**Let's study.**

- Construction of a triangle similar to the given triangle
 - * To construct a triangle, similar to the given triangle, bearing the given ratio with the sides of the given triangle.
 - (i) When vertices are distinct
 - (ii) When one vertex is common
- Construction of a tangent to a circle.
 - * To construct a tangent at a point on the circle.
 - (i) Using centre of the circle.
 - (ii) Without using the centre of the circle.
 - * To construct tangents to the given circle from a point outside the circle.

**Let's recall.**

In the previous standard you have learnt the following constructions. Let us recall those constructions.

- To construct a line parallel to a given line and passing through a given point outside the line.
- To construct the perpendicular bisector of a given line segment.
- To construct a triangle whose sides are given.
- To divide a given line segment into given number of equal parts
- To divide a line segment in the given ratio.
- To construct an angle congruent to the given angle.

In the ninth standard you have carried out the activity of preparing a map of surroundings of your school. Before constructing a building we make its plan. The surroundings of a school and its map, the building and its plan are similar to each other. We need to draw similar figures in Geography, architecture, machine drawing etc. A triangle is the simplest closed figure. We shall learn how to construct a triangle similar to the given triangle.





Let's learn.

Construction of Similar Triangle

To construct a triangle similar to the given triangle, satisfying the condition of given ratio of corresponding sides.

The corresponding sides of similar triangles are in the same proportion and the corresponding angles of these triangles are equal. Using this property, a triangle which is similar to the given triangle can be constructed.

Ex. (1) $\Delta ABC \sim \Delta PQR$, in ΔABC , $AB = 5.4$ cm, $BC = 4.2$ cm, $AC = 6.0$ cm.
 $AB : PQ = 3 : 2$. Construct ΔABC and ΔPQR .

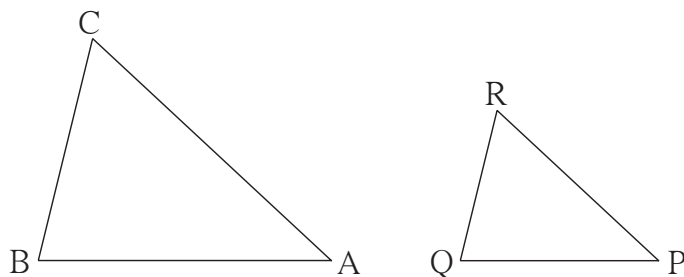


Fig. 4.1
Rough Figure

Construct ΔABC of given measure.

ΔABC and ΔPQR are similar.

\therefore their corresponding sides are proportional.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{3}{2} \dots\dots\dots (I)$$

As the sides AB , BC , AC are known, we can find the lengths of sides PQ , QR , PR .

Using equation [I]

$$\frac{5.4}{PQ} = \frac{4.2}{QR} = \frac{6.0}{PR} = \frac{3}{2}$$

$\therefore PQ = 3.6$ cm, $QR = 2.8$ cm and $PR = 4.0$ cm



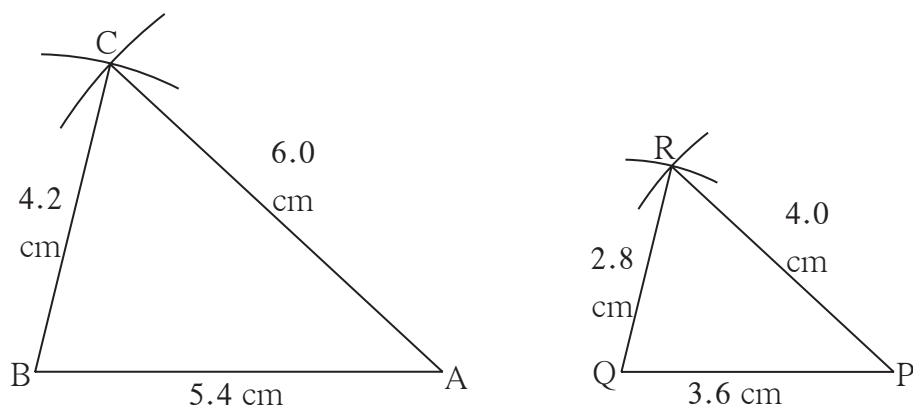


Fig. 4.2

For More Information

While drawing the triangle similar to the given triangle, sometimes the lengths of the sides we obtain by calculation are not easily measurable by a scale. In such a situation we can use the construction ‘To divide the given segment in the given number of equal parts’.

For example, if length of side AB is $\frac{11.6}{3}$ cm, then by dividing the line segment of length 11.6 cm in three equal parts, we can draw segment AB.

If we know the lengths of all sides of ΔPQR , we can construct ΔPQR .

In the above example (1) there was no common vertex in the given triangle and the triangle to be constructed. If there is a common vertex, it is convenient to follow the method in the following example.

Ex.(2) Construct any ΔABC . Construct $\Delta A'BC'$ such that $AB : A'B = 5:3$ and $\Delta ABC \sim \Delta A'BC'$

Analysis : As shown in fig 4.3 , let the points B, A, A' and B, C, C' be collinear.

$$\Delta ABC \sim \Delta A'BC' \therefore \angle ABC = \angle A'BC'$$

$$\frac{AB}{A'B} = \frac{BC}{BC'} = \frac{AC}{A'C'} = \frac{5}{3}$$

\therefore sides of ΔABC are longer than corresponding sides of $\Delta A'BC'$.

\therefore the length of side BC' will be equal to 3 parts out of 5 equal parts of side BC. So if we construct ΔABC , point C' will be on the side BC, at a distance equal to 3 parts from B. Now A' is the point of intersection of AB and a line through C' , parallel to CA.

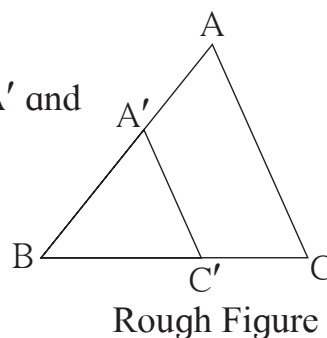


Fig. 4.3

Rough Figure

$$\frac{BA'}{BA} = \frac{BC'}{BC} = \frac{3}{5} \text{ i.e., } \frac{BA}{BA'} = \frac{BC}{BC'} = \frac{5}{3} \text{ Taking inverse}$$

Steps of construction :

- (1) Construct any ΔABC .
- (2) Divide segment BC in 5 equal parts.
- (3) Name the end point of third part of seg BC as C' $\therefore BC' = \frac{3}{5} BC$
- (4) Now draw a line parallel to AC through C' . Name the point where the parallel line intersects AB as A' .
- (5) $\Delta A'BC'$ is the required triangle similar to ΔABC

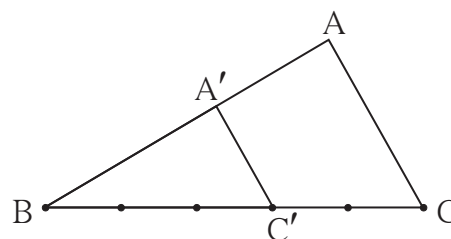


Fig. 4.4

Note : To divide segment BC , in five equal parts, it is convenient to draw a ray from B , on the side of line BC in which point A does not lie.

Take points T_1, T_2, T_3, T_4, T_5 on the ray such that

$$BT_1 = T_1T_2 = T_2T_3 = T_3T_4 = T_4T_5$$

Join T_5C and draw lines parallel to T_5C through T_1, T_2, T_3, T_4 .

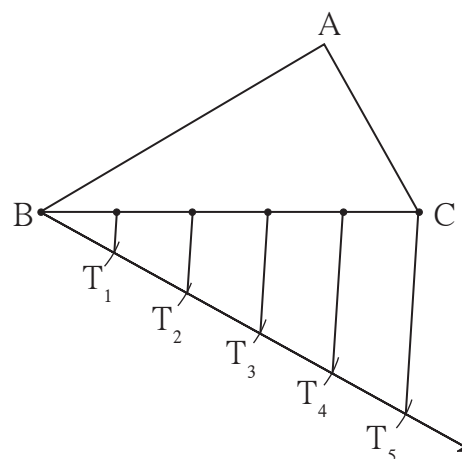


Fig. 4.5



Let's think

$\Delta A'BC'$ can also be constructed as shown in the adjoining figure.

What changes do we have to make in steps of construction in that case ?

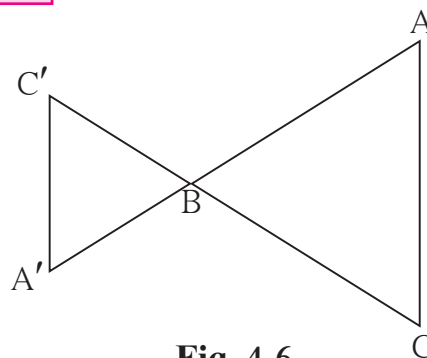


Fig. 4.6

Ex. (3) Construct $\Delta A'BC'$ similar to ΔABC such that $AB:A'B = 5:7$

Analysis : Let points B, A, A' as well as points B, C, C' be collinear.

$\Delta ABC \sim \Delta A'BC'$ and $AB : A'B = 5:7$

\therefore sides of ΔABC are smaller than sides of $\Delta A'BC'$

and $\angle ABC \cong \angle A'BC'$

Let us draw a rough figure with these

considerations. Now $\frac{BC}{BC'} = \frac{5}{7}$

\therefore If seg BC is divided into 5 equal parts, then seg BC' will be 7 times each part of seg BC.

\therefore let us divide side BC of ΔABC in 5 equal parts and locate point C' at a distance equal to 7 such parts from B on ray BC. A line through point C' parallel to seg AC is drawn it will intersect ray BA at point A'. According to the basic proportionality theorem we will get $\Delta A'BC'$ as described.

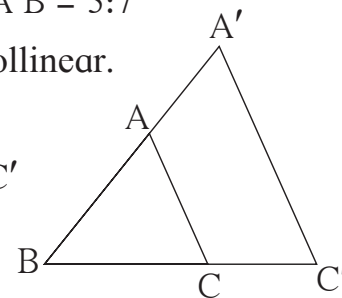


Fig. 4.7
Rough Figure

Steps of construction :

- (1) Construct any ΔABC .
- (2) Divide segment BC into 5 equal parts. Fix point C' on ray BC such that length of BC' is seven times of each equal part of seg BC
- (3) Draw a line parallel to side AC, through C'. Name the point of intersection of the line and ray BA as A'.

We get the required $\Delta A'BC'$ similar to ΔABC .

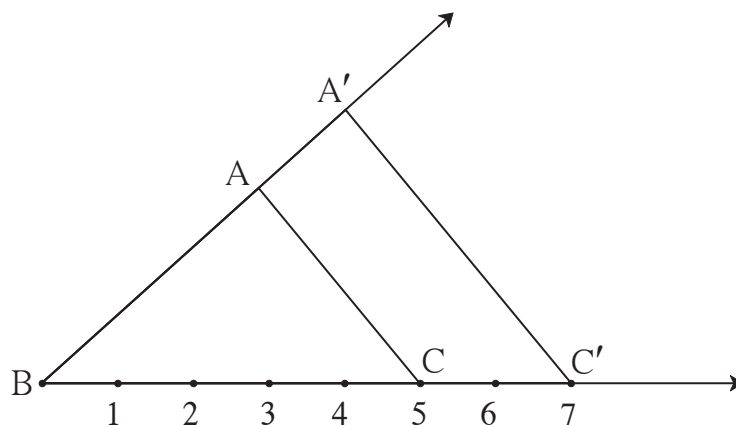


Fig. 4.8

Practice set 4.1

1. $\triangle ABC \sim \triangle LMN$. In $\triangle ABC$, $AB = 5.5$ cm, $BC = 6$ cm, $CA = 4.5$ cm.
Construct $\triangle ABC$ and $\triangle LMN$ such that $\frac{BC}{MN} = \frac{5}{4}$.
2. $\triangle PQR \sim \triangle LTR$. In $\triangle PQR$, $PQ = 4.2$ cm, $QR = 5.4$ cm, $PR = 4.8$ cm.
Construct $\triangle PQR$ and $\triangle LTR$, such that $\frac{PQ}{LT} = \frac{3}{4}$.
3. $\triangle RST \sim \triangle XYZ$. In $\triangle RST$, $RS = 4.5$ cm, $\angle RST = 40^\circ$, $ST = 5.7$ cm.
Construct $\triangle RST$ and $\triangle XYZ$, such that $\frac{RS}{XY} = \frac{3}{5}$.
4. $\triangle AMT \sim \triangle AHE$. In $\triangle AMT$, $AM = 6.3$ cm, $\angle TAM = 50^\circ$, $AT = 5.6$ cm.
 $\frac{AM}{AH} = \frac{7}{5}$. Construct $\triangle AHE$.

Construction of a tangent to a circle at a point on the circle

(i) Using the centre of the circle.

Analysis :

Suppose we want to construct a tangent l passing through a point P on the circle with centre C . We shall use the property that a line perpendicular to the radius at its outer end is a tangent to the circle. If CP is a radius with point P on the circle, line l through P and perpendicular to CP is the tangent at P . For this we will use the construction of drawing a perpendicular to a line through a point on it.

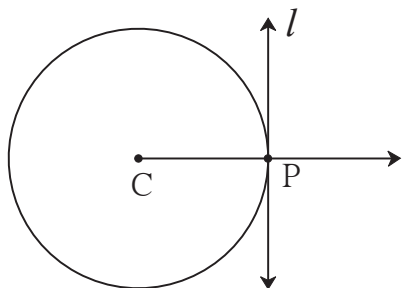


Fig. 4.9

For convenience we shall draw ray CP

Steps of construction

- (1) Draw a circle with centre C .
Take any point P on the circle.
- (2) Draw ray CP .
- (3) Draw line l perpendicular to ray CX through point P .
Line l is the required tangent to the circle at point ' P '.

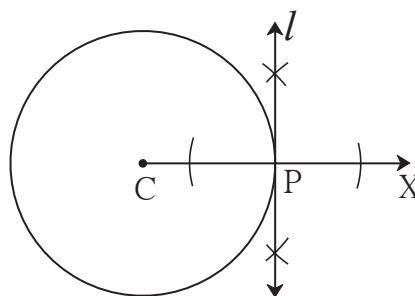


Fig. 4.10

ii) Without using the centre of the circle.

Example: Construct a circle of any radius. Take any point C on it. Construct a tangent to the circle without using centre of the circle.

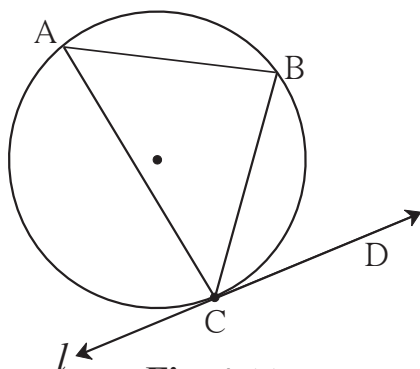


Fig. 4.11

Analysis :

As shown in the figure, let line l be the tangent to the circle at point C. Line CB is a chord and $\angle CAB$ is an inscribed angle. Now by tangent- secant angle theorem, $\angle CAB \cong \angle BCD$.

By converse of tangent- secant theorem, if we draw the line CD such that, $\angle CAB \cong \angle BCD$, then it will be the required tangent.

Steps of Construction :

- (1) Draw a circle of a suitable radius. Take any point C on it.
- (2) Draw chord CB and an inscribed $\angle CAB$.
- (3) With the centre A and any convenient radius draw an arc intersecting the sides of $\angle BAC$ in points M and N.
- (4) Using the same radius and centre C, draw an arc intersecting the chord CB at point R.
- (5) Taking the radius equal to $d(MN)$ and centre R, draw an arc intersecting the arc drawn in the previous step. Let D be the point of intersection of these arcs. Draw line CD. Line CD is the required tangent to the circle.

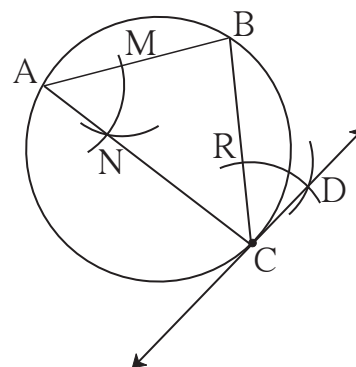


Fig. 4.12

Note that $\angle MAN$ and $\angle BCD$ in the above figure are congruent. If we draw seg MN and seg RD, then $\triangle MAN$ and $\triangle RCD$ are congruent by SSS test.

$$\therefore \angle MAN \cong \angle BCD$$

To construct tangents to a circle from a point outside the circle.

Analysis :

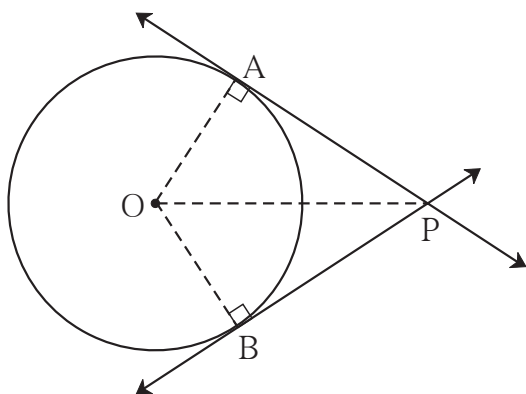


Fig. 4.13

As shown in the figure let P be a point in the exterior of the circle. Let PA and PB be the tangents to the circle with the centre O, touching the circle in points A and B respectively. So if we find points A and B on the circle, we can construct the tangents PA and PB. If OA and OB are the radii of the circle, then $OA \perp$ line PA and $OB \perp$ line PB.

Δ OAP and OBP are right angled triangles and seg OP is their common hypotenuse. If we draw a circle with diameter OP, then the points where it intersects the circle with centre O, will be the positions of points A and B respectively, because angle inscribed in a semicircle is a right angle.

Steps of Construction

- (1) Construct a circle of any radius with centre O.
- (2) Take any point P in the exterior of the circle.
- (3) Draw segment OP. Draw perpendicular bisector of seg OP to get its midpoint M.

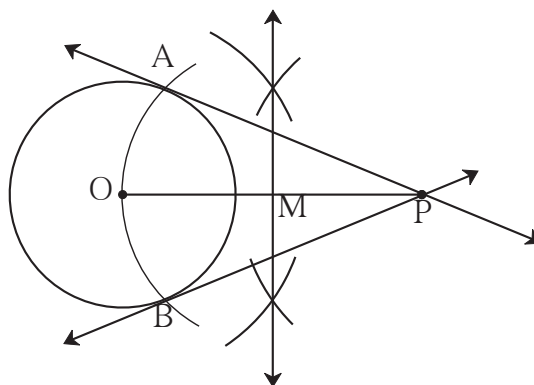


Fig. 4.14

- (4) Draw a circle with radius OM and centre M
- (5) Name the points of intersection of the two circles as A and B.
- (6) Draw line PA and line PB.

Practice set 4.2

1. Construct a tangent to a circle with centre P and radius 3.2 cm at any point M on it.
2. Draw a circle of radius 2.7 cm. Draw a tangent to the circle at any point on it.
3. Draw a circle of radius 3.6 cm. Draw a tangent to the circle at any point on it without using the centre.
4. Draw a circle of radius 3.3 cm Draw a chord PQ of length 6.6 cm. Draw tangents to the circle at points P and Q. Write your observation about the tangents.

5. Draw a circle with radius 3.4 cm. Draw a chord MN of length 5.7 cm in it. Construct tangents at point M and N to the circle.
6. Draw a circle with centre P and radius 3.4 cm. Take point Q at a distance 5.5 cm from the centre. Construct tangents to the circle from point Q.
7. Draw a circle with radius 4.1 cm. Construct tangents to the circle from a point at a distance 7.3 cm from the centre.

◆◆◆◆◆ Problem set 4 ◆◆◆◆◆

1. Select the correct alternative for each of the following questions.
 - (1) The number of tangents that can be drawn to a circle at a point on the circle is
(A) 3 (B) 2 (C) 1 (D) 0
 - (2) The maximum number of tangents that can be drawn to a circle from a point outside it is
(A) 2 (B) 1 (C) one and only one (D) 0
 - (3) If $\triangle ABC \sim \triangle PQR$ and $\frac{AB}{PQ} = \frac{7}{5}$, then
(A) $\triangle ABC$ is bigger. (B) $\triangle PQR$ is bigger.
(C) Both triangles will be equal. (D) Can not be decided.
 2. Draw a circle with centre O and radius 3.5 cm. Take point P at a distance 5.7 cm from the centre. Draw tangents to the circle from point P.
 3. Draw any circle. Take any point A on it and construct tangent at A without using the centre of the circle.
 4. Draw a circle of diameter 6.4 cm. Take a point R at a distance equal to its diameter from the centre. Draw tangents from point R.
 5. Draw a circle with centre P. Draw an arc AB of 100° measure. Draw tangents to the circle at point A and point B.
 6. Draw a circle of radius 3.4 cm and centre E. Take a point F on the circle. Take another point A such that E-F-A and FA = 4.1 cm. Draw tangents to the circle from point A.
 7. $\triangle ABC \sim \triangle LBN$. In $\triangle ABC$, AB = 5.1 cm, $\angle B = 40^\circ$, BC = 4.8 cm, $\frac{AC}{LN} = \frac{4}{7}$. Construct $\triangle ABC$ and $\triangle LBN$.
 8. Construct $\triangle PYQ$ such that, PY = 6.3 cm, YQ = 7.2 cm, PQ = 5.8 cm. If $\frac{YZ}{YO} = \frac{6}{5}$, then construct $\triangle XYZ$ similar to $\triangle PYQ$.
- 