16. Binomial Distribution

Ex. (1) A biased coin is tossed 6 times. The probability of heads on any toss is 0.3. Let X denote the number of heads that comes up. Find (i) P(x=2) (ii) P(x=3) and (iii) P(1< x ≤5).

Solution : X denote the number of heads that comes up.

The probability of heads on any toss is 0.3.

Probability of success (p) = 0.3 = $\frac{3}{10}$

Probability of failure (q) = 1 – p = 1 – 0.3 = 0.7 = $\frac{7}{10}$

Clearly X ~ (n,p) with n = 6 , $p = \frac{3}{10}$ *and* $q = \frac{7}{10}$

$P(X = r) = {}^{n}C_r\, p^r q^{n-r}$ where r = 0,1,2,3,...,n.

(i) $P(X=2) = {}^{6}C_2 \left(\frac{3}{10}\right)^2 \left(\frac{7}{10}\right)^{6-2}$

$= \frac{6.5}{2.1}\left(\frac{3}{10}\right)^2 \left(\frac{7}{10}\right)^4$

$\frac{(135)(7^4)}{10^6}$

$\frac{(135)(2401)}{10^6}$

= 0.324135

(ii) P(X=3)

$= {}^{6}C_3 \left(\frac{3}{10}\right)^3 \left(\frac{7}{10}\right)^{6-3}$

$= 20\left(\frac{27}{1000}\right)\left(\frac{343}{1000}\right)$

$= \frac{540}{1000} \times \frac{343}{1000}$

$= 0.18522$

(iii) P(1< x ≤5) = P(X=2,3,4,5)

= 1 – P(X=0,1,6)

$= 1 - \{ P(X=0) + P(X=1) + P(X=6)\}$ ... ...(I)

$P(X=0) + P(X=1) + P(X=6)$

$= {}^6C_0\left(\frac{3}{10}\right)^0\left(\frac{7}{10}\right)^{6-0} + {}^6C_1\left(\frac{3}{10}\right)^1\left(\frac{7}{10}\right)^{6-1} + {}^6C_6\left(\frac{3}{10}\right)^6\left(\frac{7}{10}\right)^{6-6}$

$= (1)(1)\left(\frac{7}{10}\right)^6 + (6)\left(\frac{3}{10}\right)\left(\frac{7}{10}\right)^5 + (1)\left(\frac{3}{10}\right)^6(1)$

$= \left(\frac{7^6}{10^6}\right) + \left(\frac{(18)(7^5)}{10^6}\right) + \left(\frac{3^6}{10^6}\right)$

$= \left(\frac{7^5(7+18)+729}{10^6}\right) = \left(\frac{(16807)(25)+729}{10^6}\right)$

$= \left(\frac{420904}{10^6}\right)$ ....(II)

From (I) and (II)

$P(1< x \le 5) = 1 - \{ P(X=0) + P(X=1) + P(X=6)\}$

$= 1 - \left(\frac{420904}{10^6}\right) =$ 0.579096

Ex. (2) If the random variable X follows the Binomial Distribution with 6 trials and a probability of success equal to $\frac{1}{4}$ at each attempt then what will be the probability of (i) exactly 4 success (ii) at least one success .

Solution : The random variable X follows the Binomial Distribution with 6 trials and a probability of success equal to $\frac{1}{4}$ at each attempt.

$\therefore\ p = \frac{1}{4}$ and $q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$

Clearly $X \sim (n,p)$ with $n = 6$, $p = \frac{1}{4}$ *and* $q = \frac{3}{4}$

$P(X = r) = {}^nC_r\, p^r q^{n-r}$ where $r = 0,1,2,3,...,n.$

(i) probability of exactly 4 success

$P(X=4) = {}^6C_4\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^{6-4}$

$= 15\left(\frac{1}{256}\right)\left(\frac{9}{16}\right)$

$= \frac{15 \times 9}{256 \times 16}$

$= \frac{135}{4096}$

$= 0.03295$

(i) probability of at least one success

$P(X \geq 1) \quad = \quad P(X = 1,2,3,4,5,6)$

$= \quad 1 - P(X < 1)$

$= \quad 1 - \{ P(X=0) \}$ ... ...(I)

$= 1 - {}^6C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{6-0}$

$= 1 - 1 (1) \left(\frac{729}{4096}\right)$

$= 1 - \frac{729}{4096}$

$= 1 - 0.1779$

$\therefore P(X \geq 1) = 0.8221$

Ex. (3) The probability that a student is not a swimmer is $\frac{2}{3}$. If 5 students are randomly chosen , find the probability that (i) 4 out of them are swimmers (ii) at least four are swimmers.

Solution : X denote the number of student is a swimmer.

The probability that a student is not a swimmer is $\frac{2}{3}$.

The probability that a student is a swimmer is $1 - \frac{2}{3} = \frac{1}{3}$

$\operatorname{var}(X) = n\,p\,q \;= (10)(\frac{1}{2})(\frac{5}{2})$.

Probability of success (p) $= \frac{1}{3}$

Probability of failure (q) $= 1 - \frac{1}{3} = \frac{2}{3}$.

Ex. (4) Let $X \sim B(n,p)$ if $n = 10$ and $E(X) = 5$. Find p and S.D.(X).

Solution : $X \sim B(n,p)$, $n = 10$ and $E(X) = 5$.

$E(X) = n\,p$

$5 = 10\,p$

$p = \frac{1}{2}$

$\therefore \; q \;=\; 1 - p$

$\therefore \quad q = 1 - \frac{1}{2}$

$\therefore \quad q = \frac{1}{2}$

$\text{var}(X) = n\,p\,q$

$= (10)(\frac{1}{2})(\frac{1}{2})$

$= \frac{5}{2}$

$\text{S.D.}(X) = \sqrt{\text{var}(X)}$

$= \sqrt{\frac{5}{2}}$

Ex. (5) The probability of hitting a target in any shot is 0.2.If 10 shots are fired then find μ and S.D.(X).

Solution : Let x = number of shots hitting the target

p = probability that the target is shot

$\therefore p = 0.2 = \frac{1}{5}$ and $q = 1 - p = \frac{4}{5}$

Given $n = 10$

$\therefore x \sim B(10, \frac{1}{5})$

we know $\mu = E(x) = n \cdot p$

$\mu = 10 \times \frac{1}{5}$

$\boxed{\mu = 2}$

and $\text{S.D.}(x) = \sqrt{npq}$

$= \sqrt{10 \times \frac{1}{5} \times \frac{4}{5}}$

$= \sqrt{\frac{8}{5}} = \sqrt{1.6}$

$= 1.2649$

Ex. (6) For a Binomial Distribution the number of trials is 5 and P(X=4) = P(X=3). Find the probability of success and also obtain P(X > 2).

Solution :

We know

$P(X=r) = {}^nC_r\, p^r q^{n-r}$

Given $P(X=4) = P(X=3)$

${}^5C_4\, p^4 q^1 = {}^5C_3\, p^3 q^2$

$5p^4 q = 10 p^3 q^2$

on cancling

$p = 2q$

$p = 2(1-p) \quad \because q = 1-p$

$p = 2 - 2p$

$3p = 2$

$p = \frac{2}{3}$

$\therefore$ Probability of success is $\frac{2}{3}$

$\therefore \; q = \frac{1}{3}$

Now $P(X>2) = P(3) + P(4) + P(5)$

$= {}^5C_3\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^2 + {}^5C_4\left(\frac{2}{3}\right)^4 + \left(\frac{1}{3}\right)$

$+ {}^5C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^0$

$= \left[10 \times \frac{8}{27} \times \frac{1}{9}\right] + \left[5 \times \frac{16}{81} \times \frac{1}{3}\right] + \left[1 \times \frac{32}{243}\right]$

$= \frac{8}{243}\left[10 + 10 + 4\right]$

$= \frac{8 \times 24}{243}$

$= 0.79$

$\therefore P(X>2) = 0.79$

Ex. (7) Student A has answered that the mean of a Binomial Distribution is 18 and variance is 12, another student B answered that the mean is 18 and variance is 21. Of the two students whose answer is correct? Justify.

Solution : A student answered that the mean is 18 and Variance is 12

We know mean = np and Var = npq

$\therefore$ np = 18 and npq = 12

$18q = 12$

$q = \frac{2}{3} \qquad \therefore p = \frac{1}{3}$

$\boxed{\therefore n = 54}$ According to A.

student B answered that

mean = 18 and var = 21

$\therefore np = 18 \qquad npq = 21$

$\therefore 18 \times q = 21$

$q = \frac{7}{6}, \therefore p = -\frac{1}{6}$

$\therefore n = \frac{18}{p}$

$n = \frac{18}{-1/6}$

$n = -108$

as n is never -ve

$\therefore$ student A is correct

Ex. (8) In a group of 10 players 5 pass fitness test. Find the probability that out of the 4 players selected at random (i) exactly two will pass fitness test (ii) at least 2 will pass fitness test.

Solution : As 5 pass fitness test among 10 players

$p = \frac{1}{2}$ and $q = \frac{1}{2}$

$P(x=2) = {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2}$

$= 6 \times \frac{1}{4} \times \frac{1}{4} \Rightarrow 6 \times \frac{1}{16}$

$\boxed{P(x=2) = 3/8}$

Now $P(x \geq 2) = P(2) + P(3) + P(4)$

$= 1 - [P(0) + P(1)]$

$= 1 - \left[{}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 + {}^4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^3\right]$

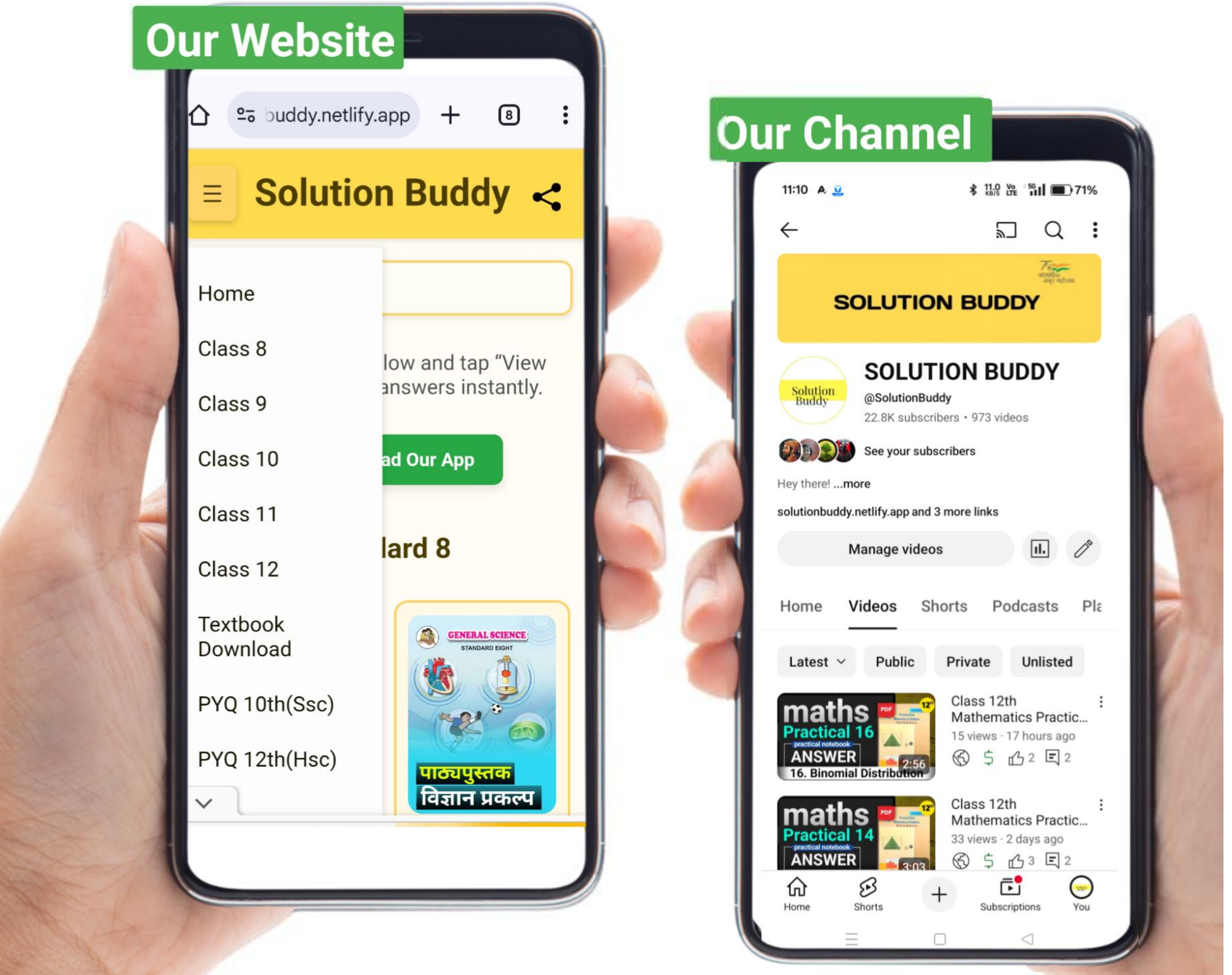
$= 1 - \left[\frac{1}{16} + \frac{4}{2}\left(\frac{1}{8}\right)\right]$

$= 1 - \left[\frac{1}{16} + \frac{4}{16}\right]$

$= 1 - \frac{5}{16}$

$\boxed{P(x \geq 2) = \frac{11}{16}}$

Sign of Teacher :



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