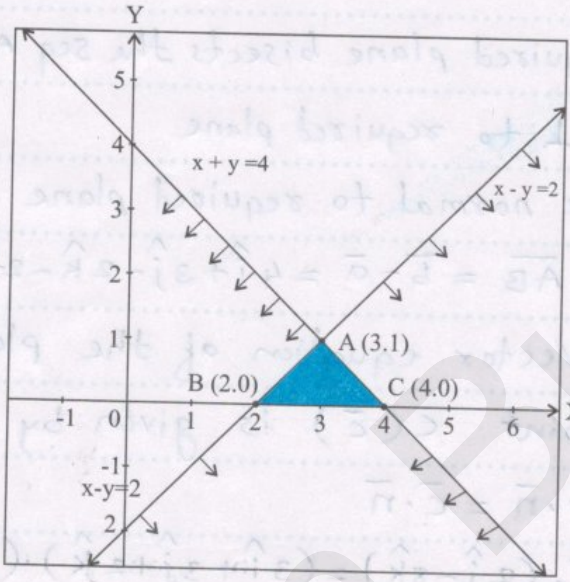
# 8. Linear Programming

**Ex.** 1) Maximize : $Z = y - 2x$ subject to $x - y \geq 2$, $x + y \leq 4$, $y \geq 0$.

**Solution** : To draw regions $x - y \geq 2$, $x + y \leq 4$, we begin by drawing the lines $x - y = 2$ and $x + y = 4$.

| Inequality | Line | Two Intercept Form | Points | Region |
|---|---|---|---|---|
| $x - y \geq 2$ | $x - y = 2$ | $\frac{x}{2} + \frac{y}{-2} = 1$ | $(2,0),(0,-2)$ | Non origin side |
| $x + y \leq 4$ | $x + y = 4$ | $\frac{x}{4} + \frac{y}{4} = 1$ | $(4,0),(0,4)$ | Origin side |




The feasible region is ABC, with corner points A(3,1), B(2,0) and C(4,0).

| Corner Points | Value of $Z = y - 2x$ |
|---|---|
| A(3,1) | $Z = 1 - 2(3) = -5$ |
| B(2,0) | $Z = 0 - 2(2) = -4$ |
| C(4,0) | $Z = 0 - 2(4) = -8$ |

The maximum value of Z is $-4$ at $x = 2, y = 0$

**Ex.** (2) Minimize : $Z = 200x + 500y$ subject to $x + 2y \geq 10$, $3x + 4y \leq 24$, $x \geq 0$, $y \geq 0$.

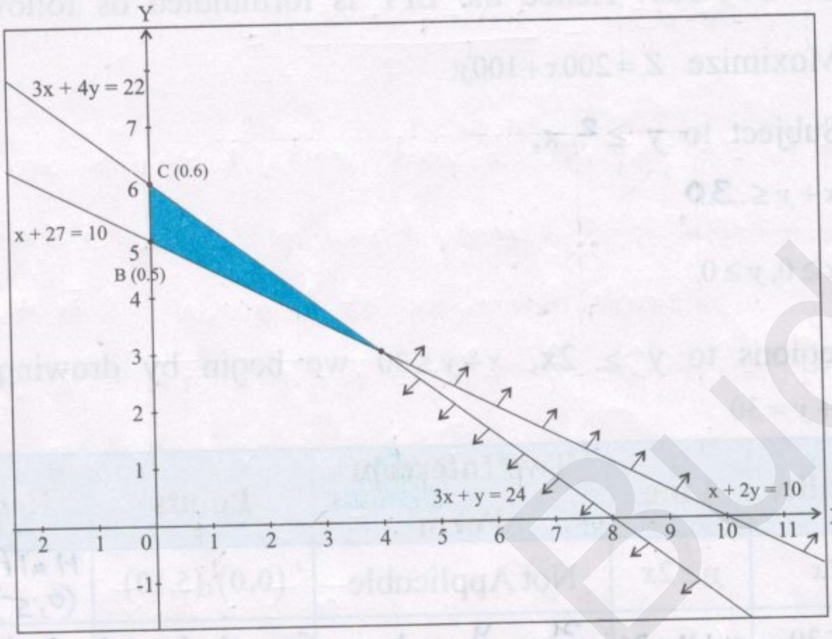


Solution Buddy

**Solution :** To draw regions $x+2y \geq 10$, $3x+4y \leq 24$, we begin by drawing the lines

$$x+2y=10, \quad 3x+4y=24.$$

| Inequality | Line | Two Intercept Form | Points | Region |
|---|---|---|---|---|
| $x+2y \geq 10$ | $x+2y=10$ | $\frac{x}{10}+\frac{y}{5}=1$ | $(10,0),(0,5)$ | Non origin side |
| $3x+4y \leq 24$ | $3x+4y=24$ | $\frac{x}{8}+\frac{y}{6}=1$ | $(8,0),(0,6)$ | Origin side |




The feasible region is ABC, with corner points A(4,3), B(0,5) and C(0,6).

| Corner Points | Value of $Z=200x+500y$ |
|---|---|
| A(4,3) | $Z=200(4)+500(3)=2300$ |
| B(0,5) | $Z=200(0)+500(5)=2500$ |
| C(0,6) | $Z=200(0)+500(6)=3000$ |

The minimum value of Z is 2300 at $x=4, y=3$.

**Ex.** (3) A carpenter makes tables and chairs. Profit per table is Rs. 200 and that per chair is Rs. 100. He should make at least two chairs per table and the total number of tables and chairs should not exceed 30. Find the maximum profit.

**Solution:** Let x be number of tables and y be number of chairs that are to be made by the carpenter. That is $x \geq 0, y \geq 0$.

Since cost of a table is Rs. 200 and cost of a chair is Rs. 100,

Total profit is $z = 200x + 100y$ , which is to be maximized.

As at least (more than or equal) 2 chairs per table

i.e. Number of chairs $\geq$ 2(Number of tables) i.e $y \geq 2x$.

Also as the total number of tables and chairs should not exceed (less than or equal) 30,

So $x + y \leq 30$. Hence the LPP is formulated as follows

Maximize $Z = 200x + 100y$

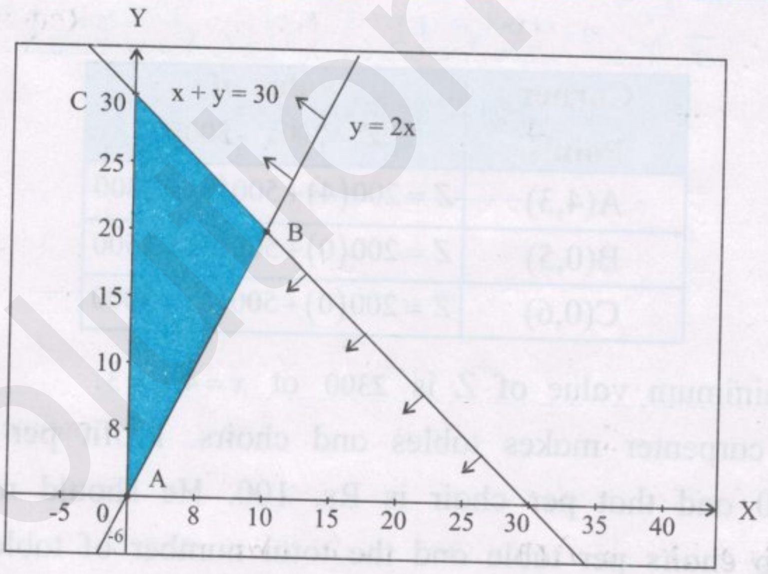
Subject to $y \geq 2x$,

$x + y \leq 30$,

$x \geq 0, y \geq 0$

To draw regions to $y \geq 2x$, $x + y \leq 30$ we begin by drawing the lines $y = 2x$, $x + y = 30$

| Inequality | Line | Two Intercept Form | Points | Region |
|---|---|---|---|---|
| $y \geq 2x$ | $y \geq 2x$ | Not Applicable | (0,0),(5,10) | Half plane (0,5) |
| $x + y \leq 30$ | $x + y = 30$ | $\frac{x}{30} + \frac{y}{30} = 1$ | (30,0), (0,30) | Origin |




The feasible region is ABC with corner points A(0,0), B(10,20) and C (0,30)



Solution Buddy

| Corner Points | Value of $Z = 200x + 100y$ |
|---|---|
| A(0, 0), | 0 |
| B(10, 20) | 4000 |
| C(0, 30) | 3000 |

The maximum value of Z is 4000 at $x = 10, y = 20$

**Ex.**4) A chemical company produces a chemical containing three basic elements A, B, C, so that it has at least 16 liters of A, 24 liters of B and 18 liters of C. This chemical is made by mixing two compounds. Each unit of compound I, has 4 liters of A, 12 liters of B and 2 liters of C. Each unit of compound II, has 2 liters of A, 2 liters of B and 6 liters of C. The cost per unit of compound I is Rs. 800 and that of compound II is Rs. 640. Formulate the problems as LPP and solve it to minimize the cost.

**Solution :** Let the company produce $x$ units of compound I and y units of compound II.

Then the total cost is $z = 800x + 640y$, this is objective function which I to be minimized.

The given information about constraints can be tabulated as follows

| | Compound I ($x$) | Compound II ($y$) | Minimum Requirement |
|---|---|---|---|
| Element A | 4 | 2 | 16 |
| Element B | 12 | 2 | 24 |
| Element C | 2 | 6 | 18 |

From the table, the constraint are $4x + 2y \geq 16, 12x + 2y \geq 24, 2x + 6y \geq 18$. Also

The LPP is formulated as follows.

Minimize Z = $800x + 640y$

Subject to

$4x+2y\geq 16$

$12x+2y\geq 24$

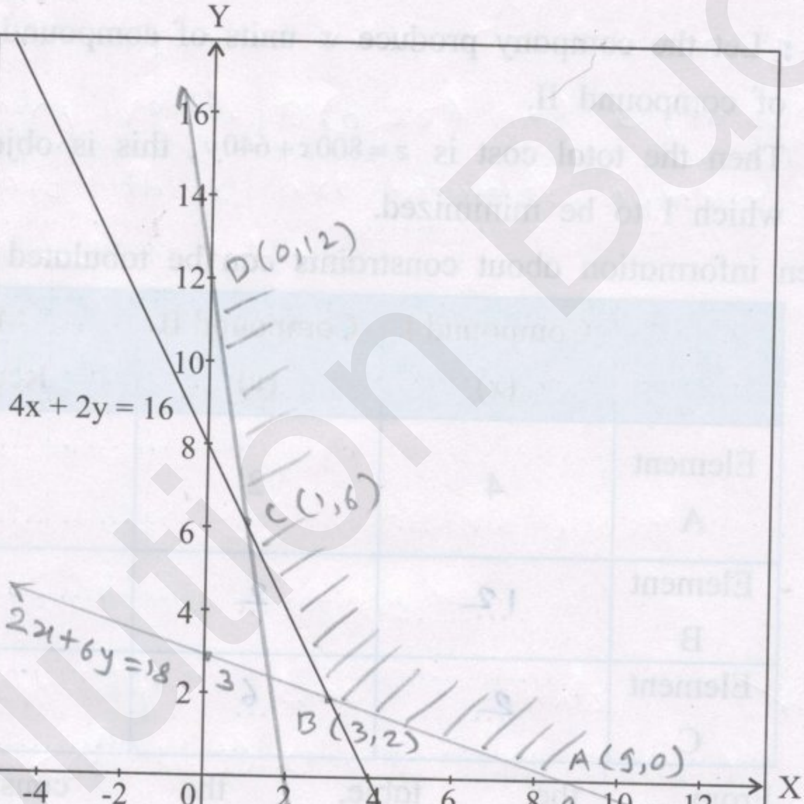
$2x+6y\geq 18$

$x,\ y\ \geq\ 0$

To draw regions $4x+2y\geq 16$, $12x+2y\geq 24$, $2x+6y\geq 18$

We begin by drawing the lines $4x+2y=16$, $12x+2y=24$, $2x+6y=18$

| Inequality | Line | Two Intercept Form | Points | Region |
|---|---|---|---|---|
| $4x+2y\geq 16$ | $4x+2y=16$ | $\frac{x}{4}+\frac{y}{8}=1$ | (4,0),(0,8) | Non origin side |
| $12x+2y\geq 24$ | $12+2y=24$ | $\frac{x}{2}+\frac{y}{12}=1$ | (2,0)(0,12) | Non origin side |
| $8x+6y\geq 18$ | $2x+6y=18$ | $\frac{x}{9}+\frac{y}{3}=1$ | (9,0)(0,3) | Non origin side |




The feasible region is ABCD with corner points A(9,0), B(3,2), C(1,6), D(0,12),



Solution Buddy

| Corner Points | Value of $z = 800x + 640y$ |
|---|---|
| A(9,0), | 7200 |
| B(3,2) | 3680 |
| C(1,6) | 4640 |
| D(0,12) | 7680 |

The minimum value of Z is 3680 at $x = 3$, $y = 2$

**Ex.** (5) Minimize : $Z = x + 2y$

subject to $x + 2y \geq 50$,

$2x - y \leq 0$,

$2x + y \leq 100$,

$x \geq 0$, $y \geq 0$

**Solution :**

| Inequality | Line | Points | Region |
|---|---|---|---|
| $x+2y \geqslant 0$ | I) $x+2y=50$ | A(50,0) | Non origin |
| | ~~$2x-y=0$~~ | B(0,25) | side |
| $2x-y \leqslant 0$ | II) $2x-y=0$ | O(0,0) | Half plane |
| | | C(10,20) | containing (0,20) |
| $2x+y \leq 100$ | III) $2x+y=100$ | A(50,0) | Origin |
| | | D(0,100) | side |

The feasible region is BCPDB

Solving (II) and (III) we get $x = 25$, $y = 50$

$\therefore P \equiv (25,50)$

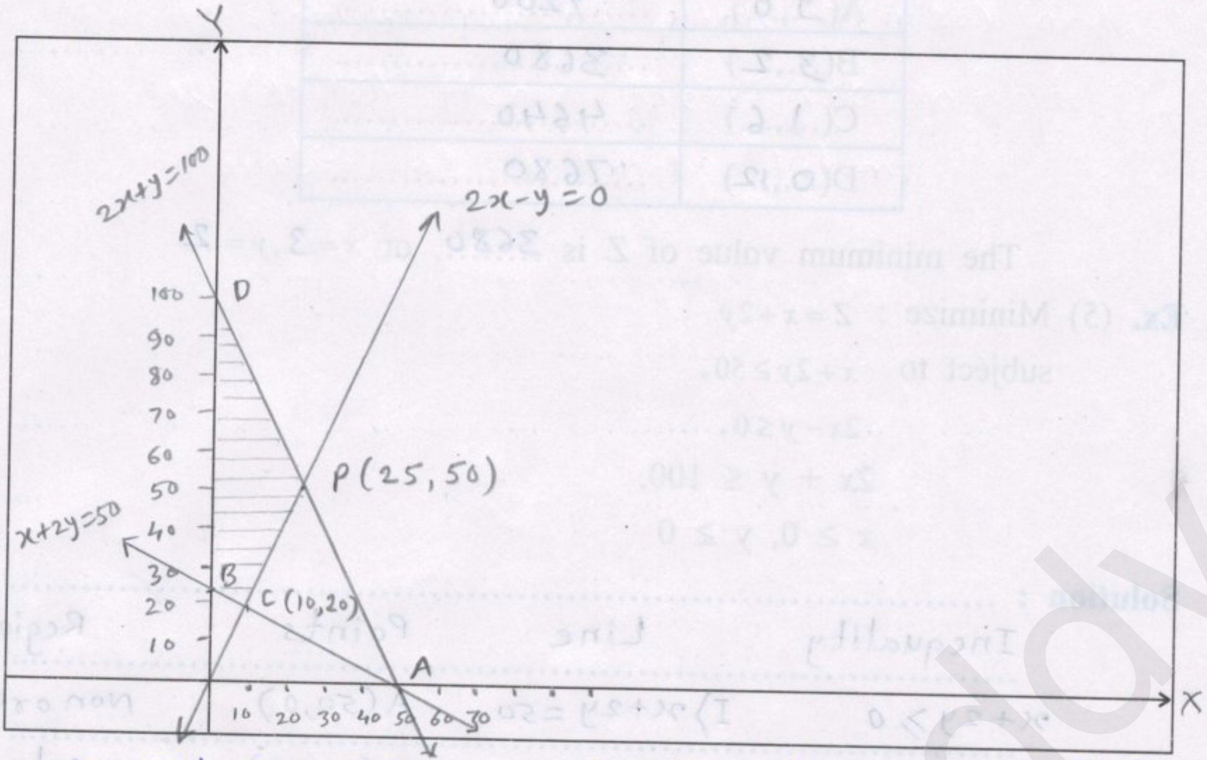
The corner points are B, C, P, D

| corner points | Value $z = x+2y$ |
|---|---|
| B(0,25) | 50 |
| C(10,20) | 50 |
| P(25,50) | 125 |
| D(0,100) | 200 |



Solution Buddy

Z has minimum value 50 at two consecutive vertices B and C




∴ Z has minimum value 50 at every point of seg joining points B(0,25) and C(10,20)

Hence there are infinite number of optional solutions

**Ex.** (6) Maximize : $Z = 3x + 9y$

subject to $x + 3y \leq 60$,

$x + y \geq 10$,

$x \leq y$,

$x \geq 0$, $y \geq 0$.

**Solution :**

| Inequality | Line | Points | Region |
|---|---|---|---|
| $x+3y \leq 60$ | I) $x+3y=60$ | A(60,0), B(0,20) | Origin side |
| $x+y \geq 10$ | II) $x+y=10$ | C(10,0), D(0,10) | Non origin side |
| $x \leq y$ | III) $x=y$ | O(0,0), E(30,30) | Half plane containing (0,30) |



Solution Buddy

The Feasible region is PQBDP

Solving (II) & (III), $x=5, y=5$

$\therefore P \equiv (5,5)$

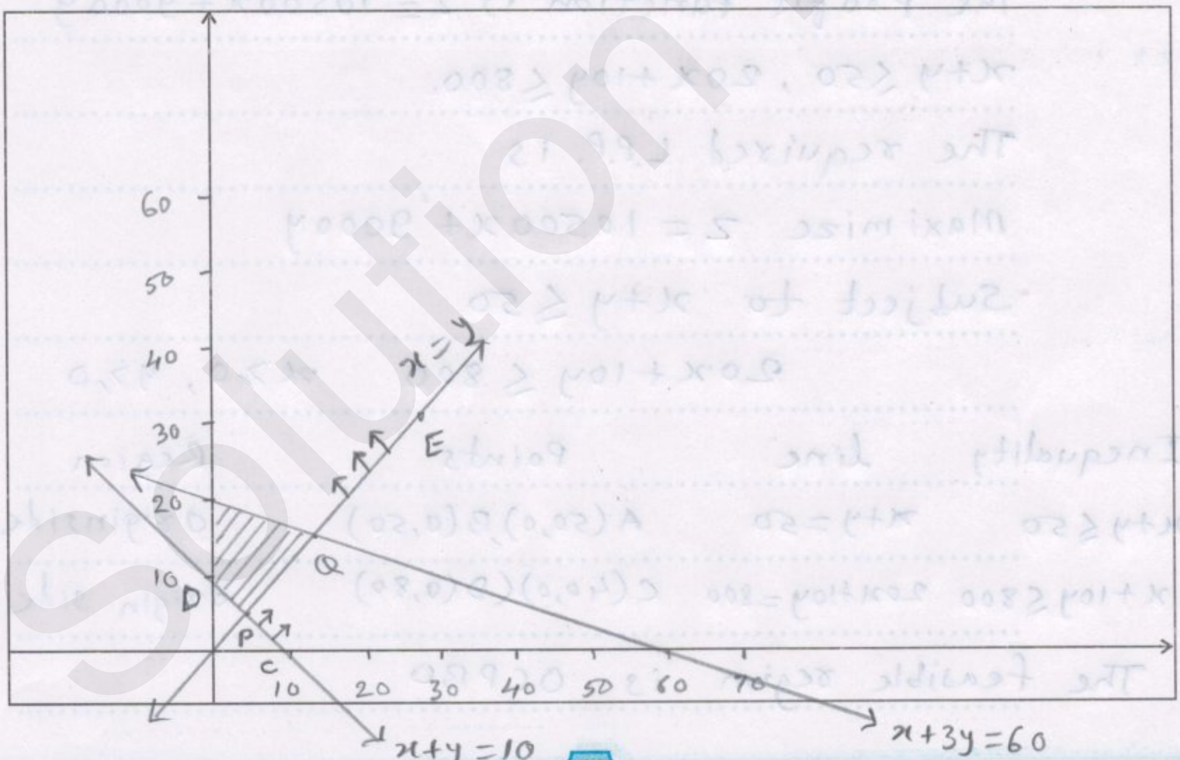
Solving (I) & (III), $x=15, y=15$

$\therefore Q \equiv (15,15)$

The corner points are P, Q, B, D

| Vertex | Value $z=3x+9y$ |
|---|---|
| P(5,5) | 60 |
| Q(15,15) | 180 |
| B(0,20) | 180 |
| D(0,10) | 90 |

$\therefore$ z has maximum value 180 at two consecutive points B and Q






Solution Buddy

∴ z has maximum value 180 at every point of segment joining points B(0,20) and Q (15,15) Hence there are infinite number of optimal solutions.

**Ex.** (7) A co-operative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y perpendicular hectare are estimated as Rs. 10,500 and RS. 9,000 respectively. To control weeds a liquid herbicide has to be used for crops X and Y at rates of 20 liter and 10 liter per hectare. Further no more than 800 liter of herbicide should be used in order to protect fish and a wild life using the pond which collects draining from this land. How much land should be allocated to each crop so as to maximize the total profit of the society ?

**Solution :**

Let $x$ and $y$ hectare land be allocated to crops X and Y respectively $x \geq 0, y \geq 0$

The profit function is $z = 10500x + 9000y$

$x+y \leq 50$, $20x+10y \leq 800$

The required L.P.P. is

Maximize $z = 10500x + 9000y$

Subject to $x+y \leq 50$

$20x+10y \leq 800 \quad x \geq 0, y \geq 0$

| Inequality | line | Points | Region |
|---|---|---|---|
| $x+y \leq 50$ | $x+y=50$ | A(50,0), B(0,50) | origin side |
| $20x+10y \leq 800$ | $20x+10y=800$ | C(40,0) D(0,80) | origin side |

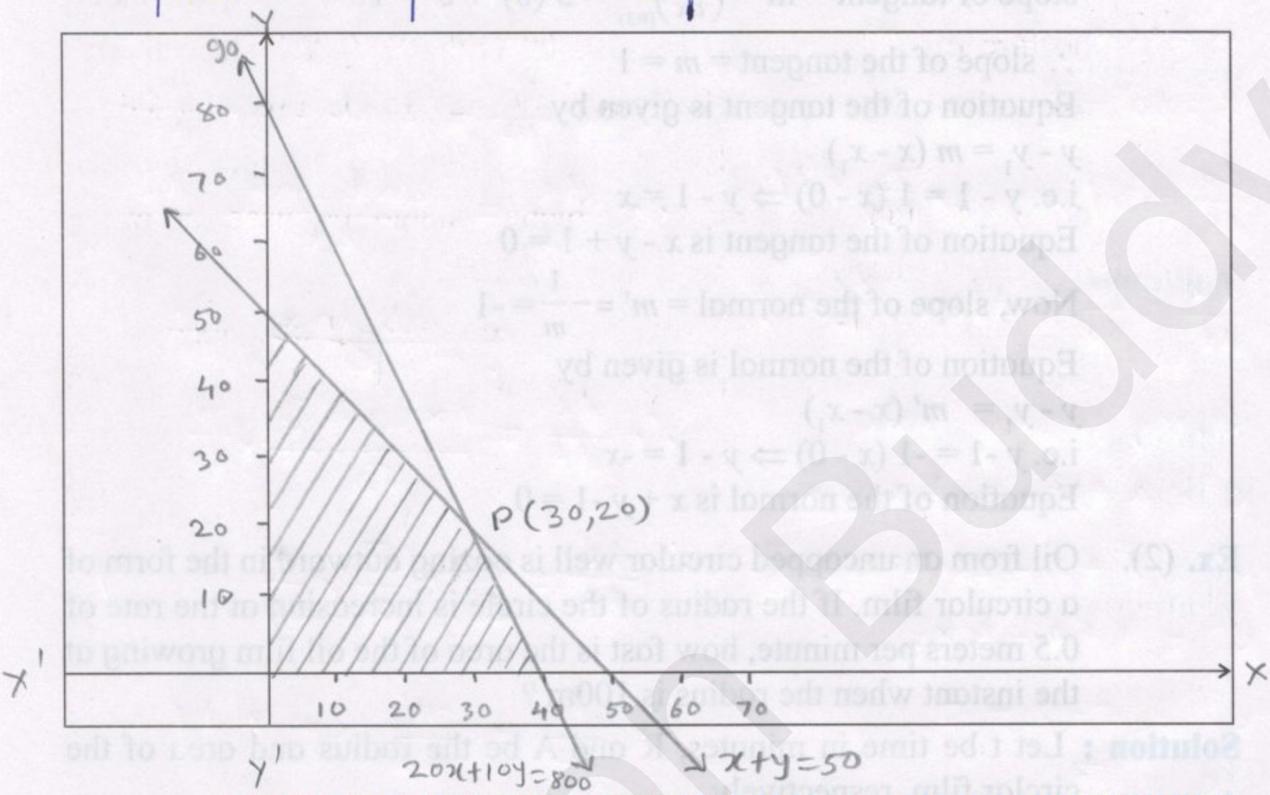
The feasible region is OCPBO

Solving $x+y=50$ and $20x+10y=800$ ie $2x+y=80$

$\therefore x=30$ , $y=20$ $\therefore P \equiv (30, 20)$

The corner points are O, C, P, B

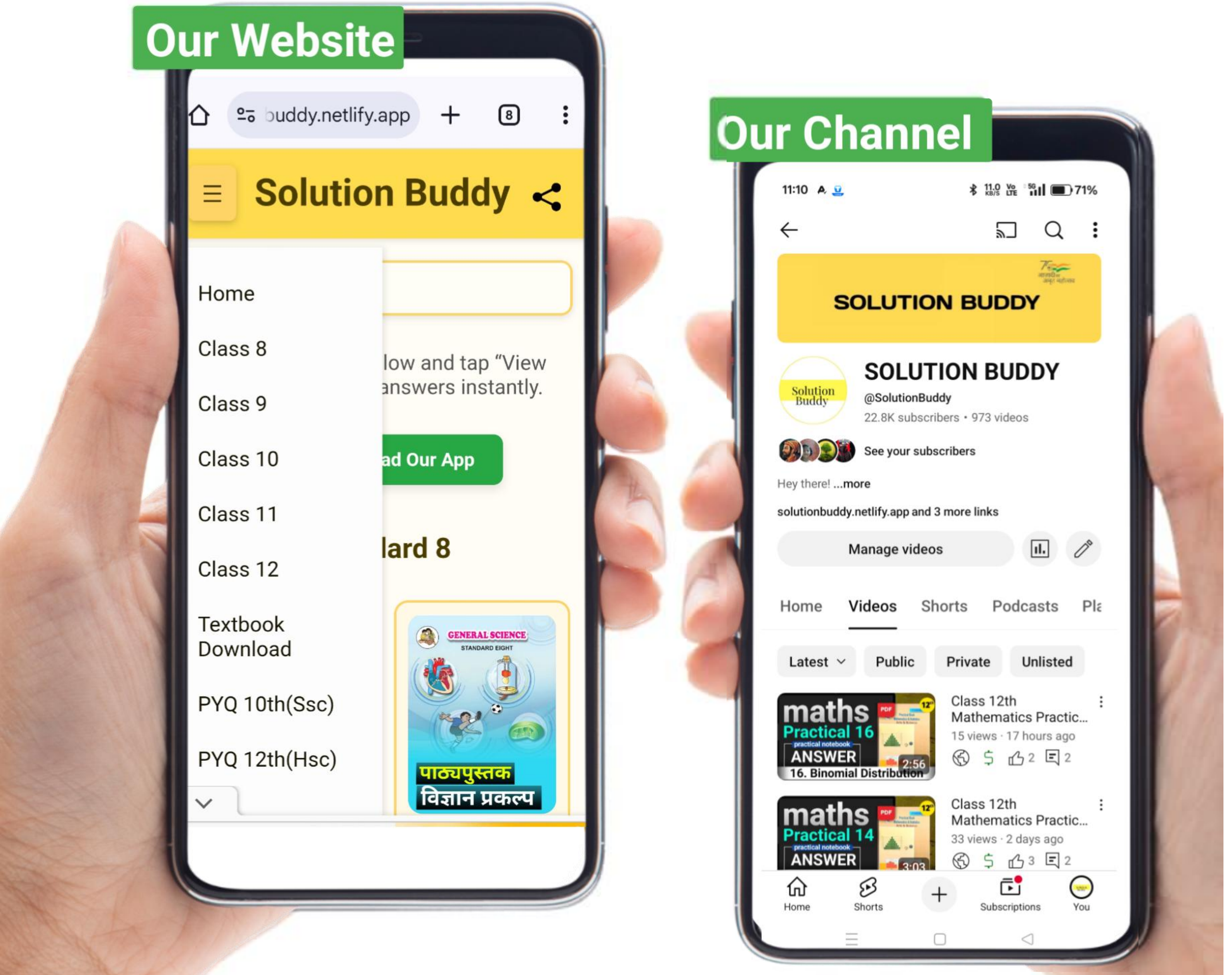
| vertex | value $Z = 10500x + 9000y$ |
|---|---|
| O (0,0) | 0 |
| C (40,0) | 420000 |
| P (30,20) | 495000 |
| B (0,50) | 450000 |




$\therefore$ The maximum value of Z is 495000 at $x=30$ and $y=20$

$\therefore$ 30 hectar and 20 hectar Land should be allocated to crop X and Y resp. for maximum profit ₹ 495000

Sign of Teacher :




📚 On Solution Buddy, You Will Get:

✔️ Exercise solutions for Class 8–12
✔️ Previous Year Question Papers (10th & 12th)
✔️ Free Textbook Downloads
✔️ Practical Solutions (Class 10, 11 & 12)
✔️Water Security Exercise & Activity Solution
✔️Defence Studies Exercise Solution
👉 Website: solutionbuddy.netlify.app
👉 YouTube: youtube.com/@solutionbuddy

# Solution Buddy