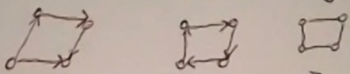
Home work due 2 weeks

MT Real on sat afternoon

160

Directed Acyclic Graphs (DAG)



1 2 3

$G=(V, A)$

Lemma 1. $\exists v \in V$ s.t. $\neg\exists (u,v) \in A$

back tracking

backtracking

Lemma 2. $\exists u \in V$ s.t. $\neg\exists (u,v) \in A$.

forward tracing

Asym

1) $\forall u \in V (\neg u < u)$

2) Trans

If $u < v$ & $v < w$ then $u < w$

Topological Ordering $<$

$\forall (u,v) \in A (u < v)$

Th. 1. G is DAG iff G has a top ordering.

Proof. 1. Suppose G has a t.o.

$V = \{v_1, v_2, \dots, v_n\}$ if $i<j$, then $v_i < v_j$

Cycle $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}) \dots (v_{i_{k-1}}, v_{i_k})$

$v_{i_1} = v_{i_k}$ $\quad v_{i_1} < v_{i_2} < v_{i_3} < \dots < v_{i_{k-1}} < v_{i_k}$

$v_{i_1} < v_{i_1}$ contradiction

$V_i \subseteq V \quad i = 1,2,3,\dots,n \qquad A_0 = A$

TO (g)

Find $v_0 \in V_i$ without incoming arrows

Put $v_i := v$

Delete v from V_i & all arrows from v

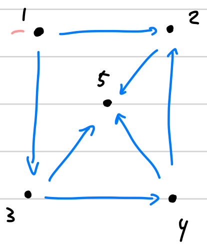
$V_{i+1} = V_i \setminus \{v\}$

$A_{i+1} = A_i \setminus \{(v, u) ; u \in V\}$

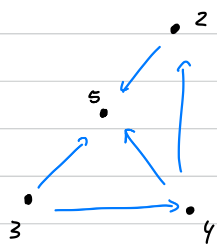
Repeat $T(g_{i+1} = (V_{i+1}, A_{i+1}))$

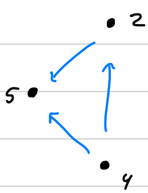
End

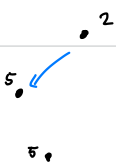
(U, V)



$\boxed{1, 3, 4, 2, 5}$?







greedy algorithms

interval scheduling problem

Resource T

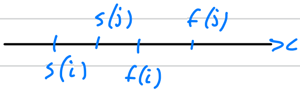
set requests $R = \{1, 2, 3, \ldots, n\}$

i $s(i)$ start time

$f(i)$ end time

$A \subseteq R$ is compatible if $\forall\, i, j \in A$ $\left(\begin{array}{l} \neg\, s(i) < f(j) \\ \neg\, s(j) < f(i) \end{array}\right)$

schedule



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overlaping

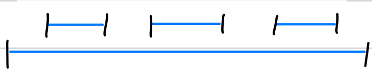
$A \subseteq R$ is optimal if it is comp & maximal

Criterion

Counter examples

a) $s(i)$ is the best

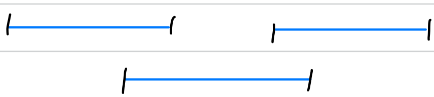
a)



b) $|i| = f(i) - s(i)$

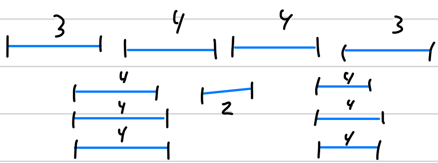
the least

b)



c) the fewest # of incompatable requests

c)



d) ???

best one???

Algorithm ISP

$A_0 = \emptyset$, $R_0 = R$, $j = 0$

While $R_j \neq \emptyset$

  choose $i \in R_j$ with the least $f(i)$

  Add i to A_j $(A_{j+1} = A_j \cup \{i\})$

  Delete all $k \in R_j$ incompatible with i

  $(R_{j+1} = R_j \setminus \{ k \in R_j \text{ s.t. } k \,\&\, i \text{ incomp}\})$

End while

return A_f as the solution

Prop 1 A_f is compatible

Prop 2 If a schedule B is opt, then $|B| = |A_f|$

Proof $A_f = \{i_1, i_2, \ldots, i_k\}$ $B\ \{j_1, j_2, \ldots, j_m\}$

Lemma 1 For any $r \leq k$ $(f(i_r) \leq f(j_r))$.

Proof by induction

1) $r = 1$ $f(i_1) \leq f(j_1)$

2) $r > 1$ assume that L is proved for $r-1$,

that is $f(i_{r-1}) \leq f(j_{r-1})$ $\Rightarrow f(i_{r-1}) < s(j_r)$

B is compatible $f(j_{r-1}) < s(j_r)$

$f(i_r) \leq f(j_r)$

By Principle of induction $\forall r \leq k$ $(f(i_r) \leq f(j_r))$

Contradiction

Proof of Pr 2 (cont) $m > k$

By L1, $\exists\ j_t$ s.t. $s(j_t) > f(j_k) \geq f(i_k)$

j_t is comp with i_k

So, $m = k$ $|B| = |A_t|$

Prop 3 ISP terminates

Prop 4 $T_{ISP}(n) = O(n \log n)$