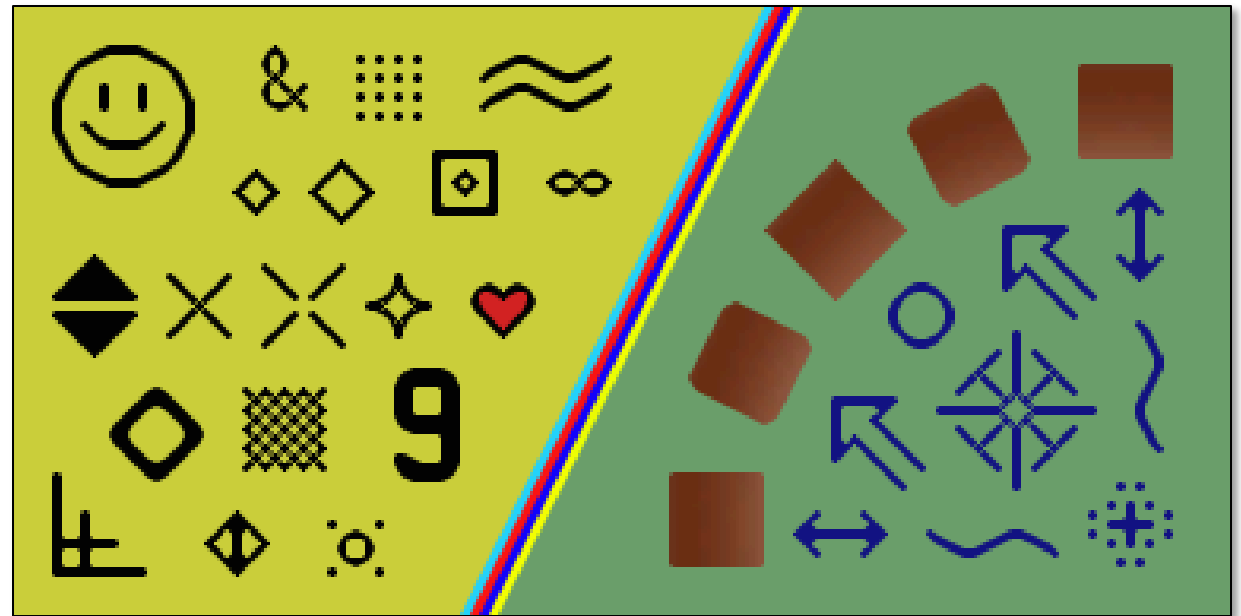
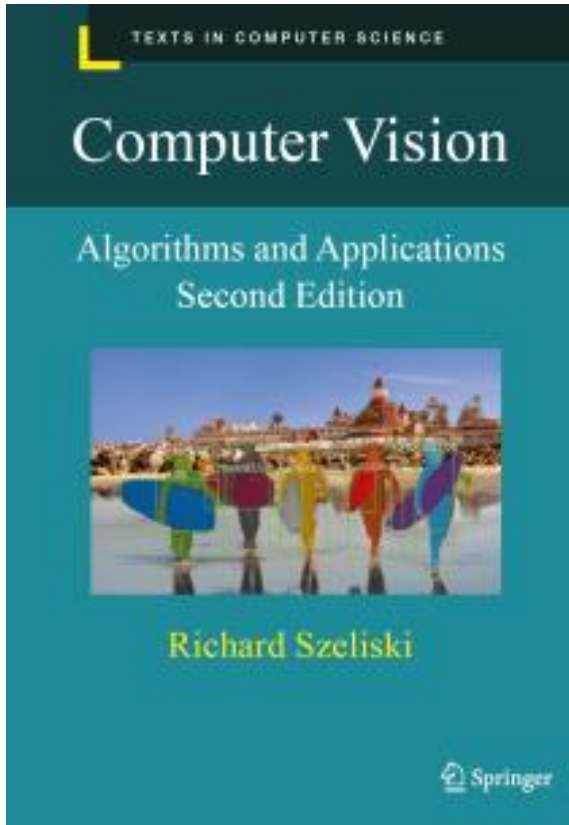


# Computer Vision

## Image Resampling & Interpolation



# Important information



## Textbook

Rick Szeliski, *Computer Vision: Algorithms and Applications* online at: <http://szeliski.org/Book/>

Many of the slides in this course are modified from the excellent class notes of similar courses offered in other schools by Noah Snavely, Prof Yung-Yu Chuang, Fredo Durand, Alyosha Efros, Bill Freeman, James Hays, Svetlana Lazebnik, Andrej Karpathy, Fei-Fei Li, Srinivasa Narasimhan, Silvio Savarese, Steve Seitz, Richard Szeliski, and Li Zhang. The instructor is extremely thankful to the researchers for making their notes available online. Please feel free to use and modify any of the slides, but acknowledge the original sources where appropriate.

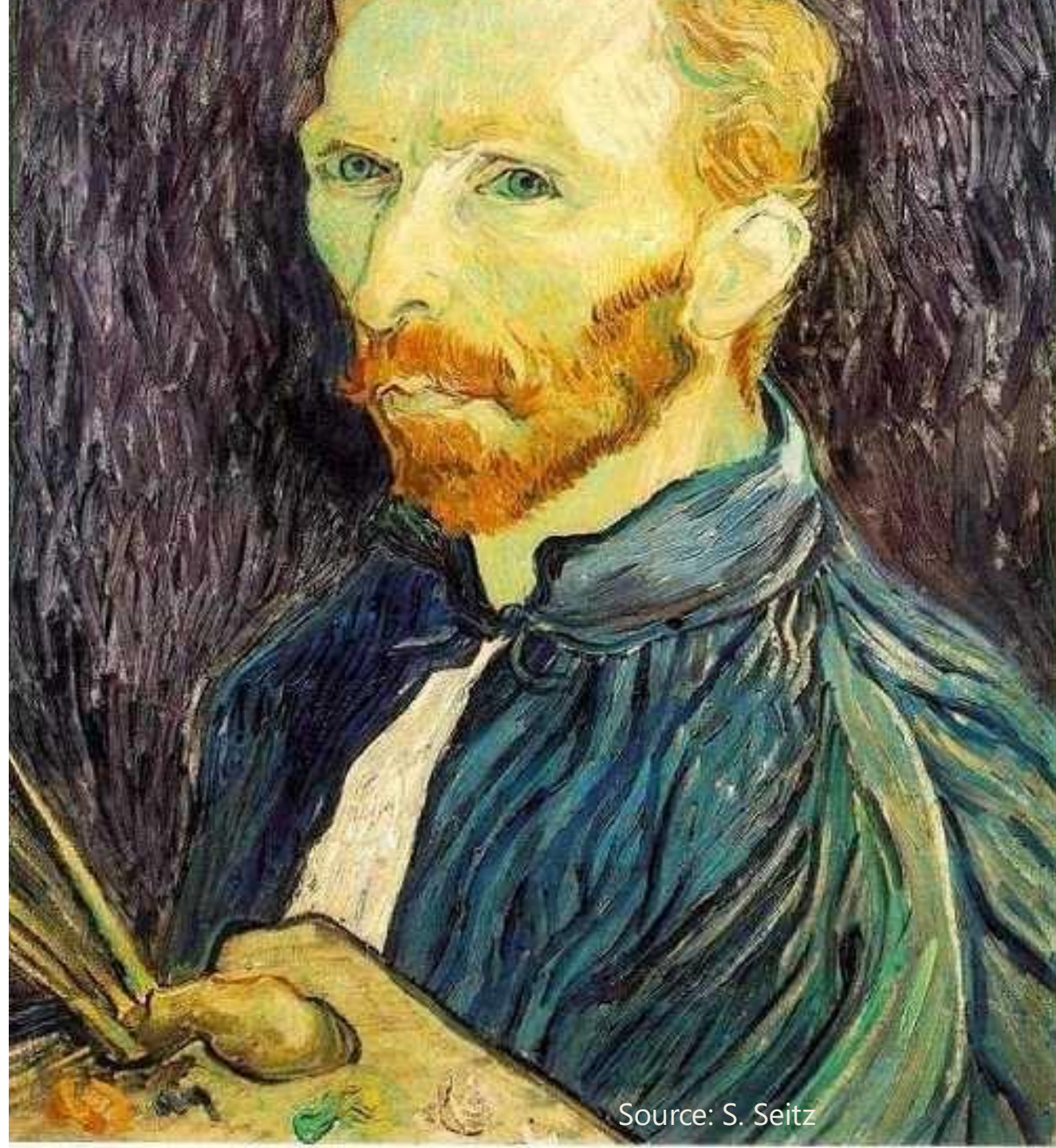
All readings are from Richard Szeliski, *Computer Vision: Algorithms and Applications*, 2nd Edition, unless otherwise noted.

# Reading

- Szeliski 2.3.1, 3.4-3.5

# Image scaling

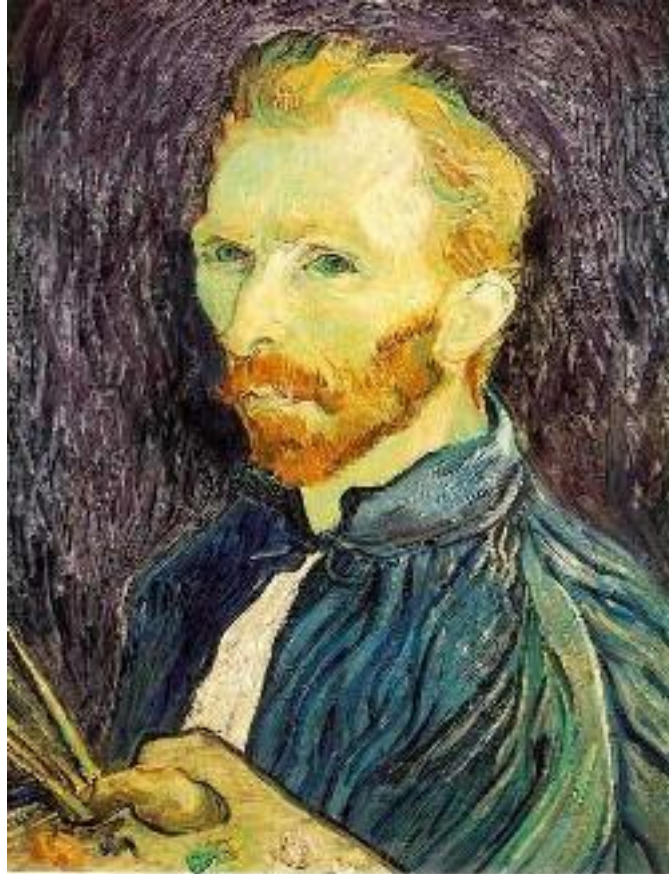
This image is too big to fit on the screen. How can we generate a half-sized version?



Source: S. Seitz



# Image sub-sampling



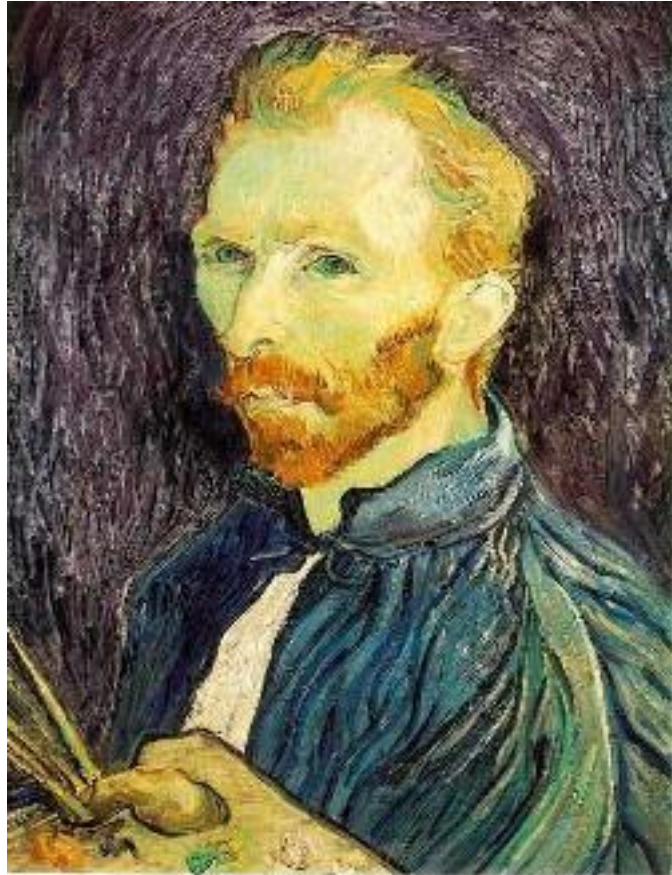
1/4



1/8

Throw away every other row and column to create a 1/2 size image  
- called *image sub-sampling*

# Image sub-sampling



1/2



1/4 (2x zoom)



1/8 (4x zoom)

Why does this look so cruffy?

Source: S. Seitz

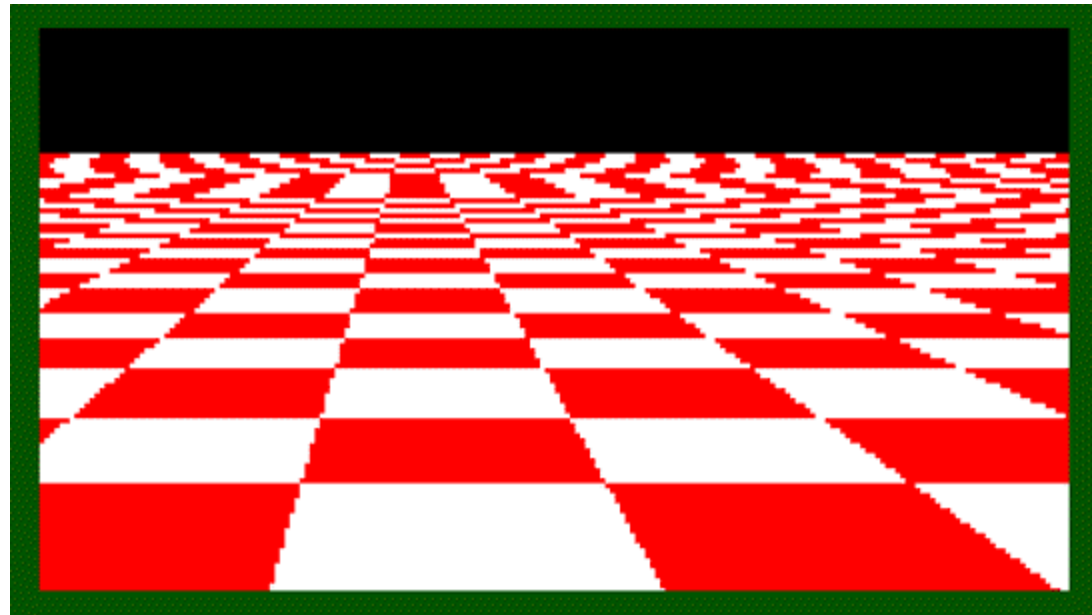


# Image sub-sampling – another example



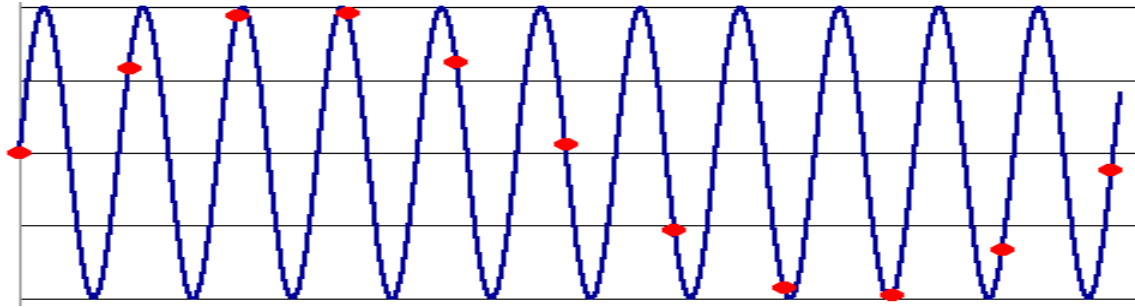
Source: F. Durand

# Even worse for synthetic images





# Aliasing



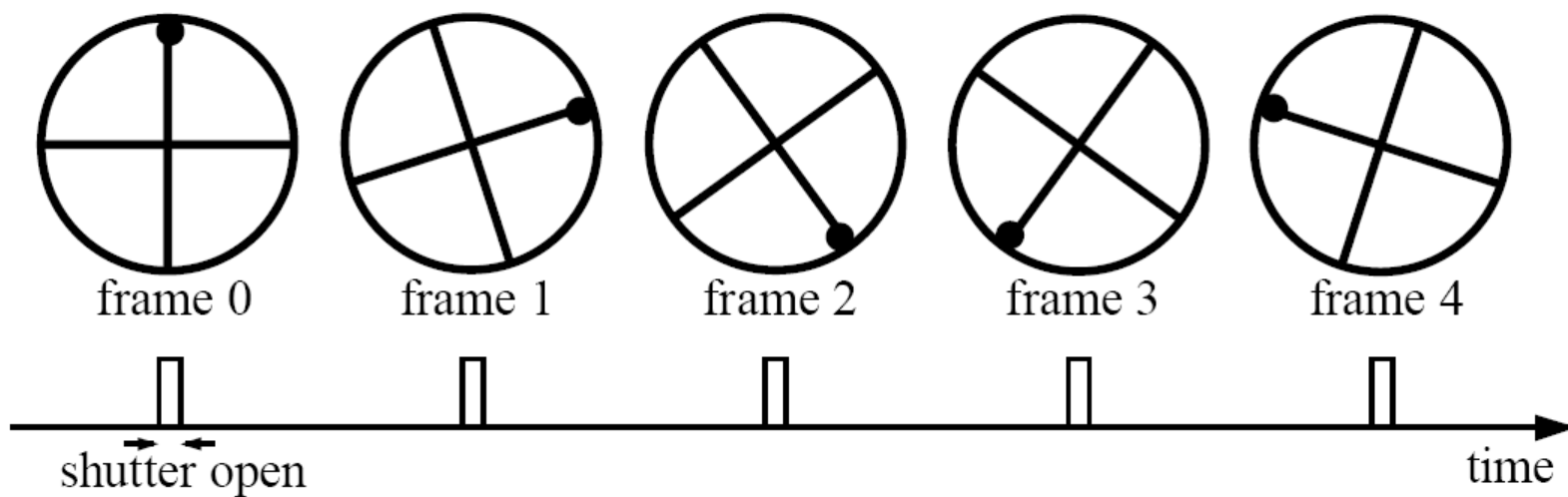
- Occurs when your sampling rate is not high enough to capture the amount of detail in your image
- Can give you the wrong signal/image—an *alias*
- To do sampling right, need to understand the structure of your signal/image
- Enter Monsieur Fourier...
  - “But what is the Fourier Transform? A visual introduction.”  
<https://www.youtube.com/watch?v=spUNpyF58BY>
- To avoid aliasing:
  - sampling rate  $\geq 2 * \text{max frequency in the image}$ 
    - said another way:  $\geq$  two samples per cycle
  - This minimum sampling rate is called the **Nyquist rate**

# Wagon-wheel effect

Imagine a spoked wheel moving to the right (rotating clockwise).

Mark wheel with dot so we can see what's happening.

If camera shutter is only open for a fraction of a frame time (frame time =  $1/30$  sec. for video,  $1/24$  sec. for film):



Without dot, wheel appears to be rotating slowly backwards!  
(counterclockwise)

# Wagon-wheel effect



[https://en.wikipedia.org/wiki/Wagon-wheel\\_effect](https://en.wikipedia.org/wiki/Wagon-wheel_effect)



# Temporal aliasing – helicopter blades

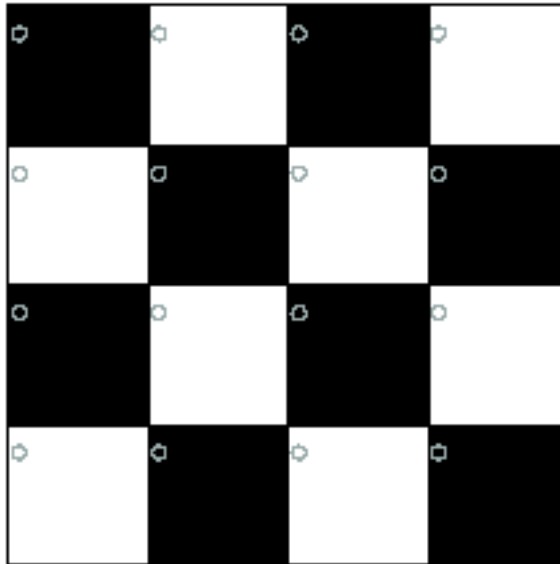
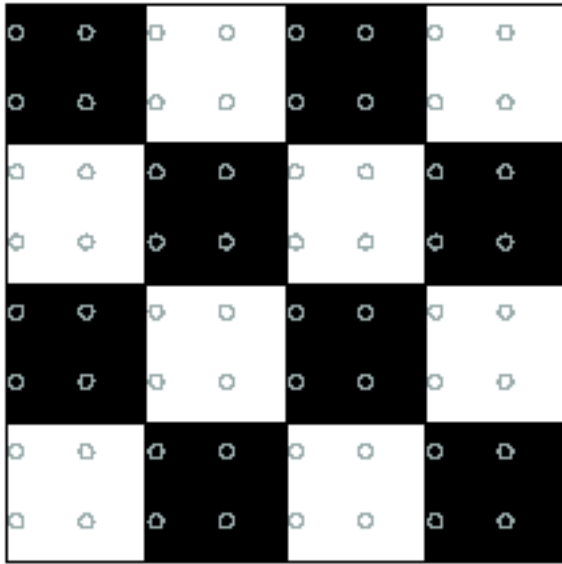


<https://www.youtube.com/watch?v=yr3ngmRuGUc>

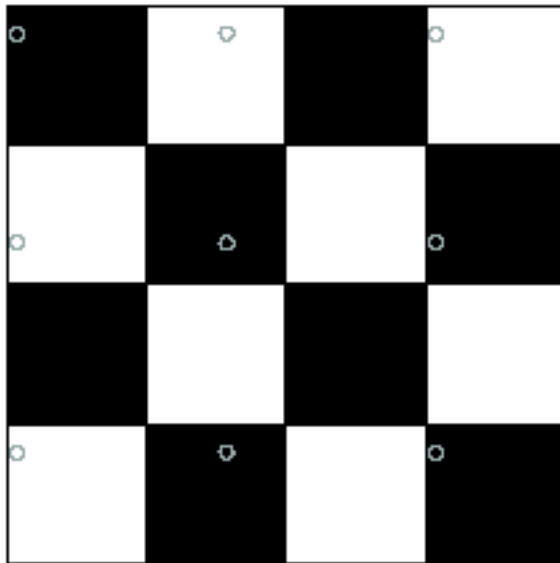
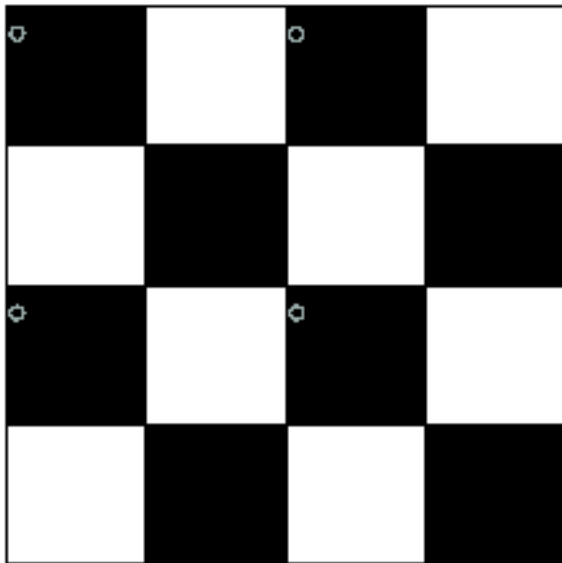
# Aliasing in practice



# Nyquist limit – 2D example



Good sampling



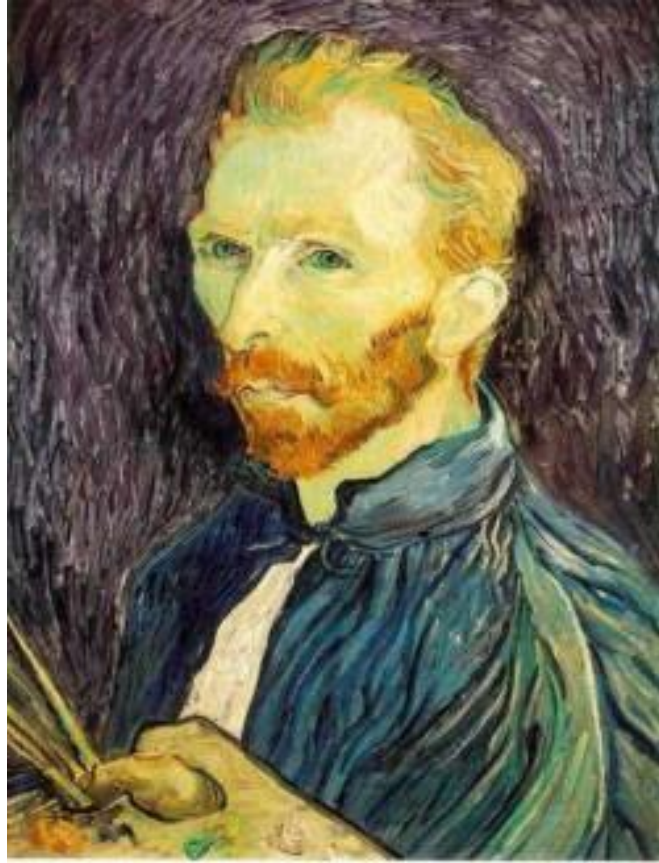
Bad sampling



# Aliasing

- When downsampling by a factor of two
  - Original image has frequencies that are too high
- How can we fix this?

# Gaussian pre-filtering



Gaussian 1/2



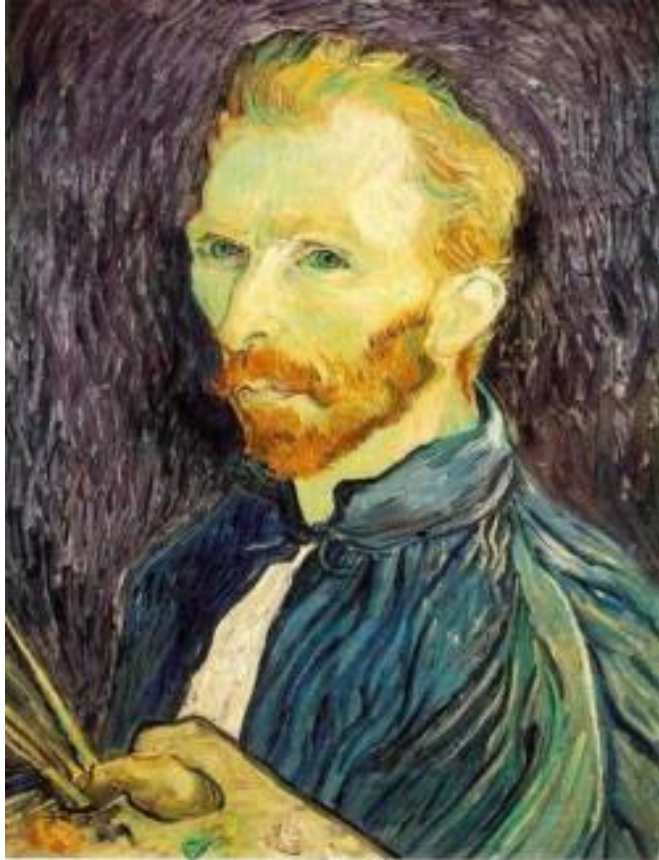
G 1/4



G 1/8

- Solution: filter the image, *then* subsample

# Subsampling with Gaussian pre-filtering



Gaussian  $1/2$



G  $1/4$

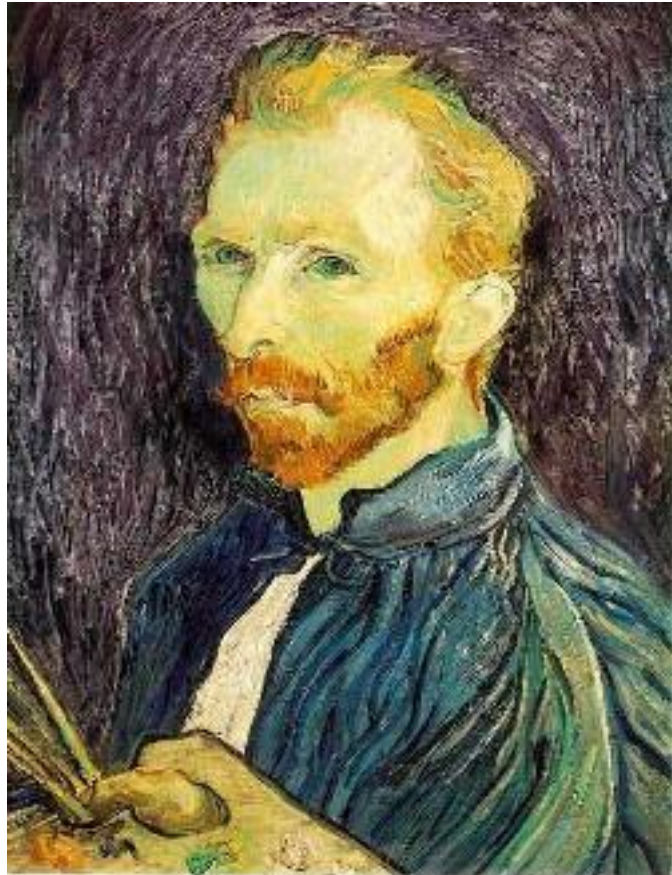


G  $1/8$

- Solution: filter the image, *then* subsample



# Compare with...



1/2



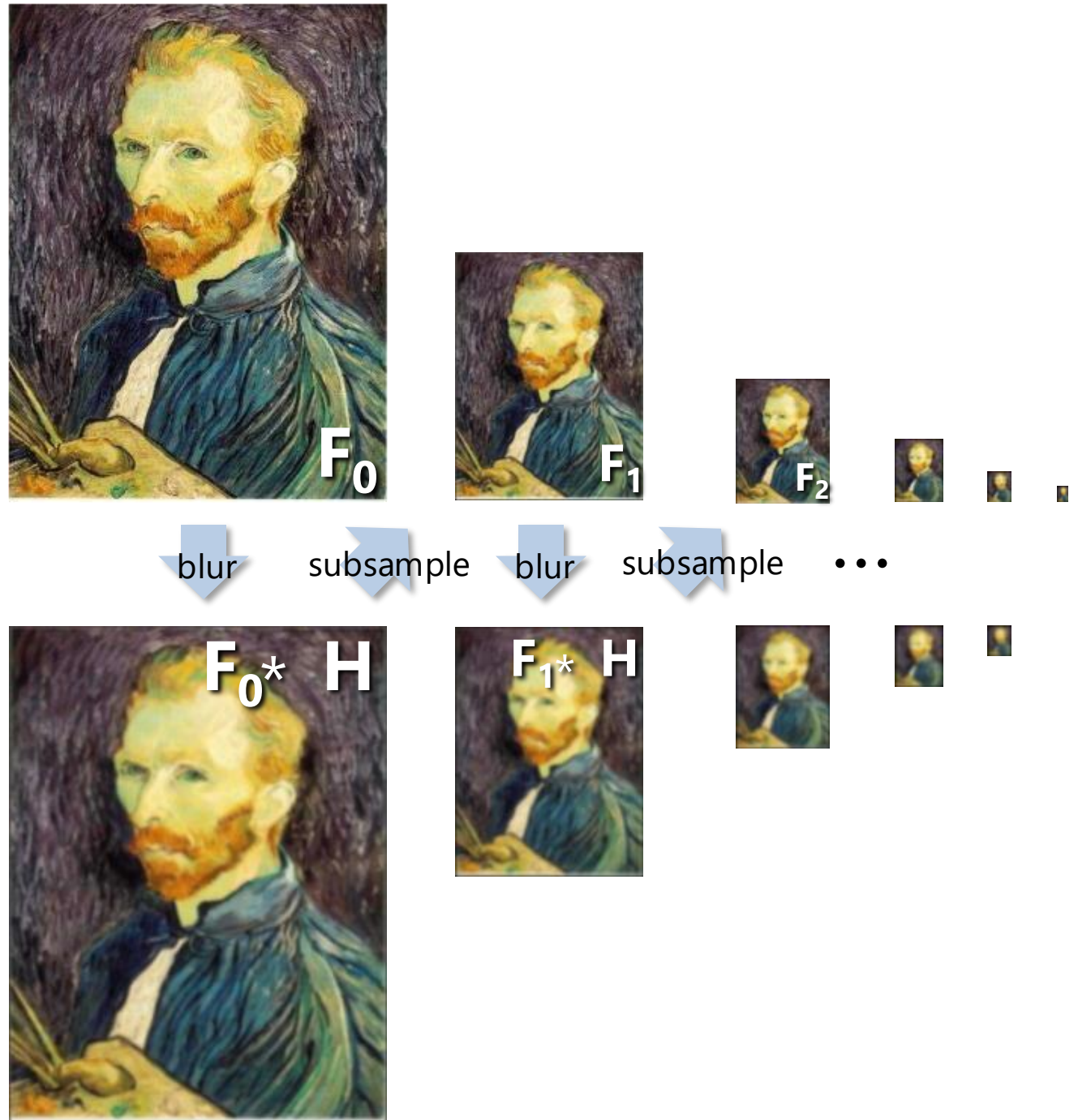
1/4 (2x zoom)



1/8 (4x zoom)

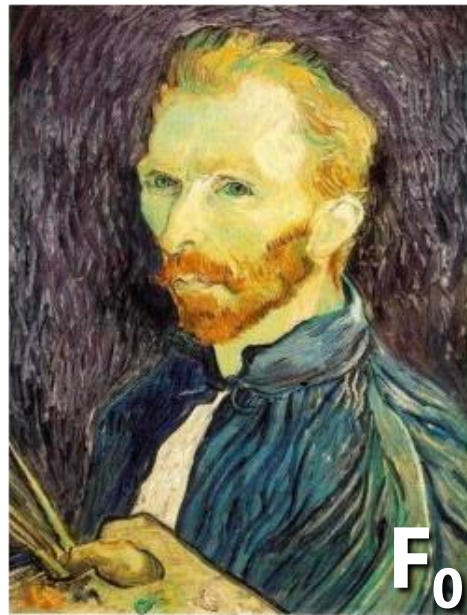
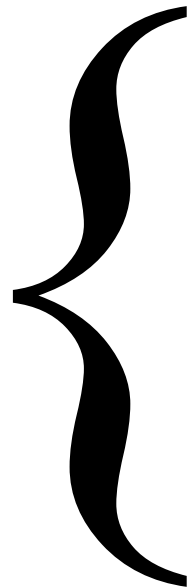
# Gaussian pre-filtering

- Solution: filter the image, *then* subsample

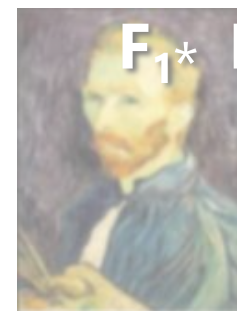
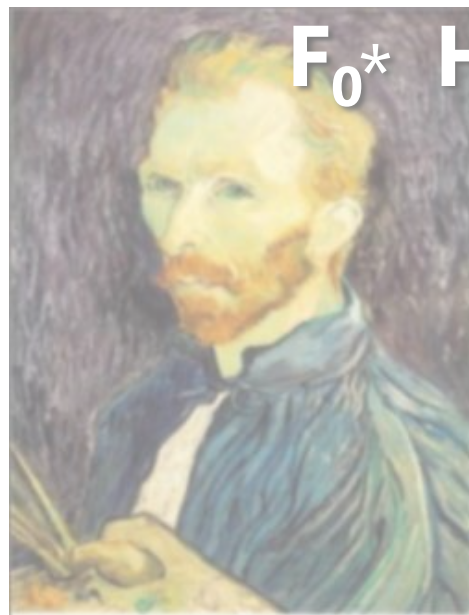




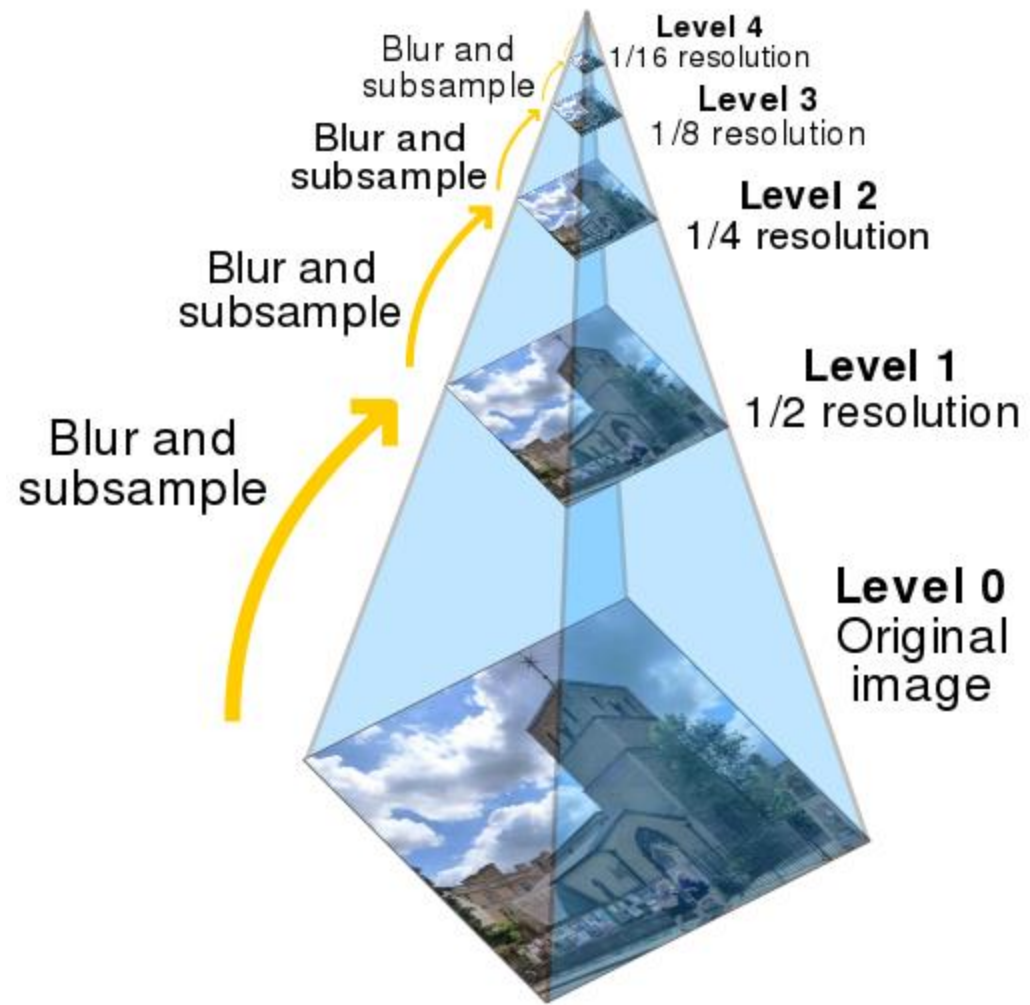
*Gaussian  
pyramid*



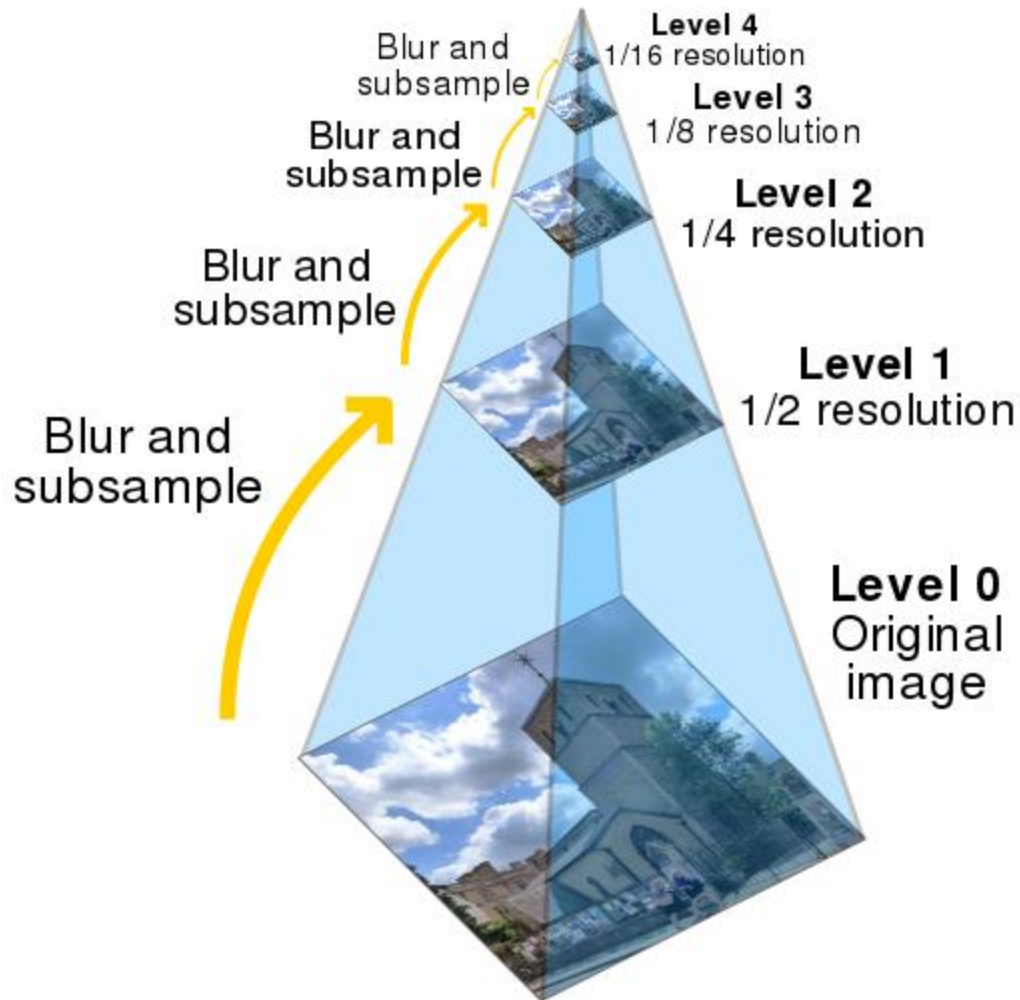
...







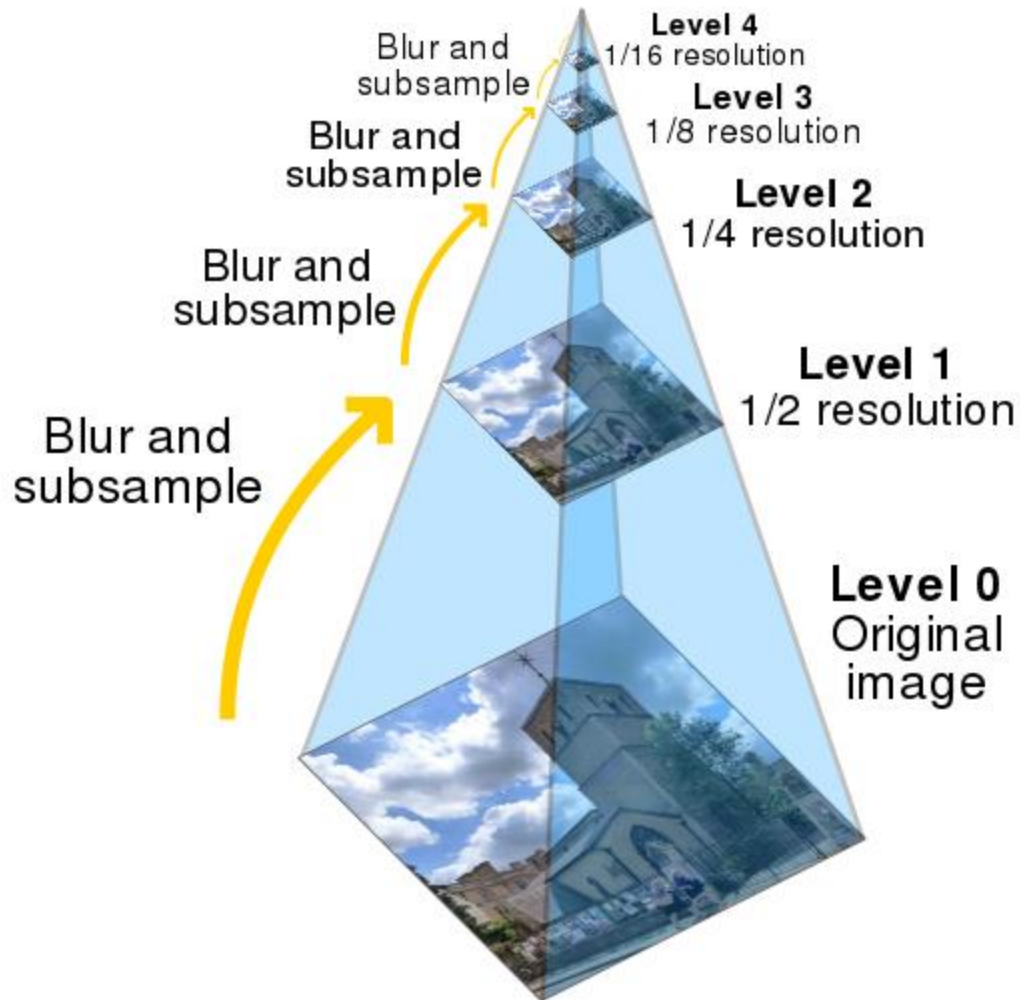
# Gaussian pyramids [Burt and Adelson, 1983]



- In computer graphics, a *mip map* [Williams, 1983]
- A precursor to *wavelet transform*

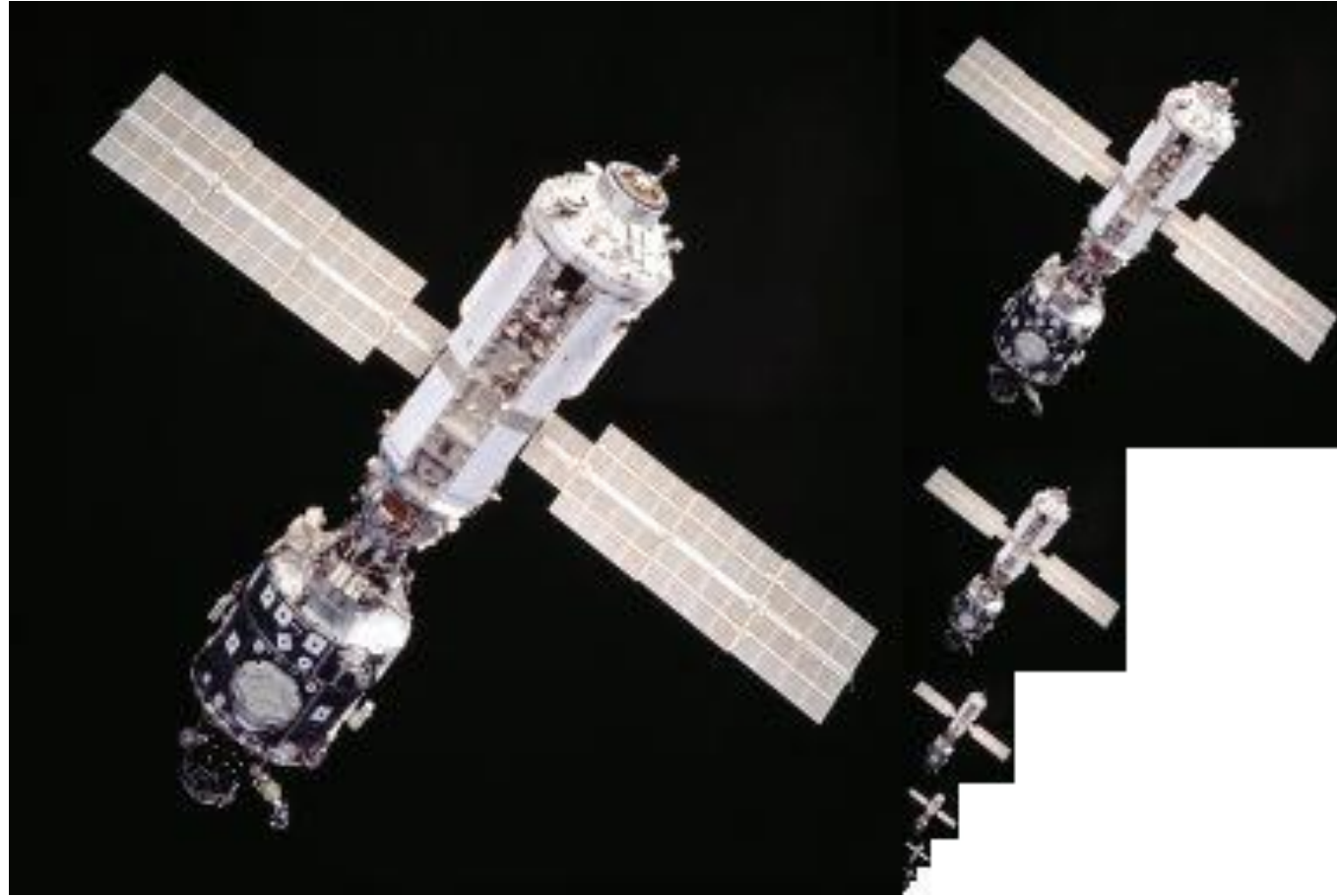
Gaussian Pyramids have all sorts of applications in computer vision

# Gaussian pyramids [Burt and Adelson, 1983]



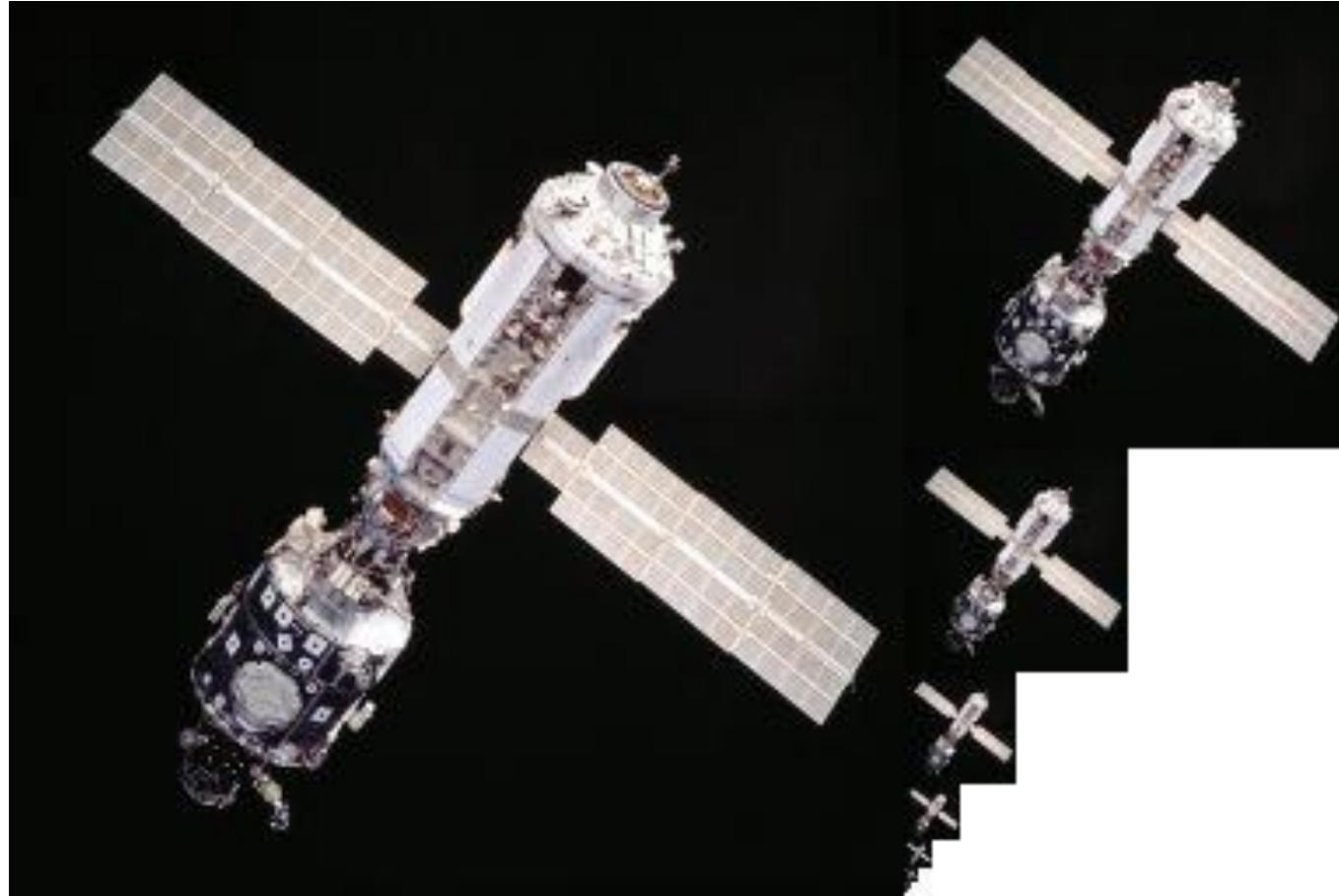
Q: How much space does a Gaussian pyramid take compared to the original image?

# Gaussian pyramid





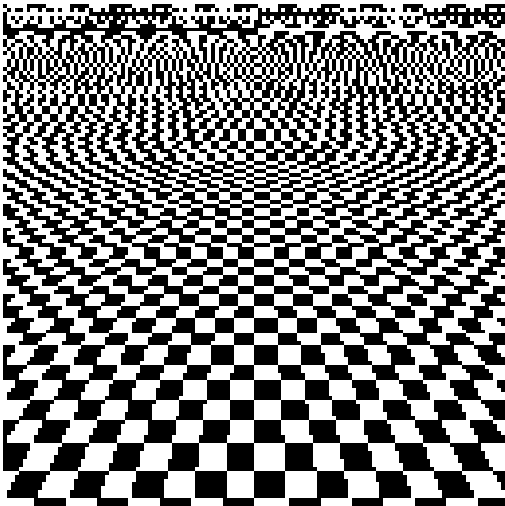
# Gaussian pyramid



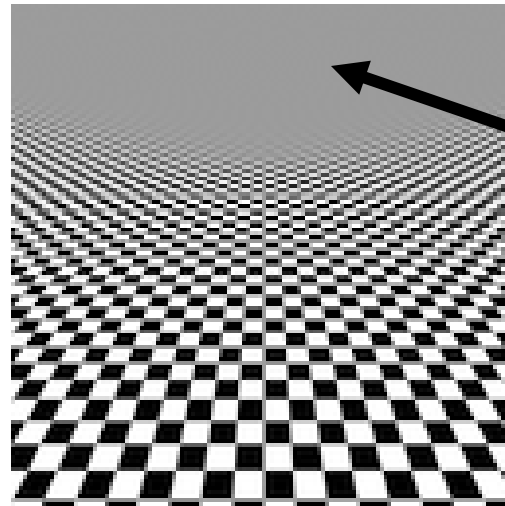
Answer:  $\frac{4}{3}$  the amount of space as the original image alone  
See [https://en.wikipedia.org/wiki/Geometric\\_series](https://en.wikipedia.org/wiki/Geometric_series)

# Back to the checkerboard

- What should happen when you make the checkerboard smaller and smaller?



Naïve subsampling



Proper prefiltering  
("antialiasing")

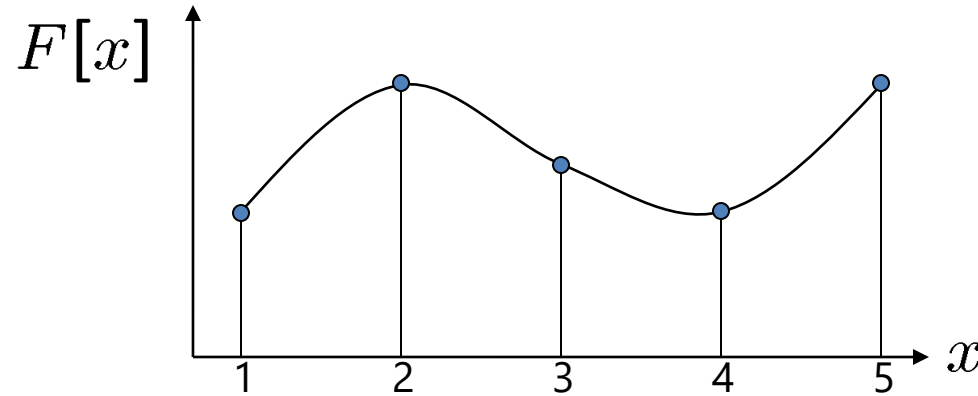
Image turns grey!  
(Average of black  
and white squares,  
because each pixel  
contains both.)

# Upsampling

- This image is too small for this screen: 
- How can we make it 10 times as big?
- Simplest approach:  
repeat each row  
and column 10 times
- ("Nearest neighbor interpolation")



# Image interpolation



$d = 1$  in this example

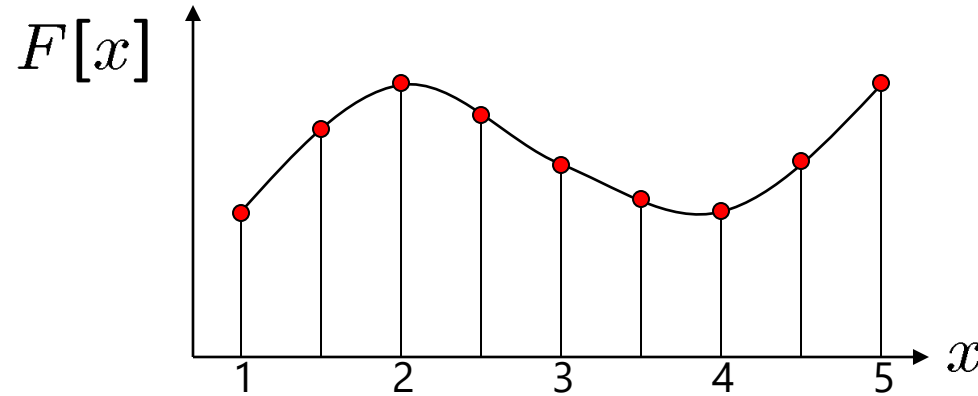
Recall that a digital images is formed as follows:

$$F[x, y] = \text{quantize}\{f(xd, yd)\}$$

- It is a discrete point-sampling of a continuous function
- If we could somehow reconstruct the original function, any new image could be generated, at any resolution and scale



# Image interpolation



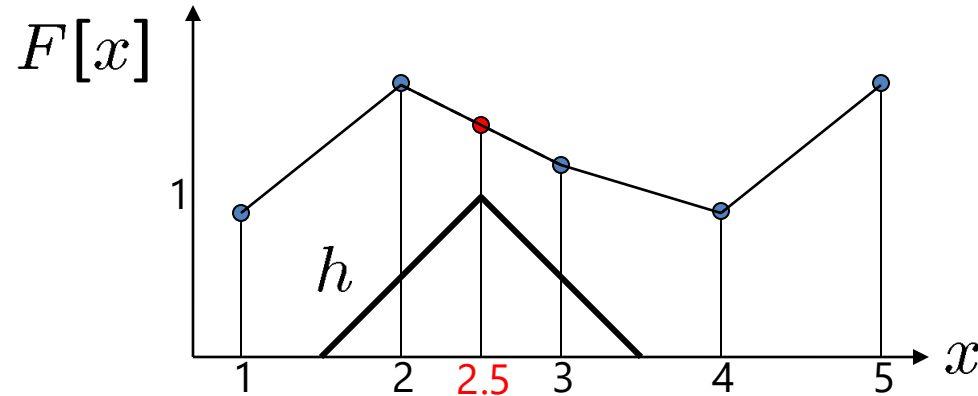
$d = 1$  in this example

Recall that a digital images is formed as follows:

$$F[x, y] = \text{quantize}\{f(xd, yd)\}$$

- It is a discrete point-sampling of a continuous function
- If we could somehow reconstruct the original function, any new image could be generated, at any resolution and scale

# Image interpolation



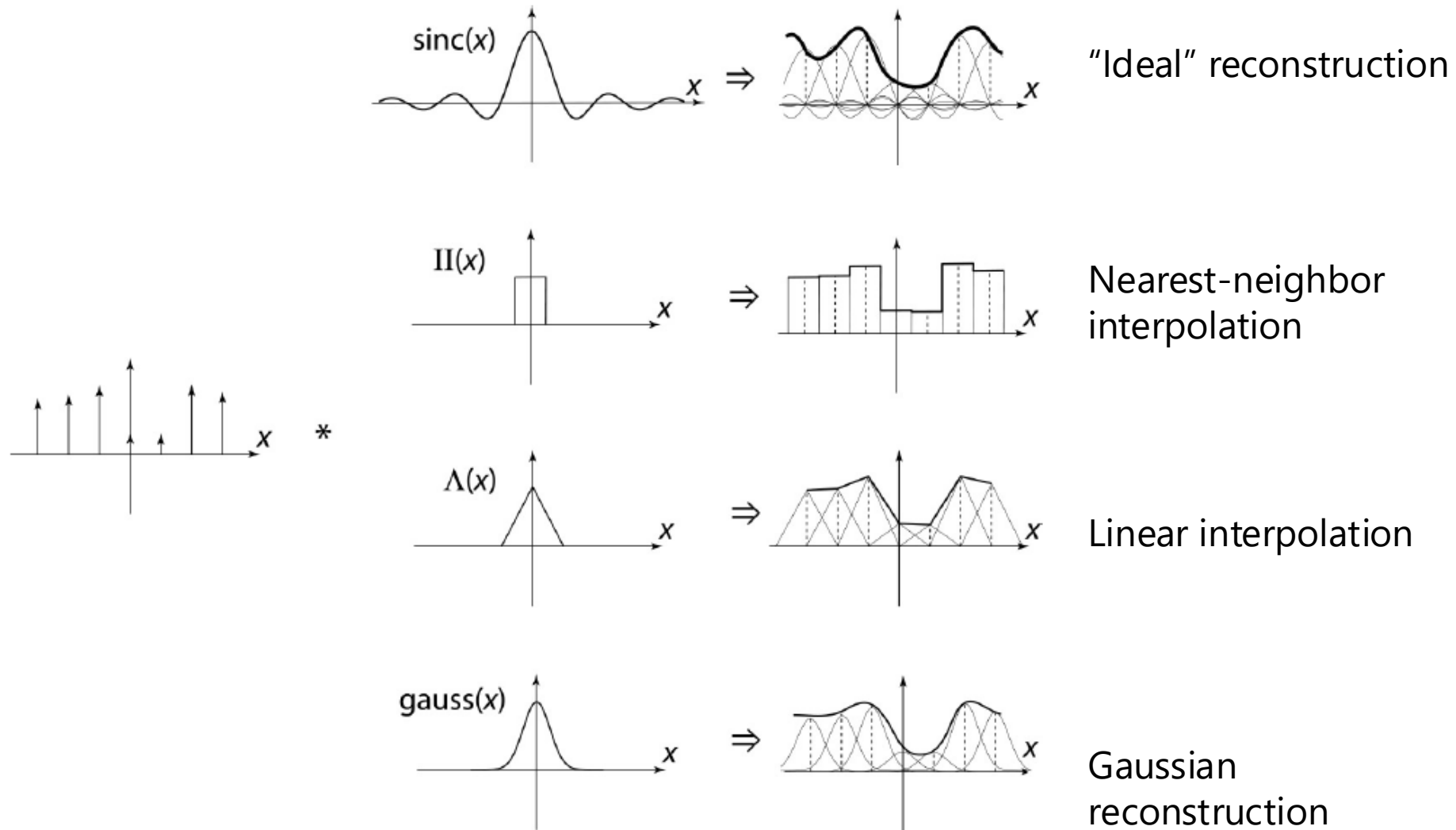
$d = 1$  in this example

- What if we don't know  $f$  ?
  - Guess an approximation:  $\tilde{f}$
  - Can be done in a principled way: filtering
  - Convert  $F$  to a continuous function:
$$f_F(x) = F\left(\frac{x}{d}\right) \text{ when } \frac{x}{d} \text{ is an integer, } 0 \text{ otherwise}$$

- Reconstruct by convolution with a *reconstruction filter*,  $h$

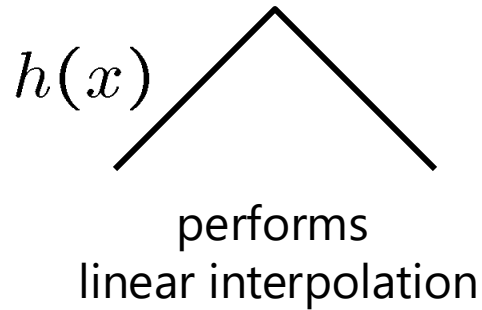
$$\tilde{f} = h * f_F$$

# Image interpolation

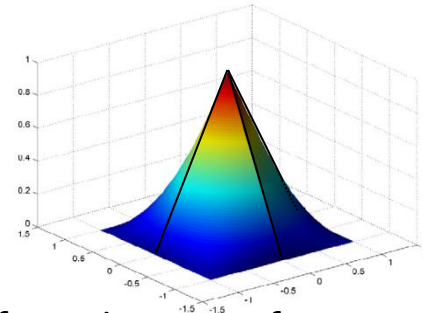


# Reconstruction filters

- What does the 2D version of this hat function look like?



$h(x, y)$



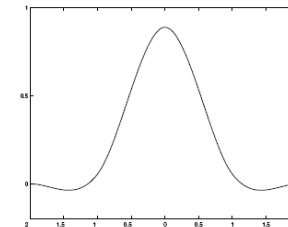
(tent function) performs  
**bilinear interpolation**

Often implemented without cross-correlation

- E.g., [http://en.wikipedia.org/wiki/Bilinear\\_interpolation](http://en.wikipedia.org/wiki/Bilinear_interpolation)

Better filters give better resampled images

- Bicubic** is common choice



Cubic reconstruction filter

$$r(x) = \frac{1}{6} \begin{cases} (12 - 9B - 6C)|x|^3 + (-18 + 12B + 6C)|x|^2 + (6 - 2B) & |x| < 1 \\ ((-B - 6C)|x|^3 + (6B + 30C)|x|^2 + (-12B - 48C)|x| + (8B + 24C)) & 1 \leq |x| < 2 \\ 0 & \text{otherwise} \end{cases}$$



# Image interpolation

Original image:  x  
10



Nearest-neighbor interpolation



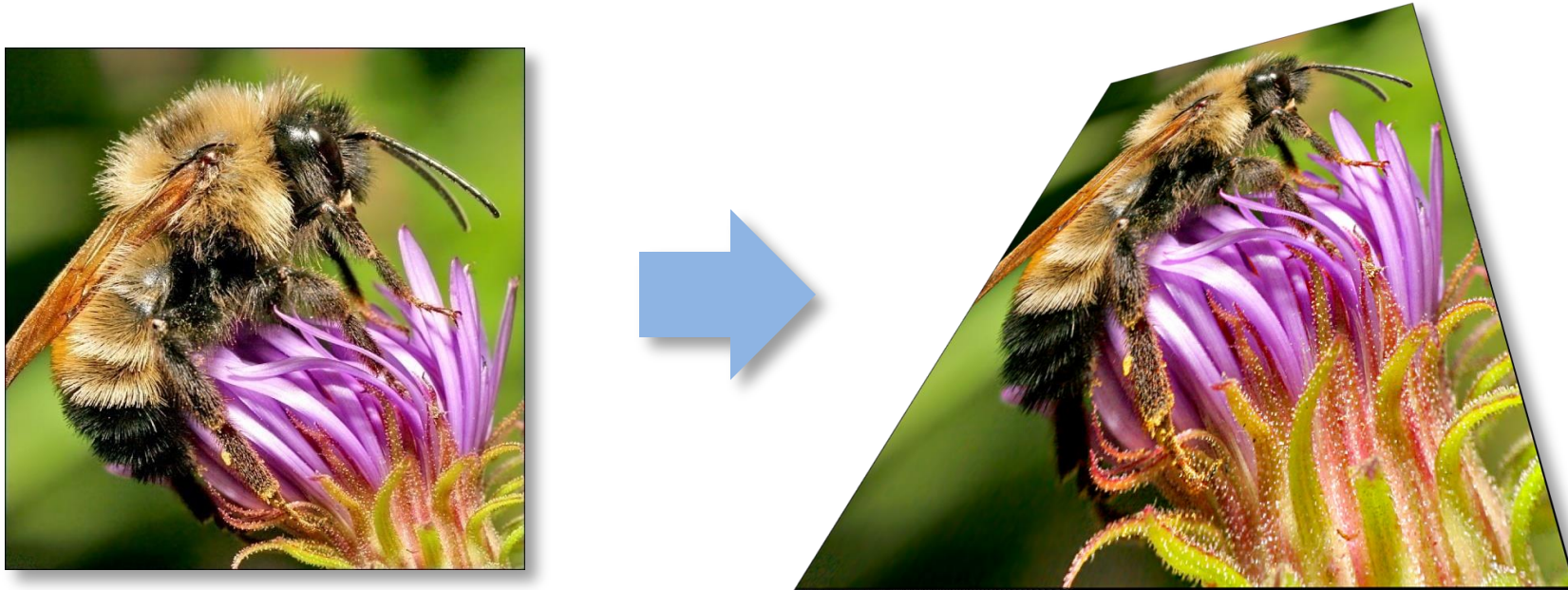
Bilinear interpolation



Bicubic interpolation

# Image interpolation

Also used for *resampling*

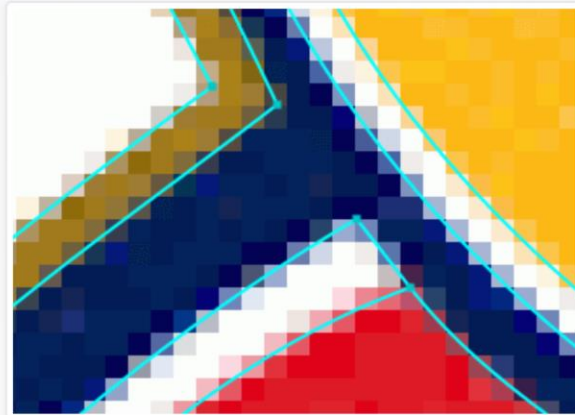


# Raster-to-vector graphics



Vector Magic

Simply the Best Auto-Tracer in the World



# Depixelating Pixel Art





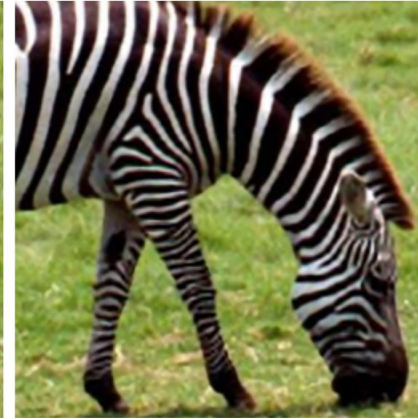
# Modern methods



(a) Bicubic



(b) SRCNN



(c) A+



(d) RAISR



(e) Bicubic



(f) SRCNN



(g) A+



(h) RAISR

From Romano, et al: RAISR: Rapid and Accurate Image Super Resolution,  
<https://arxiv.org/abs/1606.01299>

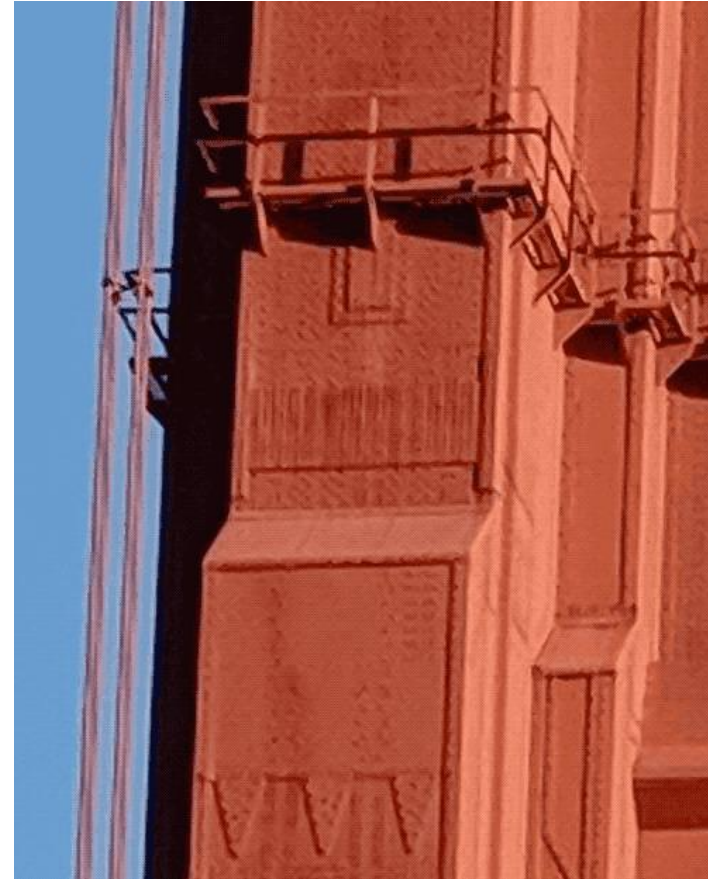
# Super-resolution with multiple images

- Can do better upsampling if you have multiple images of the scene taken with small (subpixel) shifts
- Some cellphone cameras (like the Google Pixel line) capture a **burst** of photos
- Can we use that burst for upsampling?

# Google Pixel 3 Super Res Zoom



Effect of hand tremor as seen in a cropped burst of photos, after global alignment



Example photo with and without super res zoom (smart burst align and merge)

<https://ai.googleblog.com/2018/10/see-better-and-further-with-super-res.html>



# Summary

- Key points:
  - **Subsampling an image** can cause aliasing. Better is to blur ("pre-filter") to remove high frequencies then downsample
  - If you repeatedly blur and downsample by 2x, you get a Gaussian pyramid
  - **Upsampling an image** requires interpolation. This can be posed as convolution with a "reconstruction kernel"



# Questions?