

## 1. Logic

### Let us Recall

- The converse, inverse and contrapositive of the implication  $p \rightarrow q$  are:

Converse :  $q \rightarrow p$

Inverse :  $\sim p \rightarrow \sim q$

Contrapositive :  $\sim q \rightarrow \sim p$

- Quantifiers and quantified Statements : Look at the following statements:

$p$  : "There exists an even prime number in the set of natural numbers".

$q$  : "All natural numbers are positive".

Each of them asserts a condition for some or all objects in a collection.

Words "there exists" and "for all" are called quantifiers. "There exists" is called existential quantifier and is denoted by symbol  $\exists$ . "For all" is called universal quantifier and is denoted by  $\forall$ . Statements involving quantifiers are called quantified statements. Every quantified statement corresponds to a collection and a condition. In statement  $p$  the collection is 'the set of natural numbers' and the condition is 'being even prime'.

What is the condition in the statement  $q$  ?

A statement quantified by universal quantifier  $\forall$  is true if all objects in the collection satisfy the condition. And it is false if at least one object in the collection does not satisfy the condition.

A statement quantified by existential quantifier  $\exists$  is true if at least one object in the collection satisfy the condition. And it is false if no object in the collection satisfy the condition.

Idempotent Law	$p \wedge p \equiv p, \quad p \vee p \equiv p$
Commutative Law	$p \vee q \equiv q \vee p, \quad p \wedge q \equiv q \wedge p$
Associative Law	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r \equiv p \wedge q \wedge r$
	$p \vee (q \vee r) \equiv (p \vee q) \vee r \equiv p \vee q \vee r$
Distributive Law	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
De Morgan's Law	$\sim(p \wedge q) \equiv \sim p \vee \sim q, \quad \sim(p \vee q) \equiv \sim p \wedge \sim q$
Identity Law	$p \wedge T \equiv p, \quad p \wedge F \equiv F, \quad p \vee F \equiv p, \quad p \vee T \equiv T$
Complement Law	$p \wedge \sim p \equiv F, \quad p \vee \sim p \equiv T$



Absorption Law	$p \vee (p \wedge q) \equiv p, p \wedge (p \vee q) \equiv p$
Conditional Law	$p \rightarrow q \equiv \sim p \vee q$
Biconditional Law	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p) \equiv (\sim p \vee q) \wedge (\sim q \vee p)$

**Ex. (1)** Write the converse, inverse, contrapositive and the negation of the implication: "If two sides of a triangle are congruent then it's two angles are congruent."

**Solution :**

Converse : If two angles of a triangle are congruent then it's two sides are congruent.

Inverse : If two sides of a triangle are not congruent then it's two angles are not congruent.

Contrapositive : If two angles of a triangle are not congruent then it's two sides are not congruent.

Negation : Two sides of a triangle are congruent but it's two angles are not congruent.

**Ex. (2)** Write (a) truth values and (b) negations of the following statements :

i)  $\forall x \in R, x^2$  is positive. ii)  $\exists x \in R, x^2$  is not positive.

iii) Every square is a rectangle. iv) Some parallelograms are rectangles.

**Solution :**

a) Truth values

i) false because the square of 0 is not positive

ii) true because the square of 0 is not positive

iii) true iv) true

b) Negations

i)  $\exists x \in R, x^2$  is not positive. ii)  $\forall x \in R, x^2$  is positive.

iii) There exists a square which is not a rectangle.

iv) No parallelogram is a rectangle.

**Ex. (3)** Without using truth table prove that  $\{[(p \vee q) \wedge \sim p] \rightarrow \sim q\} \equiv q \rightarrow p$

**Solution :**

$$\text{L.H.S.} = \{[(p \vee q) \wedge \sim p] \rightarrow \sim q\}$$

$$\equiv \sim[(p \vee q) \wedge \sim p] \vee \sim q \quad (\dots \text{Conditional law} \dots)$$



$$\begin{aligned}
&\equiv [\sim(p \vee q) \vee p] \vee \sim q & (\text{De-Morgan's Law}) \\
&\equiv [(\sim p \wedge \sim q) \vee p] \vee \sim q & (\text{De-Morgan's Law}) \\
&\equiv [(\sim p \vee p) \wedge (\sim q \vee p)] \vee \sim q & (\text{Distributive Law}) \\
&\equiv [(t) \wedge (\sim q \vee p)] \vee \sim q & (\text{Complement Law}) \\
&\equiv (\sim q \vee p) \vee \sim q & (\text{Identity Law}) \\
&\equiv \sim q \vee p \vee \sim q & (\text{Associative Law}) \\
&\equiv \sim q \vee p & (\text{Idempotent Law}) \\
&\equiv q \rightarrow p & (\text{Conditional Law})
\end{aligned}$$

L. H. S. = R. H. S.

**Ex. (4)** Using truth table prove that  $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$ .

**Solution :**

I	II	III	IV	V	VI	VII	VIII	IX
p	q	$\sim p$	$\sim q$	$p \leftrightarrow q$	$\sim(p \leftrightarrow q)$	$p \wedge \sim q$	$q \wedge \sim p$	$(p \wedge \sim q) \vee (q \wedge \sim p)$
T	T	F	F	T	F	F	F	F
T	F	F	T	F	T	T	F	T
F	T	T	F	F	T	F	T	T
F	F	T	T	T	F	F	F	F

From column (VI) and (IX) we conclude that

$$\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p) \dots \dots \dots$$

**Ex. (5)** Is  $\sim(p \leftrightarrow q)$  equivalent to  $(\sim p) \leftrightarrow q$  ? Justify.

**Solution :**

I	II	III	IV	V	VI
p	q	$\sim p$	$\sim p \leftrightarrow q$	$p \leftrightarrow q$	$\sim(p \leftrightarrow q)$
T	T	F	F	T	F
T	F	F	T	F	T
F	T	T	T	F	T
F	F	T	F	T	F

From column no IV and VI we conclude that  $\sim(p \leftrightarrow q) \equiv (\sim p) \leftrightarrow q$

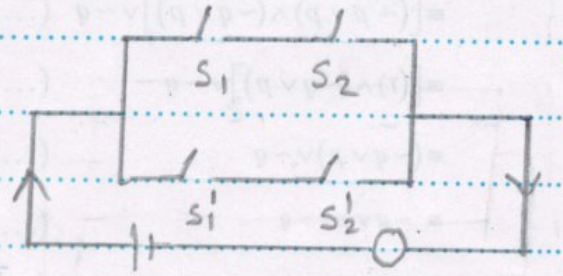


**Ex. (6)** Draw the switching circuits and prepare the input output tables for statement patterns :

a)  $(p \wedge q) \vee (\sim p \wedge \sim q)$       b)  $(p \vee q) \wedge \sim q$

**Solution :**

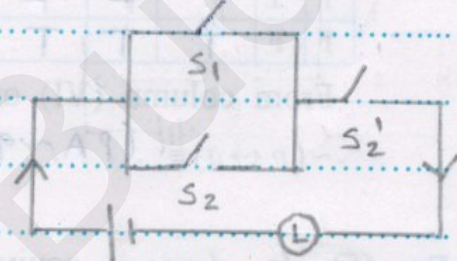
- ① Let p switch  $S_1$  is closed  
 q switch  $S_2$  is closed  
 $\sim p$   $S_1'$  is closed  
 $\sim q$   $S_2'$  is closed



p	q	$p \wedge q$	$\sim p$	$\sim q$	$(p \wedge q) \vee (\sim p \wedge \sim q)$
1	1	1	0	0	1
1	0	0	0	1	0
0	1	0	1	0	0
0	0	0	1	1	1

②

p	q	$\sim q$	$p \vee q$	$(p \vee q) \wedge \sim q$
1	1	0	1	0
1	0	1	1	1
0	1	0	1	0
0	0	1	0	0



**Ex. (7)** Using truth table prove that  $(p \wedge q) \vee r \equiv (p \vee r) \wedge (q \vee r)$

**Solution :**

p	q	r	$p \wedge q$	$(p \wedge q) \vee r$	$p \vee r$	$q \vee r$	$(p \vee r) \wedge (q \vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	T	F	F
F	T	T	F	T	T	T	T
F	T	F	F	F	F	T	F
F	F	T	F	T	T	T	T
F	F	F	F	F	F	F	F

Sign of Teacher :

From column (v) and (viii) we conclude that

$$(p \wedge q) \vee r \equiv (p \vee r) \wedge (q \vee r)$$



## 15. Probability Distribution

**Ex. (1).** A random variable  $X$  has the following probability distribution :

$X = x$	0	1	2	3	4	5	6
$P(X = x)$	$k$	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

Find (i)  $k$  (ii)  $P(X < 3)$  (iii)  $P(X \geq 2)$  (iv)  $P(0 < X < 4)$  (v)  $P(2 \leq X \leq 5)$

**Solution :** For a random variable  $X$  we have  $\sum_{i=1}^n p_i = 1$

$$\therefore k + 3k + 5k + 7k + 9k + 11k + 13k = 1$$

$$\text{i.e. } 49k = 1 \Rightarrow k = \frac{1}{49}$$

$X = x$	0	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{49}$	$\frac{3}{49}$	$\frac{5}{49}$	$\frac{7}{49}$	$\frac{9}{49}$	$\frac{11}{49}$	$\frac{13}{49}$

(i)  $k = \frac{1}{49}$

(ii)  $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= \frac{1}{49} + \frac{3}{49} + \frac{5}{49} = \frac{9}{49}$$

(iii)  $P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$

(iv)  $P(0 < X < 4) = P(X = 1) + P(X = 2) + P(X = 3)$

(v)  $P(2 \leq X \leq 5) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$

$$= \frac{5}{49} + \frac{7}{49} + \frac{9}{49} + \frac{11}{49} = \frac{32}{49}$$

**Ex. (2).** Calculate the Expected value and Variance of  $X$  if  $X$  denotes the number obtained on the uppermost face when a fair die is thrown.

**Solution :** When a fair die is thrown, the sample space is  $S = \{1, 2, 3, 4, 5, 6\}$ .

Let  $X$  denotes the number obtained on the uppermost face.

$\therefore X$  can take values 1, 2, 3, 4, 5, 6.

$$P(X = 1) = P(X = 2) = P(X = 3) = P(X = 4) = P(X = 5) = P(X = 6)$$

$$= \frac{1}{6}$$

The probability distribution is

$X = x$	1	2	3	4	5	6	Total
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1



$x_i p_i$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{6}{6}$	$\frac{21}{6} = \frac{7}{2}$
$x_i^2 \cdot p_i$	$\frac{1}{6}$	$\frac{4}{6}$	$\frac{9}{6}$	$\frac{16}{6}$	$\frac{25}{6}$	$\frac{36}{6}$	$\frac{91}{6}$

(i) Expected Value =  $E(X) = \sum_{i=1}^n x_i p_i = \frac{7}{2} = 3.5$

(ii) Variance =  $V(X) = E(X^2) - [E(X)]^2$

$$= \sum_{i=1}^n x_i^2 p_i - \left( \sum_{i=1}^n x_i p_i \right)^2$$

$$= \frac{91}{6} - \left( \frac{7}{2} \right)^2 = \frac{91}{6} - \frac{49}{4}$$

$$= \frac{182 - 147}{12}$$

$$\therefore \text{Variance} = V(X) = \frac{35}{12} = 2.9167$$

**Ex. (3).** A discrete random variable  $X$  takes the values -1, 0 and 2 with the probabilities  $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$  respectively. Find  $V(X)$  and Standard Deviation.

**Solution :** Given that the random variable  $X$  takes the values -1, 0 and 2.

The corresponding probabilities are  $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$ .

$$P(-1) = \frac{1}{4}, P(0) = \frac{1}{2} \text{ and } P(2) = \frac{1}{4}$$

Given data can be tabulated as follows

$X = x$	-1	0	2	Total
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	1
$x_i p_i$	$-\frac{1}{4}$	0	$\frac{1}{2}$	$\frac{1}{4}$
$x_i^2 p_i$	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{5}{4}$

(i) Variance =  $V(X) = E(X^2) - [E(X)]^2$

$$= \sum_{i=1}^n x_i^2 p_i - \left( \sum_{i=1}^n x_i p_i \right)^2$$

$$= \frac{5}{4} - \left( \frac{1}{4} \right)^2$$

$$= \frac{5}{4} - \frac{1}{16} = \frac{76}{64} = 1.1875$$

(ii) Standard Deviation =  $\sigma = \sqrt{V(X)} = \sqrt{1.1875} = 1.0897$

**Ex. (4)** The p. d. f. of  $X$ , find  $P(X < 1)$  and  $P(|X| < 1)$  where

$$f(x) = \frac{x+2}{18} \quad \text{if } -2 < x < 4$$

$$= 0 \quad \text{otherwise.}$$



**Solution :** Given that the p. d. f. of  $X$  is

$$f(x) = \frac{x+2}{18} \quad \text{if } -2 < x < 4$$

$$= 0 \quad \text{otherwise.}$$

$$\begin{aligned} \text{(i)} \quad P(X < 1) &= \int_{-2}^1 f(x) dx \\ &= \int_{-2}^1 \frac{x+2}{18} dx \\ &= \frac{1}{18} \int_{-2}^1 (x+2) dx \\ &= \frac{1}{18} \left[ \frac{(x+2)^2}{2} \right]_{-2}^1 \\ &= \frac{1}{36} \left[ \frac{(x+2)^2}{2} \right]_{-2}^1 \\ &= \frac{1}{36} [9 - 0] = \frac{9}{36} = \frac{1}{4} = 0.25 \end{aligned}$$

$$\text{(ii)} \quad P(|X| < 1) = P(-1 < X < 1)$$

$$\begin{aligned} &= \int_{-1}^1 \frac{x+2}{18} dx \\ &= \frac{1}{18} \int_{-1}^1 (x+2) dx \\ &= \frac{1}{18} \left[ \frac{(x+2)^2}{2} \right]_{-1}^1 \\ &= \frac{1}{36} \left[ \frac{(x+2)^2}{2} \right]_{-1}^1 \\ &= \frac{1}{36} [9 - 1] = \frac{8}{36} = \frac{2}{9} = 0.2222 \end{aligned}$$

**Ex. (5).** A random variable  $X$  has the following probability distribution :

$x$	0	1	2	3	4	5	6	7
$P(X = x)$	0	$k$	$2k$	$2k$	$3k$	$k^2$	$2k^2$	$7k^2 + k$

Find (i)  $k$  (ii)  $P(X < 3)$  (iii)  $P(X > 6)$  (iv)  $P(0 < X < 3)$  (v)  $P(2 \leq X \leq 4)$

**Solution :** Since  $P(x)$  is probability distribution of  $x$

$$\sum_{x=0}^7 P(x) = 1$$

$$\therefore P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) = 1$$

$$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$10k^2 + 9k - 1 = 0$$



$$10K^2 + 10K - K - 1 = 0$$

$$10K(K+1) - 1(K+1) = 0$$

$$\therefore (10K-1)(K+1) = 0$$

$$K = \frac{1}{10} \text{ or } K = -1$$

$K = -1$  is not possible

i)  $\therefore \boxed{K = \frac{1}{10}}$

ii)  $P(X < 3) = 0 + K + 2K$   
 $= 3K$   
 $= \frac{3}{10}$

iii)  $P(X > 6) = 7K^2 + K$   
 $= \frac{7}{10^2} + \frac{1}{10}$   
 $= \frac{17}{10}$

iv)  $P(0 < X < 3) = K + 2K$   
 $= 3K$   
 $= \frac{3}{10}$

v)  $P(2 \leq X \leq 4)$   
 $= 2K + 2K + 3K$   
 $= 7K$   
 $= \frac{7}{10}$

**Ex. (6).** The p. m. f. of a random variable  $X$  is as follows :

$X = x$	1	2	3	4
$P(x)$	$\frac{1}{30}$	$\frac{4}{30}$	$\frac{9}{30}$	$\frac{16}{30}$

Find Mean and the Variance.

**Solution:**

$X = x$	1	2	3	4	$\Sigma$
$P(x)$	$\frac{1}{30}$	$\frac{4}{30}$	$\frac{9}{30}$	$\frac{16}{30}$	$\frac{10}{30}$
$x^2 \cdot P(x)$	$\frac{1}{30}$	$\frac{16}{30}$	$\frac{81}{30}$	$\frac{256}{30}$	$\frac{59}{5}$

From the table

$$\Sigma x_i P_i = \frac{10}{3} \text{ and } \Sigma x_i^2 P_i = \frac{59}{5}$$

$$\text{mean} = E(X) = \frac{10}{3} = 3.33$$



$$\text{Variance} = V(x) = \sum x_i^2 p_i - [\sum x_i p_i]^2$$

$$= \frac{59}{5} - \left[\frac{10}{3}\right]^2$$

$$= \frac{59}{5} - \frac{100}{9}$$

$$= \frac{531 - 500}{45}$$

$$= \frac{31}{45}$$

Hence mean = 3.33

Variance = 0.6888

**Ex. (7).** From a survey of 20 families in a society, the following data was obtained :

No. of children	0	1	2	3	4
No. of families	5	11	2	0	2

For the random variable  $X$  = number of children in a randomly chosen family, Find  $E(X)$  and  $V(X)$ .

**Solution :**

$x$	0	1	2	3	4	$\Sigma$
$f$	5	11	2	0	2	$N=20$
$P(x)$	$\frac{5}{20}$	$\frac{11}{20}$	$\frac{2}{20}$	0	$\frac{2}{20}$	
$x_i p_i$	0	$\frac{11}{20}$	$\frac{4}{20}$	0	$\frac{8}{20}$	$\frac{23}{20}$
$x_i^2 p_i$	0	$\frac{11}{20}$	$\frac{8}{20}$	0	$\frac{32}{20}$	$\frac{51}{20}$

Here  $E(x) = \sum x_i p_i$

$$= \frac{23}{20}$$

$$= 1.15$$



$$V(x) = \sum x_i^2 p_i - \left[ \sum x_i p_i \right]^2$$

$$= \frac{51}{20} - \left[ \frac{23}{20} \right]^2$$

$$= \frac{51}{20} - \frac{529}{400} \Rightarrow \frac{491}{400}$$

$$V(x) = 1.2275$$

**Ex. (8).** Find the c.d.f.  $F(X)$  associated with the following p.d.f  $f(x)$  :

$$f(x) = 12x^2(1-x) \quad \text{for } 0 < x < 1$$

$$= 0 \quad \text{otherwise.}$$

Also, find  $P\left(\frac{1}{3} < X < \frac{1}{2}\right)$  by using p.d.f and c.d.f.

**Solution:**

$$F(x) = \int_0^x f(x) dx$$

$$= \int_0^x 12x^2(1-x) dx$$

$$= 12 \int_0^x (x^2 - x^3) dx$$

$$= 12 \left[ \frac{x^3}{3} - \frac{x^4}{4} \right]_0^x$$

$$= 12 \left[ \frac{4x^3 - 3x^4}{12} \right]$$

$$F(x) = 4x^3 - 3x^4 \quad - I$$

Now  $P\left(\frac{1}{3} < X < \frac{1}{2}\right)$  Using p.d.f.

$$= \int_{1/3}^{1/2} 12x^2(1-x) dx$$

$$= \left[ 4x^3 - 3x^4 \right]_{1/3}^{1/2}$$

$$= \left[ 4\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^4 \right] - \left[ 4\left(\frac{1}{3}\right)^3 - 3\left(\frac{1}{3}\right)^4 \right]$$

$$= \left[ \frac{4}{8} - \frac{3}{16} \right] - \left[ \frac{4}{27} - \frac{3}{81} \right]$$

$$= \frac{5}{16} - \frac{9}{81} = \frac{405 - 144}{1296}$$

$$= \frac{29}{144}$$

$$\boxed{F(x) = 0.2013}$$

Now  $P\left(\frac{1}{3} < X < \frac{1}{2}\right)$  by c.d.f.

$$F(x) = 4x^3 - 3x^4$$

$$= F\left(\frac{1}{2}\right) - F\left(\frac{1}{3}\right)$$

$$= \left[ 4\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^4 \right] - \left[ 4\left(\frac{1}{3}\right)^3 - 3\left(\frac{1}{3}\right)^4 \right]$$

$$= \left[ \frac{4}{8} - \frac{3}{16} \right] - \left[ \frac{4}{27} - \frac{3}{81} \right]$$

$$= \frac{5}{16} - \frac{9}{81}$$

$$= \frac{29}{144}$$

$$\boxed{F(x) = 0.2013}$$

Sign of Teacher :



- Q. 26.** A solenoid of length  $\pi$  m and 5 cm in diameter has a winding of 1000 turns and carries a current of 5A. Calculate the magnetic field at its centre along the axis.

## SECTION – D

**Attempt any THREE questions of the following :**

**[12]**

- Q. 27.** What is Ferromagnetism? Explain it on the basis of domain theory.
- Q. 28.** Obtain an expression for average power dissipated in a series LCR circuit.
- Q. 29.** Distinguish between interference and diffraction of light.  
A double slit arrangement produces interference fringes for sodium light of wavelength 589 nm, that are 0.20 degree apart. What is the angular fringe separation if the entire arrangement is immersed in water?  
( R.I. of water = 1.33 )
- Q. 30.** State Einstein's photoelectric equation and mention physical significance of each term involved in it.  
The wavelength of incident light is  $4000\text{\AA}$ . Calculate the energy of incident photon.
- Q. 31.** State any four uses of Van de Graaff generator.  
In a parallel plate air capacitor, intensity of electric field is changing at the rate of  $2 \times 10^{11}$  V/ms. If area of each plate is  $20\text{ cm}^2$ , calculate the displacement current.





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