# MAT168 HW2

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# (1)

## Original Problem

$$\zeta = 2x_1 - 6x_2 + 0x_3$$
$$x_4 = -2 + x_1 + x_2 + x_3$$
$$x_5 = 1 - 2x_1 + x_2 - x_3$$

## Setup

$$N = \begin{bmatrix} -1 & -1 & -1 \\ 2 & -1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A = [NB] = \begin{bmatrix} -1 & -1 & -1 & 1 & 0 \\ 2 & -1 & 1 & 0 & 1 \end{bmatrix}$$

$$b = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$c = \begin{bmatrix} 2 \\ -6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_B^* = \begin{bmatrix} x_4^* \\ x_5^* \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

# Iteration 1

## Step 1

$$x_B^* \ngeq 0$$

## Step 2

$$z_N^* = -c_N = \begin{bmatrix} 2 \\ -6 \\ 0 \end{bmatrix}$$

$$j = 2$$

## Step 3

$$\Delta x_B = B^{-1} N e_j$$

$$\Delta x_B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -1 & -1 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\Delta x_B = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

## Step 4

$$t = \left( max_{i \in B} \frac{\Delta x_i}{x_i^*} \right)^{-1}$$

$$t = \left( max_{i \in B} \left\{ \frac{-2}{-1}, \frac{-1}{1} \right\} \right)^{-1}$$

$$t = \frac{-1}{1},$$

**Step 5**

$i = 5$

**Step 6**

$$\Delta z_N = -\left(B^{-1}N\right)^T e_i$$

$$\Delta z_N = -\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -1 & -1 & -1 \\ 2 & -1 & 1 \end{bmatrix}\right)^T \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Delta z_N = -\begin{bmatrix} -1 & -1 & -1 \\ 2 & -1 & 1 \end{bmatrix}^T \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Delta z_N = \begin{bmatrix} 1 & -2 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Delta z_N = \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}$$

**Step 7**

$$s = \frac{z_j^*}{\Delta z_j} = \frac{-6}{1} = -6$$

**Step 8**

$x_2^* =$

# (2)

# (3)

# (4)

## Constraints

The production facilities can produce 11k units per month. Let $\{x\}_1^3$ be the packages shipped from San Francisco and $\{x\}_4^6$ be the packages shipped from Sacramento.
$x_1 + x_2 + x_3 \leq 11$
$x_4 + x_5 + x_6 \leq 11$

We have to meet the demand at the destinations which is 10k at Davis, 8k at Winters, and 4k at Woodland. Let $x_1, x_4$ be the packages shipped to Davis. Let $x_2, x_5$ be the packages shipped to Winters. Let $x_3, x_6$ be the packages shipped to Woodland.
$x_1 + x_4 \geq 10$
$x_2 + x_5 \geq 8$
$x_3 + x_6 \geq 4$

## Objective Function

We want to minimize cost of both facilities. Let vector $c$ be the cost to ship.
$min\ c^T x$
$min\ 10x_1 + 8x_2 + 12x_3 + 4x_4 + 11x_5 + 6x_6$

## Online Solver

We used this online solver: https://online-optimizer.appspot.com

### Input

var x1 >= 0;
var x2 >= 0;
var x3 >= 0;
var x4 >= 0;
var x5 >= 0;
var x6 >= 0;

```
minimize z: 10*x1 + 8*x2 + 12*x3 + 4*x4 + 11*x5 + 6*x6;

subject to c11: x1 + x2 + x3 <= 11;
subject to c12: x4 + x5 + x6 <= 11;
subject to c13: x1 + x4 >= 10;
subject to c14: x2 + x5 >= 8;
subject to c15: x3 + x6 >= 4;

end;
```

## Output Model Overview

| Label | Value |
|---|---|
| Problem type | Linear optimization |
| Objective | Minimize z |
| Optimal objective value | 146 |
| Solver Status | Optimal |
| Total number of variables | 6 |
| Continuous variables | 6 |
| Number of constraints | 6 |
| Non-binary nonzero coefficients | 18 |

## Output Model Variables

| Variable | Type | Value | Value bounds | Status | Reduced obj coef | Obj coef tol interval |
|---|---|---|---|---|---|---|
| x1 | Real | 3 | [0, Inf] | Basic | 0 | [-1, Inf] |
| x2 | Real | 8 | [0, Inf] | Basic | 0 | [-6, Inf] |
| x3 | Real | 0 | [0, Inf] | At lower bound | 0 | |
| x4 | Real | 7 | [0, Inf] | Basic | 0 | |
| x5 | Real | 0 | [0, Inf] | At lower bound | 9 | |
| x6 | Real | 4 | [0, Inf] | Basic | 0 | |

## Output Model Constraints

| Name | Lhs value | Rhs bounds | Slack | Status | Dual value | Rhs tol interval |
|---|---|---|---|---|---|---|
| c11 | 11 | [-Inf, 11] | 0 | Basic | 0 | |
| c12 | 11 | [-Inf, 11] | 0 | At upper bound | -6 | [11, 14] |
| c13 | 10 | [10, Inf] | 0 | At lower bound | 10 | [7, 10] |
| c14 | 8 | [8, Inf] | 0 | At lower bound | 8 | [0, 8] |
| c15 | 4 | [4, Inf] | 0 | At lower bound | 12 | [1, 4] |

## Output Model Log Messages

```
Reading model section from editor.mod ...
16 lines were read
```

Generating z...
Generating c11...
Generating c12...
Generating c13...
Generating c14...
Generating c15...
Model has been successfully generated
Scaling...
A: min—aij— = 1 max—aij— = 12 ratio = 12
GM: min—aij— = 0.7681450856702011 max—aij— = 1.3018373985007101 ratio = 1.6947806121350972
EQ: min—aij— = 0.6030226891555273 max—aij— = 1 ratio = 1.6583123951777
Solving the model using the simplex optimizer
GLPK Simplex Optimizer, v4.49
6 rows, 6 columns, 18 non-zeros
Preprocessing...
5 rows, 6 columns, 12 non-zeros
Scaling...
A: min—aij— = 1 max—aij— = 1 ratio = 1
Problem data seem to be well scaled
Constructing initial basis...
Size of triangular part = 5
0: obj = 0 infeas = 22 (0)
*4: obj = 209 infeas = 0 (0)
*6: obj = 146 infeas = 0 (0)
OPTIMAL SOLUTION FOUND

# Collaboration

All collaborators are listed (in alphabetical order) below:

- Anne
- Jack
- Dhruv
- Fengqin
- Zhongning
- Sterling

# Academic Integrity

On my personal integrity as a student and member of the UCD community, I have not given, nor received and unauthorized assistance on this assignment.
Signature: Andrew Jowe