used by Peacock & Dodds (1996) depends on time implicitly through the factor Δ_{nl} in the relation $k_0=k(1+\Delta_{\mathrm{nl}})^{-1/3}$. In comparison, in this Letter, I have adopted the commonly used parameter σ_8, instead of a spectral index, to characterize this time dependence, and I have shown that the scaling behavior in equation (3) and Figure 3 absorbs this dependence in C + HDM as well as LCDM models and results in the simple formula, equation (4).

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FIGURES

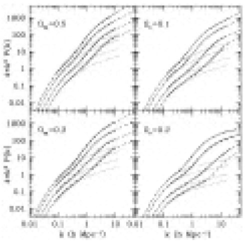
Full image (136kb) | Discussion in text

FIG. 1.—The linear and fully evolved power spectrum at various redshifts for two flat LCDM models (Ω_m =0.3 and 0.5) and two flat C + HDM models (Ω_ν =0.1 and 0.2). The solid curves are computed directly from the simulations; the dashed curves show the close approximation given by eq. (4) of this Letter; the dotted curves represent the linear power spectrum given by eqs. (1) and (2). All are normalized to the 4-yr *COBE* data and the present values of σ_8 that are 1.29 (Ω_m=0.3), 1.53 (Ω_m=0.5), 0.9 (Ω_ν=0.1), and 0.81 (Ω_ν=0.2). In each panel, the curves from the bottom up are for scale factors a=0.2, 0.33, 0.6, and 1.

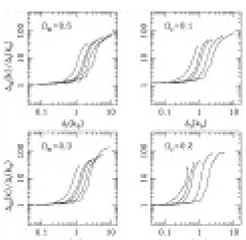
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FIG. 2.—Ratio of the nonlinear to linear density variance, $\Delta_{\rm nl}(k)/\Delta_l(k_0)$, as a function of $\Delta_l(k_0)$ at various redshifts for two LCDM and two C + HDM models. The wavenumber k_0 corresponds to the precollapsed linear value of k, where $k_0 = k(1+\Delta_{\rm nl})^{-1/3}$. In each panel, the curves from left to right correspond to a=0.2, 0.33, 0.4, 0.6, and 1.

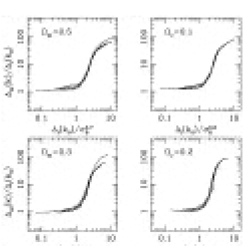
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FIG. 3.—Same as Fig. 2, but the horizontal axis represents the scaled Δ_l penta *plus-kern*>penta *plus-kern*>penta *plus-kern*>penta *plus-kern*>penta *plus-kern*>/σ_8^β , where β=0.7+10Ω_ν^2 . It shows that the time

dependence in the ratio $\Delta_{\rm nl}/\Delta_l$ in Fig. 2 is now largely removed, except at late times, in the two LCDM models (see text).

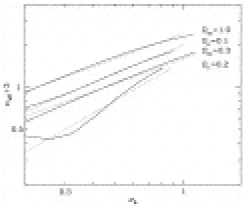 Full image (54kb) | Discussion in text

FIG. 4.—The effective spectral index, $n_{\rm eff}$, as a function of σ_8 for four models. The dotted lines illustrate that the power law $d\ln(n_{\rm eff}+3)/d\ln\sigma_8 \propto^{\beta}$ is a good approximation for $\sigma_8 \gtrsim 0.3$.