3. Trigonometric Functions – I

Let us Recall

- A solution α of a trigonometric equation is called a principal solution if $0 \le \alpha < 2\pi$.
- The general solution of $\sin\theta = \sin\alpha$ is $\theta = n\pi + (-1)^n \alpha$, where $n \in Z$.
- The general solution of $\cos\theta = \cos\alpha$ is $\theta = 2n\pi \pm \alpha$, where $n \in Z$.
- The general solution of $\tan\theta = \tan\alpha$ is $\theta = n\pi + \alpha$, where $n \in Z$.
- The general solution of $\sin^2\theta = \sin^2\alpha$ is $\theta = n\pi \pm \alpha$, where $n \in Z$.
- The general solution of $\cos^2\theta = \cos^2\alpha$ is $\theta = n\pi \pm \alpha$, where $n \in Z$.
- The general solution of $\tan^2\theta = \tan^2\alpha$ is $\theta = n\pi \pm \alpha$, where $n \in Z$.
- **The Sine Rule** : In Δ ABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$, where R is the circumradius of Δ ABC.

  Following are the different forms of the Sine rule.

  (i) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ (ii) $a = 2R\sin A, \quad b = 2R\sin B, \quad c = 2R\sin C$

  (iii) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$ (iv) $\frac{a}{b} = \frac{\sin A}{\sin B}, \frac{b}{c} = \frac{\sin B}{\sin C}$

  (v) $b\sin A = a\sin B, c\sin B = b\sin C, c\sin A = a\sin C$
- **The Cosine Rule** : In Δ ABC,

  $a^2 = b^2 + c^2 - 2bc\cos A$, $b^2 = c^2 + a^2 - 2ca\cos B$, $c^2 = a^2 + b^2 - 2ab\cos C$
- **The Projection Rule** : In Δ ABC,

  $a = b\cos C + c\cos B$, $b = c\cos A + a\cos C$, $c = a\cos B + b\cos A$
- **Half angle formulae** : In Δ ABC, if $a+b+c = 2s$ then

  (i) $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$, $\sin\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}$, $\sin\frac{C}{2} = \sqrt{\frac{(s-b)(s-a)}{ab}}$

  (ii) $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$, $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ac}}$, $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

  (iii) $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$, $\tan\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$, $\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

Ex. (1) In Δ ABC, prove that $a^3\sin(B-C)+b^3\sin(C-A)+c^3\sin(A-B)=0$.

Solution : Method I

We know that by Sine Rule, in Δ ABC, $\frac{a}{sinA}=\frac{b}{sinB}=\frac{c}{sinC}=2R$

$\therefore \quad \frac{sinA}{a}=\frac{sinB}{b}=\frac{sinC}{c}=k$

$\therefore \quad \sin A=ak, \quad \sin B=bk, \quad \sin C=ck$

By Cosine Rule, $b^2+c^2-a^2=2bc\,cosA$,

$c^2+a^2-b^2=2ca\,cosB$,

$a^2+b^2-c^2=2ab\,cosC$

Consider the expression, $a^3\sin(B-C)$,

$a^3\sin(B-C)=a^3(\sin B\cos C-\cos B\sin C)$

$= a^3(bk\cos C-ck\cos B)=ka^2(ab\cos C-ac\cos B)$

$=\frac{ka^2}{2}(2ab\,cosC-2ac\,cosB) = \frac{ka^2}{2}\left((a^2+b^2-c^2)-(c^2+a^2-b^2)\right)$

$= \frac{ka^2}{2}(2b^2-2c^2)=ka^2b^2-ka^2c^2$

$\therefore \quad a^3\sin(B-C)=k^2b^2-k^2c^2 \quad \ldots(1)$

Similarly we can prove that

$b^3\sin(C-A)=kc^2b^2-ka^2b^2 \quad \ldots(2)$

$c^3\sin(A-B)=ka^2c^2-kb^2c^2 \quad \ldots(3)$

Adding (1), (2) and (3) , we get

$a^3\sin(B-C)+b^3\sin(C-A)+c^3\sin(A-B)=0$

Method II : By using identity $\sin(A+B)\sin(A-B)=\sin^2 A-\sin^2 B$

Consider the expression, $a^3\sin(B-C)$,

$a^3\sin(B-C)=a^2a\sin(B-C)$

$=a^2k\sin A\sin(B-C)$

$=a^2k\sin(B+C)\sin(B-C)$

$=a^2k(b^2-c^2)$

$\therefore \quad a^3\sin(B-C) = ka^2b^2-ka^2c^2 \quad \ldots(1)$

Similarly we can prove that

$$b^3\sin(C-A)=kc^2b^2-ka^2b^2 \quad \ldots(2)$$

$$c^3\sin(A-B)=ka^2c^2-kb^2c^2 \ldots(3)$$

Adding (1), (2) and (3) , we get

$$a^3\sin(B-C)+b^3\sin(C-A)+c^3\sin(A-B)=0$$

Ex. (2) In Δ ABC prove that :

$$\left(c^2+b^2-a^2\right)tanA=\left(a^2+c^2-b^2\right)tanB=\left(b^2+a^2-c^2\right)tanC$$

Solution : By Cosine Rule, $a^2=b^2+c^2-2bc\cos A$

$$b^2=c^2+a^2-2ca\cos B$$

$$c^2=a^2+b^2-2ab\cos C$$

Consider the expression $\left(c^2+b^2-a^2\right)tanA$,

$$\left(c^2+b^2-a^2\right)\tan A=2bc\cos A\times\frac{\sin A}{\cos A}$$

$$=2bc\times\sin A$$

$$=2bc\times ak \quad \text{(by Sine Rule)}$$

$$=2abck$$

$$\therefore \quad \left(c^2+b^2-a^2\right)tanA=2abck \quad \ldots\ (1)$$

Similarly we can prove that

$$\left(a^2+c^2-b^2\right)tanB=2abck \qquad ...(2)$$

$$\left(b^2+a^2-c^2\right)tanC=2abck \qquad ...(3)$$

From (1), (2) and (3) , we get

$$\left(c^2+b^2-a^2\right)tanA=\left(a^2+c^2-b^2\right)tanB=\left(b^2+a^2-c^2\right)tanC$$

Ex.(3) In Δ ABC, prove that $cot\left(\frac{A}{2}\right)+cot\left(\frac{B}{2}\right)+cot\left(\frac{C}{2}\right)=\left(\frac{a+b+c}{b+c-a}\right)cot\left(\frac{A}{2}\right)$

Solution : We know that $\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$, $\quad \tan\frac{B}{2}=\sqrt{\frac{(s-a)(s-c)}{s.(s-b)}}$,

$$\tan\frac{C}{2}=\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$$

$$\text{L.H.S.} = \quad \cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}$$

$$=\frac{1}{\tan\frac{A}{2}}+\frac{1}{\tan\frac{B}{2}}+\frac{1}{\tan\frac{C}{2}}$$

$$=\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}+\sqrt{\frac{s(s-b)}{(s-a)(s-c)}}+\sqrt{\frac{s(s-c)}{(s-b)(s-a)}}$$

$$=\sqrt{\frac{s(s-a)^2}{(s-b)(s-c)(s-a)}}+\sqrt{\frac{s(s-b)^2}{(s-a)(s-c)(s-b)}}+\sqrt{\frac{s(s-c)^2}{(s-b)(s-a)(s-c)}}$$

$$=\sqrt{\frac{s}{(s-b)(s-a)(s-c)}}\left\{\sqrt{(s-a)^2}+\sqrt{(s-b)^2}+\sqrt{(s-c)^2}\right\}$$

$$=\sqrt{\frac{s}{(s-a)(s-b)(s-c)}}\left\{(s-a)+(s-b)+(s-c)\right\}$$

$$=\sqrt{\frac{s}{(s-a)(s-b)(s-c)}}\left\{3s-(a+b+c)\right\}$$

$$=\sqrt{\frac{s}{(s-b)(s-a)(s-c)}}\left\{3s-2s\right\}$$

$$=\sqrt{\frac{s}{(s-b)(s-a)(s-c)}}\times s$$

$$=\sqrt{\frac{s}{(s-b)(s-c)}}\times\frac{s}{\sqrt{s-a}}$$

$$=\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}\times\frac{s}{(s-a)}$$

$$=\frac{2s}{(2s-2a)}\times\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}$$

$$=\frac{a+b+c}{(a+b+c-2a)}\times\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}$$

$$=\frac{a+b+c}{b+c-a}\cot\frac{A}{2}$$

= R.H.S.

Ex.(4) If $0<2x<1$ and $\sin^{-1}x+\sin^{-1}2x=\frac{\pi}{3}$ then find x.

Solution : Let $\sin^{-1}x=\theta$

$\therefore\ \sin\theta=x$ and $\cos\theta=\sqrt{1-\sin^2\theta}$

As $\sin^{-1}x+\sin^{-1}2x=\frac{\pi}{3}$

$\sin^{-1}2x=\frac{\pi}{3}-\sin^{-1}x$

$\therefore\ \sin^{-1}2x=\frac{\pi}{3}-\theta$

$\therefore\ 2x=\sin\left(\frac{\pi}{3}-\theta\right)$

$\therefore\ 2x=\sin\frac{\pi}{3}\cos\theta-\cos\frac{\pi}{3}\sin\theta$

$\therefore\ 2x=\frac{\sqrt{3}}{2}.\cos\theta-\frac{1}{2}.\sin\theta$

$\therefore\ 4x=\sqrt{3}\cos\theta-\sin\theta$

$\therefore\ 4x=\sqrt{3}\sqrt{1-\sin^2\theta}-x$

$\therefore\ 5x=\sqrt{3\left(1-x^2\right)}$

$\therefore\ 25x^2=3-3x^2$

$\therefore\ 28x^2=3$

$\therefore\ x=\pm\sqrt{\frac{3}{28}}$

But $0<2x<1$, $\therefore\ x=\sqrt{\frac{3}{28}}$

Ex.(5) Find the general solution of (a) $\sin\theta+\cos\theta+1=0$ (b) $\tan^3\theta-3\tan\theta=0$

Solution : (a) Given $\sin\theta+\cos\theta+1=0$ $\therefore\ \sin\theta+\cos\theta=-1$

Solution :

(a) Given $\sin\theta+\cos\theta+1=0$

$\therefore\ \sin\theta+\cos\theta=-1$

multiplying b.s. by $\frac{1}{\sqrt{2}}$

$\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta=-\frac{1}{\sqrt{2}}$

$\cos\frac{\pi}{4}\cos\theta+\sin\frac{\pi}{4}\sin\theta=-\cos\frac{\pi}{4}$

$\cos\left(\theta-\frac{\pi}{4}\right)=\cos\left(\pi-\frac{\pi}{4}\right)$

$\because -\cos\theta=\cos(\pi-\theta)$

$\therefore\ \cos\left(\theta-\frac{\pi}{4}\right)=\cos\left(\frac{3\pi}{4}\right)$

$\cos\theta=\cos\alpha\Rightarrow\theta=2n\pi\pm\alpha,\ n\in Z$

$\therefore\ \theta-\frac{\pi}{4}=2n\pi\pm\frac{3\pi}{4},\ n\in Z$

$\therefore\ \theta-\frac{\pi}{4}=2n\pi+\frac{3\pi}{4}$

(or) $\theta-\frac{\pi}{4}=2n\pi-\frac{3\pi}{4},\ n\in Z$

$\therefore \theta = 2n\pi + \frac{3\pi}{4} + \frac{\pi}{4}$ or $\theta = 2n\frac{3\pi}{4} + \frac{\pi}{4}$, $n \in Z$

$\therefore \theta = 2n\pi + \pi$ or $\theta = 2n\pi - \frac{\pi}{2}$, $n \in Z$

(b) $\tan^3\theta - 3\tan\theta = 0$

$\therefore \tan\theta(\tan^2\theta - 3) = 0$

Solution :

Given $\tan^3\theta - 3\tan\theta = 0$

$\therefore \tan\theta(\tan^2\theta - 3) = 0$

$\tan\theta = 0$ or $\tan^2\theta - 3 = 0$

consider $\tan\theta = 0$

$\tan\theta = 0 \Rightarrow \theta = n\pi$, $n \in Z$

consider $\tan^2\theta - 3 = 0$

$\tan^2\theta = 3$

$\tan^2\theta = (\sqrt{3})^2$

$\tan^2\theta = \tan^2\frac{\pi}{3}$

$\tan^2\theta = \tan^2\alpha \Rightarrow \theta = m\pi \pm \alpha, m \in Z$

$\therefore \theta = m\pi \pm \frac{\pi}{3}$, $m \in Z$

$\therefore$ The required general sol^n are $\theta = n\pi$ or $\theta = m\pi \pm \frac{\pi}{3}$

$m, n \in Z$

Ex. (6) Using Cosine rule prove the Sine rule.

Solution : $\left(\frac{\sin A}{a}\right)^2 = \frac{\sin^2 A}{a^2}$

$= \frac{1-\cos^2 A}{a^2}$

$= \frac{1 - \left[\frac{b^2+c^2-a^2}{2bc}\right]^2}{a^2}$

$= \frac{1 - \frac{(b^2+c^2-a^2)^2}{(2bc)^2}}{a^2}$

$= \frac{\frac{(2bc)^2 - (b^2+c^2-a^2)^2}{(2bc)^2}}{a^2}$

$= \frac{(2bc+b^2+c^2-a^2)(2bc-b^2-c^2+a^2)}{4a^2b^2c^2}$

$= \frac{(b^2+2bc+c^2-a^2)(a^2-(b^2-2bc+c^2))}{4a^2b^2c^2}$

$= \frac{[(b+c)^2-a^2][a^2-(b-c)^2]}{4a^2b^2c^2}$

$= \frac{(a+b+c)(a+b-c)(b+c-a)(a-b+c)}{4a^2b^2c^2}$ —— (1)

See on Next Page

$\left(\frac{\sin B}{b}\right)^2 = \frac{\sin^2 B}{b^2} = \frac{1-\cos^2 B}{b^2}$

$= \frac{1 - \left[\frac{c^2+a^2-b^2}{2ca}\right]^2}{b^2}$

$= \frac{\frac{(2ca)^2 - (c^2+a^2-b^2)^2}{(2ca)^2}}{b^2}$

$= \frac{(2ca+c^2+a^2-b^2)(2ca-c^2-a^2+b^2)}{4a^2b^2c^2}$

$= \frac{[(c+a-b)(c+a+b)(b+c-a)(b-c+a)]}{4a^2b^2c^2}$

$= \frac{(a+b+c)(a+b-c)(b+c-a)(a-b+c)}{4a^2b^2c^2}$ —— (II)

IIIly

$\left(\frac{\sin C}{c}\right)^2 = \frac{(a+b+c)(a+b-c)(b+c-a)(a-b+c)}{4a^2b^2c^2}$ —— (III)

from (I) (II) and (III)

$\left(\frac{\sin A}{a}\right)^2 = \left(\frac{\sin B}{b}\right)^2 = \left(\frac{\sin C}{c}\right)^2$

$\therefore \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Ex. (7) Write principal solutions of $\tan 5\theta = -1$

Solution :

$\tan 5\theta = -1$

$\tan 5\theta = -\tan\frac{\pi}{4}$

$\tan 5\theta = \tan(\pi - \frac{\pi}{4})$

$[\because -\tan\theta = \tan(\pi-\theta)]$

$\therefore \tan 5\theta = \tan\frac{3\pi}{4}$

$\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha, n \in Z$

$\therefore 5\theta = n\pi + \frac{3\pi}{4}, n \in Z$

$\therefore \theta = \frac{n\pi}{5} + \frac{3\pi}{20}, n \in Z$

put $n=0$, $\theta = \frac{3\pi}{20} \in [0, 2\pi]$

Put $n=1$, $\theta = \frac{\pi}{5} + \frac{3\pi}{20} = \frac{7\pi}{20} \in [0, 2\pi]$

Put $n=2$, $\theta = \frac{2\pi}{5} + \frac{3\pi}{20} = \frac{11\pi}{20} \in [0, 2\pi]$

Put $n=3$, $\theta = \frac{3\pi}{5} + \frac{3\pi}{20} = \frac{3\pi}{4} \in [0, 2\pi]$

Put $n=4$, $\theta = \frac{4\pi}{5} + \frac{3\pi}{20} = \frac{19\pi}{20} \in [0, 2\pi]$

Put $n=5$, $\theta = \frac{5\pi}{5} + \frac{3\pi}{20} = \frac{23\pi}{20} \in [0, 2\pi]$

Put $n=6$, $\theta = \frac{6\pi}{5} + \frac{3\pi}{20} = \frac{27\pi}{20} \in [0, 2\pi]$

Put $n=7$, $\theta = \frac{7\pi}{5} + \frac{3\pi}{20} = \frac{31\pi}{20} \in [0, 2\pi]$

put $n=8$, $\theta = \frac{8\pi}{5} + \frac{3\pi}{20} = \frac{35\pi}{20} \in [0, 2\pi]$

put $n=9$, $\theta = \frac{9\pi}{5} + \frac{3\pi}{20} = \frac{39\pi}{20} \in [0, 2\pi]$

put $n=10$, $\theta = \frac{10\pi}{5} + \frac{3\pi}{20} = \frac{43\pi}{20} \notin [0, 2\pi]$

$\therefore$ The principal solutions of $\tan 5\theta = -1$ is $\left\{\frac{3\pi}{20}, \frac{7\pi}{20}, \frac{11\pi}{20}, \frac{3\pi}{4}, \frac{19\pi}{20}, \frac{23\pi}{20}, \frac{27\pi}{20}, \frac{31\pi}{20}, \frac{7\pi}{4}, \frac{39\pi}{20}\right\}$

Sign of Teacher :

Q. 26. A solenoid of length π m and 5 cm in diameter has a winding of 1000 turns and carries a current of 5A. Calculate the magnetic field at its centre along the axis.

SECTION – D

Attempt any THREE questions of the following : **[12]**

Q. 27. What is Ferromagnetism? Explain it on the basis of domain theory.

Q. 28. Obtain an expression for average power dissipated in a series LCR circuit.

Q. 29. Distinguish between interference and diffraction of light.

A double slit arrangement produces interference fringes for sodium light of wavelength 589 nm, that are 0.20 degree apart. What is the angular fringe separation if the entire arrangement is immersed in water?

(R.I. of water = 1.33)

Q. 30. State Einstein's photoelectric equation and mention physical significance of each term involved in it.

The wavelength of incident light is 4000Å. Calculate the energy of incident photon.

Q. 31. State any four uses of Van de Graaff generator.

In a parallel plate air capacitor, intensity of electric field is changing at the rate of 2×10^{11} V/ms. If area of each plate is 20 cm^2, calculate the displacement current.

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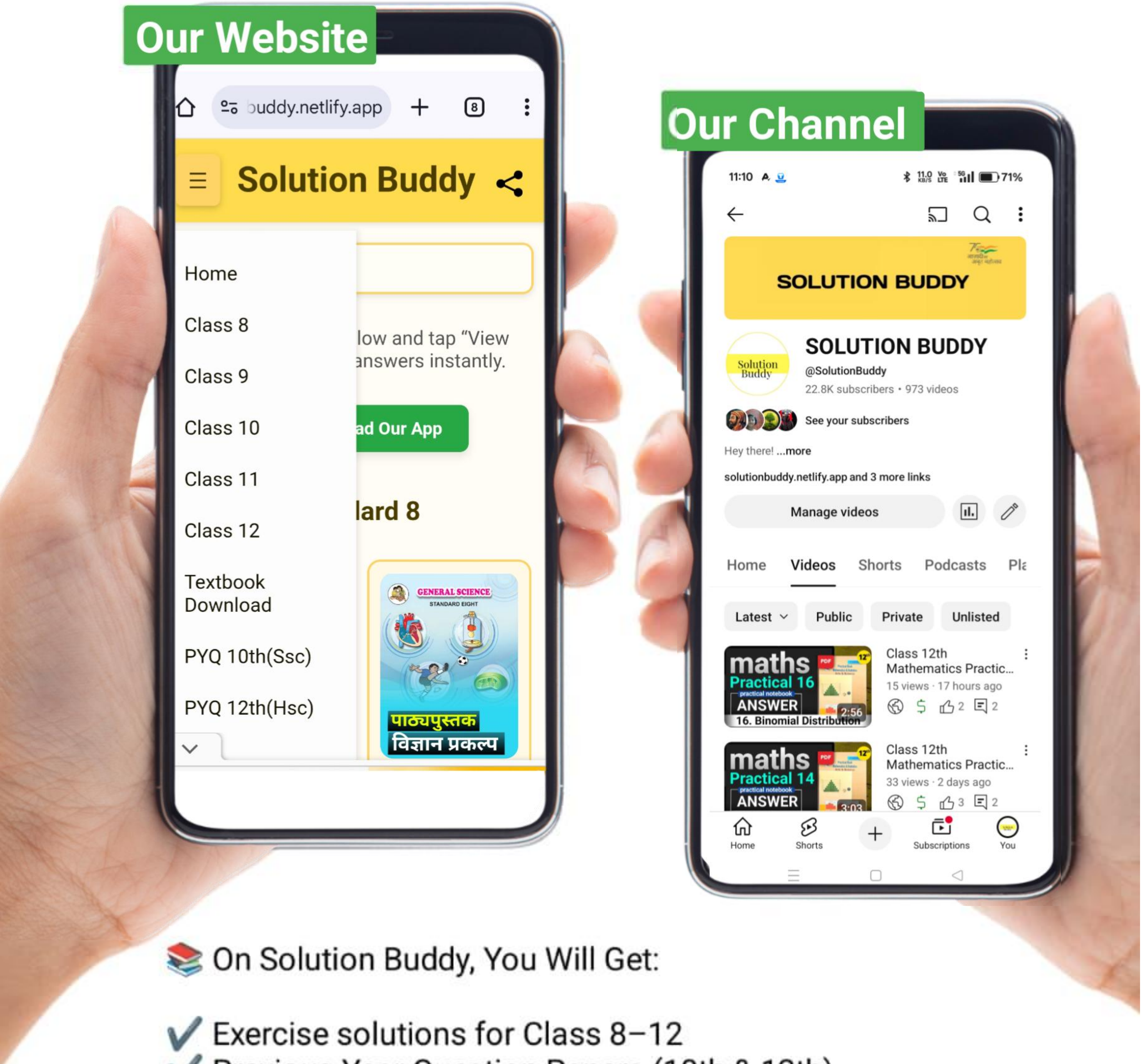

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