HW1

January 20, 2025

1 1

[4]:
```
def fib(n):
  if n == 0:
    return "0"
  elif n == 1:
    return "01"

  prevprev = "0"
  prev = "01"
  newstr = "".join([prev, prevprev])
  for i in range(2, n):
    prevprev = prev
    prev = newstr
    newstr = "".join([prev, prevprev])

  return newstr
```

[5]:
```
fib(1)
```

[5]:
```
'01'
```

[6]:
```
fib(3)
```

[6]:
```
'01001'
```

[7]:
```
for x in range(10):
  print(f"S{x} = {fib(x)}")
```

```
S0 = 0
S1 = 01
S2 = 010
S3 = 01001
S4 = 01001010
S5 = 0100101001001
S6 = 010010100100101001010
S7 = 01001010010010100101001001010010
S8 = 0100101001001010010100100101001001010010100100101001010010
```

S9 = 0100101001001010010100100101001001010010100100101001010010100100101001010010
00100101001001

2 2

[9]:
```
def seq_count(seq, idx = 0, prevElem = None, count = 0, highestCount = 0):
  if idx == len(seq):
    # take the highest of the current count and the global highest
    return max(count, highestCount)
  elif idx == 0:
    # start the recursion
    return seq_count(seq, 1, seq[0], 1, 1)

  if prevElem == seq[idx]:
    # add a count
    return seq_count(seq, idx + 1, seq[idx], count + 1, highestCount)
  else:
    # start a new count
    highestCount = max(count, highestCount)
    return seq_count(seq, idx + 1, seq[idx], 1, highestCount)
```

[10]:
```
seq_count([1, 3, 1, 1, 3, 3, 4, 4, 4])
```

[10]: 3

[11]:
```
seq_count((1, 3, 1, 1, 1, '1', 1, [3, 3, 3, 3], 3, 4, 0))
```

[11]: 3

[12]:
```
seq_count([[1], [1], [1], 1, 3, 3, 2, 3, 3, 3, 3, 2, 4, 0])
```

[12]: 4

[13]:
```
seq_count(('G', 'g', 'a', "a", '''a''', 2, 's', 's'))
```

[13]: 3

[14]:
```
seq_count([3, 1, int(True), 1, 1, 2, 3, 3])
```

[14]: 4

[15]:
```
seq_count((1, 3, None, 3, 3, 1, 3, 3, 4, 0))
```

[15]: 2

[23]:
```
# brute force
def pattern_count(seq, pattern, idx = 0, maxCount = 0):
  if idx == len(seq):
```

```
    return maxCount

  # do calculation starting from this idx
  i, j = idx, 0
  count = 0
  while i < len(seq) and seq[i] == pattern[j]:
    i += 1
    j = (j + 1) % len(pattern)
    if j == 0:
      # we have cycled through all elements in the pattern
      count += 1

  maxCount = max(count, maxCount)
  return pattern_count(seq, pattern, idx + 1, maxCount)
```

[24]:
```
pattern_count([0, 1, 2, 1, 2, 3, 1, 2, 1, 2, 1, 2, 4, 1, 2], (1, 2))
```

[24]: 3

[25]:
```
pattern_count([], [2])
```

[25]: 0

[26]:
```
pattern_count(['ab', 'ab', 'a', 'a', 'b'], 'ab')
```

[26]: 1

[27]:
```
pattern_count('CGGACTACTAGACT', 'ACT')
```

[27]: 2

[28]:
```
pattern_count((1, (1, 1, 1, 1), 2, 1, 1, 1), [1, 1])
```

[28]: 1

[29]:
```
pattern_count(['ab', 'ab', 'a', 'a', 'b'], ('ab',))
```

[29]: 2

3 3

(a)

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import binom

# (a) (i)
```

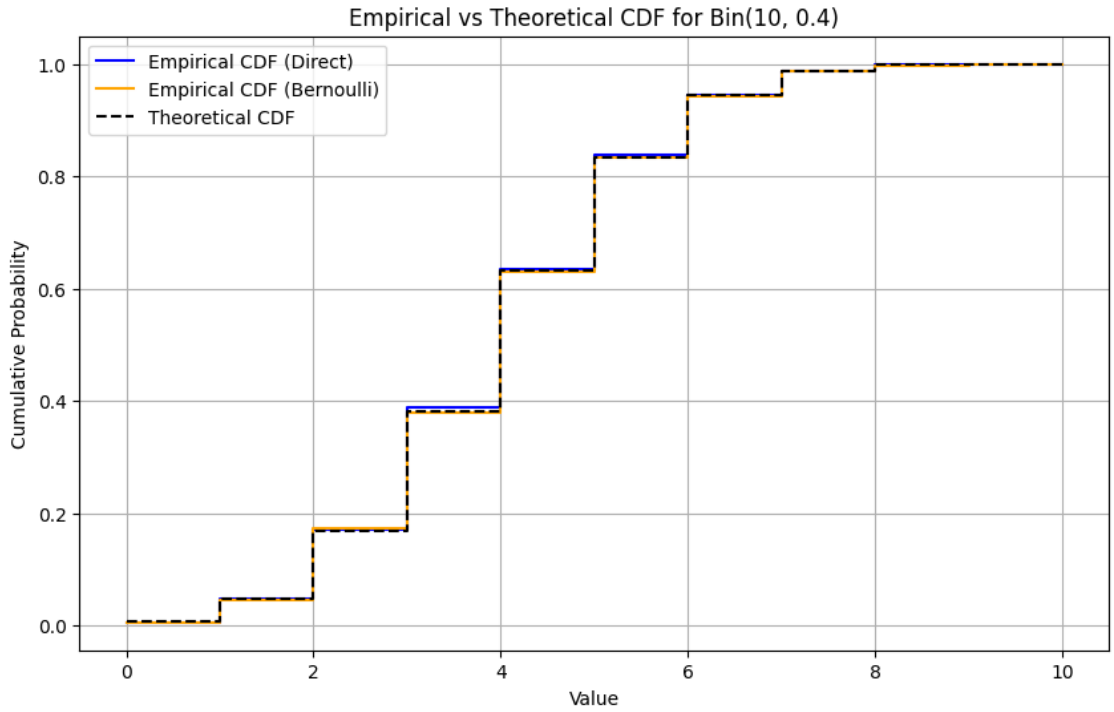
```
n, p, size = 10, 0.4, 10000
samples_direct = binom.ppf(np.random.uniform(0, 1, size), n, p)

# (a) (ii)
bernoulli_samples = np.random.binomial(1, p, size=(size, n))
samples_bernoulli = np.sum(bernoulli_samples, axis=1)

# (a) (iii)
x = np.arange(0, n + 1)
theoretical_cdf = binom.cdf(x, n, p)

empirical_cdf_direct = [np.mean(samples_direct <= val) for val in x]
empirical_cdf_bernoulli = [np.mean(samples_bernoulli <= val) for val in x]

plt.figure(figsize=(10, 6))
plt.step(x, empirical_cdf_direct, where='post', label='Empirical CDF (Direct)',␣
 ↪color='blue')
plt.step(x, empirical_cdf_bernoulli, where='post', label='Empirical CDF␣
 ↪(Bernoulli)', color='orange')
plt.step(x, theoretical_cdf, where='post', label='Theoretical CDF',␣
 ↪color='black', linestyle='--')
plt.xlabel('Value')
plt.ylabel('Cumulative Probability')
plt.title('Empirical vs Theoretical CDF for Bin(10, 0.4)')
plt.legend()
plt.grid()
plt.show()
```



(b)

[3]:
```
from scipy.stats import cauchy, norm

# (b) (i) Determine the optimal scaling factor c
x = np.linspace(-10, 10, 1000)
cauchy_pdf = cauchy.pdf(x)
normal_pdf = norm.pdf(x)
c = np.max(normal_pdf / cauchy_pdf)

# (b) (ii) Generate samples using accept-reject method
n_samples = 10000
samples = []
iterations = 0  # Track total iterations

while len(samples) < n_samples:
    # Generate candidate samples from the Cauchy distribution
    candidate_samples = cauchy.rvs(size=n_samples - len(samples))

    # Accept or reject based on scaling factor
    u = np.random.uniform(0, 1, size=n_samples - len(samples))
    accept = u < (norm.pdf(candidate_samples) / (c * cauchy.
 ↪pdf(candidate_samples)))
    samples.extend(candidate_samples[accept])
```

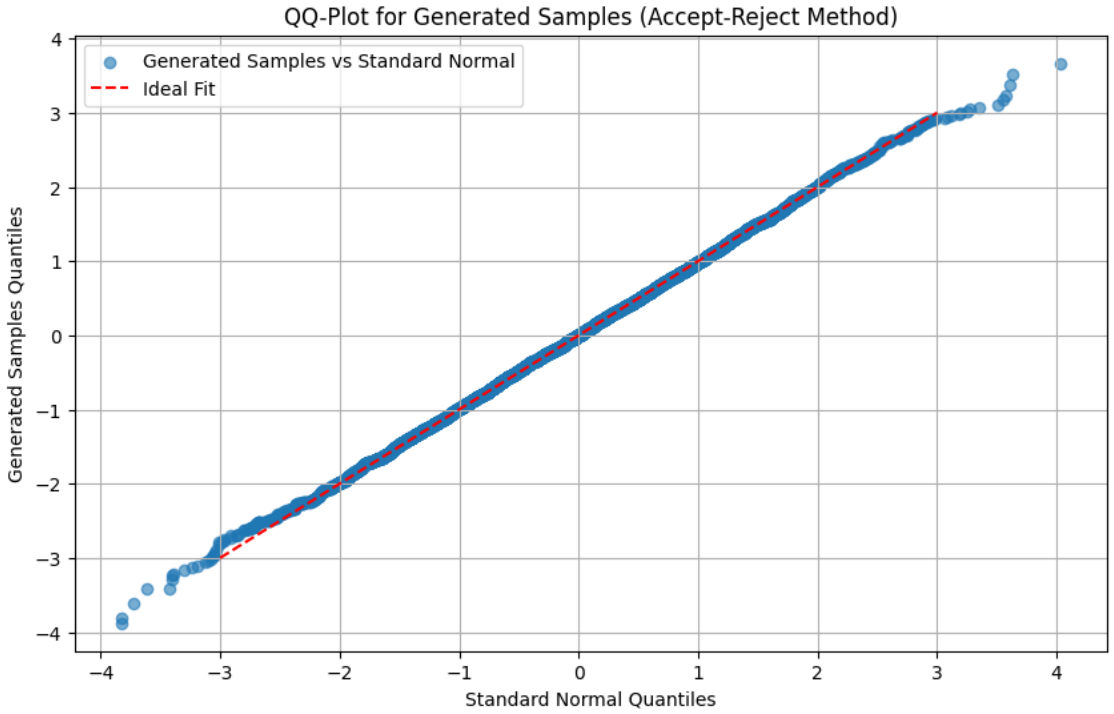
```
    iterations += len(candidate_samples)  # Increment iterations

samples = np.array(samples)

# (b) (iii) Compare estimated and theoretical acceptance probabilities
acceptance_probability_estimated = n_samples / iterations
acceptance_probability_theoretical = 1 / c
print(f"Estimated Acceptance Probability: {acceptance_probability_estimated:.
 ↪4f}")
print(f"Theoretical Acceptance Probability: {acceptance_probability_theoretical:
 ↪.4f}")

# (b) (iv) Generate QQ-plot
plt.figure(figsize=(10, 6))
norm_samples = norm.rvs(size=n_samples)
plt.scatter(np.sort(norm_samples), np.sort(samples), alpha=0.6,␣
 ↪label='Generated Samples vs Standard Normal')
plt.plot([-3, 3], [-3, 3], 'r--', label='Ideal Fit')
plt.xlabel('Standard Normal Quantiles')
plt.ylabel('Generated Samples Quantiles')
plt.title('QQ-Plot for Generated Samples (Accept-Reject Method)')
plt.legend()
plt.grid()
plt.show()
```

```
Estimated Acceptance Probability: 0.6542
Theoretical Acceptance Probability: 0.6578
```



4 4

[2]:
```
import pandas as pd
```

[103]:
```
county_age_dist = pd.read_csv("county_age_dist.csv")
fips_state = pd.read_csv("fips_state.csv", delimiter=';')
fips_county = pd.read_csv("fips_county.csv")
```

(a)

[104]:
```
county_age_dist.head()
```

[104]:

	fips	0-17	18-24	25-34	35-44	45-54	55-64	65-74	75-84	85+
0	1001	25941	11422	12315	13828	14000	12697	9594	5430	1945
1	1003	86587	37568	44133	46730	49675	52405	43252	23262	8854
2	1005	11057	6162	6603	5907	6490	6377	5255	2795	1074
3	1007	9671	5241	5788	5472	6707	5563	4270	2555	638
4	1009	25671	11360	12635	13570	14737	14123	12106	6560	2022

[105]:
```
fips_county.head()
```

[105]:

	fips	name	info
0	01000	Alabama	NaN
1	01001	Autauga County	NaN

```
2  01003   Baldwin County   NaN
3  01005   Barbour County   NaN
4  01007      Bibb County   NaN
```

[106]:
```
fips_county['fips'] = pd.to_numeric(fips_county['fips'], errors='coerce')
```

[107]:
```
fips_county = fips_county.dropna(subset=['fips'])
```

[108]:
```
fips_county['fips'] = fips_county['fips'].astype('int64')
```

[109]:
```
fips_county.head()
```

[109]:
```
   fips             name  info
0  1000          Alabama   NaN
1  1001   Autauga County   NaN
2  1003   Baldwin County   NaN
3  1005   Barbour County   NaN
4  1007      Bibb County   NaN
```

[110]:
```
data = pd.merge(county_age_dist, fips_county, on="fips")
```

[111]:
```
data.head()
```

[111]:
```
   fips   0-17  18-24  25-34  35-44  45-54  55-64  65-74  75-84   85+  \
0  1001  25941  11422  12315  13828  14000  12697   9594   5430  1945
1  1003  86587  37568  44133  46730  49675  52405  43252  23262  8854
2  1005  11057   6162   6603   5907   6490   6377   5255   2795  1074
3  1007   9671   5241   5788   5472   6707   5563   4270   2555   638
4  1009  25671  11360  12635  13570  14737  14123  12106   6560  2022

              name  info
0   Autauga County   NaN
1   Baldwin County   NaN
2   Barbour County   NaN
3      Bibb County   NaN
4    Blount County   NaN
```

[112]:
```
fips_state.head()
```

[112]:
```
   FIPS        STATE
0     1      ALABAMA
1     2       ALASKA
2     4      ARIZONA
3     5     ARKANSAS
4     6   CALIFORNIA
```

[113]:
```
data['state_fips'] = data['fips'] // 1000
```

[114]:
```
data.head()
```

[114]:
```
   fips   0-17  18-24  25-34  35-44  45-54  55-64  65-74  75-84   85+  \
0  1001  25941  11422  12315  13828  14000  12697   9594   5430  1945
1  1003  86587  37568  44133  46730  49675  52405  43252  23262  8854
2  1005  11057   6162   6603   5907   6490   6377   5255   2795  1074
3  1007   9671   5241   5788   5472   6707   5563   4270   2555   638
4  1009  25671  11360  12635  13570  14737  14123  12106   6560  2022

              name  info  state_fips
0   Autauga County   NaN           1
1   Baldwin County   NaN           1
2   Barbour County   NaN           1
3      Bibb County   NaN           1
4    Blount County   NaN           1
```

[115]:
```
data = pd.merge(data, fips_state, left_on="state_fips", right_on="FIPS",␣
↪how="inner")
```

[116]:
```
data.head()
```

[116]:
```
   fips   0-17  18-24  25-34  35-44  45-54  55-64  65-74  75-84   85+  \
0  1001  25941  11422  12315  13828  14000  12697   9594   5430  1945
1  1003  86587  37568  44133  46730  49675  52405  43252  23262  8854
2  1005  11057   6162   6603   5907   6490   6377   5255   2795  1074
3  1007   9671   5241   5788   5472   6707   5563   4270   2555   638
4  1009  25671  11360  12635  13570  14737  14123  12106   6560  2022

              name  info  state_fips  FIPS     STATE
0   Autauga County   NaN           1     1   ALABAMA
1   Baldwin County   NaN           1     1   ALABAMA
2   Barbour County   NaN           1     1   ALABAMA
3      Bibb County   NaN           1     1   ALABAMA
4    Blount County   NaN           1     1   ALABAMA
```

[117]:
```
data = data.rename(columns=lambda x: x.strip().capitalize())
data.head()
```

[117]:
```
   Fips   0-17  18-24  25-34  35-44  45-54  55-64  65-74  75-84   85+  \
0  1001  25941  11422  12315  13828  14000  12697   9594   5430  1945
1  1003  86587  37568  44133  46730  49675  52405  43252  23262  8854
2  1005  11057   6162   6603   5907   6490   6377   5255   2795  1074
3  1007   9671   5241   5788   5472   6707   5563   4270   2555   638
4  1009  25671  11360  12635  13570  14737  14123  12106   6560  2022

              Name Info  State_fips  Fips     State
0   Autauga County  NaN           1     1   ALABAMA
```

```
1   Baldwin County  NaN           1     1   ALABAMA
2   Barbour County  NaN           1     1   ALABAMA
3      Bibb County  NaN           1     1   ALABAMA
4    Blount County  NaN           1     1   ALABAMA
```

[118]:
```
data = data.drop(columns=['Info'])
data.head()
```

[118]:
```
   Fips   0-17  18-24  25-34  35-44  45-54  55-64  65-74  75-84   85+  \
0  1001  25941  11422  12315  13828  14000  12697   9594   5430  1945
1  1003  86587  37568  44133  46730  49675  52405  43252  23262  8854
2  1005  11057   6162   6603   5907   6490   6377   5255   2795  1074
3  1007   9671   5241   5788   5472   6707   5563   4270   2555   638
4  1009  25671  11360  12635  13570  14737  14123  12106   6560  2022

              Name  State_fips  Fips     State
0   Autauga County           1     1   ALABAMA
1   Baldwin County           1     1   ALABAMA
2   Barbour County           1     1   ALABAMA
3      Bibb County           1     1   ALABAMA
4    Blount County           1     1   ALABAMA
```

[119]:
```
for col in data.columns:
  data[col] = data[col].map(lambda x: x.strip() if isinstance(x, str) else x)
data.head(4)
```

[119]:
```
   Fips   0-17  18-24  25-34  35-44  45-54  55-64  65-74  75-84   85+  \
0  1001  25941  11422  12315  13828  14000  12697   9594   5430  1945
1  1003  86587  37568  44133  46730  49675  52405  43252  23262  8854
2  1005  11057   6162   6603   5907   6490   6377   5255   2795  1074
3  1007   9671   5241   5788   5472   6707   5563   4270   2555   638

              Name  State_fips  Fips     State
0   Autauga County           1     1   ALABAMA
1   Baldwin County           1     1   ALABAMA
2   Barbour County           1     1   ALABAMA
3      Bibb County           1     1   ALABAMA
```

(b)

[120]:
```
def calculate_proportions(row):
    total_pop = sum([row[f"{age_group}"] for age_group in ["0-17", "18-24",␣
 ↪"25-34", "35-44", "45-54", "55-64", "65-74", "75-84", "85+"]])
    cpe = sum([row["65-74"], row["75-84"], row["85+"]]) / total_pop
    cpy = sum([row["0-17"], row["18-24"]]) / total_pop
    return pd.Series({"CPY": cpy, "CPE": cpe})

proportions = data.apply(calculate_proportions, axis=1)
```

```
data = pd.concat([data, proportions], axis=1)

# View first 4 rows as per the test
data.head(4)
```

[120]:

	Fips	0-17	18-24	25-34	35-44	45-54	55-64	65-74	75-84	85+	\
0	1001	25941	11422	12315	13828	14000	12697	9594	5430	1945	
1	1003	86587	37568	44133	46730	49675	52405	43252	23262	8854	
2	1005	11057	6162	6603	5907	6490	6377	5255	2795	1074	
3	1007	9671	5241	5788	5472	6707	5563	4270	2555	638	

	Name	State_fips	Fips	State	CPY	CPE
0	Autauga County	1	1	ALABAMA	0.348627	0.158334
1	Baldwin County	1	1	ALABAMA	0.316346	0.192037
2	Barbour County	1	1	ALABAMA	0.332927	0.176411
3	Bibb County	1	1	ALABAMA	0.324845	0.162575