Aug 4

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Corallary 3 FFA gives a max flow

Pr 10 given max flow f it's possible to compute a min cut in $O(m)$ time

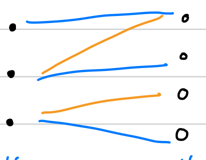
proof
1) Find g_f $O(m)$
2) Find conn comp of s in g_f $O(m)$

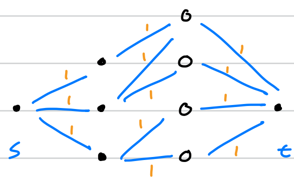
max-flow Min Cut Theory $\forall$ flow netw g, max value of an s-t flow is equal to ~~[illegible]~~t of an s-t cut

minimum ?

Biparte matching problem

$g = (V, E)$
$V = V_1 \cup V_2$
$V_1 \cap V_2 = \emptyset$
$\forall e = (u, v) \in E$
$u \in V_1$ & $v \in V_2$





$g_0 = (V_0, E_0)$
$V_0 = V \cup \{s, t\}$
$\forall e = (u,v) \in E_0$ $C(e) = 1$

matching $H = (U, D)$
if $H \subseteq g$, $U \subseteq V$ & $D \subseteq E$
$\forall (u, v), (r, l) \in U \Rightarrow u \neq r$ & $v \neq l$

$E_0 = E \cup \{(s, u) ; u \in V\} \cup \{(v, t) : v \in V_2\}$

Pr 1 If $f(e) = 1$ for $\forall e$ D, then f is at flow in g_0.

$\forall e = (E, u)$ & $\forall e\ (v, t)$ $\quad f(e) = 1$
$u \in U \cap V_1$ $\quad v \in U \cap V_2$ $\quad f(h) = 0$

suppose $\exists$ flow f w/ $V(f) = k$

Pr 2 If $\forall$ $C(e)$ are whole #s
then $\exists$ max flow f with $v(f)$ whole #

Cor 1 $\forall$ flow f in g_0, $f(e) = \{ \begin{smallmatrix} 1 \\ 0 \end{smallmatrix}$

Cor 2 If $M \subseteq E$ & $M = \{ e \in E$ s.t. $f(e) = 1 \}$,
then $|M| = k$

Pr 3 a) $\forall$ $v \in U \cap V_1$ $\exists$ only 1 edge (v, u) with $u \in U \cap V_2$
b) If $f(e) = 1$ & $e = (s, v)$, then $\exists$ only 1 (v, u) with $u \in V_2$
$f(v, u) = 1$

Follows from the consist condition

Pr 4 a) $\forall$ $u \in U \cap V_2$ $\exists$ only 1 (v, u) with $v \in U \cap V_1$
b) If $f(e)$ & $e = (u, t)$, then $\exists$ only 1 (v, u) s.t.
$v \subseteq V$ & $f(v, u) = 1$

Th 1 The size of max matching in g = to the value of max flow in g_0

$g \rightarrow g_0 \xrightarrow{FFA} f_{max} \rightarrow$ max matching

$|V_1| = |V_2| = n$

H is max matching $|D| = n$
(u, D)

$A \subseteq V_1$ $\quad \Gamma(A) = \{ v \in V_2$ s.t. $\exists$ $u \in A$ $(u, v) \in E \}$

Th 2 If $|V_1| = |V_2|$, then either $\exists$ a perfect match or

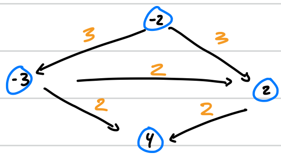
$\exists$ $A \subseteq V_1$ s.t. $|\Gamma(A)| < |A|$

Circulation with demands

$g = (V, E)$ demands $d: V \to \mathbb{Z}$
integers

$S = \{ v \in V \text{ s.t } d(v) < 0 \}$
supply

$T = \{ V \in V \text{ s.t. } d(V) < 0 \}$
demand



sources v $d(v) < 0$

sinks u $d(u) > 0$

capacity $c: E \to \mathbb{R}^{++}$

circulation $f: E \to \mathbb{R}^{+}$

Feasible circulation

1) capacity cond $0 \le f(e) \le C(e)$ $\forall e \in E$

2) demand cond $\forall v \in V$ $f^{in}(V) - f^{out}(V) = d(V)$

Pr 1 If $\exists$ feas circ f with dem d, then $\sum_{V \in V} d(V) = 0$

Proof $\sum_{v} d(V) = \sum_{v \in V} \left(f^{in}(V) - f^{out}(V) \right)$

$f(e)$ $\underset{u}{\bullet} \xrightarrow{e} \underset{v}{\bullet}$

Color 1 $\sum_{d(V) > 0} d(V) = \sum_{d(V) < 0} (-d(V)) = D$