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HW 5

Problem. On the planet Aleph at the place Sonton in the wild mountains of Bonton, robots extract a precious liquid adroline to be transported to the Earth. However, spaceships cannot come to the wild mountains of Bonton and it is necessary to transport adroline from Sonton to the spaceport Sinkon. To do this, robots built a pipeline from Sonton to Sinkon, but due to the environment, it was impossible to construct a direct pipeline and the assembled pipeline had the structure of a network in which adroline flew. Due to the technical conditions, the capacities of the edges of this network are whole numbers. After a technological innovation, the capacities of the edges were increased by 5 units.

Suppose (A, B) was a minimum s-t cut with respect to these capacities before the innovation. Will (A, B) still be a minimum s-t cut with respect to the new capacities after the innovation?

Let
S = Sonton
T = Sinkton

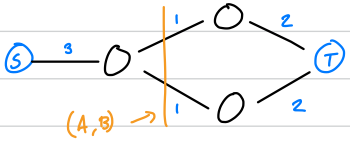

figure 1

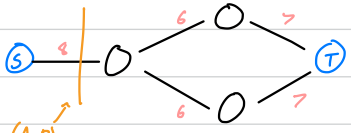

figure 2

For above graph (A,B) could be the orange line as seen in figure 1 because that cut has a value of 2.
After the technological innovations (aka adding 5 capacity to each edge) the old cut (A,B) would have a value of 12 which is greater than 4, so the (A,B) cut would no longer be a minimum s-t cut.
∴ (A,B) will not always be a minimum cut with respect to the new capacities after innovation

But one exception would be if a graph had all edges with the same capacity. If this was the case then (A,B) would be the cut with the minimum ammount of edges and it would not change after all edges were increased in capacity by 5 because all edges have the same weight still.