# 10. Electrostatics

**Can you recall?**

1. Have you experienced a shock while getting up from a plastic chair and shaking hand with your friend?
2. Ever heard a crackling sound while taking out your sweater in winter?
3. Have you seen the lightning striking during pre-monsoon weather?

## 10.1 Introduction:

Electrostatics deals with static electric charges, the forces between them and the effects produced in the form of electric fields and electric potentials. We have already studied some aspects of electrostatics in earlier standards. In this Chapter we will review some of them and then go on to study some aspects in details.

Current electricity, which plays a major role in our day to day life, is produced by moving charges. Charges are present everywhere around us though their presence can only be felt under special circumstances. For example, when we remove our sweater in winter on a dry day, we hear some crackling sound and the sweater appears to stick to our body. This is because of the electric charges produced due to friction between our body and the sweater. Similarly, the lightening that we see in the sky is also due to the flow of large amount of electric charges that develop on the clouds due to friction.

## 10.2 Electric Charges:

Historically, opposite electric charges were known to the Greeks in the 600 BC. They realized that equal and opposite charges develop on amber and fur when rubbed against each other. Now we know that electric charge is a basic property of elementary particles of which matter is made of. These elementary particles are proton, neutron, and electron. Atoms are made of these particles and matter is made from atoms. A proton is considered to be positively charged and electron to be negatively charged. Neutron is electrically neutral, i.e., it has no charge. An atomic nucleus is made up of protons and neutrons and hence is positively charged. Negatively charged electrons surround the nucleus so as to make an atom electrically neutral. Thus, most matter around us is electrically neutral.

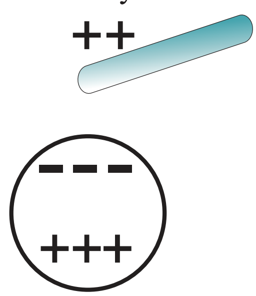


Fig. 10.1 a

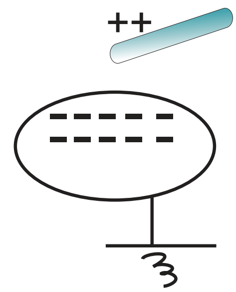


Fig. 10.1 b

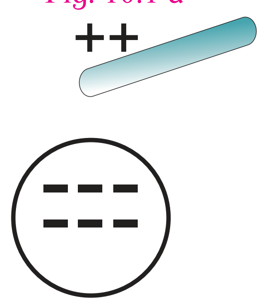


Fig. 10.1 c

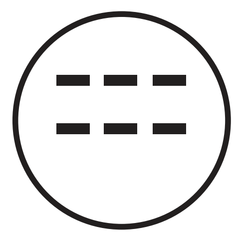


Fig. 10.1 d

Fig. 10.1 (a): Insulated conductor

Fig. 10.1 (b): +ve charge is neutralized by electron from Earth

Fig. 10.1 (c): Earthing is removed -ve charge still stays on the conductor due to +ve charged rod

Fig. 10.1 (d): Rod removed -ve charge is distributed over the surface of the conductor

When certain dissimilar substances, like fur and amber or comb and dry hair are rubbed against each other, electrons get transferred to the other substance making them charged. The substance receiving electrons develops a negative charge while the other is left with an equal amount of positive charge. This can be called charging by conduction as charges are transfered from one body to another. Charges can be separated by other means as well, like chemical reactions (in cells), convection (in clouds), diffusion (in living cells) etc.

If an uncharged conductor is brought near a charged body, (not in physical contact) the nearer side of the conductor develops opposite charge to that on the charged body and the far side of the conductor develops charge similar to that on the charged body. This is called induction. This happens because the electrons in a conductor are free and can move easily in presence of a charged body. This can be seen from Fig. 10.1.

A charged body attracts or repels electrons in a conductor depending on whether the charge on the body is positive or negative respectively. Positive and negative charges are redistributed and are accumulated at the ends of the conductor near and away from the changed body. From the above discussion it can be inferred that there are only two types of charges found in nature, namely, positive and negative charges. In induction, there is no transfer of charges between the charged body and the conductor. So when the charged body is moved away from the conductor, the charges in the conductor are free again.

**Can you tell?**

1. When a petrol or a diesel tanker is emptied in a tank, it is grounded.
2. A thick chain hangs from a petrol or a diesel tanker and it is in contact with ground when the tanker is moving.

## 10.3 Basic Properties of Electric Charge:

### 10.3.1 Additive Nature of Charge:

Electric charge is additive, similar to mass. The total electric charge on an object is equal to the algebraic sum of all the electric charges distributed on different parts of the object.

It may be pointed out that while taking the algebraic sum, the sign (positive or negative) of the electric charges must be taken into account. Thus if two bodies have equal and opposite charges, the net charge on the system of the two bodies is zero. This is similar to that in case of atoms where the nucleus is positively charged and this charge is equal to the negative charge

**Gold Leaf Electroscope:**

This is a classic instrument for detecting presences of electric charge. A metal disc is connected to one end of a narrow metal rod and a thin piece of gold leaf is fixed to the other end. The whole of this part of the electroscope is insulted from the body of the instrument. A glass front prevents air draughts but allows to observe the effect of charge on the leaf.

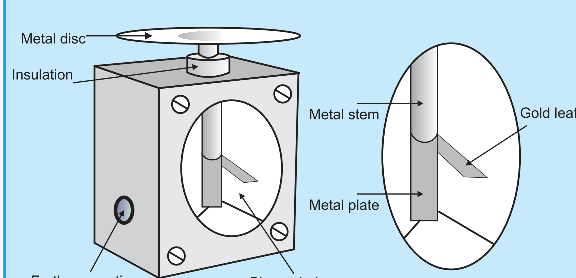
When a charge is put on the disc at the top it spreads down to the plate and leaf moves away from the plate. This happens because similar charges repel. The more the charge on the disc, more is the separation of the leaf from the plate.

The leaf can be made to fall again by touching the disc. This is done by earthing the electroscope. An earth terminal prevents the case from accumulating any stray charge. The electroscope can be charged in two ways.

(a) by contact- a charged rod is brought in contract with the disc and charge is transferred to the electroscope. This method gives the gold leaf the same charge as that on the conductor. This is not a very effective method of charging the electroscope.

(b) by induction- a charged rod is brought close to the disc (not touching it) and the electroscope is earthed. The rod is then removed. This method give the gold leaf opposite charges.

The following diagrams show how the charges spread to the gold leaf and lift it.



of the electrons making the atoms electrically neutral .

It is interesting to compare the additive property of charge with that of mass.

1) The masses of the particles constituting an object are always positive, whereas the charges distributed on different parts of the abject may be positive or negative.
2) The total mass of an object is always positive whereas, the total charge on the object may be positive, zero or negative.

### 10.3.2 Quantization of Charge:

The minimum value of the charge on an electron as determined by the Milikan's oil drop experiment is $e = 1.6\times10^{-19}$ C. This is called the elementary charge. Here, C stands for coulomb which is the unit of charge in SI system. Unit of charge is defind in article 10.4.3. Since protons (+ve) and electrons (-ve) are the charged particles constituting matter, the charge on an object must be an integral multiple of $\pm e$. $q = \pm ne$, where n is an integer.

Further, charge on an object can be increased or decreased in multiples of e. It is because, during the charging process an integral number of electrons can be transferred from one body to the other body. This is known as **quantization of charge** or discrete nature of charge.

The discrete nature of electric charge is usually not observable in practice. It is because the magnitude of the elementary electric charge, e, is extremely small. Due to this, the number of elementary charges involved in charging an object becomes extremely large. Suppose, for example, when a glass rod is rubbed with silk, a charge of the order of one μC ($10^{-6}$ C) appears on the glass rod or silk. Since elementary charge $e = 1.6\times10^{-19}$ C, the number of elementary charges on the glass rod (or silk) is given by

$$n = \frac{10^{-6}\ \mathrm{C}}{1.6\times10^{-19}\mathrm{C}} = 6.25\times10^{12}$$

Since it is a tremendously large number, the quantization of charge is not observed and one usually observes a continuous variation of charge.

**Example 10.1:** How much positive and negative charge is present in 1gm of water? How many electrons are present in it? Given, molecular mass of water is 18.0 g.

**Solution:** Molecular mass of water is 18.0 gm, that means the number of molecules in 18.0 gm of water is $6.02\times10^{23}$.

$\therefore$ Number of molecules in 1gm of water $= 6.02\times10^{23}/18$. One molecule of water ($H_2O$) contains two hydrogen atoms and one oxygen atom. Thus the number of electrons in $H_2O$ is sum of the number of electrons in $H_2$ and oxygen. There are 2 electrons in $H_2$ and 8 electrons oxygen.

$\therefore$ Number of electrons in $H_2O = 2+8 = 10$.

Total number of protons / electrons in 1.0 gm of water $= \dfrac{6.02\times10^{23}}{18}\times10 = 3.34\times10^{23}$

Total positive charge $= 3.34\times10^{23}\times$ charge on a proton

$= 3.34\times10^{23}\times1.6\times10^{-19}\ \mathrm{C} = 5.35\times10^{4}\ \mathrm{C}$

This positive charge is balanced by equal amount of negative charge so that the water molecule is electrically neutral.

**Do you know ?**

According to recent advancement in physics, it is now believed that protons and neutrons are themselves built out of more elementary units called quarks. They are of six types, having fractional charge (-1/3)e or +(2/3)e. A proton or a neutron consists of a combination of three quarks. It may be clearly understood that even in the quark model, quantization of charge is not affected. It is only the step size of the charge that decreases from e to e/3. Quarks are always present in bound states and no free quarks are known to exist.
In modern day experiments it is possible to observe the discrete nature of charge in very sensitive divides such as single electron transistor

### 10.3.3 Conservation of Charge:

We know that when a glass rod is rubbed with silk, it becomes positively charged and silk becomes negatively charged. The amount of positive charge on glass rod is found to be exactly the same as negative charge on silk. Thus, the systems of glass rod and silk together possesses zero net charge after rubbing.

Result and conclusion of this experiment can be generalized and we can say that "in any given physical process, charge may get transferred from one part of the system to another, but the total charge in the system remains constant" or, **for an isolated system total charge cannot be created nor destroyed.** In simple words, the total charge of an isolated system is always conserved.

### 10.3.4 Forces between Charges:

It was observed in carefully conducted experiments with charged objects that they experience force when brought close (not touching) to each other. This force can be attractive or repulsive. **Like charges repel each other and unlike charges attract each other.** Figure 10.2 describes this schematically. This is the reason for charging by induction as described in section 10.2 and Fig. 10.1.

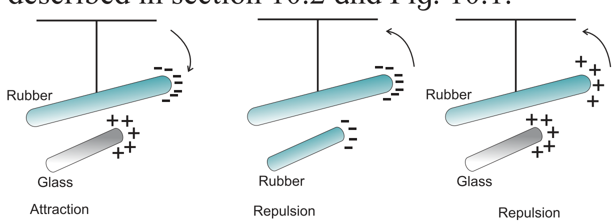



Fig. 10.2: Attractive and repulsive force.

## 10.4 Coulomb's Law:

The electric interaction between two charged bodies can be expressed in terms of the forces they exert on each other. Coulomb (1736-1806) made the first quantitative investigation of the force between electric charges. He used point charges at rest to study the interaction. A point charge is a charge whose dimensions are negligibly small compared to its distance from another bodies. **Coulomb's law is a fundamental law governing interaction between charges at rest.**

### 10.4.1 Scalar form of Coulomb's Law:

**Statement :** *The force of attraction or repulsion between two point charges at rest is directly proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between them.* This force acts along the line joining the two charges.

Let $q_1$ and $q_2$ be two point charges at rest with respect to each other and separated by a distance r. The magnitude F of the force between them is given by,

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = K\frac{q_1 q_2}{r^2} \qquad \text{--- (10.1)}$$

where K is the constant of proportionality. Its magnitude depends on the units in which F, $q_1$, $q_2$ and r are expressed and also on the properties of the medium around the charges.

The force between the two charges will be attractive if they are unlike (one positive and one negative). The force will be repulsive if charges are similar (both positive or both negative). Figure 10.3 describes this schematically.

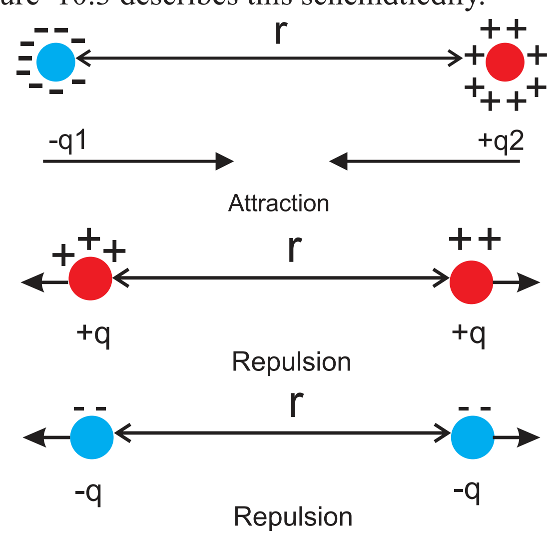



Fig. 10.3: Coulomb's law.

### 10.4.2 Relative Permittivity or Dielectric Constant:

While discussing the coulomb's law it was assumed that the charges are held stationery in vacuum. When the charges are kept in a material medium, such as water, mica or parafined paper, the medium affects the force between the charges. The force between the two charges placed in a medium may be written as,

$$F_{med} = \frac{1}{4\pi\varepsilon}\left(\frac{q_1 q_2}{r^2}\right) \qquad \text{--- (10.2)}$$

where ε is called the absolute permittivity of the medium. The force between the same two charges placed in free space or vacuum at distance r is given by,

$$F_{vac} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r^2}\right) \qquad \text{--- (10.3)}$$

Dividing Eq. (10.3) by (10.2)

$$\frac{F_{vac}}{F_{med}} = \frac{\frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r^2}\right)}{\frac{1}{4\pi\varepsilon}\left(\frac{q_1 q_2}{r^2}\right)} = \frac{\varepsilon}{\varepsilon_0}$$

The ratio $\frac{\varepsilon}{\varepsilon_0}$ is the relative permittivity or dielectric constant of the medium and is denoted by $\varepsilon_r$ or K.

$$\text{K or } \varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = \frac{F_{vac}}{F_{med}} \qquad \text{--- (10.4)}$$

Thus,

(i) $\varepsilon_r$ is the ratio of absolute permittivity of a medium to the permittivity of free space.

(ii) $\varepsilon_r$ is the ratio of the force between two point charges placed a certain distance apart in free space or vacuum to the force between the same two point charges when placed at the same distance in the given medium.

$\varepsilon_r$ is a dimensionless quantity.

(iii) $\varepsilon_r$ is also called specific inductive capacity.

The force between two point charges $q_1$ and $q_2$ placed at a distance r in a medium of relative permittivity $\varepsilon_r$, is given by

$$F = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1 q_2}{r^2} \qquad \text{--- (10.5)}$$

For water, $\varepsilon_r$ = 80 then from Eq. (10.4)

$$\frac{F_{vac}}{F_{water}} = \varepsilon_r = 80$$

$$F_{water} = \frac{F_{vac}}{80}$$

This means that when two point charges are placed some distance apart in water, the force between them is reduced to $\left(\frac{1}{80}\right)^{th}$ of the force between the same two charges placed at the same distance in vacuum.

**Thus, a material medium reduces the force between charges by a factor of $\varepsilon_r$, its relative permittivity**.

While using Eq. (10.5) we assume that the medium is homogeneous, isotropic and infinitely large.

### 10.4.3 Definition of Unit Charge from the Coulomb's Law:

The force between two point charges $q_1$ and $q_2$, separated by a distance r in free space, is written by using Eq. (10.2),

$$F = \frac{1}{4\pi\varepsilon_0}\times\frac{q_1 q_2}{r^2} = 9\times 10^9 \times \frac{q_1 q_2}{r^2}$$

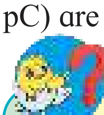
$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

If $q_1 = q_2 = 1C$ and r = 1.0 m

Then $F = 9.0 \times 10^9$ N

From this, we define, coulomb (C) the unit of charge in SI units.

**One coulomb is the amount of charge which, when placed at a distance of one metre from another charge of the same magnitude in vacuum, experiences a force of $9.0 \times 10^9$ N**. This force is a tremendously large force realisable in practical situations. It is, therefore, necessary to express the charge in smaller units for practical purpose. Subunits of coulomb are used in electrostatics. For example, micro-coulomb ($10^{-6}$ C, μC), nano-coulomb ($10^{-9}$ C, nC) or pico-coulomb. ($10^{-12}$ C, pC) are normally used units.



**Do you know ?**

Force between two charges of 1.0 C each, separated by a distance of 1.0 m is $9.0\times10^9$ N or, about 10 million metric tonne. A normal truck-load is about 10 metric tonne. So, this force is equivalent to about one million truck-loads. A tremendously large force indeed !

**Example 10.2:** Charge on an electron is $1.6\times10^{-19}$ C. How many electrons are required to accumulate a charge of one coulomb?

**Solution:** $1.6\times10^{-19}$ C = 1 electron

$$\therefore 1C = \frac{1}{1.6\times10^{-19}} \text{ electrons}$$

$$= 0.625\times10^{19} = 6.25\times10^{18} \text{ electrons}$$

$6.25\times10^{18}$ electrons are required to accumulate a charge of one coulomb.

**It is now possible to measure a very small amount of current in otto-amperes which measures flow of single electron.**

### 10.4.4 Coulomb's Law in Vector Form:

As shown in Fig 10.4, $q_1$ and $q_2$ are two similar point charges situated at points A and B. $r_{12}$ is the distance of separation between them. $\vec{F}_{21}$ denotes the force exerted on $q_2$ by $q_1$

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{\left|\hat{r}_{21}\right|^2} \times \hat{r}_{21} \qquad \text{--- (10.6)}$$

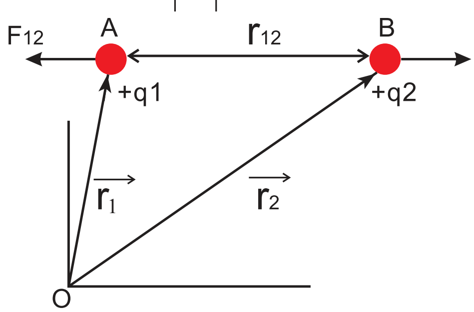



Fig. 10.4: Coulomb's law in vector form

$\hat{r}_{21}$ is the unit vector along $\overline{AB}$, away from B. Similarly, the force $\vec{F}_{12}$ exerted on $q_1$ by $q_2$ is given by

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{\left|r_{12}\right|^2} \times \hat{r}_{12} \qquad \text{--- (10.7)}$$

$\hat{r}_{12}$ is the unit vector along $\overline{BA}$, away from A. $\vec{F}_{12}$ acts on $q_1$ at A and is directed along BA, away from A. The unit vectors $\hat{r}_{12}$ and $\hat{r}_{21}$ are oppositely directed i.e., $\hat{r}_{12} = -\hat{r}_{21}$ hence,

$$\vec{F}_{21} = -\vec{F}_{12}$$

Thus, the two charges experience force of equal magnitude and opposite in direction. These two forces form an action- reaction pair. **As $\vec{F}_{21}$ and $\vec{F}_{12}$ act along the line joining the two charges, the electrostatic force is a central force.**

**Example 10.3:** Calculate and compare the electrostatic and gravitational forces between two protons which are $10^{-15}$ m apart. Value of G = $6.674\times10^{-11}$ $m^3$ $kg^{-1}$ $s^{-2}$ and mass of the proton is $1.67\times10^{-27}$ kg

**Solution:** The electrostatic force between the protons is given by $F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$

Here, $q_1 = q_2 = +1.6\times10^{-19}$ C, $r = 10^{-15}$m

$$\therefore F_e = 9\times10^9 \frac{(1.6\times10^{-19})(1.6\times10^{-19})}{(10^{-15})^2}$$

$$= 9\times1.6\times1.6\times10^1 \text{ N}$$

$$F_e = 2.3\times10^2 \text{ N} \qquad \text{--- (10.8.a)}$$

The gravitational force between the protons is given by

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$= \frac{6.674\times10^{-11}\times1.67\times10^{-27}\times1.67\times10^{-27}}{(10^{-15})^2}$$

$$F_g = 1.86\times10^{-34} \text{ N} \qquad \text{--- (10.8.b)}$$

Comparing 10.8. (a) and 10.8.(b)

$$\frac{F_e}{F_g} = \frac{2.30\times10^{-2}\text{ N}}{1.86\times10^{-34}\text{ N}} = 1.23\times10^{36}$$

Thus, the electrostatic force is about 36 orders of magnitude stronger than the gravitational force.

**Comparison of gravitational and electrostatic forces:**

**Similarities**

1. Both forces obey inverse square law : $F \propto \frac{1}{r^2}$
2. Both are central forces : act along the line joining the two objects.

**Differences**

1. Gravitational force between two objects is always attractive while electrostatic force between two charges can be either attractive or repulsive depending on the nature of charges.
2. Gravitational force is about 36 orders of magnitude weaker than the electrostatic force.

### 10.5 Principle of Superposition:

The principle of superposition states that *when a number of charges are interacting, the resultant force on a particular charge is given by the vector sum of the forces exerted by individual charges.*

Consider a number of point charges $q_1$, $q_2$, $q_3$ ------- kept at points $A_1$, $A_2$, $A_3$--- as shown in Fig. 10.5. The force exerted on the charge $q_1$ by $q_2$ is $\vec{F}_{12}$. The value of $\vec{F}_{12}$ is calculated by ignoring the presence of other charges. Similarly, we find $\vec{F}_{13}$ , $\vec{F}_{14}$ etc, one at a time, using the coulomb's law.

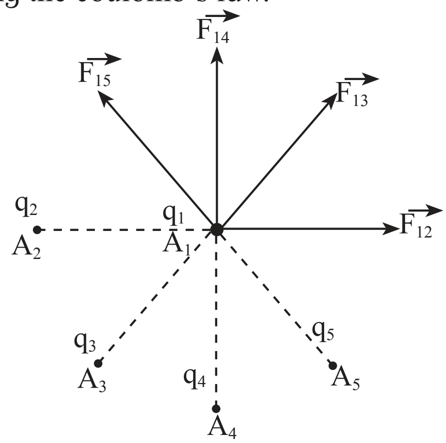



Fig. 10.5: Principle of superposition.

Total force $\vec{F}_1$ on charge $q_1$ is the vector sum of all such forces.

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + ...$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{\left|\vec{r}_{12}\right|^2}\hat{r}_{12} + \frac{q_1 q_3}{\left|\vec{r}_{13}\right|^2}\hat{r}_{13} + ...\right],$$

where $\hat{r}_{12}, \hat{r}_{13}$ etc., are unit vectors directed to $q_1$ from $q_2$, $q_3$ etc., and $r_{12}$, $r_{13}$, $r_{14}$,etc., are the distances from $q_1$ to $q_2$, $q_3$ etc respectively.

Let there be N point charges $q_1$, $q_2$,$q_3$ etc., $q_N$. The force $\vec{F}$ exerted by these charges on a test charge $q_0$ can be written using the summation notation Σ as follows,

$$\vec{F}_{test} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ---- + \vec{F}_N \quad \text{--- (10.9)}$$

$$= \sum_{n=1}^{N} F_n = \frac{1}{4\pi\varepsilon_0} \sum_{n=1}^{N} \frac{q_0 q_n}{r_{0n}^2}\hat{r}_{0n} \quad \text{--- (10.10)}$$

Where $\hat{r}_{0n}$ is a unit vector directed from the $n^{th}$ charge to the test charge $q_o$ and $r_{0n}$ is the separation between them, $\vec{r}_{0n} = r_{0n}\, r_{0n}$

**Can you tell?**

Three charges, *q* each, are placed at the vertices of an equilateral triangle. What will be the resultant force on charge *q* placed at the centroid of the triangle?

**Example 10.4:** Three charges of 2μC, 3μC and 4μC are placed at points A, B and C respectively, as shown in Fig. a. Determine the force on A due to other charges.

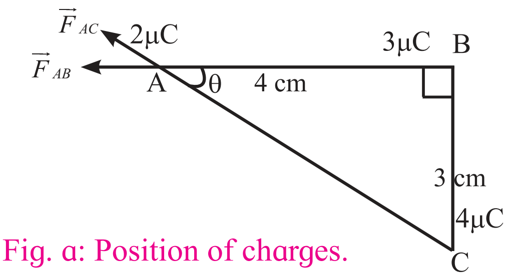



Fig. a: Position of charges.

**Solution :** Given,

AB = 4.0 cm, BC = 3.0 cm

$\therefore$ AC = $\sqrt{4^2+3^2}$ = 5.0 cm

Magnitude of force $\vec{F}_{AB}$ on A due to B is,

$$\vec{F}_{AB} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2\times10^{-6}\times3\times10^{-6}}{(4\times10^{-2})^2}$$

$$= \frac{9\times10^9\times6}{16\times10^{-4}}\times10^{-12}$$

$$= 3.37\times10$$

$$= 33.7 \text{ N}$$

This force acts at point A and is directed along $\overrightarrow{BA}$ (Fig. (b)).

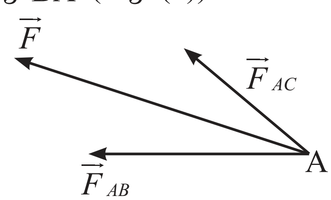



Fig. b: Forces acting at point A.

Magnitude of force $\vec{F}_{AC}$ on A due to C is,

$$\vec{F}_{AC} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2\times10^{-6}\times4\times10^{-6}}{(5\times10^{-2})^2}$$

$$= \frac{9\times10^9\times8.0\times10^{-12}}{25\times10^{-4}}$$

$$= \frac{72}{25}\times10 = 28.8 \text{ N}$$

This force acts at point A and is directed along $\overrightarrow{CA}$ . (Fig. 10.6.(b))

$$\vec{F} = \vec{F}_{AB} + \vec{F}_{AC}$$

Magnitude of resultant force is,

$$F = \left[F_{AC}^2 + F_{AB}^2 + 2F_{AC}\cdot F_{AB}\cdot\cos\theta\right]^{1/2}$$

$$= 59.3 \text{ N}$$

Using Eq. (2.10)

calculaling tan α, α =16.93$^0$

Direction of the resultant force is 16.9° north of west. (Fig. c)

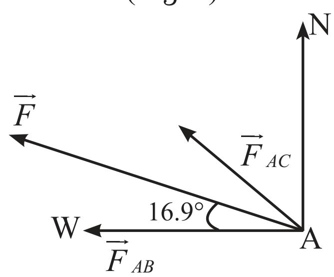



Fig. c: Direction of the resultant force.

### 10.6 Electric Field:

Space around a charge $Q$ gets modified so that when a test charge is brought in this region, it experiences a coulomb force. *This region around a charged object in which coulomb force is experienced by another charge is called electric field.*

Mathematically, electric field is defined as the force experienced per unit charge. Let $Q$ and $q$ be two charges separated by a distance r.

The coulomb force between them is given by $\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\hat{r}$, where, $\hat{r}$ is the unit vector along the line joining $Q$ to $q$.

Therefore, electric field due to charge $Q$ is given by,

$$\vec{E} = \frac{\vec{F}}{q} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} \qquad \text{--- (10.11)}$$

The coulomb force acts across an empty space (vacuum) and does not need any intervening medium for its transmission.

**The electric field exists around a charge irrespective of the presence of other charges.**

Since the coulomb force is a vector, the electric field of a charge is also a vector and is directed along the direction of the coulomb force, experienced by a test charge.

A precise definition of electric field is: Electric field is the force experienced by a test charge in presence of the given charge at the given distance from it.

$$E = \lim_{q\to 0}\frac{\vec{F}}{q}$$

Test charge is a positive charge so small in magnitude that it dose not affect the surroundings of the given charge.

The magnitude of electric field at a distance r from a point charge $Q$ is same at all points on the surface of a sphere of radius r as shown in Fig. 10.6. Its direction is along the radius of the sphere, pointing away from its centre if the charge is positive.

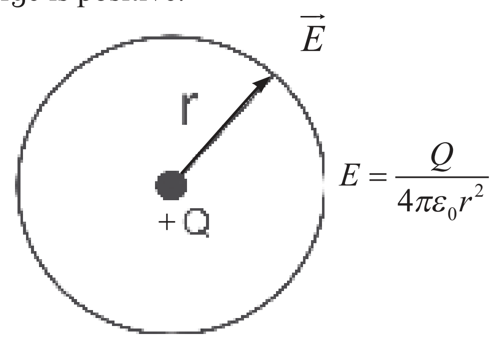



Fig. 10.6: Electric field due to a point charge (+Q).

SI unit of electric intensity is newton per coulomb ($NC^{-1}$). Practically, electric field is expressed in volt per metre($Vm^{-1}$). This is discussed in article 10.6.2.

Dimensional formula of E is,

$$E = \frac{F}{q_0} = \frac{[LMT^{-2}]}{[IT]}$$

$$E = [LMT^{-3}I^{-1}]$$

#### 10.6.1 Electric Field Intensity due to a Point Charge in a Material Medium:

Consider a point charge $q$ placed at point O in a medium of dielectric constant $K$ as shown in Fig. 10. 7.

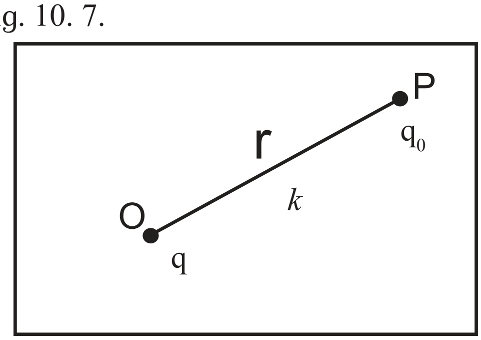



Fig. 10.7: Field in a material medium.

Consider the point P in the electric field of point charge $q$ at distance r from it. A test charge $q_0$ placed at the point P will experience a force which is given by the Coulomb's law,

$$\vec{F} = \frac{1}{4\pi\varepsilon_0 K}\frac{q\,q_0}{r^2}\hat{r}$$

where $\hat{r}$ is the unit vector in the direction of force i.e., along OP.

By the definition of electric field intensity

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{r^2}\hat{r}$$

The direction of $\vec{E}$ will be along OP when q is positive and along PO when q is negative.

The magnitude of electric field intensity in a medium is given by

$$E = \frac{1}{4\pi\varepsilon_0 K}\frac{q}{r^2} \qquad \text{--- (10.12 )}$$

For air or vacuum $K = 1$ then

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

The coulomb force between two charges and electric field $E$ of a charge both follow the inverse square law, ($F\propto 1/r^2$, $E\propto 1/r^2$) Fig. 10.8.

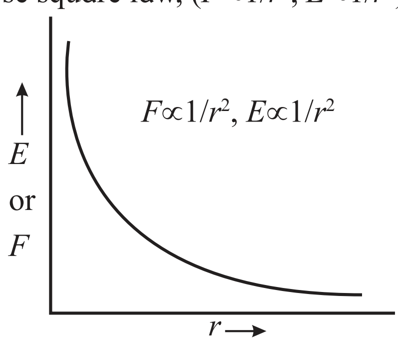



Fig. 10.8: Variation of Coulomb force/ Electric field due to a point charge.

1. **Uniform electric field:** A uniform electric field is a field whose magnitude and direction is same at all points. For example, field between two parallel plates. Fig 10.9.a
2. **Non uniform electric field:** A field whose magnitude and direction is not the same at all points. For example, field due to a point charge. In this case, the magnitude of field is same at distance r from the point charge in any direction but the direction of the field is not same. Fig 10.9.b

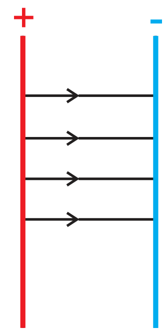



Fig. 10.9 (a): uniform electric field.

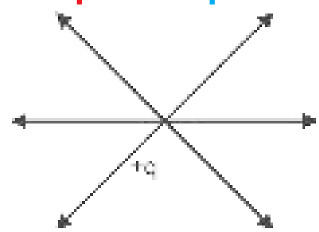



Fig. 10.9 (b): non uniform electric field.

### 10.6.2 Practical Way of Calculating Electric Field

A pair of charged parallel plates is arranged as shown in Fig. 10.10. The electric field between them is uniform. A potential difference V is applied between two parallel plates separated by a distance '$d$'. The electric field between them is directed from plate A to plate B as shown.

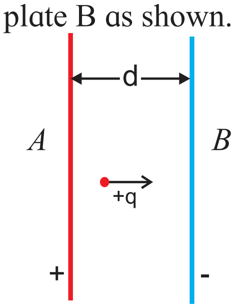



Fig 10.10: Electric field between two parallel plates.

A charge $+q$ placed between the plates experiences a force F due to the electric field. If we have to move the charge against the direction of field, i.e., towards the positive plate, we have to do some work on it. If we move the charge $+q$ from the negative plate B to the positive plate A, the work done against the field is $W = Fd$; where '$d$' is the separation between the plates. The potential difference $V$ between the two plates is given by

$W = Vq$, but $W = Fd$

$\therefore Vq = Fd \;\therefore\; F/q = V/d = E$

$\therefore$ Electric field can be defined as

$$E = V/d \qquad \text{--- (10.13)}$$

**This is the commonly used definition of electric field.**

**Example 10.5:** Gap between two electrodes of the spark-plug used in an automobile engine is 1.25 mm. If the potential of 20 V is applied across the gap, what will be the magnitude of electric field between the electrodes?

**Solution:**

$$E = \frac{V}{d}$$

$$E = \frac{20V}{1.25\times10^{-3}m} = 16\times10^{-3} = 1.6\times10^{4}\,\frac{V}{m}$$

This electric field is sufficient to ionize the gaseous mixture of fuel compressed in the cylinder and ignite it.

**Can you tell?**

Why a small voltage can produce a reasonably large electric field?

**Example 10.6:** Three point charges are placed at the vertices of a right isosceles triangle as shown in the Fig. a. What is the magnitude and direction of the resultant electric field at point P which is the mid point of its hypotenuse?

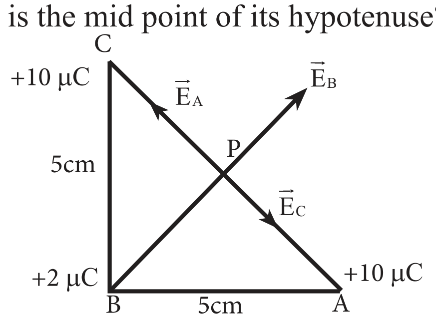



Fig (a): Position of charges.

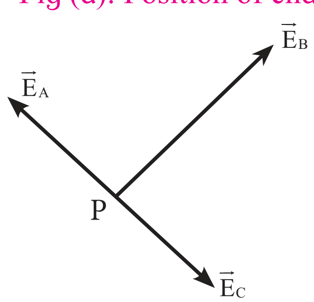



Fig (b): Electric field at point P.

**Solution:** Electric field is the force an a unit positive charge, the fields at P due to the charges at A, B and C are shown in the Fig. b. $\vec{E}_A$ is the field at P due to charge at A and $\vec{E}_C$ is the field at P due to charge at C. Since P is the midpoint of AC and the fields at A and C are equal in magnitudes and are opposite in direction, $\vec{E}_A = -\vec{E}_C$ . $\vec{E}_A + \vec{E}_C = 0$ . Thus, the field at P is only to the charge at B and can be written as

$$\vec{E}_P = \vec{E}_B = \frac{2\times10^{-6}}{4\pi\varepsilon_0 (BP)^2}$$

$$\vec{E}_P = \frac{2\times10^{-6}\times9\times10^{9}}{(5/\sqrt{2})^2}$$

$$= \frac{2\times9\times10^{3}\times2}{25}$$

$$= \frac{36}{25}\times10^{3}$$

$$= 1.44\times10^{15}\ \text{NC}^{-1}\ \text{along}\ \overrightarrow{BP}$$

To calculate BP

$$|\overrightarrow{BP}| = (BA)\cos(45)^\circ = \frac{5}{\sqrt{2}}$$

**Example 10.7:** A simplified model of hydrogen atom consists of an electron revolving about a proton at a distance of $5.3\times10^{-11}$m. The charge on a proton is $+1.6\times10^{-19}$ C. Calculate the intensity of the electric field due to proton at this distance.

**Solution:**

$$E = \frac{q}{4\pi\varepsilon_o r^2}$$

$$q = +1.6\times10^{-19}\ C$$

$$r = 5.3\times10^{-11}\ m$$

$$\frac{1}{4\pi\varepsilon_o} = 9.0\times10^{9}\ Nm^2C^{-1}$$

$$E = \frac{1.6\times10^{-19}}{(5.3\times10^{-11})^2}\times9.0\times10^{9} = 5.1\times10^{11}\ NC^{-1}$$

The force between electron and proton in hydrogen atom can be calculated by using the electric field. We have, $E = \frac{F}{q}\ \therefore F = qE$

$F = -1.6\times10^{-19}\ \text{C}\times 5.1\times10^{11}\ \text{NC}^{-1}$

$= -8.16\times10^{8}$ N.

This force is attractive.

Using the Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

$$= 9.0\times10^{9}\ Nm^2C^{-1}\times\frac{(-1.6\times10^{-19}C)\times(+1.6\times10^{-19}C)}{(5.3\times10^{-11}m)^2}$$

$$= -8.6\times10^{8}\ N$$

**Knowing electric field at a point is useful to estimate the force experienced by a charge at that point.**

### 10.6.3 Electric Lines of Force:

Michael Faraday (1791-1867) introduced the concept of lines of force for visualising electric and magnetic fields. *An electric line of force is an imaginary curve drawn in such a way that the tangent at any given point on this curve gives the direction of the electric field at that point.* See Fig.10.11. If a test charge is placed in an electric field it would be acted upon by a force at every point in the field and will move along a path. **The path along which the unit positive charge moves is called a line of force.**

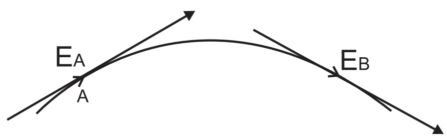



Fig. 10.11: Electric line of force.

A line of force is defined as a curve such that the tangent at any point to this curve gives the direction of the electric field at that point.

The density of field lines indicates the strength of electric fields at the given point in space. Figure 10.12.

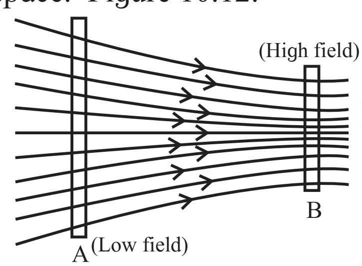



Fig. 10.12: density of field lines and strength of electric field.

**Characteristics of electric lines of force**

(1) The lines of force originate from a positively charged object and terminate on a negatively charged object.

(2) The lines of force neither intersect nor meet each other, as it will mean that electric field has two directions at a single point.

(3) The lines of force leave or terminate on a conductor normally.

(4) The lines of force do not pass through conductor i.e. electric field inside a conductor is always zero, but they pass through insulators.

(5) Magnitude of the electric field intensity is proportional to the number of lines of force per unit area of the surface held perpendicular to the field.

(6) Electric lines of force are crowded in a region where electric intensity is large.

(7) Electric lines of force are widely separated from each other in a region where electric intensity is small

(8) The lines of force of an uniform electric field are parallel to each other and are equally spaced.

The lines of force are purely a geometric construction which help us visualise the nature of electric field in a region. **The lines of force have no physical existence.**

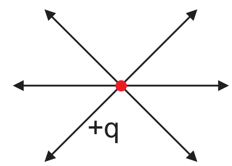



Fig. 10.13 (a): Lines of force due to positive charge.

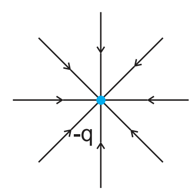



Fig. 10.13 (b): Lines of force due to negative charge.

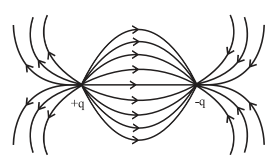



Fig. 10.13 (c): Lines of force due to opposite charge.

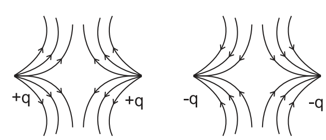



Fig. 10.13; (d): Lines of force due to similar charge.

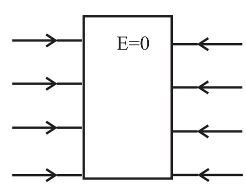



Fig. 10.13 (e): Lines of force terminate on a conductor.

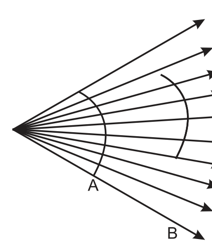



Fig. 10.13 (f): Intensity of a electric field is more at point A and less at B. More lines cross the area at A and less at the same area at B.

Fig. 10.13: The lines of force due to various geometrical arrangement of electrical charges.

**Can you tell?**

Lines of force are imaginary, can they have any practical use?

## 10.7 Electric Flux:

As discussed previously, the number of lines of force per unit area is the intensity of the electric field $\vec{E}$.

$$\therefore E = \frac{\text{Number of lines of force}}{\text{Area enclosing the lines of force}} \quad \text{- (10.14)}$$

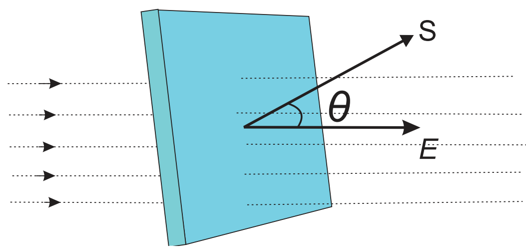



Fig. 10.14: Flux through area S.

Number of lines of force = $(E)\cdot$(Area) When the area is inclined at an angle θ with the direction of electric field, Fig. 10.14, the electric flux can be calculated as follows.

Let the angle between electric field $\vec{E}$ and area vector $d\vec{S}$ be θ, then the electric flux passing through area $dS$ is given by

$d\phi$ = (component of $dS$ along $\vec{E}$)·(area of $d\vec{S}$)

$$d\phi = E\,(dS\cos\theta)$$

$$d\phi = EdS\cos\theta$$

$$d\phi = \vec{E}\cdot d\vec{S} \quad \text{--- (10.15)}$$

Total flux through the entire surface

$$\Phi = \int d\phi = \int_s \vec{E}.d\vec{S} = \vec{E}.\vec{S} \quad \text{--- (10.16)}$$

The SI unit of electric flux can be calculated using,

$\Phi = \vec{E}\cdot\vec{S}$ =(V/m) m$^2$ =Vm

## 10.8 Gauss' Law:

Karl Friedrich Gauss (1777-1855) one of the greatest mathematician of all times, formulated a law expressing the relationship between the electric charge and its electric field which is called the Gauss' law. Gauss' law is analongous to Coulomb's law in the sense that it too expresses the relationship between electric field and electric charge. Gauss' law provides equivalent method for finding electric intensity. It relates values of field at a closed surface and the total charges enclosed by that surface.

Consider a closed surface of any shape which encloses number of positive electric charges (Fig. 10.15). To prove Gauss' theorem, imagine a small charge +q present at a point O inside closed surface. Imagine an infinitesimal area dA of the given irregular closed surface.

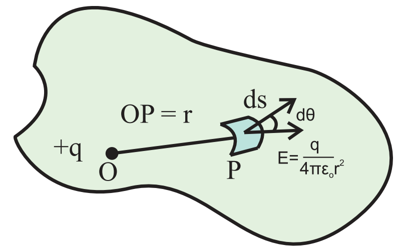



Fig. 10.15: Gauss' law.

The magnitude of electric field intensity at point P on dS due to charge +q at point O is,

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r^2}\right)$$

The direction of E is away from point O. Let θ be the angle subtended by normal drawn to area $dS$ and the direction of E. Electric flux, $d\phi$, passing through area $dS$, $= E\cos\theta\, dS$

$$= \frac{q}{4\pi\varepsilon_0 r^2}\cos\theta\, dS$$

$$= \left(\frac{q}{4\pi\varepsilon_0}\right)\left(\frac{dS\cos\theta}{\text{r}^2}\right)$$

$$d\phi = \left(\frac{q}{4\pi\varepsilon_0}\right)d\omega \quad \text{--- (10.17)}$$

where, $d\omega = \dfrac{dS\cos\theta}{r^2}$ is the solid angle subtended by area dS at point O.

Total electric flux, $\phi_E$, crossing the given closed surface can be obtained by integrating Eq. (10.17) over its area. Thus,

$$\Phi_E = \int d\phi = \int_s \vec{E}.\vec{ds} = \int \frac{q}{4\pi\varepsilon_0}\,d\omega = \frac{q}{4\pi\varepsilon_0}\int d\omega$$

But $\int d\omega = 4\pi =$ solid angle subtended by entire closed surface at point O

$$\text{Total flux} = \frac{q}{4\pi\varepsilon_0}(4\pi)$$

$$\Phi_E = \int_s \vec{E}\cdot\vec{ds} = +q/\varepsilon_0 \quad \text{--- (10.18)}$$

This is true for every electric charge enclosed by a given closed surface.

Total flux due to charge $q_1$, over the given closed surface = + $q_1/\varepsilon_0$

Total flux due to charge $q_2$, over the given closed surface = + $q_2/\varepsilon_0$

Total flux due to charge $q_n$, over the given closed surface = + $q_n/\varepsilon_0$

Positive sign in Eq. (10.18) indicates that the flux is directed outwards, away form the charge. If the charge is negative, the flux will be is directed inwards as shown in Fig 10.16 (b). If a charge is outside the closed surface the net flux through it will be zero Fig 10.16 (c).

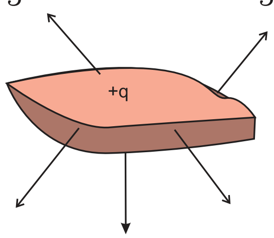



Fig. 10.16 (a): Flux due to positive charge.

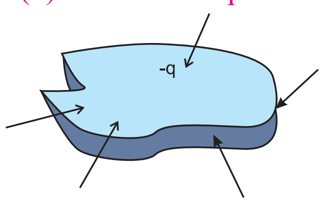



Fig. 10.16 (b): Flux due to negative charge.

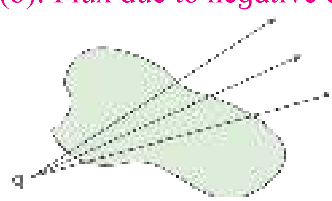



Fig. 10.16 (c): Flux due to charge outside a closed surface is zero.

According to the superposition principle, the total flux $\phi$ due to all charges enclosed within the given closed surface is

$$\Phi_E = \frac{q_1}{\varepsilon_0} + \frac{q_2}{\varepsilon_0} + \frac{q_3}{\varepsilon_0} + \text{----} + \frac{q_n}{\varepsilon_0} = \sum_{i=1}^{i=n} \frac{q_i}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

**Statement of Gauss' law**

The flux of the net electric field through a closed surface equals the net charge enclosed by the surface divided by $\varepsilon_0$

$$\int \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0}$$

where Q is the total charge within the surface.

Gauss' law is applicable to any closed surface of regular or irregular shape.

**Example 10.8:** A charge of 5.0 C is kept at the centre of a sphere of radius 1 m. What is the flux passing through the sphere? How will this value change if the radius of the sphere is doubled?

**Solution:** Flux per unit area is given by Eq. 10.16.

According to Gauss law, the total flux through the sphere $\Phi = \int \vec{E}.\vec{ds}$, where the integration is over the surface of the sphere. As the electric field is same all over the sphere i.e. $|\bar{E}|$ = constant and the direction of $\bar{E}$ as well as that of $d\bar{s}$ is along the radius, we get

$$\text{flux} = \Phi = |E|\, 4\pi R^2$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2} = 9\times10^9 \times \frac{5.0\,\text{C}}{(1.0\,\text{m})^2}$$

$$E = 9\times10^9 \times 5 = 4.5\times10^{10}\,\text{NC}^{-1}$$

$$\phi = \vec{E}\cdot\vec{S}$$

$$\therefore \text{flux} = 4.5\times10^{10} \times 4\pi(1)^2$$

$$= 5.65\times10^{11}\,\text{Vm}$$

Thus the total flux is independent of radius.

$E \propto 1/r^2$, and area $\propto r^2$. This can also be seen from Gauss' law, where the net flux crossing a closed surface is equal to $q/\varepsilon_0$ where q is the net charge inside the closed surface. As the charge inside the sphere is unchanged, the flux passing through a sphere of any radius is the same. Thus, if the radius of the sphere is increased by a factor of 2, the net flux passing through its surface remains unchanged. As shown in Fig. 10.17, same number of lines of force cross both the surfaces. The total flux is independent of shape of the closed surface because Eq. 10.18 does not involve any radius.

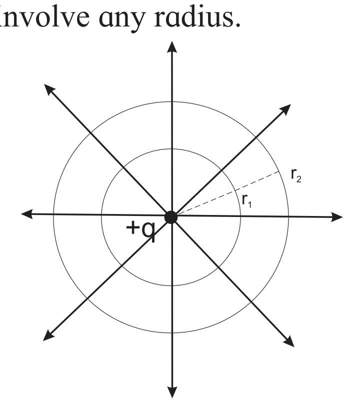



Fig. 10.17: Flux is independent of the shape and size of closed surface.

**Do you know ?**

**Gaussian surface**

All the lines of force originating from a point charge penetrate an imaginary three dimensional surface. The total flux $\Phi_E = q/\varepsilon_0$. The same number of lines of force will cross the surface of any shape. The total flux through both the surfaces is the same. Calculating flux involves calculating $\int \vec{E} \cdot \vec{ds}$, hence it is convenient to consider a regular surface surrounding the given charge distribution. A surface enclosing the given charge distribution and symmetric about it is a Gaussian surface.

For example. if we have a point charge the Gaussian surface will be a sphere. If the charge distribution is linear, the Gaussian surface would be a cylinder with the charges distributed along its axis. Gaussian surface offers convenience of calculating the integral $\int \vec{E} \cdot \vec{ds}$.

Remember that a Gaussian surface is purely imaginary and does not exist physically.

## 10.9 Electric Dipole:

A pair of equal and opposite charges separated by a finite distance is called an electric dipole. It is shown in Fig. (10.18).

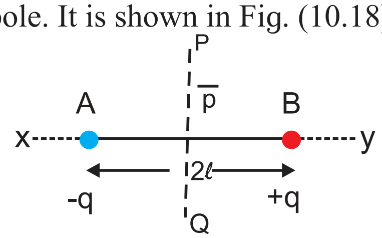



Fig. 10.18: Electric dipole. x-y axial line, P-Q equatorial line.

Line joining the two charges is called the dipole axis. A line passing through the dipole axis is called **axial line**. A line passing through the centre of the dipole and perpendicular to the axial line is called the **equatorial line** as shown in Fig. 10.18.

Strength of a dipole is measured in terms of a quantity called the **dipole moment**. Let q be the magnitude of each charge and $2\vec{l}$ be the distance from negative charge to positive charge. Then the product q $(2\vec{l}\,)$ is called the dipole moment $\vec{p}$.

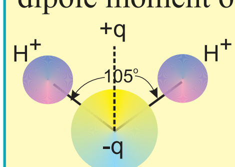
Dipole moment is defined as

$$\vec{p} = q(2\vec{l}\,) \qquad \text{--- (10.19)}$$

A dipole moment is a vector whose magnitude is q (2$l$) and the direction is from the negative to the positive charge. The unit of dipole moment is Columb-meter (Cm) or Debye (D). 1D = $3.33\times10^{-30}$ Cm. If two charges +e and -e are separated by 1.0$A^0$, the dipole moment is $1.6\times10^{-29}$ Cm or 4.8 D. For example, a water molecule has a permanent dipole moment of

**Natural dipole:**

The water molecule is non-linear, i.e., the two hydrogen atoms and one oxygen atom are not in a straight line. The two hydrogen-oxygen bonds in water molecule are at an angle of 105°. The positive charge of a water molecule is effectively concentrated on the hydrogen side and the negative charge on the oxygen side of the molecule. Thus, the positive and negative charges of the water molecule are inherently separated by a small distance. This separation of positive and negative charges gives rise to the permanent dipole moment of a water molecule.




Molecules of water, ammonia, sulphur di-oxide, sodium chloride etc. have an inherent separation of centers of positive and negative charges. Such molecules are called polar molecules.

**Polar molecules are the molecules in which the center of positive charge and the negative charge is naturally separated.**

Molecules such as $H_2$, $Cl_2$, $CO_2$ $CH_4$ and many others have their positive and negative charges effectively centered at the same point and are called non-polar molecules.

**Non-polar molecules are the molecules in which the center of positive charge and the negative charge is one and the same.** They do not have a permanent electric dipole. When an external electric field is applied to such molecules the centers of positive and negative charge are displaced and a dipole is induced.

$6.172\times10^{-30}$ Cm or 1.85 D. Its direction is from oxygen to hydrogen. See box on Natural dipole.

### 10.9.1 Couple Acting on an Electric Dipole in a Uniform Electric Field:

Consider an electric dipole placed in a uniform electric field E. The axis of electric dipole makes an angle θ with the direction of electric field as shown in Fig. 10.19 a.

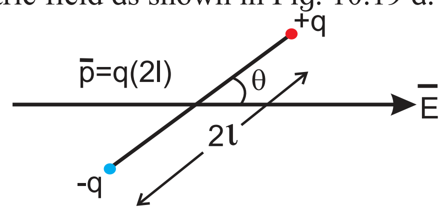



Fig. 10.19 (a): Dipole in uniform electric field.

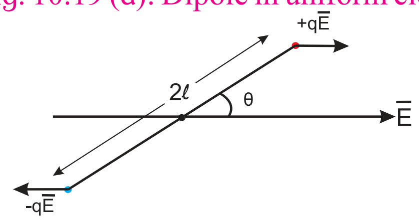



Fig. 10.19 (b): Couple acting on a dipole.

Figure 10.19. b shows the couple acting on an electric dipole in uniform electric field.

The force acting on charge - q at A is

$\vec{F}_A = -q\vec{E}$ in the direction opposite to that of $\vec{E}$ and the force acting on charge +q at B is $\vec{F}_B = +q\vec{E}$ in the direction of $\vec{E}$. Since $\vec{F}_A = -\vec{F}_B$, the two equal and opposite forces separated by a distance form a couple. Moment of the couple is called torque and is defined by $\vec{\tau} = \vec{d}\times\vec{F}$ where, d is the perpendicular distance between the two equal and opposite forces.

∴Magnitude of Torque =

Magnitude of force × Perpendicular distance

∴ Torque on the dipole = $\vec{\tau} = \overrightarrow{BP}\times q\vec{E}$

$\therefore \tau = qE2l\sin\theta$ --- (10.20)

but $\vec{p} = q\times 2\vec{l}$

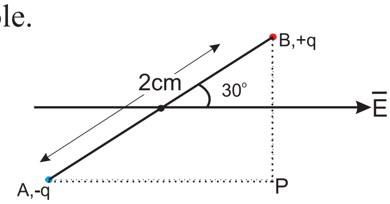
$\therefore \tau = pE\sin\theta$ --- (10.21)

In vector form $\vec{\tau} = \vec{p}\times\vec{E}$ --- (10.22)

If θ =90° sin θ =1, then $\tau = pE$

When the axis of electric dipole is perpendicular to uniform electric field, torque of the couple acting on the electric dipole is maximum, i.e., $\tau = pE$. It θ = 0 then τ = 0, this is the minimum torque on the dipole. Torque tends to align the dipole axis along the direction of electric field.

**Example 10.9:** An electric dipole of length 2.0 cm is placed with its axis making an angle of 30° with a uniform electric field of $10^5$ N/C. as shown in figure. If it experiences a torque of $10\sqrt{3}$ Nm, calculate the magnitude of charge on the dipole.




**Solution:** Given

$$\tau = 10\sqrt{3}, Nm,\ \ E = 10^5\ N/C,$$

$$2l = 2.0\times10^{-2}\,m,\ \ \theta = 30^0$$

$$\tau = qE2l\sin\theta$$

$$10\sqrt{3}Nm = q10^5\ N/C2.0\times10^{-2}\,m\left(\frac{1}{2}\right)$$

$$\therefore q = \frac{10\sqrt{3}}{10^3} = \sqrt{3}\times10^{-2}C$$

$$= 1.73\times10^{-2}C$$

### 10.9.2 Electric Intensity at a Point due to an Electric Dipole:

**Case 1 :** At a point on the axis of a dipole.

Consider an electric dipole consisting of two charges -q and +q separated by a distance $2l$ as shown in Fig. 10.20. Let P be a point at a distance r from the centre C of the dipole. The electric intensity $\vec{E}_a$ at P due to the dipole is the vector sum of the field due to the charge - q at A and + q at B.

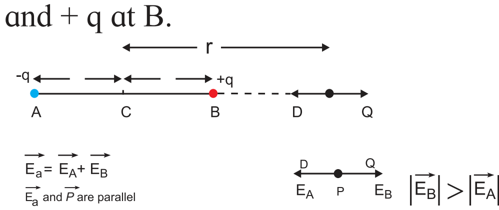



Fig. 10.20: Electric field of a dipole along its axis.

Electric field intensity at P due to the charge -q at A

$$= \vec{E}_A = \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{(r+l)^2}\ \widehat{PD}$$

where $\widehat{PD}$ is unit vector directed along $\overrightarrow{PD}$

Electric intensity at P due to charge +q at B

$$= \vec{E}_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-l)^2}\ \widehat{PQ}$$

where $\widehat{PQ}$ is a unit vector directed along $\overrightarrow{PQ}$.

The magnitude of $\vec{E}_B$ is greater than that of $\vec{E}_A$. (Because BP < AB)

Resultant field $\vec{E}_a$ at P on the axis, due to the dipole is

$$\vec{E}_a = \vec{E}_B + \vec{E}_A$$

The magnitude of $\vec{E}_a$ is given by

$$|\vec{E}_a| = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(r-l)^2} - \frac{q}{(r+l)^2}\right]$$

$$|\vec{E}_a| = \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2 + l^2 + 2lr - r^2 + 2lr - l^2}{(r^2 - l^2)^2}\right]$$

$$|\vec{E}_a| = \frac{2(2lq)r}{4\pi\varepsilon_0(r^2 - l^2)^2}$$

But $2lq$ = p, the dipole moment

$$|\vec{E}_a| = \frac{1}{4\pi\varepsilon_0}\frac{2pr}{(r^2 - l^2)^2} \quad \text{--- (10.23)}$$

$\vec{E}_a$, is directed along PQ, which is the direction of the dipole moment $\vec{p}$ i.e. from the negative to the positive charge, parallel to the axis. If r>> $l$, $l^2$ can be neglected compared to $r^2$,

$$\left|\vec{E}_a\right| = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} \quad \text{--- (10.24)}$$

The field will be along the direction of the dipole moment $\vec{p}$.

**Case 2:** At a point on the equatorial line. As shown in Fig. 10.21 (a)

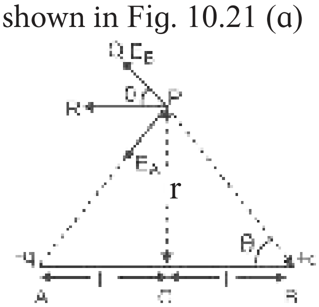


Fig. 10.21 (a): Electric field of a dipole at a point on the equatorial line.

Electric field at point P due to charge -q at A is: $\vec{E}_A = \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{(AP)^2}\widehat{PA}$

where $\widehat{PA}$ is the unit vector direction along $\overrightarrow{PA}$.

Similarly, Electric field at P due to charge + q at B is: $\vec{E}_B = \frac{1}{4\pi\varepsilon_0}\frac{(+q)}{(BP)^2}\widehat{PQ}$

where $\widehat{PQ}$ is the unit vector directed along $\overrightarrow{PQ}$ or $\overrightarrow{BP}$

Electric field at $\vec{P}$ is the, sum of $\vec{E}_A$ and $\vec{E}_B$

$$\therefore \vec{E}_{eq} = \vec{E}_A + \vec{E}_B$$

Consider $\Delta$ ACP

$(AP)^2 = (PC)^2 + (AC)^2 = r^2 + l^2 = (BP)^2$

$$\therefore \left|\vec{E}_A\right| = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2 + l^2)} \quad \text{--- (10.25)}$$

$$\left|\vec{E}_B\right| = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2 + l^2)} \quad \text{--- (10.26)}$$

$$\left|\vec{E}_A\right| = \left|\vec{E}_B\right|$$

The resultant of fields $\vec{E}_A$ and $\vec{E}_B$ acting at point P can be calculated by resolving these vectors $\vec{E}_A$ and $\vec{E}_B$ along the equatorial line and along a direction perpendicular to it.

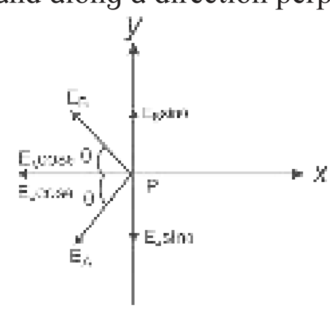


Fig. 10.21 (b): Components of the field at point P.

Consider Fig. 10.21 (b). Let the *y*-axis coincide with the equator of the dipole *x*-axis will be parallel to dipole axis, as shown. The origin is at point P.

The *y*-components of $E_A$ and $E_B$ are $E_A\sin\theta$ and $E_B\sin\theta$ respectively. They are equal in magnitude but opposite in direction and cancel each other. There is no contribution from them towards the resultant.

The *x*-components of $E_A$ and $E_B$ are $E_A\cos\theta$ and $E_B\cos\theta$ respectively. They are of equal magnitude and are in the same direction

$$\therefore |\vec{E}_{eq}| = E_A\cos\theta + E_B\cos\theta \quad \text{--- (10.27)}$$

By using Eq. 10.25 and 10.26

$$\left|\vec{E}_{eq}\right| = 2E_A\cos\theta$$

$$= 2\left(\frac{q}{4\pi\varepsilon_0(r^2 + l^2)}\right)\frac{l}{\sqrt{r^2 + l^2}}$$

$$= \frac{2ql}{4\pi\varepsilon_0(r^2 + l^2)^{3/2}}$$

If $r>>l$ then $l^2$ is very small compared to $r^2$

$$\left|\vec{E}_{eq}\right| = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} \qquad \text{--- (10.28)}$$

The direction of this field is along $-\vec{p}$ (anti-parallel to $\vec{p}$) as shown in Fig. 10.21 (c).

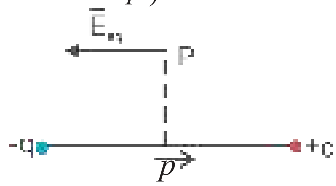



Fig. 10.21 (c): Electric field at point P is anti-parallel to $\vec{p}$.

Comparing Eq. 10.28 and 10.24 we find that the electric intensity at an axial point is twice that at a point on the equatorial position, lying at the same distance from the centre of the dipole.

## 10.10 Continuous Charge Distribution:

A system of charges can be considered as a continuous charge distribution, if the charges are located very close together, compared to their distances from the point where the intensity of electric field is to be found out.

The charge distribution is continuous in the sense that, a system of closely spaced charges is equivalent to a total charge which is continuously distributed along a line or a surface or a volume. To find the electric field due to continuous charge distribution, we define following terms for different types of charge distribution.

**(a) Linear charge density ($\lambda$).**

As shown in Fig. 10.22 charge q is uniformly distributed along a liner conductor of length $l$. The linear charge density $\lambda$ is defined as,

$$\lambda = \frac{q}{l} \qquad \text{--- (10.29)}$$

SI unit of $\lambda$ is (C/m).

For example, charge distributed uniformly on a straight thin rod or a thin nylon thread. If the charge is not distributed uniformly over the length of thin conductor then charge dq on small element of length $dl$ can be written as $dq = \lambda dl$

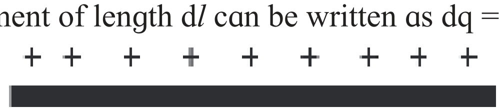


Fig. 10.22: Linear charge.

**(b) Surface charge density ($\sigma$)**

Suppose a charge $q$ is uniformly distributed over a surface of area $A$. As shown in Fig. 10.23, then the surface charge density $\sigma$ is defined as

$$\sigma = \frac{q}{A} \qquad \text{--- (10.30)}$$

SI unit of $\sigma$ is (C/m$^2$)

For example, charge distributed uniformly on a thin disc or a synthetic cloth. If the charge is not distributed uniformally over the surface of a conductor, then charge $dq$ on small area element $dA$ can be written as $dq = \sigma\, dA$.

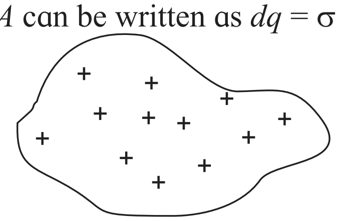


Fig. 10.23: Surface charge.

**(c) Volume charge density ($\rho$)**

Suppose a charge $q$ is uniformly distributed throughout a volume $V$, then the volume charge density $\rho$ is defined as the charge per unit volume.

$$\rho = \frac{q}{V} \qquad \text{--- (10.31)}$$

S.I. unit of $\rho$ is (C/m$^3$)

For example, charge on a solid plastic sphere or a solid plastic cube.

If the charge is not distributed uniformaly over the volume of a material, then charge $dq$ over small volume element $dV$ can be written as $dq = \rho\, dV$.

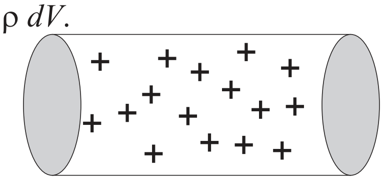


Fig. 10.24:Volume charge.

Electric field due to a continuous charge distribution can be calculated by adding electric fields due to all these small charges.

**Can you tell?**

The surface charge density of Earth is $\sigma$ = -1.33 nC/m$^2$. That is about $8.3\times10^9$ electrons per square meter. If that is the case why don't we feel it?

**Do you know ?**

**Static charge can be useful**

Static charges can be created whenever there is a friction between an insulator and other object. For example, when an insulator like rubber or ebonite is rubbed against a cloth, the friction between them causes electrons to be transferred from one to the other. This property of insulators is used in many applications such as Photocopier, Inkjet printer, Panting metal panels, Electrostatic precipitation/separators etc.

**Static charge can be harmful**

i. When charge transferred from one body to other is very large sparking can take place. For example lightning in sky.
ii. Sparking can be dangerous while refuelling your vehicle.
iii. One can get static shock if charge transferred is large.
iv. Dust or dirt particles gathered on computer or TV screens can catch static charges and can be troublesome.

**Precautions against static charge**

i. Home appliances should be grounded.
ii. Avoid using rubber soled footwear.
iii. Keep your surroundings humid. (dry air can retain static charges).

**Internet my friend**

1. https://www.physicsclassroom.com>class
2. https://courses.lumenlearing.com>elect
3. https://www.khanacademy.org>science.
4. https://www.topper.com>guides>physics

## Exercises

**1. Choose the correct option.**

i. A positively charged glass rod is brought close to a metallic rod isolated from ground. The charge on the side of the metallic rod away from the glass rod will be
(A) same as that on the glass rod and equal in quantity
(B) opposite to that on the glass of and equal in quantity
(C) same as that on the glass rod but lesser in quantity
(D) same as that on the glass rod but more in quantity

ii. An electron is placed between two parallel plates connected to a battery. If the battery is switched on, the electron will
(A) be attracted to the +ve plate
(B) be attracted to the -ve plate
(C) remain stationary
(D) will move parallel to the plates

iii. A charge of + 7 $\mu$C is placed at the centre of two concentric spheres with radius 2.0 cm and 4.0 cm respectively. The ratio of the flux through them will be
(A) 1:4 (B) 1:2
(C) 1:1 (D) 1:16

iv. Two charges of 1.0 C each are placed one meter apart in free space. The force between them will be
(A) 1.0 N (B) $9\times10^{9}$ N
(C) $9\times10^{-9}$ N (D) 10 N

v. Two point charges of +5 $\mu$C are so placed that they experience a force of $80\times10^{-3}$ N. They are then moved apart, so that the force is now $2.0\times10^{-3}$ N. The distance between them is now
(A) 1/4 the previous distance
(B) double the previous distance
(C) four times the previous distance
(D) half the previous distance

vi. A metallic sphere A isolated from ground is charged to +50 $\mu$C. This sphere is brought in contact with other isolated metallic sphere B of half the radius of sphere A. The charge on the two sphere will be now in the ratio
(A) 1:2 (B) 2:1
(C) 4:1 (D) 1:1

vii. Which of the following produces uniform electric field?

(A) point charge
(B) linear charge
(C) two parallel plates
(D) charge distributed an circular any

viii. Two point charges of A = +5.0 μC and B = -5.0 μC are separated by 5.0 cm. A point charge C = 1.0 μC is placed at 3.0 cm away from the centre on the perpendicular bisector of the line joining the two point charges. The charge at C will experience a force directed towards
(A) point A
(B) point B
(C) a direction parallel to line AB
(D) a direction along the perpendicular bisector

**2. Answer the following questions.**

i. What is the magnitude of charge on an electron?
ii. State the law of conservation of charge.
iii. Define a unit charge.
iv. Two parallel plates have a potential difference of 10V between them. If the plates are 0.5 mm apart, what will be the strength of electric charge.
v. What is uniform electric field?
vi. If two lines of force intersect of one point. What does it mean?
vii. State the units of linear charge density.
viii. What is the unit of dipole moment?
ix. What is relative permittivity?

**3. Solve numerical examples.**

i. Two small spheres 18 cm apart have equal negative charges and repel each other with the force of $6\times10^{-3}$ N. Find the total charge on both spheres.
[Ans: q = $2.94\times10^{-10}$ C]

ii. A charge +q exerts a force of magnitude -0.2 N on another charge -2q. If they are separated by 25.0 cm, determine the value of q.
[Ans: q = 0.8333 μC]

iii. Four charges of + $6\times10^{-8}$ C each are placed at the corners of a square whose sides a are 3 cm each. Calculate the resultant force on each charge and show its direction on a digram drawn to scale.
[Ans: $6.89\times10^{-2}$ N]

iv. The electric field in a region is given by $\vec{E}$ = 5.0 kN/C. Calculate the electric flux Through a square of side 10.0 cm in the following cases
(a) the square is along the XY plane
[Ans: = $5.0\times10^{-2}$ Vm]
(b) The square is along XZ plane
[Ans: Zero]
(c) The normal to the square makes an angle of $45^0$ with the Z axis.
[Ans: $3.5\times10^{-2}$ Vm]

v. Three equal charges of $10\times10^{-8}$ C respectively, each located at the corners of a right triangle whose sides are 15 cm, 20 cm and 25 cm respectively. Find the force exerted on the charge located at the 90° angle.
[Ans: $4.59.\times10^{-3}$ N]

vi. A potential difference of 5000 volt is applied between two parallel plates 5 cm a part a small oil drop having a charge of $9.6\times10^{-19}$ C falls between the plates. Find (a) electric field intensity between the plates and (b) the force on the oil drop.
[Ans: (a) $1.0.\times10^{5}$ N/C
(b) $9.6.\times10^{-14}$ N]

vii. Calculate the electric field due to a charge of $-8.0\times10^{-8}$ C at a distance of 5.0 cm from it.
[Ans: $2.88\times10^{-2}$ N/C]

***