Chapter 12: Electromagnetic induction

EXERCISES [PAGES 286 - 287]

Exercises | Q 1.1 | Page 286

Choose the correct option:
A circular coil of 100 turns with a cross-sectional area of 1 m^2 is kept with its plane perpendicular to the magnetic field of 1 T. The magnetic flux linkage is

1. 1 Wb
2. **100 Wb**
3. 50 Wb
4. 200 Wb

SOLUTION

100 Wb

Exercises | Q 1.2 | Page 286

Choose the correct option:
A conductor rod of length (l) is moving with velocity (v) in a direction normal to a uniform magnetic field (B). What will be the magnitude of induced emf produced between the ends of the moving conductor?

BLv

BLv^2

$\frac{1}{2}Blv$

$\frac{2Bl}{v}$

SOLUTION

BLv

Exercises | Q 1.3 | Page 286

Choose the correct option:
Two inductor coils with inductance 10 mH and 20 mH are connected in series. What is the resultant inductance of the combination of the two coils?

20 mH

30 mH

10 mH

$\frac{20}{3}$ mH

SOLUTION

20 mH

Exercises | Q 1.4 | Page 286

Choose the correct option

A current through a coil of self-inductance 10 mH increases from 0 to 1 A in 0.1 s. What is the induced emf in the coil?

1. **0.1 V**
2. 1 V
3. 10 V
4. 0.01 V

SOLUTION

0.1 V

Exercises | Q 1.5 | Page 286

Choose the correct option:

What is the energy required to build up current of 1 A in an inductor of 20 mH?

1. **10 mJ**
2. 20 mJ
3. 20 J
4. 10 J

SOLUTION

10 mJ

Exercises | Q 2.1 | Page 286

Answer in brief:

What do you mean by electromagnetic induction?

SOLUTION

The phenomenon of production of emf in a conductor or circuit by a changing magnetic flux through the circuit is called electromagnetic induction.

Exercises | Q 2.2 | Page 286
Answer in brief.
State and explain Lenz’s law in light of the principle of conservation of energy.

SOLUTION

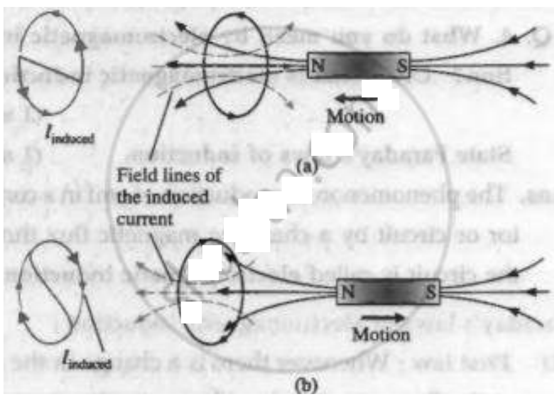
Lenz's law: The direction of the induced current is such as to oppose the change that produces it.

The change that induces a current may be

1. the motion of a conductor in a magnetic field or
2. the change of the magnetic flux through a stationary circuit.

Explanation: Consider Faraday's magnet-and coil experiment. If the bar magnet is moved towards the coil with its N-pole facing the coil, as in the shown first figure, the number of magnetic lines of induction (pointing to the left) through the coil increases.

The induced current in the coil sets up a magnetic field of its own pointing to the right (as given by Amperes right-hand rule) to oppose the growing flux due to the magnet. Hence, to move the magnet towards the coil against this repulsive flux of the induced current, we must do work. The work done shows up as electric energy in the coil.



When the magnet is withdrawn, with its N-pole still facing the coil, the number of magnetic lines of induction (pointing left) through the coil decreases. The induced current reverses its direction to supplement the decreasing flux with its own, as shown in the second figure. Facing the coil along with the magnet, the induced current is in the clockwise sense. The electric energy in the coil comes from the work done to withdraw the magnet, now against an attractive force. Thus, we see that Lenz's law is a consequence of the law of conservation of energy.

Exercises | Q 2.3 | Page 286
Answer in brief.
What are eddy currents? State applications of eddy currents.

SOLUTION

Whenever a conductor or a part of it is moved in a magnetic field "cutting" magnetic field lines, or placed in a changing magnetic field, the free electrons in the bulk of the metal start circulating in closed paths equivalent to current-carrying loops. These loop currents resemble eddies in a fluid stream and are hence called eddy or Foucault currents [after Jean Bernard Leon Foucault (1819-68), French physicist, who first detected them].

Applications:

1. **Dead-beat galvanometer:** A pivoted moving-coil galvanometer used for measuring current has the coil wound on a light aluminum frame. The rotation of the metal frame in the magnetic field produces eddy currents in the frame which opposes the rotation and the coil is brought to rest quickly. This makes the galvanometer dead-beat.

2. **Electric brakes:** When a conducting plate is pushed into a magnetic field, or pulled out, very quickly, the interaction between the eddy currents in the moving conductor and the field retards the motion. This property of eddy currents is used as a method of braking in vehicles.

Exercises | Q 2.4 | Page 286

Answer in brief:
If a copper disc swings between the poles of a magnet, the pendulum comes to rest very quickly. Explain the reason. What happens to the mechanical energy of the pendulum?

SOLUTION

As the copper disc enters and leaves the magnetic field, the changing magnetic flux through it induces eddy current in the disc. In both cases, Fleming's right-hand rule shows that opposing magnetic force damps the motion. After a few swings, the mechanical energy becomes zero and the motion comes to a stop. Joule heating due to the eddy current warms up the disc. Thus, the mechanical energy of the pendulum is transformed into thermal energy.

Exercises | Q 2.5 | Page 286

Answer in brief:
Explain why the inductance of two coils connected in parallel is less than the inductance of either coil.

SOLUTION

Assuming that their mutual inductance can be ignored, the equivalent inductance of a parallel combination of two coils is given by

$$\frac{1}{L_{parallel}} = \frac{1}{L_1} + \frac{1}{L_2} \text{ or } L_{parallel} = \frac{L_1 L_2}{L_1 + L_2}$$

Hence, the equivalent inductance is less than the inductance of either coil.

Exercises | Q 3 | Page 286

In a Faraday disc dynamo, a metal disc of radius R rotates with an angular velocity ω about an axis perpendicular to the plane of the disc and passing through its center. The disc is placed in a magnetic field B acting perpendicular to the plane of the disc. Determine the induced emf between the rim and the axis of the disc.

SOLUTION

Suppose a thin conducting disc of radius R is rotated anticlockwise, about its axis, in a plane perpendicular to a uniform magnetic field of induction $\vec{B}$ (see the figure in the above Note for reference). $\vec{B}$ points downwards. Let the constant angular speed of the disc be ω.

Consider an infinitesimal element of radial thickness dr at a distance r from the rotation axis. In one rotation, the area traced by the element is dA = 2πrdr. Therefore, the time rate at which the element traces out the area is

$\frac{dA}{dt}$ = frequency of rotation x dA = fdA

where f = $\frac{\omega}{2\pi}$ is the frequency of rotation.

$$\therefore \frac{dA}{dt} = \frac{\omega}{2\pi}(2\pi rdr) = \omega rdr$$

The total emf induced between the axle and the rim of the rotating disc is

$$|e| = \int B\frac{dA}{dt} = \int_0^R B\omega dr = B\omega\int_0^R rdr = B\omega\frac{R^2}{2}$$

For anticlockwise rotation in $\vec{B}$ pointing down, the axle is at a higher potential.

Exercises | Q 4 | Page 286

A horizontal wire 20 m long extending from east to west is falling with a velocity of 10 m/s normal to the Earth's magnetic field of 0.5×10^{-4} T. What is the value of induced emf in the wire?

SOLUTION

Data: l = 20 m, v = 10 m/s, B = 0.5×10^{-4} T

The magnitude of the induced emf,

$|e| = Blv = (5 \times 10^{-5})(20)(10) = 10^{-2}$ V = 10 mV

Exercises | Q 5 | Page 286

A metal disc of radius 30 cm spins at 20 revolution per second about its transverse symmetry axis in a uniform magnetic field of 0.20 T. The field is parallel to the axis of rotation. Calculate

(a) the area swept out per second by the radius of the disc

(b) the flux cut per second by a radius of the disc

(c) the induced emf between the axle and rim of the disc.

SOLUTION

Data: R = 0.3 m, f = 20 rps, B = 0.2 T

(a) The area swept out per unit time by a given radius = (the frequency of rotations) x (the area swept out per rotation) = $f(\pi r^2)$

= (20)(3.142 × 0.09) = 5.656 m^2

(b) The time rate at which a given radius cuts magnetic flux

$$= \frac{d\phi_m}{dt} = B\,f\left(\pi r^2\right)$$

= (0.2)(5.656) = 1.131 Wb/s

(c) The magnitude of the induced emf,

$$|e| = \frac{d\phi_m}{dt} = 1.131 \text{ V}$$

Exercises | Q 6 | Page 286

A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 10 A in 0.2 s, what is the change of flux linkage with the other coil?

SOLUTION

Data: M = 1.5 H, I_{1i} = 0, I_{lf} = 10 A, Δt = 0.2 s

The flux linked per unit turn with the second coil due to current I_1 in the first coil is

$\Phi_{21} = MI_1$

Therefore, the change in the flux due to change in I_1 is

$\Delta\,\Phi_{21} = M(\Delta\, I_1) = M(I_{1f} - I_{1i}) = 1.5(10 - 0) = 15$ Wb

Exercises | Q 7 | Page 287

A long solenoid has turns per unit length 1.5 x 10^3 per meter and area of cross-section 25 cm^2. A coil C of 150 turns is wound tightly around the center of the solenoid. For a current of 3 A in the solenoid, calculate

(a) the magnetic flux density at the centre of the solenoid
(b) the flux linkage in the coil C
(c) the average emf induced in coil C if the current in the solenoid is reversed in the direction in a time of 0.5 s. [$\mu_0 = 4\pi \times 10^{-7}$ H/m]

SOLUTION

Data: $n = 1.5 \times 10^3\ m^{-1}$, $A = 25 \times 10^{-4}\ m^2$,

$N_C = 150$, $I = 3A$, $\Delta t = 0.5$ s,

$\mu_0 = 4\pi \times 10^{-7}$ H/m

(a) Magnetic flux density inside the solenoid,

$$B = \mu_0 nI = (4\pi \times 10^{-7})(1500)(3)$$

$= 5.656 \times 10^{-3}$ T = 5.656 mT

(b) Flux per unit turn through the coils of the solenoid, $\Phi_m = BA$

Since the coil C is wound tightly over the solenoid, the flux linkage of C is

$N_C\Phi_m = N_CBA = (150)\ (5.656 \times 10^{-3})\ (25 \times 10^{-4})$

$= 2.121 \times 10^{-3}$ Wb = 2.121 mWb

(c) Initial flux through coil C,

$\Phi_i = N_C\Phi_m = 2.121 \times 10^{-3}$ Wb

Reversing the current in the solenoid reverses the flux through coil C, the magnitude remaining the same. But since the flux enters through the other face of the coil, the final flux through C is

$\Phi_i = -\ 2.121 \times 10^{-3}$ Wb

Therefore, the average emf induced in coil C,

$$e = \frac{\phi_f - \phi_i}{\Delta t}$$

$$= \frac{(-2.121 - 2.121) \times 10^{-3}}{0.5}$$

$$= 2 \times 4.242 \times 10^{-3} = 8.484 \times 10^{-3}\ V = 8.484\ mV$$

Exercises | Q 8 | Page 287

A search coil having 2000 turns with area 1.5 cm^2 is placed in a magnetic field of 0.60 T. The coil is moved rapidly out of the field in a time of 0.2 s. Calculate the induced emf in the search coil.

SOLUTION

Data: N = 2000, A_i= 1.5 × 10^{-4} m^2, A_f = 0,

B = 0.6 T, Δt = 0.2s

Initial flux, $N\Phi_i$ = NBA_i =2000(0.6)(1.5 × 10^{-4}) = 0.18 Wb

Final flux, $N\Phi_i$ = 0, since the coil is withdrawn out of the field.

Induced emf e = $-\mathbf{N}\frac{\triangle\phi_\mathrm{m}}{\triangle\mathrm{t}} = -\mathbf{N}\frac{\phi_\mathrm{f}-\phi_\mathrm{i}}{\triangle\mathrm{f}}$

∴ e = - $\frac{0-0.18}{0.2} = 0.9$ V

Exercises | Q 9 | Page 287

An aircraft of wing span of 50 m flies horizontally in the Earth's magnetic field of 6 x 10^{-5} T at a speed of 400 m/s. Calculate the emf generated between the tips of the wings of the aircraft.

SOLUTION

Data: l = 50 m, B = 6 x 10^{-5} T, v = 400 m/s

The magnitude of the induced emf,

|e| = Blv = (6 × 10^{-5})(400)(50) = 1.2 V

Exercises | Q 10 | Page 287

A stiff semi-circular wire of radius R is rotated in a uniform magnetic field B about an axis passing through its ends. If the frequency of rotation of wire is f, calculate the amplitude of alternating emf induced in the wire.

SOLUTION

In one rotation, the wire traces out a circle of radius R, i.e., an area A = πR^2.

Therefore, the rate at which the wire traces out the area is

$$\frac{dA}{dt} = \text{frequency or rotation} \times A = fA$$

If the angle between the uniform magnetic field $\vec{B}$ and the rotation axis is θ, the magnitude of the induced emf is

$$|e| = B\frac{dA}{dt}\cos\theta = BfA\cos\theta = Bf(\pi R^2)\cos\theta$$

so that the required amplitude is equal to $Bf(\pi R^2)$.

Exercises | Q 11 | Page 287

Calculate the induced emf between the ends of an axle of a railway carriage 1.75 m long traveling on level ground with a uniform velocity of 50 kmph. The vertical component of Earth's magnetic field (Bv) is 5×10^{-5} T.

SOLUTION

Data: l = 1.75 m, v = 50 km/h = $50 \times \frac{5}{18}$ m/s,

$B_V = 5 \times 10^{-5}$ T

The area swept out by the wing per unit time= lv.

∴ The magnetic flux cut by the wing per unit time

$$= \frac{d\phi_m}{dt} = B_v(lv)$$

$$= (5 \times 10^{-5})(1.75)\left(50 \times \frac{5}{18}\right) = 121.5 \times 10^{-5}$$

= 1.215 m Wb/s

Therefore, the magnitude of the induced emf,

|e| = 1.215 mV

Exercises | Q 12 | Page 287

The mutual inductance of two coils is 10 mH. If the current in one of the coil changes from 5 A to 1 A in 0.2 s, calculate the emf induced in the other coil. Also calculate the induced charge flowing through the coil if its resistance is 5 Ω.

SOLUTION

Data: M = 10 mH = 10^{-2} H, I_{1i} = 5 A, I_{1f} = 1 A, Δ t = 0.2 s, R = 5 Ω

The mutually induced emf in coil 2 due to the changing current in coil 1,

$$e_{21} = -M\frac{\triangle I_1}{\triangle t} = -M\frac{I_{1f} - I_{1i}}{\triangle t}$$

$$= -\left(10^{-2}\right)\left(\frac{1-5}{0.2}\right) = 0.2 \text{ V}$$

If ΔQ_2 is the charge that flows through coil 2 due to the changing current in coil 1, the induced current in coil 2 is

$$I_2 = \frac{\triangle Q_2}{\triangle t} = \frac{e_2}{R_2}$$

$$\therefore \Delta Q_2 = \frac{e_2}{R_2}\triangle t = \frac{0.2}{5}(0.2) = \frac{0.04}{5}$$

= 0.008 C = 8 mC

Exercises | Q 13 | Page 287

An emf of 96.0 mV is induced in the windings of a coil when the current in a nearby coil is increasing at the rate of 1.20 A/s. What is the mutual inductance (M) of the two coils?

SOLUTION

$|e_2| = 9.6 \times 10^{-2}$ V, dI_1/dt = 1.2 A/s

$$|e_2| = \frac{MdI_1}{dt}$$

Mutual inductance,

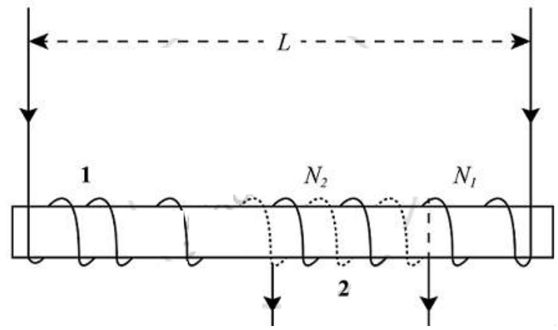
$$M = \frac{|e_2|}{dI_1/dt} = \frac{9.6\times10^{-2}}{1.2} = 8\times10^{-2} \text{ H}$$

= 80 mH

Exercises | Q 14 | Page 287

A long solenoid of length l, cross-sectional area A and having N_1 turns (primary coil), has a small coil of N_2 turns (secondary coil) wound about its center. Determine the Mutual inductance (M) of the two coils.

SOLUTION



$$\phi_2 = N_2 B_1 A$$

$$\phi_2 = N_2 \times \frac{\mu_0 N_1 I_1}{L} \times A$$

$$\phi_2 = \frac{\mu_0 N_1 N_2 A}{L} I_1$$

Comparing with $\phi_2 = MI_1$

We get $M_2 = \dfrac{\mu_0 N_1 N_2 A}{L}$

Exercises | Q 15 | Page 287

The primary and secondary coils of a transformer each have an inductance of 200×10^{-6} H. The mutual inductance between the windings is 4×10^{-6} H. What percentage of the flux from one coil reaches the other?

SOLUTION

Data: $L_p = L_S = 2 \times 10^{-4}$ H, $M = 4 \times 10^{-6}$ H

$$M = K\sqrt{L_p L_S}$$

The coupling coefficient is

$$K = \frac{M}{\sqrt{L_p L_S}} = \frac{4 \times 10^{-6}}{\sqrt{\left(2 \times 10^{-4}\right)^2}} = \frac{4 \times 10^{-6}}{2 \times 10^{-4}} = 2 \times 10^{-2}$$

Therefore, the percentage of flux of the primary reaching the secondary is 0.02 × 100% = 2 %

Exercises | Q 16 | Page 287

A toroidal ring made from a bar, of length 1 m and diameter 1 cm, bent into a circle. It is wound tightly with 100 turns per centimeter. If the permeability of the bar is equal to that of free space, calculate

(a) the magnetic field inside the bar when the current through the turns is 100 A

(b) the self-inductance of the coil.

SOLUTION

Data: l = 1 m, d = 1 cm, n = 100 cm^{-1} = 10^4 m^{-1}, I = 100 A, $\mu_0 = 4\pi \times 10^{-7}$ H/m

The radius of cross section, r = d/2 = 0.5 cm

$= 5 \times 10^{-3}$ m

(a) Magnetic field inside the toroid,

$B = \mu_0 n I = (4\pi \times 10^{-7})(10^4)(100)$

= 0.4 × 3.142 = 1.257 T

(b) Self inductance of the toroid,

$$L = \mu_{02}\pi R n^2 A = \mu_0 n^2 l A = \mu_0 n^2 l \left(\pi r^2\right)$$

$$= \left(4\pi \times 10^{-7}\right)\left(10^4\right)^2(1)\left[\pi\left(5 \times 10^{-3}\right)^2\right]$$

$$= \pi^2 \times 10^{-3} = 9.87 \times 10^{-3} \text{H} = 9.87 \text{ mH}$$

Exercises | Q 17 | Page 287

A uniform magnetic field B, pointing upward fills a circular region of radius s in a horizontal plane. If B changes with time, find the induced emf.

SOLUTION

The area of the region, $A = \pi s^2$, remains constant while B = B(t) is a function of time. Therefore, the induced emf,

$$e = -\frac{d\phi_m}{dt} = -\frac{d}{dt}(BA) = -A\frac{dB(t)}{dt} = -\pi s^2 \frac{dB(t)}{dt}$$