Jul 30 140

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Prop 2 If $c: E \to N$, then $f: E \to N$ & $C_f: E \to N$

Prop 3 If P is a simple s-t path in g_f,
then $v(f') = v(f) + \underset{\text{bottleneck}(P,f)}{b}$ & $v(f') > v(f)$

Proof Let e be 1st edge in P
$e = (s, r)$. $f'(e) = f(e) + b$ & $b > 0 \Rightarrow f'(e) > f(e)$

$v(f') = \sum\limits_{d \text{ out of } s} f'(d) = \sum\limits_{d \text{ outs of } s} f(d) + b = v(f) + b > v(f') > v(f)$

Let $v(g) = C$ $\quad v(g) = \sum\limits_{C \text{ out of } s} c(e)$

Prop 4 FFA terminates in at most c iterations of the while loop

Corollary FFA terminates

Let $|E| = m$ & $|V| = n$ $\quad m \geq \frac{n}{2}$

$O(m+n) = O(m)$

Pr 5 $T_{FFA}(m+n) = O(m\,c)$

Proof To build g_f $\quad \forall v \in V$ $\quad R_1 = \{(v, u);\ v \in V\}$

$O(n)$ $\quad R_2 = \{(w, v);\ v \in V\}$

to find s.p. P $\quad O(m+n) = O(m)$
Augment (P, f) $\quad O(n)$
Updating g_f to $g_{f'}$ $\quad O(m)$

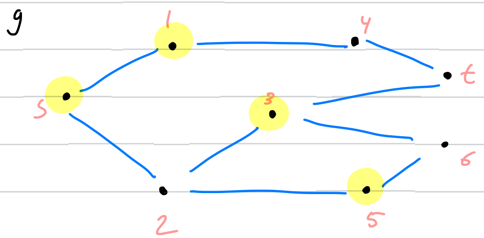
$\Big\}\ O(m)$

$g = (V, E)$????

$g = (V, E)$ An s-t cut is (A, B) s.t.

$A, B \subseteq V$ & $A \cup B = V$ & $A \cap B = \emptyset$

$s \in A$ & $t \in B$



$A = \{s, 1, 3, 5\}$ s-t cut

$B = \{t, 2, 4, 6\}$

$C = \{s, 1, 2, 3, 4\}$ not s-t cut

$D = \{t, 2, 4, 5\}$

Pr 6 $\forall$ flow f $\forall$ s-t cut (A, B)

$$v(f) = f^{out}(A) - f^{in}(A)$$

$$f^{out}(A) = \sum_{e \text{ out } v \in A} f(e) \qquad f^{in}(A) = \sum_{e \text{ in } v \in A} f(e)$$

Proof $v(f) = \sum_{e \text{ out } s} f(e)$

Consistency

$\forall v \neq s$ $\quad \sum_{e \text{ in } v} f(e) = \sum_{e \text{ out } v} f(e)$

$$\sum_{e \text{ out } v \in A} f(e) - \sum_{e \text{ in } v \in A} f(e)$$

Prop 7 $\quad \forall$ flow f $\forall$ s.t. cut (A,B) $\qquad v(f) = \sum_{e \text{ in } t} f(e)$

$$v(f) = f^{in}(B) - f^{out}(B)$$

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Capacity of (A,B)

$$c(A,B) = \sum_{e \text{ out } A} c(e)$$

Prop 8 $\quad \forall$ flow f $\forall$ s.t. cut (A,B)

$$v(f) \leq c(A,B)$$

Proof $\quad v(f) = f^{out}(A) - f^{in}(A) \leq f^{out}(A) \leq \sum_{e \text{ out } A} c(e)$

Prop 9 $\quad$ If $\neg \exists$ s-t path in G_f then $\exists$ s-t cut (A^*, B^*) s.t. $v(f) = c(A^*, B^*)$

Proof $\quad A^* = \{v \in V$ s.t. $\exists$ s-v path in $G_f\}$

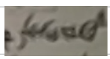
$B^* = V \setminus A^*$

(A^*, B^*) is an s-t cut $\quad s \in A^*$ & $t \in B^*$

$A^* \cup B^*$ $\qquad A^* \cap B^* = \emptyset$

$e = (u,v)$ s.t. $u \in A^*$ & $v \in B^*$ $\Rightarrow f(e) = c(e)$

Assume $f(e) \neq c(e)$ $\quad$ e is for use in G_f

??? 

$s \to \to \to \underset{u}{\bullet} \to v$ $\qquad \Rightarrow \exists$ path from s to v

$$v(f) = f^{out}(A^*) - f^{in}(A^*) = \sum_{e \text{ out } A^*} f(e) - \sum_{e \text{ in } A^*} f(e) = \sum_{c \text{ out } A^* \& s^*} f(c) - 0 = ???$$

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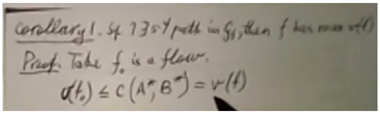

Corollary 1 If ¬∃ s.t. path in G_f, then f has max $v(f)$

Proof Take f_0 is a flow

$$v(f_0) \le c(A^*, B^*) = v(f)$$

Corollary 2 (A^*, B^*) has min Capacity

Proof $c(A, B) \ge v(f) = c(A^*, B^*)$

Flow Networks Chapter