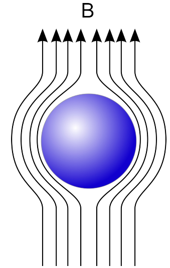
Superconducting qubits

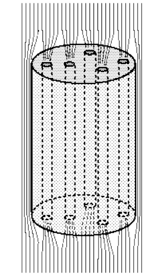
Superconductors: macroscopic properties

-- Infinite conductivity (at $\omega = 0$)

-- Meissner effect (perfect diamagnetism)



The expulsion of magnetic field costs energy. A strong enough field will penetrate the material, destroying superconductivity. In type I superconductors, this happens abruptly at some critical H_c . In type II superconductors, there is an intermediate regime, $H_{c1} < H < H_{c2}$, where field penetrates in the form of quantized vortices.



Superconducting materials

-- Conventional superconductors (described by the Bardeen-Cooper-Schrieffer theory)

Hg: $T_c = 4.15\ K$ Pb: $T_c = 7.19\ K$

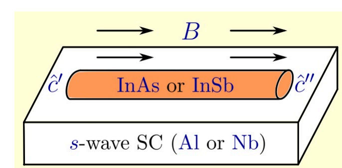
NbTi: $T_c = 10\ K, \quad H_{c2}(0) = 15\ T$ (used in magnets)

Al: $T_c = 1.2\ K, \quad H_c(0) = 0.01\ T$ (used in superconducting qubits)

-- Cuprates, e.g. $YBa_2Cu_3O_{7-\delta}$: $T_c = 95\ K, \quad H_{c2}(0) > 100\ T$ (used in high-field magnets)

-- Exotic: UPt_2Al_3 (ferromagnetic)

Sr_2RuO_4, "Majorana wires" (topological)



Some microscopic properties

-- The state of electrons in a superconductor is characterized by a superconducting phase φ , a real variable defined modulo 2π that varies in space and time

-- $\varphi(r)$ can vary significantly at distances $\gtrsim \xi$, where ξ is the coherence length. At distances much shorter than ξ , $\varphi(r)$ is almost constant.

In pure aluminium, ξ is about 1600 nm. It decreases in thin films and in the presence of defects. In qubit design, superconducting islands (connected to each other by Josephson junctions) are usually smaller than ξ .

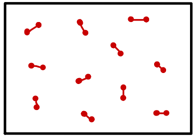
-- A small superconducting island is characterized by a wavefunction $\psi(\varphi)$

Elementary operators: $e^{\pm i\varphi}$, $\hat{N} = i\frac{\partial}{\partial\varphi}$ (the number of electron pairs minus some constant)

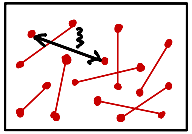
Electric charge of the island: $Q = -2e\,N$ (the constant is chosen such that *N* corresponds to the total charge)

Charge eigenstates: $|N\rangle = \int \psi_N(\varphi)\ |\varphi\rangle\, d\varphi$, $\psi_N(\varphi) = \frac{e^{-iN\varphi}}{\sqrt{2\pi}}$

Naive picture of Cooper pairs:



A more realistic picture:

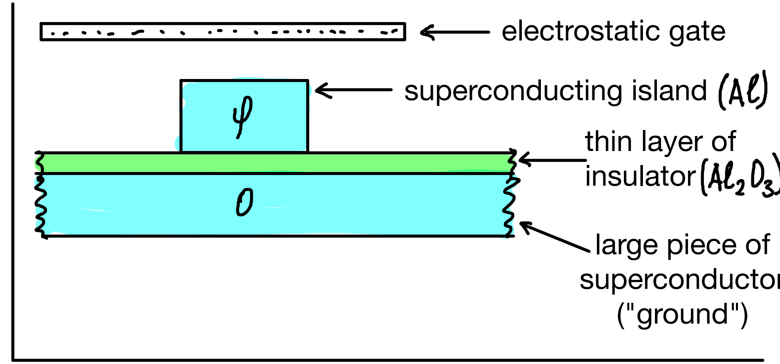


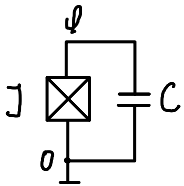
-- In addition, an island may contain Bogolubov quasiparticles, which are fermions and have excitation energy $\geqslant \Delta$ ($\Delta \approx 2K$ in aluminium). They contribute to *N* indirectly:

$$\psi(2\pi) = (-1)^{N_{qp}}\psi(0) \iff N \equiv \frac{N_{qp}}{2} \pmod 1$$

The simplest superconducting circuit: Cooper pair box

Superconducting qubits must be operated at very low temperature, $T \ll \Delta$, such that there are no Bogolyubov quasiparticles. In practice, $T \sim 20\,mK$.





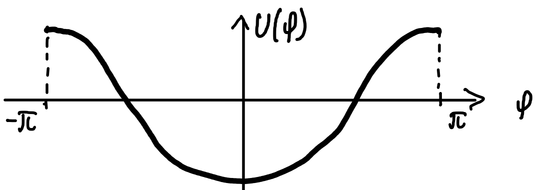
$$\boxed{H = \underbrace{V(\varphi)}_{\text{Josephson term}} + \underbrace{\frac{(2e)^2}{2C}(\hat{N} - N_g)^2}_{\text{Coulomb term}}}$$

Josephson term : $\boxed{V(\varphi) = -J\cos\varphi}$

Coulomb term

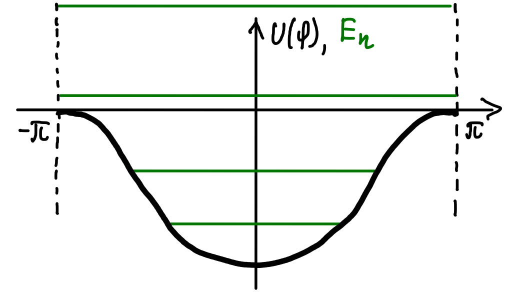
$$\cos\varphi = \frac{1}{2}\Big(\underbrace{e^{i\varphi}}_{\substack{\text{electron pair}\\ \text{tunnels out}}} + \underbrace{e^{-i\varphi}}_{\substack{\text{electron pair}\\ \text{tunnels in}}}\Big), \qquad e^{\pm i\varphi}|N\rangle = |N \mp 1\rangle$$

$$E_{\text{Coulomb}} = \frac{Q^2}{2C} + \underbrace{V_g}_{\substack{\text{external}\\ \text{potential}}} Q = \frac{(Q - V_g C)^2}{2C} - \underbrace{\frac{C V_g^2}{2}}_{\substack{\text{constant term}\\ \text{is neglected}}}$$



Properties of the Cooper pair box Hamiltonian

$$H = U(\varphi) + \frac{(2e)^2}{2C}\left(i\frac{\partial}{\partial\varphi} - N_g\right)^2, \qquad U(\varphi) = -J\cos\varphi$$



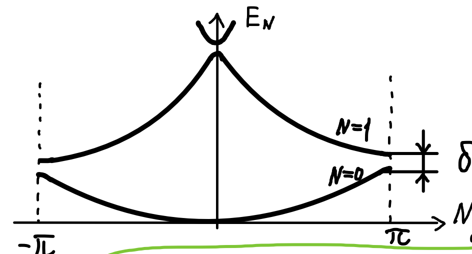
Parameters with dimension of energy:

Josephson energy $E_J = J$

Coulomb energy $E_c = \dfrac{e^2}{2C}$

Coulomb-dominated regime: $E_c \gg E_J$

$$E_N \approx 4E_c(N - N_g)^2$$



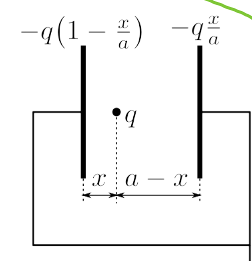
$$\hat{U} = -\frac{J}{2}\left(e^{i\varphi} + e^{-i\varphi}\right)$$

$$\delta E = 2\left|\langle 1|\hat{U}|0\rangle\right| = J$$

Technical problem:

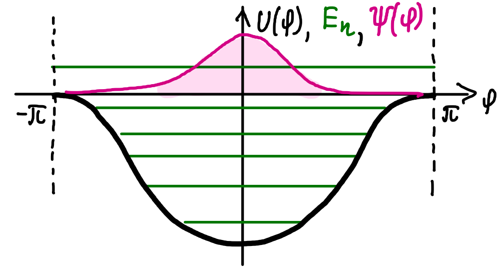
The offset charge N_g of a manufactured device is unpredictable. It can be tuned by electrical gating but drifts over time.
This is because some uncontrolled charges (or, at least, dipoles) are present in the system and even move.

Classical electrostatics problem: a charge inside a capacitor. The induced charges on the capacitor plates depend on the position of the inserted charge q.



Josephson-dominated regime: $E_J \gg E_C$

$$H = -\underbrace{J}_{E_J}\cos\varphi + \underbrace{\frac{(2e)^2}{2C}}_{4E_C}\left(i\frac{\partial}{\partial\varphi} - N_g\right)^2$$



-- The wavefunctions of low-energy states are concentrated away from $\varphi = \pi$

$\Rightarrow$ energy levels are insensitive to N_g

because it can be eliminated by a gauge transformation $\psi \to \tilde{\psi}$

$$e^{iN_g\varphi}\left(i\frac{\partial}{\partial\varphi} - N_g\right)\psi(\varphi) = i\frac{\partial}{\partial\varphi}\underbrace{\left(e^{iN_g\varphi}\psi(\varphi)\right)}_{\tilde{\psi}(\varphi)}$$

-- The Hamiltonian can be approximated by a harmonic oscillator: $\cos\varphi \approx 1 - \frac{\varphi^2}{2}$

$$H \approx \frac{E_J}{2}\varphi^2 + 4E_C\left(i\frac{\partial}{\partial\varphi}\right)^2 + const,$$

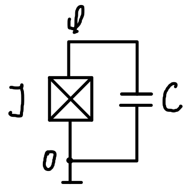
energy levels: $E_n \approx \omega_0\left(n + \frac{1}{2}\right)$

Josephson plasma frequency: $\omega_0 = \sqrt{8E_JE_C} = 2e\sqrt{\frac{J}{C}}$ (independent of the contact area)

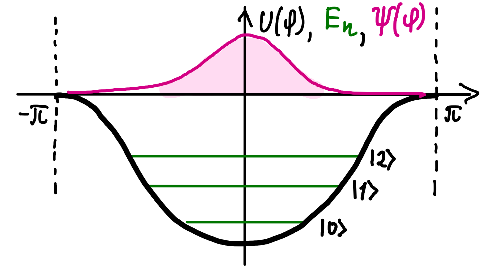
Typical value: $\hbar\omega_0 \sim 1\,K$, $\frac{\omega_0}{2\pi} \sim 20\,GHz = 2\cdot 10^{10}\,s^{-1}$

Some qubit designs

Slightly anharmonic oscillator (transmon)



$$H = -\underbrace{J}_{E_J}\cos\varphi + \underbrace{\frac{(2e)^2}{2C}}_{4E_C}\left(i\frac{\partial}{\partial\varphi} - N_g\right)^2,$$

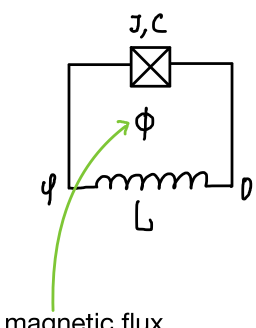


$E_2 - E_1 < E_1 - E_0$

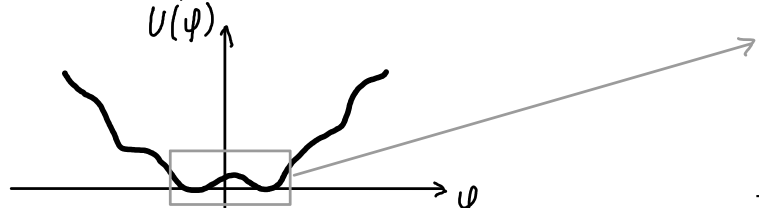
In the Josephson-dominated regime, $\boxed{E_J \gg E_C}$, the eigenvalue sensitivity to N_g is exponentially suppressed

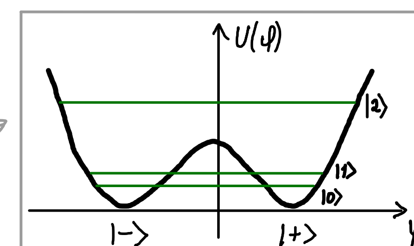
(The transmon includes a separate capacitor in addition to the intrinsic capacitance of the Josephson junction. This reduces the oscillation frequency down to ~ 5 GHz.)

Double-well potential (flux qubit, or fluxonium)



$$H = \underbrace{\frac{\varphi^2}{2(2e)^2 L} - J\cos\left(\varphi - \frac{2\pi\Phi}{\Phi_0}\right)}_{U(\varphi)} + \frac{(2e)^2}{2C}\left(i\frac{\partial}{\partial\varphi} - N_g\right)^2$$

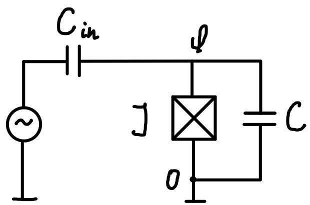




The potential is symmetric if $\Phi = \frac{1}{2}\Phi_0$

Single-qubit gates (for the transmon)

Applying a microwave signal



$$H(t) = -J\cos\varphi + \frac{(2e)^2}{2(C+C_{in})}\hat{N}^2 + V(t)\,\frac{2e\,C_{in}}{C+C_{in}}\hat{N},$$

$$[\hat{N}, \varphi] = i, \qquad \cos\varphi \approx 1 - \frac{\varphi^2}{2} + \underbrace{\frac{\varphi^4}{24}}_{\text{anharmonism}} - \cdots$$

Near-resonance signal:

$$V(t) \propto \cos(\omega t)$$

$$\omega \approx \omega_{10} := E_1 - E_0$$

First, let us consider a harmonic oscillator and ignore all constant terms and factors:

$$H(t) \approx \omega_{osc}\, b^+ b + 2v\cos(\omega t)(b + b^+)$$

The eigenstates $|n\rangle$ are close to the harmonic oscillator eigenstates, but the resonance occurs only for the transition $|0\rangle \leftrightarrow |1\rangle$. Therefore, we may project the Hamiltonian on the corresponding two-dimensional subspace. (The role of resonance will become clear after passing to a rotating frame.)

$$\boxed{H_{eff}(t) = \omega_{10}\,|1\rangle\langle 1| + 2v\cos(\omega t)\big(|0\rangle\langle 1| + |1\rangle\langle 0|\big)}$$

Passing to a rotating frame (similar to the interaction representation)

$$H(t) = \omega_{10} |1\rangle\langle 1| + v\left(e^{i\omega t} + e^{-i\omega t}\right)\left(|0\rangle\langle 1| + |1\rangle\langle 0|\right)$$

We aim to simplify the problem, assuming that $|\omega - \omega_{10}| \ll \omega$

Evolution operator in the rotating frame: $\boxed{\breve{U} := e^{iH_0 t} U, \quad \text{where} \quad H_0 = \omega |1\rangle\langle 1|}$

$$\frac{dU}{dt} = -iHU \Rightarrow \frac{d\breve{U}}{dt} = -i\breve{H}\breve{U},$$

where $\breve{H} = i\frac{d\breve{U}}{dt}\breve{U}^{-1} = i\left(\frac{de^{iH_0 t}}{dt}U + e^{iH_0}\frac{dU}{dt}\right)U^{-1}e^{-iH_0 t} = \boxed{e^{iH_0 t}(-H_0 + H)e^{-iH_0 t}}$

$$\breve{H} = (\omega_{10} - \omega)|1\rangle\langle 1| + v\left(\left(1 + \cancel{e^{-2i\omega t}}\right)|0\rangle\langle 1| + \left(\cancel{e^{2i\omega t}} + 1\right)|1\rangle\langle 0|\right)$$

oscillating terms may be neglected because they average out, whereas the effect of constant terms accumulates

$$\boxed{\breve{H} \approx (\omega_{10} - \omega)|1\rangle\langle 1| + v\left(|0\rangle\langle 1| + |1\rangle\langle 0|\right)}$$

If $|\omega_{10} - \omega|$ is greater than v, then the effect of the microwave signal is small. Similarly, higher oscillator levels can be neglected if $|\omega_{21} - \omega| \gg v$.

Rabi oscillations exactly at the resonance $(\omega = \omega_{10})$

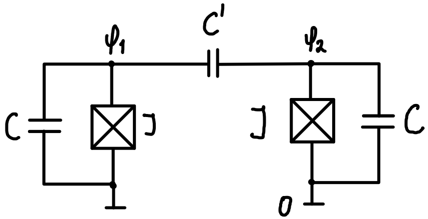
$$\breve{H} = v\left(|0\rangle\langle 1| + |1\rangle\langle 0|\right) = v\,\sigma^x \quad \Rightarrow \quad \breve{U}(t) = e^{-i\breve{H}t} = \begin{pmatrix} \cos(vt) & -i\sin(vt) \\ -i\sin(vt) & \cos(vt) \end{pmatrix}$$

By applying microwave pulses of various durations, one can implement all such gates.

Rotations about the *y* axis are implemented by phase-shifted pulses:

$$V(t) \propto \sin\omega t = \cos\left(\omega t - \frac{\pi}{2}\right)$$

Toward the implementation of two-qubit gates: coupling two qubits



$$H = -J\cos\varphi_1 - J\cos\varphi_2 + \frac{1}{2(2e)^2}\left(\hat{N}_1, \hat{N}_2\right)\underbrace{\begin{pmatrix} C+C' & -C' \\ -C' & C+C' \end{pmatrix}}_{\text{capacitance matrix}}^{-1}\begin{pmatrix} \hat{N}_1 \\ \hat{N}_2 \end{pmatrix}$$

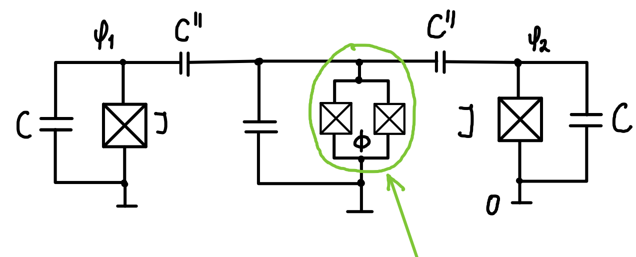
$$\hat{N}_1 = i\frac{\partial}{\partial\varphi_1}, \quad \hat{N}_2 = i\frac{\partial}{\partial\varphi_2}$$

Hamiltonian projected onto the qubit subspace

$$H_{eff} = -\frac{\omega_{10}}{2}\left(\sigma_1^z + \sigma_2^z\right) + \mathcal{V}\,\sigma_1^x\sigma_2^x$$

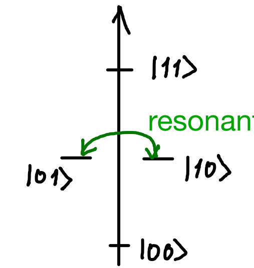
Tunable coupler

The effective coupling $\mathcal{V}$ is tuned to 0 in the idle state. It can be turned on for a specific time to perform a gate



Superconducting quantum interference device (SQUID) with a variable magnetic flux Φ works as a tunable Josephson junction

Passing to a rotating frame and ignoring non-resonant terms



$$H = \underbrace{-\frac{\omega_{10}}{2}\left(\sigma_1^z + \sigma_2^z\right)}_{H_0} + v\,\underbrace{\sigma_1^x \sigma_2^x}_{|00\rangle\langle 11| + |01\rangle\langle 10| + |10\rangle\langle 01| + |11\rangle\langle 00|}$$

$$\breve{H} = e^{iH_0 t}(H - H_0)\,e^{-iH_0 t} = v\Big(\cancel{e^{-2i\omega_{10}t}|00\rangle\langle 11|} + \underbrace{|01\rangle\langle 10| + |10\rangle\langle 01|}_{\text{keeping only resonant matrix elements}} + \cancel{e^{2i\omega_{10}t}|11\rangle\langle 00|}\Big)$$

$$\boxed{\breve{H} \approx v\left(|01\rangle\langle 10| + |10\rangle\langle 01|\right)}$$

The resulting gate

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(vt) & -i\sin(vt) & 0 \\ 0 & -i\sin(vt) & \cos(vt) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

State of the art

Google's Sycamore quantum processor:

-- 53 physical qubits (error correction has not been implemented yet)

-- effective coupling during two-qubit gates: $\frac{V_{eff}}{2\pi} \approx -20\ MHz$

$\Rightarrow$ time to implement iSWAP = $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ is 12.5 ns

-- two-qubit gate error $\sim 0.5 - 1\%$