15. Probability Distribution

Ex. (1). A random variable X has the following probability distribution :

$X = x$	0	1	2	3	4	5	6
$P(X = x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

Find (i) k (ii) $p(X < 3)$ (iii) $p(X \geq 2)$ (iv) $P(0 < X < 4)$ (v) $P(2 \leq X \leq 5)$

Solution : For a random variable X we have $\sum_{i=1}^{n} p_i = 1$

$\therefore k + 3k + 5k + 7k + 9k + 11k + 13k = 1$

i.e. $49k = 1 \Rightarrow k = \frac{1}{49}$

$X = x$	0	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{49}$	$\frac{3}{49}$	$\frac{5}{49}$	$\frac{7}{49}$	$\frac{9}{49}$	$\frac{11}{49}$	$\frac{13}{49}$

(i) $k = \frac{1}{49}$

(ii) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$= \frac{1}{49} + \frac{3}{49} + \frac{5}{49} = \frac{9}{49}$

(iii) $P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$

(iv) $P(0 < X < 4) = P(X = 1) + P(X = 2) + P(X = 3)$

(v) $P(2 \leq X \leq 5) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$

$= \frac{5}{49} + \frac{7}{49} + \frac{9}{49} + \frac{11}{49} = \frac{32}{49}$

Ex. (2). Calculate the Expected value and Variance of X if X denotes the number obtained on the uppermost face when a fair die is thrown.

Solution : When a fair die is thrown, the sample space is $S = \{1, 2, 3, 4, 5, 6\}$.

Let X denotes the number obtained on the uppermost face.

$\therefore X$ can take values 1, 2, 3, 4, 5, 6.

$P(X = 1) = P(X = 2) = P(X = 3) = P(X = 4) = P(X = 5) = P(X = 6)$

$= \frac{1}{6}$

The probability distribution is

$X = x$	1	2	3	4	5	6	Total
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1

$x_i p_i$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{6}{6}$	$\frac{21}{6}=\frac{7}{2}$
$x_i^2 p_i$	$\frac{1}{6}$	$\frac{4}{6}$	$\frac{9}{6}$	$\frac{16}{6}$	$\frac{25}{6}$	$\frac{36}{6}$	$\frac{91}{6}$

(i) Expected Value $= E(X) = \sum_{i=1}^{n} x_i, p_i = \frac{7}{2} = 3.5$

(ii) Variance $= V(X) = E(X^2) - [E(X)]^2$

$$= \sum_{i=1}^{n} x_i^2, p_i - \left(\sum_{i=1}^{n} x_i, p_i\right)^2$$

$$= \frac{91}{6} - \left(\frac{7}{2}\right)^2 = \frac{91}{6} - \frac{49}{4}$$

$$= \frac{182-147}{12}$$

$\therefore$ Variance $= V(X) = \frac{35}{12} = 2.9167$

Ex. (3). A discrete random variable X takes the values -1, 0 and 2 with the probabilities $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$ respectively. Find $V(X)$ and Standard Deviation.

Solution : Given that the random variable X takes the values -1, 0 and 2.

The corresponding probabilities are $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$.

$P(-1) = \frac{1}{4}$, $P(0) = \frac{1}{2}$ and $P(2) = \frac{1}{4}$

Given data can be tabulated as follows

$X = x$	-1	0	2	Total
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	1
$x_i p_i$	-1/4	0	1/2	1/4
$x^2_i p_i$	1/4	0	1/4	5/4

(i) Variance $= V(X) = E(X^2) - [E(X)]^2$

$$= \sum_{i=1}^{n} x_i^2 p_i - \left(\sum_{i=1}^{n} x_i p_i\right)^2$$

$$= \frac{5}{4} - \left(\frac{1}{4}\right)^2$$

$$= \frac{5}{4} - \frac{1}{16} = 1.1875$$

(ii) Standard Deviation $= \sigma = \sqrt{V(X)} = 1.0897$

Ex. (4) The p. d. f. of X, find $P(X < 1)$ and $P(|X| < 1)$ where

$f(x) = \frac{x+2}{18}$ if $-2 < x < 4$

$= 0$ otherwise.

Solution : Given that the p. d. f. of X is

$$f(x) = \frac{x+2}{18} \quad \text{if } -2 < x < 4$$
$$= 0 \quad \text{otherwise.}$$

(i) $P(X < 1) = \int_{-2}^{1} f(x)dx$

$= \int_{-2}^{1} \_\_\_\_ dx$

$= \frac{1}{18} \int_{-2}^{1} (\ldots\ldots\ldots\ldots) dx$

$= \frac{1}{18} \left[\frac{(\ldots\ldots\ldots)^2}{\ldots\ldots}\right]_{-2}^{1}$

$= \frac{1}{36} \left[\frac{(\ldots\ldots\ldots)^2}{\ldots\ldots}\right]_{-2}^{1}$

$= \frac{1}{36} [\ldots - \ldots] = \frac{\ldots}{36} = \frac{\ldots}{\ldots} = \ldots\ldots\ldots$

(i) $P(|X| < 1) = P(-1 < X < 1)$

$= \int_{-1}^{1} \_\_\_\_ dx$

$= \frac{1}{18} \int_{-1}^{1} (\ldots\ldots\ldots) dx$

$= \frac{1}{18} \left[\frac{(\ldots\ldots\ldots)^2}{\ldots\ldots}\right]_{-1}^{1}$

$= \frac{1}{36} \left[(\ldots\ldots\ldots)^2\right]_{-1}^{1}$

$= \frac{1}{36} [\ldots\ldots - \ldots\ldots] = \frac{\ldots}{36} = \frac{2}{\ldots} = \ldots\ldots\ldots$

Ex (5). **A random variable X has the following probability distribution :**

x	0	1	2	3	4	5	6	7
$P(X = x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find (i) k (ii) $P(X < 3)$ (iii) $P(X > 6)$ (iv) $P(0 < X < 3)$ (v) $P(2 \leq X \leq 4)$

Solution : Since P(X) is Probability distribution of x

$\sum_{x=0} [P(x)] = 1$

$P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) = 1$

$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$10k^2 + 9k - 1 = 0$

$10K^2 + 10K - K - 1 = 0$

$10K(K+1) - 1(K+1) = 0$

$\therefore (10K-1)(K+1) = 0$

$K = \frac{1}{10}$ or $K = -1$

$K = -1$ Is Not Possible

(i) $K = 1/10$

(ii) $P(x<3) = 0 + K + 2K$

$= 3K$

$= 3/10$

(iii) $P(x>6) = 7K^2 + K$

$= \frac{7}{10^2} + \frac{1}{10}$

$= \frac{17}{100}$

(iv) $P(0<x<3) = K + 2K$

$= 3K$

$= \frac{3}{10}$

(v) $P(2 \le x \le 4)$

$= 2K + 2K + 3K$

$= 7K$

$= \frac{7}{10}$

Ex. (6). The p. m. f. of a random variable X is as follows :

$X = x$	1	2	3	4
$P(x)$	$\frac{1}{30}$	$\frac{4}{30}$	$\frac{9}{30}$	$\frac{16}{30}$

Find Mean and the Variance.

Solution:

$X = x$	1	2	3	4	Σ
$P(x)$	1/30	4/30	9/30	16/30	10/3
$x^2 P(x)$	1/30	16/30	9/30	256/30	59/5

From the table

$\Sigma x_i p_i = \frac{10}{3}$ and $\Sigma x_i^2 p_i = 59/5$

Mean $= E(x) = \frac{10}{3} = 3.33$

Variance $= V(x) = \sum x_i^2 p_i - [\sum x_i p_i]^2$

$= \frac{59}{5} - \left[\frac{10}{3}\right]^2$

$= \frac{59}{5} - \frac{100}{9}$

$= \frac{531 - 500}{45}$

$V(x) = \frac{31}{14}$

Hence mean $= 3.33$

Variance $= 0.6888$

Ex. (7). From a survey of 20 familie[illegible] the following data was obtained :

No. of children	0	1	4
No. of families	5	11	2

For the random variable X [illegible] children in a randomly chosen family, Find $E(X)$ and $V(X)$.

Solution :

x	0	1	2	3	4	Σ
f	5	11	2/20	0	2	20
$P(x)$	5/25	11/20	4/20	0	2/20	.
$x_i p_i$	0	1/20	4/20	0	8/20	23/20
$x_i^2 p_i$	0	11/20	8/20	0	32/20	51/20

Hence $E(x) = \sum x_i p_i$

$= \frac{23}{20}$

$= 1.15$

$V(X) = \sum x_i^2 p_i - [\sum x_i p_i]^2$

$= \frac{51}{20} - \left[\frac{23}{20}\right]^2$

$= \frac{51}{20} - \frac{529}{400} = \frac{491}{400}$

$V(X) = 1.2275$

Cancelled

Ex. (8). **Find the c.d.f. $F(X)$ associated with the following p.d.f $f(x)$:**

$f(x) = 12x^2(1 - x)$ **for** $0 < x < 1$

$= 0$ **otherwise.**

Also, find $P\left(\frac{1}{3} < X < \frac{1}{2}\right)$ **by using p.d.f and c.d.f.**

Solution :

Sign of Teacher :

Q. 26. A solenoid of length π m and 5 cm in diameter has a winding of 1000 turns and carries a current of 5A. Calculate the magnetic field at its centre along the axis.

SECTION – D

Attempt any THREE questions of the following : **[12]**

Q. 27. What is Ferromagnetism? Explain it on the basis of domain theory.

Q. 28. Obtain an expression for average power dissipated in a series LCR circuit.

Q. 29. Distinguish between interference and diffraction of light.

A double slit arrangement produces interference fringes for sodium light of wavelength 589 nm, that are 0.20 degree apart. What is the angular fringe separation if the entire arrangement is immersed in water?

(R.I. of water = 1.33)

Q. 30. State Einstein's photoelectric equation and mention physical significance of each term involved in it.

The wavelength of incident light is 4000Å. Calculate the energy of incident photon.

Q. 31. State any four uses of Van de Graaff generator.

In a parallel plate air capacitor, intensity of electric field is changing at the rate of 2×10^{11} V/ms. If area of each plate is 20 cm^2, calculate the displacement current.

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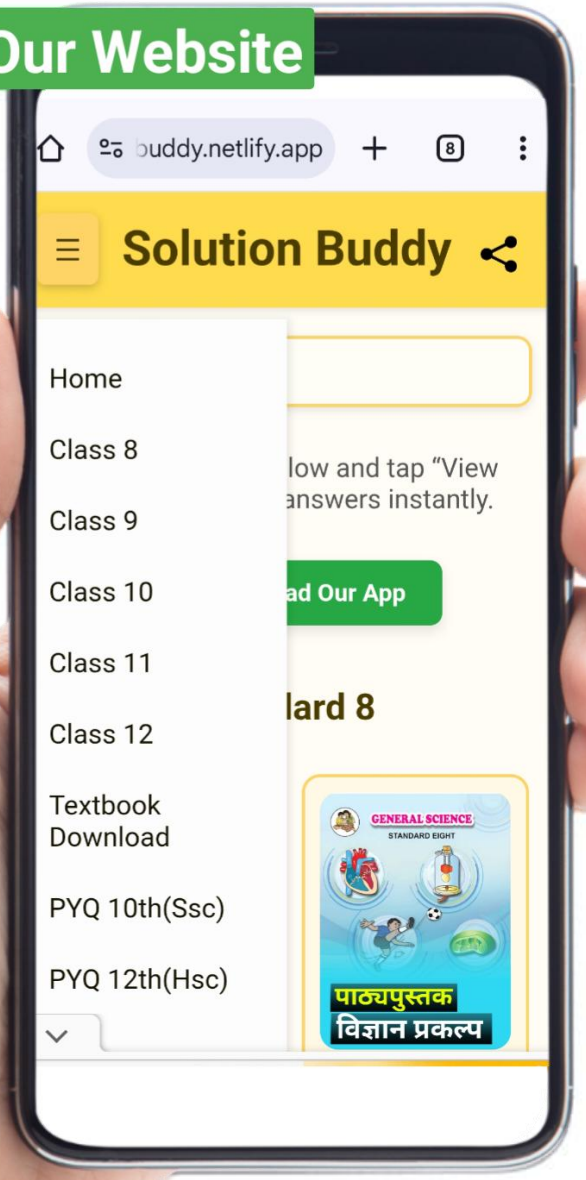

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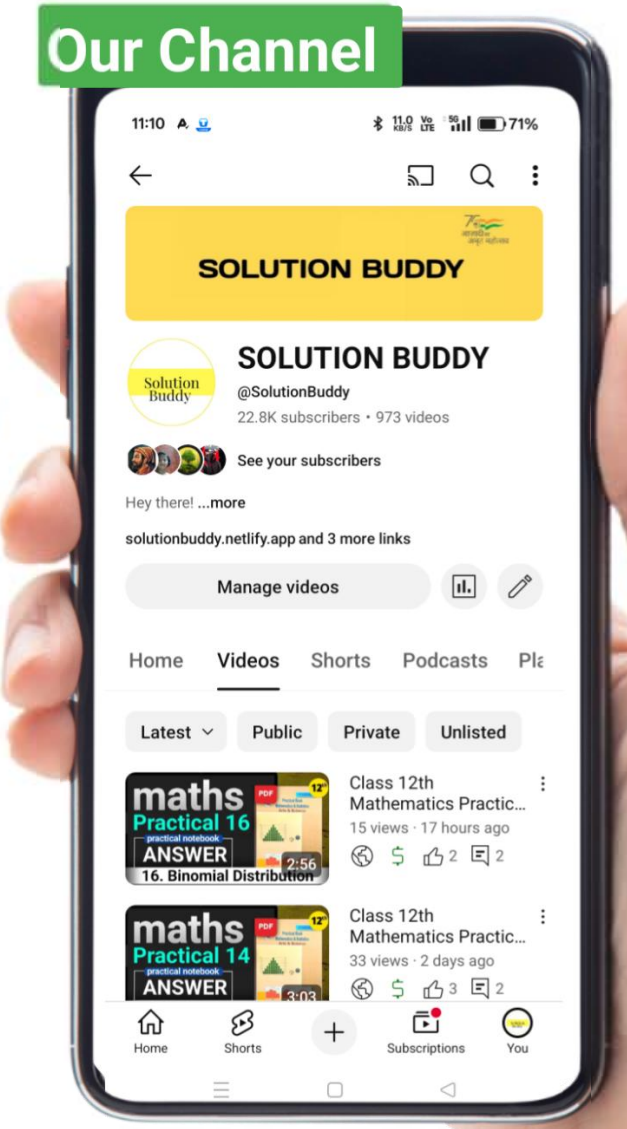

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