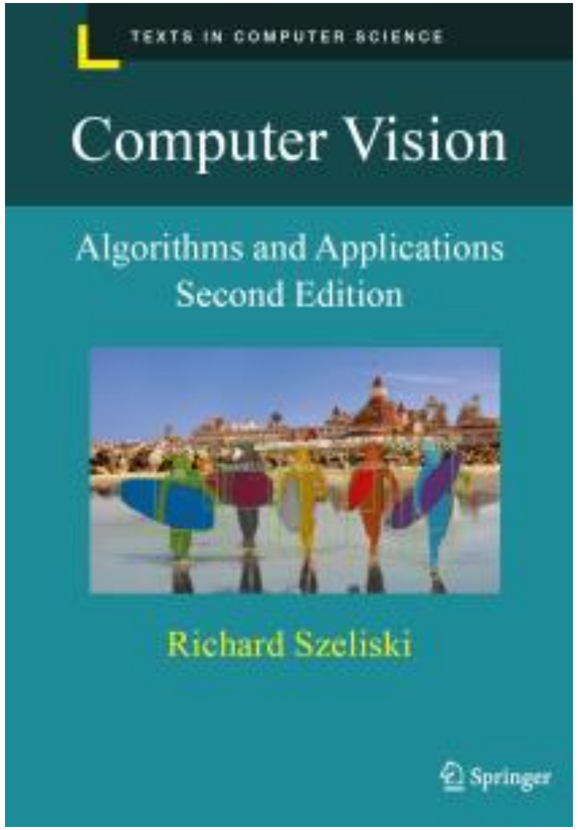
Computer Vision

Image alignment

http://www.wired.com/gadgetlab/2010/07/camera-software-lets-you-see-into-the-past/

Important information



Textbook

Rick Szeliski, *Computer Vision: Algorithms and Applications* online at: http://szeliski.org/Book/

Many of the slides in this course are modified from the excellent class notes of similar courses offered in other schools by Noah Snavely, Prof Yung-Yu Chuang, Fredo Durand, Alyosha Efros, Bill Freeman, James Hays, Svetlana Lazebnik, Andrej Karpathy, Fei-Fei Li, Srinivasa Narasimhan, Silvio Savarese, Steve Seitz, Richard Szeliski, and Li Zhang. The instructor is extremely thankful to the researchers for making their notes available online. Please feel free to use and modify any of the slides, but acknowledge the original sources where appropriate.

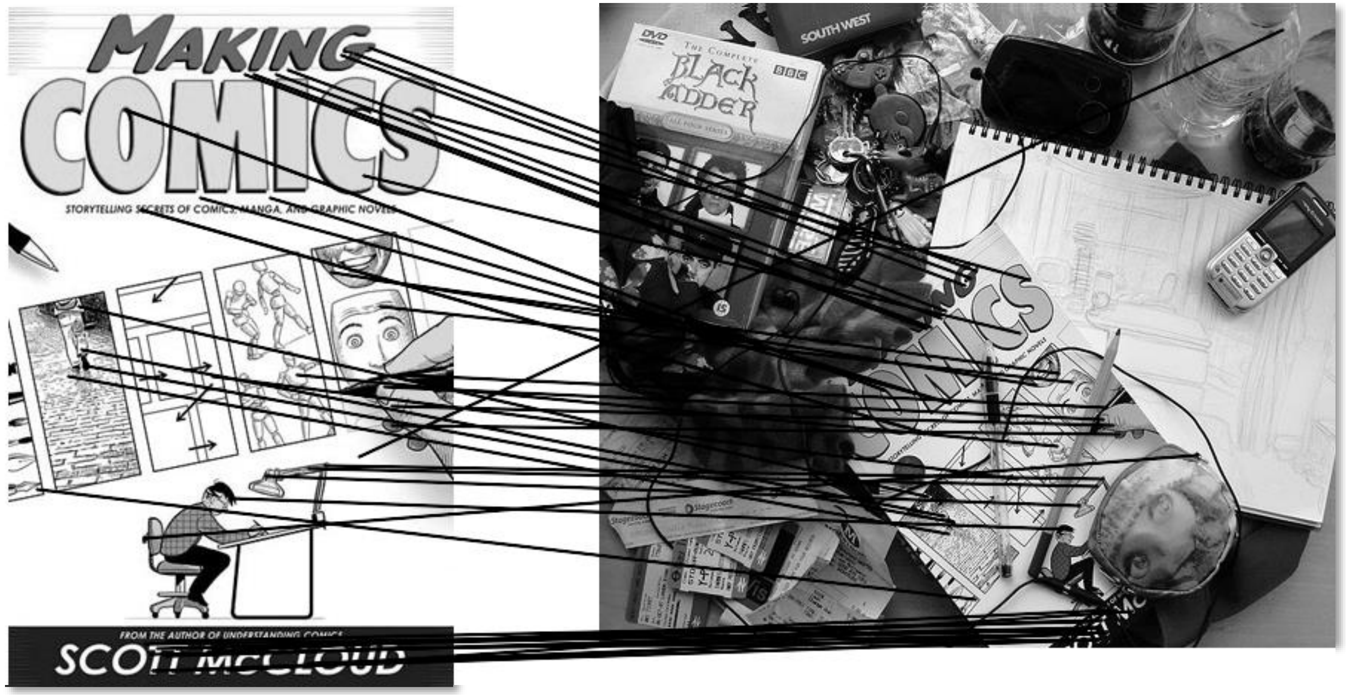
All readings are from Richard Szeliski, Computer Vision: Algorithms and Applications, 2nd Edition, unless otherwise noted.

Reading

- Szeliski (2^{nd} edition): Chapter 8.1

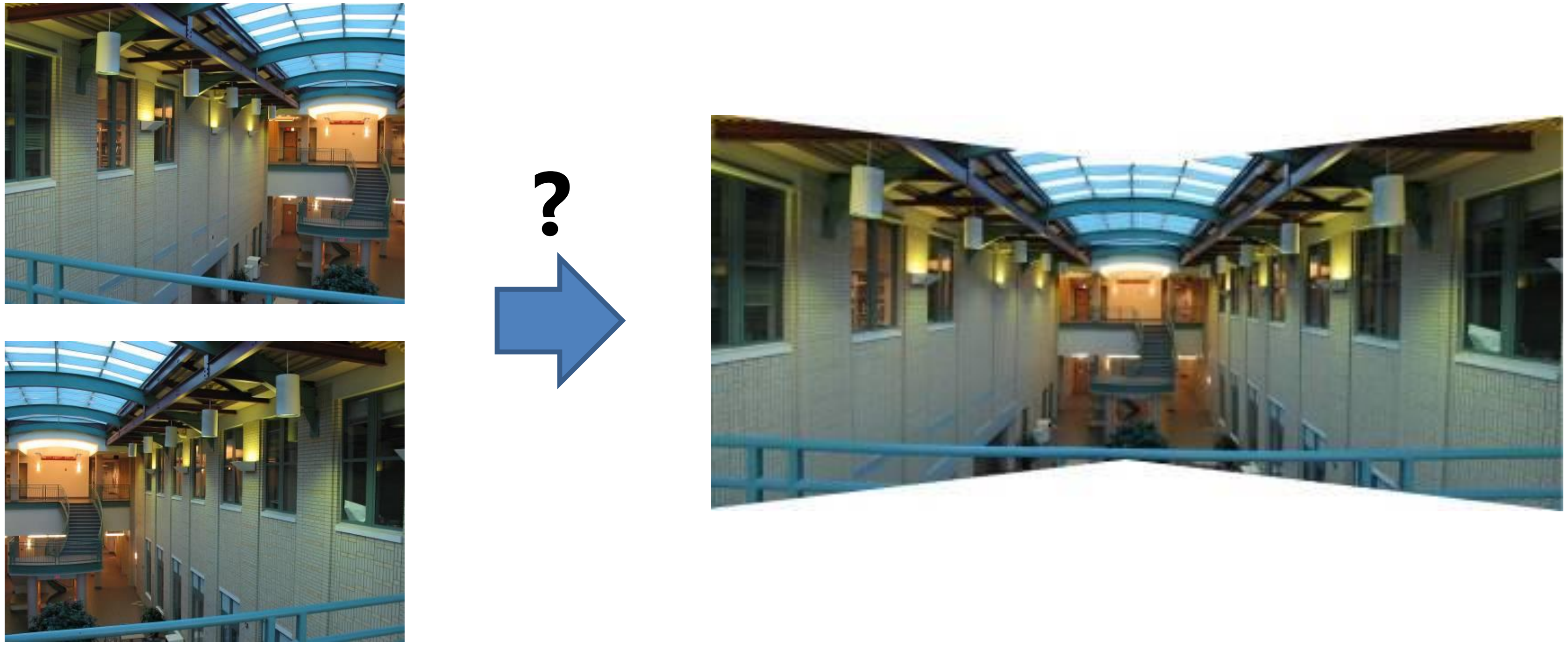
Computing transformations

- Given a set of matches between images A and B
  - How can we compute a transform T from A to B (e.g., an affine transform)?

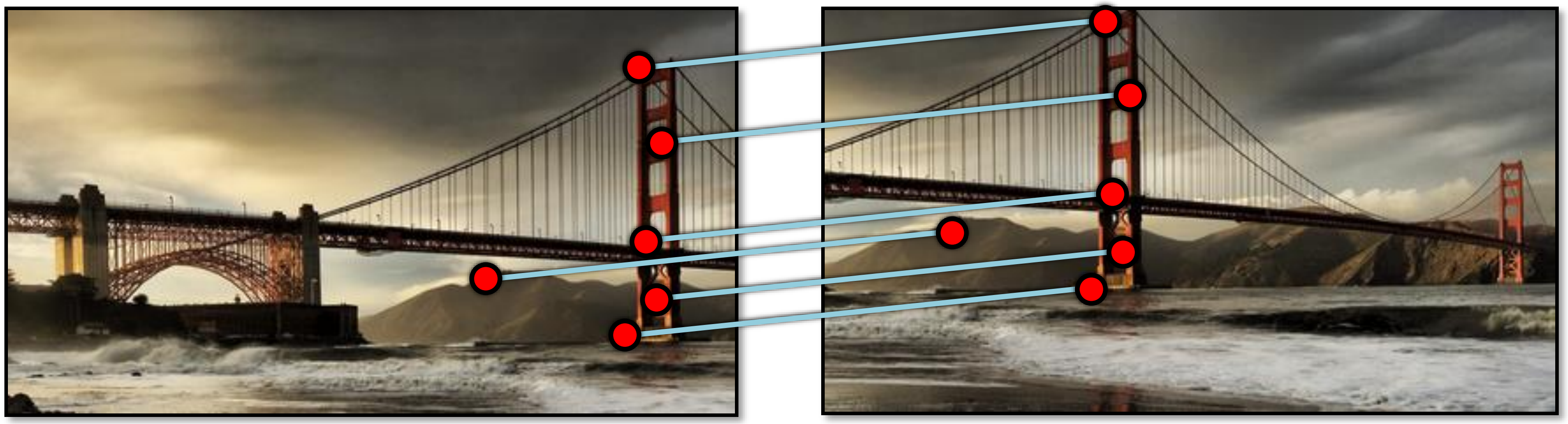


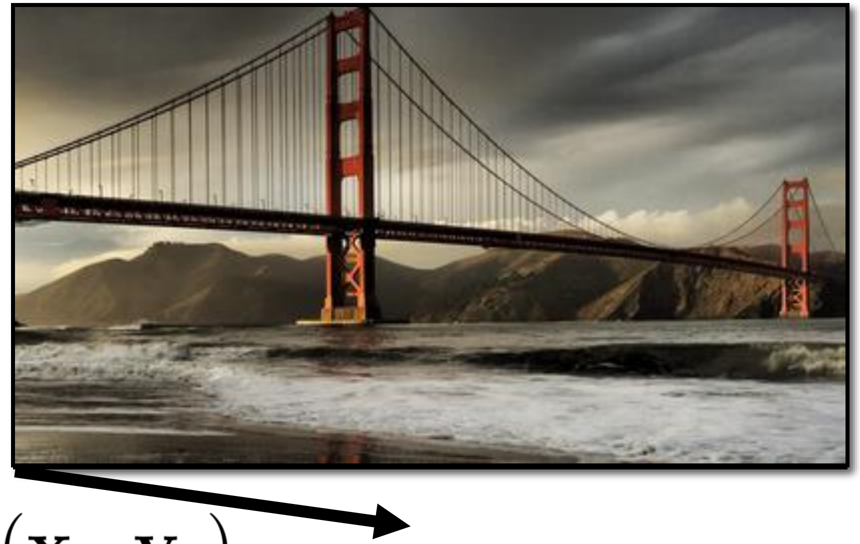
  - Find transform T that best “agrees” with the matches

Computing transformations



Simple case: translations

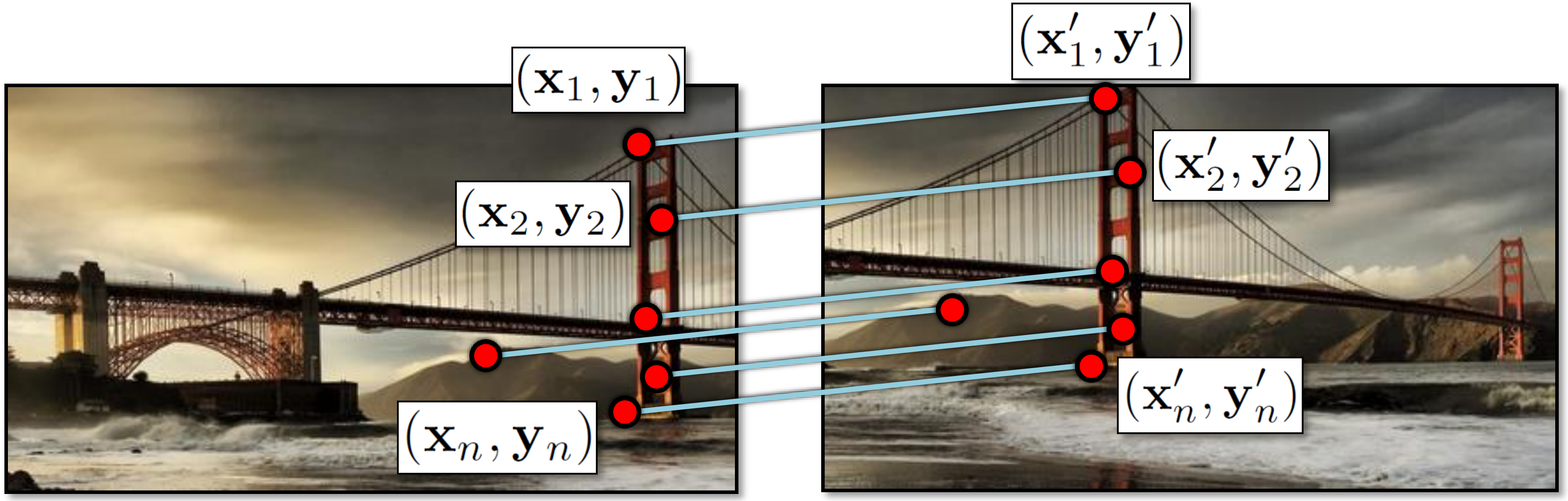




$(\mathbf{x}_t, \mathbf{y}_t)$

How do we solve for $(\mathbf{x}_t, \mathbf{y}_t)$ **?**

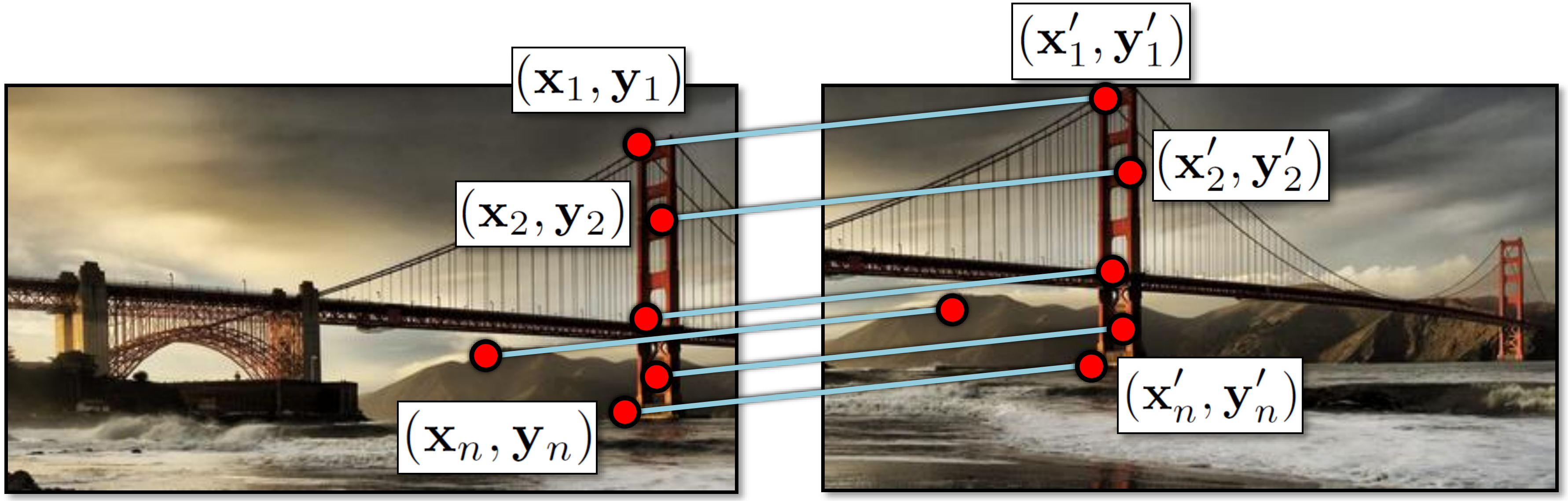
Simple case: translations



Displacement of match i = $(\mathbf{x}'_i - \mathbf{x}_i, \mathbf{y}'_i - \mathbf{y}_i)$

$$(\mathbf{x}_t, \mathbf{y}_t) = \left(\frac{1}{n} \sum_{i=1}^{n} \mathbf{x}'_i - \mathbf{x}_i, \frac{1}{n} \sum_{i=1}^{n} \mathbf{y}'_i - \mathbf{y}_i \right)$$

Another view

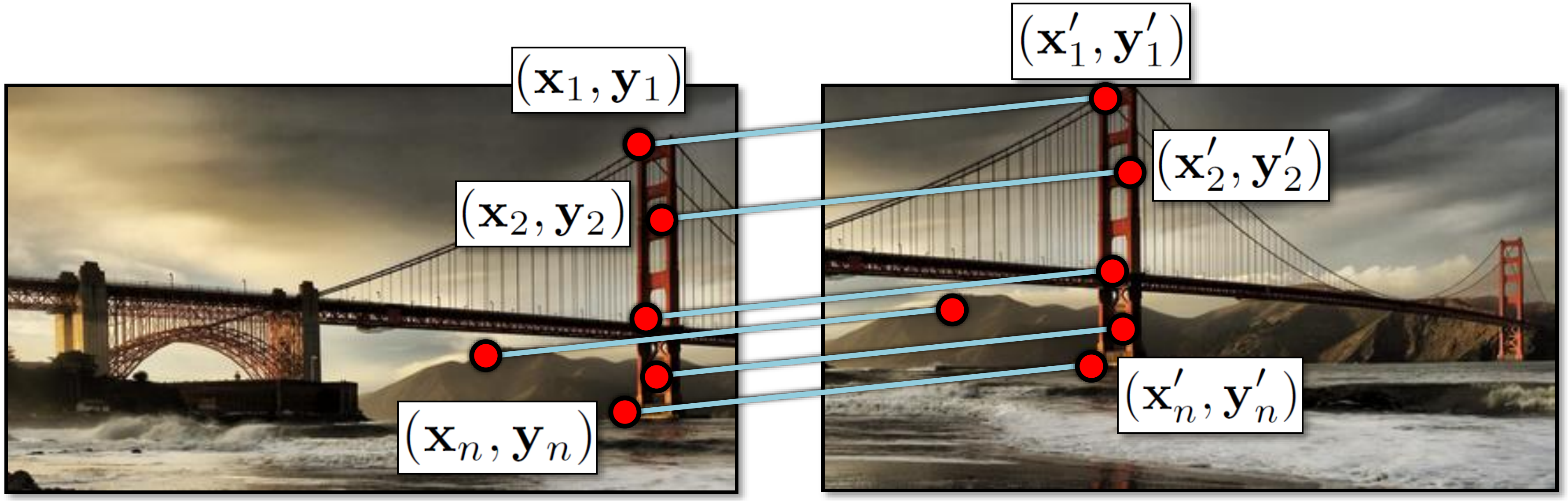


$$\mathbf{x}_i + \mathbf{x_t} = \mathbf{x}'_i$$

$$\mathbf{y}_i + \mathbf{y_t} = \mathbf{y}'_i$$

- System of linear equations
  - What are the knowns? Unknowns?
  - How many unknowns? How many equations (per match)?

Another view



$$\mathbf{x}_i + \mathbf{x_t} = \mathbf{x}_i'$$
$$\mathbf{y}_i + \mathbf{y_t} = \mathbf{y}_i'$$

- Problem: more equations than unknowns
  - "Overdetermined" system of equations
  - We will find the *least squares* solution

Least squares formulation

- For each point $(\mathbf{x}_i, \mathbf{y}_i)$

$$
\begin{aligned}
\mathbf{x}_i + \mathbf{x_t} &= \mathbf{x}'_i \\
\mathbf{y}_i + \mathbf{y_t} &= \mathbf{y}'_i
\end{aligned}
$$

- we define the *residuals* as

$$
\begin{aligned}
r_{\mathbf{x}_i}(\mathbf{x}_t) &= (\mathbf{x}_i + \mathbf{x}_t) - \mathbf{x}'_i \\
r_{\mathbf{y}_i}(\mathbf{y}_t) &= (\mathbf{y}_i + \mathbf{y}_t) - \mathbf{y}'_i
\end{aligned}
$$

Least squares formulation

- Goal: minimize sum of squared residuals

$$C(\mathbf{x}_t, \mathbf{y}_t) = \sum_{i=1}^{n} \left(r_{\mathbf{x}_i}(\mathbf{x}_t)^2 + r_{\mathbf{y}_i}(\mathbf{y}_t)^2 \right)$$

- "Least squares" solution
- For translations, is equal to mean (average) displacement

Least squares formulation

- Can also write as a matrix equation

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ & \vdots & \\ 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\substack{\mathbf{A} \\ 2n \times 2}} \underbrace{\begin{bmatrix} x_t \\ y_t \end{bmatrix}}_{\substack{\mathbf{t} \\ 2 \times 1}} = \underbrace{\begin{bmatrix} x'_1 - x_1 \\ y'_1 - y_1 \\ x'_2 - x_2 \\ y'_2 - y_2 \\ \vdots \\ x'_n - x_n \\ y'_n - y_n \end{bmatrix}}_{\substack{\mathbf{b} \\ 2n \times 1}}$$

$$A^T A = \begin{bmatrix} n & 0 \\ 0 & n \end{bmatrix}$$

$$(A^T A)^{-1} = \begin{bmatrix} \frac{1}{n} & 0 \\ 0 & \frac{1}{n} \end{bmatrix}$$

Least squares

$$\mathbf{A}\mathbf{t} = \mathbf{b}$$

- Find **t** that minimizes

$$||\mathbf{A}\mathbf{t} - \mathbf{b}||^2$$

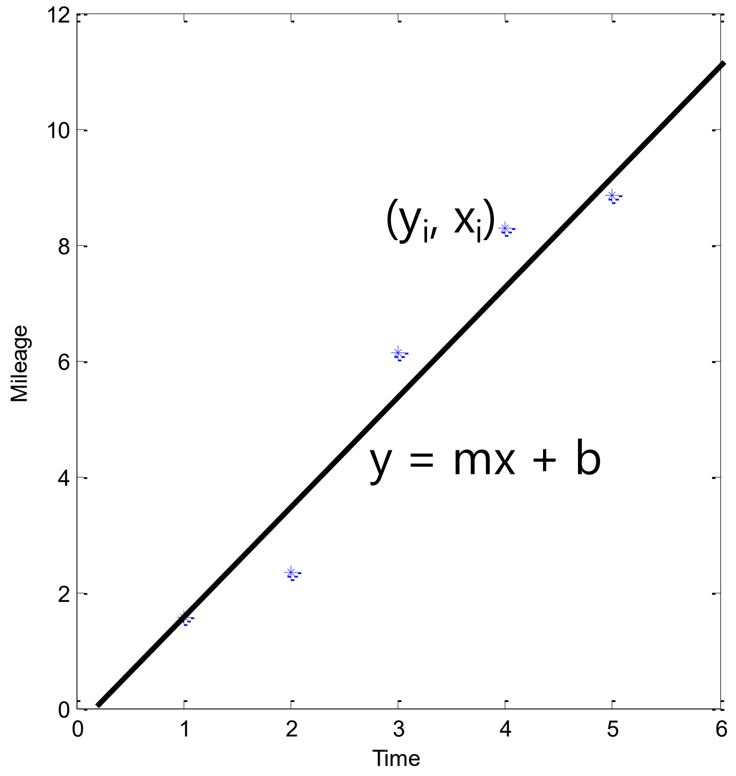
- To solve, form the *normal equations*

$$\mathbf{A}^{\mathrm{T}}\mathbf{A}\mathbf{t} = \mathbf{A}^{\mathrm{T}}\mathbf{b}$$

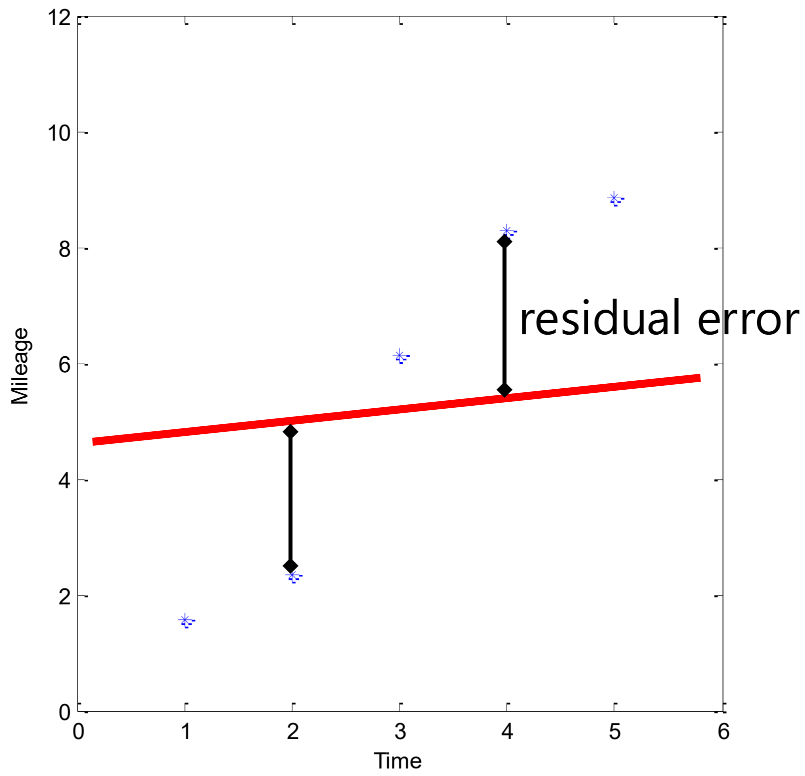
$$\mathbf{t} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$$

Questions?

Least squares: linear regression



Linear regression



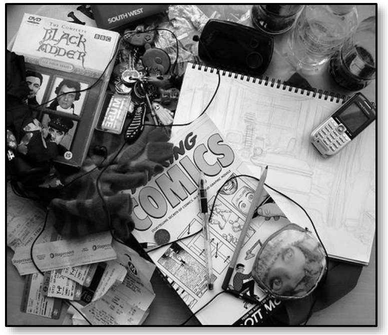
$$\mathrm{Cost}(m, b) = \sum_{i=1}^{n} |y_i - (mx_i + b)|^2$$

Linear regression

$$\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \\ x_n & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

Affine transformations

$$
\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}
$$



- How many unknowns?
- How many equations per match?
- How many matches do we need?

Affine transformations

- Residuals:

$$
\begin{aligned}
r_{x_i}(a,b,c,d,e,f) &= (ax_i + by_i + c) - x_i' \\
r_{y_i}(a,b,c,d,e,f) &= (dx_i + ey_i + f) - y_i'
\end{aligned}
$$

- Cost function:

$$
C(a,b,c,d,e,f) = \\
\sum_{i=1}^{n} \left(r_{x_i}(a,b,c,d,e,f)^2 + r_{y_i}(a,b,c,d,e,f)^2 \right)
$$

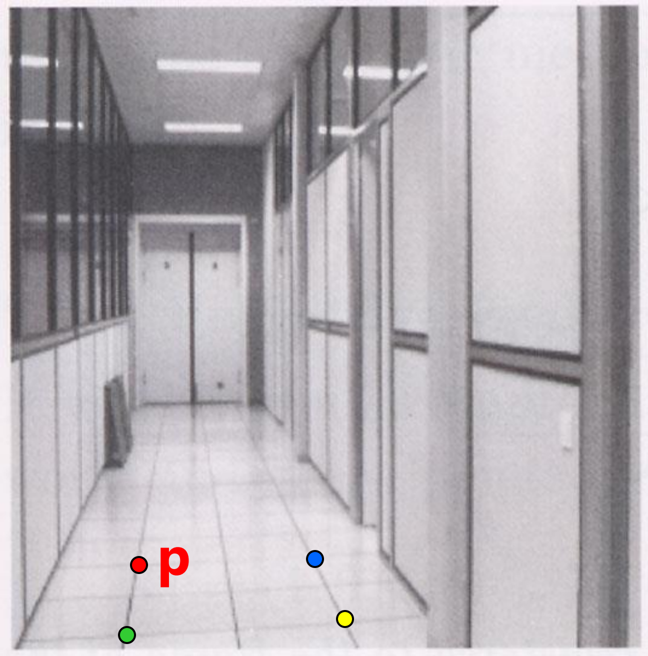
Affine transformations

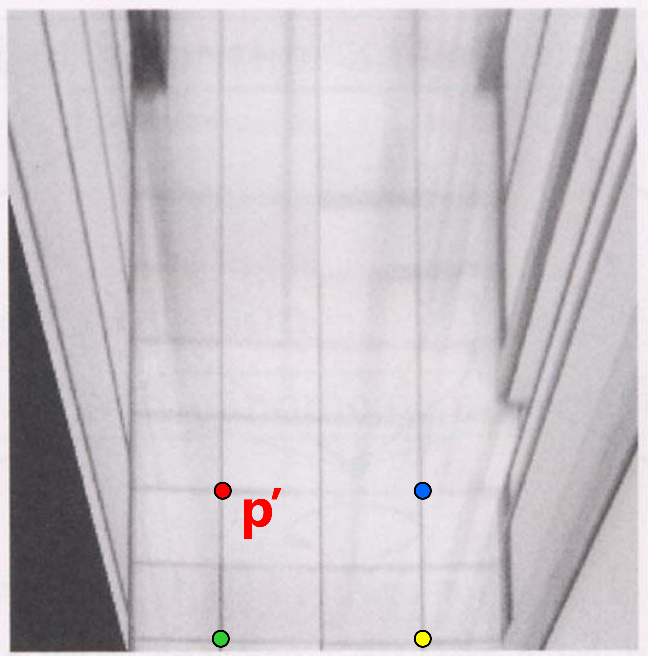
- Matrix form

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2 & y_2 & 1 \\ & & & \vdots & & \\ x_n & y_n & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_n & y_n & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} x'_1 \\ y'_1 \\ x'_2 \\ y'_2 \\ \vdots \\ x'_n \\ y'_n \end{bmatrix}$$

$$\underset{2n \times 6}{\mathbf{A}} \quad \underset{6 \times 1}{\mathbf{t}} = \underset{2n \times 1}{\mathbf{b}}$$

Homographies





To unwarp (rectify) an image

- solve for homography **H** given **p** and **p′**
- solve equations of the form: w**p′** = **Hp**
  - linear in unknowns: w and coefficients of **H**
  - H is defined up to an arbitrary scale factor
  - how many matches are necessary to solve for **H**?

Minimum number of matches to compute homography = 4

Solving for homographies

$$\begin{bmatrix} x'_i \\ y'_i \\ 1 \end{bmatrix} \cong \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$x'_i = \frac{h_{00}x_i + h_{01}y_i + h_{02}}{h_{20}x_i + h_{21}y_i + h_{22}}$$

$$y'_i = \frac{h_{10}x_i + h_{11}y_i + h_{12}}{h_{20}x_i + h_{21}y_i + h_{22}}$$

Not linear!

Multiplying each equation by the denominator of the RHS:

$$x'_i(h_{20}x_i + h_{21}y_i + h_{22}) = h_{00}x_i + h_{01}y_i + h_{02}$$
$$y'_i(h_{20}x_i + h_{21}y_i + h_{22}) = h_{10}x_i + h_{11}y_i + h_{12}$$

Solving for homographies

$$
\begin{aligned}
x'_i(h_{20}x_i + h_{21}y_i + h_{22}) &= h_{00}x_i + h_{01}y_i + h_{02} \\
y'_i(h_{20}x_i + h_{21}y_i + h_{22}) &= h_{10}x_i + h_{11}y_i + h_{12}
\end{aligned}
$$

$$
\begin{bmatrix}
x_i & y_i & 1 & 0 & 0 & 0 & -x'_i x_i & -x'_i y_i & -x'_i \\
0 & 0 & 0 & x_i & y_i & 1 & -y'_i x_i & -y'_i y_i & -y'_i
\end{bmatrix}
\begin{bmatrix}
h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22}
\end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \end{bmatrix}
$$

Solving for homographies

$$\underbrace{\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1 x_1 & -x'_1 y_1 & -x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1 x_1 & -y'_1 y_1 & -y'_1 \\ & & & & & \vdots & & & \\ x_n & y_n & 1 & 0 & 0 & 0 & -x'_n x_n & -x'_n y_n & -x'_n \\ 0 & 0 & 0 & x_n & y_n & 1 & -y'_n x_n & -y'_n y_n & -y'_n \end{bmatrix}}_{\substack{\mathbf{A} \\ 2n \times 9}} \underbrace{\begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix}}_{\substack{\mathbf{h} \\ 9}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{\substack{\mathbf{0} \\ 2n}}$$

Defines a least squares problem minimize $\|\mathbf{A}\mathbf{h} - \mathbf{0}\|^2$

- Since $\mathbf{h}$ is only defined up to scale, solve for unit vector $\hat{\mathbf{h}}$
- Solution: $\hat{\mathbf{h}}$ = eigenvector of $\mathbf{A}^T\mathbf{A}$ with smallest eigenvalue
- Works with 4 or more points

Recap: Two Common Optimization Problems

Problem statement

minimize $\|\mathbf{Ax}-\mathbf{b}\|^2$

(least squares solution to $\mathbf{Ax}=\mathbf{b}$)

Solution

$\mathbf{x}=(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}$

$\mathbf{x}=\mathbf{A}\backslash\mathbf{b}$ (matlab)

Problem statement

minimize $\mathbf{x}^T\mathbf{A}^T\mathbf{Ax}$ s.t. $\mathbf{x}^T\mathbf{x}=1$

(non−trivial lsq solution to $\mathbf{Ax}=0$)

Solution

$[\mathbf{v},\lambda]=\mathrm{eig}(\mathbf{A}^T\mathbf{A})$

$\lambda_1<\lambda_{2..n}: \mathbf{x}=\mathbf{v}_1$

Computing transformations

Questions?

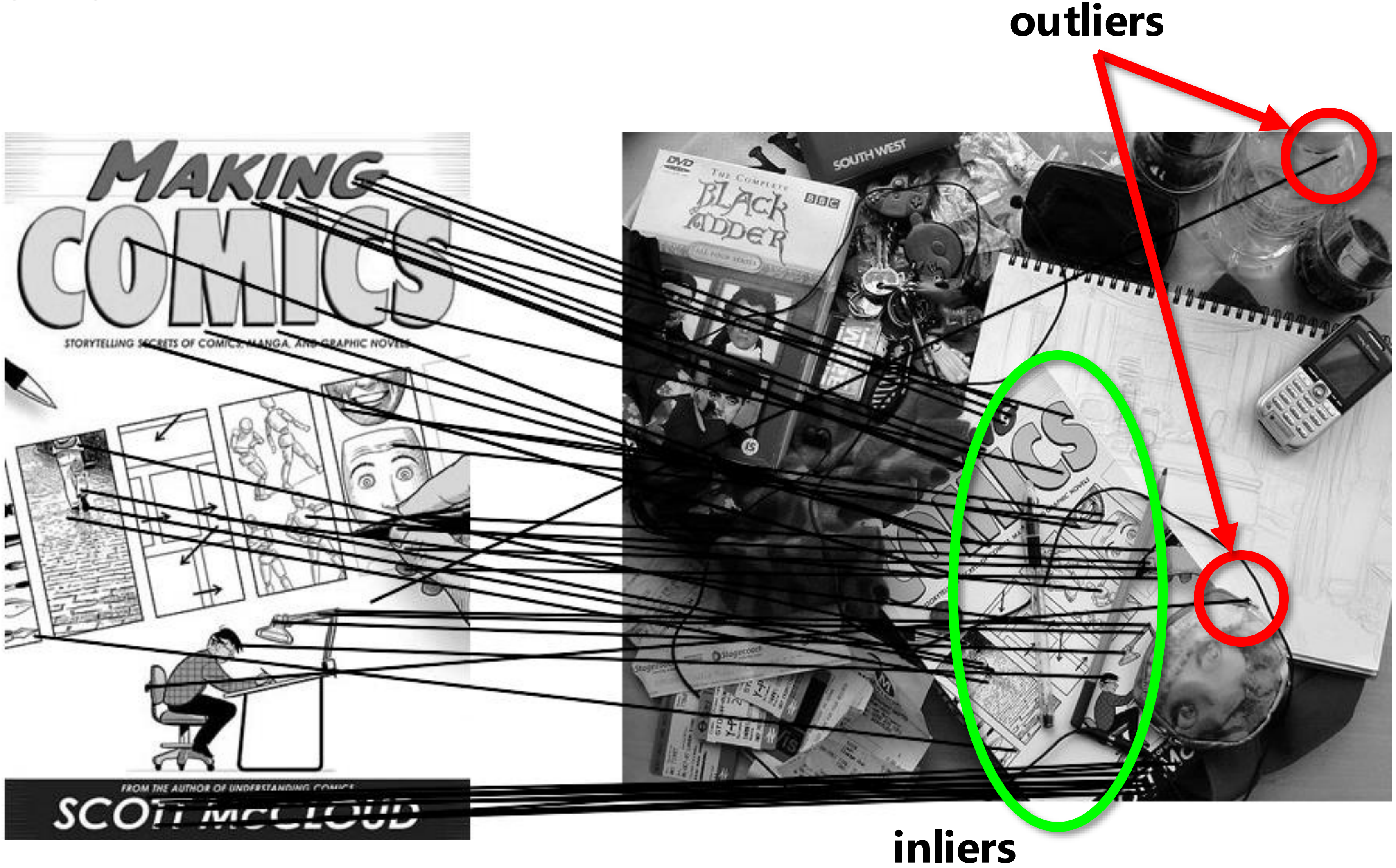
Image alignment algorithm

Given images A and B

1. Compute image features for A and B
2. Match features between A and B
3. Compute homography between A and B using least squares on set of matches

What could go wrong?

Outliers



outliers
MAKING
COMICS
STORYTELLING SECRETS OF COMICS, MANGA, AND GRAPHIC NOVELS
FROM THE AUTHOR OF UNDERSTANDING COMICS
SCOTT McCLOUD
DVD
THE COMPLETE
BLACK ADDER
BBC
SOUTH WEST
inliers