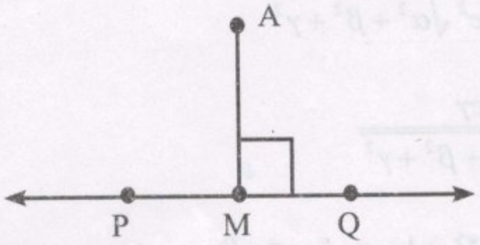
7. Line and Plane

Ex. (1) Find the coordinates of the foot of the perpendicular drawn from $A(1,2,1)$ to the line joining $P(1,4,6)$ and $Q(5,4,4)$.

Solution : Let M be the foot of the perpendicular drawn from A to line PQ.



Let $k:1$ be the ratio in which M divides PQ.

$\therefore \quad M \equiv \left(\dfrac{5k+1}{k+1}, \dfrac{4k+4}{k+1}, \dfrac{4k+6}{k+1}\right)$

$\therefore$ Direction ratios of AM are $\dfrac{5k+1}{k+1}-1, \dfrac{4k+4}{k+1}-2, \dfrac{4k+6}{k+1}-1$

i. e. $\dfrac{4k}{k+1}, \dfrac{2k+2}{k+1}, \dfrac{3k+5}{k+1}$

And the direction ratios of PQ are $4,0,-2$

As $AM \perp PQ$, $(4)\times\dfrac{4k}{k+1}+(0)\times\dfrac{2k+2}{k+1}+(-2)\times\dfrac{3k+5}{k+1}=0$

$\therefore \quad 16k-6k-10=0$

$\therefore \quad k=1$

The co-ordinates of M are $(3,4,5)$.

AM $= \sqrt{(3-1)^2+(4-2)^2+(5-1)^2} = \sqrt{4+4+16} = \sqrt{24} = 2\sqrt{6}$ unit.

Ex. (2) If θ is the angle between the line $\dfrac{x-x_1}{a}=\dfrac{y-y_1}{b}=\dfrac{z-z_1}{c}$ and the plane $\alpha x+\beta y+\gamma z+\delta=0$ then prove that

$$sin\theta = \frac{a\times\alpha+b\times\beta+c\times\gamma}{\sqrt{a^2+b^2+c^2}\sqrt{\alpha^2+\beta^2+\gamma^2}}.$$

Hence find the angle between the line $x=y=z$ and the XY plane.

Solution : $\dfrac{x-x_1}{a} = \dfrac{y-y_1}{b} = \dfrac{z-z_1}{c} \quad \ldots\ldots(1)$

$ax+\beta y+\gamma z+\delta=0 \quad \ldots\ldots(2)$

As the angle between line and the plane is θ, the angle between line and the normal to the plane is $\frac{\pi}{2}-\theta$. Note that $\frac{\pi}{2}-\theta$ is an acute angle.

The direction ratios of line are a,b,c and that of the normal are α,β,γ

$$\therefore \quad \cos\left(\frac{\pi}{2}-\theta\right)=\frac{a\alpha+b\beta+c\gamma}{\sqrt{a^2+b^2+c^2}\sqrt{\alpha^2+\beta^2+\gamma^2}}$$

$$\therefore \quad \sin\theta=\frac{a\alpha+b\beta+c\gamma}{\sqrt{a^2+b^2+c^2}\sqrt{\alpha^2+\beta^2+\gamma^2}}$$

Now the equation of XY plane is $z=0$

Let the angle between line $x=y=z$ and the XY plane be θ.

$$\therefore \quad \sin\theta=\frac{a\alpha+b\beta+c\gamma}{\sqrt{a^2+b^2+c^2}\sqrt{\alpha^2+\beta^2+\gamma^2}}=\frac{1(0)+1(0)+1(1)}{\sqrt{1^2+1^2+1^2}\sqrt{0^2+0^2+1^1}}=\frac{1}{3}$$

$$\therefore \quad \theta=\sin^{-1}\left(\frac{1}{3}\right)$$

Ex. (3) Find the ratio in which XY plane divides the line joining $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ and $B(-4,31,-2)$. Find the vector equation of line AB.

Solution : Let k : 1 be the ratio in which XY plane divides the line joining $A(1,28,1)$ and $B(-4,31,-2)$.

Let (h, k, 0) be the point in which XY plane cuts the line AB.

$$\therefore \quad 0=\frac{kz_2+z_1}{k+1}$$

$$\therefore \quad kz_2+z_1=\ldots 0$$

$$\therefore \quad k=-\frac{z_1}{z_2}=\ldots -\left(\frac{1}{-2}\right)=\frac{1}{2}$$

$\therefore$ XY plane divides AB internally in the ratio 1 : 2

Vector along AB is $-5\hat{i}+3\hat{j}-3\hat{k}$

$\bar{a}=\hat{i}+28\hat{j}+\hat{k}$

The equation line AB is $\bar{r}=(\hat{i}+28\hat{j}+\hat{k})+\lambda\left(-5\hat{i}+3\hat{j}-3\hat{k}\right)$

Ex. (4) Obtain coordinates of points on the line $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$, which are at 6 unit distance from the origin.

Solution : Let $P(2k, 2k, k)$ be a point on the line $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ which is at 6 unit distance from the origin.

$OP = 6$

$\therefore \quad OP^2 = 36$

$\therefore \quad (2k)^2 + (2k)^2 + (k)^2 = 36$

$\therefore 9k^2 = 36$

$\therefore k = \pm 2$

$\therefore$ There are two points on the given line which are at 6 unit distance from the origin.

Their co-ordinates are $(4, 4, 2)$ and $(-4, -4, -2)$.

Ex. (5) Find the vector and Cartesian equation of the plane passing through the points $A(2,3,1)$, $B(4,-5,3)$ and parallel to X-axis.

Solution :

Since the plane is $\perp$ to x-axis, its normal is $\perp$ to x-axis. The unit vector along x-axis is $\hat{i}$. Hence the normal vector to the plane $\perp$ to $\hat{i}$

Let $A(2,3,1)$, $B(4,-5,3)$ be the point on the plane

$\therefore \bar{a} = 2\hat{i}+3\hat{j}+\hat{k}$, $\bar{b} = 4\hat{i}-5\hat{j}+3\hat{k}$

$\overline{AB} = \bar{b}-\bar{a} = 4\hat{i}-5\hat{j}+3\hat{k}-2\hat{i}-3\hat{j}-\hat{k}$

$= 2\hat{i}-8\hat{j}+2\hat{k}$

The normal vector to the plane perpendicular to $\overline{AB}$

Since $\hat{i} \times \overline{AB}$ is $\perp$ to both $\hat{i}$ and $\overline{AB}$, it is normal vector to the plane and it is

$\bar{n} = \hat{i} \times \overline{AB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 2 & -8 & 2 \end{vmatrix}$

$= -2\hat{j} - 8\hat{k}$

The vector equation of the plane passing through $A(\bar{a})$ and $\perp$ to $\bar{n}$ is $\bar{r}\cdot\bar{n} = \bar{a}\cdot\bar{n}$

$\therefore \bar{r}(-2\hat{j}-8\hat{k}) = (2\hat{i}+3\hat{j}+\hat{k})\cdot(-2\hat{j}-8\hat{k})$

$\therefore \bar{r}(-2\hat{j}-8\hat{k}) = 0-6-8$

$\therefore \bar{r}(-2\hat{j}-8\hat{k}) = -14$

$\therefore \bar{r}(\hat{j}+4\hat{k}) = 7$

If $\bar{r} = x\hat{i}+y\hat{j}+2\hat{k}$

$\therefore (x\hat{i}+y\hat{j}+2\hat{k})\cdot(\hat{j}+4\hat{k}) = 7$

$\therefore y + 4z = 7$

This is required equation.

Ex. (6) Find the vector equation of the plane passing through the origin and containing the line $r=\left(\hat{i}+4\hat{j}+\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+\hat{k}\right)$

Solution :

Given line is $\bar{r}=(\hat{i}+4\hat{j}+\hat{k})+\lambda(\hat{i}+2\hat{j}+\hat{k})$ ——(I)

Comparing with $\bar{r}=\bar{a}+\lambda\bar{b}$

$\therefore \bar{a}=\hat{i}+4\hat{j}+\hat{k}$, $\bar{b}=\hat{i}+2\hat{j}+\hat{k}$

Since required plane passing through origin and containing line (I)

$\therefore$ required plane is $\parallel$ to $\overline{OA}$ and $\bar{b}$

$\bar{n}$ be normal to the plane

$$\therefore \bar{n}=\overline{OA}\times\bar{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 4 & 1\\ 1 & 2 & 1\end{vmatrix}$$

$\therefore \bar{n}=2\hat{i}+0\hat{j}-2\hat{k}=2\hat{i}-2\hat{k}$

$\therefore$ required vector equation of plane is

$\bar{r}\cdot\bar{n}=\bar{0}\cdot\bar{n}$

$\bar{r}\cdot(2\hat{i}-2\hat{k})=0$

$\boxed{\therefore\ \bar{r}(\hat{i}-\hat{k})=0}$

Ex. (7) Find the vector equation of the plane which bisects the segment joining $A(2,3,6)$ and $B(4,3,-2)$ at right angle.

Solution : We have $A(2,3,6)$ and $B(4,3,-2)$

Let $\bar{a}$ and $\bar{b}$ be p.v. of A and B resp.

$\bar{a} = 2\hat{i}+3\hat{j}+6k$, $\bar{b} = 4\hat{i}+3\hat{j}-2\hat{k}$

Let point $C(\bar{c})$ be midpoint of seg AB

$\therefore \bar{c} = \frac{\bar{a}+\bar{b}}{2} = \frac{2\hat{i}+3\hat{j}+6\hat{k}+4\hat{i}+3\hat{j}-2\hat{k}}{2}$

$= \frac{6\hat{i}+6\hat{j}+4\hat{k}}{2} = 3\hat{i}+3\hat{j}+2\hat{k}$

Since required plane bisects the seg AB at right angle

$\therefore$ AB is $\perp$ to required plane

Let $\bar{n}$ be normal to required plane

$\therefore \bar{n} = \overline{AB} = \bar{b}-\bar{a} = 4\hat{i}+3\hat{j}-2\hat{k}-2\hat{i}-3\hat{j}-6\hat{k} = 2\hat{i}-8\hat{k}$

$\therefore$ The vector equation of the plane passing through the point $C(\bar{c})$ is given by

$\bar{r}\cdot\bar{n} = \bar{c}\cdot\bar{n}$

$\bar{r}\cdot(2\hat{i}-8\hat{k}) = (3\hat{i}+3\hat{j}+2\hat{k})\cdot(2\hat{i}+0\hat{j}-8\hat{k})$

$\therefore \bar{r}\cdot(2\hat{i}-8\hat{k}) = 3(2)+3(0)+2(-8)$

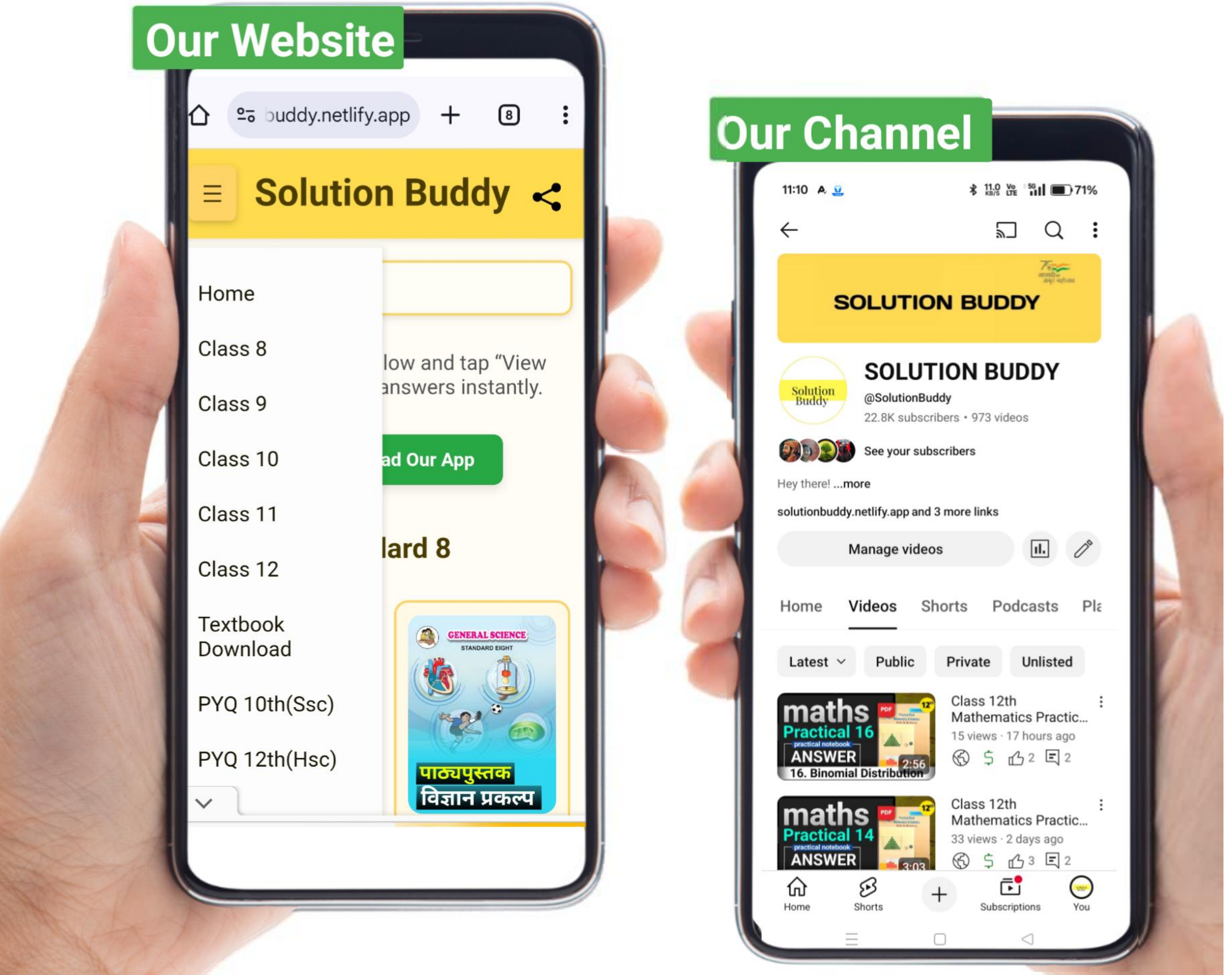
$\therefore \bar{r}\ (2\hat{i}-8\hat{k}) = 6+0-16$

$\therefore \bar{r}\ (2\hat{i}-8\hat{k}) = -10$

$\therefore \bar{r}\ (\hat{i}-4\hat{k}) = -5$

This is the required vector equation of plane

Sign of Teacher :



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