A. Activities

Carry out the following activities

1. The rate of growth of the population of a city at any time 't' is proportional to the size of the population. For a certain city it is found that the constant of proportionally is 0.04. Find the population of the city after 25 years if the initial population is 10000 by completing the following activity. (Give e = 2.7182)

Let 'x' be the population of the city at time 't'

$\therefore \frac{dx}{dt} \propto x, \therefore \frac{dx}{x} \boxed{0.04} \boxed{dt}$

But constant of proportionally is 0.04

$\frac{dx}{x} = k\,dt$

$\frac{dx}{x} = \boxed{0.04}\,dt$

$\therefore \int \frac{dx}{x} = \boxed{0.04} \int \boxed{dt}$, Integrating we get

$\log x = \boxed{0.04t} + c$ ..... (1)

i) $t = 0, x = 10000, \log(10000) = 0 + c$

$c = \log 10000$

ii) $t = 25, x = ?$

$\therefore \log x = 25 \boxed{0.04} + \log(10000)$

$\log x = \boxed{1} + \log(10000)$

$\therefore x = 10000 \times \boxed{2.7182} = \boxed{27182}$

2. The rate of decay of certain substance is directly proportional to the amount present at that instant. Initially there are 1.5 gms of certain substance and t hours later it is found that 0.5 gms are left. Find the time using the following activity.

Let M be the mass of the substance at time 't'.

$\therefore \frac{dM}{dt} \propto M$

$\therefore \frac{dM}{dt} = -kM$ (-ve sign indicates dis-integration)

$\therefore \frac{dM}{M} = -k\,dt$, Integrating we get,

$\int \frac{dM}{M} = -k \int dt$

$\therefore \log \boxed{M} = -k \boxed{t} + c$ ..... (1)

i) $t = 0, M = 1.5$

$\therefore \log 1.5 = \boxed{c}$

$\therefore c = \log \boxed{1.5}$

ii) $M = 0.5, t = ?$

$\therefore kt = \log(\boxed{1.5}) - \log(\boxed{0.5})$

$kt = \log(\boxed{1.5/0.5})$

$t = \frac{1}{K} \log \boxed{3.}$

3. From the following information find the number of years in which the city have population 4,00,000.

Given $\frac{dx}{dt} \propto x$, x : population at time 't' years

t	x
0	50000
25	10000

As $\frac{dx}{dt} \propto x$, $\therefore \frac{dx}{dt} = kx$, $\frac{dx}{x} = kdt$

Integrating we get, $\log x = k \boxed{t} + c$ ..... (1)

i) $t = 0, x = 50000$, $\therefore \log 50000 = k \boxed{0} + c$

$\therefore c = \log(50000)$

ii) $t = 25, x = 10000$

$\therefore \log(10000) = \boxed{25} k + \log \boxed{50,000}$

$\therefore \log \boxed{10/5} = 25 k$

$\therefore k = \frac{1}{25} \log \boxed{2}$

iii) $x = 400000$ we get,

$\log(400000) = \left[K = \frac{1}{25} \log \boxed{2}\right] t + \log \boxed{50000}$

$\log\left(\frac{400000}{50000}\right) = \frac{t}{25} \times \log \boxed{2}$

$\log 8 = \frac{t}{25} \log \boxed{2}$, $\frac{t}{25} = \boxed{3}$, $t = \boxed{75}$ years

B. Solve the Following

Q.1. The population of a town increasing at a rate proportional to the population at that time. If the population increases from 40 thousands to 60 thousands in 40 years, what will be the population in another 20 years?

(Given $\sqrt{\frac{3}{2}} = 1.2247$).

Solⁿ: As $\frac{dx}{dt} \propto x$, x be the population at time t

$\therefore \frac{dx}{x} = k\,dt \quad \therefore \int \frac{dx}{x} = k\int dt$

$\log x = kt + c$ ---- (i)

When $t = 0$, $x = 40,000$

$\log(40,000) = 0 + c$

$c = \log(40,000)$ --- (ii)

when $t = 40$, $x = 60,000$

$\log(60,000) = 40k + \log 40,000$

$\log\left(\frac{3}{2}\right) = 40k$

$\therefore k = \frac{1}{40}\log\left(\frac{3}{2}\right)$ ---- (iii)

when $t = 60$; we get:

$\log x = k(60) + \log(40,000)$

$\log x = \frac{60}{40}\log\left(\frac{3}{2}\right) + \log(40,000)$

$\log x = \frac{3}{2}\log\left(\frac{3}{2}\right) + \log(40,000)$

$= \log\left(\frac{3}{2}\sqrt{\frac{3}{2}}\right) + \log(40,000)$

$= \log\left(\frac{3}{2}\sqrt{\frac{3}{2}} \times 40,000\right)$

$= \log[3 \times 1.2247 \times 20,000]$

$\log x = \log[37482]$

$x = 37482$

Population in another 20 yrs i.e in 60 years will be 73,482.

Q.2. Bacteria increase at the rate proportional to the number of bacteria present. If the original number N doubles in 3 hours, find in how many hours the number of bacteria will be 4N ?

As, $\frac{dN}{dt} \propto N \quad \therefore \frac{dN}{dt} = KN$

Integrating $\log N = Kt + C$ --(i)

when $t = 0$, $N = N_0$

$\log N_0 = 0 + C$

$C = \log N_0$ ----- (ii)

when $t = 3$, $N = 2N_0$

$\log 2N_0 = 3K + \log N_0$

$\log\left(\frac{2N_0}{N_0}\right) = 3K$

$\therefore K = \frac{1}{3}\log 2$ -- (iii)

when $N = 4N_0$:

$\log 4N_0 = \left[\frac{1}{3}\log 2\right]t + \log N_0$

$\log\left[\frac{4N_0}{N_0}\right] = \frac{t}{3}[\log 2]$

$\log 4 = t/3[\log 2]$

$2\log 2 = t/3[\log 2]$

$2 = t/3$

$t = 6$ hours.

Hence, In 6 hours number of bacteriya will be 4N.

Q.3. The population of a country doubles in 60 years, in how many years will it be triple when the rate of increase is proportional to the number of inhabitants. (Given $\log_2 3 = 1.5894$)

Let P be the population of the country at time t.

As $\frac{dP}{dt} \propto P$, $\frac{dP}{dt} = KP$

Integrating,

$$\log P = Kt + c \quad \ldots\ldots (i)$$

Let $e^c = \alpha$ $\therefore P = e^{Kt} \cdot \alpha$ ......(ii)

Let Initial population at $t = 0$ $\therefore N = \alpha e^0$

$\therefore N = \alpha$.

Eq^n (ii) becomes $P = Ne^{Kt}$

when $P = 3N$.

$$3N = Ne^{Kt}$$

$$3 = e^{Kt}$$

$$\log 3 = Kt$$

$$\log 3 = \left[\frac{1}{60} \log 2\right] t$$

$$\therefore t = 60 \frac{\log 3}{\log 2}$$

$$t = 60 \times \log_2 3.$$

$$t = 60 \times 1.5894.$$

$$t = 95.364$$

$\therefore$ The population of the country will triple approximately in 95.36 years.

Sign of Teacher :