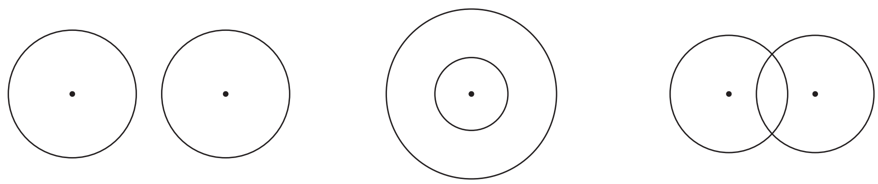
3 Circle

Let's study.

- Circles passing through one, two,three points
- Circles touching each other
- Inscribed angle and intercepted arc
- Secant tangent angle theorem
- Secant and tangent
- Arc of a circle
- Cyclic quadrilateral
- Theorem of intersecting chords

Let's recall.

You are familiar with the concepts regarding circle, like - centre, radius, diameter, chord, interior and exterior of a circle. Also recall the meanings of - congruent circles, concentric circles and intersecting circles.



congruent circles concentric circles intersecting circles

Recall the properties of chord studied in previous standard and perform the activity below.

Activity I : In the adjoining figure, seg DE is a chord of a circle with centre C. seg CF $\perp$ seg DE. If diameter of the circle is 20 cm, DE =16 cm find CF.

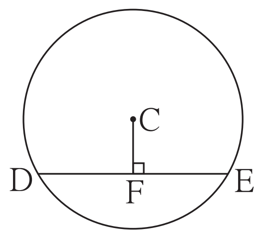


Fig. 3.1

Recall and write theorems and properties which are useful to find the solution of the above problem.

(1) The perpendicular drawn from centre to a chord ________________

(2) ________________

(3) ________________

Using these properties, solve the above problem.

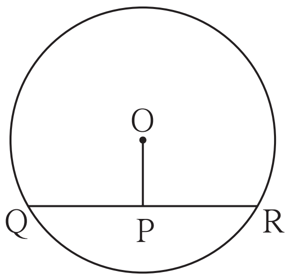


Fig. 3.2

Activity II : In the adjoining figure, seg QR is a chord of the circle with centre O. P is the midpoint of the chord QR. If QR = 24, OP = 10 , find radius of the circle.

To find solution of the problem, write the theorems that are useful.

(1) ______________________________

(2) ______________________________

Using these theorems solve the problems.

Activity III : In the adjoining figure, M is the centre of the circle and
seg AB is a diameter.
seg MS $\perp$ chord AD
seg MT $\perp$ chord AC
$\angle DAB \cong \angle CAB$.
Prove that : chord AD $\cong$ chord AC.

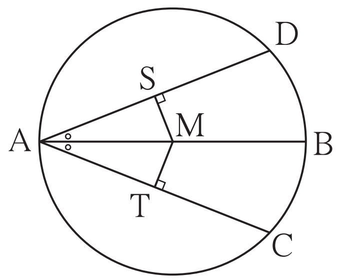


Fig. 3.3

To solve this problem which of the following theorems will you use ?

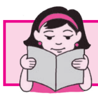
(1) The chords which are equidistant from the centre are equal in length.

(2) Congruent chords of a circle are equidistant from the centre.

Which of the following tests of congruence of triangles will be useful?

(1) SAS, (2) ASA, (3) SSS, (4) AAS, (5) hypotenuse-side test.

Using appropriate test and theorem write the proof of the above example.



Let's learn.

Circles passing through one, two, three points

In the adjoining figure, point A lies in a plane. All the three circles with centres P, Q, R pass through point A. How many more such circles may pass through point A ?

If your answer is many or innumerable, it is correct.

Infinite number of circles pass through a point.

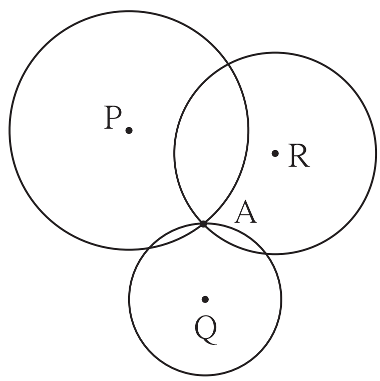


Fig. 3.4

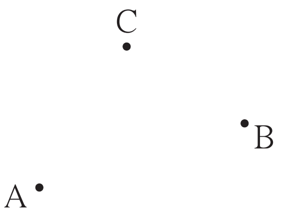


Fig. 3.5

In the adjoining figure, how many circles pass through points A and B ?

How many circles contain all the three points A, B, C ?

Perform the activity given below and try to find the answer.

ActivityI: Draw segment AB. Draw perpendicular bisector l of the segment AB. Take point P on the line l as centre, PA as radius and draw a circle. Observe that the circle passes through point B also. Find the reason. (Recall the property of perpendicular bisector of a segment.)

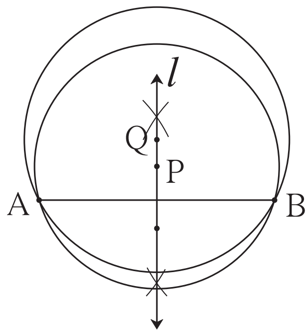


Fig. 3.6

Taking any other point Q on the line l, if a circle is drawn with centre Q and radius QA, will it pass through B ? Think.

How many such circles can be drawn, passing through A and B ? Where will their centres lie ?

Activity II : Take any three non-collinear points. What should be done to draw a circle passing through all these points ? Draw a circle passing through these points.
Is it possible to draw one more circle passing through these three points ? Think of it.

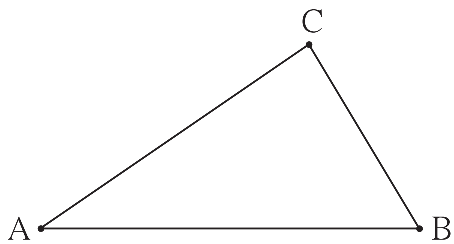


Fig. 3.7

Activity III : Take 3 collinear points D, E, F. Try to draw a circle passing through these points. If you are not able to draw a circle, think of the reason.

Let's recall.

(1) Infinite circles pass through one point.

(2) Infinite circles pass through two distinct points.

(3) There is a unique circle passing through three non-collinear points.

(4) No circle can pass through 3 collinear points.

Let's learn.

Secant and tangent

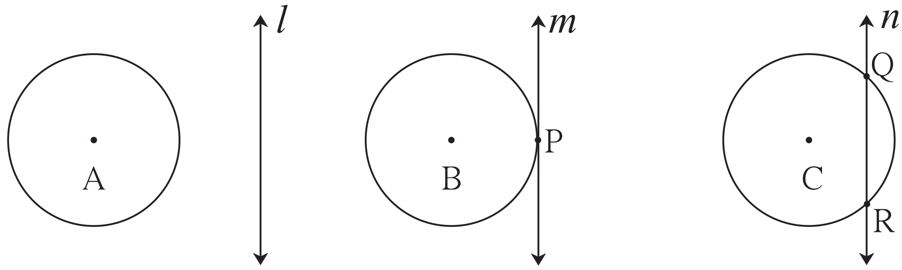


Fig. 3.8

In the figure above, not a single point is common in line l and circle with centre A.

Point P is common to both, line m and circle with centre B. Here, line m is called *a tangent* of the circle and point P is called the point of contact.

Two points Q and R are common to both, the line n and the circle with centre C.

Q and R are intersecting points of line n and the circle. Line n is called *a secant* of the circle .

Let us understand an important property of a tangent from the following activity.

Activity :

Draw a sufficiently large circle with centre O. Draw radius OP. Draw a line AB $\perp$ seg OP. It intersects the circle at points A, B. Imagine the line slides towards point P such that all the time it remains parallel to its original position. Obviously, while the line slides, points A and B approach each other along the circle. At the end, they get merged in point P, but the angle between the radius OP and line AB will remain a right angle.

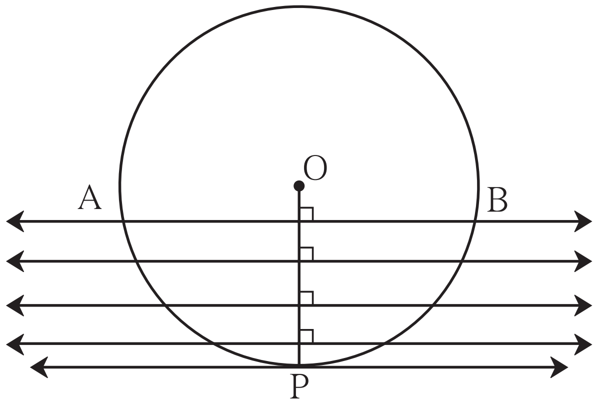


Fig. 3.9

At this stage the line AB becomes a tangent of the circle at P.

So it is clear that, the tangent at any point of a circle is perpendicular to the radius at that point.

This property is known as 'tangent theorem'.

Tangent theorem

Theorem : A tangent at any point of a circle is perpendicular to the radius at the point of contact.

There is an indirect proof of this theorem.

For more information

Given : Line l is a tangent to the circle with centre O at the point of contact A.

To prove : line $l \perp$ radius OA.

Proof : Assume that, line l is not perpendicular to seg OA.

Suppose, seg OB is drawn perpendicular to line l.

Of course B is not same as A.

Now take a point C on line l such that A-B-C and BA = BC .

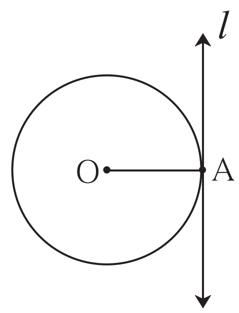


Fig. 3.10

Now in, Δ OBC and Δ OBA

seg BC $\cong$ seg BA .... (construction)

$\angle$ OBC $\cong$ $\angle$ OBA .... (each right angle)

seg OB $\cong$ seg OB

$\therefore$ Δ OBC $\cong$ Δ OBA .... (SAS test)

$\therefore$ OC = OA

But seg OA is a radius.

$\therefore$ seg OC must also be radius.

$\therefore$ C lies on the circle.

That means line l intersects the circle in two distinct points A and C.

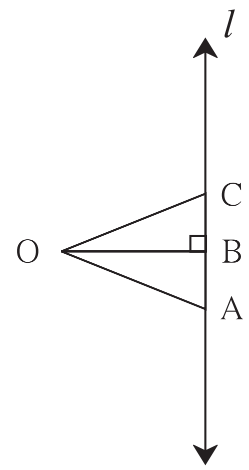


Fig. 3.11

But line l is a tangent. .... (given)

$\therefore$ it intersects the circle in only one point.

Our assumption that line l is not perpendicular to radius OA is wrong.

$\therefore$ line $l \perp$ radius OA.

Let's recall.

Which theorems do we use in proving that hypotenuse is the longest side of a right angled triangle?

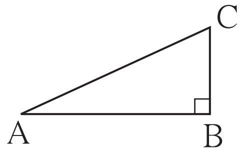


Fig. 3.12

Let's learn.

Converse of tangent theorem

Theorem: A line perpendicular to a radius at its point on the circle is a tangent to the circle.

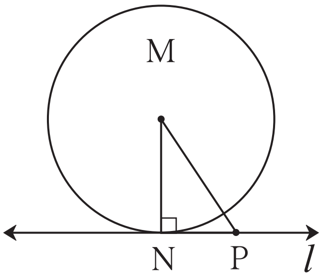


Fig. 3.13

Given : M is the centre of a circle seg MN is a radius.

Line $l \perp$ seg MN at N.

To prove : Line l is a tangent to the circle.

Proof : Take any point P, other that N, on the line l. Draw seg MP.

Now in Δ MNP, $\angle$ N is a right angle.

$\therefore$ seg MP is the hypotenuse.

$\therefore$ seg MP > seg MN.

As seg MN is radius, point P can't be on the circle.

$\therefore$ no other point, except point N, of line l is on the circle.

$\therefore$ line l intersects the circle in only one point N.

$\therefore$ line l is a tangent to the circle.

Let's discuss.

In figure 3.14 , B is a point on the circle with centre A. The tangent of the circle passing through B is to be drawn. There are infinite lines passing through the point B. Which of them will be the tangent ? Can the number of tangents through B be more than one ?

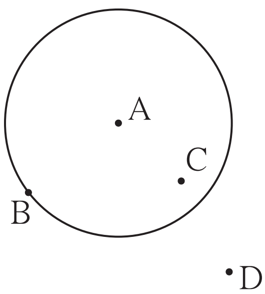


Fig. 3.14

Point C lies in the interior of the circle. Can you draw tangents to the circle through C ?

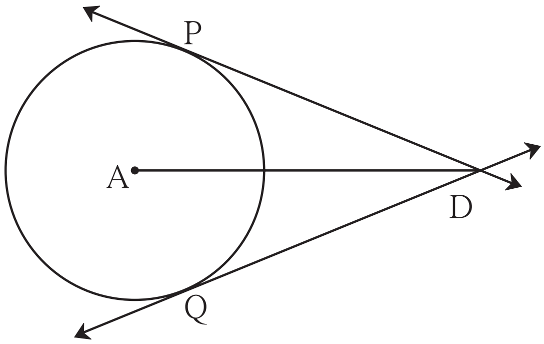


Fig. 3.15

Point D is in the exterior of the circle. Can you draw a tangent to the circle through D? If yes, how many such tangents are possible? From the discussion you must have understood that two tangents can be drawn to a circle from the point outside the circle as shown in the figure.

In the adjoing figure line DP and line DQ. touch the circle at points P and Q. Seg DP and seg DQ are called tangent segments.

Tangent segment theorem

Theorem : Tangent segments drawn from an external point to a circle are congruent.

Observe the adjoining figure. Write 'given' and 'to prove.'

Draw radius AP and radius AQ and complete the following proof of the theorem.

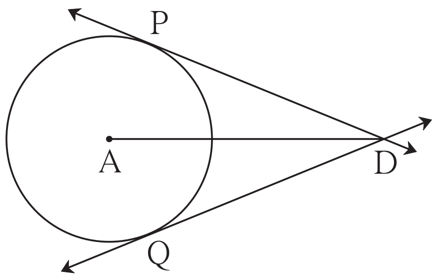


Fig. 3.16

Proof : In Δ PAD and Δ QAD,

seg PA $\cong$ ________ radii of the same circle.

seg AD $\cong$ seg AD ________________

$\angle$APD = $\angle$AQD = 90° ...... tangent theorem

$\therefore$ Δ PAD $\cong$ Δ QAD ________________

$\therefore$ seg DP $\cong$ seg DQ ________________

Solved Examples

Ex. (1) In the adjoining figure circle with centre D touches the sides of $\angle$ ACB at A and B. If $\angle$ ACB = 52°, find measure of $\angle$ ADB.

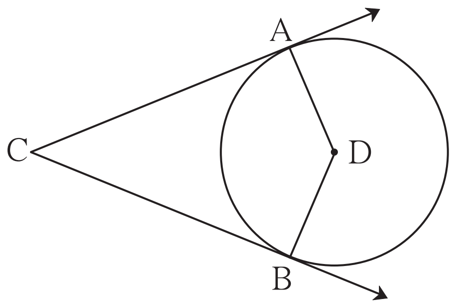


Fig. 3.17

Solution : The sum of all angles of a quadrilateral is 360°.

$\therefore$ $\angle$ ACB + $\angle$ CAD + $\angle$ CBD + $\angle$ ADB = 360°

$\therefore$ 52° + 90° + 90° + $\angle$ ADB = 360° .... Tangent theorem

$\therefore$ $\angle$ ADB + 232° = 360°

$\therefore$ $\angle$ ADB = 360° − 232° = 128°

Eg. (2) Point O is the centre of a circle. Line *a* and line *b* are parallel tangents to the circle at P and Q. Prove that segment PQ is a diameter of the circle.

Solution : Draw a line *c* through O which is parallel to line *a*. Draw radii OQ and OP.

Now, $\angle$ OPT = 90° ..... Tangent theorem

$\therefore \angle$SOP = 90°... Int. angle property ... (I)

line *a* || line *c* ..... construction

line *a* || line b ..... given

$\therefore$ line b || line c

$\therefore \angle$SOQ = 90°... Int. angle property ...(II)

$\therefore$ From (I) and (II),

$\angle$ SOP + $\angle$ SOQ = 90° + 90° = 180°

$\therefore$ ray OP and ray OQ are opposite rays.

$\therefore$ P, O, Q are collinear points.

$\therefore$ seg PQ is a diameter of the circle.

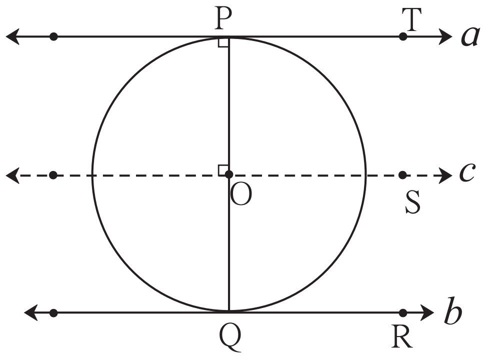
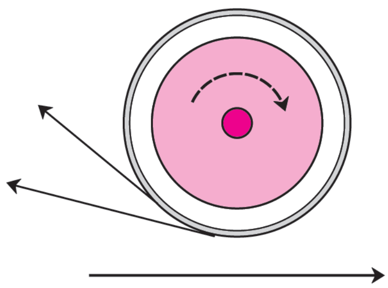


Fig. 3.18

When a motor cycle runs on a wet road in rainy season, you may have seen water splashing from its wheels. Those splashes are like tangents of the circle of the wheel. Find out the reason from your science teacher.

Observe the splinters escaping from a splintering wheel in Diwali fire works and while sharpning a knife. Do they also look like tangents ?


Direction of running wheel

Remember this !

(1) Tangent theorem : The tangent at any point of a circle is perpendicular to the radius through the point of contact.

(2) A line perpendicular to a radius at its point on the circle, is a tangent to the circle.

(3) Tangent segments drawn from an external point to a circle are congruent.

Practice set 3.1

1. In the adjoining figure the radius of a circle with centre C is 6 cm, line AB is a tangent at A. Answer the following questions.

(1) What is the measure of $\angle$CAB ? Why ?

(2) What is the distance of point C from line AB? Why ?

(3) d(A,B)= 6 cm, find d(B,C).

(4) What is the measure of $\angle$ABC ? Why ?

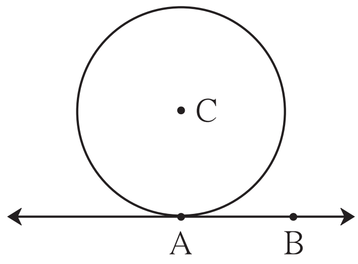


Fig. 3.19

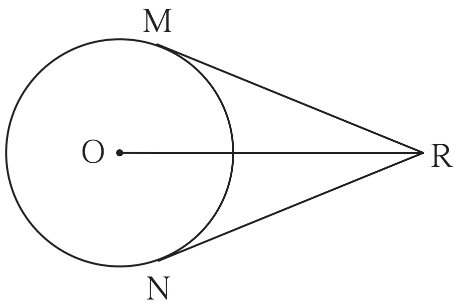


Fig. 3.20

2. In the adjoining figure, O is the centre of the circle. From point R, seg RM and seg RN are tangent segments touching the circle at M and N. If (OR) = 10 cm and radius of the circle = 5 cm, then

(1) What is the length of each tangent segment ?

(2) What is the measure of $\angle$MRO ? (3) What is the measure of $\angle$ MRN ?

3. Seg RM and seg RN are tangent segments of a circle with centre O. Prove that seg OR bisects $\angle$MRN as well as $\angle$MON.

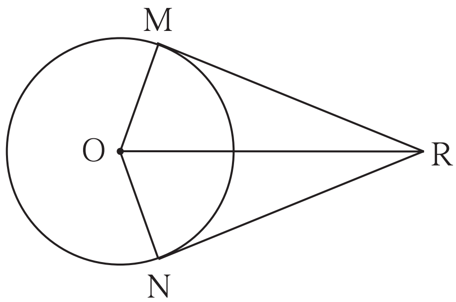


Fig. 3.21

4. What is the distance between two parallel tangents of a circle having radius 4.5 cm ? Justify your answer.

ICT Tools or Links

With the help of Geogebra software, draw a circle and its tangents from a point in its exterior. Check that the tangent segments are congruent.

Let's learn.

Touching circles

Activity I :

Take three collinear points X-Y-Z as shown in figure 3.22. Draw a circle with centre X and radius XY.

Draw another circle with centre Z and radius YZ.

Note that both the circles intersect each other at the single point Y.

Draw a line through point Y and perpendicular to seg XZ.

Note that this line is a common tangent of the two circles.

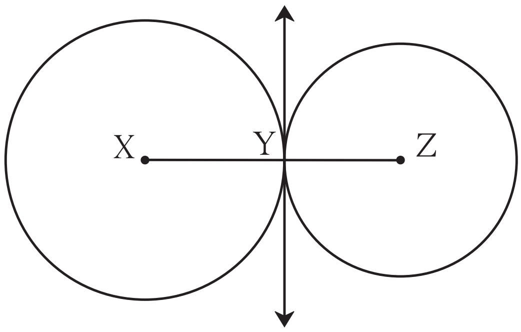


Fig. 3.22

Activity II :

Take points Y-X-Z as shown in the figure 3.23.

Draw a circle with centre Z and radius ZY.

Also draw a circle with centre X and radius XY.

Note that both the circles intersect each other at the point Y.

Draw a line perpendicular to seg YZ through point Y, that is the common tangent for the circles.

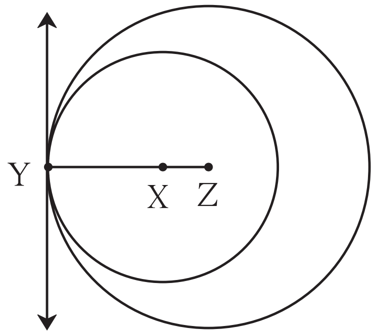


Fig. 3.23

You must have understood, the circles in both the figures are coplaner and intersect at one point only. Such circles are said to be circles touching each other.

Touching circles can be defined as follows.

If two circles in the same plane intersect with a line in the plain in only one point, they are said to be touching circles and the line is their common tangent. The point common to the circles and the line is called their common point of contact.

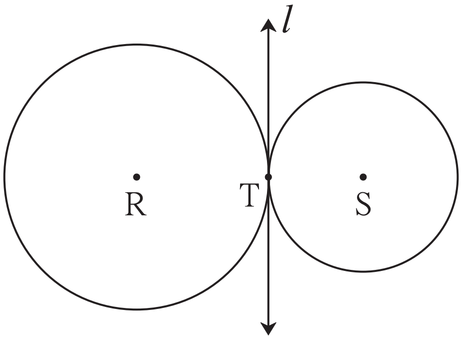


Fig. 3.24

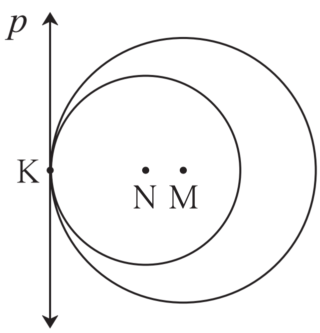


Fig. 3.25

In figure 3.24, the circles with centres R and S touch the line l in point T. So they are two touching circles with l as common tangent. They are touching externally.

In figure 3.25 the circles with centres M, N touch each other internally and line p is their common tangent.

Let's think.

(1) The circles shown in figure 3.24 are called externally touching circles. why ?

(2) The circles shown in figure 3.25 are called internally touching circles. why ?

(3) In figure 3.26, the radii of the circles with centers A and B are 3 cm and 4 cm respectively. Find -

(i) d(A,B) in figure 3.26 (a) (ii) d(A,B) in figure 3.26 (b)

Theorem of touching circles

Theorem : If two circles touch each other, their point of contact lies on the line joining their centres.

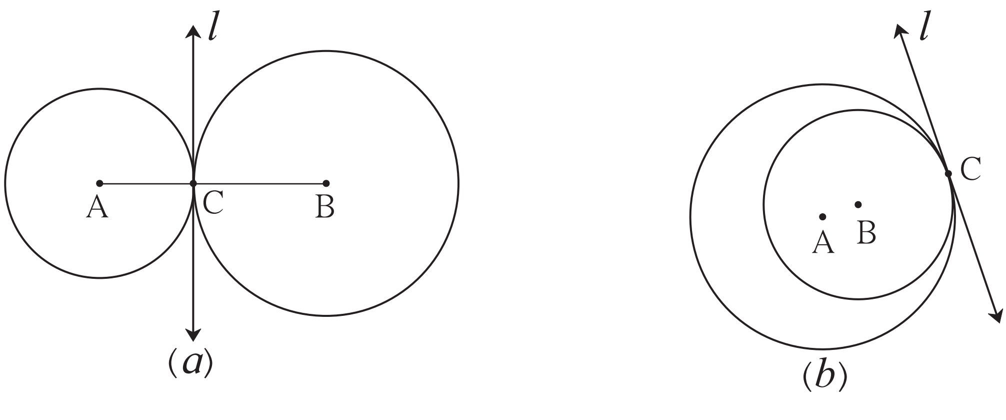


Fig. 3.26

Given : C is the point of contact of the two circles with centers A, B.

To prove : Point C lies on the line AB.

Proof : Let line l be the common tangent passing through C, of the two touching circles. line $l \perp$ seg AC, line $l \perp$ seg BC. $\therefore$ seg AC $\perp$ line l and seg BC $\perp$ line l. Through C, only one line perpendicular to line l can be drawn. $\therefore$ points C, A, B are collinear.

Remember this !

(1) The point of contact of the touching circles lies on the line joining their centres.

(2) If the circles touch each other externally, distance between their centres is equal to the sum of their radii.

(3) The distance between the centres of the circles touching internally is equal to the difference of their radii.

Practice set 3.2

1. Two circles having radii 3.5 cm and 4.8 cm touch each other internally. Find the distance between their centres.

2. Two circles of radii 5.5 cm and 4.2 cm touch each other externally. Find the distance between their centres.

3. If radii of two circles are 4 cm and 2.8 cm. Draw figure of these circles touching each other - (i) externally (ii) internally.

4. In fig 3.27, the circles with centres P and Q touch each other at R. A line passing through R meets the circles at A and B respectively. Prove that -

(1) seg AP || seg BQ,

(2) Δ APR ~ Δ RQB, and

(3) Find $\angle$ RQB if $\angle$ PAR = 35°

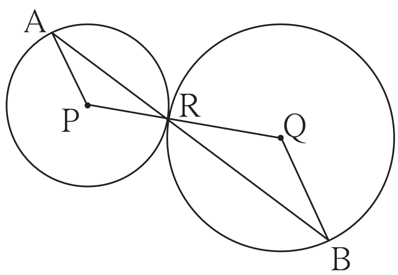


Fig. 3.27

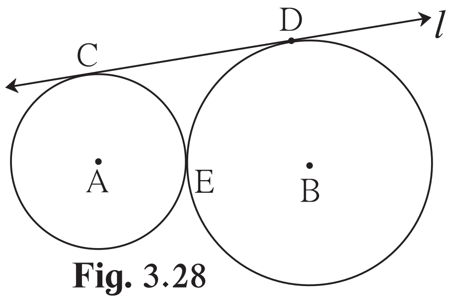


Fig. 3.28

5*. In fig 3.28 the circles with centres A and B touch each other at E. Line l is a common tangent which touches the circles at C and D respectively. Find the length of seg CD if the radii of the circles are 4 cm, 6 cm.

Let's recall.

Arc of a circle

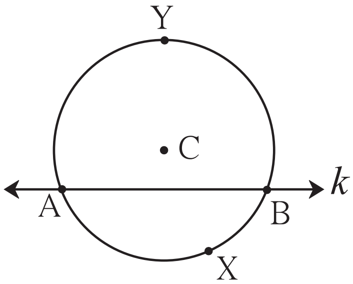


Fig. 3.29

A secant divides a circle in two parts. Any one of these two parts and the common points of the circle and the secant constitute an **arc of the circle.**

The points of intersection of circle and secant are called the end points of the arcs.

In figure 3.29, due to secant k we get two arcs of the circle with centre C– arc AYB, arc AXB.

If the centre of a circle is on one side of the secant then the arc on the side of the centre is called **'major arc'** and the arc which is on the other side of the centre is called **'minor arc'**. In the figure 3.29 arc AYB is a major arc and arc AXB is a minor arc. If there is no confusion then the name of a minor arc is written using its end points only. For example, the arc AXB in figure 3.29, is written as arc AB.

Here after, we are going to use the same convention for writing the names of arcs.

Central angle

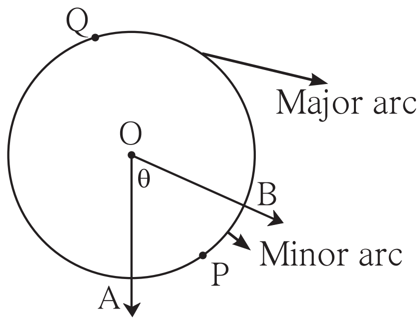


Fig. 3.30

When the vertex of an angle is the centre of a circle, it is called a central angle. In the figure 3.30, O is the centre of a circle and $\angle$AOB is a central angle.

Like secant, a central angle also divides a circle into two arcs.

Measure of an arc

To compare two arcs, we need to know their measures. Measure of an arc is defined as follows.

(1) Measure of a minor arc is equal to the measure of its corresponding central angle. In figure 3.30 measure of central $\angle$ AOB is θ.

$\therefore$ measure of minor arc APB is also θ.

(2) Measure of major arc = 360° – measure of corresponding minor arc.

In figure 3.30 measure of major arc AQB = 360° – measure of minor arc APB

$= 360^\circ - \theta$

(3) Measure of a semi circular arc, that is of a semi circle is 180°.

(4) Measure of a complete circle is 360°.

Let's learn.

Congruence of arcs

When two coplanar figures coincide with each other, they are called congruent figures. We know that two angles of equal measure are congruent.

Similarily, are two arcs of the same measure congruent ?

Find the answer of the question by doing the following activity.

Activity :

Draw two circles with centre C, as shown in the figure. Draw $\angle$ DCE, $\angle$ FCG of the same measure and $\angle$ ICJ of different measure.

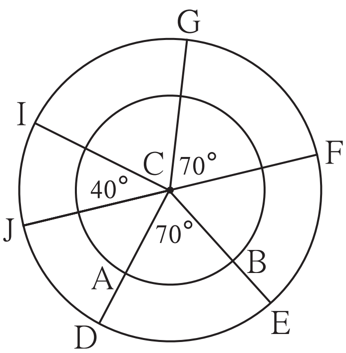


Fig. 3.31

Arms of $\angle$ DCE intersect inner circle at A and B.

Do you notice that the measures of arcs AB and DE are the same ? Do they coincide ? No, definitely not.

Now cut and seperate the sectors C-DE; C-FG and C-IJ. Check whether the arc DE, arc FG and arc IJ coincide with each other.

Did you notice that equality of measures of two arcs is not enough to make the two arcs congruent ? Which additional condition do you think is neccessary to make the two arcs congruent ?

From the above activity -

Two arcs are congruent if their measures and radii are equal.

'Arc DE and arc GF are congruent' is written in symbol as arc DE $\cong$ arc GF.

Property of sum of measures of arcs

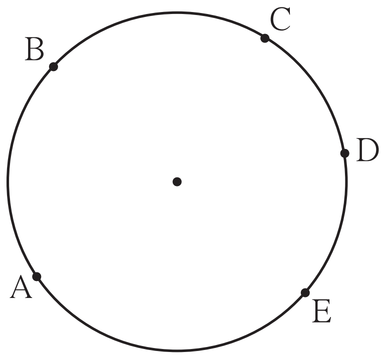


Fig. 3.32

In figure 3.32, the points A, B, C, D, E are concyclic. With these points many arcs are formed. There is one and only one common point C to arc ABC and arc CDE. So measure of arc ACE is the sum of measures of arc ABC and arc CDE.

m(arc ABC) + m(arc CDE) = m(arc ACE)

But arc ABC and arc BCE have many points in common. [All points on arc BC.]
So m(arc ABE) $\neq$ m(arc ABC) + m(arc BCE).

Theorem : The chords corresponding to congruent arcs of a circle (or congruent circles) are congruent.

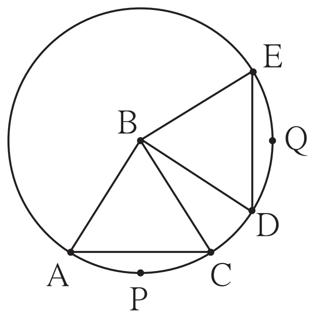


Fig. 3.33

Given : In a circle with centre B arc APC $\cong$ arc DQE

To Prove : Chord AC $\cong$ chord DE

Proof : (Fill in the blanks and complete the proof.)

In Δ ABC and Δ DBE,

side AB $\cong$ side DB (........)

side .... $\cong$ side .... (........)

$\angle$ABC $\cong$ $\angle$DBE measures of congruent arcs

$\therefore$ Δ ABC $\cong$ Δ DBE (.......)

$\therefore$ chord AC $\cong$ chord DE (......)

Theorem : Corresponding arcs of congruent chords of a circle (or congruent circles) are congruent.

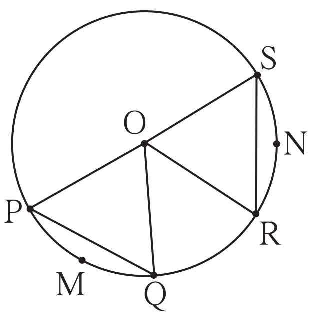


Fig. 3.34

Given : O is the centre of a circle chord PQ $\cong$ chord RS.

To prove : Arc PMQ $\cong$ arc RNS

Proof : Consider the following statements and write the proof.

Two arcs are congruent if their measures and radii are equal. Arc PMQ and arc RNS are arcs of the same circle, hence have equal radii.

Their measures are same as the measures of their central angles. To obtain central angles we have to draw radii OP, OQ, OR, OS.

Can you show that Δ OPQ and Δ ORS are congruent ?

Prove the above two theorems for congruent circles.

Let's think.

- While proving the first theorem of the two, we assume that the minor arc APC and minor arc DQE are congruent. Can you prove the same theorem by assuming that corresponding major arcs congruent ?
- In the second theorem, are the major arcs corresponding to congruent chords congruent ? Is the theorem true, when the chord PQ and chord RS are diameters of the circle ?

Solved Examples

Ex. (1) A, B, C are any points on the circle with centre O.

(i) Write the names of all arcs formed due to these points.

(ii) If m arc (BC) = 110° and m arc (AB) = 125°, find measures of all remaining arcs.

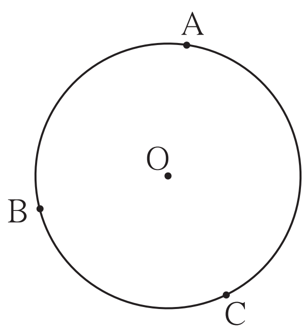


Fig. 3.35

Solution : (i) Names of arcs -

arc AB, arc BC, arc AC, arc ABC, arc ACB, arc BAC

(ii) m(arc ABC) = m(arc AB) + m(arc BC)

$= 125° + 110° = 235°$

m (arc AC) = 360° − m (arc ACB)

$= 360° - 235° = 125°$

Similarly, m(arc ACB) = 360° − 125° = 235°

and m(arc BAC) = 360° − 110° = 250°

Ex. (2) In the figure 3.36 a rectangle PQRS is inscribed in a circle with centre T. Prove that, (i) arc PQ $\cong$ arc SR
(ii) arc SPQ $\cong$ arc PQR

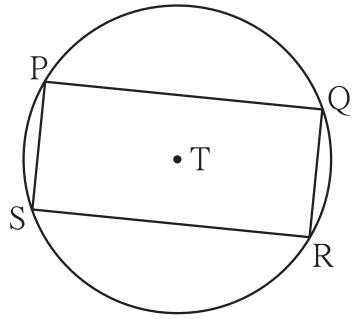


Fig. 3.36

Solution : (i) $\square$ PQRS in a rectangle.

$\therefore$ chord PQ $\cong$ chord SR .... opposite sides of a rectangle

$\therefore$ arc PQ $\cong$ arc SR ...... arcs corresponding to congruent chords.

(ii) chord PS $\cong$ chord QR ..... Opposite sides of a rectangle

$\therefore$ arc SP $\cong$ arc QR ..... arcs corresponding to congruent chords.

$\therefore$ measures of arcs SP and QR are equal

Now, m(arc SP) + m(arc PQ) = m(arc PQ) + m(arc QR)

$\therefore$ m(arc SPQ) = m(arc PQR)

$\therefore$ arc SPQ $\cong$ arc PQR

Remember this !

(1) An angle whose vertex is the centre of a circle is called a central angle.
(2) Definition of measure of an arc – (i) The measure of a minor arc is the measure of its central angle. (ii) Measure of a major arc = 360° – measure of its corresponding minor arc. (iii) measure of a semicircle is 180°.
(3) When two arcs are of the same radius and same measure, they are congruent.
(4) When only one point C is common to arc ABC, and arc CDE of the same circle, m(arc ABC) + m(arc CDE) = m(arc ACE)
(5) Chords of the same or congruent circles are equal if the related arcs are congruent.
(6) Arcs of the same or congruent circles are equal if the related chords are congruent.

Practice set 3.3

1. In figure 3.37, points G, D, E, F are concyclic points of a circle with centre C.
$\angle$ ECF = 70°, m(arc DGF) = 200° find m(arc DE) and m(arc DEF).

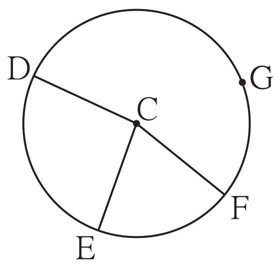


Fig. 3.37

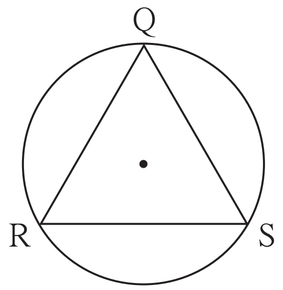


Fig. 3.38

2*. In fig 3.38 Δ QRS is an equilateral triangle. Prove that,

(1) arc RS $\cong$ arc QS $\cong$ arc QR

(2) m(arc QRS) = 240°.

3. In fig 3.39 chord AB $\cong$ chord CD, Prove that,

arc AC $\cong$ arc BD

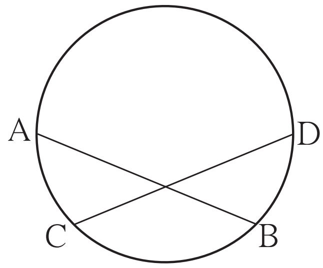


Fig. 3.39

Let's learn.

We have learnt some properties relating to a circle and points as well as lines (tangents). Now let us learn some properties regarding circle and angles with the help of some activities.

Activity I :

Draw a sufficiently large circle of any radius as shown in the figure 3.40. Draw a chord AB and central $\angle$ ACB. Take any point D on the major arc and point E on the minor arc.

(1) Measure $\angle$ADB and $\angle$ACB and compare the measures.

(2) Measure $\angle$ADB and $\angle$AEB. Add the measures.

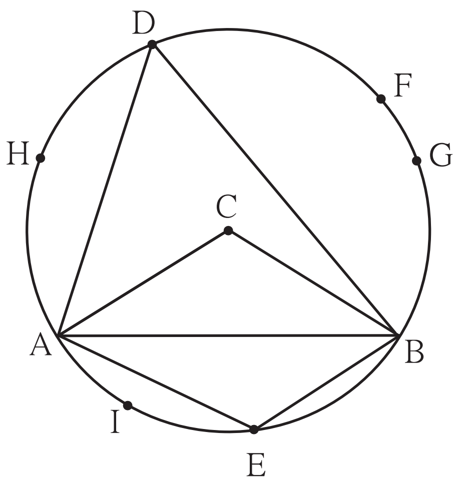


Fig. 3.40

(3) Take points F, G, H on the arc ADB. Measure $\angle$AFB, $\angle$AGB, $\angle$AHB. Compare these measures with each other as well as with measure of $\angle$ADB.

(4) Take any point I on the arc AEB. Measure $\angle$AIB and compare it with $\angle$AEB.

From the activity you must have noticed-

(1) The measure $\angle$ ACB is twice the measure of $\angle$ ADB.

(2) The sum of the measures of $\angle$ ADB and $\angle$ AEB is 180°.

(3) The angles $\angle$ AHB, $\angle$ ADB, $\angle$ AFB and $\angle$ AGB are of equal measure.

(4) The measure of $\angle$ AEB and $\angle$ AIB are equal.

Activity II :

Draw a sufficiently large circle with centre C as shown in the figure 3.41. Draw any diameter PQ. Now take points R,S,T on both the semicircles. Measure $\angle$PRQ, $\angle$ PSQ, $\angle$PTQ. Note that each is a right angle.

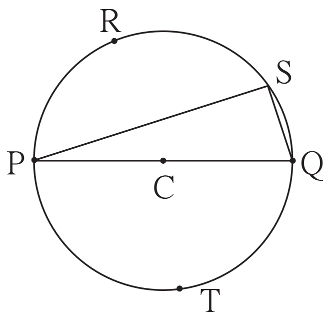


Fig. 3.41

The properties you saw in the above activities are theorems that give relations between circle and angles.

Let us learn some definitions required to prove the theorems.

Inscribed angle

In figure 3.42, C is the centre of a circle. The vertex D, of $\angle$PDQ lies on the circle. The arms of $\angle$PDQ intersect the circle at A and B. Such an angle is called an angle inscribed in the circle or in the arc.

In figure 3.42, $\angle$ADB is inscribed in the arc ADB.

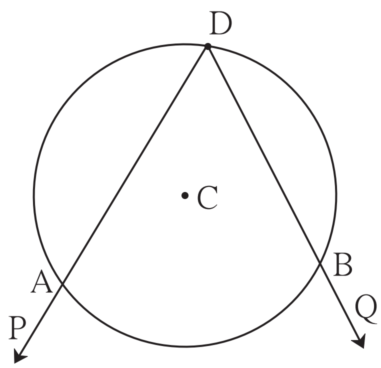


Fig. 3.42

Intercepted arc

Observe all figures (i) to (vi) in the following figure 3.43.

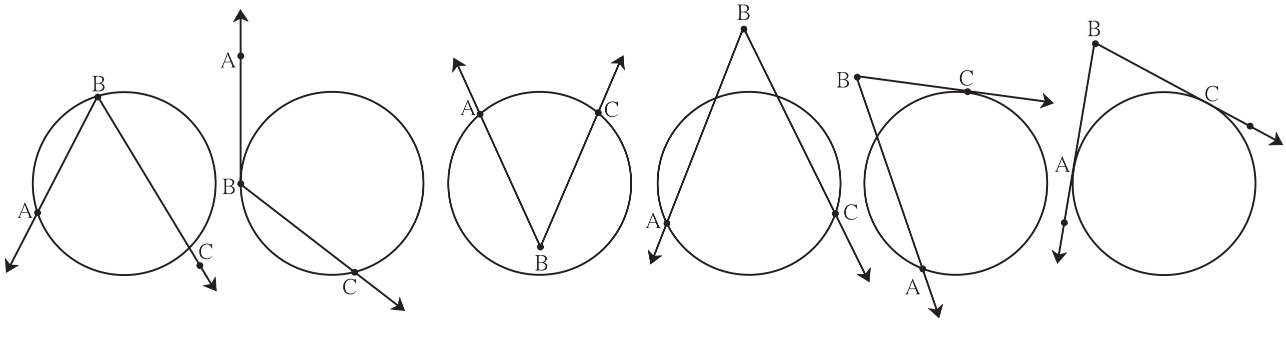


Fig. 3.43

In each figure, the arc of a circle that lies in the interior of the $\angle$ ABC is an arc intercepted by the $\angle$ ABC. The points of intersection of the circle and the angle are end points of that intercepted arc. Each side of the angle has to contain an end point of the arc.

In figures 3.43 (i), (ii) and (iii) only one arc is intercepted by that angle; and in (iv), (v) and (vi), two arcs are intercepted by the angle.

Also note that, only one side of the angle touches the circle in (ii) and (v), but in (vi) both sides of the angle touch the circle.

In figure 3.44, the arc is not intercepted arc, as arm BC does not contain any end point of the arc.

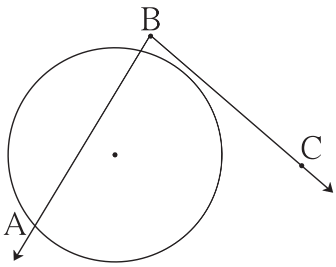


Fig. 3.44

Inscribed angle theorem

The measure of an inscribed angle is half of the measure of the arc intercepted by it.

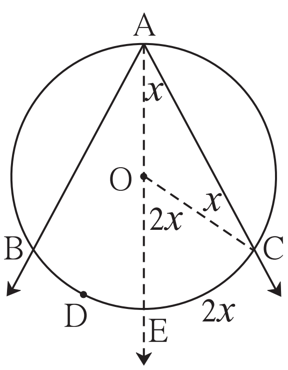


Fig. 3.45

Given : In a circle with centre O, $\angle$ BAC is inscribed in arc BAC. Arc BDC is intercepted by the angle.

To prove: $\angle BAC = \frac{1}{2}\, m(\text{arc BDC})$

Construction : Draw ray AO. It intersects the circle at E. Draw radius OC.

Proof : In Δ AOC,

side OA $\cong$ side OC ...... radii of the same circle.

$\therefore$ $\angle$ OAC = $\angle$ OCA ..... theorem of isosceles triangle.

Let $\angle$ OAC = $\angle$ OCA = x ...... (I)

Now, $\angle$ EOC = $\angle$ OAC + $\angle$ OCA .... exterior angle theorem of a triangle.

$= x^\circ + x^\circ = 2x^\circ$

But $\angle$ EOC is a central angle.

$\therefore$ m(arc EC) = $2x^\circ$ .... definition of measure of an arc ..... (II)

$\therefore$ from (I) and (II).

$\angle$ OAC = $\angle$ EAC = $\frac{1}{2}$ m(arc EC) ..... (III)

Similarly, drawing seg OB, we can prove $\angle$EAB = $\frac{1}{2}$ m(arc BE)...... (IV)

$\therefore$$\angle$EAC + $\angle$EAB = $\frac{1}{2}$ m(arc EC) + $\frac{1}{2}$ m(arc BE).... from (III) and (IV)

$\therefore$ $\angle$BAC = $\frac{1}{2}$ [m(arc EC) + m(arc BE)]

$= \frac{1}{2}$ [m(arc BEC)] = $\frac{1}{2}$ [m(arc BDC)] ..... (V)

Note that we have to consider three cases regarding the position of the centre of the circle and the inscribed angle. The centre of the circle lies (i) on one of the arms of the angle (ii) in the interior of the angle (iii) in the exterior of the angle. Out of these, first two are proved in (III) and (V). We will prove now the third one.

In figure 3.46,

$\angle$ BAC = $\angle$ BAE – $\angle$ CAE

$= \frac{1}{2}$ m(arc BCE) – $\frac{1}{2}$ m(arc CE)

...... from (III)

$= \frac{1}{2}$ [m(arc BCE) – m(arc CE)]

$= \frac{1}{2}$ [m(arc BC)] ...... (VI)

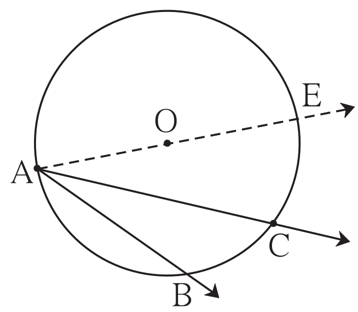


Fig. 3.46

The above theorem can also be stated as follows.

The measure of an angle subtended by an arc at a point on the circle is half of the measure of the angle subtended by the arc at the centre.

The corollaries of the above theorem can also be stated in similar language.

Corollaries of inscribed angle theorem

1. Angles inscribed in the same arc are congruent.

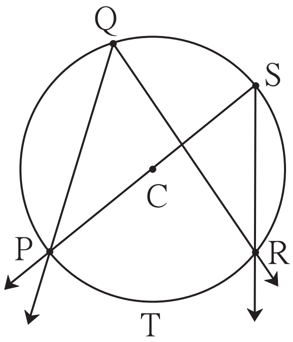


Fig. 3.47

Write 'given' and 'to prove' with the help of the figure 3.47.

Think of the answers of the following questions and write the proof.

(1) Which arc is intercepted by $\angle$ PQR ?

(2) Which arc is intercepted by $\angle$ PSR ?

(3) What is the relation between an inscribed angle and the arc intersepted by it ?

2. Angle inscribed in a semicircle is a right angle.

With the help of figure 3.48 write 'given', 'to prove' and 'the proof'.

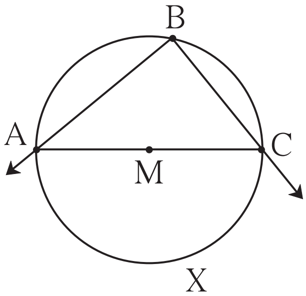


Fig. 3.48

Cyclic quadrilateral

If all vertices of a quadrilateral lie on the same circle then it is called a cyclic quadrilateral.

Theorem of cyclic quadrilateral

Theorem: Opposite angles of a cyclic quadrilateral are supplementry.

Fill in the blanks and complete the following proof.

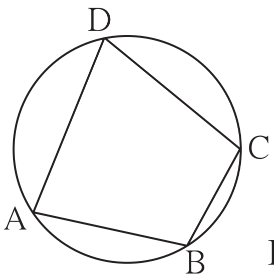


Fig. 3.49

Given : $\square$ $\boxed{}$ is cyclic.

To prove : $\angle B + \angle D = \boxed{}$

$\boxed{} + \angle C = 180^\circ$

Proof : Arc ABC is intercepted by the inscribed angle $\angle$ ADC.

$\therefore \angle ADC = \frac{1}{2}\boxed{}$ .... (I)

Similarly, $\boxed{}$ is an inscribed angle. It intercepts arc ADC.

$\therefore \boxed{} = \frac{1}{2} \, m(\text{arc ADC})$ ..... (II)

$\therefore m\angle ADC + \boxed{} = \frac{1}{2} \boxed{} + \frac{1}{2} \, m(\text{arc ADC})$ ..... from (I) & (II)

$= \frac{1}{2} \left[\boxed{} + m(\text{arc ADC}) \right]$

$= \frac{1}{2} \times 360^\circ$ .... arc ABC and arc ADC constitute a complete circle.

$= \boxed{}$

Similarly we can prove, $\angle A + \angle C = \boxed{}$.

Corollary of cyclic quadrilateral theorem

An exterior angle of a cyclic quadrilateral is congruent to the angle opposite to its adjacent interior angle.

Write the proof of the theorem yourself.

Let's think.

In the above theorem, after proving $\angle B + \angle D = 180^\circ$, can you use another way to prove $\angle A + \angle C = 180^\circ$?

Converse of cyclic quadrilateral theorem

Theorem : If a pair of opposite angles of a quadrilateral is supplementary, the quadrilateral is cyclic.

Try to prove this theorem by 'indirect method'. From the above converse, we know that if opposite angles of a quadrilateral are supplementary then there is a circumcircle for the quadrilateral.

For every triangle there exists a circumcircle but there may not be a circumcircle for every quadrilateral.

The above converse gives us the condition to ensure the existence of circumcircle of a quadrilateral.

With one more condition four non-collinear points are concyclic. It is stated in the following theorem.

Theorem : If two points on a given line subtend equal angles at two distinct points which lie on the same side of the line, then the four points are concyclic.

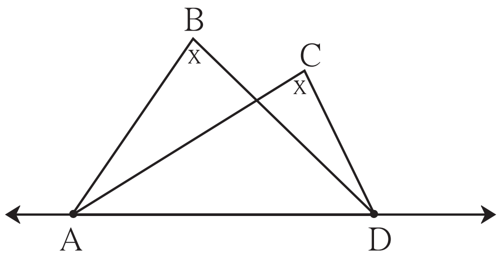


Fig. 3.50

Given : Points B and C lie on the same side of the line AD. $\angle ABD \cong \angle ACD$

To prove: Points A, B, C, D are concyclic. (That is, $\square$ABCD is cyclic.)

This theorem can be proved by 'indirect method'.

Let's think.

The above theorem is converse of a certain theorem. State it.

Solved Examples

Ex. (1) In figure 3.51, chord LM $\cong$ chord LN

$\angle L = 35°$ find

(i) m(arc MN)

(ii) m(arc LN)

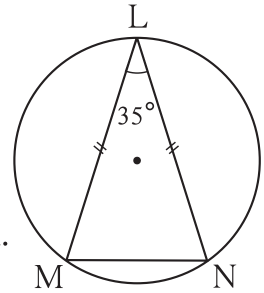


Fig. 3.51

Solution : (i) $\angle L = \frac{1}{2}\, m(\text{arc MN})$ ......inscribed angle theorem.

$\therefore\ 35 = \frac{1}{2}\, m(\text{arc MN})$

$\therefore\ 2 \times 35 = m(\text{arc MN}) = 70°$

(ii) $m(\text{arc MLN}) = 360° - m(\text{arc MN})$ ...... definition of measure of arc

$= 360° - 70° = 290°$

Now, chord LM $\cong$ chord LN

$\therefore$ arc LM $\cong$ arc LN

but $m(\text{arc LM}) + m(\text{arc LN}) = m(\text{arc MLN}) = 290°$ ..... arc addition property

$m(\text{arc LM}) = m(\text{arc LN}) = \frac{290°}{2} = 145°$

or, (ii) chord LM $\cong$ chord LN

$\therefore\ \angle M = \angle N$ ...... isosceles trinangle theorem.

$\therefore\ 2\angle M = 180° - 35° = 145°$

$\therefore\ \angle M = \frac{145°}{2}$

Now, $m(\text{arc LN}) = 2 \times \angle M$

$$= 2 \times \frac{145^\circ}{2} = 145^\circ$$

Ex. (2) In figure 3.52, chords PQ and RS intersect at T.

(i) Find $m(\text{arc SQ})$ if $m\angle STQ = 58^\circ$, $m\angle PSR = 24^\circ$.

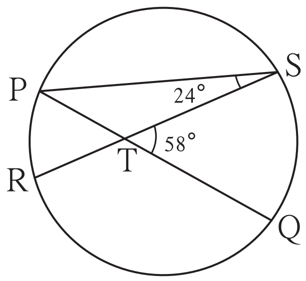


Fig. 3.52

(ii) Verify,

$\angle STQ = \frac{1}{2}[m(\text{arc PR}) + m(\text{arc SQ})]$

(iii) Prove that :

$\angle STQ = \frac{1}{2}[m(\text{arc PR}) + m(\text{arc SQ})]$

for any measure of $\angle STQ$.

(iv) Write in words the property in (iii).

Solution : (i) $\angle SPQ = \angle SPT = 58^\circ - 24^\circ = 34^\circ$ .... exterior angle theorem.

$m(\text{arc QS}) = 2\angle SPQ = 2 \times 34^\circ = 68^\circ$

(ii) $m(\text{arc PR}) = 2\angle PSR = 2 \times 24^\circ = 48^\circ$

Now, $\frac{1}{2}[m(\text{arc PR}) + m(\text{arc SQ})] = \frac{1}{2}[48 + 68]$

$$= \frac{1}{2} \times 116 = 58^\circ$$

$$= \angle STQ$$

(iii) Fill in the blanks and complete the proof of the above property.

$\angle STQ = \angle SPQ + \boxed{}$ .... exterior angle theorem of a triangle

$= \frac{1}{2} m(\text{arc SQ}) + \boxed{}$ .... inscribed angle theorem

$= \frac{1}{2}[\boxed{} + \boxed{}]$

(iv) If two chords of a circle intersect each other in the interior of a circle then the measure of the angle between them is half the sum of measures of arcs intercepted by the angle and its opposite angle.

Ex. (3) Prove that, if two lines containing chords of a circle intersect each other outside the circle, then the measure of angle between them is half the difference in measures of the arcs intercepted by the angle.

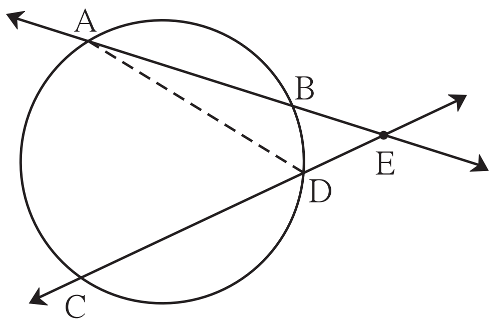


Fig. 3.53

Given : Chords AB and CD intersect at E in the exterior of the circle.

To prove: $\angle AEC = \frac{1}{2}[m(\text{arc AC}) - m(\text{arc BD})]$

Construction: Draw seg AD.

Consider angles of Δ AED and its exterior angle and write the proof.

Remember this !

(1) The measure of an inscribed angle is half the measure of the arc intercepted by it.

(2) Angles inscribed in the same arc are congruent.

(3) Angle inscribed in a semicircle is a right angle.

(4) If all vertices of a quadrilateral lie on the same circle then the quadrilateral is called a cyclic quadrilateral.

(5) Opposite angles of a cyclic quadrilateral are supplementary.

(6) An exterior angle of a cyclic quadrilateral is congruent to the angle opposite to its adjacent interior angle.

(7) If a pair of opposite angles of a quadrilateral is supplementary, then the quadrilateral is cyclic.

(8) If two points on a given line subtend equal angles at two different points which lie on the same side of the line, then those four points are concyclic.

(9) In figure 3.54,

(i) $\angle AEC = \frac{1}{2}[m(\text{arc AC}) + m(\text{arc DB})]$

(ii) $\angle CEB = \frac{1}{2}[m(\text{arc AD}) + m(\text{arc CB})]$

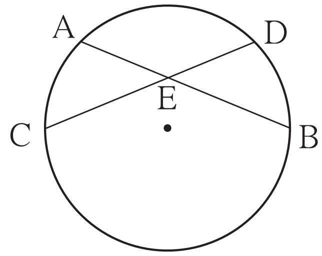


Fig. 3.54

(10) In figure 3.55,

$\angle BED = \frac{1}{2}\ [m(\text{arc BD}) - m(\text{arc AC})]$

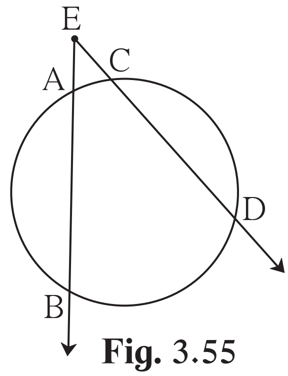


Fig. 3.55

Practice set 3.4

1. In figure 3.56, in a circle with centre O, length of chord AB is equal to the radius of the circle. Find measure of each of the following.

(1) $\angle AOB$ (2) $\angle ACB$

(3) arc AB (4) arc ACB.

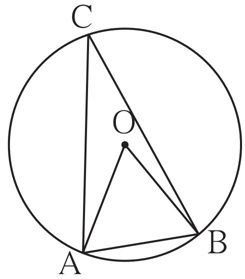


Fig. 3.56

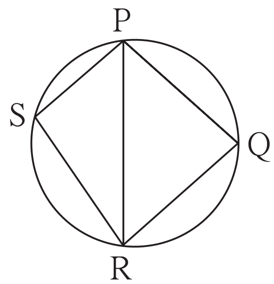


Fig. 3.57

2. In figure 3.57, $\square$ PQRS is cyclic. side PQ $\cong$ side RQ. $\angle PSR = 110^\circ$, Find–

(1) measure of $\angle PQR$

(2) m(arc PQR)

(3) m(arc QR)

(4) measure of $\angle PRQ$

3. $\square$ MRPN is cyclic, $\angle R = (5x - 13)^\circ$, $\angle N = (4x + 4)^\circ$. Find measures of $\angle R$ and $\angle N$.

4. In figure 3.58, seg RS is a diameter of the circle with centre O. Point T lies in the exterior of the circle. Prove that $\angle RTS$ is an acute angle.

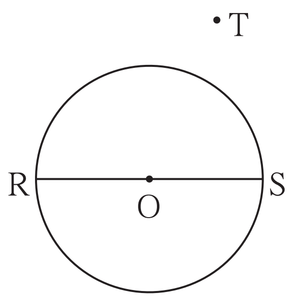


Fig. 3.58

5. Prove that, any rectangle is a cyclic quadrilateral.

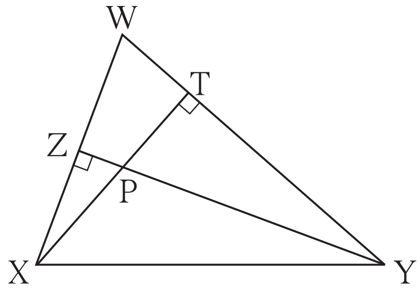


Fig. 3.59

6. In figure 3.59, altitudes YZ and XT of Δ WXY intersect at P. Prove that,

(1) $\square$ WZPT is cyclic.

(2) Points X, Z, T, Y are concyclic.

7. In figure 3.60, m(arc NS) = 125°, m(arc EF) = 37°, find the measure $\angle$NMS.

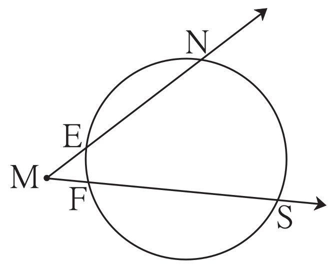


Fig. 3.60

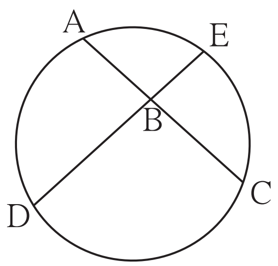


Fig. 3.61

8. In figure 3.61, chords AC and DE intersect at B. If $\angle$ ABE = 108°, m(arc AE) = 95°, find m(arc DC) .

Let's learn.

Activity :

Draw a circle as shown in figure 3.62. Draw a chord AC. Take any point B on the circle. Draw inscribed $\angle$ABC, measure it and note the measure.

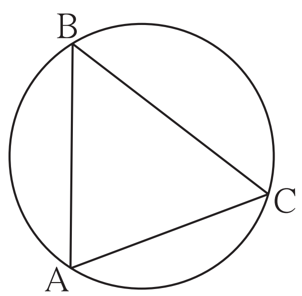


Fig. 3.62

Now as shown in figure 3.63, draw a tangent CD of the same circle, measure angle $\angle$ACD and note the measure.

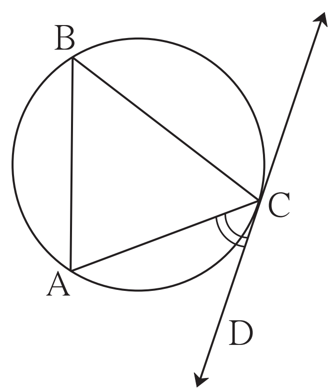


Fig. 3.63

You will find that $\angle ACD = \angle ABC$.

You know that $\angle ABC = \frac{1}{2}\ m(\text{arc } AC)$

From this we get $\angle ACD = \frac{1}{2}\ m(\text{arc } AC)$.

Now we will prove this property.

Theorem of angle between tangent and secant

If an angle has its vertex on the circle, its one side touches the circle and the other intersects the circle in one more point, then the measure of the angle is half the measure of its intercepted arc.

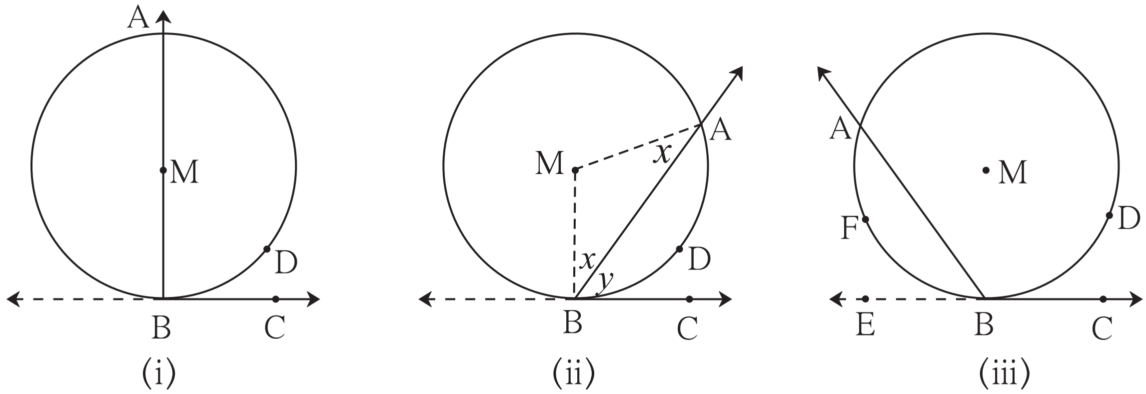


Fig. 3.64

Given : Let $\angle ABC$ be an angle, where vertex B lies on a circle with centre M. Its side BC touches the circle at B and side BA intersects the circle at A. Arc ADB is intercepted by $\angle ABC$.

To prove: $\angle ABC = \frac{1}{2}\ m(\text{arc } ADB)$

Proof : Consider three cases.

(1) In figure 3.64 (i), the centre M lies on the arm BA of $\angle ABC$,

$\angle ABC = \angle MBC = 90°$ ..... tangent theorem (I)

arc ADB is a semicircle.

$\therefore\ m(\text{arc } ADB) = 180°$ ..... definition of measure of arc (II)

From (I) and (II)

$\angle ABC = \frac{1}{2}\ m(\text{arc } ADB)$

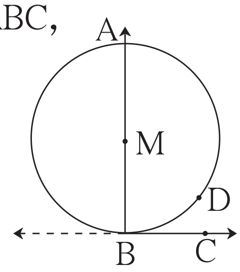


Fig. 3.64(i)

(2) In figure 3.64 (ii) centre M lies in the exterior of $\angle ABC$,

Draw radii MA and MB.

Now, $\angle MBA = \angle MAB$ ..... isosceles triangle theorem

$\angle MBC = 90°$ ..... tangent theorem..... (I) let $\angle$ MBA = $\angle$ MAB = x and $\angle$ ABC = y.

$\angle$ AMB = 180 − $(x + x)$ = 180 − $2x$

$\angle$ MBC = $\angle$ MBA + $\angle$ ABC = $x + y$

$\therefore x + y = 90°$ $\quad \therefore 2x + 2y = 180°$

In Δ AMB, $2x + \angle$ AMB = 180°

$\therefore 2x + 2y = 2x + \angle$ AMB

$\therefore 2y = \angle$ AMB

$\therefore y = \angle \text{ABC} = \frac{1}{2}\angle \text{AMB} = \frac{1}{2}\, m(\text{arc ADB})$

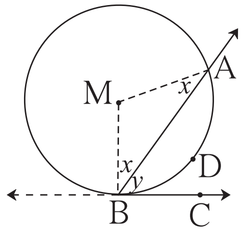


Fig. 3.64(ii)

(3) With the help of fig 3.64 (iii),

Fill in the blanks and write proof.

Ray $\boxed{\qquad\qquad}$ is the opposite ray of ray BC.

Now, $\angle\text{ABE} = \frac{1}{2}\, m(\qquad)$ ...... proved in (ii).

$\therefore 180 - \boxed{\qquad\qquad} = \frac{1}{2}\, m(\text{arc AFB})$ ..... linear pair

$= \frac{1}{2}\,[360 - m(\boxed{\qquad\qquad})]$

$\therefore 180 - \angle\text{ABC} = 180 - \frac{1}{2}\, m(\text{arc ADB})$

$\therefore -\angle\text{ABC} = -\frac{1}{2}\, m(\boxed{\qquad\qquad})$

$\therefore \angle\text{ABC} = \frac{1}{2}\, m(\text{arc ADB})$

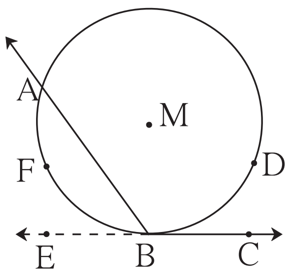


Fig. 3.64(iii)

Alternative statement of the above theorem.

In the figure 3.65, line AB is a secant and line BC is a tangent. The arc ADB is intercepted by $\angle$ ABC. Chord AB divides the circle in two parts. These are opposite arcs of each other.

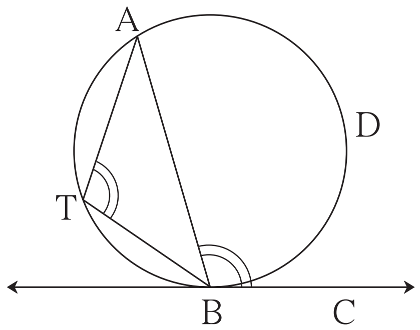


Fig. 3.65

Now take any point T on the arc opposite to arc ADB .

From the above theorem,

$\angle \text{ABC} = \frac{1}{2}\, m\ (\text{arc ADB}) = \angle \text{ATB}$.

$\therefore$ **the angle between a tangent of a circle and a chord drawn from the point of contact is congruent to the angle inscribed in the arc opposite to the arc intercepted by that angle.**

Converse of theorem of the angle between tangent and secant

A line is drawn from one end point of a chord of a circle and if the angle between the chord and the line is half the measure of the arc intercepted by that angle then that line is a tangent to the circle.

In figure 3.66,

If $\angle PQR = \frac{1}{2} m(\text{arc PSQ})$,

[or $\angle PQT = \frac{1}{2} m(\text{arc PUQ})$]

then line TR is a tangent to the circle.

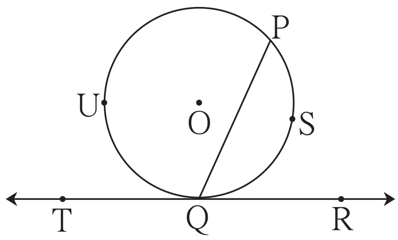


Fig. 3.66

This property is used in constructing a tangent to the given circle.

An indirect proof of this converse can be given.

Theorem of internal division of chords

Suppose two chords of a circle intersect each other in the interior of the circle, then the product of the lengths of the two segments of one chord is equal to the product of the lengths of the two segments of the other chord.

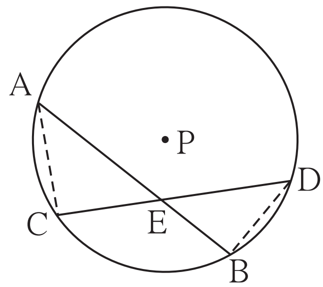


Fig. 3.67

Given : Chords AB and CD of a circle with centre P intersect at point E.

To prove: $AE \times EB = CE \times ED$

Construction : Draw seg AC and seg DB.

Proof : In Δ CAE and Δ BDE,

$\angle AEC \cong \angle DEB$ ..... opposite angles

$\angle CAE \cong \angle BDE$ ..... angles inscribed in the same arc

$\therefore \Delta CAE \sim \Delta BDE$ ..... AA test

$\therefore \frac{AE}{DE} = \frac{CE}{BE}$ ..... corresponding sides of similar triangles

$\therefore AE \times EB = CE \times ED$

Let's think.

We proved the theorem by drawing seg AC and seg DB in figure 3.67, Can the theorem be proved by drawing seg AD and segCB, instead of seg AC and seg DB?

For more information

In figure 3.67 point E divides the chord AB into segments AE and EB. AE × EB is the area of a rectangle having sides AE and EB. Similarly E divides CD into segments CE and ED. CE × ED is the area of a rectangle of sides CE and ED. We have proved that AE × EB = CE × ED.

So the above theorem can be stated as, 'If two chords of a circle intersect in the interior of a circle then the area of the rectangle formed by the segments of one chord is equal to the area of similar rectangle formed by the other chord.'

Theorem of external division of chords

If secants containing chords AB and CD of a circle intersect outside the circle in point E, then AE × EB = CE × ED.

Write 'given' and 'to prove' with the help of the statement of the theorem and figure 3.68.

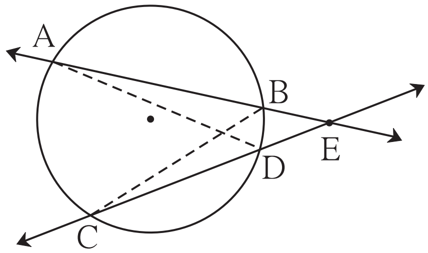


Fig. 3.68

Construction : Draw seg AD and seg BC.

Fill in the blanks and complete the proof.

Proof : In Δ ADE and Δ CBE,

$\angle$ AED ≅ $\boxed{}$ .... common angle

$\angle$ DAE ≅ $\angle$ BCE ....($\boxed{}$)

∴ Δ ADE ~ $\boxed{}$ ....($\boxed{}$)

$\therefore \dfrac{(AE)}{\boxed{}} = \dfrac{\boxed{}}{\boxed{}}$ .... corresponding sides of similar triangles

∴ $\boxed{}$ = CE × ED

Tangent secant segments theorem

Point E is in the exterior of a circle. A secant through E intersects the circle at points A and B, and a tangent through E touches the circle at point T, then $EA \times EB = ET^2$.

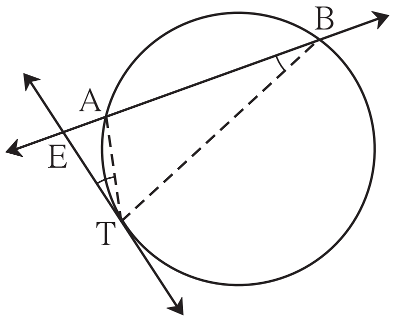


Fig. 3.69

Write 'given' and 'to prove' with reference to the statement of the theorem.

Construction : Draw seg TA and seg TB.

Proof : In Δ EAT and Δ ETB,

$\angle AET \cong \angle TEB$ .... common angle

$\angle ETA \cong \angle EBT$... tangent secant theorem

$\therefore \Delta EAT \sim \Delta ETB$ ..... AA similarity

$\therefore \frac{ET}{EB} = \frac{EA}{ET}$ ..... corresponding sides

$\therefore EA \times EB = ET^2$

Remember this !

(1) In figure 3.70,

$AE \times EB = CE \times ED$

This property is known as theorem of chords intersecting inside the circle.

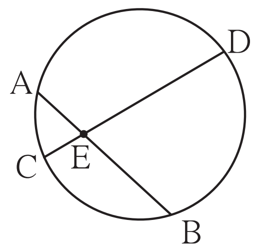


Fig. 3.70

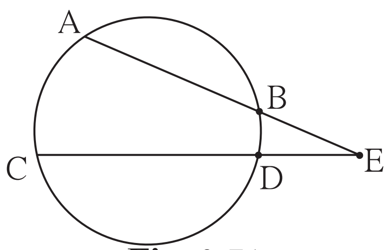


Fig. 3.71

(2) In figure 3.71,

$AE \times EB = CE \times ED$

This property is known as theorem of chords intersecting outside the circle.

(3) In figure 3.72,

$EA \times EB = ET^2$

This property is known as tangent secant segments theorem.

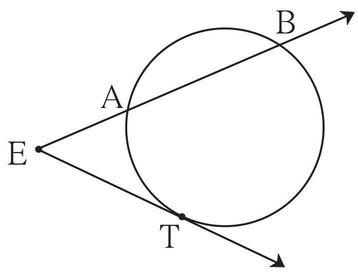


Fig. 3.72

Ex. (1) In figure 3.73, seg PS is a tangent segment. Line PR is a secant. If PQ = 3.6, QR = 6.4, find PS.

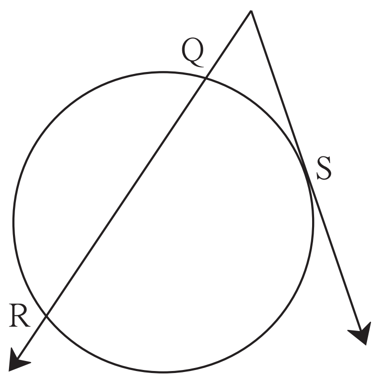


Fig. 3.73

Solution : $PS^2 = PQ \times PR$ .... tangent secant segments theorem

$= PQ \times (PQ + QR)$

$= 3.6 \times [3.6 + 6.4]$

$= 3.6 \times 10$

$= 36$

$\therefore PS = 6$

Ex. (2) In figure 3.74, chord MN and chord RS intersect each other at point P. If PR = 6, PS = 4, MN = 11 find PN.

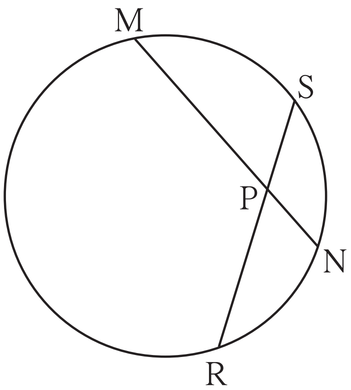


Fig. 3.74

Solution : By theorem on interesecting chords,

$PN \times PM = PR \times PS$... (I)

let PN = x. $\therefore$ PM = $11 - x$

substituting the values in (I),

$x\,(11 - x) = 6 \times 4$

$\therefore\ 11x - x^2 - 24 = 0$

$\therefore\ x^2 - 11x + 24 = 0$

$\therefore\ (x - 3)\,(x - 8) = 0$

$\therefore\ x - 3 = 0$ or $x - 8 = 0$

$\therefore\ x = 3$ or $x = 8$

$\therefore$ PN = 3 or PN = 8

Ex. (3) In figure 3.75, two circles intersect each other in points X and Y. Tangents drawn from a point M on line XY touch the circles at P and Q.

Prove that, seg PM ≅ seg QM.

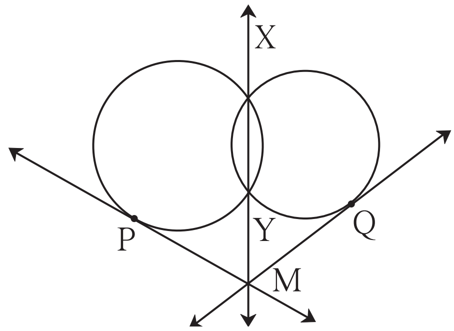


Fig. 3.75

Solution : Fill in the blanks and write proof.

Line MX is a common .... of the two circles.

$\therefore PM^2 = MY \times MX$ ..... (I)

Similarly .... = ... × .... , tangent secant segment theorem ......(II)

$\therefore$ From (I) and (II) ...... = QM^2

$\therefore$ PM = QM

seg PM ≅ seg QM

Ex. (4)

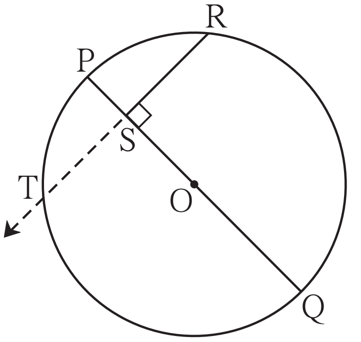


Fig. 3.76

In figure 3.76, seg PQ is a diameter of a circle with centre O. R is any point on the circle.

seg RS ⊥ seg PQ.

Prove that, SR is the geometric mean of PS and SQ.

[That is, $SR^2 = PS \times SQ$]

Solution : Write the proof with the help of the following steps.

(1) Draw ray RS. It intersects the circle at T.

(2) Show that RS = TS.

(3) Write a result using theorem of intersection of chords inside the circle.

(4) Using RS = TS complete the proof.

Let's think.

(1) In figure 3.76, if seg PR and seg RQ are drawn, what is the nature of Δ PRQ?

(2) Have you previously proved the property proved in example (4) ?

Practice set 3.5

1. In figure 3.77, ray PQ touches the circle at point Q. PQ = 12, PR = 8, find PS and RS.

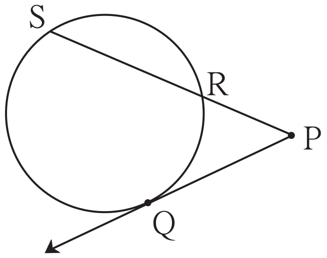


Fig. 3.77

2. In figure 3.78, chord MN and chord RS intersect at point D.
   (1) If RD = 15, DS = 4, MD = 8 find DN
   (2) If RS = 18, MD = 9, DN = 8 find DS

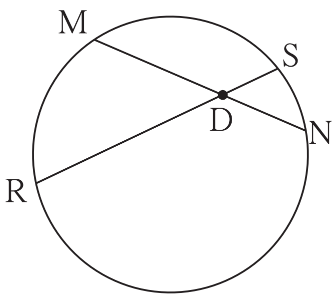


Fig. 3.78

3. In figure 3.79, O is the centre of the circle and B is a point of contact. seg OE ⊥ seg AD, AB = 12, AC = 8, find
   (1) AD (2) DC (3) DE.

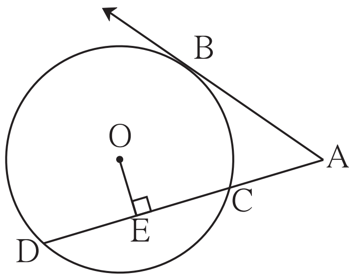


Fig. 3.79

4. In figure 3.80, if PQ = 6, QR = 10, PS = 8 find TS.

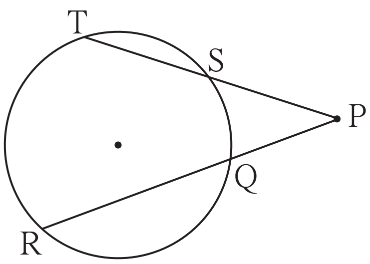


Fig. 3.80

5. In figure 3.81, seg EF is a diameter and seg DF is a tangent segment. The radius of the circle is r. Prove that, DE × GE = $4r^2$

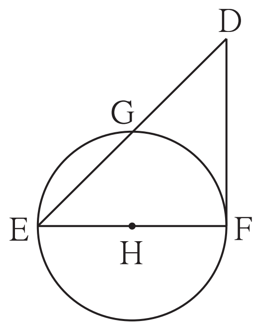


Fig. 3.81

Problem set 3

1. Four alternative answers for each of the following questions are given. Choose the correct alternative.

(1) Two circles of radii 5.5 cm and 3.3 cm respectively touch each other. What is the distance between their centers ?

(A) 4.4 cm (B) 8.8 cm (C) 2.2 cm (D) 8.8 or 2.2 cm

(2) Two circles intersect each other such that each circle passes through the centre of the other. If the distance between their centres is 12, what is the radius of each circle ?

(A) 6 cm (B) 12 cm (C) 24 cm (D) can't say

(3) A circle touches all sides of a parallelogram. So the parallelogram must be a, ...

(A) rectangle (B) rhombus (C) square (D) trapezium

(4) Length of a tangent segment drawn from a point which is at a distance 12.5 cm from the centre of a circle is 12 cm, find the diameter of the circle.

(A) 25 cm (B) 24 cm (C) 7 cm (D) 14 cm

(5) If two circles are touching externally, how many common tangents of them can be drawn?

(A) One (B) Two (C) Three (D) Four

(6) $\angle$ACB is inscribed in arc ACB of a circle with centre O. If $\angle$ACB = 65°, find m(arc ACB).

(A) 65° (B) 130° (C) 295° (D) 230°

(7) Chords AB and CD of a circle intersect inside the circle at point E. If AE = 5.6, EB = 10, CE = 8, find ED.

(A) 7 (B) 8 (C) 11.2 (D) 9

(8) In a cyclic $\square$ABCD, twice the measure of $\angle$ A is thrice the measure of $\angle$C. Find the measure of $\angle$C ?

(A) 36 (B) 72 (C) 90 (D) 108

9)★ Points A, B, C are on a circle, such that m(arc AB) = m(arc BC) = 120°. No point, except point B, is common to the arcs. Which is the type of Δ ABC ?

(A) Equilateral triangle (B) Scalene triangle

(C) Right angled triangle (D) Isosceles triangle

(10) Seg XZ is a diameter of a circle. Point Y lies in its interior. How many of the following statements are true ?

(i) It is not possible that $\angle$ XYZ is an acute angle.

(ii) $\angle$ XYZ can't be a right angle.

(iii) $\angle$ XYZ is an obtuse angle.

(iv) Can't make a definite statement for measure of $\angle$ XYZ.

(A) Only one (B) Only two (C) Only three (D) All

2. Line l touches a circle with centre O at point P. If radius of the circle is 9 cm, answer the following.

(1) What is d(O, P) = ? Why ?

(2) If d(O, Q) = 8 cm, where does the point Q lie ?

(3) If d(PQ) = 15 cm, How many locations of point R are line on line l ? At what distance will each of them be from point P ?

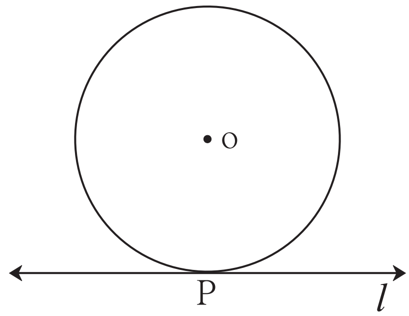


Fig. 3.82

3. In figure 3.83, M is the centre of the circle and seg KL is a tangent segment.

If MK = 12, KL = $6\sqrt{3}$ then find -

(1) Radius of the circle.

(2) Measures of $\angle$ K and $\angle$ M.

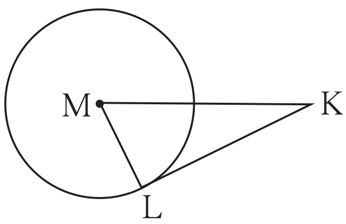


Fig. 3.83

4. In figure 3.84, O is the centre of the circle. Seg AB, seg AC are tangent segments. Radius of the circle is r and l(AB) = r, Prove that, $\square$ABOC is a square.

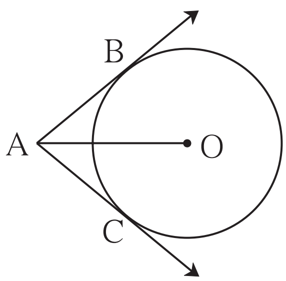


Fig. 3.84

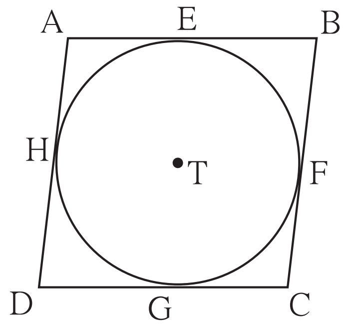


Fig. 3.85

5. In figure 3.85, $\square$ABCD is a parallelogram. It circumscribes the circle with cnetre T. Point E, F, G, H are touching points. If AE = 4.5, EB = 5.5, find AD.

6. In figure 3.86, circle with centre M touches the circle with centre N at point T. Radius RM touches the smaller circle at S. Radii of circles are 9 cm and 2.5 cm. Find the answers to the following questions hence find the ratio MS:SR.

(1) Find the length of segment MT

(2) Find the length of seg MN

(3) Find the measure of $\angle$ NSM.

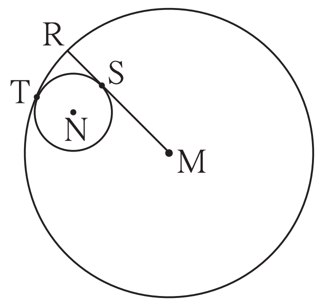


Fig. 3.86

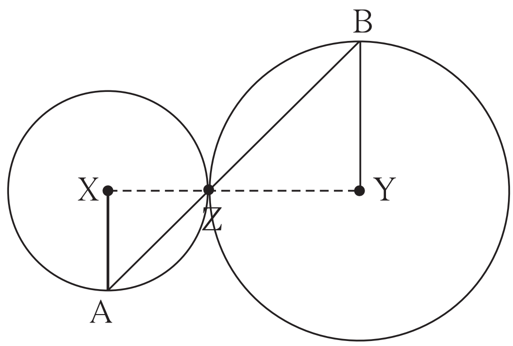


Fig. 3.87

7. In the adjoining figure circles with centres X and Y touch each other at point Z. A secant passing through Z intersects the circles at points A and B respectively. Prove that, radius XA || radius YB.

Fill in the blanks and complete the proof.

Construction : Draw segments XZ and ....

Proof :By theorem of touching circles, points X, Z, Y are ....

$\therefore$ $\angle$ XZA $\cong$ .... opposite angles

Let $\angle$ XZA = $\angle$ BZY = a ..... (I)

Now, seg XA $\cong$ seg XZ .... (..........)

$\therefore$ $\angle$ XAZ = ... = a .... (isosceles triangle theorem) (II)

similarly, seg YB $\cong$ .... (..........)

$\therefore$ $\angle$ BZY = ... = a .... (..........) (III)

$\therefore$ from (I), (II), (III),

$\angle$ XAZ = ...

$\therefore$ radius XA || radius YB .... (..........)

8. In figure 3.88, circles with centres X and Y touch internally at point Z . Seg BZ is a chord of bigger circle and it itersects smaller circle at point A.
Prove that, seg AX || seg BY.

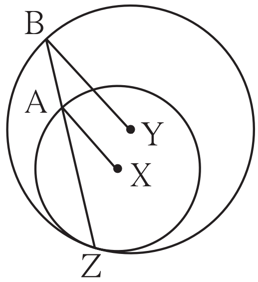


Fig. 3.88

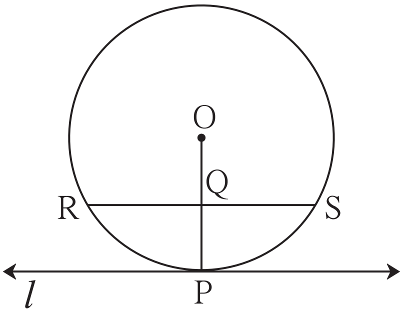


Fig. 3.89

9. In figure 3.89, line *l* touches the circle with centre O at point P. Q is the mid point of radius OP. RS is a chord through Q such that chords RS || line *l*. If RS = 12 find the radius of the circle.

10★. In figure 3.90, seg AB is a diameter of a circle with centre C. Line PQ is a tangent, which touches the circle at point T.
seg AP $\perp$ line PQ and seg BQ $\perp$ line PQ.
Prove that, seg CP $\cong$ seg CQ.

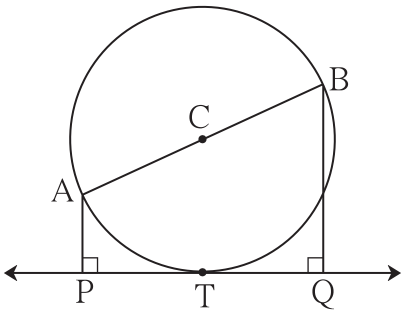


Fig. 3.90

11★. Draw circles with centres A, B and C each of radius 3 cm, such that each circle touches the other two circles.

12★. Prove that any three points on a circle cannot be collinear.

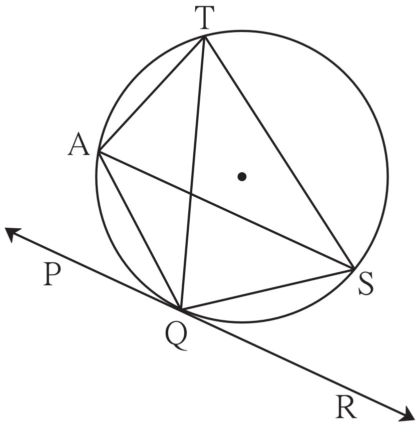


Fig. 3.91

13. In figure 3.91, line PR touches the circle at point Q. Answer the following questions with the help of the figure.

(1) What is the sum of $\angle$TAQ and $\angle$ TSQ ?

(2) Find the angles which are congruent to $\angle$AQP.

(3) Which angles are congruent to $\angle$QTS ?

(4) $\angle$TAS = 65°, find the measure of $\angle$TQS and arc TS.

(5) If $\angle$AQP = 42°and $\angle$SQR = 58° find measure of $\angle$ATS.

14. In figure 3.92, O is the centre of a circle, chord PQ $\cong$ chord RS

If $\angle$ POR = 70°

and (arc RS) = 80°, find –

(1) m(arc PR)

(2) m(arc QS)

(3) m(arc QSR)

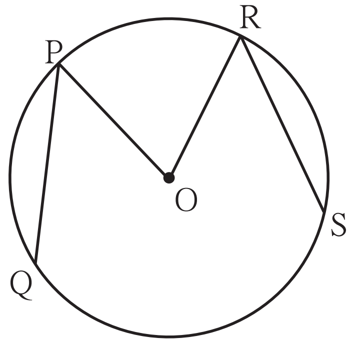


Fig. 3.92

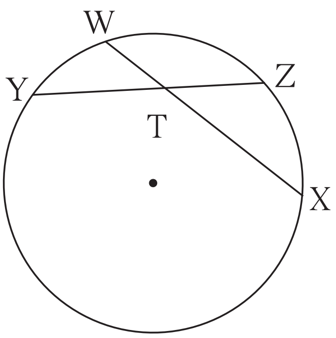


Fig. 3.93

15. In figure 3.93, m(arc WY) = 44°, m(arc ZX) = 68°, then

(1) Find the measure of $\angle$ ZTX.

(2) If WT = 4.8, TX = 8.0, YT = 6.4, find TZ.

(3) If WX = 25, YT = 8, YZ = 26, find WT.

16. In figure 3.94,

(1) m(arc CE) = 54°,

m(arc BD) = 23°, find measure of $\angle$CAE.

(2) If AB = 4.2, BC = 5.4,

AE = 12.0, find AD

(3) If AB = 3.6, AC = 9.0,

AD = 5.4, find AE

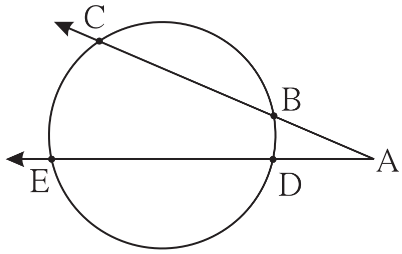


Fig. 3.94

17. In figure 3.95, chord EF || chord GH. Prove that, chord EG $\cong$ chord FH.

Fill in the blanks and write the proof.

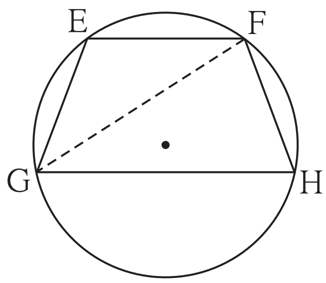


Fig. 3.95

Proof : Draw seg GF.

$\angle$ EFG = $\angle$ FGH .... $\boxed{}$ (I)

$\angle$ EFG = $\boxed{}$ .... inscribed angle theorem (II)

$\angle$ FGH = $\boxed{}$ .... inscribed angle theorem (III)

$\therefore$ m(arc EG) = $\boxed{}$ from (I), (II), (III).

chord EG $\cong$ chord FH .... $\boxed{}$

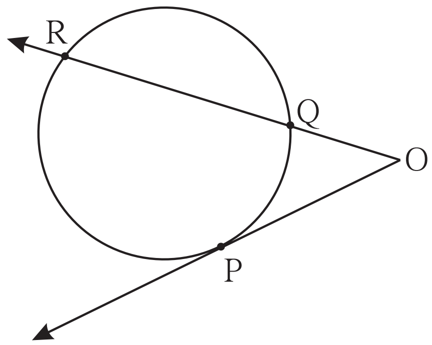


Fig. 3.96

18. In figure 3.96 P is the point of contact.

(1) If m(arc PR) = 140°,

$\angle$ POR = 36°,

find m(arc PQ)

(2) If OP = 7.2, OQ = 3.2,

find OR and QR

(3) If OP = 7.2, OR = 16.2,

find QR.

19. In figure 3.97, circles with centres C and D touch internally at point E. D lies on the inner circle. Chord EB of the outer circle intersects inner circle at point A. Prove that, seg EA $\cong$ seg AB.

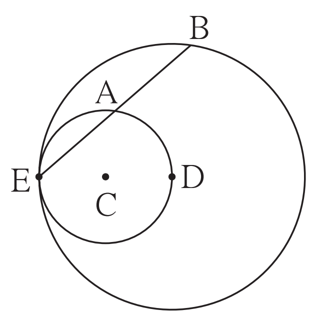


Fig. 3.97

20. In figure 3.98, seg AB is a diameter of a circle with centre O . The bisector of $\angle$ ACB intersects the circle at point D. Prove that, seg AD $\cong$ seg BD. Complete the following proof by filling in the blanks.

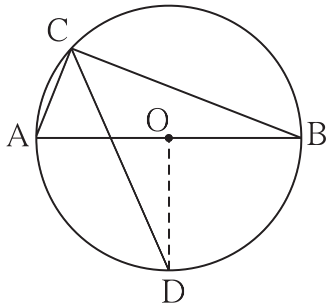


Fig. 3.98

Proof : Draw seg OD.

$\angle$ ACB = $\boxed{}$ .... angle inscribed in semicircle

$\angle$ DCB = $\boxed{}$ .... CD is the bisector of $\angle$ C

m(arc DB) = $\boxed{}$ .... inscribed angle theorem

$\angle$ DOB = $\boxed{}$ .... definition of measure of an arc (I)

seg OA $\cong$ seg OB .... $\boxed{}$ (II)

$\therefore$ line OD is $\boxed{}$ of seg AB .... From (I) and (II)

$\therefore$ seg AD $\cong$ seg BD

21. In figure 3.99, seg MN is a chord of a circle with centre O. MN = 25, L is a point on chord MN such that ML = 9 and d(O,L) = 5. Find the radius of the circle.

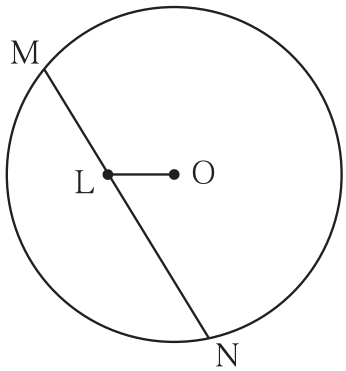


Fig. 3.99

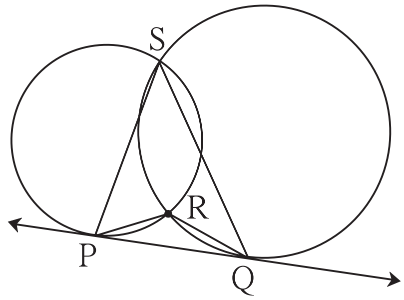


Fig. 3.100

22★. In figure 3.100, two circles intersect each other at points S and R. Their common tangent PQ touches the circle at points P, Q. Prove that, $\angle$ PRQ + $\angle$ PSQ = 180°

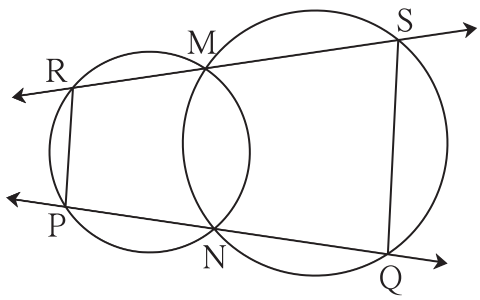


Fig. 3.101

23★. In figure 3.101, two circles intersect at points M and N. Secants drawn through M and N intersect the circles at points R, S and P, Q respectively.
Prove that : seg SQ || seg RP.

24★. In figure 3.102, two circles intersect each other at points A and E. Their common secant through E intersects the circles at points B and D. The tangents of the circles at points B and D intersect each other at point C.
Prove that □ ABCD is cyclic.

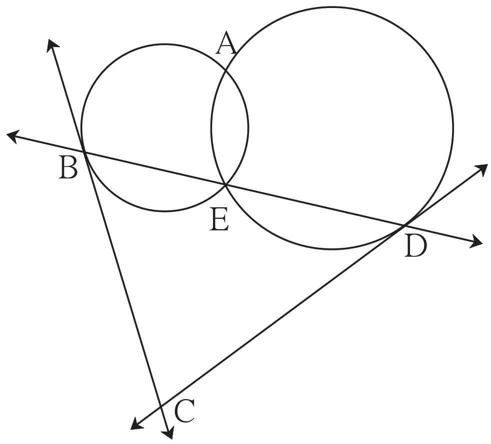


Fig. 3.102

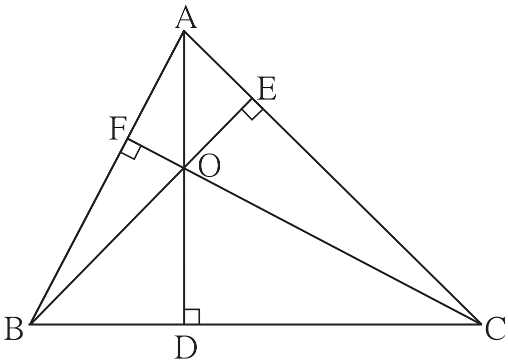


Fig. 3.103

25★. In figure 3.103, seg AD ⊥ side BC, seg BE ⊥ side AC, seg CF ⊥ side AB. Ponit O is the orthocentre. Prove that , point O is the incentre of Δ DEF.

ICT Tools or Links

Use the geogebra to verify the properties of chords and tangents of a circle.