STA 135 (Spring 2025) Homework 1

Due April 18th 9:00 am PST. Please submit your HW online to CANVAS.

Problem 1

Suppose that

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \sim \mathcal{N}_3(\boldsymbol{\mu}, \Sigma),$$

where

$$\boldsymbol{\mu} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} 6 & 1 & -2 \\ 1 & 13 & 4 \\ -2 & 4 & 4 \end{pmatrix}.$$

(a). (10 pts) Find the distribution of $z = 2y_1 - y_2 + 3y_3$.

(b). (10 pts) Find the joint distribution of y_1 and y_3.

(c). (10 pts) Find the joint distribution y_1, y_3 and $0.5(y_1 + y_3)$.

Problem 2

Suppose $\boldsymbol{y}$ and $\boldsymbol{x}$ are subvectors, each 2×1 that

$$\begin{pmatrix} \boldsymbol{y} \\ \boldsymbol{x} \end{pmatrix} \sim \mathcal{N}_4(\boldsymbol{\mu}, \Sigma),$$

where

$$\boldsymbol{\mu} = \left(\begin{array}{r} 2 \\ -1 \\ \hline 3 \\ 1 \end{array}\right), \qquad \boldsymbol{\Sigma} = \left(\begin{array}{rr|rr} 7 & 3 & -3 & 2 \\ 3 & 6 & 0 & 4 \\ \hline -3 & 0 & 5 & -2 \\ 2 & 4 & -2 & 4 \end{array}\right).$$

(20 pts) Find the conditional distribution of $\boldsymbol{y}|\boldsymbol{x}$.