16. Random Variables

A. Activities :

1) Given p. d. f. of a continuous r. v. X. as $f(x) = \frac{x^2}{3}$, $-1 < x < 2$

$= 0$, otherwise

$\therefore$ c. d. f. of X is given by

$$F(x) = \int_{\boxed{-1}}^{x} f(y)dy$$

$$= \int_{-1}^{\boxed{x}} \frac{y^2}{3} dy = \left[\frac{y^3}{\boxed{9}}\right]_{-1}^{x}$$

$$F(x) = \frac{x^3}{\boxed{9}} + \frac{\boxed{1}}{9} \quad \forall x \in R$$

Now, $P(X<1) = F(1) = \frac{(1)^3}{9} + \frac{\boxed{1}}{\boxed{9}} = \frac{2}{9}$

$P(X < -2) = \boxed{F(-2)}$

$$P(X > 0) = 1 - P(\boxed{X \leq 0})$$

$$= \boxed{1} - F(0)$$

$$= 1 - \left(\frac{\boxed{0}}{9} + \frac{1}{\boxed{9}}\right) = \frac{\boxed{8}}{9}$$

$$= \boxed{8} /9$$

2) Given the function

$$f(x) = \frac{x^2}{3} \quad , -1 < x < 2$$

$$= 0 \quad , \text{ otherwise}$$

(i) $f(x)$ is the p. d. f. of r. v. X if

(a) $f(x) \boxed{\geq} 0.\ \forall X \in R$ and

(b) $\int_{-1}^{2} f(x)dx = \boxed{1}$

$\because f(x) = \frac{x^2}{3}, f(x) \leq \boxed{1} \ \forall x \in R$

Now, $\int_{-1}^{2} f(x)dx = \int_{-1}^{2} \frac{x^2}{\boxed{3}} dx$

$$= \left[\frac{x^{\boxed{3}}}{\boxed{9}}\right]_{-1}^{2} = \frac{\boxed{8}}{9} - \frac{(-1)^3}{\boxed{9}}$$

$= \frac{8}{9} + \frac{1}{9} = \boxed{1}$

Thus, $f(x)$ is the p. d. f. of X

(ii) $P(0 < X < 1) = \int_{\boxed{0}}^{1} f(x)dx$

$= \int_{0}^{\boxed{1}} \frac{x^2}{3} \boxed{dx} = \left[\frac{\boxed{x^3}}{9}\right]_0^1 = \frac{\boxed{1}}{9}$

3)

x_i	p_i	$p_i x_i$	$X_i^2 p_i$
0	0.45	0	$\boxed{0}$
1	0.35	$\boxed{0.35}$	0.35
2	0.15	0.30	$\boxed{0.60}$
3	0.03	$\boxed{0.09}$	0.27
4	0.02	0.08	$\boxed{0.32}$
Total	$\boxed{1}$	0.82	$\boxed{1.54}$

$\therefore E(X) = \Sigma \boxed{x_i p_i} = \boxed{0.82}$

$V(X) = \Sigma \boxed{x_i^2 p_i} - (\Sigma x_i \ p_i)^{\boxed{2}}$

$= \boxed{1.54} - (0.82)^2$

$= 1.54 - \boxed{0.6724} = \boxed{0.8676}$

4) X is a discrete r. v. with p. m. f.

x	0	1	2	3	0
P (X = x)	0.1	0.2	0.3	0.15	0.25

(i) $P(X < 1) = P(X = \boxed{0}) = \boxed{0.1}$

(ii) $P(X \leq 3) = P(X = 3) + P(X < \boxed{3})$

$= 0.15 + \boxed{0.6} = 0.75$

(iii) $P(1 < X < 4) = P(X = \boxed{2}) + P(X = 3)$

$= 0.3 + \boxed{0.15} = \boxed{0.45}$

(iv) $P(2 \leq X \leq 3) = P(X = \boxed{2}) + P(X = \boxed{3})$

$= \boxed{0.3} + \boxed{0.15} = 0.45$

B. Solve the Following

Q.1. Three coins are tossed simultaneously . X is the number of heads .Find expected value and variance of X.

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$.

$n = 3$, $p = 1/2$, $q = 1 - p \Rightarrow q = 1/2$.

Expected Value $= E(X) = np = 3\left(\frac{1}{2}\right) = \left(\frac{3}{2}\right) = 1.5$

Variance of $X = Var(X) = npq = 3\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 0.75$

OR.

X = the no. of heads.

$\therefore$ Range set $X = \{0, 1, 2, 3\}$.

$X = x$	$P[X=x]$	$x_i p_i$	$x_i^2 p_i$
0	1/8	0	0
1	3/8	3/8	3/8
2	3/8	6/8	12/8
3	1/8	3/8	9/8
Total	1	12/8	24/8

$\therefore E(X) = \sum x_i p_i = \frac{12}{8} = 1.5$

$V(X) = \sum x_i^2 p_i - (x_i p_i)^2 = \frac{24}{8} - (1.5)^2 = 3 - 2.25 = 0.75$

Q.2. The probability distribution of X is as follows

x	0	1	2	3	4
P [X = x]	0	k	2k	2k	k

Find i) k , ii) P [X < 2] iii) P [X ≥3] iv) P [1 ≤ X < 4] v) F (2)

(i) The table gives probability distribution and $\sum_{0}^{4} p_i = 1$

$\therefore 0 + K + 2K + 2K + K = 1$.

$6K = 1$.

$\therefore K = 1/6$ or $K = 0.167$.

(ii) $P[X < 2] = P[X=0] + P[X=1]$

$= 0 + K$.

$= K$.

$= 0.167$.

(iii) $P[X \geq 3] = P[X=3] + P[X=4] = 2K + K.$

$= 3K.$

$= 3(1/6) = 0.5$

(iv) $P[1 \leq X < 4] = P[X=1] + P[X=2] + P[X=3]$

$= K + 2K + 2K.$

$= 5K.$

$= 5(1/6) = 0.833$

(v) $F(2) = P[X \leq 2] = P[X=0] + P[X=1] + P[X=2] = 0 + K + 2K$

$= 3K = 3(1/6) = 0.5$

Q.3. Find k if the following is p.d.f.of a r.v. X

$f(x) = k x^2 (1 - x)$ for $0 < x < 1$

$= 0$, otherwise.

Sol^n: Since, $f(x)$ is the P.d.f of r.v. X.

$$\int_0^1 Kx^2(1-x)\,dx = 1$$

$$\therefore K \int_0^1 (x^2 - x^3)\,dx = 1.$$

$$K\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = 1$$

$$\therefore K\left[\frac{1}{3} - \frac{1}{4} - \frac{0}{3} - \frac{0}{4}\right] = 1$$

$$\therefore K\left[\frac{1}{3} - \frac{1}{4}\right] = 1.$$

$$\therefore K\left(\frac{1}{12}\right) = 1.$$

$$\therefore \boxed{K = 12}$$

Sign of Teacher :