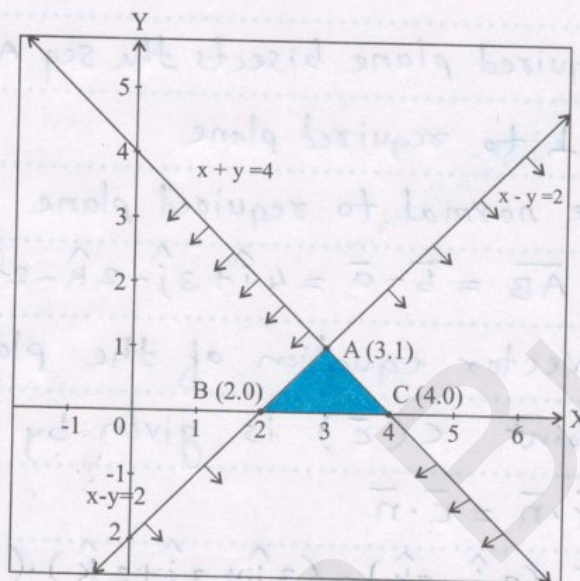


## 8. Linear Programming

**Ex. 1)** Maximize :  $Z = y - 2x$  subject to  $x - y \geq 2$ ,  $x + y \leq 4$ ,  $y \geq 0$ .

**Solution :** To draw regions  $x - y \geq 2$ ,  $x + y \leq 4$ , we begin by drawing the lines  $x - y = 2$  and  $x + y = 4$ .

Inequality	Line	Two Intercept Form	Points	Region
$x - y \geq 2$	$x - y = 2$	$\frac{x}{2} + \frac{y}{-2} = 1$	$(2, 0), (0, -2)$	Non origin side
$x + y \leq 4$	$x + y = 4$	$\frac{x}{4} + \frac{y}{4} = 1$	$(4, 0), (0, 4)$	Origin side



The feasible region is ABC, with corner points A(3,1), B(2,0) and C(4,0).

Corner Points	Value of $Z = y - 2x$
A(3,1)	$Z = 1 - 2(3) = -5$
B(2,0)	$Z = 0 - 2(2) = -4$
C(4,0)	$Z = 0 - 2(4) = -8$

The maximum value of  $Z$  is  $-4$  at  $x = 2, y = 0$

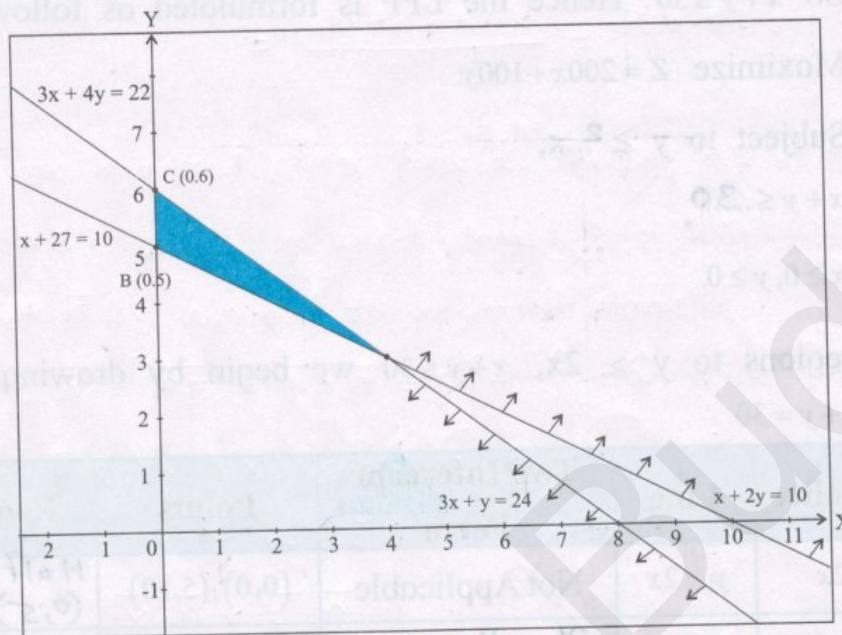
**Ex. (2)** Minimize :  $Z = 200x + 500y$  subject to  $x + 2y \geq 10$ ,  $3x + 4y \leq 24$ ,  $x \geq 0$ ,  $y \geq 0$ .



**Solution :** To draw regions  $x+2y \geq 10$ ,  $3x+4y \leq 24$ , we begin by drawing the lines

$$x+2y=10, \quad 3x+4y=24.$$

Inequality	Line	Two Intercept Form	Points	Region
$x+2y \geq 10$	$x+2y=10$	$\frac{x}{10} + \frac{y}{5} = 1$	$(10,0), (0,5)$	Non origin side
$3x+4y \leq 24$	$3x+4y=24$	$\frac{x}{8} + \frac{y}{6} = 1$	$(8,0), (0,6)$	Origin side



The feasible region is ABC, with corner points A(4,3), B(0,5) and C(0,6).

Corner Points	Value of $Z = 200x + 500y$
A(4,3)	$Z = 200(4) + 500(3) = 2300$
B(0,5)	$Z = 200(0) + 500(5) = 2500$
C(0,6)	$Z = 200(0) + 500(6) = 3000$

The minimum value of  $Z$  is 2300 at  $x=4, y=3$ .

**Ex. (3)** A carpenter makes tables and chairs. Profit per table is Rs. 200 and that per chair is Rs. 100. He should make at least two chairs per table and the total number of tables and chairs should not exceed 30. Find the maximum profit.



**Solution:** Let  $x$  be number of tables and  $y$  be number of chairs that are to be made by the carpenter. That is  $x \geq 0, y \geq 0$ .

Since cost of a table is Rs. 200 and cost of a chair is Rs. 100,

$$Z = 200x + 100y$$

Total profit is....., which is to be maximized.

As at least (more than or equal) 2 chairs per table

i.e. Number of chairs  $\geq 2$ (Number of tables) i.e  $y \geq 2x$ .

Also as the total number of tables and chairs should not exceed (less than or equal) 30,

So  $x + y \leq 30$ . Hence the LPP is formulated as follows

Maximize  $Z = 200x + 100y$

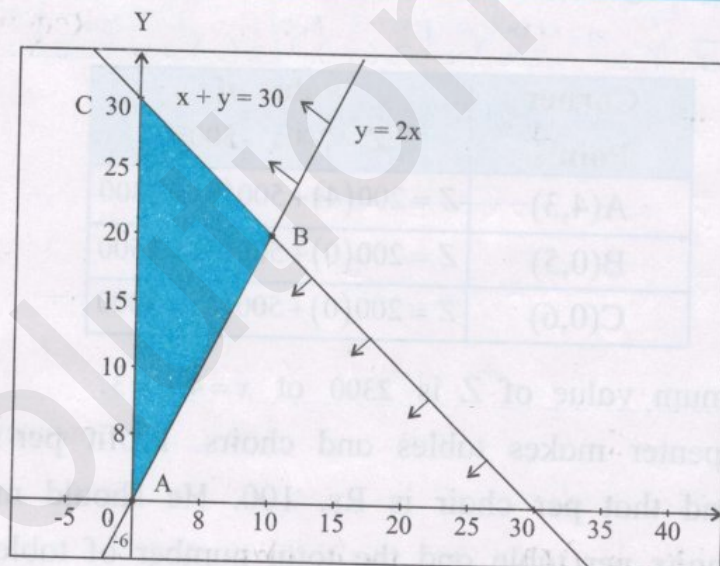
Subject to  $y \geq 2x$ ,

$$x + y \leq 30,$$

$$x \geq 0, y \geq 0$$

To draw regions to  $y \geq 2x$ ,  $x + y \leq 30$  we begin by drawing the lines  $y = 2x$ ,  $x + y = 30$

Inequality	Line	Two Intercept Form	Points	Region
$y \geq 2x$	$y \geq 2x$	Not Applicable	$(0,0), (5,10)$	Half plane $(0,5) \dots$
$x + y \leq 30$	$x + y = 30$	$\frac{x}{30} + \frac{y}{30} = 1$	$(30,0) \dots (0,30)$	Origin



The feasible region is ABC with corner points A(0,0), B(10,20) and C(0,30)



Corner Points	Value of $Z = 200x + 100y$
A(0,0),	0
B(10,20)	4000
C(0,30)	3000

The maximum value of  $Z$  is 4000 at  $x = 10, y = 20$

**Ex.4)** A chemical company produces a chemical containing three basic elements A, B, C, so that it has at least 16 liters of A, 24 liters of B and 18 liters of C. This chemical is made by mixing two compounds. Each unit of compound I, has 4 liters of A, 12 liters of B and 2 liters of C. Each unit of compound II, has 2 liters of A, 2 liters of B and 6 liters of C. The cost per unit of compound I is Rs. 800 and that of compound II is Rs. 640. Formulate the problems as LPP and solve it to minimize the cost.

**Solution :** Let the company produce  $x$  units of compound I and  $y$  units of compound II.

Then the total cost is  $z = 800x + 640y$ , this is objective function which I to be minimized.

The given information about constraints can be tabulated as follows

	Compound I (x)	Compound II (y)	Minimum Requirement
Element A	4	2	16
Element B	12	2	24
Element C	2	6	18

From the table, the constraint are  $4x + 2y \geq 16, 12x + 2y \geq 24, 2x + 6y \geq 18$ . Also

The LPP is formulated as follows.

Minimize  $Z = 800x + 640y$



Subject to

$$4x + 2y \geq 16$$

$$12x + 2y \geq 24$$

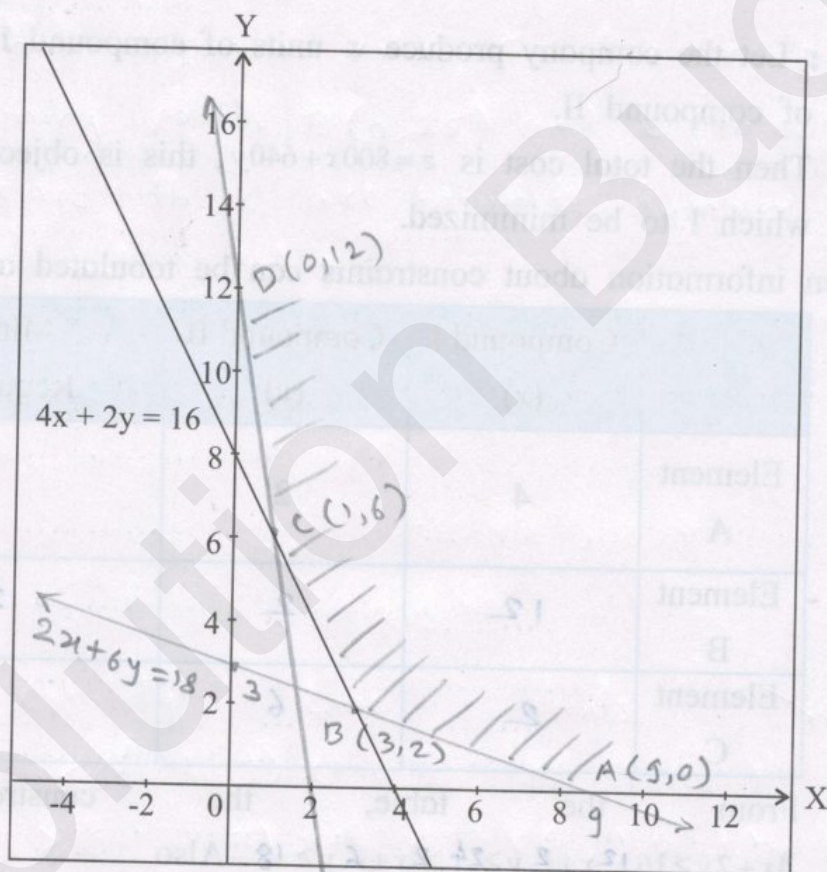
$$2x + 6y \geq 18$$

$$x, y \geq 0$$

To draw regions  $4x + 2y \geq 16$ ,  $12x + 2y \geq 24$ ,  $2x + 6y \geq 18$

We begin by drawing the lines  $4x + 2y = 16$ ,  $12x + 2y = 24$ ,  $2x + 6y = 18$

Inequality	Line	Two Intercept Form	Points	Region
$4x + 2y \geq 16$	$4x + 2y = 16$	$\frac{x}{4} + \frac{y}{8} = 1$	$(4, 0), (0, 8)$	Non origin side
$12x + 2y \geq 24$	$12x + 2y = 24$	$\frac{x}{2} + \frac{y}{12} = 1$	$(2, 0), (0, 12)$	Non origin side
$2x + 6y \geq 18$	$2x + 6y = 18$	$\frac{x}{9} + \frac{y}{3} = 1$	$(9, 0), (0, 3)$	Non origin side



The feasible region is ABCD with corner points A(9, 0), B(3, 2), C(1, 6), D(0, 12),



Corner Points	Value of $z = 800x + 640y$
A(1, 0),	.....7200.....
B(3, 2)	....3680.....
C(1, 6)	....4640.....
D(0, 12)	....7680.....

The minimum value of  $Z$  is 3680 at  $x=3, y=2$ .

**Ex. (5)** Minimize :  $Z = x + 2y$   
 subject to  $x + 2y \geq 50$ ,  
 $2x - y \leq 0$ ,  
 $2x + y \leq 100$ ,  
 $x \geq 0, y \geq 0$

**Solution :**

Inequality	Line	Points	Region
$x + 2y \geq 0$	I) $x + 2y = 50$	A(50, 0)	Non origin
	<del><math>2x - y = 0</math></del>	B(0, 25)	side
$2x - y \leq 0$	II) $2x - y = 0$	O(0, 0)	Half plane
		C(10, 20)	containing (0, 20)
$2x + y \leq 100$	III) $2x + y = 100$	A(50, 0)	origin
		D(0, 100)	side

The feasible region is BCPDB

Solving (II) and (III) we get  $x=25, y=50$

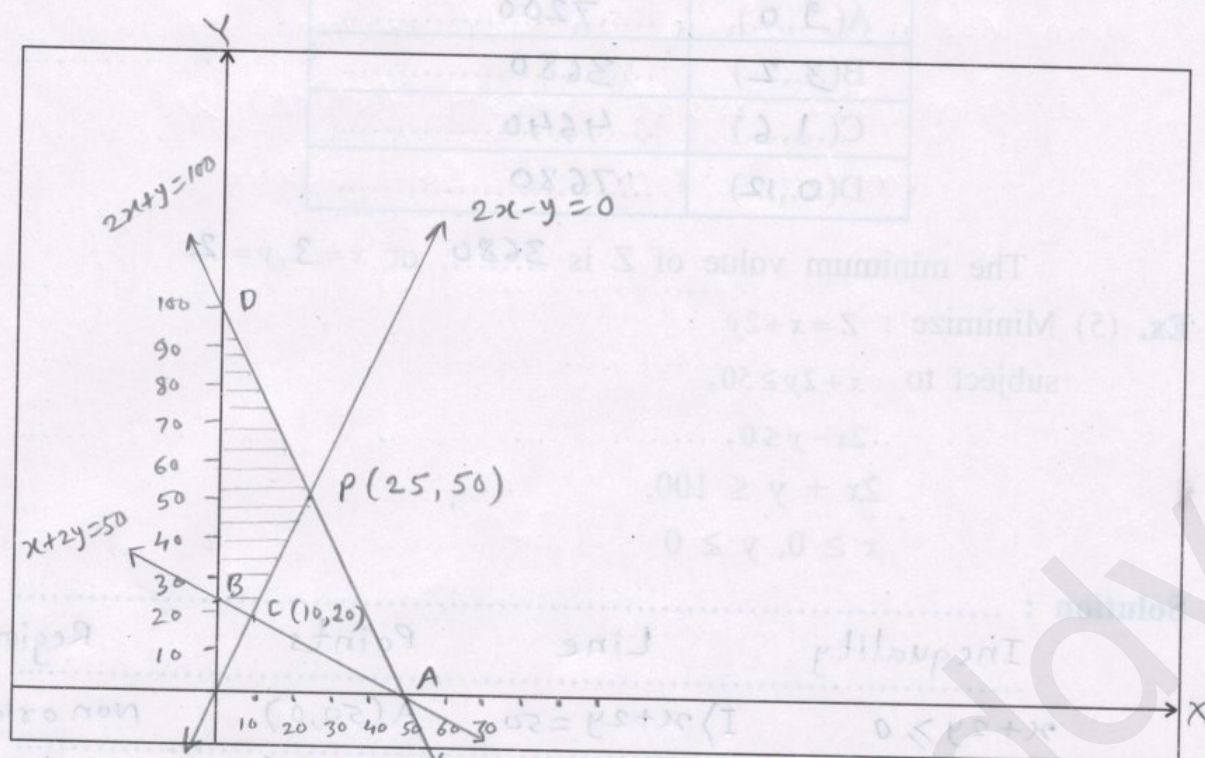
$\therefore P \equiv (25, 50)$

The corner points are B, C, P, D

corner points	value $z = x + 2y$
B(0, 25)	50
C(10, 20)	50
P(25, 50)	125
D(0, 100)	200



$Z$  has minimum value 50 at two consecutive vertices B and C



$\therefore Z$  has minimum value 50 at every point of seg joining points B(0, 25) and C(10, 20)

Hence there are infinite number of optimal solutions

**Ex. (6)** Maximize :  $Z = 3x + 9y$

subject to  $x + 3y \leq 60$ ,

$x + y \geq 10$ ,

$x \leq y$ ,

$x \geq 0, y \geq 0$ .

**Solution :**

Inequality	Line	Points	Region
$x + 3y \leq 60$	I) $x + 3y = 60$	A(60, 0), B(0, 20)	Origin side
$x + y \geq 10$	II) $x + y = 10$	C(10, 0), D(0, 10)	Non origin side
$x \leq y$	III) $x = y$	O(0, 0), E(30, 30)	Half plane containing (0, 30)



The Feasible region is PQBDP

solving (II) & (III),  $x=5, y=5$

$$\therefore P \equiv (5, 5)$$

Solving (I) & (III),  $x=15, y=15$

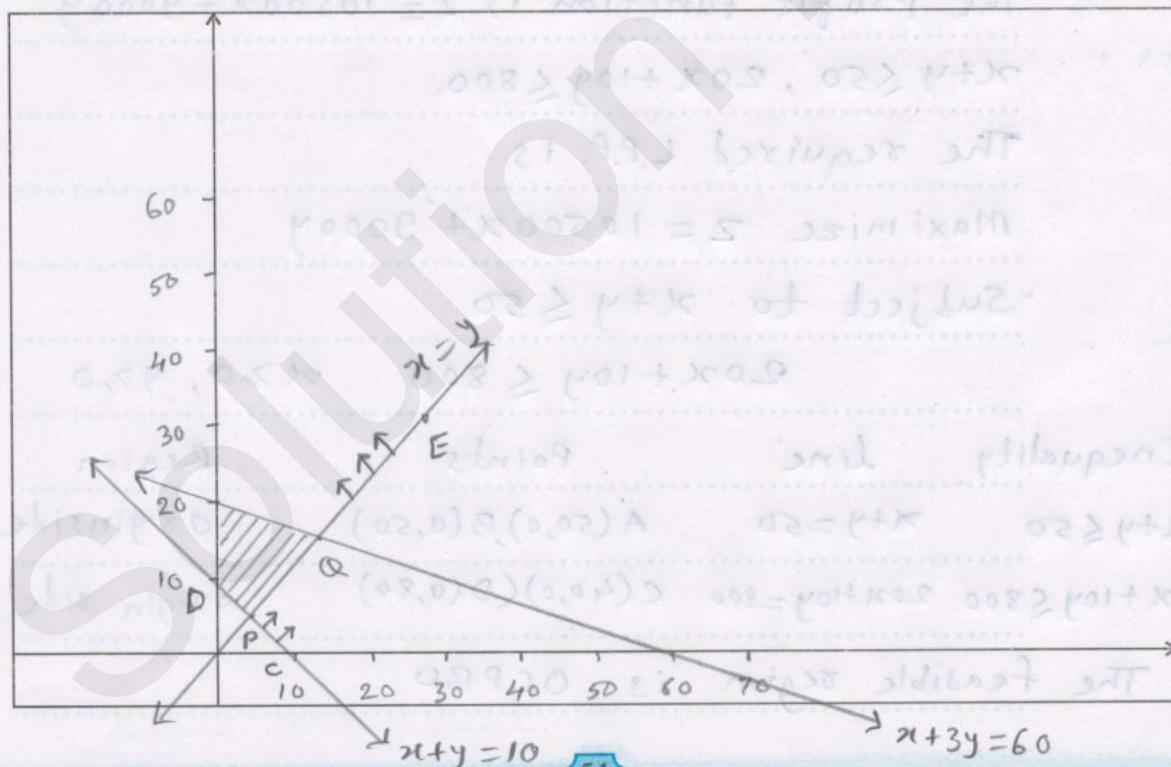
$$\therefore Q \equiv (15, 15)$$

The corner points are P, Q, B, D

Vertex	Value $z = 3x + 9y$
P(5, 5)	60
Q(15, 15)	180
B(0, 20)	180
D(0, 10)	90

$\therefore z$  has maximum value 180 at

two consecutive points B and Q





$\therefore z$  has maximum value 180 at every point of segment joining points  $B(0,20)$  and  $Q(15,15)$  Hence there are infinite number of optimal solutions.

**Ex. (7)** A co-operative society of farmers has 50 hectare of land to grow two crops X and Y. The profit from crops X and Y per hectare are estimated as Rs. 10,500 and RS. 9,000 respectively. To control weeds a liquid herbicide has to be used for crops X and Y at rates of 20 liter and 10 liter per hectare. Further no more than 800 liter of herbicide should be used in order to protect fish and a wild life using the pond which collects draining from this land. How much land should be allocated to each crop so as to maximize the total profit of the society ?

**Solution :**

Let  $x$  and  $y$  hectare land be allocated to crops X and Y respectively  $x \geq 0, y \geq 0$

The profit function is  $z = 10500x + 9000y$

$x + y \leq 50, 20x + 10y \leq 800$

The required L.P.P. is

Maximize  $z = 10500x + 9000y$

Subject to  $x + y \leq 50$

$20x + 10y \leq 800 \quad x \geq 0, y \geq 0$

Inequality	line	Points	Region
$x + y \leq 50$	$x + y = 50$	$A(50,0), B(0,50)$	origin side
$20x + 10y \leq 800$	$20x + 10y = 800$	$C(40,0), D(0,80)$	origin side

The feasible region is OCPBO

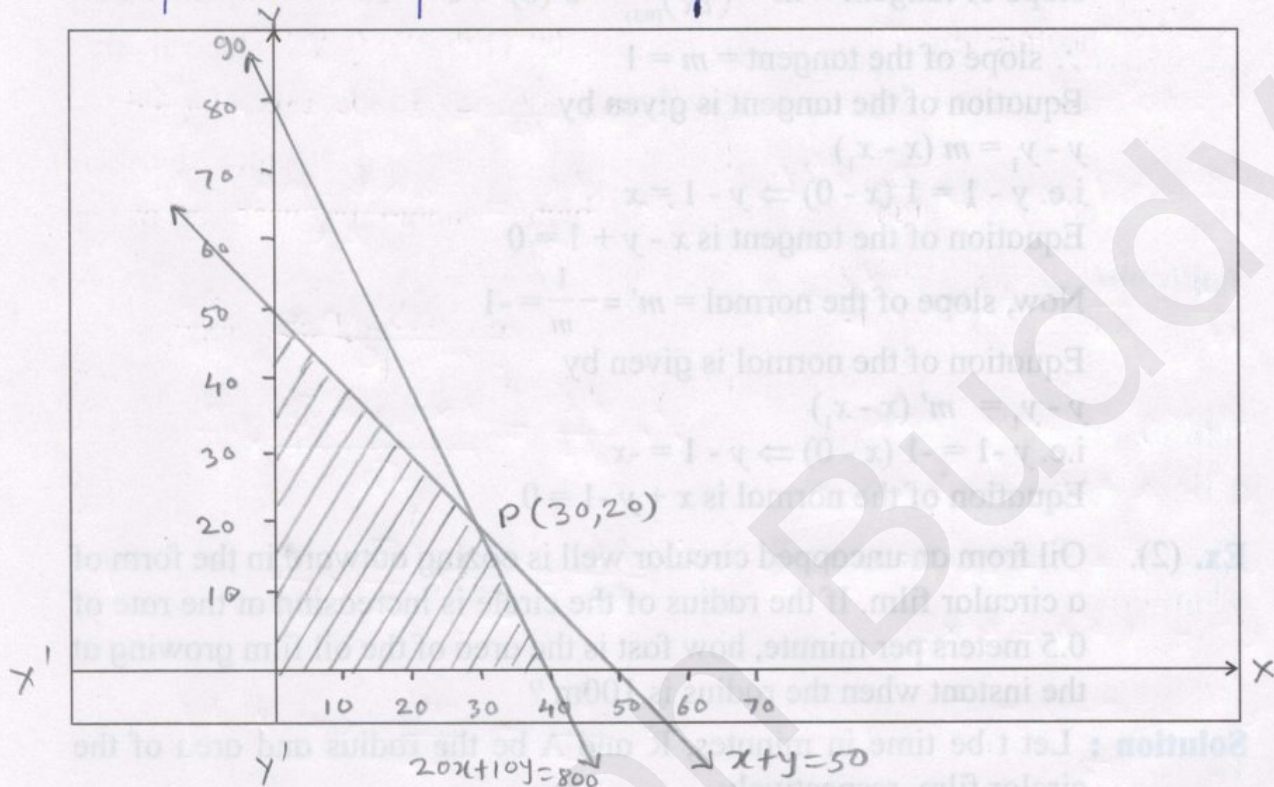


Solving  $x+y=50$  and  $20x+10y=800$  i.e.  $x+y=80$

$$\therefore x=30, y=20 \quad \therefore P \equiv (30, 20)$$

The corner points are O, C, P, B

Vertex	value	$z = 10500x + 9000y$
O(0,0)	0	
C(40,0)	420000	
P(30,20)	495000	
B(0,50)	450000	



$\therefore$  The maximum value of  $z$  is 495000 at  $x=30$  and  $y=20$

$\therefore$  30 hectares and 20 hectares Land should be allocated to crop  $x$  and  $y$  resp. for maximum profit ₹ 495000

Sign of Teacher :



- Q. 26.** A solenoid of length  $\pi$  m and 5 cm in diameter has a winding of 1000 turns and carries a current of 5A. Calculate the magnetic field at its centre along the axis.

## SECTION – D

**Attempt any THREE questions of the following :**

**[12]**

- Q. 27.** What is Ferromagnetism? Explain it on the basis of domain theory.
- Q. 28.** Obtain an expression for average power dissipated in a series LCR circuit.
- Q. 29.** Distinguish between interference and diffraction of light.  
A double slit arrangement produces interference fringes for sodium light of wavelength 589 nm, that are 0.20 degree apart. What is the angular fringe separation if the entire arrangement is immersed in water?  
( R.I. of water = 1.33 )
- Q. 30.** State Einstein's photoelectric equation and mention physical significance of each term involved in it.  
The wavelength of incident light is  $4000\text{\AA}$ . Calculate the energy of incident photon.
- Q. 31.** State any four uses of Van de Graaff generator.  
In a parallel plate air capacitor, intensity of electric field is changing at the rate of  $2 \times 10^{11}$  V/ms. If area of each plate is  $20\text{ cm}^2$ , calculate the displacement current.





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