11. Maxima and Minima

A. Activities

Carry out the following activities.

1) Find the maximum value of the products of square of first number and the other number if sum of two number is 6 by completing the following activity.

The two numbers be x and y

$\therefore x + y = \boxed{6}$, $\therefore y = \boxed{6} - x$ from the given condition: the product of square of first number and the other number $= x^2y = x^2(\boxed{6} - x)$

Let $f(x) = x^2(\boxed{6} - x) = \boxed{6}x^2 - x^3$

Now $f'(x) = \frac{d}{dx}(\boxed{6}x^2 - x^3) = \boxed{12}x - 3x^2$

And $f''(x) = \frac{d}{dx}(\boxed{12}x - 3x^2) = \boxed{12} - 6x$

$f'(x) = 0$ gives $\boxed{12}x - 3x^2 = 0$

$\therefore 3x(\boxed{4} - x) = 0$

$\therefore x = 0$ or $x = \boxed{4}$

a) $f''(0) = \boxed{12} - 6(0)\boxed{>}0$

$\therefore$ f has $\boxed{\text{minimum}}$ value of $x = 0$

b) Now $f''(4) = 12 - 6\boxed{4} = \boxed{-24} < 0$

f has a maximum value at $x = 4$

When $x = 4$, $y = \boxed{2}$

$\therefore$ maxima value $= x^2y = (4)^2\boxed{2} = \boxed{32}$

2) A manufacturer can sell x items at a price of ₹ (280 - x) each. The cost of producing x items is $(x^2 + 40x + 35)$. Find the number of items to be sold so that the manufacturer can make maximum profit by complating the following activity.

Let the number of items sold be x

The profit $= p(x) =$ s.p. - c.p.

$(280-x)x$

$= \boxed{} - (x^2 + 40x + 35)$

$= \boxed{280}x - \boxed{x^2} - x^2 - 40 - 35$

$p(x) = 240x - \boxed{2}x^2 - 35$

$\therefore p'(x) = 240(1) - \boxed{2}\ 2x - \boxed{0}$

$p'(x) = 0$ gives $x = \boxed{60}$

Now $p''(x) = 0 - \boxed{4} \times 1 = \boxed{-4} < 0$

p(x) is maximum for $x = 60$

Hence, the number of items sold for maximum profit are $\boxed{60}$

3) If Mr. Pritesh orders x chairs at the price $p = 2x^2 - 12x - 192$ per chair. Show that the cost of the deal is minimum using following activity.

Let Mr. Pritesh order x chairs. Then the total price of x chairs

$= p.x = (2x^2 - 12x - 192)\,x$

Let $f(x) = \boxed{2x^3 - 12x^2 - 192x}$

$f'(x) = \boxed{6}\,x^2 - 24x - 192$

Nos $f''(x) = \boxed{12}\,x - 24$

$f'(x) = 0$ gives $x^2 - \boxed{4}\,x - 32 = 0$

$\therefore (x - \boxed{8})(x + \boxed{4}) = 0$

$\therefore x = \boxed{8}$ as $x = -4$

As $f''(8) = \boxed{72} > 0$

$\therefore$ f is minimum for $x = 8$

$\therefore$ Mr. Pritesh should order 8 chairs for minimum cost of the deal.

4) Find the minimum value of $4x^2 + \frac{1}{x}$ using following activity

$f(x) = 4x^2 + \frac{1}{x}$

$f'(x) = \boxed{8}\,x - \frac{1}{\boxed{x^2}}$, $f''(x) = \boxed{8} + \frac{\boxed{2}}{x^3}$

Now $f'(x) = 0$ gives $8x - \frac{1}{x^2} = 0$

$\therefore 8x^3 - \boxed{1} = 0$

$\therefore x^3 = \frac{1}{\boxed{8}}$, $x = \frac{1}{\boxed{2}}$

When $x = \frac{1}{2}$, $f''(x) = 8 + \frac{2}{\boxed{(1/2)}^3} = \boxed{24} > 0$

$\therefore$ f has $\boxed{\text{minimum}}$ at $x = \frac{1}{2}$

The minimum value of f at $x = \frac{1}{2}$

is $f(\frac{1}{2}) = [4(\frac{1}{2})^2 + \frac{1}{(1/2)}]$

$= \boxed{4} \times \frac{1}{4} + \boxed{2}$

$= \boxed{3}$

B. Solve the Following

Q.1. Find the maximum & minimum value of
$f(x) = x^3 - 3x^2 - 24x + 5$.

Solⁿ:- Given- $f(x) = x^3 - 3x^2 - 24x + 5$

$\therefore \quad f'(x) = 3x^2 - 6x - 24$

& $\quad f''(x) = 6x - 6$

For maxima & minima

$f'(x) = 0$

$\therefore \quad 3x^2 - 6x - 24 = 0$

$3(x^2 - 2x - 8) = 0$

$x^2 - 2x - 8 = 0$

$x^2 - 4x + 2x - 8 = 0$

$x(x-4) + 2(x-4) = 0$

$(x-4)(x+2) = 0$

$\therefore \quad x - 4 = 0$ & $x + 2 = 0$

$x = 4$ & $x = -2$

i) For $x = 4$, $f''(x) = 24 - 6 = 18 > 0$

$\therefore$ $f(x)$ is minimum at $x = 4$

So, minimum value $= f(4)$

$= (4)^3 - 3(4)^2 - 24(4) + 5$

$= 64 - 48 - 96 + 5$

$= -75$

2) For $x = -2$, $f''(x) = -12 - 6 = -18 < 0$

$\therefore$ $f(x)$ is maximum at $x = -2$

So, maximum value $= f(-2) = (-2)^3 - 3(-2)^2 - 24(-2) + 5$

$= -8 - 12 + 48 + 5 = \underline{33}$

Ans: $f(x)$ has max. value 33 at $x = -2$ & min. value -75 at $x = 4$

Q.2. A wire of length 120cm is bent in the form of a rectangle. Find its dimensions if the area of the rectangle is maximum.

Let x & y be the length & breath respectively of rectangle.

$\therefore \quad 2(x+y) = 120 \Rightarrow x + y = 60$

$\Rightarrow y = 60 - x \quad$ —(1)

Now, Area of rectangle $= x \cdot y$

$= x(60 - x)$

From (1)

$\therefore f(x) = 60x - x^2$

$f'(x) = 60 - 2x$ & $f''(x) = -2$

Now, for extrem value.

$f'(x) = 0$

$60 - 2x = 0 \Rightarrow 2x = 60 \Rightarrow x = 30$

Now, $\left[f''(x)\right]_{at\ x=30} = -2 < 0$

$\therefore$ Area of rectangle is maximum when $x = 30$ and $y = 30$

Hence the dimensions of rectangle are 30cm × 30cm

Q.3. A rectangle sheet of paper has its area 24 sq. Meters. The margin at the top & the bottom are 75cm each & the sides 50cm each. What are the dimensions of the paper if the area of the printed space is maximum?

Let x be the length & y be the breadth of rectangle

& Area of rectangle = 24

$\therefore x \cdot y = 24 \Rightarrow y = \frac{24}{x}$

Area of Printed space $= (x - 1.5)(y - 1)$

$= xy - x - 1.5y + 1.5$

$= x\left(\frac{24}{x}\right) - x - 1.5\left(\frac{24}{x}\right) + 1.5$

$= 24 - x - \frac{36}{x} + 1.5$

$= -x - \frac{36}{x} + 25.5$

$\therefore f'(x) = -1 + \frac{36}{x^2}$ & $f''(x) = -\frac{72}{x^3}$

For maximum $f'(x) = 0$

$x^2 = 36 \Rightarrow \boxed{x = 6m}$

& $y = \frac{24}{6} = 4m$

$\boxed{y = 4m}$

Q.4. An open box is to be cut out of piece of square card board of side 18 cm by cutting off equal squares from the corners and turning up the sides. Find the maximum volume of the box.

Solⁿ: Volume of box $= (18-2x)(18-2x)\cdot x$

$f(x) = (324 - 72x + 4x^2)x$

$f(x) = 324x - 72x^2 + 4x^3$

$\therefore f'(x) = 324 - 144x + 12x^2$

& $f''(x) = -144 + 24x$

Now for max. volume

$f'(x) = 0 \Rightarrow 324 - 144x + 12x^2 = 0$

$\Rightarrow x^2 - 12x + 27 = 0$

$\Rightarrow (x-9)(x-3) = 0$

$x = 9$ & 3

Now, $[f''(x)]_{x=9} = -144 + 24\times 9 = 72 > 0$

& $[f''(x)]_{x=3} = -144 + 24\times 3 = -72 < 0$

$\therefore f(x)$ is max. at $x = 3$

Hence, max. volume $= (18-2\times3)(18-2\times3)\times3$

$= (18-6)(18-6)\times 3$

$= 12\times12\times3$

$= 432\ cm^3$

Ans.

Sign of Teacher :