

## 17. Binomial Distribution

### A. Activities

- 1) For a binomial distribution mean is 6 and variance is 2.

$$\therefore \text{Mean} = E(X) = 6$$

$$\text{variance} = npq = 2$$

$$\therefore \frac{V(X)}{E(X)} = \frac{npq}{np} = \frac{2}{6}$$

$$\therefore q = \frac{1}{3}$$

$$\therefore p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\therefore E(x) = np = 6$$

$$\therefore n \times \frac{2}{3} = 6$$

$$\therefore 2n = 18 \quad \therefore n = 9$$

- 2) Given  $X \sim B(3, 0.3)$

$$\therefore n = 3 \text{ and } p = 0.3, q = 0.7$$

The p. m. f. of X is

$$P(X=x) = {}^n C_x p^x q^{n-x}$$

$$\text{i) } P(X=3) = {}^3 C_3 (0.3)^3 (0.7)^0 = 0.027$$

$$\text{ii) } P(X \geq 2) = P(X=2) + P(X=3)$$

$$= {}^3 C_2 (0.3)^2 (0.7)^1 + {}^3 C_3 (0.3)^3 (0.7)^0 \\ = 0.189 + 0.027 = 0.216$$

$$\text{iii) } P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= {}^3 C_0 (0.3)^0 (0.7)^3 + {}^3 C_1 (0.3)^1 (0.7)^2$$

$$+ {}^3 C_2 (0.3)^2 (0.7)^1$$

$$= 0.343 + 0.441 + 0.189 = 0.973$$

- 3) Given that  $n = 10, p = 2/5$

$$\therefore q = 1 - \frac{2}{5} = 3/5$$

$$\therefore \text{Mean} = n p = 10 \times \frac{2}{5} = 4$$

$$\text{Variance} = n [p] q$$

$$= \boxed{10} \cdot \frac{2}{5} \times \frac{\boxed{3}}{5} = \boxed{\frac{12}{5}}$$

### B. Solve the Following

Q.1. If a fair die is rolled 5 times, what is the probability that two of rolls will show 1's?

Soln: ... For Binomial Distribution,

$$X \sim B(n, p) \text{ with } n=5, p=\frac{1}{6}$$

$$\therefore q = 1-p = 1-\frac{1}{6} = \frac{5}{6}$$

... The pmf of  $X$  is

$$P(X=x) = {}^n C_x p^x q^{n-x}$$

$$\therefore P(X=x) = {}^5 C_x \left(\frac{1}{6}\right)^x \left(\frac{5}{6}\right)^{5-x}$$

$$\therefore P(X=2) = {}^5 C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{5-2}$$

$$= \frac{5!}{2!(5-2)!} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3$$

$$= \frac{5 \times 4 \times 3!}{(2 \times 1)(3!) \times 6} \left(\frac{1}{6}\right) \left(\frac{125}{216}\right)$$

$$= \frac{10 \times 125}{7776} = 0.16075$$

$$\therefore \boxed{P(X=2) = 0.16075}$$

Q.2. Find the probability of guessing correctly at least six of the ten answers in a True or False objective test

Soln: Let  $X$  denotes the number of correct answers.

$$P(\text{Answer is correct}) = p = \frac{1}{2}$$

$$\therefore q = 1-p = 1-\frac{1}{2} = \frac{1}{2}$$

Given  $n=10$ , for Binomial Distribution,

$$X \sim B(10, \frac{1}{2})$$

The pmf of  $X$  is  $P(X=x) = {}^{10} C_x p^x q^{10-x}$

$$\begin{aligned}
 &= P(\text{Atleast six answers are correct}) = P(X \geq 6) \\
 &= P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10) \\
 &= {}^{10}C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^4 + {}^{10}C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3 + {}^{10}C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 + {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^1 + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^0 \\
 &= \left(\frac{1}{2}\right)^{10} \left[ {}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10} \right] \\
 &= \frac{1}{2^{10}} \left[ {}^{11}C_7 + {}^{11}C_9 + 1 \right] \quad \left\{ \because {}^nC_1 + {}^nC_{n+1} = {}^{n+1}C_{n+1} \right\} \\
 &= \frac{1}{2^{10}} \left[ \frac{11!}{7!4!} + \frac{11!}{9!2!} + 1 \right] = \frac{1}{2^{10}} \left[ \frac{11 \times 10 \times 9 \times 8}{24} + \frac{11 \times 10}{2} + 1 \right] \\
 &= \frac{1}{2^{10}} [330 + 55 + 1] = \frac{386}{1024} = \boxed{\frac{193}{512}}
 \end{aligned}$$

Q.3. It is observed that, it rains on 12 days out of 30 days .Find the probability that it rains exactly 3 days of the week.

Soln: Let  $X$  : No. of days it rains in a week.

$$\therefore X = \{0, 1, 2, \dots, 7\}$$

$p$ : Probability that it rains

$$\therefore p = \frac{12}{30} = \frac{2}{5}$$

$$\therefore q = 1 - p = 1 - \frac{2}{5} = \frac{3}{5} \text{ and } n = 7$$

$$\therefore P(X=x) = {}^7C_x \left(\frac{2}{5}\right)^x \left(\frac{3}{5}\right)^{7-x}$$

$\therefore P(\text{Rains exactly 3 days of week}) = P(X=3)$

$$= {}^7C_3 \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^{7-3}$$

$$= {}^7C_3 \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^4$$

$$= \frac{7!}{3!4!} \times \frac{8}{125} \times \frac{81}{625}$$

$$= \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 4!} \times \frac{8 \times 8}{125 \times 62} = \frac{22680}{78125}$$

$$\therefore P(X=3) = 0.2903$$

Hence the probability that it rains exactly 3 days of the week is 0.2903

Sign of Teacher :