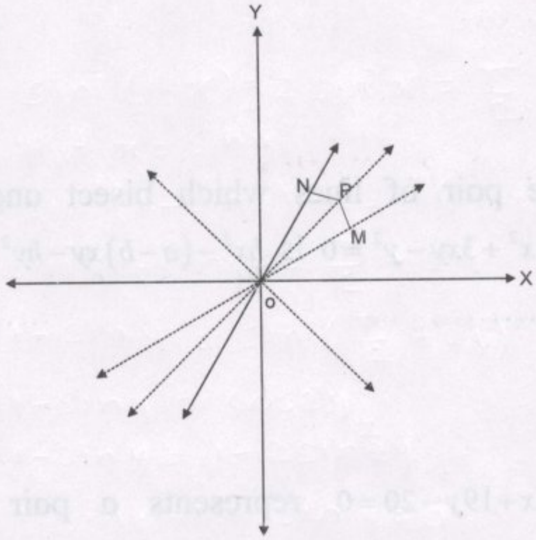
5. Pair of straight lines

Ex. (1) Find the joint equation of bisectors of angles between lines represented by $ax^2 + 2hxy + by^2 = 0$. Hence write the joint equation of the pair of lines which bisect angles between the lines given by $x^2 + 3xy - y^2 = 0$.



Solution : Let m_1 and m_2 be the slopes of lines represented by the equation

$ax^2 + 2hxy + by^2 = 0$.

$\therefore m_1 + m_2 = -\dfrac{2h}{b}$ and $m_1 m_2 = \dfrac{a}{b}$.

Let their separate equations be $m_1 x - y = 0$ and $m_2 x - y = 0$.

Let $P(x_1, y_1)$ be any point on one of the angle bisectors.

Let PM and PN be perpendiculars drawn from P to the lines $m_1 x - y = 0$ and $m_2 x - y = 0$.

$\therefore$ PM = PN

$$\therefore \left|\frac{m_1 x_1 - y_1}{\sqrt{m_1^2 + 1}}\right| = \left|\frac{m_2 x_1 - y_1}{\sqrt{m_2^2 + 1}}\right|$$

$$\therefore (m_1 x_1 - y_1)^2 (m_2^2 + 1) = (m_2 x_1 - y_1)^2 (m_1^2 + 1)$$

$$\therefore (m_1^2 x_1^2 - 2m_1 x_1 y_1 + y_1^2)(m_2^2 + 1) = (m_2^2 x_1^2 - 2m_2 x_1 y_1 + y_1^2)(m_1^2 + 1)$$

$$\therefore m_1^2 m_2^2 x_1^2 - 2m_1 m_2^2 x_1 y_1 + m_2^2 y_1^2 + m_1^2 x_1^2 - 2m_1 x_1 y_1 + y_1^2$$

$$= m_1^2 m_2^2 x_1^2 - 2m_1^2 m_2 x_1 y_1 + m_1^2 y_1^2 + m_2^2 x_1^2 - 2m_2 x_1 y_1 + y_1^2$$

$$\therefore \quad \left(m_1^2 - m_2^2\right) x_1^2 + 2m_1 m_2 \left(m_1 - m_2\right) x_1 y_1 - 2\left(m_1 - m_2\right) x_1 y_1 - \left(m_1^2 - m_2^2\right) y_1^2 = 0$$

$$\therefore \quad \left(m_1 + m_2\right) x_1^2 + 2m_1 m_2 x_1 y_1 - 2x_1 y_1 - \left(m_1 + m_2\right) y_1^2 = 0$$

$$\therefore \quad \left(\frac{2h}{b}\right) x_1^2 + \frac{2a}{b} x_1 y_1 - 2x_1 y_1 - \left(-\frac{2h}{b}\right) y_1^2 = 0$$

$$\therefore \quad -2hx_1^2 + 2\left(a-b\right) x_1 y_1 + 2hy_1^2 = 0$$

$$\therefore \quad hx_1^2 - \left(a-b\right) x_1 y_1 - hy_1^2 = 0$$

$$\therefore \quad hx^2 - \left(a-b\right) xy - hy^2 = 0$$

The joint equation of the pair of lines which bisect angles between the lines given by $x^2 + 3xy - y^2 = 0$ is $hx^2 - (a-b)xy - hy^2 = 0$

i .e. $\frac{3}{2}x^2 - (2)xy - \frac{3}{2}y^2 = 0$

$3x^2 - 4xy - 3y^2 = 0$

Ex. (2) Show that $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$ represents a pair of intersecting lines. Find the acute angle between them. Also find the co-ordinates of their point of intersection.

Solution : We have $2x^2 - xy - 3y^2 = (2x - 3y)(x + y)$

Suppose $2x^2 - xy - 3y^2 - 6x + 19y - 20 = (2x - 3y + c)(x + y + k)$

$\therefore \quad 2x^2 - xy - 3y^2 - 6x + 19y - 20 = 2x^2 - xy - 3y^2 + (c + 2k)x + (c - 3k)y + ck$

On comparing coefficients, we get

$c + 2k = -6 \quad \ldots \quad (1)$

$c - 3k = 19 \quad \ldots \quad (2)$

$ck = -20 \quad \ldots \quad (3)$

Solving (1) and (2) we get $c = 4$ and $k = -5$

They satisfy equation (3) also.

$\therefore$ Given general equation can be factorized as

$(2x - 3y + 4)(x + y - 5) = 0$

$\therefore$ Given equation represents a pair of intersecting lines.

The acute angle between them is given by

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right| = \left|\frac{\sqrt{(2h)^2 - 4ab}}{a+b}\right| = \left|\frac{\sqrt{(-1)^2 - 4(2)(-3)}}{2-3}\right| = 5$$

$\therefore\ \theta = \tan^{-1}(5)$

Their point of intersection is given by $\left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right) = \left(\frac{11}{5}, \frac{14}{5}\right)$

Ex. (3) If the line $4x+6y+7=0$ coincides with one of the lines represented by $4x^2+2xy-6y^2+11x-y+\lambda=0$ then find the value of λ and the slope of the other line.

Solution : Method I :

As $4x^2+2xy-6y^2=(4x+6y)(x-y)$, let the equation of the other line be $x-y+c=0$.

Their joint equation is $(4x+6y+7)(x-y+c)=0$

$\therefore\ 4x^2+2xy-6y^2+(7+4c)x+(-7+6c)y+7c=0$

Comparing with given joint equation, we get

$7+4c=11$, $-7+6c=-1$ and $\lambda=7c$

Solving first two equations we get, $c=1$.

$\therefore\ \lambda=7$ and the equation of the other line is $x-y+1=0$.

$\therefore$ The slope of the other line is $-\left(\frac{1}{-1}\right)=1$

Method II :

Co-ordinates of every point on the line $4x+6y+7=0$ satisfy the joint equation.

$A\left(-\frac{7}{4}, 0\right)$ is a point on the line $4x+6y+7=0$.

It satisfy the equation $4x^2+2xy-6y^2+11x-y+\lambda=0$.

$\therefore\ 4\left(-\frac{7}{4}\right)^2+0-0+11\left(-\frac{7}{4}\right)-0+\lambda=0$

$\therefore\ \lambda=7$.

The joint equation is $4x^2+2xy-6y^2+11x-y+7=0$

Slope of the line $4x+6y+7=0$ is $\frac{-4}{6}=-\frac{2}{3}$

Let slope of the other line be m_1.

$\therefore\ -\frac{2}{3}$ and m_1 are the roots of the equation $6m^2-2m-4=0$.

$\therefore -\frac{2}{3}+m_1=-\frac{-2}{6}$ $\qquad \therefore -\frac{2}{3}+m_1=\frac{1}{3}$

$\therefore\ m_1=1$

$\therefore$ The slope of the other line is . 1 .

Ex. (4) Find the joint equation of the pair of lines passing through A(2, 3), each of which make angle 30^0 with the Y-axis.

Solution : Lines make angles 30^0 with the Y-axis.

$\therefore$ Their inclinations are 60^0 and . 120^0.

$\therefore$ Their slopes are $\tan 60^0 = \sqrt{3}$ and tan. 120^0 . $= -\sqrt{3}$

They pass through A(2, 3).

Their separate equations are $(y-3) = \sqrt{3}(x-2)$ and $(y-3) = -\sqrt{3}(x-2)$

$\therefore$ $\sqrt{3}(x-2)-(y-3) = 0$ and $\sqrt{3}(x-2)+(y-3)$.

$\therefore$ Their joint equation is

$\left(\sqrt{3}(x-2)-(y-3)\right)\left(\sqrt{3}(x-2)+(y-3)\right)=0$

$\therefore$ $3(x-2)^2-(y-3)^2=0$

$\therefore$ $3(x^2-4x+4)-(y^2-6y+9)=0$

$\therefore$ $3x^2-12x+12-y^2+6y-9=0$

$\therefore$ $3x^2-y^2-12x+6y+3=0$

This is the required joint equation.

Ex. (5) If lines represented by the equatio $ax^2+2hxy+by^2+2gx+2fy+c=0$ intersect each other then show that the co-ordinates of their point of intersection are $\left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right)$.

Solution :

Given joint eq^n is $ax^2+2hxy+by^2+2gx+2fy+c=0$

Consider $ax+hy+g=0$ — (I)

$hx+by+f=0$ — (II)

Let above two lines intersect each other

solving eq^n (I) and (II)

$$\frac{x}{\begin{vmatrix} h & g \\ b & f \end{vmatrix}} = \frac{-y}{\begin{vmatrix} a & g \\ h & f \end{vmatrix}} = \frac{1}{\begin{vmatrix} a & h \\ h & b \end{vmatrix}}$$

$$\therefore \frac{x}{hf-bg} = \frac{-y}{af-gh} = \frac{1}{ab-h^2}$$

$$\therefore \frac{x}{hf-bg} = \frac{1}{ab-h^2}$$

$$\frac{y}{gh-af} = \frac{1}{ab-h^2}$$

$$\therefore x = \frac{hf-bg}{ab-h^2}, y = \frac{gh-af}{ab-h^2}$$

$\therefore$ The coordinates of point of intersection are

$$\left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right)$$

Ex. (6) Δ OAB is formed by the lines $x^2 - 4xy + y^2 = 0$ and $x + y - 2 = 0$. Find the equation of the median drawn from O.

Solution :

Let $A \equiv (x_1, y_1)$, $B \equiv (x_2, y_2)$

By mid point formula $P \equiv \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

The coordinates of A and B can be obtained by solving $x^2 - 4xy + y^2 = 0$ and $x + y - 2 = 0$ simultaneously

put $y = 2 - x$ in $x^2 - 4xy + y^2 = 0$

$$x^2 - 4x(2-x) + (2-x)^2 = 0$$

$$x^2 - 8x + 4x^2 + 4 - 4x + x^2 = 0$$

$$6x^2 - 12x + 4 = 0$$

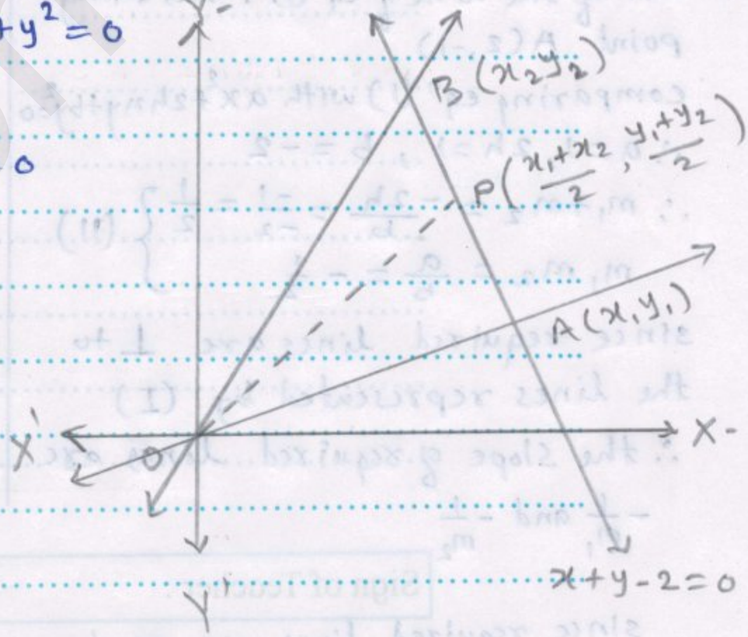
$$\therefore 3x^2 - 6x + 2 = 0$$

it is Q.E. with two roots

$$x_1 + x_2 = \frac{-(-6)}{3} = 2$$

$$x_1 + x_2 = 2$$

$$\frac{x_1 + x_2}{2} = 1$$



The co ordinate of the point P is 1

Point P is on line $x+y-2=0$

$\therefore 1+y-2=0$

$y-1=0$

$y=1$

$\therefore P \equiv (1,1)$

for median OP, $O \equiv (0,0)$

$P \equiv (1,1)$

$\therefore$ the eq^n of median OP is

$$\frac{y-0}{1-0} = \frac{x-0}{1-0}$$

$y = x$

$\therefore x-y=0$ is the required eq^n of median.

Ex. (7) Show that one of the lines represented by $x^2+xy-2y^2=0$ passes through the point $A(2,-1)$. Find the joint equation of lines passing through $A(2,-1)$ and perpendicular to the lines represented by the equation $x^2+xy-2y^2=0$.

Solution :

Given joint eq^n is $x^2+xy-2y^2=0$ —(I)

put $A(2,-1)$ in (I)

$\therefore$ LHS $= x^2+xy-2y^2$

$= 4-2-2 = 0$

LHS $=$ RHS

$\therefore$ one of the line of eq^n (I) passes through point $A(2,-1)$

comparing eq^n (I) with $ax^2+2hxy+by^2=0$

$\therefore a=1,\ 2h=1,\ b=-2$

$$\left.\begin{aligned} \therefore m_1+m_2 &= \frac{-2h}{b} = \frac{-1}{-2} = \frac{1}{2} \\ m_1 m_2 &= \frac{a}{b} = -\frac{1}{2} \end{aligned}\right\} \text{(II)}$$

since required lines are $\perp$ to the lines represented by (I)

$\therefore$ the slope of required lines are $-\frac{1}{m_1}$ and $-\frac{1}{m_2}$

Sign of Teacher :

since required lines are passing through $A(2,-1) = (x_1, y_1)$

eq^n of the lines are

$y-y_1 = m(x-x_1)$

$(y+1) = -\frac{1}{m_1}(x-2)$ & $y+1 = -\frac{1}{m_2}(x-2)$

$(x-2)+m_1(y+1)=0$ & $(x-2)+m_2(y+1)=0$

$\therefore$ Joint eq^n is

$(x-2)^2+m_2(x-2)(y+1)+m_1(x-2)(y+1)$
$m_1m_2(y+1)^2=0$

$\therefore (x-2)^2+\frac{1}{2}(x-2)(y+1)-\frac{1}{2}(y+1)^2=0$ using — II

$2(x-2)^2+(x-2)(y+1)-(y+1)^2=0$

$\therefore 2x^2-8x+8+xy+x-2y-2-y^2-2y-1=0$

$\therefore 2x^2+xy-y^2-7x-4y+5=0$

Q. 26. A solenoid of length π m and 5 cm in diameter has a winding of 1000 turns and carries a current of 5A. Calculate the magnetic field at its centre along the axis.

SECTION – D

Attempt any THREE questions of the following : [12]

Q. 27. What is Ferromagnetism? Explain it on the basis of domain theory.

Q. 28. Obtain an expression for average power dissipated in a series LCR circuit.

Q. 29. Distinguish between interference and diffraction of light.

A double slit arrangement produces interference fringes for sodium light of wavelength 589 nm, that are 0.20 degree apart. What is the angular fringe separation if the entire arrangement is immersed in water?

(R.I. of water = 1.33)

Q. 30. State Einstein's photoelectric equation and mention physical significance of each term involved in it.

The wavelength of incident light is 4000Å. Calculate the energy of incident photon.

Q. 31. State any four uses of Van de Graaff generator.

In a parallel plate air capacitor, intensity of electric field is changing at the rate of 2×10^{11} V/ms. If area of each plate is 20 cm^2, calculate the displacement current.

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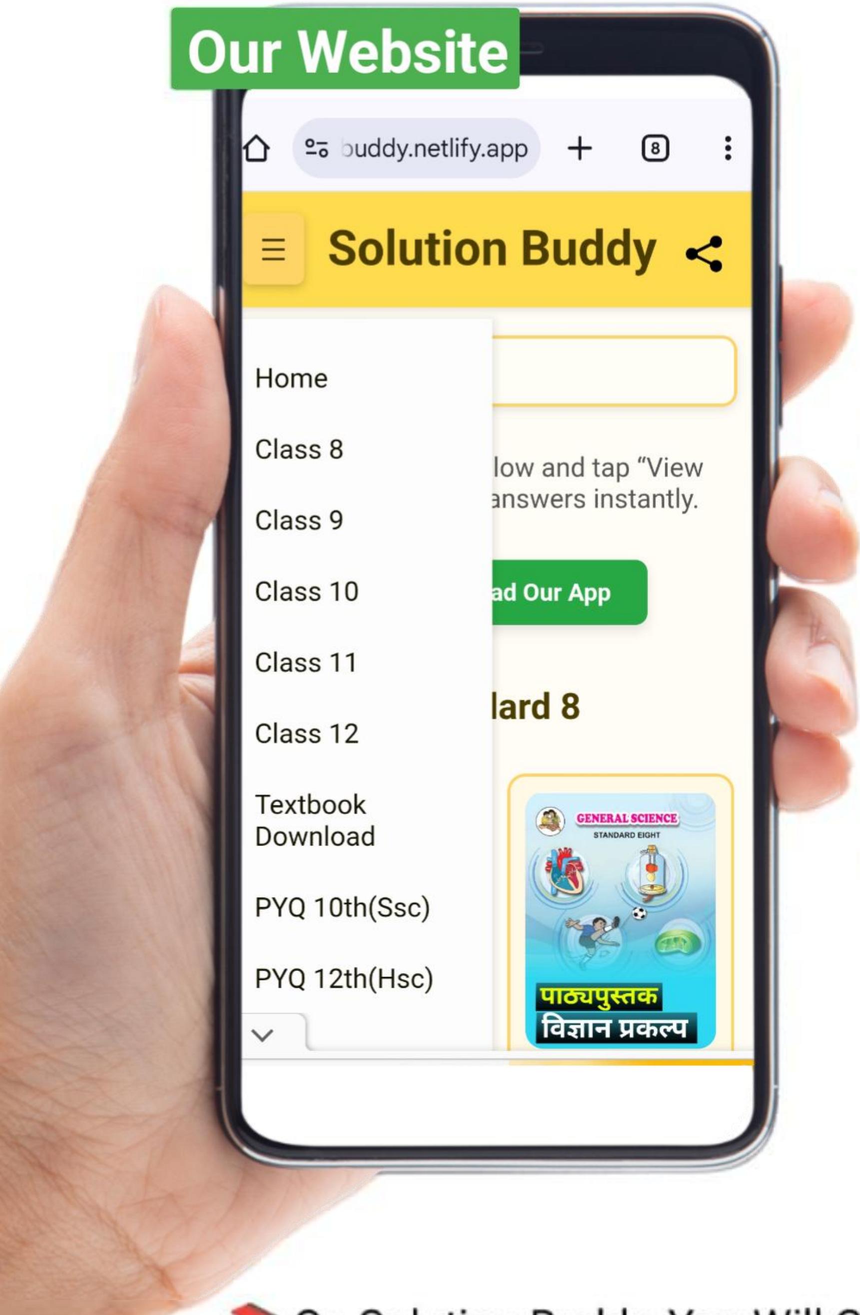

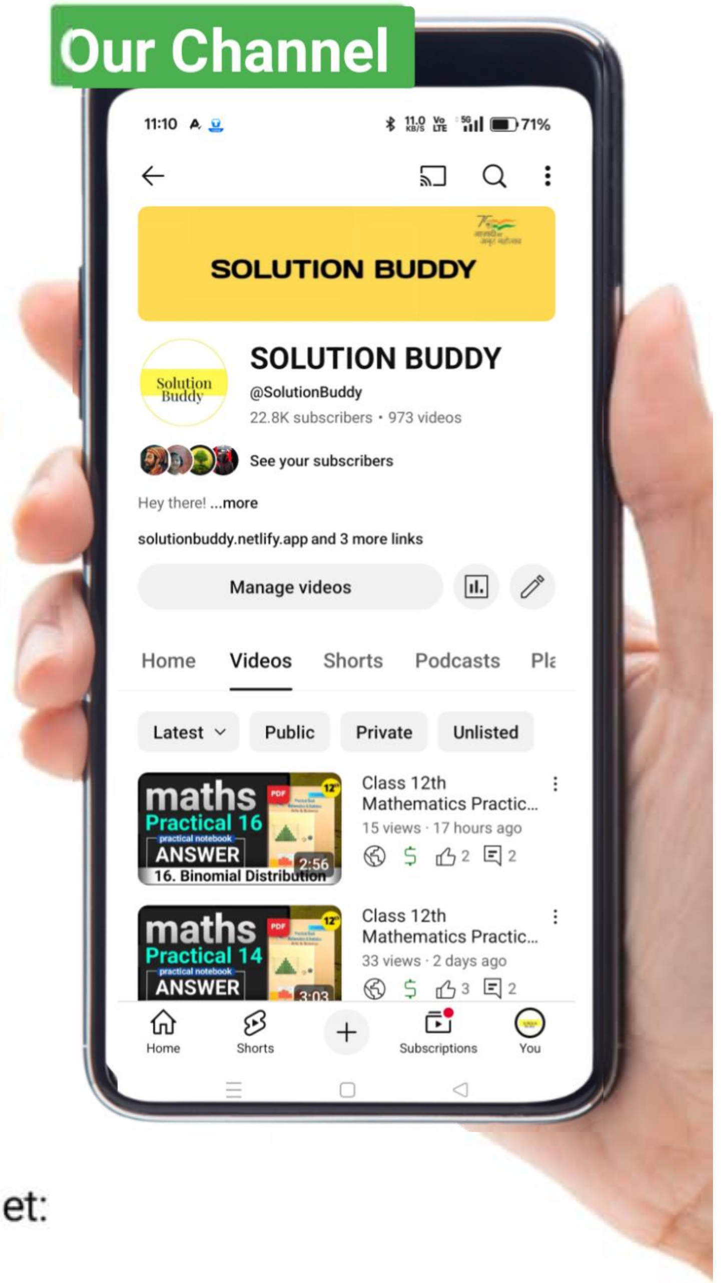

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