

13. Linear Programming Problems

C. Activities.

Ex. 1. Ganesh wants to plan his diet to fulfill his requirements of fats, proteins, and carbohydrates at the minimum cost. Two diet plans are available and the following table gives information about these diet plans

Diet Plan	Units		Carbohydrates	Per Unit Cost (Rs.)
	Fats	Proteins		
I	3	4	8	60
II	2	3	5	50
Minimum Requirement	88	120	230	

Formulate the problem as a linear programming problem and obtain the optimum solution.

Solution. Let x be the number of units of diet I and y be the number of units of diet II that Ganesh consumes.

The problem is to minimize $z = 60x + 50y$ subject to

$$3x + 2y \geq 88 \quad (1)$$

$$4x + 3y \geq 120 \quad (2)$$

$$8x + 5y \geq 230 \quad (3)$$

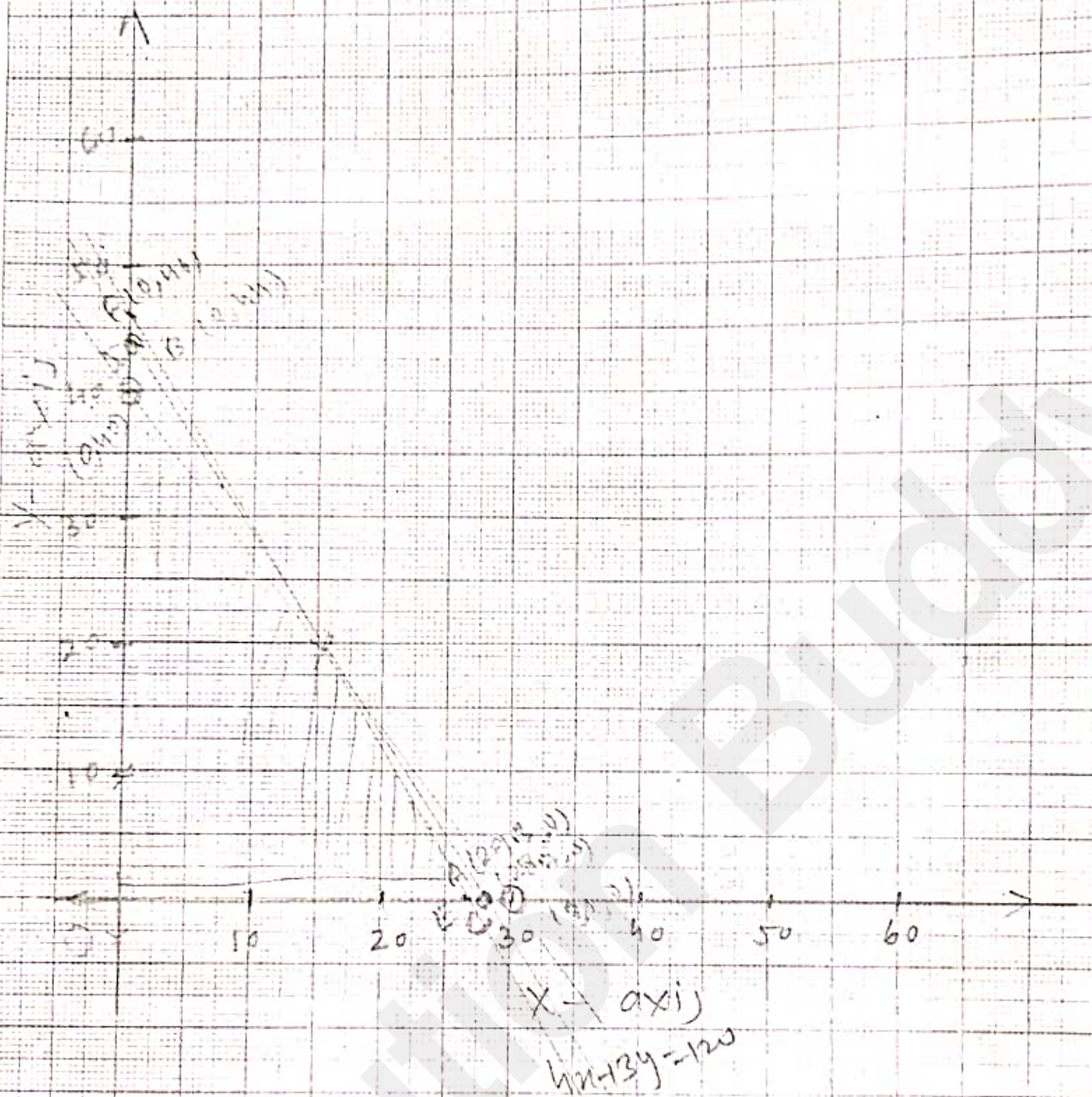
$$x \geq 0 \quad (4)$$

$$y \geq 0 \quad (5)$$

Draw graph of the feasible region. Find the vertices and complete the following table. (Please see Page 62)

Inequation	Equation	Intersection of line with x-axis	Intersection of line with y-axis	Region
$3x + 2y \geq 88$	$3x + 2y = 88$	A (29.3, 0)	B (0, 44)	Non-origin
$4x + 3y \geq 120$	$4x + 3y = 120$	C (30, 0)	D (0, 40)	Non-origin
$8x + 5y \geq 230$	$8x + 5y = 230$	E (28.75, 0)	F (0, 46)	Non-origin

Scale -
2 cm = 10 units



Vertex	Lines through vertex		Value of z
(24, 8)	(1)	(2)	1840
(20, 14)	(1)	(3)	1900
(0, 40)	(2)	(4)	2000
(28.7, 0)	(3)	(5)	1725
(0, 0)	(4)	(5)	0

Ex. 2. A firm manufactures products of two types P1 and P2. The per unit profits are Rs. 3 on P1 and Rs. 4 on P2. Every product is processed on two machines A and B. P1 requires 2 minutes of processing time on machine A and 3 minutes on B. P2 requires 3 minutes on A and 2 minutes on B. Machine A is available for 7 hours and 30 minutes while machine B is available for 8 hours and 30 minutes during a working day. Formulate the problem as a linear programming problem and solve it.

Solution : Let x be the number of units of P1 and y be the number of units of P2 produced. Then the problem is to

Maximize $z = 3x + 4y$ subject to

$$2x + 3y \leq 450 \quad (1)$$

$$x + y \leq 510 \quad (2)$$

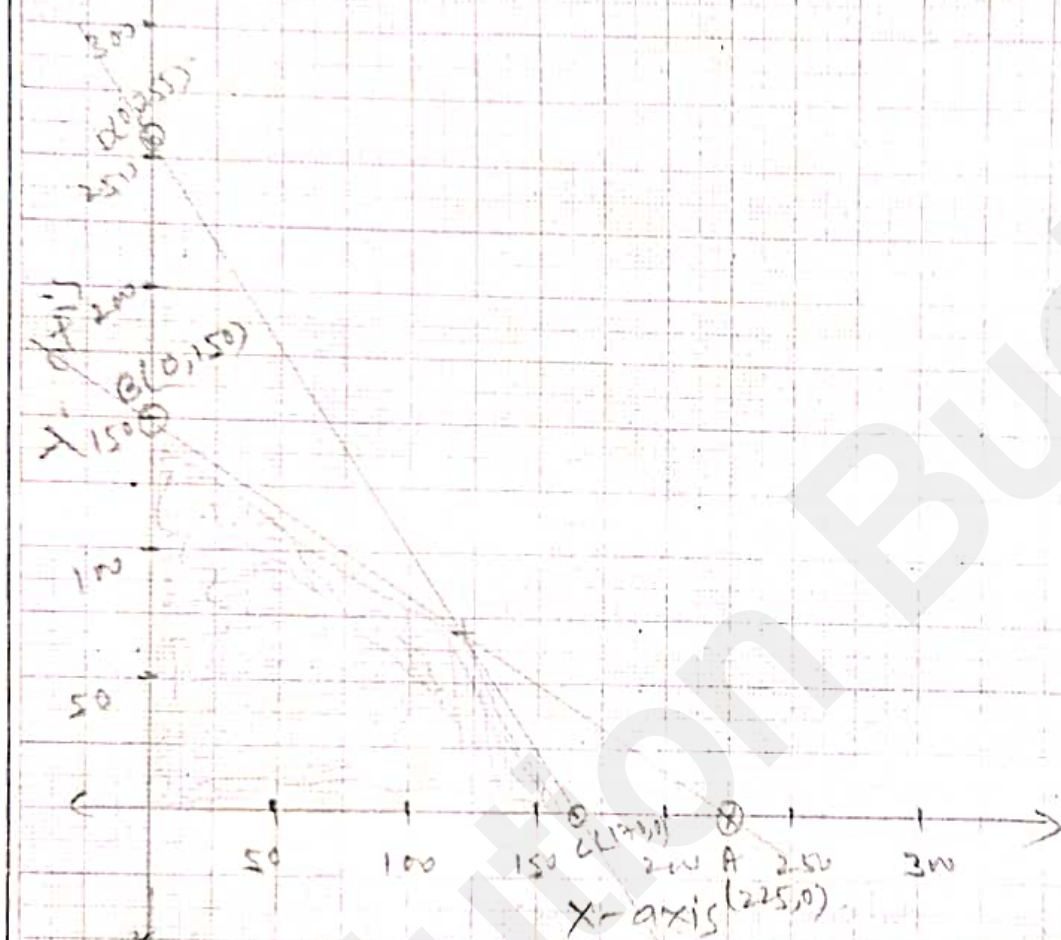
$$x \geq 0 \quad (3)$$

$$y \geq 0 \quad (4)$$

Draw the graph of the feasible region. Complete the following table. (Please see Page 64)

Inequation	Equation	Intersection of line with x-axis	Intersection of line with y-axis	Region
$2x + 3y \leq 450$	$2x + 3y = 450$	A (225, 0)	B (0, 150)	Origin side
$3x + 2y \leq 510$	$3x + 2y = 510$	C (170, 0)	D (0, 255)	Origin side
$x \geq 0$	$x = 0$	-	-	R.H.S of y-axis
$y \geq 0$	$y = 0$	-	-	Above x-axis

Scale
2 cm = 50 km/h



Q.2. Solve the following Linear Programming Problem graphically :

Minimize $Z = 4x + 2y$

$3x + y \geq 27$, $x + y \geq 21$, $x + 2y \geq 30$, $x \geq 0$, $y \geq 0$.

Solⁿ

Construct the table as follows:-

Inequality	Equation	Double Intercept form	Point	Region
$3x + y \geq 27$	$3x + y = 27$	$\frac{x}{9} + \frac{y}{27} = 1$	A(9,0), B(0,27)	Non-origin
$x + y \geq 21$	$x + y = 21$	$\frac{x}{21} + \frac{y}{21} = 1$	C(21,0), D(0,21)	"
$x + 2y \geq 30$	$x + 2y = 30$	$\frac{x}{30} + \frac{y}{15} = 1$	E(30,0), F(0,15)	"
$x \geq 0$	$x = 0$	-	-	R.H. of y-axis
$y \geq 0$	$y = 0$	-	-	Above x-axis

Shaded portion E G H B is the feasible region whose vertices are E(30,0), G, H & B(0,27)

G is the intersection point of lines $x + y = 21$ (1) & $x + 2y = 30$ (2)
Now, on solving (1) & (2) we get - $x = 12$, $y = 9$
 $\therefore \boxed{G(12, 9)}$

H is the intersection point of lines $3x + y = 27$ (3) & $x + y = 21$ (4)
Now, on solving (3) & (4) we get - $x = 3$, $y = 18$
 $\therefore \boxed{H(3, 18)}$

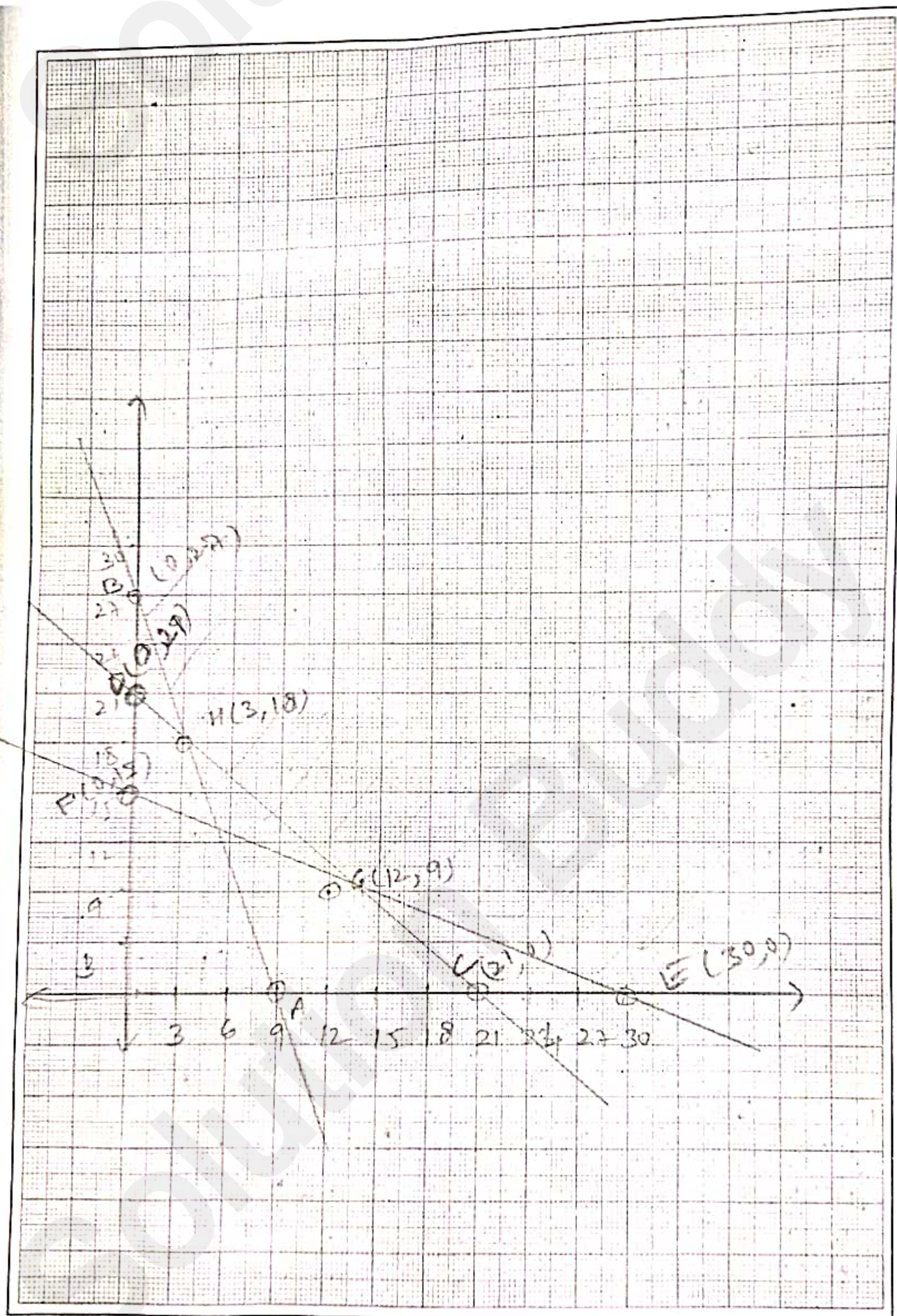
Here, $Z = 4x + 2y$

Now, we will find minimum value of Z as follows-

Feasible points	Value of $Z = 4x + 2y$
B(0,27)	$Z = 4 \times 0 + 2 \times 27 = 54$
<u>H(3,18)</u>	<u>$Z = 48$</u>
G(12,9)	$Z = 66$
E(30,0)	$Z = 120$

Z is minimum when $x = 3$, $y = 18$

$\therefore Z$ has minimum value 48 at H(3,18)



Q.3. A carpenter makes chairs and tables. Profits are ₹ 140/- per chair and ₹ 210/- per table. Both products are processed on three machines: Assembling, Finishing and Polishing. The time required for each product (in hours) and total time available in hours for each machine are as follows:

	Chair	Table	Available time
Assembling machine	3	3	36
Finishing machine	5	2	50
Polishing machine	2	6	60

Find the number of chairs and tables to be made so as to get maximum profit.

Solⁿ:

Let x be the no. of chairs and y be the no. of tables

Formulated as: maximize $Z = 140x + 210y$

subject to constraint: $3x + 3y \leq 36$, $5x + 2y \leq 50$
 $2x + 6y \leq 60$, $x, y \geq 0$

Inequation	Equation	Double Intercept form	Points	Region
$3x + 3y \leq 36$	$3x + 3y = 36$	$\frac{x}{12} + \frac{y}{12} = 1$	A(12, 0) B(0, 12)	Origin side
$5x + 2y \leq 50$	$5x + 2y = 50$	$\frac{x}{10} + \frac{y}{25} = 1$	C(10, 0) D(0, 25)	"
$2x + 6y \leq 60$	$2x + 6y = 60$	$\frac{x}{30} + \frac{y}{10} = 1$	E(30, 0) F(0, 10)	"
$x \geq 0$	$x = 0$	—	—	R.H.S. of y-axis
$y \geq 0$	$y = 0$	—	—	Above x-axis

Shaded portion OFGHC is the feasible region whose vertices are O(0, 0), F(0, 10), G, H, and C(10, 0), &

Now, G is point, intersection of lines $x + 3y = 30$ — (1)
 $x + y = 12$ — (2)

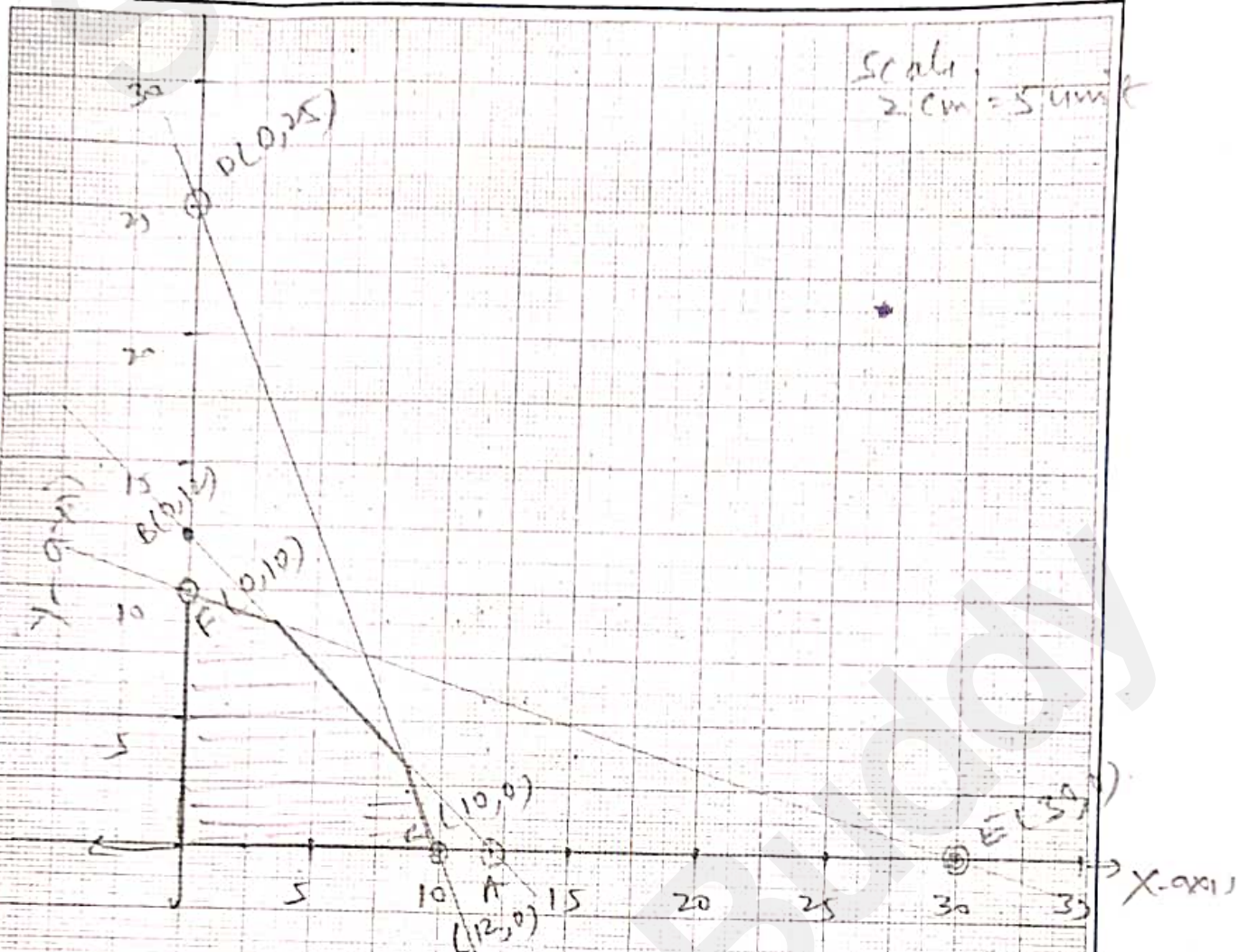
on solving (1) & (2) we get- $x = 3, y = 9$

$\therefore \boxed{G(3, 9)}$

H is point, intersection of lines $x + y = 12$ & $5x + 2y = 50$
 on solving — $x = \frac{26}{3}, y = \frac{10}{3} \therefore \boxed{H(\frac{26}{3}, \frac{10}{3})}$

Now, we will find maximum value of Z .

Scale:
2 cm = 5 units



Feasible points

value of $Z = 140x + 210y$

$O(0,0)$

$$Z = 140 \times 0 + 210 \times 0 = 0$$

$F(0,10)$

$$Z = 2100$$

$G(3,9)$

$$Z = 2310$$

$H\left(\frac{26}{3}, \frac{10}{3}\right)$

$$Z = 1913.33$$

$C(10,0)$

$$Z = 1400$$

$\therefore Z$ has max. value 2310 at $(3,0)$

hence max. profit is 2310 when x (no. of chairs) is 3 & y (no. of tables) is 9.

Sign of Teacher :