9. Applications of Derivatives – I

Ex. (1). Find the equations of the tangent and normal line to the curve $y = x^3 + e^x$ at $x = 0$.

Solution : Equation of the curve is $y = x^3 + e^x$ ... (1)

For , $x = 0$ from (1) we get,

$y = 0^3 + e^0 = 1$ $\therefore$ the point is (0, 1)

Diff. (1) *w.r.t.x* we get,

$$\frac{dy}{dx} = 3x^2 + e^x$$

slope of tangent = m = $\left(\frac{dy}{dx}\right)_{(0,1)} = 3\,(0)^2 + e^0 = 1$

$\therefore$ slope of the tangent = $m = 1$

Equation of the tangent is given by

$y - y_1 = m\,(x - x_1)$

i.e. $y - 1 = 1\,(x - 0) \Rightarrow y - 1 = x$

Equation of the tangent is $x - y + 1 = 0$

Now, slope of the normal = $m' = -\frac{1}{m} = -1$

Equation of the normal is given by

$y - y_1 = m'\,(x - x_1)$

i.e. $y - 1 = -1\,(x - 0) \Rightarrow y - 1 = -x$

Equation of the normal is $x + y - 1 = 0$

Ex. (2). Oil from an uncapped circular well is oozing outward in the form of a circular film. If the radius of the circle is increasing at the rate of 0.5 meters per minute, how fast is the area of the oil film growing at the instant when the radius is 100m ?

Solution : Let t be time in minutes, R and A be the radius and area of the circlar film, respectively.

We know that, $A = \pi R^2$

Diff. we get,

$$\frac{dA}{dt} = \pi\frac{d}{dt}(R^2) = 2\pi R\frac{dR}{dt} \quad \dots (1)$$

It is known that, $\frac{dR}{dt} = 0.5$ meter/min

How fast the oil film is growing is given by (1)

$$\frac{dA}{dt} = 2\pi\,(R)\,(0.5) = \pi\,R$$

When R = 100 meter we get,

$$\frac{dA}{dt} = 100\pi \text{ sq. meter}$$

asteroid

Ex. (3). Find equation of the tangent and normal to the locus of the astroid given by $x = a\cos^3 t$ and $y = a\sin^3 t$ at the point $t = \frac{\pi}{4}$.

Solution : The parametric equations of the curve are

$x = a\cos^3 t$ ... (1) $\quad y = a\sin^3 t$ ... (2)

When, $t = \frac{\pi}{4}$, ...

from (1), $x_1 = a\cos^3\left(\frac{\pi}{4}\right) = a\left(\cos\frac{\pi}{4}\right)^3 = \frac{a}{2\sqrt{2}}$

from (II), $y_1 = a\sin^3\left(\frac{\pi}{4}\right) = a\left(\sin\frac{\pi}{4}\right)^3 = \frac{a}{2\sqrt{2}}$

Therefore the point is $\left(\frac{a}{2\sqrt{2}}, \frac{a}{2\sqrt{2}}\right)$.

Differentiate (1) *w.r.t.t.* we get,

$\frac{dx}{dt} = a(3)(\cos^2 t)(-\sin t) = -3a(\cos^2 t)(\sin t)$

Differentiate (2) *w.r.t.t.* we get,

$\frac{dy}{dt} = a(3)(\sin^2 t)(\cos t) = 3a(\sin^2 t)(\cos t)$

Now, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3\sin^2 t\cos t}{-3\cos^2 t\sin t} = -\tan t$

Slope of tangent at $t = \frac{\pi}{4}$ = is $m = -\tan\left(\frac{\pi}{4}\right) = -1$

Slope of normal at $t = \frac{\pi}{4}$ is $m' = -\frac{1}{m} \quad -\frac{1}{(-1)} = 1$

Equation of the tangent is given by

$y - y_1 = m(x - x_1)$

$y - \frac{a}{2\sqrt{2}} = (-1)\left(x - \frac{a}{2\sqrt{2}}\right)$

$\Rightarrow y - \frac{a}{2\sqrt{2}} = -x + \frac{a}{2\sqrt{2}} \Rightarrow x + y - \frac{a}{\sqrt{2}} = 0$

Equation of the normal is given by

$y - y_1 = m'(x - x_1)$

$y - \frac{a}{2\sqrt{2}} = (1)\left(x - \frac{a}{2\sqrt{2}}\right)$

$\Rightarrow y - \frac{a}{2\sqrt{2}} = x - \frac{a}{2\sqrt{2}}$

$\Rightarrow (-x) + y - \frac{a}{2\sqrt{2}} + \frac{a}{2\sqrt{2}} = y - x = 0.$

Ex. (4). Find the approximate value of sin 179°. Given 1° = 0.0175°.

Solution : Let $f(x) = \sin x$ ... (I)

Differentiate *w.r.t.x.*

$f'(x) = \cos x$ ..... (II)

Now, 179° = 180° -1°

$= \pi - 0.0175^c$

Let $a = \pi$, $h = -0.0175^c$

For $x = a = \pi$, from (I) we get

$f(a) = f(\pi) = \sin(\pi) = 0$ ... (III)

For , $x = a = \pi$ from (II) we get ... (IV)

We have , $f(a+h) \doteqdot f(a) + hf'(a)$

-0.0175 $f(\pi + (\ldots)) \doteqdot f(\pi) + (-0.0175)\, f'(\pi)$

$f(179^\circ) \doteqdot (\ldots 0 \ldots) - (0.0175)(-1)$ … [From (III) and (IV)]

Note : As θ gets smaller and smaller $\sin\theta \doteqdot \theta$ [we know, $\sin(180-\theta) = \sin\theta$]

$\sin(179^\circ) = \sin(180^\circ - 1^\circ) = \sin 1^\circ \doteqdot 1^\circ \doteqdot 0.0175^c$

Ex. (5). A triangle has two sides a = 1cm and b = 2 cm. How fast is the third side c increasing when the angle α between the given sides is 60° and is increasing at the rate of 3° per second ? [Hint : Use cosine rule]

Solution :

$a = 1\text{cm},\ b = 2\text{cm}$

$\alpha = 60^\circ = \frac{\pi}{3}$

$\frac{d\alpha}{dt} = 3^\circ/\text{sec} = 3 \times \frac{\pi}{180}$ rad/sec

$\frac{d\alpha}{dt} = \frac{\pi}{60}$ rad/sec, $\frac{dc}{dt} = ?$

Using cosine rule

$c^2 = a^2 + b^2 - 2ab\cos\alpha$

$c^2 = 1^2 + 2^2 - 2(1)(2)\cos\frac{\pi}{3}$

$c^2 = 1 + 4 - 4 \times \frac{1}{2}$

$\therefore c^2 = 5 - 2 = 3$

$c = \sqrt{3}$

$c^2 = a^2 + b^2 - 2ab\cos\alpha$

diff w.r.to t.

$\therefore 2c\frac{dc}{dt} = 0 + 0 - 2ab \times -\sin\alpha \frac{d\alpha}{dt}$

where a, b are constant

$\therefore 2c\frac{dc}{dt} = 2ab\sin\alpha\frac{d\alpha}{dt}$

$\therefore \frac{dc}{dt} = \frac{2ab\sin\alpha}{2c}\frac{d\alpha}{dt}$

$= \frac{ab\sin\alpha}{c}\frac{d\alpha}{dt}$

$\therefore \frac{dc}{dt} = \frac{1 \times 2\sin\frac{\pi}{3}}{\sqrt{3}} \times \frac{\pi}{60}$ cm rad/sec

$\therefore \frac{dc}{dt} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{2} \times \frac{\pi}{60}$ cm.rad/sec

$\therefore \frac{dc}{dt} = \frac{\pi}{60}$ cm.rad/sec

Ex. (6). The surface area of a spherical balloon is increasing at the rate of $2\text{cm}^2/\text{sec}$. At what rate is the volume of the balloon is increasing, when the radius of the balloon is 6 cm?

Solution :

Let r, S, V be the radius, surface area, volume of spherical ballon resp.

$\frac{ds}{dt} = 2\ cm^2/sec,\ r = 6\,cm$

$s = 4\pi r^2$

diff w.r.t o r

$\frac{ds}{dt} = 4\pi \times 2r \frac{dr}{dt}$

$\therefore 2 = 8\pi r \frac{dr}{dt}$

$\therefore \frac{2}{8\pi r} = \frac{dr}{dt}$

$\therefore \frac{dr}{dt} = \frac{1}{4\pi r}$ —(I)

$V = \frac{4}{3}\pi r^3$

diff w.r.t o t

$\frac{dv}{dt} = \frac{4}{3}\pi \times 3r^2 \frac{dr}{dt}$

$\therefore \frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\therefore \frac{dv}{dt} = 4\pi r^2 \times \frac{1}{4\pi r}$ from (I)

$\therefore \frac{dv}{dt} = r$

$\therefore \frac{dv}{dt} = 6\ cm^3/sec$

$\therefore$ The volume is increasing at the rate of $6\ cm^3/sec$

Ex. (7). The displacement s of a particle at time t is given by $s = 2t^3 - 5t^2 + 4t - 3$.

Find (i) the time when the acceleration is $14\ cm/sec^2$.

(ii) the velocity and the displacement at that time.

Solution :

$s = 2t^3 - 5t^2 + 4t - 3$

diff w.r.t. t

$v = \frac{ds}{dt} = 6t^2 - 10t + 4$

diff w.r.t t

$a = \frac{d^2s}{dt^2} = 12t - 10$

(i) $a = \frac{d^2s}{dt^2} = 14\ cm/sec^2$

$\therefore 14 = 12t - 10$

$\therefore 14 + 10 = 12t$

$24 = 12t$

$t = 2\ sec$

ii) $v = \frac{ds}{dt} = 6t^2 - 10t + 4$

$v_{t=2} = \left(\frac{ds}{dt}\right)$

$= 6(2)^2 - 10(2) + 4$

$= 24 - 20 + 4$

$= 8\ cm/sec$

$s = 2t^3 - 5t^2 + 4t - 3$

$= 2(2)^3 - 5(2)^2 + 4(2) - 3$

$= 16 - 20 + 8 - 3$

$= 1\ cm$

Sign of Teacher :

Q. 26. A solenoid of length π m and 5 cm in diameter has a winding of 1000 turns and carries a current of 5A. Calculate the magnetic field at its centre along the axis.

SECTION – D

Attempt any THREE questions of the following : **[12]**

Q. 27. What is Ferromagnetism? Explain it on the basis of domain theory.

Q. 28. Obtain an expression for average power dissipated in a series LCR circuit.

Q. 29. Distinguish between interference and diffraction of light.

A double slit arrangement produces interference fringes for sodium light of wavelength 589 nm, that are 0.20 degree apart. What is the angular fringe separation if the entire arrangement is immersed in water?

(R.I. of water = 1.33)

Q. 30. State Einstein's photoelectric equation and mention physical significance of each term involved in it.

The wavelength of incident light is 4000Å. Calculate the energy of incident photon.

Q. 31. State any four uses of Van de Graaff generator.

In a parallel plate air capacitor, intensity of electric field is changing at the rate of 2×10^{11} V/ms. If area of each plate is 20 cm^2, calculate the displacement current.

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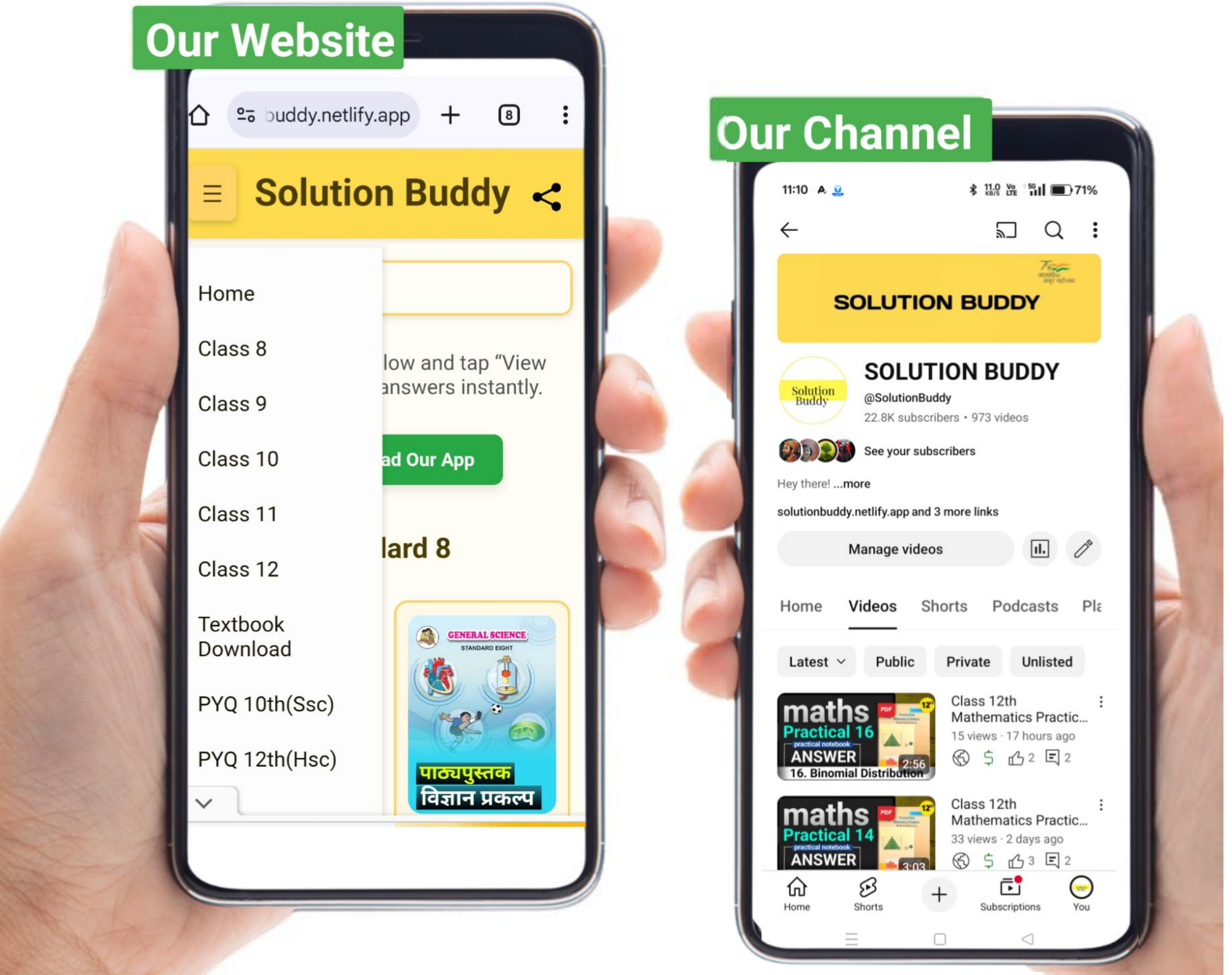

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