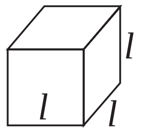
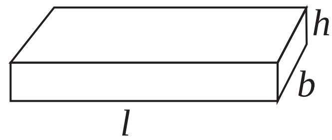
16 Surface area and Volume

Let's recall.

Total surface area of a cuboid = $2(l \times b + b \times h + l \times h)$

Total surface area of a cube = $6l^2$





1 m = 100 cm 1 sq m = 100 × 100 sq cm = 10000 sq cm = 10^4 sq cm

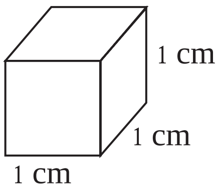
1 cm = 10 mm 1 sq cm = 10 × 10 sq mm = 100 sq mm = 10^2 sq mm

Let's learn.

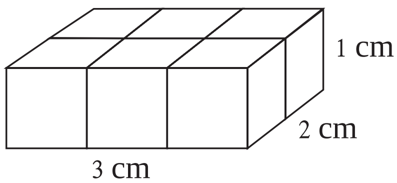
Cuboid, cube, cylinder etc are three dimensional solid figures. **These solid figures occupy some space** . **The measure of the space occupied by a solid is called the volume of the solid**.

Standard unit of volume

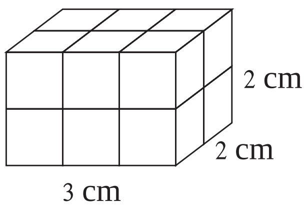
The cube with side 1 cm is shown in the adjoining figure. The space occupied by this cube is a standard unit of volume. It is written as 1 cubic centimeter. In short it is written as 1 cc or 1 cm^3.



Activity I : Get some cubes with side 1 cm. Arrange 6 such cubes as shown in figure. We get a cuboid of length 3 cm, breadth 2 cm and height 1 cm. Note that the volume of the cuboid is 3 × 2 × 1 = 6 cc.



Activity II : The length, breadth and height of the adjoining cuboid is 3 cm, 2 cm and 2 cm respectively. In this cuboid there are 3 × 2 × 2 = 12 cubes of volume 1 cc each.



From this we get the formula,

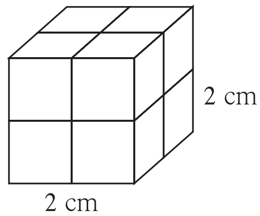
volume of a cuboid = length × breadth × height.

Taking l for length, b for breadth and h for height,

Volume of a cuboid = $l \times b \times h$

Activity III :

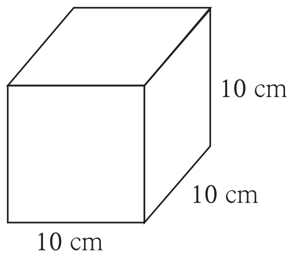
In the adjoing figure 8 cubes each of volume 1 cc are arranged. By this arrangement we get a cube of side 2 cm. Note that volume of this cube $= 2 \times 2 \times 2 = 2^3$



From this if the side of cube is l then **volume of the cube** $= \boldsymbol{l \times l \times l = l^3}$.

Volume of liquid: Space occupied by a liquid in the container is its volume. We know that units used for measuring the volume of liquid are mililitre and litre.

An empty cube of side 10 cm is shown in the adjoining figure.



Its volume $= 10 \times 10 \times 10 = 1000$ cc. If this cube is filled with water the volume of water will also be 1000 cc. This volume is called 1 litre .

We know that 1 litre = 1000 mililitre

$\therefore$ 1 litre = 1000 cc = 1000 ml, hence 1 cc = 1 ml

The volume of water filled in a cube of side 1 cm is 1 ml.

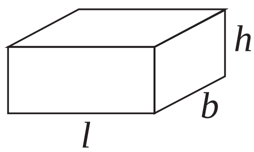
Solved Examples

Ex. (1) Find how many litre of water will a cuboidal fish tank contain if its length, breadth and height are 1 m, 40 cm and 50 cm respectively.

Solution: The water contained in the tank is equal to volume of the tank.

Length of tank = 1m = 100cm, breadth = 40cm,

height = 50cm.



$\therefore$ Volume of the tank $= l \times b \times h = 100 \times 40 \times 50 = 200000$cc,

Volume of water in the tank = 200000 cc $= \dfrac{200000}{1000} = 200$ litre

$(\because\ 1000 \text{ cc} = 1l)$

$\therefore$ Tank will contain 200 litre of water

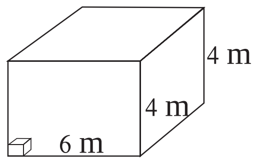
Ex. (2) The length and height of a cuboidal warehouse is 6m, 4m and 4m respectively. How many cube shaped boxes of side 40 cm will fill the warehouse completely ?

Solution : When all the boxes are arranged to fill the warehouse completely the total volume of all boxes equals the volume of the warehouse. To solve the example we will consider the following steps.

(1) Find volume of warehouse

(2) Find volume of a box.

(3) Find the number of boxes



Step (1): Length of warehouse = 6 m = 600 cm, breadth = height = 4 m = 400cm

Volume fo warehouse = $l \times b \times h$ = 600 × 400 × 400 cc

Step (2): Volume of a box = $l^3 = (40)^3$ = 40 × 40 × 40 cc

Step (3): Number of boxes = $\dfrac{\text{volume of warehouse}}{\text{volume of a box}} = \dfrac{600 \times 400 \times 400}{40 \times 40 \times 40}$ = 1500

∴ 1500 boxes will fill the warehouse completely.

Ex. (3) If 5 litre molten mixture of khoa and sugar is poured in a tray it fills to its full capacity. Find the length of the tray if its breadth is 40 cm and height is 2.5 cm

Solution: To solve the example fill the empty boxes with suitable numbers.

Step (1) : Capacity of tray = 5 litre = $\boxed{}$ cc (∵ 1 litre = 1000 cc)

Step (2) : Volume of mixture = $\boxed{}$ cc

Step (3) : Volume of rectangular tray = volume of mixture

$l \times b \times h = \boxed{}$ cc

length × 40 × 2.5 = $\boxed{}$ cc, ∴ length = $\dfrac{\boxed{}}{100}$ = 50 cm

Now I know.

- Volume of cuboid = length × breadth × height = $l \times b \times h$
- Volume of cube = side^3 = l^3

Practice Set 16.1

1. Find the volume of a box if its length, breadth and height are 20 cm, 10.5 cm and 8 cm respectively.

2. A cuboid shape soap bar has volume 150 cc. Find its thickness if its length is 10 cm and breadth is 5 cm.

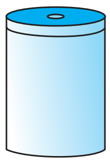
3. How many bricks of length 25 cm, breadth 15 cm and height 10 cm are required to build a wall of length 6 m, height 2.5 m and breadth 0.5 m?

4. For rain water harvesting a tank of length 10 m, breadth 6 m and depth 3m is built. What is the capacity of the tank ? How many litre of water can it hold?

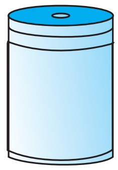
Let's learn.

Surface area of cylinder

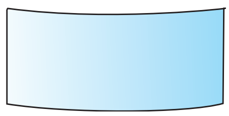
Take a cylinder shaped container.Take a rectangular paper sheet whose breadth is equal to the height of the container. Wrap the paper around the container such that it will exactly cover the curved surface area of container. Cut off the remaining paper.



Cylindrical container



Paper wraped



Length = Circmference of the circle

Unwrap the covered paper. It is a rectangle. Area of this rectangle equals the area of curved surface of cylineder.

Length of the rectangle is the circumference of circle and breadth is height of the cylinder.

Curved surface area of cylinder = Area of rectangle = length × breadth

= circumference of the base of the cylinder × height of the cylinder

$= 2\pi r \times h = 2\pi rh$

The upper and the lower surface of the closed cylinder is circular

$\therefore$ total surface area of closed cylinder = curved surface area + area of upper surface + area of lower surface

$\therefore$ total surface area = curved surface area + 2 × area of circular faces

$$= 2\pi rh + 2\pi r^2 = 2\pi r\,(h + r)$$

Solved examples

Ex. (1) A water tank of cylindrical shape has diameter 1 m and height 2 m. Tank is closed with lid. The tank is to be painted internally and externally including the lid.

Find the expenditure if the rate of painting is ₹ 80 per sqm. ($\pi = 3.14$)

Solution: The tank is to be painted internally and externally. It means that the area to be painted is twice the total surface area.

The diameter of base of cylinder is 1 m

$\therefore$ radius is 0.5 m and height of cylinder is 2 m.

$\therefore$ total surface area of cylinder $= 2\pi r\,(h + r) = 2 \times 3.14 \times 0.5\,(2.0 + 0.5)$

$= 2 \times 3.14 \times 0.5 \times 2.5 = 7.85$ sqm

$\therefore$ the area of the surface to be painted $= 2 \times 7.85 = 15.70$ sqm

$\therefore$ the total expenditure of painting the tank $= 15.70 \times 80 =$ ₹1256.

Ex. (2) A zinc sheet is of length 3.3 m and breadth 3 m. How many pipes of length 30 cm and radius 3.5 cm can be made from this sheet?

Solution: Area of rectangular sheet = length $\times$ breadth

$= 3.3 \times 3$ sqm $= 330 \times 300$ sqcm

Length of pipe is same as the height of cylinder $= h = 30$ cm

Radius of pipe = Radius of base of cylinder $= r = 3.5$ cm,

The sheet required for a pipe = curved surface area of one pipe

$$= 2\pi rh = 2 \times \frac{22}{7} \times \frac{35}{10} \times \frac{30}{1}$$

$= 2 \times 22 \times 15 \quad = 660$ sqcm

$$\text{Number of pipes made from the sheet} = \frac{\text{Area of sheet}}{\text{Curved surface area of a pipe}}$$

$$= \frac{330 \times 300}{660} = 150$$

$\therefore$ from the zinc sheet 150 pipes can be made.

Practice Set 16.2

1. In each example given below, radius of base of a cylinder and and its height are given. Then find the curved surface area and total surface area.

   (1) $r = 7$ cm, $h = 10$ cm (2) $r = 1.4$ cm, $h = 2.1$ cm (3) $r = 2.5$ cm, $h = 7$ cm

   (4) $r = 70$ cm, $h = 1.4$ cm (5) $r = 4.2$ cm, $h = 14$ cm

2. Find the total surface area of a closed cylindrical drum if its diameter is 50 cm and height is 45 cm. ($\pi = 3.14$)

3. Find the area of base and radius of a cylinder if its curved surface area is 660 sqcm and height is 21 cm

4. Find the area of the sheet required to make a cyclindrical container which is open at one side and whose diameter is 28 cm and height is 20 cm. Find the approximate area of the sheet required to make a lid of height 2 cm for this container.

Let's learn.

Volume of a cylinder

To find how much water a tank of cylinderical shape can hold, we have to find the volume of the tank.

A common formula for volume of any prism = area of base × height

Base of a cylinder is circular. $\therefore$ volume of a cylinder = $\pi r^2 h$

Solved Examples

Ex. (1) Find the volume of a cylinder whose height is 10 cm and radius of base is 5 cm ($\pi = 3.14$)

Solution : Radius of base of cylinder $r = 5$ cm and height $h = 10$ cm

volume of cylinder $= \pi r^2 h = 3.14 \times 5^2 \times 10 = 3.14 \times 25 \times 10 = 785$ cc.

Ex. (2) Height of a cylindrical drum is 56 cm. Find the radius of the drum if the capacity of that drum is 70.4 litre.($\pi = \frac{22}{7}$)

Solution : Let the radius of cylindrical drum be = r

capacity of drum = volume of drum $= 70.4 \times 1000$ cc

1litre = 1000 ml $\therefore$ 70.4 litre = 70400 ml

$$\therefore \text{ volume of water} = \pi r^2 h = 70400$$

$$\therefore r^2 = \frac{70400}{\pi h} = \frac{70400 \times 7}{22 \times 56} = \frac{70400}{22 \times 8} = \frac{8800}{22} = 400$$

$\therefore r = 20$, $\therefore$ radius of the drum is 20 cm.

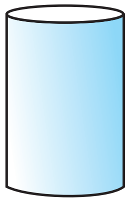
Ex. (3) Radius of a solid copper cylinder is 4.2 cm and its height is 16 cm. How many discs of diameter 1.4 cm and thickness 0.2 cm can be made from this cylinder melting it.

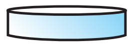
Solution: Radius of the base of the cylinder = R = 4.2 cm

height of the cylinder = H = 16 cm

Volume of cylinder = $\pi R^2 H = \pi \times 4.2 \times 4.2 \times 16.0$

Radius of the base of the disc = $1.4 \div 2 = 0.7$ cm





Thickness of disc = height of disc = 0.2 cm

Volume of a disc = $\pi r^2 h = \pi \times 0.7 \times 0.7 \times 0.2$

Let n discs be made from the molten cylinder.

$\therefore$ $n \times$ volume of one disc = volume of cylinder

$$n = \frac{\text{Volume of cylinder}}{\text{Volume of one disc}} = \frac{\pi R^2 H}{\pi r^2 h} = \frac{R^2 H}{r^2 h} = \frac{4.2 \times 4.2 \times 16}{0.7 \times 0.7 \times 0.2}$$

$$= \frac{42 \times 42 \times 160}{7 \times 7 \times 2} = 6 \times 6 \times 80 = 2880$$

$\therefore$ 2880 discs will be made from the cylinder.

Now I know.

Curved surface area of cylinder = $2\pi rh$

Total surface area of cylinder = $2\pi r(h + r)$

Volume of cylinder = $\pi r^2 h$

Practice Set 16.3

1. Find the volume of the cylinder if height (h) and radius of the base (r) are as given below.

(1) r = 10.5 cm, h = 8 cm (2) r = 2.5 m, h = 7 m

(3) r = 4.2 cm, h = 5 cm (4) r = 5.6 cm, h = 5 cm

2. How much iron is needed to make a rod of length 90 cm and diametar 1.4 cm?

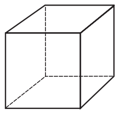
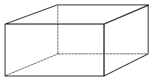
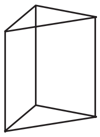
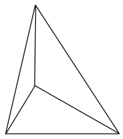
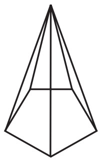
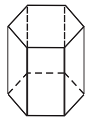
3. How much water will a tank hold if the interior diameter of the tank is 1.6 m and its depth is 0.7 m ?

4. Find the volume of the cylinder if the circumference of the cylinder is 132 cm and height is 25 cm.

Euler's Formula :

Leonard Euler, a great mathematician, at a very young age discovered an interesting formula regarding the faces, vertices and edges of solid figures.

Count and write the faces, vertices and edges of the following figures and complete the table. From the table verify Euler's formula, F + V = E + 2.

Name	Cube	Cuboid	Triangular Prism	Triangular pyramid	Pentagonal pyramid	Hexagonal prism
Shapes						
Faces (F)	6					8
Vertices (V)	8					12
Edges (E)		12			10	

Answers

Practice Set 16.1

1. 1680 c cm **2.** 3 cm **3.** 2000 bricks **4.** 1,80,000 litre.

Practice Set 16.2

1. (1) 440 sq cm, 748 sq cm (2) 18.48 sq cm, 30.80 sq cm

(3) 110 sq cm, 149.29 sq cm (4) 616 sq cm, 31416 sq cm

(5) 369.60 sq cm, 480.48 sq cm

2. 10,990 sq cm **3.** 5 cm, 78.50 sq cm

4. 2376 sq cm, the approximate area for lid is 792 sq cm.

Practice Set 16.3

1. (1) 2772 c cm (2) 137.5 c m (3) 277.2 c cm (4) 492.8 c cm

2. 138.6 c cm **3.** 1408 litre **4.** 34650 c cm

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