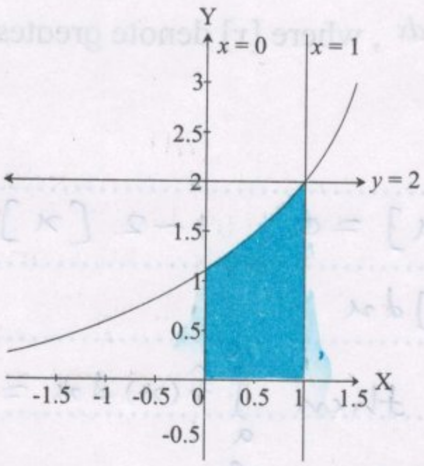
13. Application of definite integration

Ex. (1) Find the area of the region bounded by the curve $y = 2^x$ and the lines x = 0 and y = 1.



Solution : The equation of the carves are $y = 2^x$ and y = 2.

Solving equations we get x = 1.

Point of intersection of the curve is (1,2).

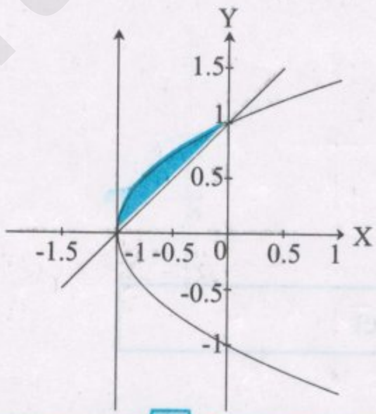
Required area (A) = $\int_0^1 (2)\,dx - \int_0^1 2^x\,dx$

$$= [2x]_0^1 - \left[\frac{2^x}{\log 2}\right]_0^1$$

$$= [2-0] - \left[\frac{2^1 - 2^0}{\log 2}\right]$$

$$= \left[2 - \frac{1}{\log 2}\right] \text{ sq.units}$$

Ex. (2) Find the area of the region enclosed by the curves $y = \sqrt{x+1}$ and $y = x + 1$.



Solution : The equation of the curves are $y=\sqrt{x+1}$ and $y=x+1$.

Solving these equations , we get $x+1=\sqrt{x+1}$

$x^2+x=0$

$x(x+1)=0$

$x=0 \quad or \quad x=-1$

$\therefore\ y=1 \ and \ y=0$

Therefore , the point of intersection of the curves are $(-1,0)$ and $(0,1)$.

$\therefore$ Required area (A) $=\int_{-1}^{0}\sqrt{x+1}\,dx-\int_{-1}^{0}(x+1)\,dx$

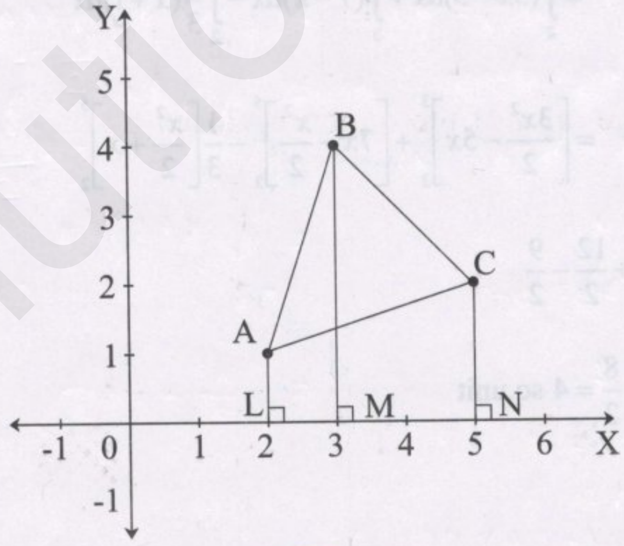
$$=\left[\frac{2}{3}(x+1)^{\frac{3}{2}}\right]_{-1}^{0}-\left[\frac{x^2}{2}+x\right]_{-1}^{0}$$

$$=\left[\frac{2}{3}(1)^{\frac{3}{2}}-0\right]-\left[0-\left(\frac{1}{2}-1\right)\right]$$

$$=\left[\frac{2}{3}\right]-\left[\frac{1}{2}\right]$$

$$=\left[\frac{1}{6}\right] \text{ sq. units}$$

Ex. (3) Find the area of the triangle formed by the vertices (2,1) , (3,4) and (5,2).



Solution : A(2,1) , B(3,4) and C(5,2) are the vertices of the triangle.

Equation of AB is $y-1=\left(\frac{4-1}{3-2}\right)(x-2)$

$$y-1=\left(\frac{3}{1}\right)(x-2)$$

$$y-1=(3x-6)$$

$$3x-y=5 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(I)}$$

Equation of AC is $y-1=\left(\frac{2-1}{5-2}\right)(x-2)$

$$y-1=\left(\frac{1}{3}\right)(x-2)$$

$$3y-3=x-2$$

$$x-3y=-1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(II)}$$

Equation of BC is

$$y-4=\left(\frac{-2}{2}\right)(x-3)$$

$$y-4=-x+3$$

$$x+y=7 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(III)}$$

Area of ΔABC = A(regionALMBA) + A(regionBCNMB) − A(regionACNLA)

$$=\int_2^3 (3x-5)dx+\int_3^5 (7-x)dx-\int_2^5 \frac{1}{3}(x+1)dx$$

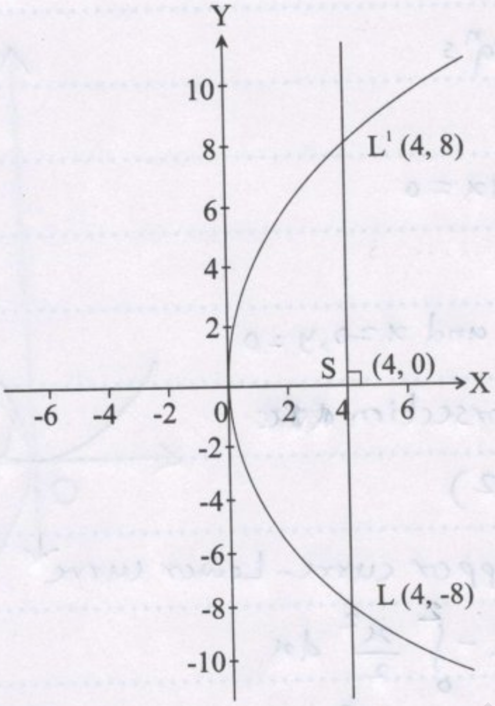
$$=\left[\frac{3x^2}{2}-5x\right]_2^3+\left[7x-\frac{x^2}{2}\right]_3^5-\frac{1}{3}\left[\frac{x^2}{2}+x\right]_2^5$$

$$=\frac{5}{2}+\frac{12}{2}-\frac{9}{2}$$

$$=\frac{8}{2}=4 \text{ sq.unit}$$

Ex. (4) Find the area of the region bounded by the parabola $y^2 = 16x$ and its latus rectum.

Solution : The equation of the parabola is $y^2 = 16x$.



The equation of parabola is $y^2 = 16x$

comparing with $y^2 = 4ax$

$\therefore 4a = 16$, $\boxed{a = 4}$

focus $S(a, 0) \equiv S(4, 0)$

$y^2 = 16x \Rightarrow y = 4\sqrt{x}$

$A(\text{region } OL'SLO) = 2A(\text{region } OL'SO)$

$= 2\int_0^4 y\, dx$

$= 2\int_0^4 4\sqrt{x} \Rightarrow 8\int_0^4 x^{\frac{1}{2}}\, dx$

$= 8\left[\frac{x^{3/2}}{\frac{3}{2}}\right] \Rightarrow 8 \times \frac{2}{3}\left[x^{3/2}\right]_0^4$

$= \frac{16}{3}\left[4^{3/2} - 0^{3/2}\right] = \frac{16}{3} \times 8$

Required Area $= \frac{128}{3}$ sq. units

Ex. (5) Find the area of the region lying between the parabolas $y^2 = 2x$ and $x^2 = 2y$.

Solution :

we have $y^2 = 2x$ and $x^2 = 2y$ $\therefore y = \sqrt{2x}$, $y = \frac{x^2}{2}$

equating these eq^{n}s

$x^2 = 2\sqrt{2x}$

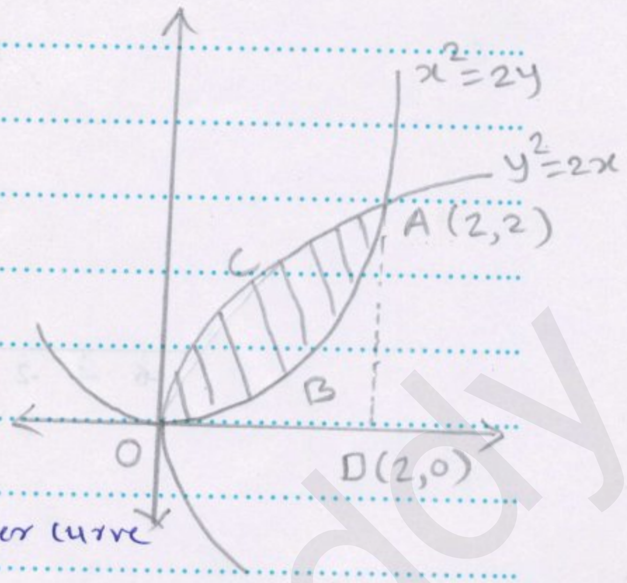
$x^4 = 8x \Rightarrow x^4 - 8x = 0$

$x = 2$, $x = 0$

when $x = 2, y = 2$ and $x = 0, y = 0$

The point of intersection are

$O(0,0)$, $A(2,2)$



Required area = Upper curve - Lower curve

$= \int_0^2 \sqrt{2}\sqrt{x} - \int_0^2 \frac{x^2}{2} dx$

$= \sqrt{2}\int_0^2 x^{\frac{1}{2}} dx - \frac{1}{2}\int_0^2 x^2 dx \Rightarrow \sqrt{2} \times \frac{2}{3}\left[x\sqrt{x}\right]_0^2 - \frac{1}{2}\left[\frac{x^3}{3}\right]_0^2$

$= \frac{2\sqrt{2}}{3}\left[2\sqrt{2} - 0\sqrt{0}\right] - \frac{1}{2}\left[\frac{8}{3} - 0\right] \Rightarrow \frac{8}{3} - \frac{4}{3}$

$= \frac{4}{3}$ Sq units

Ex. (6) Find the area bounded by the curve $y = x^2$ and the line $y = x + 6$.

Solution :

we have $y = x^2$ and $y = x + 6$

equating the equations

$x^2 = x + 6$ $\therefore x^2 - x - 6 = 0$

$x^2 - 3x + 2x - 6 = 0$

$x(x-3) + 2(x-3) = 0$

$\therefore x = -2$ and $x = 3$

when $x = -2$, $y = 4$

when $x = 3$, $y = 9$

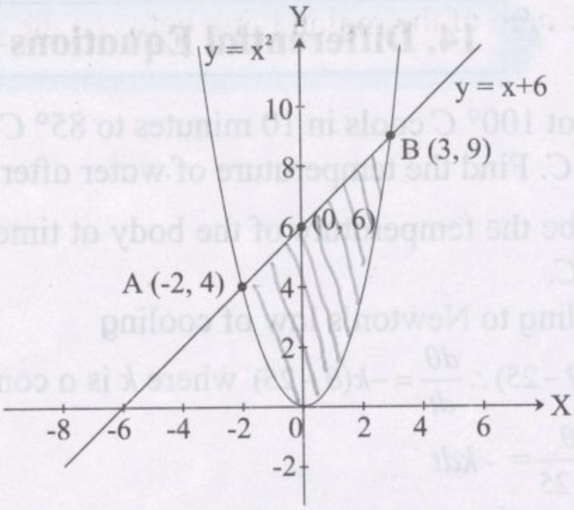
$\therefore$ The points are $A(-2,4)$, $B(3,9)$

Required area = region(OBAO)

$= \int_{-2}^{3} y\,dx - \int_{-2}^{3} y\,dx$

$= \int_{-2}^{3} (x+6)\,dx - \int_{-2}^{3} x^2\,dx$

$= \left[\frac{x^2}{2} + 6x\right]_{-2}^{3} - \left[\frac{x^3}{3}\right]_{-2}^{3}$



$= \frac{9}{2} + 18 - \left(\frac{4}{2} - 12\right) - \left[\frac{27}{3} - \left(-\frac{8}{3}\right)\right]$

$= \left(\frac{9}{2} + 28\right) - \frac{35}{3} \Rightarrow \frac{65}{2} - \frac{35}{3}$

$= \frac{125}{6}$ sq units.

Ex. (7) Find the area of the region enclosed by the parabola $y^2 = 16x$ and the chord BC where B(1,4) and C(9,12).

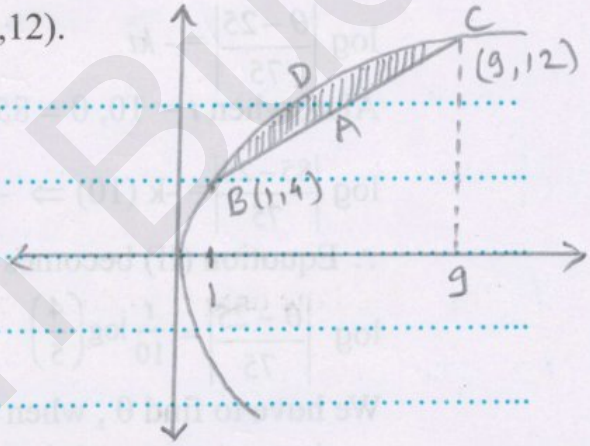
Solution :

we have $y^2 = 16x$, $B(1,4)$, $C(9,12)$

eqⁿ of line BC is

$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$ $\therefore \frac{x-1}{9-1} = \frac{y-4}{12-4}$

$\frac{x-1}{8} = \frac{y-4}{8}$ $\therefore \boxed{y = x+3}$



Required area = Upp Curve − Low Cur.

$= \int_1^9 y\,dx - \int_1^9 y\,dx$

$= \int_1^9 4\sqrt{x}\,dx - \int_1^9 x+3\,dx$

$= 4\left[\frac{x^{3/2}}{3/2}\right]_1^9 - \left[\frac{x^2}{2} + 3x\right]_1^9$

$= 4 \times \frac{2}{3}\left[9^{3/2} - 1\right] - \left[\frac{81}{2} + 27 - \frac{1}{2} - 3\right]$

$= \frac{8}{3}\left[27 - 1\right] - (64)$

$= \frac{8}{3} \times 26 - 64$

$= \frac{16}{3}$ sq units.

Sign of Teacher :

Q. 26. A solenoid of length π m and 5 cm in diameter has a winding of 1000 turns and carries a current of 5A. Calculate the magnetic field at its centre along the axis.

SECTION – D

Attempt any THREE questions of the following : **[12]**

Q. 27. What is Ferromagnetism? Explain it on the basis of domain theory.

Q. 28. Obtain an expression for average power dissipated in a series LCR circuit.

Q. 29. Distinguish between interference and diffraction of light.

A double slit arrangement produces interference fringes for sodium light of wavelength 589 nm, that are 0.20 degree apart. What is the angular fringe separation if the entire arrangement is immersed in water?

(R.I. of water = 1.33)

Q. 30. State Einstein's photoelectric equation and mention physical significance of each term involved in it.

The wavelength of incident light is 4000Å. Calculate the energy of incident photon.

Q. 31. State any four uses of Van de Graaff generator.

In a parallel plate air capacitor, intensity of electric field is changing at the rate of 2×10^{11} V/ms. If area of each plate is 20 cm^2, calculate the displacement current.

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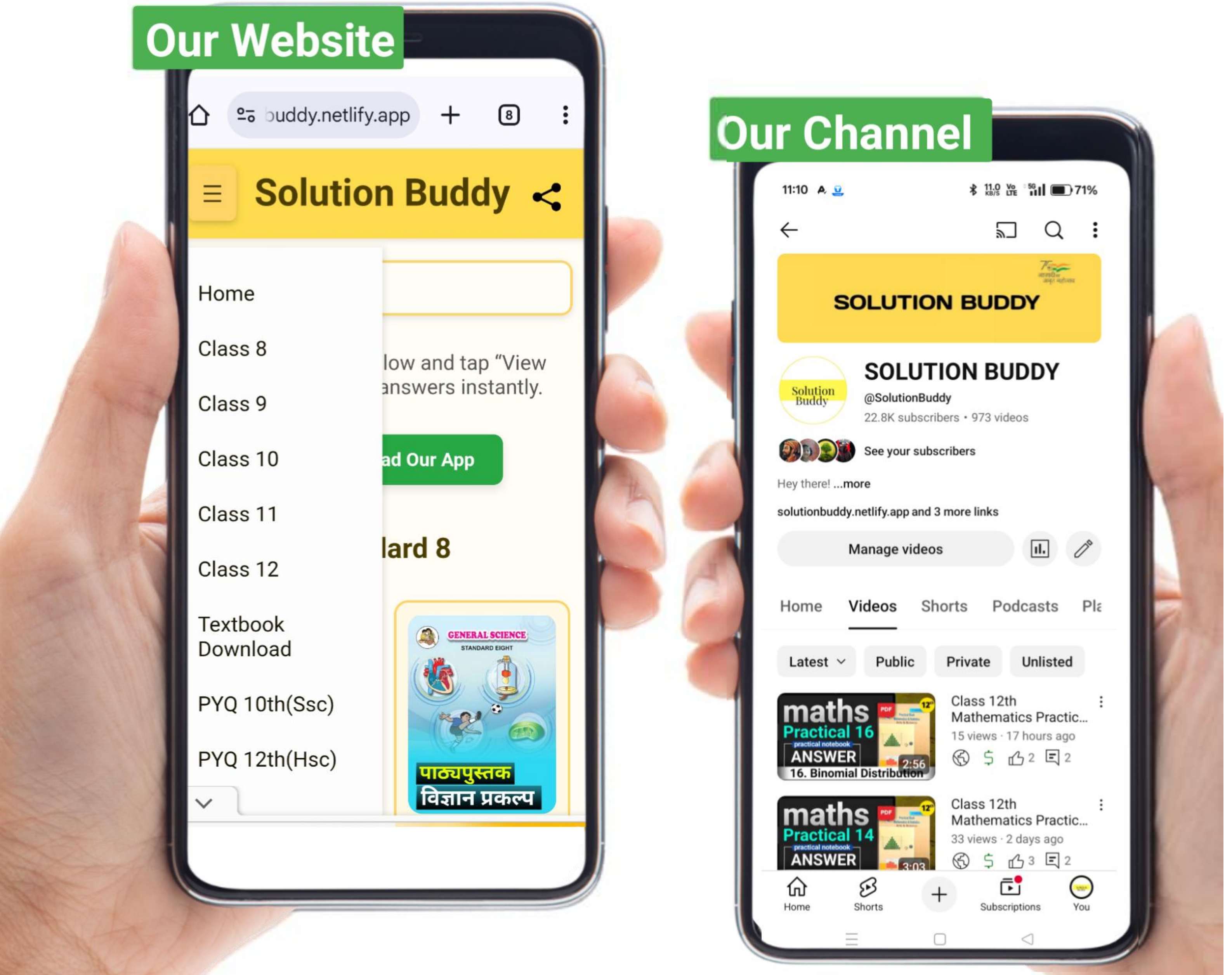

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