# 1

**(a)** img

It looks like a normal distribution with inverted concavity. As we increase $\mu$, it shifts right on the x-axis.

**(b)**

$$L(\mu; x) = \prod_{i=1}^{n} \frac{1}{4} exp\left(-\frac{|x_i - \mu|}{2}\right)$$

$$l(\mu; x) = \sum_{i=1}^{n} \left(log\left(\frac{1}{4}\right) - \frac{|x_i - \mu|}{2}\right)$$

The function is not differentiable over the range of possible parameters when $x = \mu$.

**(c)** To maximize $l$, we want to minimize:

$$\sum_{i=1}^{n} |x_i - \mu|$$

We can minimize this by selecting the following:

$$\hat{\mu}_{mle} = median(x_1, ..., x_n)$$

# 2

### (a) img

As we increase $\lambda$ from $\lambda = 1$, the concavity gets steeper. The graph also reflects over the y-axis.

### (b)

$$L(\lambda; x) = \prod_{i=1}^{n} \frac{\lambda}{2} |x_i|^{\lambda-1}$$

$$L(\lambda; x) = \left(\frac{\lambda}{2}\right)^n \prod_{i=1}^{n} |x_i|^{\lambda-1}$$

$$l(\lambda; x) = n\ log\left(\frac{\lambda}{2}\right) + \sum_{i=1}^{n} (\lambda - 1) log|x_i|$$

$$l(\lambda; x) = n\ log\left(\frac{\lambda}{2}\right) + (\lambda - 1) \sum_{i=1}^{n} log|x_i|$$

Function is differentiable over the range of all parameters.

### (c)

$$\frac{\delta l(\lambda; x)}{\delta \lambda} = \frac{\frac{1}{2}}{\frac{\lambda}{2}} n + \sum_{i=1}^{n} log|x_i|$$

$$\frac{\delta l(\lambda; x)}{\delta \lambda} = \frac{n}{\lambda} + \sum_{i=1}^{n} log|x_i| = 0$$

$$\sum_{i=1}^{n} log|x_i| = -\frac{n}{\lambda}$$

$$\hat{\lambda}_{mle} = -\frac{n}{\sum_{i=1}^{n} log|x_i|}$$

### (d)

$$log(f(\lambda; X)) = log(\lambda) - log(2) + (\lambda - 1) log|X|$$

$$\frac{\delta log(f(\lambda; X))}{\delta \lambda} = \frac{1}{\lambda} + log|X|$$

$$\frac{\delta^2 log(f(\lambda; X))}{\delta \lambda^2} = -\frac{1}{\lambda^2}$$

$$I(\lambda) = -E\left[-\frac{1}{\lambda^2}\right]$$

$$I(\lambda) = \frac{1}{\lambda^2}$$

**(e)**

$$\sqrt{n}(\hat{\lambda}_{mle} - \lambda) \xrightarrow{d} N\left(0, \lambda^2\right)$$

## 3

### (a)

$$L(\theta; x) = \prod_{i=1}^{n} (x_i + 1)\theta^2 (1-\theta)^{x_i}$$

$$L(\theta; x) = \theta^{2n} \prod_{i=1}^{n} (x_i + 1)(1-\theta)^{x_i}$$

$$l(\theta; x) = 2n\ log(\theta) + \sum_{i=1}^{n} (log(x_i + 1) + x_i\ log(1-\theta))$$

$$\frac{\delta l(\theta; x)}{\delta \theta} = \frac{2n}{\theta} + \sum_{i=1}^{n} \frac{-x_i}{1-\theta} = 0$$

$$\frac{2n}{\theta} = \sum_{i=1}^{n} \frac{x_i}{1-\theta}$$

$$2n(1-\theta) = \theta \sum_{i=1}^{n} x_i$$

$$2n = 2n\theta + \theta \sum_{i=1}^{n} x_i$$

$$2n = \left(2n + \sum_{i=1}^{n} x_i\right) \theta$$

$$\hat{\theta}_{mle} = \frac{2n}{2n + \sum_{i=1}^{n} x_i}$$

$$\hat{\theta}_{mle} = \frac{2}{2 + \bar{X}}$$

### (b)

$$log(f(\theta; X)) = log(X + 1) + 2\ log(\theta) + X\ log(1-\theta)$$

$$\frac{\delta log(f(\theta; X))}{\delta \theta} = \frac{2}{\theta} - \frac{X}{1-\theta}$$

$$\frac{\delta^2 log(f(\theta; X))}{\delta \theta^2} = -\frac{2}{\theta^2} - \frac{X}{(1-\theta)^2}$$

$$I(\theta) = -E\left[-\frac{2}{\theta^2} - \frac{X}{(1-\theta)^2}\right]$$

$$I(\theta) = E\left[\frac{2}{\theta^2} + \frac{X}{(1-\theta)^2}\right]$$

$$I(\theta) = \frac{2}{\theta^2} + \frac{2 * \frac{1-\theta}{\theta}}{(1-\theta)^2}$$

$$I(\theta) = \frac{2}{\theta^2} + \frac{2(1-\theta)}{\theta(1-\theta)^2}$$

$$I(\theta) = \frac{2}{\theta^2} + \frac{2}{\theta(1-\theta)}$$

$$I(\theta) = \frac{2(1-\theta)}{\theta^2(1-\theta)} + \frac{2\theta}{\theta^2(1-\theta)}$$

$$I(\theta) = \frac{2}{\theta^2(1-\theta)}$$

**(c)**

$$\sqrt{n}(\hat{\theta}_{mle} - \theta) \xrightarrow{d} N\left(0,\ \frac{\theta^2(1-\theta)}{2}\right)$$