

1 Linear Equations in Two Variables



Let's study.

- Methods of solving linear equations in two variables – graphical method, Cramer's method
- Equations that can be transformed in linear equation in two variables
- Application of simultaneous equations



Let's recall.

Linear equation in two variables

An equation which contains two variables and the degree of each term containing variable is one, is called a linear equation in two variables.

$ax + by + c = 0$ is the general form of a linear equation in two variables; a, b, c are real numbers and a, b are not equal to zero at the same time.

Ex. $3x - 4y + 12 = 0$ is the general form of equation $3x = 4y - 12$

Activity : Complete the following table

No.	Equation	Is the equation a linear equation in 2 variables ?
1	$4m + 3n = 12$	Yes
2	$3x^2 - 7y = 13$	
3	$\sqrt{2}x - \sqrt{5}y = 16$	
4	$0x + 6y - 3 = 0$	
5	$0.3x + 0y - 36 = 0$	
6	$\frac{4}{x} + \frac{5}{y} = 4$	
7	$4xy - 5y - 8 = 0$	

Simultaneous linear equations

When we think about two linear equations in two variables at the same time, they are called simultaneous equations.

Last year we learnt to solve simultaneous equations by eliminating one variable. Let us revise it.

Ex. (1) Solve the following simultaneous equations.

$$(1) \quad 5x - 3y = 8; \quad 3x + y = 2$$

Solution :

Method I : $5x - 3y = 8$. . . (I)

$$3x + y = 2 \quad \dots \quad \text{(II)}$$

Multiplying both sides of equation (II) by 3.

$$9x + 3y = 6 \quad \dots \quad \text{(III)}$$

$$5x - 3y = 8 \quad \dots \quad \text{(I)}$$

Now let us add equations (I) and (III)

$$5x - 3y = 8$$

$$+ 9x + 3y = 6$$

$$\hline 14x = 14$$

$$\therefore x = 1$$

substituting $x = 1$ in equation (II)

$$3x + y = 2$$

$$\therefore 3 \times 1 + y = 2$$

$$\therefore 3 + y = 2$$

$$\therefore y = -1$$

solution is $x = 1, y = -1$; it is also written as $(x, y) = (1, -1)$

Method (II)

$$5x - 3y = 8 \quad \dots \quad \text{(I)}$$

$$3x + y = 2 \quad \dots \quad \text{(II)}$$

Let us write value of y in terms of x from equation (II) as

$$y = 2 - 3x \quad \dots \quad \text{(III)}$$

Substituting this value of y in equation (I).

$$5x - 3y = 8$$

$$\therefore 5x - 3(2 - 3x) = 8$$

$$\therefore 5x - 6 + 9x = 8$$

$$\therefore 14x - 6 = 8$$

$$\therefore 14x = 8 + 6$$

$$\therefore 14x = 14$$

$$\therefore x = 1$$

Substituting $x = 1$ in equation (III).

$$y = 2 - 3x$$

$$\therefore y = 2 - 3 \times 1$$

$$\therefore y = 2 - 3$$

$$\therefore y = -1$$

$x = 1, y = -1$ is the solution.

Ex. (2) Solve : $3x + 2y = 29$; $5x - y = 18$

Solution : $3x + 2y = 29$. . . (I) and $5x - y = 18$. . . (II)

Let's solve the equations by eliminating 'y'. Fill suitably the boxes below.

Multiplying equation (II) by 2.

$$\therefore 5x \times \boxed{} - y \times \boxed{} = 18 \times \boxed{}$$

$$\therefore 10x - 2y = \boxed{} \dots \text{ (III)}$$

Let's add equations (I) and (III)

$$\begin{array}{r} 3x + 2y = 29 \\ + \quad \boxed{} - \boxed{} = \boxed{} \\ \hline \boxed{} = \boxed{} \end{array} \quad \therefore x = \boxed{}$$

Substituting $x = 5$ in equation (I)

$$3x + 2y = 29$$

$$\therefore 3 \times \boxed{} + 2y = 29$$

$$\therefore \boxed{} + 2y = 29$$

$$\therefore 2y = 29 - \boxed{}$$

$$\therefore 2y = \boxed{} \qquad \therefore y = \boxed{}$$

$(x, y) = (\text{ }, \text{ })$ is the solution.

Ex. (3) Solve : $15x + 17y = 21$; $17x + 15y = 11$

Solution : $15x + 17y = 21$. . . (I)

$$17x + 15y = 11 \quad \dots \quad (\text{II})$$

In the two equations above, the coefficients of x and y are interchanged. While solving such equations we get two simple equations by adding and subtracting the given equations. After solving these equations, we can easily find the solution.

Let's add the two given equations.

$$\begin{array}{r} 15x + 17y = 21 \\ + \quad 17x + 15y = 11 \\ \hline 32x + 32y = 32 \end{array}$$

Dividing both sides of the equation by 32.

$$x + y = 1 \dots (III)$$

Now, let's subtract equation (II) from (I)

$$\begin{array}{r} 15x + 17y = 21 \\ - \\ 17x + 15y = 11 \\ \hline -2x + 2y = 10 \end{array}$$

dividing the equation by 2.

$$-x + y = 5 \dots (IV)$$

Now let's add equations (III) and (IV).

$$\begin{array}{r} x + y = 1 \\ + \\ -x + y = 5 \\ \hline \therefore 2y = 6 \quad \therefore y = 3 \end{array}$$

Place this value in equation (III).

$$x + y = 1$$

$$\therefore x + 3 = 1$$

$$\therefore x = 1 - 3 \quad \therefore x = -2$$

$(x, y) = (-2, 3)$ is the solution.

Practice Set 1.1

(1) Complete the following activity to solve the simultaneous equations.

$$5x + 3y = 9 \text{ -----(I)}$$

$$2x - 3y = 12 \text{ ----- (II)}$$

Let's add equations (I) and (II).

$$\begin{array}{r} 5x + 3y = 9 \\ + \\ 2x - 3y = 12 \\ \hline \end{array}$$

$$\boxed{} x = \boxed{}$$

$$x = \frac{\boxed{}}{\boxed{}}$$

$$x = \boxed{}$$

Place $x = 3$ in equation (I).

$$5 \times \boxed{} + 3y = 9$$

$$3y = 9 - \boxed{}$$

$$3y = \boxed{}$$

$$y = \frac{\boxed{}}{3}$$

$$y = \boxed{}$$

\therefore Solution is $(x, y) = (\boxed{}, \boxed{})$.

2. Solve the following simultaneous equations.

- (1) $3a + 5b = 26$; $a + 5b = 22$ (2) $x + 7y = 10$; $3x - 2y = 7$
 (3) $2x - 3y = 9$; $2x + y = 13$ (4) $5m - 3n = 19$; $m - 6n = -7$
 (5) $5x + 2y = -3$; $x + 5y = 4$ (6) $\frac{1}{3}x + y = \frac{10}{3}$; $2x + \frac{1}{4}y = \frac{11}{4}$
 (7) $99x + 101y = 499$; $101x + 99y = 501$
 (8) $49x - 57y = 172$; $57x - 49y = 252$



Let's recall.

Graph of a linear equation in two variables

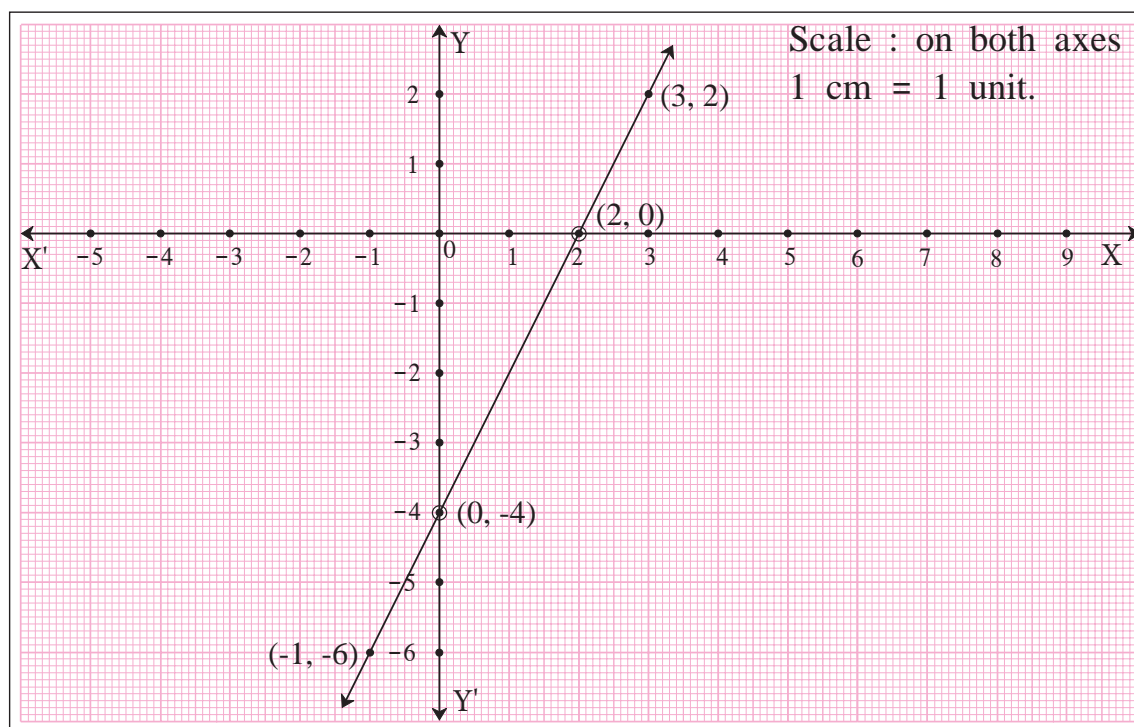
In the 9th standard we learnt that the graph of a linear equation in two variables is a straight line. The ordered pair which satisfies the equation is a solution of that equation. The ordered pair represents a point on that line.

Ex. Draw graph of $2x - y = 4$.

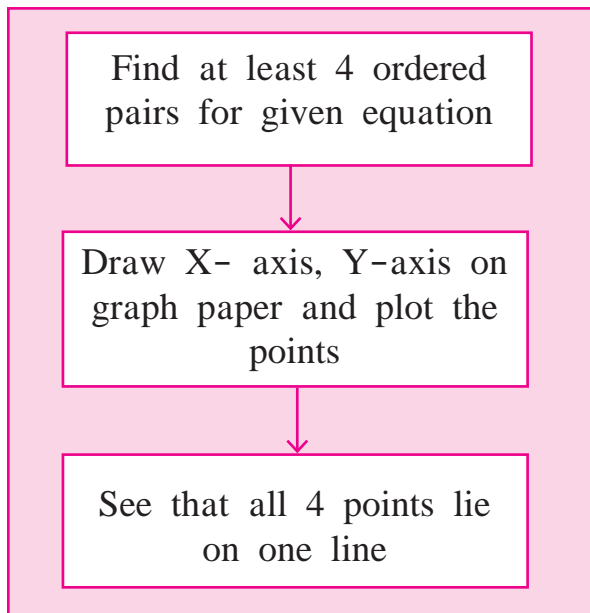
Solution : To draw a graph of the equation let's write 4 ordered pairs.

x	0	2	3	-1
y	-4	0	2	-6
(x, y)	(0, -4)	(2, 0)	(3, 2)	(-1, -6)

To obtain ordered pair by simple way let's take $x = 0$ and then $y = 0$.



Steps to follow for drawing a graph of linear equation in two variables.



Two points are sufficient to represent a line, but if co-ordinates of one of the two points are wrong then you will not get a correct line.

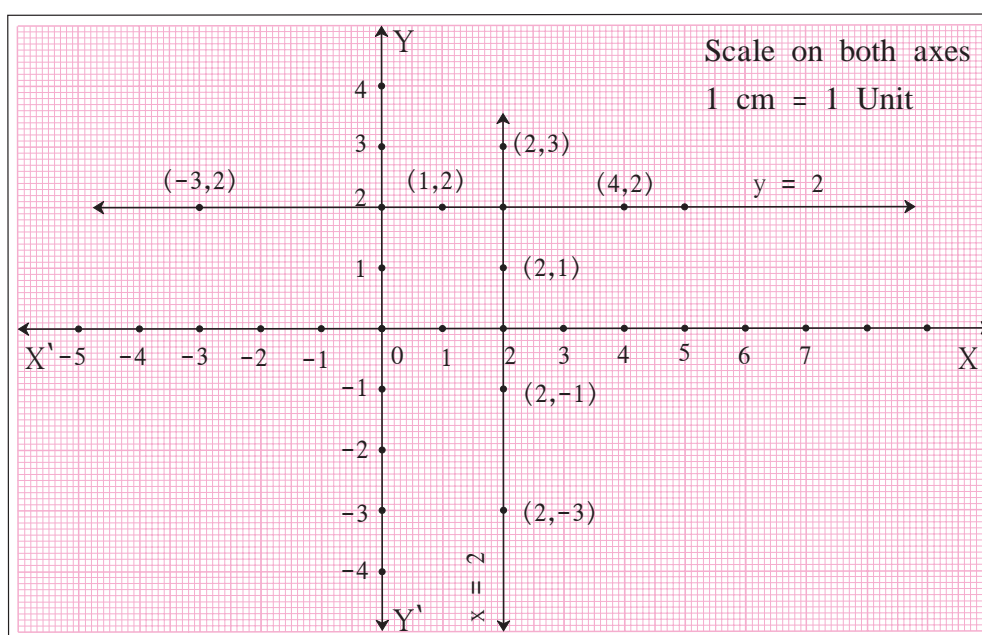
If you plot three points and if they are non collinear then it is understood that one of the points is wrongly plotted. But it is not easy to identify the incorrect point.

If we plot four points, it is almost certain that three of them will be collinear.

A linear equation $y = 2$ is also written as $0x + y = 2$. The graph of this line is parallel to X- axis; as for any value of x , y is always 2.

x	1	4	-3
y	2	2	2
(x, y)	(1, 2)	(4, 2)	(-3, 2)

Similarly equation $x = 2$ is written as $x + 0y = 2$ and its graph is parallel to Y-axis.





Let's learn.

Graphical method

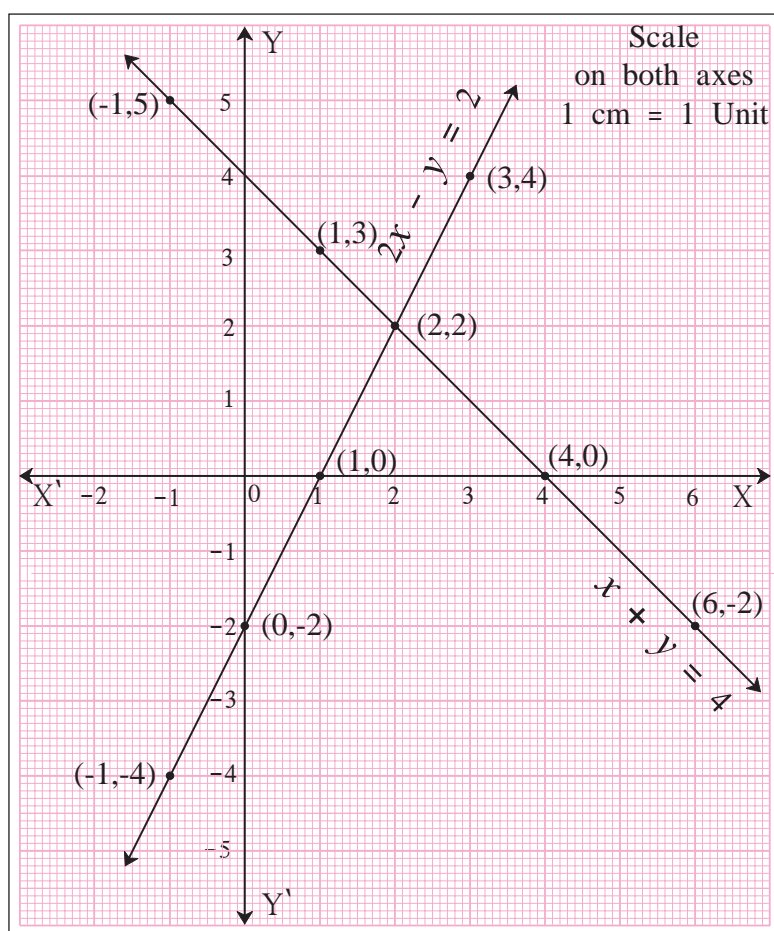
Ex. Let's draw graphs of $x + y = 4$, $2x - y = 2$ and observe them.

$$x + y = 4$$

x	-1	4	1	6
y	5	0	3	-2
(x, y)	(-1, 5)	(4, 0)	(1, 3)	(6, -2)

$$2x - y = 2$$

x	0	1	3	-1
y	-2	0	4	-4
(x, y)	(0, -2)	(1, 0)	(3, 4)	(-1, -4)



Each point on the graph satisfies the equation. The two lines intersect each other at (2, 2).

Hence ordered pair (2, 2) i.e. $x = 2$, $y = 2$ satisfies the equations $x + y = 4$ and $2x - y = 2$.

The values of variables that satisfy the given equations, give the solution of given equations.

\therefore the solution of given equations $x + y = 4$, $2x - y = 2$ is $x = 2$, $y = 2$.

Let's solve these equations by method of elimination.

$$x + y = 4 \quad \dots \quad (I)$$

$$2x - y = 2 \quad \dots \quad (II)$$

Adding equations (I) and (II) we get,

$$3x = 6 \quad \therefore \quad x = 2$$

substituting this value in equation (I)

$$x + y = 4$$

$$\therefore 2 + y = 4$$

$$\therefore y = 2$$

Activity (I) : Solve the following simultaneous equations by graphical method.

- Complete the following tables to get ordered pairs.

$$x - y = 1$$

x	0		3	
y		0		-3
(x, y)				

$$5x - 3y = 1$$

x	2			-4
y		8	-2	
(x, y)				

- Plot the above ordered pairs on the same co-ordinate plane.
- Draw graphs of the equations.
- Note the co-ordinates of the point of intersection of the two graphs. Write solution of these equations.

Activity II : Solve the above equations by method of elimination. Check your solution with the solution obtained by graphical method.



Let's think.

The following table contains the values of x and y co-ordinates for ordered pairs to draw the graph of $5x - 3y = 1$

x	0	$\frac{1}{5}$	1	-2
y	$-\frac{1}{3}$	0	$\frac{4}{3}$	$-\frac{11}{3}$
(x, y)	$(0, -\frac{1}{3})$	$(\frac{1}{5}, 0)$	$(1, \frac{4}{3})$	$(-2, -\frac{11}{3})$

- Is it easy to plot these points ?
- Which precaution is to be taken to find ordered pairs so that plotting of points becomes easy ?

Practice Set 1.2

1. Complete the following table to draw graph of the equations -

(I) $x + y = 3$ (II) $x - y = 4$

$$x + y = 3$$

x	3		
y		5	3
(x, y)	(3, 0)		(0, 3)

$$x - y = 4$$

x		-1	0
y	0		-4
(x, y)			(0, -4)

2. Solve the following simultaneous equations graphically.

(1) $x + y = 6$; $x - y = 4$

(2) $x + y = 5$; $x - y = 3$

(3) $x + y = 0$; $2x - y = 9$

(4) $3x - y = 2$; $2x - y = 3$

(5) $3x - 4y = -7$; $5x - 2y = 0$

(6)★ $2x - 3y = 4$; $3y - x = 4$



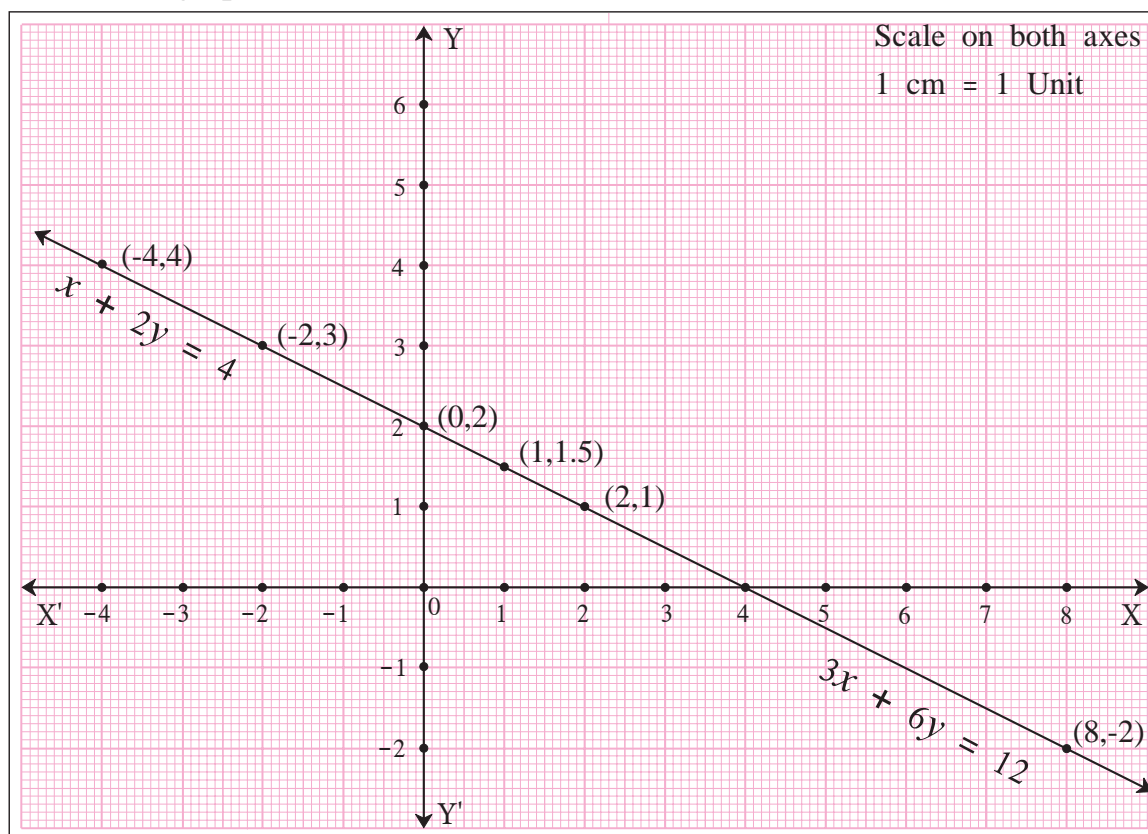
Let's discuss.

To solve simultaneous equations $x + 2y = 4$; $3x + 6y = 12$ graphically, following are the ordered pairs.

$x + 2y = 4$			
x	-2	0	2
y	3	2	1
(x, y)	$(-2, 3)$	$(0, 2)$	$(2, 1)$

$3x + 6y = 12$			
x	-4	1	8
y	4	1.5	-2
(x, y)	$(-4, 4)$	$(1, 1.5)$	$(8, -2)$

Plotting the above ordered pairs, graph is drawn. Observe it and find answers of the following questions.



- (1) Are the graphs of both the equations different or same ?
- (2) What are the solutions of the two equations $x + 2y = 4$ and $3x + 6y = 12$?
How many solutions are possible ?
- (3) What are the relations between coefficients of x , coefficients of y and constant terms in both the equations ?
- (4) What conclusion can you draw when two equations are given but the graph is only one line ?

Now let us consider another example.

Draw graphs of $x - 2y = 4$, $2x - 4y = 12$ on the same co-ordinate plane. Observe it. Think of the relation between the coefficients of x , coefficients of y and the constant terms and draw the inference.



ICT Tools or Links.

Use Geogebra software, draw X- axis, Y-axis. Draw graphs of simultaneous equations.



Let's learn.

Determinant

$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is a determinant. (a, b) , (c, d) are rows and $\begin{pmatrix} a \\ c \end{pmatrix}$, $\begin{pmatrix} b \\ d \end{pmatrix}$ are columns.

Degree of this determinant is 2, because there are 2 elements in each column and 2 elements in each row. Determinant represents a number which is $(ad-bc)$.

$$\text{i.e. } \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad-bc$$

$ad-bc$ is the value of determinant $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

Determinants, usually, are represented with capital letters as A, B, C, D, etc.

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Ex. Find the values of the following determinants.

$$(1) A = \begin{vmatrix} 5 & 3 \\ 7 & 9 \end{vmatrix}$$

$$(2) N = \begin{vmatrix} -8 & -3 \\ 2 & 4 \end{vmatrix}$$

$$(3) B = \begin{vmatrix} 2\sqrt{3} & 9 \\ 2 & 3\sqrt{3} \end{vmatrix}$$

Solution :

$$(1) A = \begin{vmatrix} 5 & 3 \\ 7 & 9 \end{vmatrix} = (5 \times 9) - (3 \times 7) = 45 - 21 = 24$$

$$(2) N = \begin{vmatrix} -8 & -3 \\ 2 & 4 \end{vmatrix} = [(-8) \times (4)] - [(-3) \times 2] = -32 - (-6) \\ = -32 + 6 = -26$$

$$(3) B = \begin{vmatrix} 2\sqrt{3} & 9 \\ 2 & 3\sqrt{3} \end{vmatrix} = [2\sqrt{3} \times 3\sqrt{3}] - [2 \times 9] = 18 - 18 = 0$$



Let's learn.

Determinant method (Cramer's Rule)

Using determinants, simultaneous equations can be solved easily and in less space. This method is known as determinant method. This method was first given by a Swiss mathematician Gabriel Cramer, so it is also known as Cramer's method.

To use Cramer's method, the equations are written as $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$.

$$a_1x + b_1y = c_1 \quad \dots \quad (I)$$

$$a_2x + b_2y = c_2 \quad \dots \quad (II)$$

Here x and y are variables, a_1, b_1, c_1 and a_2, b_2, c_2 are real numbers, $a_1b_2 - a_2b_1 \neq 0$

Now let us solve these equations.

Multiplying equation (I) by b_2 .

$$a_1b_2x + b_1b_2y = c_1b_2 \quad \dots \quad (III)$$

Multiplying equation (II) by b_1 .

$$a_2b_1x + b_2b_1y = c_2b_1 \quad \dots \quad (IV)$$

Subtracting equation (IV) from (III)

$$a_1 b_2 x + b_1 b_2 y = c_1 b_2$$

$$\begin{array}{r} - \\ a_2 b_1 x - b_2 b_1 y = c_2 b_1 \\ \hline \end{array}$$

$$(a_1 b_2 - a_2 b_1) x = c_1 b_2 - c_2 b_1$$

$$x = \frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1} \dots \dots \text{(V)}$$

$$\text{Similarly } y = \frac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1} \dots \dots \text{(VI)}$$

To remember and write the expressions

$c_1 b_2 - c_2 b_1$, $a_1 b_2 - a_2 b_1$, $a_1 c_2 - a_2 c_1$ we use the determinants.

Now $a_1 x + b_1 y = c_1$ and $a_2 x + b_2 y = c_2$ We can write 3 columns. $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$, $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

The values x , y in equation (V), (VI) are written using determinants as follows

$$x = \frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1} = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}},$$

$$y = \frac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1} = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

To remember let us denote $D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$, $D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$, $D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$

$$\therefore x = \frac{D_x}{D}, \quad y = \frac{D_y}{D}$$

For writing D , D_x , D_y remember the order of columns $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$, $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$.

From the equations,

$a_1 x + b_1 y = c_1$
 and $a_2 x + b_2 y = c_2$ we get the columns $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$, $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$.

- In D the column of constants $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$ is omitted.
- In D_x the column of the coefficients of x , $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ is replaced by $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$.
- In D_y the column of the coefficients of y , $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ is replaced by $\begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$.



Let's remember!

Cramer's method to solve simultaneous equations.

Write given equations in the form $ax + by = c$.

Find the values of determinants D , D_x and D_y

Using, $x = \frac{D_x}{D}$ and $y = \frac{D_y}{D}$
 find values of x , y .

Gabriel Cramer

(31 July, 1704 to 4 January, 1752)

This Swiss mathematician was born in Geneva. He was very well versed in mathematics, since childhood. At the age of eighteen, he got a doctorate. He was a professor in Geneva.



Solved Example

Ex. (1) Solve the following simultaneous equations using Cramer's Rule.

$$5x + 3y = -11 ; 2x + 4y = -10$$

Solution : Given equations

$$5x + 3y = -11$$

$$2x + 4y = -10$$

$$D = \begin{vmatrix} 5 & 3 \\ 2 & 4 \end{vmatrix} = (5 \times 4) - (2 \times 3) = 20 - 6 = 14$$

$$D_x = \begin{vmatrix} -11 & 3 \\ -10 & 4 \end{vmatrix} = (-11) \times 4 - (-10) \times 3 = -44 - (-30) \\ = -44 + 30 = -14$$

$$D_y = \begin{vmatrix} 5 & -11 \\ 2 & -10 \end{vmatrix} = 5 \times (-10) - 2 \times (-11) = -50 - (-22) \\ = -50 + 22 = -28$$

$$x = \frac{D_x}{D} = \frac{-14}{14} = -1 \quad \text{and} \quad y = \frac{D_y}{D} = \frac{-28}{14} = -2$$

$\therefore (x, y) = (-1, -2)$ is the solution.

Activity 1 : To solve the simultaneous equations by determinant method, fill in the blanks

$$y + 2x - 19 = 0 ; 2x - 3y + 3 = 0$$

Solution : Write the given equations in the form $ax + by = c$

$$2x + y = 19$$

$$2x - 3y = -3$$

$$D = \begin{vmatrix} \boxed{} & \boxed{} \\ 2 & -3 \end{vmatrix} = [\boxed{} \times (-3)] - [2 \times (\boxed{})] = \boxed{} - (\boxed{}) \\ = \boxed{} - \boxed{} = \boxed{}$$

$$D_x = \begin{vmatrix} 19 & \boxed{} \\ \boxed{} & -3 \end{vmatrix} = [19 \times (\boxed{})] - [(\boxed{}) \times (\boxed{})] = \boxed{} - \boxed{} \\ = \boxed{}$$

$$D_y = \begin{vmatrix} \boxed{} & 19 \\ 2 & \boxed{} \end{vmatrix} = [(\boxed{}) \times (\boxed{})] - [(\boxed{}) \times (\boxed{})]$$

$$= \boxed{} - \boxed{} = \boxed{}$$

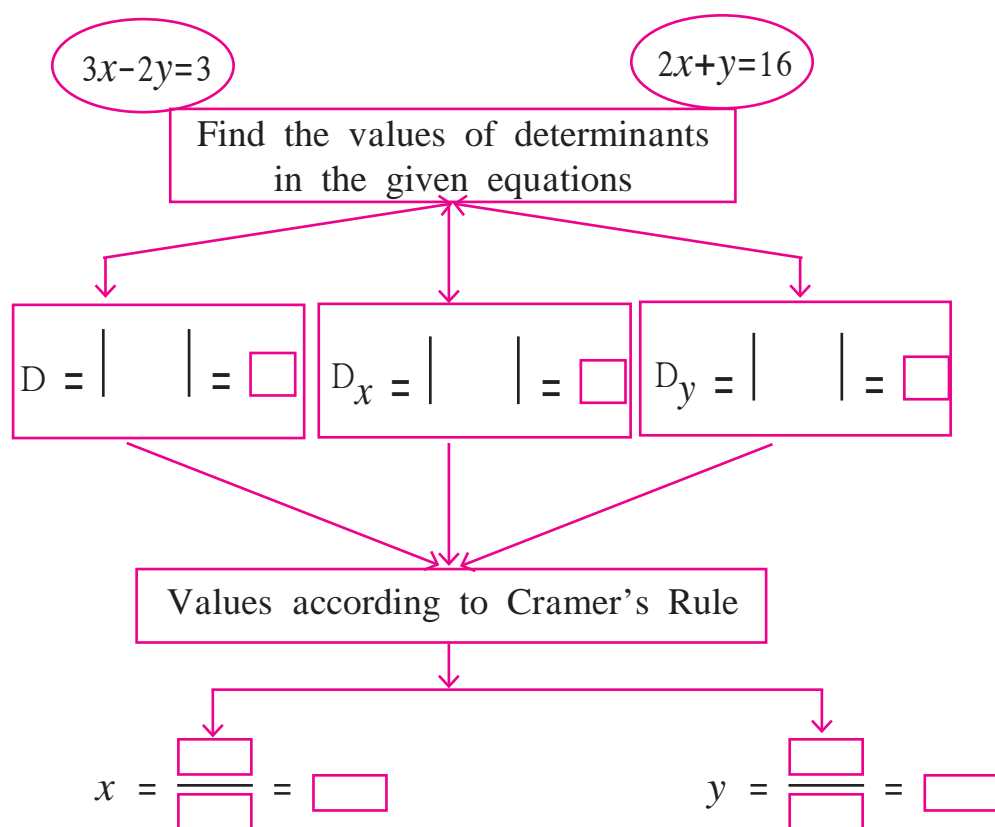
By Cramer's Rule -

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D}$$

$$\therefore x = \frac{\boxed{}}{\boxed{}} = \boxed{} \qquad y = \frac{\boxed{}}{\boxed{}} = \boxed{}$$

$$\therefore (x, y) = (\boxed{}, \boxed{}) \text{ is the solution of the given equations.}$$

Activity 2 : Complete the following activity -



$$\therefore (x, y) = (\boxed{}, \boxed{}) \text{ is the solution.}$$

**Let's think.**

- What is the nature of solution if $D = 0$?
- What can you say about lines if common solution is not possible?

Practice Set 1.3

1. Fill in the blanks with correct number

$$\begin{vmatrix} 3 & 2 \\ 4 & 5 \end{vmatrix} = 3 \times \boxed{} - \boxed{} \times 4 = \boxed{} - 8 = \boxed{}$$

2. Find the values of following determinants.

$$(1) \begin{vmatrix} -1 & 7 \\ 2 & 4 \end{vmatrix} \quad (2) \begin{vmatrix} 5 & 3 \\ -7 & 0 \end{vmatrix} \quad (3) \begin{vmatrix} \frac{7}{3} & \frac{5}{3} \\ \frac{3}{2} & \frac{1}{2} \end{vmatrix}$$

3. Solve the following simultaneous equations using Cramer's rule.

(1) $3x - 4y = 10$; $4x + 3y = 5$ (2) $4x + 3y - 4 = 0$; $6x = 8 - 5y$

(3) $x + 2y = -1$; $2x - 3y = 12$ (4) $6x - 4y = -12$; $8x - 3y = -2$

(5) $4m + 6n = 54$; $3m + 2n = 28$ (6) $2x + 3y = 2$; $x - \frac{y}{2} = \frac{1}{2}$

**Let's learn.**

Equations reducible to a pair of linear equations in two variables

Activity : Complete the following table.

Equation	No. of variables	whether linear or not
$\frac{3}{x} - \frac{4}{y} = 8$	2	Not linear
$\frac{6}{x-1} + \frac{3}{y-2} = 0$	<input type="text"/>	<input type="text"/>
$\frac{7}{2x+1} + \frac{13}{y+2} = 0$	<input type="text"/>	<input type="text"/>
$\frac{14}{x+y} + \frac{3}{x-y} = 5$	<input type="text"/>	<input type="text"/>



Let's think.

In the above table the equations are not linear. Can you convert the equations into linear equations ?



Let's remember!

We can create new variables making a proper change in the given variables. Substituting the new variables in the given non-linear equations, we can convert them in linear equations.

Also remember that the denominator of any fraction of the form $\frac{m}{n}$ cannot be zero.

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Solve:

Ex. (1) $\frac{4}{x} + \frac{5}{y} = 7$; $\frac{3}{x} + \frac{4}{y} = 5$

Solution : $\frac{4}{x} + \frac{5}{y} = 7$; $\frac{3}{x} + \frac{4}{y} = 5$

$$4\left(\frac{1}{x}\right) + 5\left(\frac{1}{y}\right) = 7 \dots \text{(I)}$$

$$3\left(\frac{1}{x}\right) + 4\left(\frac{1}{y}\right) = 5 \dots \text{(II)}$$

Replacing $\left(\frac{1}{x}\right)$ by m and $\left(\frac{1}{y}\right)$ by n in equations (I) and (II), we get
 $4m + 5n = 7 \dots \text{(III)}$

$$3m + 4n = 5 \dots \text{(IV)}$$

On solving these equations we get

$$m = 3, n = -1$$

$$\begin{aligned} \text{Now, } m &= \frac{1}{x} \quad \therefore 3 = \frac{1}{x} \quad \therefore x = \frac{1}{3} \\ n &= \frac{1}{y} \quad \therefore -1 = \frac{1}{y} \quad \therefore y = -1 \end{aligned}$$

\therefore Solution of given simultaneous equations is $(x, y) = \left(\frac{1}{3}, -1\right)$

Ex.(2) $\frac{4}{x-y} + \frac{1}{x+y} = 3$; $\frac{2}{x-y} - \frac{3}{x+y} = 5$

Solution : $\frac{4}{x-y} + \frac{1}{x+y} = 3$; $\frac{2}{x-y} - \frac{3}{x+y} = 5$

$$4\left(\frac{1}{x-y}\right) + 1\left(\frac{1}{x+y}\right) = 3 \dots \text{(I)}$$

$$2\left(\frac{1}{x-y}\right) - 3\left(\frac{1}{x+y}\right) = 5 \dots \text{(II)}$$

Replacing $\left(\frac{1}{x-y}\right)$ by a and $\left(\frac{1}{x+y}\right)$ by b we get

$$4a + b = 3 \dots \text{(III)}$$

$$2a - 3b = 5 \dots \text{(IV)}$$

On solving these equations we get, $a = 1$ $b = -1$

But $a = \left(\frac{1}{x-y}\right)$, $b = \left(\frac{1}{x+y}\right)$

$$\therefore \left(\frac{1}{x-y}\right) = 1, \left(\frac{1}{x+y}\right) = -1$$

$$\therefore x - y = 1 \dots \text{(V)}$$

$$x + y = -1 \dots \text{(VI)}$$

Solving equation (V) and (VI) we get $x = 0$, $y = -1$

\therefore Solution of the given equations is $(x, y) = (0, -1)$

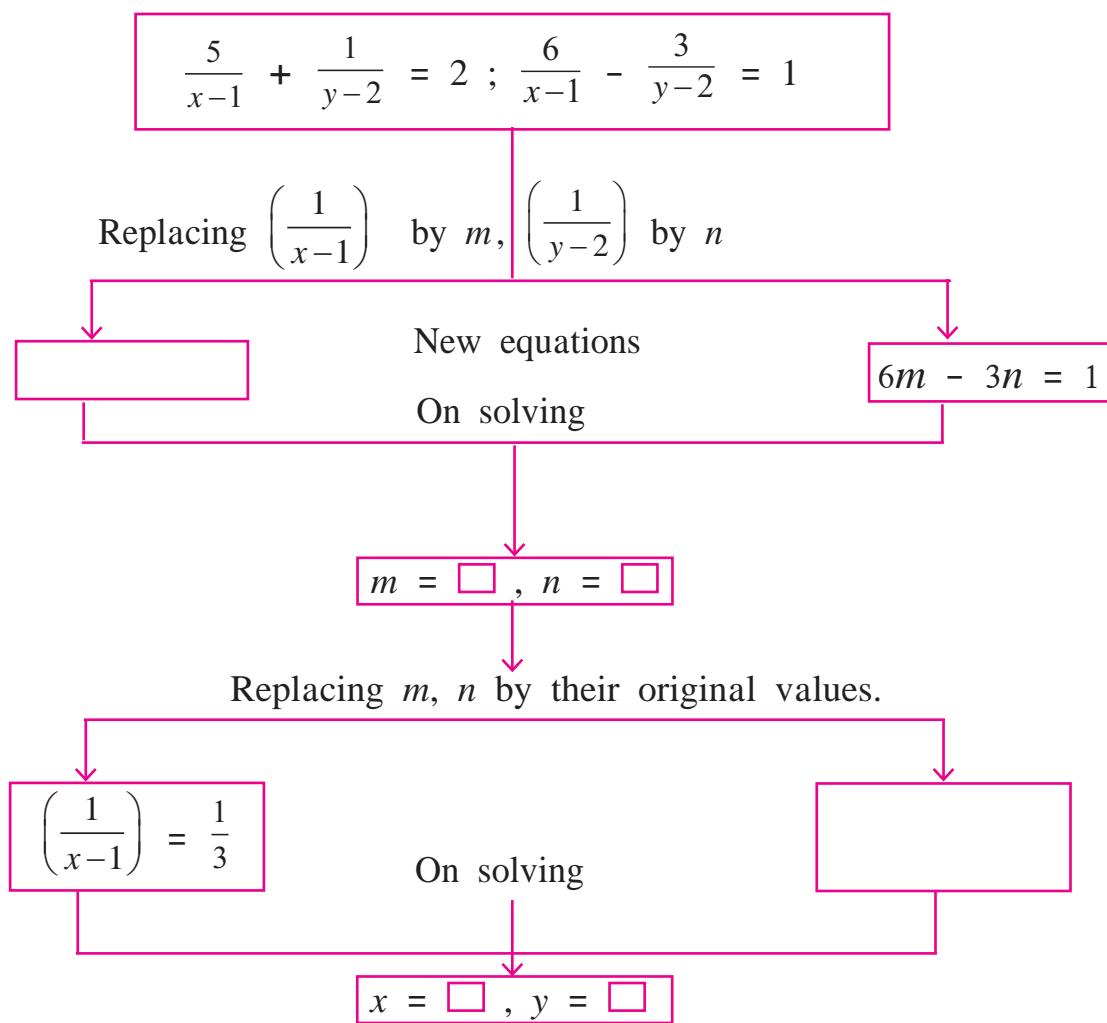


Let's think.

In the above examples the simultaneous equations obtained by transformation are solved by elimination method.

If you solve these equations by graphical method and by Cramer's rule will you get the same answers ? Solve and check it.

Activity : To solve given equations fill the boxes below suitably.



$\therefore (x, y) = (\quad , \quad)$ is the solution of the given simultaneous equations.

Practice Set 1.4

1. Solve the following simultaneous equations.

(1) $\frac{2}{x} - \frac{3}{y} = 15 ; \frac{8}{x} + \frac{5}{y} = 77$

(2) $\frac{10}{x+y} + \frac{2}{x-y} = 4 ; \frac{15}{x+y} - \frac{5}{x-y} = -2$

(3) $\frac{27}{x-2} + \frac{31}{y+3} = 85 ; \frac{31}{x-2} + \frac{27}{y+3} = 89$

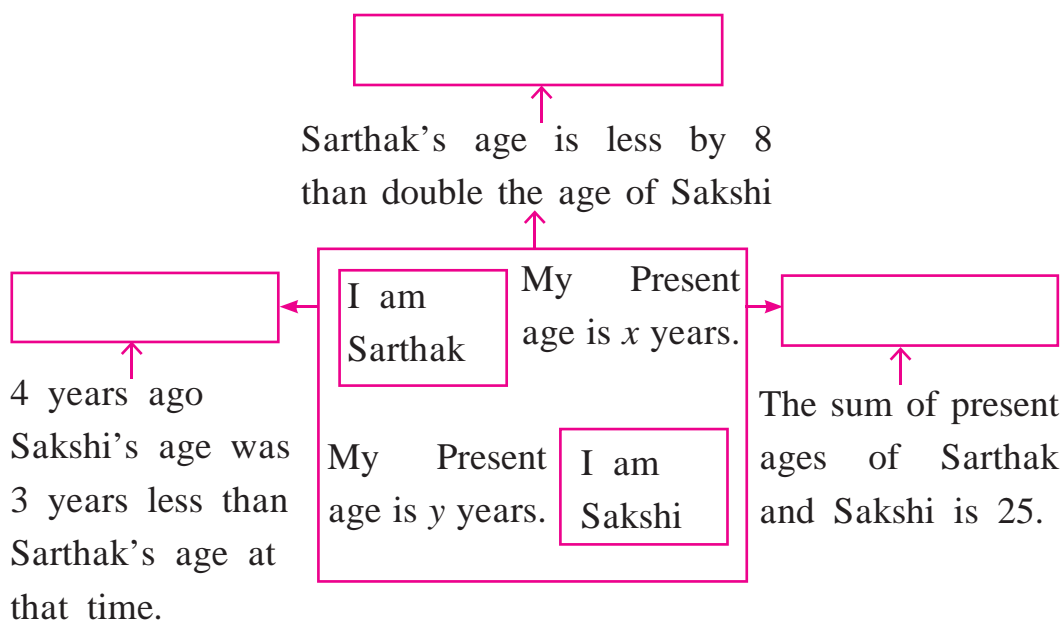
(4) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4} ; \frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = -\frac{1}{8}$



Let's learn.

Application of Simultaneous equations

Activity : There are some instructions given below. Frame the equations from the information and write them in the blank boxes shown by arrows.



Ex. (1) The perimeter of a rectangle is 40 cm. The length of the rectangle is more than double its breadth by 2. Find length and breadth.

Solution : Let length of rectangle be x cm and breadth be y cm.

From first condition -

$$2(x + y) = 40$$

$$x + y = 20 \dots (I)$$

From 2nd condition -

$$x = 2y + 2$$

$$\therefore x - 2y = 2 \dots (II)$$

Let's solve eq. (I), (II) by determinant method

$$x + y = 20$$

$$x - 2y = 2$$

$$D = \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = [1 \times (-2)] - (1 \times 1) = -2 - 1 = -3$$

$$D_x = \begin{vmatrix} 20 & 1 \\ 2 & -2 \end{vmatrix} = [20 \times (-2)] - (1 \times 2) = -40 - 2 = -42$$

$$D_y = \begin{vmatrix} 1 & 20 \\ 1 & 2 \end{vmatrix} = (1 \times 2) - (20 \times 1) = 2 - 20 = -18$$

$$x = \frac{D_x}{D} \text{ and } y = \frac{D_y}{D}$$

$$\therefore x = \frac{-42}{-3} \text{ and } y = \frac{-18}{-3}$$

$$\therefore x = 14, \quad y = 6$$

\therefore Length of the rectangle is 14 cm and breadth is 6 cm.

Ex. (2)

Sale ! Sale !! Sale !!! only for 2 days



I have some analogue wrist watches and some digital wrist watches. I am going to sell them at a discount

Sale of 1st day

Analogue watch = 11

Digital watch = 6

Received amount = ₹ 4330

Sale of the 2nd day

Analogue watch = 22

Digital watch = 5

Received amount = ₹ 7330

Find selling price of wrist watch of each type.

Solution : Let selling price of each analogue watch be ₹ x

Selling price of each digital watch be ₹ y

From first condition -

$$11x + 6y = 4330 \dots (I)$$

from 2nd condition -

$$22x + 5y = 7330 \dots (II)$$

multiplying equation (I) by 2 we get,

$$22x + 12y = 8660 \dots (III)$$

subtract equation (III) from equation (II).

$$\begin{array}{r} 22x + 5y = 7330 \dots (II) \\ - \\ 22x + 12y = 8660 \dots (III) \\ \hline -7y = -1330 \end{array}$$

$$\therefore y = 190$$

Substitute this value of y in equation (I)

$$11x + 6y = 4330$$

$$\therefore 11x + 6(190) = 4330$$

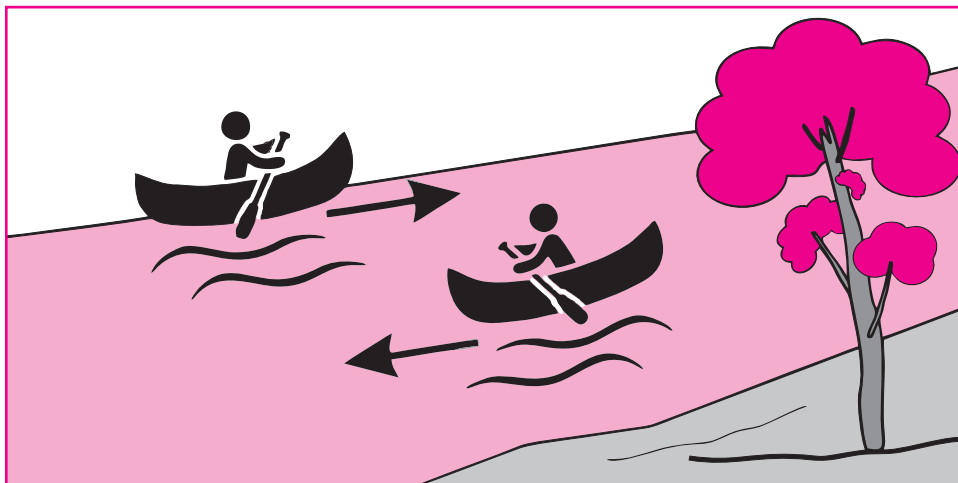
$$\therefore 11x + 1140 = 4330$$

$$\therefore 11x = 3190$$

$$\therefore x = 290$$

\therefore selling price of each analogue watch is ₹ 290 and
of each digital watch is ₹ 190.

Ex. (3)



A boat travels 16 km upstream and 24 km downstream in 6 hours.

The same boat travels 36 km upstream and 48 km downstream in 13 hours.

Find the speed of water current and speed of boat in still water.

Solution : Let the speed of the boat in still water be x km/hr and the speed of water current be y km/hr

\therefore speed of boat in downstream = $(x + y)$ km/hr.

and that in upstream = $(x - y)$ km/hr.

Now distance = speed \times time \therefore time = $\frac{\text{distance}}{\text{speed}}$

Time taken by the boat to travel 16 km upstream = $\frac{16}{x - y}$ hours and it takes $\frac{24}{x + y}$ hours to travel 24 km downstream.

from first condition -

$$\frac{16}{x - y} + \frac{24}{x + y} = 6 \dots (I)$$

from 2nd condition

$$\frac{36}{x - y} + \frac{48}{x + y} = 13 \dots (II)$$

By replacing $\frac{1}{x - y}$ by m and $\frac{1}{x + y}$ by n we get

$$16m + 24n = 6 \dots (III)$$

$$36m + 48n = 13 \dots (IV)$$

Solving equations (III) and (IV) $m = \frac{1}{4}$, $n = \frac{1}{12}$

Replacing m, n by their original values we get

$$x - y = 4 \dots (V) \quad x + y = 12 \dots (VI)$$

Solving equations (V), (VI) we get $x = 8$, $y = 4$

\therefore speed of the boat in still water is 8 km/hr. and speed of water current is 4 km/hr.

Ex. (4) A certain amount is equally distributed among certain number of students. Each would get ₹ 2 less if 10 students were more and each would get ₹ 6 more if 15 students were less. Find the number of students and the amount distributed.

Solution : Let the number of students be x and amount given to each student be ₹ y .

\therefore Total amount distributed is xy

From the first condition we get,

$$(x + 10)(y - 2) = xy$$

$$\therefore xy - 2x + 10y - 20 = xy$$

$$\therefore -2x + 10y = 20$$

$$\therefore -x + 5y = 10 \dots (I)$$

From the 2nd condition we get,

$$(x - 15)(y + 6) = xy$$

$$\therefore xy + 6x - 15y - 90 = xy$$

$$\therefore 6x - 15y = 90$$

$$\therefore 2x - 5y = 30 \dots (II)$$

Adding equations (I) and (II)

$$\begin{array}{r} -x + 5y = 10 \\ + \quad 2x - 5y = 30 \\ \hline \end{array}$$

$$x = 40$$

Substitute this value of x in equation (I)

$$-x + 5y = 10$$

$$\therefore -40 + 5y = 10$$

$$\therefore 5y = 50$$

$$\therefore y = 10$$

Total amount distributed is $= xy = 40 \times 10 = ₹ 400$.

$\therefore ₹ 400$ distributed equally among 40 students.

Ex. (5) A three digit number is equal to 17 times the sum of its digits; If the digits are reversed, the new number is 198 more than the old number ; also the sum of extreme digits is less than the middle digit by unity. Find the original number.

Solution : Let the digit in hundreds place be x and that in unit place be y .

H	T	unit
x	$x + y + 1$	y

$$\begin{aligned}\therefore \text{the three digit number is } & 100x + 10(x + y + 1) + y \\ & = 100x + 10x + 10y + 10 + y = 110x + 11y + 10\end{aligned}$$

$$\text{the sum of the digits in the given number} = x + (x + y + 1) + y = 2x + 2y + 1$$

\therefore From first condition

$$\text{Given number} = 17 \times (\text{sum of the digits})$$

$$\therefore 110x + 11y + 10 = 17 \times (2x + 2y + 1)$$

$$\therefore 110x + 11y + 10 = 34x + 34y + 17$$

$$\therefore 76x - 23y = 7 \dots \text{(I)}$$

The number obtained by reversing the digits

$$= 100y + 10(x + y + 1) + x = 110y + 11x + 10$$

$$\text{Given number} = 110x + 11y + 10$$

From 2nd condition, Given number + 198 = new number.

$$110x + 11y + 10 + 198 = 110y + 11x + 10$$

$$99x - 99y = -198$$

$$x - y = -2$$

$$\therefore x = y - 2 \dots \text{(II)}$$

Substitute this value of x in equation (I).

$$\therefore 76(y - 2) - 23y = 7$$

$$\therefore 76y - 152 - 23y = 7$$

$$53y = 159$$

$\therefore y = 3$ \therefore the digit in units place is $= 3$

Substitute this value in equation (II)

$$x = y - 2$$

$$\therefore x = 3 - 2 = 1$$

$\therefore x = 1$ \therefore The digit in hundred's place is 1

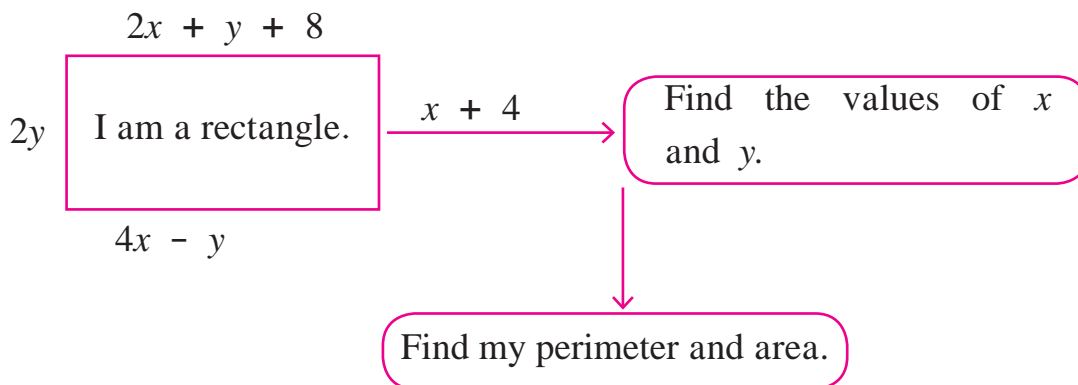
the digit in ten's place is $3 + 1 + 1 = 5$

\therefore the number is 153.

Practice Set 1.5

(1) Two numbers differ by 3. The sum of twice the smaller number and thrice the greater number is 19. Find the numbers.

(2) Complete the following.



(3) The sum of father's age and twice the age of his son is 70. If we double the age of the father and add it to the age of his son the sum is 95. Find their present ages.

(4) The denominator of a fraction is 4 more than twice its numerator. Denominator becomes 12 times the numerator, if both the numerator and the denominator are reduced by 6. Find the fraction.

(5) Two types of boxes A, B are to be placed in a truck having capacity of 10 tons. When 150 boxes of type A and 100 boxes of type B are loaded in the truck, it weighs 10 tons. But when 260 boxes of type A are loaded in the truck, it can still accommodate 40 boxes of type B, so that it is fully loaded. Find the weight of each type of box.

(6) [★] Out of 1900 km, Vishal travelled some distance by bus and some by aeroplane. Bus travels with average speed 60 km/hr and the average speed of aeroplane is 700 km/hr. It takes 5 hours to complete the journey. Find the distance, Vishal travelled by bus.

Problem Set - 1

1. Choose correct alternative for each of the following questions

(1) To draw graph of $4x + 5y = 19$, Find y when $x = 1$.

- (A) 4 (B) 3 (C) 2 (D) -3

(2) For simultaneous equations in variables x and y , $D_x = 49$, $D_y = -63$, $D = 7$ then what is x ?

- (A) 7 (B) -7 (C) $\frac{1}{7}$ (D) $-\frac{1}{7}$

(3) Find the value of $\begin{vmatrix} 5 & 3 \\ -7 & -4 \end{vmatrix}$

- (A) -1 (B) -41 (C) 41 (D) 1

(4) To solve $x + y = 3$; $3x - 2y - 4 = 0$ by determinant method find D .

- (A) 5 (B) 1 (C) -5 (D) -1

(5) $ax + by = c$ and $mx + ny = d$ and $an \neq bm$ then these simultaneous equations have -

- (A) Only one common solution. (B) No solution.
(C) Infinite number of solutions. (D) Only two solutions.

2. Complete the following table to draw the graph of $2x - 6y = 3$

x	-5	<input type="text"/>
y	<input type="text"/>	0
(x, y)	<input type="text"/>	<input type="text"/>

3. Solve the following simultaneous equations graphically.

(1) $2x + 3y = 12$; $x - y = 1$

(2) $x - 3y = 1$; $3x - 2y + 4 = 0$

(3) $5x - 6y + 30 = 0$; $5x + 4y - 20 = 0$

(4) $3x - y - 2 = 0$; $2x + y = 8$

(5) $3x + y = 10$; $x - y = 2$

4. Find the values of each of the following determinants.

(1) $\begin{vmatrix} 4 & 3 \\ 2 & 7 \end{vmatrix}$

(2) $\begin{vmatrix} 5 & -2 \\ -3 & 1 \end{vmatrix}$

(3) $\begin{vmatrix} 3 & -1 \\ 1 & 4 \end{vmatrix}$

5. Solve the following equations by Cramer's method.

(1) $6x - 3y = -10$; $3x + 5y - 8 = 0$

(2) $4m - 2n = -4$; $4m + 3n = 16$

(3) $3x - 2y = \frac{5}{2}$; $\frac{1}{3}x + 3y = -\frac{4}{3}$

(4) $7x + 3y = 15$; $12y - 5x = 39$

(5) $\frac{x+y-8}{2} = \frac{x+2y-14}{3} = \frac{3x-y}{4}$

6. Solve the following simultaneous equations.

(1) $\frac{2}{x} + \frac{2}{3y} = \frac{1}{6}$; $\frac{3}{x} + \frac{2}{y} = 0$ (2) $\frac{7}{2x+1} + \frac{13}{y+2} = 27$; $\frac{13}{2x+1} + \frac{7}{y+2} = 33$

(3) $\frac{148}{x} + \frac{231}{y} = \frac{527}{xy}$; $\frac{231}{x} + \frac{148}{y} = \frac{610}{xy}$ (4) $\frac{7x-2y}{xy} = 5$; $\frac{8x+7y}{xy} = 15$

(5) $\frac{1}{2(3x+4y)} + \frac{1}{5(2x-3y)} = \frac{1}{4}$; $\frac{5}{(3x+4y)} - \frac{2}{(2x-3y)} = -\frac{3}{2}$

7. Solve the following word problems.

- (1) A two digit number and the number with digits interchanged add up to 143. In the given number the digit in unit's place is 3 more than the digit in the ten's place. Find the original number.

Let the digit in unit's place is x

and that in the ten's place is y

\therefore the number = $\square y + x$

The number obtained by interchanging the digits is $\square x + y$

According to first condition two digit number + the number obtained by interchanging the digits = 143

$$\therefore \square 10y + x + \square = 143$$

$$\therefore \square x + \square y = 143$$

$$x + y = \square \dots \dots \text{(I)}$$

From the second condition,

digit in unit's place = digit in the ten's place + 3

$$\therefore x = \square + 3$$

$$\therefore x - y = 3 \dots \dots \text{(II)}$$

Adding equations (I) and (II)

$$2x = \square$$

$$x = 8$$

Putting this value of x in equation (I)

$$x + y = 13$$

$$8 + \square = 13$$

$$\therefore y = \square$$

The original number is $10y + x$

$$= \square + 8$$

$$= 58$$

- (2) Kantabai bought $1\frac{1}{2}$ kg tea and 5 kg sugar from a shop. She paid ₹ 50 as return fare for rickshaw. Total expense was ₹ 700. Then she realised that by ordering online the goods can be bought with free home delivery at the same price. So next month she placed the order online for 2 kg tea and 7 kg sugar. She paid ₹ 880 for that. Find the rate of sugar and tea per kg.

- (3) To find number of notes that Anushka had, complete the following activity.

Suppose that Anushka had x notes of ₹ 100 and y notes of ₹ 50 each

Anushka got ₹ 2500/- from Anand as denominations mentioned above
 \therefore equation I

\therefore The No. of notes (\square, \square)

If Anand would have given her the amount by interchanging number of notes, Anushka would have received ₹ 500 less than the previous amount
 \therefore equation II

- (4) Sum of the present ages of Manish and Savita is 31. Manish's age 3 years ago was 4 times the age of Savita. Find their present ages.
- ★ (5) In a factory the ratio of salary of skilled and unskilled workers is 5 : 3. Total salary of one day of both of them is ₹ 720. Find daily wages of skilled and unskilled workers.
- ★ (6) Places A and B are 30 km apart and they are on a straight road. Hamid travels from A to B on bike. At the same time Joseph starts from B on bike, travels towards A. They meet each other after 20 minutes. If Joseph would have started from B at the same time but in the opposite direction (instead of towards A) Hamid would have caught him after 3 hours. Find the speed of Hamid and Joseph.

