homework 4

Question 9.8

9.8 *An alternative approach for position estimation*

Assume a=(1,1), b=(1,-1), c=(-1,1), and d=(-1,-1). Locate node u using the noisy range estimates $l_a = 0.7, l_b = 1.5, l_c = 1.6,\ l_d = 2.2$.

(a) Calculate the centroid of the three known locations.

(b) Obtain $\boldsymbol{R}$ and $\boldsymbol{y}$ according to (9.25) and (9.26) respectively.

(c) Calculate the position estimate by using (9.28).

$$\boldsymbol{R} = \begin{bmatrix} 2(\boldsymbol{r}_1 - \bar{\boldsymbol{r}}) \\ \vdots \\ 2(\boldsymbol{r}_N - \bar{\boldsymbol{r}}) \end{bmatrix} \tag{9.25}$$

$$\boldsymbol{y} = \begin{bmatrix} \bar{l}_i^2 - l_1^2 - \bar{\boldsymbol{r}}_\delta + \|\boldsymbol{r}_1 - \bar{\boldsymbol{r}}\|^2 \\ \\ \bar{l}_i^2 - l_N^2 - \bar{\boldsymbol{r}}_\delta + \|\boldsymbol{r}_N - \bar{\boldsymbol{r}}\|^2 \end{bmatrix} \tag{9.26}$$

$$\hat{\boldsymbol{r}} = \bar{\boldsymbol{r}} + \boldsymbol{y}^T \boldsymbol{R}(\boldsymbol{R}^T \boldsymbol{R})^{-1} \tag{9.28}$$

answer 9.8a

$$\text{centroid} = \bar{r} = \frac{\sum_n x_n}{n} = (0,0)$$

answer 9.8b

$$R = \begin{bmatrix} 2(a-\bar{r}) \\ 2(b-\bar{r}) \\ 2(c-\bar{r}) \\ 2(d-\bar{r}) \end{bmatrix} = \begin{bmatrix} 2(1-0) & 2(1-0) \\ 2(1-0) & 2(-1-0) \\ 2(-1-0) & 2(1-0) \\ 2(-1-0) & 2(-1-0) \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & -2 \\ -2 & 2 \\ -2 & -2 \end{bmatrix}$$

$$\bar{l_i}^2 = \frac{1}{n}\sum_n ||l_i||^2 = \frac{1}{4}(l_a^2 + l_b^2 + l_c^2 + l_d^2) = 2.535$$

$$\bar{r_\delta} = \frac{1}{n}\sum ||r_i - \bar{r}||^2 = \frac{1}{4}(||a-\bar{r}||^2 + ||b-\bar{r}||^2 + ||c-\bar{r}||^2 + ||d-\bar{r}||^2) = 2$$

$$y = \begin{bmatrix} \bar{l_i^2} - l_1^2 - \bar{r_\delta} + ||r_1 - \bar{r}||^2 \\ \vdots \\ \bar{l_i^2} - l_N^2 - \bar{r_\delta} + ||r_N - \bar{r}||^2 \end{bmatrix} = \begin{bmatrix} 2.535 - 0.7^2 - 2 + ||a - \bar{r}||^2 \\ 2.535 - 1.5^2 - 2 + ||b - \bar{r}||^2 \\ 2.535 - 1.6^2 - 2 + ||c - \bar{r}||^2 \\ 2.535 - 2.2^2 - 2 + ||d - \bar{r}||^2 \end{bmatrix} = \begin{bmatrix} 2.535 - 0.7^2 - 2 + 2 \\ 2.535 - 1.5^2 - 2 + 2 \\ 2.535 - 1.6^2 - 2 + 2 \\ 2.535 - 2.2^2 - 2 + 2 \end{bmatrix} = \begin{bmatrix} 2.045 \\ 0.285 \\ -0.025 \\ -2.305 \end{bmatrix}$$

answer 9.8c

```
In [54]: bar_r + y.T @ R @ np.linalg.inv(R.T @ R)
Out[54]: array([0.5825, 0.505 ])

In [55]: bar_r
Out[55]: array([0, 0])

In [56]: y.T
Out[56]: array([ 2.045,  0.285, -0.025, -2.305])

In [57]: R
Out[57]:
array([[ 2,  2],
       [ 2, -2],
       [-2,  2],
       [-2, -2]])
```

$$\hat{r} = \bar{r} + y^T R (R^T R)^{-1} = [0.5825, 0.505]$$

Question 9.9

9.9 Weighted centroid computation

Three beacons are located at *a*=(1,1), *b*=(1,-1), and *c*=(-1,1). The received powers from nodes *a*, *b*, and *c* are 1.2, 1.5, and 1.7 respectively. Calculate the unknown position of the receiver through a weighted centroid computation.

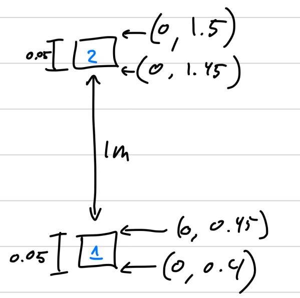
answer 9.9

$$\bar{r} = \frac{1}{1.2 + 1.5 + 1.7}(1.2(1,1) + 1.5(1,-1) + (-1,1)1.7) = \frac{1}{4.4}((1.2, 1.2) + (1.5, -1.5) + (-1.7, 1.7)) = \frac{1}{4.4}(1, 1.4)$$

Question 3

3. Using landmarks for location.
Two collinear landmarks on the line y=0 observed by a camera have linear dimension 0.05 m and the space between them is 1 m. The bottom edge of the first landmark is at (0,0.4). There are 1000 pixels in the horizontal direction and the camera looks at the landmarks in the horizontal plane. It has a 60 degree field of view. The bottom landmark occupies pixels 235-259 and the top landmark occupies pixels 714 to 732, going from left to right in the focal plane. Where is the camera?

answer 3

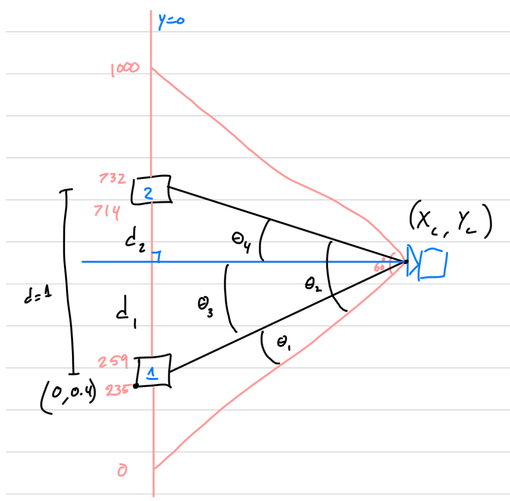


Landmark 1

Centroid 1 = $(0, \frac{1}{2}(0.4 + 0.45)) = (0, 0.425)$

Landmark 2

Centroid 2 = $(0, \frac{1}{2}(1.45 + 1.5)) = (0, 1.475)$



$\theta_1 = \frac{235+259}{2} \cdot \frac{60^\circ}{1000} = 14.82^\circ$

$\theta_2 = \frac{714+732}{2} \cdot \frac{60^\circ}{1000} = 43.38^\circ$

$\theta_3 = 30^\circ - \theta_1 = 15.18^\circ$

$\theta_4 = \theta_2 - 30^\circ = 13.38^\circ$

$\frac{d_1}{d_2} = \frac{\tan\theta_3}{\tan\theta_4} = 1.1406$

$\rightarrow d_1 = 1.1406 \cdot d_2$

$d = d_1 + d_2 = (1.1406 * d_2)d_2 = d_2(1.1406 * 1)$

$d = 1 + 2 * 0.025 = 1.05$

$d_1 = 0.5594$

$d_2 = 0.4905$

$\tan\theta_3 = \frac{d_1}{x_c} \rightarrow x_c = \frac{d_1}{\tan\theta_3} = 2.0621m$

$\tan\theta_4 = \frac{d_2}{x_c} \rightarrow x_c = \frac{d_2}{\tan\theta_4} = 2.0621m$

Both are equal ✔️

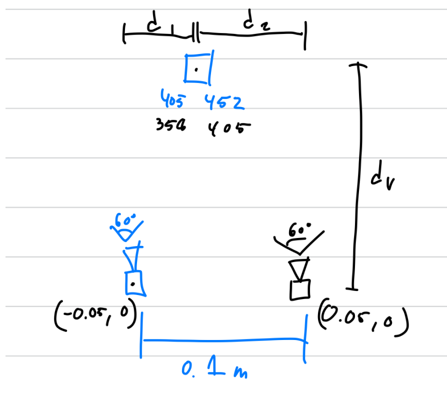
$(x_c, y_c) = (0, 0.425) + (x_c, d_1) = (2.0261, 0.9844)$

$(x_c, y_c) = (0, 1.475) + (x_c, -d_2) = (2.0261, 0.9844)$

Question 4

4. Stereo vision for relative location.
Two cameras with the same orientation view the same object in the horizontal plane. They are separated by 0.1 m. Their field of view is 60 degrees. There are 1000 pixels in the horizontal direction. In the left camera's view the object occupies pixel 405-452 while in the right camera's view it occupies 358-405. Using the centroid of the object as its center, what is the location of the object for a coordinate system in which the cameras are at (-0.05,0) and (0.05,0)?

answer 4



$\theta_1 = \frac{405+452}{2} \cdot \frac{60^\circ}{1000} = 25.71^\circ$

$\theta_{12} = 30^\circ - \theta_1 = 4.29^\circ$

$\theta_2 = \frac{358+405}{2} \cdot \frac{60^\circ}{1000} = 22.89^\circ$

$\theta_{22} = 30^\circ - \theta_2 = 7.11^\circ$

$\tan\theta_{12} = \frac{d_1}{d_v} \rightarrow d_1 = d_v \cdot \tan\theta_{12}$

$\tan\theta_{22} = \frac{d_1+0.1}{d_v}$

$d_v = \frac{0.1}{\tan\theta_{22} - \tan\theta_{12}} = 2.0113m$

$d_1 = 0.1508m$

$(d_1 + 0.05, d_v) = (-0.2008m, 2.0113m)$