Introduction to Optimization
University of California Davis
Fall 2021
Prof. Stefan C. Schonsheck

Homework 2

Due Oct. 19

1 Problem Set

1: Consider the following problems from the first homework. For each, derive the dual problem and illustrate that strong duality holds for these problems by finding the optimal solution to the dual **(You need to find the values of the slack variables as well!)** . (Hint: You do not need to use the simplex method for this problem) (15 pts each):

$$\begin{aligned} max \quad & 6x_1 + 8x_2 + 5x_3 + 9x_4 \\ s.t. \quad & 2x_1 + x_2 + x_3 + 3x_4 \leq 5 \\ & x_1 + 3x_2 + x_3 + 2x_4 \leq 3 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

$$\begin{aligned} max \quad & x_1 + 3x_2 \\ s.t. \quad & -x_1 - x_2 \leq -3 \\ & -x_1 + x_2 \leq -1 \\ & x_1 + 2x_2 \leq 4 \\ & x_1, x_2 \geq 0 \end{aligned}$$

2: Solve the following linear program (20 pts):

$$\begin{aligned} max \quad & -x_1 - 2x_2 \\ s.t. \quad & -2x_1 + 7x_2 \leq 6 \\ & -3x_1 + x_2 \leq -1 \\ & 9x_1 - 4x_2 \leq 6 \\ & x_1 - x_2 \leq 1 \\ & 7x_1 - 3x_2 \leq 6 \\ & -5x_1 - 2x_2 \leq 3 \\ & x_1, x_2 \geq 0 \end{aligned}$$

3: Use the dual-primal two-phase algorithm to solve (25 pts):

$$\begin{aligned} max \quad & 2x_1 - 6x_2 \\ s.t. \quad & -x_1 - x_2 - x_3 \leq -2 \\ & 2x_1 - x_2 + x_3 \leq 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

4: Imagine you are a college student trying to live on a small budget. You want to minimize your food costs, but you know a diet consisting solely of instant ramen and beer may be harmful to your health. You look up recommended dietary allowance of several different nutrients to determine what you should eat to be healthy. Let $i = 1...m$ denote the nutrients and let $b_i, i = 1...m$ denote the minimum daily requirement for each of the nutrients. Suppose make a list of your n favorite foods. Let j denote their index on your list and c_j be the cost of the j^{th} item. Let a_{ij} denote the amount of the i^{th} nutrient contained in the j^{th} item on your food list.

a: Formulate a linear problem to minimize your costs while still getting all of your daily required nutrients (5 pts)

b: Formulate the dual to this problem (10 pts)

c: Introduce another person to this story who is naturally interested in solving the dual problem. (Hint: Give semantic meaning to the dual problem) (10 pts)

d: Name some real-world considerations that your model leaves out (**Up to 5 bonus points**)

2 Collaboration

Please use this space to recognize any and all collaborations which assisted you on the completion of this assignment.

3 Academic Integrity

Please copy and sign the following statement of academic integrity:

> On my personal integrity as a student and member of the UCD community, I have not given, nor received and unauthorized assistance on this assignment.