Board Answer Paper: March 2020

MATHEMATICS PART – II

Q.1 (A)

i. (B) (3, 4, 5) ***[1 Mark]***

ii. (C) 8.8 ***[1 Mark]***

iii. (D) 5 ***[1 Mark]***

iv. (A) 27 cm^3 ***[1 Mark]***

Hints:

i. $5^2 = 25$

$3^2 + 4^2 = 9 + 16 = 25$

$\therefore$ $5^2 = 3^2 + 4^2$

ii. Distance between centres = 5.5 + 3.3

= 8.8

iii. Distance of (–3, 4) from origin

$= \sqrt{(-3)^2 + (4)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25} = 5$

iv. Volume of a cube = $(\text{side})^3 = (3)^3 = 27\ cm^3$

Q.1 (B)

i. Let the corresponding sides of similar triangles be s_1 and s_2.

Let A_1 and A_2 be their corresponding areas.

$s_1 : s_2 = 3 : 5$ …[Given]

$\therefore$ $\frac{s_1}{s_2} = \frac{3}{5}$ …(i)

$\frac{A_1}{A_2} = \frac{{s_1}^2}{{s_2}^2}$ …[Theorem of areas of similar triangles]

$= \left(\frac{s_1}{s_2}\right)^2$

$= \left(\frac{3}{5}\right)^2$ …[From (i)]

$= \frac{9}{25}$

$\therefore$ **Ratio of areas of similar triangles = 9 : 25** ***[1 Mark]***

ii. Diagonal of a square $= \sqrt{2} \times \text{side}$

$= \sqrt{2} \times 10 = \mathbf{10\sqrt{2}cm}$ ***[1 Mark]***

iii. □ABCD is cyclic.

Opposite angles of a cyclic quadrilateral are supplementary.

$\therefore$ $\angle B + \angle D = 180^\circ$

$\therefore$ $110^\circ + \angle D = 180^\circ$

$\therefore$ $\angle D = 180^\circ - 110^\circ = \mathbf{70^\circ}$ ***[1 Mark]***

iv. Here, $x_1 = 2, x_2 = 4, y_1 = 3, y_2 = 7$

Slope of line AB $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{7-3}{4-2} = \frac{4}{2} = \mathbf{2}$ ***[1 Mark]***

Q.2 (A)

i. $PS^2 = PQ \times \boxed{PR}$ ...[Tangent secant segments theorem]

$= PQ \times (PQ + \boxed{QR})$

$= 3.6 \times (3.6 + 6.4)$

$= 3.6 \times \boxed{10}$

$= 36$

$\therefore$ $PS = \boxed{6}$ …[By taking square roots]

[½ mark each]

ii. $1 + \tan^2 \theta = \sec^2 \theta$

$\therefore$ $1 + \tan^2 \theta = \left(\frac{25}{7}\right)^{\boxed{2}}$

$\therefore$ $\tan^2 \theta = \frac{625}{49} - \boxed{1}$

$= \frac{625 - 49}{49} = \frac{\boxed{576}}{49}$

$\therefore$ $\tan \theta = \frac{\boxed{24}}{7}$ …[By taking square roots] ***[½ mark each]***

iii.

Type of arc	Name of the arc	Measure of the arc
Minor arc	**arc AXB**	**100°**
Major arc	**arc AYB**	**260°**

[½ mark each]

Q.2 (B)

i. In ΔPQR,
$NM \parallel RQ$ …[Given]

$\therefore$ $\frac{PN}{NR} = \frac{PM}{MQ}$ …[Basic proportionality theorem]

$\therefore$ $\frac{PN}{8} = \frac{15}{10}$

$\therefore$ $PN = \frac{15}{10} \times 8$

$\therefore$ **PN = 12 units** ***[2 Marks]***

ii. In ΔMNP, $\angle MNP = 90°$ and
seg NQ $\perp$ seg MP …[Given]

$\therefore$ $NQ^2 = MQ \times QP$ …[Theorem of geometric mean]

$\therefore$ $NQ = \sqrt{MQ \times QP}$ …[Taking square root of both sides]

$= \sqrt{9 \times 4} = 3 \times 2$

$\therefore$ **NQ = 6 units** ***[2 Marks]***

iii. Line KL is the tangent to the circle at point L and seg ML is the radius.

$\therefore$ $\angle MLK = 90°$ …[Tangent theorem]

In ΔMLK, $\angle MLK = 90°$

$\therefore$ $MK^2 = ML^2 + KL^2$ …[Pythagoras theorem]

$\therefore$ $12^2 = ML^2 + \left(6\sqrt{3}\right)^2$

$\therefore$ $144 = ML^2 + 108$

$\therefore$ $ML^2 = 144 - 108$

$\therefore$ $ML^2 = 36$

$\therefore$ $ML = \sqrt{36} = 6$ units …[Taking square root of both sides]

$\therefore$ **Radius of the circle is 6 units.** ***[2 Marks]***

iv. Let $(x_1, y_1) = (22, 20)$,
$(x_2, y_2) = (0, 16)$
Let the co-ordinates of the midpoint be P (x, y).

∴ By midpoint formula,

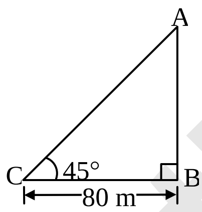
$$x = \frac{x_1 + x_2}{2} = \frac{22+0}{2} = 11$$

$$y = \frac{y_1 + y_2}{2} = \frac{20+16}{2} = \frac{36}{2} = 18$$

∴ **The co-ordinates of the midpoint of the segment joining (22, 20) and (0, 16) are (11, 18).**

[2 Marks]

v. Let AB represent the height of the church and point C represent the position of the person.
BC = 80 m
Angle of elevation = ∠ACB = 45°
In right angled ΔABC,



$\tan 45° = \frac{AB}{BC}$ …[By definition]

∴ $1 = \frac{AB}{80}$

∴ AB = 80 m

∴ **The height of the church is 80 m.** ***[2 Marks]***

Q.3 (A)

i. In ΔXDE, PQ || DE …[Given]

∴ $\frac{XP}{PD} = \frac{\boxed{\mathbf{XQ}}}{QE}$ …[Basic proportionality theorem] …(i)

In ΔXEF, QR || EF …[Given]

∴ $\frac{XQ}{\boxed{\mathbf{QE}}} = \frac{XR}{\boxed{\mathbf{RF}}}$ …(ii) $\left[\boxed{\textbf{Basic proportionality theorem}}\right]$

∴ $\frac{XP}{PD} = \frac{\boxed{\mathbf{XR}}}{\boxed{\mathbf{RF}}}$ …[From (i) and (ii)]

∴ seg PR || seg DF …[By converse of basic proportionality theorem]

[½ mark each]

ii. Slope of line $= \frac{y_2 - y_1}{x_2 - x_1}$

∴ Slope of line AB $= \frac{2-1}{8-6} = \boxed{\mathbf{\frac{1}{2}}}$ …(i)

∴ Slope of line BC $= \frac{4-2}{9-8} = \boxed{\mathbf{2}}$ …(ii)

∴ Slope of line CD $= \frac{3-4}{7-9} = \boxed{\mathbf{\frac{1}{2}}}$ …(iii)

∴ Slope of line DA $= \frac{3-1}{7-6} = \boxed{\mathbf{2}}$ …(iv)

∴ Slope of line AB = $\boxed{\textbf{Slope of line CD}}$ …[From (i) and (iii)]

∴ line AB || line CD

∴ Slope of line BC = $\boxed{\textbf{Slope of line DA}}$ …[From (ii) and (iv)]

∴ line BC || line DA
Both the pairs of opposite sides of the quadrilateral are parallel.

∴ □ABCD is a parallelogram.

[½ mark each]

Q.3 (B)

i. In ΔPQR, point S is the midpoint of side QR. …[Given]

$\therefore$ seg PS is the median.

$\therefore$ $PQ^2 + PR^2 = 2\,PS^2 + 2\,SR^2$ …[Apollonius theorem]

$\therefore$ $11^2 + 17^2 = 2\,(13)^2 + 2\,SR^2$

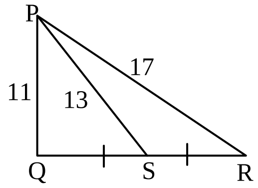
$\therefore$ $121 + 289 = 2\,(169) + 2\,SR^2$

$\therefore$ $410 = 338 + 2\,SR^2$

$\therefore$ $2\,SR^2 = 410 - 338$

$\therefore$ $2\,SR^2 = 72$

$\therefore$ $SR^2 = \frac{72}{2} = 36$



$\therefore$ $SR = \sqrt{36}$ …[Taking square root of both sides]

$= 6$ units

Now, $QR = 2\,SR$ …[S is the midpoint of QR]

$= 2 \times 6$

$\therefore$ **QR = 12 units** ***[3 Marks]***

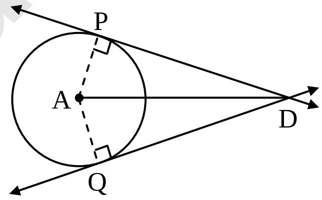
ii. **Given:** A is the centre of the circle.

Tangents through external point D touch the circle at the points P and Q.

To prove: seg DP $\cong$ seg DQ

Construction: Draw seg AP and seg AQ.

Proof:



In ΔPAD and ΔQAD,

seg PA $\cong$ seg QA …[Radii of the same circle]

seg AD $\cong$ seg AD …[Common side]

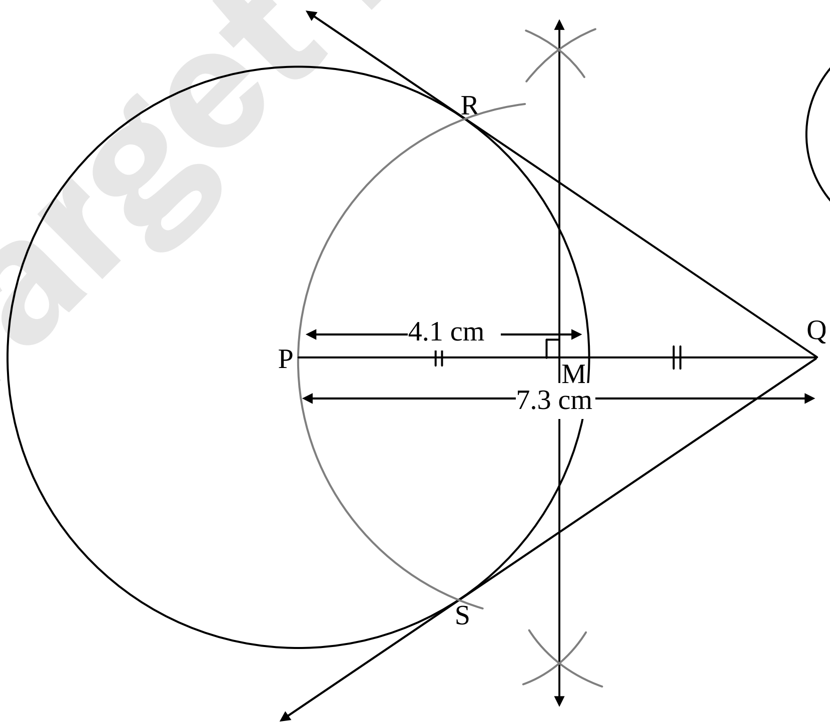
$\angle APD = \angle AQD = 90^\circ$ …[Tangent theorem]

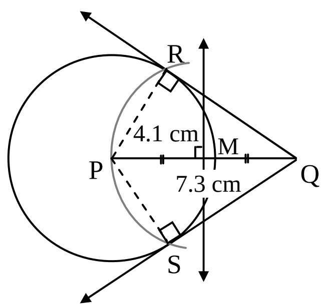
$\therefore$ $\Delta PAD \cong \Delta QAD$ …[By Hypotenuse side test]

$\therefore$ seg DP $\cong$ seg DQ …[Corresponding sides of congruent triangles]

[3 Marks]

iii.





Rough Figure

[3 Marks]

iv. Volume of cuboid = $l \times b \times h$

$= 16 \times 11 \times 10$

$= 1760 \text{ cm}^3$

Thickness of coin (H) = 2 mm

= 0.2 cm ...[$\because$ 1 cm = 10 mm]

Diameter of coin (D) = 2 cm

$\therefore$ Radius of coin (R) = $\frac{D}{2} = \frac{2}{2}$ = 1 cm

$\therefore$ Volume of one coin $= \pi R^2 H$

$= 3.14 \times 1^2 \times 0.2$

$= 0.628 \text{ cm}^3$

Number of coins that were made $= \frac{\text{Volume of cuboid}}{\text{Volume of one coin}}$

$= \frac{1760}{0.628}$

$= 2802.5477$

≈ 2802

$\therefore$ **2802 coins were made by melting the cuboid.**

[3 Marks]

Q.4

i. seg PQ || seg BC and AB is their transversal.

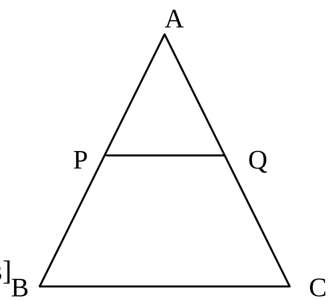
$\therefore$ $\angle APQ \cong \angle ABC$ …(i)[Corresponding angles]

In ΔAPQ and ΔABC,

$\angle APQ \cong \angle ABC$ …[From (i)]

$\angle PAQ \cong \angle BAC$ …[Common angle]

$\therefore$ $\Delta APQ \sim \Delta ABC$ …[By AA test of similarity]



$\frac{A(\Delta APQ)}{A(\Delta ABC)} = \frac{AP^2}{AB^2}$ …(ii)[By theorem of areas of similar triangles]

$A(\Delta APQ) = \frac{1}{2} A(\Delta ABC)$ ...[$\because$ Seg PQ divides ΔABC into two parts of equal areas.]

$\therefore$ $\frac{A(\Delta APQ)}{A(\Delta ABC)} = \frac{1}{2}$ …(iii)

$\therefore$ $\frac{AP^2}{AB^2} = \frac{1}{2}$ …[From (ii) and (iii)]

$\therefore$ $\frac{AP}{AB} = \frac{1}{\sqrt{2}}$ …[Taking square root of both sides]

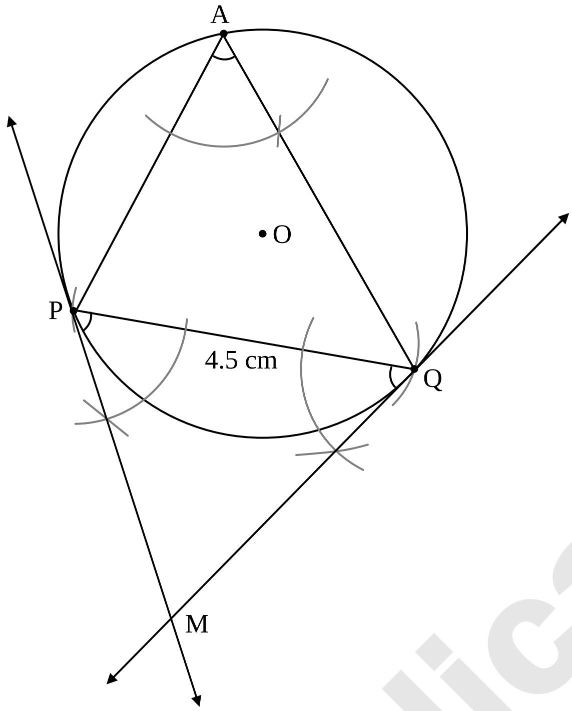
$\therefore$ $1 - \frac{AP}{AB} = 1 - \frac{1}{\sqrt{2}}$ …[Subtracting both sides from 1]

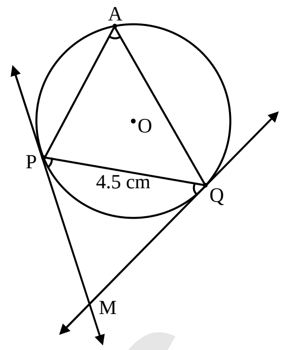
$\therefore$ $\frac{AB - AP}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$

$\therefore$ $\mathbf{\frac{BP}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}}$ …[A–P–B]

[4 Marks]

ii.





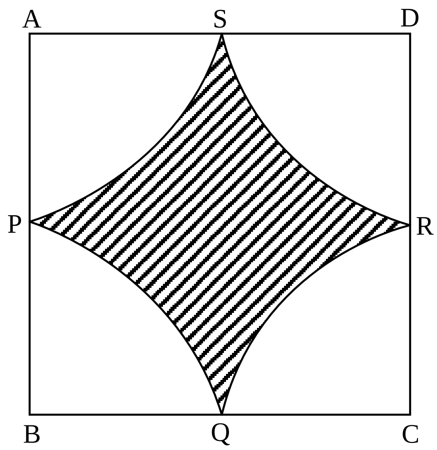
Rough Figure

[4 Marks]

iii. Area of square ABCD $= (\text{side})^2$

$= (50)^2$

$= 2500 \text{ m}^2$



Radius of sector A–SP $= \dfrac{1}{2} \times 50 = 25\text{m}$

$\theta = 90°$ …[Angle of a square]

Area of sector A–SP $= \dfrac{\theta}{360} \times \pi r^2$

$= \dfrac{90}{360} \times \dfrac{22}{7} \times (25)^2$

$= \left(\dfrac{1}{4} \times \dfrac{13750}{7}\right) \text{ m}^2$

A(shaded region) = Area of square ABCD – 4(Area of sector A–SP)

$= 2500 - 4\left(\dfrac{1}{4} \times \dfrac{13750}{7}\right)$

$= 2500 - \dfrac{13750}{7}$

$= \dfrac{17500 - 13750}{7}$

$= \dfrac{3750}{7}$

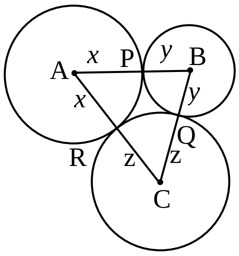
$\approx 535.71\text{m}^2$

$\therefore$ **Area of the shaded region is 535.71m^2.**

[4 Marks]

Q.5

i. Let AP = AR = x
BP = BQ = y
CQ = CR = z
[Radii of the same circle]



AP + BP = AB …[A–P–B]
∴ $x + y = 3$ …(i)
BQ + CQ = BC …[B–Q–C]
∴ $y + z = 3$ …(ii)
AR + CR = AC …[A–R–C]
∴ $x + z = 4$ …(iii)
Adding equations (i), (ii) and (iii), we get
$x + y + y + z + x + z = 3 + 3 + 4$
∴ $2x + 2y + 2z = 10$
∴ $2(x + y + z) = 10$
∴ $x + y + z = 5$ …(iv)
Substituting equation (i) in equation (iv), we get
$3 + z = 5$
∴ $z = 5 - 3$
∴ z = 2 cm
Substituting equation (ii) in equation (iv), we get
$x + 3 = 5$
∴ $x = 5 - 3$
∴ x = 2 cm
Substituting equation (iii) in equation (iv), we get
$y + 4 = 5$
∴ $y = 5 - 4$
∴ y = 1 cm
∴ **The radii of circles with centres A, B, C are 2 cm, 1 cm and 2 cm respectively.**

[3 Marks]

ii. $\sin\theta + \sin^2\theta = 1$ …[Given]
∴ $\sin\theta = 1 - \sin^2\theta$
∴ $\sin\theta = \cos^2\theta$ …[$\because 1 - \sin^2\theta = \cos^2\theta$]
∴ $\sin^2\theta = \cos^4\theta$ …[Squaring both the sides]
∴ $1 - \cos^2\theta = \cos^4\theta$ …[$\because \sin^2\theta = 1 - \cos^2\theta$]
∴ $1 = \cos^2\theta + \cos^4\theta$
∴ $\boldsymbol{\cos^2\theta + \cos^4\theta = 1}$

[3 Marks]