

## Chapter 5: Application of Definite Integration

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### EXERCISE 5.1 [PAGE 187]

#### Exercise 5.1 | Q 1.1 | Page 187

Find the area of the region bounded by the following curves, X-axis and the given lines:  
 $y = 2x$ ,  $x = 0$ ,  $x = 5$

#### SOLUTION

$$\text{Required area} = \int_0^5 y \cdot dx, \text{ where } y = 2x$$

$$= \int_0^5 2x \cdot dx$$

$$= \left[ \frac{2x^2}{2} \right]_0^5$$

$$= 25 - 0$$

$$= 25 \text{ sq units.}$$

#### Exercise 5.1 | Q 1.2 | Page 187

Find the area of the region bounded by the following curves, X-axis and the given lines:  
 $x = 2y$ ,  $y = 0$ ,  $y = 4$

#### SOLUTION

$$\text{Required area} = \int_0^4 x \cdot dy, \text{ where } x = 2y$$

$$= \int_0^4 2y \cdot dy$$

$$= \left[ \frac{2y^2}{2} \right]_0^4$$

$$= 16 - 0$$

$$= 16 \text{ sq units.}$$

**Exercise 5.1 | Q 1.3 | Page 187**

Find the area of the region bounded by the following curves, X-axis and the given lines :  
 $x = 0, x = 5, y = 0, y = 4$

**SOLUTION**

$$\text{Required area} = \int_0^5 y \cdot dx, \text{ where } y = 4$$

$$= \int_0^5 4 \cdot dx$$

$$= [4x]_0^5$$

$$= 20 - 0$$

$$= 20 \text{ sq units.}$$

**Exercise 5.1 | Q 1.4 | Page 187**

Find the area of the region bounded by the following curves, X-axis and the given lines :  
 $y = \sin x, x = 0, x = \pi/2$

**SOLUTION**

$$\text{Required area} = \int_0^{\frac{\pi}{2}} y \cdot dx, \text{ where } y = \sin x$$

$$= \int_0^{\frac{\pi}{2}} \sin x \cdot dx$$

$$= [-\cos x]_0^{\frac{\pi}{2}}$$

$$= -\cos \frac{\pi}{2} + \cos 0$$

$$= 0 + 1$$

$$= 1 \text{ sq unit.}$$

**Exercise 5.1 | Q 1.5 | Page 187**

Find the area of the region bounded by the following curves, X-axis and the given lines:  
 $xy = 2, x = 1, x = 4$

**SOLUTION**

For  $xy = 2$ ,  $y = \frac{2}{x}$ .

$$\text{Required area} = \int_1^4 y \cdot dx, \text{ where } y = \frac{2}{x}$$

$$= \int_1^4 \frac{2}{x} \cdot dx$$

$$= [2 \log|x|]_1^4$$

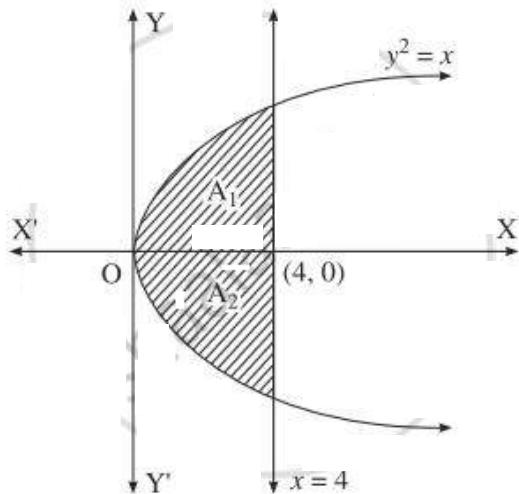
$$= 2 \log 4 - 2 \log 1$$

$$= 2 \log 4 - 0$$

$$= 2 \log 4 \text{ sq units.}$$

**Exercise 5.1 | Q 1.6 | Page 187**

Find the area of the region bounded by the following curves, X-axis and the given lines :  
 $y^2 = x$ ,  $x = 0$ ,  $x = 4$

**SOLUTION**

The required area consists of two bounded regions  $A_1$  and  $A_2$  which are equal in areas.

For  $y^2 = x$ ,  $y = \sqrt{x}$

Required area =  $A_1 + A_2 = 2A_1$

$$= 2 \int_0^4 y \cdot dx, \text{ where } y = \sqrt{x}$$

$$= 2 \int_0^4 \sqrt{x} \cdot dx$$

$$= 2 \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4$$

$$= 2 \left[ \frac{2}{3} (4)^{\frac{3}{2}} - 0 \right]$$

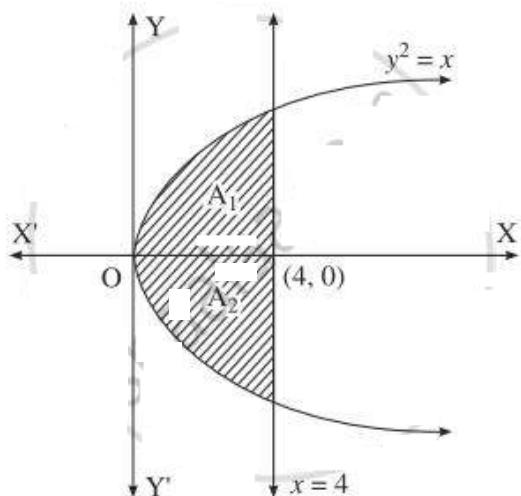
$$= 2 \left[ \frac{2}{3} (2^2)^{\frac{3}{2}} \right]$$

$$= \frac{32}{3} \text{ sq units.}$$

### Exercise 5.1 | Q 1.7 | Page 187

Find the area of the region bounded by the following curves, X-axis and the given lines:  
 $y^2 = 16x$ ,  $x = 0$ ,  $x = 4$

#### SOLUTION



The required area consists of two bounded regions  $A_1$  and  $A_2$  which are equal in areas.

For  $y^2 = x$ ,  $y = \sqrt{x}$

Required area =  $A_1 + A_2 = 2A_1$

$$= 2 \int_0^4 y \cdot dx, \text{ where } y = \sqrt{x}$$

$$= 2 \int_0^4 \sqrt{x} \cdot dx$$

$$= 2 \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0$$

$$= 2 \left[ \frac{2}{3} (4)^{\frac{3}{2}} - 0 \right]$$

$$= 2 \left[ \frac{2}{3} (2^2)^{\frac{3}{2}} \right]$$

$$= \frac{128}{3} \text{ sq units.}$$

### Exercise 5.1 | Q 2.1 | Page 187

Find the area of the region bounded by the parabola:  $y^2 = 16x$  and its latus rectum.

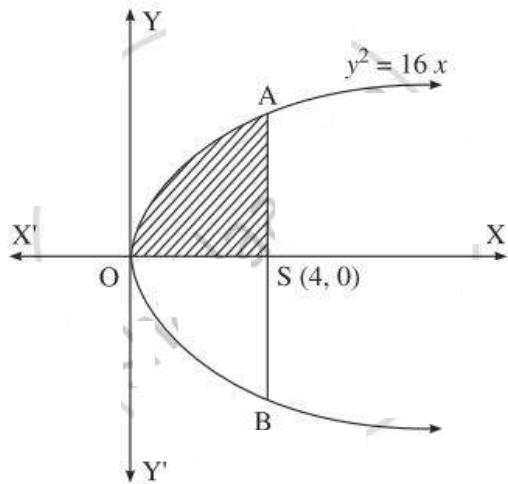
#### SOLUTION

Comparing  $y^2 = 16x$  with  $y^2 = 4ax$ , we get

$$4a = 16$$

$$\therefore a = 4$$

$$\therefore \text{focus is } S(a, 0) = (4, 0)$$



$$\text{For } y^2 = 16x, y = 4\sqrt{x}$$

Required area = area of the region OBSAO  
 $= 2[\text{area of the region OSAO}]$

$$\begin{aligned}
 &= 2 \int_0^4 y \cdot dx, \text{ where } y = 4\sqrt{x} \\
 &= 2 \int_0^4 4\sqrt{x} \cdot dx \\
 &= 8 \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4 \\
 &= 8 \left[ \frac{2}{3} (4)^{\frac{3}{2}} - 0 \right] \\
 &= 8 \left[ \frac{2}{3} (2^2)^{\frac{3}{2}} \right] \\
 &= \frac{128}{3} \text{ sq units.}
 \end{aligned}$$

### Exercise 5.1 | Q 2.2 | Page 187

Find the area of the region bounded by the parabola:  $y = 4 - x^2$  and the X-axis.

#### SOLUTION

The equation of the parabola is  $y = 4 - x^2$   
 $\therefore x^2 = 4 - y$ , i.e.  $(x - 0)^2 = -(y - 4)$

It has vertex at P(0, 4)

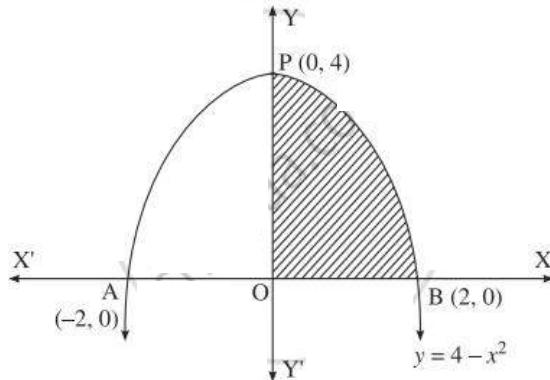
For points of intersection of the parabola with X-axis,  
we put  $y = 0$  in its equation.

$$\therefore 0 = 4 - x^2$$

$$\therefore x^2 = 4$$

$$\therefore x = \pm 2.$$

$\therefore$  the parabola intersect the X-axis at A (-2, 0) and B(2, 0)



Required area = area of the region APBOA  
 $= 2[\text{area of the region OPBO}]$

$$= 2 \int y \cdot dx, \text{ where } y = 4 - x^2$$

$$= 2 \int_0^2 (4 - x^2) \cdot dx$$

$$= 8 \int_0^2 1 \cdot dx - 2 \int_0^2 x^2 \cdot dx$$

$$= 8[x]_0^2 - 2 \left[ \frac{x^3}{3} \right]_0$$

$$= 8(2 - 0) - \frac{2}{3}(8 - 0)$$

$$= 16 - \frac{16}{3}$$

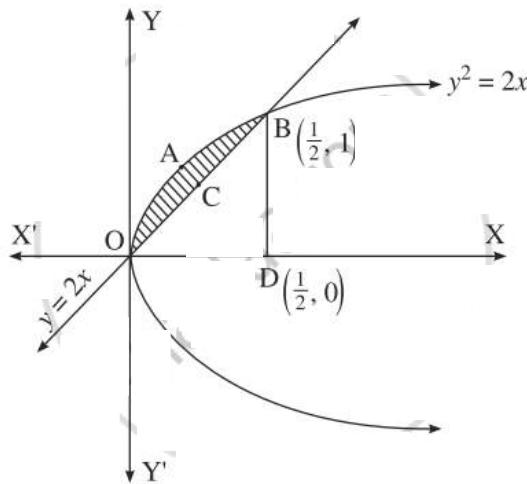
$$= \frac{32}{3} \text{ sq units.}$$

**Exercise 5.1 | Q 3.1 | Page 187**

Find the area of the region included between:  $y^2 = 2x$  and  $y = 2x$

**SOLUTION**

The vertex of the parabola  $y^2 = 2x$  is at the origin  $O = (0, 0)$ .



To find the points of intersection of the line and the parabola, equating the values of  $2x$  from both the equations we get,

$$\therefore y^2 = y$$

$$\therefore y^2 - y = 0$$

$$\therefore y(y - 1) = 0$$

$$\therefore y = 0 \text{ or } y = 1$$

$$\text{When } y = 0, x = \frac{0}{2} = 0$$

$$\text{When } y = 1, x = \frac{1}{2}$$

$\therefore$  the points of intersection are  $O(0, 0)$  and  $B\left(\frac{1}{2}, 1\right)$

Required area = area of the region OABCO

= area of the region OABDO – area of the region OCBDO

Now, area of the region OABDO

= area under the parabola  $y^2 = 2x$  between  $x = 0$  and  $x = \frac{1}{2}$

$$\begin{aligned}
&= \int_0^{\frac{1}{2}} y \cdot dx, \text{ where } y = \sqrt{2}x \\
&= \int_0^{\frac{1}{2}} \sqrt{2}x dx \\
&= \sqrt{2} \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{1}{2}} \\
&= \sqrt{2} \left[ \frac{2}{3} \left( \frac{1}{2} \right)^{\frac{3}{2}} - 0 \right] \\
&= \sqrt{2} \left[ \frac{2}{3} \cdot \frac{1}{2\sqrt{2}} \right] \\
&= \frac{1}{3}
\end{aligned}$$

Area of the region OCBDO

= area under the line y

=  $2x$  between x

$$= 0 \text{ and } x = \frac{1}{2}$$

$$= \int_0^{\frac{1}{2}} y \cdot dx, \text{ where } y = 2x$$

$$= \int_0^{\frac{1}{2}} 2x \cdot dx$$

$$= \left[ \frac{2x^2}{2} \right]_0^{\frac{1}{2}}$$

$$= \frac{1}{4} - 0$$

$$= \frac{1}{4}$$

$\therefore$  required area

$$= \frac{1}{3} = \frac{1}{4}$$

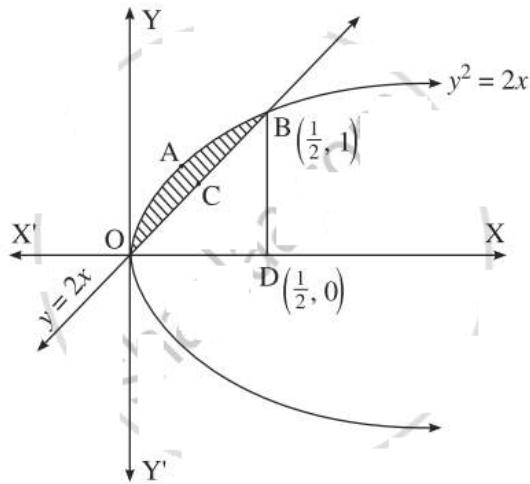
$$= \frac{1}{12} \text{ sq unit.}$$

### Exercise 5.1 | Q 3.2 | Page 187

Find the area of the region included between:  $y^2 = 4x$ , and  $y = x$

#### SOLUTION

The vertex of the parabola  $y^2 = 4x$  is at the origin  $O = (0, 0)$ .



To find the points of intersection of the line and the parabola, equaling the values of  $4x$  from both the equations we get,

$$\therefore y^2 = y$$

$$\therefore y^2 - y = 0$$

$$\therefore y(y - 1) = 0$$

$$\therefore y = 0 \text{ or } y = 1$$

$$\text{When } y = 0, x = \frac{0}{2} = 0$$

$$\text{When } y = 1, x = \frac{1}{2}$$

$\therefore$  the points of intersection are O(0, 0) and B $\left(\frac{1}{2}, 1\right)$

Required area = area of the region OABCO

= area of the region OABDO – area of the region OCBDO

Now, area of the region OABDO

$$= \text{area under the parabola } y^2 = 4x \text{ between } x = 0 \text{ and } x = \frac{1}{2}$$

$$= \int_0^{\frac{1}{2}} y \cdot dx, \text{ where } y = \sqrt{x}$$

$$= \int_0^{\frac{1}{2}} \sqrt{2x} dx$$

$$= \sqrt{2} \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{1}{2}}$$

$$= \sqrt{2} \left[ \frac{2}{3} \left( \frac{1}{2} \right)^{\frac{3}{2}} - 0 \right]$$

$$= \sqrt{2} \left[ \frac{2}{3} \cdot \frac{1}{2\sqrt{2}} \right]$$

$$= \frac{1}{3}$$

Area of the region OCBDO

= area under the line y

= 2x between x

$$= 0 \text{ and } x = \frac{1}{2}$$

$$= \int_0^{\frac{1}{2}} y \cdot dx, \text{ where } y = x$$

$$= \int_0^{\frac{1}{2}} 2x \cdot dx$$

$$= \left[ \frac{2x^2}{2} \right]_0^{\frac{1}{2}}$$

$$= \frac{4}{1} - 0$$

$$= \frac{4}{3}$$

$\therefore$  required area

$$= \frac{4}{1} = \frac{4}{3}$$

$$= \frac{8}{3} \text{ sq units.}$$

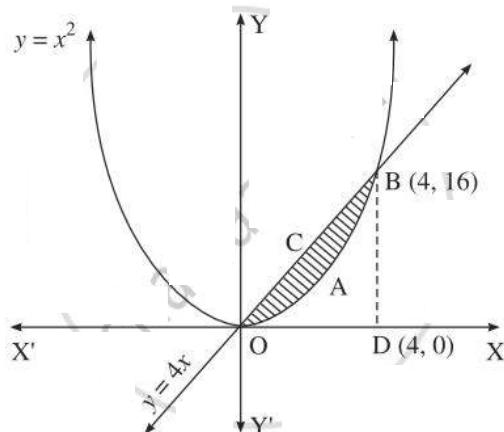
### Exercise 5.1 | Q 3.3 | Page 187

Find the area of the region included between:  $y = x^2$  and the line  $y = 4x$

#### SOLUTION

The vertex of the parabola  $y = x^2$  is at the origin O(0, 0)

To find the points of the intersection of the line and the parabola.



Equating the values of  $y$  from the two equations, we get

$$\begin{aligned}x^2 &= 4x \\ \therefore x^2 - 4x &= 0 \\ \therefore x(x - 4) &= 0 \\ \therefore x = 0, x &= 4\end{aligned}$$

When  $x = 0, y = 4(0) = 0$   
When  $x = 4, y = 4(4) = 16$

$\therefore$  the points of intersection are O(0, 0) and B(4, 16)  
Required area = area of the region OABCO

= (area of the region ODBCO) – (area of the region ODBAO)  
Now, area of the region ODBCO

= area under the line  $y = 4x$  between  $x = 0$  and  $x = 4$

$$= \int_0^4 y \cdot dx, \text{ where } y = 4x$$

$$= \int_0^4 4x \cdot dx$$

$$= 4 \int_0^4 x \cdot dx$$

$$= 4 \left[ \frac{x^2}{2} \right]_0^4$$

$$= 2(16 - 0)$$

$$= 32$$

Area of the region ODBAO

= area under the parabola  $y = x^2$  between  $x = 0$  and  $x = 4$

$$= \int_0^4 y \cdot dx, \text{ where } y = x^2$$

$$= \int_0^4 x^2 \cdot dx$$

$$\begin{aligned}
 &= \left[ \frac{x^3}{3} \right]_0^4 \\
 &= \frac{1}{3}(64 - 0) \\
 &= \frac{64}{3}
 \end{aligned}$$

$\therefore$  required area

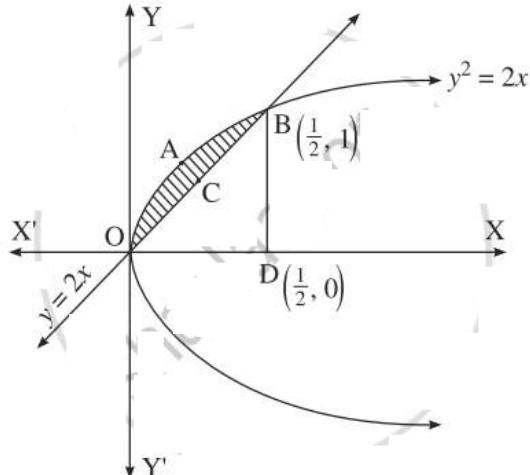
$$\begin{aligned}
 &= 32 - \frac{64}{3} \\
 &= \frac{32}{3} \text{ sq units.}
 \end{aligned}$$

### Exercise 5.1 | Q 3.4 | Page 187

Find the area of the region included between:  $y^2 = 4ax$  and the line  $y = x$

#### SOLUTION

The vertex of the parabola  $y^2 = 4ax$  is at the origin O = (0, 0).



To find the points of intersection of the line and the parabola, equating the values of  $4ax$  from both the equations we get,

$$\begin{aligned}
 &\therefore y^2 = y \\
 &\therefore y^2 - y = 0 \\
 &\therefore y(y - 1) = 0 \\
 &\therefore y = 0 \text{ or } y = 1
 \end{aligned}$$

$$\text{When } y = 0, x = \frac{0}{2} = 0$$

$$\text{When } y = 1, x = \frac{1}{2}$$

$\therefore$  the points of intersection are O(0, 0) and B $\left(\frac{1}{2}, 1\right)$

Required area = area of the region OABCO

= area of the region OABDO – area of the region OCBDO

Now, area of the region OABDO

$$= \text{area under the parabola } y^2 = 4ax \text{ between } x = 0 \text{ and } x = \frac{1}{2}$$

$$= \int_0^{\frac{1}{2}} y \cdot dx, \text{ where } y = \sqrt{2x}$$

$$= \int_0^{\frac{1}{2}} \sqrt{2}x dx$$

$$= \sqrt{2} \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{\frac{1}{2}}$$

$$= \sqrt{2} \left[ \frac{2}{3} \left( \frac{1}{2} \right)^{\frac{3}{2}} - 0 \right]$$

$$= \sqrt{2} \left[ \frac{2}{3} \cdot \frac{1}{2\sqrt{2}} \right]$$

$$= \frac{1}{3}$$

Area of the region OCBDO

= area under the line y

= 4ax between x

$$= 0 \text{ and } x = \frac{1}{4ax}$$

$$= \int_0^{\frac{1}{2}} y \cdot dx, \text{ where } y = x$$

$$= \int_0^{\frac{1}{2}} 2x \cdot dx$$

$$= \left[ \frac{2x^2}{2} \right]_0^{\frac{1}{2}}$$

$$= \frac{4}{3} - 0$$

$$= \frac{2a^2}{1}$$

$\therefore$  required area

$$= \frac{4}{3} = \frac{2a^2}{1}$$

$$= \frac{8a^2}{3} \text{ sq units.}$$

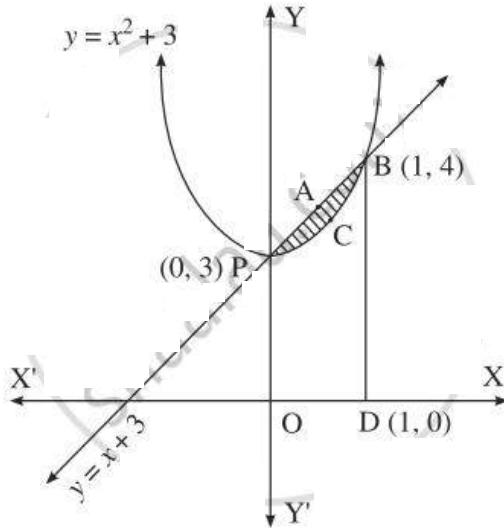
### Exercise 5.1 | Q 3.5 | Page 187

Find the area of the region included between:  $y = x^2 + 3$  and the line  $y = x + 3$

#### SOLUTION

The given parabola is  $y = x^2 + 3$ , i.e.  $(x - 0)^2 = y - 3$

$\therefore$  its vertex is P(0, 3).



To find the points of intersection of the line and the parabola. Equating the values of y from both the equations, we get

$$x^3 + 3 = x + 3$$

$$\therefore x^2 - x = 0$$

$$\therefore x(x - 1) = 0$$

$$\therefore x = 0 \text{ or } x = 1$$

$$\text{When } x = 0, y = 0 + 3 = 3$$

$$\text{When } x = 1, y = 1 + 3 = 4$$

$\therefore$  the points of intersection are P(0, 3) and B(1, 4) Required area = area of the region PABCP

= area of the region OPABDO – area of the region OPCBDO

Now, area of the region OPABDO

= area under the line  $y = x + 3$  between  $x = 0$  and  $x = 1$

$$= \int_0^1 y \cdot dx, \text{ where } y = x + 3$$

$$= \int_0^1 (x + 3) \cdot dx$$

$$= \int_0^1 x \cdot dx + 3 \int_0^1 1 \cdot dx$$

$$\begin{aligned}
 &= \left[ \frac{x^2}{2} \right]_0^1 + 3[x]_0^1 \\
 &= \left( \frac{1}{2} - 0 \right) + 3(1 - 0) \\
 &= \frac{7}{2}
 \end{aligned}$$

Area of the region OPCBDO

= area under the parabola  $y = x^2 + 3$  between  $x = 0$  and  $x = 1$

$$= \int_0^1 y \cdot dx, \text{ where } y = x^2 + 3$$

$$= \int_0^1 (x^2 + 3) \cdot dx$$

$$= \int_0^1 x^2 \cdot dx + 3 \int_0^1 1 \cdot dx$$

$$= \left[ \frac{x^3}{3} \right]_0^1 + 3[x]_0^1$$

$$= \left( \frac{1}{3} - 0 \right) + 3(1 - 0)$$

$$= \frac{10}{3}$$

$$\therefore \text{required area} = \frac{7}{2} - \frac{10}{3}$$

$$= \frac{21 - 20}{6}$$

$$= \frac{1}{6} \text{ sq unit.}$$

#### MISCELLANEOUS EXERCISE 5 [PAGES 188 - 190]

##### Miscellaneous Exercise 5 | Q 1.01 | Page 188

Choose the correct option from the given alternatives :

The area bounded by the region  $0 \leq x \leq 5$  and  $2 \leq y \leq 5$  is given by

1. 12 sq units
2. 8 sq units
3. 25 sq units
4. 32 sq units

**SOLUTION**

12 sq units.

Miscellaneous Exercise 5 | Q 1.02 | Page 188

**Choose the correct option from the given alternatives :**

The area of the region enclosed by the curve  $y = \frac{1}{x}$ , and the lines  $x = e$ ,  $x = e^2$  is given by

1 sq unit

- $\frac{1}{2}$  sq unit  
 $\frac{3}{2}$   
 $\frac{3}{2}$  sq units  
 $\frac{5}{2}$   
 $\frac{5}{2}$  sq units

**SOLUTION**

1 sq unit.

Miscellaneous Exercise 5 | Q 1.03 | Page 188

**Choose the correct option from the given alternatives :**

The area bounded by the curve  $y = x^3$ , the X-axis and the lines  $x = -2$  and  $x = 1$  is

- 9 sq units  
 $-\frac{15}{4}$  sq units  
 $\frac{15}{4}$  sq units  
 $\frac{17}{4}$  sq units

**SOLUTION**

$-\frac{15}{4}$  sq units.

**Choose the correct option from the given alternatives :**

The area enclosed between the parabola  $y^2 = 4x$  and line  $y = 2x$  is

$$\frac{2}{3} \text{ sq units}$$

$$\frac{1}{3}$$

$$\frac{1}{3} \text{ sq unit}$$

$$\frac{1}{4}$$

$$\frac{3}{4} \text{ sq unit}$$

**SOLUTION**

$$\frac{1}{3} \text{ sq unit.}$$

**Choose the correct option from the given alternatives :**

The area of the region bounded between the line  $x = 4$  and the parabola  $y^2 = 16x$  is

$$\frac{128}{3} \text{ sq units}$$

$$\frac{108}{3}$$

$$\frac{118}{3} \text{ sq units}$$

$$\frac{218}{3}$$

**SOLUTION**

$$\frac{128}{3} \text{ sq units.}$$

**Choose the correct option from the given alternatives :**

The area of the region bounded by  $y = \cos x$ , Y-axis and the lines  $x = 0, x = 2\pi$  is

- 1 sq unit
- 2 sq units
- 3 sq units
- 4 sq units**

**SOLUTION**

4 sq units.

**Choose the correct option from the given alternatives :**

The area bounded by the parabola  $y^2 = 8x$ , the X-axis and the latus rectum is

- $\frac{31}{3}$  sq units
- $\frac{32}{3}$  sq units
- $\frac{32\sqrt{2}}{3}$  sq units
- $\frac{16}{3}$  sq units

**SOLUTION**

$\frac{32}{3}$  sq units.

**Choose the correct option from the given alternatives :**

The area under the curve  $y = 2\sqrt{x}$ , enclosed between the lines  $x = 0$  and  $x = 1$  is

4 sq units

$\frac{3}{4}$  sq unit

$\frac{2}{3}$

$\frac{3}{4}$  sq unit

$\frac{4}{3}$

$\frac{4}{3}$  sq units

**SOLUTION**

$\frac{4}{3}$  sq units.

Miscellaneous Exercise 5 | Q 1.09 | Page 189

**Choose the correct option from the given alternatives :**

The area of the circle  $x^2 + y^2 = 25$  in first quadrant is

$\frac{25\pi}{4}$  sq units

$5\pi$  sq units

5 sq units

3 sq units

**SOLUTION**

$\frac{25\pi}{4}$  sq units.

Miscellaneous Exercise 5 | Q 1.1 | Page 189

**Choose the correct option from the given alternatives :**

The area of the region bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is

ab sq units

$\pi ab$  sq units

$\frac{\pi}{ab}$  sq units

$\pi a^2$  sq units

**SOLUTION**

$\pi ab$  sq units.

Miscellaneous Exercise 5 | Q 1.11 | Page 189

**Choose the correct option from the given alternatives :**

The area bounded by the parabola  $y^2 = x$  and the line  $2y = x$  is

$$\frac{4}{3} \text{ sq unit}$$

$$1 \text{ sq unit}$$

$$\frac{2}{3} \text{ sq unit}$$

$$\frac{1}{3} \text{ sq unit}$$

**SOLUTION**

$$\frac{4}{3} \text{ sq unit.}$$

Miscellaneous Exercise 5 | Q 1.12 | Page 189

**Choose the correct option from the given alternatives :**

The area enclosed between the curve  $y = \cos 3x$ ,  $0 \leq x \leq \frac{\pi}{6}$  and the X-axis is

$$\frac{1}{2} \text{ sq unit}$$

$$1 \text{ sq unit}$$

$$\frac{2}{3} \text{ sq unit}$$

$$\frac{1}{3} \text{ sq unit}$$

**SOLUTION**

$$\frac{1}{3} \text{ sq unit.}$$

Miscellaneous Exercise 5 | Q 1.13 | Page 189

**Choose the correct option from the given alternatives :**

The area bounded by  $y = \sqrt{x}$  and the  $x = 2y + 3$ , X-axis in first quadrant is  
 $2\sqrt{3}$  sq units

**9 sq units**

$\frac{34}{3}$  sq units

18 sq units

**SOLUTION**

9 sq units.

Miscellaneous Exercise 5 | Q 1.14 | Page 189

**Choose the correct option from the given alternatives :**

The area bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  and the line  $\frac{x}{a} + \frac{y}{b} = 1$  is

$(\pi ab - 2ab)$  sq units

$\left(\frac{\pi ab}{4} - \frac{ab}{2}\right)$  sq units

$(\pi ab - ab)$  sq units

$\pi ab$  sq units

**SOLUTION**

$\left(\frac{\pi ab}{4} - \frac{ab}{2}\right)$  sq units.

Miscellaneous Exercise 5 | Q 1.15 | Page 189

**Choose the correct option from the given alternatives :**

The area bounded by the parabola  $y = x^2$  and the line  $y = x$  is

- $\frac{1}{2}$  sq unit
- $\frac{1}{3}$  sq unit
- $\frac{1}{6}$  sq unit
- $\frac{1}{12}$  sq unit

**SOLUTION**

$$\frac{1}{6} \text{ sq unit.}$$

Miscellaneous Exercise 5 | Q 1.16 | Page 189

**Choose the correct option from the given alternatives :**

The area enclosed between the two parabolas  $y^2 = 4x$  and  $y = x$  is

- $\frac{16}{3}$  sq units
- $\frac{32}{3}$  sq units
- $\frac{8}{3}$  sq units
- $\frac{3}{4}$  sq units

**SOLUTION**

$$\frac{8}{3} \text{ sq units.}$$

Miscellaneous Exercise 5 | Q 1.17 | Page 190

**Choose the correct option from the given alternatives :**

The area bounded by the curve  $y = \tan x$ , X-axis and the line  $x = \frac{\pi}{4}$  is

- $\frac{1}{2} \log 2$  sq units
- $\log 2$  sq units

$2 \log 2$  sq units

$3 \cdot \log 2$  sq units

**SOLUTION**

$$\frac{1}{2} \log 2 \text{ sq units.}$$

Miscellaneous Exercise 5 | Q 1.18 | Page 190

**Choose the correct option from the given alternatives :**

The area of the region bounded by  $x^2 = 16y$ ,  $y = 1$ ,  $y = 4$  and  $x = 0$  in the first quadrant, is

$\frac{7}{3}$  sq units

$\frac{3}{8}$

$\frac{3}{64}$  sq units

$\frac{64}{3}$

$\frac{3}{56}$  sq units

$\frac{56}{3}$  sq units

**SOLUTION**

$$\frac{56}{3} \text{ sq units.}$$

Miscellaneous Exercise 5 | Q 1.19 | Page 190

**Choose the correct option from the given alternatives :**

The area of the region included between the parabolas  $y^2 = 4ax$  and  $x^2 = 4ay$ , ( $a > 0$ ) is given by

$\frac{16a^2}{3}$  sq units

$\frac{8a^2}{3}$

$\frac{64}{3}$  sq units

$\frac{3}{56}$

$\frac{56}{3}$  sq units

**SOLUTION**

$$\frac{16a^2}{3} \text{ sq units.}$$

**Choose the correct option from the given alternatives :**

The area of the region included between the line  $x + y = 1$  and the circle  $x^2 + y^2 = 1$  is

$$\left(\frac{\pi}{2} - 1\right) \text{sq units}$$

$$(\pi - 2) \text{ sq units}$$

$$\left(\frac{\pi}{4} - \frac{1}{2}\right) \text{sq units}$$

$$\left(\pi - \frac{1}{2}\right) \text{sq units}$$

**SOLUTION**

$$\left(\frac{\pi}{4} - \frac{1}{2}\right) \text{sq units}$$

**Solve the following :**

Find the area of the region bounded by the following curve, the X-axis and the given lines :  $0 \leq x \leq 5, 0 \leq y \leq 2$

**SOLUTION**

$$\text{Required area} = \int_0^5 y \cdot dx, \text{ where } y = 2$$

$$= \int_0^5 2 \cdot dx = [2x]_0^5$$

$$= 2 \times 5 - 0$$

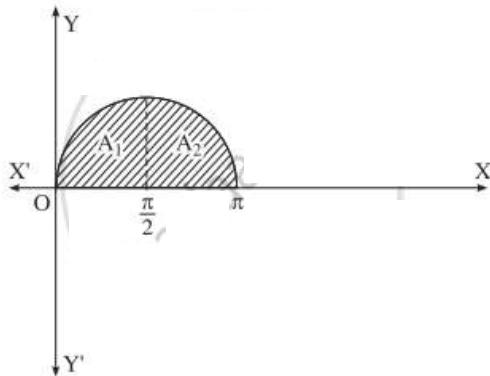
$$= 10 \text{ sq units.}$$

**Solve the following :**

Find the area of the region bounded by the following curve, the X-axis and the given lines :  $y = \sin x, x = 0, x = \pi$

### SOLUTION

The curve  $y = \sin x$  intersects the X-axis at  $x = 0$  and  $x = \pi$  between  $x = 0$  and  $x = \pi$ .



Two bounded regions  $A_1$  and  $A_2$  are obtained. Both the regions have equal areas.  
 $\therefore$  required area  $= A_1 + A_2 = 2A_1$

$$\begin{aligned} &= 2 \int_0^{\frac{\pi}{2}} y \cdot dx, \text{ where } y = \sin x \\ &= 2 \int_0^{\frac{\pi}{2}} \sin x \cdot dx \\ &= 2[-\cos x]_0^{\frac{\pi}{2}} \\ &= 2\left[-\cos \frac{\pi}{2} - \cos 0\right] \\ &= 2(-0 + 1) \\ &= 2 \text{ sq units.} \end{aligned}$$

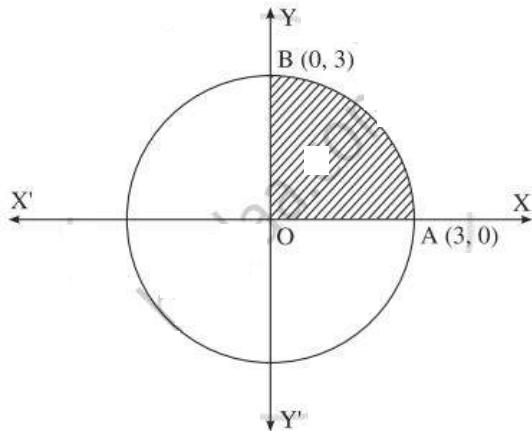
Miscellaneous Exercise 5 | Q 2.02 | Page 190

**Solve the following :**

Find the area of the circle  $x^2 + y^2 = 9$ , using integration.

### SOLUTION

By the symmetry of the circle, its area is equal to 4 times the area of the region OABO. Clearly for this region, the limits of integration are 0 and 3.



From the equation of the circle,  $y^2 = 9 - x^2$ .

In the first quadrant,  $y > 0$

$$\therefore y = \sqrt{9 - x^2}$$

$\therefore$  area of the circle = 4 (area of the region OABO)

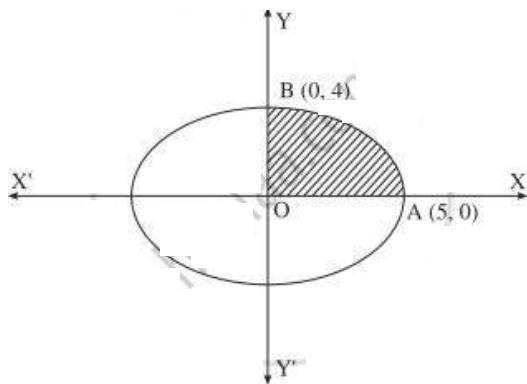
$$\begin{aligned} &= 4 \int_0^3 y \cdot dx = 4 \int_0^3 \sqrt{9 - x^2} \cdot dx \\ &= 4 \left[ \frac{x}{2} \sqrt{9 - x^2} + \frac{9}{2} \sin^{-1}\left(\frac{x}{3}\right) \right]_0^3 \\ &= 4 \left[ \frac{3}{2} \sqrt{9 - 9} + \frac{9}{2} \sin^{-1}\left(\frac{3}{3}\right) \right] - 4 \left[ \frac{0}{2} \sqrt{9 - 0} + \frac{9}{2} \sin^{-1}(0) \right] \\ &= 4 \cdot \frac{9}{2} \cdot \frac{\pi}{2} \\ &= 9\pi \text{ sq units.} \end{aligned}$$

Miscellaneous Exercise 5 | Q 2.03 | Page 190

**Solve the following :**

Find the area of the ellipse  $\frac{x^2}{25} + \frac{y^2}{16} = 1$  using integration

**SOLUTION**



By the symmetry of the ellipse, its area is equal to 4 times the area of the region OABO. Clearly for this region, the limits of integration are 0 and 5.

From the equation of the ellipse

$$\frac{y^2}{16} = 1 - \frac{x^2}{25} = \frac{25 - x^2}{25}$$

$$\therefore y^2 = \frac{16}{25} (25 - x^2)$$

In the first quadrant  $y > 0$

$$\therefore y = \frac{4}{5} \sqrt{25 - x^2}$$

$\therefore$  area of the ellipse = 4 (area of the region OABO)

$$\begin{aligned}
 &= 4 \int_0^5 y \cdot dx \\
 &= \int_0^5 \frac{4}{5} \sqrt{25 - x^2} \cdot dx \\
 &= \frac{16}{5} \int_0^5 \sqrt{25 - x^2} \cdot dx \\
 &= \frac{16}{5} \left[ \frac{x}{2} \sqrt{25 - x^2} + \frac{25}{2} \sin^{-1}\left(\frac{x}{5}\right) \right]_0^5 \\
 &= \frac{16}{5} \left( \frac{5}{2} \sqrt{25 - 25} + \frac{25}{2} \sin^{-1}(1) \right) - \frac{16}{5} \left[ \frac{5}{2} \sqrt{25 - 0} + \frac{25}{2} \sin^{-1}(0) \right]
 \end{aligned}$$

$$= \frac{16}{5} \times \frac{25}{2} \times \frac{\pi}{2}$$

=  $20\pi$  sq units.

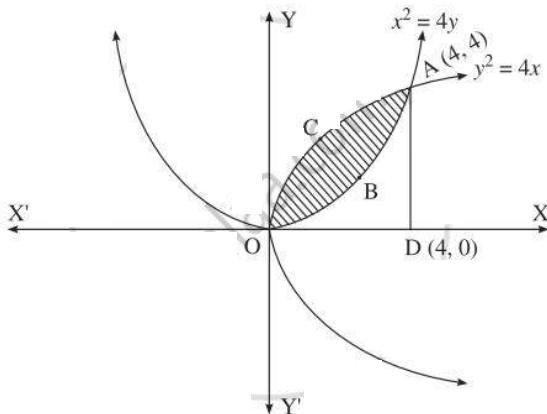
### Miscellaneous Exercise 5 | Q 2.04 | Page 190

**Solve the following :**

Find the area of the region lying between the parabolas :

$$y^2 = 4x \text{ and } x^2 = 4y$$

**SOLUTION**



For finding the points of intersection of the two parabolas, we equate the values of  $y^2$  from their equations.

$$\text{From the equation } x^2 = 4y, y = \frac{x^2}{4}$$

$$\therefore y = \frac{x^4}{16}$$

$$\therefore \frac{x^4}{16} = 4x$$

$$\therefore x^4 - 64x = 0$$

$$\therefore x(x^3 - 64) = 0$$

$$\therefore x = 0 \text{ or } x^3 = 64$$

$$\text{i.e. } x = 0 \text{ or } x = 4$$

$$\text{When } x = 0, y = 0$$

$$\text{When } x = 4, y = \frac{4^2}{4} = 4$$

$\therefore$  the points of intersection are O(0, 0) and A(4, 4).

Required area = area of the region OBACO

$$= [\text{area of the region ODACO}] - [\text{area of the region ODABO}]$$

Now, area of the region ODACO

$$= \text{area under the parabola } y^2 = 4x,$$

i.e.  $y = 2\sqrt{x}$  between  $x = 0$  and  $x = 4$

$$= \int_0^4 2\sqrt{x} \cdot dx$$

$$= \left[ 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4$$

$$= 2 \times \frac{2}{3} \times 4^{\frac{3}{2}} - 0$$

$$= \frac{4}{3} \times (2^3)$$

$$= \frac{32}{3}$$

Area of the region ODABO

$$= \text{area under the parabola } x^2 = 4y,$$

$$\text{i.e. } y = \frac{x^2}{4} \text{ between } x = 0 \text{ and } x = 4$$

$$= \int_0^4 \frac{1}{4} x^2 \cdot dx$$

$$= \frac{1}{4} \left[ \frac{x^3}{3} \right]_0^4$$

$$= \frac{1}{4} \left( \frac{64}{3} - 0 \right)$$

$$= \frac{16}{3}$$

$$\therefore \text{required area} = \frac{32}{3} - \frac{16}{3}$$

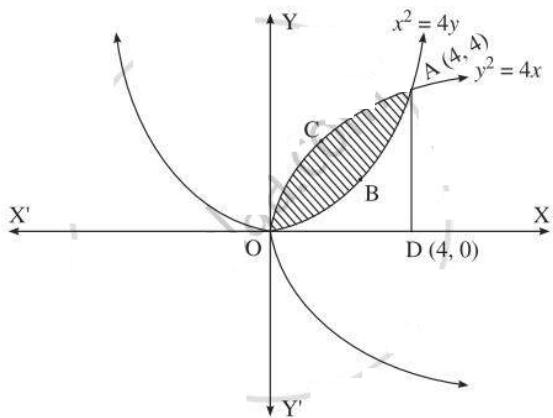
$$= \frac{16}{3} \text{ sq units.}$$

### Miscellaneous Exercise 5 | Q 2.04 | Page 190

**Solve the following :**

Find the area of the region lying between the parabolas :  $y^2 = x$  and  $x^2 = y$ .

**SOLUTION**



For finding the points of intersection of the two parabolas, we equate the values of  $y^2$  from their equations.

From the equation  $x^2 = y$ ,  $y = \frac{x^2}{y}$

$$\therefore y = \frac{x^2}{y}$$

$$\therefore \frac{x^2}{y} = x$$

$$\therefore x^2 - y = 0$$

$$\therefore x(x^3 - y) = 0$$

$$\therefore x = 0 \text{ or } x^3 = y$$

i.e.  $x = 0$  or  $x = 4$

When  $x = 0$ ,  $y = 0$

When  $x = 4$ ,  $y = \frac{4^2}{4} = 4$

$\therefore$  the points of intersection are O(0, 0) and A(4, 4).

Required area = area of the region OBACO

$$= [\text{area of the region ODACO}] - [\text{area of the region ODABO}]$$

Now, area of the region ODACO

= area under the parabola  $y^2 = 4x$ ,

i.e.  $y = 2\sqrt{x}$  between  $x = 0$  and  $x = 4$

$$= \int_0^4 2\sqrt{x} \cdot dx$$

$$= \left[ 2 \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4$$

$$= 2 \times \frac{2}{3} \times 4^{\frac{3}{2}} - 0$$

$$= \frac{4}{3} \times (2^3)$$

$$= \frac{32}{3}$$

Area of the region ODABO

= area under the parabola  $x^2 = 4y$ ,

i.e.  $y = \frac{x^2}{3}$  between  $x = 0$  and  $x = 4$

$$= \int_0^4 \frac{1}{3}x^2 \cdot dx$$

$$= \frac{1}{3} \left[ \frac{x^3}{3} \right]_0^4$$

$$= \frac{1}{3} \left( \frac{y}{3} - 0 \right)$$

$$= \frac{y}{3}$$

$$\therefore \text{required area} = \frac{1}{3} - \frac{y}{3}$$

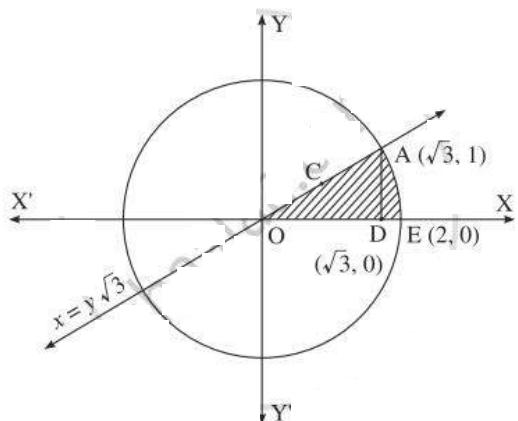
$$= \frac{1}{3} \text{ sq units.}$$

### Miscellaneous Exercise 5 | Q 2.05 | Page 190

**Solve the following :**

Find the area of the region in first quadrant bounded by the circle  $x^2 + y^2 = 4$  and the X-axis and the line  $x = y\sqrt{3}$ .

**SOLUTION**



For finding the point of intersection of the circle and the line, we solve

$$x^2 + y^2 = 4 \quad \dots(1)$$

$$\text{and } x = y\sqrt{3} \quad \dots(2)$$

$$\text{From (2), } x^2 = 3y$$

$$\text{From (1), } x^2 = 4 - y^2$$

$$\therefore 3y^2 = 4 - y^2$$

$$\therefore 4y^2 = 4$$

$$\therefore y^2 = 1$$

$\therefore y = 1$  in the first quadrant.

$$\text{When } y = 1, x = 1 \times \sqrt{3} = \sqrt{3}$$

$\therefore$  the circle and the line intersect at  $A(\sqrt{3}, 1)$  in the first quadrant

Required area = area of the region OCADO

= area of the region OCADO + area of the region DAED

Now, area of the region OCADO

= area under the line  $x = y\sqrt{3}$

i.e.  $y = \frac{x}{\sqrt{3}}$  between  $x = 0$  and  $x = \sqrt{3}$

$$\begin{aligned}
&= \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}} \cdot dx \\
&= \left[ \frac{x^2}{2\sqrt{3}} \right]_0^{\sqrt{3}} \\
&= \frac{3}{2\sqrt{3}} - 0 \\
&= \frac{\sqrt{3}}{2}
\end{aligned}$$

Area of the region DAED

$$\begin{aligned}
&= \text{area under the circle } x^2 + y^2 = 4 \text{ i.e. } y = +\sqrt{4 - x^2} \text{ (in the first quadrant) between } x = \sqrt{3} \text{ and } x = 2 \\
&= \int_{\sqrt{3}}^2 \sqrt{4 - x^2} \cdot dx \\
&= \left[ \frac{x}{2} \sqrt{4 - x^2} + \frac{4}{2} \sin^{-1} \left( \frac{x}{2} \right) \right]_{\sqrt{3}}^2 \\
&= \left[ \frac{2}{2} \sqrt{4 - 4} + 2 \sin^{-1}(1) \right] - \left[ \frac{\sqrt{3}}{2} \sqrt{4 - 3} + 2 \sin^{-1} \frac{\sqrt{3}}{2} \right] \\
&= 0 + 2 \left( \frac{\pi}{2} \right) - \frac{\sqrt{3}}{2} - 2 \left( \frac{\pi}{3} \right) \\
&= \pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} \\
&= \frac{\pi}{3} - \frac{\sqrt{3}}{2} \\
&\therefore \text{required area} = \frac{\sqrt{3}}{2} + \left( \frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) \\
&= \frac{\pi}{3} \text{ sq units.}
\end{aligned}$$

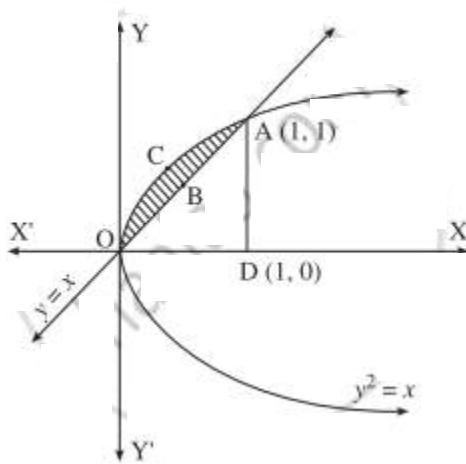
### Miscellaneous Exercise 5 | Q 2.06 | Page 190

**Solve the following :**

Find the area of the region bounded by the parabola  $y^2 = x$  and the line  $y = x$  in the first quadrant.

### SOLUTION

To obtain the points of intersection of the line and the parabola, we equate the values of  $x$  from both the equations.



$$\begin{aligned}\therefore y^2 &= y \\ \therefore y^2 - y &= 0 \\ \therefore y(y - 1) &= 0 \\ \therefore y &= 0 \text{ or } y = 1\end{aligned}$$

When  $y = 0, x = 0$   
When  $y = 1, x = 1$

$\therefore$  the points of intersection are  $O(0, 0)$  and  $A(1, 1)$ . Required area of the region OCABO  
 $=$  area of the region OCADO – area of the region OBADO

Now, area of the region OCADO

$=$  area under the parabola  $y^2 = x$  i.e.  $y = \pm\sqrt{x}$  (in the first quadrant) between  $x = 0$  and  $x = 1$

$$= \int_0^1 \sqrt{x} \cdot dx$$

$$= \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^1$$

$$= \frac{2}{3} \times (1 - 0)$$

$$= \frac{2}{3}$$

Area of the region OBADO

= area under the line  $y = x$  between  $x = 0$  and  $x = 1$

$$= \int_0^1 x \cdot dx$$

$$= \left[ \frac{x^2}{2} \right]_0^1$$

$$= \frac{1}{2} - 0$$

$$= \frac{2}{3}$$

$$\therefore \text{required area} = \frac{2}{3} - \frac{1}{2}$$

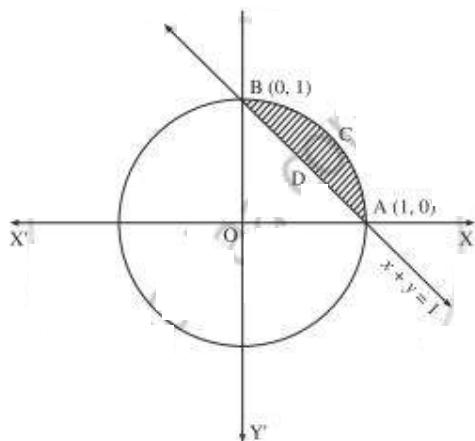
$$= \frac{1}{6} \text{ sq unit.}$$

### Miscellaneous Exercise 5 | Q 2.07 | Page 190

**Solve the following :**

Find the area enclosed between the circle  $x^2 + y^2 = 1$  and the line  $x + y = 1$ , lying in the first quadrant.

**SOLUTION**



Required area = area of the region ACBDA

= (area of the region OACBO) – (area of the region OADBO)

Now, area of the region OACBO

= area under the circle  $x^2 + y^2 = 1$  between  $x = 0$  and  $x = 1$

$$= \int_0^1 \cdot dx, \text{ where } y^2 = 1 - x^2,$$

i.e.  $y = \sqrt{1 - x^2}$ , as  $y > 0$

$$= \int_0^1 \sqrt{1 - x^2} \cdot dx$$

$$= \left[ \frac{x}{2} \sqrt{1 - x^2} + \frac{1}{2} \sin^{-1}(x) \right]_0^1$$

$$= \frac{1}{2} \sqrt{1 - 1} + \frac{1}{2} \sin^1 1 - 0$$

$$= \frac{1}{2} \times \frac{\pi}{2}$$

$$= \frac{\pi}{4}$$

Area of the region OADBO

= area under the line  $x + y = 1$  between  $x = 0$  and  $x = 1$

$$= \int_0^1 y \cdot dx, \text{ where } y = 1 - x$$

$$= \int_0^1 (1 - x) \cdot dx$$

$$= \left[ x - \frac{x^2}{2} \right]_0^1$$

$$= 1 - \frac{1}{2} - 0$$

$$= \frac{1}{2}$$

$$\therefore \text{required area} = \left( \frac{\pi}{4} - \frac{1}{2} \right) \text{sq units.}$$

### Miscellaneous Exercise 5 | Q 2.08 | Page 190

**Solve the following :**

Find the area of the region bounded by the curve  $(y - 1)^2 = 4(x + 1)$  and the line  $y = x - 1$ .

#### SOLUTION

The equation of the curve is  $(y - 1)^2 = 4(x + 1)$

This is a parabola with vertex at A(-1, 1).

To find the points of intersection of the line  $y = x - 1$  and the parabola.

Put  $y = x - 1$  in the equation of the parabola, we get

$$(x - 1 - 1)^2 = 4(x + 1)$$

$$\therefore x^2 - 4x + 4 = 4x + 4$$

$$\therefore x^2 - 8x = 0$$

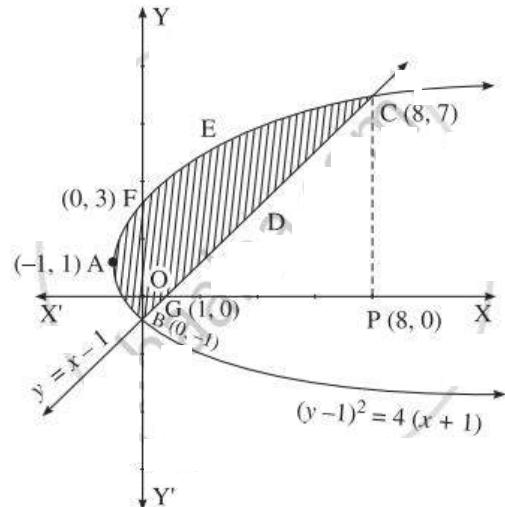
$$\therefore x(x - 8) = 0$$

$$\therefore x = 0, x = 8$$

$$\text{When } x = 0, y = 0 - 1 = -1$$

$$\text{When } x = 8, y = 8 - 1 = 7$$

$\therefore$  the points of intersection are B(0, -1) and C(8, 7)



To find the points where the parabola  $(y - 1)^2 = 4(x + 1)$  cuts the Y-axis.

Put  $x = 0$  in the equation of the parabola, we get

$$(y - 1)^2 = 4(0 + 1) = 4$$

$$\therefore y - 1 = \pm 2$$

$$\therefore y - 1 = 2 \text{ or } y - 1 = -2$$

$$\therefore y = 3 \text{ or } y = -1$$

$\therefore$  the parabola cuts the Y-axis at the points B(0, -1) and F(0, 3).

To find the point where the line  $y = x - 1$  cuts the X-axis. Put  $y = 0$  in the equation of the line, we get

$$\begin{aligned}x - 1 &= 0 \\ \therefore x &= 1 \\ \therefore \text{the line cuts the X-axis at the point } G(1, 0).\end{aligned}$$

Required area = area of the region BFAB + area of the region OGDCEFO + area of the region OBGO

Now, area of the region BFAB

= area under the parabola  $(y - 1)^2 = 4(x + 1)$ , Y-axis from  $y = -1$  to  $y = 3$

$$\begin{aligned}&= \int_{-1}^3 x \cdot dy, \text{ where } x + 1 = \frac{(y - 1)^2}{4}, \text{ i.e. } x = \frac{(y - 1)^2}{4} - 1 \\ &= \int_{-1}^3 \left[ \frac{(y - 1)^2}{4} - 1 \right] \cdot dy \\ &= \left[ \frac{1}{4} \cdot \frac{(y - 1)^3}{3} - y \right]_{-1}^3 \\ &= \left[ \left\{ \frac{1}{12}(3 - 1)^3 - 3 \right\} - \left\{ \frac{1}{12}(-1 - 1)^3 - (-1) \right\} \right] \\ &= \frac{8}{12} - 3 + \frac{8}{12} - 1 \\ &= \frac{16}{12} - 4 \\ &= \frac{4}{3} - 4 \\ &= -\frac{8}{3}\end{aligned}$$

Since, area cannot be negative, area of the region BFAB

$$= \left| -\frac{8}{3} \right|$$

$$= \frac{8}{3} \text{ sq units.}$$

Area of the region OGDCEFO

= area of the region OPCEFO – area of the region GPCDG

$$= \int_0^8 y \cdot dx, \text{ where } (y - 1)^2$$

$$= 4(x + 1), \text{ i.e. } y = 2\sqrt{x + 1} + 1 - \int_1^8 y \cdot dx, \text{ where } y = x - 1$$

$$= \int_0^8 [2\sqrt{x + 1} + 1] \cdot dx - \int_1^8 (x - 1) \cdot dx$$

$$= \left[ \frac{2 \cdot (x + 1)^{\frac{3}{2}}}{\frac{3}{2}} + x \right]_0^8 - \left[ \frac{x^2}{2} - x \right]_1^8$$

$$= \left[ \frac{4}{3}(9)^{\frac{3}{2}} + 8 - \frac{4}{3}(1)^{\frac{3}{2}} - 0 \right] - \left[ \left( \frac{64}{2} - 8 \right) - \left( \frac{1}{2} - 1 \right) \right]$$

$$= \left( 36 + 8 - \frac{4}{3} \right) - \left( 24 + \frac{1}{2} \right)$$

$$= 44 - \frac{4}{3} - 24 - \frac{1}{2}$$

$$= 20 - \left( \frac{4}{3} + \frac{1}{2} \right)$$

$$= 20 - \frac{11}{6}$$

$$= \frac{109}{6} \text{ sq units.}$$

$$\text{Area of region OBGO} = \int_0^1 y \cdot dx, \text{ where } y = x - 1$$

$$= \int_0^1 (x - 1) \cdot dx$$

$$= \left[ \frac{x^2}{2} - x \right]_0^1$$

$$= \frac{1}{2} - 1 - 0$$

$$= -\frac{1}{2}$$

Since, area cannot be negative,

$$\text{area of the region} = \left| -\frac{1}{2} \right| = \frac{1}{2} \text{ sq unit.}$$

$$\therefore \text{required area} = \frac{8}{3} + \frac{109}{6} + \frac{1}{2}$$

$$= \frac{16 + 109 + 3}{6}$$

$$= \frac{128}{6}$$

$$= \frac{64}{3} \text{ sq units.}$$

### Miscellaneous Exercise 5 | Q 2.09 | Page 190

**Solve the following :**

Find the area of the region bounded by the straight line  $2y = 5x + 7$ , X-axis and  $x = 2$ ,  $x = 5$ .

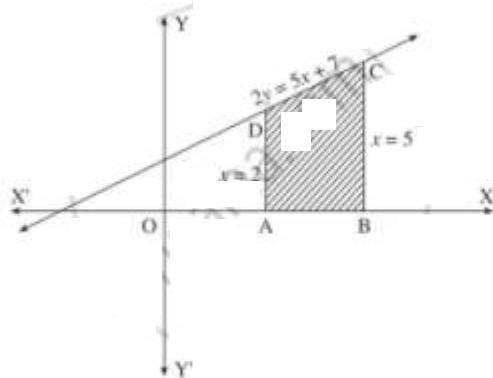
### SOLUTION

The equation of the line is  $2y = 5x + 7$ ,

$$\text{i.e., } y = \frac{5}{2}x + \frac{7}{2}$$

Required area = area of the region ABCDA

= area under the line  $y = \frac{5}{2}x + \frac{7}{2}$  between  $x = 2$  and  $x = 5$



$$= \int_2^5 \left( \frac{5}{2}x + \frac{7}{2} \right) \cdot dx$$

$$= \frac{5}{2} \cdot \int_2^5 x \cdot dx + \frac{7}{2} \int_2^5 1 \cdot dx$$

$$= \frac{5}{2} \left[ \frac{x^2}{2} \right]_2^5 + \frac{7}{2} [x]_2^5$$

$$= \frac{5}{2} \left[ \frac{25}{2} - \frac{4}{2} \right] + \frac{7}{2} [5 - 2]$$

$$= \frac{5}{2} \times \frac{21}{2} + \frac{21}{2}$$

$$= \frac{105}{4} + \frac{42}{4}$$

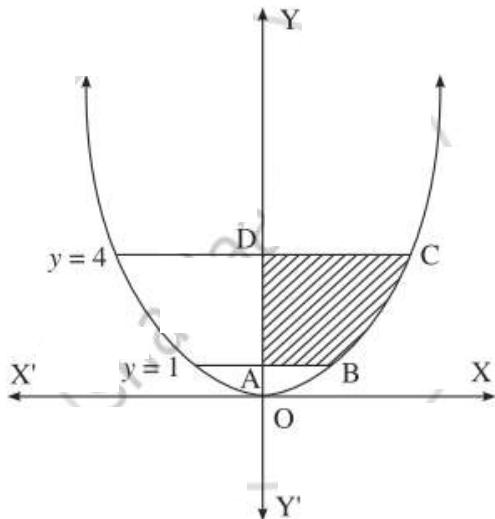
$$= \frac{147}{4} \text{ sq units.}$$

**Miscellaneous Exercise 5 | Q 2.10 | Page 190**

**Solve the following :**

Find the area of the region bounded by the curve  $y = 4x^2$ , Y-axis and the lines  $y = 1$ ,  $y = 4$ .

**SOLUTION**



By symmetry of the parabola, the required area is 2 times the area of the region ABCD.

From the equation of the parabola,  $x^2 = \frac{y}{4}$

the first quadrant,  $x > 0$

$$\therefore x = \frac{1}{2}\sqrt{y}$$

$$\therefore \text{required area} = \int_1^4 x \cdot dy$$

$$= \frac{1}{2} \int_1^4 \sqrt{y} \cdot dy$$

$$= \frac{1}{2} \left[ \frac{y^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^4$$

$$= \frac{1}{2} \times \frac{2}{3} \left[ 4^{\frac{3}{2}} - 1^{\frac{3}{2}} \right]$$

$$= \frac{1}{3} \left[ (2^2)^{\frac{3}{2}} - 1 \right]$$

$$= \frac{1}{3} [8 - 1]$$

$$= \frac{7}{3} \text{ sq units.}$$