Chapter 4

4.1. SOP form: $f = \overline{x}_1 x_2 + \overline{x}_2 x_3$
POS form: $f = (\overline{x}_1 + \overline{x}_2)(x_2 + x_3)$

4.2. SOP form: $f = x_1 \overline{x}_2 + x_1 x_3 + \overline{x}_2 x_3$
POS form: $f = (x_1 + x_3)(x_1 + \overline{x}_2)(\overline{x}_2 + x_3)$

4.3. SOP form: $f = \overline{x}_1 x_2 x_3 \overline{x}_4 + x_1 x_2 \overline{x}_3 x_4 + \overline{x}_2 x_3 x_4$
POS form: $f = (\overline{x}_1 + x_4)(x_2 + x_3)(\overline{x}_2 + \overline{x}_3 + \overline{x}_4)(x_2 + x_4)(x_1 + x_3)$

4.4. SOP form: $f = \overline{x}_2 \overline{x}_3 + \overline{x}_2 \overline{x}_4 + x_2 x_3 x_4$
POS form: $f = (\overline{x}_2 + x_3)(x_2 + \overline{x}_3 + \overline{x}_4)(\overline{x}_2 + x_4)$

4.5. SOP form: $f = \overline{x}_3 \overline{x}_5 + \overline{x}_3 x_4 + x_2 x_4 \overline{x}_5 + \overline{x}_1 x_3 \overline{x}_4 x_5 + x_1 x_2 \overline{x}_4 x_5$
POS form: $f = (\overline{x}_3 + x_4 + x_5)(\overline{x}_3 + \overline{x}_4 + \overline{x}_5)(x_2 + \overline{x}_3 + \overline{x}_4)(x_1 + x_3 + x_4 + \overline{x}_5)(\overline{x}_1 + x_2 + x_4 + \overline{x}_5)$

4.6. SOP form: $f = \overline{x}_2 x_3 + \overline{x}_1 x_5 + \overline{x}_1 x_3 + \overline{x}_3 \overline{x}_4 + \overline{x}_2 x_5$
POS form: $f = (\overline{x}_1 + \overline{x}_2 + \overline{x}_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_4)(x_3 + \overline{x}_4 + x_5)$

4.7. SOP form: $f = x_3 \overline{x}_4 \overline{x}_5 + \overline{x}_3 \overline{x}_4 x_5 + x_1 x_4 x_5 + x_1 x_2 x_4 + x_3 x_4 x_5 + \overline{x}_2 x_3 x_4 + x_2 \overline{x}_3 x_4 \overline{x}_5$
POS form: $f = (x_3 + x_4 + x_5)(\overline{x}_3 + x_4 + \overline{x}_5)(x_1 + \overline{x}_2 + \overline{x}_3 + \overline{x}_4 + x_5)$

4.8. $f = \sum m(0, 7)$
$f = \sum m(1, 6)$
$f = \sum m(2, 5)$
$f = \sum m(0, 1, 6)$
$f = \sum m(0, 2, 5)$
etc.

4.9. $f = x_1 x_2 x_3 + x_1 x_2 x_4 + x_1 x_3 x_4 + x_2 x_3 x_4$

4.10. SOP form: $f = x_1 x_2 \overline{x}_3 + x_1 \overline{x}_2 x_4 + x_1 x_3 \overline{x}_4 + \overline{x}_1 x_2 x_3 + \overline{x}_1 x_3 x_4 + x_2 \overline{x}_3 x_4$
POS form: $f = (x_1 + x_2 + x_3)(x_1 + x_2 + x_4)(x_1 + x_3 + x_4)(x_2 + x_3 + x_4)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3 + \overline{x}_4)$
The POS form has lower cost.

4.11. The statement is false. As a counter example consider $f(x_1, x_2, x_3) = \sum m(0, 5, 7)$.
Then, the minimum-cost SOP form $f = x_1 x_3 + \overline{x}_1 \overline{x}_2 \overline{x}_3$ is unique.
But, there are two minimum-cost POS forms:
$f = (x_1 + \overline{x}_3)(\overline{x}_1 + x_3)(x_1 + \overline{x}_2)$ and
$f = (x_1 + \overline{x}_3)(\overline{x}_1 + x_3)(\overline{x}_2 + x_3)$

4.12. If each circuit is implemented separately:
$f = \overline{x}_1\overline{x}_4 + \overline{x}_1 x_2 x_3 + x_1\overline{x}_2 x_4$ Cost$= 15$
$g = \overline{x}_1\overline{x}_3\overline{x}_4 + \overline{x}_2 x_3\overline{x}_4 + x_1\overline{x}_3 x_4 + x_1 x_2 x_4$ Cost $= 21$

In a combined circuit:
$f = \overline{x}_2 x_3\overline{x}_4 + \overline{x}_1\overline{x}_3\overline{x}_4 + x_1\overline{x}_2\overline{x}_3 x_4 + \overline{x}_1 x_2 x_3$
$g = \overline{x}_2 x_3\overline{x}_4 + \overline{x}_1\overline{x}_3\overline{x}_4 + x_1\overline{x}_2\overline{x}_3 x_4 + x_1 x_2 x_4$
The first 3 product terms are shared, hence the total cost is 31.

4.13. If each circuit is implemented separately:
$f = \overline{x}_1 x_2 x_4 + x_2 x_4 x_5 + x_3\overline{x}_4\overline{x}_5 + \overline{x}_1\overline{x}_2\overline{x}_4 x_5$ Cost $= 22$
$g = \overline{x}_3\overline{x}_5 + \overline{x}_4\overline{x}_5 + \overline{x}_1\overline{x}_2\overline{x}_4 + \overline{x}_1 x_2 x_4 + x_2 x_4 x_5$ Cost $= 24$

In a combined circuit:
$f = \overline{x}_1 x_2 x_4 + x_2 x_4 x_5 + x_3\overline{x}_4\overline{x}_5 + \overline{x}_1\overline{x}_2\overline{x}_4 x_5$
$g = \overline{x}_1 x_2 x_4 + x_2 x_4 x_5 + x_3\overline{x}_4\overline{x}_5 + \overline{x}_1\overline{x}_2\overline{x}_4 x_5 + \overline{x}_3\overline{x}_5$

The first 4 product terms are shared, hence the total cost is 31. Note that in this implementation $f \subseteq g$, thus g can be realized as $g = f + \overline{x}_3\overline{x}_5$, in which case the total cost is lowered to 28.

4.14. $f = (x_3 \uparrow g) \uparrow ((g \uparrow g) \uparrow x_4)$ where $g = (x_1 \uparrow (x_2 \uparrow x_2)) \uparrow ((x_1 \uparrow x_1) \uparrow x_2)$

4.15. $\overline{f} = (((x_3 \downarrow x_3) \downarrow g) \downarrow ((g \downarrow g) \downarrow (x_4 \downarrow x_4))$, where
$g = ((x_1 \downarrow x_1) \downarrow x_2) \downarrow (x_1 \downarrow (x_2 \downarrow x_2))$. Then, $f = \overline{f} \downarrow \overline{f}$.

4.16. $f = (g \uparrow k) \uparrow ((g \uparrow g) \uparrow (k \uparrow k))$, where $g = (x_1 \uparrow x_1) \uparrow (x_2 \uparrow x_2) \uparrow (x_5 \uparrow x_5)$
and $k = (x_3 \uparrow (x_4 \uparrow x_4)) \uparrow ((x_3 \uparrow x_3) \uparrow x_4)$

4.17. $\overline{f} = (g \downarrow k) \downarrow ((g \downarrow g) \downarrow (k \downarrow k))$, where $g = x_1 \downarrow x_2 \downarrow x_5$
and $k = ((x_3 \downarrow x_3) \downarrow x_4) \downarrow (x_3 \downarrow (x_4 \downarrow x_4))$. Then, $f = \overline{f} \downarrow \overline{f}$.

4.18. $f = \overline{x}_1(x_2 + x_3)(x_4 + x_5) + x_1(\overline{x}_2 + x_3)(\overline{x}_4 + x_5)$

4.19. $f = x_1\overline{x}_3\overline{x}_4 + x_2\overline{x}_3\overline{x}_4 + x_1 x_3 x_4 + x_2 x_3 x_4 = (x_1 + x_2)\overline{x}_3\overline{x}_4 + (x_1 + x_2)x_3 x_4$
This requires 2 OR and 2 AND gates.

4.20. $f = x_1 \cdot g + \overline{x}_1 \cdot \overline{g}$, where $g = \overline{x}_3 x_4 + x_3\overline{x}_4$

4.21 $f = g \cdot h + \overline{g} \cdot \overline{h}$, where $g = x_1 x_2$ and $h = x_3 + x_4$

4.22. Let $D(0, 20)$ be 0 and $D(15, 26)$ be 1. Then decomposition yields:
$g = x_5(\overline{x}_1 + x_2)$
$f = (\overline{x}_3\overline{x}_4 + x_3 x_4)g + \overline{x}_3 x_4\overline{g} = x_3 x_4 g + \overline{x}_3\overline{x}_4 g + \overline{x}_3 x_4\overline{g}$
Cost $= 9 + 18 = 27$

The optimal SOP form is:
$f = \overline{x}_3 x_4 \overline{x}_5 + \overline{x}_1 x_3 x_4 x_5 + x_1 \overline{x}_2 \overline{x}_3 x_4 + \overline{x}_1 \overline{x}_3 \overline{x}_4 x_5 + x_2 \overline{x}_3 \overline{x}_4 x_5 + x_2 x_3 x_4 x_5$
Cost = $7 + 29 = 36$

4.23. Note that $X \# Y = X \cdot \overline{Y}$. Therefore,

$$\begin{aligned} (A \cdot B)\#C &= A \cdot B \cdot \overline{c} \\ (A\#C) \cdot (B\#C) &= A \cdot \overline{C} \cdot B \cdot \overline{C} \\ &= A \cdot B \cdot \overline{C} \end{aligned}$$

Similarly,

$$\begin{aligned} (A + B)\#C &= (A + B) \cdot \overline{C} \\ &= A \cdot \overline{C} + B \cdot \overline{C} \\ (A\#C) + (B\#C) &= A \cdot \overline{C} + B \cdot \overline{C} \end{aligned}$$

4.24. The initial cover is $C^0 = \{0000, 0011, 0100, 0101, 0111, 1000, 1001, 1111\}$.
Using the $*$-product get the prime implicants
P = {00x0, 0x00, x000, 010x, 01x1, 100x, x111}.
The minimum cover is $C_{minimum}$ = {00x0, 010x, 100x, x111}, which corresponds to $f = \overline{x}_1 \overline{x}_2 \overline{x}_4 + \overline{x}_1 x_2 \overline{x}_3 + x_1 \overline{x}_2 \overline{x}_3 + x_2 x_3 x_4$.

4.25. The initial cover is C^0 = {0x0x0, 110xx, x1101, 1001x, 11110, 01x10, 0x011}.
Using the $*$-product get the prime implicants
P = {0x0x0, xx01x, x1x10, 110xx, x10x0, 11x01, x1101}.
The minimum cover is $C_{mimimum}$ = {0x0x0, xx01x, x1x10, 110xx, x1101}, which corresponds to $f = \overline{x}_1 \overline{x}_3 \overline{x}_5 + \overline{x}_3 x_4 + x_2 x_4 \overline{x}_5 + x_1 x_2 \overline{x}_3 + x_2 x_3 \overline{x}_4 x_5$.

4.26. The initial cover is C^0 = {00x0, 100x, x010, 1111, 00x1, 011x}.
Using the $*$-product get the prime implicants P = {00xx, 0x1x, x00x, x0x0, x111}.
The minimum-cost cover is $C_{minimum}$ = {x00x, x0x0, x111}, which corresponds to $f = \overline{x}_2 \overline{x}_3 + \overline{x}_2 \overline{x}_4 + x_2 x_3 x_4$.

4.27. Expansion of $\overline{x}_1 \overline{x}_2 \overline{x}_3$ gives $\overline{x}_1$.
Expansion of $\overline{x}_1 \overline{x}_2 x_3$ gives $\overline{x}_1$.
Expansion of $\overline{x}_1 x_2 \overline{x}_3$ gives $\overline{x}_1$.
Expansion of $x_1 x_2 x_3$ gives $x_2 x_3$.
The set of prime implicants comprises $\overline{x}_1$ and $x_2 x_3$.

4.28. Expansion of $\overline{x}_1 x_2 \overline{x}_3 x_4$ gives $x_2 \overline{x}_3 x_4$ and $\overline{x}_1 x_2 x_4$.
Expansion of $x_1 x_2 \overline{x}_3 x_4$ gives $x_2 \overline{x}_3 x_4$.
Expansion of $x_1 x_2 x_3 \overline{x}_4$ gives $x_3 \overline{x}_4$.
Expansion of $\overline{x}_1 x_2 x_3$ gives $\overline{x}_1 x_3$.
Expansion of $\overline{x}_2 x_3$ gives $\overline{x}_2 x_3$.
The set of prime implicants comprises $x_2 \overline{x}_3 x_4$, $\overline{x}_1 x_2 x_4$, $x_3 \overline{x}_4$, $\overline{x}_1 x_3$, and $\overline{x}_2 x_3$.

4.29. Representing both functions in the form of Karnaugh map, it is easy to show that $f = g$. The minimum cost SOP expression is
$f = g = \overline{x}_2\overline{x}_3\overline{x}_5 + \overline{x}_2 x_3\overline{x}_4 + x_1 x_3 x_4 + x_1 x_2 x_4 x_5$.

4.30. The cost of the circuit in Figure P4.2 is 11 gates and 30 inputs, for a total of 41. The functions implemented by the circuit can also be realized as

$$\begin{aligned} f &= \overline{x}_1\overline{x}_2\overline{x}_4 + x_2\overline{x}_3\overline{x}_4 + \overline{x}_1 x_3 x_4 + x_1 x_4 \\ g &= \overline{x}_1\overline{x}_2\overline{x}_4 + x_2\overline{x}_3\overline{x}_4 + \overline{x}_1 x_3 x_4 + \overline{x}_2 x_4 + x_3\overline{x}_4 \end{aligned}$$

The first three product terms in f and g are the same; therefore, they can be shared. Then, the cost of implementing f and g is 8 gates and 24 inputs, for a total of 32.

4.31. The cost of the circuit in Figure P4.3 is 11 gates and 26 inputs, for a total of 37. The functions implemented by the circuit can also be realized as

$$\begin{aligned} f &= (\overline{x}_2 \uparrow x_4) \uparrow (\overline{x}_1 \uparrow x_2 \uparrow x_3) \uparrow (x_1 \uparrow \overline{x}_2 \uparrow x_3) \uparrow (\overline{x}_2 \uparrow \overline{x}_3) \\ g &= (\overline{x}_2 \uparrow x_4) \uparrow (\overline{x}_1 \uparrow x_2 \uparrow x_3) \uparrow (x_1 \uparrow \overline{x}_2 \uparrow x_3) \uparrow (\overline{x}_1 \uparrow \overline{x}_1) \end{aligned}$$

The first three NAND terms in f and g are the same; therefore, they can be shared. Then, the cost of implementing f and g is 7 gates and 20 inputs, for a total of 27.