1. Logic

Let us Recall

- The converse, inverse and contrapositive of the implication $p \to q$ are:

Converse : $q \to p$

Inverse : $\sim p \to \sim q$

Contrapositive : $\sim q \to \sim p$

- Quantifiers and quantified Statements : Look at the following statements:

p : "There exists an even prime number in the set of natural numbers".

q : "All natural numbers are positive".

Each of them asserts a condition for some or all objects in a collection. Words "there exists" and "for all" are called quantifiers. "There exists" is called existential quantifier and is denoted by symbol $\exists$. "For all" is called universal quantifier and is denoted by $\forall$. Statements involving quantifiers are called quantified statements. Every quantified statement corresponds to a collection and a condition. In statement p the collection is 'the set of natural numbers' and the condition is 'being even prime'.

What is the condition in the statement q ?

A statement quantified by universal quantifier $\forall$ is true if all objects in the collection satisfy the condition. And it is false if at least one object in the collection does not satisfy the condition.

A statement quantified by existential quantifier $\exists$ is true if at least one object in the collection satisfy the condition. And it is false if no object in the collection satisfy the condition.

Idempotent Law	$p \wedge p \equiv p$, $\quad p \vee p \equiv p$
Commutative Law	$p \vee q \equiv q \vee p \quad p \wedge q \equiv q \wedge p$
Associative Law	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r \equiv p \wedge q \wedge r$
	$p \vee (q \vee r) \equiv (p \vee q) \vee r \equiv p \vee q \vee r$
Distributive Law	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
De Morgan's Law	$\sim(p \wedge q) \equiv \sim p \vee \sim q$, $\sim(p \vee q) \equiv \sim p \wedge \sim q$
Identity Law	$p \wedge T \equiv p$, $p \wedge F \equiv F$, $p \vee F \equiv p$, $p \vee T \equiv T$
Complement Law	$p \wedge \sim p \equiv F$, $p \vee \sim p \equiv T$

Absorption Law	$p \vee (p \wedge q) \equiv p,\ p \wedge (p \vee q) \equiv p$
Conditional Law	$p \rightarrow q \equiv \sim p \vee q$
Biconditional Law	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p) \equiv (\sim p \vee q) \wedge (\sim q \vee p)$

Ex. (1) Write the converse, inverse, contrapositive and the negation of the implication : "If two sides of a triangle are congruent then it's two angles are congruent."

Solution :

Converse : If two angles of a triangle are congruent then it's two sides are congruent.

Inverse : If two sides of a triangle are not congruent then it's two angles are not congruent.

Contrapositive : If two angles of a triangle are not congruent then it's two sides are not congruent.

Negation : Two sides of a triangle are congruent but it's two angles are not congruent.

Ex. (2) Write (a) truth values and (b) negations of the following statements :

i) $\forall x \in R, x^2$ is positive. ii) $\exists x \in R, x^2$ is not positive.

iii) Every square is a rectangle. iv) Some parallelograms are rectangles.

Solution :

a) Truth values

i) false because the square of 0 is not positive

ii) true because the square of 0 is not positive

iii) true iv) true

b) Negations

i) $\exists x \in R, x^2$ is not positive. ii) $\forall x \in R, x^2$ is positive.

iii) There exists a square which is not a rectangle.

iv) No parallelogram is a rectangle.

Ex. (3) Without using truth table prove that $\{[(p \vee q) \wedge \sim p] \rightarrow \sim q\} \equiv q \rightarrow p$

Solution :

L.H.S. $= \{[(p \vee q) \wedge \sim p] \rightarrow \sim q\}$

$\equiv \sim[(p \vee q) \wedge \sim p] \vee \sim q$ (.....Conditional law.....)

$\equiv\left[\sim(p\vee q)\vee p\right]\vee\sim q$ (De-Morgan's Law)

$\equiv\left[(\sim p\wedge\sim q)\vee p\right]\vee\sim q$ (De-Morgan's Law)

$\equiv\left[(\sim p\vee p)\wedge(\sim q\vee p)\right]\vee\sim q$ (Distributive Law)

$\equiv\left[(t)\wedge(\sim q\vee p)\right]\vee\sim q$ (Complement Law)

$\equiv(\sim q\vee p)\vee\sim q$ (Identity Law)

$\equiv\ \sim q\vee p\vee\sim q$ (Associative law)

$\equiv\ \sim q\vee p$ (Idempotent Law)

$\equiv q\to p$ (Conditional Law)

L. H. S. = R. H. S.

Ex. (4) Using truth table prove that $\sim(p\leftrightarrow q)\equiv(p\wedge\sim q)\vee(q\wedge\sim p)$.

Solution :

I	II	III	IV	V	VI	VII	VIII	IX
p	q	$\sim p$	$\sim q$	$p\leftrightarrow q$	$\sim(p\leftrightarrow q)$	$p\wedge\sim q$	$q\wedge\sim p$	$(p\wedge\sim q)\vee(q\wedge\sim p)$
T	T	F	F	T	F	F	F	F
T	F	F	T	F	T	T	F	T
F	T	T	F	F	T	F	T	T
F	F	T	T	T	F	F	F	F

From column (VI) and (IX) we conclude that

$\sim(p\leftrightarrow q)\equiv(p\wedge\sim q)\vee(q\wedge\sim p)$

Ex. (5) Is $\sim(p\leftrightarrow q)$ equivalent to $(\sim p)\leftrightarrow q$? Justify.

Solution :

I	II	III	IV	V	VI
p	q	$\sim p$	$\sim p\leftrightarrow q$	$p\leftrightarrow q$	$\sim(p\leftrightarrow q)$
T	T	F	F	T	F
T	F	F	T	F	T
F	T	T	T	F	T
F	F	T	F	T	F

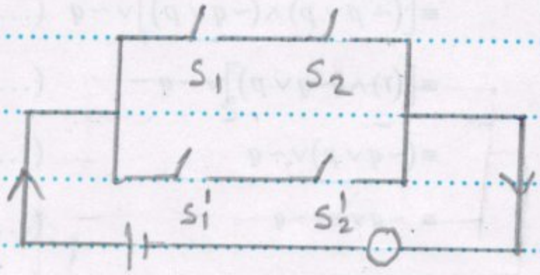
From column no IV and VI we conclude that $\sim(p\leftrightarrow q)\equiv(\sim p)\leftrightarrow q$

Ex. (6) Draw the switching circuits and prepare the input output tables for statement patterns :

a) $(p \wedge q) \vee (\sim p \wedge \sim q)$ b) $(p \vee q) \wedge \sim q$

Solution :

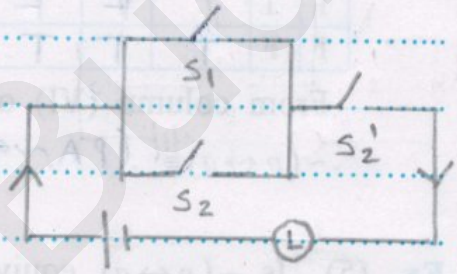
(a) Let p switch S_1 is closed
q switch S_2 is closed
$\sim p$ S_1' is closed
$\sim q$ S_2' is closed



p	q	$p\wedge q$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$(p\wedge q)\vee(\sim p\wedge \sim q)$
1	1	1	0	0	0	1
1	0	0	0	1	0	0
0	1	0	1	0	0	0
0	0	0	1	1	1	1

(b)

p	q	$\sim q$	$p\vee q$	$(p\vee q)\wedge \sim q$
1	1	0	1	0
1	0	1	1	1
0	1	0	1	0
0	0	1	0	0



Ex. (7) Using truth table prove that $(p \wedge q) \vee r \equiv (p \vee r) \wedge (q \vee r)$

Solution :

p	q	r	$p\wedge q$	$(p\wedge q)\vee r$	$p\vee r$	$q\vee r$	$(p\vee r)\wedge(q\vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	T	F	F
F	T	T	F	T	T	T	T
F	T	F	F	F	F	T	F
F	F	T	F	T	T	T	T
F	F	F	F	F	F	F	F

Sign of Teacher :

From column (V) and (VIII) we conclude that $(p\wedge q)\vee r \equiv (p\vee r)\wedge(q\vee r)$

15. Probability Distribution

Ex. (1). A random variable X has the following probability distribution :

$X = x$	0	1	2	3	4	5	6
$P(X = x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

Find (i) k (ii) $p(X < 3)$ (iii) $p(X \geq 2)$ (iv) $P(0 < X < 4)$ (v) $P(2 \leq X \leq 5)$

Solution : For a random variable X we have $\sum_{i=1}^{n} p_i = 1$

$\therefore k + 3k + 5k + 7k + 9k + 11k + 13k = 1$

i.e. $49k = 1 \Rightarrow k = \frac{1}{49}$

$X = x$	0	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{49}$	$\frac{3}{49}$	$\frac{5}{49}$	$\frac{7}{49}$	$\frac{9}{49}$	$\frac{11}{49}$	$\frac{13}{49}$

(i) $k = \frac{1}{49}$

(ii) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$= \frac{1}{49} + \frac{3}{49} + \frac{5}{49} = \frac{9}{49}$

(iii) $P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$

(iv) $P(0 < X < 4) = P(X = 1) + P(X = 2) + P(X = 3)$

(v) $P(2 \leq X \leq 5) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$

$= \frac{5}{49} + \frac{7}{49} + \frac{9}{49} + \frac{11}{49} = \frac{32}{49}$

Ex. (2). Calculate the Expected value and Variance of X if X denotes the number obtained on the uppermost face when a fair die is thrown.

Solution : When a fair die is thrown, the sample space is $S = \{1, 2, 3, 4, 5, 6\}$.

Let X denotes the number obtained on the uppermost face.

$\therefore X$ can take values 1, 2, 3, 4, 5, 6.

$P(X = 1) = P(X = 2) = P(X = 3) = P(X = 4) = P(X = 5) = P(X = 6)$

$= \frac{1}{6}$

The probability distribution is

$X = x$	1	2	3	4	5	6	Total
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1

$x_i.p_i$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{6}{6}$	$\frac{21}{6}=\frac{7}{2}$
$x_i^2.p_i$	$\frac{1}{6}$	$\frac{4}{6}$	$\frac{9}{6}$	$\frac{16}{6}$	$\frac{25}{6}$	$\frac{36}{6}$	$\frac{91}{6}$

(i) Expected Value = $E(X) = \sum_{i=1}^{n} x_i, p_i = \frac{7}{2} = 3.5$

(ii) Variance = $V(X) = E(X^2) - [E(X)]^2$

$$= \sum_{i=1}^{n} x_i^2, p_i - \left(\sum_{i=1}^{n} x_i, p_i\right)^2$$

$$= \frac{91}{6} - \left(\frac{7}{2}\right)^2 = \frac{91}{6} - \frac{49}{4}$$

$$= \frac{182-147}{12}$$

$\therefore$ Variance = V(X) = $\frac{35}{12} = 2.9167$

Ex. (3). A discrete random variable X takes the values -1, 0 and 2 with the probabilities $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$ respectively. Find $V(X)$ and Standard Deviation.

Solution : Given that the random variable X takes the values -1, 0 and 2.

The corresponding probabilities are $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$.

$P(-1) = \frac{1}{4}$, $P(0) = \frac{1}{2}$ and $P(2) = \frac{1}{4}$

Given data can be tabulated as follows

$X = x$	-1	0	2	Total
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	1
$x_i p_i$	$-\frac{1}{4}$	0	$\frac{1}{2}$	$\frac{1}{4}$
$x_i^2 p_i$	$\frac{1}{4}$	0	1	$\frac{5}{4}$

(i) Variance = $V(X) = E(X^2) - [E(X)]^2$

$$= \sum_{i=1}^{n} x_i^2 p_i - \left(\sum_{i=1}^{n} x_i p_i\right)^2$$

$$= \frac{5}{4} - \left(\frac{1}{4}\right)^2$$

$$= \frac{5}{4} - \frac{1}{16} = \frac{76}{64} = 1.1875$$

(ii) Standard Deviation = $\sigma = \sqrt{V(X)} = \sqrt{1.1875} = 1.0897$

Ex. (4) The p. d. f. of X, find $P(X < 1)$ and $P(|X| < 1)$ where

$$f(x) = \frac{x+2}{18} \quad \text{if } -2 < x < 4$$
$$= 0 \quad \text{otherwise.}$$

Solution : Given that the p. d. f. of X is

$$f(x) = \frac{x+2}{18} \quad \text{if } -2 < x < 4$$
$$= 0 \quad \text{otherwise.}$$

(i) $P(X<1) = \int_{-2}^{1} f(x)dx$

$= \int_{-2}^{1} \frac{x+2}{18}\,dx$

$= \frac{1}{18}\int_{-2}^{1} (x+2)\,dx$

$= \frac{1}{18}\left[\frac{(x+2)^2}{2}\right]_{-2}^{1}$

$= \frac{1}{36}\left[(x+2)^2\right]_{-2}^{1}$

$= \frac{1}{36}[9-0] = \frac{9}{36} = \frac{9}{36} = 0.25$

(i) $P(|X|<1) = P(-1<X<1)$

$= \int_{-1}^{1} \frac{x+2}{18}\,dx$

$= \frac{1}{18}\int_{-1}^{1} (x+2)dx$

$= \frac{1}{18}\left[\frac{(x+2)^2}{2}\right]_{-1}^{1}$

$= \frac{1}{36}\left[(x+2)^2\right]_{-1}^{1}$

$= \frac{1}{36}[9 - 1] = \frac{8}{36} = \frac{2}{9} = 0.2222$

Ex. (5). A random variable X has the following probability distribution :

x	0	1	2	3	4	5	6	7
$P(X = x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find (i) k (ii) $P(X<3)$ (iii) $P(X>6)$ (iv) $P(0<X<3)$ (v) $P(2 \le X \le 4)$

Solution : Since $P(x)$ is probability distribution of x

$\sum_{x=0}^{7} P(x) = 1$

$\therefore P(0)+P(1)+P(2)+P(3)+P(4)+P(5)+P(6)+P(7) = 1$

$0+K+2K+2K+3K+K^2+2K^2+7K^2+K = 1$

$10K^2+9K-1 = 0$

$10K^2 + 10K - K - 1 = 0$

$10K(K+1) - 1(K+1) = 0$

$\therefore (10K-1)(K+1) = 0$

$K = \frac{1}{10}$ or $K = -1$

$K = -1$ is not possible

i) $\therefore \boxed{K = \frac{1}{10}}$

ii) $P(x<3) = 0 + K + 2K$

$= 3K$

$= \frac{3}{10}$

iii) $P(x>6) = 7K^2 + K$

$= \frac{7}{10^2} + \frac{1}{10}$

$= \frac{17}{10}$

iv) $P(0 < x < 3) = K + 2K$

$= 3K$

$= \frac{3}{10}$

v) $P(2 \le x \le 4)$

$= 2K + 2K + 3K$

$= 7K$

$= \frac{7}{10}$

Ex. (6). The p. m. f. of a random variable X is as follows :

$X = x$	1	2	3	4
$P(x)$	$\frac{1}{30}$	$\frac{4}{30}$	$\frac{9}{30}$	$\frac{16}{30}$

Find Mean and the Variance.

Solution :

$X = x$	1	2	3	4	Σ
$p(x)$	1/30	4/30	9/30	16/30	10/3
$x^2 \cdot p(x)$	1/30	16/30	81/30	256/30	59/5

from the table

$\Sigma x_i p_i = \frac{10}{3}$ and $\Sigma x_i^2 p_i = \frac{59}{5}$

mean $= E(x) = \frac{10}{3} = 3.33$

$$\text{Variance} = V(x) = \sum x_i^2 p_i - \left[\sum x_i p_i\right]^2$$

$$= \frac{59}{5} - \left[\frac{10}{3}\right]^2$$

$$= \frac{59}{5} - \frac{100}{9}$$

$$= \frac{531 - 500}{45}$$

$$= \frac{31}{45}$$

Hence mean = 3.33

Variance = 0.6888

Ex. (7). From a survey of 20 families in a society, the following data was obtained :

No. of children	0	1	2	3	4
No. of families	5	11	2	0	2

For the random variable X = number of children in a randomly chosen family, Find $E(X)$ and $V(X)$.

Solution :

x	0	1	2	3	4	Σ
f	5	11	2	0	2	$N = 20$
$P(x)$	$\frac{5}{25}$	$\frac{11}{20}$	$\frac{2}{20}$	0	$\frac{2}{20}$	
$x_i p_i$	0	$\frac{11}{20}$	$\frac{4}{20}$	0	$\frac{8}{20}$	$\frac{23}{20}$
$x_i^2 p_i$	0	$\frac{11}{20}$	$\frac{8}{20}$	0	$\frac{32}{20}$	$\frac{51}{20}$

Here $E(x) = x_i p_i$

$$= \frac{23}{20}$$

$$= 1.15$$

$$V(x) = \Sigma x_i^2 p_i - [\Sigma x_i p_i]^2$$

$$= \frac{51}{20} - \left[\frac{23}{20}\right]^2$$

$$= \frac{51}{20} - \frac{529}{400} \Rightarrow \frac{491}{400}$$

$$V(x) = 1.2275$$

Ex. (8). Find the c.d.f. $F(X)$ associated with the following p.d.f $f(x)$:

$f(x) = 12x^2(1 - x)$ for $0 < x < 1$

$= 0$ otherwise.

Also, find $P\left(\frac{1}{3} < X < \frac{1}{2}\right)$ by using p.d.f and c.d.f.

Solution :

$$F(x) = \int_0^x f(x)\, dx$$

$$= \int_0^x 12x^2(1-x)\, dx$$

$$= 12\int_0^x (x^2 - x^3)\, dx$$

$$= 12\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^x$$

$$= 12\left[\frac{4x^3 - 3x^4}{12}\right]$$

$$F(x) = 4x^3 - 3x^4 \quad \text{— I}$$

Now $p\left(\frac{1}{3} < x < \frac{1}{2}\right)$ Using p.d.f.

$$= \int_{1/3}^{1/2} 12x^2(1-x)\, dx$$

$$= \left[4x^3 - 3x^4\right]_{1/3}^{1/2}$$

$$= \left[4\left(\tfrac{1}{2}\right)^3 - 3\left(\tfrac{1}{2}\right)^4\right] - \left[4\left(\tfrac{1}{3}\right)^3 - 3\left(\tfrac{1}{3}\right)^4\right]$$

$$= \left[\frac{4}{8} - \frac{3}{16}\right] - \left[\frac{4}{27} - \frac{3}{81}\right]$$

$$= \frac{5}{16} - \frac{9}{81} = \frac{405-144}{1296}$$

$$= \frac{29}{144}$$

$$\boxed{F(x) = 0.2013}$$

Now $p\left(\frac{1}{3} < x < \frac{1}{2}\right)$ by c.d.f.

$$F(x) = 4x^3 - 3x^4$$

$$= F\left(\tfrac{1}{2}\right) - F\left(\tfrac{1}{3}\right)$$

$$= \left[4\left(\tfrac{1}{2}\right)^3 - 3\left(\tfrac{1}{2}\right)^4\right] - \left[4\left(\tfrac{1}{3}\right)^3 - 3\left(\tfrac{1}{3}\right)^4\right]$$

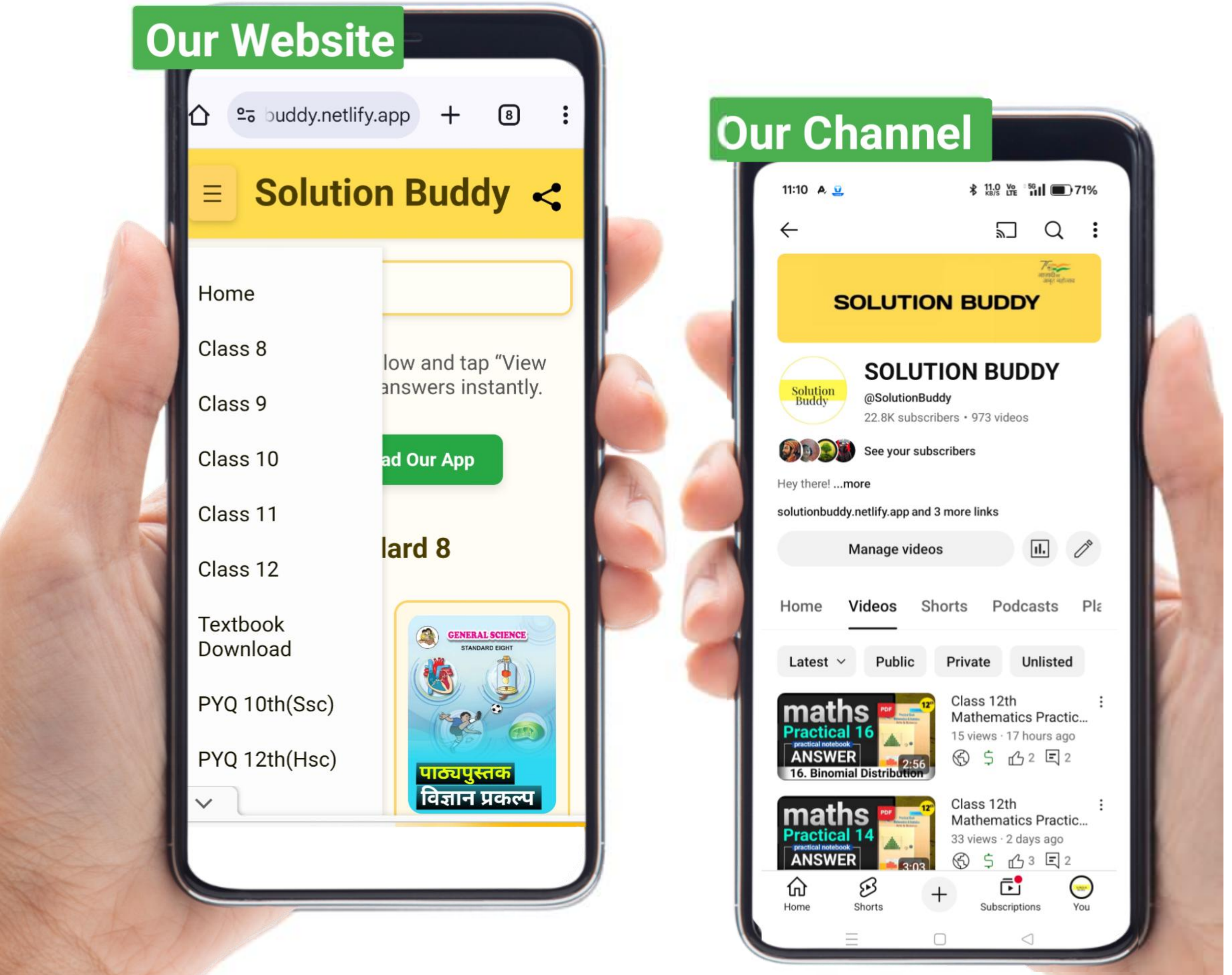
$$= \left[\frac{4}{8} - \frac{3}{16}\right] - \left[\frac{4}{27} - \frac{3}{81}\right]$$

$$= \frac{5}{16} - \frac{9}{81}$$

$$= \frac{29}{144}$$

$$\boxed{F(x) = 0.2013}$$

Sign of Teacher :



📚 On Solution Buddy, You Will Get:

✔️ Exercise solutions for Class 8–12
✔️ Previous Year Question Papers (10th & 12th)
✔️ Free Textbook Downloads
✔️ Practical Solutions (Class 10, 11 & 12)
✔️ Water Security Exercise & Activity Solution
✔️ Defence Studies Exercise Solution
👉 Website: solutionbuddy.netlify.app
👉 YouTube: youtube.com/@solutionbuddy

Solution Buddy