18. Poisson Distribution

A. Activities

1) Given X ~ P (1/2)

$\therefore$ Th p. m. f. of X is $p(x) = \dfrac{e^{-\boxed{m}} m^{\boxed{x}}}{\boxed{x!}}, x = 0, 1, 2...$

$\therefore P(x = 3) = \dfrac{\boxed{e}^{-1/2} . (1/2)^{\boxed{3}}}{3!}$

$= \dfrac{0.6065 \times \boxed{1/8}}{\boxed{6}} = \boxed{0.0126}$

2) Given mean $= \boxed{m} = 1.5$

$\therefore$ The p. m. f. of X is

$P(x) = \dfrac{e^{-m} . \boxed{m^x}}{\boxed{x!}}, x = 0, 1, 2, ...$

(i) $\therefore P(X = 0) = e^{\boxed{-1.5}} = 0.2231$

(ii) $P(X \geq 3) = 1 - P(X < \boxed{3})$

$= \boxed{1} - [P(X = 0) + P(X = \boxed{1}) + P(X = 2)$

$= 1 - \left[e^{\boxed{-1.5}} + \boxed{1.5} . e^{-1.5} + \dfrac{\boxed{e}^{-1.5} (\boxed{1.5})^2}{\boxed{2!}} \right]$

$= \boxed{1} - [\boxed{0.2231} + 0.3346 + \boxed{0.2509}]$

$= \boxed{1} - 0.8087$

$= \boxed{0.1913}$

B. Solve the Following

Q.1. If X has Poisson distribution with parameter m & if

P [X = 2] = P [X = 3], Find P [X ≥ 2]. [Given $e^{-3} = 0.0497$]

Soln : As X follows Poisson distribution,

$\therefore P(X = x) = \dfrac{e^{-m} m^x}{x!}$

Now given,

$P(X = 2) = P(X = 3)$

$\therefore \dfrac{e^{-m} m^2}{2!} = \dfrac{e^{-m} m^3}{3!}$

$\therefore \dfrac{m^2}{2} = \dfrac{m^3}{6} \qquad \therefore \boxed{m = 3}$

$P(X \geq 2) = 1 - P(X < 2) = 1 - [P(X=0) + P(X=1)]$

$= 1 - \left[\frac{e^{-3} 3^0}{0!} + \frac{e^{-3} 3^1}{1!}\right] = 1 - [0.0497 + 3 \times 0.0497]$

$= 1 - [0.1988] = 0.8012$

$\therefore \boxed{P(X \geq 2) = 0.8012}$

Q.2. If X has a Poisson distribution with variance 2.

Find i) $P[X = 4]$ ii) $P[X \leq 4]$ iii) mean of X . [Given $e^{-2} = 0.1353$]

Soln Variance $= 2 \Rightarrow m = 2$ $\therefore \boxed{P(X = x) = \frac{e^{-2} 2^x}{x!}}$

i) $P(X = 4) = \frac{e^{-2} (2)^4}{4!}$

$= \frac{0.1353 \times 16}{4 \times 3 \times 2 \times 1}$

$= 0.0451 \times 2$

$\therefore \boxed{P(X = 4) = 0.0902}$

ii) $P(X \leq 4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= \left[\frac{e^{-2} 2^0}{0!} + \frac{e^{-2} 2^1}{1!} + \frac{e^{-2} 2^2}{2!} + \frac{e^{-2} 2^3}{3!} + \frac{e^{-2} 2^4}{4!}\right]$

$\therefore P(X \leq 4) = e^{-2}\left[1 + 2 + 2 + \frac{8}{6} + \frac{16}{24}\right]$

$= e^{-2}\left[5 + \frac{12}{6}\right]$

$= e^{-2} \times 7 = 0.1353 \times 7$

$\therefore \boxed{P(X \leq 4) = 0.9471}$

iii) Mean $=$ Variance $= m = 2$

$\therefore \boxed{\text{Mean} = 2}$

Q.3. In a town 10 accidents take place in a span of 50 days .Assuming that the number of accidents follows Poisson distribution . Find the probability that there will be 3 or more accidents per day. [Given $e^{-0.2} = 0.3187$]

Soln: Here $m = \frac{10}{50} = 0.2$

$\therefore$ Poisson Distribution is $X \sim P(m)$

The pmf of X is $P(X = x) = \frac{e^{-m} m^x}{x!}$; $x = 0, 1, 2, \ldots$

$\therefore P(\text{3 or more accident per day}) = P(X \geq 3)$

$\therefore P(X \geq 3) = 1 - P(X < 3) = 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - \left[\frac{e^{-0.2}(0.2)^0}{0!} + \frac{e^{-0.2}(0.2)^1}{1!} + \frac{e^{-0.2}(0.2)^2}{2!}\right]$

$= 1 - [(0.8187) + (0.2)(0.8187) + (0.02) \times (0.8187)]$

$= 1 - [0.8187 + 0.16374 + 0.016374]$

$= 1 - 0.9988 = 0.0012$

$\therefore \boxed{P(X \geq 3) = 0.0012}$

Sign of Teacher :