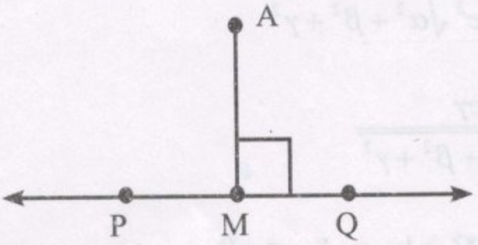
7. Line and Plane

Ex. (1) Find the coordinates of the foot of the perpendicular drawn from $A(1,2,1)$ to the line joining $P(1,4,6)$ and $Q(5,4,4)$.

Solution : Let M be the foot of the perpendicular drawn from A to line PQ.



Let $k : 1$ be the ratio in which M divides PQ.

$\therefore \quad M \equiv \left(\dfrac{5k+1}{k+1}, \dfrac{4k+4}{k+1}, \dfrac{4k+6}{k+1}\right)$

$\therefore$ Direction ratios of AM are $\dfrac{5k+1}{k+1}-1, \dfrac{4k+4}{k+1}-2, \dfrac{4k+6}{k+1}-1$

i. e. $\dfrac{4k}{k+1}, \dfrac{2k+2}{k+1}, \dfrac{3k+5}{k+1}$

And the direction ratios of PQ are $4,0,-2$

As $AM \perp PQ$, $(4)\times\dfrac{4k}{k+1}+(0)\times\dfrac{2k+2}{k+1}+(-2)\times\dfrac{3k+5}{k+1}=0$

$\therefore \quad 16k-6k-10=0$

$\therefore \quad k=1$

The co-ordinates of M are $(3,4,5)$.

AM $= \sqrt{(3-1)^2+(4-2)^2+(5-1)^2} = \sqrt{4+4+16} = \sqrt{24} = 2\sqrt{6}$ unit.

Ex. (2) If θ is the angle between the line $\dfrac{x-x_1}{a}=\dfrac{y-y_1}{b}=\dfrac{z-z_1}{c}$ and the plane $\alpha x+\beta y+\gamma z+\delta=0$ then prove that

$$sin\theta = \frac{a\times\alpha+b\times\beta+c\times\gamma}{\sqrt{a^2+b^2+c^2}\sqrt{\alpha^2+\beta^2+\gamma^2}}.$$

Hence find the angle between the line $x=y=z$ and the XY plane.

Solution : $\dfrac{x-x_1}{a} = \dfrac{y-y_1}{b} = \dfrac{z-z_1}{c} \quad \ldots\ldots(1)$

$ax+\beta y+\gamma z+\delta=0 \quad \ldots\ldots(2)$

As the angle between line and the plane is θ, the angle between line and the normal to the plane is $\frac{\pi}{2}-\theta$. Note that $\frac{\pi}{2}-\theta$ is an acute angle.

The direction ratios of line are a,b,c and that of the normal are α,β,γ

$$\therefore \quad \cos\left(\frac{\pi}{2}-\theta\right)=\frac{a\alpha+b\beta+c\gamma}{\sqrt{a^2+b^2+c^2}\sqrt{\alpha^2+\beta^2+\gamma^2}}$$

$$\therefore \quad \sin\theta=\frac{a\alpha+b\beta+c\gamma}{\sqrt{a^2+b^2+c^2}\sqrt{\alpha^2+\beta^2+\gamma^2}}$$

Now the equation of XY plane is $z=0$

Let the angle between line $x=y=z$ and the XY plane be θ.

$$\therefore \quad \sin\theta=\frac{a\alpha+b\beta+c\gamma}{\sqrt{a^2+b^2+c^2}\sqrt{\alpha^2+\beta^2+\gamma^2}}=\frac{1(0)+1(0)+1(1)}{\sqrt{1^2+1^2+1^2}\sqrt{0^2+0^2+1^1}}=\frac{1}{3}$$

$$\therefore \quad \theta=\sin^{-1}\left(\frac{1}{3}\right)$$

Ex. (3) Find the ratio in which XY plane divides the line joining $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $B(-4,31,-2)$. Find the vector equation of line AB.

Solution : Let k : 1 be the ratio in which XY plane divides the line joining $A(1,28,1)$ and $B(-4,31,-2)$.

Let (h, k, 0) be the point in which XY plane cuts the line AB.

$$\therefore \quad 0=\frac{kz_2+z_1}{k+1}$$

$$\therefore \quad kz_2+z_1=\ldots 0$$

$$\therefore \quad k=-\frac{z_1}{z_2}=\ldots -\left(\frac{1}{-2}\right)=\frac{1}{2}$$

$\therefore$ XY plane divides AB internally in the ratio … 1 : … 2

Vector along AB is $-5\hat{i}+3\hat{j}-3\hat{k}$

$\bar{a}=\hat{i}+28\hat{j}+\hat{k}$

The equation line AB is $\bar{r}=(\ldots\hat{i}+28\hat{j}+\hat{k}\ldots)+\lambda\left(-5\hat{i}+3\hat{j}-3\hat{k}\right)$

Ex. (4) Obtain coordinates of points on the line $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$, which are at 6 unit distance from the origin.

Solution : Let $P(2k, 2k, k)$ be a point on the line $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ which is at 6 unit distance from the origin.

$OP = 6$

$\therefore \quad OP^2 = 36$

$\therefore \quad (2k)^2 + (2k)^2 + (k)^2 = 36$

$\therefore 9k^2 = 36$

$\therefore \quad k = \pm 2$

$\therefore$ There are two points on the given line which are at 6 unit distance from the origin.

Their co-ordinates are $(4, 4, 2)$ and $(-4, -4, -2)$.

Ex. (5) Find the vector and Cartesian equation of the plane passing through the points $A(2,3,1)$, $B(4,-5,3)$ and parallel to X-axis.

Solution :

Since the plane is $\perp$ to x-axis, its normal is $\perp$ to x-axis. The unit vector along x-axis is $\hat{i}$. Hence the normal vector to the plane $\perp$ to $\hat{i}$

Let $A(2,3,1)$, $B(4,-5,3)$ be the point on the plane

$\therefore \bar{a} = 2\hat{i}+3\hat{j}+\hat{k}$, $\bar{b} = 4\hat{i}-5\hat{j}+3\hat{k}$

$\overline{AB} = \bar{b}-\bar{a} = 4\hat{i}-5\hat{j}+3\hat{k}-2\hat{i}-3\hat{j}-\hat{k}$

$= 2\hat{i}-8\hat{j}+2\hat{k}$

The normal vector to the plane perpendicular to $\overline{AB}$

Since $\hat{i}\times\overline{AB}$ is $\perp$ to both $\hat{i}$ and $\overline{AB}$, it is normal vector to the plane and it is

$\bar{n} = \hat{i}\times\overline{AB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 2 & -8 & 2 \end{vmatrix}$

$= -2\hat{j}-8\hat{k}$

The vector equation of the plane passing through $A(\bar{a})$ and $\perp$ to $\bar{n}$ is $\bar{r}\cdot\bar{n} = \bar{a}\cdot\bar{n}$

$\therefore \bar{r}(-2\hat{j}-8\hat{k}) = (2\hat{i}+3\hat{j}+\hat{k})\cdot(-2\hat{j}-8\hat{k})$

$\therefore \bar{r}(-2\hat{j}-8\hat{k}) = 0-6-8$

$\therefore \bar{r}(-2\hat{j}-8\hat{k}) = -14$

$\therefore \bar{r}(\hat{j}+4\hat{k}) = 7$

If $\bar{r} = x\hat{i}+y\hat{j}+z\hat{k}$

$\therefore (x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{j}+4\hat{k}) = 7$

$\therefore y+4z = 7$

This is required equation.

Ex. (6) Find the vector equation of the plane passing through the origin and containing the line $r=\left(\hat{i}+4\hat{j}+\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+\hat{k}\right)$

Solution :

Given line is $\bar{r}=(\hat{i}+4\hat{j}+\hat{k})+\lambda(\hat{i}+2\hat{j}+\hat{k})$ —— (I)

Comparing with $\bar{r}=\bar{a}+\lambda\bar{b}$

$\therefore \bar{a}=\hat{i}+4\hat{j}+\hat{k}$, $\bar{b}=\hat{i}+2\hat{j}+\hat{k}$

since required plane passing through origin and containing line (I)

$\therefore$ required plane is $\parallel$ to $\overline{OA}$ and $\bar{b}$

$\bar{n}$ be normal to the plane

$$\therefore \bar{n}=\overline{OA}\times\bar{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 4 & 1\\ 1 & 2 & 1\end{vmatrix}$$

$\therefore \bar{n}=2\hat{i}+0\hat{j}-2\hat{k}=2\hat{i}-2\hat{k}$

$\therefore$ required vector equation of plane is

$\bar{r}\cdot\bar{n}=\bar{0}\cdot\bar{n}$

$\bar{r}\cdot(2\hat{i}-2\hat{k})=0$

$\boxed{\therefore\ \bar{r}(\hat{i}-\hat{k})=0}$

Ex. (7) Find the vector equation of the plane which bisects the segment joining $A(2,3,6)$ and $B(4,3,-2)$ at right angle.

Solution : We have $A(2,3,6)$ and $B(4,3,-2)$

Let $\bar{a}$ and $\bar{b}$ be p.v. of A and B resp.

$\bar{a} = 2\hat{i}+3\hat{j}+6k$, $\bar{b} = 4\hat{i}+3\hat{j}-2\hat{k}$

Let point $C(\bar{c})$ be midpoint of seg AB

$\therefore \bar{c} = \dfrac{\bar{a}+\bar{b}}{2} = \dfrac{2\hat{i}+3\hat{j}+6\hat{k}+4\hat{i}+3\hat{j}-2\hat{k}}{2}$

$= \dfrac{6\hat{i}+6\hat{j}+4\hat{k}}{2} = 3\hat{i}+3\hat{j}+2\hat{k}$

Since required plane bisects the seg AB at right angle

$\therefore$ AB is $\perp$ to required plane

Let $\bar{n}$ be normal to required plane

$\therefore \bar{n} = \overline{AB} = \bar{b}-\bar{a} = 4\hat{i}+3\hat{j}-2\hat{k}-2\hat{i}-3\hat{j}-6\hat{k} = 2\hat{i}-8\hat{k}$

$\therefore$ The vector equation of the plane passing through the point $C(\bar{c})$ is given by

$\bar{r}\cdot\bar{n} = \bar{c}\cdot\bar{n}$

$\bar{r}\cdot(2\hat{i}-8\hat{k}) = (3\hat{i}+3\hat{j}+2\hat{k})\cdot(2\hat{i}+0\hat{j}-8\hat{k})$

$\therefore \bar{r}\cdot(2\hat{i}-8\hat{k}) = 3(2)+3(0)+2(-8)$

$\therefore \bar{r}\ (2\hat{i}-8\hat{k}) = 6+0-16$

$\therefore \bar{r}\ (2\hat{i}-8\hat{k}) = -10$

$\therefore \bar{r}\ (\hat{i}-4\hat{k}) = -5$

This is the required vector equation of plane

Sign of Teacher :

Q. 26. A solenoid of length π m and 5 cm in diameter has a winding of 1000 turns and carries a current of 5A. Calculate the magnetic field at its centre along the axis.

SECTION – D

Attempt any THREE questions of the following : **[12]**

Q. 27. What is Ferromagnetism? Explain it on the basis of domain theory.

Q. 28. Obtain an expression for average power dissipated in a series LCR circuit.

Q. 29. Distinguish between interference and diffraction of light.

A double slit arrangement produces interference fringes for sodium light of wavelength 589 nm, that are 0.20 degree apart. What is the angular fringe separation if the entire arrangement is immersed in water?

(R.I. of water = 1.33)

Q. 30. State Einstein's photoelectric equation and mention physical significance of each term involved in it.

The wavelength of incident light is 4000Å. Calculate the energy of incident photon.

Q. 31. State any four uses of Van de Graaff generator.

In a parallel plate air capacitor, intensity of electric field is changing at the rate of 2×10^{11} V/ms. If area of each plate is 20 cm^2, calculate the displacement current.

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