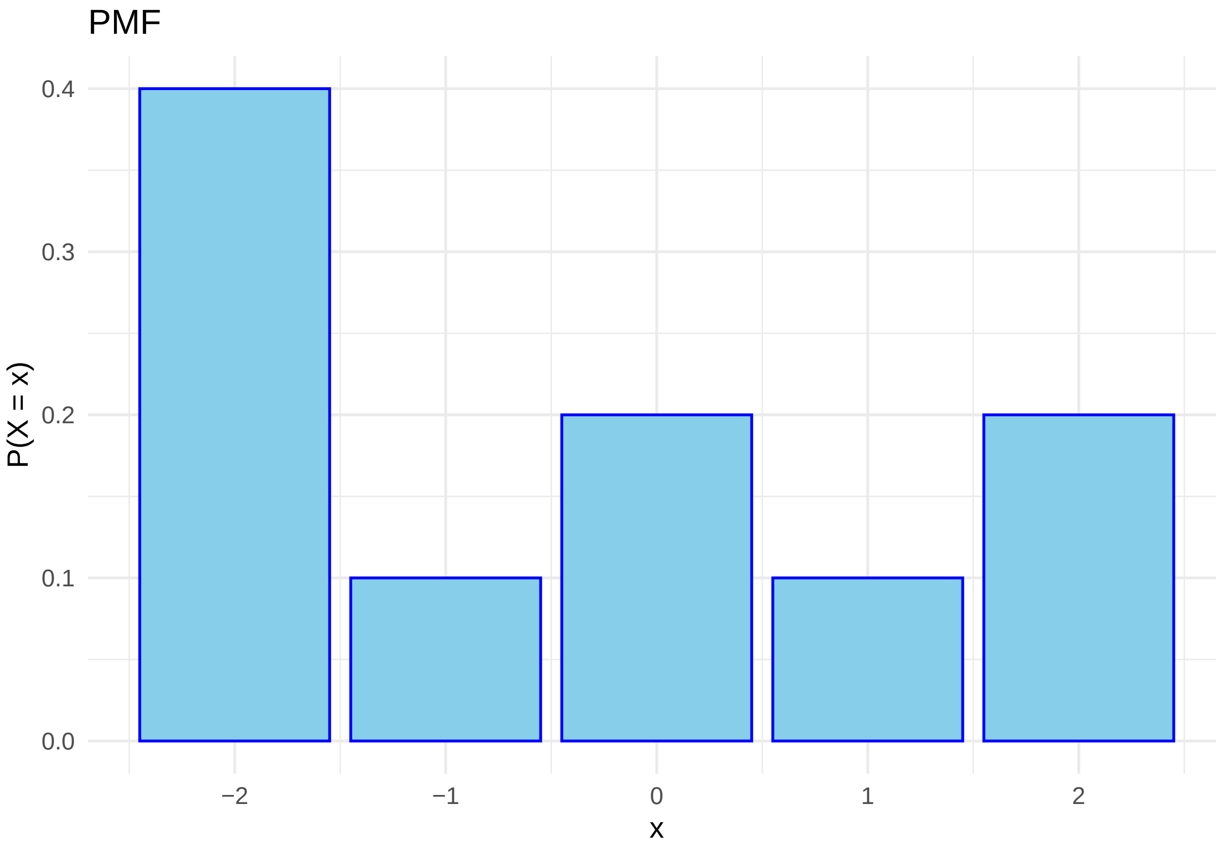
1

$$F_Y(y) = P(Y \leq y)$$
$$F_Y(y) = P(2 - X \leq y)$$
$$F_Y(y) = P(2 - y \leq X)$$
$$F_Y(y) = 1 - P(X \leq 2 - y)$$
$$F_Y(y) = 1 - F_X(2 - y)$$
$$f_Y(y) = f_X(2 - y)$$
$$f_Y(y) = \frac{1}{2}$$

2

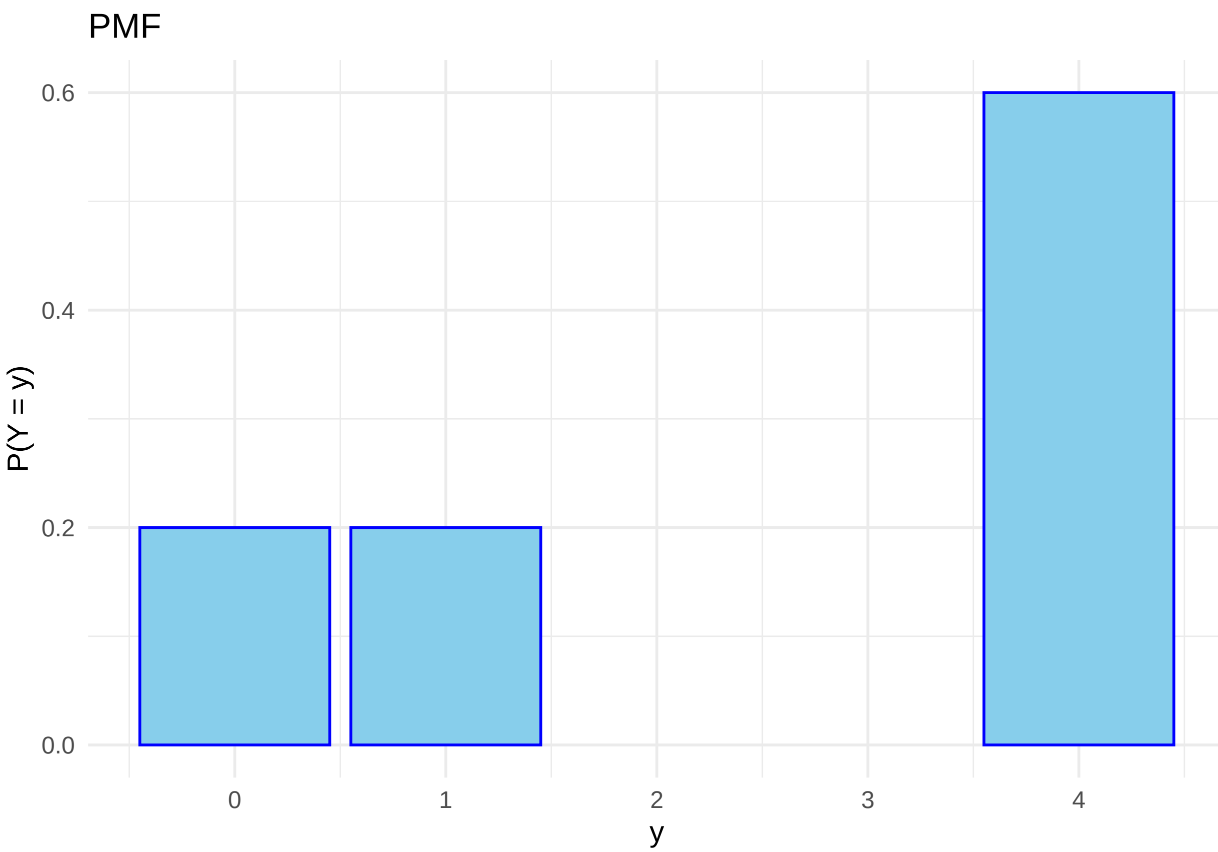


(a)

(b)

$$F_Y(y) = P(y \leq Y)$$
$$F_Y(y) = P(y \leq X^2)$$
$$F_Y(y) = P(\sqrt{y} \leq X)$$
$$F_Y(y) = F_X(\sqrt{y})$$
$$f_Y(y) = f_X(\sqrt{y}) * \frac{1}{2\sqrt{y}}$$

y	f_Y
0	0.2
1	0.2
4	0.6



(c)

$$E[Y] = 0 * 0.2 + 1 * 0.2 + 4 * 0.6$$
$$E[Y] = 2.6$$

3

(a)

$$f_X(x) = e^{-x},\ 0 \leq X < \infty$$
$$Y = e^{-X}$$

The support for Y is:

$$0 < Y \leq 1$$

(b)

$$F_Y(y) = P(Y \leq y)$$
$$F_Y(y) = P(e^{-X} \leq y)$$
$$F_Y(y) = P(-X \leq ln(y))$$

$$F_Y(y) = P(X > -ln(y))$$
$$F_Y(y) = 1 - P(X \leq -ln(y))$$
$$F_Y(y) = 1 - F_X(-ln(y))$$
$$f_Y(y) = \frac{f_X(-ln(y))}{y}$$
$$f_Y(y) = \frac{y}{y}$$
$$f_Y(y) = 1$$

4

(a)

$$f_X(x) = 3e^{-3x},\ 0 \leq X < \infty$$
$$Y = X + 4$$

The support for Y is:

$$4 \leq Y < \infty$$

(b)

$$F_Y(y) = P(Y \leq y)$$
$$F_Y(y) = P(X + 4 \leq y)$$
$$F_Y(y) = P(X \leq y - 4)$$
$$F_Y(y) = F_X(y - 4)$$
$$f_Y(y) = f_X(y - 4)$$
$$f_Y(y) = 3e^{-3(y-4)}$$

5

(a)

$$f_X(x) = 5e^{-5x},\ 0 \leq X < \infty$$
$$Y = \frac{1}{X}$$

The support for Y is:

$$0 < Y < \infty$$

(b)

$$F_Y(y) = P(Y \leq y)$$
$$F_Y(y) = P\left(\frac{1}{X} \leq y\right)$$
$$F_Y(y) = P\left(\frac{1}{y} \leq X\right)$$
$$F_Y(y) = 1 - P\left(X \leq \frac{1}{y}\right)$$
$$F_Y(y) = 1 - F_X\left(\frac{1}{y}\right)$$
$$f_Y(y) = -f_X\left(\frac{1}{y}\right) * \frac{-1}{y^2}$$
$$f_Y(y) = 5e^{-\frac{5}{y}} * \frac{1}{y^2}$$
$$f_Y(y) = \frac{5e^{-\frac{5}{y}}}{y^2}$$

6

$$F_Y(y) = P(Y \leq y)$$
$$F_Y(y) = P(e^{2X} \leq y)$$
$$F_Y(y) = P(2X \leq ln(y))$$
$$F_Y(y) = F_X(\frac{ln(y)}{2})$$
$$f_Y(y) = f_X(\frac{ln(y)}{2}) * \frac{1}{2y}$$
$$f_Y(y) = \frac{1}{\sqrt{2\pi}}\ exp\left(\frac{x^2}{2}\right) * \frac{1}{2y}$$

7

(a)

$$f_X(x) = \frac{1}{4},\ X \in [-2, 2]$$
$$F_X(x) = \frac{1}{4}(x+2)$$
$$Y = X^2$$

The support for Y is:

$$0 \leq Y \leq 4$$

(b)

$$Y \in [0, 4]$$
$$F_Y(y) = P(Y \leq y)$$
$$F_Y(y) = P(X^2 \leq y)$$
$$F_Y(y) = P(-\sqrt{y} \leq X \leq \sqrt{y})$$
$$F_Y(y) = P(X \leq \sqrt{y}) - P(X \leq -\sqrt{y})$$
$$F_Y(y) = F_X(\sqrt{y}) - F_X(-\sqrt{y})$$
$$F_Y(y) = \frac{1}{4}(\sqrt{y} + 2) - \frac{1}{4}(-\sqrt{y} + 2)$$
$$F_Y(y) = \frac{\sqrt{y}}{2}$$

(c)

$$f_Y(y) = \frac{1}{4\sqrt{y}}$$

(d)

$$E[Y] = \int_0^4 \frac{y}{4\sqrt{y}}\ dy$$
$$E[Y] = \frac{1}{4}\int_0^4 \sqrt{y}\ dy$$
$$E[Y] = \frac{1}{4}\left[\frac{2y^{\frac{3}{2}}}{3}\right]_0^4$$
$$E[Y] = \frac{1}{6}\left[y^{\frac{3}{2}}\right]_0^4$$
$$E[Y] = \frac{1}{6}(4)^{\frac{3}{2}}$$
$$E[Y] = \frac{4}{3}$$

$$E[X^2] = \int_{-2}^{2} \frac{x^2}{4}\ dx$$
$$E[X^2] = \frac{1}{4}\int_{-2}^{2} x^2\ dx$$
$$E[X^2] = \frac{1}{12}\left[x^3\right]_{-2}^{2}$$
$$E[X^2] = \frac{8+8}{12}$$
$$E[X^2] = \frac{4}{3}$$

$$(E[X])^2 = \left(\int_{-2}^{2} \frac{x}{4}\ dx\right)^2$$
$$(E[X])^2 = \left(\frac{1}{4}\int_{-2}^{2} x\ dx\right)^2$$

$$(E[X])^2 = \left(\frac{1}{8}\left[x^2\right]_{-2}^{2}\right)^2$$

$$(E[X])^2 = \left(\frac{1}{8} * 0\right)^2$$

$$(E[X])^2 = 0$$

$$E[X^2] \neq (E[X])^2$$

8

(a)

$$f_X(x) = \frac{1}{3},\ X \in [-1, 2]$$

$$F_X(x) = \frac{1}{3}(x+1)$$

$$Y = |X|$$

The support for Y is:

$$Y \in [0, 2]$$

(b)

$$F_Y(y) = P(Y \leq y)$$

$$F_Y(y) = P(|X| \leq y)$$

Case where $y \in [0, 1]$:

$$F_Y(y) = P(-y \leq X \leq y)$$

$$F_Y(y) = P(X \leq y) - P(X \leq -y)$$

$$F_Y(y) = \frac{1}{3}(y+1) - \frac{1}{3}(-y+1)$$

$$F_Y(y) = \frac{1}{3}y + \frac{1}{3}y$$

$$F_Y(y) = \frac{2}{3}y$$

Case where $y \in [1, 2]$:

$$F_Y(y) = P(-1 \leq X \leq y)$$

$$F_Y(y) = P(X \leq y) - P(X \leq -1)$$

$$F_Y(y) = P(X \leq y) - P(X \leq -1)$$

$$F_Y(y) = F_X(y) - F_X(-1)$$

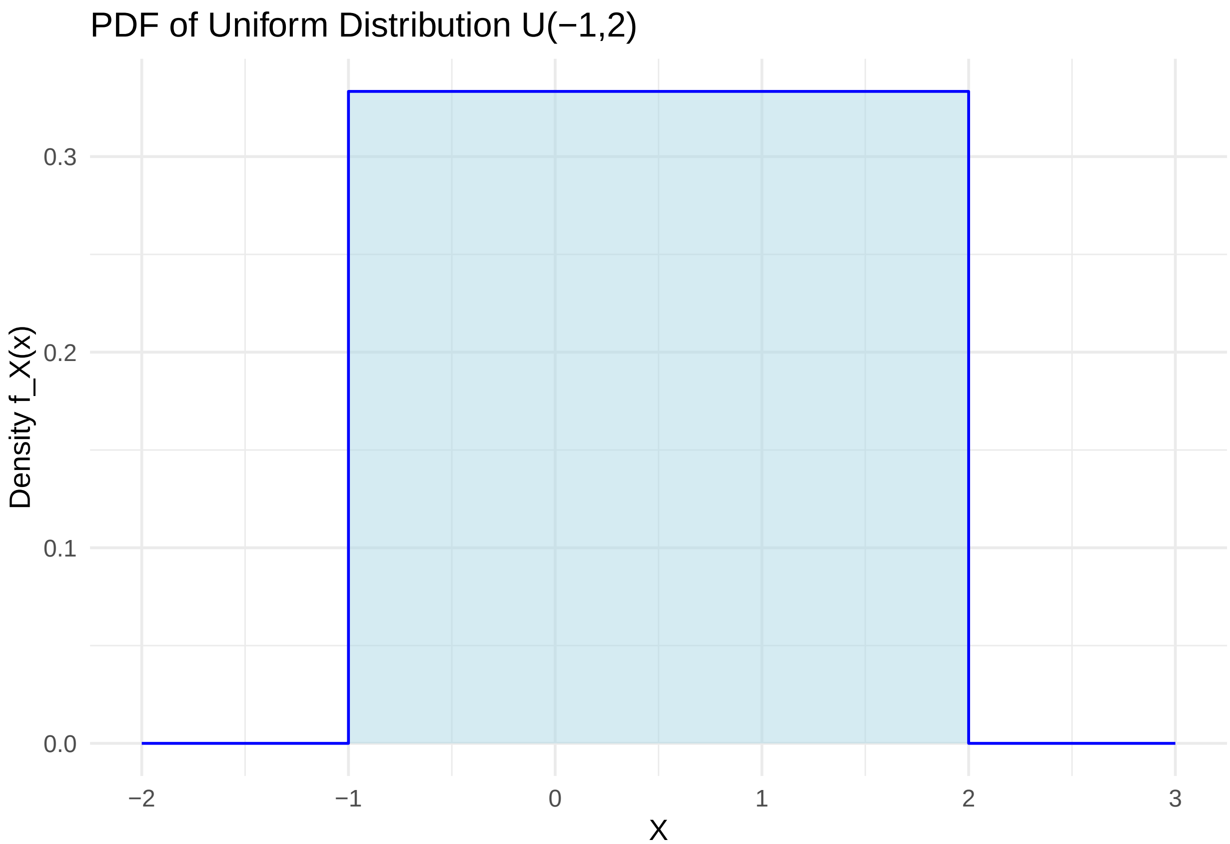
$$F_Y(y) = \frac{y+1}{3}$$

Put it together:

$$F_Y(y) = \begin{cases} \frac{2}{3}y & \text{if } y \in [0, 1] \\ \frac{y+1}{3} & \text{if } y \in [1, 2] \end{cases}$$

(c)

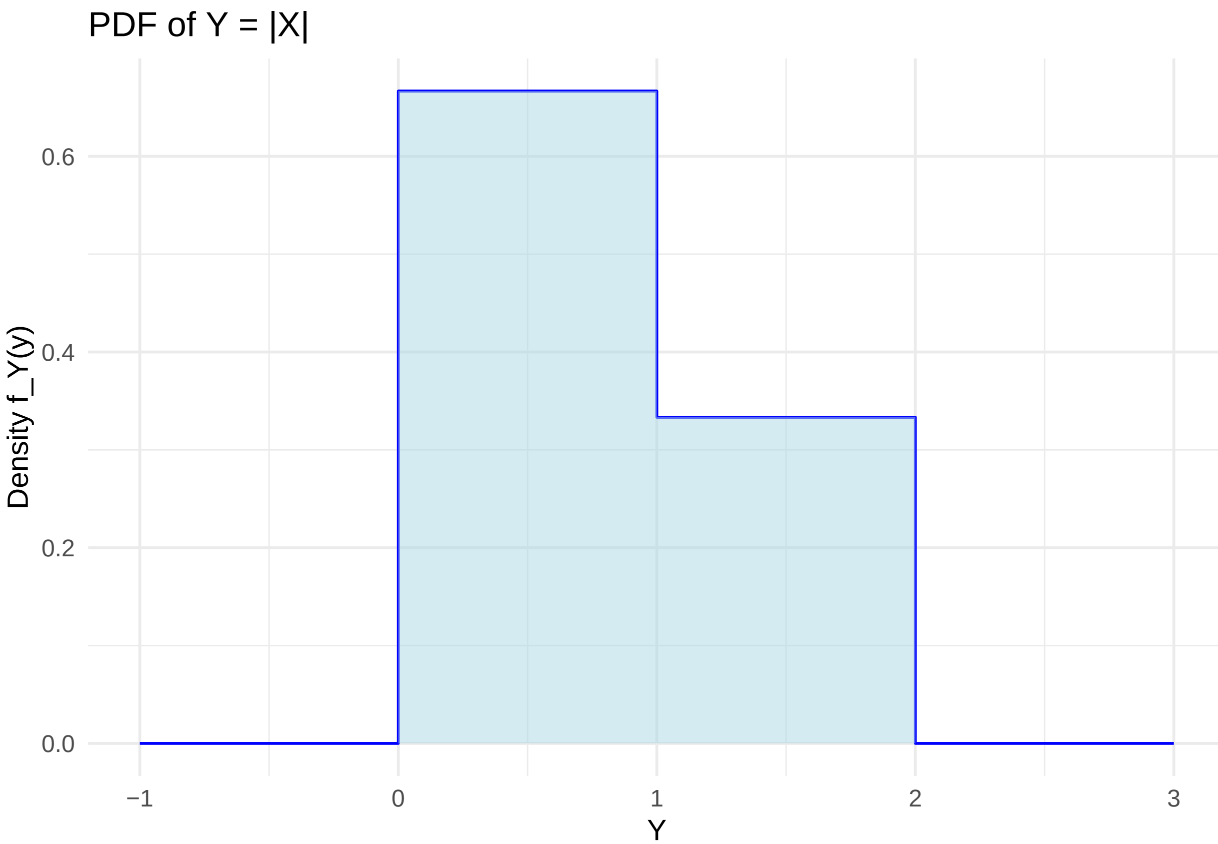
$$f_Y(y) = \begin{cases} \frac{2}{3} & \text{if } y \in [0,1] \\ \frac{1}{3} & \text{if } y \in [1,2] \end{cases}$$



(d)

```
## Warning in geom_segment(aes(x = 0, xend = 1, y = 2/3, yend = 2/3), color = "blue"): All aesthetics ha
## i Please consider using `annotate()` or provide this layer with data containing a single row.

## Warning in geom_segment(aes(x = 1, xend = 2, y = 1/3, yend = 1/3), color = "blue"): All aesthetics ha
## i Please consider using `annotate()` or provide this layer with data containing a single row.
```



9

(a)

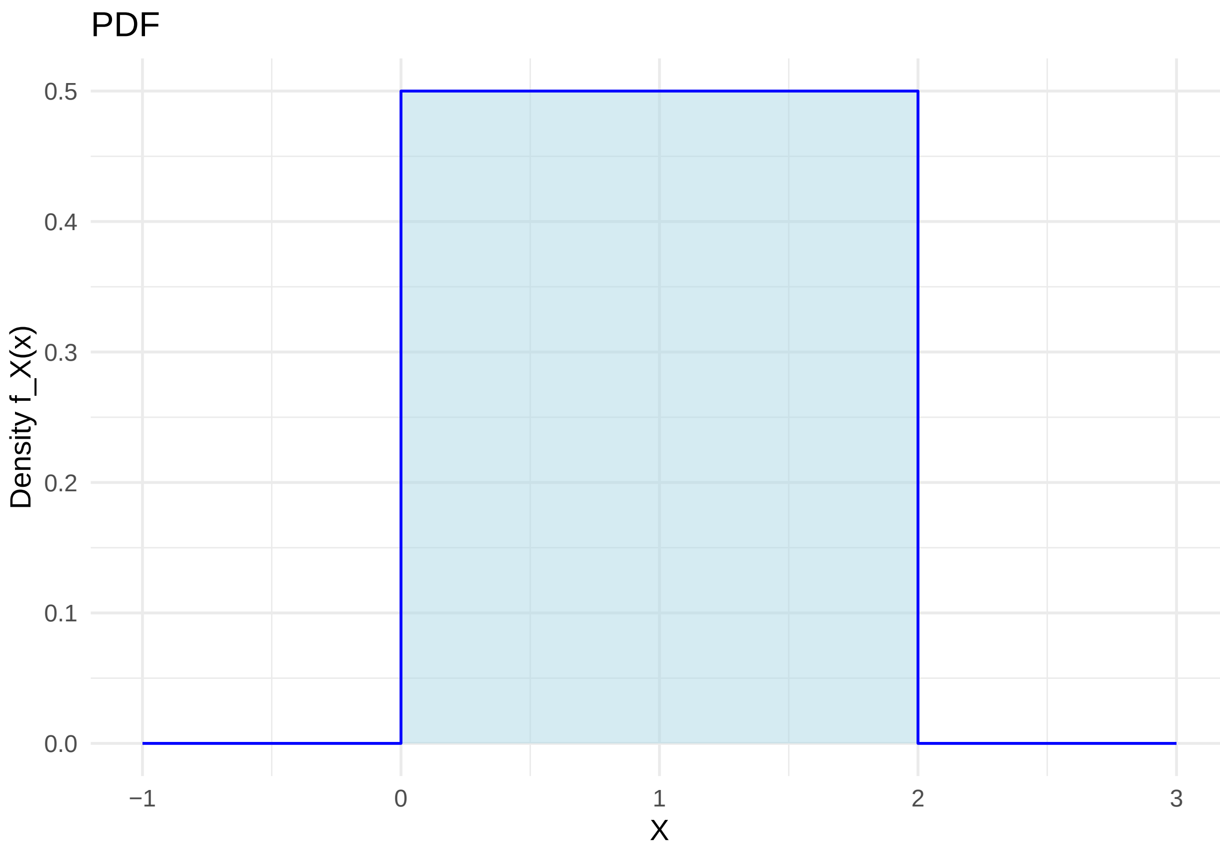
$$f_X(x) = \frac{1}{2},\ X \in [0, 2]$$
$$Y = 3X - 2$$

The support for Y is:

$$Y \in [-2, 4]$$

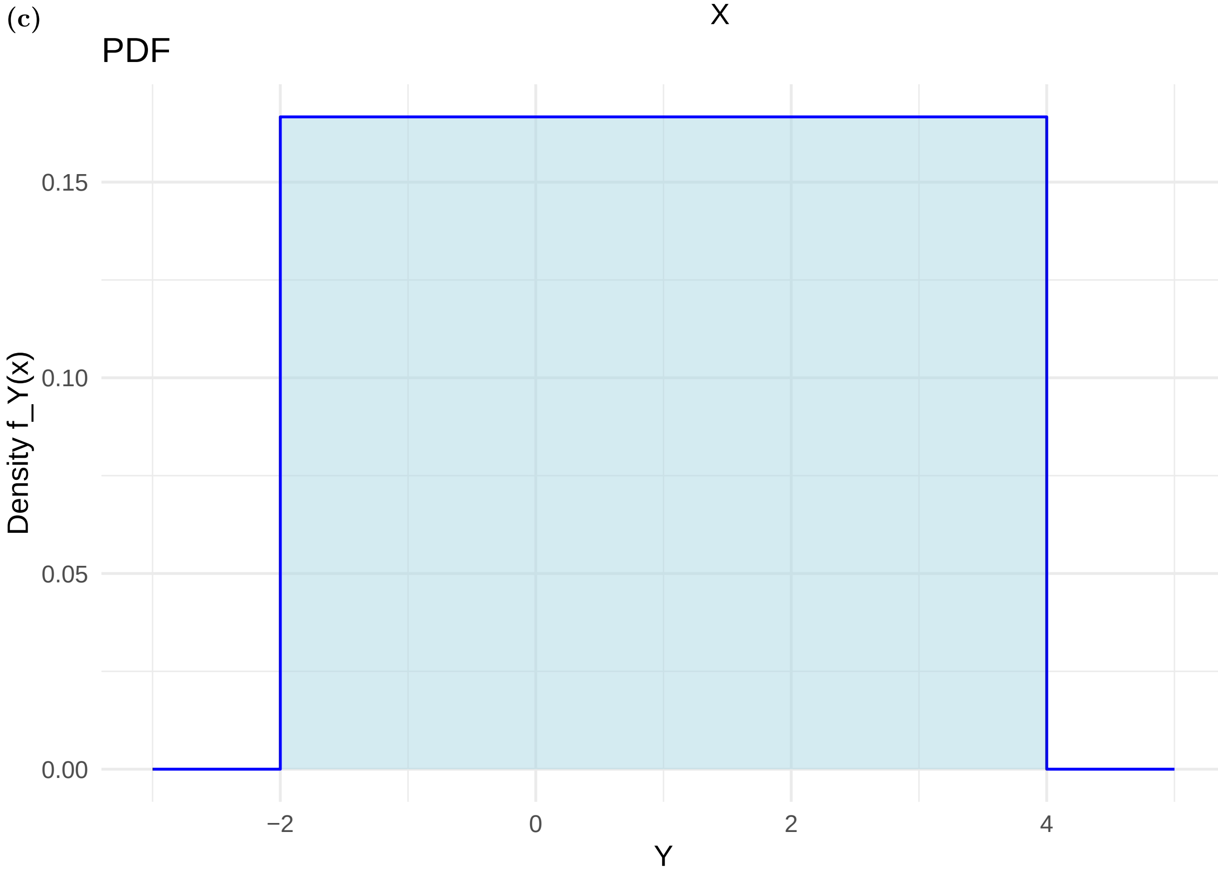
(b)

$$F_Y(y) = P(Y \leq y)$$
$$F_Y(y) = P(3X - 2 \leq y)$$
$$F_Y(y) = P(3X \leq y + 2)$$
$$F_Y(y) = P(X \leq \frac{y+2}{3})$$
$$F_Y(y) = F_X(\frac{y+2}{3})$$
$$f_Y(y) = \frac{1}{3} f_X(\frac{y+2}{3})$$
$$f_Y(y) = \frac{1}{3} * \frac{1}{2}$$
$$f_Y(y) = \frac{1}{6}$$



PDF
Density f_X(x)
0.0
0.1
0.2
0.3
0.4
0.5
−1
0
1
2
3
X

(c)



PDF
Density f_Y(x)
0.00
0.05
0.10
0.15
−2
0
2
4
Y

10

(a)

$$M_X(t) = E[e^{tX}] = \sum_x e^{tx} P(X = x)$$

$$M_X(t) = e^{-1t} * 0.4 + e^{0t} * 0.2 + e^{1t} * 0.4$$

$$M_X(t) = e^{-t} * 0.4 + 0.2 + e^{t} * 0.4$$

$$M_X(t) = 0.4e^{t} + 0.4e^{-t} + 0.2$$

(b)

$$M_Y(t) = [M_X(t)]^2$$

$$M_Y(t) = [0.4e^{t} + 0.4e^{-t} + 0.2]^2$$

$$M_Y(t) = 0.16e^{-2t} + 0.16e^{-t} + 0.36 + 0.16e^{t} + 0.16e^{2t}$$

(c)

y	f_Y
-2	0.16
-1	0.16
0	0.36
1	0.16
2	0.16