A. Activities

1. Carry out the following activity.

p	q	r	$q \to r$	$r \to p$	$(q \to r) \vee (r \to p)$
T	T	T	T	T	T
T	T	F	F	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	F	T
F	T	F	F	T	T
F	F	T	T	F	T
F	F	F	T	T	T

The given statement pattern is ..tautology...

2. Using algebra of statements, complete the following activity to show

$[p \vee (\sim p \vee \sim q)] \wedge [p \vee (q \wedge r)] \equiv p \vee (q \wedge r)$

LHS $\equiv [p \vee (\sim p \vee \sim q)] \wedge [p \vee (q \wedge r)]$

$\equiv [(p \vee \sim p) \vee \sim q] \wedge [p \vee (q \wedge r)]$ (Associative law)

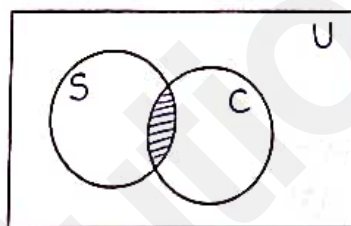$\equiv [(t \vee \sim q) \wedge [p \vee (q \wedge r)]$ (Complement law)

$\equiv t \wedge [p \vee q \wedge r]$ (Identity law)

$\equiv p \vee [q \wedge r]$ (Identity law)

$\equiv RHS$

3. Complete the following venn diagrams.

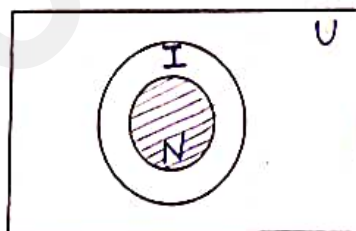i) Some share brokers are chartered accountants.

Where U : Set of all human beings.
S : Set of all share brokers.
C : Set of all chartered accountants

(denote the corresponding sets and represent region)

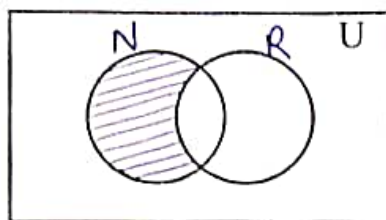ii) 'Some integers are natural numbers'

Where U : Set of all real numbers.
I : Set of all integers.
N : Set of all natural numbers

(Show the sets and shade the region at the appropriate place)

iii) Some non residant Indians are not rich

Where U : Set of all Indians.

N : Set of all non resident Indians.

R : Set of all rich Indians.

(Show/represent the appropriate region)

4. Determine the truth value of the following statement by completing the activity. Satara is in Maharashtra or $3 \times 7 = 21$

Statements	Truth Values
p : Satara is in Maharashtra	T
q : $3 \times 7 = 21$	T

The symbolic form of given statement is $p \vee q$. Its truth value is

$\boxed{T}$

5. a) Find the negation of the following by completing the activity.

$\sim [(p \leftrightarrow q) \vee (\sim q \to \sim r)]$

$\equiv \sim [p \leftrightarrow q] \wedge \sim [\sim q \to \sim r]$ (Negation of disjunction)

$\equiv [(p \wedge \boxed{\sim q}) \vee (q \wedge \boxed{\sim p})] \wedge [\sim q \wedge \sim (\boxed{\sim r})]$

(Negation of implication and Negation of biconditional)

$\equiv [(p \wedge \sim q) \vee (\boxed{q} \wedge \sim p)] \wedge [\sim q \wedge \boxed{r}]$

(Negation of negation)

b) Write the dual of the following by completing the following activity.

i) Dual of $(\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$ is $(\sim p \boxed{\vee} q) \boxed{\wedge}$

$(p \vee \sim q) \boxed{\wedge} (\sim p \vee \boxed{\sim q})$

ii) Dual of $(p \to q) \vee (q \to p)$ is dual of $(\sim p \vee q) \boxed{\wedge} (\sim q \boxed{\vee} p)$

B. Solve the Following

Q.1. Using truth table, examine whether $(p \to q) \leftrightarrow (\sim p \vee q)$ is a tautology, contradiction or neither.

p	q	p→q	~p	~p∨q	(p→q)↔(~p∨q)
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

∴ From the given table, all the entries in the last column are True (T's)

∴ The given statement pattern is a tautology.

Q.2. Prove using truth table that : $(p \leftrightarrow q) \equiv [(\sim p \vee q) \wedge (\sim q \vee p)]$

1	2	3	4	5	6	7	8
p	q	$p \leftrightarrow q$	$\sim p$	$\sim p \vee q$	$\sim q$	$\sim q \vee p$	$[(\sim p \vee q) \wedge (\sim q \vee p)]$
T	T	T	F	T	F	T	T
T	F	F	F	F	T	T	F
F	T	F	T	T	F	F	F
F	F	T	T	T	T	T	T

From the above truth table, all entries in 3rd and 8th column are identical.

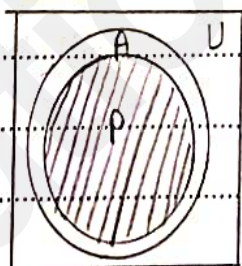∴ $(p \leftrightarrow q) \equiv [(\sim p \vee q) \wedge (\sim q \vee p)]$

Q.3. Represent the following statements by Venn diagrams.

(a) All poets are great artists.

Let U : The set of all human beings.

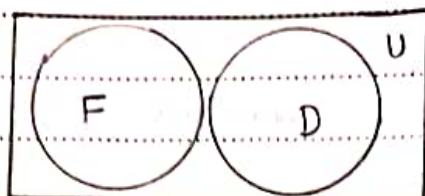A : The set of all great artists.

P : The set of all poets.

P ⊂ A

(b) No Filmstar is a director.

Let U : The set of all human beings.

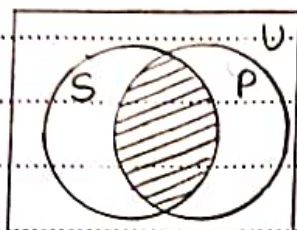F : The set of all filmstars.

D : The set of all directors.

$$F \cap D = \emptyset$$

(c) Some policemen are scholars.

Let U: The set of all human beings.

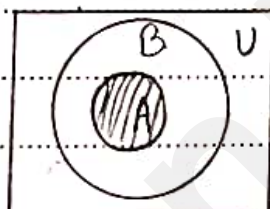S: Thet set of all scholars.,

P: The set of all policemen.

$$S \cap P \neq \emptyset$$

(d) If the person is a teacher then the person is sincere.

Let U: The set of all human beings.

A: The set of all teachers.

B: The set of all sincere persons..
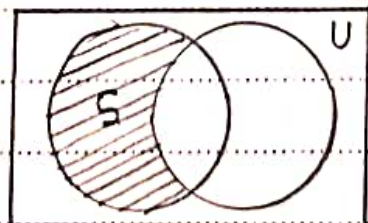

$$A \subset B$$

(e) Every sincere person may not be a doctor.

Let U: The set of all human beings

S: The set of all sincere persons

D: The set of all doctors

$$S - D \neq \emptyset$$

Q.4: Consider the following statements.

(a) If a man is rich then he buys a car.

(b) If a man is not rich then he does not buy a car.

(c) If a man buys a car then he is rich.

(d) If a man does not buy a car, then he is not rich.

Identify the statements having the same meaning.

Let p: A man is rich ; q: A man buys a car.

∴ Symbolic forms of the above statements are

(a) $p \rightarrow q$ (b) $\sim p \rightarrow \sim q$ (c) $q \rightarrow p$ (d) $\sim q \rightarrow \sim p$

∵ Every conditional statement is logically equivalent to its contrapositive , ∴ $p \rightarrow q \equiv \sim q \rightarrow \sim p$

         & $q \rightarrow p \equiv \sim p \rightarrow \sim q$

∴ Statement ⓐ and ⓓ are same and ⓑ and ⓒ are same.

Q.5. State the dual of each of the following statements by applying the principle of duality.

(a) If Shantanu passes in Accountancy, then Pratik passes in Mathematics.

(b) $p \vee (q \vee r) \equiv [(p \wedge q) \vee (r \vee s)]$

(c) $(p \rightarrow q) \vee (q \rightarrow p)$

(d) If Darshana drives the car, then Payal will walk.

(a) We know that, $p \rightarrow q \equiv \sim p \vee q$

∴ We can write given statement as : Shantanu does not pass in accountancy or Pratik passes in Mathematics.

∴ Dual is : Shantanu does not pass in Accountancy and Pratik passes in Mathematics.

(b) $p \wedge (q \wedge r) \equiv [(p \vee q) \wedge (r \wedge s)]$

(c) $(p \rightarrow q) \vee (q \rightarrow p) \equiv (\sim p \vee q) \vee (\sim q \vee p)$

∴ Dual is : $(\sim p \wedge q) \wedge (\sim q \wedge p)$

(d) Darshana does not drive the car and Payal will walk.

Sign of Teacher :