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CS 180 Midterm

Problem 1. There is a multiverse W of disconnected universes U_i ($i = 1, 2, \dots, n$) and it was found between which of these universes, it is possible to establish connections. Creation of each connection demands a lot of resources. Physicists and engineers constructed a function that estimates the price of each possible connection.

(50 pts). Build an algorithm that allows connecting the largest number of the universes for the minimal price. Prove that it is correct and estimate its complexity.

(50 pts). If it is impossible to do this, prove it.

(10 pts). Is it always possible to connect all universes in W? Explain your answer.

$U_1, U_2, \dots, U_n \in W$

sort all connections between universes in increasing order of price

while the number of edges <= n-1 and there are avaliable connections that do not form loops:

add the lowest price connection which does not create a loop to a new or existing mutiverse, W_i

if two multiverses share a connection then merge the multiverses

end while

add the multiverses W_i (where $i <= n$) with the most universes to a list, A

if $|A| > 1$

sort the tied multiverses in increasing order

return the multiverse with the least price

else

return the only multiverse in the list

endif

This algorithm is essentially Kruskal's algorithm but modified to also handle disconnected graphs.

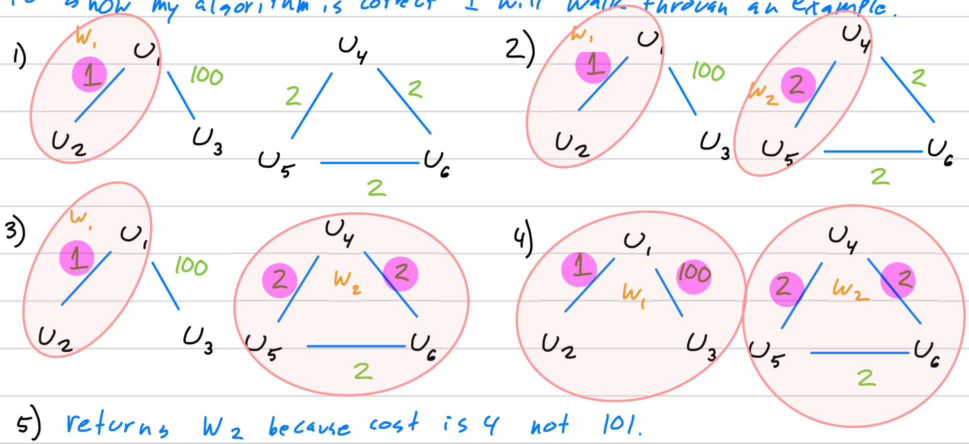
The part that will take the most time is the creation of the multiverses W_i this will take at most $\binom{n}{2}$ steps to check each edge which is $\boxed{O(n^2)}$.

This algorithm will add the least expensive edge until a minimal spanning tree is formed or until a minimal spanning forrest is formed. Then it will pick the minimal spanning tree with the most vertexes or if there is a tie it picks the one with the most universes + least cost.

● = it is included in the W_i

○ + W_i => which multiverse the universes are part of

To show my algorithm is correct I will walk through an example.



5) returns W_2 because cost is 4 not 101.

Which is correct because W_2 has 3 universes with cost 4 & W_1 has 3 universes with cost 101

(10 pts). Is it always possible to connect all universes in W? Explain your answer.

It is not always possible to connect all universes because there could be universes in W that are in no way connected to other universes in W. ∴ there will not always be a minimal spanning tree that connects all universes in W

Problem 2 (15 pts). On the planet Alphaomega, there are n spaceships and n persons having the rank of a spaceship captain. Each captain has the preference list of spaceships and the crew of each spaceship has the preference list of captains. The goal is to find a Stable Spaceship Matching of pairs (c, s).

Decide whether the following statement is true or false.

In every instance of the Stable Spaceship Matching, there is a stable matching containing a pair (c, s) *such that, at least, one of them is ranked third on the preference list of the other.*

If it is true, give a short explanation and design an algorithm.

If it is false, give a counterexample and explain that it is correct.

Let $n = 3$

Captain(c)	pref list most to least(s)	Crew(s)	pref list most to least(c)
1	1, 2, 3	1	1, 2, 3
2	2, 3, 1	2	2, 3, 1
3	3, 1, 2	3	3, 1, 2

∴ the stable matching is

$(c_1, s_1), (c_2, s_2), (c_3, s_3)$

∴ this is a counterexample because $c_1, c_2, \& c_3$ are top on the preference list of each ship and the same is true for $s_1, s_2, \& s_3$ in relation to each captain.

∴ $\boxed{\text{this statement is false}}$

Problem 3 (25 pts). Take the following functions and arrange them in descending order of growth rate indicating when functions have the same order of growth rate.

1) $\sqrt[3]{n}$ $\Rightarrow O(n^{1/3})$

2) $\log_{10} 2^n$ $\Rightarrow n \cdot \log_{10} 2 = O(n)$

3) $\sqrt[4]{n^3}$ $\Rightarrow O(n^{3/4})$

4) $\sqrt{n^n}$ $\Rightarrow O(n^{n/2})$

5) $\log_{10}{}^2 n + 5n$ $\Rightarrow O((\log_2 n)^2) + O(n) = O(n)$

6) $5n + 5^n$ $\Rightarrow O(5^n)$

7) $n^{7/2}$ $\Rightarrow O(n^{7/2})$

8) $\log_{10}(n^5 + 5n)$ $< \log_2(n^5+5n) < 5 \log_2(n) \Rightarrow O(\log_2 n)$

$n^a > n$ if $a > 1$

$n^a < n$ if $a < 1$

$n^{n/2} = (n^{1/2})^n > 5^n$

$n^a > \log_2 n$ $\forall a$

$O(n^{n/2}) > O(5^n) > O(n^{7/2}) > O(n) > O(n^{3/4}) > O(n^{1/3}) > O(\log_2 n)$

4 6 7 5 & 2 3 1 8

decending order of growth rate

$\boxed{4, 6, 7, 5 = 2, 3, 1, 8}$

biggest smallest