BOARD ANSWER PAPER: MARCH 2022

MATHEMATICS PART - II

Q.1 (A)

i. (A) 48°

ii. (C) 6 cm

iii. (D) (–5, 3)

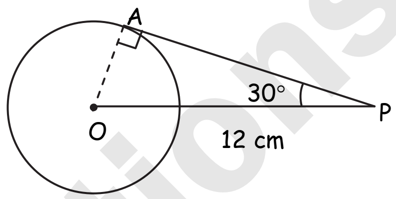
iv. (B) 1

Hints:

i. $\triangle ABC \sim \triangle DEF \Rightarrow m\angle A = m\angle D$

$\Rightarrow m\angle D = 48^\circ$

ii. In $\triangle AOP$, $\sin 30^\circ = \dfrac{AO}{OP}$



$\therefore$ $\dfrac{1}{2} = \dfrac{AO}{12}$

$\therefore$ AO = 6 cm

iii. According to the given conditions,
Y-coordinate of the point B must be 3.

iv. $2 \tan 45^\circ - 2 \sin 30^\circ$

$= 2(1) - 2\left(\dfrac{1}{2}\right) = 2 - 1 = 1$

Q.1 (B)

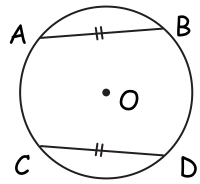
i. In $\triangle$ ABC, $\angle ABC = 90^\circ$, $\angle BAC = \angle BCA = 45^\circ$ …[Given]

$\therefore$ $AB = \dfrac{1}{\sqrt{2}}\ AC$ …[By $45^\circ - 45^\circ - 90^\circ$ Theorem]

$\therefore$ $AB = \dfrac{1}{\sqrt{2}} \times 9\sqrt{2}$

$\therefore$ AB = 9 units

ii. In a circle with centre O,
chord AB $\cong$ chord CD



$\therefore$ m(arc AB) $\cong$ m(arc CD)
…[Corresponding arcs of congruent chords of a circle are congruent]

$\therefore$ m(arc CD) = 120°

iii. Let $A(x_1, y_1) = A(4, -3)$
$B(x_2, y_2) = B(7, 5)$
$C(x_3, y_3) = C(-2, 1)$

$\therefore$ By centroid formula,

$y = \dfrac{y_1 + y_2 + y_3}{3} = \dfrac{-3+5+1}{3} = \dfrac{3}{3} = 1$

$\therefore$ The Y-co-ordinate of the centroid of the given triangle is 1.

iv. $\sin\theta = \cos\theta$

$\therefore$ $\dfrac{\sin\theta}{\cos\theta} = 1$

$\therefore$ $\tan\theta = 1$

We know that, $\tan 45^\circ = 1$

$\therefore$ $\tan\theta = \tan 45^\circ$

$\therefore$ $\theta = 45^\circ$

Q.2 (A)

i. In $\triangle ABP$ and $\triangle CDP$,

$\frac{AP}{CP} = \frac{BP}{DP}$ ...[$\boxed{\textbf{Given}}$]

$\angle APB \cong \boxed{\angle CPD}$ ...[Vertically opposite angles]

$\therefore$ $\boxed{\triangle ABP} \sim \triangle CDP$ ...[$\boxed{\textbf{SAS}}$ test of similarity]

ii. $\triangle ABC$ is $\boxed{\textbf{right angled}}$ triangle.

$\therefore$ By Pythagoras theorem,

$AB^2 + BC^2 = AC^2$

$\therefore$ $5^2 + BC^2 = 13^2$

$\therefore$ $25 + BC^2 = \boxed{169}$

$\therefore$ $BC^2 = 169 - 25 = \boxed{144}$

$\therefore$ $BC = \boxed{\textbf{12 units}}$

iii. L.H.S. = $\cot\theta + \tan\theta$

$= \frac{\cos\theta}{\sin\theta} + \frac{\boxed{\sin\theta}}{\cos\theta}$

$= \frac{\boxed{\cos^2\theta} + \sin^2\theta}{\sin\theta \times \cos\theta}$

$= \frac{1}{\sin\theta \times \cos\theta}$ ... $\because \boxed{\sin^2\theta + \cos^2\theta = 1}$

$= \frac{1}{\sin\theta} \times \frac{1}{\cos\theta}$

$= \boxed{\text{cosec}\theta} \times \sec\theta$

$\therefore$ L.H.S. = R.H.S.

Q.2 (B)

i. $\Delta ABC \sim \Delta PQR$ ...[Given]

$\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AB^2}{PQ^2}$ ...[Theorem of areas of similar triangles]

$\therefore$ $\frac{A(\Delta ABC)}{125} = \left(\frac{AB}{PQ}\right)^2$

$\therefore$ $A(\Delta ABC) = \left(\frac{4}{5}\right)^2 \times 125$

$\therefore$ $A(\Delta ABC) = \frac{16}{25} \times 125$

$= 16 \times 5 = 80$

$\therefore$ $A(\Delta ABC) = 80 \text{ cm}^2$

ii. Chords AD and CE intersect externally at point B.

$\therefore$ $m\angle DBE = \frac{1}{2}$ [m(arc DXE) – m(arc AYC)]

$= \frac{1}{2}(105^\circ - 47^\circ)$

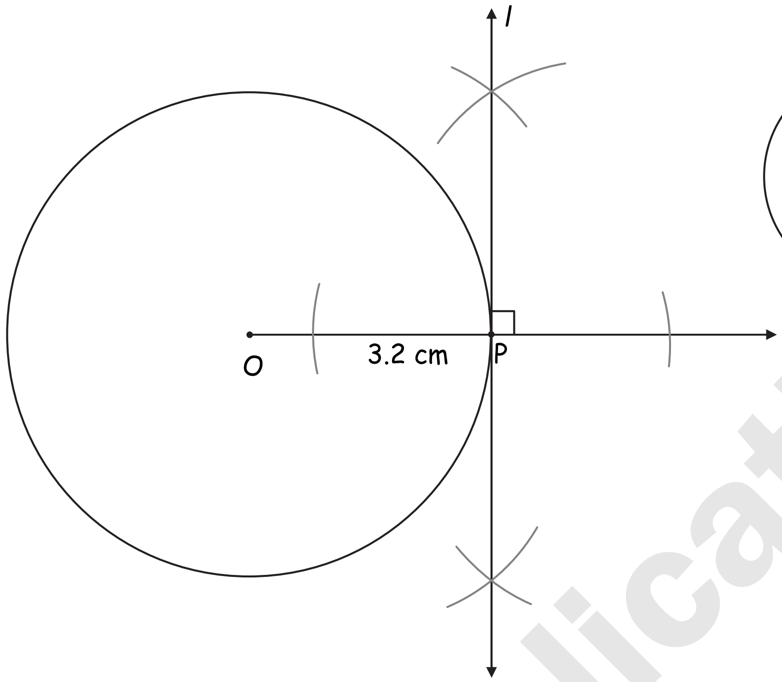
$= \frac{1}{2} \times 58^\circ$

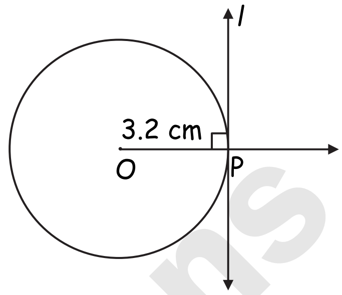
$\therefore$ $m\angle DBE = 29^\circ$

iii. **Analysis:**

seg OP $\perp$ line l ...[Tangent is perpendicular to radius]

The perpendicular to seg OP at point P will give the required tangent at P.





Rough Figure

iv. $\sin\theta = \frac{11}{61}$ ...[Given]

We know that,

$\sin^2\theta + \cos^2\theta = 1$

$\therefore$ $\left(\frac{11}{61}\right)^2 + \cos^2\theta = 1$

$\therefore$ $\frac{121}{3721} + \cos^2\theta = 1$

$\therefore$ $\cos^2\theta = 1 - \frac{121}{3721}$

$\therefore$ $\cos^2\theta = \frac{3721 - 121}{3721}$

$\therefore$ $\cos^2\theta = \frac{3600}{3721}$

$\therefore$ $\boldsymbol{\cos\theta = \frac{60}{61}}$ ...[Taking square root of both sides]

v. Here, AB = 9 cm, BC = 40 cm, AC = 41 cm

$\therefore$ $41^2 = 1681$

$9^2 + 40^2 = 1681$

i.e., $AB^2 + BC^2 = AC^2$

$\therefore$ (9, 40, 41) is a Pythagorean triplet.

$\therefore$ ΔABC is a right angled triangle.

Q.3 (A)

i. In $\triangle PTS$,

$\angle SPQ = \angle STQ - \boxed{\angle PST}$ ...[$\because$ Exterior angle theorem]

$= 58^\circ - 24^\circ$

$\therefore$ $\angle SPQ = 34^\circ$

$\therefore$ $m(\text{arc } QS) = 2 \times \angle SPQ = 2 \times \boxed{34}^\circ = 68^\circ$ ...[$\because$ $\boxed{\text{Inscribed angle theorem}}$]

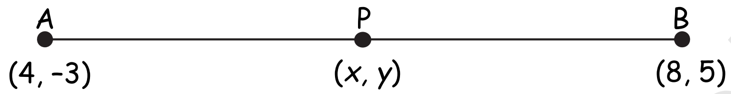
Similarly $m(\text{arc } PR) = 2\angle PSR = \boxed{48}^\circ$

$\therefore$ $\frac{1}{2}[m(\text{arc } QS) + m(\text{arc } PR)] = \frac{1}{2} \times \boxed{116}^\circ = 58^\circ$ ...(I)

but $\angle STQ = 58^\circ$ ...(II)[given]

$\therefore$ $\frac{1}{2}[m(\text{arc } PR) + m(\text{arc } QS)] = \boxed{\angle STQ}$ ...[from (I) and (II)]

ii.



$\therefore$ By section formula,

$x = \frac{mx_2 + nx_1}{\boxed{m + n}}$, $\quad y = \frac{\boxed{my_2 + ny_1}}{m + n}$

$\therefore$ $x = \frac{3 \times 8 + 1 \times 4}{3 + 1}$, $\quad y = \frac{3 \times 5 + 1 \times (-3)}{3 + 1}$

$= \frac{\boxed{24} + 4}{4}$ $\quad = \frac{\boxed{15} - 3}{4}$

$\therefore$ $x = \boxed{7}$ $\quad \therefore$ $y = \boxed{3}$

Q.3 (B)

i. $2\,AX = 3\,BX$ ...[Given]

$\therefore$ $\frac{AX}{BX} = \frac{3}{2}$

$\therefore$ $\frac{AX + BX}{BX} = \frac{3 + 2}{2}$ ... [By componendo]

$\therefore$ $\frac{BA}{BX} = \frac{5}{2}$ ... (i)[A–X–B]

In $\triangle BCA$ and $\triangle BYX$,

$\left.\begin{array}{l}\angle BCA \cong \angle BYX \\ \angle BAC \cong \angle BXY\end{array}\right\}$ ... [Corresponding angles]

$\therefore$ $\triangle BCA \sim \triangle BYX$ ... [By AA test of similarity]

$\therefore$ $\frac{BA}{BX} = \frac{AC}{XY}$ ... [Corresponding sides of similar triangles]

$\therefore$ $\frac{5}{2} = \frac{AC}{9}$ ... [From (i)]

$\therefore$ $AC = \frac{9 \times 5}{2}$

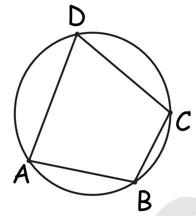
$\therefore$ $AC = 22.5$ units

ii. **Given:** □ABCD is cyclic.

To prove: $\angle B + \angle D = 180°$, $\angle A + \angle C = 180°$

Proof:

Arc ABC is intercepted by the inscribed angle $\angle$ADC.



$\therefore$ $\angle ADC = \frac{1}{2}$m(arc ABC) ...(i)

Similarly, $\angle$ABC is an inscribed angle.

It intercepts arc ADC.

$\therefore$ $\angle ABC = \frac{1}{2}$ m(arc ADC) ...(ii)

$\therefore$ $\angle ADC + \angle ABC$

$= \frac{1}{2}$ m(arc ABC) + $\frac{1}{2}$ m(arc ADC) ...[Adding (i) and (ii)]

$= \frac{1}{2}\left[\text{m(arc ABC)} + \text{m(arc ADC)}\right]$

$= \frac{1}{2} \times 360°$ ...$\left[\begin{array}{l}\text{Arc ABC and arc ADC} \\ \text{constitute a complete circle}\end{array}\right]$

$= 180°$

Similarly we can prove, $\angle A + \angle C = 180°$

iii. $\triangle ABC \sim \triangle PQR$ ...[Given]

$\therefore$ $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ...(i)[Corresponding sides of similar triangles]

But $\frac{AB}{PQ} = \frac{3}{2}$ ...(ii)[Given]

$\therefore$ $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{3}{2}$ ...[From (i) and (ii)]

$\therefore$ $\frac{5.4}{PQ} = \frac{4.2}{QR} = \frac{6}{PR} = \frac{3}{2}$

$\therefore$ $\frac{5.4}{PQ} = \frac{3}{2}$

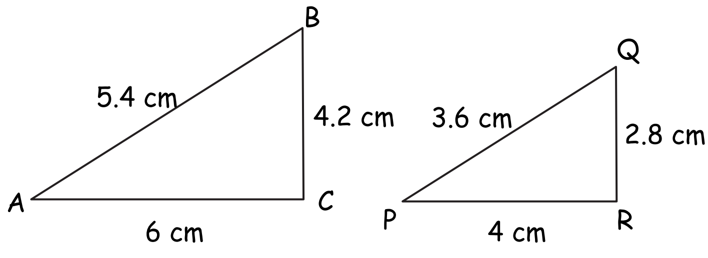
$\therefore$ $PQ = \frac{5.4 \times 2}{3} = 3.6$ cm

Also, $\frac{4.2}{QR} = \frac{3}{2}$

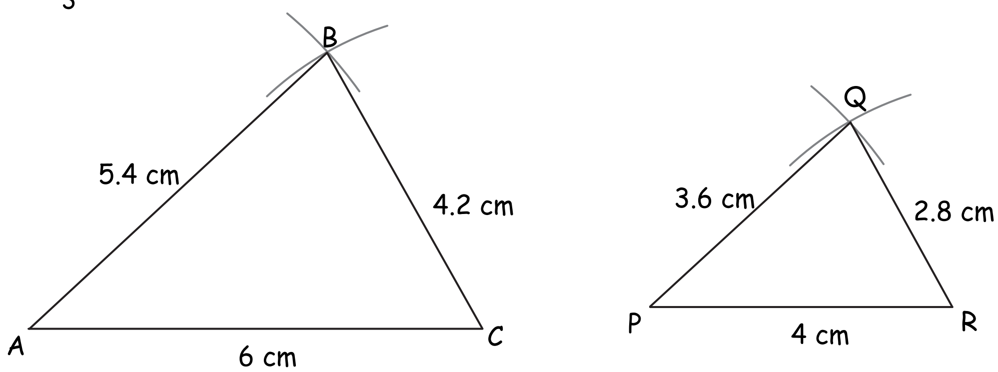
$\therefore$ $QR = \frac{4.2 \times 2}{3} = 2.8$ cm

and $\frac{6}{PR} = \frac{3}{2}$

$\therefore$ $PR = \frac{6 \times 2}{3} = 4$ cm



Rough Figure



iv. L.H.S. = $\frac{\tan A}{\left(1+\tan^2 A\right)^2} + \frac{\cot A}{\left(1+\cot^2 A\right)^2}$

$= \frac{\tan A}{\left(\sec^2 A\right)^2} + \frac{\cot A}{\left(\operatorname{cosec}^2 A\right)^2}$ ... $\left[\because 1+\tan^2\theta = \sec^2\theta,\ 1+\cot^2\theta = \operatorname{cosec}^2\theta\right]$

$= \frac{\tan A}{\sec^4 A} + \frac{\cot A}{\operatorname{cosec}^4 A}$

$= \tan A \times \frac{1}{\sec^4 A} + \cot A \times \frac{1}{\operatorname{cosec}^4 A}$

$= \frac{\sin A}{\cos A} \times \cos^4 A + \frac{\cos A}{\sin A} \times \sin^4 A$

$= \sin A \cos^3 A + \cos A \sin^3 A$

$= \sin A \cos A (\cos^2 A + \sin^2 A)$

$= \sin A \cos A (1)$ ...$[\because \sin^2\theta + \cos^2\theta = 1]$

$= \sin A \cos A$

= R.H.S.

$\therefore$ $\frac{\tan A}{\left(1+\tan^2 A\right)^2} + \frac{\cot A}{\left(1+\cot^2 A\right)^2} = \sin A \cos A$

Q.4

i. □ABCD is a parallelogram ...[Given]

$\therefore$ side AB || side CD ...[Opposite sides of a parallelogram]

$\therefore$ side AB || side CP ...[C – P – D]

and BP is their transversal.

$\therefore$ $\angle CPB \cong \angle ABP$ ...[Alternate angles]

$\therefore$ $\angle CPX \cong \angle ABX$ ...(i)[P – X – B}

In $\triangle PXC$ and $\triangle BXA$,

$\angle PXC \cong \angle BXA$ ...[Vertically opposite angles]

$\angle CPX \cong \angle ABX$ ...[From (i)]

$\therefore$ $\triangle PXC \sim \triangle BXA$ ...[By AA test of similarity]

$\therefore$ $\frac{CX}{AX} = \frac{XP}{XB} = \frac{AB}{CP}$ ...(ii)[corresponding sides of similar triangles]

Note that:

seg AB $\cong$ seg CD ...(iii) [$\because$ □ABCD is a parallelogram]

seg CP = $\frac{1}{2}$ seg CD ...(iv) [Point P is the mid-point of side CD]

$\therefore$ seg CP = $\frac{1}{2}$ seg AB ...(v) [From (iii) and (iv)]

$\therefore$ $\frac{CX}{AX} = \frac{XP}{XB} = \frac{AP}{CB} = \frac{2}{1}$ ...[From (ii) and (v)]

$\therefore$ $\frac{CX}{AX} = \frac{2}{1}$

$\therefore$ $\frac{CX+AX}{AX} = \frac{2+1}{2}$ ...[By componendo]

$\therefore$ $\frac{AC}{AX} = \frac{3}{2}$

$\therefore$ 3AX = 2AC

Hence proved

ii. **Given:** seg AB and seg AD are tangent segment drawn to a circle with centre C from exterior point A.

To prove: $\angle A = \frac{1}{2}$[m(arc BYD) – m(arc BXD)]

Proof:

In □ABCD,

$\angle A + \angle B + \angle C + \angle D = 360^\circ$ ...[Sum of the measures of the quadrilateral is 360°]

$\therefore$ $\angle A + 90^\circ + \angle C + 90^\circ = 360^\circ$ ...[Tangent theorem]

$\therefore$ $\angle A + \angle C = 360^\circ - 90^\circ - 90^\circ$

$\therefore$ $\angle A + \angle C = 180^\circ$ ...(i)

Note that m(arc BXD) = $\angle C$ ...(ii)[Definition of measure of minor arc]

$\therefore$ $\angle A$ + m(arc BXD) = 180° ...[From (i) and (ii)]

$\therefore$ $\angle A$ = 180° – m(arc BXD) ...(iii)

Also, m(arc BXD) + m(arc BYD) = 360° ...[Measure of circle is 360°]

$\therefore$ $\frac{\text{m(arc BXD)}}{2} + \frac{\text{m(arc BYD)}}{2} = 180^\circ$ ...(iv)[Divide both side by 2]

$\therefore$ $\angle A = \frac{\text{m(arc BXD)}}{2} + \frac{\text{m(arc BYD)}}{2}$ – m(arc BXD) ...[From (iii) and (iv)]

$\therefore$ $\angle A = \frac{1}{2}$[m(arc BYD) – m(arc BXD)]

Hence proved.

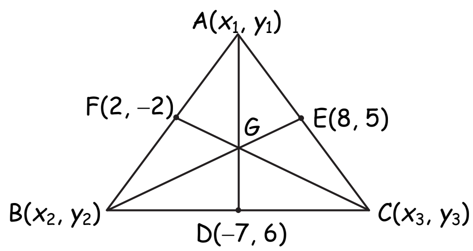
iii. Suppose A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are the vertices of the triangle.

D (–7, 6), E (8, 5) and F (2, –2) are the midpoints of sides BC, AC and AB respectively.

Let G be the centroid of $\triangle$ABC.

D is the midpoint of seg BC.

By midpoint formula,



Co-ordinates of D = $\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$

$\therefore$ $(-7, 6) = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$

$\therefore$ $\frac{x_2 + x_3}{2} = -7$ and $\frac{y_2 + y_3}{2} = 6$

$\therefore$ $x_2 + x_3 = -14$ ...(i) and

$y_2 + y_3 = 12$ ...(ii)

E is the midpoint of seg AC.

By midpoint formula,

Co-ordinates of E = $\left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$

$\therefore$ $(8, 5) = \left(\frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2}\right)$

$\therefore$ $\frac{x_1 + x_3}{2} = 8$ and $\frac{y_1 + y_3}{2} = 5$

$\therefore$ $x_1 + x_3 = 16$ ...(iii) and

$y_1 + y_3 = 10$ ...(iv)

F is the midpoint of seg AB.

By midpoint formula,

Co-ordinates of F = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

∴ $(2, -2) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

∴ $\frac{x_1 + x_2}{2} = 2$ and $\frac{y_1 + y_2}{2} = -2$

∴ $x_1 + x_2 = 4$ …(v) and

$y_1 + y_2 = -4$ …(vi)

Adding (i), (iii) and (v),

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = -14 + 16 + 4$

∴ $2x_1 + 2x_2 + 2x_3 = 6$

∴ $x_1 + x_2 + x_3 = 3$ …(vii)

Adding (ii), (iv) and (vi),

$y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 12 + 10 - 4$

∴ $2y_1 + 2y_2 + 2y_3 = 18$

∴ $y_1 + y_2 + y_3 = 9$ …(viii)

G is the centroid of △ABC.

By centroid formula,

Co-ordinates of G $= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

$= \left(\frac{3}{3}, \frac{9}{3}\right)$ …[From (vii) and (viii)]

$= (1, 3)$

∴ The co-ordinates of the centroid of the triangle are (1, 3).

Q.5

i. $(a^2 + b^2)^2 = a^4 + 2a^2b^2 + b^4$ …(i)

$(a^2 - b^2)^2 = a^4 - 2a^2b^2 + b^4$ …(ii)

$(2ab)^2 = 4a^2b^2$ …(iii)

Now, $(a^4 + 2a^2b^2 + b^4) = (a^4 - 2a^2b^2 + b^4) + 4a^2b^2$

∴ $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$ … [From (i), (ii) and (iii)]

∴ $[(a^2 + b^2), (a^2 - b^2), (2ab)]$ is a Pythagorean triplet.

∴ The triangle with sides $(a^2 + b^2)$, $(a^2 - b^2)$ and $(2ab)$ is a right angled triangle.

a. Let a = 2, b = 1

$a^2 + b^2 = 2^2 + 1^2 = 4 + 1 = 5$

$a^2 - b^2 = 2^2 - 1^2 = 4 - 1 = 3$

$2ab = 2 \times 2 \times 1 = 4$

∴ (5, 3, 4) is a Pythagorean triplet.

b. Let a = 4, b = 3

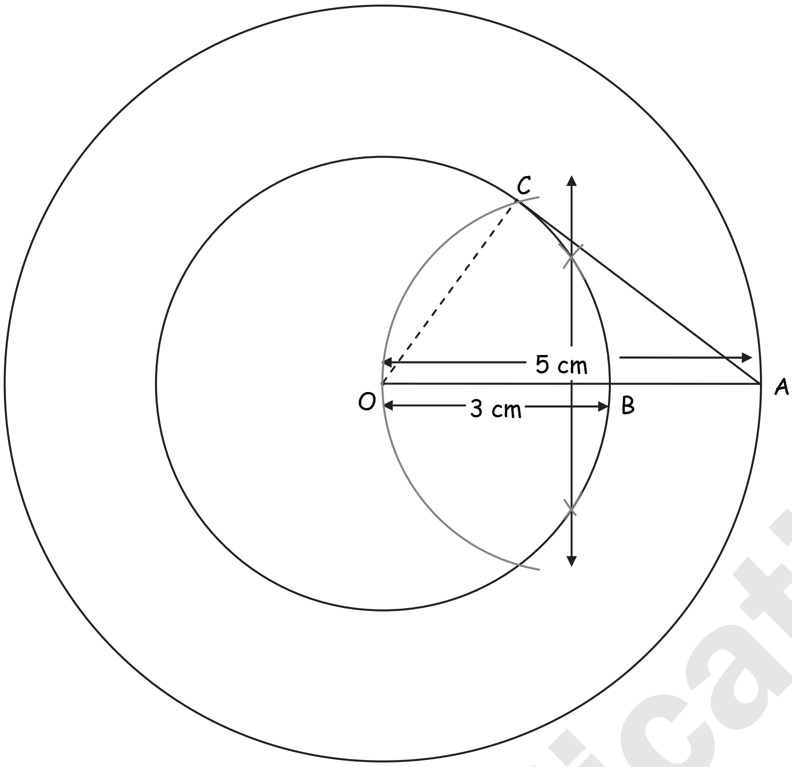
$a^2 + b^2 = 4^2 + 3^2 = 16 + 9 = 25$

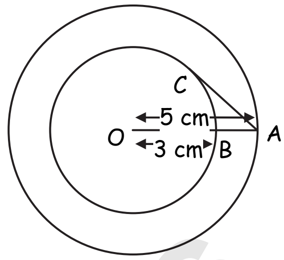
$a^2 - b^2 = 4^2 - 3^2 = 16 - 9 = 7$

$2ab = 2 \times 4 \times 3 = 24$

∴ (25, 7, 24) is a Pythagorean triplet.

ii.





Rough figure

Length of the tangent segment is 4 cm.

By Pythagoras theorem:

Tangent $CA \perp$ radius OC ...[Tangent theorem]

$\therefore$ In $\triangle AOC$, $\angle C = 90^\circ$

$OA = 5$ cm ...[Radius of the bigger circle]

$OC = 3$ cm ...[Radius of the smaller circle]

$\therefore$ By Pythagoras Theorem, we get

$OA^2 = OC^2 + AC^2$

$\therefore$ $(5)^2 = (3)^2 + AC^2$

$\therefore$ $25 = 9 + AC^2$

$\therefore$ $AC^2 = 25 - 9$

$\therefore$ $AC^2 = 16$

$\therefore$ $AC = 4$ cm

$\therefore$ Length of the tangent segment is 4 cm.