Quiz 2

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x	y	$\hat{y}$	e_i
16	199	201.150	-2.150
16	205	201.150	3.850
16	196	201.150	-5.150
16	200	201.150	-1.150
24	218	217.425	0.575
24	220	217.425	2.575
24	215	217.425	-2.425
24	223	217.425	5.575
32	237	233.700	3.300
32	234	233.700	0.300
32	235	233.700	1.300
32	230	233.700	-3.700
40	250	249.975	0.025
40	248	249.975	-1.975
40	253	249.975	3.025
40	246	249.975	-3.975

$n = 16$

$\bar{y} = 225.5625$

$\bar{x} = 28$

$b_1 = 2.0344$

$b_0 = 168.6$

$MSE = 10.4589$

$s^2(b_1) = 0.0082$

$t_1 = 22.5057$

$p = 0$

Given $\alpha = 0.01$ (type I error), the decision rule is that if the p value is smaller than α, we reject the null hypothesis. Otherwise, we fail to reject the null hypothesis.

Since $p < \alpha$, we reject the null hypothesis. Therefore, $\beta_1 \neq 0$.

Appendix

```r
# plug in data
x <- c(16, 16, 16, 16, 24, 24, 24, 24, 32, 32, 32, 32, 40, 40, 40, 40)
y <- c(199, 205, 196, 200, 218, 220, 215, 223,
       237, 234, 235, 230, 250, 248, 253, 246)

# find the number of samples
x.n <- length(x)

# find the means
y.b <- mean(y)
x.b <- mean(x)

# find the numerator and denominator for later calculations
n <- sum((x - x.b) * y)
d <- sum((x - x.b) ** 2)

# find the predicted b0 and b1 values
b1 <- n / d
b0 <- y.b - x.b * b1

# find the fitted y value
y_fitted <- b0 + b1 * x

# find the residual
residual <- y - y_fitted

# find the mse
mse <- sum(residual ** 2) / (x.n - 2)

# find the variance of b1
s_squared_b1 <- mse / d

# find the t-statistic of b1
t1 <- b1 / sqrt(s_squared_b1)

# find the p-value of the t-statistic
p <- (1 - pt(t1, x.n - 2, ncp = 0)) * 2

# format a table for nice printing
df <- data.frame(x = x, y = y)
df[["$\\hat y$"]] <- y_fitted
df[["$e_i$"]] <- residual
```