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Statistics



Let's study.

- Measures of a central tendency-
mean, median and mode from grouped frequency table.
- Graphical representation of statistical data -
histogram, frequency polygon, pie diagram

Statistics is useful in many fields of life: for example, agriculture, economics, commerce, medicine, botany, biotechnology, physics, chemistry, education, sociology, administration etc. An experiment can have many outcomes. To assess the possibility of possible outcomes, one has to carry out the experiment on a large scale and keep the record meticulously. Possibilities of different outcomes can be assessed using the record. For this purpose, rules are formulated in statistics.

Francis Galton (1822-1911) has done much of fundamental work in statistics. He used to prepare questionnaires, distribute them among people and request them to fill them up. He collected information from a number of people and recorded their backgrounds, financial situations, likes and dislikes, health etc. on a large scale. By that time, it was known that the fingerprints of different people are different. He collected finger-prints of a large number of people and invented a method of their classification. Using statistical methods, he showed that the possibility of finger prints of two different people being identical is nearly zero. This result made it possible to identify a person from his finger-prints. This method of identifying criminals was accepted in the judiciary. He had done much work in the field of anthropology of humans and other animals also.



Francis Galton



Let's recall.

We usually find a specific property in the numerical data collected in a survey that the scores have a tendency to cluster around a particular score. This score is a representative number of the group. The number is called the measure of central tendency.

In the previous standards we have studied the measures of central tendency, namely the mean, median and mode, for ungrouped data.

Activity 1 : Measure the height in cm of all students in your class. We find that the heights of many students cluster near a specific number.

Activity 2 : Collect a number of fallen leaves of a peepal tree. Distribute the leaves among the students and ask them to measure the lengths of them. Record the lengths. We notice that their lengths tend to cluster around a number.

Now we are going to do some more study of the mean, median and mode. Let us know the symbols and the terminology required for it.

$$\text{The mean of statistical data} = \frac{\text{The sum of all scores}}{\text{Total no. of scores}} = \frac{\sum_{i=1}^N x_i}{N}$$

(Here x_i is the i^{th} score)

Mean is denoted by \bar{X} and it represents the average of the given data.

$$\bar{X} = \frac{\sum_{i=1}^N x_i}{N}$$



Let's learn.

Mean from classified frequency distribution

When the number of scores in a data is large, it becomes tedious to write all numbers in the above formula and take their sum. So we use some different methods to find the sum.

Sometimes, the large data collected from an experiment is presented in a table in the grouped form. In such a case, we cannot find the exact mean of statistical data. Hence, let us study a method which gives the approximate mean, or a number nearby.

Direct method

Let us study the method by an example.

Ex. : The following table shows the frequency distribution of the time required for each worker to complete a work . From the table find the mean time required to complete the job for a worker.

| Time (Hrs.) for each to complete the work | 15-19 | 20-24 | 25-29 | 30-34 | 35-39 |
|---|-------|-------|-------|-------|-------|
| No. of workers | 10 | 15 | 12 | 8 | 5 |

Solution :

- (1) Vertical columns are drawn as shown in the table.
- (2) Classes are written in the first column.
- (3) The class mark x_i is in the second column.
- (4) In the third column, the number of workers, that is frequency (f_i) is written.
- (5) In the fourth column, the product ($x_i \times f_i$) for each class is written.
- (6) Then $\sum_{i=1}^N x_i f_i$ is written.
- (7) The mean is found using the formula

| Class (Time-hours) | Class mark x_i | Frequency (No. of Workers) f_i | Class mark \times Frequency $x_i f_i$ |
|-----------------------|---------------------|--|---|
| 15-19 | 17 | 10 | 170 |
| 20-24 | 22 | 15 | 330 |
| 25-29 | 27 | 12 | 324 |
| 30-34 | 32 | 8 | 256 |
| 35-39 | 37 | 5 | 185 |
| Total | | $\sum f_i = 50$ | $\sum x_i f_i = 1265$ |

$$\text{Mean} = \bar{X} = \frac{\sum x_i f_i}{N} = \frac{1265}{50} = 25.3 \quad \because \sum f_i = N$$

The mean time required to complete the work for a worker = 25.3 hrs. (Approx)

Solved Examples

Ex. (1) The percentage of marks of 50 students in a test is given in the following table.
Find the mean of the percentage.

| Percentage of marks | 0-20 | 20-40 | 40-60 | 60-80 | 80-100 |
|---------------------|------|-------|-------|-------|--------|
| No. of students | 3 | 7 | 15 | 20 | 5 |

Solution : The following table is prepared as per steps.

| Class (Percentage of marks) | Class mark x_i | Frequency (No. of students) f_i | Class mark \times frequency $x_i f_i$ |
|--------------------------------|---------------------|---|---|
| 0-20 | 10 | 3 | 30 |
| 20-40 | 30 | 7 | 210 |
| 40-60 | 50 | 15 | 750 |
| 60-80 | 70 | 20 | 1400 |
| 80-100 | 90 | 5 | 450 |
| Total | | $N = \sum f_i = 50$ | $\sum x_i f_i = 2840$ |

$$\begin{aligned} \bar{X} &= \frac{\sum x_i f_i}{\sum f_i} \\ &= \frac{2840}{50} \\ &= 56.8 \\ \therefore \text{The mean of the percentage} &= 56.8 \end{aligned}$$

Ex. (2) The maximum temperatures in °C of 30 towns, in the last summer, is shown in the following table. Find the mean of the maximum temperatures.

| | | | | | |
|--------------|-------|-------|-------|-------|-------|
| Max. temp. | 24-28 | 28-32 | 32-36 | 36-40 | 40-44 |
| No. of towns | 4 | 5 | 7 | 8 | 6 |

Solution :

| Class (Temp. °C) | Class mark x_i | Frequency (No. of towns) f_i | Class mark × frequency $x_i f_i$ |
|---------------------|---------------------|--------------------------------------|-------------------------------------|
| 24-28 | 26 | 4 | 104 |
| 28-32 | 30 | 5 | 150 |
| 32-36 | 34 | 7 | 238 |
| 36-40 | 38 | 8 | 304 |
| 40-44 | 42 | 6 | 252 |
| Total | | $N = \sum f_i = 30$ | $\sum x_i f_i = 1048$ |

$$\text{Mean} = \bar{X} = \frac{\sum x_i f_i}{\sum f_i} = \frac{1048}{30} = 34.9^\circ\text{C}$$

Assumed mean method

In the examples solved above, we see that some times the product $x_i f_i$ is large. Hence it becomes difficult to calculate the mean by direct method. So let us study another method, called the 'assumed mean method'. Finding the mean becomes simpler if we use addition and division in this method.

For example, we have to find the mean of the scores 40, 42, 43, 45, 47 and 48.

The observation of the scores reveals that the mean of the data is more than 40. So let us assume that the mean is 40. $40-40 = 0$, $42 - 40 = 2$, $43-40 = 3$, $45-40 = 5$, $47 - 40 = 7$, $48 - 40 = 8$ These are called 'deviations'. Let us find their mean. Adding this mean to the assumed mean, we get the mean of the data.

That is, mean = assumed mean + mean of the deviations

$$\bar{X} = 40 + \left(\frac{0+2+3+5+7+8}{6} \right) = 40 + \frac{25}{6} = 40 + 4\frac{1}{6} = 44\frac{1}{6}$$

Using the symbols-

A- for assumed mean; d - for deviation and \overline{d} - for the mean of the deviations, the formula for mean of the given data can be briefly written as $\overline{X} = A + \overline{d}$.

Let us solve the same example taking 43 as assumed mean. For this, let us find the deviations by subtracting 43 from each score.

$$40 - 43 = -3, 42 - 43 = -1, 43 - 43 = 0, 45 - 43 = 2, 47 - 43 = 4, 48 - 43 = 5$$

$$\text{The sum of the deviations} = -3 -1 + 0 + 2 + 4 + 5 = 7$$

$$\text{Now, } \overline{X} = A + \overline{d}$$

$$= 43 + \left(\frac{7}{6}\right) \quad (\text{as the number of deviations is 6})$$

$$= 43 + 1\frac{1}{6} = 44\frac{1}{6}$$

Note that; use of assumed mean method reduces the work of calculations.

Also note that; taking any score, or any other convenient number as assumed mean does not change the mean of the data.

Ex. : The daily sale of 100 vegetable vendors is given in the following table. Find the mean of the sale by assumed mean method.

| Daily sale (Rupees) | 1000-1500 | 1500-2000 | 2000-2500 | 2500-3000 |
|---------------------|-----------|-----------|-----------|-----------|
| No. of vendors | 15 | 20 | 35 | 30 |

Solution : Assumed mean = A = 2250, $d_i = x_i - A$ is the deviation.

| Class Daily sale (Rupees) | Class mark x_i | $d_i = x_i - A$ $= x_i - 2250$ | Frequency (No. of vendors) f_i | Frequency \times deviation $f_i d_i$ |
|------------------------------|------------------------|-----------------------------------|--|--|
| 1000-1500 | 1250 | -1000 | 15 | -15000 |
| 1500-2000 | 1750 | -500 | 20 | -10000 |
| 2000-2500 | 2250 \rightarrow A | 0 | 35 | 0 |
| 2500-3000 | 2750 | 500 | 30 | 15000 |
| Total | | | $N = \sum f_i = 100$ | $\sum f_i d_i = -10000$ |

The table is prepared according to the following steps :-

- (1) Assumed mean, A is chosen as 2250. (Generally, the class mark of the class having maximum frequency is chosen as the assumed mean.)
 - (2) Classes of sale are written in the first column.
 - (3) Class marks are written in the second column.
 - (4) Values of $d_i = x_i - A = x_i - 2250$ are written in the third column.
 - (5) In the fourth column, the number of vendors and their sum is written as $\sum f_i$.
 - (6) In the fifth column, the product $(f_i \times d_i)$ and their sum is written as $\sum f_i d_i$.
- \bar{d} and \bar{X} are calculated using the formulae.

$$\bar{d} = \frac{\sum f_i d_i}{\sum f_i} = -\frac{10000}{100} = -100 \quad \therefore \text{mean } \bar{X} = A + \bar{d} = 2250 - 100 = 2150$$

The mean of sale is ₹ 2150.

Activity :- Solve the above example by direct method.

Solved Examples

Ex. (1) The following table shows the frequency table of daily wages of 50 workers in a trading company. Find the mean wages of a worker, by assumed mean method.

| Daily Wages (Rs) | 200-240 | 240-280 | 280-320 | 320-360 | 360-400 |
|----------------------------|---------|---------|---------|---------|---------|
| Frequency (No. of workers) | 5 | 10 | 15 | 12 | 8 |

Solution : Let us take the assumed mean $A = 300$.

| Class (₹ Wage) | Class mark x_i | $d_i = x_i - A$ $d_i = x_i - 300$ | Frequency (No. of workers) f_i | Frequency \times Deviation $f_i d_i$ |
|----------------|---------------------|--------------------------------------|----------------------------------|--|
| 200-240 | 220 | -80 | 5 | -400 |
| 240-280 | 260 | -40 | 10 | -400 |
| 280-320 | 300 $\rightarrow A$ | 0 | 15 | 0 |
| 320-360 | 340 | 40 | 12 | 480 |
| 360-400 | 380 | 80 | 8 | 640 |
| Total | | | $\sum f_i = 50$ | $\sum f_i d_i = 320$ |

$$\overline{d} = \frac{\sum f_i d_i}{\sum f_i} = \frac{320}{50} = 6.4$$

$$\begin{aligned}\text{Mean, } \overline{X} &= A + \overline{d} \\ &= 300 + 6.4 \\ &= 306.40\end{aligned}$$

The mean of daily wages = 306.40 ₹

Step deviation method

We studied the direct method and assumed mean method to find the mean. Now we study one more method which reduces the calculations still further.

- Find the values of d_i as $d_i = x_i - A$ and write in the column.
- If we can find g , the G.C.D. of all d_i easily, we create a column for all u_i where $u_i = \frac{d_i}{g}$
- Find the mean \overline{u} of all u_i .
- Using the formula $\overline{X} = A + \overline{u} g$, find the mean of the data.

Example : The amount invested in health insurance by 100 families is given in the following frequency table. Find the mean of investments using step deviation method.

| Amount invested (₹) | 800-1200 | 1200-1600 | 1600-2000 | 2000-2400 | 2400-2800 | 2800-3200 |
|---------------------|----------|-----------|-----------|-----------|-----------|-----------|
| No. of families | 3 | 15 | 20 | 25 | 30 | 7 |

Solution : Assumed mean $A = 2200$ observing all ' d_i 's $g = 400$.

| Class (Insurance ₹) | Class Mark x_i | $d_i = x_i - A$ $= x_i - 2200$ | $u_i = \frac{d_i}{\mathcal{J}}$ | Frequency (No. of families) f_i | $f_i u_i$ |
|------------------------|------------------------|-----------------------------------|---------------------------------|--|----------------------|
| 800–1200 | 1000 | –1200 | –3 | 3 | –9 |
| 1200–1600 | 1400 | –800 | –2 | 15 | –30 |
| 1600–2000 | 1800 | –400 | –1 | 20 | –20 |
| 2000–2400 | 2200 → A | 0 | 0 | 25 | 0 |
| 2400–2800 | 2600 | 400 | 1 | 30 | 30 |
| 2800–3200 | 3000 | 800 | 2 | 7 | 14 |
| Total | | | | $\sum f_i = 100$ | $\sum f_i u_i = -15$ |

The above table is made using the following steps.

- (1) The classes of investment are written in the first column.
- (2) The values of x_i are written in the second column.
- (3) The values of $d_i = x_i - A$ are written in the third column.
- (4) The G.C.D of all values of d_i is 400. Therefore $\mathcal{J} = 400$.
- (5) The corresponding frequencies are written in the fifth column.
- (6) The product $f_i \times u_i$ for each class is written in the sixth column.

The mean of u_i is found by the following formula.

$$\overline{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{-15}{100} = -0.15$$

$$\overline{X} = A + \overline{u} \mathcal{J}$$

$$= 2200 + (-0.15) (400)$$

$$= 2200 + (-60.00)$$

$$= 2200 - 60 = 2140$$

∴ The mean of investments in health insurance = ₹ 2140.

Activity : Solve the above example by direct method and by assumed mean method and see that the mean found by any method is the same.

Solved Example

Ex. (1) The following table shows the funds collected by 50 students for flood affected people. Find the mean of the funds.

| Fund (Rupees) | 0-500 | 500-1000 | 1000-1500 | 1500-2000 | 2000-2500 | 2500-3000 |
|-----------------|-------|----------|-----------|-----------|-----------|-----------|
| No. of students | 2 | 4 | 24 | 18 | 1 | 1 |

If the number of scores in two consecutive classes is very low, it is convenient to club them. So, in the above example, we club the classes 0 - 500, 500 - 1000 and 2000 - 2500, 2500 - 3000. Now the new table is as follows

| Fund (Rupees) | 0-1000 | 1000-1500 | 1500-2000 | 2000-3000 |
|-----------------|--------|-----------|-----------|-----------|
| No. of students | 6 | 24 | 18 | 2 |

Solution : Let $A = 1250$, examining all d_i , $g = 250$.

| Class Fund (₹) | Class mark x_i | $d_i = x_i - A = x_i - 1250$ | $u_i = \frac{d_i}{g}$ | Frequency f_i | $f_i u_i$ |
|----------------|------------------|------------------------------|-----------------------|-----------------|---------------------|
| 0-1000 | 500 | -750 | -3 | 6 | -18 |
| 1000-1500 | 1250 → A | 0 | 0 | 24 | 0 |
| 1500 - 2000 | 1750 | 500 | 2 | 18 | 36 |
| 2000-3000 | 2500 | 1250 | 5 | 2 | 10 |
| Total | | | | $\sum f_i = 50$ | $\sum f_i u_i = 28$ |

$$\bar{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{28}{50} = 0.56,$$

$$\bar{u} g = 0.56 \times 250 = 140$$

$$\bar{X} = A + g \bar{u} = 1250 + 140 = 1390$$

∴ the average of the funds is ₹ 1390.

Activity -

1. Solve the above example by direct method.
2. Verify that the mean calculated by assumed mean method is the same.
3. Find the mean in the above example by taking $A = 1750$.

Practice Set 6.1

1. The following table shows the number of students and the time they utilized daily for their studies. Find the mean time spent by students for their studies by direct method.

| | | | | | |
|-----------------|-----|-----|-----|-----|------|
| Time (hrs.) | 0-2 | 2-4 | 4-6 | 6-8 | 8-10 |
| No. of students | 7 | 18 | 12 | 10 | 3 |

2. In the following table, the toll paid by drivers and the number of vehicles is shown. Find the mean of the toll by 'assumed mean' method.

| | | | | | |
|-----------------|---------|---------|---------|---------|---------|
| Toll (Rupees) | 300-400 | 400-500 | 500-600 | 600-700 | 700-800 |
| No. of vehicles | 80 | 110 | 120 | 70 | 40 |

3. A milk centre sold milk to 50 customers. The table below gives the number of customers and the milk they purchased. Find the mean of the milk sold by direct method.

| | | | | | |
|-------------------|-----|-----|-----|-----|-----|
| Milk Sold (Litre) | 1-2 | 2-3 | 3-4 | 4-5 | 5-6 |
| No. of Customers | 17 | 13 | 10 | 7 | 3 |

4. A frequency distribution table for the production of oranges of some farm owners is given below. Find the mean production of oranges by 'assumed mean' method.

| | | | | | |
|------------------------------|-------|-------|-------|-------|-------|
| Production (Thousand rupees) | 25-30 | 30-35 | 35-40 | 40-45 | 45-50 |
| No. of farm owners | 20 | 25 | 15 | 10 | 10 |

5. A frequency distribution of funds collected by 120 workers in a company for the drought affected people are given in the following table. Find the mean of the funds by 'step deviation' method.

| | | | | | |
|----------------|-------|----------|-----------|-----------|-----------|
| Fund (Rupees) | 0-500 | 500-1000 | 1000-1500 | 1500-2000 | 2000-2500 |
| No. of workers | 35 | 28 | 32 | 15 | 10 |

6. The following table gives the information of frequency distribution of weekly wages of 150 workers of a company. Find the mean of the weekly wages by 'step deviation' method.

| | | | | |
|-----------------------|-----------|-----------|-----------|-----------|
| Weekly wages (Rupees) | 1000-2000 | 2000-3000 | 3000-4000 | 4000-5000 |
| No. of workers. | 25 | 45 | 50 | 30 |



Let's recall.

There was a science exhibition in a city for two days. A school sent two boys and two girls to participate in the exhibition. There were ten hotels, within a distance of one kilometer, from the venue of exhibition. Their rates of meals, in the ascending order were rupees 40, 45, 60, 65, 70, 80, 90, 100 and 500. They had to choose one of them for the dinner.

The average of rates in all the hotels was ₹ $\frac{1130}{10} = 113$.

Which hotel do you think they chose? Except the rate ₹ 500, all others were less than ₹ 113. The students decided to choose a hotel having medium rate. The first day they chose the hotel with rate ₹ 70 and on the next day, the hotel with the rate ₹ 80/-.

This example shows that sometimes the median is used instead of the mean.

In the previous standard we have studied the concept of a median.

- If the numbers in a data are arranged in the ascending order, the number at the middle position is called the median of the data.
- The median divides the array of numbers in two equal parts, that is the number of scores below and above the median is equal.
- The scores are written as $k_1 \leq k_2 \leq k_3 \dots \dots \leq k_n$.
- If the number of scores is odd, then the $\frac{n+1}{2}$ th score is the median of the data. That is, the number of scores below as well as above $k_{\frac{n+1}{2}}$ is $\frac{n-1}{2}$; verify the fact by taking $n = 2m + 1$.
- If the number of the scores is even, then the mean of the middle two terms is the median. This is because the number of terms below $k_{\frac{n}{2}}$ and above $k_{\frac{n+2}{2}}$ is equal, which is $\frac{n-2}{2}$. Verify this by taking $n = 2m$.
- Hence the mean of $\frac{n}{2}$ th and $\frac{n+2}{2}$ th term is the median of the data.

Ex. (1) In 32, 33, 38, 40, 43, 48, 50; the fourth number is at the middle. Hence the median of the data is 40

Ex. (2) In 61, 62, 65, 66, 68, 70, 74, 75 ; the number of scores is 8, that is even.

Therefore, the fourth and the fifth numbers are at the middle, which are 66 and 68. Hence the median = $\frac{66+68}{2} = 67$



Let's learn.

Median for grouped frequency distribution

When the number of scores in a data is large, it is difficult to arrange them in ascending order. In such case, the data is divided into groups. So let us study, with an example, how the median of grouped data is found.

Ex. The scores 6, 8, 10.4, 11, 15.5, 12, 18 are grouped in the following table.

| Class | Tally Marks | Frequency |
|-------|-------------|-----------|
| 6-10 | | 2 |
| 11-15 | | 2 |
| 16-20 | | 1 |

| Class | Tally Marks | Frequency |
|-----------|-------------|-----------|
| 5.5-10.5 | | 3 |
| 10.5-15.5 | | 2 |
| 15.5-20.5 | | 2 |

We could not record the scores 10.4 and 15.5 in the first table, as they cannot be placed in any of the classes 6-10, 11-15, 16-20. We know that in such a case the classes are made continuous.

For this, in the first table, the lower class limits are reduced by 0.5 and the upper class limits are increased by 0.5 and the second table is prepared. In the second table, the score 15.5 is placed in the class 15.5-20.5.

Note that if the method of making groups is changed, the frequency distribution may change.



Let's remember!

In the above table, the class mark of 6-10 is $= \frac{6+10}{2} = \frac{16}{2} = 8$;

Similarly, the class mark of 5.5-10.5 is $= \frac{5.5+10.5}{2} = \frac{16}{2} = 8$.

This shows that, if the classes are made continuous, the class marks do not change

Solved Example :

The following table shows frequency distribution of marks of 100 students of 10th class which they obtained in a practice examination. Find the median of the marks.

| Marks in exam | 0-20 | 20-40 | 40-60 | 60-80 | 80-100 |
|-----------------|------|-------|-------|-------|--------|
| No. of students | 4 | 20 | 30 | 40 | 6 |

Solution : $N = 100$

$\therefore \frac{N}{2} = 50$. Hence the 50th number will be the approximate median. Hence we have to find out the class which contains the 50th term. Writing the cumulative frequencies less than the upper limit, we can find it.

So, let us prepare less than cumulative frequency distribution table.

| Class (Student's marks) | No. of students f_i | Cumulative frequency less than the upper limit cf |
|-----------------------------|--------------------------|--|
| 0-20 | 4 | 4 |
| 20-40 | 20 | 24 |
| 40-60 | 30 | 54 |
| 60-80 | 40 | 94 |
| 80-100 | 6 | 100 |

- From the table, the 50th score is in the class 40-60. The class which contains the median, is called the **median class**. So, here 40-60 is the median class.
- The lower class limit of 40-60 is 40. Its frequency is 30.
- Out of the first 50 scores, 24 scores are less than 40. The remaining $50 - 24 = 26$ are in class (40-60). The 50th score in that class is estimated as follows.
- 26 out of 30 scores in the class 40-60, are upto the 50th score and the class interval is 20. So it is assumed that, the 50th score is more than 40 by $\frac{26}{30} \times 20$.

$$\therefore \text{it is approximately } 40 + \frac{26}{30} \times 20 = 40 + \frac{52}{3} = 57\frac{1}{3}.$$

$$\therefore \text{median} = 57\frac{1}{3}$$

- We can formulate this as follows,

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f} \right] \times h$$

In the formula,

L = Lower class limit of the median class,

N = Sum of frequencies

h = Class interval of the median class,

f = Frequency of the median class

cf = Cumulative frequency of the class preceding the median class.

In the above example; $\frac{N}{2} = 50$, $cf = 24$, $h = 20$, $f = 30$, $L = 40$,

$$\begin{aligned} \text{Median} &= L + \left[\frac{\frac{N}{2} - cf}{f} \right] \times h \quad \text{(Formula)} \\ &= 40 + \left(\frac{50 - 24}{30} \right) \times 20 \\ &= 40 + \frac{26 \times 20}{30} \\ &= 40 + 17\frac{1}{3} \\ &= 57\frac{1}{3} \end{aligned}$$



Let's remember!

- ◆ If the given classes are not continuous, we have to make them continuous to find out the median.
- ◆ It is difficult to write the scores in the ascending order when the number of scores is large. So the data is classified into groups. It is not possible to find the exact median of a classified data, but the approximate median is found by the formula.

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f} \right] \times h$$

Solved Examples

Ex. (1) Observe the following frequency distribution table. It shows the distances travelled by 60 public transport buses in a day. Find the median of the distance travelled.

| Daily distance travelled (in Km) | 200–209 | 210–219 | 220–229 | 230–239 | 240–249 |
|----------------------------------|---------|---------|---------|---------|---------|
| No. of buses | 4 | 14 | 26 | 10 | 6 |

Solution : (1) The classes in the table are not continuous.

The upper class limit of a class and the lower class of its succeeding class differ by 1.

∴ Let us subtract $1 \div 2 = 0.5$ from the lower class limit of each class and add to the upper class limit of each class, and make the classes continuous.

(2) Make a column of cumulative frequency 'less than' in the new table showing the continuous classes.

| Given Class | Continuous classes | Frequency f_i | Cumulative frequency less than |
|-------------|--------------------|--------------------|--------------------------------|
| 200–209 | 199.5–209.5 | 4 | 4 |
| 210–219 | 209.5–219.5 | 14 | 18 $\rightarrow cf$ |
| 220–229 | 219.5–229.5 | 26 $\rightarrow f$ | 44 |
| 230–239 | 229.5–239.5 | 10 | 54 |
| 240–249 | 239.5–249.5 | 6 | 60 |

Here, total of frequencies = $\sum f_i = N = 60$ ∴ $\frac{N}{2} = 30$.

∴ 30th score is the approximate median.

First 18 scores are less than 219.5 and the remaining, $30 - 18 = 12$ scores are in the class 219.5 – 229.5. Therefore, 219.5 – 229.5 is the median class. The cumulative frequency of the class 219.5–229.5 is 44.

In the formula,

L = Lower class limit = 219.5, h = Class interval of the median class = 10

cf = The frequency of the class preceding the median class = 18,

f = The frequency of the median class = 26

$$\text{Median} = L + \left[\frac{\frac{N}{2} - cf}{f} \right] \times h$$

$$\begin{aligned}
 \therefore \text{Median} &= 219.5 + \left(\frac{30-18}{26} \right) \times 10 \\
 &= 219.5 + \left(\frac{12 \times 10}{26} \right) \\
 &= 219.50 + 4.62 \\
 &= 224.12
 \end{aligned}$$

\therefore The median of the distance travelled is = 224.12 Km

Ex. (2) The following table shows the ages of persons who visited a museum on a certain day. Find the median age of the persons visiting the museum.

| Age (Years) | No. of persons |
|--------------|----------------|
| Less than 10 | 3 |
| Less than 20 | 10 |
| Less than 30 | 22 |
| Less than 40 | 40 |
| Less than 50 | 54 |
| Less than 60 | 71 |

Solution : The given cumulative frequency table is of the 'less than' form. So, we will have to decide the true class limits first. We know that, the 'less than' cumulative frequency is associated with the upper class limits. The upper class limit of the first class is 10. The age of any person is a positive number, so the first class must be 0-10. The upper class limit of the next class is 20, so the second class must be 10-20. In this way, make the classes of interval 10. In this way the last class is 50-60. So the given table can now be rewritten as follows.

| Age (years) | Class | No. of persons (Frequency) | Cumulative frequency Less than |
|--------------|-------|-------------------------------|-----------------------------------|
| Less than 10 | 0-10 | 3 | 3 |
| Less than 20 | 10-20 | $10 - 3 = 7$ | 10 |
| Less than 30 | 20-30 | $22 - 10 = 12$ | $22 \rightarrow cf$ |
| Less than 40 | 30-40 | $40 - 22 = 18 \rightarrow f$ | 40 |
| Less than 50 | 40-50 | $54 - 40 = 14$ | 54 |
| Less than 60 | 50-60 | $71 - 54 = 17$ | 71 |

Here $N = 71 \therefore \frac{N}{2} = 35.5$ and $h = 10$

The number 35.5 is in the class 30–40, hence it is the median class. The cumulative frequency of its preceding class is 22, $\therefore cf = 22, L = 30, f = 18$.

$$\begin{aligned}\text{Median} &= L + \left[\frac{\frac{N}{2} - cf}{f} \right] \times h \\ &= 30 + (35.5 - 22) \frac{10}{18} \\ &= 30 + (13.5) \frac{10}{18} \\ &= 30 + 7.5 \\ &= 37.5\end{aligned}$$

\therefore the median age of the persons visiting the museum is = 37.5 years

Practice Set 6.2

- The following table shows classification of number of workers and the number of hours they work in a software company. Find the median of the number of hours they work.

| | | | | |
|--------------------|------|-------|-------|-------|
| Daily No. of hours | 8–10 | 10–12 | 12–14 | 14–16 |
| Number of workers | 150 | 500 | 300 | 50 |

- The frequency distribution table shows the number of mango trees in a grove and their yield of mangoes. Find the median of data.

| | | | | | |
|----------------|--------|---------|---------|---------|---------|
| No. of Mangoes | 50–100 | 100–150 | 150–200 | 200–250 | 250–300 |
| No. of trees | 33 | 30 | 90 | 80 | 17 |

- The following table shows the classification of number of vehicles and their speeds on Mumbai-Pune express way. Find the median of the data.

| | | | | | | |
|----------------------------------|-------|-------|-------|-------|-------|-------|
| Average Speed of Vehicles(Km/hr) | 60–64 | 64–69 | 70–74 | 75–79 | 79–84 | 84–89 |
| No. of vehicles | 10 | 34 | 55 | 85 | 10 | 6 |

4. The production of electric bulbs in different factories is shown in the following table. Find the median of the productions.

| No. of bulbs produced (Thousands) | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 | 80-90 | 90-100 |
|-----------------------------------|-------|-------|-------|-------|-------|-------|--------|
| No. of factories | 12 | 35 | 20 | 15 | 8 | 7 | 8 |



Let's learn.

Mode for grouped frequency distribution

We know that the score repeating maximum number of times in a data is called the mode of the data.

For example, a company manufactures bicycles of different colours. To know which colour is most wanted, the company needs to know the mode. If a company manufactures many items, it may want to know which item sells most. In such cases, the mode is needed.

We have learnt the method of finding the mode of an ungrouped data.

Now let us study the method of estimation of mode of grouped data.

The following formula is used for the purpose.

$$\text{Mode} = L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h$$

In the above formula,

L = Lower class limit of the modal class.

f_1 = Frequency of the modal class.

f_0 = Frequency of the class preceding the modal class.

f_2 = Frequency of the class succeeding the modal class.

h = Class interval of the modal class.

Let us see, with an example, how the mode is estimated using the above formula.

Solved Examples

Ex.(1) The classification of children according to their ages, playing on a ground is shown in the following table. Find the mode of ages of the children.

| Age-group of children (Yrs) | 6-8 | 8-10 | 10-12 | 12-14 | 14-16 |
|--------------------------------|-----|----------------------|----------------------|----------------------|-------|
| No. of children | 43 | $58 \rightarrow f_0$ | $70 \rightarrow f_1$ | $42 \rightarrow f_2$ | 27 |

From the table, we note that the maximum number of children is of the age-group 10-12. So the modal class is 10-12.

Solution : Here $f_1 = 70$, and modal class is 10-12.

\therefore in the given example,

L = Lower class limit of the modal class = 10

h = Class interval of the modal class = 2

f_1 = Frequency of the modal class = 70

f_0 = Frequency of the class preceding the modal class = 58

f_2 = Frequency of the class succeeding the modal class = 42

$$\begin{aligned}
 \text{Mode} &= L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h \\
 &= 10 + \left[\frac{70 - 58}{2(70) - 58 - 42} \right] \times 2 \\
 &= 10 + \left[\frac{12}{140 - 100} \right] \times 2 \\
 &= 10 + \left[\frac{12}{40} \right] \times 2 \\
 &= 10 + \frac{24}{40} \\
 &= 10 + 0.6 \\
 &= 10.6
 \end{aligned}$$

\therefore the mode of the ages of children playing on the ground is 10.6 Years.

Ex. (2) The following frequency distribution table shows the classification of the number of vehicles and the volume of petrol filled in them. Find the mode of the volume.

| | | | | | |
|-----------------------|-----|-----|-----|-------|-------|
| Petrol filled (Litre) | 1-3 | 4-6 | 7-9 | 10-12 | 13-15 |
| No. of vehicle | 33 | 40 | 27 | 18 | 12 |

Solution : The given classes are not continuous. So, let us make them continuous and rewrite the table.

| Class | Continuous classes | Frequency |
|-------|--------------------|----------------------|
| 1-3 | 0.5-3.5 | $33 \rightarrow f_0$ |
| 4-6 | 3.5-6.5 | $40 \rightarrow f_1$ |
| 7-9 | 6.5-9.5 | $27 \rightarrow f_2$ |
| 10-12 | 9.5-12.5 | 18 |
| 13-15 | 12.5-15.5 | 12 |

From the above table, the modal class is 3.5-6.5

$$\begin{aligned}
 \text{Mode} &= L + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h \\
 \text{Mode} &= 3.5 + \left[\frac{40 - 33}{2(40) - 33 - 27} \right] \times h \\
 &= 3.5 + \left[\frac{7}{80 - 60} \right] \times 3 \\
 &= 3.5 + \frac{21}{20} \\
 &= 3.5 + 1.05 \\
 &= 4.55
 \end{aligned}$$

\therefore The mode of the volume of petrol filled is = 4.55 litre.

Practice Set 6.3

1. The following table shows the information regarding the milk collected from farmers on a milk collection centre and the content of fat in the milk, measured by a lactometer. Find the mode of fat content.

| Content of fat (%) | 2-3 | 3-4 | 4-5 | 5-6 | 6-7 |
|------------------------|-----|-----|-----|-----|-----|
| Milk collected (Litre) | 30 | 70 | 80 | 60 | 20 |

2. Electricity used by some families is shown in the following table. Find the mode for use of electricity.

| Use of electricity (Unit) | 0-20 | 20-40 | 40-60 | 60-80 | 80-100 | 100-120 |
|---------------------------|------|-------|-------|-------|--------|---------|
| No. of families | 13 | 50 | 70 | 100 | 80 | 17 |

3. Grouped frequency distribution of supply of milk to hotels and the number of hotels is given in the following table. Find the mode of the supply of milk.

| Milk (Litre) | 1-3 | 3-5 | 5-7 | 7-9 | 9-11 | 11-13 |
|---------------|-----|-----|-----|-----|------|-------|
| No. of hotels | 7 | 5 | 15 | 20 | 35 | 18 |

4. The following frequency distribution table gives the ages of 200 patients treated in a hospital in a week. Find the mode of ages of the patients.

| Age (years) | Less than 5 | 5-9 | 10-14 | 15-19 | 20-24 | 25-29 |
|-----------------|-------------|-----|-------|-------|-------|-------|
| No. of patients | 38 | 32 | 50 | 36 | 24 | 20 |

Activity :-

- Find the mean weight of 20 students in your class.
- Find the mode of sizes of shirts of students in your class
- Every student in your class should measure his/her own pulse rate, note the pulse rates of all students and find the mode of the pulse rate.
- Measure the height of every student in the class, prepare a grouped frequency distribution table and find the median of the heights.



Let's remember!

We have studied the central tendencies mean, median and mode. Before selecting any of these measures, we have to know the purpose of its selection clearly.

Suppose, we have to judge the performance of five divisions of standard 10 in the internal examination. For the purpose, we have to find the 'mean' of marks of students in each division.

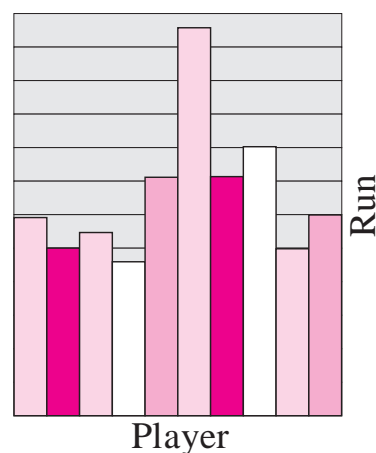
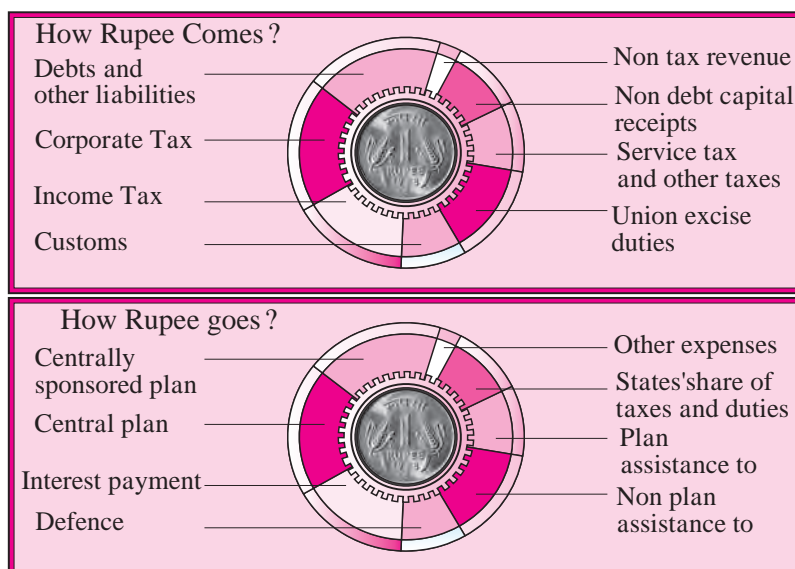
If we have to make two groups of students in a division based on their marks in the examination, we have to find the 'median' of their marks

If a 'bachat' group producing chinks wants to know about the colour of chinks having maximum demand, it will have to choose the 'mode'.

Pictorial representation of statistical data

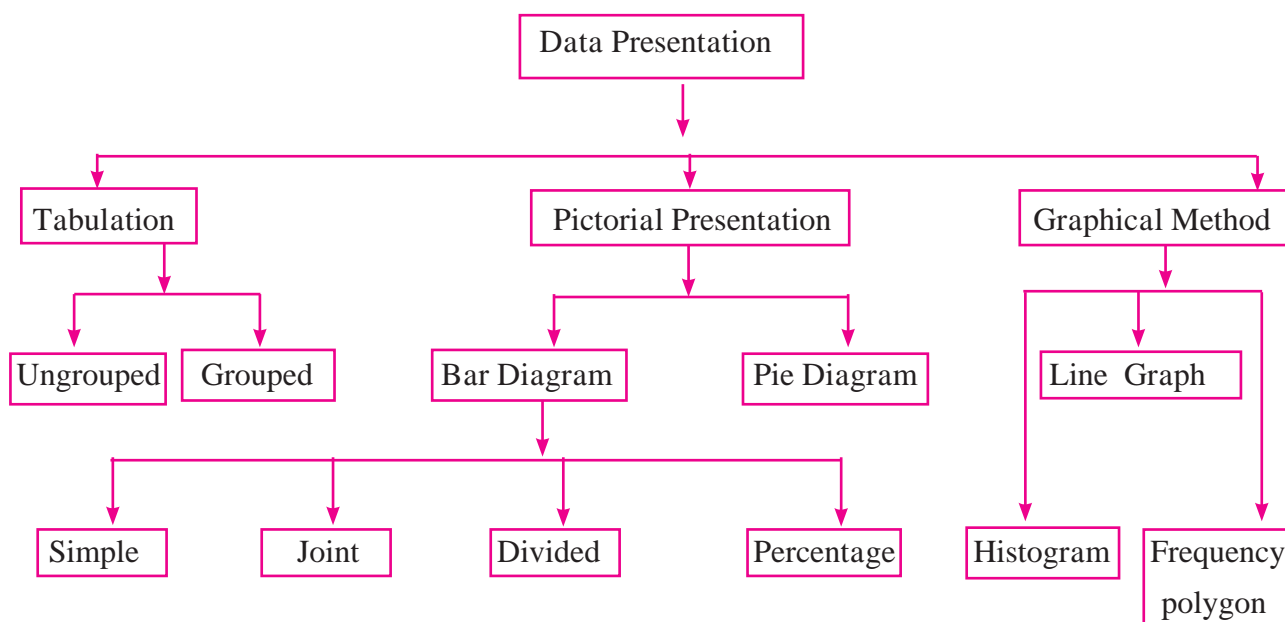
The mean, median or mode of a numerical data or analysis of the data is useful to draw some useful inferences.

We know that tabulation is one of the methods of representing numerical data in brief. But a table does not quickly reveal some aspects of the data. A common man is interested in the important aspects of a data. For example, annual budget, information about a game, etc. Let us think of another way of data representation for the purpose.



Presentation of data

Pictorial and graphical presentation are attractive methods of data interpretation. The tree chart below shows different methods of data interpretation.



We have studied some of these methods and graphs in previous standards. Now we will learn a histogram, a frequency polygon and a pie diagram.

Florence Nightingale (1820-1910) The lady is considered as an idol in the field of nursing. She was devoted to the work of caring for the wounded and the sick. In the crimean war, she nursed wounded soldiers and saved their lives. She is also known for her fundamental work in the field of statistics. She kept a systematic record of the conditions of wounded soldiers, treatments given to them and the results of the treatments and deduced important conclusions. The cause of the death of soldiers was more often a disease like typhoid or cholera and not the wounds in the war. The causes of the diseases were lack of cleanliness of the surrounding, polluted water and crowded dwelling of the patients. Florence exhibited the information in the form of graphs, and pie charts to convince the people. She showed that proper treatments and observing the rules of cleanliness decreases the death rate considerably. The municipalities accepted her observations, that to maintain the hygiene of town, good drainage system and clean drinking water for everyone are necessary. Her work established that systematic records and the statistical methods are useful in drawing reliable inferences.





Let's learn.

Histogram

Study the following example to know about a histogram and how to draw it.

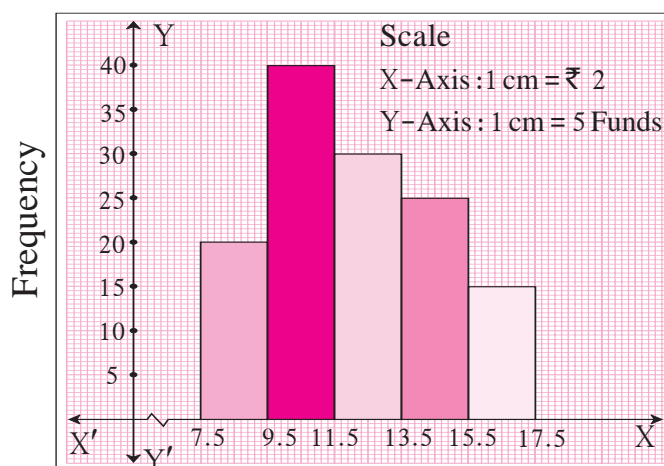
Ex : The table below shows the net asset value (NAV) per unit of mutual funds of some companies.

Draw a histogram representing the information.

| NAV (₹) | 8-9 | 10-11 | 12-13 | 14-15 | 16-17 |
|---------------------|-----|-------|-------|-------|-------|
| No. of mutual funds | 20 | 40 | 30 | 25 | 15 |

Solution : The given classes are not continuous. Lets make the classes continuous.

| Continuous Classes | 7.5-9.5 | 9.5-11.5 | 11.5-13.5 | 13.5-15.5 | 15.5-17.5 |
|--------------------|---------|----------|-----------|-----------|-----------|
| Frequency | 20 | 40 | 30 | 25 | 15 |




Classes
fig 6.1

Method of drawing a histogram :

1. If the given classes are not continuous, make them continuous. Such classes are called extended class intervals.
2. Show the classes on the X- axis with a proper scale.
3. Show the frequencies of the Y- axis with a proper scale.
4. Taking each class as the base, draw rectangles with heights proportional to the frequencies.

Note :

On the X-axis, a mark ‘—’ is called the krink mark and it is shown between the origin and the first class. It means, there are no observations upto the first class. The mark can be used on the Y- axis also, if needed. This enables us to draw a graph of optimum size.

Practice Set 6.4

1. Draw a histogram of the following data.

| | | | | |
|------------------------|---------|---------|---------|---------|
| Height of student (cm) | 135–140 | 140–145 | 145–150 | 150–155 |
| No. of students | 4 | 12 | 16 | 8 |

2. The table below shows the yield of jowar per acre. Show the data by histogram.

| | | | | | |
|--------------------------|-----|-----|-----|-----|-------|
| Yield per acre (quintal) | 2–3 | 4–5 | 6–7 | 8–9 | 10–11 |
| No. of farmers | 30 | 50 | 55 | 40 | 20 |

3. In the following table, the investment made by 210 families is shown. Present it in the form of a histogram.

| | | | | | |
|---------------------------------|-------|-------|-------|-------|-------|
| Investment (Thousand Rupees) | 10–15 | 15–20 | 20–25 | 25–30 | 30–35 |
| No. of families | 30 | 50 | 60 | 55 | 15 |

4. Time allotted for the preparation of an examination by some students is shown in the table. Draw a histogram to show the information.

| | | | | | |
|-----------------|-------|--------|---------|---------|---------|
| Time (minutes) | 60–80 | 80–100 | 100–120 | 120–140 | 140–160 |
| No. of students | 14 | 20 | 24 | 22 | 16 |



Let's learn.

Frequency polygon

The information in a frequency table can be presented in various ways. We have studied a histogram. A frequency polygon is another way of presentation.

Let us study two methods of drawing a frequency polygon.

(1) With the help of a histogram (2) Without the help of a histogram.

(1) We shall use the histogram in figure 6.1 to learn the method of drawing a frequency polygon.

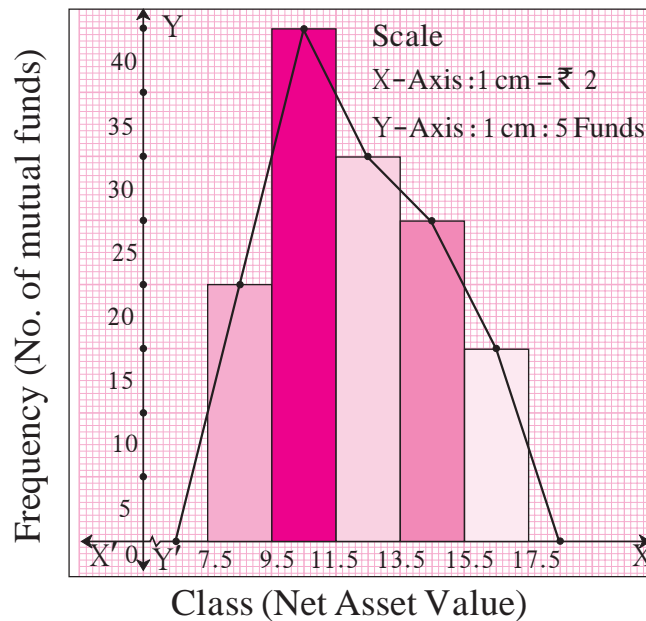


Fig. 6.2

1. Mark the mid-point of upper side of each rectangle in the histogram.
2. Assume that a rectangle of zero height exists preceeding the first rectangle and mark its mid-point. Similarly, assume a rectangle succeeding the last rectangle and mark its mid-point.
3. Join all mid-points in order by line segments.

The closed figure so obtained is the frequency polygon.

- (2) Observe the following table. It shows how the coordinates of points are decided to draw a frequency polygon, without drawing a histogram.

| Class | Continuous class | Class mark | Frequency | Coordinates of points |
|---------|------------------|------------|-----------|-----------------------|
| 6 - 7 | 5.5 - 7.5 | 6.5 | 0 | (6.5, 0) |
| 8 - 9 | 7.5 - 9.5 | 8.5 | 20 | (8.5, 20) |
| 10 - 11 | 9.5 - 11.5 | 10.5 | 40 | (10.5, 40) |
| 12 - 13 | 11.5 - 13.5 | 12.5 | 30 | (12.5, 30) |
| 14 - 15 | 13.5 - 15.5 | 14.5 | 25 | (14.5, 25) |
| 16 - 17 | 15.5 - 17.5 | 16.5 | 15 | (16.5, 15) |
| 18 - 19 | 17.5 - 19.5 | 18.5 | 0 | (18.5, 0) |

The points corresponding to the coordinates in the fifth column are plotted. Joining them in order by line segments, we get a frequency polygon. The polygon is shown in figure 6.3. Observe it.

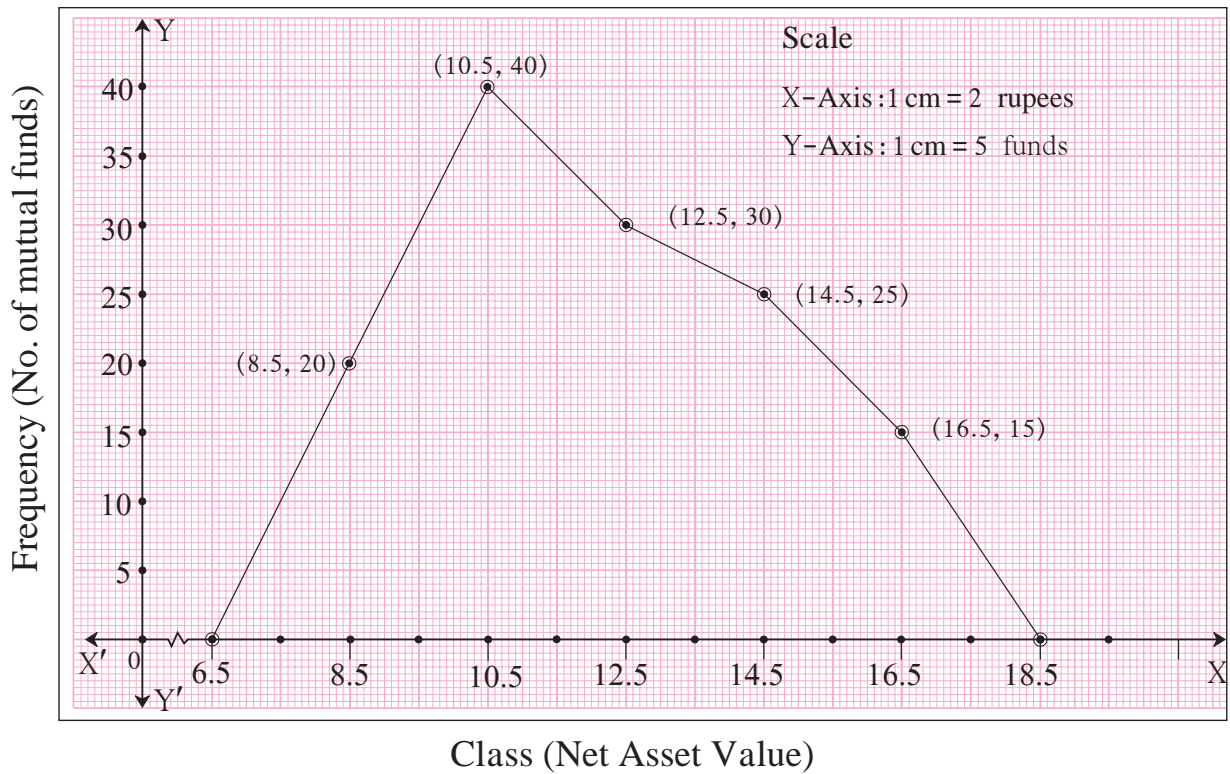


fig 6.3

solved Examples

Ex. (1) Answer the following questions based on the frequency polygon given in the adjacent figure.

- (1) Write frequency of the class 50-60.
- (2) State the class whose frequency is 14.
- (3) State the class whose class mark is 55.
- (4) Write the class in which the frequency is maximum.
- (5) Write the classes whose frequencies are zero.

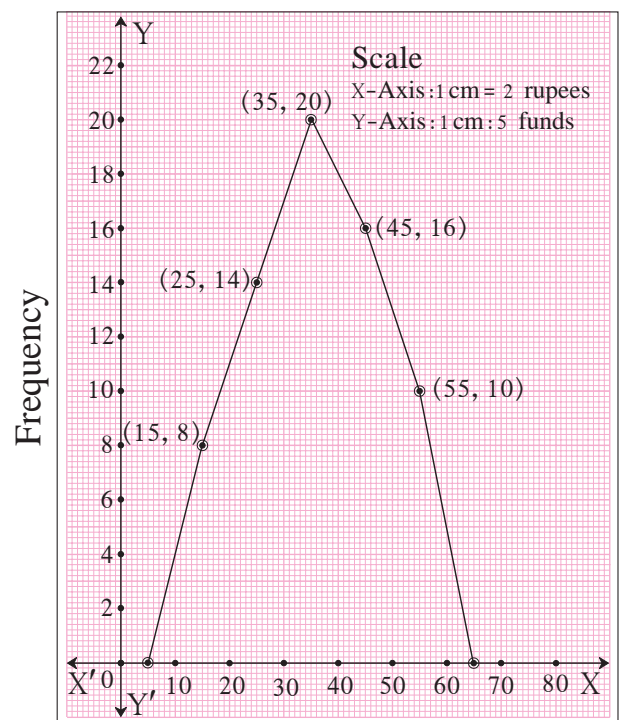


fig 6.4

Solution :

- (1) The class marks are on the X- axis. The point whose x - coordinate is 55 (as the mid- point of the class 50–60 is 55.) y -coordinate is 10. So, the frequency of the class 50–60 is 10.
- (2) The frequencies are shown on the Y-axis. The x -coordinate of the point whose y - coordinate is 14, is 25. Note the mark 14 on the Y- axis . The class mark of the class 20–30 is 25. Hence, the frequency of the class 20–30 is 14.
- (3) The class mark of the class 50–60 is 55.
- (4) The frequency is shown on the Y-axis. On the polygon the maximum value of the y - coordinate is 20. Its corresponding x - coordinate is 35, which is the mark of the class 30–40. Therefore, the maximum frequency is in the class 30–40.
- (5) The frequencies of the classes 0–10 and 60–70 are zero.

Ex. (2) The following table shows the weights of children and the number of children.

Draw a frequency polygon showing the information.

| Weight of children (kg) | 18–19 | 19–20 | 20–21 | 21–22 | 22–23 | 23–24 |
|-------------------------|-------|-------|-------|-------|-------|-------|
| No. of children | 4 | 13 | 15 | 19 | 17 | 6 |

Let us prepare a table showing the co-ordinates necessary to draw a frequency polygon.

| Class | 18–19 | 19–20 | 20–21 | 21–22 | 22–23 | 23–24 |
|-----------------------|-----------|-----------|-----------|-----------|-----------|----------|
| Class mark | 18.5 | 19.5 | 20.5 | 21.5 | 22.5 | 23.5 |
| Frequency | 4 | 13 | 15 | 19 | 17 | 6 |
| Coordinates of points | (18.5, 4) | (19.5,13) | (20.5,15) | (21.5,19) | (22.5,17) | (23.5,6) |

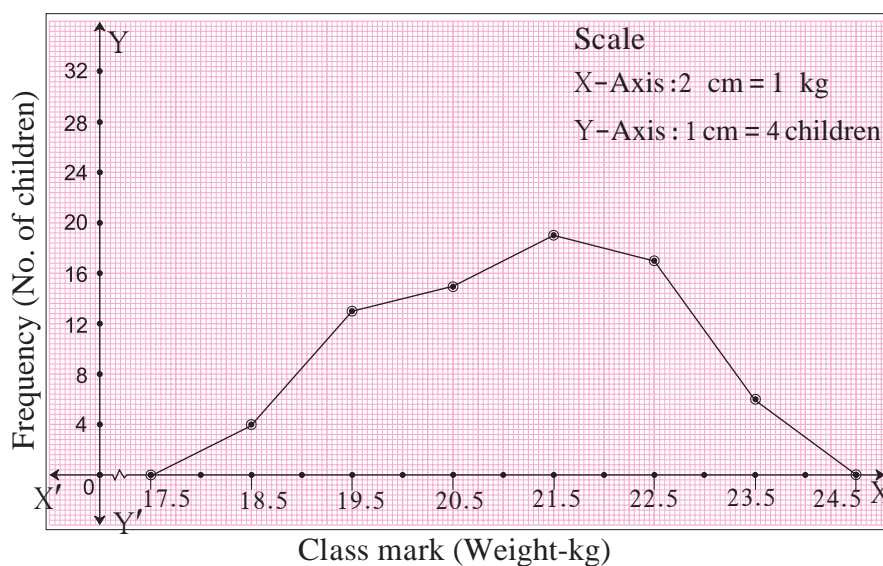


Fig. 6.5

Practice Set 6.5

1. Observe the following frequency polygon and write the answers of the questions below it.

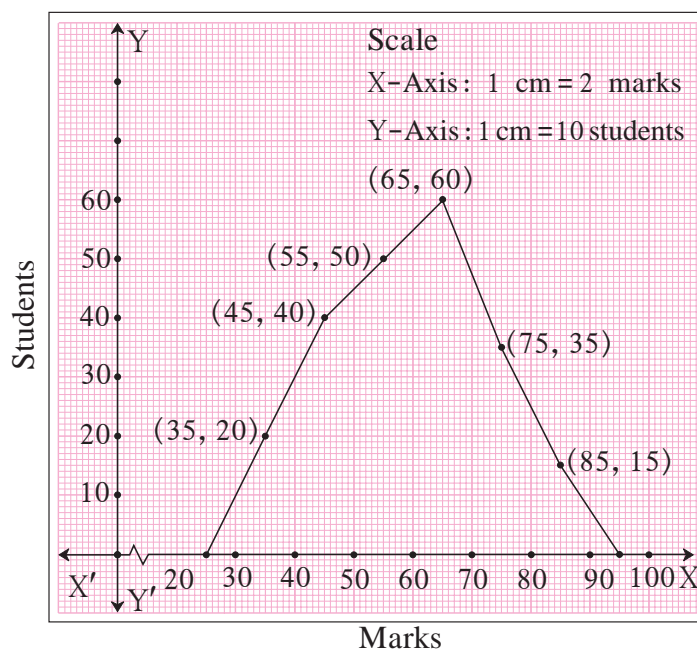


fig 6.6

- (1) Which class has the maximum number of students?
- (2) Write the classes having zero frequency.
- (3) What is the class-mark of the class, having frequency of 50 students?
- (4) Write the lower and upper class limits of the class whose class mark is 85.
- (5) How many students are in the class 80–90?

2. Show the following data by a frequency polygon.

| Electricity bill (₹) | 0–200 | 200–400 | 400–600 | 600–800 | 800–1000 |
|----------------------|-------|---------|---------|---------|----------|
| Families | 240 | 300 | 450 | 350 | 160 |

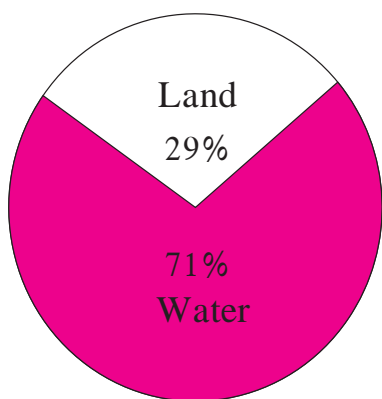
3. The following table shows the classification of percentages of marks of students and the number of students. Draw a frequency polygon from the table.

| Result (Percentage) | 30–40 | 40–50 | 50–60 | 60–70 | 70–80 | 80–90 | 90–100 |
|---------------------|-------|-------|-------|-------|-------|-------|--------|
| No. of students | 7 | 33 | 45 | 65 | 47 | 18 | 5 |

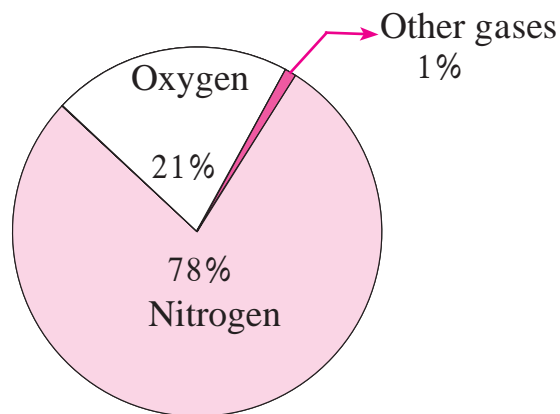


Pie diagram

In the previous standards, we have seen the following figures in Geography and Science. Such graphs are called pie diagrams.



Proportion of land and water on the earth



Proportion of constituents of air

fig 6.7

In a pie diagram, the numerical data is shown in a circle. Different components of a data are shown by proportional sectors of the circle.

In figure 6.8, seg OA and seg OB are radii of a circle with centre O.

$\angle AOB$ is the central angle.

The shaded region O - AXB is a sector of the circle.

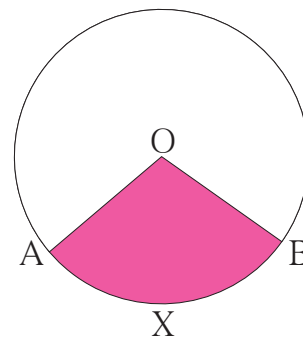


fig 6.8



Let's learn.

Reading of Pie diagram

The following example illustrates how a pie chart gives information at a glance. 120 students of standard 10 were asked which game they like. The information obtained is shown in the adjacent pie diagram. Answers to the question as-

'Which game is liked the most'

'What percentage of students like kho-kho?'

'What percentage of students like kabaddi?'

can be obtained from the pie diagram at a glance.

Observe one more pie diagram.

Figure 6.10 shows the annual financial planning of a school. From the pie diagram we see that

- 45% of the amount is reserved for educational equipment.
- 35% of the amount is shown for games.
- 10% of the amount is kept for sanitation.
- 10% of the amount is reserved for environment.

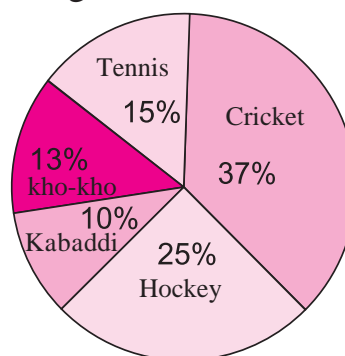


fig 6.9

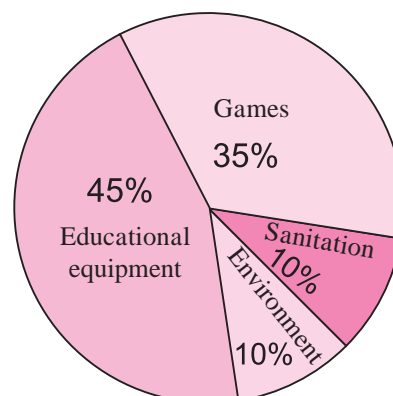


fig 6.10

In this way, we get information at a glance from a pie diagram.

Let us have more information about a pie diagram.

Many times we find information of different types in newspapers given in the form of pie diagrams. For example, the annual budget, performance of different nations in olympic games, etc.

Now we shall see, by examples, how to interpret the information from a pie diagram.

Example :

As deduced from a survey, the classification of skilled workers is shown in the pie diagram (fig 6.11). If the number of workers in the production sector is 4500, answer the following questions.

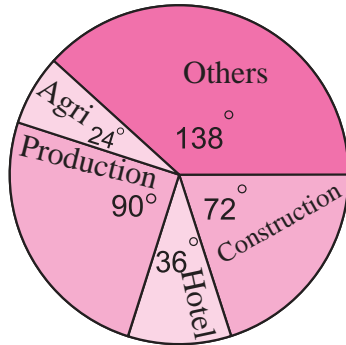


fig 6.11

- (i) What is the total number of skilled workers in all fields?
- (ii) What is the number of skilled workers in the field of constructions?
- (iii) How many skilled workers are in agriculture?
- (iv) Find the difference between the numbers of workers in the field of production and construction.

Solution : (i) Suppose, the total number of skilled workers in all fields is x .

\therefore the central angle for x persons is $= 360^\circ$

Central angle for number of persons in production field

$$= \frac{\text{Number of persons in production field}}{x} \times 360$$

$$\therefore 90 = \frac{4500}{x} \times 360$$

$$\therefore x = 18000$$

\therefore total number of skilled workers in all the fields together $= 18000$.

(ii) The angle shown for construction sector $= 72^\circ$.

$$\therefore 72 = \frac{\text{Number of persons in construction}}{18000} \times 360$$

$$\begin{aligned} \therefore \text{number of persons in construction field} &= \frac{72 \times 18000}{360} \\ &= 3600 \end{aligned}$$

(iii) The central angle for agriculture field is 24° .

$$\therefore 24 = \frac{\text{Number of workers in agriculture}}{\text{total skilled workers}} \times 360$$

$$24 = \frac{\text{Number of workers in agriculture}}{18000} \times 360$$

$$\begin{aligned} \therefore \text{number of workers in agriculture} &= \frac{24 \times 18000}{360} \\ &= 1200 \end{aligned}$$

(iv) The difference between angles relating fields of production and construction
 $= 90^\circ - 72^\circ = 18^\circ$.

\therefore The difference between the central angles =

$$\frac{\text{Difference between numbers of workers in the fields}}{\text{Total number of skilled workers}} \times 360$$

$$18 = \frac{\text{Difference between the numbers of workers in the fields}}{18000} \times 360$$

$$\text{Difference between the numbers of workers in the two fields} = \frac{18 \times 18000}{360}$$

$$= 900$$



Let's remember!

- Every component of a data is shown by a sector associated with it.
- The measure of the central angle of the sector is in proportion with the number of scores in that component.
- The measure of central angle (θ) = $\frac{\text{Number of scores in component}}{\text{Total number of scores}} \times 360^\circ$
- A circle of suitable radius should be drawn. Divide the circle in sectors such that the measure of central angle of each sector is proportional to the number of scores in its corresponding component in the data.



Let's learn.

To draw a Pie diagram

We have seen how to read a pie diagram. Now let us learn to draw it.

1. To draw a pie diagram, the whole circle is divided into sectors proportional to the components of the data
2. The measure of central angle of each sector is found by the following formula.

The measure of central angle of sector θ

$$= \frac{\text{Number of scores in the components}}{\text{Total number of scores}} \times 360$$

A circle of a suitable radius is drawn. Then it is divided into sectors such that, the number of sectors is equal to the number of components in the data.

Let us understand the method through examples.

Solved Examples

Ex. (1) In a bicycle shop, number of bicycles purchased and choice of their colours was as follows. Find the measures of sectors of a circle to show the information by a pie diagram.

Solution : In all 36 bicycles were purchased. Out of them 10 bicycles were white coloured.

∴ the measure of sector showing white coloured bicycles

$$= \frac{\text{Number of white bicycles}}{\text{Total number of bicycles}} \times 360$$

$$= \frac{10}{36} \times 360 = 100$$

The measures of angles of sector relating to bicycles of other colours can be calculated similarly which are shown in the adjacent table.

| Colour | Number of bicycles | Central angle of the sector |
|--------|--------------------|--|
| White | 10 | $\frac{10}{36} \times 360^\circ = 100^\circ$ |
| Black | 9 | $\frac{9}{36} \times 360^\circ = 90^\circ$ |
| Blue | 6 | 60° |
| Grey | 7 | 70° |
| Red | 4 | 40° |
| Total | 36 | 360° |

Ex. (2) The following table shows the daily supply of electricity to different places in a town. Show the information by a pie diagram.

| Places | Factories | Houses | Roads | Shops | Offices | Others |
|--|-----------|--------|-------|-------|---------|--------|
| Supply of electricity (Thousand units) | 24 | 14 | 7 | 5 | 6 | 4 |

Solution : The total supply of electricity is 60,000 units. Let us find the measures of central angles and show in the table.

| Supply of electricity | Unit | Measure of central angle |
|-----------------------|------|--|
| Factories | 24 | $\frac{24}{60} \times 360 = 144^\circ$ |
| Houses | 14 | $\frac{14}{60} \times 360 = 84^\circ$ |
| Roads | 7 | $\frac{7}{60} \times 360 = 42^\circ$ |
| Shops | 5 | $\frac{5}{60} \times 360 = 30^\circ$ |
| Offices | 6 | $\frac{6}{60} \times 360 = 36^\circ$ |
| Others | 4 | $\frac{4}{60} \times 360 = 24^\circ$ |
| Total | 60 | 360° |

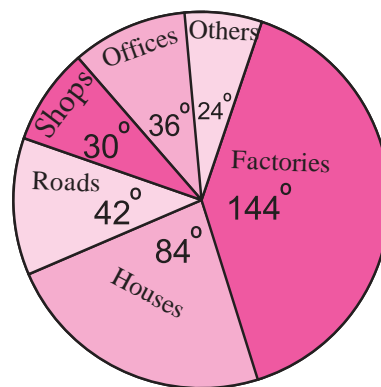


fig 6.12

Steps of drawing pie chart :

- (1) As shown in the figure, a circle and a radius is drawn. Then the sectors having measures of angles in the table, (144° , 84° , 42° , 30° , 36° , and 24°) were drawn one by one, in the clockwise direction. (While drawing the sectors one by one, we can change their order.)
- (2) The components of the data were recorded in the sectors.

Activity :

The monthly expenditure of a family on different items is shown in the following table. Calculate the related central angles and draw a pie chart.

| Different items | Percentage of expenditure | Measure of central angle |
|-----------------|---------------------------|---------------------------------------|
| Food | 40 | $\frac{40}{100} \times 360 = \square$ |
| Cloting | 20 | $\square \times \square = \square$ |
| House rent | 15 | $\square \times \square = \square$ |
| Education | 20 | $\square \times \square = \square$ |
| Expenditure | 05 | $\square \times \square = \square$ |
| Total | 100 | 360° |

Practice Set 6.6

1. The age group and number of persons, who donated blood in a blood donation camp is given below. Draw a pie diagram from it.

| Age group (Yrs) | 20-25 | 25-30 | 30-35 | 35-40 |
|-----------------|-------|-------|-------|-------|
| No. of persons | 80 | 60 | 35 | 25 |

2. The marks obtained by a student in different subjects are shown. Draw a pie diagram showing the information.

| Subject | English | Marathi | Science | Mathematics | Social science | Hindi |
|---------|---------|---------|---------|-------------|----------------|-------|
| Marks | 50 | 70 | 80 | 90 | 60 | 50 |

3. In a tree plantation programme, the number of trees planted by students of different classes is given in the following table. Draw a pie diagram showing the information.

| Standard | 5 th | 6 th | 7 th | 8 th | 9 th | 10 th |
|--------------|------|------|------|------|------|-------|
| No. of trees | 40 | 50 | 75 | 50 | 70 | 75 |

4. The following table shows the percentages of demands for different fruits registered with a fruit vendor. Show the information by a pie diagram.

| Fruits | Mango | Sweet lime | Apples | Cheeku | Oranges |
|-----------------------|-------|------------|--------|--------|---------|
| Percentages of demand | 30 | 15 | 25 | 20 | 10 |

5. The pie diagram in figure 6.13 shows the proportions of different workers in a town. Answer the following questions with its help.

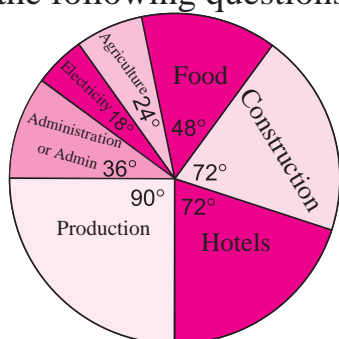


fig 6.13

- (1) If the total workers is 10,000; how many of them are in the field of construction?
- (2) How many workers are working in the administration?
- (3) What is the percentage of workers in production?

6. The annual investments of a family are shown in the adjacent pie diagram. Answer the following questions based on it.

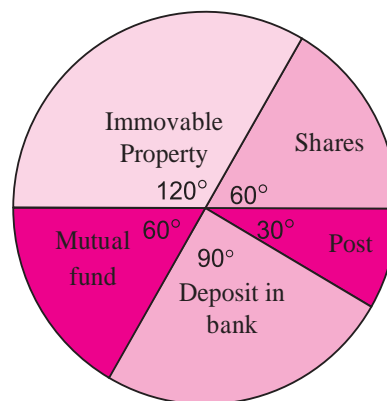


fig 6.14

- (1) If the investment in shares is ₹ 2000/, find the total investment.
- (2) How much amount is deposited in bank?
- (3) How much more money is invested in immovable property than in mutual fund?
- (4) How much amount is invested in post?

Miscellaneous Problems – 6

1. Find the correct answer from the alternatives given.

- (1) The persons of O- blood group are 40%. The classification of persons based on blood groups is to be shown by a pie diagram. What should be the measures of angle for the persons of O- blood group?

(A) 114°

(B) 140°

(C) 104°

(D) 144°

- (2) Different expenditures incurred on the construction of a building were shown by a pie diagram. The expenditure ₹ 45,000 on cement was shown by a sector of central angle of 75° . What was the total expenditure of the construction ?

(A) 2,16,000 (B) 3,60,000 (C) 4,50,000 (D) 7,50,000

- (3) Cumulative frequencies in a grouped frequency table are useful to find . . .

(A) Mean (B) Median (C) Mode (D) All of these

- (4) The formula to find mean from a grouped frequency table is $\bar{X} = A + \frac{\sum f_i u_i}{\sum f_i} \times h$
In the formula $u_i = \dots$

(A) $\frac{x_i + A}{h}$ (B) $(x_i - A)$ (C) $\frac{x_i - A}{h}$ (D) $\frac{A - x_i}{h}$

- (5)

| | | | | |
|---------------------------------|-------|-------|-------|-------|
| Distance Covered per litre (km) | 12-14 | 14-16 | 16-18 | 18-20 |
| No. of cars | 11 | 12 | 20 | 7 |

The median of the distances covered per litre shown in the above data is in the group

(A) 12-14 (B) 14-16 (C) 16-18 (D) 18-20

- (6)

| | | | | |
|--------------------------------------|-----|-----|-----|-------|
| No. of trees planted by each student | 1-3 | 4-6 | 7-9 | 10-12 |
| No. of students | 7 | 8 | 6 | 4 |

The above data is to be shown by a frequency polygon. The coordinates of the points to show number of students in the class 4-6 are

(A) (4, 8) (B) (3, 5) (C) (5, 8) (D) (8, 4)

2. The following table shows the income of farmers in a grape season. Find the mean of their income.

| | | | | | | |
|-----------------------------|-------|-------|-------|-------|-------|-------|
| Income (Thousand Rupees) | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
| Farmers | 10 | 11 | 15 | 16 | 18 | 14 |

3. The loans sanctioned by a bank for construction of farm ponds are shown in the following table. Find the mean of the loans.

| | | | | | |
|------------------------|-------|-------|-------|-------|-------|
| Loan (Thousand rupees) | 40-50 | 50-60 | 60-70 | 70-80 | 80-90 |
| No. of farm ponds | 13 | 20 | 24 | 36 | 7 |

4. The weekly wages of 120 workers in a factory are shown in the following frequency distribution table. Find the mean of the weekly wages.

| | | | | |
|--------------------------|--------|-----------|-----------|-----------|
| Weekly wages (Rupees) | 0–2000 | 2000–4000 | 4000–6000 | 6000–8000 |
| No. of workers | 15 | 35 | 50 | 20 |

5. The following frequency distribution table shows the amount of aid given to 50 flood affected families. Find the mean of the amount of aid.

| | | | | | |
|-----------------------------------|-------|-------|-------|-------|--------|
| Amount of aid (Thosand rupees) | 50–60 | 60–70 | 70–80 | 80–90 | 90–100 |
| No. of families | 7 | 13 | 20 | 6 | 4 |

6. The distances covered by 250 public transport buses in a day is shown in the following frequency distribution table. Find the median of the distances.

| | | | | | |
|---------------|---------|---------|---------|---------|---------|
| Distance (km) | 200–210 | 210–220 | 220–230 | 230–240 | 240–250 |
| No. of buses | 40 | 60 | 80 | 50 | 20 |

7. The prices of different articles and demand for them is shown in the following frequency distribution table. Find the median of the prices.

| | | | | | |
|-----------------|--------------|-------|-------|-------|--------|
| Price (Rupees) | 20 less than | 20–40 | 40–60 | 60–80 | 80–100 |
| No. of articles | 140 | 100 | 80 | 60 | 20 |

8. The following frequency table shows the demand for a sweet and the number of customers. Find the mode of demand of sweet.

| | | | | | |
|---------------------------|-------|---------|---------|----------|-----------|
| Weight of sweet (gram) | 0–250 | 250–500 | 500–750 | 750–1000 | 1000–1250 |
| No. of customers | 10 | 60 | 25 | 20 | 15 |

9. Draw a histogram for the following frequency distribution.

| | | | | | | |
|------------------------------|-------|-------|--------|---------|---------|---------|
| Use of electricity (Unit) | 50–70 | 70–90 | 90–110 | 110–130 | 130–150 | 150–170 |
| No. of families | 150 | 400 | 460 | 540 | 600 | 350 |

10. In a handloom factory different workers take different periods of time to weave a saree. The number of workers and their required periods are given below. Present the information by a frequency polygon.

| | | | | | | |
|----------------|------|-------|-------|-------|-------|-------|
| No. of days | 8-10 | 10-12 | 12-14 | 14-16 | 16-18 | 18-20 |
| No. of workers | 5 | 16 | 30 | 40 | 35 | 14 |

11. The time required for students to do a science experiment and the number of students is shown in the following grouped frequency distribution table. Show the information by a histogram and also by a frequency polygon.

| | | | | | | |
|--|-------|-------|-------|-------|-------|-------|
| Time required for experiment (minutes) | 20-22 | 22-24 | 24-26 | 26-28 | 28-30 | 30-32 |
| No. of students | 8 | 16 | 22 | 18 | 14 | 12 |

12. Draw a frequency polygon for the following grouped frequency distribution table.

| | | | | | | |
|-------------------------|-------|-------|-------|-------|-------|-------|
| Age of the donor (Yrs.) | 20-24 | 25-29 | 30-34 | 35-39 | 40-44 | 45-49 |
| No. of blood donors | 38 | 46 | 35 | 24 | 15 | 12 |

13. The following table shows the average rainfall in 150 towns. Show the information by a frequency polygon.

| | | | | | |
|-----------------------|------|-------|-------|-------|--------|
| Average rainfall (cm) | 0-20 | 20-40 | 40-60 | 60-80 | 80-100 |
| No. of towns | 14 | 12 | 36 | 48 | 40 |

14. Observe the adjacent pie diagram. It shows the percentages of number of vehicles passing a signal in a town between 8 am and 10 am

(1) Find the central angle for each type of vehicle.

(2) If the number of two-wheelers is 1200, find the number of all vehicles.

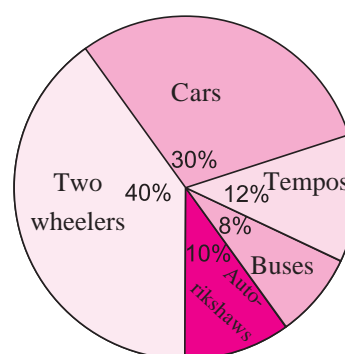


fig 6.15

15. The following table shows causes of noise pollution. Show it by a pie diagram.

| | | | | |
|--------------|---------|--------------------|----------|--------|
| Construction | Traffic | Aircraft take offs | Industry | Trains |
| 10% | 50% | 9% | 20% | 11% |

16. A survey of students was made to know which game they like. The data obtained in the survey is presented in the adjacent pie diagram. If the total number of students are 1000,

- (1) How many students like cricket?
- (2) How many students like football?
- (3) How many students prefer other games?

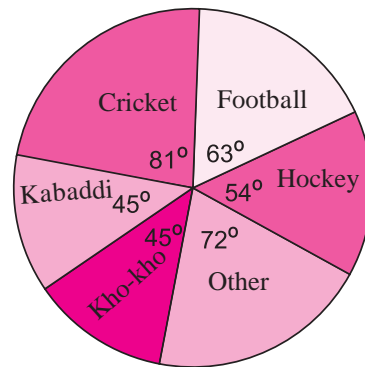


fig 6.16

17. Medical check up of 180 women was conducted in a health centre in a village. 50 of them were short of haemoglobin, 10 suffered from cataract and 25 had respiratory disorders. The remaining women were healthy. Show the information by a pie diagram.
18. On an environment day, students in a school planted 120 trees under plantation project. The information regarding the project is shown in the following table. Show it by a pie diagram.

| Tree name | Karanj | Behada | Arjun | Bakul | Kadunimb |
|--------------|--------|--------|-------|-------|----------|
| No. of trees | 20 | 28 | 24 | 22 | 26 |

