Minimization of DFA Using Equivalence Theorem

Step-01:

- Eliminate all the dead states and inaccessible states from the given DFA (if any).

Dead State

All those non-final states which transit to itself for all input symbols in ∑ are called as dead states.

Inaccessible State

All those states which can never be reached from the initial state are called as inaccessible states.

Step-02:

Draw a state transition table for the given DFA.

- Transition table shows the transition of all states on all input symbols in Σ.

Step-03:

Now, start applying equivalence theorem.

- Take a counter variable k and initialize it with value 0.
- Divide Q (set of states) into two sets such that one set contains all the non-final states and other set contains all the final states.
- This partition is called P_0.

Step-04:

- Increment k by 1.
- Find P_k by partitioning the different sets of P_{k-1} .
- In each set of P_{k-1} , consider all the possible pair of states within each set and if the two states are distinguishable, partition the set into different sets in P_k.

Two states q_1 and q_2 are distinguishable in partition P_k for any input symbol 'a',

if δ (q_1, a) and δ (q_2, a) are in different sets in partition P_{k-1}.

Step-05:

- Repeat step-04 until no change in partition occurs.
- In other words, when you find $P_k = P_{k-1}$, stop.

Step-06:

- All those states which belong to the same set are equivalent.
- The equivalent states are merged to form a single state in the minimal DFA.

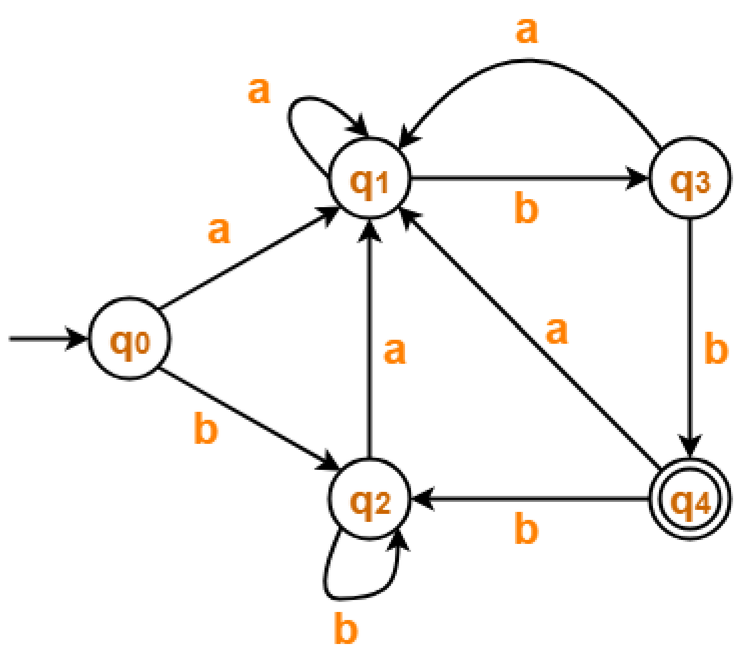
Number of states in Minimal DFA

= Number of sets in P_k

PRACTICE PROBLEMS BASED ON MINIMIZATION OF DFA-

Problem-01:

Minimize the given DFA-



Solution-

Step-01:

The given DFA contains no dead states and inaccessible states.

Step-02:

Draw a state transition table-

	a	b
→q0	q1	q2
q1	q1	q3

q2	q1	q2
q3	q1	*q4
***q4**	q1	q2

Step-03:

Now using Equivalence Theorem, we have-

P_0 = { q_0 , q_1 , q_2 , q_3 } { q_4 }

P_1 = { q_0 , q_1 , q_2 } { q_3 } { q_4 }

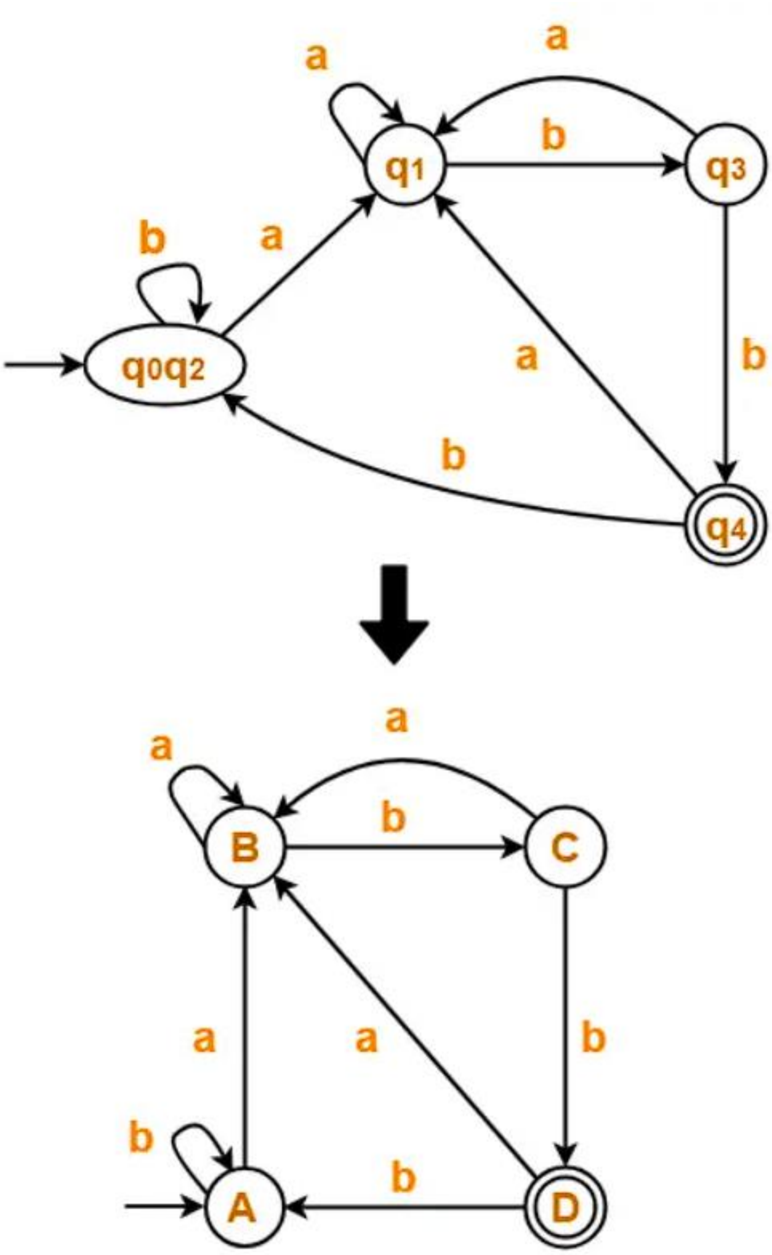
P_2 = { q_0 , q_2 } { q_1 } { q_3 } { q_4 }

P_3 = { q_0 , q_2 } { q_1 } { q_3 } { q_4 }

Since $P_3 = P_2$, so we stop.

From P_3, we infer that states q_0 and q_2 are equivalent and can be merged together.

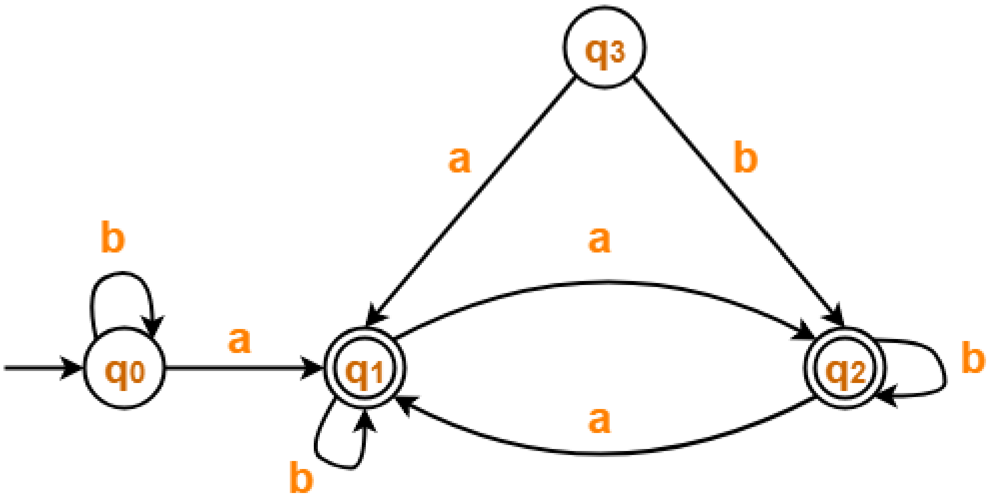
So, Our minimal DFA is-



Minimal DFA

Problem-02:

Minimize the given DFA-

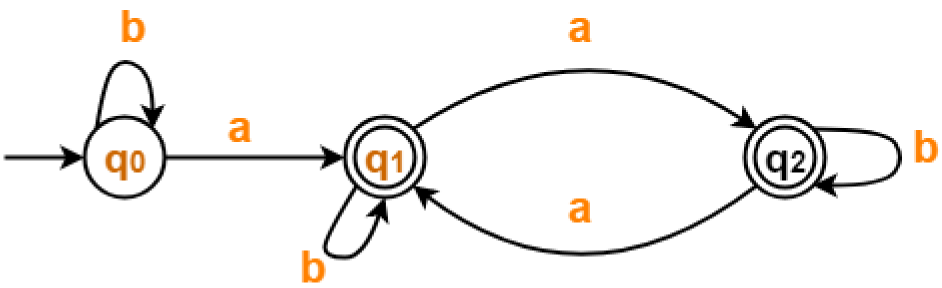


Solution-

Step-01:

- State q_3 is inaccessible from the initial state.
- So, we eliminate it and its associated edges from the DFA.

The resulting DFA is-



Step-02:

Draw a state transition table-

	a	b

→q0	*q1	q0
***q1**	*q2	*q1
***q2**	*q1	*q2

Step-03:

Now using Equivalence Theorem, we have-

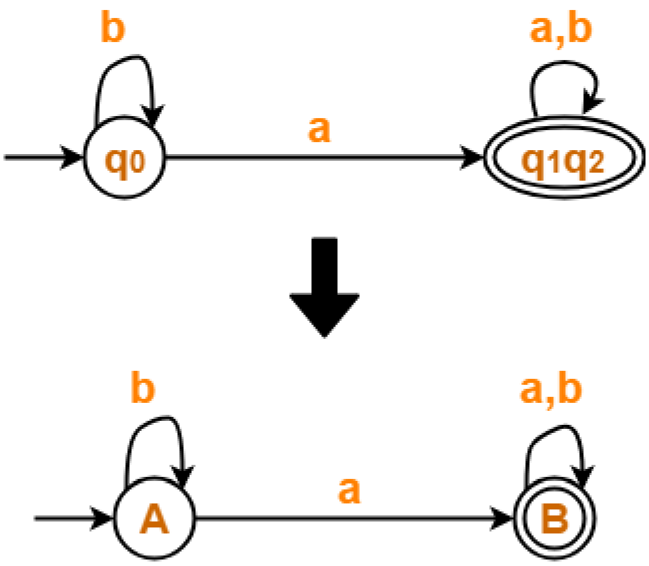
$P_0 = \{ q_0 \} \{ q_1 , q_2 \}$

$P_1 = \{ q_0 \} \{ q_1 , q_2 \}$

Since $P_1 = P_0$, so we stop.

From P_1, we infer that states q_1 and q_2 are equivalent and can be merged together.

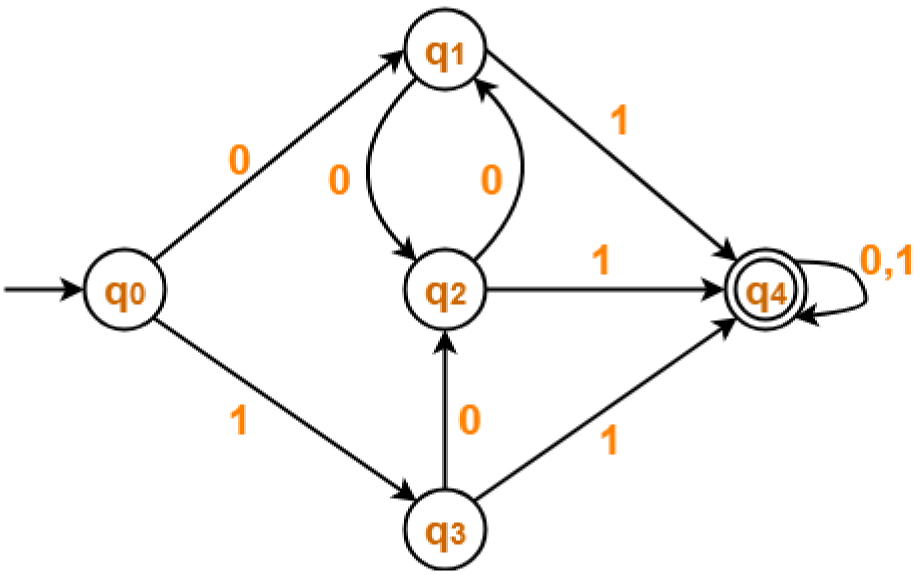
So, Our minimal DFA is-



Minimal DFA

Problem-03:

Minimize the given DFA-



Solution-

Step-01:

The given DFA contains no dead states and inaccessible states.

Step-02:

Draw a state transition table-

	0	**1**
→q0	q1	q3

q1	q2	*q4
q2	q1	*q4
q3	q2	*q4
***q4**	*q4	*q4

Step-03:

Now using Equivalence Theorem, we have-

$P_0 = \{ q_0 , q_1 , q_2 , q_3 \} \{ q_4 \}$

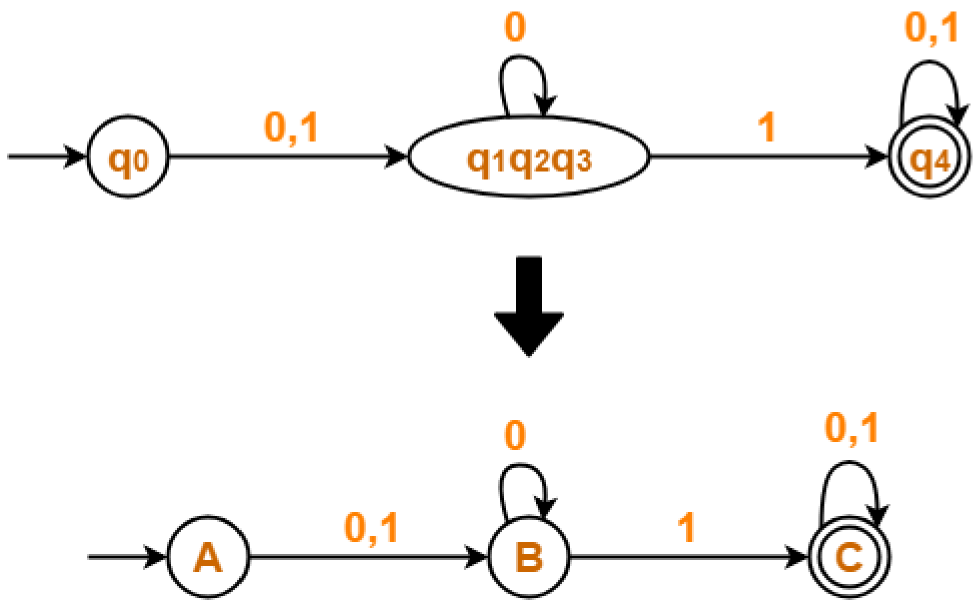
$P_1 = \{ q_0 \} \{ q_1 , q_2 , q_3 \} \{ q_4 \}$

$P_2 = \{ q_0 \} \{ q_1 , q_2 , q_3 \} \{ q_4 \}$

Since $P_2 = P_1$, so we stop.

From P_2, we infer that states q_1 , q_2 and q_3 are equivalent and can be merged together.

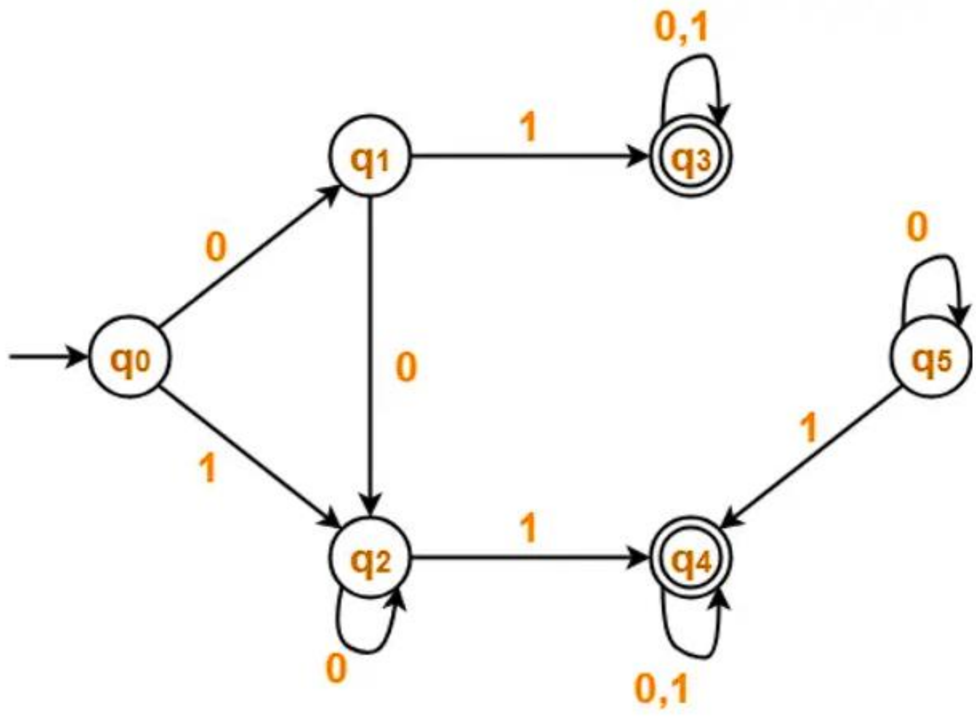
So, Our minimal DFA is-



Minimal DFA

Problem-04:

Minimize the given DFA-

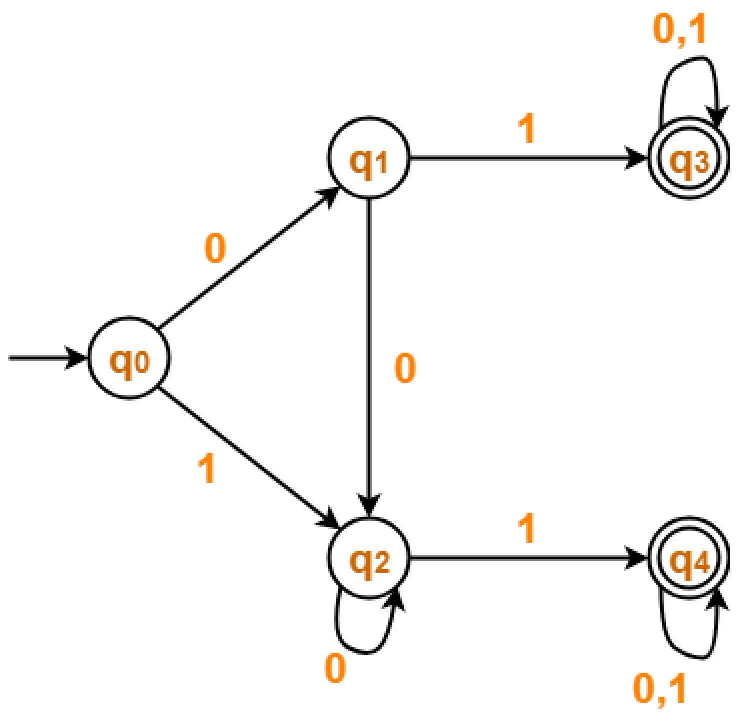


Solution-

Step-01:

- State q_5 is inaccessible from the initial state.
- So, we eliminate it and its associated edges from the DFA.

The resulting DFA is-



Step-02:

Draw a state transition table-

	0	**1**
→q0	q1	q2
q1	q2	*q3
q2	q2	*q4
***q3**	*q3	*q3
***q4**	*q4	*q4

Step-03:

Now using Equivalence Theorem, we have-

$P_0 = \{ q_0 , q_1 , q_2 \} \{ q_3 , q_4 \}$

$P_1 = \{ q_0 \} \{ q_1 , q_2 \} \{ q_3 , q_4 \}$

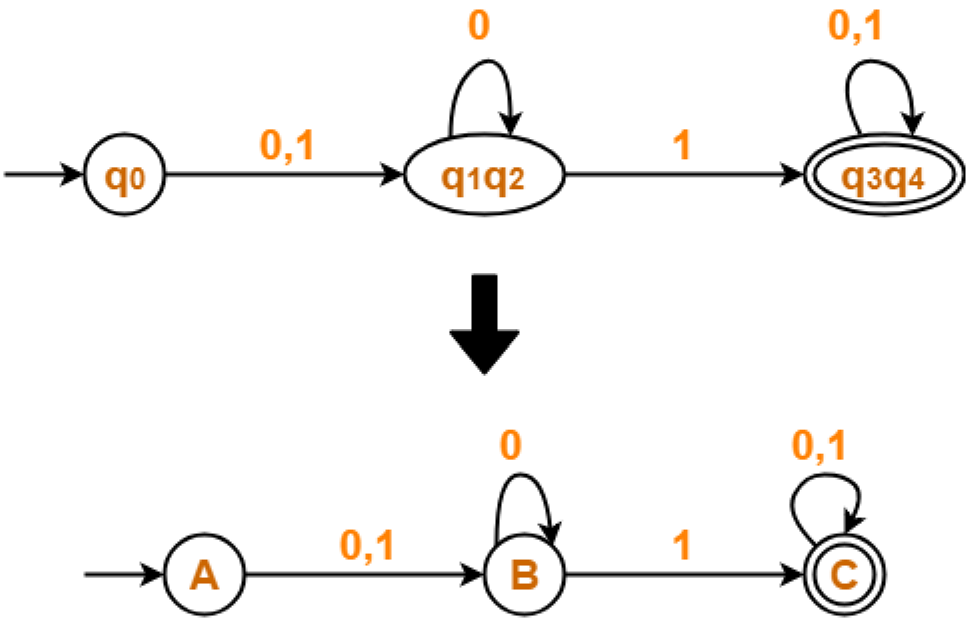
$P_2 = \{ q_0 \} \{ q_1 , q_2 \} \{ q_3 , q_4 \}$

Since $P_2 = P_1$, so we stop.

From P_2, we infer-

- States q_1 and q_2 are equivalent and can be merged together.
- States q_3 and q_4 are equivalent and can be merged together.

So, Our minimal DFA is-



Minimal DFA