Reduction to Max-Flow

$\mathcal{G} = (V, E) \to \mathcal{G}' = (V', E')$

$V' = V \cup \{s^*, t^*\}$ $E' = E \cup \{(s^*, v)$ s.t. v is a source in $\mathcal{G}\}$ ($d(v) < 0$)
$\cup \{(u, t^*)$ s.t. u is a sink in $\mathcal{G}\}$ ($d(u) > 0$)

$c(s^*, v) = -d(v)$
$c(u, t^*) = d(u)$

Pr 2 $\forall$ flow f in $\mathcal{G}'$ $v(f) \le D$

Proof Take cut (A, B) s.t. $A = \{s^*\}$, B V'
$v(f) \le c(A, B) = D$

Pr 3 a) If f is a feas. circ. then it extended to a flow f in $\mathcal{G}'$ by
$f'(s^*, v) = -d(v)$, $f'(u, t^*) = d(u)$ & $\forall e \in E\ (f'(e) = f(e))$

b) $v(f') = D$

Corollary 2 f' is the max flow

Pr 4 If f' is a max flow with $v(f') = D$, then f is a feasible circulation

$$\sum_{e \text{ into } v} f(e) = \sum_{e \text{ out } v} f'(e) = \sum_{\substack{e \text{ out } v \\ \text{in } \mathcal{G}}} f'(e) + d(v)$$

$d(v) > 0$

$f^{in}(v) = f^{out}(v) + d(v) \Rightarrow f^{in}(v) - f^{out}(v) = d(v)$

$d(v) < 0$

$f^{out}(v) = \sum_{e \text{ in } V_{in}} f(e) - d(v) \Rightarrow f^{out}(V) - f^{in}(V) = -d(v)$

$f^{in}(v) - f^{out}(v) = d(v)$

Relations between $\mathbb{P}$ and NP

problem $A \in \mathbb{P}$ if $\exists$ pol bounded deterministic alg A_0

$T_{A_0}(n) = O(p(n)) = O(n^k)$ s.t. solves A

problem $B \in NP$ if $\exists$ pol. bound. nondeterm alg B_0 s.t. solves B

$a_1, a_2, a_3, \dots, a_n$ a n

compare a with a_i 1

$P = NP$?

tractible if from P

A reducible to B

B_0 solves B

initial data I_A for A

$$\begin{tikzcd} I_B \arrow[r] & B_0 \arrow[r] & \text{sol } B \arrow[d, "T_2"] \\ I_A \arrow[u, "T_1"] & & \text{sol } A \end{tikzcd}$$

A is poly-Time reducible to B if $T_1, T_2 \in P_{alg}$

$A \le_P B$

Pr1 If $B \in P$ & $A \le_P B$, then $A \in P$.

Cor If $A \le_P B$ & $A \notin P$, then $B \notin P$

Pr 2

$P \subseteq NP$

NP - hard problem B

if $\forall$ problem $D \in NP$ $(D \leq_P B)$

NP - complete problem E

if E is NP - hard & $E \in NP$

SAT satisfying problem

Boolean Variables $X_1, X_2, \ldots, X_n$

term X_i or $\bar{X}_i$ $\quad l = l(c)$

clause $c = t_1 \vee t_2 \vee t_3 \vee \ldots \vee t_l$

B formula

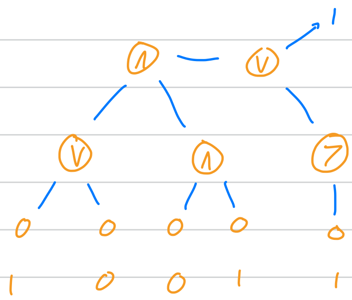
in conjunctive normal form $b = c_1 \wedge c_2 \wedge \ldots \wedge c_k$

$l(c_i) = 3$ 3- SAT is NP - complete

Independent set problem Is NP - complete

$g = (V, E)$ $u \subseteq V$ is independent in g

Circuit satisfiability CS

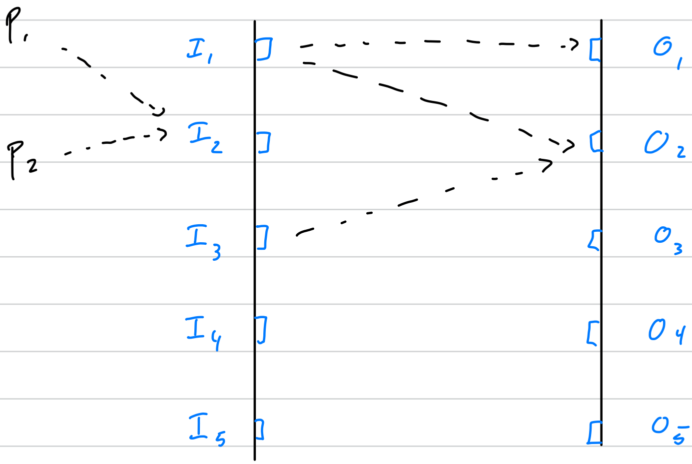


Algorithms designed to run forever without stopping

Internet router

moving packets avoiding conjestion

$P \left(I(P), O(P) \right)$



Final

① flow networks

Monday 10 AM OH