Chapter 13: AC Circuits

EXERCISES [PAGES 304 - 305]

Exercises | Q 1.1 | Page 304

Choose the correct option.
If the rms current in a 50 Hz AC circuit is 5A, the value of the current 1/300 seconds after its value becomes zero is

- $5\sqrt{2}$ A
- **$5\sqrt{\frac{3}{2}}$ A**
- $\frac{5}{6}$ A
- $\frac{5}{\sqrt{2}}$ A

SOLUTION

$5\sqrt{\frac{3}{2}}$ **A**

Explanation:

The expression of sinusoidal current is $I = I_p \sin\omega t$

$I = \left(I_{rms} \times \sqrt{2}\right) \sin(2\pi \times 50 \times 1/300)$

$= 5\sqrt{2} \sin \pi/3 = 5\sqrt{2} \times \frac{\sqrt{3}}{2}$

$= 5\sqrt{\frac{3}{2}}$ A

Exercises | Q 1.2 | Page 304

Choose the correct option.
A resistor of 500 Ω and inductance of 0.5 H is in series with an AC source which is given by V = 1002 sin (1000 t). The power factor of the combination is

$\frac{1}{\sqrt{2}}$

$\frac{1}{\sqrt{3}}$

0.5

0.6

SOLUTION

$\frac{1}{\sqrt{2}}$

Explanation:

Given: L= 0.5H, R=500Ω

On comparing source voltage with V = $V_0\sin(\omega t)$

We get $\omega = 1000\ s^{-1}$

Inductive reactance $X_L = \omega L = 1000 \times 0.5 = 500\Omega$

Power factor $\cos\phi = \frac{R}{\sqrt{X_L^2 + R^2}}$

$\therefore \cos\phi = \frac{500}{\sqrt{(500)^2 + (500)^2}}$

Or $\cos\phi = \frac{500}{500\sqrt{2}} = \frac{1}{\sqrt{2}}$

Exercises | Q 1.3 | Page 304

Choose the correct option.

In a circuit L, C and R are connected in series with an alternating voltage of frequency f. the current leads the voltage by 450. The value of C is

$$\frac{1}{\pi f(2\pi fL - R)}$$

$$\frac{1}{2\pi f(2\pi fL - R)}$$

$$\frac{1}{\pi f(2\pi fL + R)}$$

$$\frac{1}{2\pi f(2\pi fL + R)}$$

SOLUTION

$$\frac{1}{2\pi f(2\pi fL - R)}$$

Exercises | Q 1.4 | Page 304

Choose the correct option.
In an AC circuit, e and i are given by e = 150 sin (150t) V and i = 150 sin $\left(150t + \frac{\pi}{3}\right)$ A. the power dissipated in the circuit is

1. 106W
2. 150W
3. **5625W**
4. Zero

SOLUTION

5625W

Explanation:

Compare V = 150 sin (150 t) with V = $V_0 \sin \omega t$, we get V_0 = 150 V

Compare I = 150 sin $\left(150t + \frac{\pi}{3}\right)$ with

I = $I_0 \sin(\omega t + \varnothing)$, we get

$I_0 = 150$ A, $ø = \frac{\pi}{3} = 60$

The power dissipated in ac circuit is

$$P = \frac{1}{2} V_0 I_0 \cos ø = \frac{1}{2} \times 150 \times 150 \times \cos 60$$

$$= \frac{1}{2} \times 150 \times 150 \times \frac{1}{2} = 5625\text{W}$$

Exercises | Q 1.5 | Page 304

Choose the correct option.
In a series LCR circuit, the phase difference between the voltage and the current is 45°. Then the power factor will be

1. 0.607
2. **0.707**
3. 0.808
4. 1

SOLUTION

0.707

Explanation:

Here, $\phi = 45°$

In series LCR circuit, power factor = $\cos\phi$

$$\therefore \cos\phi = \frac{1}{\sqrt{2}} = 0.707$$

Exercises | Q 2.1 | Page 304

Answer in brief.
An electric lamp is connected in series with a capacitor and an AC source is glowing with a certain brightness. How does the brightness of the lamp change on increasing the capacitance?

SOLUTION

Impedance, Z = $\sqrt{R^2 + \frac{1}{\omega^2 C^2}}$, where R is the resistance of the lamp, w is the angular frequency of AC and C is the capacitance of the capacitor connected in series with the AC source and the lamp. When C is increased, $\frac{1}{\omega^2 C^2}$ decreases. Hence, Z increases. R Power factor, cos Φ = $\frac{R}{Z}$

As Z increases, the power factor decreases.
Now, the average power over one cycle,

$$P_{av} = V_{rms} i_{rms} \cos\phi$$
$$= V_{rms}\left(\frac{V_{rms}}{Z}\right)\cos\phi$$
$$= \frac{V_{rms}^2}{Z}\cos\phi$$

∴ P_{av} decreases as z increases and cos¢ decreases. As the current through the lamp $\left(\frac{V_{rms}}{Z}\right)$ decreases, the brightness of the lamp will decrease when C is increased.

Exercises | Q 2.2 | Page 304

Answer in brief.
The total impedance of a circuit decreases when a capacitor is added in series with Land R. Explain why.

SOLUTION

For an LR circuit, the impedance,

$Z_{LR} = \sqrt{R^2 + X_L^2}$, where X_L is the reactance of the inductor.
When a capacitor of capacitance C is added in series with L and R, the impedance,

$Z_{LCR} = \sqrt{R^2 + (X_L - X_C)^2}$ because in the case of an inductor the current lags behind the voltage by a phase angle of π/2 rad while in the case of a capacitor the current leads the voltage by a phase angle of π/2 rad. The decrease in net reactance decreases the total impedance $(Z_{LCR} < Z_{LR})$.

Exercises | Q 2.3 | Page 304

Answer in brief.
For a very high-frequency AC supply, a capacitor behaves like a pure conductor. Why?

SOLUTION

The reactance of a capacitor is, $X_C = \frac{1}{2\pi fC}$, where f is the frequency of the AC supply and C is the capacitance of the capacitor. For very high frequency, f, X_C is very small. Hence, for a very high-frequency AC supply, a capacitor behaves like a pure conductor.

Exercises | Q 2.4 | Page 304

Answer in brief.
What is wattles current?

SOLUTION

The current that does not lead to energy consumption, hence zero power consumption, is called wattless current.
In the case of a purely inductive circuit or a purely capacitive circuit, the average power consumed over a complete cycle is zero and hence the corresponding alternating current in the circuit is called wattless current.

Exercises | Q 2.5 | Page 304

Answer in brief.
What is the natural frequency of the LC circuit? What is the reactance of this circuit at this frequency?

SOLUTION

The natural frequency of LC circuit is $\frac{1}{2\pi\sqrt{LC}}$, where L is the inductance and C is the capacitance. The reactance of this circuit at this frequency is

$$\frac{1}{2\pi fC - \frac{1}{2\pi fL}} = \frac{1}{\frac{2\pi C}{2\pi\sqrt{LC}} - \frac{1}{\frac{2\pi L}{2\pi\sqrt{LC}}}}$$

$$= \frac{1}{\sqrt{\frac{C}{L}} - \sqrt{\frac{C}{L}}} = \frac{1}{\text{zero}} = \infty$$

Exercises | Q 3 | Page 304

In a series LR circuit $X_L = R$ and power factor of the circuit is P_1. When capacitor with capacitance C such that $X_L = X_C$ is put in series, the power factor becomes P_2. Calculate P_1/P_2.

SOLUTION

Given: Power factor P_1 (When X_L=R)

New power factor P_2 (When X_L = X_C)

$$P_1 = \frac{R}{Z}$$

$$\Rightarrow P_1 = \frac{R}{\sqrt{R^2 + X^2}} = \frac{R}{\sqrt{2R^2}} = \frac{1}{\sqrt{2}}$$

$$P_2 = \frac{R}{Z}$$

$$\Rightarrow P_2 = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = 1$$

Thus, $\frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$

Exercises | Q 4 | Page 304

When an AC source is connected to an ideal inductor show that the average power supplied by the source over a complete cycle is zero.

SOLUTION

In an AC circuit containing only an ideal inductor, the current i lags behind the emf e by a phase angle of π/2 rad. Here, for $e = e_0 \sin \omega t$, we have,

$i = i_0 \sin (\omega t - \pi/2)$

Instantaneous power, P = ei

$= (e_0 \sin \omega t)[i_0(\sin \omega t \cos \pi/2 - \cos \omega t \sin \pi/2)]$

$= - e_0 i_0 \sin \omega t \cos \omega t$ as $\cos \pi/2 = 0$ and $\sin \pi/2 = 1$

Average power over one cycle,

$$P_{av} = \frac{\text{work done in one cycle}}{\text{time for one cycle}}$$

$$= \frac{\int_0^T P\ dt}{T} = \frac{-\int_0^T e_0 i_0 \sin \omega t \cos \omega t dt}{T}$$

$$= \frac{-e_0 i_0}{T} \int_0^T \sin \omega t \cos \omega t\ dt$$

Now, $\int_0^T \sin \omega t \cos \omega t\ dt = 0$

$\therefore P_{av} = 0$

[**Note:** For reference, see the answer to Q. 6. The proof should be written as part of the answer.]

Exercises | Q 5 | Page 304

Prove that an ideal capacitor in an AC circuit does not dissipate power

SOLUTION

In an AC circuit containing only an ideal capacitor, the current i leads the emf e by a phase angle of π/2 rad.

Here, for $e = e_0 \sin wt$, we have, $i = i_0 \sin (\omega t + \pi/2)$ Instantaneous power, $P = ei$

$= (e_0 \sin \omega t)[i_0 (\sin \omega t \cos \pi/2 + \cos \omega t \sin \pi/2)]$

$= e_0 i_0 \sin \omega t \cos \omega t$ as $\cos \pi/2 = 0$ and $\sin \pi/2 = 1$

Average power over one cycle, P_{av}

$$= \frac{\text{work done in one cycle}}{\text{time for one cycle}}$$

$$= \frac{\int_0^T P\ dt}{T} = \frac{\int_0^T e_0 i_0 \sin \omega t \cos \omega t dt}{T}$$

$$= \frac{e_0 i_0}{T} = \int_0^T \sin \omega t \cos \omega t\ dt$$

Now, $\int_0^T \sin\omega t \cos\omega t \, dt = 0$

∴ Pav = 0, i.e., the circuit does not dissipate power.

[**Note:** For reference, see the answer Q.6. The proof should be wriiten as part of the answer.]

Exercises | Q 6 | Page 304

(a) An emf $e = e_0 \sin \omega t$ applied to a series LCR circuit derives a current $I = I_0 \sin\omega t$ in the circuit. Deduce the expression for the average power dissipated in the circuit.
(b) For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain.

SOLUTION

Instantaneous power,

$P = ei = (e_0 \sin \omega t) [i_0 \sin (\omega t \pm \Phi)]$

$= e_0 i_0 \sin \omega t (\sin \omega t \cos \Phi \pm \cos \omega t \sin \Phi)$

$= e_0 i_0 \cos \Phi \sin^2 \omega t \pm e_0 i_0 \sin \Phi \sin \omega t \cos \omega t$

Average power over one cycle,

$$P_{av} = \frac{\text{work done in one cycle}}{\text{time for one cycle}}$$

$$= \frac{\int_0^T P \, dt}{T}$$

$$= \frac{\int_0^T \left[e_0 i_0 \cos\phi \sin^2 \omega t \ \pm \ e_0 i_0 \sin\phi \sin\omega t\right] dt}{T}$$

$$= \frac{e_0 i_0}{T}\left[\cos\phi \int_0^T \sin^2 \omega t \, dt \pm \sin\phi \int_0^T \sin\omega t \cos\omega t dt\right]$$

Now, $\int_0^T \sin^2 \omega t \, dt = \int_0^T \left(\frac{1 - \cos 2\omega t}{2}\right) dt$

$$= \int_0^T \frac{1}{2}dt - \int_0^T \frac{\cos 2\omega t}{2}dt = \frac{T}{2} - \frac{1}{2}\left(\frac{\sin 2\omega t}{2\omega}\right)_0^T$$

$$= \frac{T}{2} - \frac{1}{4\omega}(\sin 2\omega T - \sin 0)$$

$$= \frac{T}{2} - \frac{1}{4\omega}\left[\sin 2\left(\frac{2\pi}{T}\right)T - 0\right]$$

$$= \frac{T}{2} - \frac{1}{4\omega}[0 - 0] = \frac{T}{2}$$

Also,

$$\int_0^T \sin \omega t \cos \omega t dt = \frac{1}{2}\int_0^T \sin 2\omega t dt = \frac{1}{2}\left[\frac{-\cos 2\omega t}{2\omega}\right]_0^T$$

$$= -\frac{1}{4\omega}\left[\cos 2\left(\frac{2\pi}{T}\right)T - \cos 0\right] = -\frac{1}{4\omega}[1 - 1] = 0$$

Hence, $P_{av} = \frac{e_0 i_0}{T}\cos\phi \times \frac{T}{2} = \frac{e_0 i_0}{2}\cos\phi$

$$= \frac{e_0}{\sqrt{2}} \cdot \frac{i_0}{\sqrt{2}}\cos\phi$$

$= e_{rms} i_{rms} \cos\phi = e_{rms} i_{rms}\left(\frac{R}{Z}\right)$, where the impedance Z = $\sqrt{R^2 + (X_L - X_C)^2}$

(b) $P_{av} = e_{rms} i_{rms} \cos\phi$

The factor cos Φ is called as power factor. For circuits used for transporting electric power, a low power factor means the power available on transportation is much less than $e_{rms} i_{rms}$. It means there is significant loss of power during transportation.

Exercises | Q 7 | Page 304

A device Y is connected across an AC source of emf e = e_0 sin ωt. The current through Y is given as i = i_0 sin (ωt + π/2).

a. Identify the device Y and write the expression for its reactance.

b. Draw graphs showing a variation of emf and current with time over one cycle of AC for Y.

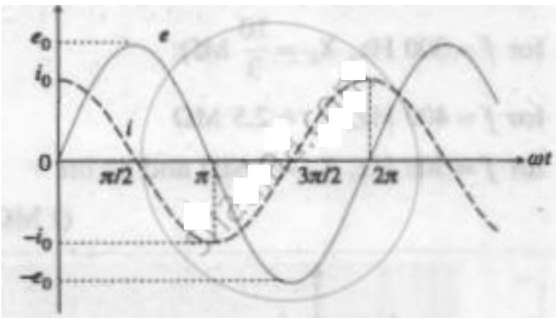
c. How does the reactance of the device Y vary with the frequency of the AC? Show graphically.

d. Draw the phasor diagram for device Y.

SOLUTION

1. The device Y is a capacitor. Its reactance is $X_c = \frac{1}{\omega C}$, where ω is the angular frequency of the applied emf and C is the capacitance of the capacitor.

2.



3. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$. Thus $X_C \propto \frac{1}{f}$, where f is the frequency of AC.

Suppose C = $\left(\frac{1000}{2\pi}\right)$pF

For f =100 Hz, X_C = 1 × 10^7 Ω = 10 M Ω;

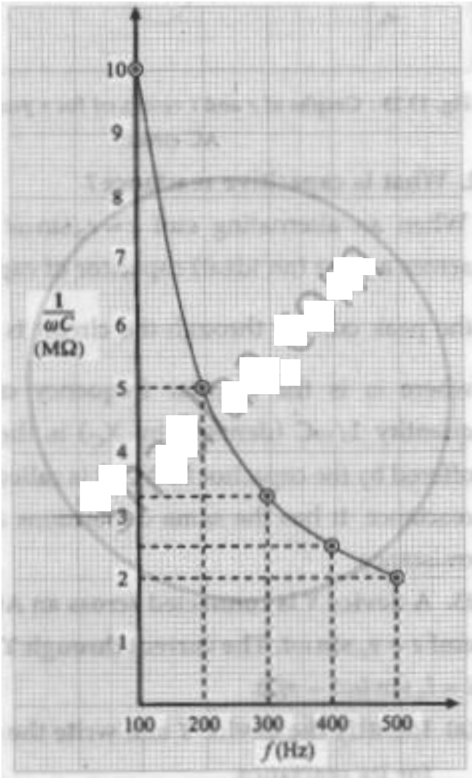
for f = 200 Hz, X_C = 5 M Ω;

for f = 300 Hz, X_C = $\frac{10}{3}$ M Ω;

for f = 400 Hz, X_C = 2.5 M Ω

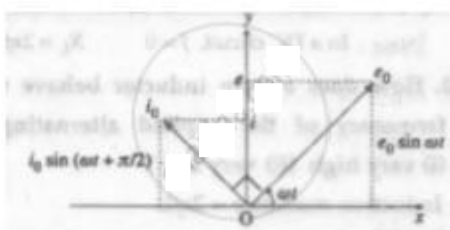
for f = 500 Hz, X_C = 2 M Ω and so on.

(1 M Ω = 10^6 Ω)



Variation of $\frac{1}{\omega C}$ with f

4. Phasor diagram for a purely capacitive circuit



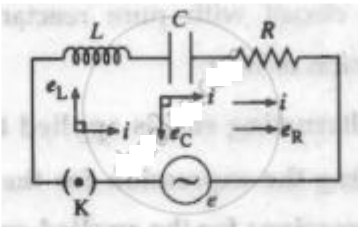
The phasor representing the peak emf (e_0) makes an angle wt in an anticlockwise direction with respect to the horizontal axis. As the current leads the voltage by 90°, the phasor representing the peak current (i_0) is turned 90° anticlockwise with respect to the phasor representing emf e_0. The projections of these phasors on the vertical axis give instantaneous values of e and i.

Exercises | Q 8 | Page 305

Derive an expression for the impedance of an LCR circuit connected to an AC power supply.

SOLUTION

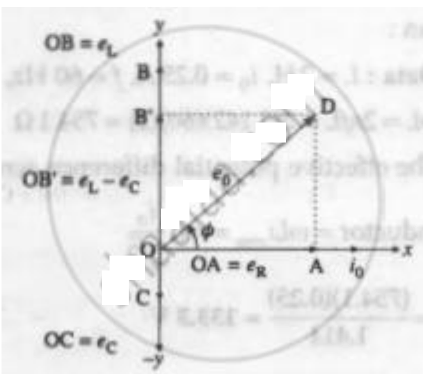
The following figure shows an inductor of inductance L, capacitor of capacitance C, a resistor of resistance R, key Kand source (power supply) of alternating emf (e) connected to form a dosed series circuit.



LCR Series circuit

We assume the inductor, capacitor and resistor to be ideal. As these are connected in series, at any instant, they carry the same current $i = i_0 \sin \omega t$.

The voltage across the resistor, $e_R = Ri$, is in phase with the current. The voltage across the inductor, $e_L = X_L i$, leads the current by π/2 rad and that across the capacitor, $e_C = X_C i$, lags behind the current by π/2 rad. This is shown in the following phasor diagram.



From this figure, $e_0^2 = e_R^2 + (e_L - e_C)^2$

$= R^2 i_0^2 + (X_l i_0 - X_C i_0)^2 = i_0^2\left[R^2 + (X_L - X_C)^2\right]$

$\therefore e_0 = i_0\sqrt{R^2 + (X_L - X_C)^2} = i_0 Z$, where

$Z = \dfrac{e_0}{i_0} = \sqrt{R^2 + (X_L - X_C)^2}$ is the effective resistance of the circuit. It is called the impedance.

Exercises | Q 9 | Page 305

Compare resistance and reactance.

SOLUTION

1. Resistance is opposition to flow of charges (current) and appears in a DC circuit as well as in an AC circuit. The term reactance appears only in an AC circuit. It occurs when an inductor and/or a capacitor is used.

2. In a purely resistive circuit, current and voltage are always in phase. When reactance is not zero, there is a nonzero phase difference between current and voltage.

3. Resistance does not depend on the frequency of AC. Reactance depends on the frequency of AC. In the case of an inductor, reactance increases linearly with frequency. In the case of a capacitor, reactance decreases as the frequency of AC increases; it is inversely proportional to frequency.

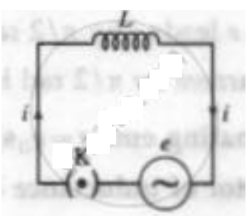
4. Resistance gives rise to the production of Joule heat in a component. In a circuit with pure reactance, there is no production of heat.

Exercises | Q 10 | Page 305

Show that in an AC circuit containing a pure inductor, the voltage is ahead of current by π/2 in phase.

SOLUTION

The following figure shows an AC source, generating a voltage $e = e_0 \sin wt$, connected to a key K and a pure inductor of inductance L to form a closed circuit.



An AC source connected to an inductor

On closing the key K, an emf is induced in the inductor as the magnetic flux linked with it changes with time. This emf opposes the applied emf and according to the laws of electromagnetic induction by Faraday and Lenz, we have,

$$e' = -L\frac{di}{dt} \quad ....(1)$$

where e' is the induced emf and i is the current through the inductor. To maintain the current, e and e' must be equal in magnitude and opposite in direction.

According to Kirchhoff's voltage law, as the resistance of the inductor is assumed to be zero, we have, $e = -e' = L\frac{di}{dt}$ ....(2)

$$\therefore \frac{di}{dt} = \frac{e}{L} = \frac{e_0 \sin \omega t}{L}$$

$$\therefore \int di = \int \frac{e_0 \sin \omega t}{L} dt$$

$$\therefore i = -\frac{e_0}{\omega L} \cos \omega t + C$$

where C is the constant of integration. C must be time independent and have the dimension of current. As e oscillates about zero, i also oscillates about zero and hence there cannot be any time independ ent component of current.

$\therefore C = 0$

$$\therefore i = -\frac{e_0}{\omega L} \cos \omega t = -\frac{e_0}{\omega L} \sin\left(\frac{\pi}{2} - \omega t\right)$$

$$\therefore i = \frac{e_0}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) \quad .....(3)$$

as $\sin(-\theta) = -\sin\theta$

From Eq. (3), $i_{peak} = i_0 = \frac{e_0}{\omega L}$

$$\therefore i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right) \quad .....(4)$$

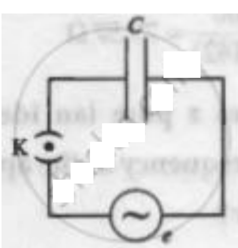
Comparison of this equation with $e = e_0 \sin wt$ shows that e leads i by π/2 rad, i.e., the voltage is ahead of current by π/2 rad in phase.

Exercises | Q 11 | Page 305

An AC source generating a voltage $e = e_0 \sin \omega t$ is connected to a capacitor of capacitance C. Find the expression for the current i flowing through it. Plot a graph of e and i versus ωt.

SOLUTION

The following figure shows an AC source, generating a voltage $e = e_0 \sin wt$, connected to a capacitor of capacitance C. The plates of the capacitor get charged due to the applied voltage. As the alternating voltage is reversed in each half cycle, the



An AC source connected to a capacitor

capacitor is alternately charged and discharged. If q is the charge on the capacitor, the corresponding potential difference across the plates of the capacitor is V = $\frac{q}{C}$ $\therefore$ q = CV. q and V are functions of time, with V = e = e_0 sin ωt.

The instantaneous current in the circuit is i = $\frac{dq}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt}$

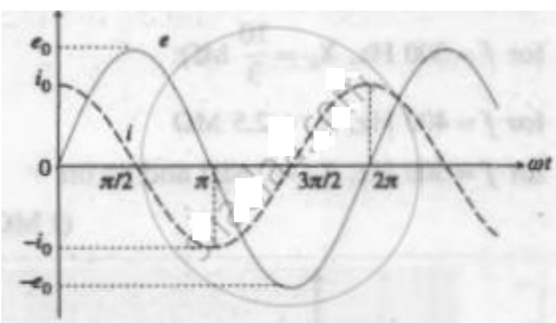
$= C\frac{d}{dt}(e_0 \sin \omega t) = \omega C e_0 \cos \omega t$

$\therefore$ i = $\frac{e_0}{(1/\omega C)} \sin\left(\omega t + \frac{\pi}{2}\right) = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$

where $i_0 = \frac{e_0}{1/\omega C}$ is the peak value of the current.

ωt (rad)	ωt + π/2 (rad)	e = e_0 sin ωt	i = i_0 sin $\left(\omega t + \frac{\pi}{2}\right)$
0	$\frac{\pi}{2}$	0	i_0
$\frac{\pi}{2}$	π	e_0	0
π	$\frac{3\pi}{2}$	0	- i_0
$\frac{3\pi}{2}$	2π	- e_0	0
2π	2π + $\frac{\pi}{2}$	0	i_0

The above table shows gives the values of e and i for different values of ωt and the following figure shows graphs of e and i versus ωt. i leads e by a phase angle of π/2 rad.



Graphs of t and i venus wt for a purely capacitive AC circuit

Exercises | Q 12 | Page 305

If the effective current in a 50 cycle AC circuit is 5 A, what is the peak value of current? What is the current 1/600 after if was zero?

SOLUTION

Data: f = 50 Hz, i_{rms} = 5 A, t = $\frac{1}{600}$ s

The peak value of the current,

$$i_0 = i_{rms}\sqrt{2} = (5)(1.414) = 7.07\text{A}$$

$$i = i_0 \sin(2\pi ft)$$

$$= 7.07 \sin\left[2\pi(50)\left(\frac{1}{600}\right)\right]$$

$$= 7.07 \sin\left(\frac{\pi}{6}\right) = (7.07)(0.5)$$

= 3.535 A

Exercises | Q 13 | Page 305

A light bulb is rated 100W for 220 V AC supply of 50 Hz. Calculate

a. resistance of the bulb.
b. the rms current through the bulb

SOLUTION

Data: Power (V_{rms} i_{rms}) = 100 W, V_{rms} = 220 V, f = 50 Hz

The rms current through the bulb,

$$i_{rms} = \frac{\text{power}}{V_{rms}} = \frac{100}{220} = 0.4545\text{A}$$

The resistance of the bulb,

$$R = \frac{V_{rms}}{i_{rms}} = \frac{220}{100/220} = (22)(22) = 484\ \Omega$$

Exercises | Q 14 | Page 305

A 15.0 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what will happen to the capacitive reactance and the current?

SOLUTION

Data: C= 15 μF = 15 x 10^{-6} F, V_{rms} = 220V, f = 50 Hz,

The capacitive reactance = $\frac{1}{2\pi fC}$

$$= \frac{1}{2(3.142)(50)\left(15 \times 10^{-6}\right)} = \frac{100 \times 100}{(3.142)(15)}$$

= 212.2 Ω

$$i_{rms} = \frac{V_{rms}}{\text{capacitive reactance}} = \frac{220}{212.2} = 1.037 \text{ A}$$

$$i_{peak} = i_{rms}\sqrt{2} = (1.037)(1.414) = 1.466\text{A}$$

If the frequency is doubled, the capacitive reactance will be halved and the current will be doubled.

Exercises | Q 15 | Page 305

An AC circuit consists of only an inductor of inductance 2 H. If the current is represented by a sine wave of amplitude 0.25 A and frequency 60 Hz, calculate the effective potential difference across the inductor (π = 3.142)

SOLUTION

Data: L = 2H, i_0 = 0.25A, f = 60 Hz, n = 3.142

ωL = 2πfL = 2(3.142)(60)(2) = 754.1 Ω

The effective potential difference across the inductor = $\omega L i_{rms} = \omega L \frac{i_0}{\sqrt{2}}$

$$= \frac{(754.1)(0.25)}{1.414} = 133.3\text{V}$$

Exercises | Q 16 | Page 305

Alternating emf of e = 220 sin 100 πt is applied to a circuit containing an inductance of (1/π) henry. Write an equation for instantaneous current through the circuit. What will be the reading of the AC galvanometer connected in the circuit?

SOLUTION

Data: e = 220 sin 100 πt, L = $\left(\frac{1}{\pi}\right)$H

Comparing e = 220 sin 100 πt with

e = e_0 sin ωt, we get

ω = 100 π

∴ ωL = (100 π)$\left(\frac{1}{\pi}\right)$ = 100 Ω

∴ The instantaneous current through the circuit

= i = $\frac{e_0}{\omega L}\sin\left(100\pi t - \frac{\pi}{2}\right)$

$= \frac{220}{100}\sin\left(100\pi t - \frac{\pi}{2}\right)$

= 2.2 sin (100πt - π/2) in ampere

$i_{rms} = \frac{i_0}{\sqrt{2}} \frac{2.2}{1.414} = \mathbf{1.556A}$ is the reading of the AC galvanometer connected in the circuit.

Exercises | Q 17 | Page 305

A 25 μF capacitor, a 0.10 H inductor, and a 25Ω resistor are connected in series with an AC source whose emf is given by e = 310 sin 314 t (volt). What is the frequency, reactance, impedance, current, and phase angle of the circuit?

SOLUTION

Data: C = 25 μF = 25 x 10^{-6} F, L = 0.10 H, R = 25Ω, e = 310 sin (314 t)[volt]

Comparing e = 310 sin (314 t) with

e = e_0 sin (2πft), we get,

the frequency of the alternating emf as

f = $\frac{314}{2\pi} = \frac{314}{2(3.14)}$ = 50 Hz

Reactance = $\left|\omega L-\frac{1}{\omega C}\right|=\left|2\pi fL-\frac{1}{2\pi fC}\right|$

$=\left|2(3.14)(50)(0.10)-\frac{1}{2(3.14)(50)\left(25\times10^{-6}\right)}\right|$

$=\left|31.4-\frac{100\times10^{2}}{(3.14)(25)}\right|$ = |31.4 - 127.4|

= 96 Ω

$$Z^{2}=R^{2}+\left(\omega L-\frac{1}{\omega C}\right)^{2}=(25)^{2}+(96)^{2}=9841\Omega^{2}$$

∴ Impedance, Z = $\sqrt{9841}\Omega$ = 99.2 Ω

Peak current, $i_{0}=\frac{e_{0}}{Z}=\frac{310}{99.2}$A

∴ $i_{rms}=\frac{i_{0}}{\sqrt{2}}=\frac{310}{(1.414)(99.2)}$A = 2.21 A

cos Φ = $\frac{R}{Z}=\frac{25}{99.2}=0.2520$

∴ Phase angle,

Φ = $\cos^{-1}(0.2520)$ = 75.40° = 1.316 rad

Exercises | Q 18 | Page 305

A capacitor of 100 μF, a coil of resistance 50Ω, and an inductance 0.5 H are connected in series with a 110 V-50Hz source. Calculate the rms value of current in the circuit.

SOLUTION

Data: C = 100 μf =100 x 10^{-6} F = 10^{-4} F,

R = 50Ω, L = 0.5H, f = 50 Hz, V_{rms} = 110V

∴ ωL = 2πfL = 2(3.142)(50)(0.5) = 157.1Ω

and $\frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2(3.142)(50)(10^{-4})}$

$= \frac{100}{3.142} = 31.83\ \Omega$

$$Z^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = (50)^2 + (157.1 - 31.83)^2$$

= 2500 + 15700 = 18200 Ω^2

∴ Impedance, Z = $\sqrt{18200}$ Ω = 134.9 Ω

The rms value of the current in the circuit,

$i_{rms} = \frac{V_{rms}}{Z} = \frac{110}{134.9}$ A = 0.8154 A

Exercises | Q 19 | Page 305

Find the capacity of a capacitor which when put in series with a 10Ω resistor makes the power factor equal to 0.5. Assume an 80V-100Hz AC supply.

SOLUTION

Here, C=?, R = 10Ω, cosϕ = 0.5,

E_v = 80V, v = 100Hz

As cosϕ = $\frac{R}{Z}$, Z = $\frac{R}{\cos\phi} = \frac{10}{0.5}$ = 20

As R^2 + $X_C^2 = Z^2$

∴ XC = $\sqrt{Z^2 - R^2} = \sqrt{20^2 - 10^2} = 10\sqrt{3}$

$$\frac{1}{\omega C} = 10\sqrt{3},$$

$$C = \frac{1}{\omega 10\sqrt{3}} = \frac{1}{2\pi \times 100 \times 10\sqrt{3}}$$

$$= 9.2 \times 10^{-5}\ F$$

Exercises | Q 20 | Page 305

Find the time required for a 50 Hz alternating current to change its value from zero to the rms value.

SOLUTION

Data: f = 50 Hz, $i = \frac{i_0}{\sqrt{2}}$ $\therefore \frac{i}{i_0} = \frac{1}{\sqrt{2}}$

$i = i_0 \sin \omega t$

$$\therefore \sin \omega t = \frac{i}{i_0} = \frac{1}{\sqrt{2}}$$

$$\therefore \omega t = \frac{\pi}{4}\ \text{rad}$$

$$\therefore 2\pi f t = \frac{\pi}{4}$$

$$\therefore t = \frac{1}{8f} = \frac{1}{8(50)} = \frac{1}{400}$$

$$= \frac{1000 \times 10^{-3}}{400} = 2.5 \times 10^{-3} s$$

Exercises | Q 21 | Page 305

Calculate the value of the capacity in picofarad, which will make 101.4 microhenry inductance to oscillate with a frequency of one megahertz.

SOLUTION

Data: $f_r = 10^6$ Hz, $L = 101.4 \times 10^{-6}$ H

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore f_r^2 = \frac{1}{4\pi^2 LC}$$

$$\therefore C = \frac{1}{4\pi^2 f_r^2 L} = \frac{1}{4(3.142)^2 \left(10^6\right)^2 \left(101.4 \times 10^{-6}\right)}$$

$$= \frac{10000 \times 10^{-10}}{4\left(3.142^2\right)(101.4)} = 2.497 \times 10^{-10} F$$

$= 249.7 \times 10^{-12}$ F = 249.7 picofarad

Exercises | Q 22 | Page 305

A 10 μF capacitor is charged to a 25 volt of potential. The battery is disconnected and a pure 100 m H coil is connected across the capacitor so that LC oscillations are set up. Calculate the maximum current in the coil.

SOLUTION

Data: $C = 10\ \mu F = 10 \times 10^{-6}\ F = 10^{-5} F$,

$L = 100\ mH = 100 \times 10^{-3}\ H = 10^{-1}\ H$, $V = 25$ V

The energy stored in the electric field in the capacitor $= \frac{1}{2} CV^2$

The energy stored in the magnetic field in the inductor $= \frac{1}{2} Li^2$

$$\frac{1}{2} CV^2 = \frac{1}{2} Li^2$$

$$\therefore i^2 = \frac{C}{L} V^2 = \frac{10^{-5}}{10^{-1}} (25)^2$$

$\therefore i = 25 \times 10^{-2}$ A = 0.25 A

Exercises | Q 23 | Page 305

A 100 μF capacitor is charged with a 50 V source supply. Then source supply is removed and the capacitor is connected across an inductance, as a result of which 5A current flows through the inductance. Calculate the value of the inductance.

SOLUTION

Here, $C = 100\mu F = 100 \times 10^{-4} F$,

$V = 50$ volt, $I = 5A$, $L = ?$

The energy stored in the electric field in the capacitor = $\frac{1}{2}CV^2$

The energy stored in the magnetic field in the inductor = $\frac{1}{2}Li^2$

As energy stored in inductor = energy stored in capacitor,

$$\therefore \frac{1}{2}CV^2 = \frac{1}{2}Li^2$$

$$\therefore L = C\frac{V^2}{i^2}$$

$$\therefore L = C\left(\frac{V}{i}\right)^2$$

$$= 10^{-4}\left(\frac{50}{5}\right)^2 = 10^{-4} \times 10^2 = 10^{-2}H$$