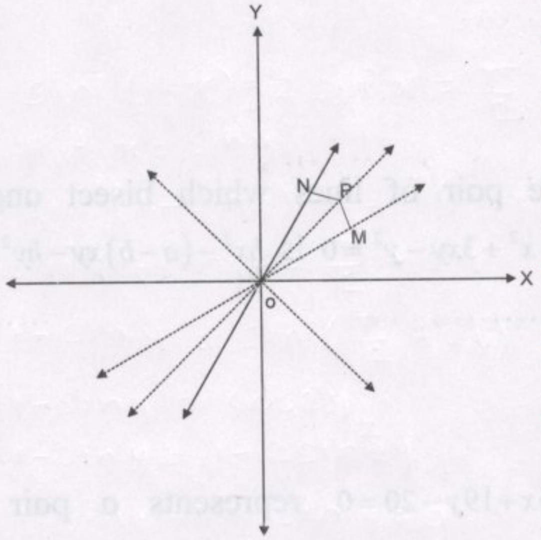
5. Pair of straight lines

Ex. (1) Find the joint equation of bisectors of angles between lines represented by $ax^2+2hxy+by^2=0$. Hence write the joint equation of the pair of lines which bisect angles between the lines given by $x^2+3xy-y^2=0$.



Solution : Let m_1 and m_2 be the slopes of lines represented by the equation

$ax^2+2hxy+by^2=0$.

$\therefore m_1+m_2=-\dfrac{2h}{b}$ and $m_1m_2=\dfrac{a}{b}$.

Let their separate equations be $m_1x-y=0$ and $m_2x-y=0$.

Let $P(x_1,y_1)$ be any point on one of the angle bisectors.

Let PM and PN be perpendiculars drawn from P to the lines $m_1x-y=0$ and $m_2x-y=0$.

$\therefore$ PM = PN

$$\therefore \left|\frac{m_1x_1-y_1}{\sqrt{m_1^2+1}}\right|=\left|\frac{m_2x_1-y_1}{\sqrt{m_2^2+1}}\right|$$

$$\therefore (m_1x_1-y_1)^2(m_2^2+1)=(m_2x_1-y_1)^2(m_1^2+1)$$

$$\therefore (m_1^2x_1^2-2m_1x_1y_1+y_1^2)(m_2^2+1)=(m_2^2x_1^2-2m_2x_1y_1+y_1^2)(m_1^2+1)$$

$$\therefore m_1^2m_2^2x_1^2-2m_1m_2^2x_1y_1+m_2^2y_1^2+m_1^2x_1^2-2m_1x_1y_1+y_1^2$$

$$= m_1^2 m_2^2 x_1^2 - 2m_1^2 m_2 x_1 y_1 + m_1^2 y_1^2 + m_2^2 x_1^2 - 2m_2 x_1 y_1 + y_1^2$$

$$\therefore \quad (m_1^2 - m_2^2) x_1^2 + 2m_1 m_2 (m_1 - m_2) x_1 y_1 - 2(m_1 - m_2) x_1 y_1 - (m_1^2 - m_2^2) y_1^2 = 0$$

$$\therefore \quad (m_1 + m_2) x_1^2 + 2m_1 m_2 x_1 y_1 - 2x_1 y_1 - (m_1 + m_2) y_1^2 = 0$$

$$\therefore \quad \left(\frac{2h}{b}\right) x_1^2 + \frac{2a}{b} x_1 y_1 - 2x_1 y_1 - \left(-\frac{2h}{b}\right) y_1^2 = 0$$

$$\therefore \quad -2hx_1^2 + 2(a-b) x_1 y_1 + 2hy_1^2 = 0$$

$$\therefore \quad hx_1^2 - (a-b) x_1 y_1 - hy_1^2 = 0$$

$$\therefore \quad hx^2 - (a-b) xy - hy^2 = 0$$

The joint equation of the pair of lines which bisect angles between the lines given by $x^2 + 3xy - y^2 = 0$ is $hx^2 - (a-b)xy - hy^2 = 0$

i .e. $\frac{3}{2}x^2 - (2)xy - \frac{3}{2}y^2 = 0$

$3x^2 - 4xy - 3y^2 = 0$

Ex. (2) Show that $2x^2 - xy - 3y^2 - 6x + 19y - 20 = 0$ represents a pair of intersecting lines. Find the acute angle between them. Also find the co-ordinates of their point of intersection.

Solution : We have $2x^2 - xy - 3y^2 = (2x - 3y)(x + y)$

Suppose $2x^2 - xy - 3y^2 - 6x + 19y - 20 = (2x - 3y + c)(x + y + k)$

$\therefore \quad 2x^2 - xy - 3y^2 - 6x + 19y - 20 = 2x^2 - xy - 3y^2 + (c + 2k)x + (c - 3k)y + ck$

On comparing coefficients, we get

$c + 2k = -6 \quad \ldots \quad (1)$

$c - 3k = 19 \quad \ldots \quad (2)$

$ck = -20 \quad \ldots \quad (3)$

Solving (1) and (2) we get $c = 4$ and $k = -5$

They satisfy equation (3) also.

$\therefore$ Given general equation can be factorized as

$(2x - 3y + 4)(x + y - 5) = 0$

$\therefore$ Given equation represents a pair of intersecting lines.

The acute angle between them is given by

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right| = \left|\frac{\sqrt{(2h)^2 - 4ab}}{a+b}\right| = \left|\frac{\sqrt{(-1)^2 - 4(2)(-3)}}{2-3}\right| = 5$$

$\therefore \quad \theta = \tan^{-1}(5)$

Their point of intersection is given by $\left(\dfrac{hf-bg}{ab-h^2}, \dfrac{gh-af}{ab-h^2}\right) = \left(\dfrac{11}{5}, \dfrac{14}{5}\right)$

Ex. (3) If the line $4x+6y+7=0$ coincides with one of the lines represented by $4x^2+2xy-6y^2+11x-y+\lambda=0$ then find the value of λ and the slope of the other line.

Solution : Method I :

As $4x^2+2xy-6y^2=(4x+6y)(x-y)$, let the equation of the other line be $x-y+c=0$.

Their joint equation is $(4x+6y+7)(x-y+c)=0$

$\therefore \quad 4x^2+2xy-6y^2+(7+4c)x+(-7+6c)y+7c=0$

Comparing with given joint equation, we get

$7+4c=11$, $-7+6c=-1$ and $\lambda=7c$

Solving first two equations we get, $c=1$.

$\therefore \quad \lambda=7$ and the equation of the other line is $x-y+1=0$.

$\therefore$ The slope of the other line is $-\left(\frac{1}{-1}\right)=1$

Method II :

Co-ordinates of every point on the line $4x+6y+7=0$ satisfy the joint equation.

$A\left(-\frac{7}{4}, 0\right)$ is a point on the line $4x+6y+7=0$.

It satisfy the equation $4x^2+2xy-6y^2+11x-y+\lambda=0$.

$\therefore \quad 4\left(-\frac{7}{4}\right)^2+0-0+11\left(-\frac{7}{4}\right)-0+\lambda=0$

$\therefore \quad \lambda=7$.

The joint equation is $4x^2+2xy-6y^2+11x-y+7=0$

Slope of the line $4x+6y+7=0$ is $\frac{-4}{6}=-\frac{2}{3}$

Let slope of the other line be m_1.

$\therefore -\frac{2}{3}$ and m_1 are the roots of the equation $6m^2-2m-4=0$.

$\therefore -\frac{2}{3}+m_1=-\frac{-2}{6} \qquad \therefore -\frac{2}{3}+m_1=\frac{1}{3}$

$\therefore \quad m_1=1$

$\therefore$ The slope of the other line is 1.

Ex. (4) Find the joint equation of the pair of lines passing through A(2, 3), each of which make angle 30^0 with the $Y-$axis.

Solution : Lines make angles 30^0 with the $Y-$axis.

$\therefore$ Their inclinations are 60^0 and 120^0.

$\therefore$ Their slopes are $\tan 60^0 = \sqrt{3}$ and $\tan 120^0 = -\sqrt{3}$

They pass through A(2, 3).

Their separate equations are $(y-3)=\sqrt{3}(x-2)$ and $(y-3)=-\sqrt{3}(x-2)$

$\therefore$ $\sqrt{3}(x-2)-(y-3) = 0$ and $\sqrt{3}(x-2)+(y-3)$.

$\therefore$ Their joint equation is

$\left(\sqrt{3}(x-2)-(y-3)\right)\left(\sqrt{3}(x-2)+(y-3)\right)=0$

$\therefore$ $3(x-2)^2-(y-3)^2=0$

$\therefore$ $3(x^2-4x+4)-(y^2-6y+9)=0$

$\therefore$ $3x^2-12x+12-y^2+6y-9=0$

$\therefore$ $3x^2-y^2-12x+6y+3=0$

This is the required joint equation.

Ex. (5) If lines represented by the equatio $ax^2+2hxy+by^2+2gx+2fy+c=0$ intersect each other then show that the co-ordinates of their point of intersection are $\left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right)$.

Solution :

Given joint eq^n is $ax^2+2hxy+by^2+2gx+2fy+c=0$

Consider $ax+hy+g=0$ — (I)

$hx+by+f=0$ — (II)

Let above two lines intersect each other

solving eq^n (I) and (II)

$$\frac{x}{\begin{vmatrix} h & g \\ b & f \end{vmatrix}} = \frac{-y}{\begin{vmatrix} a & g \\ h & f \end{vmatrix}} = \frac{1}{\begin{vmatrix} a & h \\ h & b \end{vmatrix}}$$

$$\therefore \frac{x}{hf-bg} = \frac{-y}{af-gh} = \frac{1}{ab-h^2}$$

$$\therefore \frac{x}{hf-bg} = \frac{1}{ab-h^2}$$

$$\frac{y}{gh-af} = \frac{1}{ab-h^2}$$

$$\therefore x = \frac{hf-bg}{ab-h^2}, \; y = \frac{gh-af}{ab-h^2}$$

$\therefore$ The coordinates of point of intersection are

$$\left(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2}\right)$$

Ex. (6) Δ OAB is formed by the lines $x^2-4xy+y^2=0$ and $x+y-2=0$.

Find the equation of the median drawn from O.

Solution :

Let $A \equiv (x_1, y_1)$, $B \equiv (x_2, y_2)$

By mid point formula $P \equiv \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

The coordinates of A and B can be obtained by solving $x^2-4xy+y^2=0$ and $x+y-2=0$ simultaneously

put $y=2-x$ in $x^2-4xy+y^2=0$

$$x^2-4x(2-x)+(2-x)^2=0$$

$$x^2-8x+4x^2+4-4x+x^2=0$$

$$6x^2-12x+4=0$$

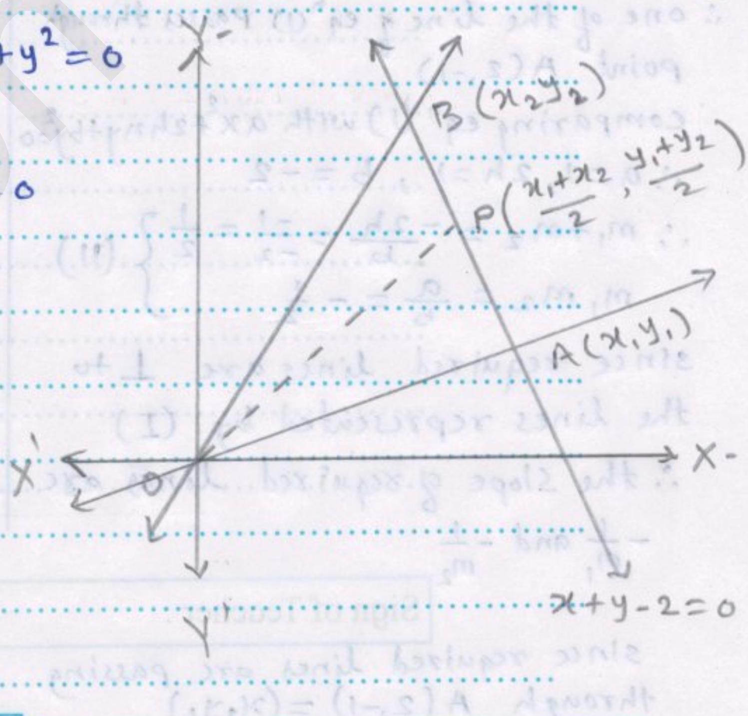
$$\therefore 3x^2-6x+2=0$$

it is Q.E. with two roots

$$x_1+x_2 = \frac{-(-6)}{3} = 2$$

$$x_1+x_2=2$$

$$\frac{x_1+x_2}{2} = 1$$



The co-ordinate of the point P is 1

Point P is on line $x+y-2=0$

$\therefore 1+y-2=0$

$y-1=0$

$y=1$

$\therefore P \equiv (1,1)$

for median OP, $O \equiv (0,0)$

$P \equiv (1,1)$

$\therefore$ the eq^n of median OP is

$\frac{y-0}{1-0} = \frac{x-0}{1-0}$

$y = x$

$\therefore x-y=0$ is the required eq^n of median.

Ex. (7) Show that one of the lines represented by $x^2+xy-2y^2=0$ passes through the point $A(2,-1)$. Find the joint equation of lines passing through $A(2,-1)$ and perpendicular to the lines represented by the equation $x^2+xy-2y^2=0$.

Solution :

Given joint eq^n is $x^2+xy-2y^2=0$ —(I)

put $A(2,-1)$ in (I)

$\therefore$ LHS $= x^2+xy-2y^2$

$= 4-2-2=0$

LHS = RHS

$\therefore$ one of the line of eq^n (I) passes through point $A(2,-1)$

comparing eq^n (I) with $ax^2+2hxy+by^2=0$

$\therefore a=1,\ 2h=1,\ b=-2$

$$\left.\begin{aligned} \therefore m_1+m_2 &= \frac{-2h}{b} = \frac{-1}{-2} = \frac{1}{2} \\ m_1 m_2 &= \frac{a}{b} = -\frac{1}{2} \end{aligned}\right\} \text{(II)}$$

since required lines are $\perp$ to the lines represented by (I)

$\therefore$ the slope of required lines are $-\frac{1}{m_1}$ and $-\frac{1}{m_2}$

since required lines are passing through $A(2,-1)=(x_1,y_1)$

eq^n of the lines are

$y-y_1 = m(x-x_1)$

$(y+1) = -\frac{1}{m_1}(x-2)$ & $y+1 = -\frac{1}{m_2}(x-2)$

$(x-2)+m_1(y+1)=0$ & $(x-2)+m_2(y+1)=0$

$\therefore$ Joint eq^n is

$(x-2)^2+m_2(x-2)(y+1)+m_1(x-2)(y+1)$
$m_1m_2(y+1)^2=0$

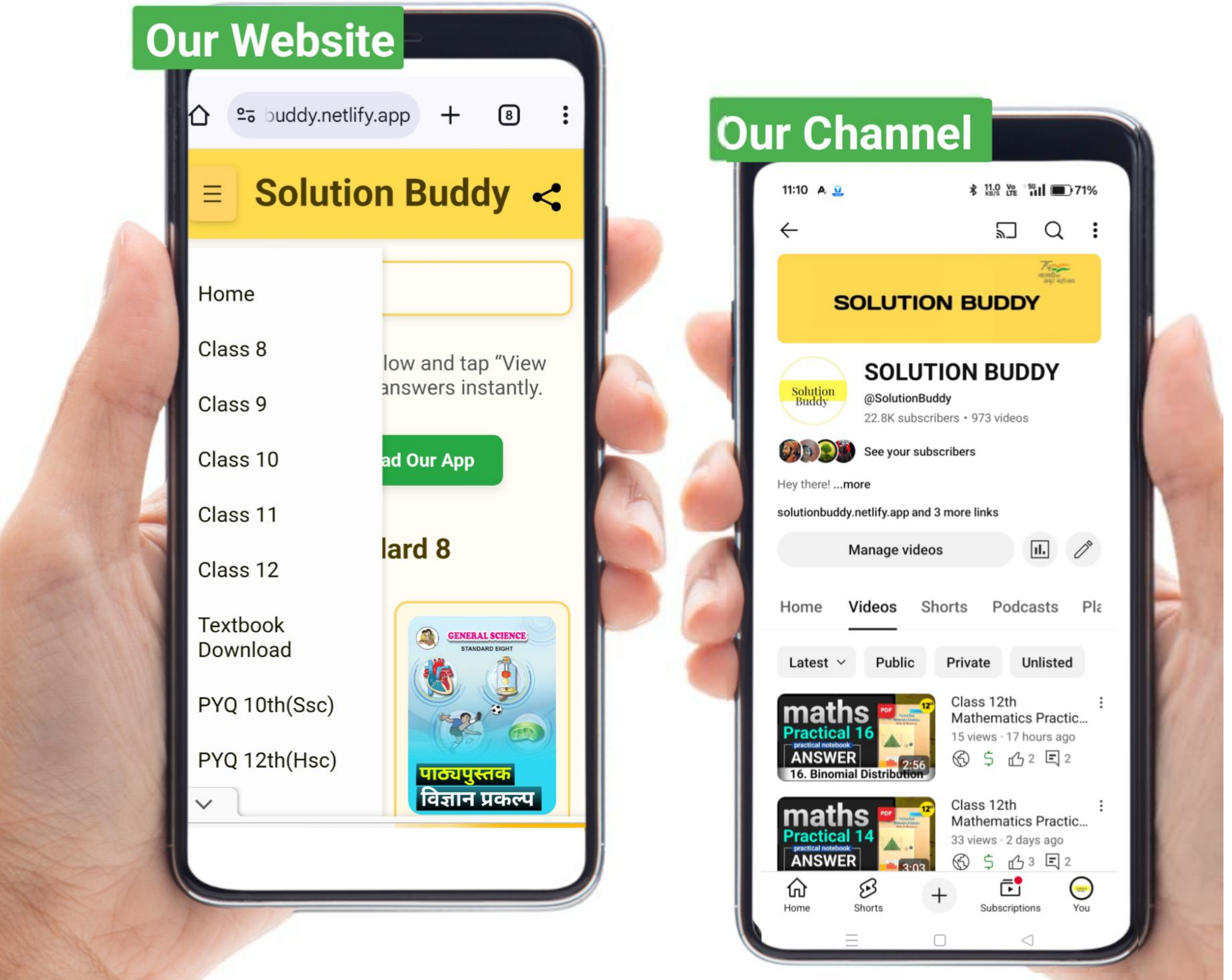
$\therefore (x-2)^2+\frac{1}{2}(x-2)(y+1)-\frac{1}{2}(y+1)^2=0$ using — II

$2(x-2)^2+(x-2)(y+1)-(y+1)^2=0$

$\therefore 2x^2-8x+8+xy+x-2y-2-y^2-2y-1=0$

$\therefore 2x^2+xy-y^2-7x-4y+5=0$

Sign of Teacher :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