15 Area

Let's recall.

We know that, if sides of closed polygon are given in the units cm, m, km then their areas are in the units sq cm, sq m and sq km respectively; because the area is measured by squares.

(1) Area of square = side^2

(2) Area of rectangle = length $\times$ breadth

(3) Area of right angled triangle $= \frac{1}{2} \times$ product of sides making right angle

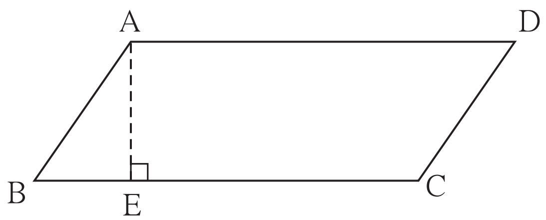
(4) Area of triangle $= \frac{1}{2} \times$ base $\times$ height

Let's learn.

Area of a parallelogram

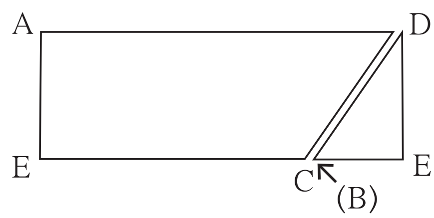
Activity :

- Draw a big enough parallelogram ABCD on a paper as shown in the figure.



Draw perpendicular AE on side BC.

Cut the right angled Δ AEB. Join it with the remaining part of $\square$ ABCD as shown in the figure.

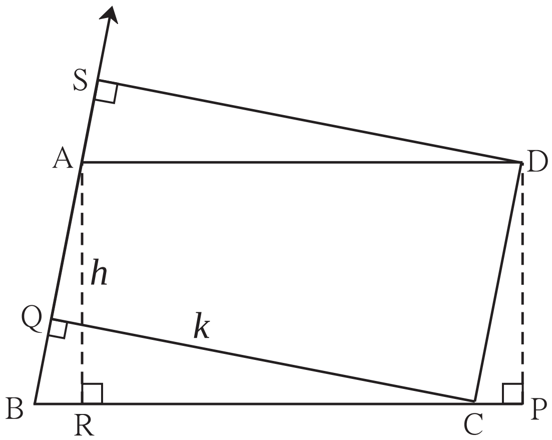


Note that the new figure formed is a rectangle.

- The rectangle is formed from the parallelogram So areas of both the figures are equal.
- Base of parallelogram is one side (length) of the retangle and its height is the other side (breadth) of the rectangle.

$\therefore$ **Area of parallelogram = base $\times$ height**

Remember that, if we consider one of the parallel sides of a parallelogram as a base then the distance between these parallel sides is the height of the parallelogram corresponding to the base.



$\square$ ABCD is a parallelogram.
seg DP $\perp$ side BC, seg AR $\perp$ side BC.
If side BC is a base then
height = l(AR) = l(DP) = h.
If seg CQ $\perp$ side AB and if we consider seg AB as a base then corresponding height is l(QC) = k.
$\therefore$ A($\square$ABCD) = l(BC) $\times$ h = l(AB) $\times$ k.

Solved Examples

Ex. (1) If base of a parallelogram is 8 cm and height is 5 cm then find its area.

Solution: Area of a parallelogram = base $\times$ height = 8 $\times$ 5

= 40

$\therefore$ area of the parallelogram is 40 sq.cm

Ex. (2) If Area of a parallelogram is 112 sq cm and base of it is 10 cm then find its height

Solution: Area of a parallelogram = base $\times$ height $\therefore$ 112 = 10 $\times$ height

$$\frac{112}{10} = \text{height}$$

$\therefore$ height of the parallelogram is 11.2 cm

Practice Set 15.1

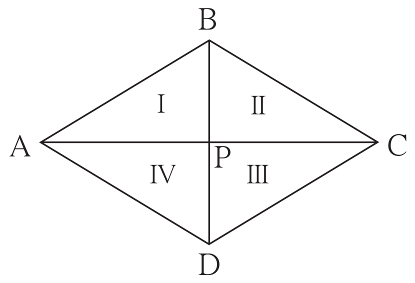
1. If base of a parallelogram is 18 cm and its height is 11 cm, find its area.

2. If area of a parallelogram is 29.6 sq cm and its base is 8 cm, find its height.

3. Area of a parallelogram is 83.2 sq cm. If its height is 6.4 cm, find the length of its base.

Area of a rhombus

Activity: Draw a rhombus as shown in the adjacent figure. We know that diagonals of a rhombus are perpendicular bisectors of each other.



Let $l(AC) = d_1$ and $l(BD) = d_2$

☐ ABCD is a rhombus. Its diagonals intersect in the point P. So we get four congruent right angled triangles. Sides of each right angled triangle are $\frac{1}{2}\ l(AC)$ and $\frac{1}{2}\ l(BD)$. Areas of all these four triangles are equal.

$l(AP) = l(PC) = \frac{1}{2}\ l(AC) = \frac{d_1}{2}$,

and $l(BP) = l(PD) = \frac{1}{2}\ l(BD) = \frac{d_2}{2}$

$$\therefore \quad \text{Area of rhombus ABCD} = 4 \times A(\Delta\ APB)$$

$$= 4 \times \frac{1}{2} \times l(AP) \times l(BP)$$

$$= 2 \times \frac{d_1}{2} \times \frac{d_2}{2}$$

$$= \frac{1}{2} \times d_1 \times d_2$$

$\therefore$ area of a rhombus $= \frac{1}{2} \times$ product of lengths of diagonals.

Solved Examples

Ex. (1) Lengths of the diagonals of a rhombus are 11.2 cm and 7.5 cm respectively. Find the area of rhombus.

Solution: Area of a rhombus $= \frac{1}{2} \times$ product of lengths of the diagonals

$$= \frac{1}{2} \times \frac{11.2}{1} \times \frac{7.5}{1} = 5.6 \times 7.5$$

$$= 42 \text{ sq cm}$$

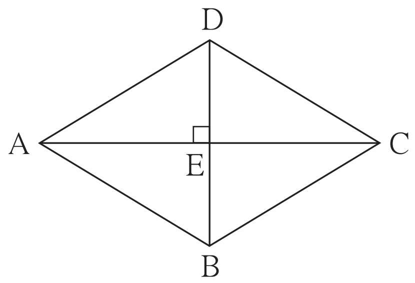
Ex. (2) Area of a rhombus is 96 sq cm. One of the diagonals is 12 cm find the length of its side.

Solution: Let $\square$ ABCD be a rhombus.

Diagonal BD is of length 12 cm.

Area of the rhombus is 96 sq cm.

So first find the length of diagonal AC.



Area of a rhombus $= \frac{1}{2} \times$ product of lengths of diagonals

$$\therefore\ 96 = \frac{1}{2} \times 12 \times l(AC) = 6 \times l(AC)$$

$$\therefore\ l(AC) = 16 \text{ cm}$$

Let E be the point of intersection of diagonals of a rhombus. Diagonals are perpendicular bisectors of each other.

$\therefore$ in Δ ADE, $m\angle E = 90°$,

$l(DE) = \frac{1}{2} l(DB) = \frac{1}{2} \times 12 = 6$; | $l(AE) = \frac{1}{2} l(AC) = \frac{1}{2} \times 16 = 8$

Using Pythagoras theorem we get,

$$l(AD)^2 = l(AE)^2 + l(DE)^2 = 8^2 + 6^2$$

$$= 64 + 36 = 100$$

$\therefore\ l(AD) = 10$ cm

$\therefore$ side of the rhombus is 10 cm.

Practice Set 15.2

1. Lengths of the diagonals of a rhombus are 15cm and 24 cm, find its area.

2. Lengths of the diagonals of a rhombus are 16.5 cm and 14.2 cm, find its area.

3. If perimeter of a rhombus is 100 cm and length of one diagonal is 48 cm, what is the area of the quadrilateral?

4★. If length of a diagonal of a rhombus is 30 cm and its area is 240 sq cm, find its perimeter.

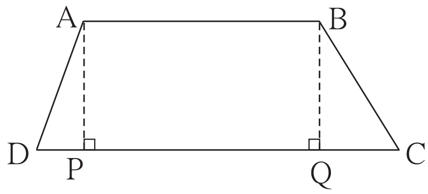
Area of a trapezium

Activity : Draw a trapezium ABCD on the paper such that seg AB || seg DC.

Draw seg AP ⊥ seg DC and

seg BQ ⊥ side DC .

let $l(AP) = l(BQ) = h$



Height of the trapezium is the distance between the parallel sides. After drawing the perpendiculars in □ ABCD, its area is divided into 3 parts. Out of these Δ APD and Δ BQC are right angled triangles.

□ ABQP is a rectangle. points P and Q are on seg DC.

Area of trapezium ABCD

$= A(\Delta\ APD) + A(\square ABQP) + A(\Delta BQC)$

$= \frac{1}{2} \times l(DP) \times h + l(PQ) \times h + \frac{1}{2}\ \ l(QC) \times h$

$= h\ [\frac{1}{2}\ l(DP) + l(PQ) + \frac{1}{2}\ l(QC)]$

$= \frac{1}{2} \times h\ [l(DP) + 2l(PQ) + l(QC)]$

$= \frac{1}{2} \times h\ [l(DP) + l(PQ) + l(AB) + l(QC)]\ \ldots \because\ l(PQ) = l(AB)$

$= \frac{1}{2} \times h\ [l(DP) + l(PQ) + l(QC) + l(AB)]$

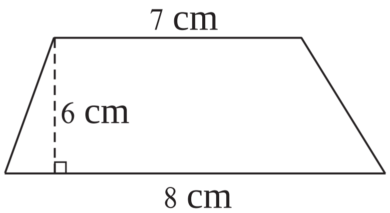
$= \frac{1}{2} \times h\ [l(DC) + l(AB)]$

$A(\square ABCD) = \frac{1}{2}$ (sum of the lengths of parallel sides) $\times\ h$

$\therefore$ Area of the trapezium $= \frac{1}{2} \times$ sum of the lengths of parallel sides $\times$ height

Solved Example

Ex. (1) In a trapezium, if distance between parallel sides is 6 cm and lengths of the parallel sides are 7 cm and 8 cm respectively then find the area of the trapezium.



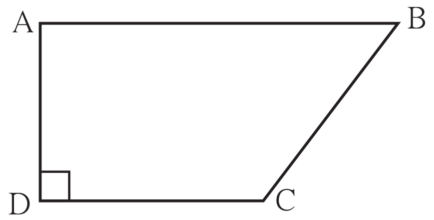
Solution : Distance between parallel sides = height of the trapezium = 6 cm

Area of the trapezium = $\frac{1}{2}$ × (sum fo the lengths of parallel sides) × height

$$= \frac{1}{2}\ (7 + 8) \times 6 = 45 \text{ sq cm}$$

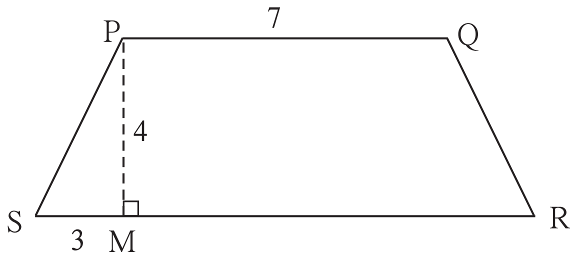
Practice Set 15.3

1. In ☐ ABCD, l(AB) = 13 cm, l(DC) = 9 cm, l(AD) = 8 cm, find the area of ☐ ABCD.



2. Length of the two parallel sides of a trapezium are 8.5 cm and 11.5 cm respectively and its height is 4.2 cm, find its area.

3★. ☐ PQRS is an isosceles trapezium l(PQ) = 7 cm. seg PM ⊥ seg SR, l(SM) = 3 cm, Distance between two parallel sides is 4 cm, find the area of ☐ PQRS

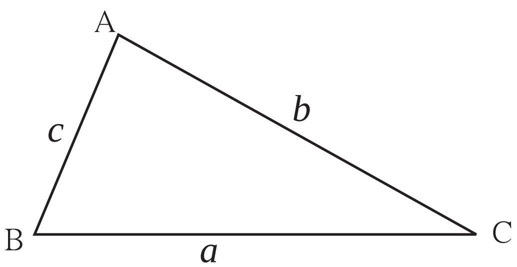


Let's learn.

Area of a Triangle

We know that, Area of a triangle = $\frac{1}{2}$ base × height.

Now we will see how to find the area of a triangle if height is not given but lengths of the three sides of a triangle are given.



Let a, b, c be the lengths of sides BC, AC and AB respectively of Δ ABC.

Let us find the semiperimeter of the triangle.

semiperimeter = $s = \frac{1}{2}\ (a + b + c)$

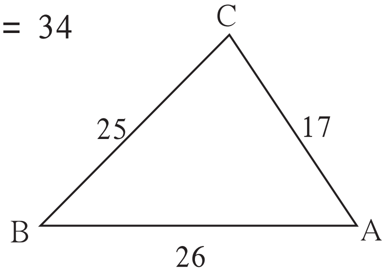
Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

This formula is known as Heron's Formula.

Ex. (1) If the sides of a triangle are 17 cm, 25 cm and 26 cm, find the area of the triangle.

Solution: Here, $a = 17$, $b = 25$, $c = 26$

semiperimeter $= s = \frac{a+b+c}{2} = \frac{17+25+26}{2} = \frac{68}{2} = 34$



Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{34(34-17)(34-25)(34-26)}$

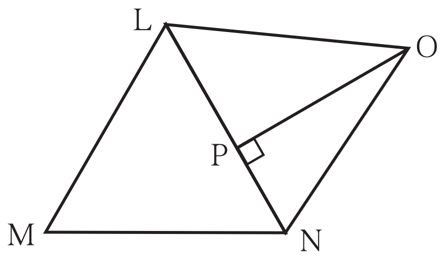
$= \sqrt{34 \times 17 \times 9 \times 8}$

$= \sqrt{17 \times 2 \times 17 \times 3 \times 3 \times 2 \times 2 \times 2}$

$= \sqrt{17^2 \times 2^2 \times 2^2 \times 3^2}$

$= 17 \times 2 \times 2 \times 3 = 204$ sq cm

Ex. (2) The figure of a plot and its measures are given.
l(LM) = 60 m. l(MN) = 60 m.
l(LN) = 96 m. l(OP) = 70 m.
find the area of the plot.



Solution: In the figure we get two triangles, Δ LMN and Δ LNO . We know the lengths of all sides of Δ LMN so by using Heron's formula we will find the area of this triangle. In Δ LNO, side LN is the base and l(OP) is the height. We will find the area of Δ LNO.

Semiperimeter of Δ LMN , s $= \frac{60+60+96}{2} = \frac{216}{2} = 108$ m

$\therefore$ Area of Δ LMN $= \sqrt{108(108-60)(108-60)(108-96)}$

$= \sqrt{108 \times 48 \times 48 \times 12}$

$= \sqrt{12 \times 9 \times 48 \times 48 \times 12}$

A(Δ LMN) $= 12 \times 3 \times 48 = 1728$ sq m

A(Δ LNO) $= \frac{1}{2}$ base $\times$ height

$= \frac{1}{2} \times 96 \times 70 = 96 \times 35 = 3360$ sq m

Area of $\square$ LMNO = A(Δ LMN) + A(Δ LNO)

= 1728 + 3360

= 5088 sq m

Area of the plot LMNO is 5088 sq m

Now I know.

Area of a parallelogram = base × height

Area of a rhombus = $\frac{1}{2}$ × product of lengths of diagonals

Area of a trapezium = $\frac{1}{2}$ × sum of the lengths of parallel sides × height

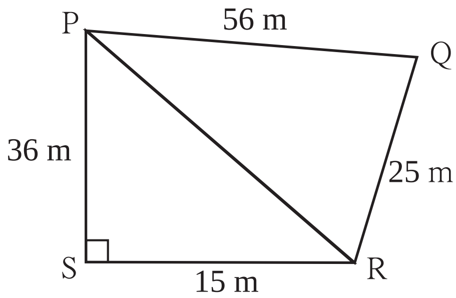
If sides of Δ ABC are a, b, c then Heron's formula for finding the area of trangle is as follows

$$A(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where } s = \frac{a+b+c}{2}$$

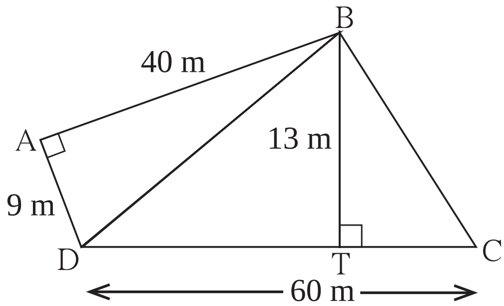
Practice Set 15.4

1. Sides of a triangle are cm 45 cm, 39 cm and 42 cm, find its area.

2. Look at the measures shown in the adjacent figure and find the area of $\square$ PQRS.



3. Some measures are given in the adjacent figure, find the area of $\square$ABCD.

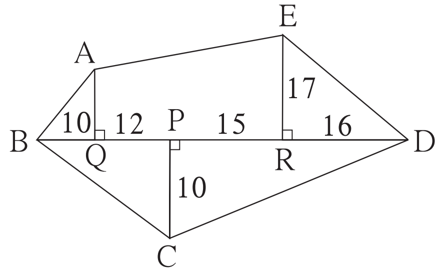


Let's learn.

Area of figures having irregular shape

Generally the plots, fields ect. are of the shape of irregular polygons. These polygons can be divided into triangles and specific quadrilaterals. Study the following examples to know how the polygons are divided and their areas are calculated.

Ex. Adjacent figure is a polygon ABCDE. All given measures are in metre. Find the area of the given figure.



Proof: Here Δ AQB, Δ ERD are right angled triangles and $\square$ AQRE is a trapezium. Base and height of Δ BCD is given. Now let us find the area of each figure.

$$A(\Delta\ AQB) = \frac{1}{2} \times l(BQ) \times l(AQ) = \frac{1}{2} \times 10 \times 13 = 65 \text{ sq m}$$

$$A(\Delta\ ERD) = \frac{1}{2} \times l(RD) \times l(ER) = \frac{1}{2} \times 16 \times 17 = 136 \text{ sq m}$$

$$A(\square\ AQRE) = \frac{1}{2}\ [l(AQ) + l(ER)] \times l(QR)$$

$$= \frac{1}{2}\ [13 + 17] \times (12 + 15)$$

$$= \frac{1}{2} \times 30 \times 27 = 15 \times 27 = 405 \text{ sq m}$$

$$l(BD) = l(BP) + l(PD) = 10 + 12 + 15 + 16 = 53 \text{ m}$$

$$A(\Delta\ BCD) = \frac{1}{2} \times l(BD) \times l(PC) = \frac{1}{2} \times 53 \times 10 = 265 \text{ sq m}$$

$\therefore$ Area of polygon ABCDE

$= A(\Delta\ AQB) + A(\square AQRE) + A(\Delta\ ERD) + A(\Delta\ BCD)$

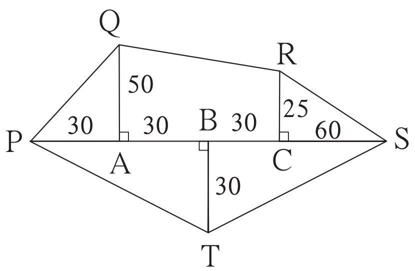
$= 65 + 405 + 136 + 265$

$= 871$ sq m

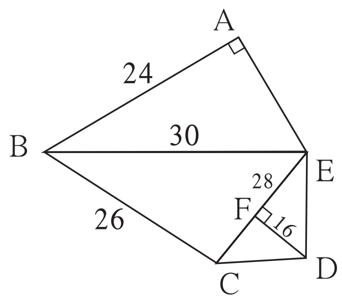
Practice Set 15.5

1. Find the areas of given plots. (All measures are in metres.)

(1)

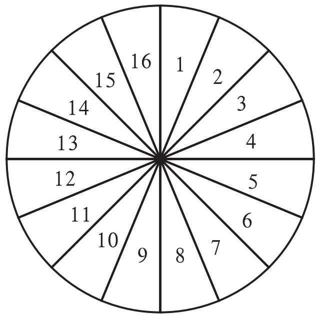


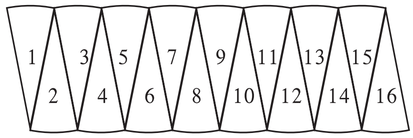
(2)



Area of a circle

Activity : Draw a circle on a card sheet. Cut the circle from the sheet. Divide the circular paper into 16 or 32 equal parts by paper folding or make 18 or 20 equal parts by dividing 360° in equal parts. Then get the sectors by cutting them along the radii. Join all these sectors as shown in the figure. We get nearly a rectangle. As we go on increasing the number of parts of the circle,the shape of the figure is more and more like that of a rectangle.





Circumference of a circle = $2\pi r$

$\therefore$ length of the rectangle is πr, that is semicircumference and breadth is r.

$\therefore$ Area of the circle = area of rectangle = length $\times$ breadth = $\pi r \times r$

$\therefore$ Area of the circle = πr^2

Solved Examples

Ex. (1) If radius of a circle is 21 cm then find its area.

Solution: Area of circle = πr^2

$$= \frac{22}{7} \times 21^2$$

$$= \frac{22}{7} \times \frac{21}{1} \times \frac{21}{1} \quad = 66 \times 21 \quad = 1386 \text{ sq cm.}$$

Ex. (2) Area of a circular ground is 3850 sq m. Find the radius of the circular ground.

Solution : Area of the circular ground = πr^2

$$3850 = \frac{22}{7} \times r^2$$

$$r^2 = \frac{3850 \times 7}{22} \qquad r^2 = 1225 \qquad r = 35 \text{ m.}$$

$\therefore$ Radius of the ground is 35 m.

Practice Set 15.6

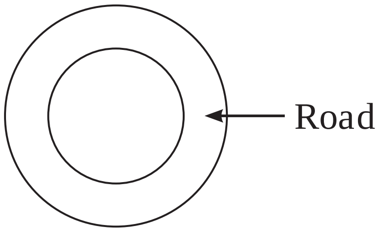
1. Radii of the circles are given below, find their areas.

(1) 28 cm (2) 10.5 cm (3) 17.5 cm

2. Areas of some circles are given below find their diameters.

(1) 176 sq cm (2) 394.24 sq cm (3) 12474 sq cm

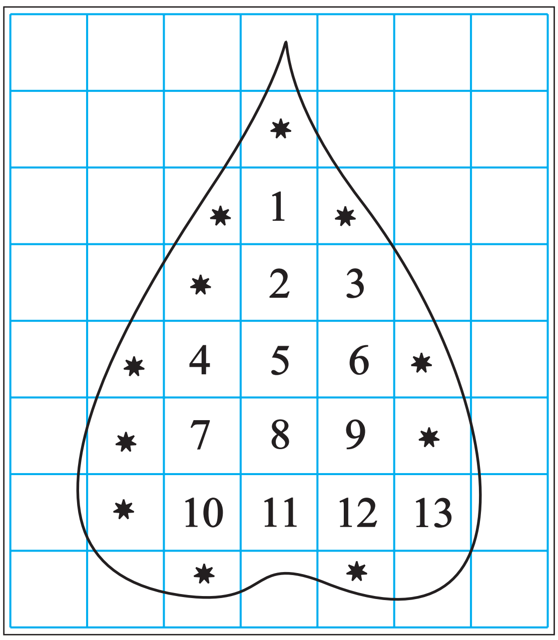
3. Diameter of the circular garden is 42 m. There is a 3.5 m wide road around the garden. Find the area of the road.



4. Find the area of the circle if its circumfence is 88 cm.

To find the approximate area of irregular figure.

Area of the irregular closed figure can be calculated approximately by using graph paper. Draw an outline of the given piece of object on the graph paper with pencil. Let us learn from the activity how the area of the given figure is calculated by counting the number of squares on the graph paper.



(1) The number of complete squares of area 1 sq cm = 13

$\therefore$ Area of those squares is

= 13 sq cm

(2) In the figure the number of parts having area more than $\frac{1}{2}$ sq cm but less than 1 sq cm = 11

$\therefore$ Area of these parts = 11 sq cm

(3) From the figure, number of parts having area $\frac{1}{2}$ sq cm = 0

$\therefore$ Area of these parts = 0 sq cm

(4) From the figure count the parts having area less than $\frac{1}{2}$ sqcm such parts are not to be considered. $\therefore$ Area of these parts = 0 sq cm

$\therefore$ Total area $= 13 + 11 + 0 + 0 = 24$ sq cm

Activity : Draw a circle of radius 28mm. Draw any one triangle and draw a trapezium on the graph paper. Find the area of these figures by counting the number of small squares on the graph paper. Verify your answers using formula for area of these figures.

Observe that the smaller the squares of graph paper, better is the approximation of area.

For more information :

Our nation has adopted the decimal system of measurement. So in the revenue department areas of lands are recorded in decimal units, Are and Hectare

100 sq m = 1 are, 100 are = 1 hectare = 10,000 sq m

In practice often area of land is measured in ‘Guntha’ or ‘Acre’.

1 Guntha land area is approximately 1 are. Which is nearly 100 sq m. 1 acre area is nearly 0.4 hectare.

Answers

Practice Set 15.1 **1.** 198 sq cm **2.** 3.7 cm **3.** 13 cm

Practice Set 15.2 **1.** 180 sq cm **2.** 117.15 sq cm **3.** 336 sq cm **4.** 68 cm

Practice Set 15.3 **1.** 88 sq cm **2.** 42 cm **3.** 40 sq cm

Practice Set 15.4 **1.** 756 sq cm **2.** 690 sq cm **3.** 570 sq cm

Practice Set 15.5 **1.** 6000 sq m **2.** 776 sq m

Practice Set 15.6 **1.** (1) 2464 sq cm (2) 346.5 sq cm (3) 962.5 sq cm

2. (1) $2\sqrt{56}$ cm (2) 22.4 cm (3) 126 cm

3. 500.50 sq m **4.** 616 sq cm

D4SRYJ