Note: $Binomial(n, p)$ can be interpreted as counting the number of red marbles in a sample of size n taken with replacement

p = fraction of red marbles in jar

Note: A $Binomial(n, b)$ can be represented as $X_1 + ... + X_n$ where $X_1, ..., X_n$ are independent bernoulli(p) random variables

$X \sim Poisson(\lambda),\ \lambda > 0$

X is an approximation to a binomial(n, p) random variable where $\lambda = np$ and n is large, p is small, $n \to \infty,\ p \to 0$, λ fixed

$$p_X(k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Recall any pmf must be

1. non-negative
2. have values sum to 1

Let's check

$$\sum_{k=0}^{\infty} p_X(k)$$

$$\sum_{k=0}^{\infty} \frac{\lambda^k e^{-\lambda}}{k!}$$

$$e^{-\lambda} \sum_{k=0}^{\infty} \frac{\lambda^k}{k!}$$

$$e^{-\lambda} e^{\lambda} = 1$$

Let's see how the $Poisson(\lambda)$ pmf arises from the $Bionomial(n, p)$ when $\lambda = np,\ n \to \infty,\ p \to 0,\ \lambda$ fixed

$$\binom{n}{k} p^k (1-p)^{n-k} = \binom{n}{k} \left(\frac{\lambda}{n}\right)^k (1 - \frac{\lambda}{n})^{n-k}$$

$$= \frac{n!}{k!(n-k)!} \frac{\lambda^k}{n^k} (1 - \frac{\lambda}{n})^n (1 - \frac{\lambda}{n})^k$$

$$= \frac{\lambda^k}{k!} \frac{n(n-1)...1}{(n-k)(n-k-1)...1} \frac{1}{n^k} (1 - \frac{\lambda}{n})^n (1 - \frac{\lambda}{n})^k$$

$$\approx \frac{e^{-\lambda} \lambda^k}{k!}$$

Continuous Random Variables

e.g. heights, weights, time, volume, ...

Def: A continuous random variable is is one such that there is a probability density function f_X such that

$$P(a \le X \le b) = \int_a^b f_X(x)\ dx;\ a, b \in \mathbb{R};\ \int_{-\infty}^{\infty} f_X(x)\ dx = 1$$

Also, we must have $f_X(x) \ge 0,\ \forall x$

Since X is continuous, there is no way to hit a specific number since there are infinite possibilities, so:

$$\int_c^c f_X(x)\ dx = 0$$

Cumulative distribution function given f_X

$$F_X(x) = \int_{-\infty}^{x} f_X(u)\ du$$

Likewise:

$$f_X(x) = \frac{d}{d_x} F_X(x) = F'_X(x)$$

Note - we can calculate $P(a \le X \le b) = F_X(b) - F_X(a)$

Example: $X \sim Uniform[a, b],\ a \le b$

$$f_X(x) = \begin{cases} h & \text{if } x \in [a, b] \\ 0 & \text{otherwise} \end{cases}$$

Let's find the cdf

$$F_X(x) = \int_{-\infty}^{x} f_x(u)\ du = \frac{1}{b-a} \int_a^x du = \frac{x-a}{b-a}$$

Consider $Uniform[0, 1]$

$$F_X(x) = x,\ x \in [0, 1]$$