5 Straight Line

Let's Study

- Locus of a points in a co-ordinate plane
- Equations of line in different forms
- Angle between two lines, perpendicular and parallel lines
- Distance of a point from a line
- Family of lines

Let's Recall

We are familiar with the properties of straight lines, the bisector of an angle, circle and triangles etc.

We will now introduce co-ordinate geometry in the study of a plane. Every point has got pair of co-ordinates and every pair of co-ordinate gives us a point in the plane.

We will use this and study the curves with the help of co-ordinates of the points.

What is the perpendicular bisector of a segment ? .... A line.

What is the bisector of an angle ? ....A ray.

These geometrical figures are sets of points in plane which satisfy certain conditions.

- The perpendicular bisector of a segment is the set of points in the plane which are equidistant from the end points of the segment. This set is a line.
- The bisector of an angle is the set of points in the plane which are equidistant from the arms of the angle. This set is a ray.

Activity : Draw segment AB of length 6 cm. Plot a few points which are equidistant from A and B. Verify that they are collinear.

Let's Learn

5.1 Locus : A set of points in a plane which satisfy certain geometrical condition (or conditions) is called a locus.

L = {P | P is a point in the plane and P satisfies given geometrical condition}

Here P is the representative of all points in L. L is called the *locus* of point P. Locus is a set of points.

The locus can also be described as the route of a point which moves while satisfying required conditions. eg. planets in solar system.

Illustration :

- The perpendicular bisector of segment AB is the set

  M = { P | P is a point such that PA=PB}.
- The bisector of angle AOB is the set :

  D = { P | P is a point such that P is equidistant from OA and OB }

  = { P | $\angle$ POA = $\angle$ POB }

Verify that the sets defined above are the same.

The plural of locus is loci.

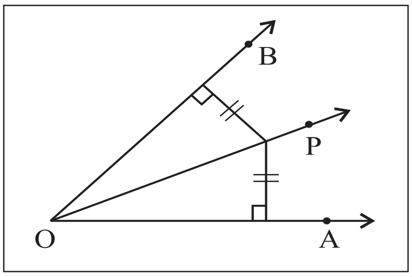


Fig. 5.1

- The circle with center O and radius 4 is the set L = { P | OP = 4}

5.1.1 Equation of Locus : If the set of points, whose co-ordinates satisfy a certain equation in x and y, is the same as the set of points on a locus, then the equation is said to be the equation of the locus.

SOLVED EXAMPLES

Ex.1 We know that the y co-ordinate of every point on the X-axis is zero and this is true for points on the X-axis only. Therefore the equation of the X-axis is $y = 0$.

Ex.2 Let L = {P | OP = 4}. Find the equation of L.

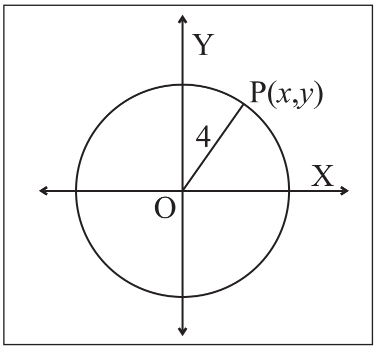


Fig. 5.2

Solution : L is the locus of points in the plane which are at 4 unit distance from the origin.

Let P(x, y) be any point on the locus L.

As OP = 4, $OP^2 = 16$

$\therefore\ (x - 0)^2 + (y - 0)^2 = 16$

$\therefore\ x^2 + y^2 = 16$

This is the equation of locus L.

The locus is seen to be a circle

Ex.3 Find the eqation of the locus of points which are equidistant from A(–3, 0) and B(3, 0). Identify the locus.

Solution : Let P(x, y) be any point on the required locus.

P is equidistant from A and B.

$\therefore$ PA = PB

$\therefore\ PA^2 = PB^2$

$\therefore\ (x + 3)^2 + (y - 0)^2 = (x - 3)^2 + (y - 0)^2$

$\therefore\ x^2 + 6x + 9 + y^2 = x^2 - 6x + 9 + y^2$

$\therefore\ 12x = 0$

$\therefore\ x = 0$. The locus is the Y-axis.

5.1.2 Shift of Origin : Let O′(h, k) be a point in the XY plane and the origin be shifted to O′. Let O′X′, O′Y′ be the new co-ordinate axes through O′ and parallel to the axes OX and OY respectively.

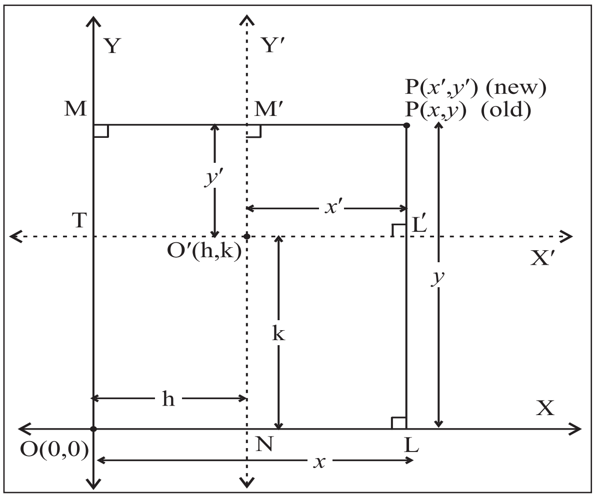


Fig. 5.3

Let (x, y) be the co-ordinates of P referred to the co-ordinates axes OX, OY and (x', y') be the co-ordinates of P referred to the co-ordinate axes O′X′, O′Y′. To find relations between (x, y) and (x', y').

Draw PL ⊥ OX and suppose it intersects O′X′ in L′.

Draw PM ⊥ OY and suppose it intersects O′Y′ in M′.

Let O′Y′ meet line OX in N and O′X′ meet OY in T.

$\therefore$ ON = h, OT = k, OL = x, OM = y,

O′L′ = x', O′M′ = y'

Now x = OL = ON + NL = ON + O′ L′

$= h + x'$

and $y = OM = OT + TM = OT + O'M' = k + y'$

$\therefore x = x' + h, \ y = y' + k$

These equations are known as the formulae for shift of origin.

Note that the new co-ordinates can also be given by (X, Y) or (u,v) in place of (x',y').

SOLVED EXAMPLES

Ex. 1) If the origin is shifted to the point O'(3, 2) the directions of the axes remaining the same, find the new co-ordinates of the points

(a)A(4, 6) (b) B(2,–5).

Solution : We have $(h, k) = (3,2)$

$x = x' + h, \ y = y' + k$

$\therefore x = x' + 3.$ and $y = y' + 2$ .... (1)

(a) $(x, y) = (4, 6)$

$\therefore$ From (1), we get $4 = x^1 + 3, \ 6 = y^1 + 2$

$\therefore x' = 1$ and $y' = 4$.

New co-ordinates of A are (1, 4)

(ii) $(x, y) = (2,-5)$

from (1), we get $2 = x' + 3, \ -5 = y' + 2$

$\therefore x' = -1$ and $y' = -7$. New co-ordinates of B are (–1,–7)

Ex. 2) The origin is shifted to the point (–2, 1), the axes being parallel to the original axes. If the new co-ordinates of point A are (7, –4), find the old co-ordinates of point A.

Solution : We have $(h, k) = (-2, 1)$ and if new co-ordinates are (X,Y).

$x = X + h, \ y = Y + k$

$\therefore x = X - 2, \ y = Y + 1$

$(X, Y) = (7, -4)$

we get $x = 7 - 2 = 5, \quad y = -4 + 1 = -3.$

$\therefore$ Old co-ordinates A are (5, –3)

Ex. 3) Obtain the new equation of the locus $x^2 - xy - 2y^2 - x + 4y + 2 = 0$ when the origin is shifed to (2, 3), the directions of the axes remaining the same.

Solution : Here $(h, k) = (2, 3)$ and if new co-ordinates are (X,Y).

$\therefore x = X + h, \quad y = Y + k$ gives

$\therefore x = X + 2, \quad y = Y + 3$

The given equation

$x^2 - xy - 2y^2 - x + 4y + 2 = 0$ becomes

$(X+2)^2 - (X + 2)(Y + 3) - 2(Y + 3)^2 - (X + 2) + 4(Y + 3) + 2 = 0$

$\therefore X^2 - XY - 2Y^2 - 10Y - 8 = 0$

This is the new equation of the given locus.

EXERCISE 5.1

1. If A(1,3) and B(2,1) are points, find the equation of the locus of point P such that PA = PB.

2. A(–5, 2) and B(4, 1). Find the equation of the locus of point P, which is equidistant from A and B.

3. If A(2, 0) and B(0, 3) are two points, find the equation of the locus of point P such that AP = 2BP.

4. If A(4, 1) and B(5, 4), find the equation of the locus of point P if $PA^2 = 3PB^2$.

5. A(2, 4) and B(5, 8), find the equation of the locus of point P such that $PA^2 - PB^2 = 13$.

6. A(1, 6) and B(3, 5), find the equation of the locus of point P such that segment AB subtends right angle at P. ($\angle APB = 90°$)

7. If the origin is shifted to the point O′(2, 3), the axes remaining parallel to the original axes, find the new co-ordinates of the points

   (a) A(1, 3) (b) B(2, 5)

8. If the origin is shifted to the point O′(1, 3) the axes remaining parallel to the original axes, find the old co-ordinates of the points

(a) C(5, 4) (b) D(3, 3)

9. If the co-ordinates A(5, 14) change to B(8, 3) by shift of origin, find the co-ordinates of the point where the origin is shifted.

10. Obtain the new equations of the following loci if the origin is shifted to the point O'(2, 2), the direction of axes remaining the same :

(a) $3x - y + 2 = 0$

(b) $x^2 + y^2 - 3x = 7$

(c) $xy - 2x - 2y + 4 = 0$

(d) $y^2 - 4x - 4y + 12 = 0$

5.2 Straight Line : The simplest locus in a plane is a line. The characteristic property of this locus is that if we find the slope of a segment joining any two points on this locus, then the slope is constant.

If a line meets the X-axis in the point A (a, 0), then 'a' is called the X-intercept of the line. If it meets the Y-axis in the point B(0, b) then 'b' is called the Y-intercept of the line.

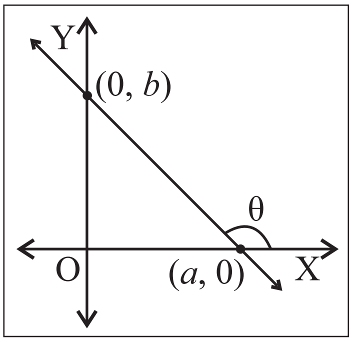


Fig. 5.4

Remarks :

(1) A line parallel to X-axis has no X-intercept.

(2) A line parallel to Y-axis has no Y-intercept.

Let's Think :

- Can intercept of a line be zero ?
- Can intercept of a line be negative ?

5.1.2 Inclination of a line : The smallest angle made by a line with the positive direction of the X-axis measured in anticlockwise sense is called the inclination of the line. We denote inclination by θ. Clearly $0° \leq \theta < 180°$.

Remark : Two lines are parallel if and only if they have the same inclination.

The inclination of the X-axis and a line parallel to the X-axis is Zero. The inclination of the Y-axis and a line parallel to the Y-axis is 90^0.

5.2.2 Slope of a line : If the inclination of a line is θ then $\tan\theta$ (if it exist) is called the slope of the line. We denote it by m. $\therefore\ m = \tan\theta$.

Activity : If A(x_1, y_1), B(x_2, y_2) are any two points on a non-vertical line whose inclination is θ then verify that

$$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_1 \neq x_2.$$

The slope of the Y-axis is not defined. Similarly the slope of a line parallel to the Y-axis is not defined. The slope of the X-axis is 0. The slope of a line parallel to the X-axis is also 0.

Remark : Two lines are parallel if and only if they have the same slope.

SOLVED EXAMPLES

Ex. 1) Find the slope of the line whose inclination is 60°.

Solution : The tangent ratio of the inclination of a line is called the slope of the line.

Inclination $\theta = 60°$.

$\therefore$ slope $= \tan\theta = \tan 60° = \sqrt{3}$.

Ex. 2) Find the slope of the line which passes through the points A(2, 4) and B(5, 7).

Solution : The slope of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Slope of the line AB $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7-4}{5-2} = 1$

Note that $x_1 \neq x_2$.

Ex. 3) Find the slope of the line which passes through the origin and the point A(-4, 4).

Solution : The slope of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Here A(–4, 4) and O(0, 0).

Slope of the line OA $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{0-4}{0+4} = -1.$

Note that $x_1 \neq x_2$.

5.2.3 Perpendicular Lines : We know that the co-ordinate axes are perpendicular to each other. Similarly a horizontal line and a vertical line are perpendicular to each other. Slope of one of them is zero whereas the slope of the other one is not defined.

Let us obtain a relation between slopes of non-vertical lines.

Theorem : Non-vertical lines having slopes m_1 and m_2 are perpendicular to each other if and only if $m_1 \times m_2 = -1$.

Proof : Let α and β be inclinations of lines having slopes m_1 and m_2. As lines are non vertical $\alpha \neq \dfrac{\pi}{2}$ and $\beta \neq \dfrac{\pi}{2}$

$\therefore \tan\alpha = m_1$ and $\tan\beta = m_2$

From Fig. 5.5 and 5.6 we have,

$\alpha - \beta = 90^0$ or $\alpha - \beta = -90^0$

$\alpha - \beta = \pm 90^0$

$\therefore \cos(\alpha - \beta) = 0$

$\therefore \cos\alpha \cos\beta + \sin\alpha \sin\beta = 0$

$\therefore \sin\alpha \sin\beta = -\cos\alpha \cos\beta$

$\therefore \tan\alpha \tan\beta = -1$

$\therefore m_1 m_2 = -1$

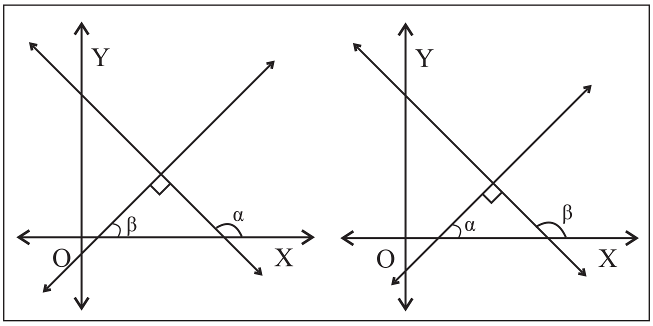


Fig. 5.5 Fig. 5.6

SOLVED EXAMPLES

Ex. 1) Show that line AB is perpendicular to line BC, where A(1, 2), B(2, 4) and C(0, 5).

Solution : Let slopes of lines AB and BC be m_1 and m_2 respectively.

$\therefore m_1 = \dfrac{4-2}{2-1} = 2$ and

$m_2 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-4}{0-2} = -\dfrac{1}{2}$

Now $m_1 \times m_2 = 2 \times \left(-\dfrac{1}{2}\right) = -1$

$\therefore$ Line AB is perpendicular to line BC.

Ex. 2) A(1,2), B(2,3) and C(–2,5) are vertices of Δ ABC. Find the slope of the altitude drawn from A.

Solution : The slope of line BC is

$m_1 = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{5-3}{-2-2} = -\dfrac{2}{4} = -\dfrac{1}{2}$

Altitude drawn from A is perpendicular to BC.

If m_2 is the slope of the altitude from A then $m_1 \times m_2 = -1$.

$$\therefore\ m_2 = \frac{-1}{m_1} = 2.$$

The slope of the altitude drawn from A is 2.

5.2.4 Angle between intersecting lines :

We have obtained relation between slopes of lines which are perpendicualr to each other. If given lines are not perpendicular to each other then how to find angle between them? Let us derive formula to find the acute angle between intersecting lines.

Theorem : If θ is the acute angle between non-vertical lines having slopes m_1 and m_2 then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Proof : Let α and β be the inclinations of non-vertical lines having slopes m_1 and m_2. $\alpha \neq 90°$ and $\beta \neq 90°$.

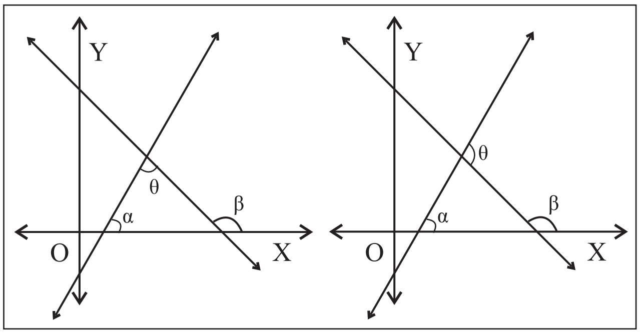


Fig. 5.7 Fig. 5.8

$\therefore\ \tan\alpha = m_1$ and $\tan\beta = m_2$

From Fig. 5.7 and 5.8, we observe that

$\theta = \beta - \alpha$ or $\theta = \pi - (\beta - \alpha)$

$\therefore\ \tan\theta = \tan(\beta - \alpha)$ or $\tan\theta = \tan\{\pi - (\beta - \alpha)\}$
$= -\tan(\beta - \alpha)$

$\therefore\ \tan\theta = |\tan(\beta - \alpha)| = |\tan(\alpha - \beta)|$

$$\therefore\ \tan\theta = \left|\frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}\right| = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Note that as θ is the acute angle, lines are not perpendicular to each other. Hence $m_1\, m_2 \neq -1$.

$\therefore\ 1 + m_1\, m_2 \neq 0$

SOLVED EXAMPLES

Ex. 1) Find the acute angle between lines having slopes 3 and −2.

Solution : Let $m_1 = 3$ and $m_2 = -2$.

Let θ be the acute angle between them.

$$\therefore\ \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{3-(-2)}{1+(3)(-2)}\right| = \left|\frac{5}{-5}\right| = 1$$

$\therefore\ \theta = 45°$

The acute angle between lines having slopes 3 and −2 is 45°.

Ex. 2) If the angle between two lines is 45° and the slope of one of the lines is $\frac{1}{2}$, find the slope of the other line.

Solution : If θ is the acute angle between lines having slopes m_1 and m_2 then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Given $\theta = 45°$.

Let $m_1 = \frac{1}{2}$. Let m_2 be the slope of the other line.

$$\tan 45° = \left|\frac{\frac{1}{2} - m_2}{1 + \left(\frac{1}{2}\right)m_2}\right| \qquad \therefore\ 1 = \left|\frac{1 - 2m_2}{2 + m_2}\right|$$

$$\therefore\ \frac{1-2m_2}{2+m_2} = 1 \quad \text{or} \quad \frac{1-2m_2}{2+m_2} = -1$$

$$\therefore\ m_2 = 3 \quad \text{or} \quad -\frac{1}{3}$$

There are two lines which satisfy the given conditions.

EXERCISE 5.2

1. Find the slope of each of the following lines which passes through the points :
   (a) A(2,–1), B(4,3) (b) C(–2,3), D(5,7)
   (c) E(2,3), F(2,–1) (d) G(7,1), H(–3,1)

2. If the X and Y-intercepts of line L are 2 and 3 respectively then find the slope of line L.

3. Find the slope of the line whose inclination is 30°.

4. Find the slope of the line whose inclination is $\frac{\pi}{4}$.

5. A line makes intercepts 3 and 3 on the co-ordiante axes. Find the inclination of the line.

6. Without using Pythagoras theorem show that points A(4,4), B(3, 5) and C(–1, –1) are the vertices of a right angled triangle.

7. Find the slope of the line which makes angle of 45° with the positive direction of the Y-axis measured anticlockwise.

8. Find the value of k for which points P(k,–1),Q(2,1) and R(4,5) are collinear.

9. Find the acute angle between the X-axis and the line joining points A(3,–1) and B(4,–2).

10. A line passes through points A(x_1, y_1) and B(h, k). If the slope of the line is m then show that $k - y_1 = m(h - x_1)$.

11. If points A(h, 0), B(0, k) and C(a, b) lie on a line then show that $\frac{a}{h}+\frac{b}{k}=1$.

5.3 Equation of line in standard forms : An equation in x and y which is satisfied by the co-ordinates of all points on a line and no other points is called the equation of the line.

The y co-ordinate of every point on the X-axis is 0 and this is true only for points on the X-axis. Therefore, the equation of the X-axis is $y = 0$

The x co-ordiante of every point on the Y-axis is 0 and this is true only for points on the Y-axis. Therefore, the equation of the Y-axis is $x = 0$.

The equation of any line parallel to the Y-axis is of the type $x = k$ (where k is a constant) and the equation of any line parallel to the X-axis is of the type $y = k$. This is all about vertical and horizontal lines.

Let us obtain equations of non-vertical and non -horizontal lines in different forms:

5.3.1 Point-slope Form : To find the equation of the line having slope m and which passes through the point A(x_1, y_1).

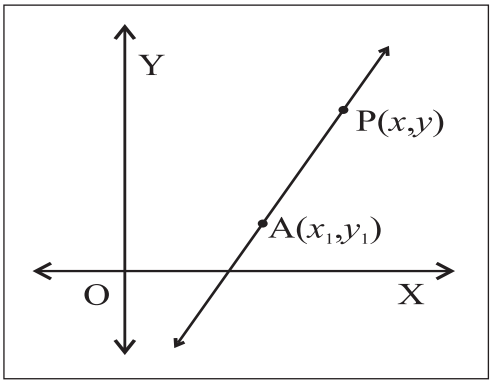


Fig. 5.9

Proof : Let L be the line passing through the point A(x_1, y_1) and which has slope m.

Let P(x, y) be any point on the line L other than A.

Then the slope of line L = $\frac{y-y_1}{x-x_1}$.

But the slope of line L is m. (given)

$\therefore \frac{y-y_1}{x-x_1} = m$

$\therefore$ The equation of the line L is

$$(y - y_1) = m \quad (x - x_1)$$

The equation of the line having slope m and passing through $A(x_1, y_1)$ is $(y - y_1) = m\ (x - x_1)$.

Remark : In particular if the line passes through the origin O(0,0) and has slope m, then its equation is $y - 0 = m\ (x - 0)$

$\therefore\ y = mx$

Ex. Find the equation of the line passing through the point A(2, 1) and having slope −3.

Soln. : Given line passes through the point A(2, 1) and slope of the line is −3.

The equation of the line having slope m and passing through $A(x_1, y_1)$ is $(y - y_1) = m\ (x - x_1)$.

The equation of the required line is

$y - 1 = -3(x - 2)$

$\therefore\ y - 1 = -3x + 6$

$\therefore\ 3x + y - 7 = 0$

5.3.2 Slope-Intercept form :

To find the equation of line having slope m and which makes intercept c on the Y-axis.

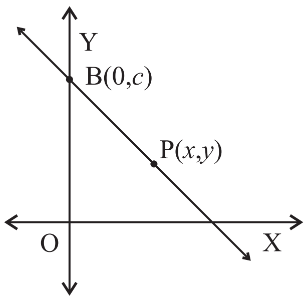


Fig. 5.10

Proof : Let L be the line with slope m and which makes Y-intercept c. Line L meets the Y-axis in the point C(0, c).

Let P(x, y) be any point on the line other than C. Then the slope of the line L is

$$\frac{y-c}{x-0} = m \text{ (given)}$$

$$\therefore\ \frac{y-c}{x} = m$$

$\therefore\ y = mx + c$

This is the equation of line L.

The equation of line having slope m and which makes intercept c on the Y-axis is $y = mx + c$.

Ex. Obtain the equation of line having slope 3 and which makes intercept 4 on the Y-axis.

Solution : The equation of line having slope m and which makes intercept c on the Y-axis is

$y = mx + c.$

$\therefore$ the equation of the line giving slope 3 and making Y-intercept 4 is $y = 3x + 4$.

5.3.3 Two-points Form :

To find the equation of line which passes through points $A(x_1, y_1)$ and $B(x_2, y_2)$.

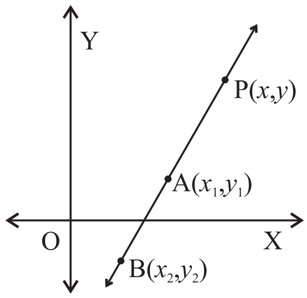


Fig. 5.11

Proof : $A(x_1, y_1)$ and $B(x_2, y_2)$ are the two given points on the line L. Let P(x, y) be any point on the line L, other than A and B.

Now points A and P lie on the line L.

$$\text{The slope of line L} = \frac{y-y_1}{x-x_1} \quad .... (1)$$

Also points A and B lie on the line L.

$$\therefore\ \text{Slope of line L} = \frac{y_2-y_1}{x_2-x_1} \quad .... (2)$$

From (1) and (2) we get $\dfrac{y-y_1}{x-x_1} = \dfrac{y_2-y_1}{x_2-x_1}$

$$\therefore\ \frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}$$

This is the equation of line L.

As line is non-vertical and non-horizontal, $x_1 \neq x_2$ and $y_1 \neq y_2$.

The equation of the line which passes through points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$

Ex. Obtain the equation of the line passing through points A(2, 1) and B(1, 2).

Solution : The equation of the line which passes through points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}.$$

$\therefore$ The equation of the line passing through points A(2, 1) and B(1, 2) is $\frac{x-2}{1-2}=\frac{y-1}{2-1}$

$$\therefore \quad \frac{x-2}{-1}=\frac{y-1}{1}$$

$$\therefore \ x - 2 = -y + 1$$

$$\therefore \ x + y - 3 - 0$$

5.3.4 Double-Intercept form :

To find the equation of the line which makes non-zero intercepts a and b on the co-ordinate axes.

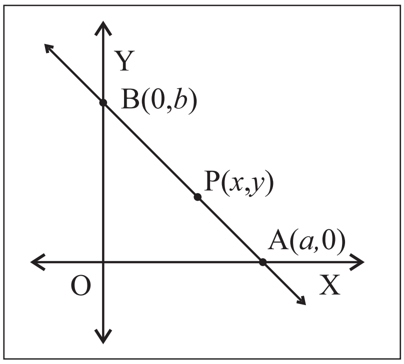


Fig. 5.12

Let a be the X-intercept and b be the Y-intercept of line L.

$\therefore$ Line L meets the X-axis in A(a, 0) and the Y-axis in B(0, b).

Let P(x, y) be any point on line L other than A and B.

$\therefore$ The slope of line L = slope of line AP

$$= \frac{y-0}{x-a}=\frac{y}{x-a}$$

$\therefore$ The slope of the line L = Slope of AB

$$= \frac{b-0}{0-a} = \frac{-b}{a}$$

$$\therefore \quad \frac{y}{x-a} = \frac{-b}{a} \qquad \therefore \quad \frac{x}{a}+\frac{y}{b}=1$$

This is the equation of line L.

The equation of the line which makes intercepts a and b on the co-ordiante axes is $\frac{x}{a}+\frac{y}{b}=1$ (a, b $\neq$ 0)

Ex. Obtain the equation of the line which makes intercepts 3 and 4 on the co-ordiante axes.

Solution : The equation of the line which makes intercepts a and b on the co-ordiante axes $\frac{x}{a}+\frac{y}{b}=1$

The equation of the line which makes intercepts 3 and 4 on the co-ordiante axes is $\frac{x}{3}+\frac{y}{4}=1$

$$\therefore \quad 4x+3y-12=0.$$

5.3.5 Normal Form :

Let L be a line and segment ON be the perpendicular (normal) drawn from the origin to line L.

If ON = p and ray ON makes angle α with the positive X$-$axis then to find the equation of line L.

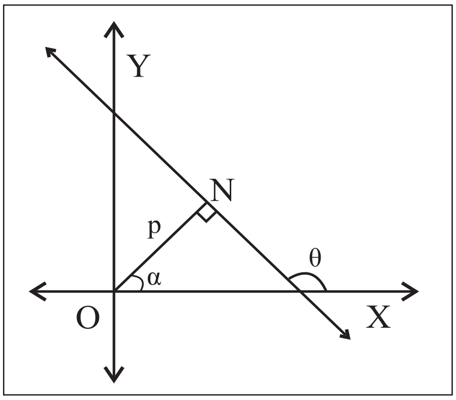


Fig. 5.13

From Fig. 5.13 we observe that N is

(p cosα, p sinα)

Therefore slope of ON is $\dfrac{p\sin\alpha - 0}{pco\,s\,\alpha - 0} = \tan\alpha$

As ON ierpendicular to line L,

Slope of line L = tanθ = – cot α

And it passes through (p cosα, p sinα)

∴ By the point – slope form, the equation of line L is,

$$y - p\ \sin\alpha = \frac{-\cos\alpha}{\sin\alpha}\left(x - p\cos\alpha\right)$$

$y\ \sin\alpha - p\ \sin^2\alpha = x\ \cos\alpha + p\ \cos^2\alpha$

$x\ \cos\alpha + y\ \sin\alpha = (\sin^2\alpha\ \cos^2\alpha)$

$x\ \cos\alpha + y\ \sin\alpha = p\ (p > o)$

The equation of the line, the normal to which from the origin has length p and the normal makes angle α with the positive directions of the X-axis, is $x\,cos\alpha + y\sin\alpha = p$.

SOLVED EXAMPLES

Ex. 1) The perpendicular drawn from the origin to a line has length 5 and the perpendicular makes angle with the positive direction of the X-axis. Find the equation of the line.

Solution : The perpendicular (normal) drawn from the origin to a line has length 5.

$\therefore\ p = 5$

The perpendicular (normal) makes angle 30° with the positive direction of the X-axis.

$\therefore\ \theta = 30°$

The equation of the required line is x cosα + y sinα = p

$\therefore\ x\ \cos 30° + y\ \sin 30° = p$

$$\therefore\ \frac{\sqrt{3}}{2}x + \frac{1}{2}y = 5$$

$$\therefore \sqrt{3}x + y - 10 = 0$$

Ex. 2) Reduce the equation $\sqrt{3}x - y - 2 = 0$ into normal form. Find the values of p and α.

Solution : Comparing $\sqrt{3}x - y - 2 = 0$ with $ax + by + c = 0$ we get $a = \sqrt{3}$, $b = -1$ and $c = -2$.

$$\sqrt{a^2 + b^2} = \sqrt{3+1} = 2$$

Divide the given equation by 2.

$$\frac{\sqrt{3}}{2}x - \frac{1}{2}y = 1$$

∴ cos330° x=sin330° y = 1 is the required normal form of the given equation.

$p = 1$ and $\theta = 330^o$.

Ex. 3) Find the equation of the line :

(i) parallel to the X -axis and 3 unit below it,

(ii) passing through the origin and having inclination 30°

(iii) passing through the point A(5,2) and having slope 6

(iv) passing through the points A(2–1) and B(5,1)

(v) having slope $-\dfrac{3}{4}$ and y –intercept 5,

(vi) making intercepts 3 and 6 on the co-ordinate axes.

(vii) passing through the point $N\left(-2,3\right)$ and the segment of the line intercepted between the co–ordinate axes is bisected at N.

Solution : (i) Equation of line parallel to the X-axis is of the form : $y = k$,

$\therefore$ the equation of the required line is

$y = 3$

(ii) Equation of line through the origin and having slope m is of the form : $y = mx$.

slope = m = tan = tan 30^0 = $\sqrt{3}$

$\therefore$ the equation of the required line is

$y = \sqrt{3}x \qquad \therefore \quad \sqrt{3}x - y = 0$

(iii) By using the point–slope form d

$y - y_1 = m\,(x - x_1)$

slope = m = –6

Equation of the required line is

$(y-2) = -6\,(x+5)$

$6x + y + 28 = 0$

(iv) By using the two points form

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

Here $(x_1, y_1) = (2 - 1)$; $(x_2, y_2) = (5,1)$

$\therefore$ the equation of the required line is

$\frac{x-2}{5-2} = \frac{y+1}{1+1}$

$\therefore \quad 2(x-2) = 3(y+1)$

$\therefore \quad 2x - 3y - 7 = 0$

(v) By using the slope intercept form $y = mx + c$

Given $m = -\frac{3}{4}$, $c = 5$

$\therefore$ the equation of the required line is

$y = -\frac{3}{4}x + 5 \qquad \therefore \quad 3x + 4y - 20 = 0$

(vi) By using the double intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

x –intercept = $a = 3$; y –intercept = b = 6 .

the equation of the required line is $\frac{x}{3} + \frac{y}{6} = 1$

$2x + y - 6 = 0$

(vii) Let the given line meet the X –axis in $A(a,0)$ and the Y –axis in $B(0,b)$.

The mid–point of AB is

$\left(\frac{a+0}{2}, \frac{0+b}{2}\right)$ = (–2, 3)

$\therefore \quad \frac{a}{2} = -2$ and $\frac{b}{2} = 3$

$\therefore \quad a = -4 \qquad b = 6$

$\therefore$ By using the double intercept form :

$\frac{x}{a} + \frac{y}{b} = 1 \qquad \frac{x}{-4} + \frac{y}{6} = 1 \quad \therefore \quad 3x - 2y + 12 = 0$

An interesting property of a straight line.

Consider any straight line in a plane. It makes two parts of the points which are not on the line.

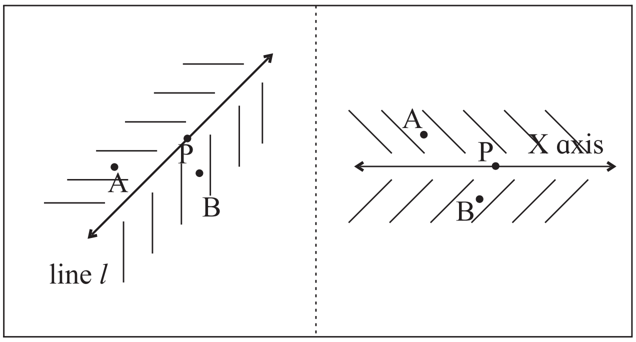


Fig. 5.14

Thus the plane is divided into 3 parts, the points on the line, points on one side of the line and points on the other side of the line.

If the line is given by $ax+by+c = 0$, then for all points (x_1,y_1) on one side of the line $ax_1+by_1+c > 0$ and for all points (x_2,y_2) on the other side of the line, $ax_2+by_2+c < 0$.

For example, consider the line given by $y - 2x - 3 = 0$. Points P(–2,0), Q(–2,4), R($\frac{1}{2}$,5) lie on one side and at each of those points, $y - 2x - 3 > 0$. The points A(0,0), B($\frac{1}{2}$,3), C(8,4) lie on the other side of the line and at each of those points $y - 2x - 3 < 0$.

Activity :

Draw the straight lines given by $2y + x = 5$, $x = 1$, $6y - x + 1 = 0$ give 4 points on each side of the lines and check the property stated above.

EXERCISE 5.3

1. Write the equation of the line :
   a) parallel to the X–axis and at a distance of 5 unit form it and above it.
   b) parallel to the Y– axis and at a distance of 5 unit form it and to the left of it.
   c) parallel to the X– axis and at a distance of 4 unit form the point (–2, 3).
2. Obtain the equation of the line :
   a) parallel to the X–axis and making an intercept of 3 unit on the Y–axis.
   b) parallel to the Y–axis and making an intercept of 4 unit on the X–axis.
3. Obtain the equation of the line containing the point :
   a) $A(2,-3)$ and parallel to the Y–axis.
   b) $B(4,-3)$ and parallel to the X–axis.
4. Find the equation of the line
   a) passing through the points $A(2,0)$ and $B(3,4)$.
   b) passing through the points $P(2,1)$ and $Q(2,-1)$
5. Find the equation of the line
   a) containing the origin and having inclination 60°.
   b) passing through the origin and parallel to AB, where A is (2,4) and B is (1,7).
   c) having slope $\frac{1}{2}$ and containing the point (3,–2).
   d) containing the point $A(3,5)$ and having slope $\frac{2}{3}$.
   e) containing the point $A(4,3)$ and having inclination 120°.
   f) passing through the origin and which bisects the portion of the line $3x+y=6$ intercepted between the co−ordinate axes.
6. Line $y=mx+c$ passes through points A(2,1) and B(3,2). Determine m and c.
7. Find the equation of the line having inclination 135° and making X-intercept 7.
8. The vertices of a triangle are $A(3,4)$, $B(2,0)$ and $C(-1,6)$. Find the equations of the lines containing
   (a) side BC (b) the median AD
   (c) the mid points of sides AB and BC.
9. Find the x and y intercepts of the following lines :
   (a) $\frac{x}{3}+\frac{y}{2}=1$ (b) $\frac{3x}{2}+\frac{2y}{3}=1$
   (c) $2x-3y+12=0$

10. Find equations of lines which contains the point $A(1,3)$ and the sum of whose intercepts on the co–ordinate axes is zero.

11. Find equations of lines containing the point A(3,4) and making equal intercepts on the co–ordinates axes.

12. Find equations of altitudes of the triangle whose vertices are $A(2,5)$, $B(6,-1)$ and $C(-4,-3)$.

13. Find the equations of perpendicular bisectors of sides of the triangle whose vertices are $P(-1,8)$, $Q(4,-2)$ and $R(-5,-3)$.

14. Find the co-ordinates of the orthocenter of the triangle whose vertices are $A(2,-2)$, $B(1,1)$ and $C(-1,0)$.

15. $N(3,-4)$ is the foot of the perpendicular drawn from the origin to line L. Find the equation of line L.

5.4 General form of equation of a line: We can write equation of every line in the form ax+by+c=0

This form of equation of a line is called the general form.

The general form of $y=3x+2$ is $3x-y+2=0$

The general form of $\frac{x}{2}+\frac{y}{3}=1$ is $3x+2y-6=0$

The slope of the line $ax+by+c=0$ is $-\frac{a}{b}$ if $b\neq 0$.

The X–intercept is $-\frac{c}{a}$ if $a\neq 0$.

The Y-intercept is $-\frac{c}{b}$ if $b\neq 0$.

Remark : If a=0 then the line is parallel to the X-axis. It does not make intercept on the X-axis.

If b=0 then the line is parallel to the Y-axis. It does not make intercept on the Y-axis.

SOLVED EXAMPLES

Ex. 1) Find the slope and intercepts made by the following lines :

(a) $x+y+10=0$ (b) $2x+y+30=0$

(c) $x+3y-15=0$

Solution : (a) Comparing equation $x+y+10=0$ with $ax+by+c=0$,

we get $a=1$, $b=1$, $c=10$

$\therefore$ Slope of this line $= -\frac{a}{b} = -1$

The X–intercept is $-\frac{c}{a} = -\frac{10}{1} = -10$

The Y-intercept is $= -\frac{c}{b} = -\frac{10}{1} = -10$

(b) Comparing equation $2x+y+30=0$

with $ax+by+c=0$.

we get $a=2$, $b=1$, $c=30$

$\therefore$ Slope of this line $= -\frac{a}{b} = -2$

The X–intercept is $-\frac{c}{a} = -\frac{30}{2} = -15$

The Y-intercept is $-\frac{c}{b} = -\frac{30}{1} = -30$

(c) Comparing equation $x+3y-15=0$ with $ax+by+c=0$.

we get $a=1$, $b=3$, $c=-15$

$\therefore$ Slope of this line $= -\frac{a}{b} = -\frac{1}{3}$

The $x-$ intercept is $-\frac{c}{a} = -\frac{-15}{1} = 15$

The $y-$ intercept is $-\frac{c}{b} = -\frac{-15}{3} = 5$

Ex. 2) Find the acute angle between the following pairs of lines :

a) $12x-4y=5$ and $4x+2y=7$

b) $y=2x+3$ and $y=3x+7$

Solution : (a) Slopes of lines $12x-4y=5$ and $4x+2y=7$ are $m_1=3$ and $m_2=2$.

If θ is the acute angle between lines having slope m_1 and m_2 then

$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$\therefore \tan\theta = \left|\frac{3-(-2)}{1+(3)(-2)}\right| = \left|\frac{5}{-5}\right| = 1$$

$\therefore \tan\theta=1 \qquad \therefore \theta=45°$

(b) Slopes of lines $y=2x+3$ and $y=3x+7$ are $m_1 = 2$ and $m_2 = 3$

The acute angle θ between lines having slopes m_1 and m_2 is given by

$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$\therefore \tan\theta = \left|\frac{2-3}{1+(2)(3)}\right| = \left|\frac{-1}{7}\right| = \frac{1}{7}$$

$$\therefore \theta = \tan^{-1}\left(\frac{1}{7}\right).$$

Ex. 3) Find the acute angle between the lines $y-\sqrt{3}x+1=0$ and $\sqrt{3}y-x+7=0$.

Solution : Slopes of the given lines are $m_1=\sqrt{3}$ and $m_2=\frac{1}{\sqrt{3}}$.

The acute angle θ between lines having slopes m_1 and m_2 is given by

$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$\tan\theta = \frac{1}{\sqrt{3}} = \left|\frac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+\sqrt{3}\times\frac{1}{\sqrt{3}}}\right| = \left|\frac{\sqrt{3}-\frac{1}{\sqrt{3}}}{1+1}\right| = \left|\frac{\sqrt{3}-\frac{1}{\sqrt{3}}}{2}\right|$$

$$= \left|\frac{1-3}{2\sqrt{3}}\right| = \left|\frac{-1}{\sqrt{3}}\right| = \frac{1}{\sqrt{3}}$$

$$\therefore \tan\theta = \frac{1}{\sqrt{3}} \qquad \therefore \theta = 30^0$$

Ex. 4) Show that following pairs of lines are perpendicular to each other.

a) $2x-4y=5$ and $2x+y=17$.

b) $y=2x+23$ and $2x+4y=27$

Solution : (i) Slopes of lines $2x-4y=5$ and $2x+y=17$ are $m_1=\frac{1}{2}$ and $m_2= -2$

Since $m_1.m_2 = \frac{1}{2}\times(-2) = -1$, given lines are perpendicular to each other.

(ii) Slopes of lines $y = 2x + 23$ and $2x+4y = 27$ are $m_1=-\frac{1}{2}$ and $m_2= 2$.

Since $m_1.m_2 = -\frac{1}{2}\times(2) = -1$, given lines are perpendicular to each other.

Ex. 5) Find equations of lines which pass through the origin and make an angle of 45° with the line $3x - y = 6$.

Solution : Slope of the line $3x - y = 6$. is 3.

Let m be the slope of one of the required lines. The angle between these lines is 45°.

$$\therefore \quad \tan 45^0 = \left|\frac{m-3}{1+(m)(3)}\right|$$

$$\therefore \quad 1 = \left|\frac{m-3}{1+3m}\right| \qquad \therefore \quad |1+3m| = |m-3|$$

$$\therefore \quad 1+3m = m-3 \quad \text{or} \quad 1+3m = -(m-3)$$

$$\therefore \quad m = -2 \quad \text{or} \quad \frac{1}{2}$$

Slopes of required lines are $m_1 = -2$ and $m_2 = \frac{1}{2}$

Required lines pass through the origin.

$\therefore$ Their equations are $y = -2x$ and

$$y = \frac{1}{2}x$$

$$\therefore \quad 2x + y = 0 \quad \text{and} \quad x - 2y = 0$$

Ex. 6) A line is parallel to the line $2x + y = 7$ and passes through the origin. Find its equation.

Solution : Slope of the line $2x + y = 7$ is -2.

Required line passes through the origin.

$\therefore$ It's equation is $y = -2x$

$\therefore$ $2x + y = 0$.

Ex. 7) A line is parallel to the line $x + 3y = 9$ and passes through the point $A(2,7)$. Find its equation.

Solution : Slope of the line $x + 3y = 9$ is $-\frac{1}{3}$

Required line passes through the point $A(2,7)$.

$\therefore$ It's equation is given by the formula $(y - y_1) = m(x - x_1)$

$$\therefore \quad (y-7) = -\frac{1}{3}(x-2)$$

$$\therefore \quad 3y - 21 = x + 2$$

$$\therefore \quad x + 3y = 23.$$

Ex. 8) A line is perpendicular to the line $3x + 2y - 1 = 0$ and passes through the point $A(1,1)$. Find its equation.

Solution : Slope of the line $3x + 2y - 1$ is $-\frac{3}{2}$.

Required line is perpendicular it.

The slope of the required line is $\frac{2}{3}$.

Required line passes through the point $A(1,1)$.

$\therefore$ It's equation is given by the formula $(y - y_1) = m(x - x_1)$

$$\therefore \quad (y-1) = \frac{2}{3}(x-1)$$

$$\therefore \quad 3y - 3 = 2x - 2$$

$$\therefore \quad 2x - 3y + 1 = 0.$$

Note:

Point of intersection of lines :

The co-ordinates of the point of intersection of two intersecting lines can be obtained by solving their equations simultaneously.

Ex. 9) Find the co-ordinates of the point of intersection of lines $x + 2y = 3$ and $2x - y = 1$.

Solution : Solving equations $x + 2y = 3$ and $2x - y = 1$ simultaneously, we get $x = 1$ and $y = 1$.

$\therefore$ Given lines intersect in point $(1,1)$.

Ex. 10) Find the equation of line which is parallel to the X-axis and which passes through the point of intersection of lines $x+2y=6$ and $2x-y=2$

Solution : Solving equations $x+2y=6$ and $2x-y=2$ simultaneously, we get $x = 2$ and $y = 2$.

$\therefore$ The required line passes through the point $(2,2)$.

As it is parallel to the X-axis, its equation is $y = 2$.

5.4.1 The distance of the Origin from a Line :

The distance of the origin from the line $ax+by+c=0$ is given by $p=\left|\frac{c}{\sqrt{a^2+b^2}}\right|$

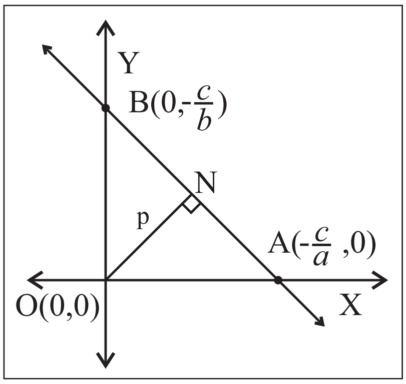


Fig. 5.15

Proof : Let A and B be the points where line $ax+by+c=0$ cuts the co-ordinate axes.

$\therefore$ A $\left(-\frac{c}{a},0\right)$ and B $\left(0,-\frac{c}{b}\right)$

OA = $\left|\frac{c}{a}\right|$ and OB = $\left|\frac{c}{b}\right|$

By Pythagoras theorem $AB^2 = OA^2 + OB^2$

$$AB^2=\left(\frac{c}{a}\right)^2+\left(\frac{c}{b}\right)^2 = c^2\left(\frac{a^2+b^2}{a^2b^2}\right)$$

$$\therefore \quad AB=\left|\frac{c}{ab}\right|\sqrt{a^2+b^2}$$

Now,

$$\text{Area of } \Delta OAB = \frac{1}{2}AB\times ON$$

$$=\frac{1}{2}\left|\frac{c}{ab}\right|\sqrt{a^2+b^2}\times p \quad \dots \text{(I)}$$

$$\text{But, Area of } \Delta OAB = \frac{1}{2}OA\times OB$$

$$=\frac{1}{2}\left|\frac{c}{a}\right|\left|\frac{c}{b}\right|=\left|\frac{c^2}{2ab}\right| \quad \dots \text{(II)}$$

From (I) and (II), we get

$$p=\left|\frac{c}{\sqrt{a^2+b^2}}\right|$$

5.4.2 The distance of the point (x_1,y_1) from a line:

The distance of the point P (x_1,y_1) from line $ax+by+c=0$ is given by

$$p=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

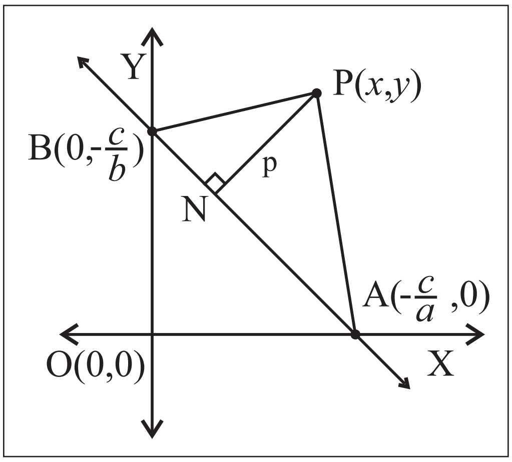


Fig. 5.16

Proof : If line $ax+by+c=0$ cuts co-ordiante axes in B and C respectively then B is $\left(-\frac{c}{a},0\right)$

and C is $\left(0,-\frac{c}{b}\right)$.

Let PM be perpendicular to $ax+by+c=0$.

Let $\text{PM}=p$

$$A(\Delta PBC)=\frac{1}{2}BC\times PM=\frac{1}{2}p\sqrt{\frac{c^2}{a^2}+\frac{c^2}{b^2}}=\frac{pc\sqrt{a^2+b^2}}{2ab} \quad .(I)$$

Now $P(x_1,y_1)$, $B\left(-\frac{c}{a},0\right)$ and $C\left(0,-\frac{c}{b}\right)$

are vertices of PBC.

$$\therefore A(\Delta PBC)=\frac{1}{2}\begin{vmatrix}1 & 1 & 1\\ x_1 & -\frac{c}{a} & 0\\ y_1 & 0 & -\frac{c}{b}\end{vmatrix}=\frac{1}{2}\left|x_1\frac{c}{b}+y_1\frac{c}{a}+\frac{c^2}{ab}\right| \quad .\ .(II)$$

From (I) and (II) we get

$$\frac{pc\sqrt{a^2+b^2}}{2ab}=\frac{1}{2}\left|x_1\frac{c}{b}+y_1\frac{c}{a}+\frac{c^2}{ab}\right|$$

$$\therefore pc\sqrt{a^2+b^2}=|acx_1+bcy_1+c^2|$$

$$\therefore p\sqrt{a^2+b^2}=|ax_1+by_1+c|$$

$$\therefore \quad p=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

5.4.3 The distance between two parallel lines :

Theorem : The distance between the parallel lines $ax+by+c_1=0$ and $ax+by+c_2=0$ is give

by $p=\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$

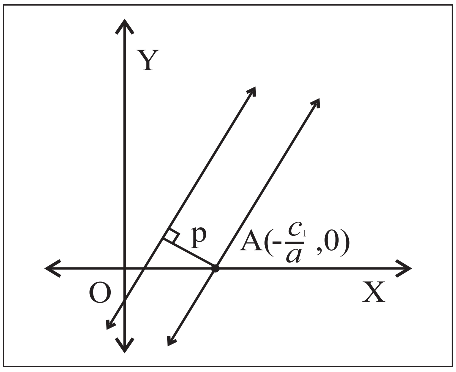


Fig. 5.17

Proof : To find the distance between parallel lines, we take any one point on any one of these two lines and find its distance from the other line.

$A\left(-\frac{c_1}{a},0\right)$ is a point on the first line.

Its distance from the line $ax+by+c_2=0$ is given by

$$p=\left|\frac{a\left(-\frac{c_1}{a}\right)+b(0)+c_2}{\sqrt{a^2+b^2}}\right|=\left|\frac{-c_1+c_2}{\sqrt{a^2+b^2}}\right|=\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$$

SOLVED EXAMPLES

Ex. 1) Find the distance of the origin from the line $3x+4y+15=0$

Solution : The distance of the origin from the line $ax+by+c=0$ is given by

$$p=\left|\frac{c}{\sqrt{a^2+b^2}}\right|$$

$\therefore$ The distance of the origin from the line $3x+4y+15=0$ is given by

$$p=\left|\frac{15}{\sqrt{3^2+4^2}}\right| = \frac{15}{5} = 3$$

Ex. 2) Find the distance of the point $P(2,5)$ from the line $3x+4y+14=0$

Solution : The distance of the point $P(x_1,y_1)$ from the line $ax+by+c=0$ is given by

$$p=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

$\therefore$ The distance of the point $P(2,5)$ from the line $3x+4y+14=0$ is given by

$$p=\left|\frac{3(2)+4(5)+14}{\sqrt{3^2+4^2}}\right|=\frac{40}{5}=8$$

Ex. 3) Find the distance between the parallel lines $6x+8y+21=0$ and $3x+4y+7=0$.

Solution : We write equation $3x+4y+7=0$ as $6x+8y+14=0$ in order to make the coefficients of x and coefficients of y in both equations to be same.

Now by using formula $p=\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$

We get the distance between the given parallel lines as

$$p=\left|\frac{21-14}{\sqrt{6^2+8^2}}\right|=\frac{7}{10}$$

5.4.4 Family of Lines : A set of lines which have a common property is called a family of lines. Consider the set of all lines passing through the origin. Equation of each of these lines is of the form $y=mx$. This set of lines is a family of lines. Different values of m give different lines.

Consider the set of all lines which pass through the point $A(2,-3)$. Equation of each of these lines is of the form $(y+3)=m(x-2)$. This set is also form a family of lines.

Consider the set of all lines which are parallel to the line $y=x$. They all have the same slope.

The **set** of all lines which pass through a fixed point or which are parallel to each other is a family of lines.

Interpretation of $u + kv = 0$: Let $u\equiv a_1x+b_1y+c_1$ and $v\equiv a_2x+b_2y+c_2$

Equations $u=0$ and $v=0$ represent two lines. Equation $u+kv=0, k\in R$ represents a family of lines.

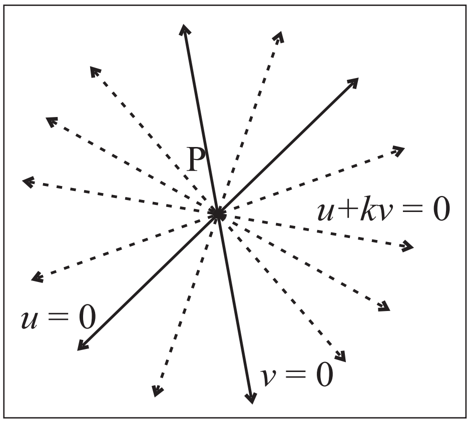


Fig. 5.18

In equation $u+kv=0$ let

$u\equiv a_1x+b_1y+c_1$, $v\equiv a_2x+b_2y+c_2$

We get $(a_1x+b_1y+c_1)+k(a_2x+b_2y+c_2)=0$

$$(a_1+ka_2)x+(b_1+kb_2)y+(c_1+kc_2)=0$$

Which is a first degree equation in x and y. Hence it represents a straight line.

i) If lines $u=0$ and $v=0$ intersect each other in $P(x_1,y_1)$ then $a_1x_1+b_1y_1+c_1=0$ and $a_2x_1+b_2y_1+c_2=0$

Therefore

$$\left(a_1x_1+b_1y_1+c_1\right)+k\left(a_2x_1+b_2y_1+c_2\right)=0+k0=0$$

Thus line $u+kv=0$ passes through the point of intersection of lines u = 0 and v = 0 for every real value of k.

ii) If lines $u=0$ and $v=0$ are parallel to each other then their slope is same.

$$\therefore\ -\frac{a_1}{b_1}=-\frac{a_2}{b_2}$$

$\therefore$ each ratio $= -\dfrac{a_1+ka_2}{b_1+kb_2}$

= slope of the line u+kv=0

$\therefore$ Slopes of lines $u=0,\ v=0$ and $u+kv=0$ are the same.

$\therefore$ Line $u+kv=0$ is parallel to lines $u=0, v=0$.

SOLVED EXAMPLES

Ex. 1) Find the equation of the line which passes through the point of intersection of lines $x+2y+6=0$, $2x-y=2$ and which makes intercept 5 on the Y-axis.

Solution : As the required line passes through the point of intersection of lines $x+2y+6=0$ and $2x-y=2$, its equation is of the form $u+k\,v=0$.

$$\therefore\ \left(x+2y+6\right)+k\left(2x-y-2\right)=0$$

$$\therefore\ \left(1+2k\right)x+\left(2-1k\right)y+\left(6-2k\right)=0$$

The Y-intercept of this line is given 5.

$$\therefore\ -\frac{6-2k}{2-k}=5 \qquad \therefore\ -6+2k=10-5k$$

$$\therefore\ 7k=16 \qquad \therefore\ k=\frac{16}{7}$$

$\therefore$ the equation of the required line is

$$\therefore\ \left(x+2y+6\right)+\frac{16}{7}\left(2x-y-2\right)=0$$

$$\therefore\ \left(7x+14y+42\right)+\left(32x-16y-32\right)=0$$

$$\therefore\ 39x-2y+10=0$$

Ex. 2) Find the equation of line which passes through the point of intersection of lines $3x+2y-6=0$, $x+y+1=0$ and the point $A\left(2,1\right)$.

Solution : Since the required line passes through the point of intersection of lines $3x+2y-6=0$ and $x+y+1=0$, its equation is of the form $u+k\,v=0$.

$$\therefore\ (3x+2y-6)+k\left(x+y+1\right)=0$$

$$\therefore\ (3+k)x+\left(2+k\right)y+\left(-6+k\right)=0$$

This line passes through the point A(2,1).

$\therefore$ $\left(2,1\right)$ satisfy this equation.

$$\therefore\ (3+k)\left(2\right)+\left(2+k\right)\left(1\right)+\left(-6+k\right)=0$$

$$\therefore\ 4k+2=0 \qquad \therefore\ k=-\frac{1}{2}$$

$\therefore$ The equation of the required line is

$$(3x+2y-6)+\left(-\frac{1}{2}\right)\left(x+y+1\right)=0$$

$$5x+3y-13=0$$

EXERCISE 5.4

1) Find the slope, X-intercept, Y-intercept of each of the following lines.

a) $2x+3y-6=0$ b) $3x-y-9=0$

c) $x+2y=0$

2) Write each of the following equations in $ax+by+c=0$ form.

a) $y=2x-4$ b) $y=4$

c) $\frac{x}{2}+\frac{y}{4}=1$ d) $\frac{x}{3}-\frac{y}{2}=0$

3) Show that lines $x-2y-7=0$ and $2x-4y+15=0$ are parallel to each other.

4) Show that lines $x-2y-7=0$ and $2x+y+1=0$ are perpendicular to each other. Find their point of intersection.

5) If the line $3x+4y=p$ makes a triangle of area 24 square unit with the co-ordinate axes then find the value of p.

6) Find the co-ordinates of the foot of the perpendicular drawn from the point A(–2,3) to the line $3x-y-1=0$.

7) Find the co-ordinates of the circumcenter of the triangle whose vertices are $A(-2,3), B(6,-1), C(4,3)$.

8) Find the co-ordinates of the orthocenter of the triangle whose vertices are A(3,–2), B(7,6), C (–1,2)

9) Show that lines $3x-4y+5=0, 7x-8y+5=0,$ and $4x+5y-45=0$ are concurrent. Find their point of concurrence.

10) Find the equation of the line whose X-intercept is 3 and which is perpendicular to the line $3x-y+23=0$.

11) Find the distance of the origin from the line $7x+24y-50=0$.

12) Find the distance of the point $A(-2,3)$ from the line $12x-5y-13=0$.

13) Find the distance between parallel lines $4x-3y+5=0$ and $4x-3y+7=0$

14) Find the distance between parallel lines $9x+6y-7=0$. and $3x+2y+6=0$

15) Find points on the line $x+y-4=0$ which are at one unit distance from the line $x+y-2=0$

16) Find the equation of the line parallel to the X-axis and passing through the point of intersection of lines $x+y-2=0$ and $4x+3y=10$.

17) Find the equation of the line passing through the point of intersection of lines $x+y-2=0$ and $2x-3y+4=0$ and making intercept 3 on the X-axis.

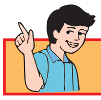
18) If A(4,3), B(0,0), and C(2,3) are the vertices of Δ ABC then find the equation of bisector of angle BAC.

19) D(–1,8), E(4,–2), F(–5,–3) are midpoints of sides BC, CA and AB of Δ ABC. Find

(i) equations of sides of Δ ABC.

(ii) co-ordinates of the circumcenter of Δ ABC.

20) O(0, 0), A(6,0) and B(0,8) are vertices of a triangle. Find the co-ordinates of the incenter of ΔOAB.



Let's Remember

- **Locus :** A set of points in a plane which satisfy certain geometrical condition (or conditions) is called a locus.
- **Equation of Locus :** Every point in XY plane has Cartesian co-ordinates. An equation which is satisfied by co-ordinates of all points on the locus and which is not satisfied by the co-ordinates of any point which does not lie on the locus is called the equation of the locus.
- **Inclination of a line :** The smallest angle θ made by a line with the positive direction of the X-axis, measured in anticlockwise sense, is called the inclination of the line. Clearly $0° \leq \theta \leq 180°$.
- **Slope of a line :** If θ is the inclination of a line then $\tan\theta$ (if it exist) is called the slope of the line.

  If $A(x_1, y_1)$, $B(x_2, y_2)$ be any two points on the line whose inclination is θ then

  $\tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1}$ (if $x_1 \neq x_2$)
- **Perpendicular and parallel lines :** Lines having slopes m_1 and m_2 are perpendicular to each other if and only if $m_1 m_2 = -1$.

  Two lines are parallel if and only if they have the same slope.
- **Angle between intersecting lines :** If θ is the acute angle between lines having slopes m_1 and m_2 then $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$
- **Equations of line in different forms :**
- Slope point form : $(y - y_1) = m(x - x_1)$
- Slope intercept form : $y = mx + c$
- Two points form : $\dfrac{x - x_1}{x_2 - x_1} = \dfrac{y - y_1}{y_2 - y_1}$
- Double intercept form : $\dfrac{x}{a} + \dfrac{y}{b} = 1$
- Normal form : $x\cos\alpha + y\sin\alpha = p$
- General form : $ax + by + c = 0$
- **Distance of a point from a line :**
- The distance of the origin from the line $ax + by + c = 0$ is given by $p = \left|\dfrac{c}{\sqrt{a^2 + b^2}}\right|$
- The distance of the point $P(x_1, y_1)$ from line $ax + by + c = 0$ is given by

  $$p = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$
- **The distance between the Parallel lines :** The distance between the parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is give by $p = \left|\dfrac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$

MISCELLANEOUS EXERCISE - 5

(I) Select the correct option from the given alternatives.

1) If A is (5,–3) and B is a point on the x–axis such that the slope of line AB is –2 then $B \equiv$

(A) (7,2) (B) $(\frac{7}{2},0)$

(C) $(0,\frac{7}{2})$ (D) $(\frac{2}{7},0)$

2) If the point (1,1) lies on the line passing through the points (a,0) and (0,b), then $\frac{1}{a}+\frac{1}{b}$ =

(A) –1 (B) 0 (C) 1 (D) $\frac{1}{ab}$

3) If A(1,–2), B(–2,3) and C(2,–5) are the vertices of ΔABC, then the equation of the median BE is

(A) $7x+13y+47=0$ (B) $13x+7y+5=0$

(C) $7x-13y+5=0$ (D) $13x-7y-5=0$

4) The equation of the line through (1,2), which makes equal intercepts on the axes, is

(A) $x+y=1$ (B) $x+y=2$

(C) $x+y=4$ (D) $x+y=3$

5) If the line $kx+4y=6$ passes through the point of intersection of the two lines $2x+3y=4$ and $3x+4y=5$, then k =

(A) 1 (B) 2 (C) 3 (D) 4

6) The equation of a line, having inclination 120° with positive direction of X–axis, which is at a distance of 3 units from the origin is

(A) $\sqrt{3}x \pm y+6=0$ (B) $\sqrt{3}x+y\pm 6=0$

(C) $x+y=6$ (D) $x+y=-6$

7) A line passes through (2,2) and is perpendicular to the line $3x+y=3$. Its y–interecpt is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) 1 (D) $\frac{4}{3}$

8) The angle between the line $\sqrt{3}x-y-2=0$ and $x-\sqrt{3}y+1=0$ is

(A) 15° (B) 30° (C) 45° (D) 60°

9) If $kx+2y-1=0$ and $6x-4y+2=0$ are identical lines, then determine k.

(A) –3 (B) $-\frac{1}{3}$ (C) $\frac{1}{3}$ (D) 3

10) Distance between the two parallel lines $y=2x+7$ and $y=2x+5$ is

(A) $\frac{\sqrt{2}}{\sqrt{5}}$ (B) $\frac{1}{\sqrt{5}}$ (C) $\frac{\sqrt{5}}{2}$ (D) $\frac{2}{\sqrt{5}}$

(II) Answer the following questions.

1) Find the value of k

a) if the slope of the line passing through the points $P(3,4)$, $Q(5,k)$ is 9.

b) the points $A(1,3), B(4,1), C(3,k)$ are collinear

c) the point P(1,k) lies on the line passing through the points A(2, 2) and B(3, 3).

2) Reduce the equation $6x+3y+8=0$ into slope-intercept form. Hence find its slope.

3) Find the distance of the origin from the line $x=-2$.

4) Does point $A(2,3)$ lie on the line $3x+2y-6=0$? Give reason.

5) Which of the following lines passes through the origin ?

(a) $x=2$ (b) $y=3$

(c) $y=x+2$ (d) $2x-y=0$

6) Obtain the equation of the line which is :

a) parallel to the X–axis and 3 unit below it.

b) parallel to the Y–axis and 2 unit to the left of it.

c) parallel to the X–axis and making an intercept of 5 on the Y–axis.

d) parallel to the Y–axis and making an intercept of 3 on the X–axis.

7) Obtain the equation of the line containing the point

(i) (2,3) and parallel to the X−axis.

(ii) (2,4) and perpendicular to the Y−axis.

8) Find the equation of the line :

a) having slope 5 and containing point $A(-1,2)$.

b) containing the point T(7,3) and having inclination 90^0.

c) through the origin which bisects the portion of the line $3x+2y=2$ intercepted between the co–ordinate axes.

9) Find the equation of the line passing through the points S(2,1)and T(2,3)

10) Find the distance of the origin from the line $12x+5y+78=0$

11) Find the distance between the parallel lines $3x+4y+3=0$ and $3x+4y+15=0$

12) Find the equation of the line which contains the point A(3,5) and makes equal intercepts on the co–ordinates axes.

13) The vertices of a triangle are A(1,4), B(2,3) and C(1,6). Find equations of

(a) the sides (b) the medians

(c) Perpendicular bisectors of sides

(d) altitudes of ΔABC.

14) Find the equation of the line which passes through the point of intersection of lines $x+y-3=0$, $2x-y+1=0$ and which is parallel X- axis.

15) Find the equation of the line which passes through the point of intersection of lines $x+y+9=0$, $2x+3y+1=0$ and which makes X-intercept 1.

16) Find the equation of the line through A $(-2,3)$ and perpendicular to the line through $S(1,2)$ and $T(2,5)$.

17) Find the X-intercept of the line whose slope is 3 and which makes intercept 4 on the Y–axis.

18) Find the distance of $P(-1,1)$ from the line $12(x+6)=5(y-2)$.

19) Line through $A(h,3)$ and $B(4,1)$ intersect the line $7x-9y-19=0$ at right angle. Find the value of h.

20) Two lines passing through $M(2,3)$ intersect each other at an angle of 45°. If slope of one line is 2, find the equation of the other line.

21) Find the Y-intercept of the line whose slope is 4 and which has X intercept 5.

22) Find the equations of the diagonals of the rectangle whose sides are contained in the lines $x = 8$, $x = 10$, $y = 11$ and $y = 12$.

23) A(1, 4), B(2,3) and C (1, 6) are vertices of ΔABC. Find the equation of the altitude through B and hence find the co-ordinates of the point where this altitude cuts the side AC of ΔABC.

24) The vertices of ΔPQR are $P(2,1)$, $Q(-2,3)$ and $R(4,5)$. Find the equation of the median through R.

25) A line perpendicular to segment joining A $(1,0)$ and B$(2,3)$ divides it internally in the ration 1:2. Find the equation of the line.

26) Find the co-ordinates of the foot of the perpendicular drawn from the point P $(-1,3)$ the line $3x-4y-16 = 0$.

27) Find points on the X-axis whose distance from the line $\frac{x}{3}+\frac{y}{4}=1$ is 4 unit.

28) The perpendicular from the origin to a line meets it at $(-2,9)$. Find the equation of the line.

29) $P(a,b)$ is the mid point of a line segment between axes. Show that the equation of the line is $\frac{x}{a}+\frac{y}{b}=2$.

30) Find the distance of the line $4x-y=0$ from the point $P(4,1)$ measured along the line making an angle of 135^0 with the positive X-axis.

31) Show that there are two lines which pass through A(3,4) and the sum of whose intercepts is zero.

32) Show that there is only one line which passes through B(5,5) and the sum of whose intercept is zero.