10. Increasing and Decreasing Functions

A. Activities

Carry out the following activities.

1) If $f(x) = x^2 - 2\log x$ is increasing. Find the value of x by completing the following activity.

$f(x) = x^2 - 2\log x$

$\therefore f'(x) = 2x - 2\boxed{1/x}$

f is increasing, if $f'(x) \boxed{>} 0$

i.e. if $2x - \dfrac{2}{\boxed{x}} \boxed{>} 0$

i.e. if $2x^2 - 2 \boxed{>} 0$

i.e. if $2x^2 \boxed{>} 2$

i.e. if $x^2 \boxed{>} 1$

i.e. if $x > \boxed{1}$ or $x < \boxed{-1}$

$\therefore$ f is increasing for $x \in (1, \boxed{\infty})$ or $x \in (\boxed{-\infty}, -1)$

2) For what values of x, $x^3 - 6x^2 - 15x + 12$ is decreasing.

$\therefore f'(x) = \boxed{3} x^2 - \boxed{12} x - \boxed{15}$

f is decreasing, if $f'(x) \boxed{<0}\ 0$

i.e. if $3x^2 - 12x - 15 \boxed{<} 0$

i.e. if $x^2 - 4x - 5 \boxed{<} 0$

i.e. $x^2 - 4x + \boxed{4} < \boxed{5+4}$

i.e. if $(x-2)^2 < \boxed{9}$

i.e. $\boxed{-3} < x - 2 < \boxed{3}$

i.e. if $\boxed{-1} < x < \boxed{5}$

$\therefore$ f is decreasing for $x \in (\boxed{-1}, \boxed{5})$

3) The total cost function for production of x articles is given as $C = 100 + 600x - 3x^2$. Find the values of x for which total cost is decreasing by completing the following activity.

$C = 100 + 600x - 3x^2$

$\therefore \dfrac{dc}{dx} = \boxed{0} + \boxed{600} - \boxed{6} x$

$= 0 + \boxed{600} - \boxed{6} x = \boxed{600} - 6x$

Total cost is decreasing, $\dfrac{dc}{dx} \boxed{<} 0$, $600 - 6x \boxed{<} 0 \therefore x \boxed{>} 100$

4) The total cost of manufacturing x articles is $C = 47x + 300x^2 - x^4$. Find x for which average cost is (i) increasing (ii) decreasing using following activity.

$C = 47x + 300x^2 - x^4$

$\therefore C_A = \frac{C}{\boxed{x}} = \frac{47x + 300x^2 - x^4}{\boxed{x}}$

$\therefore C_A = 47 + \boxed{300}x - \boxed{x^3}$

$\therefore \frac{d}{dx} C_A = \boxed{300} - 3x^2$

(i) C_A is increasing, if $\frac{d}{dx} C_A > \boxed{0}$

i.e. if $300 - 3x^2 \boxed{>} 0$

i.e. if $300 \boxed{>} 3x^2$

i.e. if $x^2 < \boxed{100}$

i.e. if $x \boxed{<} 10$

(ii) C_A is decreasing, if $\frac{d}{dx} C_A < 0$

i.e. if $300 - 3x^2 \boxed{<} 0$

i.e. if $3x^2 > \boxed{300}$

i.e. if $x^2 > \boxed{100}$

i.e. if $x > \boxed{10}$

Hence, the average cost is decreasing if $x > \boxed{10}$

B. Solve the Following

Q.1. In a factory, for production of Q articles, standing charges are Rs.500/-, labour charges are Rs.700/- and processing charges are Rs.50 Q. The price of an article is Rs.1700 - 3Q. For what value of Q the price is increasing?

Cost of production of Q articles

C = standing charges + labour charges + processing charges

$\therefore C = 500 + 700 + 50Q = 1200 + 50Q$

$\therefore$ Revenue $(R) = P.Q = (1700 - 3Q)Q = 1700Q - 3Q^2$

$\therefore$ Profit $= R - C$

$\therefore \pi = 1700Q - 3Q^2 - (1200 + 50Q)$.

$= 1650Q - 3Q^2 - 1200$

$\therefore$ differentiating w.r.t. Q on both side,

$\therefore \frac{d\pi}{dQ} = 1650 - 6Q$

If profit is increasing then $\frac{d\pi}{dQ} > 0$

$\therefore 1650 - 6Q > 0$ ie $1650 > 6Q$

$\therefore \quad Q < 275$

$\therefore$ Profit is increasing for $Q < 275$.

Q.2. Find the value of x for which

(i) $f(x) = x^3 - 6x^2 + 12x + 10$ is increasing.

Given $f(x) = x^3 - 6x^2 + 12x + 10$

$\therefore f'(x) = 3x^2 - 12x + 12 + 0$

$= 3(x^2 - 4x + 4)$

$= 3(x-2)^2 > 0$ for all $x \in \mathbb{R}, x \neq 2$

$\therefore f'(x) > 0, \ \forall x \in \mathbb{R} \setminus \{2\}$

$\therefore f$ is increasing for all $x \in \mathbb{R} \setminus \{2\}$

(ii) $f(x) = 3x^2 - 15x + 9$ is decreasing.

Given $f(x) = 3x^2 - 15x + 9$

$\therefore f'(x) = 6x - 15$

If $f(x)$ is decreasing then $f'(x) < 0$

ie. $6x - 15 < 0$

i.e $x < \frac{15}{6}$

i.e. $x < 5/2$

$\therefore f$ is decreasing if $x \in (-\infty, 5/2)$

Q.3. The demand function of commodity at price p is given by $D = 40 - \frac{5p}{8}$
Check whether it is an increasing or decreasing function.

Given, $D = 40 - \frac{5p}{8}$

diff w.r.t. p

$\therefore \frac{dD}{dp} = 0 - \frac{5}{8}(1)$

$\therefore \frac{dD}{dp} = -5/8 < 0$

$\therefore$ The given function is a decreasing function.

Q.4. For manufacturing x units, labour cost is 150 - 54x and processing cost is x^2. Price of each unit is $p = 10800 - 4x^2$. Find the values of x for which (i) total cost is decreasing (ii) revenue is increasing.

Let C be the total cost function and R be the revenue

$\therefore$ Total cost = Labour cost + processing cost

$\therefore C = 150 - 54x + x^2$

$\therefore \frac{dC}{dx} = -54 + 2x$

Since total cost C is decreasing

$\therefore \frac{dC}{dx} = -54 + 2x < 0$

$\therefore 2x < 54 \Rightarrow x < 27$.

$\therefore$ Total cost is decreasing for $x < 27$

(ii) $R = p \cdot x = (10800) - 4x^2)x = 10800x - 4x^3$

$R = 10800x - 4x^3$

diff w.r.t. x

$$\frac{dR}{dx} = 10800 - 12x^2$$

Since revenue is increasing

$$\therefore \frac{dR}{dx} = 10800 - 12x^2 > 0$$

$$\therefore 10800 > 12x^2$$

$$\therefore 900 > x^2$$

$$\therefore x < 30 \text{ or } x > -30$$

$\because$ x is number of units, hence x can't be negative.

$\therefore$ Revenue is increasing if $x < 30$

Q.5. The total cost of manufacturing x articles is $C = 47x + 300x^2 - x^4$. Find values of x for which the average cost is (i) increasing (ii) decreasing.

Total cost $= C = 47x + 300x^2 - x^4$

$$C_A = \frac{C}{x} = \frac{47x + 300x^2 - x^4}{x} = 47 + 300x - x^3$$

$$\therefore \frac{dC_A}{dx} = \frac{d}{dx}(47 + 300x - x^3) = 300 - 3x^2$$

i) $\because$ C_A is increasing $\Rightarrow \frac{dC_A}{dx} > 0$.

$$\therefore 300 - 3x^2 > 0$$

$$\therefore x^2 < 100.$$

$$\therefore x < 10 \text{ or } x > -10.$$

Since x can't be negative,

$$\therefore x < 10$$

$\therefore$ The average ~~price~~ cost C_A is increasing for $x < 10$

ii) C_A is decreasing if $\frac{dC_A}{dx} < 0$

ie $300 - 3x^2 < 0 \Rightarrow x^2 > 100$.

$\therefore x > 10$ or $x < -10$ ($\because$ x can't be negative)

$\therefore$ The average cost C_A is decreasing for $x > 10$.

Sign of Teacher :