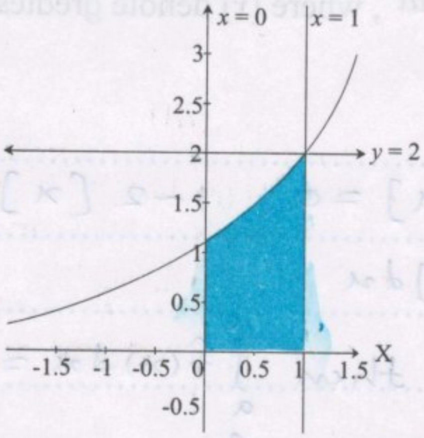
# 13. Application of definite integration

**Ex.** (1) Find the area of the region bounded by the curve $y = 2^x$ and the lines $x = 0$ and $y = 1$.




**Solution :** The equation of the carves are $y = 2^x$ and $y = 2$.

Solving equations we get $x = 1$.

Point of intersection of the curve is (1,2).

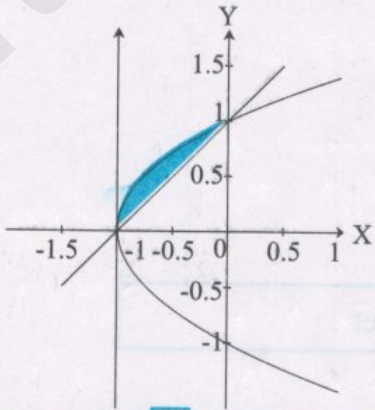
Required area (A) $= \int_0^1 (2)\,dx - \int_0^1 2^x\,dx$

$$= [2x]_0^1 - \left[\frac{2^x}{\log 2}\right]_0^1$$

$$= [2-0] - \left[\frac{2^1 - 2^0}{\log 2}\right]$$

$$= \left[2 - \frac{1}{\log 2}\right] \text{ sq.units}$$

**Ex.** (2) Find the area of the region enclosed by the curves $y = \sqrt{x+1}$ and $y = x + 1$.



**Solution :** The equation of the curves are $y=\sqrt{x+1}$ and $y=x+1$ .

Solving these equations , we get $x+1=\sqrt{x+1}$

$x^2+x=0$

$x(x+1)=0$

$x=0 \quad or \quad x=-1$

$\therefore\ y=1 \ and \ y=0$

Therefore , the point of intersection of the curves are $(-1, 0)$ and $(0, 1)$.

$\therefore$ Required area (A) $= \int_{-1}^{0}\sqrt{x+1}\,dx - \int_{-1}^{0}(x+1)\,dx$

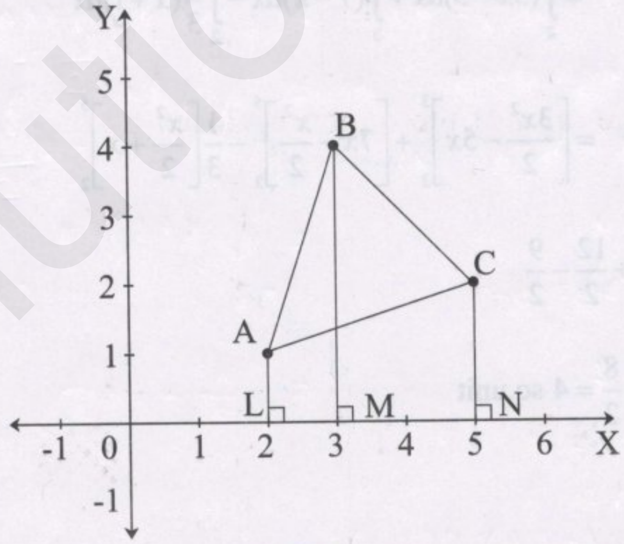
$= \left[\frac{2}{3}(x+1)^{\frac{3}{2}}\right]_{-1}^{0} - \left[\frac{x^2}{2}+x\right]_{-1}^{0}$

$= [\frac{2}{3}(1)^{\frac{3}{2}} - 0] - [0 - (\frac{1}{2}-1)]$

$= [\frac{2}{3}] - [\frac{1}{2}]$

$= [\frac{1}{6}]$ sq. units

**Ex.** (3) Find the area of the triangle formed by the vertices (2,1) , (3,4) and (5,2).



**Solution :** A(2,1) , B(3,4) and C(5,2) are the vertices of the triangle.

Equation of AB is $y-1=\left(\frac{4-1}{3-2}\right)(x-2)$

$$y-1=\left(\frac{3}{1}\right)(x-2)$$

$$y-1=(3x-6)$$

$$3x-y=5 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(I)}$$

Equation of AC is $y-1=\left(\frac{2-1}{5-2}\right)(x-2)$

$$y-1=\left(\frac{1}{3}\right)(x-2)$$

$$3y-3=x-2$$

$$x-3y=-1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(II)}$$

Equation of BC is

$$y-4=\left(\frac{-2}{2}\right)(x-3)$$

$$y-4=-x+3$$

$$x+y=7 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(III)}$$

Area of $\Delta$ABC = A(regionALMBA) + A(regionBCNMB) − A(regionACNLA)

$$=\int_2^3(3x-5)dx+\int_3^5(7-x)dx-\int_2^5\frac{1}{3}(x+1)dx$$

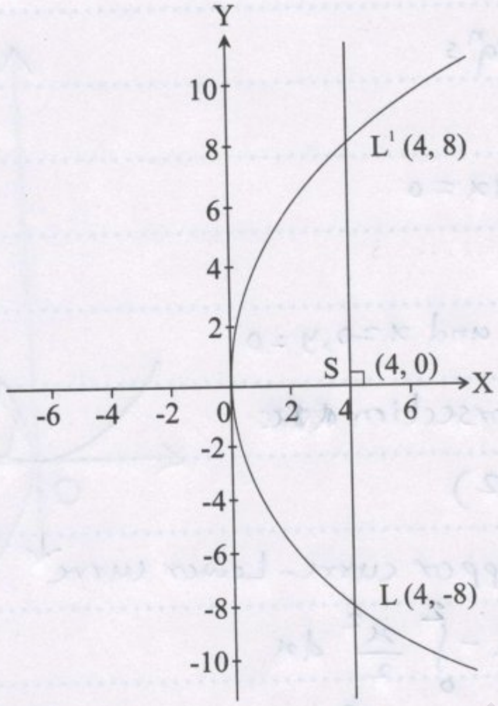
$$=\left[\frac{3x^2}{2}-5x\right]_2^3+\left[7x-\frac{x^2}{2}\right]_3^5-\frac{1}{3}\left[\frac{x^2}{2}+x\right]_2^5$$

$$=\frac{5}{2}+\frac{12}{2}-\frac{9}{2}$$

$$=\frac{8}{2}=4 \text{ sq.unit}$$

**Ex.** (4) Find the area of the region bounded by the parabola $y^2 = 16x$ and its latus rectum.

**Solution :** The equation of the parabola is $y^2 = 16x$.




The equation of parabola is $y^2 = 16x$

comparing with $y^2 = 4ax$

$\therefore 4a = 16$, $\boxed{a = 4}$

focus $S(a, 0) \equiv S(4, 0)$

$y^2 = 16x \Rightarrow y = 4\sqrt{x}$

A (region OL'SLO) = 2 A (region OL'SO)

$= 2\int_0^4 y\,dx$

$= 2\int_0^4 4\sqrt{x} \Rightarrow 8\int_0^4 x^{\frac{1}{2}}\,dx$

$= 8\left[\frac{x^{3/2}}{\frac{3}{2}}\right] \Rightarrow 8 \times \frac{2}{3}\left[x^{3/2}\right]_0^4$

$= \frac{16}{3}\left[4^{3/2} - 0^{3/2}\right] = \frac{16}{3} \times 8$

Required Area $= \frac{128}{3}$ sq. units

**Ex.** (5) Find the area of the region lying between the parabolas $y^2 = 2x$ and $x^2 = 2y$.

**Solution :**

We have $y^2 = 2x$ and $x^2 = 2y$ $\therefore y = \sqrt{2x}$ , $y = \frac{x^2}{2}$

equating these eq$^n$s

$x^2 = 2\sqrt{2x}$

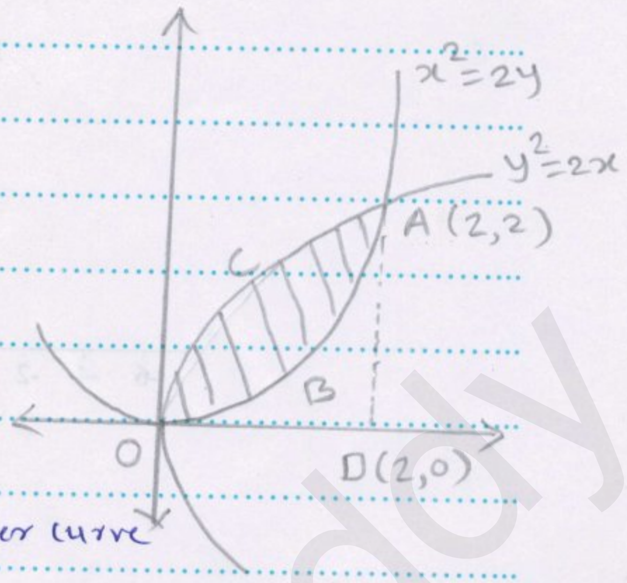
$x^4 = 8x \Rightarrow x^4 - 8x = 0$

$x = 2, x = 0$

when $x = 2, y = 2$ and $x = 0, y = 0$

The point of intersection are

$O(0,0)$ , $A(2,2)$




Required area = Upper curve − Lower curve

$= \int_0^2 \sqrt{2}\sqrt{x} - \int_0^2 \frac{x^2}{2}\, dx$

$= \sqrt{2}\int_0^2 x^{\frac{1}{2}} dx - \frac{1}{2}\int_0^2 x^2 dx \Rightarrow \sqrt{2}\times\frac{2}{3}\left[x\sqrt{x}\right]_0^2 - \frac{1}{2}\left[\frac{x^3}{3}\right]_0^2$

$= \frac{2\sqrt{2}}{3}\left[2\sqrt{2} - 0\sqrt{0}\right] - \frac{1}{2}\left[\frac{8}{3} - 0\right] \Rightarrow \frac{8}{3} - \frac{4}{3}$

$= \frac{4}{3}$ sq units

**Ex.** (6) Find the area bounded by the curve $y = x^2$ and the line y = x + 6.

**Solution :**

We have $y = x^2$ and $y = x + 6$

equating the equations

$x^2 = x + 6 \therefore x^2 - x - 6 = 0$

$x^2 - 3x + 2x - 6 = 0$

$x(x-3) + 2(x-3) = 0$

$\therefore x = -2$ and $x = 3$

when $x = -2$ , $y = 4$

when $x = 3$ , $y = 9$

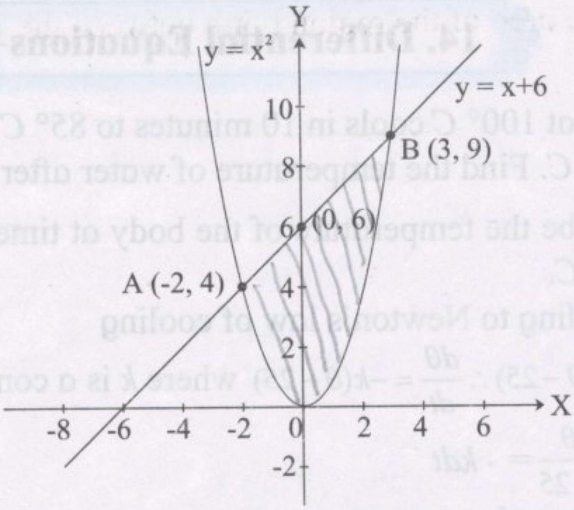
$\therefore$ The points are $A(-2,4)$, $B(3,9)$

Required area = region(OBAO)

$= \int_{-2}^{3} y\, dx - \int_{-2}^{3} y\, dx$

$= \int_{-2}^{3} (x+6)\, dx - \int_{-2}^{3} x^2 dx$

$= \left[\frac{x^2}{2} + 6x\right]_{-2}^{3} - \left[\frac{x^3}{3}\right]_{-2}^{3}$

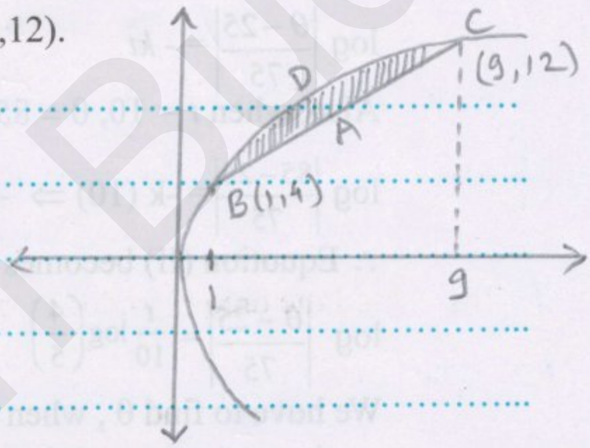



$$= \frac{9}{2} + 18 - \left(\frac{4}{2} - 12\right) - \left[\frac{27}{3} - \left(-\frac{8}{3}\right)\right]$$

$$= \left(\frac{9}{2} + 28\right) - \frac{35}{3} \Rightarrow \frac{65}{2} - \frac{35}{3}$$

$$= \frac{125}{6} \text{ sq units.}$$

**Ex.** (7) Find the area of the region enclosed by the parabola $y^2 = 16x$ and the chord BC where B(1,4) and C(9,12).




**Solution :**

we have $y^2 = 16x$, $B(1,4)$, $C(9,12)$

eqⁿ of line BC is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} \therefore \frac{x-1}{9-1} = \frac{y-4}{12-4}$$

$$\frac{x-1}{8} = \frac{y-4}{8} \therefore \boxed{y = x+3}$$

Required area = Upp Curve − Low Cur.

$$= \int_1^9 y\,dx - \int_1^9 y\,dx$$

$$= \int_1^9 4\sqrt{x}\,dx - \int_1^9 x+3\,dx$$

$$= 4\left[\frac{x^{3/2}}{3/2}\right]_1^9 - \left[\frac{x^2}{2} + 3x\right]_1^9$$

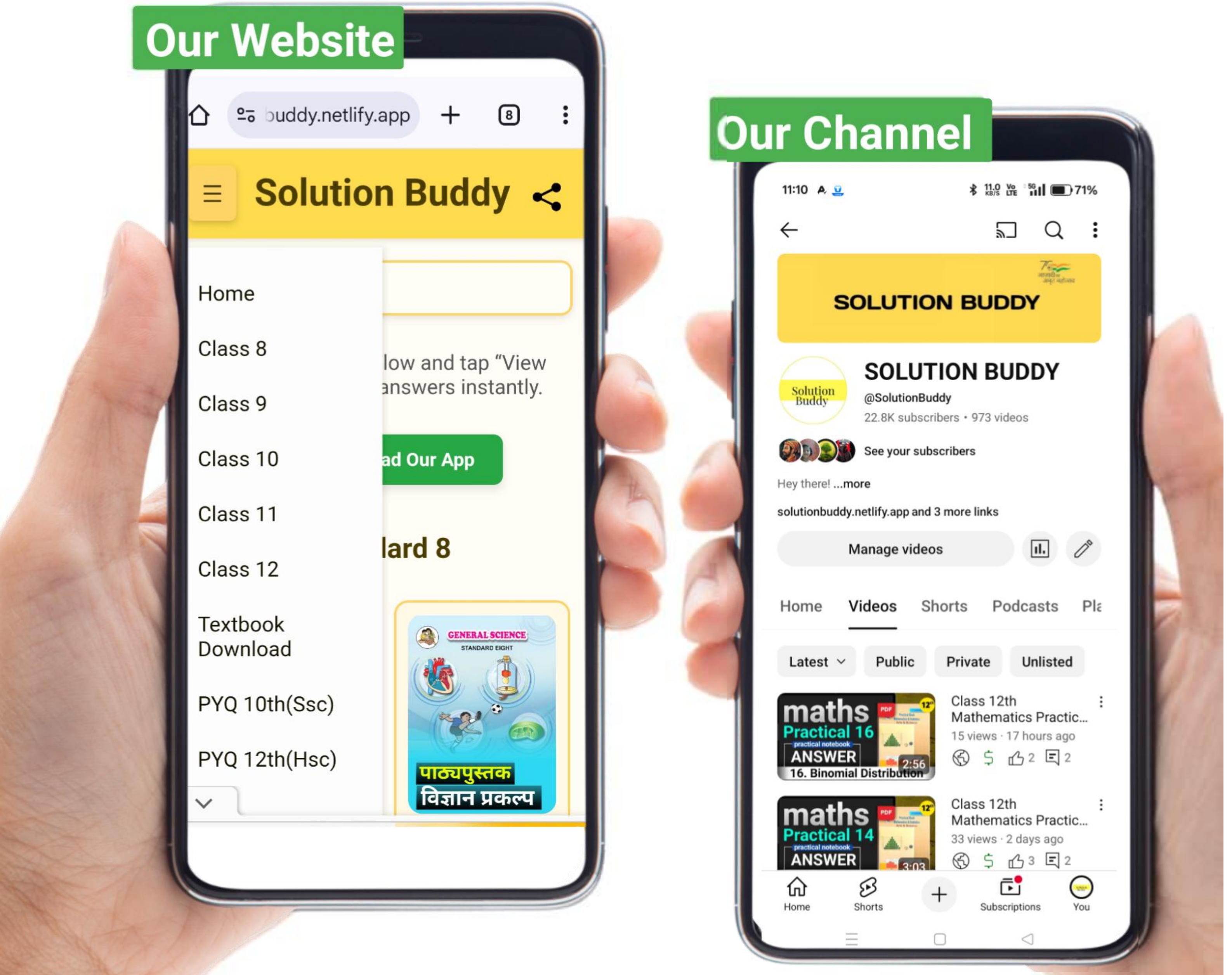
$$= 4 \times \frac{2}{3}\left[9^{3/2} - 1\right] - \left[\frac{81}{2} + 27 - \frac{1}{2} - 3\right]$$

$$= \frac{8}{3}\left[27 - 1\right] - (64)$$

$$= \frac{8}{3} \times 26 - 64$$

$$= \frac{16}{3} \text{ Sq units.}$$

Sign of Teacher :

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