14. Dual Nature of Radiation and Matter

Can you recall?

1. What is electromagnetic radiation?
2. What are the characteristics of a wave?
3. What do you mean by frequency and wave number associated with a wave?
4. What are the characteristic properties of particles of matter?
5. How do we define momentum of a particle?
6. What are the different types of energies that a particle of matter can possess?

14.1 Introduction:

In earlier chapters you have studied various optical phenomena like reflection, refraction, interference, diffraction and polarization of light. Light is electromagnetic radiation and most of the phenomena mentioned have been explained considering light as a wave. We are also familiar with the wave nature of electromagnetic radiation in other regions like X-rays, γ-rays, infrared and ultraviolet radiation and microwaves apart from the visible light. Electromagnetic radiation consists of mutually perpendicular oscillating electric and magnetic fields, both being perpendicular to the direction in which the wave and energy are travelling.

In Chapter 3 on Kinetic Theory of Gases and Radiation, you have come across spectrum of black body radiation which cannot be explained using the wave nature of radiation. Such phenomena appear during the interaction of radiation with matter and need quantum physics to explain them.

The idea of '*quantization of energy*' was first proposed by Planck to explain the black body spectrum. Planck proposed a model that says (i) energy is emitted in packets and (ii) at higher frequencies, the energy of a packet is large. Planck assumed that atoms behave like tiny oscillators that emit electromagnetic radiation only in discrete packets ($E = nh\nu$), where ν is the frequency of oscillator. The emissions occur only when the oscillator makes a jump from one quantized level of energy to another of lower energy. This model of Planck turned out to be the basis for Einstein's theory to explain the observations of experiments on photoelectric effect which we will study in the following section.

14.2 The Photoelectric Effect:

Heinrich Hertz discovered photoelectric emission in 1887 while he was working on the production of electromagnetic waves by spark discharge. He noticed that when ultraviolet light is incident on a metal electrode, a high voltage spark passes across the electrodes. Actually electrons were emitted from the metal surface. The surface which emits electrons, when illuminated with appropriate radiation, is known as a photosensitive surface.

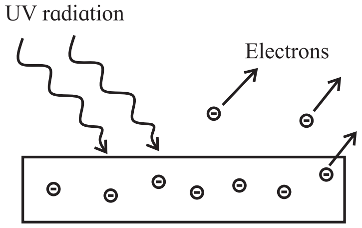


Fig. 14.1: Process of photoelectric effect.

The phenomenon of emission of electrons from a metal surface, when radiation of appropriate frequency is incident on it, as shown in Fig. 14.1, is known as photoelectric effect. For metals like zinc, cadmium, magnesium etc., ultraviolet radiation is necessary while for alkali metals, even visible radiation is sufficient.

Electrical energy can be obtained from light (electromagnetic radiation) in two ways (i) photo-emissive effect as described above and (ii) photo-voltaic effect, used in a solar cell. In the latter case, an electrical potential difference is generated in a semiconductor using solar energy.

14.2.1 Experimental Set-up of Photoelectric Effect:

A typical laboratory experimental set-up for the photoelectric effect (Fig. 14.2) consists of an evacuated glass tube with a quartz window containing a photosensitive metal plate - the emitter E and another metal plate - the collector C. The emitter and collector are connected to a voltage source whose voltage can be changed and to an ammeter to measure the current in the circuit. A potential difference of V, as measured by the voltmeter, is maintained between the emitter E (the cathode) and collector C (the anode), normally C being at a positive potential with respect to the emitter. This potential difference can be varied and C can even be at negative potential with respect to E. When the anode potential V is positive, it accelerates the electrons (hence called accelerating potential) while when the anode potential V is negative, it retards the flow of electrons (therefore known as retarding potential). A source S of monochromatic light (light corresponding to only one specific frequency) of sufficiently high frequency (short wavelength $\leq 10^{-7}$ m) is used.

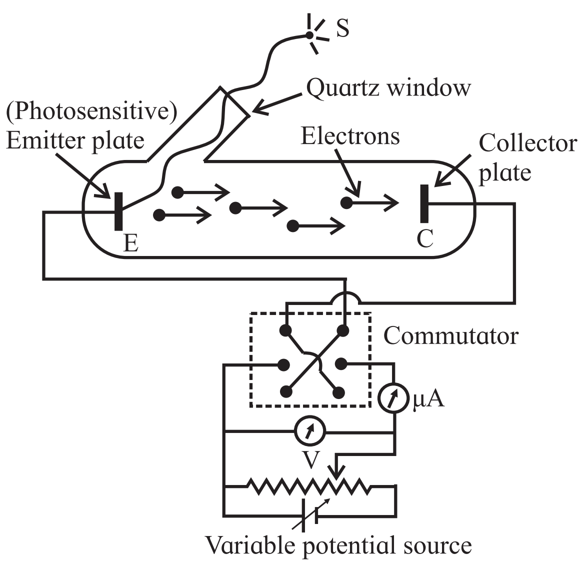


Fig. 14.2: Schematic of experimental set-up for photoelectric effect.

Light is made to fall on the surface of the metal plate E and electrons are ejected from the metal through its surface. These electrons, called photoelectrons, are collected at the collector C (photoelectron are ordinary electrons, they are given this name to indicate that they are emitted due to incident light). We now know that free electrons are available in a metal plate. They are emitted if sufficient energy (we will know more about this energy later in the Chapter) is supplied to them to overcome the barrier that keeps them inside the metal.

In the late nineteenth century, these facts were not known and scientists working on photoelectric effect performed various experiments and noted down their observations. These observations are summarized below. We will try to analyze these observations and their explanation.

14.2.2 Observations from Experiments on Photoelectric Effect:

1. When ultraviolet radiation was incident on the emitter plate, current I was recorded even if the intensity of radiation was very low. Photocurrent I was observed only if the frequency of the incident radiation was more than some threshold frequency ν_0. ν_0 was same for a given metal and was different for different metals used as the emitter. For a given frequency ν ($> \nu_0$) of the incident radiation, no matter how feeble was the light meaning however small the intensity of radiation be, electrons were always emitted.
2. There was no time lag between the incidence of light and emission of electrons. The photocurrent started instantaneously (within 10^{-9} s) on shining the radiation even if the intensity of radiation was low. As soon as the incident radiation was stopped, the flow of current stopped.
3. Keeping the frequency ν of the incident radiation and accelerating potential V fixed, if the intensity was increased, the photo current increased linearly with intensity as shown in Fig. 14.3.

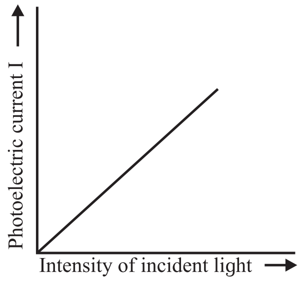


Fig. 14.3: Photocurrent as a function of incident intensity for fixed incident frequency and accelerating potential .

4. The photocurrent I could also be varied by changing the potential of the collector plate. I was dependent on the accelerating potential V (potential difference between the emitter and collector) for given incident radiation (intensity and frequency were fixed). Initially the current increased with voltage but then it remained constant. This was termed as the saturation current I_0 (Fig. 14.4). The superscripts 1, 2, 3 of I_0 refer o different intensities of the incident light.

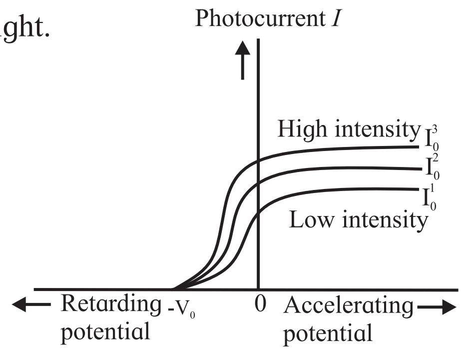


Fig. 14.4: Photocurrent as a function of accelerating potential for fixed incident frequency and different incident intensities. The superscripts 1, 2, 3 of I_0 refer to different intensities

5. Keeping the accelerating voltage and incident frequency fixed, if the intensity of incident radiation was increased, the value of saturation current also increased proportionately, e.g., if the intensity was doubled, the saturation current was also doubled.

6. The maximum kinetic energy KE_{max} (and hence the maximum velocity) of the electrons depended on the potential V for a given metal used for the emitter plate and for a given frequency of the incident radiation. If the material is changed or the frequency of the incident radiation is changed, KE_{max} changed. It did not depend on the intensity of the incident radiation. Thus, even for very small incident intensity, if the frequency of incident radiation was larger than the threshold frequency ν_0, KE_{max} from a given surface was always the same for a given incident frequency.

7. If increasingly negative potentials were applied to the collector, the photocurrent decreased and for some typical value $-V_0$, photocurrent became zero. V_0 was termed as cut-off or stopping potential. It indicated that when the potential was retarding, the photoelectrons still had enough energy to overcome the retarding (opposing) electric field and reach the collector. Value of V_0 was same for any incident intensity as long as the incident frequency was same (Fig. 14.4) but was different for different emitter materials.

8. If the frequency of incident radiation was changed keeping the intensity constant, then the saturation current remained the same but the stopping potential V_0 changed. V_0^1, V_0^2, V_0^3 are the stopping potentials for incident frequencies ν_1, ν_2, ν_3 respectively. The superscripts 1, 2, 3 of I_0 refer to different intensities of the incident light. This observation is depicted in Fig. 14.5. The stopping potential V_0 varied linearly with ν as shown in Fig. 14.6. For different metals, the slopes of such straight lines were the same but the intercepts on the frequency and stopping potential axes were different.

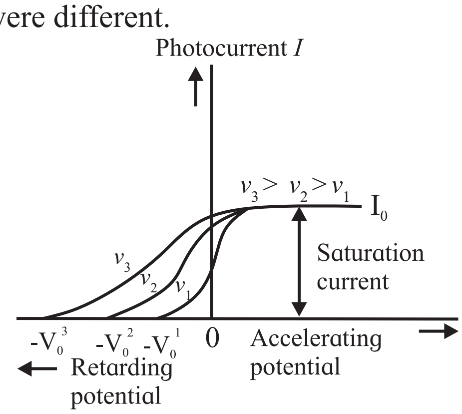


Fig. 14.5: Photocurrent as a function of accelerating potential for fixed incident intensity but different incident frequencies for the same emitter material .

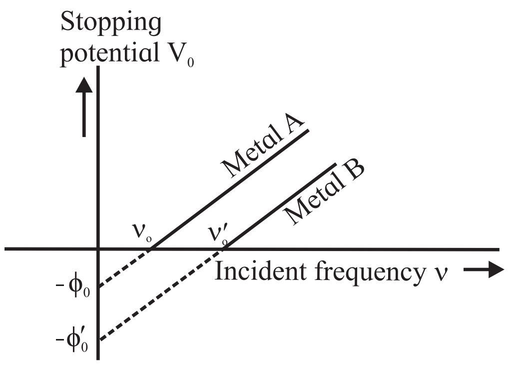


Fig. 14.6: Stopping potential as a function of frequency of incident radiation for emitters made of different metals.

9. The photocurrent and hence the number of electrons depended on the intensity but not on the frequency of incident radiation, as long as the incident frequency was larger than the threshold frequency ν_0 and the potential of anode was higher than that of cathode.

14.2.3 Failure of Wave Theory to Explain the Observations from Experiments on Photoelectric Effect:

Most of these observations could not be explained by the wave theory of electromagnetic radiation. First and foremost was the instantaneous emission of electrons on incidence of light. Wave picture would expect that the metal surface will absorb the incident energy continuously. All the electrons near the surface will absorb energy. The metal surface will require reasonable time (~ few minutes to hours) to accumulate sufficient energy to knock off electrons. Greater the intensity of incident radiation, more will be the incident energy, hence expected time required to knock off the electrons will be less. For small incident intensity, the energy incident on unit area in unit time will be small, and will take longer to knock off the electrons. These arguments were contradictory to observations.

Let us try to estimate the time that will be required for the photocurrent to start. We need to define the term 'work function' of a metal for this exercise.

We know that metals have free electrons. This fact makes metals good conductors of heat and electricity. These electrons are free to move inside the metal but are otherwise confined inside the metal. They cannot escape from the surface unless sufficient energy is supplied to them. The minimum amount of energy required to be provided to an electron to pull it out of the metal from the surface is called the **work function of the metal** and is denoted by ϕ_0. Work function depends on the properties of the metal and the nature of its surface. Values of work function of metals are generally expressed in a unit of energy called the electron volt (eV).

You have studied ionization energy of an atom. What is ionization energy to an atom is the work function to a solid which is a large collection of atoms.

Table 14.1 : Typical values of work function for some common metals.

Metal	Work function (in eV)
Potassium	2.3
Sodium	2.4
Calcium	2.9
Zinc	3.6
Silver	4.3
Aluminum	4.3
Tungsten	4.5
Copper	4.7
Nickel	5.0
Gold	5.1

Example 14.1: Radiation of intensity 0.5×10^{-4} W/m^2 falls on the emitter in a photoelectric set-up. The emitter (cathode) is made up of potassium and has an area of 5 cm^2. Let us assume that the electrons from only the surface are knocked off by the radiation. According to the wave theory, what will be the time required to notice some deflection in the microammeter

connected in the circuit? (Given the metallic radius of potassium atom is 230 pm and work function of potassium is 2.3 eV.)

Solution : Given

Intensity of radiation = 0.5×10^{-4} W/m^2,

Area of cathode = 5 cm^2 = 5×10^{-4} m^2.

Radius of potassium atom = 230 pm

= 230×10^{-12} m

Work function of potassium = 2.3 eV

= $2.3 \times 1.6 \times 10^{-19}$ J

The number N of electrons present on the surface of cathode can be approximately calculated assuming that each potassium atom contributes one electron and the radius of potassium atom is 230×10^{-12} m.

N = Area of cathode/ area covered by one atom

= $5\times10^{-4}/(3.1415\times230\times10^{-12}\times230\times10^{-12})$

= 3008×10^{12}

Incident power on the cathode is

= 0.5×10^{-4} W/m^2 $\times 5\times10^{-4}$ m^2

= 2.5×10^{-8} W

Wave theory assumes that this power distributed over the whole area of the cathode is uniformly absorbed by all the electrons. Therefore the energy absorbed by each electron in one second is

= 2.5×10^{-8} W $/3009\times10^{12} \approx 8.311\times10^{-24}$ W.

Work function of potassium is

2.30 eV = $2.30 \times 1.6 \times10^{-19}$ J

= 3.68×10^{-19} J.

Hence each electron will require minimum 3.68×10^{-19} J of energy to be knocked off from the surface of the cathode.

The time required to accumulate this energy will be

3.68×10^{-19} J / 8.31×10^{-24} W

= 4.428×10^{4} s, which is about half a day.

Secondly, since larger incident intensity implies larger energy, the electrons are expected to be emitted with larger kinetic energy. But the observation showed that the maximum kinetic energy did not depend on the incident intensity but depended on the incident frequency. According to wave theory, frequency of incident radiation has no role in determining the kinetic energy of photoelectrons. Moreover, wave theory expected photoelectrons to be emitted for any frequency if the intensity of radiation was large enough. But observations indicated that for a given metal surface, some characteristic cut-off frequency ν_0 existed below which no photoelectrons were emitted however intense the incident radiation was and photoelectrons were always emitted if incident frequency ν was greater than ν_0 even if the intensity was low.

14.2.4 Einstein's Postulate of Quantization of Energy and the Photoelectric Equation:

Planck's hypothesis of energy quantization to explain the black body radiation was extended by Einstein in 1905 to all types of electromagnetic radiations. Einstein proposed that under certain conditions, light behaves as if it was a particle and its energy is released or absorbed in bundles or quanta. He named the quantum of energy of light as photon with energy $E = h\nu$, where ν is the frequency of light and h is a constant defined by Planck in his model to explain black body radiation. It is now known as the Planck's constant and has a value 6.626×10^{-34} J s.

It may be noted that the equation

$$E = h\nu \qquad \text{--- (14.1)}$$

is a relation between a particle like property, the energy E and a wave like property, the frequency ν. Equation (14.1) is known as the Einstein's relation.

Einstein's relation (14.1) holds good for the entire electromagnetic spectrum. It says that energy of electromagnetic radiation is directly proportional to the frequency (and is inversely proportional to the wavelength since $\nu = c/\lambda$). Hence high frequency radiation means high energy radiation. Alternatively, short wavelength radiation means high energy radiation.

Example 14.2: (a) Calculate the energies of photons corresponding to ultraviolet light and red light, given that their wavelengths are 3000 Å and 7000 Å respectively. (Remember that the photon are not coloured. Colour is human perception for that frequency range.)
(b) A typical FM radio station has its broadcast frequency 98.3 MHz. What is the energy of an FM photon of this frequency?

Solution: Given

λ_{uv} = 3000 Å = 3000 × 10^{-10} m,

λ_{red} = 7000 Å = 7000 × 10^{-10} m and

ν_{FM} = 98.3 MHz = 98.3 × 10^{6} s^{-1}

We know that energy E of electromagnetic radiation of frequency ν is $h\nu$ and if λ is the corresponding wavelength, then $\lambda\nu = c$, c being the speed of electromagnetic radiation in vacuum.

Hence, $E = h\nu = \dfrac{hc}{\lambda}$

(a)

$$E = \frac{6.63\times10^{-34}\ \mathrm{J\,s}\times 3\times10^{8}\ \mathrm{m\,s^{-1}}}{3000\times10^{-10}\ \mathrm{m}}$$

$$= 6.63\times10^{-19}\ \mathrm{J} = 4.144\ \mathrm{eV}$$

for a photon corresponding to ultraviolet light and

$$E = \frac{6.63\times10^{-34}\ \mathrm{J\,s}\times 3\times10^{8}\ \mathrm{m\,s^{-1}}}{7000\times10^{-10}\ \mathrm{m}}$$

$$= 2.84\times10^{-19}\ \mathrm{J} = 1.776\ \mathrm{eV}$$

for a photon corresponding to red light.

(b) The energy of photon of FM frequency 98.3 MHz is 6.63 × 10^{-34} J s × 98.3 × 10^{6} s^{-1} = 651.73 × 10^{-28} J = 40.74 × 10^{-8} eV.

This is very small energy as compared to the photon energy in the visible range.

- Wavelength (in Å) × energy (in eV) ≈ 12500 (numerically)
- Wavelength (in nm) × energy (in eV) ≈ 1250 (numerically)

Try this

Determine the wavelengths and frequencies for photons of energies (i) 10^{-12} J, (ii) 10^{-15} J, (iii) 10^{-18} J, (iv) 10^{-21} J and (v) 10^{-24} J. Accordingly prepare a chart (along a horizontal line) of various regions of electromagnetic spectrum and identify these regions in categories that you know. Compare your results with a standard chart from any reference book or from Internet. You would notice that γ photons are the most energetic photons and their energies are ~ 10^{-13} - 10^{-12} J. This is a very small amount of energy on the human scale and therefore we do not notice individual photons along their passage.

The explanation using Einstein's postulate of quantization of energy for the observations mentioned in section 14.2.2 is given below.

1. Einstein argued that when a photon of ultraviolet radiation arrives at the metal surface and collides with an electron, it gives all of its energy $h\nu$ to the electron. The energy is gained by the electron and the photon no longer exists. If ϕ_0 is the work function of the material of the emitter plate, then electrons will be emitted if and only if the energy gained by the electrons is more than or equal to the work function i.e.,$h\nu \geq \phi_0$. Thus, a minimum or threshold frequency ν_0 (= ϕ_0/h) is required to eject electrons from the metal surface. If $\nu < \nu_0$, the photon will not have enough energy to liberate an electron. As a result, no electron will be ejected however intense the incident radiation is. Similarly if $\nu > \nu_0$, the energy will always be sufficient to eject an electron, however small the incident intensity is.
2. Energy is given by the photon to the electron as soon as the radiation is incident on the surface. The exchange of energy between the photon and electron

is instantaneous. Hence there is no time lag between the incidence of light and emission of electrons. Also when the incident radiation is stopped, there are no photons to transfer the energy to electrons, hence the photoemission stops immediately.

3. According to Einstein's proposition, if the intensity of incident radiation for a given wavelength is increased, there will be an increase in the number of energy quanta (photons) incident on unit area in unit time; the energy of each quantum being the same ($= h\nu = hc/\lambda$). Therefore larger intensity radiation will knock off more number of electrons from the surface and hence the current will be larger (if $\nu > \nu_0$). Conversely lower intensity implies less number of incident photons, hence, less number of ejected electrons and therefore lower current.

4. Once the electron is emitted from the surface, if the collector is at a higher potential than the emitter, the electric field will accelerate the electrons towards the collector. Higher is the accelerating potential, more will be number of electrons reaching the collector. Hence the photocurrent I increases with the accelerating potential initially. Moreover, since the intensity of incident radiation determines the number of photons incident on the metal surface on unit area in unit time, it determines the maximum number of electrons that can be knocked off by the incident radiation. Hence for a given intensity, increasing the accelerating potential can increase the current only till all the knocked off electrons have reached the collector. No increase can be seen in the current beyond this limit. This explains the saturation current I_0.

5. Increasing the incident intensity will increase the number of incident photons and eventually the saturation current.

6. If the frequency of incident radiation is more than the threshold frequency, then the energy ϕ_0 is used by the electron to escape from the metal surface and remaining energy of the photon becomes the kinetic energy of the electron. Depending on the energy of the electron inside the metal and other processes like collisions after emission from the surface, the maximum kinetic energy is equal to $(h\nu - \phi_0)$. Hence,

$$KE_{max} = h\nu - \phi_0 \qquad \text{--- (14.2)}$$

Equation (14.2) is known as Einstein's photoelectric equation. KE_{max} depends on the material of the emitter plate and varies linearly with the incident frequency ν; it is independent of the intensity of the incident radiation.

7. The electrons that are emitted from the metal surface have different kinetic energies. The reasons for this are many-fold: all the electrons in a solid do not possess the same energy, the electrons may be ejected from varying depths inside the metal surface, electrons may suffer collisions before they come out of the metal surface and may lose their energy etc. If V is the potential difference between the emitter and collector and the collector is at a lower potential, an electron will lose its kinetic energy in overcoming the retarding force. If the kinetic energy is not sufficient, the emitted electrons may not reach the collector and the photocurrent will be zero. If KE_{max} is the energy of the most energetic electron at the emitter surface (where its potential energy is zero) and $-V_0$ is the stopping potential, then this electron will fail to reach the collector if $KE_{max} < eV_0$, where e is the electron charge and eV_0 is the energy needed for the electron to overcome the retarding potential V_0. If the electron just fails to

reach the collector, i.e., it has lost all its kinetic energy just at the collector, $KE_{max} = eV_0$ and the photocurrent becomes zero. Equation (14.2) then explains that stopping potential V_0 depends on the incident frequency and the material of the emitter and does not depend on the incident intensity.

8. If the ejected electrons have kinetic energy more than eV_0, electrons can reach the collector, hence current flows. When the kinetic energy of the electron is less than or equal to eV_0, no current will flow. Photocurrent will become zero when $KE_{max} = eV_0$. Using $KE_{max} = eV_0$, we can write Eq. (14.2) as

$$eV_0 = h\nu - \phi_0$$

or, $$V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi_0}{e} \quad \text{--- (14.3)}$$

Above equation tells us that V_0 varies linearly with incident frequency ν, and the slope of the straight line depends on constants h and e while the intercept of the line depends on the material through ϕ_0. Thus the slope of lines in Fig. 14.6 is same and is independent of the material of the emitter but intercepts are different for different materials.

9. All the above arguments thus bring out the fact that the magnitude of photocurrent depends on the incident intensity through the number of emitted photoelectrons and the potential V of the collector but not on the incident frequency ν as long as $\nu > \nu_0$.

Thus all the observations related to the experiments on photoelectric effect were explained by Einstein's hypothesis of existence of a photon or treating light as bundles of energy. Although Einstein gave his hypothesis in 1905, it was not widely accepted by the scientific community. In 1909, when Millikan measured the charge of an electron and the value of h, calculated from Eq. (14.3), matched with the value given by Planck, the hypothesis was accepted. The work function values ϕ_0 for some metals were also confirmed from Eq. (14.3). Einstein and Millikan received Nobel prizes for their respective discoveries in 1921 and 1923 respectively.

Use your brain power

You must have seen light emitting diodes (LEDs) of different colours. In LED, electrical energy is converted into light energy corresponding to different colours. Can you tell what must be the difference in the working of LEDs of different colours.

Design an experiment using LEDs to determine the value of Planck's constant.

You might know that Nobel prize in physics for the year 2014 was awarded to Professors Isamu Akasaki, Hiroshi Amano and Shuji Nakamura for the invention of blue LEDs. They made the first blue LED in the early 1990s. Try to search on the Internet why it was difficult to make a blue LED.

According to Einstein, energy of radiation of frequency ν comes in bundles with magnitude $h\nu$. Thus energy of a light beam having n photons will be $nh\nu$, where n can take only integral values. Is it then possible to vary the incident energy continuously? Why we do not see individual photons? To understand this issue, let us consider the following example.

Example 14.3: The wavelength and power of the incident light is 4000 Å and 0.1 W respectively. What is the minimum change in the energy of the incident light? What is the number of incident photons?

Solution : Given incident intensity = 0.1 W and λ = 4000 Å = 4000 × 10^{-10} m.

The energy E of a photon of given wavelength is

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\ \mathrm{J\,s}\times 3\times10^{8}\ \mathrm{m/s}}{4000\times10^{-10}\ \mathrm{m}}$$

$$= 4.972\times10^{-19}\ \mathrm{J}$$

This is the minimum change in energy and is very small. The change in energy can therefore be considered as continuous.

Number of photons N incident per second is

$$N = \frac{0.1\,\text{W}}{4.97\times10^{-19}\,\text{J}} \approx 2.011\times10^{17}$$

The number of photons coming out is so large that human eye cannot comprehend or count it. Even if one wishes to count, say 10 photons per second, ~10^9 years will be required.

Can you tell?

- A particular metal used as a cathode in an experiment on photoelectric effect does not show photoelectric effect when it is illuminated with green light. Which of the colours in the visible spectrum are likely to generate photocurrent?

Table 14.2 : Summary of analysis of observations from experiments on photoelectric effect.

Observation	Wave theory	Photon picture
Electrons are emitted as soon as the light is incident on the metal surface.	Very intense light is needed for instantaneous emission of electrons.	Only one photon is needed to eject one electron from the metal surface and energy exchange between electron and photon is instantaneous on collision.
Very low intensity of incident light is also sufficient to generate photocurrent.	Low intensity should not give photocurrent.	Low intensity of incident light means less number of photons and not low energy photons. Hence low current will be produced.
High intensity gives larger photocurrent means higher rate of release of electrons.	High intensity means higher energy radiation and therefore more electrons are emitted.	Higher intensity means more number of photons incident in unit time, therefore more number of electrons are emitted in unit time and hence photocurrent is larger.
Increasing the intensity has no effect on the electron energy.	Higher intensity should mean electrons emitted with higher energies.	Higher intensity means higher number of incident photons per unit time. Energy of photon is same as it does not depend on the intensity.
A minimum threshold frequency is needed for photocurrent to start.	Low frequency light should release electrons but would take more time.	A photon of low frequency light will not have sufficient energy to release an electron from the surface.
Increasing the frequency of incident light increases the maximum kinetic energy of electrons.	Increasing intensity should increase the maximum kinetic energy. Maximum kinetic energy should not depend on the incident frequency.	Increasing the frequency increases the energy of the photon. Therefore electrons receive more energy which results in increasing the maximum kinetic energy.

14.3 Wave-Particle Duality of Electromagnetic Radiation:

In its interaction with matter, light behaves as if it is made up of packets of energy called quanta. Later it was confirmed from other theoretical and experimental investigations that these light quanta can have associated momentum. Hence the question came up whether a particle can be associated with light or electromagnetic radiation in general. Particle nature was confirmed by Compton in 1924 in experiments on scattering of X-rays due to electrons of matter. Summary of these results is given in the box below and you can know more about these experiments from the reference books given at the end of this book or from the links given below

- http://physics.usask.ca/~bzulkosk/modphyslab/phys251manual/compton_2009.pdf
- http://www.phys.utk.edu/labs/modphys/Compton Scattering Experiment.pdf
- http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/comptint.html

Do you know?

The particle nature of radiation is seen in black body radiation and photoelectric effect. In the former, near room temperature, the radiation is mostly in the infrared region while in the latter it is in the visible and ultraviolet region of the spectrum. The third experiment, which established that a photon possesses momentum like a particle, was Compton scattering where X-rays and γ-rays interact with matter. In 1923, A. H. Compton made a monochromatic beam of X-rays, of wavelength λ, incident on a graphite sheet and measured the intensity of the scattered rays in different directions as a function of wavelength. He found that although the incident beam consisted of a single wavelength λ, the scattered intensity was maximum at two wavelengths. One of these was same as the incident wavelength but the other λ' was larger by an amount $\Delta\lambda$. $\Delta\lambda$ is known as the Compton shift that depends on the scattering angle.

Compton explained his observations by considering incidence of X-ray beam on graphite as collision of X-ray photons with the electrons of graphite, like collision of billiard balls. Energy and momentum is transferred during the collision and scattered photons have lower energy than the incident photons. Therefore they have lower frequency or higher wavelength. The Compton shift is given by the relation

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

where θ is the scattering angle. The shift depends only on the scattering angle and not on the incident wavelength. This shift cannot be explained using wave theory. If we let the Planck's constant go to zero, we get the result expected from wave theory. This is the test to check whether the new picture is correct or not.

Compton showed that photon has an associated momentum along with the energy it carries. All photons of electromagnetic radiation of a particular frequency have the same energy and momentum. Photons are electrically neutral and are not deflected by electric or magnetic fields. Photons can have particle-like collisions with other particles such as electrons. In photon – particle collision, energy and momentum of the system are conserved but the number of photons is not conserved. Photons can be absorbed or new photons can be created. Photons can transfer their energy and momentum during collisions with particles and disappear. When we turn on light, they are created. Photon always moves with the speed of light, it is never at rest. Mass of a photon is not defined as we do for a particle in Newtonian mechanics. Its rest mass is zero (in all frames of reference).

Effects of wave nature of light were seen in experiments on interference or diffraction when the slit widths or the separation between two slits are smaller than or comparable to the wavelength of light. If the slit width is large or the spacing between slits is more, the interference or diffraction patterns will not be same and the wave nature will not be so obvious.

It was realized by scientists that some phenomena observed in experiments in the laboratory or in nature (like interference and diffraction) can be explained by considering light in particular, and electromagnetic radiation in general, as a wave. On the other hand, some other observations (like photoelectric effect and black body radiation) can be explained only if we consider electromagnetic radiation as consisting of photons with definite quantum of energy (and momentum as evident from Compton scattering experiments). Also there are some phenomena which can be explained by both the theories. It is therefore essential to consider that both the characters or behaviours hold good; *one dominates in some situations and the other works in rest of the situations*. It is necessary to keep both the physical models to explain the careful experimental observations. There is thus a need to hypothesize the dual character of light. Later it turned out that such a picture is required not only for light but for the whole electromagnetic spectrum. **This phenomenon is termed as wave-particle duality of electromagnetic radiation.**

14.4 Photo Cell:

Photo cell is a device that makes use of the photoelectric effect and converts light energy into electrical energy. Schematic of a photocell is shown in Fig. 14.7. It consists of a semi-cylindrical photosensitive metal plate E (acting as a cathode) and a wire loop collector C (acting as an anode) supported in an evacuated glass or quartz bulb. The electrodes are connected to an external circuit having a high tension battery B and a microammeter μA. Instead of a photosensitive metal plate, the photosensitive material can be pasted in the form of a thin film on the inner walls of the glass bulb.

When light of suitable wavelength falls on the cathode, photoelectrons are emitted. These electrons are attracted towards the anode due to the applied electric field. The generated photocurrent is noted from the microammeter. Photocell is used to operate control systems and in light measuring devices. Light meters in photographic cameras make use of photocell to measure the intensity of light. Photocell can also be used to switch on or off the street lights.

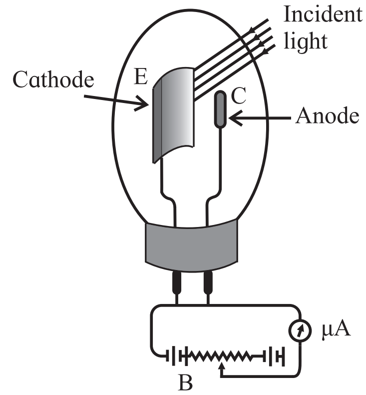


Fig. 14.7 : Schematic of a photocell.

Suppose source of ultraviolet radiation is kept near the passage or entrance of a mall or house and the light is made incident on the cathode of a photocell, photocurrent is generated. When a person passes through the passage or comes near the entrance, incident light beam is interrupted and photocurrent stops. This event can be used to operate a counter in counting devices, or to set a burglar alarm. Such an arrangement can be used to identify traffic law defaulters by setting an alarm using the photocell.

Use your brain power

Is solar cell a photocell?

14.5 De Broglie Hypothesis:

In 1924, Prince Louis de Broglie (pronounced as 'de broy') proposed, on the basis of the symmetry existing in nature, that if radiation has dual nature - sometimes wave nature dominates and sometimes particle nature, matter may also possess dual nature. Normally we talk about matter as composed of particles, but are there situations where matter seems to show wave-like properties? This will become evident from the experiments on diffraction of electrons from nickel crystals described later in this chapter.

De Broglie used the properties, frequency ν and wavelength λ, of a wave and proposed a relation to connect these with the particle properties, energy E and momentum p. The momentum p carried by a photon of energy E is given by the relation

$$p = \frac{E}{c} \qquad \text{--- (14.4)}$$

which is valid for a massless particle travelling with the speed of light c according to Einstein's special theory of relativity. Using the Einstein's relation for E,

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} \qquad \text{--- (14.5)}$$

where λ, the wavelength, is given by $\lambda\nu = c$.

De Broglie proposed that a moving material particle of total energy E and momentum p has associated with it a wave analogous to a photon. He then suggested that the wave and particle properties of matter can also be described by a relation similar to Eq. (14.5) for a photon. Thus frequency and wavelength of a wave associated with a material particle, of mass m moving with a velocity v, are given as

$\nu = E/h$ and

$$\lambda = h/p = h/mv \qquad \text{--- (14.6)}$$

He referred to these waves associated with material particles as *matter waves*. The wavelength of the matter waves, given by Eq. (14.6), is now known as de Broglie wavelength. Greater is the momentum, shorter is the wavelength. Equation (14.6) for the wavelength of matter waves is known as de Broglie relation.

For a particle of mass m moving with a velocity v, the kinetic energy

$$E_K = \frac{1}{2} m\text{v}^2 \text{ or } \text{v} = \sqrt{\frac{2E_K}{m}}.$$

Thus,

$$\lambda = \frac{h}{m\text{v}} = \frac{h}{m}\sqrt{\frac{m}{2E_K}} = \frac{h}{\sqrt{2mE_K}}$$

For a charged particle of charge q, accelerated from rest, through a potential difference V, the work done is qV. This provides the kinetic energy. Thus $E_K = qV$.

$$\therefore \lambda = \frac{h}{\sqrt{2mE_K}} = \frac{h}{\sqrt{2mqV}}.$$

This relation holds for any charged particle like electron, proton or for even charged ions where m corresponds to the mass of the charged particle. Of course, when V is very large (say in kV), so that the speed of the particle becomes close to the speed of light, such an equation will not be applicable. You will learn about other effects in such situations in higher classes.

For an electron moving through a potential difference of V (given in volts)

$$\lambda = \frac{h}{\sqrt{2m_e eV}}$$

$$= \frac{6.63\times10^{-34}\text{ J s}}{\sqrt{2\times9.11\times10^{-31}\text{kg}\times1.6\times10^{-19}\text{C}\times V\left(\text{in volts}\right)}}$$

$$= \frac{1.228\times10^{-9}}{\sqrt{V\left(\text{in volts}\right)}}\text{ m}$$

$$\text{or,}\quad \lambda\left(\text{in nm}\right) = \frac{1.228}{\sqrt{V\left(\text{in volts}\right)}} \qquad \text{--- (14.7)}$$

Example 14.4: An electron is accelerated through a potential of 120 V. Find its de Broglie wavelength.

Solution: Given V = 120 V.

We know that $\lambda = \frac{1.228}{\sqrt{V}}$ using Eq. (14.7).

$$\therefore \lambda = \frac{1.228}{\sqrt{120}} = 0.112\text{ nm.}$$

Use your brain power

Can you estimate the de Broglie wavelength of the Earth?

Can you tell?

- The expression $p = E/c$ defines the momentum of a photon. Can this expression be used for momentum of an electron or proton?

Shortly after the existence of photons (particles associated with electromagnetic waves) was postulated, it was also experimentally found that sub-atomic particles like electrons, protons, neutrons and atomic particles also exhibit wave properties. The wavelength associated with an electron of energy few eV is of the order of few Å. Therefore to observe the wave nature of electron, slit width or diffracting objects should be of same order of magnitude (few Å).

The wave property of electron was confirmed experimentally in 1927 by Davisson and Germer in America and in 1928 by George P. Thomson in England by diffraction of electrons by atoms in metals. Knowing that the size of the atoms and their spacing in crystals is of the order of few Å, they anticipated that if electrons are scattered by atoms in a crystal, the associated matter waves will interfere and will show diffraction effects. It turned out to be true in their experiments. Electrons showed constructive and destructive interference. No electrons were found in certain directions due to destructive interference while in other directions, maximum numbers of electrons were seen due to constructive interference.

Louis de Broglie received the Nobel prize in Physics in 1929 and Davisson, Germer and Thomson shared the Nobel prize in Physics in 1937. It was amazing that Sir J. J. Thomson discovered the existence of electron as a sub-atomic **particle** while his son G. P. Thomson showed that electron behaves like a **wave**.

14.6 Davisson and Germer Experiment:

A schematic of the experimental arrangement of the Davisson and Germer experiment is shown in Fig. 14.8. The whole set-up is enclosed in an evacuated chamber. It uses an electron gun - a device to produce electrons by heating a tungsten filament F using a battery B. Electrons from the gun are accelerated through vacuum to a desired velocity by applying suitable accelerating potential across a cylindrical anode and are collimated into a focused beam. This beam of electrons falls on a nickel crystal and is scattered in different directions by the atoms of the crystal. Thus, in the Davisson and Germer experiment, electrons were used in place of light waves. Scattered electrons were detected by an electron detector and the current was measured with the help of a galvanometer. By moving the detector on a circular scale that is by changing the scattering angle θ (angle between the incident and the scattered electron beams), the intensity of the scattered electron beam was measured for different values of scattering angle. Scattered intensity was not found to be uniform in all directions (as predicted by classical theory). The intensity pattern resembled a diffraction pattern with peaks corresponding to constructive interference and troughs to regions of destructive interference. Diffraction is a property of waves. Hence, above observations implied that the electrons formed a diffraction pattern on scattering and that particles could show wave-like properties.

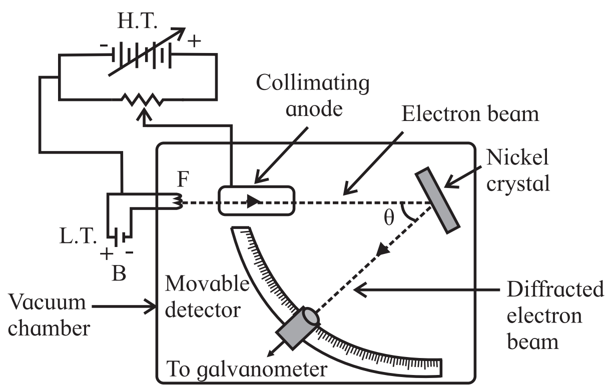


Fig. 14.8: Schematic of Davisson and Germer experiment.

Davisson and Germer varied the accelerating potential from 44 V to 68 V and observed a peak in the intensity of the scattered electrons at scattering angle of 50º for a potential of 54 V. This peak was the result of constructive interference of the electrons scattered from different layers of the regularly spaced atoms of the nickel crystal.

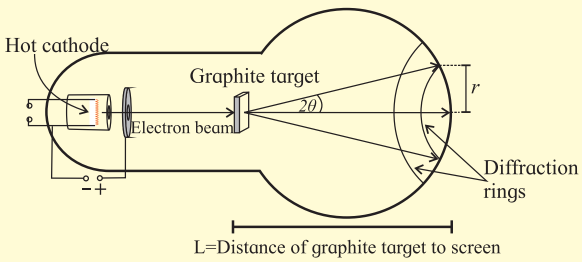
From Eq. (14.7), for V = 54 V, we get

$$\lambda = 1.228/\sqrt{54} = 0.167 \text{ nm} \quad \text{--- (14.8)}$$

From the electron diffraction measurements, the wavelength of matter waves associated with the electrons was found to be 0.165 nm. The two values of λ, obtained from the experimental results and from the theoretical de Broglie relation, were in close agreement. The Davisson and Germer experiment thus substantiated de Broglie's hypothesis of wave-particle duality and verified his relation.

Use your brain power

Diffraction results described above can be produced in the laboratory using an electron diffraction tube as shown in figure. It has a filament which on heating produces electrons. This filament acts as a cathode. Electrons are accelerated to quite high speeds by creating large potential difference between the cathode and a positive electrode. On its way, the beam of electrons comes across a thin sheet of



graphite. The electrons are diffracted by the atomic layers in the graphite and form diffraction rings on the phosphor screen. By changing the voltage between the cathode and anode, the energy, and therefore the speed, of the electrons can be changed. This will change the wavelength of the electrons and a change will be seen in the diffraction pattern. **By increasing the voltage, the radius of the diffraction rings will decrease**. **Try to explain why?**

14.7 Wave-Particle Duality of Matter:

Material particles show wave-like nature under certain circumstances. This phenomenon is known as **wave-particle duality of matter**. Frequency ω and wave number k are used to describe waves in classical theories while mass m and momentum p are used to describe particles. Wave-particle duality implies that all moving particles have an associated frequency and an associated wave number and all waves have an associated energy and an associated momentum. We come across the wave-particle duality of matter due to quantum behaviour when we are dealing with microscopic objects (sizes $\leq 10^{-6}$ m). Small order of magnitude of h sets the scale at which quantum phenomena manifest themselves.

If all the material objects in motion have an associated wavelength (and therefore an associated wave), why then we do not talk about wavelength of a child running with speed v on a pathway 2 m wide or a car moving with speed v on a road 20 m wide? To understand this, let us try to calculate these quantities.

Example 14.5: A student, weighing 45 kg, is running with a speed of 8 km per hr on a foot path 2 m wide. A small car, weighing 1200 kg, is moving with a speed of 60 km per hr on a 20 m wide road. Calculate their de Broglie wavelengths.

Solution : Given

$v_1 = 8$ km / hr $= 8 \times 10^3 /3600$ m / s and

$m_1 = 45$ kg for the student,

$v_2 = 60$ km / hr $= 60 \times 10^3 /3600$ m / s and

$m_2 = 1200$ kg for the car,

momentum $p_1 = 45 \times 8 \times 10^3 /3600$

$= 100$ kg m /s for the student and

momentum $p_2 = 1200 \times 60 \times 10^3 /3600$

$= 20000$ kg m /s for the car.

The de Broglie wavelength

$\lambda_1 = h/p_1 = 6.63 \times 10^{-34}$ J s / 100 kg m /s

$= 6.63 \times 10^{-36}$ m. for the student, and

de Broglie wavelength

$\lambda_2 = h/p_2 = 6.63 \times 10^{-34}$ J s/ 20000 kg m /s)

$= 3.32 \times 10^{-38}$ m for the car.

The wavelengths calculated in example 14.5 are negligible compared to the size of the moving objects as well as to the widths of the paths on which the objects are moving.

Therefore the wavelengths associated with macroscopic particles do not play any significant role in our everyday life and we need not consider their wave nature. Also the wavelengths for macroscopic particles are so small that they cannot be measured.

On the other hand, if we try to estimate the associated de Broglie wavelength of a moving electron passing through a small aperture of size 10^{-10} m or an oxygen molecule in air, we will find it to be significant as can be seen in the following example.

Example 14.6: Calculate the de Broglie wavelength of an electron moving with kinetic energy of 100 eV passing through a circular hole of diameter 2 Å.

Solution: Given

$E_K = 100 \text{ eV} = 100 \times 1.6 \times 10^{-19}$ J.

The speed of the electron is given by the relation $\frac{1}{2} mv^2 = 100 \times 1.6 \times 10^{-19}$ J.

$$\therefore v = \sqrt{\frac{2 \times 100 \times 1.6 \times 10^{-19} \text{ J}}{9.11 \times 10^{-31} \text{ kg}}}$$

$= 5.927 \times 10^7$ m/s and

momentum $p = 9.11 \times 10^{-31}$ kg $\times$ 0.593×10^7 m/s $= 5.40 \times 10^{-24}$ kg m/s

$\therefore$ the de Broglie wavelength $\lambda = h/p$

$= 6.63 \times 10^{-34}$ J s / 5.40×10^{-24} kg m/s

$= 1.23 \times 10^{-10}$ m $= 1.228$ Å.

The wavelength of the electron in above example is comparable to the size of the hole through which the electron is passing. The wavelength associated with this electron is same as the size of a helium atom and more than double the size of a hydrogen atom.

Use your brain power

On what scale or under which circumstances is the wave nature of matter apparent?

Photon picture allows transfer of energy and momentum in the same manner as in Newtonian mechanics. Wave nature does not modify that. Whenever wavelengths are small compared to the dimensions of slits or obstacles, or are not measurable, we can use Newtonian mechanics.

In conclusion, for both electromagnetic radiation and atomic and sub-atomic particles, particle nature is dominant during their interaction with matter. On the other hand, while traveling through space, particularly when their confinement is of same order of magnitude as their associated wavelength, the wave nature is dominant.

Do you know?

We have seen earlier that electrons are bound inside a metal surface and need some minimum energy equal to the work function to be knocked off from the surface. This energy, if provided by any means, can make the electron come out of the metal surface. Physical ways to provide this energy differentiate the physical processes involved and accordingly different devices and characterizing microscopes based on them have been designed by scientists.

- Thermionic emission : By heating to temperatures ~2000 ºC provide thermal energy.
- Field emission : By establishing strong electric fields ~10^6 V/m at the surface of a metal tip, provide electrical energy.
- Photo-electron emission : By shining radiation of suitable frequency (ultraviolet or visible) on a metal surface provide light energy.

Electron microscope:

You have learnt about resolving power and resolution of telescopes and microscopes that use the ordinary visible light. The resolution of a microscope is limited by the wavelength of the light used. The shorter the wavelength of the characterizing probe, the smaller is the limit of resolution of a microscope, i.e., the resolution of microscope is better. Better resolution can be attained by illuminating the objects to be seen by radiation of smaller wavelengths. We have seen that an electron can behave as a wave and its wavelength is much smaller than the wavelength of visible light. The wavelength can be made much smaller as it depends on the velocity and kinetic energy of the electron. An electron beam accelerated to several keV of energy will correspond to de Broglie wavelength much smaller than an angstrom, i.e., $\lambda_e << 1\times10^{-10}$ m. The resolution of this electron microscope will be several hundred times higher than that obtainable with an optical microscope.

Other advantages of electron microscopes are that (i) electrons do not penetrate the matter as visible light or X-rays do, (ii) electron beams can be more easily produced and controlled by electric and magnetic fields than electromagnetic waves and (iii) electrons can be focused like light is focused with lenses.

It was proposed in 1925 that atoms in the solids can act as diffraction centers for electron waves and can give information about the geometry or structure of solid, just as X-rays do on getting diffracted by solids. However, it took many years to realize an electron microscope for practical applications. The first electron microscope was developed by Herald Ruska in Berlin, Germany in the year 1929.

Microscopic objects, when illuminated using electron beams, yield high resolution images. Images of microscopic and nanometric objects and even of viruses have been obtained by scientists using electron microscopes, making valuable contributions to mankind.

Transmission electron microscopy can resolve very small particles. A micrograph on the cover page of this book shows tiny crystals of dimensions less than 50 nm. An electron diffraction pattern is also seen on the cover page (spot pattern). When an electron beam passes through a crystal having periodic arrangement of atoms, diffraction occurs. The crystal acts as a collection of diffraction slits for the electron beam.

Internet my friend

1. http://phet-web.colorado.edu/simulations/schrodinger/dg.jnlp
2. https://physics.info/photoelectric/
3. https://www.britannica.com/science/photoelectric-effect
4. https://www.britannica.com/science/wave-particle-duality
5. https://www.sciencedaily.com/terms/wave-particle_duality.htm
6. https://www.thoughtco.com/de-broglie-hypothesis-2699351
7. https://www.toppr.com/guides/physics/dual-nature-of-radiation-and-matter
8. http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/DavGer2.html

Exercises

1. Choose the correct answer.

i) A photocell is used to automatically switch on the street lights in the evening when the sunlight is low in intensity. Thus it has to work with visible light. The material of the cathode of the photo cell is

(A) zinc (B) aluminum
(C) nickel (D) potassium

ii) Polychromatic (containing many different frequencies) radiation is used in an experiment on photoelectric effect. The stopping potential

(A) will depend on the average wavelength
(B) will depend on the longest wavelength
(C) will depend on the shortest wavelength
(D) does not depend on the wavelength

iii) An electron, a proton, an α-particle and a hydrogen atom are moving with the same kinetic energy. The associated de Broglie wavelength will be longest for

(A) electron (B) proton
(C) α-particle (D) hydrogen atom

iv) If N_{Red} and N_{Blue} are the number of photons emitted by the respective sources of equal power and equal dimensions in unit time, then

(A) $N_{Red} < N_{Blue}$ (B) $N_{Red} = N_{Blue}$
(C) $N_{Red} > N_{Blue}$ (D) $N_{Red} \approx N_{Blue}$

v) The equation $E = pc$ is valid

(A) for all sub-atomic particles
(B) is valid for an electron but not for a photon
(C) is valid for a photon but not for an electron
(D) is valid for both an electron and a photon

2. Answer in brief.

i) What is photoelectric effect?

ii) Can microwaves be used in the experiment on photoelectric effect?

iii) Is it always possible to see photoelectric effect with red light?

iv) Using the values of work function given in Table 14.1, tell which metal will require the highest frequency of incident radiation to generate photocurrent.

v) What do you understand by the term wave-particle duality? Where does it apply?

3. Explain the inverse linear dependence of stopping potential on the incident wavelength in a photoelectric effect experiment.

4. It is observed in an experiment on photoelectric effect that an increase in the intensity of the incident radiation does not change the maximum kinetic energy of the electrons. Where does the extra energy of the incident radiation go? Is it lost? State your answer with explanatory reasoning.

5. Explain what do you understand by the de Broglie wavelength of an electron. Will an electron at rest have an associated de Broglie wavelength? Justify your answer.

6. State the importance of Davisson and Germer experiment.

7. What will be the energy of each photon in monochromatic light of frequency 5×10^{14} Hz?

[Ans : 3.315×10^{-19} J = 2.071 eV]

8. Observations from an experiment on photoelectric effect for the stopping potential by varying the incident frequency were plotted. The slope of the linear curve was found to be approximately 4.1×10^{-15} V s. Given that

the charge of an electron is 1.6×10^{-19} C, find the value of the Planck's constant h.

[Ans : 6.56×10^{-34} J s]

9. The threshold wavelength of tungsten is 2.76×10^{-5} cm. (a) Explain why no photoelectrons are emitted when the wavelength is more than 2.76×10^{-5} cm. (b) What will be the maximum kinetic energy of electrons ejected in each of the following cases (i) if ultraviolet radiation of wavelength $\lambda = 1.80 \times 10^{-5}$ cm and (ii) radiation of frequency 4×10^{15} Hz is made incident on the tungsten surface.

[Ans: 2.40 eV, 12.07 eV]

10. Photocurrent recorded in the micro ammeter in an experimental set-up of photoelectric effect vanishes when the retarding potential is more than 0.8 V if the wavelength of incident radiation is 4950 Å. If the source of incident radiation is changed, the stopping potential turns out to be 1.2 V. Find the work function of the cathode material and the wavelength of the second source.

[Ans: 1.71 eV, 4270 Å]

11. Radiation of wavelength 4500 Å is incident on a metal having work function 2.0 eV. Due to the presence of a magnetic field B, the most energetic photoelectrons emitted in a direction perpendicular to the field move along a circular path of radius 20 cm. What is the value of the magnetic field B?

[Ans. : 1.473×10^{-5} T]

12. Given the following data for incident wavelength and the stopping potential obtained from an experiment on photoelectric effect, estimate the value of Planck's constant and the work function of the cathode material. What is the threshold frequency and corresponding wavelength? What is the most likely metal used for emitter?

Incident wavelength (in Å)	2536	3650
Stopping potential (in V)	1.95	0.5

[Ans: 6.427×10^{-34} J s, 2.801 eV, 6.761×10^{14} Hz, 4438 Å, calcium]

13. Calculate the wavelength associated with an electron, its momentum and speed

(a) when it is accelerated through a potential of 54 V,

[Ans: 0.1671 nm, 39.70×10^{-25} kg m s^{-1}, 4.358×10^{6} m s^{-1}]

(b) when it is moving with kinetic energy of 150 eV.

[Ans: 0.1002 nm, 66.17×10^{-25} kg m s^{-1}, 7.263×10^{6} m s^{-1}]

14. The de Broglie wavelengths associated with an electron and a proton are same. What will be the ratio of (i) their momenta (ii) their kinetic energies?

[Ans: 1, 1836]

15. Two particles have the same de Broglie wavelength and one is moving four times as fast as the other. If the slower particle is an α-particle, what are the possibilities for the other particle?

[Ans: proton or neutron]

16. What is the speed of a proton having de Broglie wavelength of 0.08 Å?

[Ans: 49.623×10^{3} m s^{-1}]

17. In nuclear reactors, neutrons travel with energies of 5×10^{-21} J. Find their speed and wavelength.

[Ans: 2.447×10^{3} m s^{-1}, 1.622 Å]

18. Find the ratio of the de Broglie wavelengths of an electron and a proton when both are moving with the (a) same speed, (b) same energy and (c) same momentum? State which of the two will have the longer wavelength in each case?
[Ans: (a) 1836, (b) electron; 42.85, electron; (c) 1, equal]

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