1. $\max\ 2x_1 - 6x_2 + 0x_3$

$$-x_1 - x_2 - x_3 \le -2$$
$$2x_1 - x_2 + x_3 \le 1$$

Aux Problem

$\max\ -x_0$

$$-x_1 - x_2 - x_3 - x_0 \le -2$$
$$2x_1 - x_2 + x_3 - x_0 \le 1$$

$$x^*_B = b = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$z^*_N = -c_N = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$N = \{0,1,2,3\} \quad B = \{4,5\}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$N = \begin{bmatrix} -1 & -1 & -1 & -1 \\ -1 & 2 & -1 & 1 \end{bmatrix}$$

It 2

2: $i = 4$

3: $\Delta z_N = -(B^{-1}N)^T e_i$

$$= -\begin{bmatrix} -1 & -1 & -1 & -1 \\ -1 & 2 & -1 & 1 \end{bmatrix}^T \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

4: $(\max\{1/1, 1/0, 1/0, 1/0\})^{-1}$

$\Rightarrow s = 1$

5: $j = 0$

6: $\Delta x_B = B^{-1}N e_j$

$$= \begin{bmatrix} -1 & -1 & -1 & -1 \\ -1 & 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

7: $t = -2/-1 = 2$

8: $x_j^* = t$

$\Rightarrow x_0^* = 2$

$$x_B^* = x_B^* - t\,\Delta x_B$$

$$x_B^* = \begin{bmatrix} -2 \\ 1 \end{bmatrix} - 2 \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$z_i^* = 5$

$z_4^* = 1$

$z_N^* = z_N^* - 5\Delta z_N$

$$z_N^* = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

4: $B = \{0, 5\}$ $N = \{4, 1, 2, 3\}$

$$x_B^* = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad z_N^* = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix} \quad N = \begin{bmatrix} 1 & -1 & -1 & -1 \\ 0 & 2 & -1 & 1 \end{bmatrix}$$

$\underline{IT\ 2}$ → primal simplex

2: $j = 1$

3: $\Delta x_B = B^{-1} N e_j$

$$= \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -1 & -1 & -1 \\ 0 & 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{array}\right] -R_1$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -1 & 0 \\ -1 & 1 & 0 & 1 \end{array}\right] R_2+R_1$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right]$$

$$= \begin{bmatrix} -1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 & -1 \\ 0 & 2 & -1 & 1 \end{bmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{bmatrix} -1 & 1 & 1 & 1 \\ -1 & 3 & 0 & 2 \end{bmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

u: $\left(\max\{1/2, 3/3\}\right)^{-1}$

$t = 1$

s: $i = 5$

6: $\Delta z_N = -(B^{-1}N)^T e_i$

$$= -\begin{bmatrix} -1 & 1 & 1 & 1 \\ -1 & 3 & 0 & 2 \end{bmatrix}^T \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -3 \\ 0 \\ -2 \end{bmatrix}$$

7: $s = \frac{-1}{-3} = \frac{1}{3}$

8: $x_j^* = t$

$x_1^* = 1$

$$x_B^* = \begin{bmatrix} 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$z_i^* = s$

$z_s^* = \frac{1}{3}$

$$z_N^* = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} - \frac{1}{3} \begin{bmatrix} 1 \\ -3 \\ 0 \\ -2 \end{bmatrix}$$

$$z_N^* = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} + \begin{bmatrix} -1/3 \\ 1 \\ 0 \\ 2/3 \end{bmatrix}$$

$$z_N^* = \begin{bmatrix} 2/3 \\ 0 \\ -1 \\ -1/3 \end{bmatrix}$$

9: $B = \{0, 1\}$ $N = \{4, 5, 2, 3\}$

$$x_B^* = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad z_N^* = \begin{bmatrix} 2/3 \\ 4/3 \\ -1 \\ -1/3 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix} \quad N = \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & -1 & 1 \end{bmatrix}$$

It 3

2: $j = 2$

3: $\Delta x_B = B^{-1} N e_j$

$$= \begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix}^{-1} \left(\begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right)$$

$$\left[\begin{array}{cc|cc} -1 & -1 & 1 & 0 \\ -1 & 2 & 0 & 1 \end{array}\right] 2R_1 + R_2$$

$$\left[\begin{array}{cc|cc} -3 & 0 & 2 & 1 \\ -1 & 2 & 0 & 1 \end{array}\right] -\tfrac{1}{3}R_1$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -2/3 & -1/3 \\ -1 & 2 & 0 & 1 \end{array}\right] R_2 + R_1$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -2/3 & -1/3 \\ -1 & 2 & 0 & 1 \end{array}\right] R_2 + R_1$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -2/3 & -1/3 \\ 0 & 2 & -2/3 & 2/3 \end{array}\right] \tfrac{1}{2}R_2$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -2/3 & -1/3 \\ 0 & 1 & -1/3 & 1/3 \end{array}\right]$$

$$= \begin{bmatrix} -2/3 & -1/3 \\ -1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & -1 & 1 \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$= \begin{bmatrix} -2/3 & -1/3 & 1 & 1/3 \\ -1/3 & 1/3 & 0 & 2/3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

4: $t = (\max\{1/2, 1/3\})^{-1}$

$t = 1$

5: $i = 0$

6: $\Delta z_N = -(B^{-1}N)^T e_i$

$$= -\begin{bmatrix} -2/3 & -1/3 & 1 & 1/3 \\ -1/3 & 1/3 & 0 & 2/3 \end{bmatrix}^T \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2/3 \\ 1/3 \\ -1 \\ -1/3 \end{bmatrix}$$

7: $s = -1/-1 = 1$

8: $x_j^* = t$

$x_2^* = 1$

$$x_B^* = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$z_i^* = 5 \Rightarrow z_0^* = 1$$

$$z_0^* = \begin{bmatrix} 2/3 \\ 4/3 \\ -1 \\ -4/3 \end{bmatrix} - \begin{bmatrix} 2/3 \\ 4/3 \\ -1 \\ -4/3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ $B = \{2, 1\}$ $N = \{4, 5, \cancel{6}, 3\}$

$$x_B^* = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad z_N^* = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 4 \\ -1 & 2 \end{bmatrix} \quad N = \begin{bmatrix} 1 & 0 & \cancel{4} & 4 \\ 0 & 1 & \cancel{4} & 1 \end{bmatrix}$$

Done with Aux

$$y^* = c_B^t B^{-1} b$$

$$= \begin{bmatrix} 2 & -6 \end{bmatrix} \begin{pmatrix} -1 & -1 \\ -1 & 2 \end{pmatrix}^{-1} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -6 \end{bmatrix} \begin{bmatrix} -2/3 & -1/3 \\ -1/3 & 1/3 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2/3 & -8/3 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$= -4/3 - 8/3 = -4$$

$$z_N^* = \begin{bmatrix} -2/3 & -1/3 \\ -1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -6 \\ 0 \end{bmatrix}$$

$$\cancel{- \begin{bmatrix} 0 \\ 0 \end{bmatrix}}$$

$$z_N^* = \begin{bmatrix} -2/3 & -1/3 \\ -1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

$$z_N^* = \begin{bmatrix} -2/3 & -1/3 & 1/3 \\ -1/3 & 1/3 & 2/3 \end{bmatrix} \begin{bmatrix} 2 \\ -6 \\ 0 \end{bmatrix}$$

$$z_N^* = \begin{bmatrix} 2/3 \\ -8/3 \\ 0 \end{bmatrix}$$

$$-4/3 + 6/3 = 2/3$$

$$-2/3 - 6/3 = -8/3$$

$$x_B^* = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad z_N^* = \begin{bmatrix} 2/3 \\ -8/3 \\ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix} \quad N = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$B = \{2, 1\} \quad N = \{4, 5, 3\}$$

2. $j = 3$

3. $\Delta x_B = B^{-1} N e_j$

$$= \begin{bmatrix} -2/3 & -1/3 & 1/3 \\ -1/3 & 1/3 & 2/3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$$

4. $\left(\max \left\{ \frac{1/3}{1}, \frac{2/3}{1} \right\} \right)^{-1}$

$t = 3/2$

5. $i = 1$

6. $\Delta z_N = -(B^{-1}N)^T e_i$

$$= -\begin{bmatrix} -2/3 & -1/3 & 1/3 \\ -1/3 & 1/3 & 2/3 \end{bmatrix}^T \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/3 \\ -1/3 \\ -2/3 \end{bmatrix}$$

7. $s = 0 / -\frac{1}{3} = 0$

8. $x_j^* = t$

$x_3^* = 3/2$

$x_B^* = x_B^* - t\Delta x_B$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} - 3/2 \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -1/2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}$$

$z_i^* = s$

$z_1^* = 0$

$z_N^* = z_N^* - s\Delta z_N$

$$= \begin{bmatrix} 2/3 \\ -8/3 \\ 0 \end{bmatrix}$$

9. $B = \{2, 3\}$ $N = \{4, 5, 1\}$

$$x_B^* = \begin{pmatrix} 1/2 \\ 3/2 \end{pmatrix} \qquad z_N^* = \begin{bmatrix} 2/3 \\ -8/3 \\ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 4 \\ -1 & 1 \end{bmatrix} \quad N = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\zeta^* = 2(0) - 6(1/2) = -3$$