1

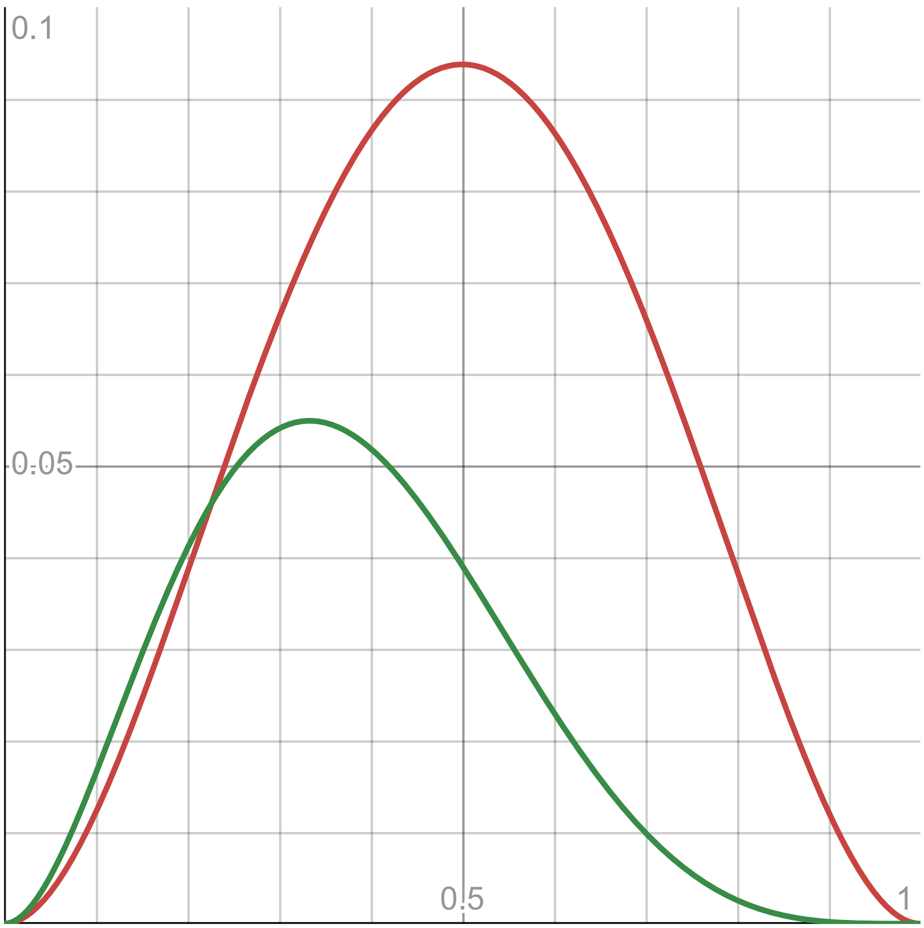

Figure 1: img

(a) As θ increased, the height of the curve decreased.

(b)

$$L(\theta) = \prod_{i=1}^{n} \frac{\theta+1}{2} X_i^2 (1-X_i)^\theta$$

$$l(\theta) = ln\left(\prod_{i=1}^{n} \frac{\theta+1}{2} X_i^2 (1-X_i)^\theta\right)$$

$$l(\theta) = \sum_{i=1}^{n} ln\left(\frac{\theta+1}{2} X_i^2 (1-X_i)^\theta\right)$$

$$l(\theta) = \sum_{i=1}^{n} \left(ln\left(\frac{\theta+1}{2}\right) + ln\left(X_i^2\right) + ln\left((1-X_i)^{\theta}\right) \right)$$

$$l(\theta) = \sum_{i=1}^{n} \left(ln\left(\frac{\theta+1}{2}\right) + 2\ ln\left(X_i\right) + \theta\ ln\left(1-X_i\right) \right)$$

$$l(\theta) = n\ ln\left(\frac{\theta+1}{2}\right) + 2\sum_{i=1}^{n} ln\left(X_i\right) + \theta \sum_{i=1}^{n} ln\left(1-X_i\right)$$

$$\frac{\delta l(\theta)}{\delta \theta} = \frac{n}{\theta+1} + \sum_{i=1}^{n} ln(1-X_i) = 0$$

$$\hat{\theta}_{MLE} = \frac{-n}{\sum_{i=1}^{n} ln(1-X_i)} - 1$$

(c)

$$\hat{\eta}_{MLE} = \sqrt{\hat{\theta}_{MLE} + 1}$$

2

(a) As ξ increased, the height of the curve decreased.

(b)

$$L(\xi) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\xi} exp\left(-\frac{1}{2}\left(\frac{X_i-2}{\xi}\right)^2\right)$$

$$l(\xi) = ln\left(\prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\xi} exp\left(-\frac{1}{2}\left(\frac{X_i-2}{\xi}\right)^2\right)\right)$$

$$l(\xi) = \sum_{i=1}^{n} \left(ln\left(\frac{1}{\sqrt{2\pi}\xi}\right) - \frac{1}{2}\left(\frac{X_i-2}{\xi}\right)^2 \right)$$

$$l(\xi) = -n\ ln\left(\sqrt{2\pi}\xi\right) - \frac{1}{2}\sum_{i=1}^{n}\left(\frac{X_i-2}{\xi}\right)^2$$

$$l(\xi) = -n\ ln\sqrt{2\pi} - n\ ln\ \xi - \frac{1}{2}\sum_{i=1}^{n}\left(\frac{X_i-2}{\xi}\right)^2$$

$$\frac{\delta l(\xi)}{\delta \xi} = -\frac{n}{\xi} + \sum_{i=1}^{n} \frac{(X_i-2)^2}{\xi^3} = 0$$

$$\frac{1}{\xi^3}\sum_{i=1}^{n}(X_i-2)^2 = \frac{n}{\xi}$$

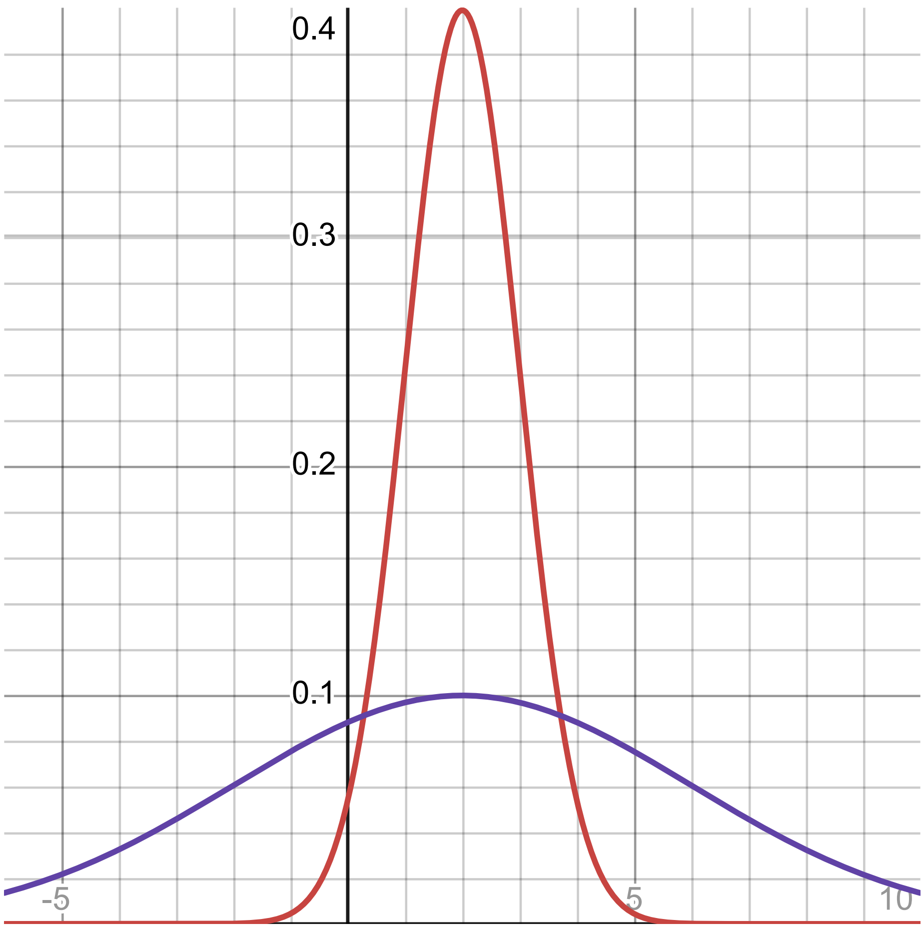

Figure 2: img

$$\sum_{i=1}^{n}(X_i - 2)^2 = n\xi^2$$

$$\hat{\xi}_{MLE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - 2)^2}$$

(c)

$$\hat{\alpha}_{MLE} = log(\hat{\xi}_{MLE})$$

3

Figure 3: img

(a) As β increases, the line is lower but is also longer.

(b)

$$L(\beta) = \begin{cases} \left(\frac{1}{\beta}\right)^n & \text{if } \beta < min(X_i) \text{ and } max(X_i) < 2\beta \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{max(X_i)}{2} < \beta < min(X_i)$$

We know that $L(\beta)$ is monotonic. To maximize $L(\beta)$, we want to minimize β.

$$\hat{\beta}_{MLE} = \begin{cases} \frac{max(X_i)}{2} & \text{if } \frac{max(X_i)}{2} < min(X_i) \\ DNE & \text{otherwise} \end{cases}$$

4

(a) As θ increases, the line is higher and also shorter.

(b)

$$L(\theta) = \begin{cases} \frac{1}{1-\theta} & \text{if } \theta < min(X_i) \text{ and } max(X_i) < 1 \\ 0 & \text{otherwise} \end{cases}$$

To maximize $L(\theta)$, we want θ to be as close to 1 as possible.

$$\hat{\theta}_{MLE} = \begin{cases} min(X_i) & \text{if } max(X_i) < 1 \\ DNE & \text{otherwise} \end{cases}$$

5

(a) As σ increases, the graph starts more towards the right and is higher.

(b)

$$F(x) = \int_{-\infty}^{x} \frac{3\sigma^3}{z^4}\, dz$$

$$F(x) = \int_{\sigma}^{x} \frac{3\sigma^3}{z^4}\, dz$$

$$F(x) = 3\sigma^3 \int_{\sigma}^{x} \frac{1}{z^4}\, dz$$

$$F(x) = \left[3\sigma^3 * \frac{-1}{3z^3}\right]_{\sigma}^{x}$$

$$F(x) = \left[\frac{-\sigma^3}{z^3}\right]_{\sigma}^{x}$$

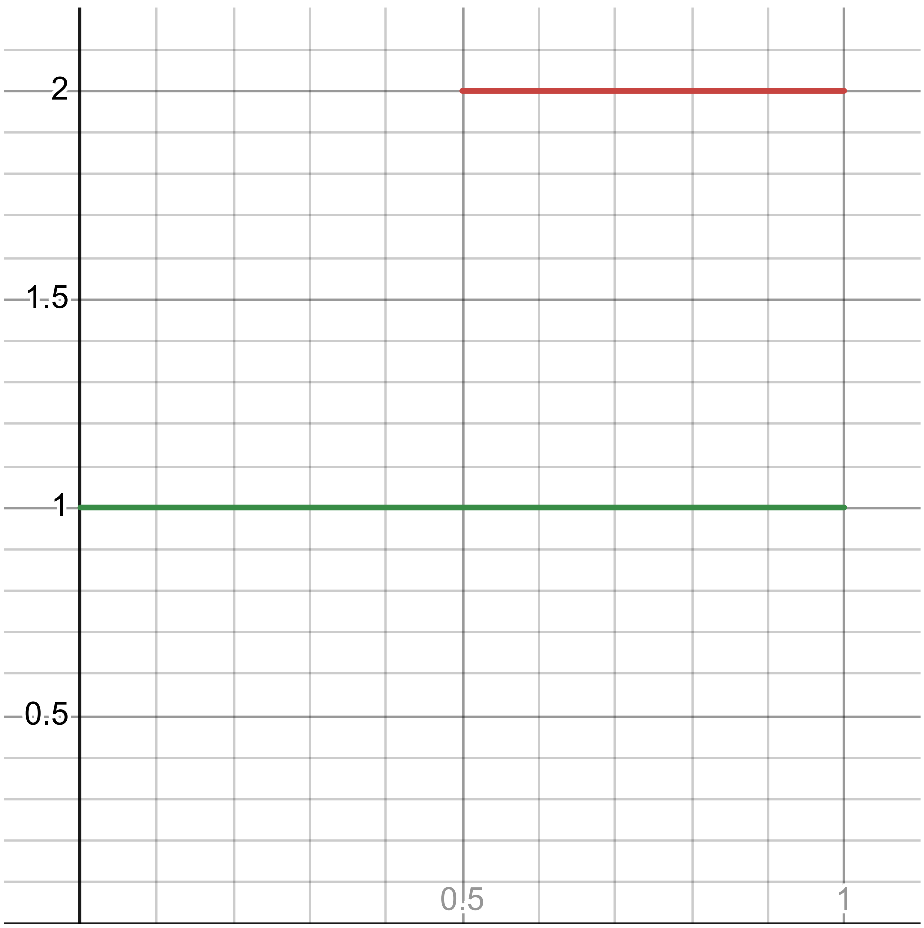

Figure 4: img

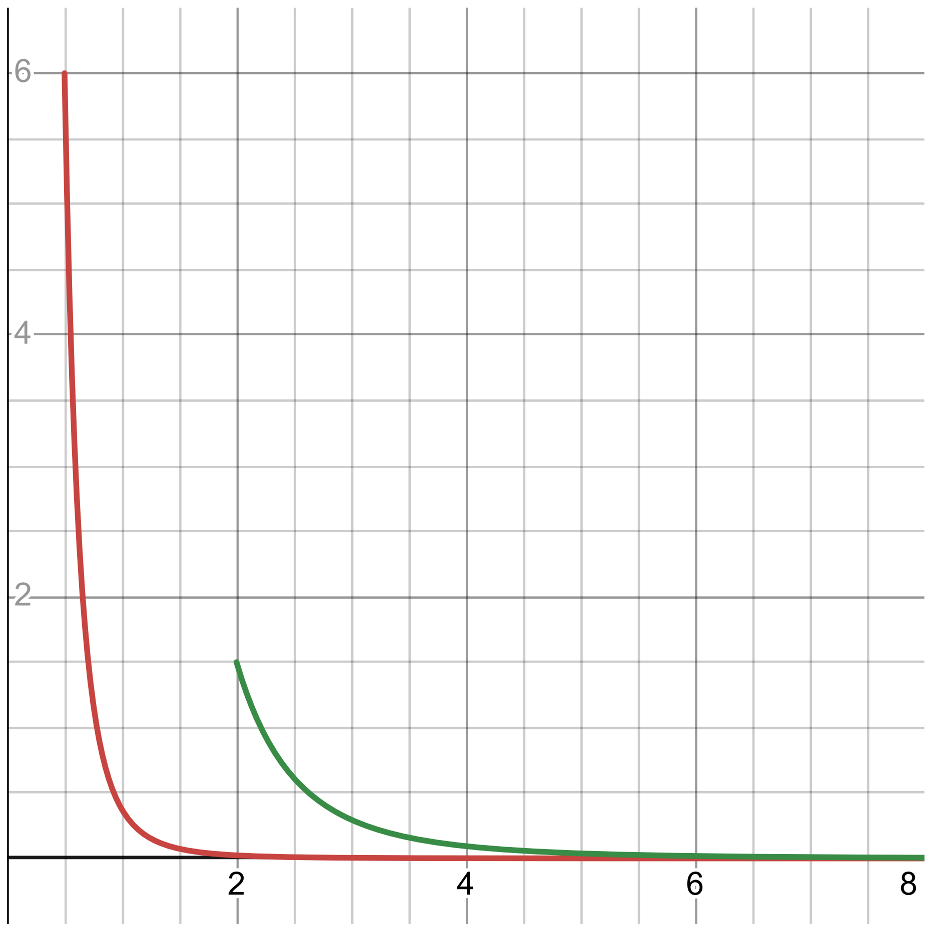

Figure 5: img

$$F(x) = \frac{-\sigma^3}{x^3} + 1$$

$$F(x) = \begin{cases} 1 - \frac{\sigma^3}{x^3} & \text{if } x > \sigma \\ 0 & \text{otherwise} \end{cases}$$

(c)

$$E[X_1] = \int_\sigma^\infty x \ \frac{3\sigma^3}{x^4} \ dx = \int_\sigma^\infty \frac{3\sigma^3}{x^3} \ dx$$

$$E[X_1] = 3\sigma^3 \int_\sigma^\infty \frac{1}{x^3} \ dx$$

$$E[X_1] = 3\sigma^3 \left[\frac{-1}{2x^2}\right]_\sigma^\infty$$

$$E[X_1] = \frac{3\sigma^3}{2\sigma^2}$$

$$E[X_1] = \frac{3\sigma}{2}$$

(d)

$$L(\sigma) = \begin{cases} \prod_{i=1}^n \frac{3\sigma^3}{X_i^4} & \text{if } min(X_i) > \sigma \\ 0 & \text{otherwise} \end{cases}$$

To maximize $L(\sigma)$, we want to maximize σ.

$$\hat{\sigma}_{MLE} = min(X_i)$$

6

$$\pi_2 = 4\pi_1$$

$$\pi_1 + \pi_2 + \pi_3 = 1$$

$$(\hat{\pi}_1)_{MLE} = \ ?$$

$$\pi_3 = 1 - \pi_2 - \pi_1$$

$$\pi_3 = 1 - 4\pi_1 - \pi_1$$

$$\pi_3 = 1 - 5\pi_1$$

$$L(\pi) = \pi_1^{Y_1}\pi_2^{Y_2}\pi_3^{Y_3}$$

$$L(\pi) = \pi_1^{Y_1}(4\pi_1)^{Y_2}(1 - 5\pi_1)^{Y_3}$$

$$l(\pi) = log(\pi_1^{Y_1}) + log((4\pi_1)^{Y_2}) + log((1 - 5\pi_1)^{Y_3})$$

$$l(\pi) = Y_1 log(\pi_1) + Y_2 log(4\pi_1) + Y_3 log(1 - 5\pi_1)$$
$$l(\pi) = Y_1 log(\pi_1) + Y_2 log(4) + Y_2 log(\pi_1) + Y_3 log(1 - 5\pi_1)$$
$$l(\pi) = (Y_1 + Y_2) log(\pi_1) + Y_3 log(1 - 5\pi_1) + Y_2 log(4)$$
$$\frac{\delta l(\pi)}{\delta \pi} = \frac{Y_1 + Y_2}{\pi_1} - \frac{5Y_3}{1 - 5\pi_1} = 0$$
$$\frac{Y_1 + Y_2}{\pi_1} = \frac{5Y_3}{1 - 5\pi_1}$$
$$(Y_1 + Y_2)(1 - 5\pi_1) = 5Y_3\pi_1$$
$$Y_1 + Y_2 - 5\pi_1 Y_1 - 5\pi_1 Y_2 = 5Y_3\pi_1$$
$$Y_1 + Y_2 = \pi_1 * 5(Y_1 + Y_2 + Y_3)$$
$$(\hat{\pi}_1)_{MLE} = \frac{Y_1 + Y_2}{5(Y_1 + Y_2 + Y_3)} = \frac{Y_1 + Y_2}{5n}$$

7

$$\pi_2 = 2\pi_1$$
$$\pi_4 = 3\pi_1$$
$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$$
$$L(\pi_1) = \pi_1^{Y_1} \pi_2^{Y_2} \pi_3^{Y_3} \pi_4^{Y_4}$$

(a)

$$(\hat{\pi}_1)_{MLE} = ?$$
$$\pi_3 = 1 - \pi_4 - \pi_2 - \pi_1$$
$$\pi_3 = 1 - 3\pi_1 - 2\pi_1 - \pi_1$$
$$\pi_3 = 1 - 6\pi_1$$
$$L(\pi_1) = \pi_1^{Y_1} (2\pi_1)^{Y_2} (1 - 6\pi_1)^{Y_3} (3\pi_1)^{Y_4}$$
$$l(\pi_1) = Y_1 log(\pi_1) + Y_2 log(2\pi_1) + Y_3 log(1 - 6\pi_1) + Y_4 log(3\pi_1)$$
$$l(\pi_1) = (Y_1 + Y_2 + Y_4) log(\pi_1) + Y_2 log(2) + Y_3 log(1 - 6\pi_1) + Y_4 log(3)$$
$$\frac{\delta l(\pi_1)}{\delta \pi_1} = \frac{Y_1 + Y_2 + Y_4}{\pi_1} - \frac{6Y_3}{1 - 6\pi_1} = 0$$
$$\frac{Y_1 + Y_2 + Y_4}{\pi_1} = \frac{6Y_3}{1 - 6\pi_1}$$
$$(Y_1 + Y_2 + Y_4)(1 - 6\pi_1) = 6Y_3\pi_1$$
$$Y_1 + Y_2 + Y_4 = 6n\pi_1$$
$$(\hat{\pi}_1)_{MLE} = \frac{Y_1 + Y_2 + Y_4}{6n}$$

(b)

$$(\hat{\pi}_3)_{MLE} = ?$$

$$\pi_1 = \frac{1-\pi_3}{6}$$

$$\pi_2 = \frac{1-\pi_3}{3}$$

$$\pi_4 = \frac{1-\pi_3}{2}$$

$$L(\pi_3) = \left(\frac{1-\pi_3}{6}\right)^{Y_1} \left(\frac{1-\pi_3}{3}\right)^{Y_2} \pi_3^{Y_3} \left(\frac{1-\pi_3}{2}\right)^{Y_4}$$

$$l(\pi_3) = Y_1 \ log\left(\frac{1-\pi_3}{6}\right) + Y_2 \ log\left(\frac{1-\pi_3}{3}\right) + Y_3 \ log(\pi_3) + Y_4 \ log\left(\frac{1-\pi_3}{2}\right)$$

$$l(\pi_3) = (Y_1+Y_2+Y_4) \ log\,(1-\pi_3) + Y_3 \ log(\pi_3) - Y_1 \ log(6) - Y_2 \ log(3) - Y_4 \ log(2)$$

$$\frac{\delta l(\pi_3)}{\delta \pi_3} = \frac{-(Y_1+Y_2+Y_4)}{1-\pi_3} + \frac{Y_3}{\pi_3} = 0$$

$$\frac{Y_3}{\pi_3} = \frac{Y_1+Y_2+Y_4}{1-\pi_3}$$

$$Y_3(1-\pi_3) = (Y_1+Y_2+Y_4)\pi_3$$

$$Y_3 = (Y_1+Y_2+Y_3+Y_4)\pi_3$$

$$(\hat{\pi}_3)_{MLE} = \frac{Y_3}{n}$$

8

$$L(\theta) = \begin{cases} \frac{1}{\theta^{27}} & \text{if } max(X_i) \le \theta \\ 0 & \text{otherwise} \end{cases}$$

$$max(X_i) = max(9.2, 3) = 9.2$$

To maximize $L(\theta)$, we want to minimize θ.

$$\hat{\theta}_{MLE} = 9.2$$