Homework 4

STA 131B | A. Farris | Winter 2025

A pdf copy of your homework solutions is due at 5pm on Friday, February 14. Submission of the pdf will be through Gradescope. You must write up and turn in your own solutions, but if you do work on them with some of your classmates, list their names as collaborators on your submission. You must explain your answers for full credit.

Problems

1. Suppose that $X \sim \mathcal{U}(0, 2)$,[1] and that $Y = 2 - X$. What is the distribution of Y?

2. Suppose that X is a discrete random variable with PMF given below, and that $Y = X^2$.

x	$f_X(x)$
-2	0.4
-1	0.1
0	0.2
1	0.1
2	0.2

   (a) Draw a picture (i.e. a histogram) of the distribution of X.

   (b) What is the distribution of Y? Draw its histogram.

   (c) What is $\mathrm{E}(Y)$?

3. Suppose that $X \sim \mathrm{Exp}(1)$, and that $Y = e^{-X}$.

   (a) What is the support of Y?

   (b) What is the PDF of Y?

4. Suppose that $X \sim \mathrm{Exp}(3)$, and that $Y = X + 4$.

   (a) What is the support of Y?

   (b) What is the PDF of Y?

5. Suppose that $X \sim \mathrm{Exp}(5)$, and that $Y = 1/X$.

   (a) What is the support of Y?

   (b) What is the PDF of Y?

6. Suppose that $X \sim \mathcal{N}(0, 1)$, [2] and that $Y = e^{2X}$. What is the PDF of Y?

[1] *Note:* The PDF for the $\mathcal{U}(a, b)$ distribution assigns constant density $f(x) = \frac{1}{b-a}$ over support $x \in (a, b)$.

[2] *Note:* The PDF for the $\mathcal{N}(0, 1)$ distribution is $f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{x^2}{2}\right)$ over support $x \in (-\infty, \infty)$.

7. Suppose that $X \sim \mathcal{U}(-2, 2)$, and that $Y = X^2$.
   (a) What is the support of Y?
   (b) What is the CDF of Y?
   (c) What is the PDF of Y?
   (d) Compute $\mathrm{E}(Y)$. Is $\mathrm{E}\left(X^2\right) = [\mathrm{E}(X)]^2$?

8. Suppose that $X \sim \mathcal{U}(-1, 2)$, and that $Y = |X|$.
   (a) What is the support of Y?
   (b) What is the CDF of Y?
   (c) What is the PDF of Y?
   (d) Draw the graphs of the PDFs for X and for Y.

9. Suppose that $X \sim \mathcal{U}(0, 2)$, and that $Y = 3X - 2$.
   (a) What is the support of Y?
   (b) What is the PDF of Y?
   (c) Draw the graphs of the PDFs for X and for Y.

10. Suppose that X_1 and X_2 are IID, discrete random variables, each with the PMF given in the table below.

x	$f_X(x)$
-1	0.4
0	0.2
1	0.4

   (a) What is the MGF of X_1?
   (b) What is the MGF of $Y = X_1 + X_2$?
   (c) Identify the PMF of $Y = X_1 + X_2$, and define it in a table (as above).