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**Properties and Applications of**

**Bose-Einstein Condensation**

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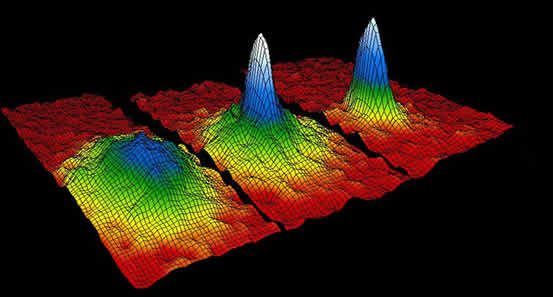


Figure 1: This is the classic image of BEC which uses a computer to generate "false color density countours.” The temperature ranges from left to right are 400nK, 200nK, and 50nK. (Cornell 93)

Bose-Einstein Condensation (BEC) is “The creation of laser-like atoms!” (Ketterle 2:44) This is done by super cooling atoms until their wave functions overlap. The term super is not used lightly here. One may consider the North Pole cold, or perhaps the farthest regions of deep space, but even that is a furnace in comparison to the temperatures needed to reach Bose Einstein Condensation. While the temperature in deep space ranges on the order of a few Kelvin, atoms reach BEC between 500 and 2 (Ketterle 120).

Much can be said about the development of BEC since its early days to its experimental discovery in 1995 by Eric Cornell, Carl Wieman, and Wolfgang Ketterle. The original paper was written in 1925 by an Indian Physicist named Satyendranath Bose as an alternate way to “derive the black-body spectrum.”(Ketterle 119) He had trouble getting it published and sent it to Einstein who was so impressed he personally translated it into German. Einstein then published several papers extending Bose’s theory to include atoms. Initially, no one, not even Einstein himself took BEC seriously. Yet, each step closer yielded new and exciting physics; like superconductivity in 1911 and the superfluidity of Helium in 1938 and again in 1972 (Ketterle 118). It took London and Tisza connecting superfluidity to BEC to resurrect this idea (Cornell 78).

In Ketterle’s Nobel Lecture, he speaks of how (just months before discovery) his NSF proposal reviewers regarded the possibility of him discovering It (129). In spite of their grim outlook they still funded him. BEC was a considered a subfield but is now considered its own field with a far reaching range of phenomenon in physics (Griffin 1). In just a few years it turned from something no one thought was possible (or if it was easily replicable) to the *coolest* thing in physics. By the time Cornell, Wieman, and Ketterle were awarded the Nobel Prize (in 2001) already 2000 articles had been written. Now, there are over 8000 articles which deal with a variety of very advanced topics.

It is helpful to see how BEC developed to understand it. When there are a large number of particles it is nearly impossible to calculate either Newton’s laws or Schrodinger’s equation (Serway 334). Currently, computers are not powerful enough to perform individual calculations of the many atoms needed for real life applications. In this case it is necessary to consider a statistical approach which deals with the probabilities of an entire group of particles. This work began largely in the late 1800s by Maxwell, Boltzmann, and Gibbs (Serway 335).

The result was Maxwell-Boltzmann statistics, yet, these statistics are not always valid. In order to check one must use the uncertainty principle. If the value of the uncertainty of particle position is much less than the “average distance between particles” then it is valid (Modern 344). One can then easily derive the following equation

as the condition to check for validity where N is the number of particles, V is the volume, m is the mass, and T is temperature (Serway 345). If this is not the case then the de Broglie wavelengths overlap and superposition occurs which makes the particles indistinguishable (Cornell 78). Then Bose Einstein Statistics must be used. The equation for the probability of finding a particle at energy E is

This is fundamental to the understanding of BEC, and can be shown pictorially as in figure 2 (Ketterle 121). At high temperatures the atoms act like billiard balls, while at low temperatures the atoms act like waves. When these waves get close enough to interact they lose their identity and form a condensate. At high energies the atoms occupy many different energy levels while at low energies the atoms occupy the lowest energy level. Bosons can do this, while fermions cannot; for fermions must obey the Pauli

Figure 2: This shows the billiard ball properties yielding to wave like properties and finally BEC as the temperature decreases (Ketterle 121).

Exclusion Principal and consequently end up occupying the lowest energy levels.

Another way to look at this is: in boson interactions, occupied energy states attract other bosons, while in fermions occupied states repel each other (Griffin 6). The difference between bosons and fermions is that bosons have integer spins while fermions have one half integer spins. Examples of bosons are the photon, deuteron, and the alpha particle while examples of fermions are the electron, proton and neutron (Serway 347). Fermions can be paired together as in cooper pairs to create an integer spin which negates the Pauli Exclusion Principal. Superconductivity can be described as a condensate of cooper pairs (Ketterle 121). Deborah, research leader of the year (at Boulder), says this could improve superconductors (Jin). Another thing which may help in the development of high temperature superconductors are optical lattices of fermions (Day).

Helium 4 for example is a boson since it has an even number (6) of fermions 2 neutrons, protons, and electrons (de Melo 45). Complications with Helium 4 do arise, due to its strong bosonic interactions, even at zero Kelvin BEC concentration would only be 10% (de Melo 46). BEC exists only within thermodynamically forbidden regions (Cornell 79). Much of the experimental difficulty comes from its location in this forbidden region (Cornell 80). This is because at these temperatures the atoms would form a solid (Leggett 309).

BEC has only been accomplished in a few types of atoms mainly Alkali metals, specifically Rubidium 97, Lithium 7, and Sodium 23 (Dalfovo 465). Laser cooling can only decrease temperature to 100 with about 1 billion atoms (Aftalion 2). The largest of these condensates is still small, on the order of 100 million atoms, while the smallest is as small as a few hundred atoms (Ketterle 120). Magnetic cooling looses several orders of magnitudes of atoms. The final condensate size is about 6 by 100 micrometers and is normally described as cigar shaped (Aftalion 3). Condensates can be as much as 100,000 times larger than ordinary atoms and can even be bigger than human cells (Collins).

We now know that in order to create a Bose-Einstein condensate the atoms of an element must be cooled to extremely low temperatures. Early attempts to achieve BEC began in 1980 with evaporative cooling on Hydrogen (Dalfovo 464). But how is this done? Generally one does this using a combination of techniques: Laser cooling; magnetic cooling; and evaporation. Using a combination of these methods it is possible to reach temperatures approaching 0K.

Magnetic cooling works due to the magnetic dipole moments of the atoms. Because of this a force is exerted on them when they are placed in a magnetic field. The most powerful form of magnetic trap is created by opposing magnetic fields which cause the atoms to circle inward, but this only affects the atoms whose dipole moments are oriented such that they desire to reach the center of the field. Atoms whose spin moments are opposite of this are ejected from the field. One disadvantage of this type of trap is that atoms that reach the center of the field where the field strength is zero have their moment flipped ejecting them from the field. This often becomes a problem when dealing with things very close to 0K like BECs. This is corrected for by either creating an optical plug which prevents the atoms from collecting at this zero point or superimposing a rotating magnetic field over the opposing fields thus preventing the field from being zero (Ketterle 128). By fine tuning this method very low temperatures can be obtained (Savage 250).

Atoms held within such a magnetic trap are then often aided in their cooling by way of laser cooling. The idea of laser cooling is simple though somewhat strange. In order to understand it one must first be comfortable with the idea that photons have momentum. Initially one might expect that transferring momentum to atoms would speed them up, not slow them down, but the momentum of a photon is very small, and in a laser trap the atoms are bombarded with photons from all sides. By bombarding them in this way they are driven towards the center where the net force on them is zero. This is called optical molasses and is used in combination with a different and even stranger idea.

By using a mere single laser beam, tuned to just below the resonance frequency of the atom, a force is exerted on the atom pulling it toward the strongest part of the E field (because of course light is an oscillating electric field paired with a magnetic one) where it tries to achieve resonance. By using these two ideas in conjunction (Page 4, Figure 3) with one another a powerful cooling trap is achieved (Savage 246).

Evaporative cooling is just what it sounds like: evaporation, except within the confines of the trap it has no limits. Part of the advantage of a magnetic trap is that the atoms don’t touch a physical surface (Cornell 81). Attempts were made (when working with liquid Hydrogen) to isolate it from the container’s walls by using liquid Helium. Probability dictates that some individual atoms will achieve enough momentum to escape the trap and when this happens they take their energy with them away from the super-cooled mass (Cornell 22). The fastest moving atoms have more kinetic energy, when they escape that kinetic energy leaves the system and the result is a lower average kinetic energy within the system. Since temperature is a measure of the average kinetic energy the result is a net reduction in temperature. The potential well these atoms have to overcome in order to escape from the cold mass needs to be reduced so that it is always just below then energy of the most excited atoms.

The end cooling technique that is used to cool most types of BECs that have been produced is referred to as a MOT (Magneto-Optical Trap). It combines all of the previously mentioned methods and is amazingly effective. It employs laser cooling and magnetic cooling in stages, and the centers of both traps must be aligned. By switching between stages and through careful tuning of the device incredibly low temperatures can be achieved (Cornell 24).

Superfluidity is a phase of matter produced by cooling a liquid below a certain temperature defined as its lambda point. At this point the liquid changes states to become *superfluid*, where it has the characteristics of a mixture of normal and superfluid characteristics. The superfluid component displays zero viscosity, zero entropy, and infinite thermal conductivity. Before going into more detail, it is a point of interest to discuss the differences between an ideal BEC and an *atomic* BEC, one which is produced experimentally in a lab. In an ideal BEC, there are no interactions between particles. It is only when there is a system of small interactions between the particles that you can have a superfluid. Atomic BECs have these small interactions, and are indeed true superfluids.

Figure 3: This shows the combination of laser cooling and magnetic evaporation (Cornell 85).

Superfluidity was first discovered in 4He and then later in 3He, most information on the subject holds its focus on these two isotopes. For ease of discussion, 4He will be the main topic discussed here, as it is most closely related to the properties of superfluid BEC. 3He has a spin of ½ (fermion) and its properties are completely different. 4He has two distinct liquid phases: He I and II. He I is a normal liquid phase, while He II is superfluid. This transition point, or lambda point, for 4He is approximately 2.17K.

Interestingly, graphing specific heat as a function of temperature resembles the Greek letter lambda. Hypothetically, if this transition occurred in a leak-proof container but in fact had an infinitesimally small leak, all of the superfluid would promptly exit through the leak. Normally, a liquid would not be able to do this because of the friction or viscosity of said liquid. Since the superfluid has zero viscosity, it can *fit* through space not normally thought of as *space*. Another example of zero viscosity can be observed if a column of evenly spaced discs are placed in a container of superfluid. If the column is spun about its center, in a normal viscous solution, the liquid between the disks should contribute to its inertia. Andronikashvilli showed that in a superfluid, a portion of the liquid contributes to the columns inertia, but a portion did not (Annett 29).

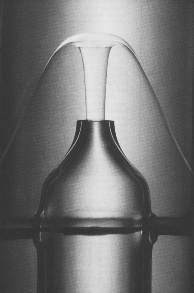


Figure 4: This is a superfluid fountain (Nature).

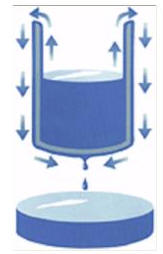
 Superfluid also has the ability to create a marvelous effect known as the “Fountain Effect” (figure 4). If a tube is placed in a container holding superfluid, and is then heated, the superfluid will shoot out from the tube creating a fountain. The cause of the fountain is related to the fluid having zero viscosity. Specifically, you can think of the entire system’s entropy as having the normal component entropy and the superfluid entropy. However, the superfluid is essentially a many-body state, and therefore has zero entropy. Because of this, it does not transfer heat and is forced up the tube creating the fountain. Similarly, if you had two containers of different heat connected by a small capillary, such that only superfluid could flow through, the heat has no mechanism for which to cause equilibrium in said containers. Another odd phenomenon observed is a superfluid left in an open container. If given time, the superfluid will actually “crawl” up along the walls of the container and completely empty itself!

Figure 5: This shows how superfluidity can appear to overcome gravity (Physicsforyou.net).

Superconductivity is a property of certain materials only occurring after being cooled to a specific temperature. For any material this is referred to as it’s “critical temperature” and is unique to that material. After reaching this point, the material abruptly changes to having zero electrical resistance and an expulsion of a magnetic field. Temperatures range from near absolute zero, as related to BEC, superfluids, etc, and up to temperatures of boiling liquid nitrogen at 77k.

Superconductivity occurs is a wide range of materials. First discovered in mercury in 1911, having zero electrical resistance can lead to some interesting results. Superconductors, once having current applied, can maintain this charge indefinitely. In order to understand why these odd phenomena occur, one must understand why resistance occurs in the first place. Normally, a conductor will have electrical resistance because of the way the electrons are moving through it. As they move through what you can picture as a lattice, they are constantly being scattered, resulting in a loss of energy in the current, and therefore, leading to resistivity.

In a superconductor, this does not happen due to what are known as cooper pairs. Cooper pairs, most simply, are a pair of electrons bound together at low temperatures. The electrons become bound because of the electron-phonon interaction. Picture an electron flowing freely in a metal lattice (as it usually does). The electron is repelled from other electrons because of their like charge, but is attracted to positive ions in the metal lattice. The attraction can alter the lattice in such a way as to attract other electrons, and at a longer distance, the electrons will “pair” up. When a cooper pair passes through the lattice, it now encounters little to no resistance. At low temperatures, cooper pairs are constantly forming, breaking, and reforming, but the net effect is zero electrical resistance. Interestingly, because the pair of electrons forms a boson, all cooper pairs tend to want to “condense” down to the same quantum state. They also have the characteristic of traveling (or flowing) without energy dissipation and can therefore be defined as a superfluid. This description of the manifestation of superconductivity is only for superconductors called “type-I”. There is another set of superconductors labeled “type-II”. All materials that have superconducting properties at “high” temperatures are in this category. Here, superconductors are characterized by moving toward a more normal state as an increasing magnetic field is applied. They are also different where type-I are usually compromised of a pure metal element, type-II are usually complex ceramics or metal alloys. Many modern superconductors are almost all comprised of complex ceramics. The ability to function at “high temperature” is relatively unknown for these type-II superconductors. Cooper pairs explain superconductivity in type-I but it fails to explain type-II.

As mentioned earlier, superconductors expel an internal magnetic field. Specifically, when a magnetic field is applied, it only penetrates a small outer layer of the superconductor. This is known as the Meissner effect. It creates a sort of “bubble” around the superconductor and is distinct from diamagnetism because it can expel changing *and* constant magnetic fields. It should also be mentioned that this effect is considerably stronger when compared to diamagnetism, and as such is used in modern applications such as magnetically-levitated trains as well as MRIs. The differences between type-I and type-II are also apparent when comparing them with this effect. Both display the effect, but act differently to an increasing magnetic field. Type-I superconductivity is destroyed when a magnetic field has reached a critical point defined “Hc”. In type-II, an initial critical point is reached “Hc1” where a mixture of normal and superconductive areas begin appear. This is caused because an increasing amount of the magnetic field is penetrating the material. At a second critical point, “Hc2”, all superconductivity is lost.

How can Type-II superconductors still have zero resistivity in a mixed stated? The answer lies in vortices. Like superfluid, type-II superconductors contain “quantum vortices”. You can think of this vortex as a circulating (like a Mobius strip) region surrounding a small “vortex core”. The proper definition of this circulating region is “supercurrent”-- flowing current which does not dissipate. Only when the vortex is “pinned” either by imperfections in the material, or other means does it remain truly superconductive. By this, it is meant that work is being prevented. The magnetic field lines are not actually passing through any “superconducting” parts of the material, but pass through “gateways”, the vortex cores comprised of non-superconducting material. You can think of the resistive, normal parts of the material as being “contained” by the supercurrent, preventing any resistivity to pass into the superconductor. This is odd in and of itself, but more interesting is the tendency for the vortices created to form triangular and square lattices, something that was predicted

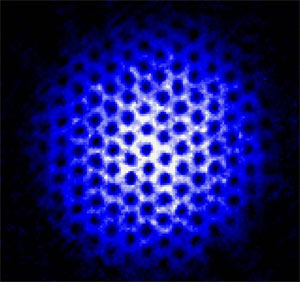


Figure 6: This is a vortex lattice (JILA NIST CU).

by Abrikosov, and is subsequently an “emergent phenomena” as described by its self organization (Annett 84). It is puzzling why such structures are formed, but it has been seen experimentally, and more recently found to not always behave in this manner. Ketterle mentions, in his noble lecture, that non-uniform (or “imperfect”) lattices have been observed (147). The exact reason is relatively mysterious, but main constituents are thermal fluctuation and random behavior that does not “evolve” with time.

Imperfect lattice structure can be somewhat explained by theories of vortex matter. Current theories suggest that vortices take on a wide range of “states” depending on temperature. At high temperatures they are thought to be liquid, a “glass” as an intermediary state, and perfectly crystalline at lower temperatures. In a liquid state, the vortices are basically free to move around, and are nearly impossible to “pin”. A glass state consists of randomly spaced vortices, but are “frozen” producing near zero resistivity, while the crystalline structures mentioned earlier as “lattice” structures, are the most desired. Consequently, this “deforming” of the lattice in a liquid state leads to “bad” pinning, and like a liquid, it can flow around any “object” attempting to stop it.

This leads to the vortex moving, and not truly having zero dissipation. A glass state can also have energy dissipation as well. Because of the thermal fluctuations, the vortices in this state can cross the energy barrier that keeps them in this state. They find a new state, one in which the vortices are no longer “frozen”. This is one of the largest problems commercial applications have, and an understanding of random behaviors, known as Quenched disorder in statistical physics, is fundamental to reducing resistivity in high-temperature superconductors. However, thermal fluctuation only occurs with a higher frequency at temperatures close to Hc2 and can therefore be fixed by lowering temperature; changing the states of the vortices themselves. Unfortunately, this reduces their effective range of function, and restricts their use in commercial applications. (Schonenberger 1-3).

*“An atom laser is a device which produces a bright, directed, and coherent beam of atoms. An ideal atom laser beam is a single frequency de Broglie matter wave, approximating a noiseless sinusoidal wave” (Savage 153)*

Atoms in a Bose-Einstein Condensate have all condensed to the same energy level and their wave functions overlap. Effectively they all share a single wave function. This is similar to the coherent light produced by a laser, which is all in phase and of the same wavelength. Atoms have been shown to have wave properties, exhibiting diffraction, and interference with Young’s double slit experiment. Because the atoms in a Bose-Einstein Condensate are all in phase with one another it seems natural that one should be able to use them to produce the atomic equivalent of a laser with them. This is the atom laser (Savage 154).

Atom lasers have certain advantages over optical lasers: the atoms need not travel at the speed of light, and the wavelengths are very short; also because the amplitude of the wave function is large the momentum of an atom laser can be known with sufficient precision to allow smaller momentums than can be produced with photons—this is due to Heisenberg’s Uncertainty Principal, which states



Figure 7: These images are a result of progressive atom laser technology from their respective university from left to right; MIT 1997, Yale 1998, and Munich 1999 (Nobel Foundation).

that the momentum and position of a particle cannot be simultaneously known with certainty. The disadvantage of atom lasers is that they must operate in near vacuum (Martellucci 56).

Atom lasers can be used in a variety of experiments similar to optical experiments. Ironically one of the ways of manipulating atom laser beams is with optical laser beams. Lenses can be formed from “detuned” lasers in the same way that they were used to form the condensate in the first place. By tuning the frequency of the laser just above or below the resonance frequency of the atoms; a force is exerted on the atoms by the electric field of the light, guiding the atoms either closer or farther away from the intense portion of the beam. In this same way it has been shown experimentally that a sheet of light can be used as a mirror for a BEC, reflecting it due to the influence of its electric field on the dipole moments of the atoms (Martellucci 99).

In practice it is actually somewhat difficult to produce a beam from a Bose-Einstein Condensate, and various methods have been employed to do this. The difficulty lies in the fact that the atoms must somehow be removed from the cooling trap in a steady and stable way without exciting them out of the BEC state (Martellucci 118). In one method which has proven effective, the atoms are transferred from a trapped state to an untrapped state at the edge of the trap. In this method atoms are allowed to leave at the bottom of the trap where they are accelerated by gravity (Martellucci 120).

In another version of the output coupler atoms are transferred from the m=-1 state to the m=0 state by a laser pulse. Because the m=0 state cannot be contained by the magnetic field, this allows the atoms to leave the trap. This laser pulse also imparts momentum to the atoms allowing the direction of the atom laser to be controlled by something other than gravity. By using a rapid enough laser pulse, matter-wave pulses which overlap sufficiently to approximate a continuous beam are produced (Martellucci 61). Once atom lasers are able to be produced more efficiently a lot of interesting applications could arise from them. Because of the small wavelength they could be used to make incredibly precise interferometers; because they are affected by gravity they could be used to make gravitational measurements; obviously as a macroscopic quantum-mechanical state they also have the potential to increase our understanding of quantum mechanics. Thoughts for the future on other applications of them are precise atomic clocks, and their obvious potential for use in the deposition of matter for nanofabrication.

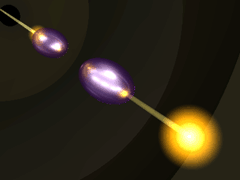


Figure 8: Dr.Hau discovered a way to stop light using a BEC and laser setup (Harvard University).

Another interesting application could be in the use of BEC to slow, or even stop light. Lene Hau was able to slow light to 17m/s (Hau 597). By stopping light, as Hau did, quantum bits can be stored which would be a new type of memory (Holloway). It would only take one atom from the lattice to store a quantum bit (Collins). What is happening in the storing of a bit is that light is coded into the condensate (in this case a sodium condensate) and it disappears, when a jolt of energy excites the atoms this information is released which conserves information (Holloway). In the future, magnetic fields could produce BEC on microchips to be used in quantum computing (Reichel).

It is curious to mention that vacuums normally contain condensates (Griffin 3). This changes our paradigm regarding what vacuums are, for if nothing is there, why does it contain a condensate? Furthermore it is quite unusual that in 2006 BEC was observed at room temperature in a gas of magnons (Demokritov 430). The reason this is so unusual is that BEC is normally associated with temperatures that are 7 orders of magnitude lower. This is evidence that BEC may be achieved many more ways than previously thought.

Some claims have even seemed bazaar, and there’s no telling what the scope of this field truly is. For instance, some say that dark matter could be a BEC effect of a congregation of matter on cosmological scales (Musser). Researchers are also looking for the densest form of matter called “color glass condensate” similar to BEC which may help explain some nuclear properties (Appell). The properties of BEC are amazing from; superfluidity (and its frictionless fountains), to superconductivity and the creation of the atom laser (it truly is the creation of laser like atoms). BEC is still too new to see too far into the future and many technological hurdles still need to be overcome. Though one thing is for sure, many exciting discoveries still lay on the horizon.

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