

# MAT168 HW1

Andrew Jowe

October 10, 2022

## Problem Set

### Question 1

#### Part 1

$$\zeta = 6x_1 + 8x_2 + 5x_3 + 9x_4$$

$$w_1 = 8 - 2x_1 - x_2 - x_3 - 3x_4$$

$$w_2 = 2 - x_1 - 3x_2 - x_3 - 2x_4$$

$$x_1, x_2, x_3, x_4, w_1, w_2 \geq 0$$

This problem is feasible since all constraints are met when  $x_1, x_2, x_3, x_4 = 0$ .

When picking the highest bound, the ratio is  $\frac{3}{2}$ . Since the ratio is positive, this problem is bounded.

#### Part 2

The objective function is unbounded since  $x_j$  ( $j \in N$  where  $N$  is the set of non-basic indexes) can infinitely increase without violating any constraints. Since the objective function is unbounded, it is also feasible.

## Part 3

We have to solve the auxiliary problem.

$$\zeta = -x_0$$

$$w_1 = -1 - 2x_1 - x_2 - x_3 - 3x_4 + x_0$$

$$w_2 = 3 - x_1 - 3x_2 - x_3 - 2x_4 + x_0$$

$$x_0, x_1, x_2, x_3, x_4, w_1, w_2 \geq 0$$

$x_0$  enters and  $w_1$  leaves.

$$\zeta = -(w_1 + 1 + 2x_1 + x_2 + x_3 + 3x_4)$$

$$\rightarrow \zeta = -w_1 - 1 - 2x_1 - x_2 - x_3 - 3x_4$$

$$x_0 = w_1 + 1 + 2x_1 + x_2 + x_3 + 3x_4$$

$$w_2 = 3 - x_1 - 3x_2 - x_3 - 2x_4 + x_0$$

$$x_0, x_1, x_2, x_3, x_4, w_1, w_2 \geq 0$$

This is a feasible dictionary, so we can move on to simplex.

A feasible solution is:

$$x_0, x_1, x_2, x_3, x_4, w_1, w_2 = 0$$

Therefore, this problem is feasible.

When picking the highest bound, the ratio is  $\frac{3}{3}$ . Since the ratio is positive, this problem is bounded.

## Question 2a

### Part 1

$$\zeta = 6x_1 + 8x_2 + 5x_3 + 9x_4$$

$$w_1 = 5 - 2x_1 - x_2 - x_3 - 3x_4$$

$$w_2 = 3 - x_1 - 3x_2 - x_3 - 2x_4$$

$$x_1, x_2, x_3, x_4, w_1, w_2 \geq 0$$

Start with the feasible solution at:

$$x_1, x_2, x_3, x_4, w_1, w_2 = 0$$

Attempt to replace non-basic variable  $x_4$ .

$$w_1 = 5 - 3x_4 \rightarrow x_4 = \frac{5}{3}$$

$$w_2 = 3 - 2x_4 \rightarrow x_4 = \frac{3}{2}$$

New solution:

$$x_1, x_2, x_3, w_1, w_2 = 0$$

$$x_4 = \frac{3}{2}$$

$x_4$  enters and  $w_2$  leaves.

Rewrite constraints.

$$w_1 = 5 - 2x_1 - x_2 - x_3 - 3x_4$$

$$2x_4 = 3 - x_1 - 3x_2 - x_3 - w_2$$

$$\rightarrow x_4 = \frac{3}{2} - \frac{1}{2}x_1 - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}w_2$$

$$x_1, x_2, x_3, x_4, w_1, w_2 \geq 0$$

Rewrite objective function.

$$\zeta = 6x_1 + 8x_2 + 5x_3 + 9\left(\frac{3}{2} - \frac{1}{2}x_1 - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}w_2\right)$$

$$\rightarrow \zeta = 6x_1 + 8x_2 + 5x_3 + \frac{27}{2} - \frac{9}{2}x_1 - \frac{27}{2}x_2 - \frac{9}{2}x_3 - \frac{9}{2}w_2$$

$$\rightarrow \zeta = \frac{27}{2} + \frac{3}{2}x_1 - \frac{11}{2}x_2 + \frac{1}{2}x_3 - \frac{9}{2}w_2$$

Attempt to replace non-basic variable  $x_1$ .

$$2x_1 = 5 - 3 * \frac{3}{2} \rightarrow x_1 = \frac{1}{2} * \frac{1}{2}$$

$$\frac{1}{2}x_1 = \frac{3}{2} - \frac{3}{2} \rightarrow x_1 = 0$$

Increasing  $x_1$  will violate a constraint. Attempt to replace non-basic variable  $x_3$ .

$$x_3 = 5 - 3 * \frac{3}{2}$$

$$\frac{1}{2}x_3 = \frac{3}{2} - \frac{3}{2} \rightarrow x_3 = 0$$

Increasing  $x_1, x_3$  will violate a constraint. Increasing  $x_2, w_2$  decreases the objective function. Therefore, the optimal solution is:

$$x_1, x_2, x_3 = 0$$

$$x_4 = \frac{3}{2}$$

## Part 2

Auxiliary problem.

$$\zeta = -x_0$$

$$-x_1 - x_2 - x_0 \leq -3$$

$$-x_1 + x_2 - x_0 \leq -1$$

$$x_1 + 2x_2 - x_0 \leq 4$$

$$x_1, x_2, x_0 \geq 0$$

Rewrite constraints.

$$x_3 = -3 + x_1 + x_2 + x_0$$

$$x_4 = -1 + x_1 - x_2 + x_0$$

$$x_5 = 4 - x_1 - 2x_2 + x_0$$

$$x_3, x_4, x_5 \geq 0$$

$x_0$  enters and  $x_3$  leaves.

$$\zeta = -(x_3 + 3 - x_1 - x_2)$$

$$\rightarrow \zeta = -x_3 - 3 + x_1 + x_2$$

$$x_0 = 3 + x_3 - x_1 - x_2$$

$$x_4 = -1 + x_1 - x_2 + (3 + x_3 - x_1 - x_2)$$

$$\rightarrow x_4 = 2 - 2x_2 + x_3$$

$$x_5 = 4 - x_1 - 2x_2 + (3 + x_3 - x_1 - x_2)$$

$$\rightarrow x_5 = 1 - 2x_1 - 3x_2 + x_3$$

$$x_1, x_2, x_0, x_3, x_4, x_5 \geq 0$$

Start with the solution at:

$$x_1, x_2, x_0, x_3, x_4, x_5 = 0$$

Attempt to replace non-basic variable  $x_1$ .

$$x_1 \leq 3$$

$$x_1 \leq \frac{1}{2}$$

$x_1$  enters and  $x_5$  leaves.

New solution is:

$$x_1 = \frac{1}{2}$$

$$x_2, x_0, x_3, x_5 = 0$$

Rewrite constraints.

$$x_0 = 3 + x_3 - \left(\frac{1}{2} - \frac{3}{2}x_2 + \frac{1}{2}x_3 - \frac{1}{2}x_5\right) - x_2$$

$$\rightarrow x_0 = \frac{5}{2} + \frac{1}{2}x_2 + \frac{1}{2}x_3 + \frac{1}{2}x_5$$

$$x_4 = 2 - 2x_2 + x_3$$

$$x_1 = \frac{1}{2} - \frac{3}{2}x_2 + \frac{1}{2}x_3 - \frac{1}{2}x_5$$

Rewrite objective function.

$$\zeta = -x_3 - 3 + \left(\frac{1}{2} - \frac{3}{2}x_2 + \frac{1}{2}x_3 - \frac{1}{2}x_5\right) + x_2$$

$$\rightarrow \zeta = -\frac{5}{2} - \frac{1}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_5$$

Dictionary is now feasible and  $-x_0$  is maximized. Do simplex.

$$\zeta = x_1 + 3x_2$$

$$\rightarrow \zeta = \frac{1}{2} + \frac{3}{2}x_2 + \frac{1}{2}x_3 - \frac{1}{2}x_5$$

$$x_0 = \frac{5}{2} + \frac{1}{2}x_2 + \frac{1}{2}x_3 + \frac{1}{2}x_5$$

$$x_4 = 2 - 2x_2 + x_3$$

$$x_1 = \frac{1}{2} - \frac{3}{2}x_2 + \frac{1}{2}x_3 - \frac{1}{2}x_5$$

## Question 3

## Question 4

Let  $\mathbf{x} = \{x\}_{i=1}^6$  be the amount imported of cotton, thread, glue, shoes, jumpsuits, hats, and  $\mathbf{c} = \{c\}_{i=1}^6$  be the cost of importing each of these raw materials and completed items.

We are producing some items in our factory. Let  $\mathbf{q} = \{q\}_{i=4}^6$  be the amount of shoes, jumpsuits, hats produced.

Manufacturing consumes materials. Let matrix  $A = [a_{i,j}]$  where  $\{j\}_1^3$  be the cotton, thread, glue needed for each product and  $\{i\}_1^3$  be the recipe for shoes, jumpsuits, and hats.

We want to minimize our total cost (import and production costs). Our objective is to minimize  $\mathbf{c} + \mathbf{q} * d$  or to maximize  $-\mathbf{c} - \mathbf{q} * d$ . We can write our objective function as:

$$\zeta = -\mathbf{c} - \mathbf{q} * d$$

Let the colonists' requirements (the minimum amount of clothing needed) be some constraints. The constraints are as shown:

$$x_4 + q_4 \geq 2n$$

$$x_5 + q_5 \geq 3n$$

$$x_6 + q_6 \geq n$$

Manufacturing is limited to the materials imported. The additional constraints are as shown:

$$q_4 * a_{1,1} + q_5 * a_{1,2} + q_6 * a_{1,3} \leq x_1$$

$$q_4 * a_{2,1} + q_5 * a_{2,2} + q_6 * a_{2,3} \leq x_2$$

$$q_4 * a_{3,1} + q_5 * a_{3,2} + q_6 * a_{3,3} \leq x_3$$

In general, whenever  $n$  increases, both  $\mathbf{x}$  and  $\mathbf{c}$  will increase too. Whenever  $n$  decreases, both  $\mathbf{x}$  and  $\mathbf{c}$  will decrease too. This makes sense because more clothing is needed when more people are in the colony, thus increasing the cost. On the other side,

less clothing is needed when less people are in the colony, thus decreasing the cost.



# Collaboration

# Academic Integrity

On my personal integrity as a student and member of the UCD community, I have not given, nor received and unauthorized assistance on this assignment.

Signature: Andrew Jowe