HW2

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1

(a)

```
A <- t(c(1, 1, 0))
mu <- c(0, 0, 0)
Sigma <- rbind(
  c(2, 1, 0),
  c(1, 4, 0),
  c(0, 0, 5)
)

z_mu <- A %*% mu
z_Sigma <- A %*% Sigma %*% t(A)
```

$$z \sim N(0, 8)$$

(b)

$$A = \begin{bmatrix} 2 & c & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 0 & c \end{bmatrix}$$

Note that z_1, z_2 are independent if and only if $Cov(z_1, z_2) = A\Sigma B^T = 0$.

Using Wolfram Alpha, we can see that:

$$A\Sigma B^T = 8 + 2c = 0$$

$$2c = -8$$

$$c = -4$$

2

(a)

```
data <- data.frame(
  y_1 = c(51, 27, 37, 42, 27, 43, 41, 38, 36, 26, 29),
  y_2 = c(36, 20, 22, 36, 18, 32, 22, 21, 23, 31, 20),
  y_3 = c(50, 26, 41, 32, 33, 43, 36, 31, 27, 31, 25),
```

```r
  y_4 = c(35, 17, 37, 34, 14, 35, 25, 20, 25, 32, 26),
  y_5 = c(42, 27, 30, 27, 29, 40, 38, 16, 28, 36, 25)
)
mu_0 <- c(30, 25, 40, 25, 30)

n <- nrow(data)
p <- ncol(data)
y_bar <- colMeans(data)
S <- cov(data)
T2 <- n * t(y_bar - mu_0) %*% solve(S) %*% (y_bar - mu_0)
```

$$T^2 = 85.3327$$

(b)

```r
alpha <- 0.05
F_critical <- qf(1 - alpha, p, n - p)
T_critical <- ((p * (n - 1)) / (n - p)) * F_critical
```

Our critical F value is 4.3874. Our critical T^2 value is 36.5615.

(c)

```r
F_stat <- ((n - p) / (p * (n - 1))) * T2
```

Our F-statistic is 10.2399, which is greater than our critical F value. Therefore, we should reject our null hypothesis.