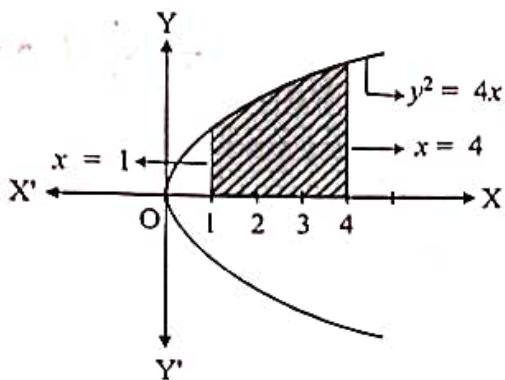


## 19. Applications of Definite Integration

### A. Activities

Carry out the following activities

- Find the area of the shaded portion using the following information



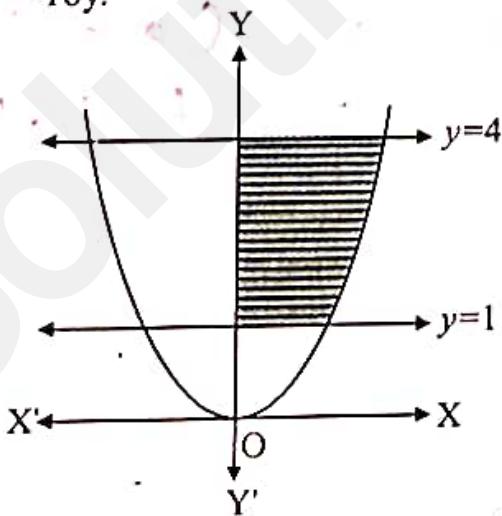
$$\begin{aligned}
 \text{Required Area} &= \int_1^4 y \, dx \\
 &= \int_1^4 2\sqrt{x} \, dx \\
 &= 2 \left[ \frac{x^{3/2}}{3/2} \right]_1^4 \\
 &= \frac{4}{3} [(4)^{3/2} - (1)^{3/2}] \\
 &= \frac{4}{3} [8 - 1] = \frac{28}{3} \text{ sq. unit}
 \end{aligned}$$

Ans

- Find the area of the region under the curve  $x = \frac{36}{25}(25 - y)^2$ , and the lines  $y = 0$ ,  $y = 5$  and the X-axis using the following activity.

$$\begin{aligned}
 \text{Required area} &= \int_0^5 x \, dy \\
 &= \int_0^5 \frac{36}{25} (25 - y^2) \, dy \\
 &= \frac{36}{25} \int_0^5 (25 - y^2) \, dy \\
 &= \frac{36}{25} \left( 25y - \frac{y^3}{3} \right)_0^5 \\
 &= 120 \text{ sq. units.}
 \end{aligned}$$

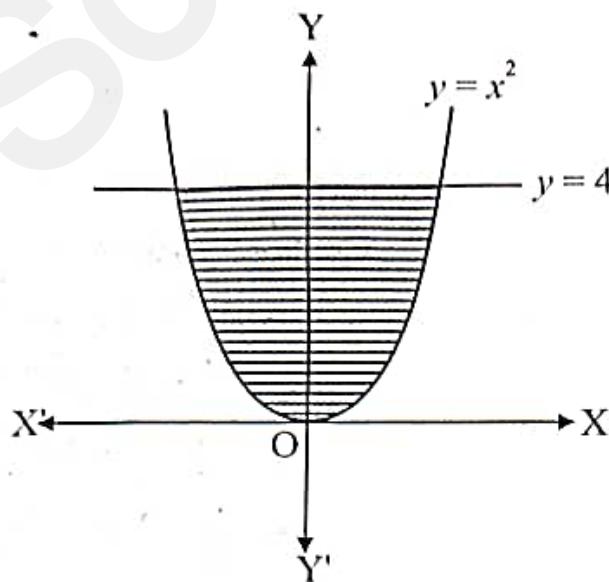
- Find the area shown in the following figure and the information given  $x^2 = 16y$ .



$$\begin{aligned}
 \text{Required Area} &= \int_1^4 x \, dy \\
 &= \int_1^4 4\sqrt{y} \, dy \\
 &= 4 \left[ \frac{y^{3/2}}{3/2} \right]_1^4 \\
 &= \frac{8}{3} [(4)^{3/2} - (1)^{3/2}] \\
 &= \frac{8}{3} (8 - 1) \\
 &= \frac{56}{3} \text{ sq. unit}
 \end{aligned}$$

Ans

4. Find the shaded area using following information.



$$\begin{aligned}
 \text{Required Area} &= a \int_0^y x dy \\
 &= 2 \int_0^4 \sqrt{y} dy \\
 &= 2 \left[ \frac{y^{3/2}}{3/2} \right]_0^4 \\
 &= \frac{4}{3} (8 - 0) = \frac{32}{3} \text{ Sq. Unit}
 \end{aligned}$$

Ans

### B. Solve the Following

Q.1. Find the area bounded by the curve  $y = 2x$ , X-axis and the lines  $x = -2$  and  $x = 4$

$$\begin{aligned}
 \text{Required area} &= \int_{-2}^4 y dx \\
 &= \int_{-2}^4 2x dx \\
 &= 2 \int_{-2}^4 x dx \\
 &= 2 \left[ \frac{x^2}{2} \right]_{-2}^4 \\
 &= 2 \left[ (4)^2 - (-2)^2 \right]
 \end{aligned}$$

$$= (8 - 4)$$

$$= 4 \text{ Sq. Unit}$$

Ans

Q.2. Find the area of the regions bounded by the following curves, the X-axis and the given lines.

a)  $y = x^2$ ,  $x = 1$ ,  $x = 3$  b)  $y^2 = 4$ ,  $x = 1$ ,  $x = 4$ .

Soln:-

$$a) A_1 = \int_1^3 y dx = \int_1^3 x^2 dx = \left[ \frac{x^3}{3} \right]_1^3$$

$$= \frac{1}{3} [(3)^3 - (1)^3] = \frac{1}{3} (27 - 1) = \frac{26}{3} \text{ sq. unit}$$

$$b) A_2 = 2 \int_1^4 y dx = 2 \int_1^4 2\sqrt{x} dx$$

$$= 4 \int_1^4 \sqrt{x} dx$$

$$= 4 \left[ \frac{x^{3/2}}{3/2} \right]_1^4$$

$$= \frac{8}{3} [(4)^{3/2} - (1)^{3/2}]$$

$$= \frac{8}{3} (8 - 1)$$

$$= \frac{56}{3} \text{ sq. unit}$$

Q.3. Find the area of the region bounded by the parabola  $y^2 = 16x$  and the line  $x = 4$

Soln:-

$$\text{Given: } y^2 = 16x \therefore y = 4\sqrt{x} \text{ (in 1st quadrant)}$$

$$\text{required area} = 2 \int_0^4 4\sqrt{x} dx$$

$$= 8 \left[ \frac{x^{3/2}}{3/2} \right]_0^4$$

$$= \frac{16}{3} [(4)^{3/2} - (0)^{3/2}]$$

$$= \frac{16}{3} \times (8 - 0)$$

$$= \frac{128}{3} \text{ sq. unit.}$$

Sign of Teacher :