

## 15. Sequencing

### A. Activities

- 1) The data given below indicate the processing time in hours of 5 jobs A, B, C, D and E on two machines  $M_1$ ,  $M_2$ ,  $M_3$

Job	A	B	C	D	E	Total
Machine $M_1$	4	13	7	12	6	42
Machine $M_2$	3	15	5	6	11	40

Using the optimal sequence along them, the optimal sequence is

$M_1 \rightarrow$	E	B	D	C	A	$M_2$
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The working table is

Job	Machine $M_1$		Machine $M_2$	
	In time	Out time	In time	Out time
E	0	6	6	17
	6	19	19	34
D	19	31	34	40
	31	38	40	45
A	38	42	45	48

$\therefore$  Total elapsed time = 48 hrs.

Idle time for machines

$$M_1 = 48 - 42 = 6 \text{ hrs.}$$

$$M_2 = 48 - 40 = 8 \text{ hrs.}$$

- 2) Given five jobs, each of which is to be processed through three machines A, B and C in the order ABC. Processing time in hours are

Job	1	2	3	4	5	Total
A	3	8	7	5	4	27

B	4	5	1	2	3	15
C	7	9	5	6	10	37

$$\text{Min (A)} = 3, \text{Max (B)} = 5, \text{Min (C)} = 5$$

since  $\text{Min (C)} = 5$   $\text{Max (B)}$  is satisfied, the problem can be converted into 5 jobs and 2 machine problem. Consider two fictitious machines

G and H

Such that

$$G = A + B$$

$$H = B + C$$

The problem is

Job	1	2	3	4	5
G	7	13	8	7	7
H	11	14	6	8	13

Using optimal sequence algorithm. We get following optimal sequence.

4	1	5	2	3
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The working table is

Job	Machine A		Machine B		Machine C	
	In time	Out time	In time	Out time	In time	Out time
4	0	5	5	7	7	13
	5	8	8	12	13	20
5	8	12	12	15	20	30
	12	20	20	25	30	39
3	20	27	27	28	39	44

$$\text{Minimum elapsed time} = 44 \text{ hr}$$

Idle time for machines

$$A = 44 - 27 = 17 \text{ hrs}$$

$$B = 44 - 15 = 29 \text{ hrs}$$

$$C = 44 - 37 = 7 \text{ hrs}$$

B. Solve the Following

Q.1. There are six jobs each of which is to be processed through the machines  $M_1$  and  $M_2$ , in the order  $M_1 M_2$ . The processing time is given in hours. Determine the sequence of these jobs which minimizes the total elapsed time and the minimum elapsed time. Also find the idle times for the machines  $M_1$  and  $M_2$ :

Jobs	1	2	3	4	5	6
Processing time on $M_1$	5	9	4	7	8	5
Processing time on $M_2$	8	4	8	3	6	6

$\min(M_{i1}, M_{i2}) = 3$ , which corresponds to Job 4 is

Processed in last.

					4
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Problem reduce to 5 jobs.

$\min(M_{i1}, M_{i2}) = 4$  which

covers to  $M_1$  &  $M_2$ .

3				2	4
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The Problem reduce to 3 jobs.

$\min(M_{i1}, M_{i2}) = 5$  which

corresponds to  $M_1$ .

	1	6		2	4
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	6	1		2	4
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The remaining must be

Processed next to Job 6 or Job 1.

3	1	6	5	2	4
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3	6	1	5	2	4
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Total elapsed time  $\Rightarrow$

Job Sequence	$M_1$		$M_2$		Idle time for $M_2$
	In	Out	In	Out	
3	0	4	4	12	4
1	4	9	12	20	0
6	9	14	20	26	0
5	14	22	26	32	0
2	22	31	32	36	0
4	31	38	38	41	2

The min total elapsed time  $T = 41$  hrs.

Idle time for  $M_1 = 41 - 38 = 3$  hrs.

Idle time for  $M_2 = 4 + 2 = 6$  hrs.

Q.2. Find the sequence that minimizes the total elapsed time required to complete the following tasks. The table below gives the processing time in hours. Also find the minimum elapsed time and idle times for both the machines:

Jobs	1	2	3	4	5
$M_1$	3	7	4	5	7
$M_2$	6	2	7	3	4



$\min(M_{i1}, M_{i2}) = 2$  which correspond to  $M_2$ .  $\therefore$  Job 2 is processed in last

				2
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Problem reduce to 4 jobs. Here

$\min(M_{i1}, M_{i2}) = 3$  which correspond to both  $M_1$  and  $M_2$

1			4	2
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Now Problem reduce to 2 jobs

$\min(M_{i1}, M_{i2}) = 4$  which correspond to both  $M_1$  &  $M_2$

Job 3 is processed first next to Job 1 and Job 5 is

Processed in last next to

Job 4. Thus the optimal seq is.

1	3	5	4	2
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Total elapsed time  $\Rightarrow$

Job Sequence	$M_1$		$M_2$		Idle time for $M_2$
	In	Out	In	Out	
1	0	3	3	9	3
3	3	7	9	16	0
5	7	14	16	20	0
4	14	19	20	23	3
2	19	26	26	28	0

Total elapsed time  $T = 28$  hours

Idle time for  $M_2 = 6$  Hours.

Idle time for  $M_1$

$= T - \{ \text{Sum of Pro. time of all five jobs on } M_1 \}$

$= 28 - 26$

$= 2$  Hours.

Q.3. Suppose there are five jobs each must go through machines  $M_1$ ,  $M_2$  and  $M_3$  in the order  $M_1 M_2 M_3$ . Processing times in hours are given below. Determine the optimal sequence and total elapsed time. Also find the idle times for three machines:

Jobs	1	2	3	4	5
$M_1$	8	5	4	6	5
$M_2$	6	2	9	7	4
$M_3$	10	13	11	10	12

Here  $\min M_1 = 4$ ,  $\max M_2 = 9$ ,  $\min M_3 = 10$ .

Since  $\min M_3 \geq \max M_2$  is satisfied, the problem can be converted into two machines problem.

Let  $G$  and  $H$  be two fictitious machines such that  $G = M_1 + M_2$  and  $H = M_2 + M_3$ . Then the Problem written as

Jobs	1	2	3	4	5
G	14	7	13	13	9
H	16	15	20	17	16

Observe that  $\text{Min}(G, H) = 7$  corresponds to job 2 on G  
 $\therefore$  Job 2 is placed first in sequence.

then sequence problem reduced to

Jobs	1	3	4	5
G	14	13	13	9
H	16	20	17	16

Now

$\text{Min}(G, H) = 9$  corresponds to job 5

2	5			
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on machine G which is placed in second in sequence. then Problem reduced to

Jobs	1	3	4
G	14	13	13
H	16	20	17

Now,  $\text{Min}(G, H) = 13$  corresponds to job 3 on machine G as well as job 4 on remaining job 1 placed last in the sequence. we consider the optimal sequence as:

2	5	3	4	1
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Total elapsed time  $\Rightarrow$

Jobs	M <sub>1</sub>		M <sub>2</sub>		M <sub>3</sub>	
	In	Out	In	Out	In	Out
2 (5, 2, 13)	0	5	5	7	7	20
5 (5, 4, 12)	5	10	10	14	20	32
3 (4, 4, 11)	10	14	14	23	32	43
4 (6, 7, 10)	14	20	23	30	43	53
1 (8, 6, 10)	20	28	30	36	53	63

Total elapsed time (T) = 63 hours

Idle time for M<sub>1</sub> = 63 - 28 = 35 hours

Idle time for M<sub>2</sub> = (63 - 36) + 5 + 3 = 35 hours

Idle time for M<sub>3</sub> = 11 hours

Sign of Teacher :