4. Trigonometric Functions - II



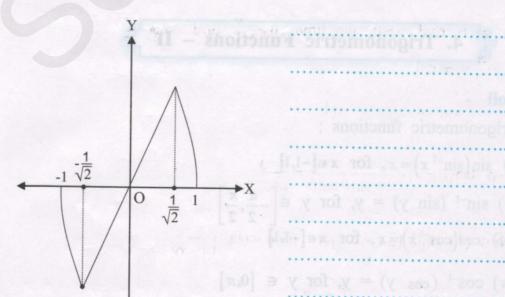
- Inverse Trigonometric functions :
 - (i) $\sin(\sin^{-1} x) = x$, for $x \in [-1,1]$
 - (ii) $\sin^{-1} (\sin y) = y$, for $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$
 - (iii) $\cos(\cos^{-1} x) = x$, for $x \in [-1,1]$
 - (iv) $\cos^{-1} (\cos y) = y$, for $y \in [0, \pi]$
 - (v) $\tan(\tan^{-1}x) = x$, for $x \in R$ (vi) $\tan^{-1}(\tan y) = y$, for $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
 - (vii) $\sec(\sec^{-1}x) = x$, for $x \in R (-1,1)$
 - (viii) $\sec^{-1}(\sec y) = y$, for $y \in [0,\pi] \left\{\frac{\pi}{2}\right\}$
 - (ix) $\cot(\cot^{-1}x)=x$, for $x \in R$ (x) $\cot^{-1}(\cot y) = y$, for $y \in (0,\pi)$
 - (xi) $\csc(\csc^{-1}x) = x$, for $x \in R (-1,1)$
 - (xii) $\operatorname{cosec}^{-1}$ (cosec y) = y, for $y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \{0\}$
- Ex. (1) Show that $\sin^{-1}(2x\sqrt{1-x^2}) = 2\sin^{-1}x$ if $-\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

Solution: Let $\sin^{-1} x = \theta$

$$\therefore \sin \theta = x, \quad x \in [-1,1],$$

$$\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Given
$$-\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$$
,



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$$\therefore \sin\left(-\frac{\pi}{4}\right) \le \sin\theta \le \sin\left(\frac{\pi}{4}\right)$$

$$\therefore -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$$
(3.0) By (101 - 4 = 0) 100 (x) ABX 101 $x = (x + 100) \times 0$ (x)

$$\therefore -\frac{\pi}{2} \le 2\theta \le \frac{\pi}{2}$$

$$\therefore 2\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].$$

$$L.H.S = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$$

$$=\sin^{-1}(2\sin\theta\cos\theta)$$

$$=\sin^{-1}(\sin 2\theta)$$

$$=2\theta$$
 As $2\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$= 2\sin^{-1} x = R.H.S.$$

Ex. (2) If
$$x > 0, y > 0$$
 then prove that $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \left(\frac{x - y}{1 + xy} \right)$

Solution: Let
$$\tan^{-1} x = \theta$$
 and $\tan^{-1} y = \phi$

$$\therefore \tan \theta = x, \tan \phi = y$$

As x>0 and y>0, we have $0<<\frac{\pi}{2}$ and $0<\phi<\frac{\pi}{2}$.

$$\therefore -\frac{\pi}{2} < -\phi < 0$$

$$\therefore \quad -\frac{\pi}{2} < \theta - \phi < \frac{\pi}{2} \qquad \dots (1)$$

Also
$$\tan(\theta - \phi) = \frac{\tan\theta - \tan\phi}{1 + \tan\theta \tan\phi} = \frac{x - y}{1 + xy}$$
 ... (2)

From (1) and (2) we get
$$\theta - \phi = \tan^{-1} \left(\frac{x - y}{1 + xy} \right)$$

$$\therefore \tan^{-1} x - \tan^{-1} y = \tan^{-1} \left(\frac{x - y}{1 + xy} \right)$$

Ex. (3) Prove that for all $x \in \mathbb{R}$

(a)
$$\cot^{-1}(-x) = \pi - \cot^{-1}(x)$$

(b)
$$\tan^{-1}(-x) = -\tan^{-1}(x)$$

Solution : (a) To prove that $\cot^{-1}(-x) = \pi - \cot^{-1}(x)$

Let
$$\cot^{-1}(-x) = \theta$$

$$\therefore \cot \theta = . \quad ? \quad \text{Where} \quad -x \in R, 0 < \theta < \pi$$

$$\therefore -\cot \theta = . \quad \chi$$

$$\therefore \cot(\pi - \theta) = x, x \in R$$

Since,
$$0 < \theta < \pi$$

$$\therefore 0 < \pi - \theta < .$$
 $\boxed{1}$.

Which implies $\cot(\pi - \theta) = x$ and $x \in R, 0 < \pi - \theta < \pi$

$$\therefore \pi - \theta = \cot^{-1} x$$

$$\therefore \theta = \pi - . \cot^{-1} x$$

$$\therefore \cot^{-1}\left(-x\right) = \pi - \cot^{-1}x$$

(b) To prove that $\tan^{-1}(-x) = -\tan^{-1}(x)$

Let
$$tan^{-1}(-x) = \theta$$

$$\therefore$$
 tan. $\bigcirc = -x$ where $-x \in \mathbb{R}, -\frac{\pi}{2} < . \bigcirc . < \frac{\pi}{2}$

$$:-tano=x$$

$$\therefore \tan(. - . \Theta_{\cdot}) = x$$

$$\therefore \quad \tan \theta = x, \qquad x \in R \text{ and } -\frac{\pi}{2} < -\theta < \frac{\pi}{2}$$

$$\therefore - \bigcirc = \tan^{-1} x$$

-.
$$\tan^{-1} x = \tan^{-1} x$$
 | $\tan x = 0$ | \tan

$$\therefore \tan^{-1}(-x) = -\tan^{-1}x$$

Ex. (4) Prove that :
$$\tan^{-1} \left[\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right] = \frac{\pi}{4} + \theta$$
 if $\theta \in \left(-\frac{\pi}{4}, \frac{\pi}{4} \right)$

Solution:
$$L.H.S. = \tan^{-1} \left[\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right]$$

$$= \tan^{-1} \left[\frac{1 + \tan \theta}{1 - \tan \theta} \right]$$

$$= \tan^{-1} \left[\frac{\tan \left(\frac{\pi}{2 + 1} \right) + \tan \theta}{1 - \tan \left(\frac{\pi}{2 + 1} \right) \tan \theta} \right]$$

We have,
$$\tan^{-1}(\tan \theta) = \theta$$
 for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. - - - - - - (2)

Since
$$-\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

$$\therefore -\frac{\pi}{4} + \frac{\pi}{4} < \theta + \frac{\pi}{4} < \frac{\pi}{4} + \frac{\pi}{4}$$

$$\therefore \quad \bigcirc \quad <\theta+\frac{\pi}{4}<\quad \boxed{1}$$

From equation (1) we get,

$$L.H.S. = \tan^{-1} \left[\tan \left(\frac{\pi}{4} + \theta \right) \right]$$

$$= \frac{\pi}{4} + \Theta \quad \text{From equation (2)}.$$

Thus,
$$\tan^{-1} \left[\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right] = \frac{\pi}{4} + \theta$$
 for $\theta \in \left(-\frac{\pi}{4}, \frac{\pi}{4} \right)$

(LHS = SIM (SIM (-11-20))

KHESSETH THE SERVE **Ex.** (5) If $\sin^{-1}(1-x)-2\sin^{-1}x = \frac{\pi}{2}$ then find the value of x.

Caluston 2HA = 2HA 1	siny =0 (07) 2 siny-1=0
$\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$	$Siny = 0 og Siny = \frac{1}{2}$
Put x= siny	" X=0 01 X=1 " X=Siny
$sin^{-1}(1-siny)-2sin^{-1}(siny)=\frac{\pi}{2}$: $sin^{-1}(1-siny)=\frac{\pi}{2}+2y$	Let $n=\frac{1}{2}$
: $1-\sin y = \sin \left(\frac{\pi}{2} + 2y\right)$	LHS = Sin (1-x) - 25in x
	$= \sin^{1}(1-\frac{1}{2})-2\sin^{1}(\frac{1}{2})$
$1-\sin y = \cos 2y$ $1-\sin y = 1-2\sin^2 y$	$= \sin^{1}(\frac{1}{2}) - 2 \sin^{1}(\frac{1}{2})$
1-siny-1+2sin2y=0	$= \frac{\pi}{6} - 2x \frac{\pi}{6} $ $= -\frac{\pi}{6}$ $= -\frac{\pi}{6}$ is not sol ⁿ $= -\frac{\pi}{6}$ is not sol ⁿ
$2\sin^2 y - \sin y = 0$	$= -\frac{\pi}{2}$ is not sol
siny. (.2siny-1).=0	LHS + RHS 10000 X = 0
Ex. (6) Show that $\sin^{-1}(2x\sqrt{1-x^2}) = -2\pi +$	$-2\cos^{-1}x$ if $-1 \le x \le -\frac{1}{\sqrt{2}}$

Solution:	- T ≤20≤T	
Solution: we know that	Here n = 20 does not	$-1 \leq \chi \leq -\frac{1}{2}$
sin" (sinx)=x,=1/2 (x < 1/2	Satisfy - 7 < x < 7	-sin # <sino<-sin#< td=""></sino<-sin#<>
LHS = Sin (2x 11-x2 -(1)		2 - 4
put x=sino => 0=sin'x	we have to find the	$\frac{\pi}{2} < \theta < -\frac{\pi}{4}$
-15251, -726057	value of x tox which	MIN 2
LHS = sin1 (2sine sin2)	-프 <x<코< td=""><td>- TI 120 5- T</td></x<코<>	- TI 120 5- T
= 3in (25ino Jcos20)	. LHs = sin (sinea)	T < -20 < T
= sin (2 sin 0 · (050)		
= sin (sin 20)	= sin [-sin(1+20)]	
where - 1 40 5 7	= sin [sin [-(11+20)]]	
multiplying.by.2bis	= sin [sin (- T-20)]	igië .
	25	

- T+ T-205-17+17.	LHS = - 11-2515 x
- 1 ≤ - 11-20 ≤ 6	$= - \pi - 2 \left(\frac{\pi}{2} - \cos \lambda \right)$
: LHS = Sin (sin (-71-20))	$("sin'x + cos'x = \frac{\pi}{2})$
where - 1 < - 11 - 20 < 0	$LHS = -\Pi - \Pi + 2\cos^2 x$
= 1-20 from (I)	$= -2 \pi + 2 \cos^{3} x$
	= xe ni 2 s - (x-1) ni 2
Ex. (7) Prove that : $\tan^{-1} \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = -\frac{\theta}{2}$,	if $\theta \in (-\pi, 0)$
T = 10 10 1 C = (Pro	sin (+-sing) -22in (si
	: LHS = tan (- tan 0)
(1) mas = (1) 1 (1) (1) (1) (1) (1) (1) (1) (1) (=-tan' (tan @)
$= \tan^{-1}\left(\frac{12\sin^2\frac{\omega}{2}}{2\cos^2\frac{\omega}{2}}\right)$: tan -n = - tan u
$= \tan^{-1}\left(\sqrt{\tan^2 \frac{\sigma}{2}}\right)$	0-600-600
0 =)0.00	LHS = - 2
$= \tan^{-1}\left(\pm \tan \frac{0}{2}\right)$: LHS = RHS
Since (-II,0)	
	Solution :
dividing by 2 on b.s.	Sin (sinot) = 0 = 2 (ot (2) LHS = Sin (201 11- x2 = (1)
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2 11 In TV quadrant	- ا هـ عدوا نِ سَلِيْ وَهُ وَ عِلْ اللَّهِ اللَّهِ عَلَيْهِ اللَّهِ اللَّهِ عَلَيْهِ اللَّهِ اللَّهِ اللَّهِ ا
3-> 23 × F	
THE SIN CHARLES THE STATE OF THE	C. Consent MONAL MARKET
-7 1 2 CO mid	= sin (sin 20)
Sign of Tagebox:	
Sign of Teacher:	multiplying by 2 bs





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