MAT168 HW1

Andrew Jowe

October 9, 2022

Problem Set

Question 1

Part 1

$\zeta = 6x_1 + 8x_2 + 5x_3 + 9x_4$
$w_1 = 8 - 2x_1 - x_2 - x_3 - 3x_4$
$w_2 = 2 - x_1 - 3x_2 - x_3 - 2x_4$
$x_1, x_2, x_3, x_4, w_1, w_2 \geq 0$
This problem is feasible since all constraints are met when $x_1, x_2, x_3, x_4 = 0$.
When picking the highest bound, the ratio is $\frac{3}{2}$. Since the ratio is positive, this problem is bounded.

Part 2

The objective function is unbounded since x_j ($j \in N$ where N is the set of non-basic indexes) can infinitely increase without violating any constraints. Since the objective function is unbounded, it is also feasible.

Part 3

We have to solve the auxiliary problem.
$\zeta = -x_0$
$w_1 = -1 - 2x_1 - x_2 - x_3 - 3x_4 + x_0$
$w_2 = 3 - x_1 - 3x_2 - x_3 - 2x_4 + x_0$
$x_0, x_1, x_2, x_3, x_4, w_1, w_2 \geq 0$
x_0 enters and w_1 leaves.
$\zeta = -(w_1 + 1 + 2x_1 + x_2 + x_3 + 3x_4)$
$\rightarrow \zeta = -w_1 - 1 - 2x_1 - x_2 - x_3 - 3x_4$
$x_0 = w_1 + 1 + 2x_1 + x_2 + x_3 + 3x_4$
$w_2 = 3 - x_1 - 3x_2 - x_3 - 2x_4 + x_0$
$x_0, x_1, x_2, x_3, x_4, w_1, w_2 \geq 0$
This is a feasible dictionary, so we can move on to simplex method.
A feasible solution is:
$x_0, x_1, x_2, x_3, x_4, w_1, w_2 = 0$
Therefore, this problem is feasible.
When picking the highest bound, the ratio is $\frac{3}{3}$. Since the ratio is positive, this problem is bounded.

Question 2a

Part 1

$\zeta = 6x_1 + 8x_2 + 5x_3 + 9x_4$
$w_1 = 5 - 2x_1 - x_2 - x_3 - 3x_4$
$w_2 = 3 - x_1 - 3x_2 - x_3 - 2x_4$
$x_1, x_2, x_3, x_4, w_1, w_2 \geq 0$
By eyeballing the objective function and constraints, no ratio is negative, so the objective function is not unbounded on this iteration.
Start with the feasible solution at $x_1, x_2, x_3, x_4, w_1, w_2 = 0$
Attempt to replace non-basic variable x_4.
$w_1 = 5 - 3x_4 \rightarrow x_4 = \frac{5}{3}$
$w_2 = 3 - 2x_4 \rightarrow x_4 = \frac{3}{2}$
$x_1, x_2, x_3, w_1, w_2 = 0$
$x_4 = \frac{3}{2}$
x_4 enters and w_2 leaves.
Rewrite objective function.
$\zeta = 6x_1 + 8x_2 + 5x_3 + 9(3 - x_1 - 3x_2 - x_3 - w_2)$
$\rightarrow \zeta = 6x_1 + 8x_2 + 5x_3 + 27 - 9x_1 - 27x_2 - 9x_3 - 9w_2$
$\rightarrow \zeta = -3x_1 - 19x_2 - 4x_3 - 9w_2 + 27$
Rewrite constraints.
$w_1 = 5 - 2x_1 - x_2 - x_3 - 3x_4$
$2x_4 = 3 - x_1 - 3x_2 - x_3 - w_2$
$x_1, x_2, x_3, x_4, w_1, w_2 \geq 0$
Decreasing x_1, x_2, x_3, w_2 would violate a constraint. Increasing x_1, x_2, x_3, w_2 would only decrease the objective function. Therefore, the optimal solution is at:
$x_1, x_2, x_3 = 0$
$x_4 = \frac{3}{2}$

Part 2

We have to do the auxiliary problem since there exists b_i ($i \in B$ where B is is the set of basic indexes) such that $b_i < 0$.
$\zeta = -x_0$
$-x_1 - x_2 - x_0 \leq -3$
$-x_1 + x_2 - x_0 \leq -1$
$x_1 + 2x_2 - x_0 \leq 4$
$x_1, x_2, x_0 \geq 0$
Rewrite constraints.
$x_3 = -3 + x_1 + x_2 + x_0$
$x_4 = 4 - x_1 - 2x_2 + x_0$
$x_3, x_4 \geq 0$
x_0 enters and x_3 leaves.
$\zeta = -(x_3 + 3 - x_1 - x_2)$
$\rightarrow \zeta = -x_3 - 3 + x_1 + x_2$
$x_0 = 3 + x_3 - x_1 - x_2$
$x_4 = 4 - x_1 - 2x_2 + x_0$
$x_1, x_2, x_0, x_3, x_4 \geq 0$
Start with the solution $x_1, x_2, x_0, x_3, x_4 = 0$.
Attempt to replace non-basic variable x_1.
$x_1 \leq 3$
$x_1 \leq 4$
x_1 enters and x_0 leaves.
New solution is:
$x_1 = 3$
$x_2, x_0, x_3, x_4 = 0$
Rewrite constraints.

$x_1 = 3 + x_3 - x_0 - x_2$
$x_4 = 4 - (3 + x_3 - x_0 - x_2) - 2x_2 + x_0$
$\rightarrow x_4 = 4 - 3 - x_3 + x_0 + x_2 - 2x_2 + x_0$
$\rightarrow x_4 = 1 - x_3 - x_2 + 2x_0$
$x_1, x_2, x_0, x_3, x_4 \geq 0$
Rewrite objective function.
$\zeta = -x_3 - 3 + (3 + x_3 - x_0 - x_2) + x_2$
$\rightarrow \zeta = 0 - x_0$
Do simplex.
$\zeta = 3 + x_3 - x_2 + 3x_2$
$\rightarrow \zeta = 3 + x_3 + 2x_2$
$x_1 = 3 + x_3 - x_2$
$x_4 = 1 - x_3 - x_2$
$x_1, x_2, x_0, x_3, x_4 \geq 0$
Attempt to replace non-basic variable x_3.
$-x_3 + 3 \leq 3 \rightarrow x_3 \geq 0$
$x_3 \leq 1$
x_3 enters and x_4 leaves.
New solution is:
$x_1 = 3$
$x_3 = 1$
$x_2, x_0, x_4 = 0$
Rewrite constraints:
$x_1 = 3 + x_3 - x_2$
$x_3 = 1 - x_2 - x_4$
$x_1, x_2, x_0, x_3, x_4 \geq 0$
Rewrite objective function:
$\zeta = 3 + 1 - x_2 - x_4 + 2x_2$
$\rightarrow \zeta = 4 - x_2 - x_4 + 2x_2$
Attempt to replace non-basic variable x_2.
$x_2 + 3 - 1 \leq 3 \rightarrow x_2 \leq 1$
$x_2 + 1 \leq 1 \rightarrow x_2 \leq 0$
We cannot increase x_2 since that would violate our constraint. Increasing any other variables would decrease our objective function. The most optimal solution is:
$x_1 = 3$
$x_2 = 0$

Question 3

Question 4

Let $\mathbf{x} = \{x\}_{i=1}^{6}$ = amount imported of cotton, thread, glue, shoes, jumpsuits, hats, and $\mathbf{c} = \{c\}_{i=1}^{6}$ be the cost of importing each of these raw materials and completed items.
We are producing some items in our factory. Let $\mathbf{q} = \{q\}_{i=4}^{6}$ = amount of shoes, jumpsuits, hats produced.
Manufacturing consumes materials. Let matrix $A = [a_{i,j}]$ where $\{j\}_1^3$ = cotton, thread, glue needed for each product and $\{i\}_1^3$ = recipe for shoes, jumpsuits, and hats.
We want to minimize our total cost (import and production costs). Our objective is to minimize $\mathbf{c} + \mathbf{q} * d$ or to maximize $-\mathbf{c} - \mathbf{q} * d$. We can write our objective function as:
$\zeta = -\mathbf{c} - \mathbf{q} * d$
Let the colonists' requirements (the minimum amount of clothing needed) be some constraints. The constraints are as shown:
$x_4 + q_4 \geq 2n$
$x_5 + q_5 \geq 3n$
$x_6 + q_6 \geq n$
Manufacturing is limited to the materials imported. The additional constraints are as shown:
$q_4 * a_{1,1} + q_5 * a_{1,2} + q_6 * a_{1,3} \leq x_1$
$q_4 * a_{2,1} + q_5 * a_{2,2} + q_6 * a_{2,3} \leq x_2$
$q_4 * a_{3,1} + q_5 * a_{3,2} + q_6 * a_{3,3} \leq x_3$
In general, whenever n increases, both $\mathbf{x}$ and $\mathbf{c}$ will increase too. Whenever n decreases, both $\mathbf{x}$ and $\mathbf{c}$ will decrease too. This makes sense because more clothing is needed when more people are in the colony, thus increasing the cost. On the other side, less clothing is needed when less people are in the colony, thus decreasing the cost.

Collaboration

Academic Integrity

On my personal integrity as a student and member of the UCD community, I have not given, nor received and unauthorized assistance on this assignment.
Signature: Andrew Jowe