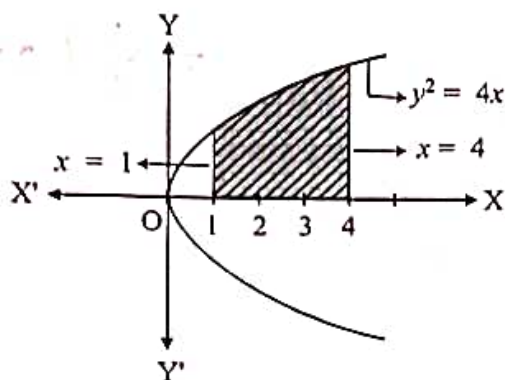


19. Applications of Definite Integration

A. Activities

Carry out the following activities

1. Find the area of the shaded portion using the following information

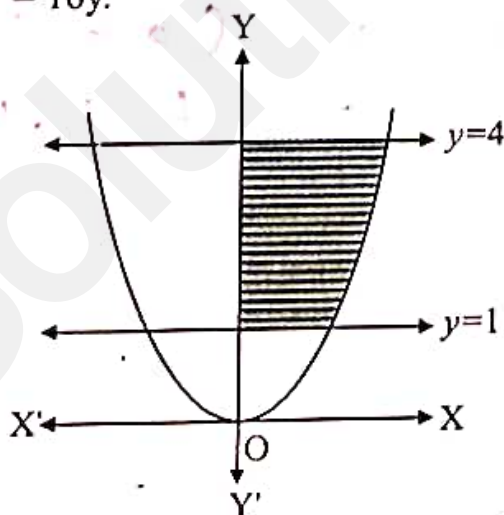


$$\begin{aligned}
 \text{Required Area} &= \int_1^4 y \, dx \\
 &= \int_1^4 2\sqrt{x} \, dx \\
 &= 2 \left[\frac{x^{3/2}}{3/2} \right]_1^4 \\
 &= \frac{4}{3} [(4)^{3/2} - (1)^{3/2}] \\
 &= \frac{4}{3} [8 - 1] = \frac{28}{3} \text{ sq. unit} \\
 &\quad \text{Ans}
 \end{aligned}$$

2. Find the area of the region under the curve $x = \frac{36}{25} (25 - y)^2$, and the lines $y = 0$, $y = 5$ and the X-axis using the following activity.

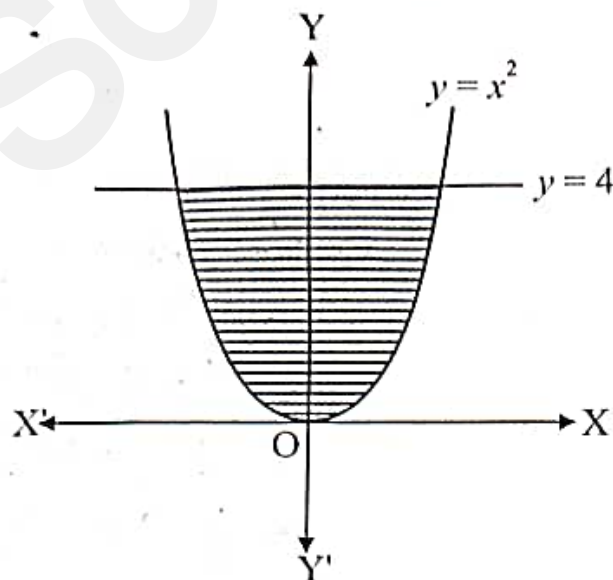
$$\begin{aligned}
 \text{Required area} &= \int_0^5 x \, dy \\
 &= \int_0^5 \frac{36}{25} (25 - y)^2 \, dy \\
 &= \frac{36}{25} \int_0^5 (25 - y)^2 \, dy \\
 &= \frac{36}{25} \left(25y - \frac{y^3}{3} \right)_0^5 \\
 &= \boxed{120} \text{ sq. units.}
 \end{aligned}$$

3. Find the area shown in the following figure and the information given $x^2 = 16y$.



$$\begin{aligned}
 \text{Required Area} &= \int_1^4 x \, dy \\
 &= \int_1^4 4\sqrt{y} \, dy \\
 &= 4 \left[\frac{y^{3/2}}{3/2} \right]_1^4 \\
 &= \frac{8}{3} [(4)^{3/2} - (1)^{3/2}] \\
 &= \frac{8}{3} (8 - 1) \\
 &= \frac{56}{3} \text{ sq. unit} \\
 &\quad \text{Ans}
 \end{aligned}$$

4. Find the shaded area using following information.



$$\begin{aligned}
 \text{Required Area} &= \int_0^4 x \, dy \\
 &= 2 \int_0^4 \sqrt{y} \, dy \\
 &= 2 \left[\frac{y^{3/2}}{3/2} \right]_0^4 \\
 &= \frac{4}{3} (8 - 0) = \frac{32}{3} \text{ sq. unit} \\
 &\quad \underline{\text{Ans}}
 \end{aligned}$$

B. Solve the Following

Q.1. Find the area bounded by the curve $y = 2x$, X-axis and the lines $x = -2$ and $x = 4$

$$\begin{aligned}
 \text{Required area} &= \int_{-2}^4 y \, dx \\
 &= \int_{-2}^4 2x \, dx \\
 &= 2 \int_{-2}^4 x \, dx \\
 &= 2 \left[\frac{x^2}{2} \right]_{-2}^4 \\
 &= \frac{2}{2} [(4)^2 - (-2)^2] \\
 &= (8 - 4) \\
 &= 4 \text{ sq. unit.} \\
 &\quad \underline{\text{Ans}}
 \end{aligned}$$

Q.2. Find the area of the regions bounded by the following curves, the X-axis and the given lines.

a) $y = x^2$, $x = 1$, $x = 3$ b) $y^2 = 4$, $x = 1$, $x = 4$.

Soln

$$\begin{aligned} \text{a) } A_c &= \int_1^3 y \, dx = \int_1^3 x^2 \, dx = \left[\frac{x^3}{3} \right]_1^3 \\ &= \frac{1}{3} [(3)^3 - (1)^3] = \frac{1}{3} (27 - 1) = \frac{26}{3} \text{ sq. unit} \end{aligned}$$

$$\begin{aligned} \text{b) } A_c &= 2 \int_1^4 y \, dx = 2 \int_1^4 2 \sqrt{x} \, dx \\ &= 4 \int_1^4 \sqrt{x} \, dx \\ &= 4 \left[\frac{x^{3/2}}{3/2} \right]_1^4 \\ &= \frac{8}{3} [(4)^{3/2} - (1)^{3/2}] \\ &= \frac{8}{3} (8 - 1) \\ &= \frac{56}{3} \text{ sq. unit} \end{aligned}$$

Q.3. Find the area of the region bounded by the parabola $y^2 = 16x$ and the line $x = 4$

Soln

Given:- $y^2 = 16x \therefore y = 4\sqrt{x}$ (in 1st quadrant)

$$\begin{aligned} \text{Required area} &= 2 \int_0^4 4\sqrt{x} \, dx \\ &= 8 \left[\frac{x^{3/2}}{3/2} \right]_0^4 \\ &= \frac{16}{3} [(4)^{3/2} - (0)^{3/2}] \\ &= \frac{16}{3} \times (8 - 0) \\ &= \frac{128}{3} \text{ sq. unit.} \end{aligned}$$

Sign of Teacher :