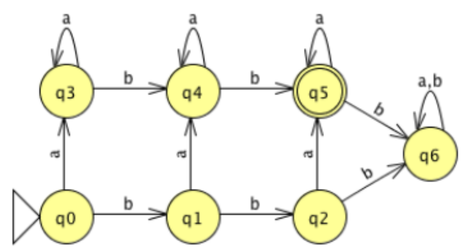
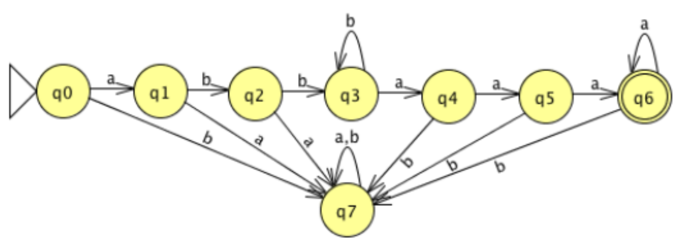
Deterministic Finite State Automata Examples

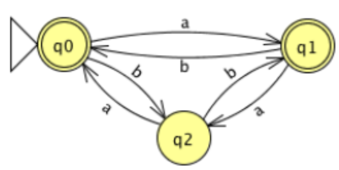
1. For $\Sigma = \{a, b\}$, construct DFA that accepts all strings with at least one a and exactly two b's.
   Ans. The following graph represents the DFA $M = (\{q_0, q_1, \ldots, q_6\}, \{a, b\}, \delta, q_0, \{q_5\})$ that accepts all strings with at least one a and exactly two b's, where δ is described as in the graph.



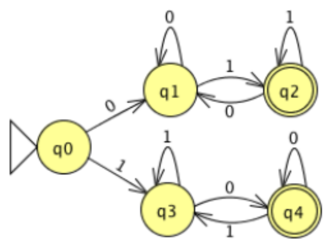
2. Given DFA for the language $L = \{ab^n a^m : n \geq 2, m \geq 3\}$.
   Ans. The following graph represents the DFA $M = (\{q_0, q_1, \ldots, q_7\}, \{a, b\}, \delta, q_0, \{q_6\})$ that accepts $L = \{ab^n a^m : n \geq 2, m \geq 3\}$, where δ is described as in the graph.



3. For $\Sigma = \{a, b\}$, find DFA for the language $L = \{w : (n_a(w) + 2n_b(w)) \bmod 3 < 2\}$.
   Ans. The following graph represents the DFA $M = (\{q_0, q_1, q_2\}, \{a, b\}, \delta, q_0, \{q_0, q_1\})$ that accepts $L = \{w : (n_a(w) + 2n_b(w)) \bmod 3 < 2\}$, where δ is described as in the graph.

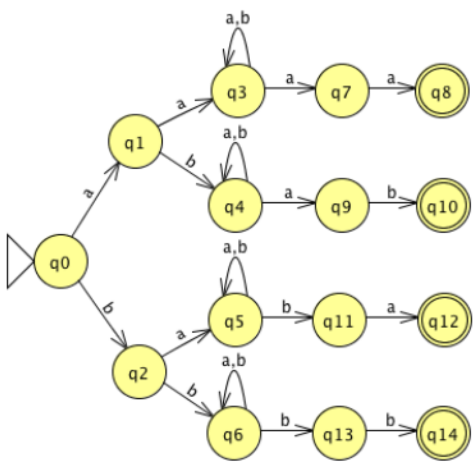


4. Construct an accepting DFA for the set of strings on $\{0, 1\}$ defined by the requirement: The leftmost symbol differs from the right most one.
   Ans. The following graph represents the DFA $M = (\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \delta, q_0, \{q_2, q_4\})$



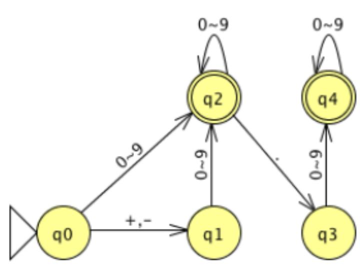
that accepts all strings that the leftmost symbol differs from the right most one, where δ is described as in the graph.

5. Show that the language $L = \{vwv : v, w \in \{a, b\}^*, |v| = 2\}$ is regular.
Ans. The following shows an NFA $M = (\{q_0, q_1, q_2, \ldots, q_{14}\}, \{a, b\}, \delta, q_0, \{q_8, q_{10}, q_{12}, q_{14}\})$ that accepts $L = \{vwv : v, w \in \{a, b\}^*, |v| = 2\}$, where δ is described as in the graph.



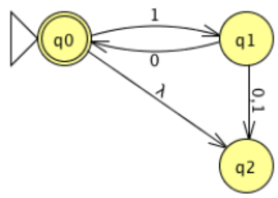
By theorem, we know that if L is accepted by an NFA, there exists a DFA that accepts L. Therefore, L is regular.

6. Show that the set of all real numbers in $\mathbb{C}$ is a regular language.
Ans. The following shows an NFA $M = (\{q_0, q_1, \ldots, q_4\}, \{+, -, ., 0, 1, \ldots, 9\}, \delta, q_0, \{q_2, q_4\})$ that accepts the set of all real numbers in $\mathbb{C}$, where δ is described as in the graph.



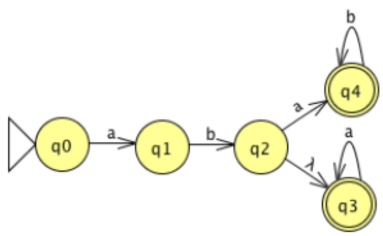
By theorem, we know that if L is accepted by an NFA, there exists a DFA that accepts L. Therefore, L is regular.

7. For the NFA in the following figure, find $\delta^*(q_0, 100)$, $\delta^*(q_1, 01)$, $\delta^*(q_0, 1010)$, and $\delta^*(q_1, 00)$.

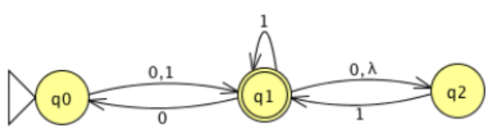


Ans. $\delta^*(q_0, 100) = \emptyset$, $\delta^*(q_1, 01) = \emptyset$, $\delta^*(q_0, 1010) = \{q_0, q_2\}$, and $\delta^*(q_1, 00) = \emptyset$. λ

8. Design an NFA with no more than five states for the set $\{abab^n : n \geq 0\} \cup \{aba^n : n \geq 0\}$.
Ans. The following is an NFA $M = (\{q_0, q_1, \ldots, q_4\}, \{a, b\}, \delta, q_0, \{q_3, q_4\})$ that accepts the set $\{abab^n : n \geq 0\} \cup \{aba^n : n \geq 0\}$, where δ is described as in the graph.

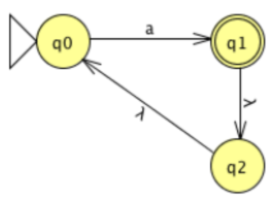


9. Which of the strings 00, 01001, 10010, 000, 0000 are accepted by the following NFA?



Ans. 01001 and 000 are accepted.

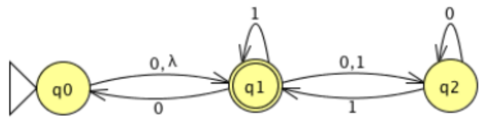
10. What is the complement of the language accepted by the following NFA?



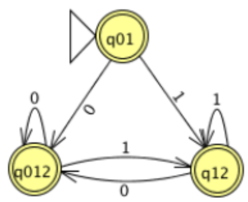
Ans. This NFA accepts $L = \{a^n : n \geq 1\}$. The complement of L is

$$\overline{L} = \{a^n : n < 1\} = \{a^0\} = \{\lambda\}.$$

11. Convert the following NFA into an equivalent DFA.



Ans. The following is the result DFA $M = (\{q_{01}, q_{012}, q_{12}\}, \{0,1\}, \delta, q_{01}, \{q_{01}, q_{012}, q_{12}\})$, where δ is described as in the graph.



12. Show that if L is regular, so is L^R.
Ans. Informally, the following procedure shows that we can always construct a finite accepter M_{L^R} that accepts L^R for a given finite accepter M_L that accepts L.

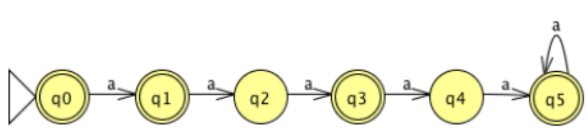
- Reverse all transitions in M_L.
- Add a new initial stare q_s and generate λ-transitions from q_s to each of the final states in M_L.
- Turn all final states of M_L into normal states of M_{L^R} and turn the initial states of M_L into a final state of M_{L^R}.

Formally, let $M_L = (Q_L, \Sigma, \delta_L, q_0, F_L)$ be an NFA that accepts L. The NFA M_{L^R} defined below accepts L^R.

(a) $M_{L^R} = (Q_L \cup \{q_s\}, \Sigma, \delta_{L^R}, q_s, \{q_0\})$ and $q_s \notin Q_L$.
(b) $\delta_L(q_i, a) = q_j \iff \delta_{L^R}(q_j, a) = q_i$ for all $a \in \Sigma$ and $q_i, q_j \in Q_L$.
(c) $\delta_{L^R}(q_s, \lambda) = q$ for all $q \in F_L$.

From (b), if $w \in L(M_L)$, $\delta_L^*(q_0, w) = q_j \in F_L \iff \delta_L^*(q_j, w) = q_0$. From (c), $\delta_{L^R}(q_s, \lambda) = q_j$ for all $q_j \in F_L$. Thus, we have that for all $w \in L(M_L)$, $w^R \in L(M_{L^R})$.

13. Find minimal DFA for the language $L = \{a^n : n \neq 2 \text{ and } n \neq 4\}$. You have to prove that the result is minimal.
Ans. The following is the result DFA $M = (\{q_0, q_1, \ldots, q_5\}, \{a\}, \delta, q_0, \{q_0, q_1, q_3, q_5\})$, where δ is described as in the graph.



By using the mark procedure, we can finally partition the state set as $\{q_0\}$, $\{q_1\}$, $\{q_2\}$, $\{q_3\}$, $\{q_4\}$, and $\{q_5\}$. Thus, such a DFA is minimal.