Homework 1
Due Oct. 5

1 Problem Set

1: Determine if the following problems are bounded or unbounded and then determine if they are feasible or infeasible (5pts each):

$$\begin{aligned}
max \quad & 6x_1 + 8x_2 + 5x_3 + 9x_4 \\
s.t. \quad & 2x_1 + x_2 + x_3 + 3x_4 \leq 8 \\
& x_1 + 3x_2 + x_3 + 2x_4 \leq 2 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}$$

$$\begin{aligned}
max \quad & 6x_1 + 8x_2 + 5x_3 + 9x_4 \\
s.t. \quad & 2x_1 + x_2 + x_3 + 3x_4 \geq 50 \\
& x_1 + 3x_2 + x_3 + 2x_4 \geq 1 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}$$

$$\begin{aligned}
max \quad & 6x_1 + 8x_2 + 5x_3 + 9x_4 \\
s.t. \quad & 2x_1 + x_2 + x_3 + 3x_4 \leq -1 \\
& x_1 + 3x_2 + x_3 + 2x_4 \leq 3 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}$$

2.a: Solve following **by hand** using the simplex method (18 pts each):

$$\begin{aligned}
max \quad & 6x_1 + 8x_2 + 5x_3 + 9x_4 \\
s.t. \quad & 2x_1 + x_2 + x_3 + 3x_4 \leq 5 \\
& x_1 + 3x_2 + x_3 + 2x_4 \leq 3 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}$$

$$\begin{aligned}
max \quad & x_1 + 3x_2 \\
s.t. \quad & -x_1 - x_2 \leq -3 \\
& -x_1 + x_2 \leq -1 \\
& x_1 + 2x_2 \leq 4 \\
& x_1, x_2 \geq 0
\end{aligned}$$

2.b: The first problem has 4 variables and only 2 linear constraints (in addition to the non-negativity condition). The second has 2 variables, but 4 constraints. Which do you think is more difficult to solve and why (4 pts)?

3: Solve the Klee-Minty problem, as described in the notes/text, for n=3. Use any pivot rule (maximum coefficient pivot may be slow) (15 pts).

4: Suppose that you have been put in charge of managing the clothing supply for a small, newly established space colony. The colony has n colonists each of whom needs at least two pairs of shoes, three jumpsuits, and a hat. You can make each of these products from some combination of cotton, thread, and glue, all of which you need to import. You can also import already made clothing for a fixed cost. For each item that you produce in the factory, you also incur a cost of d per item (a pair of shoes, jumpsuit, and hat each count as an 'item') for the use of electricity in your factory. Write down (but do not solve) a linear program to minimize the cost of acquiring all of the clothing which your colony requires. You may assume that you are allowed to make and import fractional quantities of each of these goods. (20 pts)

Hint: Let $\mathbf{x} = \{x_i\}_{i=1}^6$ = amount imported of cotton, thread, glue, shoes, jumpsuits, hats, and $\mathbf{c} = \{c_i\}_{i=1}^6$ be the cost of importing each of these raw materials and completed items

Bonus:(Up to 5 pts): What can you say about this problem as n changes?

2 Collaboration

Please use this space to recognize any and all collaborations that assisted you in the completion of this assignment.

3 Academic Integrity

Please copy and sign the following statement of academic integrity:

On my personal integrity as a student and member of the UCD community, I have not given, nor received and unauthorized assistance on this assignment.