MAT168 HW2

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(1)

Original Problem

$$\zeta = 2x_1 - 6x_2 + 0x_3$$
$$x_4 = -2 + x_1 + x_2 + x_3$$
$$x_5 = 1 - 2x_1 + x_2 - x_3$$

Given

$$N = \begin{bmatrix} -1 & -1 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A = [NB] = \begin{bmatrix} -1 & -1 & -1 & 1 & 0 \\ 2 & -1 & 1 & 0 & 1 \end{bmatrix}$$

$$b = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$c = \begin{bmatrix} 2 \\ -6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_B = \begin{bmatrix} x_4^* \\ x_5^* \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$z_N = \begin{bmatrix} -2 \\ 6 \\ 0 \end{bmatrix}$$

$$\boldsymbol{B} = \{4, 5\}, \boldsymbol{N} = \{1, 2, 3\}$$

Constraint Equations

$$B^{-1}N = \begin{bmatrix} 1 & 1 & 1 \\ -2 & 1 & -1 \end{bmatrix}$$

Range for c_2

Setup

$$\Delta c = [0, 1, 0, 0, 0]$$
$$\Delta c_B^T = [0, 0]$$
$$\Delta c_N^T = [0, 1, 0]$$

Please note the indices for these variables correspond to:

$$\Delta c = \{1, 2, 3, 4, 5\}$$

$$\Delta c_B^T = \boldsymbol{B} = \{4, 5\}$$
$$\Delta c_N^T = \boldsymbol{N} = \{1, 2, 3\}$$

Calculate Δz_N

$$\Delta z_N = (B^{-1}N)^T \Delta c_B - \Delta c_N$$

$$\Delta z_N = \begin{bmatrix} 1 & 1 & 1 \\ -2 & 1 & -1 \end{bmatrix}^T \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\Delta z_N = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

Solve for t

Since $z_N + t\Delta z_N \geq 0$:

$$\begin{bmatrix} 2 \\ -6 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Then:

$$-6 - t \geq 0$$

$$t \leq -6$$

$$t \in (-\infty, -6]$$

Change Original Problem

To make the problem more feasible, we can change c_2 to equal 1 since $-7 \in (-\infty, -6]$. Our new problem is:

$$\zeta = 2x_1 + x_2 + 0x_3$$

$$x_4 = -2 + x_1 + x_2 + x_3$$

$$x_5 = 1 - 2x_1 + x_2 - x_3$$

Dual Simplex Iteration 1

Step 1

The dictionary isn't optimized yet because:

$$x_B^* \ngeq 0$$

Step 2

Pick $i \in \{i \in \boldsymbol{B} : x_i^* < 0\}$:

$$i = 4$$

Step 3

$$\Delta z_N = -(B^{-1}N)^T e_i$$

$$\Delta z_N = -\begin{bmatrix} 1 & 1 & 1 \\ -2 & 1 & -1 \end{bmatrix}^T \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Delta z_N = \begin{bmatrix} -1 & 2 \\ -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Delta z_N = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

Step 4

$$s = \left(max_{j \in N} \frac{\Delta z_j}{z_j^*} \right)^{-1}$$

$$s = \left(max \left\{ \frac{-1}{-2}, \frac{-1}{6}, \frac{-1}{0} \right\} \right)^{-1}$$

$$s = \left(\frac{1}{2} \right)^{-1}$$

$$s = 2$$

Step 5

Pick j that corresponds to the max chosen for s:

$$j = 1$$

Step 6

$$\Delta x_B = B^{-1} N e_i$$

$$\Delta x_B = \begin{bmatrix} 1 & 1 & 1 \\ -2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta x_B = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

Step 7

$$t = \frac{x_i^*}{\Delta x_i} = \frac{-2}{1}$$

Step 8

$$x_B^* = x_B^* - t\Delta x_B$$

$$x_B^* = \begin{bmatrix} -2 \\ 1 \end{bmatrix} - (-2) \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$x_B^* = \begin{bmatrix} -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

$$x_B^* = \begin{bmatrix} 0 \\ -3 \end{bmatrix}$$

$$x_i^* = t$$

$$x_4^* = -2$$

$$x_B^* = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$z_N^* = z_N^* - s\Delta z_N$$

$$z_N^* = \begin{bmatrix} -2 \\ 6 \\ 0 \end{bmatrix} - 2 \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

$$z_N^* = \begin{bmatrix} -2 \\ 6 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$$

$$z_N^* = \begin{bmatrix} 0 \\ 8 \\ 2 \end{bmatrix}$$

$$z_j^* = s$$

$$z_1^* = 2$$

$$z_N^* = \begin{bmatrix} 2 \\ 8 \\ 2 \end{bmatrix}$$

Step 9

$$\boldsymbol{B} = \{1,5\},\ \boldsymbol{N} = \{4,2,3\}$$

$$B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$N = \begin{bmatrix} 1 & -1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

Dual Simplex Iteration 2

Step 1

The dictionary isn't optimized yet because:

$$x_B^* \ngeq 0$$

Step 2

Pick $i \in \{i \in \boldsymbol{B} : x_i^* < 0\}$:

$$i = 5$$

Step 3

$$\Delta z_N = -(B^{-1}N)^T e_i$$

$$\Delta z_N = -\left(\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \right)^T \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Please note that:

$$\left[\begin{array}{cc|cc} -1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array} \right] \rightarrow -R_1$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -1 & 0 \\ 2 & 1 & 0 & 1 \end{array} \right] \rightarrow R_2 - 2R_1$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 \end{array} \right]$$

Therefore:

$$\Delta z_N = -\left(\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \right)^T \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Delta z_N = -\begin{bmatrix} -1 & 1 & 1 \\ 2 & -3 & -1 \end{bmatrix}^T \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Delta z_N = -\begin{bmatrix} -1 & 2 \\ 1 & -3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Delta z_N = \begin{bmatrix} 1 & -2 \\ -1 & 3 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Delta z_N = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

Step 4

$$s = \left(max_{j \in N} \frac{\Delta z_j}{z_j^*} \right)^{-1}$$

$$s = \left(max \left\{ \frac{-2}{2}, \frac{3}{8}, \frac{1}{2} \right\} \right)^{-1}$$

$$s = \left(\frac{1}{2} \right)^{-1}$$

$$s = 2$$

Step 5

Pick j that corresponds to the max chosen for s:

$$j = 3$$

Step 6

$$\Delta x_B = B^{-1} N e_i$$

$$\Delta x_B = \begin{bmatrix} -1 & 1 & 1 \\ 2 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta x_B = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

Step 7

$$t = \frac{x_i^*}{\Delta x_i} = -\frac{3}{2}$$

Step 8

$$x_B^* = x_B^* - t\Delta x_B$$
$$x_B^* = \begin{bmatrix} -2 \\ -3 \end{bmatrix} - \frac{-3}{2} \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$
$$x_B^* = \begin{bmatrix} -2 \\ -3 \end{bmatrix} + \begin{bmatrix} -\frac{3}{2} \\ 3 \end{bmatrix}$$
$$x_B^* = \begin{bmatrix} -\frac{7}{2} \\ 0 \end{bmatrix}$$

$$x_i^* = t$$
$$x_5^* = -\frac{3}{2}$$
$$x_B^* = \begin{bmatrix} -\frac{7}{2} \\ -\frac{3}{2} \end{bmatrix}$$

$$z_N^* = z_N^* - s\Delta z_N$$
$$z_N^* = \begin{bmatrix} 2 \\ 8 \\ 2 \end{bmatrix} - 2 \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$
$$z_N^* = \begin{bmatrix} 2 \\ 8 \\ 2 \end{bmatrix} + \begin{bmatrix} 4 \\ -6 \\ -2 \end{bmatrix}$$
$$z_N^* = \begin{bmatrix} 6 \\ 2 \\ 0 \end{bmatrix}$$

$$z_j^* = s$$
$$z_3^* = 2$$
$$z_N^* = \begin{bmatrix} 6 \\ 2 \\ 2 \end{bmatrix}$$

Step 9

$$\boldsymbol{B} = \{1, 3\},\ \boldsymbol{N} = \{4, 2, 5\}$$
$$B = \begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}$$
$$N = \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Dual Simplex Iteration 3

Step 1

The dictionary isn't optimized yet because:

$$x_B^* \ngeq 0$$

Step 2

Pick $i \in \{i \in \boldsymbol{B} : x_i^* < 0\}$:

$$i = 1$$

Step 3

$$\Delta z_N = -(B^{-1}N)^T e_i$$
$$\Delta z_N = -\left(\begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \right)^T \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Please note that:

$$\left[\begin{array}{cc|cc} -1 & -1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{array}\right] \rightarrow R_1 + R_2$$
$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 \end{array}\right] \rightarrow R_2 - 2R_1$$
$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 0 & 1 & -2 & -1 \end{array}\right]$$

Therefore:

$$\Delta z_N = -\left(\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \right)^T \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Delta z_N = -\begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & -1 \end{bmatrix}^T \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Delta z_N = -\begin{bmatrix} 1 & -2 \\ -2 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Delta z_N = \begin{bmatrix} -1 & 2 \\ 2 & -3 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Delta z_N = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

Step 4

$$s = \left(max_{j \in N} \frac{\Delta z_j}{z_j^*} \right)^{-1}$$

$$s = \left(max \left\{ \frac{-1}{6}, \frac{2}{2}, \frac{-1}{0} \right\} \right)^{-1}$$

$$s = \left(\frac{2}{2} \right)^{-1}$$

$$s = 1$$

Step 5

Pick j that corresponds to the max chosen for s:

$$j = 2$$

Step 6

$$\Delta x_B = B^{-1} N e_i$$

$$\Delta x_B = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\Delta x_B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

Step 7

$$t = \frac{x_i^*}{\Delta x_i} = \frac{-\frac{7}{2}}{-2} = \frac{7}{4}$$

Step 8

$$x_B^* = x_B^* - t\Delta x_B$$
$$x_B^* = \begin{bmatrix} -\frac{7}{2} \\ -\frac{3}{2} \end{bmatrix} - \frac{7}{4} \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$
$$x_B^* = \begin{bmatrix} -\frac{7}{2} \\ -\frac{3}{2} \end{bmatrix} + \begin{bmatrix} \frac{7}{2} \\ -\frac{21}{4} \end{bmatrix}$$
$$x_B^* = \begin{bmatrix} 0 \\ -\frac{27}{4} \end{bmatrix}$$

$$x_i^* = t$$
$$x_1^* = \frac{7}{4}$$
$$x_B^* = \begin{bmatrix} \frac{7}{4} \\ -\frac{27}{4} \end{bmatrix}$$

$$z_N^* = z_N^* - s\Delta z_N$$
$$z_N^* = \begin{bmatrix} 6 \\ 6 \\ 2 \end{bmatrix} - 1 \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$
$$z_N^* = \begin{bmatrix} 6 \\ 6 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$
$$z_N^* = \begin{bmatrix} 7 \\ 4 \\ 3 \end{bmatrix}$$

$$z_j^* = s$$
$$z_2^* = 1$$
$$z_N^* = \begin{bmatrix} 7 \\ 1 \\ 3 \end{bmatrix}$$

Step 9

$$\boldsymbol{B} = \{2,3\},\ \boldsymbol{N} = \{4,1,5\}$$
$$B = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$
$$N = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

Dual Simplex Iteration 4

Step 1

The dictionary isn't optimized yet because:

$$x_B^* \ngeq 0$$

Step 2

Pick $i \in \{i \in \boldsymbol{B} : x_i^* < 0\}$:

$$i = 3$$

Step 3

$$\Delta z_N = -(B^{-1}N)^T e_i$$
$$\Delta z_N = -\left(\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \right)^T \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Please note that:

$$\left[\begin{array}{cc|cc} -1 & -1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{array}\right] \to R_1 + R_2$$

$$\left[\begin{array}{cc|cc} -2 & 0 & 1 & 1 \\ -1 & 1 & 0 & 1 \end{array}\right] \to -\frac{1}{2} R_1$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ -1 & 1 & 0 & 1 \end{array}\right] \to R_2 + R_1$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \end{array}\right]$$

Therefore:

$$\Delta z_N = -\left(\begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix}\right)^T \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Delta z_N = -\begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} \end{bmatrix}^T \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Delta z_N = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{3}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Delta z_N = \begin{bmatrix} \frac{1}{2} \\ -\frac{3}{2} \\ -\frac{1}{2} \end{bmatrix}$$

Step 4

$$s = \left(max_{j \in N} \frac{\Delta z_j}{z_j^*}\right)^{-1}$$

$$s = \left(max \left\{ \frac{\frac{1}{2}}{5}, \frac{-\frac{3}{2}}{-1}, \frac{-\frac{1}{2}}{3} \right\}\right)^{-1}$$

$$s = \left(\frac{3}{2}\right)^{-1}$$

$$s = \frac{2}{3}$$

Step 5

Pick j that corresponds to the max chosen for s:

$$j = 1$$

Step 6

$$\Delta x_B = B^{-1} N e_i$$

$$\Delta x_B = \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\Delta x_B = \begin{bmatrix} -\frac{1}{2} \\ \frac{3}{2} \end{bmatrix}$$

Step 7

$$t = \frac{x_i^*}{\Delta x_i} = \frac{-\frac{3}{4}}{\frac{3}{2}} = -\frac{1}{2}$$

Step 8

$$x_i^* = t$$
$$x_3^* = -\frac{1}{2}$$

$$x_B^* = x_B^* - t\Delta x_B$$
$$x_B^* = \begin{bmatrix} \frac{3}{4} \\ \frac{3}{2} \end{bmatrix} - \frac{3}{4}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$z_j^* = s$$
$$z^*$$

$$z_N^* = z_N^* - s\Delta z_N$$
$$z_N^* = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} - 1\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$$

Last

(2)

(3)

Optimal Dictionary

$$\zeta = 13 - 3x_2 - x_4 - x_6$$
$$x_3 = 1 + x_2 + 3x_4 - 2x_6$$
$$x_1 = 2 - 2x_2 - 2x_4 + x_6$$
$$x_5 = 1 + 5x_2 + 2x_4$$

Given

$$B = \{3, 1, 5\},\ N = \{2, 4, 6\}$$
$$c = [5, 4, 3, 0, 0, 0]$$
$$z_N^* = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$
$$B = \begin{bmatrix} 1 & 3 & -2 \\ -2 & -2 & 1 \\ 5 & 2 & 0 \end{bmatrix}$$

Constraint Equations

$$B^{-1}N = \begin{bmatrix} 1 & 3 & -2 \\ -2 & -2 & 1 \\ 5 & 2 & 0 \end{bmatrix}$$

Range for c_2

Setup

$$\Delta c = [0, 1, 0, 0, 0, 0]$$
$$\Delta c_B^T = [0, 0, 0]$$
$$\Delta c_N^T = [0, 1, 0]$$

Please note the indices for these variables correspond to:

$$\Delta c = \{1, 2, 3, 4, 5, 6\}$$
$$\Delta c_B^T = B = \{3, 1, 5\}$$
$$\Delta c_N^T = N = \{2, 4, 6\}$$

Calculate Δz_N

$$\Delta z_N = (B^{-1}N)^T \Delta c_B - \Delta c_N$$
$$\Delta z_N = \begin{bmatrix} 1 & 3 & -2 \\ -2 & -2 & 1 \\ 5 & 2 & 0 \end{bmatrix}^T \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$
$$\Delta z_N = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

Solve for t

Since $z_N^* + t\Delta z_N \geq 0$:

$$\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Then:

$$1 - t \geq 0$$

$$t \leq 1$$

$$t \in (-\infty, 1]$$

Range for c_3

Setup

$$\Delta c = [0, 0, 1, 0, 0, 0]$$
$$\Delta c_B^T = [1, 0, 0]$$
$$\Delta c_N^T = [0, 0, 0]$$

Please note the indices for these variables correspond to:

$$\Delta c = \{1, 2, 3, 4, 5, 6\}$$

$$\Delta c_B^T = B = \{3, 1, 5\}$$
$$\Delta c_N^T = N = \{2, 4, 6\}$$

Calculate Δz_N

$$\Delta z_N = (B^{-1}N)^T \Delta c_B - \Delta c_N$$

$$\Delta z_N = \begin{bmatrix} 1 & 3 & -2 \\ -2 & -2 & 1 \\ 5 & 2 & 0 \end{bmatrix}^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta z_N = \begin{bmatrix} 1 & -2 & 5 \\ 3 & -2 & 2 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta z_N = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$$

Solve for t

Since $z_N^* + t\Delta z_N \geq 0$:

$$\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Then:

$$3 + t \geq 0$$

$$t \geq -3$$

$$1 + 3t \geq 0$$

$$t \geq \frac{-1}{3}$$

$$1 - 2t \geq 0$$

$$t \leq \frac{1}{2}$$

$$t \in \left[\frac{-1}{3}, \frac{1}{2}\right]$$

(4)

Constraints

The production facilities can produce 11k units per month. Let $\{x\}_1^3$ be the packages shipped from San Francisco and $\{x\}_4^6$ be the packages shipped from Sacramento.
$x_1 + x_2 + x_3 \leq 11$
$x_4 + x_5 + x_6 \leq 11$

We have to meet the demand at the destinations which is 10k at Davis, 8k at Winters, and 4k at Woodland. Let x_1, x_4 be the packages shipped to Davis. Let x_2, x_5 be the packages shipped to Winters. Let x_3, x_6 be the packages shipped to Woodland.
$x_1 + x_4 \geq 10$
$x_2 + x_5 \geq 8$
$x_3 + x_6 \geq 4$

Objective Function

We want to minimize cost of both facilities. Let vector c be the cost to ship.
$min\ c^T x$
$min\ 10x_1 + 8x_2 + 12x_3 + 4x_4 + 11x_5 + 6x_6$

Online Solver

We used this online solver: https://online-optimizer.appspot.com

Input

var x1 >= 0;
var x2 >= 0;
var x3 >= 0;
var x4 >= 0;
var x5 >= 0;
var x6 >= 0;

minimize z: 10*x1 + 8*x2 + 12*x3 + 4*x4 + 11*x5 + 6*x6;

subject to c11: x1 + x2 + x3 <= 11;
subject to c12: x4 + x5 + x6 <= 11;
subject to c13: x1 + x4 >= 10;
subject to c14: x2 + x5 >= 8;
subject to c15: x3 + x6 >= 4;

end;

Output Model Overview

Label	Value
Problem type	Linear optimization
Objective	Minimize z
Optimal objective value	146
Solver Status	Optimal
Total number of variables	6
Continuous variables	6
Number of constraints	6
Non-binary nonzero coefficients	18

Output Model Variables

Variable	Type	Value	Value bounds	Status	Reduced obj coef	Obj coef tol interval
x1	Real	3	[0, Inf]	Basic	0	[-1, Inf]
x2	Real	8	[0, Inf]	Basic	0	[-6, Inf]
x3	Real	0	[0, Inf]	At lower bound	0	
x4	Real	7	[0, Inf]	Basic	0	
x5	Real	0	[0, Inf]	At lower bound	9	
x6	Real	4	[0, Inf]	Basic	0	

Output Model Constraints

Name	Lhs value	Rhs bounds	Slack	Status	Dual value	Rhs tol interval
c11	11	[-Inf, 11]	0	Basic	0	
c12	11	[-Inf, 11]	0	At upper bound	-6	[11, 14]
c13	10	[10, Inf]	0	At lower bound	10	[7, 10]
c14	8	[8, Inf]	0	At lower bound	8	[0, 8]
c15	4	[4, Inf]	0	At lower bound	12	[1, 4]

Output Model Log Messages

Reading model section from editor.mod ...
16 lines were read
Generating z...
Generating c11...
Generating c12...
Generating c13...
Generating c14...
Generating c15...
Model has been successfully generated
Scaling...
A: min—aij— = 1 max—aij— = 12 ratio = 12
GM: min—aij— = 0.7681450856702011 max—aij— = 1.3018373985007101 ratio = 1.6947806121350972
EQ: min—aij— = 0.6030226891555273 max—aij— = 1 ratio = 1.6583123951777
Solving the model using the simplex optimizer
GLPK Simplex Optimizer, v4.49
6 rows, 6 columns, 18 non-zeros
Preprocessing...
5 rows, 6 columns, 12 non-zeros
Scaling...
A: min—aij— = 1 max—aij— = 1 ratio = 1
Problem data seem to be well scaled
Constructing initial basis...

Size of triangular part = 5
0: obj = 0 infeas = 22 (0)
*4: obj = 209 infeas = 0 (0)
*6: obj = 146 infeas = 0 (0)
OPTIMAL SOLUTION FOUND

Collaboration

All collaborators are listed (in alphabetical order) below:

- Anne
- Jack
- Dhruv
- Fengqin
- Zhongning
- Sterling

Academic Integrity

On my personal integrity as a student and member of the UCD community, I have not given, nor received and unauthorized assistance on this assignment.
Signature: Andrew Jowe