homework1

From the textbook (see Bruin Learn): Questions 3.9, 4.1, 4.4

Question 3.9 - *Geological layers*

Consider example 3.13. The distances to the detectors are `x1=100m` and `x2=200m`.

The propagation time to node 1 and node 2 is `0.6675s` and `0.67s` respectively. Compute

the values of `d` and `v`.

Example 3.13 Location of geological layers

Depths of geological layers can be determined by using an array of sensors on the surface. A source (such as an explosive) is excited at a known time and location. The signals are then observed at each of the node locations. The various layers have different propagation velocities, and so to solve the equations for both depth and velocity an array is required.

For example, suppose the first layer has a depth of d, and the first two detectors are at ranges of x_1 and x_2 from the source, as depicted in Figure 3.11.

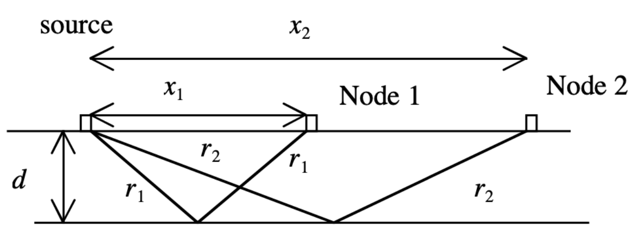


Figure 3.11 Reflections from a single geological layer

The reflected path from the source to node 1 has length $2r_1$, while from the source to 2 it is $2r_2$ where $r_1^2 = (x_1/2)^2 + d^2$ and $r_2^2 = (x_2/2)^2 + d^2$.

The propagation time to node 1 is $2r_1/v$, and the time to node 2 is $2r_2/v$. Eliminating r_1 and r_2, there are two non-linear equations in d and v.

An important issue in seismic measurements is the coupling of the measurement device to the medium. A buried seismometer has much superior sensitivity to one that is simply placed on the surface. Typically a seismometer or geophone must be buried at least 10 cm or firmly attached to a rigid structure for good results. In any case the seismic channel can be decomposed into the components depicted in Figure 3.12: coupling between source and medium, the dispersive medium, an additive noise source, and the coupling between the medium and the sensor. The noise level is highly environmentally dependent. Vehicular traffic, construction activity, and ocean waves all contribute seismic noise. In an application such as identification of a vehicle based on its seismic signature, one must deal with the problem of estimating not only the signature of the vehicle but also the coupling of the vehicle to the medium and the noise due to other vehicles. Of course, the coupling of the sensor to the medium can be determined by a calibration procedure.

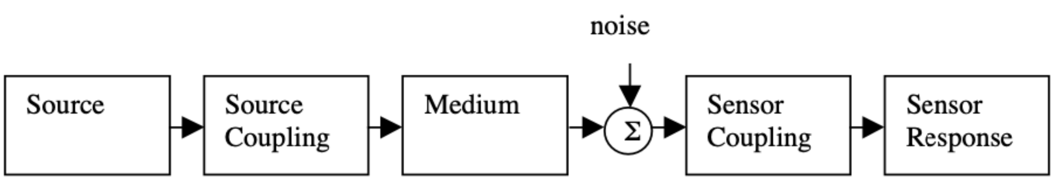


Figure 3.12 Model of signal propagation

answer 3.9

$$r_1^2 = \left(\frac{x_1}{2}\right)^2 + d^2$$
$$r_2^2 = \left(\frac{x_2}{2}\right)^2 + d^2$$

$$0.6675 = \frac{2r_1}{v}$$
$$0.67 = \frac{2r_2}{v}$$

$$0.6675 = \frac{2\sqrt{\left(\frac{x_1}{2}\right)^2 + d^2}}{v} = \frac{2\sqrt{\left(\frac{100}{2}\right)^2 + d^2}}{v}$$
$$0.67 = \frac{2\sqrt{\left(\frac{x_2}{2}\right)^2 + d^2}}{v} = \frac{2\sqrt{\left(\frac{200}{2}\right)^2 + d^2}}{v}$$

$$0.6675 * v = 2\sqrt{\left(\frac{100}{2}\right)^2 + d^2} \rightarrow v = \frac{2\sqrt{\left(\frac{100}{2}\right)^2 + d^2}}{0.6675}$$
$$0.67 * v = 2\sqrt{\left(\frac{200}{2}\right)^2 + d^2} \rightarrow v = \frac{2\sqrt{\left(\frac{200}{2}\right)^2 + d^2}}{0.67}$$

$$\frac{2\sqrt{\left(\frac{200}{2}\right)^2 + d^2}}{0.67} = \frac{2\sqrt{\left(\frac{100}{2}\right)^2 + d^2}}{0.6675}$$

$$d = \sqrt{\frac{13333.25}{0.013375}} = 998.438m$$
$$v = 2995.323\frac{m}{s}$$

Question 4.1 - *Accelerometer system design and system scale estimate*

Consider the design of an accelerometer that is intended to meet specific acceleration sensitivity goals over a specified bandwidth given a position sensor sensitivity. The designer may adjust mass, spring constant, proof mass value and resonance quality factor to achieve goals.

Question 4.1a

a) First, consider an accelerometer with an electronic displacement sensor having a position sensitivity of 1 pm/(Hz)$^{1/2}$. For a target acceleration sensitivity of 10^{-5} m/s^2/(Hz)$^{1/2}$ in the bandwidth from 0.001 to 1 kHz, find the largest sensor resonance frequency that may meet this objective while ignoring the effect of thermal noise.

answer 4.1a

$$\text{position sensitivity} = \frac{1\text{pm}}{\text{Hz}^{\frac{1}{2}}}$$
$$\text{target acceleration sensitivity} = 10^{-5}\frac{\frac{m}{s^2}}{\text{Hz}^{\frac{1}{2}}}$$
$$0.001\text{kHz} \leq \text{bandwidth} \leq 1\text{kHz}$$

low-frequency responsivity is given by

$$\text{resonant frequency} = \omega_0 = \sqrt{\frac{k}{m}}$$
$$\text{deflection of the proof mass} = \delta D$$
$$\text{acceleration} = a$$

$$\frac{\delta D}{a} = \frac{1}{\omega_0^2} \;\forall\; \omega < \omega_0$$

$$\begin{aligned} \omega_0^2 &= \frac{a}{\delta D} = \frac{10^{-5}}{10^{-12}} = 10^7 \\ \omega_0 &= \sqrt{10^7} \approx 3,162.277 \\ f_0 &= \frac{\omega_0}{2 * \pi} = 503.292 \text{ Hz} \end{aligned}$$

Question 4.1b

b) Now, include the effect of thermal noise for operation at 300K and compute the required proof mass value for this accelerometer for Q values of 1, 100, and 10^4 such that the thermal noise contribution does not exceed the sensitivity target.

answer 4.1b

$$k_b = 1.38 * 10^{-23} \text{ J/K}$$
$$T = 300K$$
$$\omega_0 = 3,162.227 \text{ rad/sec}$$

$$\text{thermal noise equivalent acceleration} = TNEA = \sqrt{\frac{4 k_b T \omega_0}{MQ}}$$
$$= \sqrt{\frac{4 * 1.38 * 10^{-23} * 300 * 3,162.227}{M\{1, 100, 10^4\}}}$$

set `TNEA` equal to `10^-5`

$$10^{-5} = \sqrt{\frac{4 * 1.38 * 10^{-23} * 300 * 3,162.227}{M\{1, 100, 10^4\}}}$$

$$\begin{cases} Q = 1 \rightarrow M = 5.236 * 10^{-7} \text{ kg} \\ Q = 100 \rightarrow M = 5.236 * 10^{-9} \text{ kg} \\ Q = 10^4 \rightarrow M = 5.236 * 10^{-11} \text{ kg} \end{cases}$$

Question 4.1c

c) If this mass were to be composed of a planar Si structure, of thickness 1μ, what would be the required area of this structure for each Q value?

answer 4.1c

Density of Si = `2.3- grams/cm^3`

$$A = \frac{M}{\text{density} * \text{thickness}}$$

$$\begin{cases} Q = 1 \to A = \frac{5.236*10^{-7}\text{kg}}{1*10^{-9}*2.3\frac{g}{cm^3}} = 2.276 * 10^{-4}\ \text{m}^2 \\ Q = 10 \to A = \frac{5.236*10^{-9}\text{kg}}{1*10^{-9}*2.3\frac{g}{cm^3}} = 2.276 * 10^{-6}\ \text{m}^2 \\ Q = 10000 \to A = \frac{5.236*10^{-11}\text{kg}}{1*10^{-9}*2.3\frac{g}{cm^3}} = 2.276 * 10^{-8}\ \text{m}^2 \end{cases}$$

Question 4.1d

d) What would be the required spring constant value to achieve the specified resonance frequency above and with masses corresponding to each Q values?

answer 4.1d

$$\omega_0 = \sqrt{\frac{k}{m}}$$
$$k = \omega_0^2 * m$$
$$\omega_0 = 3,162.227\ \text{rad/sec}$$

$$\begin{cases} Q = 1, m = 5.236 * 10^{-7}\ \text{kg} \to k = 5.236\frac{N}{m} \\ Q = 100, m = 5.236 * 10^{-9}\ \text{kg} \to k = 5.236 * 10^{-2}\frac{N}{m} \\ Q = 10000, m = 5.236 * 10^{-11}\ \text{kg} \to k = 5.236 * 10^{-4}\frac{N}{m} \end{cases}$$

Question 4.4 - *Signal dependent temperature coefficients*

A silicon pressure microsensor system employs a piezoresistive strain sensor for diaphragm deflection having a responsivity to displacement of $\alpha = 1\text{V}/\mu$ (at T = 300K). Further, this displacement is related to pressure with a pressure-dependent deflection of $K = 0.01\mu/\text{N/m}^2$. This is followed by an amplifier having a gain G = 10 (at T = 300K). This amplifier further shows an input-referred offset potential, $V_{offset} = 0$ at 300K. Each of these characteristics include temperature coefficients. These temperature coefficients are listed here:

α	10^{-2}/K

K	10^{-4}/K
G	$- 10^{-3}$/K
V_{offset}	- 10μV/K

Question 4.4a

a) Consider that the pressure sensor is exposed to no pressure difference. Find an expression for its output signal in terms of input pressure and temperature. Compute the temperature coefficient that describes its operation.

answer 4.4a

$$\begin{aligned}
V_{offset} &= 10\mu V(300K - T) \\
&= 10^{-5}(300 - T) \\
G &= 10(1 - 10^{-3}(T - 300)) \\
V_{out} &= P_{in} \propto KG * V_{offset}G \\
&= V_{offset}G \\
&= (10^{-5}(300 - T))(10(1 - 10^{-3}(T - 300)))
\end{aligned}$$

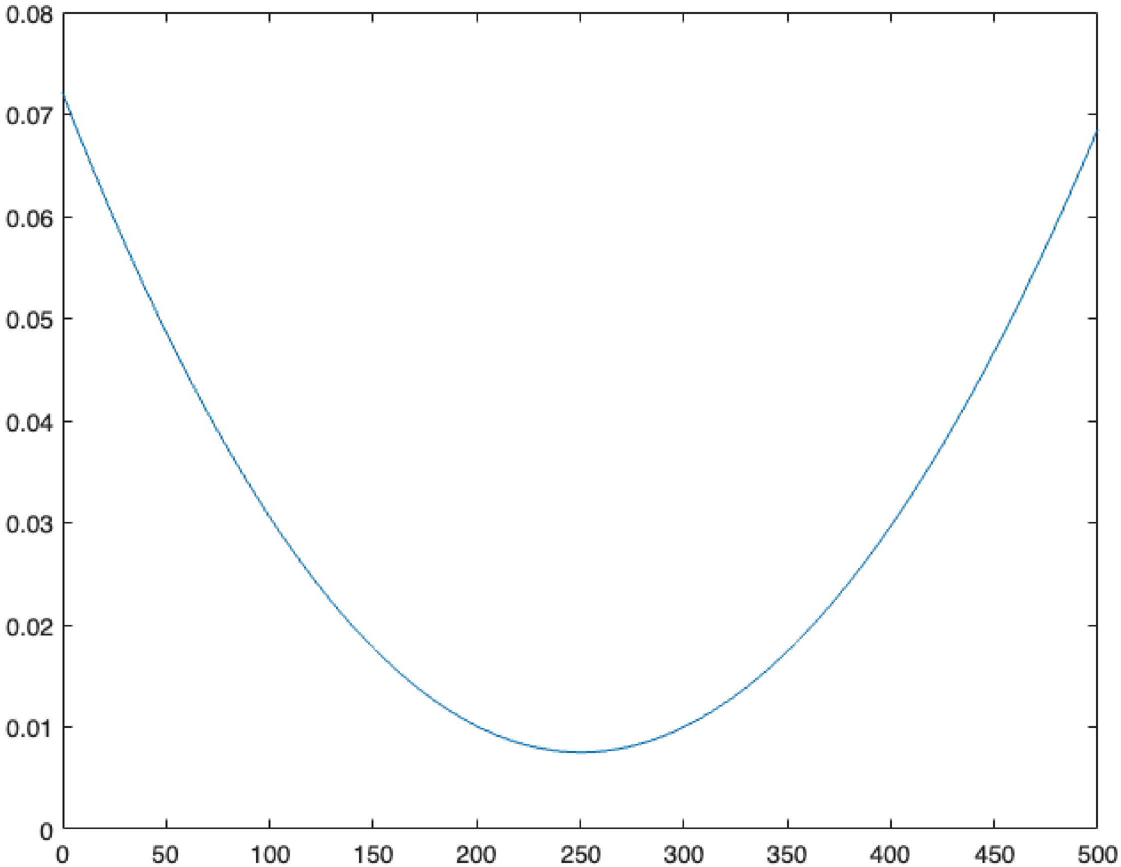
Question 4.4b

b) Consider that the pressure sensor is exposed to a pressure difference signal of 0.1 N/m^2. Find an expression for the dependence of its output signal on temperature and plot this. Estimate the temperature coefficient of the entire sensor system responsivity in the neighborhood of the specific temperature values of 300K and 350K.

answer 4.4b

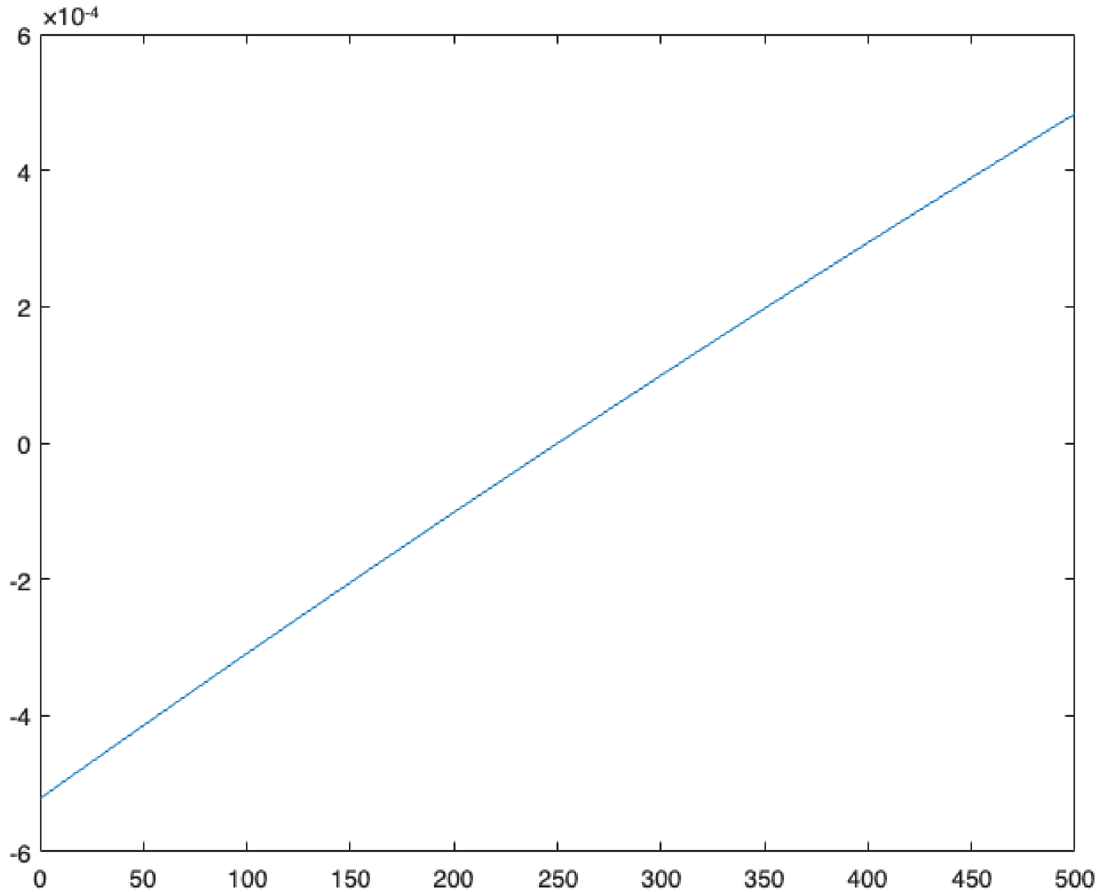
$$\begin{aligned}
V_{offset} &= -10^{-5} * (T - 300) \\
G &= 10 - 10^{-3} * (T - 300) \\
\alpha &= 1 + 10^{-2} * (T - 300) \\
K &= 0.01 + 10^{-4} * (T - 300) \\
V_{out} &= P_{in}\alpha KG + V_{offset}G \\
&= 0.1(1 + 10^{-2} * (T - 300))(0.01 + 10^{-4} * (T - 300))(10 - 10^{-3} * (T - 300)) \\
&+ (-10^{-5} * (T - 300))(10 - 10^{-3} * (T - 300))
\end{aligned}$$

```
clf; clear;
x = [0:0.1:500];
y = 0.1.*(1 + 0.01.*(x-300))
.*(0.01+0.0001.*(x-300))
.*(10-0.001.*(x-300))
+ (10- 0.001.*(x-300))
.*(-0.00001.*(x-300));
plot(x,y)
```



V_out vs temp(k)

```
clf; clear;
syms x;
a = [0:0.1:500];
y = 0.1.*(1 + 0.01.*(x-300))
.*(0.01+0.0001.*(x-300))
.*(10-0.001.*(x-300))
+ (10- 0.001.*(x-300))
.*(-0.00001.*(x-300));
plot(a,vpa(subs(diff(y),x,a)));
```



dV_out/dT vs temp(k)

```
clf; clear;
syms x
y = 0.1.*(1 + 0.01.*(x-300))
.*(0.01+0.0001.*(x-300))
.*(10-0.001.*(x-300))
+ (10- 0.001.*(x-300))
.*(-0.00001.*(x-300));
disp(vpa(subs(diff(y),x, 300)));
disp(vpa(subs(diff(y),x, 350)));
```

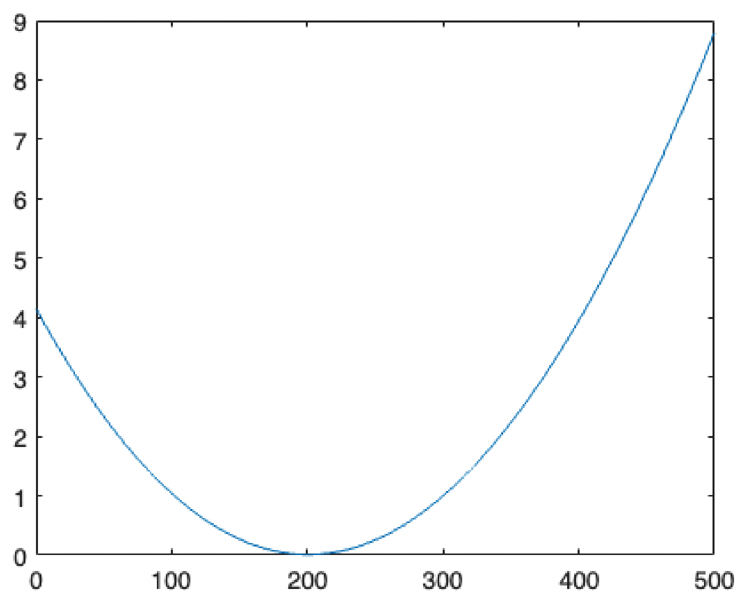
$$\text{temp coef 300k} = 0.000099$$
$$\text{temp coef 350k} = 0.00019725$$

Question 4.4c

c) Consider that the pressure sensor is exposed to a pressure difference signal of 10 N/m^2. Find an expression for the dependence of its output signal on temperature and plot this. Estimate the temperature coefficient of the entire sensor system responsivity in the neighborhood of the specific temperature values of 300K and 350K.

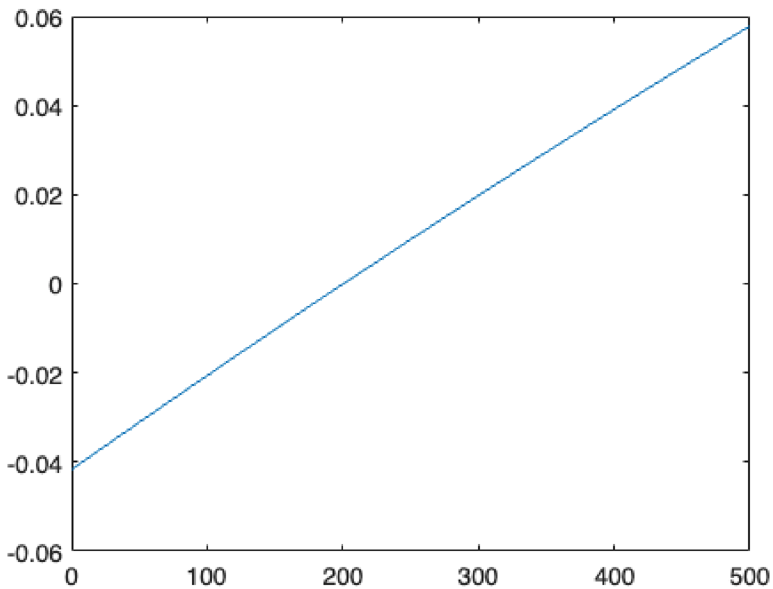
answer 4.4c

```
clf; clear;
x = [0:0.1:500];
y = 10.*(1 + 0.01.*(x-300))
.*(0.01+0.0001.*(x-300))
.*(10-0.001.*(x-300))
+(10- 0.001.*(x-300))
.*(-0.00001.*(x-300));
plot(x,y)
```



V_out vs temp(k)

```
clf; clear;
syms x;
a = [0:0.1:500];
y = 10.*(1 + 0.01.*(x-300))
.*(0.01+0.0001.*(x-300))
.*(10-0.001.*(x-300))
+ (10- 0.001.*(x-300))
.*(-0.00001.*(x-300));
plot(a,vpa(subs(diff(y),x,a)));
```



dV_out/dT vs temp(k)

```
clf; clear;
syms x
y = 10.*(1 + 0.01.*(x-300))
.*(0.01+0.0001.*(x-300))
.*(10-0.001.*(x-300))
+ (10- 0.001.*(x-300))
.*(-0.00001.*(x-300));
disp(vpa(subs(diff(y),x, 300)));
disp(vpa(subs(diff(y),x, 350)));
```

$$\text{temp coef } 300\text{K} = 0.0198$$
$$\text{temp coef } 350\text{K} = 0.029526$$

Question 4.4d

d) Discuss why this temperature coefficient is not linear and why it depends on input signal.

answer 4.4d

The temperature coefficient is not linear because the parameters are all dependent on `T`. When we multiply all the parameters our output is polynomial.