

Def 6 Root of a tree

Def 7 Parent - child

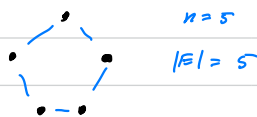
ancestor - descendant

If  $V_1$  is a parent of  $V_2$  &  $V_2$  is ancestor of  $V_3$   
then  $V_1$  is ancestor of  $V_3$

Proposition 1 If a tree has  $n$  nodes, then it has  $n-1$  edges  
 $n-1$  childs  $\Rightarrow n-1$  edges

Proposition 2  $g = (V, E) \quad |V| = n$

- 1)  $g$  is connected
- 2)  $g$  has no cycles
- 3)  $|E| = n-1$



Def 8 s-e connectivity

$s, t \in V$   $g$  is s-t connected if  $\exists$  path from  $s$  to  $t$

Def 9 Independent set  $X \subseteq V$

$\forall u, v \in X \quad \neg (u, v) \in E$

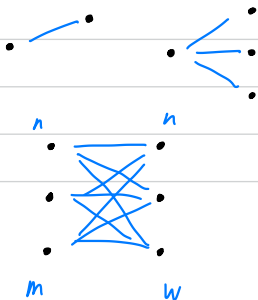
Def 10 Biparte graph  $g$

$V = V_1 \cup V_2$  &  $V_1, V_2$  are independent sets



Def 11 Matchings in biparte graphs  $g$

Def 12 Degree of a node



Def 13 Spanning tree of  $g$

is a tree that includes all nodes from  $g$  & subgraph

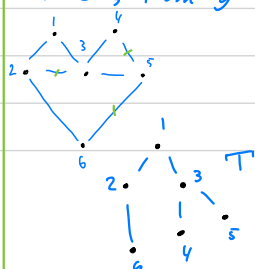
$$R_1 = \{1, 3\}$$

$$R_2 = \{1, 2, 3\}$$

$$(1, 2), (1, 3)$$

$$R_3 = \{1, 2, 3, 4, 5, 6\}$$

$$(1, 2), (1, 3), (2, 6), (3, 4), (3, 5)$$



$$g = (V, E)$$

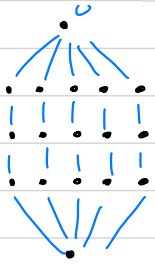
### problems

- 1) Find  $v \in V$  with property  $P$   $P(v) = 1$
- 2) Find connected component of  $v \in V$   $P(u) = 0$
- 3) Check  $V-U$  connectivity

Breadth-first search (BFS)

Depth-first search (DFS)

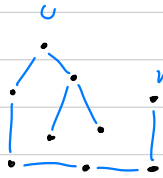
### Depth-first



DFS - 4 st

BFS - 16 st

$P(v) = 1$

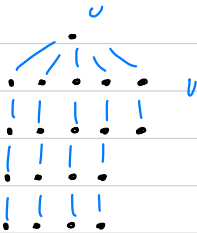


DFS - 5 st

BFS - 8 st

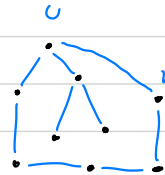
$v$

### Breadth-first



BFS - 5 st

DFS - 16 st



BFS - 3 st

DFS - 5 st

when 7  $u-v$  path ... ???

only for connected graph

$$g = (V, E)$$

$R$  is the set of tested vertices

**BFS**

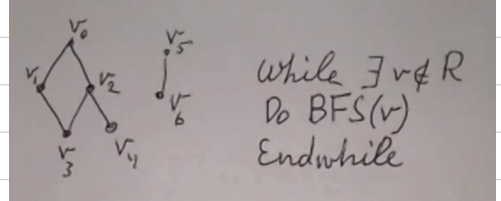
$$R = \{v_0\} \quad v_0 \in V$$

while  $\exists (u, v) \in E$  <sub>with</sub>  $u \in R$  &  $v \notin R$

Do  $A(v)$  &  $v \in R$

End while

for disconnected graph



**DFS**

DFS(u):

Mark  $u$  exp(u) &  $u \in R$  &  $A(u)$

For  $\forall (u, v) \in E$

If  $v \neq \text{exp}(u)$

then Rec DFS(v)

End if

End for

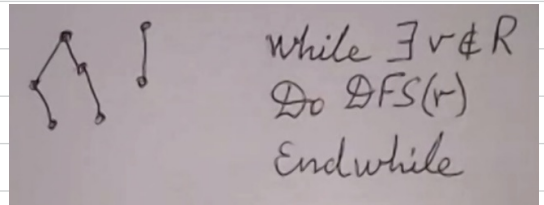
$\exists W \in C_g(v_0)$  but  $w \notin R$

path from  $v_0$  to  $W$

$v_1, v_2, \dots, W$

$v_i \in R$  &  $v_{i+1} \notin R$

tests all vertices  
in the connected  
component  $v_0$



**Def 14** Spanning forest of  $g$