15. Probability Distribution

Ex. (1). A random variable X has the following probability distribution :

$X = x$	0	1	2	3	4	5	6
$P(X = x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

Find (i) k (ii) $p(X < 3)$ (iii) $p(X \geq 2)$ (iv) $P(0 < X < 4)$ (v) $P(2 \leq X \leq 5)$

Solution : For a random variable X we have $\sum_{i=1}^{n} p_i = 1$

$\therefore k + 3k + 5k + 7k + 9k + 11k + 13k = 1$

i.e. $49k = 1 \Rightarrow k = \frac{1}{49}$

$X = x$	0	1	2	3	4	5	6
$P(X = x)$	$\frac{1}{49}$	$\frac{3}{49}$	$\frac{5}{49}$	$\frac{7}{49}$	$\frac{9}{49}$	$\frac{11}{49}$	$\frac{13}{49}$

(i) $k = \frac{1}{49}$

(ii) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= \frac{1}{49} + \frac{3}{49} + \frac{5}{49} = \frac{9}{49}$$

(iii) $P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6)$

(iv) $P(0 < X < 4) = P(X = 1) + P(X = 2) + P(X = 3)$

(v) $P(2 \leq X \leq 5) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$

$$= \frac{5}{49} + \frac{7}{49} + \frac{9}{49} + \frac{11}{49} = \frac{32}{49}$$

Ex. (2). Calculate the Expected value and Variance of X if X denotes the number obtained on the uppermost face when a fair die is thrown.

Solution : When a fair die is thrown, the sample space is $S = \{1, 2, 3, 4, 5, 6\}$.

Let X denotes the number obtained on the uppermost face.

$\therefore X$ can take values 1, 2, 3, 4, 5, 6.

$P(X = 1) = P(X = 2) = P(X = 3) = P(X = 4) = P(X = 5) = P(X = 6)$

$= \frac{1}{6}$

The probability distribution is

$X = x$	1	2	3	4	5	6	Total
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1

$x_i.p_i$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{6}{6}$	$\frac{21}{6}=\frac{7}{2}$
$x_i^2.p_i$	$\frac{1}{6}$	$\frac{4}{6}$	$\frac{9}{6}$	$\frac{16}{6}$	$\frac{25}{6}$	$\frac{36}{6}$	$\frac{91}{6}$

(i) Expected Value = $E(X) = \sum_{i=1}^{n} x_i, p_i = \frac{7}{2} = 3.5$

(ii) Variance = $V(X) = E(X^2) - [E(X)]^2$

$$= \sum_{i=1}^{n} x_i^2, p_i - \left(\sum_{i=1}^{n} x_i, p_i\right)^2$$

$$= \frac{91}{6} - \left(\frac{7}{2}\right)^2 = \frac{91}{6} - \frac{49}{4}$$

$$= \frac{182-147}{12}$$

$\therefore$ Variance = V(X) = $\frac{35}{12} = 2.9167$

Ex. (3). A discrete random variable X takes the values -1, 0 and 2 with the probabilities $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$ respectively. Find $V(X)$ and Standard Deviation.

Solution : Given that the random variable X takes the values -1, 0 and 2.

The corresponding probabilities are $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$.

$P(-1) = \frac{1}{4}$, $P(0) = \frac{1}{2}$ and $P(2) = \frac{1}{4}$

Given data can be tabulated as follows

$X = x$	-1	0	2	Total
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	1
$x_i p_i$	$-\frac{1}{4}$	0	1/2	1/4
$x_i^2 p_i$	1/4	0	1	5/4

(i) Variance = $V(X) = E(X^2) - [E(X)]^2$

$$= \sum_{i=1}^{n} x_i^2 p_i - \left(\sum_{i=1}^{n} x_i p_i\right)^2$$

$$= \frac{5}{4} - \left(\frac{1}{4}\right)^2$$

$$= \frac{5}{4} - \frac{1}{16} = 1.1875$$

(ii) Standard Deviation = $\sigma = \sqrt{V(X)} = 1.0897$

Ex. (4) The p. d. f. of X, find P(X < 1) and $P(|X| < 1)$ where

$$f(x) = \frac{x+2}{18} \quad \text{if } -2 < x < 4$$
$$= 0 \quad \text{otherwise.}$$

Solution : Given that the p. d. f. of X is

$$f(x) = \frac{x+2}{18} \quad \text{if } -2 < x < 4$$
$$= 0 \quad \text{otherwise.}$$

(i) $P(X < 1) = \int_{-2}^{1} f(x)dx$

$$= \int_{-2}^{1} \text{\_\_\_\_} \, dx$$

$$= \frac{1}{18} \int_{-2}^{1} (\ldots\ldots\ldots\ldots) \, dx$$

$$= \frac{1}{18} \left[\frac{(\ldots\ldots\ldots)^2}{\text{\_\_\_\_}} \right]_{-2}^{1}$$

$$= \frac{1}{36} \left[\frac{(\ldots\ldots\ldots)^2}{\text{\_\_\_\_}} \right]_{-2}^{1}$$

$$= \frac{1}{36} [\ldots - \ldots] = \frac{}{36} = \text{\_\_\_\_} = \ldots\ldots\ldots$$

(i) $P(|X| < 1) = P(-1 < X < 1)$

$$= \int_{-1}^{1} \text{\_\_\_\_} \, dx$$

$$= \frac{1}{18} \int_{-1}^{1} (\ldots\ldots\ldots) dx$$

$$= \frac{1}{18} \left[\frac{(\ldots\ldots)^2}{\text{\_\_\_\_}} \right]_{-1}^{1}$$

$$= \frac{1}{36} \left[(\ldots\ldots)^2 \right]_{-1}^{1}$$

$$= \frac{1}{36} [\ldots - \ldots] = \frac{}{36} = \frac{2}{} = \ldots\ldots$$

~~Ex~~ (5). A random variable X has the following probability distribution :

x	0	1	2	3	4	5	6	7
$P(X = x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find (i) k (ii) $P(X < 3)$ (iii) $P(X > 6)$ (iv) $P(0 < X < 3)$ (v) $P(2 \leq X \leq 4)$

Solution : Since P(X) Is Probability distibution of x

$\sum_{x=0} \ldots [P(x)] = 1$

$\therefore P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) = 1$

$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$10k^2 + 9k - 1 = 0$

$10k^2 + 10k - k - 1 = 0$

$10k(k+1) - 1(k+1) = 0$

$\therefore (10k-1)(k+1) = 0$

$k = \frac{1}{10}$ or $k = -1$

$k = -1$ Is Not Possible

(i) $k = 1/10$

(ii) $P(x<3) = 0 + k + 2k$

$= 3k$

$= 3/10$

(iii) $P(x>6) = 7k^2 + k$

$= \frac{7}{10^2} + \frac{1}{10}$

$= \frac{17}{100}$

(iv) $P(0<x<3) = k + 2k$

$= 3k$

$= \frac{3}{10}$

(v) $P(2 \le x \le 4)$

$= 2k + 2k + 3k$

$= 7k$

$= \frac{7}{10}$

Ex. (6). The p. m. f. of a random variable X is as follows :

$X = x$	1	2	3	4
$P(x)$	$\frac{1}{30}$	$\frac{4}{30}$	$\frac{9}{30}$	$\frac{16}{30}$

Find Mean and the Variance.

Solution :

$X = x$	1	2	3	4	Σ
$P(x)$	1/30	4/30	9/30	16/30	10/3
$x^2 P(x)$	1/30	16/30	81/30	256/30	59/5

From the table

$\Sigma x_i p_i = \frac{10}{3}$ and $\Sigma x_i^2 p_i = 59/5$

Mean $= E(x) = \frac{10}{3} = 3.33$

Variance $= V(x) = \sum x_i^2 p_i - [\sum x_i p_i]^2$

$= \frac{59}{5} - \left[\frac{10}{3}\right]^2$

$= \frac{59}{5} - \frac{100}{9}$

$= \frac{531 - 500}{45}$

$V(x) = \frac{31}{45}$

Hence mean $= 3.33$

Variance $= 0.6888$

Ex. (7). From a survey of 20 famili[illegible] the following data was obtained :

No. of children	0	1	[illegible]	[illegible]	4
No. of families	5	11	[illegible]	[illegible]	2

For the random variable X [illegible] children in a randomly chosen family, Find $E(X)$ and $V(X)$.

Solution :

x	0	1	2	3	4	Σ
f	5	11	2/20	0	2	20
$P(x)$	5/25	11/20	4/20	0	2/20	.
$x_i p_i$	0	1/20	4/20	0	8/20	23/20
$x_i^2 p_i$	0	11/20	8/20	0	32/20	5/20

Here $E(x) = x_i p_i$

$= \frac{23}{20}$

$= 1.15$

$$V(X) = \sum x_i^2 p_i - \left[\sum x_i p_i\right]^2$$

$$= \frac{51}{20} - \left[\frac{23}{20}\right]^2$$

$$= \frac{51}{20} - \frac{529}{400} = \frac{491}{400}$$

$V(X) = 1.2275$

Cancelled

Ex. (8). Find the c.d.f. $F(X)$ associated with the following p.d.f $f(x)$:

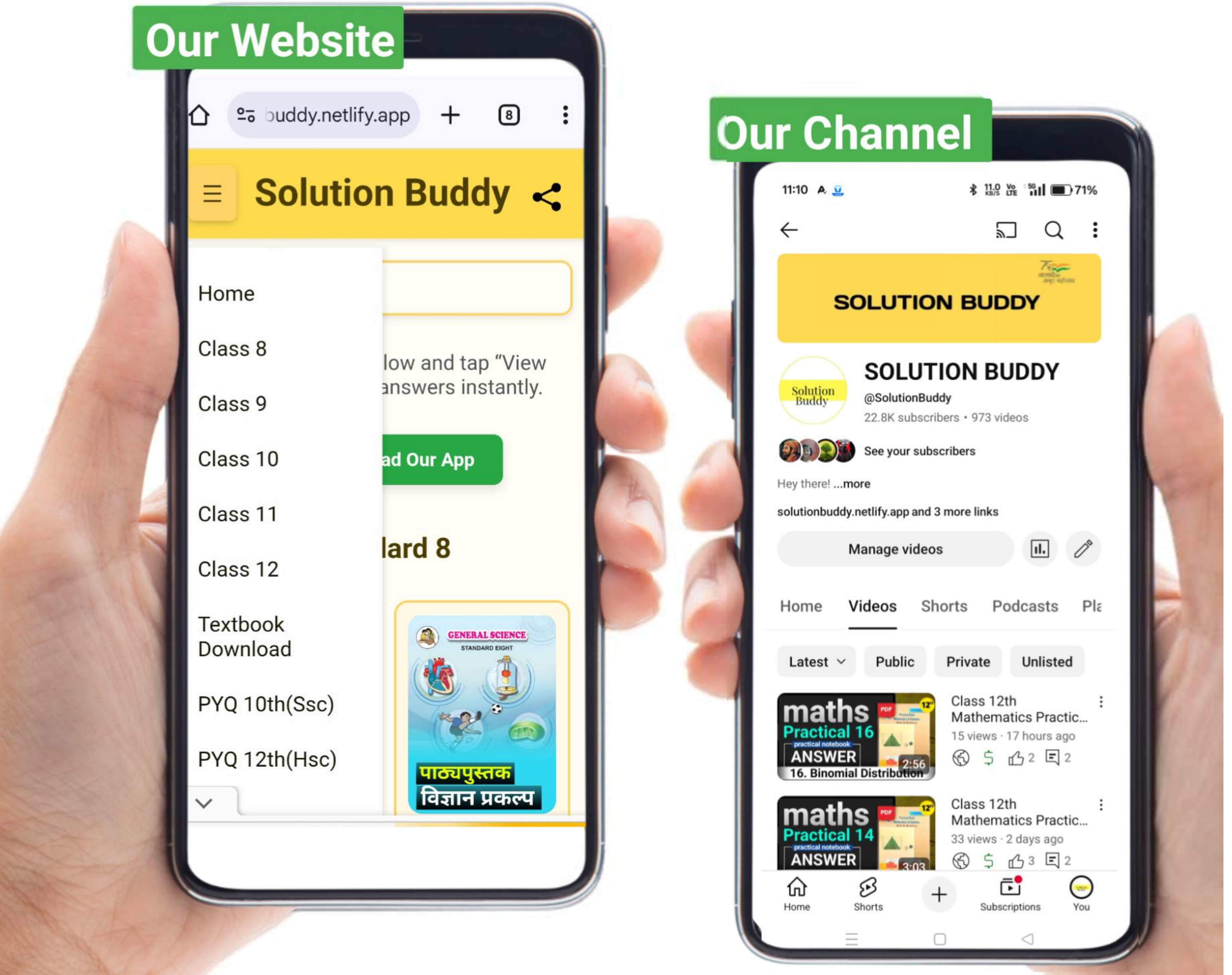
$f(x) = 12x^2(1 - x)$ for $0 < x < 1$

$= 0$ otherwise.

Also, find $P\left(\frac{1}{3} < X < \frac{1}{2}\right)$ by using p.d.f and c.d.f.

Solution :

Sign of Teacher :



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