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Hw 1

Problem 1 (25 pts). What is the difference between an algorithm and a program?

An algorithm is a finite constructive structure that defines a process for solving a problem. In other words, an algorithm is a set of instructions on how a computer could solve a problem.

A program is a set of instructions, that comply with the rules of a specific programming language, which are written to complete a specific task with a computer.

A program is essentially like an implementation of an algorithm.

Problem 2 (75 pts). Men, women and yupi live on the planet Alphaomega. Their family pattern is a triple that consists of a man, a woman and a yupi. Three sets are given: M includes n men, W includes n women and Y includes n yupi. A matching is a set H of ordered triples of the form (m, w, y) with the property that each member of M , each member of W and each member of Y appears in at most one triple from H . A matching H is called *perfect* if each member of M , each member of W and each member of Y appears exactly in one triple from H .

Assume that each man ranks all women and all yupi, each woman ranks all men and all yupi, and each yupi ranks all women and all men.

Two triples (m, w, y) and (m', w', y') form an *instability* in a matching H if one of the following conditions is true:

- (1) m prefers w' to w and w' prefers m to m'
- (2) m prefers y' to y and y' prefers m to m'
- (3) y prefers w' to w and w' prefers y to y'

A matching H is called *stable* if it does not have instabilities.

Decide whether the following statement is true or false.

There is an algorithms that solves the Stable Matching Problem for every instance of this problem.

False

If it is true, design an algorithm for building a stable perfect matching. Note that when you design an algorithm, you have to prove that it solves the necessary problem

If it is false, give a counterexample.

$$\text{Let } m + m' \in M, w + w' \in W, y + y' \in Y \quad + \quad n=2$$

Preferences					All possible marriages (None of which are stable)	not stable b/c
table		M	W	Y		
①	m	NA	$w > w'$	$y' > y$	$(m, w, y), (m', w', y')$	1 + 6
②	m'	NA	$w > w'$	$y > y'$	$(m, w, y'), (m', w', y)$	3 + 5
③	w	$m > m'$	NA	$y > y'$	$(m, w', y'), (m', w, y)$	1 + 3
④	w'	$m > m'$	NA	$y > y'$	$(m, w, y), (m', w', y')$	1 + 3
⑤	y	$m > m'$	$w > w'$	NA		
⑥	y'	$m > m'$	$w > w'$	NA		

This example of preferences shows that each possible matchings is unstable

Which means that there is no algorithm that could build this stable matching