STA 135 (Spring 2025) Homework 2

Due on May 2nd 9:00 am PST. Please submit your HW online to CANVAS.

Problem 1

Suppose that

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \sim \mathcal{N}_3(\boldsymbol{\mu}, \Sigma),$$

where

$$\boldsymbol{\mu} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{pmatrix}.$$

(a). (6 pts) Find the distribution of $z = y_1 + y_2$.

(b). (14 pts) Determine the constant c such that $z_1 = 2y_1 + cy_2$ and $z_2 = 2y_1 + cy_3$ are independent.

Problem 2

Subject Number	y_1	y_2	y_3	y_4	y_5
1	51	36	50	35	42
2	27	20	26	17	27
3	37	22	41	37	30
4	42	36	32	34	27
5	27	18	33	14	29
6	43	32	43	35	40
7	41	22	36	25	38
8	38	21	31	20	16
9	36	23	27	25	28
10	26	31	31	32	36
11	29	20	25	26	25

Figure 1: Data for Problem 2

Assuming that

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{pmatrix} \sim \mathcal{N}_5(\boldsymbol{\mu}, \Sigma),$$

and Σ is unknown. Suppose we observe some data as in Figure 1. Using the above data and choose some proper statistic to test the following null hypothesis

$$\mathbf{H}_0 : \boldsymbol{\mu} = \begin{pmatrix} 30 \\ 25 \\ 40 \\ 25 \\ 30 \end{pmatrix}.$$

(a). (10 pts) What is the value of the testing statistic?

(b). (10 pts) What is the critical value under $\alpha = 0.05$? You can only use R to compute the quantile of some F-distribution.

(c). (5 pts) Should you accept or reject the null hypothesis?