Line and Plane

EXERCISE 6.1 [PAGES 200 - 201]

Exercise 6.1 | Q 1 | Page 200

Find the vector equation of the line passing through the point having position vector $-2\hat{i} + \hat{j} + \hat{k}$ and parallel to vector $4\hat{i} - \hat{j} + 2\hat{k}$.

Solution:

The vector equation of the line passing through $A(\bar{a})$ and parallel to the vector $\bar{b}$ is $\bar{r} = \bar{a} + \lambda\bar{b}$, where λ is a scalar.

$\therefore$ the vector equation of the line passing through the point having position vector $-2\hat{i} + \hat{j} + \hat{k}$ and parallel to the vector $4\hat{i} - \hat{j} + 2\hat{k}$ is

$$\bar{r} = \left(-2\hat{i} + \hat{j} + \hat{k}\right) + \lambda\left(4\hat{i} - \hat{j} + 2\hat{k}\right).$$

Exercise 6.1 | Q 2 | Page 200

Find the vector equation of the line passing through points having position vector $3\hat{i} + 4\hat{j} - 7\hat{k}$ and $6\hat{i} - \hat{j} + \hat{k}$.

Solution:

The vector equation of the line passing through the $A(\bar{a})$ and $B(\bar{b})$ is $\bar{r} = \bar{a} + \lambda(\bar{b} - \bar{a})$, λ is a scalar.

$\therefore$ the vector equation of the line passing through the points having position vector $3\hat{i} + 4\hat{j} - 7\hat{k}$ and $6\hat{i} - \hat{j} + \hat{k}$ is

$$\bar{r} = \left(3\hat{i} + 4\hat{j} - 7\hat{k}\right) + \lambda\left[\left(6\hat{i} - \hat{j} + \hat{k}\right) - \left(3\hat{i} + 4\hat{j} - 7\hat{k}\right)\right]$$

i.e. $\bar{r} = \left(3\hat{i} + 4\hat{j} - 7\hat{k}\right) + \lambda\left(3\hat{i} - 5\hat{j} + 8\hat{k}\right)$.

Exercise 6.1 | Q 3 | Page 200

Find the vector equation of line passing through the point having position vector $5\hat{i} + 4\hat{j} + 3\hat{k}$ and having direction ratios $-3, 4, 2$.

Solution:

Let A be the point whose position vector is a = $5\hat{i} + 4\hat{j} + 3\hat{k}$.

Let $\bar{b}$ be the vector parallel to the line having direction ratio = –3, 4, 2

Then, $\bar{b} = -3\hat{i} + 4\hat{j} + 2\hat{k}$

The vector equation of the line passing through $A(\bar{a})$ and parallel to $\bar{b}$ is $\bar{r} = \bar{a} + \lambda\bar{b}$, where λ is a scalar.

∴ the required vector equation of the line is

$\bar{r} = 5\hat{i} + 4\hat{j} + 3\hat{k} + \lambda\left(-3\hat{i} + 4\hat{j} + 2\hat{k}\right)$.

Exercise 6.1 | Q 4 | Page 200

Find the vector equation of the line passing through the point having position vector $\hat{i} + 2\hat{j} + 3\hat{k}$ and perpendicular to vectors $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - \hat{j} + \hat{k}$.

Solution:

Let $\bar{b} = \hat{i} + \hat{j} + \hat{k}$ and $\bar{c} = 2\hat{i} - \hat{j} + \hat{k}$

The vector perpendicular to the vectors $\bar{b}$ and $\bar{c}$ is given by

$$\bar{b} \times \bar{c} = \begin{vmatrix} \hat{i} \\ 1\ 1\ \ 1 \\ 2\ \ -1\ \ 1 \end{vmatrix}$$

= $\hat{i}(1+1) - \hat{j}(1-2) + \hat{k}(-1-2)$

= $2\hat{i} + \hat{j} - 3\hat{k}$

Since the line is perpendicular to the vector $\bar{b}$ and $\bar{c}$, it is parallel to $\bar{b} \times \bar{c}$.

The vector equation of the line passing through $A(\bar{a})$ and parallel to $\bar{b} \times \bar{c}$ is $\bar{r} = \bar{a} + \lambda(\bar{b} \times \bar{c})$, where λ is a scalar.

Here, $\bar{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

Hence, the vector equation of the required line is $\bar{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - 3\hat{k}\right)$.

Exercise 6.1 | Q 5 | Page 200

Find the vector equation of the line passing through the point having position vector $-\hat{i}-\hat{j}+2\hat{k}$ and parallel to the line $\bar{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(3\hat{i}+2\hat{j}+\hat{k}\right)$.

Solution:

Let A be point having position vector $\bar{a}=-\hat{i}-\hat{j}+2\hat{k}$

The required line is parallel to the line

$$\bar{r}=\left(\hat{i}+2\hat{j}+3\hat{k}+\lambda\left(3\hat{i}+2\hat{j}+\hat{k}\right)\right.$$

$\therefore$ it is parallel to the vector

$$\bar{b}=3\hat{i}+2\hat{j}+\hat{k}$$

The vector equation of the line passing through $A(\bar{a})$ and parallel to $\bar{b}$ is $\bar{r}=\bar{a}+\lambda\bar{b}$ where λ is a scalar.

$\therefore$ the required vector equation of the line is

$$\bar{r}=\left(-\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(3\hat{i}+2\hat{j}+\hat{k}\right).$$

Exercise 6.1 | Q 6 | Page 200

Find the Cartesian equations of the line passing through A(– 1, 2, 1) and having direction ratios 2, 3, 1.

Solution: The cartesian equations of the line passing through $(x_1, y_1, z_1)$and having direction ratiosa,b,c are and having direction ratios a,b,c are

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

$\therefore$ the cartesian equations of the line passing through the point (–1, 2, 1) and having direction ratios 2, 3, 1 are

$$\frac{x-(-1)}{2}=\frac{y-2}{3}=\frac{z-1}{1}$$

i.e. $\frac{x+1}{2}-\frac{y-2}{3}=\frac{z-1}{1}$.

Exercise 6.1 | Q 7 | Page 200

Find the Cartesian equations of the line passing through A(2, 2,1) and B(1, 3, 0).

Solution: The cartesian equations of the line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) are

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

Here, (x_1, y_1, z_1) ≡ (2, 2, 1) and (x_2, y_2, z_2) ≡ (1, 3, 0)

∴ the required cartesian equations are

$$\frac{x-2}{1-2}=\frac{y-2}{3-2}=\frac{z-1}{0-1}$$

i.e. $\frac{x-2}{-1}=\frac{y-2}{1}=\frac{z-1}{-1}$.

Exercise 6.1 | Q 8 | Page 200

A(– 2, 3, 4), B(1, 1, 2) and C(4, –1, 0) are three points. Find the Cartesian equations of the line AB and show that points A, B, C are collinear.

Solution: We find the cartesian equations of the line AB. The cartesian equations of the line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) are

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

Here, (x_1, y_1, z_1) ≡ (-2, 3, 4) and (x_2, y_2, z_2) ≡ (4, -1, 0)

∴ the required cartesian equations of the line AB are

$$\frac{x-(-2)}{4-(-2)}=\frac{y-3}{-1-3}=\frac{z-4}{0-4}$$

$$\therefore \frac{x+2}{6}=\frac{y-3}{-4}=\frac{z-4}{-4}$$

$$\therefore \frac{x+2}{3}=\frac{y-3}{-2}=\frac{z-4}{-2}$$

C = (4, -1, 0)

For x = 4, $\frac{x+2}{3}=\frac{4+2}{3}$ = 2

For y = −1, $\dfrac{y-3}{-2} = \dfrac{-1-3}{-2}$ = 2

For z = 0, $\dfrac{z-4}{-2} = \dfrac{0-4}{-2}$ = 2

∴ coordinates of C satisfy the equations of the line AB.

∴ C lies on the line passing through A and B.

Hence, A, B, C are collinear.

Exercise 6.1 | Q 9 | Page 200

Show that the lines given by

$$\frac{x+1}{-10} = \frac{y+3}{-1} = \frac{z-4}{1} \text{ and } \frac{x+10}{-1} = \frac{y+1}{-3} = \frac{z-1}{4}$$

intersect. Also, find the coordinates of their point of intersection.

Solution: The equations of the lines are

$$\frac{x+1}{-10} = \frac{y+3}{-1} = \frac{z-4}{1} = \lambda \qquad \text{...(say)...(1)}$$

and $\dfrac{x+10}{-1} = \dfrac{y+1}{-3} = \dfrac{z-1}{4} = \mu$ ...(say)...(2)

From (1), x = – 1 – 10λ, y = – 3 – λ , z = 4 + λ

∴ the coordinates of any point on the line (1) are (–1 – 10λ, – 3 – λ, 4 + λ)

From (2), x = – 10 – μ, y = – 1 – 3μ, z = 1 + 4μ

∴ the coordinates of any point on the line (2) are (– 10 – μ, – 1 – 3μ, 1 + 4μ)

Lines (1) and (2) intersect, if (– 1 – 10λ, – 3 – λ, 4 + λ) = (– 10 – μ, – 1 – 3μ, 1 + 4μ)

∴ the equation – 1 – 10λ = – 10 – μ, – 3 – λ = –1 – 3μ and 4 + λ = 1 + 4μ are simultaneously true.

Solving the first two equations, we get, λ = 1, and μ = 1.

These values of λ and μ satisfy the third equation also.

∴ the lines intersect.

Putting λ = 1 in (– 1 – 10λ, – 3 – λ, 4 + λ) or μ = 1 in (– 10 – μ, –1 – 3μ, 1 + 4μ), we get the point of intersection (–11, – 4, 5).

Exercise 6.1 | Q 10 | Page 200

A line passes through (3, –1, 2) and is perpendicular to lines $\bar{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$ and $\bar{r} = \left(2\hat{i} + \hat{j} - 3\hat{k}\right) + \mu\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$. Find its equation.

Solution:

The line $\bar{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$ is parallel to the vector $\bar{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ and the line $\bar{r} = \left(2\hat{i} + \hat{j} - 3\hat{k}\right) + \mu\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$ is parallel to the vector. $\bar{c} = \hat{i} - 2\hat{j} + 2\hat{k}$.

The vector perpendicular to the vectors $\bar{b}$ and $\bar{c}$ is given by

The vector perpendicular to the vectors $\bar{b}$ and $\bar{c}$ is given by

$$\bar{b} \times \bar{c} = \begin{vmatrix} \hat{i} & & \\ 2 & -2 & 1 \\ 1 & -2 & 2 \end{vmatrix}$$

$= \hat{i}(-4+2) - \hat{j}(4-1) + \hat{k}(-4+2)$
$= -2\hat{i} - 3\hat{j} - 2\hat{k}$

Since the required line is perpendicular to the given lines,
it is perpendicular to both $\bar{b}$ and $\bar{c}$.
$\therefore$ It is parallel to $\bar{b} \times \bar{c}$
The equation of the line passing through $A(\bar{a})$ and parallel to $\bar{b}$ and $\bar{c}$ is
$\bar{r} = \bar{a} + \lambda\left(\bar{b} \times \bar{c}\right)$, where λ is a scalar.
Here, $\bar{a} = 3\hat{i} - \hat{j} + 2\hat{k}$
$\therefore$ the equation of the required line is
$\bar{r} = \left(3\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(-2\hat{i} - 3\hat{j} - 2\hat{k}\right)$
or
$\bar{r} = \left(3\hat{i} - \hat{j} + 2\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 2\hat{k}\right)$, where μ = −λ.

Exercise 6.1 | Q 11 | Page 201

Show that the line $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+4}{-2}$ passes through the origin.

Solution:

The equation of the line is

$$\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+4}{-2}$$

The coordinates of the origin O are (0, 0, 0)

For x = 0, $\frac{x-2}{1} = \frac{0-2}{1} = -2$

For y = 0, $\frac{y-4}{2} = \frac{0-4}{2} = -2$

For z = 0, $\frac{z+4}{-2} = \frac{0+4}{-2} = -2$

∴ coordinates of the origin O satisfy the equation ofthe line.

Hence, the line passes through the origin.

EXERCISE 6.2 [PAGE 207]

Exercise 6.2 | Q 1 | Page 207

Find the length of the perpendicular (2, –3, 1) to the line

$$\frac{x+1}{2} = \frac{y-3}{3} = \frac{z+1}{-1}.$$

Solution1:

Let PM be the perpendicular drawn from the point P(2, –3, 1) to the line

$$\frac{x+1}{2} = \frac{y-3}{3} = \frac{z+1}{-1} = \lambda \quad \text{...(Say)}$$

The coordinates of any point on the line are given by

$x = -1 + 2\lambda$, $y = 3 + 3\lambda$, $z = -1 - \lambda$

Let the coordinates of M be

$(-1 + 2\lambda, 3 + 3\lambda, -1 - \lambda)$ ...(1)

The direction ratios of PM are

$-1 + 2\lambda - 2, 3 + 3\lambda + 3, -1 - \lambda - 1$

i.e. $2\lambda - 3, 3\lambda + 6, -\lambda - 2$

The direction ratios of the given line are 2, 3, –1.
Since PM is perpendicular to the given line, we get
$2(2\lambda - 3) + 3(3\lambda + 6) - 1(-\lambda - 2) = 0$
$\therefore 4\lambda - 6 + 9\lambda + 18 + \lambda + 2 = 0$
$\therefore 14\lambda + 14 = 0$
$\therefore \lambda = -1.$
Put $\lambda = -1$ in (1), the coordinates of M are
$(-1 - 2, 3 - 3, -1 + 1)$ i.e. $(-3, 0, 0)$.
∴ length of perpendicular from P to the given line
= PM

$$= \sqrt{(-3-2)^2 + (0+3)^2 + (0-1)^2}$$

$$= \sqrt{(25+9+1)}$$

$$= \sqrt{35}\text{units.}$$

Solution2:

We know that the perpendicular distance from the point

$\mathrm{P}\left|\bar{\alpha}\right|$ to the line $\bar{\mathrm{r}} = \bar{\mathrm{a}} + \lambda\bar{\mathrm{b}}$ is given by

$$\sqrt{\left|\bar{\alpha} - \bar{\mathrm{a}}\right|^2 - \left[\frac{\left(\bar{\alpha} - \bar{a}\right).\bar{b}}{\left|\bar{\mathrm{b}}\right|}\right]^2} \quad \ldots(1)$$

Here, $\bar{\alpha} = 2\hat{\mathrm{i}} - 3\hat{\mathrm{j}} + \hat{\mathrm{k}}, \bar{\mathrm{a}} = -\hat{\mathrm{i}} + 3\hat{\mathrm{j}} - \hat{\mathrm{k}}, \bar{\mathrm{b}} = 2\hat{\mathrm{i}} + 3\hat{\mathrm{j}} - \hat{\mathrm{k}}$

$$\therefore \bar{\alpha} - \bar{\mathrm{a}} = \left(2\hat{\mathrm{i}} - 3\hat{\mathrm{j}} + \hat{\mathrm{k}}\right) - \left(-\hat{\mathrm{i}} + 3\hat{\mathrm{j}} - \hat{\mathrm{k}}\right)$$

$$= 3\hat{\mathrm{i}} - 6\hat{\mathrm{j}} + 2\hat{\mathrm{k}}$$

$\therefore \left|\bar{\alpha} - \bar{a}\right|^2 = 3^2 + (-6)^2 + 2^2 = 9 + 36 + 4 = 49$

Also, $\left(\bar{\alpha} - \bar{a}\right).\bar{b} = \left(3\hat{i} - 6\hat{j} + 2\hat{k}\right).\left(2\hat{i} + 3\hat{j} - \hat{k}\right)$

= (3)(2) + (– 6)(3) + (2)(– 1)

= 6 – 18 – 2

= – 14

$\left|\bar{b}\right| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14}$

Substitutng these values in (1), we get

length of perpendicular from P to given line

= PM

$= \sqrt{49 - \left(\frac{-14}{\sqrt{14}}\right)^2}$

$= \sqrt{49 - 14}$

$= \sqrt{35}\text{ units.}$

Exercise 6.2 | Q 2 | Page 207

Find the co-ordinates of the foot of the perpendicular drawn from the point $2\hat{i} - \hat{j} + 5\hat{k}$ to the line $\bar{r} = \left(11\hat{i} - 2\hat{j} - 8\hat{k}\right) + \lambda\left(10\hat{i} - 4\hat{j} - 11\hat{k}\right)$. Also find the length of the perpendicular.

Solution:

Let M be the foot of perpendicular drawn from the point $P\left(2\hat{i} - \hat{j} + 5\hat{k}\right)$ on the line

$\bar{r} = \left(11\hat{i} - 2\hat{j} - 8\hat{k}\right) + \lambda\left(10\hat{i} - 4\hat{j} - 11\hat{k}\right).$

Let the position vector of the point M be

$\left(11\hat{i} - 2\hat{j} - 8\hat{k}\right) + \lambda\left(10\hat{i} - 4\hat{j} - 11\hat{k}\right)$

$= (11 + 10\lambda)\hat{i} + (-2 - 4\lambda)\hat{j} + (-8 - 11\lambda)\hat{k}$.

Then $\overline{PM}$ = Position vector of M – Position vector of P

$$= \left[(11 + 10\lambda)\hat{i} + (-2 - 4\lambda)\hat{j} + (-8 - 11\lambda)\hat{k}\right] - \left(2\hat{i} - \hat{j} + 5\hat{k}\right)$$

$$= (9 + 10\lambda)\hat{i} + (-1 - 4\lambda)\hat{j} + (-13 - 11\lambda)\hat{k}$$

Since PM is perpendicular to the given line which is parallel to

$\bar{b} = 10\hat{i} - 4\hat{j} - 11\hat{k}$,

$\overline{PM} \perp^r \bar{b}$

$\therefore$ $\overline{PM}.\bar{b}$ = 0

$$\therefore \left[(9 + 10\lambda)\hat{i} + (-1 - 4\lambda) - 11(-13 - 11\lambda)\hat{k}\right].\left(10\hat{i} - 4\hat{j} - 11\hat{k}\right) = 0$$

$\therefore$ 10(9 + 10λ) – 4(–1 – 4λ) – 11(13 – 11λ) = 0

$\therefore$ 90 + 100λ + 4 + 16λ + 143 + 121λ = 0

$\therefore$ 237λ + 237 = 0

$\therefore$ λ = – 1

Putting this value of λ, we get the position vector of M as $\hat{i} + 2\hat{j} + 3\hat{k}$.

$\therefore$ coordinates of the foot of perpendicular M are (1, 2, 3).

Now, $\overline{PM} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - \left(2\hat{i} - \hat{j} + 5\hat{k}\right)$

= $-\hat{i} + 3\hat{j} - 2\hat{k}$

$\therefore$ $\left|\overline{PM}\right| = \sqrt{(-1)^2 + (3)^2 + (-2)^2}$

= $\sqrt{1 + 9 + 4}$

= $\sqrt{14}$

Hence, the coordinates of the foot of perpendicular are (1, 2, 3) and length of perpendicular = $\sqrt{14}\text{units}$.

Exercise 6.2 | Q 3 | Page 207

Find the shortest distance between the lines
$\bar{r} = \left(4\hat{i} - \hat{j}\right) + \lambda\left(\hat{i} + 2\hat{j} - 3\hat{k}\right)$ and $\bar{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) + \mu\left(\hat{i} + 4\hat{j} - 5\hat{k}\right)$

Solution:

We know that the shortest distance between the skew lines

$\bar{r} = \bar{a}_1 + \lambda\bar{b}$ and $\bar{r} = \bar{a}_2 + \mu\bar{b}_2$ is given by d = $\dfrac{\left|(\bar{a}_2 - \bar{a}_1).\left(\bar{b}_1 \times \bar{b}_2\right)\right|}{\left|\bar{b}_1 \times \bar{b}_2\right|}$.

Here , $\bar{a}_1 = 4\hat{i} - \hat{j}, \bar{a}_2 = \hat{i} - \hat{j} + 2\hat{k},$

$\bar{b}_1 = \hat{i} + 2\hat{j} - 3\hat{k}, \bar{b}_2 = \hat{i} + 4\hat{j} - 5\hat{k}$

$$\therefore \bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix}$$

= $(-10 + 12)\hat{i} - (-5 + 3)\hat{j} + (4 - 2)\hat{k}$

= $2\hat{i} + 2\hat{j} + 2\hat{k}$

and

$\bar{a}_2 - \bar{a}_1 = \left(\hat{i} - \hat{j} + 2\hat{k}\right) - \left(4\hat{i} - \hat{j}\right)$

= $-3\hat{i} + 2\hat{k}$

$\therefore (\bar{a}_2 - \bar{a}_2).\left(\bar{b}_1 \times \bar{b}_2\right) = \left(-3\hat{i} + 2\hat{k}\right).\left(2\hat{i} + 2\hat{j} + 2\hat{k}\right)$

= −3(2) + 0(2) + 2(2)

= − 6 + 0 + 4

= −2

and

$\left|\bar{b}_1 \times \bar{b}_2\right| = \sqrt{2^2 + 2^2 + 2^2}$

= $\sqrt{4 + 4 + 4}$

= $2\sqrt{3}$

∴ required shortest distance between the given lines

$= \left|\frac{-2}{2\sqrt{3}}\right|$

$= \frac{1}{\sqrt{3}}\text{units.}$

Exercise 6.2 | Q 4 | Page 207

Find the shortest distance between the lines

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

Solution:

The shortest distance between the lines

$\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1} \text{ and } \frac{x-x_1}{l_2} = \frac{y-y_2}{m_{12}} = \frac{z-z_2}{n_2}$ is give n by

$$d = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1n_2-m_2n_1)^2+(l_2n_1-1_1n_2)^2+(l_1m_2-l_2m_1)^2}}$$

The equation of the given lines are

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

∴ $x_1 = -1$, $y_1 = -1$, $z_1 = -1$, $x_2 = 3$, $y_2 = 5$, $z_2 = 7$,

$l_1 = 7$, $m_1 = -6$, $n_1 = 1$, $l_2 = 1$, $m_2 = -2$, $n_2 = 1$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = \begin{vmatrix} 4 & 6 & 8 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$= 4(-6+2) - 6(7-1) + 8(-14+6)$

$= -16 - 36 - 64$

$= -116$

and

$(m_1n_2 – m_2n_1)^2 + (l_2n_1 – l_1n_2)^2 + (l_1m_2 - l_2m_1)^2$

= (–6 + 2)2 + (1 – 7)2 + (1 – 7)2 + (–14 + 6)

= 16 + 36 + 64

= 116

Hence, the required shortest distance between the given lines

$= \left|\frac{-116}{\sqrt{116}}\right|$

$= \sqrt{116}$

$= 2\sqrt{29}$units.

Exercise 6.2 | Q 5 | Page 207

Find the perpendicular distance of the point (1, 0, 0) from the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ Also find the co-ordinates of the foot of the perpendicular.

Solution:

Let PM be the perpendicular drawn from the point (1, 0, 0) to the line

$\frac{x+1}{2} = \frac{y-3}{-3} = \frac{z+10}{8} = \lambda$ ...(Say)

The coordinates of any point on the line are given by x = –1 + 2λ, y = 3 + 3λ, z = 8 – λ

Let the coordinates of M be

(–1 + 2λ, 3 + 3λ, –1 – λ) ...(1)

The direction ratios of PM are

–1 + 2λ –2, 3 + 3λ + 3, –1 – λ –1

i.e. 2λ –3, 3λ + 6, – λ –2

The direction ratios of the given line are 2, 3, 8.

Since PM is perpendicular to the given line, we get

2(2λ – 3) + 3(3λ + 6) – 1(– λ – 2) = 0

∴ 4λ – 6 + 9λ + 18 + λ + 2 = 0

∴ 14λ + 14 = 0

$\therefore \lambda = -1$.

Put $\lambda = -1$ in (1), the coordinates of M are

$(-1-2, 3-3, -1+1)$ i.e. $(-3, 0, 0)$.

$\therefore$ length of perpendicular from P to the given line

= PM

$$= \sqrt{(-3-2)^2 + (0+3)^2 + (0-1)^2}$$

= sqrt((25 + 9 + 1)

$$= \sqrt{35}\text{units.}$$

Alternative Method :

We know that the perpendicular distance from the point

$P|\overline{\infty}|$ to the lin $\bar{r} = \bar{a} + \lambda\bar{b}$ is given by

$$\sqrt{|\overline{\infty} - \bar{a}|^2 - \left[\frac{(\overline{oo} - \bar{a}).\bar{b}}{|\bar{b}|}\right]^2} \quad ...(1)$$

Here, $\overline{\infty} = 2\hat{i} - 3\hat{j} + \hat{k}, \bar{a} = -\hat{i} + 3\hat{j} - \hat{k}, \bar{b} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\therefore \overline{\infty} - \bar{a} = \left(2\hat{i} - 3\hat{j} + \hat{k}\right) - \left(-\hat{i} + 3\hat{j} - \hat{k}\right)$

$= 3\hat{i} - 6\hat{j} + 2\hat{k}$

$\therefore |\overline{\infty} - \bar{a}|^2 = 3^2 + (-6)2 + 2^2 = 9 + 36 + 4 = 49$

Also, $(\overline{\infty} - \bar{a}).\bar{b} = \left(3\hat{i} - 6\hat{j} + 2\hat{k}\right).\left(2\hat{i} + 3\hat{j} - \hat{k}\right)$

= (3)(2) + (– 6)(3) + (2)(– 1)

= 6 – 18 – 2

= −14

$$|\bar{b}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14}$$

Substitutng tese values in (1), w get

length of perpendicular from P to given line

= PM

$$= \sqrt{49 - \left(-\frac{14}{\sqrt{14}}\right)^2}$$
$$= \sqrt{49 - 14}$$
$$= \sqrt{35}\text{units}$$
or
$$2\sqrt{6}\text{units}, (3, -4, 2).$$

Exercise 6.2 | Q 6 | Page 207

A(1, 0, 4), B(0, -11, 13), C(2, -3, 1) are three points and D is the foot of the perpendicular from A to BC. Find the co-ordinates of D.

Solution: Equation of the line passing through the points ($x_{,1}$, y_1, z_1) and (x_2, y_2, z_2) is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

∴ the equation of the line BC passing through the points B (0, –11, 13) and C)2, –3,1) is

$$\frac{x - 0}{2 - 0} = \frac{y + 11}{-3 + 11} = \frac{z - 13}{1 - 13}$$

i.e. $x(2) = \frac{y + 11}{8} = \frac{z - 13}{-12} = \lambda$ ..(Say)

AD is the perpendicular from the point A(1, 0, 4) to the line BC.

The coordinates of any point on the line BC are given by

x = 2λ, y = –11 + 8λ, z = 13 – 12λ

Let the coordinates of D be (2λ, – 11 + 8λ, 13 – 12λ) ...(1)

∴ the direcion ratio of AD are

2λ – 1, λ 11 + 8λ – 0, 13 – 12λ – 4

i.e. 2λ – 1, – 11 + 8λ, 9 – 12λ

The direction ratios of the line BC are 2, 8, – 12.

Since AD is perpendicular to BC, we get

2(2λ – 1) + 8(– 11 + 8λ) – 12(9 – 12λ) = 0

∴ 4λ – 2 – 88 + 64λ – 108 + 144λ = 0

∴ 212λ – 198 = 0

$\therefore \lambda = \dfrac{198}{212} = \dfrac{99}{106}$

Putting $\lambda = \dfrac{99}{106}$ in (1), the coordinates of D are

$\left(\dfrac{198}{106}, -11 + \dfrac{792}{106}, 13 - \dfrac{1188}{106}\right)$

i.e. $\left(\dfrac{198}{106}, \dfrac{-374}{106}, \dfrac{190}{106}\right)$,

i.e. $\left(\dfrac{99}{53}, \dfrac{-187}{53}, \dfrac{95}{53}\right)$.

Exercise 6.2 | Q 7.1 | Page 207

By computing the shortest distance, determine whether following lines intersect each other.

$\bar{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} + \hat{k}\right)$ and $\bar{r} = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} + \hat{j} - \hat{k}\right)$

Solution: The shortest distance between the lines

$\bar{r} = \bar{a}_1 + \lambda\bar{b}_1$ and $\bar{r} = \bar{a}_2 + \mu\bar{b}_2$ is given by

d = $\left|\dfrac{(\bar{a}_2 - \bar{a}_1).\left(\bar{b}_1 \times \bar{b}_2\right)}{\left|\bar{b}_1 \times \bar{b}_2\right|}\right|$.

Here, $\bar{a}_1 = \hat{i} - \hat{j}, \bar{a}_2 = 2\hat{i} - \hat{j}, \bar{b}_1 = 2\hat{i} + \hat{k}, \bar{b}_2 = \hat{i} + \hat{j} - \hat{k}$.

$\therefore \bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$

= $(0 - 1)\hat{i} - (-2 - 1)\hat{j} + (2 - 0)\hat{k}$

= $-\hat{i} + 3\hat{j} + 2\hat{k}$

And

$$\bar{a}_2 - \bar{a}_1 = \left(2\hat{i} - \hat{j}\right) - \left(\hat{i} - \hat{j} - \hat{i}\right)$$

$$\therefore (\bar{a}_2 - \bar{a}_1).(\bar{b}_1 \times \bar{b}_2) = \hat{i}.\left(-\hat{i} + 3\hat{j} + 2\hat{k}\right)$$

= 1(– 1) + 0(3) + 0(2)

= – 1

and

$$\left|\bar{b}_1 \times \bar{b}_2\right| = \sqrt{(-1)^2 + 3^2 + 2^2}$$

= $\sqrt{1 + 9 + 4}$

= $\sqrt{14}$

∴ the shortest distance between the given lines

= $\left|\frac{-1}{\sqrt{14}}\right|$

= $\frac{1}{\sqrt{14}}$ unit

Hence, the given line do not intersect.

Exercise 6.2 | Q 7.2 | Page 207

By computing the shortest distance, determine whether following lines intersect each other.

$$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5} \text{ and } \frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3}$$

Solution:

The shortest distance between the lines

$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3}$ is give n by

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1n_2 - m_2n_1)^2 + (l_2n_1 - 1_1n_2)^2 + (l_1m_2 - l_2m_1)^2}}$$

The equation of the given lines are

$$\frac{x-5}{4}=\frac{y-7}{-5}=\frac{z+3}{-5} \text{ and } \frac{x-8}{7}=\frac{y-7}{1}=\frac{z-5}{3}$$

$\therefore x_1 = -1, y_1 = -1, z_1 = -1, x_2 = 3, y_2 = 5, z_2 = 7,$

$l_1 = 7, m_1 = -6, n_1 = 1, l_2 = 1, m_2 = -2, n_2 = 1$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = \begin{vmatrix} 4 & 6 & 8 \\ 4 & -5 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

= 4(– 6 + 2) –6(7 –1) + 8(–14 + 6)

= – 16 – 36 –64

= – 116

and

$(m_1n_2 - m_2n_1)^2 + (l_2n_1 - l_1n_2)^2 + (l_1m_2 - l_2m_1)^2$

= (–6 + 2)2 + (1 – 7)2 + (1 – 7)2 + (–14 + 6)

= 16 + 36 + 64

= 116

Hence, the required shortest distance between the given lines

$= \left|\frac{-116}{\sqrt{116}}\right|$

$= \sqrt{116}$

$= 2\sqrt{29}\text{units}$

or

The shortest distance between the lines

$= \frac{282}{\sqrt{3830}}\text{units}$

Hence, the given lines do not intersect.

Exercise 6.2 | Q 8 | Page 207

If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect each other, then find k.

Solution:

The lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$

intersect, if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$

The equations of the given lines are

$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$

∴ $x_1 = 1$, $y_1 = -1$, $z_1 = 1$, $x_2 = 3$, $y_2 = k$, $z_2 = 0$,

$l_1 = 2$, $m_1 = 3$, $n_1 = 4$, $l_2 = 1$, $m_2 = 2$, $n_2 = 1$.

Since these lines intersect, we get

$$\begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

∴ 2(3 – 8) –(k + 1)(2 – 4)– 1(4 –3) = 0
∴ –10 + 2(k + 1) –1 = 0
∴ 2(k + 1) =11

∴ k + 1 1/12

∴ k = 9/2.

EXERCISE 6.3 [PAGE 216]

Exercise 6.3 | Q 1 | Page 216

Find the vector equation of a plane which is at 42 unit distance from the origin and which is normal to the vector $2\hat{i} + \hat{j} - 2\hat{k}$

Solution:

If $\hat{n}$ is a unit vector along the normal and p i the length of the perpendicular from origin to the plane, then the vector equation of the plane $\bar{r}.\hat{n} = p$

Here, $\bar{n} = 2\hat{i} + \hat{j} - 2\hat{k}$ and p = 42

$\therefore |\bar{n}| = \sqrt{2^2 + 1^2 + (-2)^2}$

= $\sqrt{9}$

= 3

$\hat{n} = \frac{\bar{n}}{|\bar{n}|}$

= $\frac{1}{3}\left(2\hat{i} + \hat{j} - 2\hat{k}\right)$

$\therefore$ th vector equation of the required plane is

$\bar{r}.\left[\frac{1}{3}\left(2\hat{i} + \hat{j} - 2\hat{k}\right)\right]$ = 42

i.e. $\bar{r}.\left(2\hat{i} + \hat{j} - 2\hat{k}\right)$ = 126.

Exercise 6.3 | Q 2 | Page 216

Find the perpendicular distance of the origin from the plane 6x – 2y + 3z – 7 = 0.

Solution: The equation of the plane is

6x – 2y + 3z – 7 = 0

∴ its vector equation is

$\bar{r}.\left(6\hat{i} - 2\hat{j} + 3\hat{k}\right)$ = 7 ...(1)

where $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore \bar{n} = 6\hat{i} - 2\hat{j} + 3\hat{k}$ is normal to the plane.

$|\bar{n}| = \sqrt{6^2 + (-2)^2 + 3^2}$

= $\sqrt{49}$

= 7

Unit vector along $\bar{\mathrm{n}}$ is

$$\hat{\mathrm{n}} = \frac{\bar{\mathrm{n}}}{|\bar{\mathrm{n}}|} = \frac{6\hat{\mathrm{i}} - 2\hat{\mathrm{j}} + 3\hat{\mathrm{k}}}{7}$$

Dividing bothsides of (1) by 7, we get

$$\bar{\mathrm{r}}.\left(\frac{6\hat{\mathrm{i}} - 2\hat{\mathrm{j}} + 3\hat{\mathrm{k}}}{7}\right) = \frac{7}{7}$$

$\therefore$ $\bar{\mathrm{r}}.\hat{\mathrm{n}}$= 1

Comparing with normal form of equation of the plane $\hat{\mathrm{r}}.\hat{\mathrm{n}} = p$ it follows that length of perpendicular from origin is 1 unit.

Alternative Method :

The equation of the plane is 6x – 2y + 3z – 7 = 0

i.e. $\left(\frac{6}{6^2 + (-2)^2 + 3}\right)x - \left(\frac{2}{\sqrt{6^2} + (-2)^2 + 3^2}\right)y + \left(\frac{3}{\sqrt{6^2 + (-2)^2 + 3^2}}\right)z = \frac{7}{\sqrt{6^2 + (-2)^2 + 3}}$

i.e. $\frac{6}{7}x - \frac{2}{7}y + \frac{3}{7}z = \frac{7}{7}$ = 1

This is the normal form of the equation of plane.

$\therefore$ perpendicular distance of the origin frm the plane is p = 1 unit.

Exercise 6.3 | Q 3 | Page 216

Find the coordinates of the foot of the perpendicular drawn from the origin to the plane 2x + 6y – 3z = 63.

Solution:

The equation of the plane is 2x + 6y - 3z = 63.

Dividing each term by

$\sqrt{2^2 + 6^2 + (-3)^2}$

= $\sqrt{49}$

= 7,

we get

$\frac{2}{7}x + \frac{6}{7}y - \frac{3}{7}z = \frac{63}{7}$ = 9

This is the normal form of the equation of plane.

$\therefore$ the direction cosines of the perpendicular drawn from the origin to the plane are

l = $\frac{2}{7}, m = \frac{6}{7}, n = -\frac{3}{7}$

and length of perpendicular from origin to the plane is p = 9.

$\therefore$ the coordinates of the foot of the perpendicular from the origin to the plane are

(lp, mp, np)i.e. $\left(\frac{18}{7}, \frac{54}{7}, -\frac{27}{7}\right)$

Exercise 6.3 | Q 4 | Page 216

Reduce the equation $\bar{\mathrm{r}}.\left(3\hat{\mathrm{i}} + 4\hat{\mathrm{j}} + 12\hat{\mathrm{k}}\right)$ to normal form and hence find

(i) the length of the perpendicular from the origin to the plane

(ii) direction cosines of the normal.

Solution:

The normal form of equation of a plane is $\bar{\mathrm{r}}.\hat{\mathrm{n}} = p$ where $\hat{\mathrm{n}}$ is unit vector along the normal and p is the length of perpendicular drawn from origin to the plane.

Given pane is $\bar{\mathrm{r}}.\left(3\hat{\mathrm{i}} + 4\hat{\mathrm{j}} + 12\hat{\mathrm{k}}\right)$ = 78 ...(1)

$\bar{\mathrm{n}} = 3\hat{\mathrm{i}} + 4\hat{\mathrm{j}} + 12\hat{\mathrm{k}}$ is normal to the plane

$\therefore |\bar{\mathrm{n}}| = \sqrt{3^2 + 4^2 + 12^2} = \sqrt{169}$ = 13

Dividing both sides of (1) by 13, get

$$\bar{\mathrm{r}}.\left(\frac{3\hat{\mathrm{i}} + 4\hat{\mathrm{j}} + 12\hat{\mathrm{k}}}{13}\right) = \frac{78}{13}$$

i.e. $\bar{\mathrm{r}}.\left(\frac{3}{13}\hat{\mathrm{i}} + \frac{4}{13}\hat{\mathrm{j}} + \frac{12}{13}\hat{\mathrm{k}}\right)$ = 6

This is the normal form of the equation of plane.

Comparing with $\bar{\mathrm{r}}.\hat{\mathrm{n}} = p$,

(i) the length of the perpendicular from the origin to plane is 6.

(ii) direction cosines of the normal are $\frac{3}{13}, \frac{4}{13}, \frac{12}{13}$.

Exercise 6.3 | Q 5 | Page 216

Find the vector equation of the plane passing through the point having position vector $\hat{i} + \hat{j} + \hat{k}$ and perpendicular to the vector $4\hat{i} + 5\hat{j} + 6\hat{k}$.

Solution:

The vector equation of the plane passing through the point $A(\bar{a})$ and perpendicular to the vector $\bar{n}$ is $\bar{r}.\bar{n} = \bar{a}.\bar{n}$

Here,

$\bar{a} = \hat{i} + \hat{j} + \hat{k}$,

$\bar{n} = 4\hat{i} + 5\hat{j} + 6\hat{k}$

$\therefore \bar{a}.\bar{n} = \left(\hat{i} + \hat{j} + \hat{k}\right).\left(4\hat{i} + 5\hat{j} + 6\hat{k}\right)$

= (1)(4) + (1)(5) + (1)(6)

= 4 + 5 + 6

= 15

∴ the vector equation of the required plane is

$\bar{r}.\left(4\hat{i} + 5\hat{j} + 6\hat{k}\right)$ = 15.

Exercise 6.3 | Q 6 | Page 216

Find the Cartesian equation of the plane passing through A(-1, 2, 3), the direction ratios of whose normal are 0, 2, 5.

Solution: The Cartesian equation of the plane passing through (x_1, y_1, z_1), the direction ratios of whose normal are a, b, c, is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

∴ the cartesian equation of the required plane is

0(x + 1) + 2(y – 2) + 5(z – 3) = 0

i.e. 0 + 2y - 4 + 5z – 15 = 0

i.e. 2y + 5z = 19.

Exercise 6.3 | Q 7 | Page 216

Find the Cartesian equation of the plane passing through A(7, 8, 6) and parallel to the XY plane.

Solution: The Cartesian equation of the plane passing through (x_1, y_1, z_1), the direction ratios of whose normal are a, b, c, is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

The required plane is parallel to XY-plane.

∴ it is perpendicular to Z-axis i.e. Z-axis is normal to the plane. Z-axis has direction ratios 0, 0, 1.

The plane passes through (7, 8, 6).

∴ the cartesian equation of the required plane is

$0(x - 7) + 0(y - 8) + 1(z - 6) = 0$

i.e. $z = 6$.

Exercise 6.3 | Q 8 | Page 216

The foot of the perpendicular drawn from the origin to a plane is M(1,0,0). Find the vector equation of the plane.

Solution:

The vector equation of the plane passing through $A(\bar{a})$ and perpendicular to $\bar{n}$ is $\bar{r}.\bar{n} = \bar{a}.\bar{n}$.

M(1,0,0) is the foot of the perpendicular drawn from origin to the plane. Then the plane is passing through M and is perpendicular to OM.

If $\bar{m}$ is the position vector of M, then $\bar{m} = \hat{i}$.

Normal to the plane is

$\bar{n} = \overline{OM} = \hat{i}$

$\bar{m}.\bar{n} = \hat{i}.\hat{i} = 1$

∴ the vector equation of the required plane is

$\bar{r}.\hat{i} = 1$.

Exercise 6.3 | Q 9 | Page 216

Find the vector equation of the plane passing through the point A(− 2, 7, 5) and parallel to vector $4\hat{i} - \hat{j} + 3\hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$.

Solution:

The vector equation of the plane passing through the point $A(\bar{a})$ and parallel to the vectors $\bar{b}$ and $\bar{c}$ is

$\bar{r}.\left(\bar{b} \times \bar{c}\right) = \bar{a}.\left(\bar{b} \times \bar{c}\right)$ ...(1)

Here, $\bar{a} = -2\hat{i} + 7\hat{j} + 5\hat{k}$

$\bar{b} = 4\hat{i} - \hat{j} + 3\hat{k}$,

$\bar{c} = \hat{i} + \hat{j} + \hat{k}$

$$\therefore \bar{b} \times \bar{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ 1 & 1 & 1 \end{vmatrix}$$

= $(-1-3)\hat{i} - (4-3)\hat{j} + (4+1)\hat{k}$

= $-4\hat{i} - \hat{j} + 5\hat{k}$

$\therefore \bar{a}.\left(\bar{b} \times \bar{c}\right) = \left(-2\hat{i} + 7\hat{j} + 5at\mathrm{k}\right).\left(-4\hat{i} - \hat{j} + 5\hat{k}\right)$

= (– 2)(– 4) + (7)(–1) + (5)(5)

= 8 – 7 + 2

= 26

$\therefore$ From (1), the vector equation of the required plane is $\bar{r}.\left(-4\hat{i} - \hat{j} + 5\hat{k}\right)$ = 26.

Exercise 6.3 | Q 10 | Page 216

Find the cartesian equation of the plane

$\bar{r} = \left(5\hat{i} - 2\hat{j} - 3\hat{k}\right) + \lambda\left(\hat{i} + \hat{j} + \hat{k}\right) + \mu\left(\hat{i} - 2\hat{j} + 3\hat{k}\right)$.

Solution:

The equation $\bar{r} = \bar{a} + \lambda\bar{b} + \mu\bar{c}$ represents a plane passing through a point having position vector $\bar{a}$ and parallel to vectors $\bar{b}$ and $\bar{c}$.

Here,

$\bar{a} = 5\hat{i} - 2\hat{j} - 3\hat{k}$,

$\bar{b} = \hat{i} + \hat{j} + \hat{k}$,

$\bar{c} = \hat{i} - 2\hat{j} + 3\hat{k}$

$$\therefore \bar{b} \times \bar{c} = \begin{vmatrix} \hat{i} & & \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix}$$

= $(3 + 2)\hat{i} - (3 - 1)\hat{j} + (-2 - 1)\hat{k}$

= $5\hat{i} - 2\hat{j} - 3\hat{k}$

= $\bar{a}$

Also,

$\bar{a}.\left(\bar{b} \times \bar{c}\right)$

= $\bar{a}.\bar{a} = |\bar{a}|^2$

= $(5)^2 + (-2)^2 + (3)^2$

= 38

The vector equation of the plane passing through A$(\bar{a})$ and parallel to $\bar{b}$ and $\bar{c}$ is

$\bar{r}.\left(\bar{b} \times \bar{c}\right) = \bar{a}.\left(\bar{b} \times \bar{c}\right)$

$\therefore$ the vector equation of the given plane is

$\bar{r}.\left(5\hat{i} - 2\hat{j} - 3\hat{k}\right)$ = 38

If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then this equation becomes

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(5\hat{i} - 2\hat{j} - 3\hat{k}\right)$ = 38

$\therefore$ 5x – 2y – 3z = 38.

This is the cartesian equation of the required plane.

Exercise 6.3 | Q 11 | Page 216

Find the vector equation of the plane which makes intercepts 1, 1, 1 on the co-ordinates axes.

Solution:

The vector equation of the plane passing through $\mathrm{A}(\bar{a}), \mathrm{B}(\bar{b})..\mathrm{C}(\bar{c})$, where A, B, C are non-collinear is $\bar{\mathrm{r}}.\left(\overline{\mathrm{AB}} \times \overline{\mathrm{AC}}\right) = \bar{\mathrm{a}}.\left(\overline{\mathrm{AB}} \times \overline{\mathrm{AC}}\right)$ ...(1)

The required plane makes intercepts 1, 1, 1 on the coordinate axes.

∴ it passes through the three non-collinear points A =(1, 0, 0, B = (0, 1, 0), C = (0, , 1)

$\therefore \bar{\mathrm{a}} = \hat{\mathrm{i}}, \bar{\mathrm{b}} = \hat{\mathrm{j}}, \bar{\mathrm{c}} = \hat{\mathrm{k}}$

$\overline{\mathrm{AB}} = \bar{\mathrm{b}} - \bar{\mathrm{a}} = \hat{\mathrm{j}} - \hat{\mathrm{i}} = -\hat{\mathrm{i}} + \hat{\mathrm{j}}$

$\therefore \overline{\mathrm{AC}} = \bar{\mathrm{c}} - \bar{\mathrm{a}} = \hat{\mathrm{k}} - \hat{\mathrm{i}} = -\hat{\mathrm{i}} + \hat{\mathrm{k}}$

$$\therefore \overline{\mathrm{AB}} \times \overline{\mathrm{AC}} = \begin{vmatrix} \hat{\mathrm{i}} & & \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

= $(1-0)\hat{\mathrm{i}} - (-1-0)\hat{\mathrm{j}} + (0+1)\hat{\mathrm{k}}$

= $\hat{i} + \hat{\mathrm{j}} + \hat{\mathrm{k}}$

Also,

$\bar{\mathrm{a}}.\left(\overline{\mathrm{AB}} \times \overline{\mathrm{AC}}\right)$

= $\hat{\mathrm{i}}.\left(\hat{\mathrm{i}} + \hat{\mathrm{j}} + \hat{\mathrm{k}}\right)$

= 1 x 1 + 0 x 1 + 0 x 1

= 1

∴ from(1)the vector equation of the required plane is $\bar{\mathrm{r}}.\left(\hat{\mathrm{i}} + \hat{\mathrm{j}} + ha\mathrm{k}\right)$ = 1.

EXERCISE 6.4 [PAGE 220]

Exercise 6.4 | Q 1 | Page 220

Find the angle between planes r¯.(i^+j^+2k^)=13andr¯(2i^+j^+k^) = 31.

Solution:

Find the angle between planes $\bar{\mathrm{r}}.\left(\hat{\mathrm{i}}+\hat{\mathrm{j}}+2\hat{\mathrm{k}}\right)=13\ \text{and}\ \bar{\mathrm{r}}\left(2\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}}\right)$ = 31.

The acute angle θ between the planes

$\bar{\mathrm{r}}.\bar{\mathrm{n}}_1=\mathrm{d}_1\ \text{and}\ \bar{\mathrm{r}}.\bar{\mathrm{n}}_2\mathrm{d}_2$ is given by

$\cos\theta=\left|\dfrac{\bar{\mathrm{n}}_1.\bar{\mathrm{n}}_2}{|\bar{\mathrm{n}}_1||\bar{\mathrm{n}}_2|}\right|$ ...(1)

Here,

$\bar{\mathrm{n}}_1=\hat{\mathrm{i}}+\hat{\mathrm{j}}+2\hat{\mathrm{k}}$,

$\bar{\mathrm{n}}_2=2\hat{\mathrm{i}}-\hat{\mathrm{j}}+\hat{\mathrm{k}}$

∴ $\bar{\mathrm{n}}_1.\bar{\mathrm{n}}_2$

= $\left(\hat{\mathrm{i}}+\hat{\mathrm{j}}+2\hat{\mathrm{k}}\right).\left(2\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}}\right)$

= (1)(2) + (1)(– 1) + (2)(1)

=2 – 1 + 2

= 3

Also,

$|\bar{\mathrm{n}}_1|=\sqrt{1^2+1^2+2^2}=\sqrt{6}$

$|\bar{\mathrm{n}}_2|=\sqrt{2^2+(-1)^2+1^2}=\sqrt{6}$

∴ from (1), we have

$\cos\theta=\left|\dfrac{3}{\sqrt{6}\sqrt{6}}\right|$

= $\dfrac{3}{6}$

= $\dfrac{1}{2}\cos 60^\circ$

∴ θ = 60°.

Exercise 6.4 | Q 2 | Page 220

Find the acute angle between the line $\bar{r}.\left(\hat{i}+2\hat{j}+2\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}-6\hat{k}\right)$ and the plane $\bar{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)$ = 0.

Solution:

The acute angle θ between the line $\bar{r}=\bar{a}+\lambda\bar{b}$ and and the plane $\bar{r}.\bar{n}=d$ is given by

$$\sin\theta = \left|\frac{\bar{b}.\bar{n}}{|\bar{b}||\bar{n}|}\right| \quad ...(1)$$

Here, $\bar{b}=2\hat{i}+3\hat{j}-6\hat{k}, \bar{n}=2\hat{i}-\hat{j}+\hat{k}$

$\therefore \bar{b}.\bar{n}=\left(2\hat{i}+3\hat{j}-6\hat{k}\right).\left(2\hat{i}-\hat{j}+\hat{k}\right)$

= (2)(2) + (3)(– 1) + (– 6)(1)

= 4 – 3 – 6

= – 5

Also, $|\bar{b}|=\sqrt{2^2+3^2+(-6)^2}=\sqrt{49}$ = 7

$|\bar{n}|=\sqrt{2^2+(-1)^2+1^2}=\sqrt{6}$

∴ from (1), we have

$$\sin\theta = \left|\frac{-5}{7\sqrt{6}}\right| = \frac{5}{7\sqrt{6}}$$

$$\therefore \theta = \sin^{-1}\left(\frac{5}{7\sqrt{6}}\right).$$

Exercise 6.4 | Q 3 | Page 220

Show that the line $\bar{r}=\left(2\hat{j}-3\hat{k}\right)+\lambda\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and $\bar{r}=\left(2\hat{i}+6\hat{j}+3\hat{k}\right)+\mu\left(2\hat{i}+3\hat{j}+4\hat{k}\right)$ are coplanar. Find the equation of the plane determined by them.

Solution:

The lines $\bar{r} = \bar{a}_1 + \lambda_1\bar{b}_1$ and $\bar{r} = \bar{a}_2 + \lambda_2\bar{b}_2$ are coplanar If $\bar{a}_1.\left(\bar{b}_1 \times \bar{b}_2\right) = \bar{a}_2.\left(\bar{b}_1 \times \bar{b}_2\right)$

Here $\bar{a}_1 = 2\hat{j} - 3\hat{k}, \bar{a}_2 = 2\hat{i} + 6\hat{j} + 3\hat{k}$,

$\bar{b}_1 = \hat{i} + 2\hat{j} + 3\hat{k}, \bar{b}_2 = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\therefore \bar{a}_2 - \bar{a}_1 = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) - \left(2\hat{j} - 3\hat{k}\right)$

= $2\hat{i} + 4\hat{j} + 6\hat{k}$

$$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & & \\ 1 & 2 & 3 \\ 2 & 3 & \end{vmatrix}$$

= $(8-9)\hat{i} - (4-6)\hat{j} + (3-4)\hat{k}$

= $-\hat{i} + 2\hat{j} - \hat{k}$

$\therefore \bar{a}_1.\left(\bar{b}_1 \times \bar{b}_2\right) = \left(2\hat{j} - 3\hat{k}\right).\left(-\hat{i} + 2\hat{j} - \hat{k}\right)$

= 0(– 1) + 2(2) + (– 3)(– 1)

= 0 + 4 + 3

= 7

$\therefore \bar{a}_1.\left(\bar{b}_1 \times \bar{b}_2\right) = \bar{a}_2.\left(\bar{b}_1 \times \bar{b}_2\right)$

Hence, the given lines are coplanar.

The plane determined by these lines is given by

$\therefore \bar{r}.\left(\bar{b}_1 \times \bar{b}_2\right) = \bar{a}_1.\left(\bar{b}_1 \times \bar{b}_2\right)$

i.e. $\bar{r}.\left(-\hat{i} + 2\hat{j} - \hat{k}\right)$ = 7

Hence, the given lines are coplnar and the equation of the plane determined bt these lines is

$\bar{r}.\left(-\hat{i} + 2\hat{j} - \hat{k}\right)$ = 7.

Exercise 6.4 | Q 4 | Page 220

Find the distance of the point $4\hat{i} - 3\hat{j} + \hat{k}$ from the plane $\bar{r}.\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$ = 21.

Solution:

The distance of the point $\mathbf{A}(\bar{a})$ from the plane $\bar{r}.\bar{n} = p$ is given by $d = \dfrac{|\bar{a}.\bar{n} - p|}{|\bar{n}|}$ ...(1)

Here, $\bar{a} = 4\hat{i} - 3\hat{j} + \hat{k}, \bar{n} = 2\hat{i} + 3\hat{j} - 6\hat{k}$, p = 21

$\therefore \bar{a}.\bar{n} = \left(4\hat{i} - \hat{j} + \hat{k}\right).\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$

= (4)(2) + (– 3)(3) + (1)(–6)

= 8 – 9 – 6

= – 7

Also, $|\bar{n}| = \sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{49}$ = 7

∴ from (1), the required distance

= $\dfrac{|-7 - 21|}{7}$

= 4 units.

Exercise 6.4 | Q 5 | Page 220

Find the distance of the point (1, 1 –1) from the plane 3x +4y – 12z + 20 = 0.

Solution:

The distance of the point (x_1, y_1, z_1) from the plane ax + by + cz + d = 0 is

$$\left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

∴ the distance of the point (1, 1, – 1) from the plane 3x + 4y – 12z + 20 = 0 is

$$\left|\frac{3(1) + 4(1 - 12(-1) + 20)}{\sqrt{3^2 + 4^2 + (-12)^2}}\right|$$

= $\left|\dfrac{3 + 4 + 12 + 20}{\sqrt{9 + 16 + 144}}\right|$

= $\dfrac{39}{\sqrt{169}}$

= $\dfrac{39}{13}$

= 3units.

MISCELLANEOUS EXERCISE 6 A [PAGES 207 - 209]

Miscellaneous Exercise 6 A | Q 1 | Page 207

Find the vector equation of the line passing through the point having position vector $3\hat{i} + 4\hat{j} - 7\hat{k}$ and parallel to $6\hat{i} - \hat{j} + \hat{k}$.

Solution:

The vector equation of the line passing through A$(\bar{a})$ and parallel to the vector $\bar{b}$ is $\bar{r} = \bar{a} + \lambda\bar{b}$, where λ is a scalar.

∴ the vector equation of the line passing through the point having position vector

$3\hat{i} + 4\hat{j} - 7\hat{k}$ and parallel to the vector $6\hat{i} - \hat{j} + \hat{k}$ is

$\bar{r} = \left(3\hat{i} + 4\hat{j} - 7\hat{k}\right) + \lambda\left(6\hat{i} - \hat{j} + \hat{k}\right)$.

Miscellaneous Exercise 6 A | Q 2 | Page 207

Find the vector equation of the line which passes through the point (3, 2, 1) and is parallel to the vector $2\hat{i} + 2\hat{j} - 3\hat{k}$.

Solution:

The vector equation of the line passing through A$(\bar{a})$ and parallel to the vector $\bar{b}$ is $\bar{r} = \bar{a} + \lambda\bar{b}$, where λ is a scalar.

∴ the vector equation of the line passing through the point having position vector $3\hat{i} + 2\hat{j} + \hat{k}$ and parallel to the vector $2\hat{i} + 2\hat{j} - 3\hat{k}$ is $\bar{r} = \left(3\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + 2\hat{j} - 3\hat{k}\right)$.

Miscellaneous Exercise 6 A | Q 3 | Page 208

Find the Cartesian equations of the line which passes through the point (–2, 4, –5) and parallel to the line $\dfrac{x+2}{3} = \dfrac{y-3}{5} = \dfrac{z+5}{6}$.

Solution:

The line $\frac{x+2}{3} = \frac{y-3}{5} = \frac{z+5}{6}$ has direction ratios 3, 5, 6. The required line has direction ratios 3, 5, 6 as it is parallel to the given line.

It passes through the point (– 2, 4, – 5).

The cartesian equation of the line passing through (x_1, y_1, z_1) and having direction ratios a, b, c are

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

∴ the required cartesian equation of the line are

$$\frac{x-(-2)}{3} = \frac{y-4}{5} = \frac{z-(-5)}{6}$$

i.e. $\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$.

Miscellaneous Exercise 6 A | Q 4 | Page 208

Obtain the vector equation of the line $\frac{x+5}{3} = \frac{y+4}{5} = \frac{z+5}{6}$.

Solution:

The cartesian equations of the line are $\frac{x+5}{3} = \frac{y+4}{5} = \frac{z+5}{6}$.

This line is passing through the point A(– 5, – 4, – 5) and having direction ratios 3, 5, 6.

Let $\bar{a}$ be the position vector of the point A w.r.t. the origin and $\bar{b}$ be the vector parallel to the line.

Then $\bar{a} = -5\hat{i} - 4\hat{j} - 5\hat{k}$ and $\bar{b} = 3\hat{i} + 5\hat{j} + 6\hat{k}$.

The vector equation of the line passing through $A(\bar{a})$ and parallel to $\bar{b}$ is $\bar{r} = \bar{a} + \lambda\bar{b}$ where λ is a scalar.

∴ the vector equation of the required line is

$$\bar{r} = \left(-5\hat{i} - 4\hat{j} - 6\hat{k}\right) + \lambda\left(3\hat{i} + 5\hat{j} + 6\hat{k}\right).$$

Miscellaneous Exercise 6 A | Q 5 | Page 208

Find the vector equation of the line which passes through the origin and the point (5, –2, 3).

Solution:

Let $\bar{b}$ be the position vector of the point B(5, –2, 3).

Then $\bar{b} = 5\hat{i} - 2\hat{j} + 3\hat{k}$

Origin has position vector $\bar{0} = 0\hat{i} + 0\hat{j} + 0\hat{k}$.

The vector equation the line passing through $A(\bar{a})$ and $b(\bar{b})$ is $\bar{r} = \bar{a} + \lambda(\bar{b} - \bar{a})$ where λ is a scalar.

∴ the vector equation of the required line is

$$\bar{r} = \bar{0} + \lambda(\bar{b} - \bar{0}) = \lambda\left(5\hat{i} - 2\hat{j} + 3\hat{k}\right).$$

Miscellaneous Exercise 6 A | Q 6 | Page 208

Find the Cartesian equations of the line which passes through points (3, –2, –5) and (3, –2, 6).

Solution:

Let A = (3, –2, –5) and (3, –2, 6)

The direction ratios of the line AB are

3 – 3, – 2 – (– 2), 6 – (– 5) i.e. 0, 0, 11.

The parametric equations of the line passing through (x1, y1, z1)

and having direction ratios a, b, c are

x = $x_1 + a\lambda, y = y_1\ b\lambda, z = z_1 + c\lambda$

∴ The parametric equations of the line passing through (3, –2, –5)

and having direction ratios are 0, 0, 11 are

x = $3 + (0)\lambda, y = -2 + 0(\lambda), z = -5 + 11\lambda$

i.e. x = 3, y = –2, z = 11λ – 5

∴ the cartesian equations of the line are

x = 3, y = –2, z = 11λ – 5, λ is a scalar.

Miscellaneous Exercise 6 A | Q 7 | Page 208

Find the Cartesian equations of the line passing through A(3, 2, 1) and B(1, 3, 1).

Solution: The direction ratios of the line AB are 3 – 1, 2 – 3, 1 – 1 i.e. 2, – 1, 0.
The parametric equations of the line passing through (x_1, y_1, z_1) and having direction ratios a, b, c are
x = x_1 + aλ, y = y_1 + bλ, z = z_1 + cλ
∴ the parametric equations of the line passing through (3, 2, 1) and having direction ratios 2, –1, 0 are
x = 3 + 2λ, y = 2 - λ, z = 1 + 0(λ)

∴ x – 3 = 2λ, y - 2 = -λ, z = 1

∴ $\dfrac{x-3}{2} = \dfrac{y-2}{-1}$ = λ, z = 1

∴ the cartesian equations of the required line are

$$\frac{x-3}{2} = \frac{y-2}{-1}, z = 1.$$

Miscellaneous Exercise 6 A | Q 8 | Page 208

Find the Cartesian equations of the line passing through the point A(1, 1, 2) and perpendicular to the vectors

$\bar{\mathrm{b}} = \hat{\mathrm{i}} + 2\hat{\mathrm{j}} + \hat{\mathrm{k}}$ and $\bar{\mathrm{c}} = 3\hat{\mathrm{i}} + 2\hat{\mathrm{j}} - \hat{\mathrm{k}}$

Solution:

Let the required line have direction ratios p, q, r.

It is perpendicular to the vector

$\bar{\mathrm{b}} = \hat{\mathrm{i}} + 2\hat{\mathrm{j}} + \hat{\mathrm{k}}$ and $\bar{\mathrm{c}} = 3\hat{\mathrm{i}} + 2\hat{\mathrm{j}} - \hat{\mathrm{k}}$.

∴ it is perpendicular to lines whose direction ratios are 1, 2, 1 and 3, 2, – 1.

∴ p + 2q + r = 0, 3 + 2q – r = 0

$$\therefore \frac{p}{\begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix}} = \frac{q}{\begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix}} = \frac{r}{\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}}$$

$$\therefore \frac{p}{-4} = \frac{q}{4} = \frac{r}{-1}$$

$$\therefore \frac{p}{-1} = \frac{q}{1} = \frac{r}{-1}$$

∴ the required line has direction ratios –1, 1, –1.

The cartesian equations of the line passing through (x_1, y_1, z_1) and having direction ratios a, b, c are $\dfrac{x = x_1}{a} = \dfrac{y - y_1}{b} = \dfrac{z - z_1}{c}$

∴ the cartesian equation of the line passing through the point (1, 1, 2) and having directions ratios –1, 1, – 1 are

$$\frac{x-1}{-1} = \frac{y-1}{1} = \frac{z-2}{-2}.$$

Miscellaneous Exercise 6 A | Q 9 | Page 208

Find the Cartesian equations of the line which passes through the point (2, 1, 3) and perpendicular to the lines
$\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$ and $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$.

Solution:

Let the required line have direction ratios p, q, r.

It is perpendicular to the vector
$\bar{b}=\hat{i}+2\hat{j}+\hat{k}$ and $\bar{c}=3\hat{i}+2\hat{j}-\hat{k}$.

∴ it is perpendicular to lines whose direction ratios are 1, 2, 1 and 3, 2, – 1.

∴ p + 2q + r = 0, 3 + 2q – r = 0

$\therefore \frac{p}{\begin{vmatrix}2 & 1\\ 2 & -1\end{vmatrix}}=\frac{q}{\begin{vmatrix}1 & 1\\ -1 & 3\end{vmatrix}}=\frac{r}{\begin{vmatrix}1 & 2\\ 3 & 2\end{vmatrix}}$

$\therefore \frac{p}{-4}=\frac{q}{4}=\frac{r}{-1}$

$\therefore \frac{p}{2}=\frac{q}{-7}=\frac{r}{4}$

∴ the required line has direction ratios 2, –7, 4.

The cartesian equations of the line passing through (x_1, y_1, z_1) and having direction ratios a, b, c are $\frac{x=x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

∴ the cartesian equation of the line passing through the point (2, –7, 4) and having directions ratios 2, –7, 4 are
$\frac{x-2}{2}=\frac{y-1}{-7}=\frac{z-2}{4}$.

Miscellaneous Exercise 6 A | Q 10 | Page 208

Find the vector equation of the line which passes through the origin and intersect the line x – 1 = y – 2 = z – 3 at right angle.

Solution:

The given line is $\frac{x-1}{1}=\frac{y-2}{1}=\frac{z-3}{1}=\lambda$ ...(Say)

∴ coordinates of any point on the line are x = $\lambda+1, y=\lambda+2, z=\lambda+3$

∴ position vector of any point on the line is

$(\lambda+1)\hat{\mathrm{i}}+(\lambda+2)\hat{\mathrm{j}}+(\lambda+3)\hat{\mathrm{k}}$ ...(1)

If $\bar{\mathrm{b}}$ is parallel to the given line whose direction ratios are 1, 1, 1 then $\bar{\mathrm{b}}=\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}}$.

Let the required line passing through O meet the given line at M.

∴ position vector of M

+

= $\overline{\mathrm{m}}=(\lambda+1)\hat{\mathrm{i}}+(\lambda+2)\hat{\mathrm{j}}+(\lambda+3)\hat{\mathrm{k}}$ ...[By (1)]

The required line is perpendicular to given line

∴ $\overline{\mathrm{OM}}.\bar{\mathrm{b}}$ = 0

∴ $\left[(\lambda+1)\hat{\mathrm{i}}+(\lambda+2)\hat{\mathrm{j}}+(\lambda+3)\hat{\mathrm{k}}\right].\left(\hat{\mathrm{i}}+\hat{\mathrm{j}}+\hat{\mathrm{k}}\right)$ = 0

∴ $(\lambda+1)\times 1+(\lambda+2)\times 1+(\lambda+3)\times 1$ = 0

∴ $3\lambda+6$ = 0

∴ λ = – 2

∴ $\overline{\mathrm{m}}=(-2+1)\hat{\mathrm{i}}+(-2+2)\hat{\mathrm{j}}+(-2+3)\hat{\mathrm{k}}=-\hat{\mathrm{i}}+\hat{\mathrm{k}}$

The vector equation of the line passing through

$\mathrm{A}(\bar{a})$ and $\mathrm{B}(\bar{b})$ is $\bar{\mathrm{r}} = \bar{\mathrm{a}} + \lambda(\bar{\mathrm{b}} - \bar{\mathrm{a}}), \lambda$ is a scalar.

$\therefore$ the vector equation of the line passing through

$\mathrm{O}(\bar{0})$ and $\mathrm{M}(\bar{\mathrm{m}})$ is $\bar{\mathrm{r}} = \bar{0} + \lambda(\bar{\mathrm{m}} - \bar{0}) = \lambda\left(-\hat{\mathrm{i}} + \hat{\mathrm{k}}\right)$ where λ is a scalar.

Hence, vector equation of the required line is $\bar{\mathrm{r}} = \lambda\left(-\hat{\mathrm{i}} + \hat{\mathrm{k}}\right)$.

Miscellaneous Exercise 6 A | Q 11 | Page 208

Find the value of λ so that the lines

$$\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2} \text{ and } \frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$$

are at right angles.

Solution:

The equations of the given lines are

$$\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{z-3}{2} \quad \text{...(1)}$$

and

$$\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5} \quad \text{...(2)}$$

Equation (1) can be written as:

$$\frac{-(x-1)}{3} = \frac{7(y-2)}{2\lambda} = \frac{z-3}{2}$$

i.e. $\dfrac{x-1}{-3} = \dfrac{y-2}{\frac{2\lambda}{7}} = \dfrac{z-3}{2}$

The direction ratios of this line are

$$a_1 = -3, b_1 = \frac{2\lambda}{7}, c_1 = 2$$

Equation (2) can be written as :

$$\frac{-7(x-1)}{3\lambda} = \frac{y-5}{1} = \frac{-(z-6)}{5}$$

i.e. $\dfrac{x-1}{-\frac{3\lambda}{7}} = \dfrac{y-5}{1} = \dfrac{z-6}{-5}$

The direction ratios of this line are

$$a_2 = \frac{-3\lambda}{7}, b_2 = 1, c_2 = -5$$

Since the lines (1) and (2) are at right angles,

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\therefore (-3)\left(\frac{-3\lambda}{7}\right) + \left(\frac{2\lambda}{7}\right)(1) + 2(-5) = 0$$

$$\therefore \left(\frac{9\lambda}{7}\right) + \left(\frac{2\lambda}{7}\right) - 10 = 0$$

$\therefore \dfrac{11\lambda}{7} = 10$

$$\therefore \lambda = \frac{70}{11}.$$

Miscellaneous Exercise 6 A | Q 12 | Page 208

Find the acute angle between the lines

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-3}{2} \text{ and } \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{1}.$$

Solution:

Let $\bar{a}$ and $\bar{b}$ be the vectors in the direction of the lines

$$\frac{x-1}{1}=\frac{y-2}{-1}=\frac{z-3}{2} \text{ and } \frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{1}$$

respectively.

Then $\bar{a}=\hat{i}-\hat{j}+2\hat{k}, \bar{b}=2\hat{i}+\hat{j}+\hat{k}$

$\therefore \bar{a}.\bar{b}=\left(\hat{i}-\hat{j}+2\hat{k}\right).\left(2\hat{i}+\hat{j}+\hat{k}\right)$

= (1)(2) + (– 1)(1) + (2)(1)

= 2 – 1 + 2

= 3

$$|\bar{a}|=\sqrt{1^2+(-1)^2+2^2}=\sqrt{6}$$

$$\left|\bar{b}\right|=\sqrt{2^2+1^2+1^2}=\sqrt{6}$$

If θ is the angle between the lines, then

$$\cos\theta=\frac{\bar{a}.\bar{b}}{|\bar{a}|\left|\bar{b}\right|}=\frac{3}{\sqrt{6}\sqrt{6}}=\frac{1}{2}=\cos 60^\circ$$

∴ θ = 60°.

Miscellaneous Exercise 6 A | Q 13 | Page 208

Find the acute angle between the lines x = y, z = 0 and x = 0, z = 0.

Solution:

The equations x = y, z = 0 can be written as $\frac{x}{1}=\frac{y}{1}, z = 0$.

$\therefore$ the direction ratios of the line are 1, 1, 0.

The direction ratios of the line x = 0, z = 0, i.e., Y-axis are 0, 1, 0.

$\therefore$ its direction ratios are 0, 1, 0.

Let $\bar{a}$ and $\bar{b}$ be the vectors in the direction of the lines x = y, z = 0 and x = 0, z = 0.

Then $\bar{a}=\hat{i}+\hat{j}, \bar{b}=\hat{j}$

$\therefore \bar{a}.\bar{b}=\left(\hat{i}+\hat{j}\right).\hat{j}$

= (1)(0) + (1)(1) + (0)(0)

= 1

$|\bar{a}|=\sqrt{1^2+1^2}=\sqrt{2}$

$|\bar{b}|=|\hat{j}|$ = 1

If θ is the acute angle between the lines, then

$$\cos\theta = \left|\frac{\bar{a}.\bar{b}}{|\bar{a}||\bar{b}|}\right| = \left|\frac{1}{\sqrt{2}\times 1}\right| = \frac{1}{\sqrt{2}} = \cos 45^\circ.$$

$\therefore \theta = 45^\circ$.

Miscellaneous Exercise 6 A | Q 14 | Page 208

Find the acute angle between the lines x = –y, z = 0 and x = 0, z = 0.

Solution:

The equations x = – y, z = 0 can be written as $\frac{x}{1} = \frac{y}{1}$, z = 0.

∴ the direction ratios of the line are 1, 1, 0.

The direction ratios of the line x = 0, z = 0, i.e., Y-axis are 0, 1, 0.

∴ its direction ratios are 0, 1, 0.

Let $\bar{a}$ and $\bar{b}$ be the vectors in the direction of the lines x = y, z = 0 and x = 0, z = 0.

Then $\bar{a} = \hat{i} + \hat{j}, \bar{b} = \hat{j}$

∴ $\bar{a}.\bar{b} = \left(\hat{i} + \hat{j}\right).\hat{j}$

= (1)(0) + (1)(1) + (0)(0)

= 1

$|\bar{a}| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\left|\bar{b}\right| = \left|\hat{j}\right|$ = 1

If θ is the acute angle between the lines, then

cos θ = $\left|\frac{\bar{a}.\bar{b}}{|\bar{a}|\bar{b}|}\right| = \left|\frac{1}{\sqrt{2} \times 1}\right| = \frac{1}{\sqrt{2}}$ = cos 45°.

∴ θ = 45°.

Miscellaneous Exercise 6 A | Q 15 | Page 208

Find the co-ordinates of the foot of the perpendicular drawn from the point (0, 2, 3) to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.

Solution: Let P = (0, 2, 3)

Let M be the foot of the perpendicular drawn from P to the line

$$\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}=\lambda \quad ..(\text{Say})$$

The coordinates of any point on the line are given by

x = 5λ – 3, y = 2λ + 1, z = 3λ – 4

Let M = (5λ – 3, 2λ + 1, 3λ – 4) ...(1)

The direction ratios of PM are

5λ – 3 – 0, 2λ + 1 – 2, 3λ – 4 – 3

i.e. 5λ – 3λ, 2λ – 1, 3λ – 7

Since, PM is perpendicular to the line whose direction ratios atr 5, 2, 3,

5(5λ – 3) + 2(2λ – 1) + 3(3λ – 7) = 0

∴ 25λ – 15 + 4λ – 2 + 9λ – 21 = 0

∴ 38λ – 38 = 0

∴ λ = 1

Substituting λ = 1 in (1), we get

M = (5 – 3, 2 + 1, 3 – 4) = (2, 3, –1).

Hence, the coordinates of the foot of perpendicular are (2, 3, –1).

Miscellaneous Exercise 6 A | Q 16.1 | Page 208

By computing the shortest distance determine whether following lines intersect each other :

$$\bar{r}=\left(\hat{i}+\hat{j}-\hat{k}\right)+\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right) \text{ and } \bar{r}\left(2\hat{i}+2\hat{j}-3\hat{k}\right)+\mu\left(\hat{i}+\hat{j}-2\hat{k}\right)$$

Solution: The shortest distance between the lines

$\bar{r}=\bar{a}_1+\lambda\bar{b}_1 \text{ and } \bar{r}=\bar{a}_2+\mu\bar{b}_2$ is given by

$$d=\left|\frac{(\bar{a}_2-\bar{a}_1).\left(\bar{b}_1\times\bar{b}_2\right)}{\left|\bar{b}_1\times\bar{b}_2\right|}\right|.$$

Here, $\bar{a}_1=\hat{i}+\hat{j}-\hat{k}, \bar{a}_2=2\hat{i}+2\hat{j}-3\hat{k},$

$\bar{b}_1=2\hat{i}-\hat{j}+\hat{k}, \bar{b}_2=\hat{i}+\hat{j}-2\hat{k}.$

$$\therefore \bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 1 & -2 \end{vmatrix}$$

= $(2-1)\hat{i} - (-4-1)\hat{j} + (4+1)\hat{k}$

= $\hat{i} - 5\hat{j} + 5\hat{k}$

and

$$\bar{a}_2 - \bar{a}_1 = \left(2\hat{i} + 2\hat{j} - 3\hat{k}\right) - \left(\hat{i} + \hat{j} - \hat{k}\right)$$

$$\therefore (\bar{a}_2 - \bar{a}_1).\left(\bar{b}_1 \times \bar{b}_2\right) = \hat{i}.\left(2\hat{i} + 2\hat{j} - 3\hat{k}\right)$$

= 1(– 1) + 0(3) + 0(2)

= – 1

and

$$\left|\bar{b}_1 \times \bar{b}_2\right| = \sqrt{(-1)^2 + 3^2 + 2^2}$$

= $\sqrt{1+9+4}$

= $\sqrt{14}$

Shortest distance between the lines is 0.

∴ the lines intersect each other.

Miscellaneous Exercise 6 A | Q 16.2 | Page 208

Bycomputing the shortest distance determine whether the fllowng line intersect ech other : $\dfrac{x-5}{4} = \dfrac{y-7}{5} = \dfrac{z+3}{5}$ and x – 6 = y – 8 = z + 2.

Solution: The shortest distance between the lines

$\frac{x-5}{4}=\frac{y-7}{5}=\frac{z+3}{5}$ and x – 6 = y – 8 = z + 2 is given by

$$d = \frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1n_2-m_2n_1)^2+(l_2n_1-1_1n_2)^2+(l_1m_2-l_2m_1)^2}}$$

The equation of the given lines are

$\frac{x-5}{4}=\frac{y-7}{5}=\frac{z+3}{5}$ and x – 6 = y – 8 = z + 2

$\therefore x_1 = 5, y_1 = 7, z_1 = 3, x_2 = 6, y_2 = 8, z_2 = 2,$

$l_1 = 4, m_1 = 5, n_1 = 1, l_2 = 1, m_2 = –2, n_2 = 1$

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = \begin{vmatrix} 4 & 6 & 8 \\ 4 & 5 & 5 \\ -6 & -8 & 2 \end{vmatrix}$$

= 4(– 6 + 2) –6(7 –1) + 8(–14 + 6)

= – 16 – 36 –64

= – 116

and

$(m_1n_2 – m_2n_1)^2 + (l_2n_1 – l_1n_2)^2 + (l_1m_2 - l_2m_1)^2$

= (–6 + 2)2 + (1 – 7)2 + (1 – 7)2 + (–14 + 6)

= 16 + 36 + 64

= 116

Hence, the required shortest distance between the given lines

$= \left| \frac{-116}{\sqrt{116}} \right|$

$= \sqrt{116}$

$= 2\sqrt{29}\text{units}$

or

Shortest distance between the lines is 0.

∴ the lines intersect each other.

Miscellaneous Exercise 6 A | Q 17 | Page 208

If the lines

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} \text{ and } \frac{x-2}{1} = \frac{y+m}{2} = \frac{z-2}{1}$$

intersect each other, find m.

Solution:

The lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-2}{1} = \frac{y+m}{2} = \frac{z-2}{1}$ intersect, if

$$\therefore \begin{vmatrix} x_2 - x_1 & y_2 - y_2 & z - z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0 \qquad \text{...(1)}$$

Here, $(x_1, y_1, z_1) \equiv (1, -1, 1)$,

$(x_2, y_2, z_2) \equiv (2, -m, 2)$,

$a_1 = 2, b_1 = 3, c_1 = 4$,

$a_2 = 1, b_2 = 2, c_2 = 1$

Substituting these values in (1), we get

$$\begin{vmatrix} 2-1 & -m+1 & 2-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} 1 & 1-m & 1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$\therefore 1(3 - 8) - (1 - m)(2 - 4) + 1(4 - 3) = 0$

$\therefore -5 + 2 - 2m + 1 = 0$

$\therefore -2m = 2$

$\therefore m = -1.$

Miscellaneous Exercise 6 A | Q 18 | Page 208

Find the vector and Cartesian equations of the line passing through the point (–1, –1, 2) and parallel to the line 2x − 2 = 3y + 1 = 6z − 2.

Solution:

Let $\bar{a}$ be the person vector of the point A(–1, –1, 2) w.r.t. the origin.

Then $\bar{a} = -\hat{i} - \hat{j} + 2\hat{k}$

The equation of given line is

$2x - 2 = 3y + 1 = 6z - 2$

$\therefore 2(x - 1) = 3\left(y + \frac{1}{3}\right) = 6\left(z - \frac{1}{3}\right)$

$$\therefore \frac{x-1}{\left(\frac{1}{2}\right)} = \frac{y+\frac{1}{3}}{\left(\frac{1}{3}\right)} = \frac{z-\frac{1}{3}}{\left(\frac{1}{6}\right)}$$

The direction ratios of this line are

$\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$ i.e. 3, 2, 1

Let $\bar{b}$ be the vector parallel to this line.

Then $\bar{b} = 3\hat{i} + 2\hat{j} + \hat{k}$

The vector equation of the line passing through $A(\bar{a})$ and parallel to $\bar{b}$ is

$\bar{r} = \bar{a} + \lambda\bar{b}$, where λ is a scalar

∴ the vector equation of the required line is

$\bar{r} = \left(-\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + \hat{k}\right)$.

The line passes through (–1, – 1, 2) and has direction ratios 3, 2, 1

∴ the cartesian equations of the line are

$$\frac{x-(-1)}{3} = \frac{y-(-1)}{2} = \frac{z-2}{1}$$

i.e. $\frac{x+1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$.

Miscellaneous Exercise 6 A | Q 19 | Page 208

Find the direction cosines of the lines

$\bar{r} = \left(-2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j}\right)$.

Solution:

The line $\bar{r} = \left(-2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j}\right)$ is parallel to

$\bar{b} = 2\hat{i} + 3\hat{j}$.

∴ direction ratios of the line are 2, 3, 0

∴ direction cosines of the line are

$$\frac{2}{\sqrt{2^2+3^2+0}}, \frac{3}{\sqrt{2^2+3^2+0}}, 0$$

i.e. $\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}, 0.$

Miscellaneous Exercise 6 A | Q 20 | Page 208

Find the Cartesian equation of the line passing through the origin which is perpendicular to x – 1 = y – 2 = z – 1 and intersect the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$.

Solution: Let the required line have direction ratios a, b, c

Since the line passes through the origin, its cartesian equation are

$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c} \quad \text{...(1)}$$

This line is perpendicular to the line

x – 1 = y – 2 = z – 1 whose direction ratios are 1, 1, 1.

∴ a + b + c = 0 ...(2)

The lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_2} = \frac{z-z_1}{c_1}$ intersect, if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Applying this condition for the lines

$\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ and $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ we get

$$\begin{vmatrix} 1-0 & -1-0 & 1-0 \\ a & b & c \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$\therefore$ 1(4b – 3c) + 1(4a –2c) + 1(3a – 2b) = 0

$\therefore$ 4b – 3c + 4a – 2c + 3a – 2b = 0

$\therefore$ 7a + 2b – 5c = 0 ...(3)

From (2) and (3), we get

$$\frac{a}{\begin{vmatrix} 1 & 1 \\ 2 & -5 \end{vmatrix}} = \frac{b}{\begin{vmatrix} 1 & 1 \\ -5 & 7 \end{vmatrix}} = \frac{a}{\begin{vmatrix} 1 & 1 \\ 7 & 2 \end{vmatrix}}$$

$$\therefore \frac{a}{-7} = \frac{b}{12} = \frac{c}{-5}$$

$\therefore$ the required line has direction ratios –7, 12, –5.

From (1), cartesian equation of required line are

$$\frac{x}{-7} = \frac{y}{12} = \frac{z}{-5}$$

i.e. $\dfrac{x}{7} = \dfrac{y}{-12} = \dfrac{z}{5}$.

Miscellaneous Exercise 6 A | Q 21 | Page 208

Find the vector equation of the line whose Cartesian equations are y = 2 and 4x – 3z + 5 = 0.

Solution: 4x – 3z + 5 = 0 can be written as

$$4x = 3z - 5 = 3\left(z - \frac{5}{3}\right)$$

$$\therefore \frac{4x}{12} = \frac{3\left(z - \frac{5}{3}\right)}{12}$$

$$\therefore \frac{x}{3} = \frac{z - \frac{5}{3}}{4}$$

$\therefore$ the cartesian equation of the line are

$$\frac{x}{3} = \frac{z - \frac{5}{3}}{4}, y = 2.$$

This line passes through the point $\mathrm{A}\left(0, 2, \frac{5}{3}\right)$ whose position vector is $\bar{a} = 2\hat{j} + \frac{5}{3}\hat{k}$

Also the line has direction ratio 3, 0, 4.

If $\bar{b}$ is a vector parallel to the line, then $\bar{b} = 3\hat{i} + 4\hat{k}$

The vector equation of the line pasing through $\mathrm{A}(\bar{a})$ and parallel to $\bar{b}$ is $\bar{r} = \bar{a} + \lambda\bar{b}$ where λ is a scalar.

$\therefore$ the vector equation of the required line is

$$\bar{r} = \left(2\hat{j} + \frac{5}{3}\hat{k}\right) + \lambda\left(3\hat{i} + 4\hat{k}\right).$$

Miscellaneous Exercise 6 A | Q 22 | Page 209

Find the coordinates of points on th line $\frac{x-1}{1} = \frac{y-2}{-2} = \frac{z-3}{2}$ which are at the distance 3 unit from the base point A(l, 2, 3).

Solution:

The cartesian equations of the line are

$$\frac{x-1}{1} = \frac{y-2}{-2} = \frac{z-3}{2} = \lambda \quad \text{...(Say)}$$

The coordinates of any point on this line are given by

x = λ + 1y = -2λ + 2z = 2λ + 3

Let M (λ + 1, -2λ + 2, 2λ + 3) ...(1)

be the point on the in whose dstance from A(1, 2, 3) is 3 units.

$$\therefore \sqrt{\lambda + 1 - 1^2 + (-2\lambda + 2 - 2)^2 + (2\lambda + 3 - 3)^2}$$

$$\therefore \sqrt{\lambda^2 + 4\lambda^2 + 4\lambda^2} = 3$$

$$\therefore \sqrt{9\lambda^2} = 3$$

$$\therefore 9\lambda^2 = 9$$

$$\therefore \lambda = \pm 1$$

When λ = 1, M = (1 + 1, –2 + 2, 2 + 3) ...[By (1)]

i.e. M = (2, 0, 5)

When λ = –1, M = (1 – 1, 2 + 2, –2 + 3) ...[By (1)]

i.e. M = (0, 4, 1)

Hence, the coordinates of the required points are (2, 0, 5) and (0, 4, 1).

MISCELLANEOUS EXERCISE 6 B [PAGES 223 - 225]

Miscellaneous Exercise 6 B | Q 1 | Page 223

Choose correct alternatives :

If the line $\frac{x}{3} = \frac{y}{4}$ = z is perpendicular to the line $\frac{x-1}{k} = \frac{y+2}{3} = \frac{z-3}{k-1}$, then the value of k is

1. 11/4
2. **- 11/4**
3. 11/2
4. 4/11

Solution: - 11/4

Miscellaneous Exercise 6 B | Q 2 | Page 223

Choose correct alternatives :

The vector equation of line 2x – 1 = 3y + 2 = z – 2 is

Options

$$\bar{r} = \left(\frac{1}{2}\hat{i} - \frac{2}{3}\hat{j} + 2\hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$$

$$\bar{r} = \hat{i} - \hat{j} + \left(2\hat{i} + \hat{j} + \hat{k}\right)$$

$$\bar{r} = \left(\frac{1}{2}\hat{i} - \hat{j}\right) + \lambda\left(\hat{i} - 2\hat{j} + 6\hat{k}\right)$$

$$\bar{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(\hat{i} - 2\hat{j} + 6\hat{k}\right)$$

Solution:

$$\bar{r} = \left(\frac{1}{2}\hat{i} - \frac{2}{3}\hat{j} + 2\hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$$

Miscellaneous Exercise 6 B | Q 3 | Page 223

Choose correct alternatives :

The direction ratios of the line which is perpendicular to the two lines $\frac{x-7}{2} = \frac{y+17}{-3} = \frac{z-6}{1}$ and $\frac{x+5}{1} = \frac{y+3}{2} = \frac{z-6}{-2}$ are

1. **4, 5, 7**
2. 4, –5, 7
3. 4, –5, –7
4. –4, 5, 8

Solution: 4, 5, 7

Miscellaneous Exercise 6 B | Q 4 | Page 223

Choose correct alternatives :

The length of the perpendicular from (1, 6,3) to the line

$$\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$$

1. 3
2. √11)
3. **√13**
4. 5

Solution: √13

Miscellaneous Exercise 6 B | Q 5 | Page 224

Choose correct alternatives :

The shortest distance between the lines

$\bar{r}=\left(\hat{i}+2\hat{j}+\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$ and $\bar{r}=\left(2\hat{i}-\hat{j}-\hat{k}\right)+\mu\left(2\hat{i}+\hat{j}+2\hat{k}\right)$ is

1. 1/√3
2. 1/√2
3. **3/√2**
4. √3/2

Solution: 3/√2

Miscellaneous Exercise 6 B | Q 6 | Page 224

Choose correct alternatives :

The lines

$\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are

coplnar if

1. k = 1 or –1
2. **k = 0 or – 3**
3. k = ± 3
4. k = 0 or – 1

Solution: k = 0 or – 3

Miscellaneous Exercise 6 B | Q 7 | Page 224

Choose correct alternatives :

The lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-3}{6}$ are

1. perpendicular
2. **intersecting**
3. skew
4. coincident

Solution: intesecting

Miscellaneous Exercise 6 B | Q 8 | Page 224

Choose correct alternatives :

Equation of X-axis is

1. x = y = z
2. y = z
3. **y = 0, z = 0**
4. x = 0, y = 0

Solution: y = 0, z = 0

Miscellaneous Exercise 6 B | Q 9 | Page 224

Choose correct alternatives :

The angle between the lines 2x = 3y = – z and 6x = – y = – 4z is

1. 45°
2. 30°
3. 0°
4. **90°**

Solution: 90°

Miscellaneous Exercise 6 B | Q 10 | Page 224

Choose correct alternatives :

Te direction ratios of the line 3x + 1 = 6y – 2 = 1 –z are

1. 2, 1, 6
2. **2, 1, – 6**
3. 2, – 1, 6
4. – 2, 1, 6

Solution: 2, 1, – 6

Miscellaneous Exercise 6 B | Q 11 | Page 224

Choose correct alternatives :

The perpendicular distance of the plane 2x + 3y – z = k from the origin is √14 units, the value of k is

1. **14**
2. 196
3. 2√14
4. √14/2

Solution: 14

Miscellaneous Exercise 6 B | Q 12 | Page 224

Choose correct alternatives :

The angle between the planes

$\bar{r}.\left(\hat{i}-2\hat{j}+3\hat{k}\right)+4=0$ and $\bar{r}.\left(2\hat{i}+\hat{j}-3\hat{k}\right)+7=0$ is

Options

$\frac{\pi}{2}$

$\frac{\pi}{3}$

$\cos^{-1}\left(\frac{3}{4}\right)$

$\cos^{-1}\left(\frac{9}{14}\right)$

Solution:

$$\cos^{-1}\left(\frac{9}{14}\right)$$

Miscellaneous Exercise 6 B | Q 13 | Page 224

Choose correct alternatives :

If the planes $\bar{r}.\left(2\hat{i}-\lambda\hat{j}+\hat{k}\right)=3$ and $\bar{r}.\left(4\hat{i}-\hat{j}+\mu\hat{k}\right)=5$ are parallel, then the values of λ and μ are respectively

Options

- $\frac{1}{2}, -2$
- $-\frac{1}{2}, 2$
- $-\frac{1}{2}, -2$
- $\frac{1}{2}, 2$

Solution:

$$\frac{1}{2}, 2$$

Miscellaneous Exercise 6 B | Q 14 | Page 225

Choose correct alternatives :

The equation of the plane passing through (2, -1, 3) and making equal intercepts on the coordinate axes is

1. x + y + z = 1
2. x + y + z = 2
3. x + y + z = 3
4. **x + y + z = 4**

Solution: x + y + z = 4

Miscellaneous Exercise 6 B | Q 15 | Page 225

Choose correct alternatives :

Measure of angle between the plane 5x – 2y + 3z – 7 = 0 and 15x – 6y + 9z + 5 = 0 is

1. **0°**
2. 30°
3. 45°
4. 90°

Solution: 0°

Miscellaneous Exercise 6 B | Q 16 | Page 225

Choose correct alternatives :

The direction cosines of the normal to the plane 2x – y + 2z = 3 are

Options

$$\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$$

$$\frac{-2}{3}, \frac{1}{3}, \frac{-2}{3}$$

$$\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$$

$$\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$$

Solution:

$$\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$$

Miscellaneous Exercise 6 B | Q 17 | Page 225

Choose correct alternatives :

The equation of the plane passing through the points (1, -1, 1), (3, 2, 4) and parallel to Y-axis is :

1. 3x + 2z – 1 = 0
2. **3x – 2z = 1**
3. 3x + 2z + 1 = 0
4. 3x + 2z = 2

Solution: 3x – 2z = 1

Miscellaneous Exercise 6 B | Q 18 | Page 225

Choose correct alternatives :

The equation of the plane in which the line $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ lie, is

1. **17x – 47y – 24z + 172 = 0**
2. 17x + 47y – 24z + 172 = 0
3. 17x + 47y + 24z + 172 = 0
4. 17x – 47y + 24z + 172 = 0

Solution: 17x – 47y – 24z + 172 = 0

Miscellaneous Exercise 6 B | Q 19 | Page 225

If the line $\frac{x+1}{2} = \frac{y-m}{3} = \frac{z-4}{6}$ lies in the plane 3x – 14y + 6z + 49 = 0, then the value of m is

1. **5**
2. 3
3. 2
4. – 5

Solution: 5

Miscellaneous Exercise 6 B | Q 20 | Page 225

Choose correct alternatives :

The foot of perpendicular drawn from the point (0,0,0) to the plane is (4, -2, -5) then the equation of the plane is

1. 4x + y + 5z = 14
2. **4x – 2y – 5z = 45**
3. x – 2y – 5z = 10
4. 4x + y + 6z = 11

Solution: 4x – 2y – 5z = 45.

MISCELLANEOUS EXERCISE 6 B [PAGES 225 - 226]

Miscellaneous Exercise 6 B | Q 1 | Page 225

Solve the following :

Find the vector equation of the plane which is at a distance of 5 units from the origin and which is normal to the vector

$2\hat{i} + \hat{j} + 2\hat{k}$.

Solution:

If $\hat{n}$ is a unit vector along the normal and p i the length of the perpendicular from origin to the plane, then the vector equation of the plane $\bar{r}.\hat{n} = p$

Here, $\bar{n} = 2\hat{i} + \hat{j} + 2\hat{k}$ and p = 5

$\therefore |\bar{n}| = \sqrt{2^2 + 1^2 + (2)^2}$

$= \sqrt{9}$

= 3

$\hat{n} = \frac{\bar{n}}{|\bar{n}|}$

$= \frac{1}{3}\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$

$\therefore$ the vector equation of the required plane is

$\bar{r}.\left[\frac{1}{3}\left(2\hat{i} + \hat{j} + 2\hat{k}\right)\right] = 5$

i.e. $\bar{r}.\left(2\hat{i} + \hat{j} + 2\hat{k}\right) = 15$.

Miscellaneous Exercise 6 B | Q 2 | Page 225

Solve the following :

Find the perpendicular distance of the origin from the plane 6x + 2y + 3z - 7 = 0

Solution:

The distance of the point (x_1, y_1, z_1) from the plane ax + by + cz + d = 0 is $\left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$

∴ the distance of the point (1, 1, – 1) from the plane 6x + 2y + 3z – 7 = 0 is

$$\left|\frac{6(1) + 2(1 - 3(-1) + 7)}{\sqrt{3^2 + 4^2 + (-12)^2}}\right|$$

$$= \left|\frac{6 + 4 + 6 + 7}{\sqrt{9 + 16 + 144}}\right|$$

$$= \frac{23}{\sqrt{169}}$$

$$= \frac{23}{13}$$

= 1units.

Miscellaneous Exercise 6 B | Q 3 | Page 225

Solve the following :

Find the coordinates of the foot of the perpendicular drawn from the origin to the plane 2x + 3y + 6z = 49.

Solution: The equation of the plane is 2x + 3y + 6z = 49.

Dividing each term by

$\sqrt{2^2 + 3^2 + (-6)^2}$

= $\sqrt{49}$

= 7,

we get

$\frac{2}{7}x + \frac{3}{7}y - \frac{6}{7}z = \frac{49}{7}$ = 7

This is the normal form of the equation of plane.

∴ the direction cosines of the perpendicular drawn from the origin to the plane are

l = $\frac{2}{7}, m = \frac{3}{7}, n = \frac{6}{7}$

and length of perpendicular from origin to the plane is p = 7.

∴ the coordinates of the foot of the perpendicular from the origin to the plane are

$(lp, mp, np)\text{i.e.}(2, 3, 6)$

Miscellaneous Exercise 6 B | Q 4 | Page 225

Solve the following :

Reduce the equation $\bar{\text{r}}.\left(6\hat{\text{i}} + 8\hat{\text{j}} + 24\hat{\text{k}}\right)$ = 13 normal form and hence find

(i) the length of the perpendicular from the origin to the plane.

(ii) direction cosines of the normal.

Solution:

The normal form of equation of a plane is $\bar{\text{r}}.\hat{\text{n}} = p$ where $\hat{\text{n}}$ is unit vector along the normal and p is the length of perpendicular drawn from origin to the plane.

Given pane is $\bar{r}.\left(6\hat{i}+8\hat{j}+24\hat{k}\right)$ = 13 ...(1)

$\bar{n}=6\hat{i}+8\hat{j}+24\hat{k}$ is normal to the plane

$\therefore$ $|\bar{n}|=\sqrt{6^2+8^2+24^2}=\sqrt{76}$ = 13

Dividing both sides of (1) by 13, get

$$\bar{r}.\left(\frac{3\hat{i}+4\hat{j}+12\hat{k}}{13}\right)=\frac{76}{13}$$

i.e. $\bar{r}.\left(\frac{3}{13}\hat{i}+\frac{4}{13}\hat{j}+\frac{12}{13}\hat{k}\right)=\frac{1}{2}$

This is the normal form of the equation of plane.

Comparing with $\bar{r}.\hat{n}=p$,

(i) the length of the perpendicular from the origin to plane is $\frac{1}{2}$.

(ii) direction cosines of the normal are $\frac{3}{13},\frac{4}{13},\frac{12}{13}$.

Miscellaneous Exercise 6 B | Q 5 | Page 226

Solve the following :

Find the vector equation of the plane passing through the points A(1, 92, 1), B(2, 91, 93) and C(0, 1, 5).

Solution: The vector equation of the plane passing through three non-collinear points

$\mathrm{A}(\bar{a}),\mathrm{B}(\bar{b})$ and $\mathrm{C}(\bar{c})$ is$\bar{r}.\left(\overline{AB}\times\overline{AC}\right)=\bar{a}.\left(\overline{AB}\times\overline{AC}\right)$

...(1)

Here, $\bar{a}=\hat{i}-2\hat{j}+\hat{k},\bar{b}=2\hat{i}-\hat{j}-3\hat{k},\bar{c}=\hat{j}+5\hat{k}$

$$\therefore \overline{AB} = \bar{b} - \bar{a} = \left(2\hat{i} - \hat{j} - 3\hat{k}\right) - \left(\hat{i} - 2\hat{j} + \hat{k}\right)$$

$$= \hat{i} + \hat{j} - 4\hat{k}$$

$$\overline{AC} = \bar{c} - \bar{a} = \left(\hat{j} + 5\hat{k}\right) - \left(\hat{i} - 2\hat{j} + \hat{k}\right)$$

$$= \hat{i} + 3\hat{j} + 4\hat{k}$$

$$\therefore \overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{i} & & \hat{k} \\ 1 & 1 & -4 \\ -1 & 3 & 4 \end{vmatrix}$$

$$= (4 + 12)\hat{i} - (4 - 4)\hat{j} + (3 + 1)\hat{k}$$

$$= 16\hat{i} + 4\hat{k}$$

Now, $\bar{a}.\left(\overline{AB} \times \overline{AC}\right) = \left(\hat{i} - 2\hat{j} + \hat{k}\right).\left(16\hat{i} + 4\hat{k}\right)$

$= (1)(16) + (-2)(0) + (1)(4) = 20$

$\therefore$ from(1), the vector equation of the required plane is

$\bar{r}.\left(16\hat{i} + 4\hat{k}\right) = 20.$

Miscellaneous Exercise 6 B | Q 6 | Page 226

Solve the following :

Find the cartesian equation of the plane passing through A(1,-2, 3) and direction ratios of whose normal are 0, 2, 0.

Solution: The Cartesian equation of the plane passing through (x_1, y_1, z_1), the direction ratios of whose normal are a, b, c, is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

$\therefore$ the cartesian equation of the required plane is

$0(x + 1) + 2(y + 2) + 5(z - 3) = 0$

i.e. 0 + 2y - 4 + 10z – 15 = 0

i.e. y + 2 = 0.

Miscellaneous Exercise 6 B | Q 7 | Page 226

Solve the following :

Find the cartesian equation of the plane passing through A(7, 8, 6) and parallel to the plane $\bar{r}.\left(6\hat{i}+8\hat{j}+7\hat{k}\right)$ = 0.

Solution: The cartesian equation of the plane

$\bar{r}.\left(6\hat{i}+8\hat{j}+7\hat{k}\right)$ = 0 is 6x + 8y + 7z = 0

The required plane is parallel to it
∴ its cartesian equation is
6x + 8y + 7z = p ...(1)

A(7, 8, 6) lies on it and hence satisfies its equation

∴ (6)(7) + (8)(8) + (7)(6) = p

i.e., p = 42 + 64 + 42 = 148.

∴ from (1), the cartesian equation of the required plane is 6x + 8y + 7z = 148.

Miscellaneous Exercise 6 B | Q 8 | Page 226

Solve the following :

The foot of the perpendicular drawn from the origin to a plane is M(1, 2, 0). Find the vector equation of the plane.

Solution:

The vector equation of the plane passing through A$(\bar{a})$ and perpendicular to $\bar{n}$ is $\bar{r}.\bar{n}=\bar{a}.\bar{n}$.

M(1, 2, 0) is the foot of the perpendicular drawn from origin to the plane. Then the plane is passing through M and is perpendicular to OM.

If $\overline{m}$ is the position vector of M, then $\overline{m}=\hat{i}$.

Normal to the plane is

$\bar{n} = \overline{OM} = \hat{i}$

$\bar{m} . \bar{n} = \hat{i} . \hat{i}$ = 5

∴ the vector equation of the required plane is

$\bar{r} . \left(\hat{i} + 2\hat{j}\right)$ = 5.

Miscellaneous Exercise 6 B | Q 9 | Page 226

Solve the following :

A plane makes non zero intercepts a, b, c on the coordinate axes.

Show that the vector equation of the plane is

$\bar{r} . \left(bc\hat{i} + ca\hat{j} + ab\hat{k}\right)$ = abc.

Solution:

The vector equation of the plane passing through

$A(\bar{a}), B(\bar{b}) . . C(\bar{c})$, where A, B, C are non-collinear is

$\bar{r} . \left(\overline{AB} \times \overline{AC}\right) = \bar{a} . \left(\overline{AB} \times \overline{AC}\right)$ ...(1)

The required plane makes intercepts 1, 1, 1 on the coordinate axes.

∴ it passes through the three non-collinear points A =(1, 0, 0, B = (0, 1, 0), C = (0, , 1)

$\therefore \bar{a} = \hat{i}, \bar{b} = \hat{j}, \bar{c} = \hat{k}$

$\overline{AB} = \bar{b} - \bar{a} = \hat{j} - \hat{i} = -\hat{i} + \hat{j}$

$\therefore \overline{AC} = \bar{c} - \bar{a} = \hat{k} - \hat{i} = -\hat{i} + \hat{k}$

$$\therefore \overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{i} & & \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$= (1 - 0)\hat{i} - (-1 - 0)\hat{j} + (0 + 1)\hat{k}$

$= \hat{i} + \hat{j} + \hat{k}$

Also,

$\bar{a}.\left(\overline{AB} \times \overline{AC}\right)$

$= \hat{i}.\left(\hat{i} + \hat{j} + \hat{k}\right)$

= 1 x 1 + 0 x 1 + 0 x 1

= 1

$\therefore$ from(1)the vector equation of the required plane is

$\bar{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 1.$

Miscellaneous Exercise 6 B | Q 10 | Page 226

Solve the following :

Find the vector equation of the plane passing through the point A(– 2, 3, 5) and parallel to the vectors $4\hat{i} + 3\hat{k}$ and $\hat{i} + \hat{j}$.

Solution:

The vector equation of the plane passing through the point $A(\bar{a})$ and parallel to the vectors $\bar{b}$ and $\bar{c}$ is

$\bar{r}.\left(\bar{b} \times \bar{c}\right) = \bar{a}.\left(\bar{b} \times \bar{c}\right)$ ...(1)

Here, $\bar{a} = -2\hat{i} + 3\hat{j} + 5\hat{k}$

$\bar{b} = 4\hat{i} + 3\hat{k}$,

$\bar{c} = \hat{i} + \hat{j}$

$$\therefore \bar{b} \times \bar{c} = \begin{vmatrix} \hat{i} & & \hat{k} \\ 4 & 0 & 3 \\ 1 & 1 & 0 \end{vmatrix}$$

$= (-1 - 3)\hat{i} - (4 - 3)\hat{j} + (3 + 1)\hat{k}$

$= -4\hat{i} - \hat{j} + 4\hat{k}$

$$\therefore \bar{a}.\left(\bar{b} \times \bar{c}\right) = \left(-2\hat{i} + 7\hat{j} + 5\hat{k}\right).\left(-4\hat{i} - \hat{j} + 4\hat{k}\right)$$

= (– 2)(– 4) + (7)(–1) + (5)(4)

= 8 – 7 + 8

= 35

∴ From (1), the vector equation of the required plane is

$\bar{r}.\left(-3\hat{i} - 3atj + 4\hat{k}\right)$ = 35.

Miscellaneous Exercise 6 B | Q 11 | Page 226

Solve the following :

Find the cartesian equation of the plane

$\bar{r} = \lambda\left(\hat{i} + \hat{j} - \hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$.

Solution:

The equation $\bar{r} = \bar{a} + \lambda\bar{b} + \mu\bar{c}$ represents a plane passing through a point having position vector $\bar{a}$ and parallel to vectors $\bar{b}$ and $\bar{c}$.

Here,

$\bar{b} = \hat{i} + \hat{j} - \hat{k}$,

$\bar{c} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore \bar{b} \times \bar{c} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

= $(3+2)\hat{i} - (3-1)\hat{j} + (-2-1)\hat{k}$

= $5\hat{i} - 4\hat{j} - \hat{k}$

= $\bar{a}$

Also,

$\bar{a}.\left(\bar{b} \times \bar{c}\right)$

= $\bar{a}.\bar{a} = |\bar{a}|^2$

= $(5)^2 + (4)^2 + (0)^2$

= 0

The vector equation of the plane passing through A$(\bar{a})$ and parallel to $\bar{b}$ and $\bar{c}$ is

$\bar{r}.\left(\bar{b} \times \bar{c}\right) = \bar{a}.\left(\bar{b} \times \bar{c}\right)$

$\therefore$ the vector equation of the given plane is

$\bar{r}.\left(5\hat{i} - 4\hat{j} - \hat{k}\right)$ = 0

If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then this equation becomes

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(5\hat{i} - 4\hat{j} - \hat{k}\right)$ = 0

$\therefore$ 5x – 4y + z = 0.

This is the cartesian equation of the required plane.

Miscellaneous Exercise 6 B | Q 12 | Page 226

Solve the following :

Find the cartesian equations of the planes which pass through A(1, 2, 3), B(3, 2, 1) and make equal intercepts on the coordinate axes.

Solution: Case 1 : Let all the intercepts be 0.

Then the plane passes through the origin.

Then the cartesian equation of the plane is ax + by + cz = 0. ...(1)

(1, 2, 3) d (3, 2, 1) lie on the plane.

∴ a + 2b + 3c = 0 and 3a + 2b + c = 0

$$\therefore \frac{a}{\begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix}} = \frac{b}{\begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix}} = \frac{c}{\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}}$$

$$\therefore \frac{a}{-4} = \frac{b}{8} = \frac{c}{-4}$$

i.e. $\frac{a}{1} = \frac{b}{-2} = \frac{c}{1}$

∴ a, b, c are proprtional to 1, – 2, 1
∴ from (1), the required cartesian equation is x –2y + z = 0

Case 2 : Let he plane make non zero intercept p on each axis.

then its equation is $\frac{x}{p} + \frac{y}{p} + \frac{z}{p} = 1$

i.e. x + y + z = p ...(2)
Since this plane pass through (1, 2, 3) and (3, 2, 1)

∴ 1 + 2 + 3 = p and 3 + 2 + 1 = p

∴ p = 6

∴ from (2), the required cartesian equation is
x + y + z = 6
Hence, the cartesian equations of required planes are
x + y + z = 6 and x – 2y + z = 0.

Miscellaneous Exercise 6 B | Q 13 | Page 226

Solve the following :

Find the vector equation of the plane which makes equal non zero intercepts on the coordinate axes and passes through (1, 1, 1).

Solution: Case 1 : Let all the intercepts be 0.

Then the plane passes through the origin.

Then the vector equation of the plane is ax + by + cz = 0. ...(1)

(1, 1, 1) lie on the plane.

∴ 1a + 1b + 1c = 0

$$\therefore \frac{\hat{i}}{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{\hat{j}}{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{\hat{k}}{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}}$$

$$\therefore \frac{\hat{i}}{1} = \frac{\hat{j}}{1} = \frac{\hat{k}}{1}$$

i.e. $\frac{\hat{i}}{1} = \frac{\hat{j}}{1} = \frac{\hat{k}}{1}$

∴ $\hat{i}, \hat{j}, \hat{k}$ are proprtional to 1, 1, 1

∴ from (1), the required cartesian equation is x - y + z = 0

Case 2 : Let he plane make non zero intercept p on each axis.

then its equation is $\frac{\hat{i}}{p} + \frac{\hat{j}}{p} + \frac{\hat{k}}{p} = 1$

i.e. $\hat{i} + \hat{j} + \hat{k} = p$ ...(2)

Since this plane pass through (1, 1, 1)

∴ 1 + 1 + 1 = p

∴ p = 3

$\therefore$ from (2), the required cartesian equation is $\hat{i} + \hat{j} + \hat{k}$ = 3

Hence, the cartesian equations of required planes are

$\bar{r}.\left(\hat{i} + \hat{j} + \hat{k}\right)$ = 3

Miscellaneous Exercise 6 B | Q 14 | Page 226

Solve the following :

Find the angle between the planes $\bar{r}.\left(-2\hat{i} + \hat{j} + 2\hat{k}\right)$ = 17 and $\bar{r}.\left(2\hat{i} + 2\hat{j} + \hat{k}\right)$ = 71.

Solution:

The acute angle between the planes

$\bar{r}.\bar{n}_1 = d_1$ and $\bar{r}.\bar{n}_2 = d_2$ is given by

$$\cos\theta = \left|\frac{\bar{n}_1.\bar{n}_2}{|\bar{n}_1||\bar{n}_2|}\right| \quad ...(1)$$

Here,

$\bar{n}_1 = -2\hat{i} + \hat{j} + 2\hat{k}$,

$\bar{n}_2 = 2\hat{i} + 2\hat{j} + \hat{k}$

$\therefore \bar{n}_1.\bar{n}_2$

= $\left(2\hat{i} + \hat{j} + 2\hat{k}\right).\left(2\hat{i} + \hat{j} + \hat{k}\right)$

= (1)(2) + (1)(1) + (2)(1)

=2 + 1 + 2

= 5

Also,

$$|\bar{n}_1| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$$

$$|\bar{n}_2| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$$

∴ from (1), we have

$$\cos\theta = \left|\frac{3}{\sqrt{6}\sqrt{6}}\right|$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}\cos 90^\circ$$

∴ θ = 90°.

Miscellaneous Exercise 6 B | Q 15 | Page 226

Solve the following :

Find the acute angle between the line $\bar{r} = \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ and the plane $\bar{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right)$ = 23.

Solution:

The acute angle θ between the line $\bar{r} = \bar{a} + \lambda\bar{b}$ and and the plane $\bar{r}.\bar{n} = d$ is given by

$$\sin\theta = \left|\frac{\bar{b}.\bar{n}}{|\bar{b}||\bar{n}|}\right| \quad ...(1)$$

Here, $\bar{b} = \hat{i} - \hat{j} + \hat{k}, \bar{n} = 2\hat{i} - \hat{j} + \hat{k}$

$$\therefore \bar{b}.\bar{n} = \left(\hat{i} - \hat{j} + \hat{k}\right).\left(2\hat{i} - \hat{j} + \hat{k}\right)$$

= (2)(2) + (3)(– 1) + (– 6)(1)

= 4 – 3 – 6

= – 5

Also, $|\bar{b}| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{2}$ = 1

$$|\bar{n}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4}$$

∴ from (1), we have

sin θ = $\left|\dfrac{2\sqrt{2}}{-3}\right| = \dfrac{2\sqrt{2}}{3}$

∴ θ = $\sin^{-1}\left(\dfrac{2\sqrt{2}}{3}\right)$.

Miscellaneous Exercise 6 B | Q 16 | Page 226

Show that the line $\bar{r} = \left(2\hat{j} - 3\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ and $\bar{r} = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) + \mu\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$ are coplanar. Find the equation of the plane determined by them.

Solution:

The lines $\bar{r} = \bar{a}_1 + \lambda_1\bar{b}_1$ and $\bar{r} = \bar{a}_2 + \lambda_2\bar{b}_2$ are coplanar If

$$\bar{a}_1.\left(\bar{b}_1 \times \bar{b}_2\right) = \bar{a}_2.\left(\bar{b}_1 \times \bar{b}_2\right)$$

Here $\bar{a}_1 = 2\hat{j} - 3\hat{k}, \bar{a}_2 = 2\hat{i} + 6\hat{j} + 3\hat{k}$,

$$\bar{b}_1 = \hat{i} + 2\hat{j} + 3\hat{k}, \bar{b}_2 = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\therefore \bar{a}_2 - \bar{a}_1 = \left(2\hat{i} + 6\hat{j} + 3\hat{k}\right) - \left(2\hat{j} - 3\hat{k}\right)$$

= $2\hat{i} + 4\hat{j} + 6\hat{k}$

$$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & & \\ 1 & 2 & 3 \\ 2 & 3 & \end{vmatrix}$$

= $(8 - 9)\hat{i} - (4 - 6)\hat{j} + (3 - 4)\hat{k}$

= $-\hat{i} + 2\hat{j} - \hat{k}$

$\therefore \bar{a}_1.\left(\bar{b}_1 \times \bar{b}_2\right) = \left(2\hat{j} - 3\hat{k}\right).\left(-\hat{i} + 2\hat{j} - \hat{k}\right)$

= 0(– 1) + 2(2) + (– 3)(– 1)

= 0 + 4 + 3

= 7

and $\bar{a}_2.\left(\bar{b}_1 \times \bar{b}_2\right) = \left(2\hat{j} + 6\hat{j} + 3\hat{k}\right).\left(-\hat{i} + 2\hat{j} - \hat{k}\right)$

= 2(– 1) + 6(2) + 3(– 1)

= –2 + 12 – 3

= 7

$\therefore \bar{a}_1.\left(\bar{b}_1 \times \bar{b}_2\right) = \bar{a}_2.\left(\bar{b}_1 \times \bar{b}_2\right)$

Hence, the given lines are coplanar.

The plane determined by these lines is given by

$\therefore \bar{r}.\left(\bar{b}_1 \times \bar{b}_2\right) = \bar{a}_1.\left(\bar{b}_1 \times \bar{b}_2\right)$

i.e. $\bar{r}.\left(-\hat{i} + 2\hat{j} - \hat{k}\right)$ = 7

Hence, the given lines are coplnar and the equation of the plane determined bt these lines is

$\bar{r}.\left(-\hat{i} + 2\hat{j} - \hat{k}\right)$ = 7.

Miscellaneous Exercise 6 B | Q 17 | Page 226

Solve the following :

Find the distance of the point $3\hat{i} + 3\hat{j} + \hat{k}$ from the plane $\bar{r}.\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$ = 21.

Solution:

The distance of the point $A(\bar{a})$ from the plane $\bar{r}.\bar{n} = p$ is given by $d = \dfrac{|\bar{a}.\bar{n} - p|}{|\bar{n}|}$ ...(1)

Here, $\bar{a} = 3\hat{i} + 3\hat{j} + \hat{k}, \bar{n} = 2\hat{i} + 3\hat{j} + 6\hat{k}$, p = 21

$\therefore \bar{a}.\bar{n} = \left(3\hat{i} + 3\hat{j} + \hat{k}\right).\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

= (3)(2) + (3)(3) + (1)(–6)

= 6 + 6 – 6

= 6

Also, $|\bar{n}| = \sqrt{3^2 + 3^2 + (-6)^2} = \sqrt{-12}$ = 0

$\therefore$ from (1), the required distance

= $\dfrac{|-12 - 21|}{12}$

= 0 units.

Miscellaneous Exercise 6 B | Q 18 | Page 226

Solve the following :

Find the distance of the point (13, 13, – 13) from the plane 3x + 4y – 12z = 0.

Solution:

The distance of the point (x_1, y_1, z_1) from the plane ax + by + cz + d = 0 is $\left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$

∴ the distance of the point (1, 1, – 1) from the plane 3x + 4y – 12z = 0 is $\left|\frac{3(1) + 4(1 - 12(-1))}{\sqrt{3^2 + 4^2 + (-12)^2}}\right|$

= $\left|\frac{3 + 4 + 12}{\sqrt{9 + 16 + 144}}\right|$

= $\frac{19}{\sqrt{169}}$

= $\frac{19}{13}$

= 19units.

Miscellaneous Exercise 6 B | Q 19 | Page 226

Solve the following :

Find the vector equation of the plane passing through the origin and containing the line $\bar{r} = \left(\hat{i} + 4\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + \hat{k}\right)$.

Solution:

The vector equation of the plane passing through $A(\bar{a})$ and perpendicular to the vector $\bar{n}$ is $\bar{r}.\bar{n} = \bar{a}.\bar{n}$ ...(1)

We can take $\bar{a} = \bar{0}$ since the plane passes through the origin.

The point M with position vector $\bar{m} = \hat{i} + 4\hat{j} + \hat{k}$ lies on the line and hence it lies on the plane.

$\therefore \overline{OM} = \bar{m} = \hat{i} + 4\hat{j} + \hat{k}$ lies on the plane.

The plane contains the given line which is parallel to $\bar{b} = \hat{i} + 2\hat{j} + \hat{k}$.

Let $\bar{n}$ be normal to the plane. Then $\bar{n}$ is perpendicular to $\overline{OM}$ as well as $\bar{b}$

$$\therefore \bar{n} = \overline{OM} \times \bar{b} = \begin{vmatrix} \hat{i} & & \\ 1 & 4 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$= (4-2)\hat{i} - (1-1)\hat{j} + (2-4)\hat{k}$

$= 2\hat{i} - 2\hat{k}$

$\therefore$ from (1), the vector equation of the required plane is

$\bar{r}.\left(2\hat{i} - 2\hat{k}\right) = \bar{0}.\bar{n} = 0$

i.e. $\bar{r}.\left(\hat{i} - \hat{k}\right) = 0$.

Miscellaneous Exercise 6 B | Q 20 | Page 226

Solve the following :

Find the vector equation of the plane which bisects the segment joining A(2, 3, 6) and B(4, 3, –2) at right angle.

Solution:

The vector equation of the plane passing through $A(\bar{a})$ and perpendicular to the vector $\bar{n}$ is $\bar{r}.\bar{n} = \bar{a}.\bar{n}$ ...(1)

The position vectors $\bar{a}$ and $\bar{b}$ of the given points A and B are

$\bar{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ and $\bar{b} = 4\hat{i} + 3\hat{j} - 2\hat{k}$

If M is the midpoint of segment AB, the position vector $\bar{m}$ of M is given by

$$\bar{m} = \frac{\bar{a} + \bar{b}}{2}$$

$$= \frac{\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right) + \left(4\hat{i} + 3\hat{j} - 2\hat{k}\right)}{2}$$

$$= \frac{6\hat{i} + 6\hat{j} + 4\hat{k}}{2}$$

$$= 3\hat{i} + 3\hat{j} + 2\hat{k}$$

The plane passes through $M(\bar{m})$.

AB is perpendicular to the plane

If $\bar{n}$ is normal to the plane, then $\bar{n} = \overline{AB}$

$$\therefore \bar{n} = \bar{b} - \bar{a} = \left(4\hat{i} + 3\hat{j} - 2\hat{k}\right) - \left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

$$= 2\hat{i} - 8\hat{k}$$

$$\therefore \bar{m}.\bar{n} = \left(3\hat{i} + 3\hat{j} + 2\hat{k}\right).\left(2\hat{i} - 8\hat{k}\right)$$

= (3)(2) + (3)(0) + (2)(– 8)

= 6 + 0 – 16

= – 10

$\therefore$ from (1), the vector equation of the required plane is

$$\bar{r}.\bar{n} = \bar{m}.\bar{n}$$

i.e. $\bar{r}.\left(2\hat{i} - 8\hat{k}\right)$ = – 10

i.e. $\bar{r}.\left(\hat{i} - 4\hat{k}\right)$ = – 5.

Miscellaneous Exercise 6 B | Q 21 | Page 226

Solve the following :

Show that the lines x = y, z = 0 and x + y = 0, z = 0 intersect each other. Find the vector equation of the plane determined by them.

Solution:

Given lines are x = y, z = 0 and x + y = 0, z = 0.

It is clear that (0, 0, 0) satisfies both the equations.

∴ the lines intersect at O whose position vector is $\bar{0}$

Since z = 0 fr both the lines, both the lines ie in XY-plane.

Hence, we have to find equation oXY-ane.

Z-axis is perpendicular to XY-plane.

∴ normal to XY plane is $\hat{k}$.

$O\left(\bar{0}\right)$ lies on the plane.

By using $\bar{r}.\bar{n} = \bar{a}.\bar{n}$, vecttor equation of the required plane is
$\bar{r}.\hat{k} = \bar{0}.\bar{k}$

i.e. $\bar{r}.\hat{k}$ = 0.

Hence, the given lines intersect each other and the vector equation of the plane determine by them is $\bar{r}.\hat{k}$ = 0.