

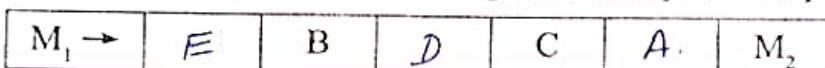
15. Sequencing

A. Activities

- 1) The data given below indicate the processing time in hours of 5 jobs A, B, C, D and E on two machines M_1 , M_2 , M_3

Job	A	B	C	D	E	Total
Machine M_1	4	13	7	12	6	42
Machine M_2	3	15	5	6	11	40

Using the optimal sequence along them, the optimal sequence is



The working table is

Job	Machine M_1		Machine M_2	
	In time	Out time	In time	Out time
E	0	6	6	17
	6	19	19	34
D	19	31	34	40
	31	38	40	45
A	38	42	45	48

$$\therefore \text{Total elapsed time} = 48 \text{ hrs.}$$

Idle time for machines

$$M_1 = 48 - 42 = 6 \text{ hrs.}$$

$$M_2 = 48 - 40 = 8 \text{ hrs.}$$

- 2) Given five jobs, each of which is to be processed through three machines A, B and C in the order ABC. Processing time in hours are

Job	1	2	3	4	5	Total
A	3	8	7	5	4	27

B	4	5	1	2	3	15
C	7	9	5	6	10	37

$$\text{Min}(A) = \boxed{3}, \text{Max}(B) = \boxed{5}, \text{Min}(C) = \boxed{5}$$

since $\text{Min}(C) = \boxed{5} \geq \text{Max}(B)$ is satisfied, the problem can be converted into 5 jobs and 2 machine problem. Consider two fictitious machines G and H

Such that

$$G = A + \boxed{B}$$

$$H = \boxed{B} + C$$

The problem is

Job	1	2	3	4	5
G	7	13	8	7	7
H	11	14	6	8	13

Using optimal sequence algorithm. We get following optimal sequence.

4	1	5	2	3.
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The working table is

Job	Machine A		Machine B		Machine C	
	In time	Out time	In time	Out time	In time	Out time
4	0	5	5	7	7	13
	5	8	8	12	13	20
5	8	12	12	15	20	30
	12	20	20	25	30	39
3	20	27	27	28	39	44

$$\text{Minimum elapsed time} = \boxed{44} \text{ hr}$$

Idle time for machines

$$A = 44 - \boxed{27} = \boxed{17} \text{ hrs}$$

$$B = \boxed{44} - 15 = 29 \text{ hrs}$$

$$C = 44 - \boxed{37} = 7 \text{ hrs}$$

B. Solve the Following

- Q.L. There are six jobs each of which is to be processed through the machines M_1 and M_2 , in the order M_1, M_2 . The processing time is given in hours. Determine the sequence of these jobs which minimizes the total elapsed time and the minimum elapsed time. Also find the idle times for the machines M_1 and M_2 :

Jobs	1	2	3	4	5	6
Processing time on M_1	5	9	4	7	8	5
Processing time on M_2	8	4	8	3	6	6

$\text{Min}(M_{12}, M_{21}) = 3$, which corresponds to M_2 . \therefore Job 4 is.....

Processed in last.....

				4
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Problem reduce to 5 jobs.....

$\text{Min}(M_{12}, M_{21}) = 4$, which.....

covers to M_1 & M_2

3			2	4
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The Problem reduce to 3 jobs.....

$\text{Min}(M_{12}, M_{21}) = 5$, which.....

corresponds to M_1

1	6		2	4
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6	1		2	4
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The remaining must be.....

Processed next to job 6 or job 1.....

3	1	6	5	2	4
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3	6	1	5	2	4
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Total elapsed time \Rightarrow

Job Sequence	M_1		M_2		Idle-time for M_2
	In	Out	In	Out	
3	0	4	4	12	4
1	4	9	12	20	0
6	9	14	20	26	0
5	14	22	26	32	0
2	22	31	32	36	0
4	31	38	38	41	2

The min total elapsed time $T = 41$ hrs.....

Idle time for $M_1 = 41 - 38 = 3$ hrs.....

Idle time for $M_2 = 4 + 2 = 6$ hrs.....

- Q.2. Find the sequence that minimizes the total elapsed time required to complete the following tasks. The table below gives the processing time in hours. Also find the minimum elapsed time and idle times for both the machines:

Jobs	1	2	3	4	5
M_1	3	7	4	5	7
M_2	6	2	7	3	4

$\text{Min}(M_{12}, M_{12}) = 2$, which corresponds to M_2 . So Job 2 is processed last.

Problem reduce to 4 jobs. Here $\text{Min}(M_{12}, M_{12}) = 3$, which corresponds to both M_1 and M_2 .

1			4	2
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Now Problem reduce to 2 jobs.

$\text{Min}(M_{12}, M_{12}) = 4$, which corresponds to both M_1 & M_2 .

Job 3 is processed first next to Job 1 and Job 5 is processed last next to

Job 4. Thus the optimal seq is:

1	3	5	4	2
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Total elapsed time \Rightarrow

Job Sequence	M ₁		M ₂		Idle time for M ₂
	In	Out	In	Out	
1	0	3	3	9	3
3	3	7	9	16	0
5	7	14	16	20	0
4	14	19	20	23	3
2	19	26	26	28	0

Total elapsed time $T = 28$ hours

Idle time for M₂ = 6 Hours.

Idle time for M₁

$$= T - \{ \text{sum of Pro. time of all five jobs on } M_1 \}$$

$$= 28 - 26$$

$$= 2 \text{ hours.}$$

Q.3. Suppose there are five jobs each must go through machines M₁, M₂ and M₃ in the order M₁M₂M₃. Processing times in hours are given below. Determine the optimal sequence and total elapsed time. Also find the idle times for three machines:

Jobs	1	2	3	4	5
M ₁	8	5	4	6	5
M ₂	6	2	9	7	4
M ₃	10	13	11	10	12

Here min M₁ = 4, Max M₂ = 9, Min M₃ = 10.

Since $\text{Min } M_3 \geq \text{Max } M_2$, is satisfied, the problem.

can be converted into two machines problem.

Let G and H be two fictitious machines such

that $G = M_1 + M_2$ and $H = M_2 + M_3$. Then the

problem written as.

Jobs	1	2	3	4	5
G	14	7	13	13	9.
H	16	15	20	17	16

Observe that $\text{Min}(G, H) = 7$ corresponds to job 2 on G

∴ Job 2 is placed first in sequence. [2]

then sequence problem reduced to:

Now,

$\text{Min}(G, H) = 9$ corresponds to job 5 [2 5]

on machine G which is placed in second in sequence. then Problem reduced to:

Jobs	1	3	4	5
G	14	13	13	9
H	16	20	17	16

Now, $\text{Min}(G, H) = 13$ corresponds to job 3 on machine G as well as job 4 on remaining job 1 placed last in the sequence. we consider the optimal sequence as: [2 5 3 4 1]

Total elapsed time :-

Jobs	M ₁		M ₂		M ₃	
	In	Out	In	Out	In	Out
2(5, 2, 13)	0	5	5	7	7	20
5(5, 4, 12)	5	10	10	14	20	32
3(4, 9, 11)	10	14	14	23	32	43
4(6, 7, 10)	14	20	23	30	43	53
1(8, 6, 10)	20	28	30	36	53.	63.

Total elapsed time (T) = 63 hours

Idle time for M₁ = 63 - 28 = 35 hours.

Idle time for M₂ = (63 - 36) + 5 + 3 = 35 hours.

Idle time for M₃ = 7 hours.

Sign of Teacher :