Computational Rocketry Lab

May 30, 2018

Introduction

This assignment will provide an introduction to simulations. The complexity of the model will increase from 1D projectile motion to a rocket propelled in a gravitational field. The main goal is to become comfortable with the idea of programming physical systems.

Warning: Unless you are extremely careful, you will likely encounter infinite loops. To cancel your program, **hold control (or command) and press c**.

Part 1

In this part you're going to calculate the maximum height of a projectile travelling under the influence of constant gravity. This is a valid approximation only when the change in distance is "small".

Here you'll convert equations of motion into code to solve a set of equations analytically and then you'll simulate it.

Here are some useful equations:

$$\frac{dx}{dt} = v = \dot{x} \tag{1}$$

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = a = \ddot{x} \tag{2}$$

$$\Delta v = at \tag{3}$$

$$v_f - v_i = gt \tag{4}$$

$$y = v_i t + \frac{1}{2}at^2 \tag{5}$$

$$F = G\frac{m_1 * m_2}{r^2} \tag{6}$$

$$F = \frac{dp}{dt} = m\frac{dv}{dt} + \frac{dm}{dt} * v \tag{7}$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \tag{8}$$

Note, in equations (1) and (2) you see Leibniz notation (left) and Newton's notation (right). Optional: See the wiki article for more information on notation.

To do this, you'll solve for t in (4) and substitute that into (5). You'll make a float and double version and compare the difference (if any). Consider what the instantaneous velocity is at the maximum height, use this value for v_f.

Next, you'll fill two other functions which break the scenario into small chunks of time. In a loop, you'll use the definitions ((1) and (2)) to update the current velocity and position. Set the simulated velocity to the initial velocity and the initial simulated height to 0 since we're only concerned with the change in height for the projectile. Add the change in velocity to the simulated velocity then add the change in height to the simulated height (this order only matters for grading purposes). You'll then create a valid condition which will exit the loop and finally return the height.

Your main loop will print the analytic versions and then the simulated versions (first double then float).

Note: For your testing purposes, there are some initial values for gravity, velocity, and time resolution. These values will change during the automated testing so switch to cin for the submission.

Part 2

In this part you're going to calculate the maximum height of a projectile travelling under the influence of a **gravitational field**. This will be valid for any change in distance, but assumes the gravitational body is a point mass which is valid for points above the surface of the planet.

You'll be comparing the simulation from before (with a constant gravitational acceleration) to your new version which will calculate gravity from the current height using the law of gravity . Using (6) and (7) you can solve for the gravitational acceleration g.

Part 3

Here you'll be simulating a rocket which burns fuel to increase its speed in a gravitational field. Use the conservation of momentum equation update the rocket's speed. To do this, calculate the change in the rocket's velocity by considering the rocket's rest frame. Add this change in velocity to the rocket's current velocity also add the change from gravity, subtract the mass, and update the height. Continue the simulation while the ratio of the rocket's mass to the original mass is grater than the ship+payload ratio. When this ratio is 0.1 (as is initially given) this means 90% of the original mass is fuel. Finally return the height at which all fuel is burned. Note that the max height is not defined if the velocity exceeds escape velocity

Further Exploration

For those of you who wish to continue, consider taking into account air resistance, rotation of the earth, the gravitational attraction of the sun and moon, and multiple burn stages.