5 Probability

Let's study.

- Probability : Introduction
- Sample space and event
- Random experiment and its outcome
- Probability of an event

Let's discuss.

Teacher : Friends, this box contains folded chits. The number of chits is exactly the same as the number of students in our class. Each student should pick one chit. Names of different plants are written on the chits. No two chits bear the same name of the plant. Let us see who gets the chit having the name 'Basil'. Make a line in the order of your roll numbers. No one will unfold the chit until the last student takes his chit.

Aruna : Sir, I am the first one in a line, but I do not want to pick a chit first, as the possibility of getting 'basil' chit from all the chits is very low.

Zarina : Sir, I am the last student in the row, I do not want to pick the chit at last as the chit containing the name 'basil' will most likely be picked up by some one else before my turn.

The first and the last student feel that for each of them, the possibility of getting the chit having the name 'basil' is very low. The above conversation indicates the thinking of less or more possibility.

We use the following words to express the possibility in our daily conversation.

- Probable
- may be
- impossible
- sure
- nearly
- 50 – 50

Read the following statements regarding predictions (possibilities for the future).

- Most probably the rain will start from today.
- The inflation is likely to rise.
- It is impossible to defeat Indian cricket team in the next match.
- I will surely get first class.
- There is no possibility of Polio infection if a child is given the polio vaccine in time.

The adjoining picture shows a ‘toss’ before a cricket match.

What are the possibilities ?

☐ or ☐

So here there are ☐ possibilities.

Activity 1 : Let each student in the class toss a coin once. What will you get?

(Teacher writes the observations on the board in a table.)

Possibilities	(H)	(T)
Number of students	. . .	. . .

Activity 2 : Ask each student to toss the same coin twice. What are the possibilities?

Possibilities	H H	HT	TH	TT
Number of students				

Activity 3 : Now throw a die, once. What are the different possibilities of getting dots on the upper face ?

Each of these is a possible result of throwing a die.

Let’s learn.

Random Experiment

The experiment in which all possible results are known in advance but none of them can be predicted with certainty and there is equal possibility for each result is known as a ‘Random experiment’.

For example, Tossing a coin, throwing a die, picking a card from a set of cards bearing numbers from 1 to 50, picking a card from a pack of well shuffled playing cards, etc.

Outcome

Result of a random experiment is known as an 'Outcome'.

Ex. (1) In a random experiment of tossing a coin - there are only two outcomes.

Head (H) or Tail (T)

(2) In a random experiment of throwing a die, there are 6 outcomes, according to the number of dots on the six faces of the die.

1 or 2 or 3 or 4 or 5 or 6.

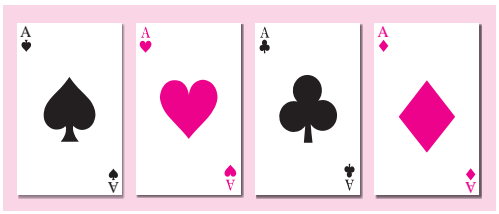
(3) In a random experiment of picking a card bearing numbers from 1 to 50, there are 50 outcomes.

(4) A card is drawn randomly from a pack of well shuffled playing cards.

There are 52 cards in a pack as shown below.

In a pack of playing cards there are 4 sets, namely heart, diamond, club and spade. In each set there are 13 cards as King, Queen, Jack, 10, 9, 8, 7, 6, 5, 4, 3, 2 and Ace.

King, Queen and Jack are known as face cards. In each pack of cards there are 4 cards of king, 4 cards of Queen and 4 cards of Jack. Thus total face cards are 12.



Ace of spade | Ace of heart | Ace of club | Ace of diamond

Equally Likely Outcomes

If a die is thrown, any of the numbers from 1, 2, 3, 4, 5, 6 may appear on the upper face. It means that each number is equally likely to occur. However, if a die is so formed that a particular face come up most often, then that die is biased. In this case the outcomes are not likely to occur equally.

Here, we assume that objects used for random experiments are fair or unbiased.

A given number of outcomes are said to be equally likely if none of them occurs

in preferance to others. For example if a coin is tossed, possibilities of getting head or tail are equal. A die, having numbers from 1 to 6 on its different faces, is thrown. Check the possibility of getting one of the numbers. Here all the outcomes are eqully likely.

Practice Set 5.1

1. How many possibilities are there in each of the following?

(1) Vanita knows the following sites in Maharashtra. She is planning to visit one of them in her summer vacation.

Ajintha, Mahabaleshwar, Lonar Sarovar, Tadoba wild life sanctuary, Amboli, Raigad, Matheran, Anandavan.

(2) Any day of a week is to be selected randomly.

(3) Select one card from the pack of 52 cards.

(4) One number from 10 to 20 is written on each card. Select one card randomly.

Let's think.

In which of the following experiments possibility of expected outcome is more?

(1) Getting 1 on the upper face when a die is thrown.

(2) Getting head by tossing a coin.

Let's learn.

Sample Space

The set of all possible outcomes of a random experiment is called the sample space. It is denoted by 'S' or 'Ω' (A greek letter 'Omega'). Each element of sample space is called a 'sample point'. The number of elements in the set 'S' is denoted by n(S). If n(S) is finite, then the sample space is said to be a finite sample space.

Following are some examples of finite sample spaces.

S. No.	Random experiment	Sample space	Number of sample points in S
1	One coin is tossed	S = {H, T)	n(S) = 2
2	Two coins are tossed	S = { HH, HT, TH, TT}	n(S) =
3	Three coins are tossed	S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}	n(S) = 8
4	A die is thrown	S = {1, 2, 3, 4, 5, 6}	n(S) =
5	Two dice are thrown	S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}	n(S) = 36
6	A card is drawn from a pack bearing numbers from 1 to 25	S = {1, 2, 3, 4, ..., 25}	n(S) =
7	A card is drawn from a well shuffled pack of 52 playing cards.	Diamond : Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King Spade : Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King Heart : Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King Club : Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King	n(S) = 52

Let's remember!

(i) The sample space for a coin tossed twice is the same as that of two coins tossed simultaneously. The same is true for three coins.

(ii) The sample space for a die rolled twice is the same as two dice rolled simultaneously.

Practice Set 5.2

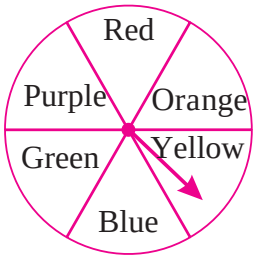
(1) For each of the following experiments write sample space 'S' and number of sample points n(S).

(1) One coin and one die are thrown simultaneously.

(2) Two digit numbers are formed using digits 2, 3 and 5 without repeating a

digits.

2. The arrow is rotated and it stops randomly on the disc. Find out on which colour it may stop.



MARCH - 2019

M	T	W	T	F	S	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

3. In the month of March 2019, find the days on which the date is a multiple of 5. (see the given page of the calender)

4. Form a 'Road safety commitee' of two, from 2 boys (B_1, B_2) and 2 girls(G_1, G_2). Complete the following activity to write the sample space.

(a) Committee of 2 boys = ☐ (b) Committee of 2 girls = ☐

(c) Committee of one boy and one girl = B_1G_1 ☐ ☐ ☐

∴ Sample space = {..., ..., ..., ..., ..., ...}

Let's learn.

Event

The outcomes satisfying particular condition are called favourable outcomes.

A set of favourable outcomes of a given sample space is an '**event**'. Event is a subset of the sample space.

Events are generally denoted by capital letteres A, B, C, D etc. For example, if two coins are tossed and A is the event of getting at least one tail, then the favourable outcomes are as follows.

A = {TT, TH, HT}

The number of elements in the event A is denoted by n(A). Here n(A) = 3.

For more information

Types of event.

(i) Certain event/Sure event
(ii) Impossible event
(iii) Simple/Elementary event
(iv) Complement of an event
(v) Mutually exclusive events
(vi) Exhaustive event

Solved Examples

Ex. (1) Two coins are tossed simultaneously. Write the sample space (S) and number of sample points n(S). Also write the following events in the set form and write the number of sample points in each event.

(i) Condition for event A : to get at least one tail.

(ii) Condition for event B : to get only one head.

(iii) Condition for event C : to get at most one tail.

(iv) Condition for event D : to get no head.

Solution : If two coins are tossed simultaneously,

S = {HH, HT, TH, TT} n(S) = 4

(i) Condition for event A : at least one head.

A = {HH, HT, TH} n(A) = 3

(ii) Condition for event B : only one head.

B = { HT, TH} n(B) = 2

(iii) Condition for event C : at most one tail.

C = {HH, HT, TH} n(C) = 3

(iv) Condition for event D : No head.

D = {TT} n(D) = 1

Ex. (2) A bag contains 50 cards. Each card bears only one number from 1 to 50. One card is drawn at randomfrom the bag. Write the sample space. Also write the events A, B and find the number of sample points in them.

(i) Condition for event A : the number on the card is divisible by 6.

(ii) Condition for event B : the number on the card is a complete square.

Solution : S = {1, 2, 3, . . . 49, 50}, n(S) = 50

(i) Condition for event A : number is divisible by 6.

A = {6, 12, 18, 24, 30, 36, 42, 48} n(A) = 8

(ii) Condition for event B : the number on the card is a complete square.

B = {1, 4, 9, 16, 25, 36, 49} n(B) = 7

Ex. (3) A sanitation committee of 2 members is to be formed from 3 boys and 2 girls. Write sample space 'S' and number of sample points n(S). Also write the following events in set form and number of sample points in the event.

(i) Condition for event A : at least one girl must be a member of the committee.

(ii) Condition for event B : Committee must be of one boy and one girl.

(iii) Condition for event C : Committee must be of boys only.

(iv) Condition for event D : At the most one girl should be a member of the committee.

Solution : Suppose B_1, B_2, B_3 are three boys and G_1, G_2 are two girls

Out of these boys and girls, a sanitation committee of two members is to be formed.

$\therefore S = \{B_1B_2, B_1B_3, B_2B_3, B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2, G_1G_2\}$ $\therefore$ n(S) = 10

(i) Condition for event A is that at least one girl should be in the committee.

$A = \{B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2, G_1G_2\}$ $\therefore$ n(A) = 7

(ii) Condition for event B is that one boy and one girl should be there in the committee.

$B = \{B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2\}$ $\therefore$ n(B) = 6

(iii) Condition for event C is that there should be only boys in the committee.

$C = \{B_1B_2, B_1B_3, B_2B_3\}$ n(C) = 3

(iv) Condition for event D is that there can be at most one girl in the committee.

$D = \{B_1B_2, B_1B_3, B_2B_3, B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2\}$ $\therefore$ n(D) = 9

Ex. (4) Two dice are rolled, write the sample space 'S' and number of sample points n(S). Also write events and number of sample points in the event according to the given condition.

(i) Sum of the digits on upper face is a prime number.

(ii) Sum of the digits on the upper face is multiple of 5.

(iii) Sum of the digits on the upper face is 25.

(iv) Digit on the upper face of the first die is less than the digit on the second die.

Solution : Sample space,

S = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6),
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6),
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6),
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)} n(S) = 36

(i) Event E : The sum of the digits on the upper face is a prime number.

E = {(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)} ∴ n(E) = 15

(ii) Event F : The sum of the digits on the upper face is a multiple of 5.

F = { (1, 4), (2, 3), (3, 2), (4, 1), (4, 6), (5, 5), (6, 4) } ∴ n(F) = 7

(iii) Event G : The sum of the digits on the upper face is 25.

G = { } = ϕ ∴ n (G) = 0

(iv) Event H : The number on upper face of first die is less than the digit on second die.

H = {(1, 2) (1, 3) (1, 4) (1, 5) (1, 6) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 4) (3, 5) (3, 6) (4, 5) (4, 6) (5, 6)} ∴ n(H) = 15

Practice Set 5.3

1. Write sample space 'S' and number of sample point n(S) for each of the following experiments. Also write events A, B, C in the set form and write n(A), n(B), n(C).

(1) One die is rolled,

Event A : Even number on the upper face.

Event B : Odd number on the upper face.

Event C : Prime number on the upper face.

(2) Two dice are rolled simultaneously,

Event A : The sum of the digits on upper faces is a multiple of 6.

Event B : The sum of the digits on the upper faces is minimum 10.

Event C : The same digit on both the upper faces.

(3) Three coins are tossed simultaneously.

Condition for event A : To get at least two heads.

Condition for event B : To get no head.

Condition for event C : To get head on the second coin.

(4) Two digit numbers are formed using digits 0, 1, 2, 3, 4, 5 without repetition of the digits.

Condition for event A : The number formed is even

Condition for event B : The number formed is divisible by 3.

Condition for event C : The number formed is greater than 50.

(5) From three men and two women, environment committee of two persons is to be formed.

Condition for event A : There must be at least one woman member.

Condition for event B : One man, one woman committee to be formed.

Condition for event C : There should not be a woman member.

(6) One coin and one die are thrown simultaneously.

Condition for event A : To get head and an odd number.

Condition for event B : To get a head or tail and an even number.

Condition for event C : Number on the upper face is greater than 7 and tail on the coin.

Let's learn.

Probability of an event

Let us think of a simple experiment. A bag contains 4 balls of the same size. Three of them are white and the fourth is black. You are supposed to pick one ball at random without seeing it. Then obviously, possibility of getting a white ball is more.

In Mathematical language, when possibility of an expected event is expressed in number, it is called 'Probability' . It is expressed as a fraction or percentage using the following formula.

For a random experiment, if sample space is 'S'and 'A' is an expected event then probability of 'A' is P(A). It is given by following formula.

$$P(A) = \frac{\text{Number of sample points in event A}}{\text{Number of sample points in sample spaces}} = \frac{n(A)}{n(S)}$$

In the above experiment, getting a white ball is event A. As there are three white balls n(A) = 3, As the number of balls is 4, n(S) = 4

$\therefore$ probability of getting a white ball is, $P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$.

Similarly, if getting black ball is event B, then n(B) = 1 $\therefore$ $P(B) = \frac{n(B)}{n(S)} = \frac{1}{4}$.

Solved Examples

Ex. (1) Find the probability of the following, when one coin is tossed.

(i) getting head (ii) getting tail

Solution : Let 'S' be the sample space.

S = {H, T} n(S) = 2

(i) Let event A be getting head

A = {H} $\therefore$ n(A) = 1

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{2}$

(ii) Let event B be getting tail

B = {T} $\therefore$ n(B) = 1

$P(B) = \frac{n(B)}{n(S)} = \frac{1}{2}$

Ex. (2) If one die is rolled then find the probability of each of the following events.

(i) Number on the upper face is prime

(ii) Number on the upper face is even.

Solution : 'S' is the sample space.

S = {1, 2, 3, 4, 5, 6} $\therefore$ n(S) = 6

(i) Event A : Prime number on the upper face.

A = {2, 3, 5} $\therefore$ n(A) = 3

$P(A) = \frac{n(A)}{n(S)}$

$\therefore P(A) = \frac{3}{6} = \frac{1}{2}$

(ii) Event B : Even number on the upper face.

$B = \{2, 4, 6\}$ $\quad \therefore n(B) = 3$

$$P(B) = \frac{n(B)}{n(S)}$$

$$\therefore P(B) = \frac{3}{6} = \frac{1}{2}$$

Ex. (3) A card is drawn from a well shuffled pack of 52 playing cards. Find the probability of each event. The card drawn is (i) a red card (ii) a face card

Solution : 'S' is the sample space. $\quad \therefore n(S) = 52$

Event A : Card drawn is a red card.

Total red cards = 13 hearts + 13 diamonds = 26

$\therefore n(A) = 26$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

Event B : Card drawn is a face card.

Total face cards = 12 $\quad \therefore n(B) = 12$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{12}{52} = \frac{3}{13}$$

Ex. (4) A box contains 5 strawberry chocolates, 6 coffee chocolates and 2 peppermint chocolates. Find the probability of each of the following events, if one of the chocolates is picked from the box at random. (i) it is a coffee chocolate.
(ii) it is a peppermint chocolate.

Solution : Sample space is 'S' and $n(S) = 5 + 6 + 2 = 13$

Event A : it is a coffee chocolate

$\therefore \quad n(A) = 6$

$$\therefore \quad P(A) = \frac{n(A)}{n(S)} = \frac{6}{13}$$

Event B : it is a peppermint chocolate

$\therefore \quad n(B) = 2$

$$\therefore \quad P(B) = \frac{n(B)}{n(S)} = \frac{2}{13}$$

Let's remember!

- The Probability is expressed as a fraction or a percentage.
- The probability of any event is from 0 to 1 or 0% to 100%.
  If E is any event, $0 \leq P(E) \leq 1$ or $0\ \% \leq P(E) \leq 100\ \%$.
  e.g. probability $\frac{1}{4}$ is written as 25 %.
- This lesson began with a discussion of 40 chits with names of plants and each of 40 students picking a chit. Only one chit had the name Basil on it. The probability of any student getting the chit of Basil is $\frac{1}{40}$. For a student standing first or last in the row, or anywhere in between, the probability is the same.

Practice Set 5.4

1. If two coins are tossed, find the probability of the following events.
   (1) Getting at least one head. (2) Getting no head.
2. If two dice are rolled simultaneously, find the probability of the following events.
   (1) The sum of the digits on the upper faces is at least 10.
   (2) The sum of the digits on the upper faces is 33.
   (3) The digit on the first die is greater than the digit on second die.
3. There are 15 tickets in a box, each bearing one of the numbers from 1 to 15. One ticket is drawn at random from the box. Find the probability of event that the ticket drawn -
   (1) shows an even number. (2) shows a number which is a multiple of 5.
4. A two digit number is formed with digits 2, 3, 5, 7, 9 without repetition. What is the probability that the number formed is
   (1) an odd number ? (2) a multiple of 5 ?
5. A card is drawn at random from a pack of well shuffled 52 playing cards. Find the probability that the card drawn is -
   (1) an ace. (2) a spade.

Problem Set 5

1. Choose the correct alternative answer for each of the following questions.

   (1) Which number cannot represent a probability ?

   (A) $\frac{2}{3}$ (B) 1.5 (C) 15 % D) 0.7

   (2) A die is rolled. What is the probability that the number appearing on upper face is less than 3 ?

   (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ D) 0

   (3) What is the probability of the event that a number chosen from 1 to 100 is a prime number ?

   (A) $\frac{1}{5}$ (B) $\frac{6}{25}$ (C) $\frac{1}{4}$ D) $\frac{13}{50}$

   (4) There are 40 cards in a bag. Each bears a number from 1 to 40. One card is drawn at random. What is the probability that the card bears a number which is a multiple of 5 ?

   (A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) $\frac{4}{5}$ D) $\frac{1}{3}$

   (5) If n(A) = 2, P(A) = $\frac{1}{5}$, then n(S) = ?

   (A) 10 (B) $\frac{5}{2}$ (C) $\frac{2}{5}$ D) $\frac{1}{3}$

2. Basketball players John, Vasim, Akash were practising the ball drop in the basket. The probabilities of success for John, Vasim and Akash are $\frac{4}{5}$, 0.83 and 58% respectively. Who had the greatest probability of success ?

3. In a hockey team there are 6 defenders , 4 offenders and 1 goalee. Out of these, one player is to be selected randomly as a captain. Find the probability of the selection that -

   (1) The goalee will be selected. (2) A defender will be selected.

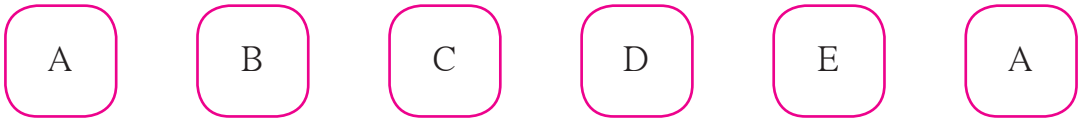
4. Joseph kept 26 cards in a cap, bearing one English alphabet on each card. One card is drawn at random. What is the probability that the card drawn is a vowel card ?

5. A balloon vendor has 2 red, 3 blue and 4 green balloons. He wants to choose one of them at random to give it to Pranali. What is the probability of the event that Pranali gets,

   (1) a red balloon (2) a blue balloon (3) a green balloon.

6. A box contains 5 red, 8 blue and 3 green pens. Rutuja wants to pick a pen at random. What is the probability that the pen is blue?

7. Six faces of a die are as shown below.



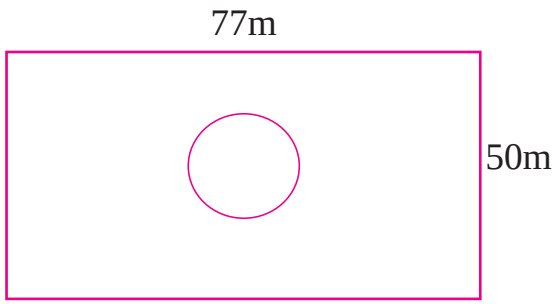
If the die is rolled once, find the probability of -

(1) 'A' appears on upper face.

(2) 'D' appears on upper face.

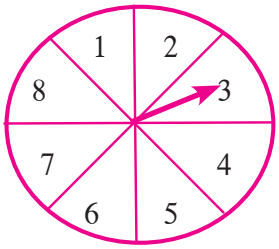
8. A box contains 30 tickets, bearing only one number from 1 to 30 on each. If one ticket is drawn at random, find the probability of an event that the ticket drawn bears (1) an odd number (2) a complete square number.

9. Length and breadth of a rectangular garden are 77 m and 50 m. There is a circular lake in the garden having diameter 14 m. Due to wind, a towel from a terrace on a nearby building fell into the garden. Then find the probability of the event that it fell in the lake.



10. In a game of chance, a spinning arrow comes to rest at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8.

All these are equally likely outcomes. Find the probability that it will rest at



(1) 8.

(2) an odd number.

(3) a number greater than 2.

(4) a number less than 9.

11. There are six cards in a box, each bearing a number from 0 to 5. Find the probability of each of the following events, that a card drawn shows,

(1) a natural number.

(2) a number less than 1.

(3) a whole number.

(4) a number is greater than 5.

12. A bag contains 3 red, 3 white and 3 green balls. One ball is taken out of the bag at random. What is the probability that the ball drawn is -

(1) red. (2) not red (3) either red or white.

13. Each card bears one letter from the word 'mathematics' The cards are placed on a table upside down. Find the probability that a card drawn bears the letter 'm'.

14. Out of 200 students from a school, 135 like Kabbaddi and the remaining students do not like the game. If one student is selected at random from all the students, find the probability that the student selected dosen't like Kabbaddi.

15.★ A two digit number is to be formed from the digits 0, 1, 2, 3, 4. Repetition of the digits is allowed. Find the probability that the number so formed is a -

(1) prime number (2) multiple of 4

(3) multiple of 11.

16.★ The faces of a die bear numbers 0, 1, 2, 3, 4, 5. If the die is rolled twice, then find the probability that the product of digits on the upper face is zero.

17. Do the following activity -

Activity I : Total number of students in your class, n(S) = ☐

Number of students from your class, wearing spectacles, n(A) = ☐

Probability of a randomly selected student wearing spectacles, P(A) = ☐

Probability of a randomly selected student not wearing spectacles, P(B) = ☐

Activity II : Decide the sample space yourself and fill in the following boxes.

Sample space

S = { }

n(S) = ☐

The condition for event A is 'getting an even number'.

A = { }

n(A) = ☐

$\therefore$ P(A) = $\frac{\square}{\square}$ = ☐

❐❐❐