

## 16. Binomial Distribution

- Ex. (1)** A biased coin is tossed 6 times. The probability of heads on any toss is 0.3. Let  $X$  denote the number of heads that comes up. Find (i)  $P(x=2)$  (ii)  $P(x=3)$  and (iii)  $P(1 < x \leq 5)$ .

**Solution :**  $X$  denote the number of heads that comes up.

The probability of heads on any toss is 0.3.

$$\text{Probability of success (p)} = 0.3 = \frac{3}{10}$$

$$\text{Probability of failure (q)} = 1 - p = 1 - 0.3 = 0.7 = \frac{7}{10}$$

Clearly  $X \sim (n, p)$  with  $n = 6$ ,  $p = \frac{3}{10}$  and  $q = \frac{7}{10}$

$P(X = r) = {}^nC_r p^r q^{n-r}$  where  $r = 0, 1, 2, 3, \dots, n$ .

$$(i) \quad P(X=2) = {}^6C_2 \left(\frac{3}{10}\right)^2 \left(\frac{7}{10}\right)^{6-2}$$

$$= \frac{6 \cdot 5}{2 \cdot 1} \left(\frac{3}{10}\right)^2 \left(\frac{7}{10}\right)^4$$

$$\frac{(135)(7^4)}{10^6}$$

$$\frac{(135)(2401)}{10^6}$$

$$= 0.324135$$

$$(ii) \quad P(X=3)$$

$$= {}^6C_3 \left(\frac{3}{10}\right)^3 \left(\frac{7}{10}\right)^{6-3}$$

$$= 20 \left(\frac{27}{1000}\right) \left(\frac{343}{1000}\right)$$

$$= \frac{540}{1000} \times \frac{343}{1000}$$

$$= 0.18522$$

$$(iii) \quad P(1 < x \leq 5) = P(X=2, 3, 4, 5)$$

$$= 1 - P(X=0, 1, 6)$$



$$= 1 - \{ P(X=0) + P(X=1) + P(X=6) \} \quad \dots \quad \dots \quad \dots (I)$$

$$P(X=0) + P(X=1) + P(X=6)$$

$$= {}^6C_0 \left( \frac{3}{10} \right)^0 \left( \frac{7}{10} \right)^{6-0} + {}^6C_1 \left( \frac{3}{10} \right)^1 \left( \frac{7}{10} \right)^{6-1} + {}^6C_6 \left( \frac{3}{10} \right)^6 \left( \frac{7}{10} \right)^{6-6}$$

$$= (1)(1) \left( \frac{7}{10} \right)^6 + (6) \left( \frac{3}{10} \right) \left( \frac{7}{10} \right)^5 + (1) \left( \frac{3}{10} \right)^6 (1)$$

$$= \left( \frac{7^6}{10^6} \right) + \left( \frac{(18)(7^5)}{10^6} \right) + \left( \frac{3^6}{10^6} \right)$$

$$= \left( \frac{7^5(7+18) + 729}{10^6} \right) = \left( \frac{(16807)(25) + 729}{10^6} \right)$$

$$= \left( \frac{420904}{10^6} \right)$$

.....(II)

From (I) and (II)

$$P(1 < x \leq 5) = 1 - \{ P(X=0) + P(X=1) + P(X=6) \}$$

$$= 1 - \left( \frac{420904}{10^6} \right) = 0.579096$$

**Ex. (2)** If the random variable X follows the Binomial Distribution with 6 trials and a probability of success equal to  $\frac{1}{4}$  at each attempt then what will be the probability of (i) exactly 4 success (ii) at least one success .

**Solution :** The random variable X follows the Binomial Distribution with 6 trials and a probability of success equal to  $\frac{1}{4}$  at each attempt.

$$\therefore p = \frac{1}{4} \text{ and } q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$$

Clearly  $X \sim (n, p)$  with  $n = 6$  ,  $p = \frac{1}{4}$  and  $q = \frac{3}{4}$

$$P(X = r) = {}^nC_r p^r q^{n-r} \text{ where } r = 0, 1, 2, 3, \dots, n.$$

(i) probability of exactly 4 success

$$P(X=4) = {}^6C_4 \left( \frac{1}{4} \right)^4 \left( \frac{3}{4} \right)^{6-4}$$

$$= 15 \left( \frac{1}{256} \right) \left( \frac{9}{16} \right)$$

$$= \frac{15 \times 9}{256 \times 16}$$



$$= \frac{135}{4096}$$

$$= 0.03295$$

(i) probability of at least one success

$$P(X \geq 1) = P(X = 1, 2, 3, 4, 5, 6)$$

$$= 1 - P(X < 1)$$

$$= 1 - \{ P(X=0) \} \quad \dots \quad \dots \quad \dots (I)$$

$$= 1 - {}^6C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{6-0}$$

$$= 1 - 1 \cdot (1) \cdot \left(\frac{729}{4096}\right)$$

$$= 1 - \frac{729}{4096}$$

$$= 1 - 0.1779$$

$$\therefore P(X \geq 1) = 0.8221$$

**Ex. (3)** The probability that a student is not a swimmer is  $\frac{2}{3}$ . If 5 students are randomly chosen, find the probability that (i) 4 out of them are swimmers (ii) at least four are swimmers.

**Solution :** X denote the number of student is a swimmer.

The probability that a student is not a swimmer is  $\frac{2}{3}$ .

The probability that a student is a swimmer is  $1 - \frac{2}{3} = \frac{1}{3}$

$$\text{var}(X) = npq = (10)\left(\frac{1}{2}\right)\left(\frac{5}{2}\right).$$

$$\text{Probability of success (p)} = \frac{1}{3}$$

$$\text{Probability of failure (q)} = 1 - \frac{1}{3} = \frac{2}{3}.$$

**Ex. (4)** Let  $X \sim B(n, p)$  if  $n=10$  and  $E(X) = 5$ . Find  $p$  and S.D.(X).

**Solution :**  $X \sim B(n, p)$ ,  $n=10$  and  $E(X) = 5$ .

$$E(X) = np$$

$$5 = 10p$$

$$p = \frac{1}{2}$$

$$\therefore q = 1 - p$$



$$\therefore q = 1 - \frac{1}{2}$$

$$\therefore q = \frac{1}{2}$$

$$\text{var}(X) = n p q$$

$$= (10) \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)$$

$$= \frac{5}{2}$$

$$\text{S.D.}(X) = \sqrt{\text{var}(X)}$$

$$= \sqrt{\frac{5}{2}}$$

**Ex. (5)** The probability of hitting a target in any shot is 0.2. If 10 shots are fired then find  $\mu$  and S.D.(X).

**Solution :** Let  $x$  = number of shots hitting the target  
 $p$  = probability that the target is shot

$$\therefore p = 0.2 = \frac{1}{5} \text{ and } q = 1 - p = \frac{4}{5}$$

$$\text{Given } n = 10$$

$$\therefore x \sim B\left(10, \frac{1}{5}\right)$$

$$\text{we know } \mu = E(x) = n \cdot p$$

$$\mu = 10 \times \frac{1}{5}$$

$$\boxed{\mu = 2}$$

$$\text{and S.D.}(x) = \sqrt{n p q}$$

$$= \sqrt{10 \times \frac{1}{5} \times \frac{4}{5}}$$

$$= \sqrt{\frac{8}{5}} = \sqrt{1.6}$$

$$= 1.2649$$



**Ex. (6)** For a Binomial Distribution the number of trials is 5 and  $P(X=4) = P(X=3)$ . Find the probability of success and also obtain  $P(X > 2)$ .

**Solution :**

We know

$$P(X=r) = {}^nC_r P^r q^{n-r}$$

$$\text{Given } P(X=4) = P(X=3)$$

$${}^5C_4 P^4 q^1 = {}^5C_3 P^3 q^2$$

$$5 P^4 q = 10 P^3 q^2$$

on canceling

$$P = 2q$$

$$P = 2(1-P) \quad \because q = 1-P$$

$$P = 2 - 2P$$

$$3P = 2$$

$$P = \frac{2}{3}$$

$\therefore$  Probability of success is  $\frac{2}{3}$

$$\therefore q = \frac{1}{3}$$

$$\text{Now } P(X > 2) = P(3) + P(4) + P(5)$$

$$= {}^5C_3 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^2 + {}^5C_4 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^1 + {}^5C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^0$$

$$+ {}^5C_5 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^0$$

$$= \left[10 \times \frac{8}{27} \times \frac{1}{9}\right] + \left[5 \times \frac{16}{81} \times \frac{1}{3}\right] + \left[1 \times \frac{32}{243}\right]$$

$$= \frac{8}{243} [10 + 10 + 4]$$

$$= \frac{8 \times 24}{243}$$

$$= 0.79$$

$$\therefore P(X > 2) = 0.79$$

**Ex. (7)** Student A has answered that the mean of a Binomial Distribution is 18 and variance is 12, another student B answered that the mean is 18 and variance is 21. Of the two students whose answer is correct? Justify.

**Solution :** A student answered that the mean is 18 and Variance is 12

We know mean =  $np$  and Var =  $npq$

$$\therefore np = 18 \quad \text{and} \quad npq = 12$$

$$18q = 12$$

$$q = \frac{2}{3}$$

$$\therefore P = \frac{1}{3}$$

$$\therefore n = 54 \quad \text{According to A.}$$



student B answered that

mean = 18 and var = 21

$$\therefore np = 18$$

$$npq = 21$$

$$\therefore 18 \times q = 21$$

$$q = \frac{7}{6}, \therefore p = -\frac{1}{6}$$

$$\therefore n = \frac{18}{p}$$

$$n = \frac{18}{-1/6}$$

$$n = -108$$

as  $n$  is never -ve

$\therefore$  student A is correct

**Ex. (8)** In a group of 10 players 5 pass fitness test. Find the probability that out of the 4 players selected at random (i) exactly two will pass fitness test (ii) at least 2 will pass fitness test.

**Solution :** As 5 pass fitness test among 10 players

$$p = \frac{1}{2} \text{ and } q = \frac{1}{2}$$

$$\begin{aligned} P(x=2) &= {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2} \\ &= 6 \times \frac{1}{4} \times \frac{1}{4} \Rightarrow 6 \times \frac{1}{16} \end{aligned}$$

$$\boxed{P(x=2) = 3/8}$$

$$\text{Now } P(x \geq 2) = P(2) + P(3) + P(4)$$

$$= 1 - [P(0) + P(1)]$$

$$= 1 - \left[ {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 + {}^4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^3 \right]$$

$$= 1 - \left[ \frac{1}{16} + \frac{4}{2} \left(\frac{1}{8}\right) \right]$$

$$= 1 - \left[ \frac{1}{16} + \frac{4}{16} \right]$$

$$= 1 - \frac{5}{16}$$

$$\boxed{P(x \geq 2) = \frac{11}{16}}$$

Sign of Teacher :

- Q. 26.** A solenoid of length  $\pi$  m and 5 cm in diameter has a winding of 1000 turns and carries a current of 5A. Calculate the magnetic field at its centre along the axis.

## SECTION – D

**Attempt any THREE questions of the following :**

**[12]**

- Q. 27.** What is Ferromagnetism? Explain it on the basis of domain theory.
- Q. 28.** Obtain an expression for average power dissipated in a series LCR circuit.
- Q. 29.** Distinguish between interference and diffraction of light.  
A double slit arrangement produces interference fringes for sodium light of wavelength 589 nm, that are 0.20 degree apart. What is the angular fringe separation if the entire arrangement is immersed in water?  
( R.I. of water = 1.33 )
- Q. 30.** State Einstein's photoelectric equation and mention physical significance of each term involved in it.  
The wavelength of incident light is  $4000\text{\AA}$ . Calculate the energy of incident photon.
- Q. 31.** State any four uses of Van de Graaff generator.  
In a parallel plate air capacitor, intensity of electric field is changing at the rate of  $2 \times 10^{11}$  V/ms. If area of each plate is  $20\text{ cm}^2$ , calculate the displacement current.





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