A. **Activities**

Carry out the following activities.

1) If the average revenue is 45 and elasticity of demand is 5. Find the marginal revenue by completing the following activity.

Given $R_A = \boxed{45}$ and $\eta = \boxed{5}$

$$\text{Now, } R_m = R_A\left(\boxed{1} - \frac{\boxed{1}}{\boxed{\eta}}\right)$$

$$= 45\left(\boxed{1} - \frac{1}{\boxed{5}}\right)$$

$$= 45\left(\frac{\boxed{4}}{\boxed{5}}\right) = \boxed{36}$$

2) Find the price by completing the following activity, if the marginal revenue is 28 and elasticity of demand is 3.

Given $R_m = 28$ and $\eta = 3$

$$\text{Since } R_m = R_A\left(\boxed{1} - \frac{\boxed{1}}{\boxed{\eta}}\right)$$

$$\therefore 28 = R_A\left(\boxed{1} - \frac{1}{\boxed{3}}\right) = \frac{\boxed{2}}{\boxed{3}} \cdot R_A$$

$$\therefore R_A = \frac{28 \times \boxed{3}}{\boxed{2}} = \boxed{42}$$

Hence, the price $= \boxed{42}$

3) If $D = 50 - 3p - p^2$. Find the elasticity of demand at $p = 5$ by completing the following activity.

$D = 50 - 3p - p^2$

$$\therefore \frac{dD}{dP} = \frac{d}{dP}(50 - 3p - p^2)$$

$$= \boxed{0} - \boxed{1} \times 3 - \boxed{2}\, p$$

$$= -3 - \boxed{2}\, p$$

$$\therefore \eta = -\frac{\boxed{P}}{\boxed{D}} \times \frac{dD}{dP} = \frac{-P}{50 - 3P - P^2} \times \left(\boxed{-3} - \boxed{2}\, P\right)$$

$$\therefore \eta = -\frac{\boxed{P}\left(\boxed{3} + \boxed{2}\, P\right)}{50 - 3P - P^2}$$

If $p = 5$, then

$$\eta = +\frac{5\,(3+\overset{2}{\cancel{2}}\times 5)}{\boxed{50}-3\boxed{5}-\boxed{5}^2} = \frac{5\times 13}{\boxed{50}-\boxed{15}-\boxed{25}}$$

$$\therefore\ \eta = +\frac{65}{\boxed{10}} = \boxed{6.5}$$

4) Comment on elasticity of demand of a commodity for $p = 200$, when demand function is $p = 400 - \frac{q^2}{2}$ using following activity.

The demand function is $p = 400 - \frac{q^2}{2}$

$$\therefore\ \frac{q^2}{2} = \boxed{400} - p$$

$$\therefore\ q^2 = 800 - \boxed{2}\ p$$

$$\therefore\ q = \sqrt{800-2p}$$

$$\therefore\ \frac{dq}{dp} = \frac{d}{dp}\left(\sqrt{800-2p}\right)$$

$$\therefore\ \frac{dq}{dp} = \frac{1}{2\sqrt{800-2P}}\,\frac{d}{dp}\left(\boxed{800}-\boxed{2}P\right)$$

$$= \frac{1}{2\sqrt{800-2P}}\left(\boxed{0}-\boxed{2}\times 1\right)$$

$$= \frac{-1}{\sqrt{800-2P}}$$

Now $\eta = \frac{-p}{q}\,.\frac{dq}{dp}$

$$\eta = \frac{-P}{\sqrt{800-2p}} \times \frac{-1}{\sqrt{\boxed{800}-\boxed{2}P}}$$

$$\eta = \frac{P}{\boxed{800}-\boxed{2}P}$$

If $p = 200$, $\eta = \frac{\boxed{200}}{\boxed{800}-\boxed{200}2} = \frac{1}{\boxed{2}}$

Since $0 < \eta < 1$, the demand is $\boxed{\text{Relatively Inelastic}}$

B. Solve the Following

Q.1. If the demand function is $D = 150 - p^2 - 3p$, find marginal revenue, average revenue and elasticity of demand for price $p = 3$.

Soln. $\because D = 150 - P^2 - 3P$

Revenue $R = P \cdot D = P(150 - P^2 - 3P)$

$= 150P - P^3 - 3P^2$

Now, marginal Revenue :

$\frac{dR}{dP} = 150 - 3P^2 - 6P$

$\therefore \left(\frac{dR}{dP}\right)_{at\ P=3} = 150 - 3(3)^2 - 6(3)$

$= 150 - 27 - 18$

$= 105$

Thus, the marginal revenue is 105 at $P = 3$

Now, $R_A = (D)_{P=3} = 150 - 3^2 - 3(3)$

$= 150 - 9 - 9 = \underline{132}$

Thus, the average revenue is 132 at $P = 3$

Now, elasticity of demand

$\eta = -\frac{P}{D} \frac{dD}{dP}$

$= \frac{-3}{132} \frac{d}{dP}(150 - P^2 - 3P)$

$= -\frac{3}{132}(\cancel{150} - 2P - 3)$

$= -\frac{3}{132} \times (-2 \times 3 - 3)$

$= -\frac{3}{132} \times -9$

$= \frac{27}{132}$

Q.2. Comment on elasticity of demand of a commodity for $p = 600$ when demand function is $p = \left(1200 - \frac{q^2}{2}\right)$.

Soln. $\because$ Elasticity of demand $\eta = -\frac{P}{D} \frac{dD}{dP}$ —(1)

Here, $P = 600$ & $P = 1200 - \frac{q^2}{2}$

$\therefore 600 = 1200 - \frac{q^2}{2}$

$\frac{q^2}{2} = 1200 - 600$

$\frac{q^2}{2} = 600$

$q^2 = 1200$

$q = 20\sqrt{3}$

Now, $p = 1200 - \frac{q^2}{2}$

~~= 1200~~

$\frac{q^2}{2} = 1200 - p$

diff. both side w. r. to 'p' –

$\frac{\cancel{2}q}{\cancel{2}} \frac{dq}{dp} = -1$

$\frac{dq}{dp} = -\frac{1}{q} = \frac{-1}{20\sqrt{3}}$

Now, from (1) $\eta = \frac{-600}{20\sqrt{3}} \times -\frac{1}{20\sqrt{3}}$

$= \frac{600}{1200} = \frac{1}{2}$

Since $0 < \eta < \frac{1}{2}$, the demand is inelastic

Q.3. Mr. Pritesh orders x mobiles at the price $p = 2x + \frac{32}{x^2} - \frac{5}{x}$

How many mobiles should he order for the most economical deal?

soln: $\because$ price of mobiles $p = 2x + \frac{32}{x^2} - \frac{5}{x}$

Total cost $C = p \cdot x$ ($\because$ x is the no. of mobiles to be ordered)

$C = \left(2x + \frac{32}{x^2} - \frac{5}{x}\right)x$

$C = 2x^2 + \frac{32}{x} - 5$

Now, $\frac{dC}{dx} = 4x - \frac{32}{x^2}$

$\because$ C is minimum

$\therefore \frac{dC}{dx} = 0 \Rightarrow 4x - \frac{32}{x^2} = 0$

$\Rightarrow 4x^3 = 32$

$\Rightarrow x^3 = 8$

$\Rightarrow x = 2$

We have, $\frac{d^2C}{dx^2} = \frac{d}{dx}\left(\frac{dC}{dx}\right) = \frac{d}{dx}\left(4x - \frac{32}{x^2}\right)$

$= 4 + \frac{64}{x^3}$

$\therefore \left(\frac{d^2C}{dx^2}\right)_{x=2} = 4 + \frac{64}{(2)^3} = \frac{32+64}{8} = \frac{96}{8} = 12 > 0$

Thus C is minimum for $x=2$, by the second derivative test

Hence 2 mobiles should be ordered for the most economical deal.

Q.4. In Shraddha's farm the cost function for output x is given by $C = \frac{x^3}{3} - 20x^2 + 70x$ Find the output for which i) Marginal cost (C_m) is minimum. ii) Average cost (C_A) is minimum.

Soln: $\because C = \frac{x^3}{3} - 20x^2 + 70x$

$\therefore C_A = \frac{C}{x} = \frac{x^2}{3} - 20x + 70$

we have marginal cost $C_m = \frac{dC}{dx} = \frac{d}{dx}\left(\frac{x^3}{3} - 20x^2 + 70x\right)$

$= \frac{\cancel{3}x^2}{\cancel{3}} - 40x + 70$

$= x^2 - 40x + 70$

Now, $\frac{dC_m}{dx} = 2x - 40$

$\because$ C_m is minimum

$\therefore \frac{dC_m}{dx} = 0 \Rightarrow 2x - 40 = 0$

$\Rightarrow 2x = 40$

$\Rightarrow \boxed{x = 20}$

Now, $\frac{d^2C_m}{dx^2} = \frac{d}{dx}\left(\frac{dC_m}{dx}\right) = \frac{d}{dx}(2x-40) = 2 > 0$

$\therefore$ marginal cost is minimum for $x=2$ by the second derivative test.

Now, $\frac{dC_A}{dx} = \frac{d}{dx}\left(\frac{x^2}{3} - 20x + 70\right) = \frac{2x}{3} - 20$

$\because$ C_A is minimum

$\therefore \frac{dC_A}{dx} = 0 \Rightarrow \frac{2x}{3} - 20 = 0$

$\Rightarrow 2x = 60$

$\Rightarrow \boxed{x = 30}$

Now, $\frac{d^2C_A}{dx^2} = \frac{d}{dx}\left(\frac{dC_A}{dx}\right) = \frac{d}{dx}\left(\frac{2x}{3} - 20\right) = \frac{2}{3} > 0$

$\therefore$ C_A is minimum for $x=30$ by the second derivative test.

Sign of Teacher :