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CS311 HW1

3-3)
a.

$g_1=2^{2^{(n+1)}}$
$g_2=2^{2^n}$
$g_3=(n+1)!$
$g_4=n!$
$g_5=e^n$
$g_6=n2^n$
$g_7=2^n$
$g_8=(3/2)^n$
$g_9=n^3$
$g_{10}=\log(n)^{\log(n)}$
$g_{11}=n^{\log(\log(n))}$
$g_{12}=\log(n)!$
$g_{13}=n^2$
$g_{14}=4^{\log(n)}$
$g_{15}=n\log(n)$
$g_{16}=\log(n!)$
$g_{17}=2^{\log(n)}$
$g_{18}=n$
$g_{19}=\mathrm{sqrt}(2)^{\log(n)}$
$g_{20}=2^{\mathrm{sqrt}(2\log(n))}$
$g_{21}=\ln(n)$
$g_{22}=\mathrm{sqrt}(\log(n))$
$g_{23}=\log^2(n)$
$g_{24}=\ln(\ln(n))$
$g_{25}=2^{\log^*(n)}$
$g_{26}=\log^*(n)$
$g_{27}=\log^*(\log(n))$
$g_{28}=\log(\log^*(n))$
$g_{29}=n^{1/\log(n)}$
$g_{30}=1$

Equivalence:

g_{15}, g_{16} $\theta(n\log(n))$: $\lim\limits_{n\to\infty}\frac{n\log(n)}{\log(n!)} \approx \lim\limits_{n\to\infty}\frac{n\log(n)}{\log(n^n)}=1$

g_{17},g_{18} $\theta(n)$: $\lim\limits_{n\to\infty}\frac{2^{\log(n)}}{n}=\lim\limits_{n\to\infty}\frac{n}{n}=1$

g_{13},g_{14} $\theta(n^2)$: $\lim\limits_{x\to\infty}\frac{4^{\log(n)}}{n^2}=\lim\limits_{x\to\infty}\frac{2^{2\log(n)}}{n^2}\lim\limits_{x\to\infty}\frac{n^2}{n^2}=1$

g_{29},g_{30} $\theta(1)$ $:\lim\limits_{x\to\infty}\frac{n^{1/\log(n)}}{1}=\lim\limits_{x\to\infty}\frac{n^{\frac{\ln(2)}{\ln(n)}}}{1}=2$. this also shows g_{29} is Omega(G_{30}) since the limit rule gives a constant factor of 2 in terms of asymptotic growth rate.

g_{10},g_{11} $\theta(\log(n)^{\log(n)})$ $:\lim\limits_{n\to\infty}\frac{n^{\log(\log(n))}}{\log(n)^{\log(n)}}=\lim\limits_{n\to\infty}\frac{\log(n)^{\log(n)}}{\log(n)^{\log(n)}}=1$

Reasoning:
First, I found the fastest growing functions and sorted them by growth rate, as there are only a few exponential growth functions. g_1 grows faster than g_2 as $2^{n+1}>2^n$ for all n, and they are not in an equivalence class because $\lim\limits_{n\to\infty}\frac{2^{2^{n+1}}}{2^{2^n}}=\lim\limits_{n\to\infty}2^{2^{n+1}}-2^n=\infty$.

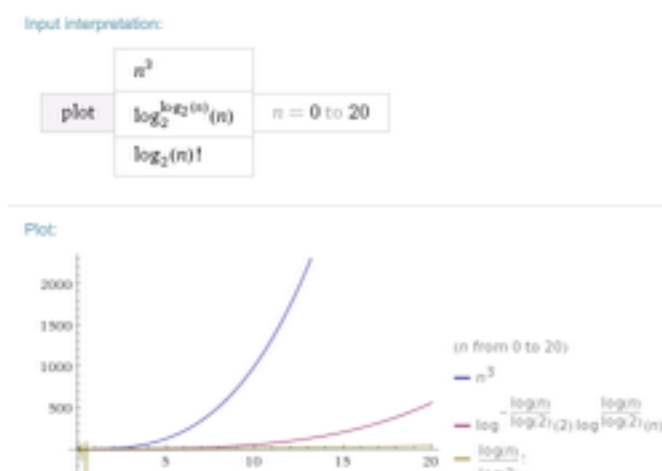
The next functions of concern are factorials, as the lower bound on a factorial growth is 2^n and upper bound is n^n . The factorial is obviously slower growing than the previous function, which grows $\lim\limits_{n\to\infty}\frac{2^{2^n}}{2^n}=\lim\limits_{n\to\infty}2^{2^n-n}$ times faster than 2^n.

The factorial grows faster than all power functions near 2^n. So, the factorial functions g_3,g_4 must be ordered after g_2 and before g_5, the next fastest growing power function, e^n.

g_5 is before g_6 because the limit rule shows that $\lim\limits_{n\to\infty}\frac{e^n}{n2^n}=\lim\limits_{n\to\infty}\frac{(\frac{e}{2})^n}{n}=\infty$.

The next three power functions, $n2^n$,2^n, and 1.5^n are self-explanatory. $n2^n$ is faster growing than 2^n by a factor of n, which also shows that they are not in an equivalence class, because theta time requires functions to differ by a constant factor that does not depend on n. 2^n has a greater base than 1.5^n , so it is faster growing.

The next order of growth is exponentials, and the fastest growing exponential function due to having the highest power is n^3. Following that, I plotted g_{10} and g_{11} relative to the functions above and below them :



Since it has already been proven that g_{10} and g_{11} are the same function, I only plotted one of them. By the plot, it is clear what the asymptotic growth ordering is by the properties of a continuous function. The limits as n->infinity of g_9/g_{10} and g_{10}/g_{12} also agree with this ordering (both diverge towards infinity).

For g_{12} and g_{13}, I used a WolframAlpha query ("lim n->inf (log_2(n)!)/(n^2)") to find that the limit tends towards infinity, meaning log(n)! grows faster than n^2.

g_{13} and g_{14} are the same function, as are 15 and 16. nlog(n) must grow faster than n due to the additional log(n) factor, but slower than n^2 since log(n) < n. The ordering is obvious now since 17 and 18 are the same function.

19 is ordered as follows since $2^{\log(n)} = n$, so with a base of sqrt(2), it must grow less than n. Though 20 has a higher base than 19, the limit of g_{19}/g_{20} as n->infinity diverges to infinity.

21 grows the fastest of the remaining functions, which are mostly logarithmic functions besides the proven constant time functions (29 and 30). This is apparent because the rest of the logarithmic functions involve application of a number of functions that result in numbers less than a single logarithm.

22 is above 23 because applying a sqrt to a number is always greater than applying a log. 23 is above 24 because log base 2 grows faster than the natural log because the natural log is base *e*.

25 is ordered as it is because the iterated logarithm grows at an extremely slow rate, to where log*(n) <= 5 for any practical purposes can almost be assumed. Although this is true, this is still a power function and it does grow faster than the iterated log itself, and trivially faster than a constant function.

26 is there because it is the first iterated log, and the other iterated logs involve applying another logarithm function, so it is trivially greater than those(27, 28).

For 27, from the definition of the iterated log, log*(log n) = log*(n) - 1, and this is $\theta(\log^*(n))$, which is clearly greater compared to 28, which is $\theta(\log(\log^*(n)))$.

29 and 30 are both constant time and 29 has already been proven to greater than 30 by a constant factor of 2.