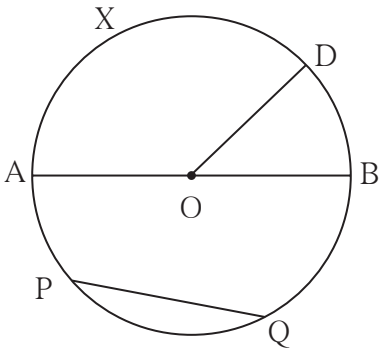
17 Circle - Chord and Arc

Let's recall.

In the adjoining figure O is the centre of the circle.

With reference to the figure fill in the blanks.



- Seg OD is ..... of the circle.
- Seg AB is ..... of the circle.
- Seg PQ is ..... of the circle.
- ..... is the central angle.
- Minor arc : arc AXD, arc BD, ..., ..., ...
- Major arc : arc PAB, arc PDQ, ... • Semicircular arc : arc ADB,....
- m(arc DB) = $m\angle$..... • m(arc DAB) = 360° – $m\angle$.....

Let's learn.

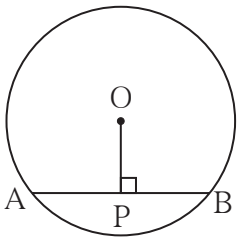
Properties of chord of a circle

Activity I :

Draw chord AB of a circle with centre O.

Draw perpendicular OP to chord AB .

Measure seg AP and seg PB.

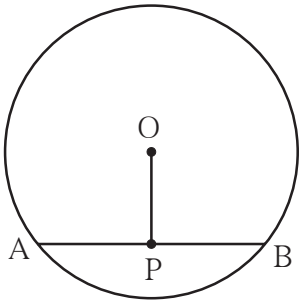


Draw five circles with different radii. Draw a chord and perpendicular from the centre to each chord in each circle. Verify with a divider that the two parts of the chords are equal. You will get the following property.

The perpendicular drawn from the centre of a circle to its chord bisects the chord.

Activity II:

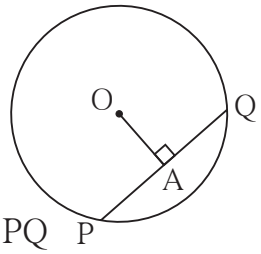
Draw five circles of different radii on a paper. Draw a chord in each circle. Find the midpoint of each chord. Join the centre of the circle and midpoint of the chord as shown in the figure. Name the chord as AB and midpoint of the chord as P. Check with set - square or protractor that ∠APO or ∠BPO are right angles.



Check whether the same result is observed for the chord of each circle. You will get the following property.

The segment joining the centre of a circle and midpoint of its chord is perpendicular to the chord.

Solved Examples



Ex. (1) In a circle with centre O, seg PQ is a chord of length 7 cm. seg OA ⊥ chord PQ, then find l(AP).

Solution: Seg OA ⊥ chord PQ, ∴ point A is midpoint of chord PQ

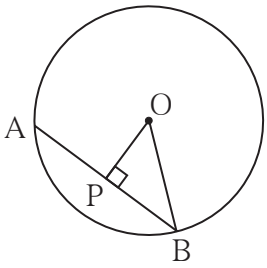
$\therefore\ l(PA) = \frac{1}{2}\ l(PQ) = \frac{1}{2} \times 7 = 3.5$ cm

Ex. (2) Radius of a circle with centre O is 10 cm. Find the length of the chord if the chord is at a distance of 6 cm from the centre.

Solution: Distance of the chord from the centre of the circle is the length of perpendicular drawn from the centre of the circle to the chord.

AB is the chord of the circle with centre O.

seg OP ⊥ chord AB.



Radius of the circle = l(OB) = 10 cm.

l(OP) = 6 cm. Δ OPB is a right angled triangle.

According to Pythagoras theorem,

$[l(OP)]^2 + [l(PB)]^2 = [l(OB)]^2$

$\therefore\ 6^2 + [l(PB)]^2 = 10^2$

$\therefore [l(PB)]^2 = 10^2 - 6^2$

$\therefore [l(PB)]^2 = (10 + 6)\ (10 - 6) = 16 \times 4 = 64$

$\therefore$ $l(PB) = 8$ cm.

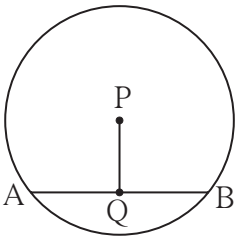
We know that, the perpendicular drawn from centre of the circle to the chord bisects the chord.

$\therefore$ $l(AB) = 2 \times l(PB) = 2 \times 8 = 16$

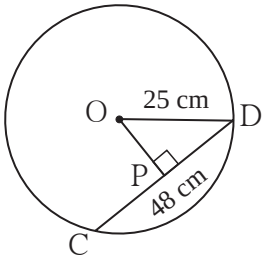
$\therefore$ length of chord AB is 16 cm.

Practice Set 17.1

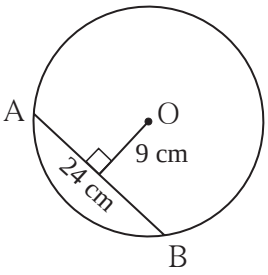
1. In a circle with centre P, chord AB is drawn of length 13 cm, seg PQ $\perp$ chord AB, then find $l(QB)$.



2. Radius of a circle with centre O is 25 cm. Find the distance of a chord from the centre if length of the chord is 48 cm.



3. O is centre of the circle. Find the length of radius, if the chord of length 24 cm is at a distance of 9 cm from the centre of the circle.

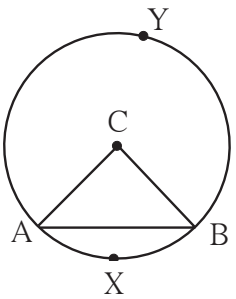


4. C is the centre of the circle whose radius is 10 cm. Find the distance of the chord from the centre if the length of the chord is 12 cm.

Let's learn.

Arcs corresponding to the chord of a circle

In the adjoining figure, seg AB is a chord of a circle with centre O. Arc AXB is minor arc and arc AYB is a major arc. These two arcs are called corresponding arcs of chord AB. Moreover chord AB is called corresponding chord of arc AXB and arc AYB.



Congruent arcs

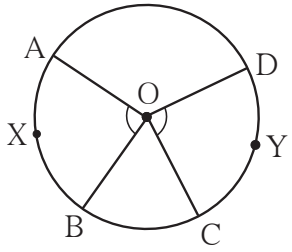
If the measures of two arcs of circle are same then two arcs are congruent.

In the circle with centre O

$\therefore$ $m\angle AOB = m\angle COD$

$\therefore$ m(arc AXB) = m(arc CYD)

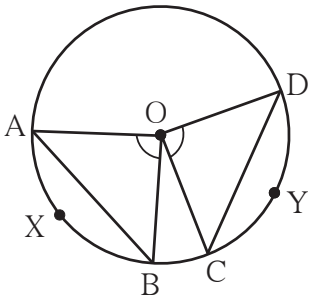
$\therefore$ arc AXB $\cong$ arc CYD



Verify this using tracing paper.

With the help of following activity find out the properties of the chord and the corresponding arc and remember them.

Activity I :



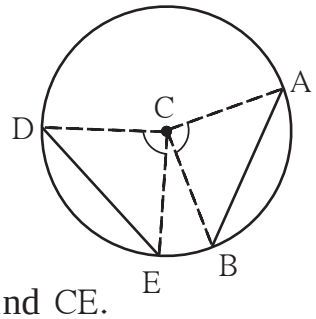
(1) Draw a circle with centre O

(2) Draw $\angle COD$ and $\angle AOB$ of same measure. You will find that the arc AXB and arc CYD are congruent.

(3) Draw chords AB and CD.

(4) Using compass experience that the length of chord AB and chord CD is also same.

Activity II :



(1) Draw a circle with centre C.

(2) Draw the congruent chords AB and DE of the circle. Draw the radii CA, CB, CD and CE.

(3) Check that $\angle ACB$ and $\angle DCE$ are congruent.

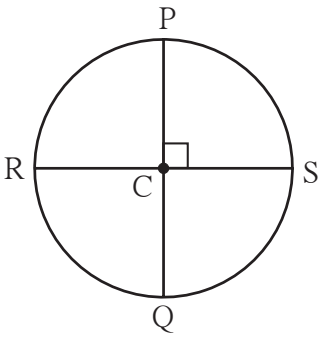
(4) Hence show that measure of arc AB and arc DE is equal. Hence these arcs are congruent.

Now I know.

The chords corresponding to congruent arcs are congruent. In a circle if two chords are congruent then their corresponding minor arcs and major arcs are congruent

Practice Set 17.2

1. The diameters PQ and RS of the circle with centre C are perpendicular to each other at C. State, why arc PS and arc SQ are congruent. Write the other arcs which are congruent to arc PS

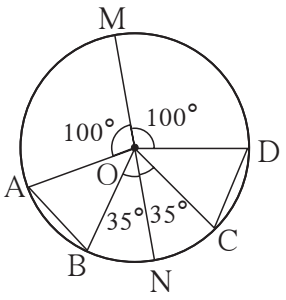


2. In the adjoining figure O is the centre of the circle whose diameter is MN. Measures of some central angles are given in the figure. Hence find the following

(1) $m\angle$ AOB and $m\angle$ COD

(2) Show that arc AB $\cong$ arc CD.

(3) Show that chord AB $\cong$ chord CD



Answers

Practice Set 17.1

1. 6.5 cm **2.** 7 cm **3.** 15 cm **4.** 8 cm

Practice Set 17.2

1. (1) Because the arcs are of equal measures, that is 90° each.

(2) arc PS $\cong$ arc PR $\cong$ arc RQ

2. (1) $m\angle AOB = m\angle COD = 45°$

(2) arc AB $\cong$ arc CD because the arcs are of equal measures that is 45° each.

(3) chord AB $\cong$ chord CD because corresponding chords of congruent arcs are congruent.