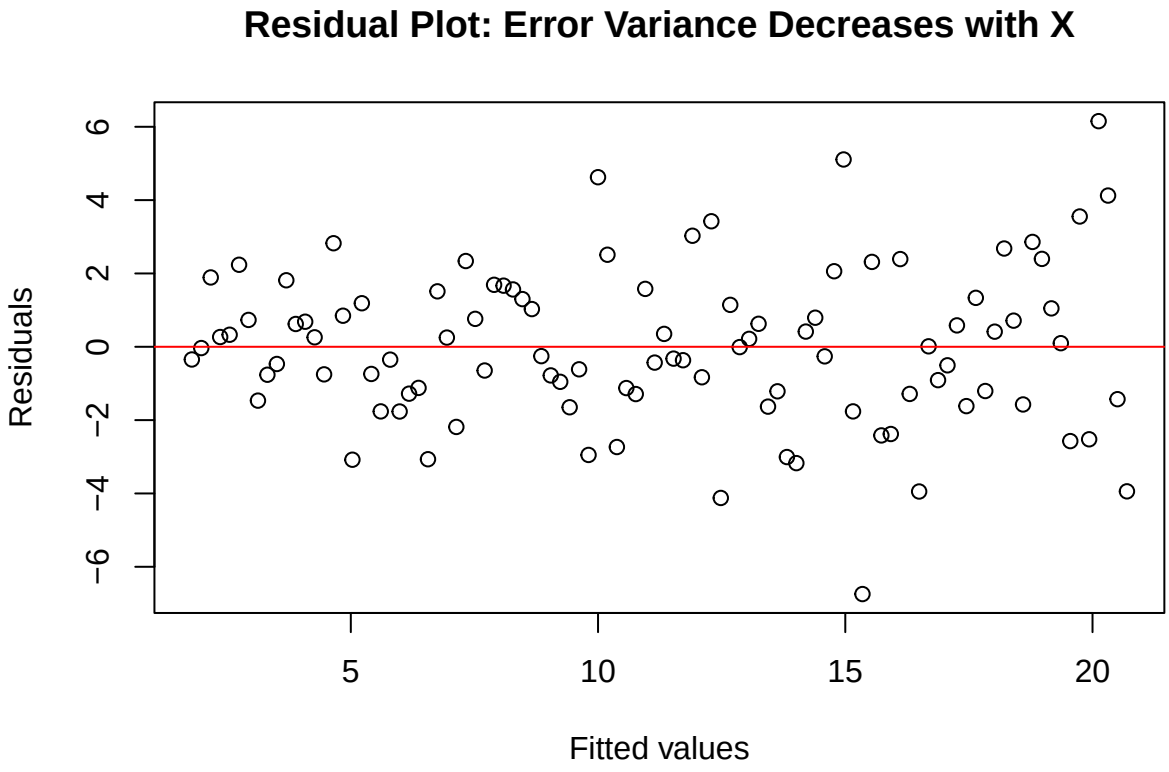
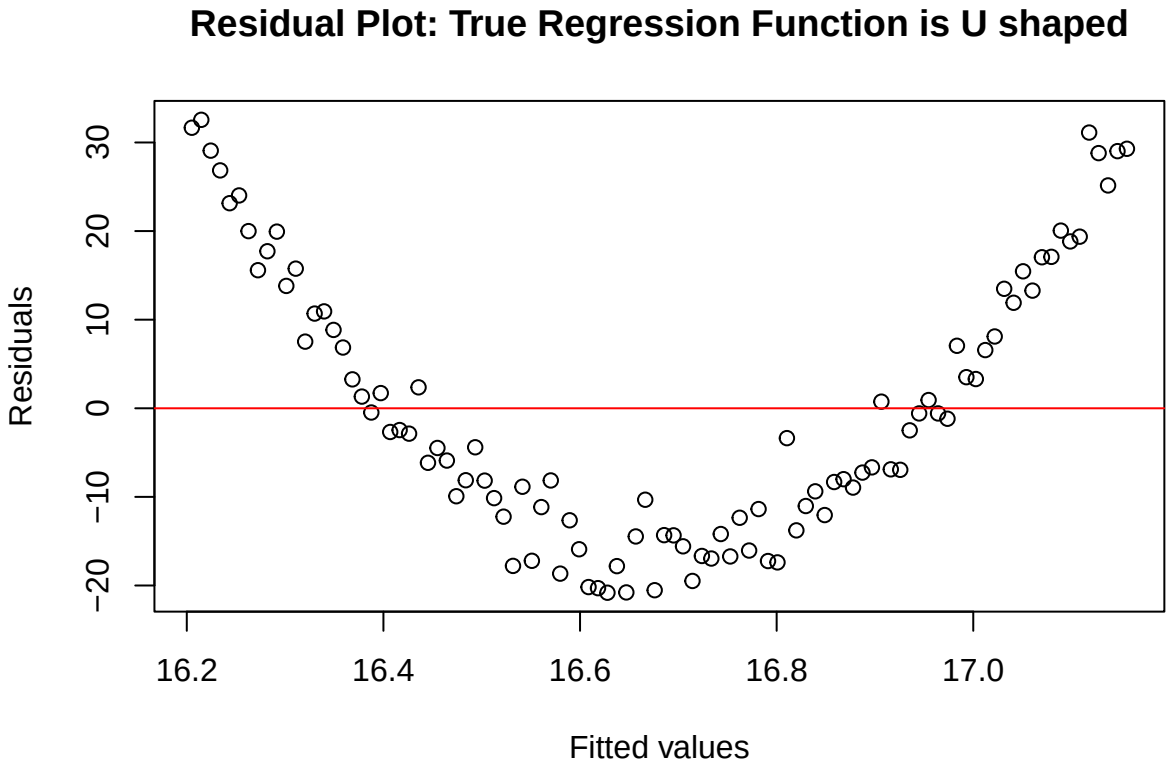
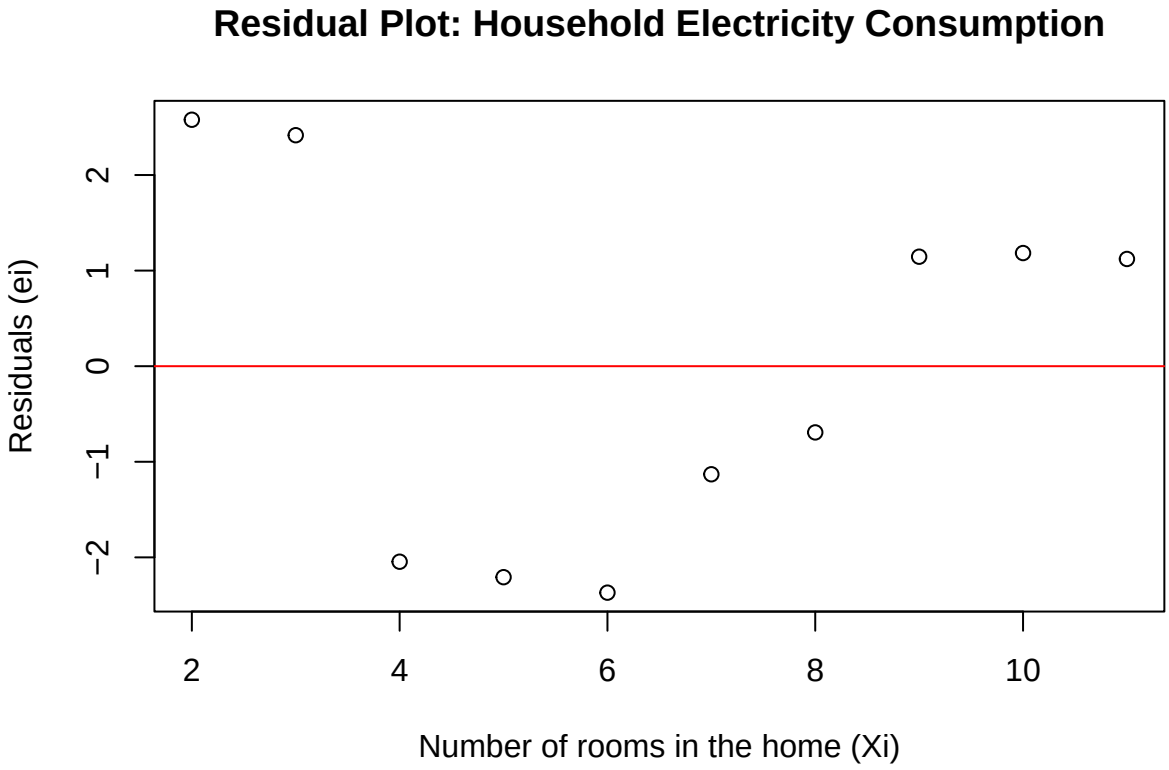
3.2


Residual Plot: Error Variance Decreases with X
Residuals
6
4
2
0
−2
−4
−6
5
10
15
20
Fitted values


Residual Plot: True Regression Function is U shaped
Residuals
30
20
10
0
−10
−20
16.2
16.4
16.6
16.8
17.0
Fitted values

3.9



Based on the plot, there does not seem to be a linear relationship between the datapoints. A transformation might alleviate the problem, as it can linearize non-linear relationships.

3.19

This difference can arise because ??

3.20

The transformation $X' = 1/X$ maintains a linear regression relationship, so the error terms are still independent and $\epsilon_{ij} \sim N(0, \sigma^2)$. However, the same does not hold after the transformation $Y' = 1/Y$, as this transformation modifies the error values.

3.23

The full model is $Y_{ij} = \mu_j + \epsilon_{ij}$

$df_F = n - c = 20 - 10 = 10$

The reduced model is $Y_{ij} = \beta_1 x_j + \epsilon_{ij}$

$df_R = n - 1 = 20 - 1 = 19$

Appendix

3.2

```r
# Case 1: Error variance decreases with X
# Generate synthetic data ?? wrong
set.seed(123)
X1 <- seq(1, 10, length.out = 100)
Y1 <- 2*X1 + rnorm(100, mean = 0, sd = X1^0.5) # Error variance decreases with X

# Fit linear regression model
model1 <- lm(Y1 ~ X1)

# Residuals for Case 1
residuals1 <- resid(model1)

# Plot residual vs. fitted values
plot(fitted(model1), residuals1, xlab = "Fitted values", ylab = "Residuals",
     main = "Residual Plot: Error Variance Decreases with X")

# Add a horizontal line at y = 0 for reference
abline(h = 0, col = "red")

# Case 2: True regression function is U shaped
# but a linear regression function is fitted
# Generate synthetic data
X2 <- seq(-5, 5, length.out = 100)
# True regression function is U shaped
Y2 <- 2*X2^2 + rnorm(100, mean = 0, sd = 3)

# Fit linear regression model
model2 <- lm(Y2 ~ X2)

# Residuals for Case 2
residuals2 <- resid(model2)

# Plot residual vs. fitted values
plot(fitted(model2), residuals2, xlab = "Fitted values", ylab = "Residuals",
     main = "Residual Plot: True Regression Function is U shaped")

# Add a horizontal line at y = 0 for reference
abline(h = 0, col = "red")
```

3.9

```r
# Provided data
x_i <- c(2, 3, 4, 5, 6, 7, 8, 9, 10, 11)
e_i <- c(3.2, 2.9, -1.7, -2.0, -2.3, -1.2, -0.9, 0.8, 0.7, 0.5)

# Fit linear regression model
```

```r
model <- lm(e_i ~ x_i)

# Get residuals
residuals <- resid(model)

# Plot residuals against Xi
plot(x_i, residuals, xlab = "Number of rooms in the home (Xi)",
     ylab = "Residuals (ei)",
     main = "Residual Plot: Household Electricity Consumption")

# Add a horizontal line at y = 0 for reference
abline(h = 0, col = "red")
```