

24.1-1.)

For lines 2-5:

Here I use the order given in Fig 24.4

1. (z,x) x.d=7 x. π =z
(z,s) s.d=2 s. π =z
(s,t) t.d=8 t. π =s
(s,y) y.d=9 y. π =s
2. (t,x) nop
(t,y) nop
(t,z) nop
(x,t) t.d=5 t. π =x
(y,x) x.d=6 x. π =y
(y,z) nop
(z,x) nop
(z,s) nop
(s,t) nop
(s,y) nop
3. (t,x) nop
(t,y) nop
(t,z) nop
(x,t) t.d=4 t. π =x
(y,x) nop
(y,z) nop
(z,x) nop
(z,s) nop
(s,t) nop
(s,y) nop
4. (t,x) nop
(t,y) nop
(t,z) nop
(x,t) nop
(y,x) nop
(y,z) nop
(z,x) nop
(z,s) nop
(s,t) nop
(s,y) nop

After the 4 iterations:

z.d=0 z. π =NIL

x.d=6 x. π =y

t.d=4 t. π =x

y.d=9 y. π =s

s.d=2 s. π =z

BELLMAN-FORD returns true starting at z.

Starting from z with $w(z,x)=4$

1. (z,x) $x.d=4$ $x.\pi=z$
(z,s) $s.d=2$ $s.\pi=z$
(s,t) $t.d=8$ $t.\pi=s$
(s,y) $y.d=9$ $y.\pi=s$
2. (t,x) nop
(t,y) nop
(t,z) nop
(x,t) $t.d=2$ $t.\pi=x$
(y,x) nop
(y,z) nop
(z,x) nop
(z,s) nop
(s,t) nop
(s,y) nop
3. (t,x) nop
(t,y) nop
(t,z) $z.d=-2$ $z.\pi=t$
(x,t) nop
(y,x) nop
(y,z) nop
(z,x) $x.d=2$ $x.\pi=z$
(z,s) $s.d=0$ $s.\pi=z$
(s,t) nop
(s,y) $y.d=7$ $y.\pi=s$
4. (t,x) nop
(t,y) nop
(t,z) nop
(x,t) $t.d=0$ $t.\pi=x$
(y,x) nop
(y,z) nop
(z,x) nop
(z,s) nop
(s,t) nop
(s,y) nop

Final values:

$z.d=-2$ $z.\pi=t$

$x.d=2$ $x.\pi=z$

$t.d=0$ $t.\pi=x$

$y.d=7$ $y.\pi=s$

$s.d=0$ $s.\pi=z$

The negative-weight cycle check on lines 5-7 will cause the algorithm to return false on the check for (t,z) because $z.d > t.d + w(t,z)$ or $-2 > 0 - 4$.

24.1-3.)

```
RELAX(u,v,w):  
  if  $v.d > u.d + w(u,v)$   
     $v.d = u.d + w(u,v)$   
     $v.\pi = u$   
  return true  
else return false
```

```
BELLMAN-FORD(G,w,s):  
  INITIALIZE-SINGLE-SOURCE(G,s)  
  changed = false  
  while (changed)  
    changed = false  
    for each edge (u,v) in G.E  
      changed = RELAX(u,v,w)  
  for each edge (u,v) in G.E  
    if  $v.d > u.d + w(u,v)$   
      return false  
  return true
```

Since it is unknown what m is, all edges must be relaxed to their final shortest-path values, which will take m iterations. On the $m+1$ th iteration, no values will change by the upper-bound property and the algorithm will terminate. The original BELLMAN-FORD makes $V-1$ iterations, which may make more iterations than $m+1$ if $m+1 < V-1$.

24.3-1.)

Assuming same-priority nodes are disambiguated in ascending alphabetical order.

Vertex s as source:

1. $u = s$
 $S = \{s\}$
 (s,t) $t.d=3$ $t.\pi=s$
 (s,y) $y.d=5$ $y.\pi=s$
2. $u = t$
 $S = \{s,t\}$
 (t,x) $x.d=9$ $x.\pi=t$
 (t,y) NOP
3. $u = y$
 $S = \{s,t,y\}$
 (y,t) NOP
 (y,x) NOP
 (y,z) $z.d=11$ $z.\pi=y$
4. $u = x$
 $S = \{s,t,y,x\}$
 (x,z) NOP
5. $u = z$
 $S = \{s,t,y,x,z\}$

(z,s) NOP

(z,x) NOP

Shortest paths:

z.d=11 z. π =y

x.d=9 x. π =t

y.d=5 y. π =s

t.d=3 t. π =s

s.d=0 s. π =NIL

Vertex z as source:

1. u = z

S = {z}

(z,s) s.d=3 s. π =z

(z,x) x.d=7 x. π =z

2. u = s

S = {z,s}

(s,t) t.d =6 t. π =s

(s,y) y.d=8 y. π =s

3. u = t

S = {z,s,t}

(t,y) NOP

(t,x) NOP

4. u = x

S = {z,s,t,x}

(x,z) NOP

5. u = y

S = {z,s,t,x,y}

(y,t) NOP

(y,x) NOP

(y,z) NOP

Shortest Paths:

y.d=8 y. π =s

t.d=6 t. π =s

x.d=7 x. π =z

s.d=3 s. π =z

z.d=0 z. π =NIL