6. Fitting of Linear Regression

V. Activities

1.

	x	y	xy	x^2	y^2
	1	2	2	1	4
	2	1	2	4	1
	3	6	18	9	36
Total	6	9	22	14	41

Number of pairs of observation (n) = 3

Mean of $x = \bar{x} = \frac{\Sigma x}{n} = \frac{6}{3} = 2$

Mean of $y = \bar{y} = \frac{\Sigma y}{n} = \frac{9}{3} = 3$

$\therefore b_{yx} = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2}$

$= \frac{3(22) - (6)(9)}{3(14) - (6)^2} = \frac{66 - 54}{44 - 36}$

$= \frac{12}{6} = 2$

Regression equation of y on x is $y - \bar{y} = b_{yx}(x - \bar{x})$

$y - 3 = 2(x - 2)$

$y - 3 = 2x - 4$

$\therefore y = 2x - 1$

Put x = 4 in above equation

$\therefore y = 2(4) - 1 = 7$

2. Mean of x = 30, $\therefore \bar{x} = 30$

Mean of y = 50, $\therefore \bar{y} = 50$

Regression coefficient of y on x = – 1.5

$\therefore b_{yx} = -1.5$

Regression equation of y on x is $y - \bar{y} = b_{yx}(x - \bar{x})$

$\therefore y - 50 = -1.5(x - 30)$

$y - 50 = -1.5x + 45$

$\therefore y = \boxed{-1.5}\, x + \boxed{95}$

To estimate y, put x = 50

$\therefore y = \boxed{-1.5}\, x + \boxed{95}$

To estimate y, put x = 50

$\therefore y = -1.5\,(\boxed{50}) + 95$

$= \boxed{20}$

B. Solve the Following

Q.1. The following table gives indices of industrial production and number of registered unemployed persons (in lakh).

Index of Production(X)	100	102	104	107	105	112	103	99
No. of unemployed(Y)	15	12	13	11	12	12	19	26

Obtain regression line of Y on X and X on Y.

X	Y	$(x-\bar{x}) = (x-104)$	$(y-\bar{y}) = (y-15)$	$(x-\bar{x})(y-\bar{y})$	$(x-\bar{x})^2$	$(y-\bar{y})^2$
100	15	-4	0	0	16	0
102	12	-2	-3	6	4	9
104	13	0	-2	0	0	4
107	11	3	-4	-12	9	16
105	12	1	-3	-3	1	9
112	12	8	-3	-24	64	9
103	19	-1	4	-4	1	16
99	26	-5	11	-55	25	121
Total → 832	120	0	0	-92	120	184

$\bar{x} = \frac{\Sigma x}{n} = \frac{832}{8} = 104$, $\bar{y} = \frac{\Sigma y}{n} = \frac{120}{8} = 15$

$b_{yx} = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{\Sigma (x-\bar{x})^2} = \frac{-92}{120} = -0.76$

$b_{xy} = \frac{\Sigma (x-\bar{x})(y-\bar{y})}{\Sigma (y-\bar{y})^2} = \frac{-92}{184} = -0.5$

(i) ∴ Regression equation Y on X

$\therefore y - \bar{y} = b_{yx}(x - \bar{x})$

$\therefore y - 15 = -0.76(x - 104)$

$\therefore y - 15 = -0.76x + 79.04$

$\therefore y = -0.76x + 94.04$

(ii) Regression equation X on Y

$x - \bar{x} = b_{xy}(y - \bar{y})$

$\therefore x - 104 = -0.5(y - 15)$

$\therefore x - 104 = -0.5y + 7.5$

$\therefore x = -0.5y + 111.5$

Q.2. The equations of the two lines of regressions are $3x+2y-26 = 0$ and $6x+y-31=0$. Find a) Means of X and Y b) Var(X) if Var(Y) = 36

(a) $3x + 2y - 26 = 0$ — (i)

$6x + y - 31 = 0$ — (ii)

eq^n (i) $- 2$(ii) we get, $3x + 2y - 26 = 0$

$-12x + 2y - 62 = 0$

$-9x + 36 = 0$

$-9x = -36$

$\therefore \bar{x} = 4$

Sub. $x = 4$ in eq^n (i), $3(4) + 2y - 26 = 0$

$2y - 14 = 0$

$\therefore \bar{y} = 7$

(b) Let $3x + 2y - 26 = 0$ be the regression eq^n y on x

$\therefore y = \frac{-3}{2}x + \frac{26}{2}$

Comparing it with $y = b_{yx} \cdot x + a$, we get

$b_{yx} = \frac{-3}{2}$

& Let $6X + Y - 31 = 0$ is the regression eq^n X on Y

$\therefore X = \frac{-1}{6} Y + \frac{31}{6} \Rightarrow b_{XY} = \frac{-1}{6}$

$\therefore r = \pm\sqrt{b_X \cdot b_Y} = \pm\sqrt{\left(\frac{-1}{6}\right)\left(\frac{-3}{2}\right)} = \pm\sqrt{\frac{1}{4}} = -\frac{1}{2} = -0.5$

$V(Y) = 36$ {Given}

i.e. $\sigma_y^2 = 36 \Rightarrow \sigma_y = 6$

Since $b_{XY} = r \cdot \frac{\sigma_x}{\sigma_y}$

$\frac{-1}{6} = -0.5 \times \frac{\sigma_x}{6}$

$\Rightarrow \sigma_x = 2 \Rightarrow \sigma_x^2 = V(X) = 4$

Q.3. Find the line of regression of X on Y for the following data:

$n = 8, \Sigma(x_i - \bar{x})^2 = 36, \Sigma(y_i - \bar{y})^2 = 44, \Sigma(x_i - \bar{x})(y_i - \bar{y}) = 24.$

$b_{XY} = \frac{\Sigma(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma(Y_i - \bar{Y})^2} = \frac{24}{44} = \frac{6}{11} = 0.54$

The Regression equation of X on Y is

$X - \bar{X} = b_{XY}(Y - \bar{Y})$

$\therefore X - \bar{X} = 0.54(Y - \bar{Y})$

Sign of Teacher :