

MAT168 HW4

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November 28, 2022

(1)

Iteration 1

Draw out original problem

l		$\boxed{0}$	$\boxed{0}$
u		6	8
		$\zeta = -x_1 + x_2 = 0$	
$-\infty$	5	$w_1 = -x_1 + x_2 = 0$	
$-\infty$	9	$w_2 = x_1 - 2x_2 = 0$	

Determine entering variable

x_2 has positive coefficient and is at lower bound, x_2 enters.

Determine leaving variable

As $x_2 \rightarrow 8$:

$$w_1 \rightarrow -\infty \leq x_2 \leq 5 \rightarrow x_2 = 5$$

$$w_2 \rightarrow -\infty \leq -2x_2 \leq 9 \rightarrow x_2 = \infty$$

w_1 leaves.

Iteration 2

Rewriting equation

l		$\boxed{0}$	$-\infty$
u		6	$\boxed{5}$
		<hr/>	
		ζ	$= 0x_1 + w_1 = 5$
0	8	x_2	$= x_1 + w_1 = 5$
$-\infty$	9	w_2	$= -x_1 - 2w_2 = -10$

Optimal

This is optimal since w_1 has positive coefficient and upper bound is chosen, and x_1 has coefficient of 0.

Solution

$$w_1 = 5$$

$$x_2 = 5$$

$$x_1 = 0$$

$$w_2 = -10$$

$$\zeta = 5$$

(2)

Primal

Given

$$c = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}$$

$$b = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$$

$$a = \begin{bmatrix} -\infty \\ -\infty \end{bmatrix}$$

$$u = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$$l = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Slack Variables

$$f = \begin{bmatrix} 0 \\ 19 \end{bmatrix}$$

$$p = \begin{bmatrix} \infty \\ \infty \end{bmatrix}$$

$$t = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

$$g = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

Dual

Equations

$$\text{minimize} \quad \begin{bmatrix} 5 \\ 9 \end{bmatrix}^T v - \begin{bmatrix} -\infty \\ -\infty \end{bmatrix}^T q + \begin{bmatrix} 6 \\ 8 \end{bmatrix}^T s - \begin{bmatrix} 0 \\ 0 \end{bmatrix}^T h$$

$$\text{subject to} \quad \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}^T (v - q) - (h - s) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Complimentary Slackness

$$f_2 = 19 \rightarrow v_2 = 0$$

$$p_i \neq 0 \rightarrow q_i = 0 \quad \forall i \in [1, m]$$

$$t_j \neq 0 \rightarrow s_j = 0 \quad \forall j \in [1, n]$$

$$g_2 = 5 \rightarrow h_2 = 0$$

Rewrite Equations Iteration 1

$$\begin{array}{ll} \text{minimize} & 5v_1^+ \\ & 0 \end{array}$$

$$\text{subject to} \quad \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix} v - h = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Rewrite Equations Iteration 2

$$\begin{array}{ll} \text{minimize} & 5v_1^+ \\ & 0 \end{array}$$

$$\begin{array}{ll} \text{subject to} & -v_1 - h_1 = -1 \\ & v_1 = 1 \end{array}$$

Solve

$$v_1 = 1$$

$$v_2 = 0$$

$$q_1, q_2 = 0$$

$$s_1, s_2 = 0$$

$$h_1, h_2 = 0$$

$$\zeta = 5v_1 = 5(1) = 5$$

Academic Integrity

On my personal integrity as a student and member of the UCD community, I have not given, nor received any unauthorized assistance on this assignment.

Andrew