Chapter 8: Electrostatics

EXERCISES [PAGES 121 - 213]

Exercises | Q 1.1 | Page 212

Choose the correct option:
A parallel plate capacitor is charged and then isolated. The effect of increasing the plate separation on charge, potential, capacitance respectively are

1. **Constant, decreases, decreases**
2. Increases, decreases, decreases
3. Constant, decreases, increases
4. Constant, increases, decreases

SOLUTION

Constant, decreases, decreases

Exercises | Q 1.2 | Page 212

Choose the correct option:
A slab of material of dielectric constant k has the same area A as the plates of a parallel plate capacitor and has a thickness (3/4d), where d is the separation of the plates. The change in capacitance when the slab is inserted between the plates is

$$C = \frac{A\varepsilon_0}{d}\left(\frac{k+3}{4k}\right)$$

$$C = \frac{A\varepsilon_0}{d}\left(\frac{2k}{k+3}\right)$$

$$C = \frac{A\varepsilon_0}{d}\left(\frac{k+3}{2k}\right)$$

$$\mathbf{C} = \frac{A\varepsilon_0}{d}\left(\frac{4k}{k+3}\right)$$

SOLUTION

$$C = \frac{A\varepsilon_0}{d}\left(\frac{4k}{k+3}\right)$$

Exercises | Q 1.3 | Page 212
Choose the correct option:
Energy stored in a capacitor and dissipated during charging a capacitor bear a ratio.

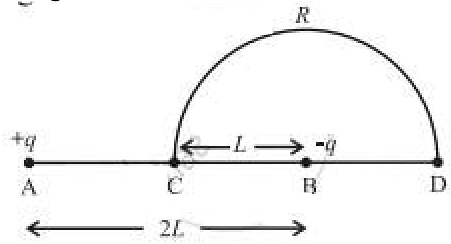
1. **1:1**
2. 1:2
3. 2:1
4. 1:3

SOLUTION

1:1

Exercises | Q 1.4 | Page 212

Choose the correct option:
Charge + q and - q are placed at points A and B respectively which are distance 2L apart. C is the midpoint of A and B. The work done in moving a charge + Q from C to D, along the semicircle CD as shown in the figure below, is



$$\frac{-qQ}{6\pi\varepsilon_0 L}$$
$$\frac{qQ}{2\pi\varepsilon_0 L}$$
$$\frac{qQ}{6\pi\varepsilon_0 L}$$
$$\frac{-qQ}{4\pi\varepsilon_0 L}$$

SOLUTION

$$\frac{-qQ}{6\pi\varepsilon_0 L}$$

Exercises | Q 1.5 | Page 121

Choose the correct answer:
A parallel-plate capacitor has circular plates of radius 8 cm and plate separation I mm. What will be the charge on the plates if a potential difference of 100 V is applied across its plates?

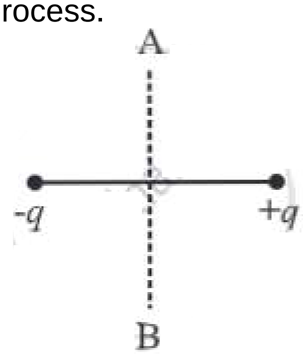
1. **1.78 × 10^{-8} C**
2. 1.78 × 10^{-5} C
3. 4.3 × 10^{4} C
4. 2 × 10^{-9} C

SOLUTION

1.78 × 10^{-8} C

Exercises | Q 2.1 | Page 212

Answer in brief:
A charge q is moved from a point A above a dipole of dipole moment p to a point B below the dipole in an equatorial plane without acceleration. Find the work done in this process.



SOLUTION

The equatorial plane of an electric dipole is an equipotential with V = 0. Therefore, no work is done in moving a charge between two points in the equatorial plane of a dipole.

Exercises | Q 2.2 | Page 212

Answer in brief:
If the difference between the radii of the two spheres of a spherical capacitor is increased, state whether the capacitance will increase or decrease.

SOLUTION

The capacitance of a spherical capacitor is C = $4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$ where a and b are the radii of the concentric inner and outer conducting shells. Hence, the capacitance decreases if the difference b - a is increased.

Exercises | Q 2.3 | Page 212

Answer in brief:
A metal plate is introduced between the plates of a charged parallel plate capacitor. What is its effect on the capacitance of the capacitor?

SOLUTION

Suppose the parallel-plate capacitor has capacitance C_0, plates of area A and separation d. Assume the metal sheet introduced has the same area A.

Case (1): Finite thickness t. Free electrons in the sheet will migrate towards the positive plate of the capacitor. Then, the metal sheet is attracted towards whichever capacitor plate is closest and gets stuck to it, so that its potential is the same as that of that plate. The gap between the capacitor plates is reduced to d - t so that the capacitance increases.

Case (2): Negligible thickness. The thin metal sheet divides the gap into two of thicknesses d_1 and d_2 of capacitances $C_1 = \varepsilon_0 A/d_1$ and $C_2 = \varepsilon_0 A/d_2$ in series.

Their effective capacitance is

$$C = \frac{C_1C_2}{C_1 + C_2} = \frac{\varepsilon_0 A}{d_1 + d_2} = \frac{\varepsilon_0 A}{d} = C_0$$

i.e., the capacitance remains unchanged.

Exercises | Q 2.4 | Page 212

Answer in brief:
The safest way to protect yourself from lightening is to be inside a car. Justify.

SOLUTION

There is a danger of lightning strikes during a thunderstorm. Because trees are taller than people and therefore closer to the clouds above, they are more likely to get hit by lightnings. Similarly, a person standing in an open ground is the tallest object and more likely to get hit by lightning. But a car with a metal body is an almost ideal Faraday cage. When a car is struck by lightning, the charge flows on the outside surface of the car to the ground but the electric field inside remains zero. This leaves the passengers inside unharmed.

Exercises | Q 2.5 | Page 212

Answer in brief:
A spherical shell of radius b with charge Q is expanded to a radius a. Find the work done by the electrical forces in the process.

SOLUTION

Consider a spherical conducting shell of radius r placed in a medium of permittivity ε. The mechanical force per unit area on the charged conductor is

$$f = \frac{F}{dS} = \frac{\sigma^2}{2\varepsilon}$$

where σ is the surface charge density on the conductor. Given the charge on the spherical shell is Q, $\sigma = Q/4\pi r^2$. The force acts outward, normal to the surface.

Suppose the force displaces a charged area element σdS through a small distance dx, then the work done by the force is

$$dW = Fdx = \left(\frac{\sigma^2}{2\varepsilon}dS\right)dx$$

During the displacement, the area element sweeps out a volume dV = dS ×dx.

Since $V = \frac{4}{3}\pi r^3$, $dV = 4\pi r^2\,dr$

$$\therefore dW = \frac{\sigma^2}{2\varepsilon}dV = \frac{1}{2\varepsilon}\left(\frac{Q}{4\pi r^2}\right)^2(4\pi r^2 dr)$$

$$= \frac{Q^2}{8\pi\varepsilon}\frac{1}{r^2}dr$$

Therefore, the work done by the force in expanding the shell from radius r = b to r = a is

$$W = \int dW = \frac{Q^2}{8\pi\varepsilon}\int_b^a \frac{1}{r^2}\,dr$$

$$= \frac{Q^2}{8\pi\varepsilon}\left[-\frac{1}{r}\right]_b^a = \frac{Q^2}{8\pi\varepsilon}\left(\frac{1}{b} - \frac{1}{a}\right)$$

This gives the required expression for the work done.

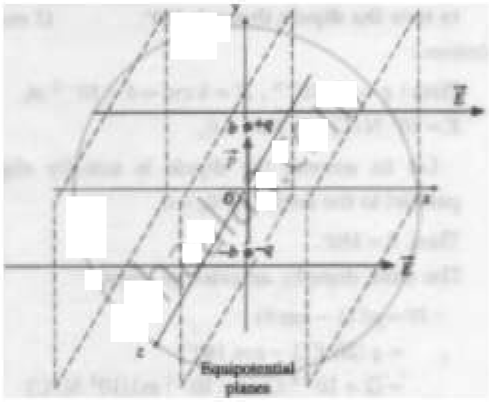
Exercises | Q 3 | Page 212

A dipole with its charges, - q and + q located at the points (O, -b, O) and (O, +b, O) is present in a uniform electric field E. The equipotential surfaces of this field are planes parallel to the yz plane.
(a) What is the direction of the electric field E?
(b) What torque would the dipole experience in this field?

SOLUTION

(a) Given, the equipotentials of the external uniform electric field are planes parallel to the yz plane, the electric field

$\vec{E} = \pm E\hat{i}$ that is, $\vec{E}$ is parallel to the x-.



A dipole in an external electric field along x-axis

(b) The above diagram, the dipole moment, $\vec{p} = q(2b)\hat{j}$

The torque on this dipole,

$$\vec{\tau} = \vec{p} \times \vec{E} = \left(2qb\hat{j}\right) \times \left(\pm E\hat{i}\right) = (2qbE)\left(\hat{j} \times \hat{i}\right)$$

Since $\hat{j} \times \hat{i} = -\bar{k}$

$$\vec{\tau} = (\pm 2qbE)\left(-\hat{k}\right) = (2qbE)\left(\pm\hat{k}\right)$$

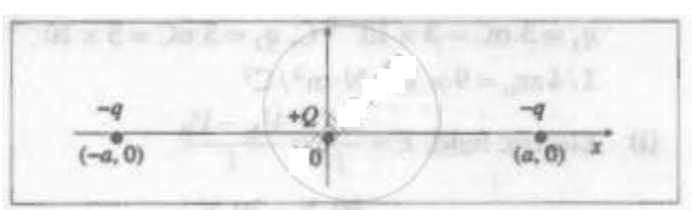
So that the magnitude of the torque is τ = 2qbE.

If $\vec{E}$ is in the direction of the + x-axis, the torque $\vec{\tau}$ is in the direction of - z-axis, while if $\vec{E}$ s in the direction of the - x-axis, the torque $\vec{\tau}$ is in the direction of + z-axis.

Exercises | Q 4 | Page 213

Three charges - q, + Q, and - q are placed at equal distance on a straight line. If the potential energy of the system of the three charges is zero, then what is the ratio of Q : q?

SOLUTION



The above figure, he line joining the charges is shown as the x-axis with the origin at the +Q charge. Let q_1 = +Q and $q_2 = q_3$ = - q. Let the two -q charges be at (- a, O) and (a, 0), since the charges are given to be equidistant.

∴ $r_{21} = r_{31}$ = a and r_{32} = 2a

The total potential energy of the system of three charges is

$$u_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{21}} + \frac{q_1q_3}{r_{31}} + \frac{q_2q_3}{r_{32}}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{-q(Q)}{a} + \frac{Q(-q)}{a} + \frac{(-q)(-q)}{2a}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[-\frac{2qQ}{a} + \frac{q^2}{2a}\right]$$

Given: $u_3 = 0$

$$\therefore \frac{2qQ}{a} = \frac{q^2}{2a}$$

$$\therefore \frac{Q}{q} = \frac{1}{4}$$

This gives the required ratio.

Exercises | Q 5 | Page 213

A capacitor has some dielectric between its plates and the capacitor is connected to a DC source. The battery is now disconnected and then the dielectric is removed. State whether the capacitance, the energy stored in it, the electric field, the charge stored, and voltage will increase, decrease, or remain constant.

SOLUTION

Assume a parallel-plate capacitor, of plate area A and plate separation d is filled with a dielectric of relative permittivity (dielectric constant) k. Its capacitance is $C = \frac{k\varepsilon_0 A}{d}$ ....(1)

If it is charged to a voltage (potential) V, the charge on its plates is Q = CV.

Since the battery is disconnected after it is charged, the charge Q on its plates, and consequently the product CV, remain unchanged.

On removing the dielectric completely, its capacitance becomes from Eq. (1),

$C' = \frac{\varepsilon_0 A}{d} = \frac{1}{k}C$ ....(2)

that is, its capacitance decreases by the factor k. Since C'V' =CV, its new voltage is

$V' = \frac{C}{C'}V = kV$ ....(3)

so that its voltage increases by the factor k.

The stored potential energy, u = $\frac{1}{2}$ QV, so that Q remaining constant, u increases by the factor k. The electric field, E = V/d, so that E also increases by a factor k.

Exercises | Q 6 | Page 212

Find the ratio of the potential differences that must be applied across the parallel and series combination of two capacitors C_1 and C_2 with their capacitances in the ratio 1 : 2 so that the energy stored in the two cases becomes the same.

SOLUTION

Data: `"C"_1/"C"_2 = 1/2, $u_1$ (for parallel) = $u_2$ (for series)`

$$\frac{C_1}{C_2} = \frac{1}{2}$$

$\therefore C_2 = 2C_1$

For the parallel combination of C_1 and C_2,

$C_p = C_1 + C_2 = 3C_1$

and charged to a potential V_1, the energy stored is

$$u_1 = \frac{1}{2}C_p V_1^2 = \frac{3}{2}C_1 V_1^2$$

For the series combination of C_1 and C_2,

$$C_s = \frac{C_1 C_2}{C_1 + C_2} = \frac{2C_1^2}{3C_1} = \frac{2}{3}C_1$$

$$u_2 = \frac{1}{2}C_sV_2^2 = \frac{1}{3}C_1V_2^2$$

$$\therefore \text{ For } u_1 = u_2, \frac{3}{2}C_1V_1^2 = \frac{1}{3}C_1V_2^2$$

$$\therefore \left(\frac{V_1}{V_2}\right)^2 = \frac{2}{9}$$

$$\therefore \frac{V_1}{V_2} = \frac{\sqrt{2}}{3} = \frac{1.414}{3} = 0.471$$

This gives the required ratio.

Exercises | Q 7 | Page 213

Two charges of magnitudes -4Q and + 2Q are located at points (2a, 0) and (Sa, 0), respectively. What is the electric flux due to these charges through a sphere of radius 4a with its center at the origin?

SOLUTION

The sphere of radius 4a encloses only the negative charge Q_1 = - 4Q. The positive charge Q_2 = + 2Q being located at a distance of 5a from the origin is outside the sphere. Only a part of the electric flux lines originating at Q_2 enters the sphere and exits entirely at other points. Hence, the electric flux through the sphere is only due to Q_1.

Therefore, the net electric flux through the sphere $= \frac{Q_1}{\varepsilon_0} = \frac{-4Q}{\varepsilon_0}$. The minus sign shows that the flux is directed into the sphere, but not radially since the sphere is not centered on Q_1.

Exercises | Q 8 | Page 213

A 6 µF capacitor is charged by a 300 V battery. If the capacitor is disconnected from the battery and connected across another uncharged 3 µF capacitors, what is the energy lost in the form of heat and radiation?

SOLUTION

Data: C = 6 µF = 6×10^{-6} F = C_1, V = 300 V C_2 = 3 µF

The electrostatic energy in the capacitor

$$= \frac{1}{2}CV^2 = \frac{1}{2}\left(6 \times 10^{-6}\right)(300)^2$$

$= 3 \times 10^{-6} \times 9 \times 10^{4} = 0.27\text{J}$

The charge on this capacitor,

$Q = CV = (6 \times 10^{-6})(300) = 1.8\text{ mC}$

When two capacitors of capacitances C_1 and C_2 are connected in parallel, the equivalent capacitance C

$= C_1 + C_2 = 6 + 3 = 9\ \mu\text{F}$

$= 9 \times 10^{-6}\text{ F}$

By conservation of charge, Q = 1.8 C.

$\therefore$ The energy of the system $= \frac{Q^2}{2C}$

$$= \frac{\left(1.8 \times 10^{-3}\right)^2}{2\left(9 \times 10^{-6}\right)} = \frac{18 \times 10^{-8}}{10^{-6}} = 0.18\text{ J}$$

The energy lost = 0.27 -0.18 = 0.09 J

Exercises | Q 9 | Page 213

One hundred and twenty five small liquid drops, each carrying a charge of 0.5 µC and of diameter 0.1 m form a bigger drop. Calculate the potential at the surface of the bigger drop.

SOLUTION

n = 125, q = 0.5 x 10^{-6} C, d = 0.1 m

The radius of each small drop, r = d/2 = 0.05 m

The volume of the larger drop being equal to the volume of the n smaller drops, the radius of the larger drop is

$$R = \sqrt[3]{n}r = \sqrt[3]{125}(0.05) = 5 \times 0.05 = 0.25\text{ m}$$

The charge on the larger drop,

$Q = nq = 125 \times (0.5 \times 10^{-6})\text{ C}$

$\therefore$ The electric potential of the surface of the larger drop,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = \left(9 \times 10^9\right) \times \frac{125 \times \left(0.5 \times 10^{-6}\right)}{0.25}$$

$= 9 \times 125 \times 2 \times 10^3 = 2.25 \times 10^6$ V

Exercises | Q 10 | Page 213

The dipole moment of a water molecule is 6.3×10^{-30} c·m. A sample of water contains 10^{21} molecules, whose dipole moments are all oriented in an electric field of strength 2.5×10^5N/C. Calculate the work to be done to rotate the dipoles from their initial orientation $\theta_1 = 0°$ to one in which all the dipoles are perpendicular to the field, $\theta_2 = 90°$.

SOLUTION

$p = 6.3 \times 10^{-30}$ C.m. $N = 10^{21}$ molecules, $E = 2.5 \times 10^5$N/C, $\theta_0 = \theta_1 = 0°$, $\theta = \theta_2 = 90°$

$W = pE(\cos\theta_0 - \cos\theta)$

The total work required to orient N dipoles is

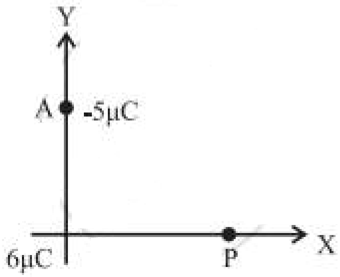
$W = NpE(\cos\theta_1 - \cos\theta_2)$

$= (10^{21})(6.3 \times 10^{-30})(2.5 \times 10^5)$

$= 15.75 \times 10^{-4}$ J = **1.575 mJ**

Exercises | Q 11 | Page 213

A charge 6 µC is placed at the origin and another charge - 5 µC is placed on the y-axis at A = (0, 6.0 m).



(a) Calculate the net electric potential at P = (8.0 m, 0).

(b) Calculate the work done in bringing a proton from infinity to the point P. What is the significance of the negative sign?

SOLUTION

$q_1 = 6 \times 10^{-6}$ C, $q_2 = -5 \times 10^{-6}$ C

$A \equiv (O, 6.0\text{m})$, $P \equiv (8.0\text{m}, 0)$, $r_1 = OP = 8$ m,

$q = e = 1.6 \times 10^{-19}$ C, $1/4\pi\varepsilon_0 = 9 \times 10^9$ N m^2/C^2

$$r_2 = AP = \sqrt{(8-0)^2 + (0-6)^2} = \sqrt{64+36} = 10 \text{ m}$$

(a) The net electric potential at P due to the system of two charges is

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$$

$$= \left(9 \times 10^9\right)\left[\frac{6 \times 10^{-6}}{8} + \frac{-5 \times 10^{-6}}{10}\right]$$

$$= \left(9 \times 10^3\right)(0.75 - 0.5)$$

$$= 2.25 \times 10^3 \text{V} = \mathbf{2.25\ kV}$$

(b) The electric potential V at the point P is the negative of the work done per unit charge, by the electric field of the system of the charges q_1 and q_2 in bringing a test charge from infinity to that point.

$$V = -\frac{W}{q_0}$$

$\therefore W = -qV = -(1.6 \times 10^{-19})(2.25 \times 10^3)$

$$\mathbf{= -3.6 \times 10^{-16}\ J = -2.25\ keV}$$

That is, in bringing the positively charged proton from a point of lower potential to a point of higher potential, the work done by the electric field on it is negative, which means that an external agent must bring the proton against the electric field of the system of the two source charges.

Exercises | Q 12 | Page 213

In a parallel plate capacitor with air between the plates, each plate has an area of 6 x 10^{-3} m^2 and the separation between the plates is 2 mm.
(a) Calculate the capacitance of the capacitor.

(b) If this capacitor is connected to 100 V supply, what would be the charge on each plate?
(c) How would charge on the plates be affected if a 2 mm thick mica sheet of k = 6 is inserted between the plates while the voltage supply remains connected?

SOLUTION

Data: k = 1 (air), $A = 6 \times 10^{-3}$ m^2, d = 2 mm $= 2 \times 10^{-3}$ m, V = 100 V, t = 2 mm = d, $k_1 = 6$,

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

(a) The capacitance of the air capacitor, $\mathbf{C_0} = \dfrac{\varepsilon_0 \mathbf{A}}{\mathbf{d}}$

$$= \frac{\left(8.85 \times 10^{-12}\right)\left(6 \times 10^{-3}\right)}{\left(2 \times 10^{-3}\right)^{-3}}$$

$= 26.55 \times 10^{-12}$ F = 26.55 pF

(b) $Q_0 = C_0V = (26.55 \times 10^{-12})(100)$

$= 26.55 \times 10^{-10}$ C = 2.655 nC

(c) The dielectric of relative permittivity k_1 completely fills the space between the plates ($\because$ t = d), so that the new capacitance is $C = k_1C_0$.

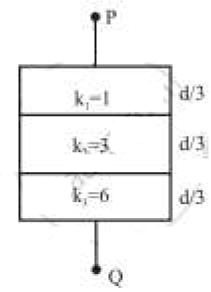
With the supply still connected, V remains the same.

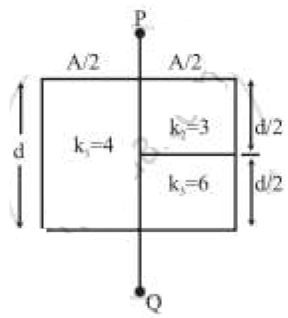
$\therefore Q = CV = kC_0V = kQ_0 = 6(2.655 \text{ nF}) = 15.93$ nC

Therefore, the charge on the plates increases.

Exercises | Q 13 | Page 213

Find the equivalent capacitance between P and Q in the following diagram. The area of each plate is A and the separation between plates is d.





SOLUTION

(i) The capacitor in the first diagram is a series combination of three capacitors of plate separations d/3 and plate areas A, with C_1 filled with air ($k_1 = 1$), C_2 filled with a dielectric of $k_2 = 3$, and C_3 filled with a dielectric of $k_3 = 6$.

$$\therefore C_1 = \frac{k_1 \varepsilon_0 A}{d/3} = \frac{3\varepsilon_0 A}{d} k_1, C_2 = \frac{k_2 \varepsilon_0 A}{d/3} = \frac{3\varepsilon_0 A}{d} k_2,$$

$$C_3 = \frac{k_3 \varepsilon_0 A}{d/3} = \frac{3\varepsilon_0 A}{d} k_3$$

$$\therefore \frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d}{3\varepsilon_0 A}\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)$$

$$\therefore C' = \frac{3\varepsilon_0 A}{d}\left(\frac{k_1 k_2 k_3}{k_1 k_2 + k_2 k_3 + k_3 k_1}\right) = \frac{3\varepsilon_0 A}{d}\left(\frac{18}{27}\right)$$

$$= \frac{2\varepsilon_0 A}{d}$$

(ii) The above second figure, a series combination of two capacitors C_2 ($k_2 = 3$) and C_3 ($k_3 = 6$), of plate areas A/2 and plate separations d/2, is in parallel with a capacitor C_1 ($k_1 = 4$) of plate area A/2 and plate separation d.

$$\therefore C_1 = \frac{k_1\varepsilon_0(A/2)}{d} = \frac{\varepsilon_0 A}{2d}k_1$$

$$C_2 = \frac{k_2\varepsilon_0(A/2)}{d/2} = \frac{\varepsilon_0 A}{d}k_2$$

$$C_3 = \frac{k_3\varepsilon_0(A/2)}{d/2} = \frac{\varepsilon_0 A}{d}k_3$$

$\therefore$ For the series combination of C_2 and C_3,

$$\frac{1}{C_4} = \frac{1}{C_2} + \frac{1}{C_3}$$

$$\therefore C_4 = \frac{C_2C_3}{C_2 + C_3} = \frac{\varepsilon_0 A}{d}\left(\frac{k_2k_3}{k_2 + k_3}\right) = \frac{\varepsilon_0 A}{d}\left(\frac{3 \times 6}{3 + 6}\right) = \frac{2\varepsilon_0 A}{d}$$

Finally, for the parallel combination of C_1 and C_4,

$$C'' = C_1 + C_4 = \frac{\varepsilon_0 A}{2d}(4) + \frac{2\varepsilon_0 A}{d} = \frac{4\varepsilon_0 A}{d}$$

Thus, the equivalent capacitances are $C' = \frac{2\varepsilon_0 A}{d}$ and $C'' = \frac{4\varepsilon_0 A}{d}$.