5.5

$$X = \begin{bmatrix} 1 & 4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}$$

$$Y = \begin{bmatrix} 16 \\ 5 \\ 10 \\ 15 \\ 13 \\ 22 \end{bmatrix}$$

$$Y'Y = \begin{bmatrix} 16 & 5 & 10 & 15 & 13 & 22 \end{bmatrix} \begin{bmatrix} 16 \\ 5 \\ 10 \\ 15 \\ 13 \\ 22 \end{bmatrix} = \begin{bmatrix} 1259 \end{bmatrix}$$

$$X'X = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 4 & 1 & 2 & 3 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 17 \\ 17 & 55 \end{bmatrix}$$

$$X'Y = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 4 & 1 & 2 & 3 & 3 & 4 \end{bmatrix} \begin{bmatrix} 16 \\ 5 \\ 10 \\ 15 \\ 13 \\ 22 \end{bmatrix} = \begin{bmatrix} 81 \\ 261 \end{bmatrix}$$

$$H^2 = \begin{bmatrix} 0.3659 & -0.1463 & 0.0244 & 0.1951 & 0.1951 & 0.3659 \\ -0.1463 & 0.6585 & 0.3902 & 0.122 & 0.122 & -0.1463 \\ 0.0244 & 0.3902 & 0.2682 & 0.1463 & 0.1463 & 0.0244 \\ 0.1951 & 0.122 & 0.1463 & 0.1707 & 0.1707 & 0.1951 \\ 0.1951 & 0.122 & 0.1463 & 0.1707 & 0.1707 & 0.1951 \\ 0.3659 & -0.1463 & 0.0244 & 0.1951 & 0.1951 & 0.3659 \end{bmatrix}$$

$$H = \begin{bmatrix} 0.3659 & -0.1463 & 0.0244 & 0.1951 & 0.1951 & 0.3659 \\ -0.1463 & 0.6585 & 0.3902 & 0.122 & 0.122 & -0.1463 \\ 0.0244 & 0.3902 & 0.2683 & 0.1463 & 0.1463 & 0.0244 \\ 0.1951 & 0.122 & 0.1463 & 0.1707 & 0.1707 & 0.1951 \\ 0.1951 & 0.122 & 0.1463 & 0.1707 & 0.1707 & 0.1951 \\ 0.3659 & -0.1463 & 0.0244 & 0.1951 & 0.1951 & 0.3659 \end{bmatrix}$$

$\therefore H^2 = H$

5.17.a

$W_1 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} Y$

$W_2 = \begin{bmatrix}1 & -1 & 0\end{bmatrix} Y$

$W_3 = \begin{bmatrix}1 & -1 & -1\end{bmatrix} Y$

5.17.b

$W = \begin{bmatrix}W_1 \\ W_2 \\ W_3\end{bmatrix}$

$E\{W\} = \begin{bmatrix}\mu_1 + \mu_2 + \mu_3 \\ \mu_1 - \mu_2 \\ \mu_1 - \mu_2 - \mu_3\end{bmatrix}$

5.24

??

6.1.a

$Y = \beta^T X + \epsilon$

$X = \begin{bmatrix}1 & x_{11} & x_{11}x_{12} \\ 1 & x_{21} & x_{21}x_{22} \\ 1 & x_{31} & x_{31}x_{32} \\ 1 & x_{41} & x_{41}x_{42}\end{bmatrix}$

$\beta = \begin{bmatrix}\beta_0 \\ \beta_1 \\ \beta_2\end{bmatrix}$

6.1.b

$log(Y) = \beta^T X + \epsilon$

$X = \begin{bmatrix}1 & x_{11} & x_{12} \\ 1 & x_{21} & x_{22} \\ 1 & x_{31} & x_{32} \\ 1 & x_{41} & x_{42}\end{bmatrix}$

$\beta = \begin{bmatrix}\beta_0 \\ \beta_1 \\ \beta_2\end{bmatrix}$

6.22.a

This is not a linear regression model. We can apply the following transformations:

$X'_{i1} = X_{i1}$

$X'_{i2} = 10^{X_{i2}}$

$X'_{i3} = X_{i1}^2$

Our linear regression model after the transformation:

$Y_i = \beta_0 + \beta_1 X'_{i1} + \beta_2 X'_{i2} + \beta_3 X'_{i3} + \epsilon_i$

6.22.b

This is not a linear regression model. It is not possible to transform this.

Example 6.9

b_0 : The intercept in the linear regression model.

b_1 : The slope of x_1 in the linear regression model.

b_2 : The slope of x_2 in the linear regression model.

Appendix

```r
# Function
r_matrix_to_mathjax <- function(mat) {
  nrow <- dim(mat)[1]

  result <- "\\begin{bmatrix}"
  for (i in 1:nrow) {
    result <- paste(result, "", paste(mat[i, ], collapse = "&"), "\\\\")
  }
  result <- paste(result, "\\end{bmatrix}")

  return(result)
}
```

```r
# 5.5
x1 <- c(4, 1, 2, 3, 3, 4)
Y <- as.matrix(c(16, 5, 10, 15, 13, 22))
X <- cbind(1, x1)

Xt <- t(X)
Yt <- t(Y)
Yt.Y <- Yt %*% Y
Xt.X <- Xt %*% X
Xt.Y <- Xt %*% Y

H <- round(X %*% solve(Xt %*% X) %*% Xt, 4)
H.2 <- round(H %*% H, 4)
```