8 Quadrilateral : Constructions and Types

Let's recall.

- Construct the triangles with given measures.

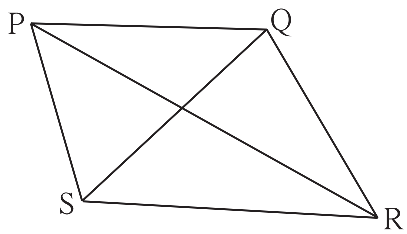
(1) Δ ABC : l (AB) = 5 cm, l (BC) = 5.5 cm, l (AC) = 6 cm

(2) Δ DEF : $m\angle$ D = 35°, $m\angle$ F = 100°, l(DF) = 4.8 cm

(3) Δ MNP : l (MP) = 6.2 cm, l (NP) = 4.5 cm, $m\angle$ P = 75°

(4) Δ XYZ : $m\angle$ Y = 90°, l (XY) = 4.2 cm, l (XZ) = 7 cm

- Every quadrilateral has 4 angles, 4 sides and 2 diagonals. So there are 10 elements of each quadrilateral.



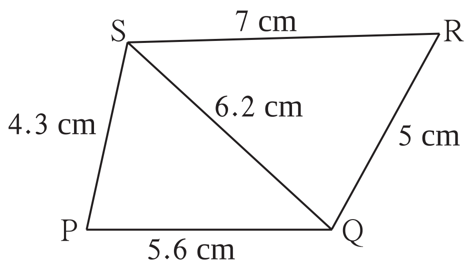
Let's learn.

Construction of a quadrilateral

We can construct a quadrilateral if we know the measures of some specific 5 elements out of 10. Constructions of triangles are the basis of constructions of quadrilaterals. This will be clear from the following examples.

(I) To construct a quadrilateral if the lengths of four sides and a diagonal is given.

Ex. Construct □ PQRS such that , l(PQ) = 5.6 cm , l(QR) = 5 cm, l(PS) = 4.3 cm, l(RS) = 7 cm, l(QS) = 6.2 cm



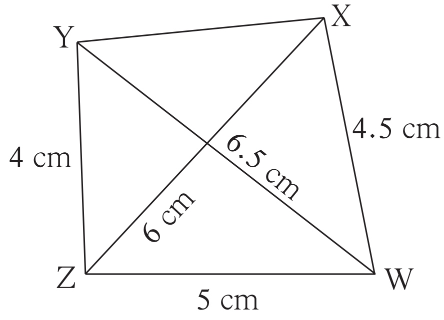
Solution : Let us draw a rough figure and show the given information in it. From the figure we see that the sides of Δ SPQ and Δ SRQ are known.

So if we costruct Δ SPQ and Δ SRQ of given measures, we get □ PQRS. Construct the given quadrilateral on your own.

(II) To construct a quadrilateral if three sides and two diagonals are given.

Ex. Construct □ WXYZ such that, l (YZ) = 4 cm, l (ZX) = 6 cm, l (WX) = 4.5 cm, l (ZW) = 5 cm, l (YW) = 6.5 cm

Solution: Let us draw a rough figure and show the given measures in it.



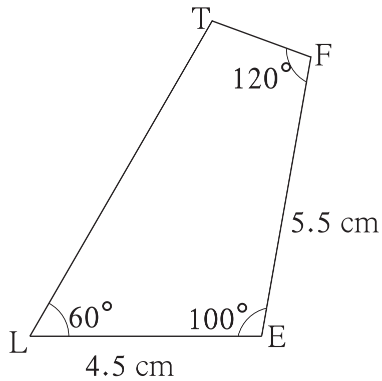
From the figure we see that all sides of Δ WXZ and Δ WZY are known. So let us draw Δ WXZ and Δ WZY using given measures. We will get □ WXYZ after drawing segment XY.

Construct this quadrilateral on your own.

(III) To construct a quadrilateral if two adjacent sides and any three angles are given.

Ex. Construct □ LEFT such that, l (EL) = 4.5 cm, l (EF) = 5.5 cm, $m\angle L = 60°$, $m\angle E = 100°$, $m\angle F = 120°$

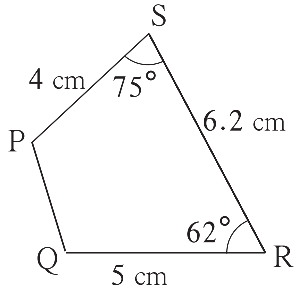
Solution: Let us show the given information in a rough figure. From the figure we see that seg LE of length 4.5 cm can be drawn and after drawing seg EF making an angle of 100° at the point E of seg LE, we get three points L, E and F. Let us draw a rays making an angle of 60° at the point L and a ray making an angle of 120° at the point F. The intersection of these two rays is point T. Now you can construct this □ LEFT.



(IV) To construct a quadrilateral if three sides and two angles included by them are given.

Ex. Construct □ PQRS such that, l (QR) = 5 cm, l (RS) = 6.2 cm, l (SP) = 4 cm, $m\angle R = 62°$, $m\angle S = 75°$

Solution: Let us draw a rough figure, show the given information in that figure. From the figure we see that after drawing seg QR, if seg RS is drawn making an angle of 62° at the point R, we can get points Q, R and S of the quadrilateral.



We will get point P on ray SP at a distance of 4 cm from S, which makes an angle of $75°$ at point S. We get $\square$ PQRS of given measure after joining points P and Q. Now you can do this construction .

Practice Set 8.1

1. Construct the following quadrilaterals of given measures.

(1) In $\square$ MORE, $l(\text{MO}) = 5.8$ cm, $l(\text{OR}) = 4.4$ cm, $m\angle \text{M} = 58°$, $m\angle \text{O} = 105°$, $m\angle \text{R} = 90°$.

(2) Construct $\square$ DEFG such that $l(\text{DE}) = 4.5$ cm, $l(\text{EF}) = 6.5$ cm, $l(\text{DG}) = 5.5$ cm, $l(\text{DF}) = 7.2$ cm, $l(\text{EG}) = 7.8$ cm.

(3) In $\square$ ABCD, $l(\text{AB}) = 6.4$ cm, $l(\text{BC}) = 4.8$ cm, $m\angle \text{A} = 70°$, $m\angle \text{B} = 50°$, $m\angle \text{C} = 140°$.

(4) Construct $\square$ LMNO such that $l(\text{LM}) = l(\text{LO}) = 6$ cm, $l(\text{ON}) = l(\text{NM}) = 4.5$ cm, $l(\text{OM}) = 7.5$ cm.

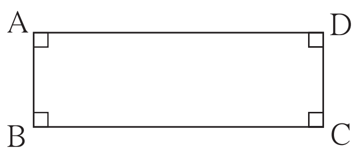
Let's recall.

By putting some conditions on sides and angles of a quadrilateral, we get different types of quadrilaterals. You already know two types of quadrilaterals, namely rectangle and square. Now we will study some more properties of these types and of some more types of quadrilaterals through activities.

Rectangle

If all angles of a quadrilateral are right angles, it is called a rectangle.

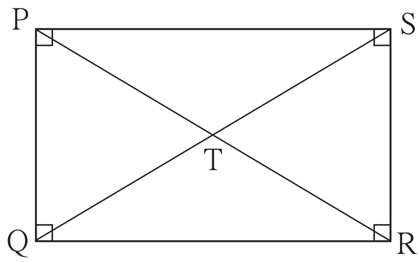
Among the five elements given to construct a quadrilateral, at least two have to be lengths of adjacent sides. You can construct a quadrilateral if two adjacent sides and three angles are given.



From the definition, we know that all angles of a rectangle are right angles. So if you know two adjacent sides, then you can construct a rectangle.

Activity I : Constuct a rectangle PQRS by taking two convenient adjacent sides. Name the point of intersetion of diagonals as T. Using divider and ruler, measure the following lengths.

(1) lengths of opposite sides, seg QR and seg PS.
(2) length of seg PQ and seg SR
(3) length of diagonals PR and QS
(4) lengths of seg PT and seg TR, which are parts of the diagonal PR.



(5) lengths of seg QT and seg TS, which are parts of the diagonal QS.

Observe the measures. Discuss about the measures obtained by your classmates. You will come to know the following properties of a rectangle through the discussion.

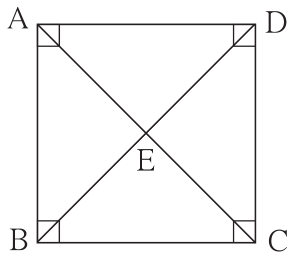
- Opposite sides of a rectangle are congruent.
- Diagonals of a rectangle are congruent.
- Diagonals of a rectangle bisect each other.

Square

If all sides and all angles of a quadrilateral are congruent, it is called a square.

Activity II : Draw a square of convenient length of side. Name the point of intersection of its diagonals as E. Using the apparatus in a compass box, measure the following lengths.

(1) lengths of diagonals AC and diagonal BD.
(2) lengths of two parts of each diagonal made by point E.
(3) all the angles made at the point E.
(4) parts of each angle of the square made by each diagonal. (e.g, $\angle$ ADB and $\angle$CDB).



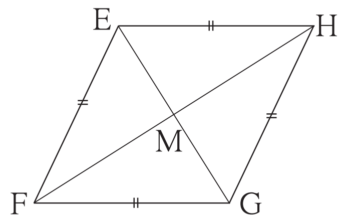
Observe the measures. Also observe the measures obtained by your classmates and discuss about them.

You will get the following properties of a square.

- Diagonals are of equal length. That is they are congruent.
- Diagonals bisect each other.
- Diagonals are perpendicular to each other.
- Diagonals bisect the opposite angles.

Rhombus

If all sides of a quadrilateral are of equal length (congruent), it is called a rhombus.



Avtivity III : Draw a rhombus EFGH by taking convenient length of side and convenient measure of an angle.

Draw its diagonals and name their point of intersection as M.

(1) Measure the opposite angles of the quadrilateral and angles at the point M.

(2) Measure the two parts of every angle made by the diagonal.

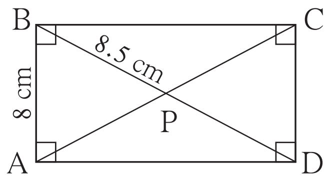
(3) Measure the lengths of both diagonals. Measure the two parts of diagonals made by point M.

From these measures you will get the following properties of a rhombus.

- Opposite angles are congruent.
- Diagonals bisect opposite angles of a rhombus.
- Diagonals bisect each other and they are perpendicular to each other.

You will see that your classmates also have got the same properties.

Solved Examples



Ex . (1) P is the point of intersection of diagonals of rectangle ABCD. (i) If l(AB) = 8 cm then l(DC) = ?, (ii) If l(BP) = 8.5 cm then find l(BD) and l(BC)

Solution: Let us draw a rough figure and show the given information in it.

(i) Opposite sides of a rectangle are congruent.

$\therefore\ l(DC) = l(AB) = 8$ cm

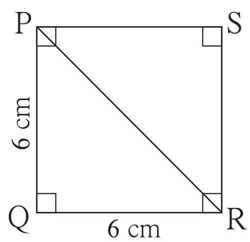
(ii) Diagonals of a rectangle bisect each other.

$\therefore\ l(BD) = 2 \times l(BP) = 2 \times 8.5 = 17$ cm

Δ BCD is a right angled triangle. Using Pythagoras theorem we get,

$l(BC)^2 = l(BD)^2 - l(CD)^2 = 17^2 - 8^2 = 289 - 64 = 225$

$\therefore\ l(BC) = \sqrt{225} = 15$ cm



Ex.(2) Find the length of a diagonal of a square of side 6 cm.

Solution: Suppose $\square$ PQRS is a square of side 6 cm.

Seg PR is a diagonal.

In Δ PQR, using Pythagoras theorem, $l(\text{PR})^2 = l(\text{PQ})^2 + l(\text{QR})^2$

$$= (6)^2 + (6)^2 = 36 + 36 = 72$$

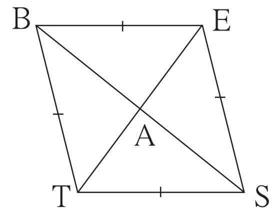
$\therefore\ l(\text{PR}) = \sqrt{72}$, $\therefore$ length of the diagonal is $\sqrt{72}$ cm.

Ex. (3) Diagonals of a rhombus BEST intersect at A.

(i) If $m\angle \text{BTS} = 110°$, then find $m\angle \text{TBS}$

(ii) If $l(\text{TE}) = 24$, $l(\text{BS}) = 70$, then find $l(\text{TS}) =$?

Solution: Let us draw rough figure of $\square$ BEST and show the point A.



(i) Opposite angles of a rhombus are congruent.

$\therefore\ m\angle \text{BES} = m\angle \text{BTS} = 110°$

Now, $m\angle \text{BTS} + m\angle \text{BES} + m\angle \text{TBE} + m\angle \text{TSE} = 360°$

$\therefore 110° + 110° + m\angle \text{TBE} + m\angle \text{TSE} = 360°$

$\therefore\ m\angle \text{TBE} + m\angle \text{TSE} = 360° - 220° = 140°$

$\therefore\ 2\,m\angle \text{TBE} = 140°$...... $\because$ Opposite angles of a rhombus are congruent.

$\therefore\ m\angle \text{TBE} = 70°$

$\therefore\ m\angle \text{TBS} = \frac{1}{2} \times 70° = 35°$ ...... $\because$ diagonal of a rhombus bisects the opposite angles

(ii) Diagonals of a rhombus are perpendicular bisectors of each other.

$\therefore$ In Δ TAS, $m\angle \text{TAS} = 90°$

$l(\text{TA}) = \frac{1}{2}\ l(\text{TE}) = \frac{1}{2} \times 24 = 12$, $l(\text{AS}) = \frac{1}{2}\ l(\text{BS}) = \frac{1}{2} \times 70 = 35$

By Pythagoras theorem,

$l(\text{TS})^2 = l(\text{TA})^2 + l(\text{AS})^2 = (12)^2 + (35)^2 = 144 + 1225 = 1369$

$\therefore\ l(\text{TS}) = \sqrt{1369} = 37$

Practice Set 8.2

1. Draw a rectangle ABCD such that $l(\text{AB}) = 6.0$ cm and $l(\text{BC}) = 4.5$ cm.

2. Draw a square WXYZ with side 5.2 cm.

3. Draw a rhombus KLMN such that its side is 4 cm and $m\angle \text{K} = 75°$.

4. If diagonal of a rectangle is 26 cm and one side is 24 cm, find the other side.

5. Lengths of diagonals of a rhombus ABCD are 16 cm and 12 cm. Find the side and perimeter of the rhombus.

6. Find the length of diagonal of a square with side 8 cm

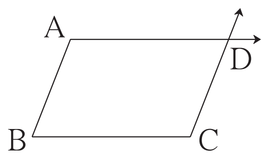
7. Measure of one angle of a rhombus is 50°, find the measures of remaining three angles.

Parallelogram

A quadrilateral having opposite sides parallel is called a parallelogram.

How can we draw a parallelogram?

Draw seg AB and seg BC making an angle of any measure as shown in the adjacent figure. You know how to draw a line parallel to a given line through a point which is outside it.



Using the method, draw a line through point C and parallel to the seg AB. Similarly, draw a line parallel to seg BC and passing through point A. Name the point of intersection of the lines as D. □ABCD is a parallelogram. We know that interior angles made by a transversal of parallel lines are supplementary. So in the above figure $m\angle A + m\angle B = 180^\circ$, $m\angle B + m\angle C = 180^\circ$, $m\angle C + m\angle D = 180^\circ$ and $m\angle D + m\angle A = 180^\circ$ so we get a property of angles of a parallelogram that–

- Adjacent angles of a parallelogram are supplementary.

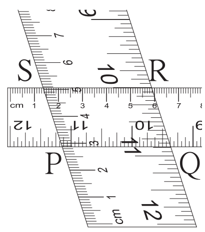
To know some more properties, draw a parallelogram PQRS as per the **activity**. Take two rulers of different widths, place one ruler horizontally and draw lines along its edges. Now place the other ruler in slant position over the lines drawn and draw lines along its edges. We get a parallelogram. Draw the diagonals of it and name the point of intersection as T.

(1) Measure the opposite angles of the parallelogram.

(2) Measure the lengths of opposite sides

(3) Measure the lengths of diagonals.

(4) Measure the lengths of parts of the diagonals made by point T.



From these measures you will get the following properties of a parallelogram.

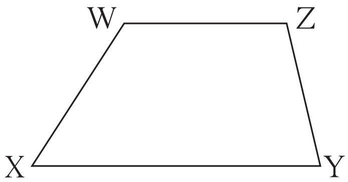
- Opposite angles are congruent.
- Opposite sides are congruent.
- Diagonals bisect each other.

Verifty the above properties by drawing some more parallelograms.

Trapezium

If only one pair of opposite sides of a quadrilateral is parallel then it is called a trapezium.

In □ WXYZ, only one pair of opposite sides, seg WZ and seg XY is parallel. So by definition □ WXYZ is a trapezium.



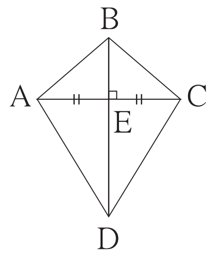
With the property of interior angles formed by two parallel lines and their transversal we get $m\angle W + m\angle X = 180°$ and $m\angle Y + m\angle Z = 180°$

In a trapezium, out of four pairs of adjacent angles, two are supplementary.

Kite

See the figure of □ ABCD. Diagonal BD of the quadrilateral bisects the diagonal AC.

If one diagonal is the perpendicular biscetor of the other diagonal then the quadrilateral is called a kite.



In the adjacent figure of □ ABCD, verify with a divider that seg AB ≅ seg CB and seg AD ≅ seg CD.

Similarly measure ∠BAD and ∠BCD and verify that they are congruent.

Thus we get two properties of a kite.

- Two pairs of adjacent sides are congruent.
- One pair of opposite angles is congruent.

Solved Examples

Ex. (1) Measures of adjacent angles of a parallelogram are $(5x - 7)°$ and $(4x + 25)°$. Find the measures of these angles.

Solution: Adjacent angles of a parallelogram are supplementary.

$\therefore (5x - 7) + (4x + 25) = 180$

$\therefore 9x + 18 = 180$

$\therefore 9x = 180 - 18 = 162$

$\therefore x = 18$

$\therefore$ measure of one angle $= (5x - 7)° = 5 \times 18 - 7 = 90 - 7 = 83°$

Measure of the other angle $= (4x + 25)° = 4 \times 18 + 25 = 72 + 25 = 97°$

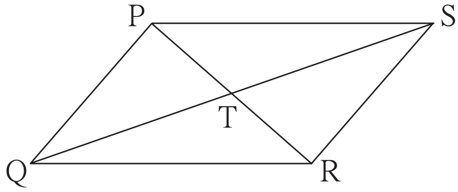
Ex. (2) $\square$ PQRS is a parallelogram. T is the point of intersection of its diagonals. Referring the figure, write the answers of questions given below.

(i) If l(PS) = 5.4 cm, then l(QR) = ?

(ii) If l(TS) = 3.5 cm, then l(QS) = ?

(iii) If $m\angle$QRS = 118°, find $m\angle$QPS.

(iv) If $m\angle$SRP = 72°, find $m\angle$RPQ.



Solution : In parallelogram PQRS,

(i) l(QR) = l(PS) = 5.4 cm .... opposite sides are congruent

(ii) l(QS) = 2 × l(TS) = 2 × 3.5 = 7 cm ...... diagonals bisect each other

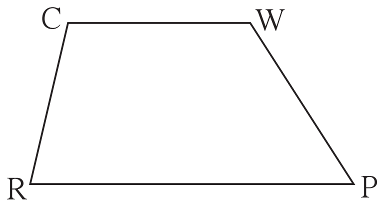
(iii) $m\angle$QPS = $m\angle$QRS = 118° ...... opposite angles are congruent

(iv) $m\angle$RPQ = $m\angle$SRP = 72° ...... alternate angles are congruent

Ex . (3) Ratio of measures of angles of $\square$ CWPR is 7:9:3:5 then find the measures of its angles and write the type of the quadrilateral.

Solution: Suppose, $m\angle C : m\angle W : m\angle P : m\angle R = 7:9:3:5$

let the measures of $\angle$C, $\angle$W, $\angle$P and $\angle$R be $7x$, $9x$, $3x$ and $5x$ respectively.



$\therefore\ 7x + 9x + 3x + 5x = 360°$

$\therefore\ 24x = 360°\quad \therefore\ x = 15$

$\therefore\ m\angle C = 7 \times 15 = 105°,\ m\angle W = 9 \times 15 = 135°$

$m\angle P = 3 \times 15 = 45°$ and $m\angle R = 5 \times 15 = 75°$

$\therefore\ m\angle C + m\angle R = 105° + 75° = 180°\quad \therefore$ side CW $\parallel$ side RP

$m\angle C + m\angle W = 105° + 135° = 240° \neq 180°$

$\therefore$ side CR is not parallel to side WP.

$\therefore$ only one pair of opposite sides of $\square$ CWPR is parallel.

$\therefore\ \square$ CWPR is a trapezium.

Practice Set 8.3

1. Measures of opposite angles of a parallelogram are $(3x-2)°$ and $(50 - x)°$. Find the measure of its each angle.

2. Referring the adjacent figure of a parallelogram, write the answers of questions given below.

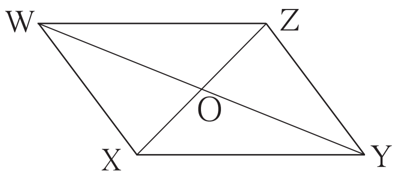
(1) If $l(WZ) = 4.5$ cm then $l(XY) = ?$

(2) If $l(YZ) = 8.2$ cm then $l(XW) = ?$

(3) If $l(OX) = 2.5$ cm then $l(OZ) = ?$

(4) If $l(WO) = 3.3$ cm then $l(WY) = ?$

(5) If $m\angle WZY = 120°$ then $m\angle WXY = ?$ and $m\angle XWZ = ?$



3. Construct a parallelogram ABCD such that $l(BC) = 7$ cm, $m\angle ABC = 40°$, $l(AB) = 3$ cm.

4. Ratio of consecutive angles of a quadrilateral is 1:2:3:4. Find the measure of its each angle. Write, with reason, what type of a quadrilateral it is.

5. Construct $\square$ BARC such that $l(BA) = l(BC) = 4.2$ cm, $l(AC) = 6.0$ cm, $l(AR) = l(CR) = 5.6$ cm

6*. Construct $\square$ PQRS, such that $l(PQ) = 3.5$ cm, $l(QR) = 5.6$ cm, $l(RS) = 3.5$ cm, $m\angle Q = 110°$, $m\angle R = 70°$.

If it is given that $\square$ PQRS is a parallelogram, which of the given information is unnecessary ?

Answers

Practice Set 8.2

4. 10 cm **5.** side 10 cm, perimeter 40 cm. **6.** $\sqrt{128}$ cm **7.** 130°, 50°, 130°

Practice Set 8.3

1. 37°, 143°, 37°, 143°

2. (1) 4.5 cm (2) 8.2 cm (3) 2.5 cm (4) 6.6 cm (5) 120°, 60°

4. 36°, 72°, 108°, 144°, a trapezium