7. Applications of Linear Regression

A. Activities

1. Given that $\bar{x} = 20$, $\bar{y} = 25$

$\sigma_x = 2$, $\sigma_y = 3$, $r = 0.75$

$\therefore b_{yx} = \boxed{r} \dfrac{\sigma_y}{\sigma_x} = \dfrac{0.75 \times \boxed{3}}{2} = \boxed{1.125}$

$b_{xy} = r \dfrac{\sigma_x}{\boxed{\sigma_y}} = \boxed{0.75} \dfrac{2}{3} = \boxed{0.5}$

Regression equation of y on x is

$\boxed{y} - \boxed{\bar{y}} = b_{yx} (\boxed{x} - \boxed{\bar{x}})$

$y - \boxed{25} = \boxed{1.125} (\boxed{x} - 20)$

$\therefore y - 25 = 1.125x - \boxed{22.5}$

$\therefore y = 1.125x + \boxed{2.5}$

Regression equation of x on y is

$\boxed{x} - \bar{x} = \boxed{b_{xy}} (y - \boxed{\bar{y}})$

$x - \boxed{20} = 0.5y - \boxed{12.5}$

$x = 0.5y + \boxed{7.5}$

2. Given : $v(x) = 9$

Regression equation of y on x is $8x - 10y + 66 = 0$ .... (i)

Regression equation of x on y is $40x - 18y - 214 = 0$ .... (ii)

To find mean values we solve equations (i) and (ii),

Multiply equation (i) by 5 and subtract equation (ii) from it.

$$
\begin{array}{l}
\quad 40x - 50y + 330 = 0 \\
(-)\ 40x\ (\pm)\ 18y - 214 = 0 \\
\hline
\end{array}
$$

$\boxed{-32}\, y + 544 = 0$

$\therefore y = \dfrac{\boxed{544}}{32} = \boxed{17}$

Substituting $y = 17$ in equation (i) we get, $8x - 10(17) + \boxed{66} = 0$

$8x = \boxed{104}$

$\therefore x = \dfrac{104}{\boxed{8}} = \boxed{13}$

$\therefore (\boxed{13} , \boxed{17})$ is the point of intersection of both the regression lines.

$\therefore \bar{x} = 13$ and $\bar{y} = \boxed{17}$

$\therefore$ Equation of y and x is

$8x - 10y + 66 = 0$

$\therefore 10y = \boxed{8} x + \boxed{66}$

$\therefore y = \dfrac{8x}{\boxed{10}} + \dfrac{\boxed{66}}{10}$

$\therefore b_{yx} = \dfrac{\boxed{8}}{\boxed{10}} = \dfrac{4}{5}$

$\therefore$ Equation of x on y is $40x - 18y - 214 = 0$

$214 + \boxed{18} y = 40 \boxed{x}$

$\therefore x = \dfrac{18\boxed{y}}{\boxed{40}} + \dfrac{\boxed{214}}{40}$

$\therefore b_{xy} = \dfrac{\boxed{18}}{\boxed{40}} = \dfrac{9}{20}$

$\therefore r = \pm\sqrt{b_{yx} \times \boxed{b_{xy}}}$

$= \boxed{+} \sqrt{\dfrac{4}{\boxed{5}} \times \dfrac{\boxed{9}}{20}} = \dfrac{\boxed{3}}{5} = \boxed{0.6}$

$\therefore v(x) = 9$

$\therefore \sigma_x = \sqrt{v(x)} = \sqrt{\boxed{9}} = \boxed{3}$

$\therefore b_{yx} = \boxed{r} \times \dfrac{\sigma_y}{\boxed{\sigma_x}}$

$\therefore \dfrac{4}{\boxed{5}} = \dfrac{\boxed{3}}{5} \times \dfrac{\boxed{\sigma_y}}{3}$

$\sigma_y = \boxed{4}$

B. Solve the Following

Q.1 For a bivariate data $\bar{x} = 53$, $\bar{y} = 28$, $b_{yx} = -1.8$ $b_{xy} = -0.2$. Find

i) Correlation coefficient between X and Y

ii) Estimate of Y for X = 50

iii) Estimate of X for Y = 25

i) $r = \pm\sqrt{b_{xy} \cdot b_{yx}} = \pm\sqrt{(-0.2)(-1.8)} = \pm\sqrt{0.36} = -0.6$

(ii) Regression eq.^{n} Y on X :	(iii) Regression eq.^{n} X on Y:
$Y - \bar{Y} = b_{yx}(X - \bar{X})$	$X - \bar{X} = b_{xy}(Y - \bar{Y})$
For $X = 50$	For $Y = 25$
$\therefore Y - 28 = (-1.8)(50 - 53)$	$\therefore X - 53 = (-0.2)(25 - 28)$
$\therefore Y = 5.4 + 28$	$\therefore X = 0.6 + 53$
$\therefore Y = 33.4$	$\therefore X = 53.6$

Q.2 You are given the following information about advertising expenditure and sales:

	Advertisement expenditure (in ₹ lakh) (X)	Sales (₹ in lakh) (Y)
Arithmetic Mean	10	90
Standard Deviation	3	12

Correlation coefficient between X and Y is 0.8

1. Obtain the two regression equations.
2. What is the likely sales when the advertising budget is ₹15 lakh?
3. What should be the advertising budget if the company wants to attend sales target of ₹120 lakhs?

Given, $\bar{X} = 10$, $\bar{Y} = 90$, $\sigma_x = 3$, $\sigma_y = 12$, $r = 0.8$

$b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y} \Rightarrow b_{xy} = 0.8 \times \frac{3}{12} = 0.2$

$b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x} \Rightarrow b_{yx} = 0.8 \times \frac{12}{3} = 3.2$

(i) Regression eq.^{n} Y on X:	Regression eq.^{n} X on Y:
$Y - \bar{Y} = b_{yx}(X - \bar{X})$	$X - \bar{X} = b_{xy}(Y - \bar{Y})$
$\therefore Y - 90 = 3.2(X - 10)$	$\therefore X - 10 = 0.2(Y - 90)$
$\therefore Y = 3.2X - 32 + 90$	$\therefore X = 0.2Y - 18 + 10$
$\therefore Y = 3.2X + 58$ ----(A)	$\therefore X = 0.2Y - 8$ -----(B)

(ii) For $X = 15$, From (A) we get,

$Y = 3.2(15) + 58 = 106 \Rightarrow$ Likely Sales is ₹106 Lakh when advertisement budget is ₹15 lakh.

(iii) For $Y = 120$

From (B), we get, $X = 0.2(120) - 8 = 24 - 8 = 16$

$\therefore$ Sales target of ₹ 102 lakh, advertisment budget must be ₹16 lakh.

Q.3 The equations of two regression lines are 2x+ 3y -6= 0 and 5x +7y- 12 = 0. Find i) Correlation coefficient ii) $\frac{\sigma_x}{\sigma_y}$

i) Let $2X + 3Y - 6 = 0$ be the regression eq.^{n} of Y on X

The eq.^{n} becomes, $3Y = -2X + 6$

$\therefore Y = -2/3\, X + 6/3$

Compairing with $Y = b_{yx} \cdot X + a$, $\Rightarrow b_{yx} = -2/3$

Let $5X + 7Y - 12 = 0$ be the regression eq.^{n} X on Y

The eq.^{n} becomes, $X = -7/5\, Y + 12/5$

Compairing with $X = b_{xy} \cdot Y + b$, $\Rightarrow b_{xy} = -7/5$

$\therefore \gamma = \pm\sqrt{b_{yx} \cdot b_{xy}} = \pm\sqrt{\left(-\frac{2}{3}\right)\left(-\frac{7}{5}\right)} = \pm\sqrt{\frac{14}{15}}$

$= \pm\sqrt{0.9333} = -0.96$

$\because b_{yx}$ & b_{xy} both are negative $\therefore \underline{\gamma = -0.96}$

ii) $b_{xy} = \gamma \cdot \frac{\sigma_x}{\sigma_y} \Rightarrow \frac{\sigma_x}{\sigma_y} = \frac{b_{xy}}{\gamma}$

$= \frac{-7/5}{-0.96}$

$= 1.4583$

$\Rightarrow \underline{\frac{\sigma_x}{\sigma_y} = 1.4583}$

Sign of Teacher :