1 - Problem 18, Page 28

We sample m items randomly without replacement.

Let $A = \{\text{at least one item in } m \text{ is defective}\}$.

$P(A) = .9$

We want to find m

This is like the birthday problem. It's easier to find the complement.

Let $A^c = \{\text{no items in } m \text{ are defective}\}$.

$P(A^c) = 1 - P(A) = .1$

Let $j \in [0, m)$ be the jth item we sample.

$P(A^c) = \Pi_j \frac{n-k-j}{n-j} = \frac{\frac{(n-k)!}{(n-k-m)!}}{\frac{n!}{(n-m)!}}$

$P(A^c) = \frac{(n-k)!(n-m)!}{(n-k-m)!n!}$

$P(A^c) = \frac{(n-k)!(n-m)!}{(n-k-m)!n!}$

$P(A^c) = \frac{(n-k)!}{n!} \frac{(n-m)!}{(n-k-m)!}$

$P(A^c) = \frac{1}{n(n-1)...(n-k+1)} \frac{(n-m)(n-m-1)...(n-m-k+1)}{1}$

$P(A^c) = \frac{(n-m)(n-m-1)...(n-m-k+1)}{n(n-1)...(n-k+1)}$

When applying the hint:

$P(A^c) = (\frac{n-m}{n})^k = .1$

Now we can solve for m:

$\frac{n-m}{n} = .1^{\frac{1}{k}}$

$n - m = .1^{\frac{1}{k}} n$

$-n + m = -.1^{\frac{1}{k}} n$

$m = n - .1^{\frac{1}{k}} n$

a: let $n = 1000, k = 10$

$m = 1000 - .1^{\frac{1}{10}} * 1000 = 206$ samples

b: let $n = 10000, k = 100$

$m = 10000 - .1^{\frac{1}{100}} * 10000 = 228$ samples

2 - Problem 46, Page 30

Let $X = \{\text{a red ball is drawn}\}$

Let $A = \{\text{drawn from Urn A}\}$, $B = \{\text{drawn from Urn B}\}$

$P(A) = P(B) = .5$

a: $P(X) = ?$

$P(X|A) = \frac{3}{3+2} = .6$

$P(X|B) = \frac{2}{2+5} = \frac{2}{7}$

$P(X) = P(X|A)P(A) + P(X|B)P(B)$

$P(X) = .6 * .5 + \frac{2}{7} * .5 = .44$

b: $P(A|X) = ?$

Bayes rule: $P(A|X)P(X) = P(X|A)P(A)$

$P(A|X) = \frac{P(X|A)P(A)}{P(X)}$

$P(A|X) = \frac{.6 * .5}{.44} = .68$

3 - Problem 54, Page 31

Let 0 represent today.

a: $P(R_1) = ?$

$P(R_0) = p, P(R_0^c) = 1 - p$

$P(R_1) = P(R_1|R_0)P(R_0) + P(R_1|R_0^c)P(R_0^c)$

$P(R_1|R_0) = \alpha, P(R_1^c|R_0) = 1 - \alpha$

$P(R_1^c|R_0^c) = \beta, P(R_1|R_0^c) = 1 - \beta$

$P(R_1) = \alpha p + (1 - \beta)(1 - p)$

$P(R_1) = \alpha p + (1 - p) - \beta(1 - p)$

$P(R_1) = \alpha p + 1 - p - \beta + \beta p$

$P(R_1) = (1 - \beta) + (\alpha + \beta - 1)p$

b: $P(R_2) = ?$

From c:

$x_2 = b + ab + a^2p$

$P(R_2) = 1 - \beta + (1 - \beta)(\alpha + \beta - 1) + (\alpha + \beta - 1)^2p$

c: $P(R_n) = ?$

From a:

$P(R_n) = (1 - \beta) + (\alpha + \beta - 1)P(R_{n-1})$

Let $x_i = P(R_i), x_0 = p$

This looks in the form of $x_n = b + ax_{n-1}$

Where $b = 1 - \beta, a = \alpha + \beta - 1$

$x_1 = b + ap$

$x_2 = b + a(b + ap) = b + ab + a^2p$

$x_3 = b + a(b + ab + a^2p) = b + ab + a^2b + a^3p$

$x_4 = b + a(b + ab + a^2b + a^3p) = b + ab + a^2b + a^3b + a^4p$

$x_n = \Sigma_{j=0}^{n-1} a^j b + a^n p$

$x_n = b\Sigma_{j=0}^{n-1} a^j + a^n p$

Let's solve the summation first:

$\Sigma_{j=0}^{n-1} a^j = \frac{1-a^n}{1-a}$

Now plug in:

$P(R_n) = b\frac{1-a^n}{1-a} + a^n p$

where $b = 1 - \beta, a = \alpha + \beta - 1$

If $a > 1$:

$P(R_\infty) = \frac{-\infty}{-C} + \infty$

$P(R_\infty) = \infty$

If $a < 1$:

$P(R_\infty) = b\frac{1}{1-a}$

4: Problem 60, Page 36

Let $A = \{\text{item produced in first shift}\}$

Let $B = \{\text{item produced in second shift}\}$

Let C = {item produced in third shift}

Since all shifts had equal productivity:

$P(A) = P(B) = P(C) = \frac{1}{3}$

Let D = {item defective}

$P(D|A) = .01$

$P(D|B) = .02$

$P(D|C) = .05$

$P(D) = P(D|A)P(A) + P(D|B)P(B) + P(D|C)P(C)$

$P(D) = .01 * \frac{1}{3} + .02 * \frac{1}{3} + .05 * \frac{1}{3} = \frac{2}{75}$

$P(C|D) = ?$

$P(C|D) = \frac{P(D|C)P(C)}{P(D)}$

$P(C|D) = \frac{.05 * \frac{1}{3}}{\frac{2}{75}} = \frac{5}{8}$

5: Problem 74, Page 33

Let's say the circuit looks like this:

xx–1–2–xx

–|—3—|–

xx–4–5–xx

Let:

A = {Node 1 fails}

B = {Node 2 fails}

C = {Node 3 fails}

D = {Node 4 fails}

E = {Node 5 fails}

$P(A) = P(B) = P(C) = P(D) = P(E) = p$

Let U = {Upper fails}

$U = A \cup B$

$P(U) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) * P(B) = p^2$

$P(U) = 2p - p^2$

Let $M = \{\text{Middle fails}\}$

$P(M) = P(C) = p$

Let $L = \{\text{Lower fails}\}$

$P(L) = P(D) + P(E) - P(D \cap E)$

$P(D \cap E) = P(D) * P(E) = p^2$

$P(L) = 2p - p^2$

Let $F = \{\text{Circuit fails}\}$

$F = U \cap M \cap L$

$P(F) = P(U) * P(M) * P(L)$

$F = (2p - p^2)^2 p$

Let $W = F^c = \{\text{Circuit works}\}$

$P(W) = 1 - P(F)$

$P(W) = 1 - (2p - p^2)^2 p$