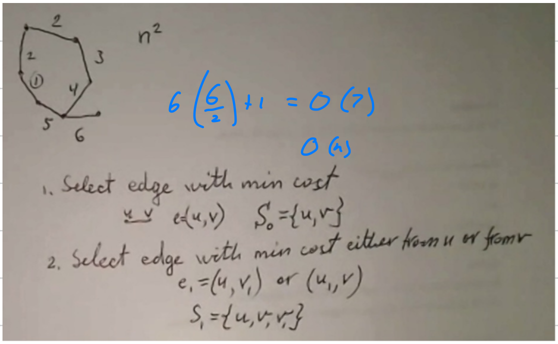
After Midterm

160

Jul 21

Kruskal's algorithm



complexity n^2

$n \log n$ $\quad l_1, l_2, l_3$

Divide + Conquer

Finding the Closest pair of points (FCPP)

$P = \{P_1, P_2, \dots, P_n\}$

$P_i = (x_i, y_i)$

$d(P_i, P_j)$

$x_i \neq x_j$ & $y_i \neq y_j \quad \forall i, j$

$P_x = \{p_1', \dots, p_n'\} \qquad x_i' < x_j'$ if $i < j$

$P_y = \{p_1'', \dots, p_n''\} \qquad y_i'' < y_j''$ if $i < j$

$Q = \{P_1', P_2', \dots, P_{n/2}'\}$

$R = \{P_{n/2+1}', P_{n/2+2}', \dots, P_n'\}$

Closest Pair (P)

construct P_x & P_y

(p_0^*, p_1^*) = Closest Pair Rec (P_x, P_y)

If $|P| = 3$, then

find the closest pair by comparing all distances

End If

If $n > 3$ then

construct Q_x, Q_y, R_x, R_y

(q_0^*, q_1^*) = closest pair Rec (Q_x, Q_y)

(r_0^*, r_1^*) = closest pair Rec (R_x, R_y)

$\delta = \min\left(d(q_0^*, q_1^*), d(r_0^*, r_1^*)\right)$

$x^* = \max\{x_i \text{ s.t. } (x_i, y_i) \in Q\}$

$L = \{(x, y) \text{ s.t. } x = x^*\}$

$S = \{p \in P \text{ s.t. } d(p, L) \le \delta\}$

Construct $S_y = \{\ldots, s, s_1, s_2, \ldots, s_{15}, \ldots\}$

For $\forall\, s \in S_y$ compute $d(s, s_j)$

$j = 1, 2, \ldots, 15$

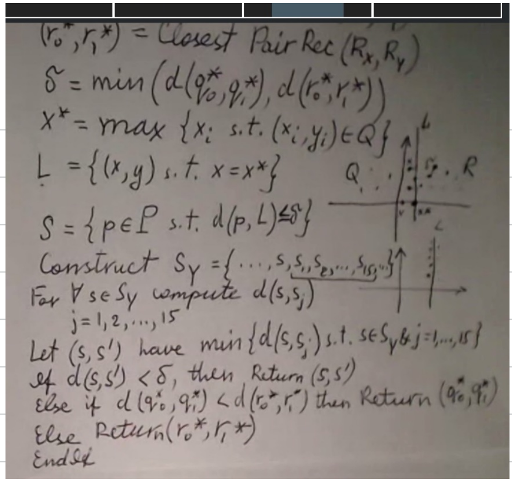
Let (s, s') have $\min\{d(s, s_j) \text{ s.t. } s \in S_y \text{ \& } j = 1, \ldots, 15$

If $d(s, s') < \delta$, then Return (s, s')

Else if $d(q_0^*, q_1^*) < d(r_0^*, r_1^*)$ then Return (q_0^*, q_1^*)

Else Return (r_0^*, r_1^*)

End If



(r_0^*, r_1^*) = Closest Pair Rec (R_x, R_y)

$\delta = \min\left(d(q_0^*, q_1^*), d(r_0^*, r_1^*)\right)$

$x^* = \max\{x_i \text{ s.t. } (x_i, y_i) \in Q\}$

$L = \{(x, y) \text{ s.t. } x = x^*\}$

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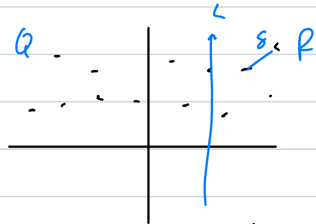
End If



Pr 1 If $q \in Q$ & $r \in R$ & $d(q,r) < \delta$, then
$d(q,L) < \delta$ & $d(r,L) < \delta$

proof

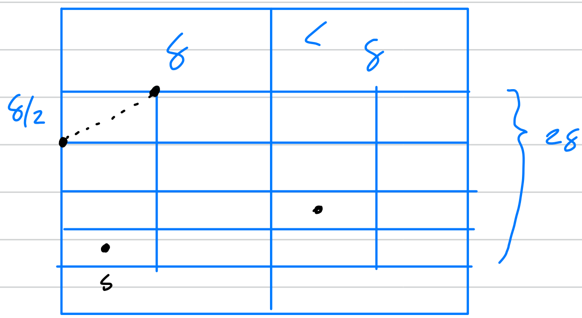
$$x_q < x^* < x_r \Rightarrow \underbrace{x^* - x_q}_{= d(q,L)} \le x_r - x_q \le d(q,r) < \delta$$

$$d(r,L) = x_r - x^* \le x_r - x_q \le d(q,r) < \delta$$

Pr 2 If $d(s,s') < \delta$ & $s, s' \in S$, then
s & s' are within 15 positions apart in S_y

Proof 1 In 1 box, not more
1 point

$$d = \frac{\delta}{2}\sqrt{2} = \frac{\sqrt{2}}{2}\delta$$



Pr 3 Closest Pair (P)
gives the closest pair

Pr 4 $T_{CP}(n) = O(n \log n)$