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HW2

Problem. Take the following functions and arrange them in ascending order of growth rate. Explain the suggested ordering.

assuming a, b, and c are greater than 0

1	$\log_2^2 n$	$= \Theta((\log_2 n)^2)$
2	$\log_{10} 2^n$	$= \frac{\log_2 2^n}{\log_2 10} = n \cdot \frac{1}{\log_2 10} = \Theta(n)$
3	$\log_2 n^3$	$= \Theta(\log_2 n)$
4	$\log_2^2 n^n$	$= \Theta((n \log_2 n)^2)$
5	$\log_2^2 n + 2n$	$= \Theta(n)$
6	$\log_{10}(10n)$	$= \Theta(\log_2 n)$
7	$a\log_{10}^2 n + b\log_{10} n + c$	$= \Theta\left(\left(\frac{\log_2 n}{\log_2 10}\right)^2\right) = \Theta((\log_2 n)^2)$
8	$\log_2^2(n + 2n)$	$= \Theta((\log_2 n)^2)$

groups

$\Theta(\log_2^2 n)$

1) $(\log_2 n)^2$

7) $\frac{a}{(\log_2 10)^2} \cdot (\log_2 n)^2$

8) $(\log_2 3n)^2$

$= (\log_2 3 + \log_2 n)^2$

$= \log_2^2 3 + 2 \cdot \log_2 3 \cdot \log_2 n + \log_2^2 n$

$\log_2^2 3n > \log_2^2 n$

$\therefore 8 > 1$

if $a < \log_2^2 10$

7, 1, 8

if $a = \log_2^2 10$ & $b < 2 \cdot \log_2 3 \cdot \log_2 n$

or $a = \log_2^2 10$ & $b = 2 \cdot \log_2 3 \cdot \log_2 n$ & $c < \log_2^2 n$

1, 7, 8

if $a > \log_2^2 10$

or $a = \log_2^2 10$ & $b > 2 \cdot \log_2 3 \cdot \log_2 n$

or $a = \log_2^2 10$ & $b = 2 \cdot \log_2 3 \cdot \log_2 n$ & $c > \log_2^2 n$

1, 8, 7

$\Theta(n)$

2) $\frac{\log_2 2^n}{\log_2 10} = \frac{1}{\log_2 10} n$

5) $\log_2^2 n + 2n$

$2n > \frac{1}{\log_2 10} n$

$\therefore 5 > 2$

$\Theta(\log_2 n)$

3) $3 \cdot \log_2 n$

6) $\log_2(10n) \cdot \frac{1}{\log_2 10}$

$\log_{10}(10n) < \log_2 n^3$

$\therefore 3 > 6$

$\Theta(\log_2 n) < \Theta(\log_2^2 n) < \Theta(n) < \Theta(n^2 \cdot \log_2^2 n)$

if $a < \log_2^2 10$

6, 3, 7, 1, 8, 2, 5, 4

if $a = \log_2^2 10$ & $b < 2 \cdot \log_2 3 \cdot \log_2 n$

or $a = \log_2^2 10$ & $b = 2 \cdot \log_2 3 \cdot \log_2 n$ & $c < \log_2^2 n$

6, 3, 1, 7, 8, 2, 5, 4

if $a > \log_2^2 10$

or $a = \log_2^2 10$ & $b > 2 \cdot \log_2 3 \cdot \log_2 n$

or $a = \log_2^2 10$ & $b = 2 \cdot \log_2 3 \cdot \log_2 n$ & $c > \log_2^2 n$

6, 3, 1, 8, 7, 2, 5, 4