A Quaternion-Based Theory of Everything: Unifying Quantum Field Theory and Gravity with Dynamic Symmetry Breaking

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Abstract

We propose a Theory of Everything (TOE) that unifies the Standard Model (SM) of particle physics and gravity in 4D spacetime using a quaternion-based framework. The measurable quantum state is a quaternion wave function $\psi = \psi_0 + \psi_1 I + \psi_2 J + \psi_3 K$, represented as a 4x4 real matrix, while the total state $Q = (M_1, M_2)$ emerges from SU(4) matrices via the Cayley-Dickson process. A dynamic relation, $\psi_0^2 = |V|^2$ with $V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$, ties the scalar and vector components, yielding SM masses (e.g., Higgs at 125 GeV) under normalization. The SM gauge fields arise from SU(4) projections, and gravity manifests as a spin-2 field via $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$. An operator q generates pilot waves through q^2, guiding fermion motion. This TOE predicts a cosmos without dark matter particles, hints at superluminal propagation, and offers testable signatures in particle physics and cosmology.

1 Introduction

Unifying quantum mechanics with general relativity (GR) remains one of the most formidable challenges in modern theoretical physics. The Standard Model (SM) of particle physics, built on the gauge group SU(3) × SU(2) × U(1), elegantly describes the strong, weak, and electromagnetic interactions, while GR governs gravitational phenomena through the Einstein field equations. Yet, these two pillars of physics clash at quantum scales: quantum field theory (QFT) encounters singularities, and GR resists conventional quantization without significant modification. This incompatibility has fueled diverse unification efforts, such as string theory, which invokes extra dimensions, and loop quantum gravity, which proposes a discrete spacetime fabric. In this paper, we introduce a novel Theory of Everything (TOE) that reconciles the SM and gravity within a 4D spacetime framework, leveraging a quaternion-based approach and avoiding the need for unobserved dimensions or particles.

Our theory centers on a quaternion wave function $\psi = \psi_0 + \psi_1 I + \psi_2 J + \psi_3 K$, represented as a 4x4 real matrix that defines the measurable quantum state, and a total quantum state $Q = (M_1, M_2) = M_1 + M_2 k$, where M_1 and M_2 are SU(4) matrices derived via the Cayley-Dickson process ($k^2 = -1$). A dynamic coupling condition, $\psi_0^2 = |V|^2$, where $V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$, links the scalar component ψ_0 to the fermion vector part, naturally yielding SM masses—such as the Higgs boson at 125 GeV—when normalized ($\psi_1^2 + \psi_2^2 + \psi_3^2 = \frac{1}{4}$). The SM gauge fields emerge from SU(4) projections within Q, fully quantized in a QFT framework, while gravity arises as a spin-2 field $h_{\mu\nu}$ from the stress-energy tensor $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$. Additionally, an operator q, also a 4x4 real matrix, acts on Q via left multiplication (Qq), producing quaternion-valued state vectors; the vector components of q^2 serve as pilot waves, guiding fermion dynamics in a manner reminiscent of de Broglie-Bohm theory.

This work unfolds across several key dimensions: Section 2 establishes the mathematical formalism, detailing the quaternion structure and SU(4) projections that underpin the unification of SM fields and gravity. Section 3 explores the Higgs mechanism and fermion quantization, elucidating how symmetry breaking generates particle masses within the quaternion framework. Section 4 outlines the quantization of gravity as a spin-2 field, integrating it with SM interactions and addressing renormalization for finite observables. Section 6 applies the theory to cosmic evolution, predicting a universe without dark matter particles, driven by Q's curvature dynamics. Section 7 investigates exotic extensions, including superluminal propagation and the potential of Q to encode data through its orthogonal structure. Finally, Section 8 provides intuitive insights, experimental tests, and comparisons with existing theories, framing this TOE as a testable unification of quantum mechanics and gravity within 4D spacetime.

2 Mathematical Framework

2.1 Quaternion Wave Function and Total State Q

The foundational entity of this theory is the quaternion wave function ψ, defined as:

$$\psi = \psi_0 + \psi_1 I + \psi_2 J + \psi_3 K, \tag{1}$$

where $\psi_0, \psi_1, \psi_2, \psi_3 \in \mathbb{R}$ are real-valued coefficients representing the scalar and vector components, respectively, and I, J, K are quaternion basis elements satisfying the algebraic relations:

$$I^2 = J^2 = K^2 = IJK = -1, \quad IJ = K, \quad JI = -K, \quad JK = I, \quad KJ = -I, \quad KI = J, \quad IK = -J. \tag{2}$$

These relations define the non-commutative quaternion algebra, which underpins the theory's unification of matter and spacetime in four dimensions. In matrix representation, ψ is expressed as a 4x4 real matrix acting on 4D vectors:

$$\psi = \begin{pmatrix} \psi_0 & -\psi_1 & -\psi_2 & -\psi_3 \\ \psi_1 & \psi_0 & -\psi_3 & \psi_2 \\ \psi_2 & \psi_3 & \psi_0 & -\psi_1 \\ \psi_3 & -\psi_2 & \psi_1 & \psi_0 \end{pmatrix}. \tag{3}$$

This matrix form arises from the quaternion basis mapped to 4x4 real matrices: the scalar 1 corresponds to the identity matrix, while I, J, K are represented by specific antisymmetric matrices ensuring the algebraic properties hold. The components $\psi_j(x)$

$(j = 0, 1, 2, 3)$ are spacetime-dependent fields, with $\psi_j = R_j(x)e^{iS_j(x)/\hbar}$, where $R_j(x) \in \mathbb{R}_+$ is the amplitude and $S_j(x) \in \mathbb{R}$ is the phase. This allows ψ to encode a relativistic four-momentum:

$$p^\mu = (\psi_0, p_1, p_2, p_3), \tag{4}$$

where $p_j = \nabla S_j$ represents the spatial momentum components derived from the phase gradients, and ψ_0 relates to the energy or scalar potential. Thus, ψ serves as the measurable state, unifying particle and field properties in a single quaternion structure.

While the quaternion wave function ψ represents the measurable state, the total quantum state of the theory is encapsulated by Q, a quaternion-like field extending over 4D spacetime. It is defined as:

$$Q = (M_1, M_2) = M_1 + M_2 k, \tag{5}$$

where $M_1, M_2 \in \mathrm{SU}(4)$ are 4x4 complex unitary matrices with determinant 1, and k is a Cayley-Dickson extension satisfying $k^2 = -1$. Classically, $Q(x)$ is a quaternion-valued function:

$$Q(x) = Q_0(x) + Q_1(x)I + Q_2(x)J + Q_3(x)K, \tag{6}$$

where $Q_0(x), Q_1(x), Q_2(x), Q_3(x) \in \mathbb{R}$ are real scalar functions, represented as a 4x4 real matrix acting on 4D vectors:

$$Q(x) = \begin{pmatrix} Q_0(x) & -Q_1(x) & -Q_2(x) & -Q_3(x) \\ Q_1(x) & Q_0(x) & -Q_3(x) & Q_2(x) \\ Q_2(x) & Q_3(x) & Q_0(x) & -Q_1(x) \\ Q_3(x) & -Q_2(x) & Q_1(x) & Q_0(x) \end{pmatrix}. \tag{7}$$

In the quantum field theory (QFT) context, $Q(x)$ becomes a 4x4 matrix-valued operator acting on a Hilbert space, unifying SM fields (via SU(4) projections) and gravitational contributions. Canonical quantization replaces the classical coefficients with operator-valued fields:

$$Q(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \sum_{j=0}^{3} \left[a_j(k) Q_j(k) e^{-ik\cdot x} + a_j^\dagger(k) Q_j^\dagger(k) e^{ik\cdot x} \right], \tag{8}$$

where:
- $\omega_k = \sqrt{c^2|\mathbf{k}|^2 + m^2}$ is the dispersion relation, with m a mass scale tied to the Higgs VEV ($v' = \psi_0 v_0$, $v_0 = 246\,\mathrm{GeV}$ when $\psi_0 = 1$),
- $a_j(k)$ and $a_j^\dagger(k)$ are annihilation and creation operators for the j-th quaternion component, satisfying:

$$[a_j(k), a_{j'}^\dagger(k')] = (2\pi)^3 2\omega_k \delta_{jj'} \delta^3(\mathbf{k} - \mathbf{k}'), \quad [a_j(k), a_{j'}(k')] = 0,$$

- $Q_j(k)$ are 4x4 basis matrices (e.g., $Q_0 = I_{4\times4}$, $Q_1 = -I$ on off-diagonal terms, etc.), adjusted for momentum k.

The operator $Q(x)$ retains its 4x4 structure, but each element becomes a field operator, enabling creation and annihilation of states (e.g., gluons, W bosons, fermions, gravitons). Classically, $Q_j(x) \in \mathbb{R}$ ensures a deterministic configuration, while quantization introduces fluctuations, with the classical limit recovered via expectation values $\langle Q(x) \rangle$. Q thus represents the full quantum state space spanned by M_1 and M_2, dynamically evolving to encode all physical degrees of freedom in the TOE.

2.2 Operator q and transformations

The operator q is a pivotal entity in the theory, acting on the total quantum state Q to generate transformed states and guide fermion dynamics via pilot waves. It is defined as a quaternion-valued operator:

$$q = q_0 + q_1 I + q_2 J + q_3 K, \tag{9}$$

where $q_0, q_1, q_2, q_3 \in \mathbb{R}$ are spacetime-dependent coefficients, represented as a 4x4 real matrix:

$$q = \begin{pmatrix} q_0 & -q_1 & -q_2 & -q_3 \\ q_1 & q_0 & -q_3 & q_2 \\ q_2 & q_3 & q_0 & -q_1 \\ q_3 & -q_2 & q_1 & q_0 \end{pmatrix}. \tag{10}$$

In the quantum context, $q(x)$ is promoted to a field operator:

$$q(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \sum_{j=0}^{3} \left[b_j(k) q_j(k) e^{-ik\cdot x} + b_j^\dagger(k) q_j^\dagger(k) e^{ik\cdot x} \right], \tag{11}$$

where $\omega_k = \sqrt{c^2|\mathbf{k}|^2 + m^2}$, $b_j(k)$ and $b_j^\dagger(k)$ are annihilation and creation operators satisfying:

$$[b_j(k), b_{j'}^\dagger(k')] = (2\pi)^3 2\omega_k \delta_{jj'} \delta^3(\mathbf{k} - \mathbf{k}'), \quad [b_j(k), b_{j'}(k')] = 0, \tag{12}$$

and $q_j(k)$ are basis matrices akin to those of Q. The operator q acts on Q via left multiplication:

$$\tilde{q} = Qq, \tag{13}$$

transforming a 4D vector $q = (q_0, q_1, q_2, q_3)^T$ into $\tilde{q} = (\tilde{q}_0, \tilde{q}_1, \tilde{q}_2, \tilde{q}_3)^T$:

$$\tilde{q}_0 = Q_0 q_0 - Q_1 q_1 - Q_2 q_2 - Q_3 q_3, \tag{14}$$
$$\tilde{q}_1 = Q_1 q_0 + Q_0 q_1 - Q_3 q_2 + Q_2 q_3, \tag{15}$$
$$\tilde{q}_2 = Q_2 q_0 + Q_3 q_1 + Q_0 q_2 - Q_1 q_3, \tag{16}$$
$$\tilde{q}_3 = Q_3 q_0 - Q_2 q_1 + Q_1 q_2 + Q_0 q_3, \tag{17}$$

forming a new quaternion-valued vector $\tilde{q} = \tilde{q}_0 + \tilde{q}_1 I + \tilde{q}_2 J + \tilde{q}_3 K$, representing transformed SM field states (e.g., fermions or gauge bosons).

The square of the operator, $q^2 = qq$, is computed via matrix multiplication:

$$q^2 = \begin{pmatrix} q_0^2 - q_1^2 - q_2^2 - q_3^2 & -2q_0q_1 & -2q_0q_2 & -2q_0q_3 \\ 2q_0q_1 & q_0^2 - q_1^2 - q_2^2 - q_3^2 & -2q_0q_3 & 2q_0q_2 \\ 2q_0q_2 & 2q_0q_3 & q_0^2 - q_1^2 - q_2^2 - q_3^2 & -2q_0q_1 \\ 2q_0q_3 & -2q_0q_2 & 2q_0q_1 & q_0^2 - q_1^2 - q_2^2 - q_3^2 \end{pmatrix}, \tag{18}$$

or in quaternion form:

$$q^2 = (q_0^2 - q_1^2 - q_2^2 - q_3^2) + 2q_0q_1 I + 2q_0q_2 J + 2q_0q_3 K, \tag{19}$$

verified by quaternion algebra:
- scalar part $q_0^2 + q_1^2(-1) + q_2^2(-1) + q_3^2(-1) = q_0^2 - q_1^2 - q_2^2 - q_3^2$,
- vector part $2q_0(q_1 I + q_2 J + q_3 K)$, with cross terms (e.g., $q_1 q_2 K - q_2 q_1 K$) canceling. The vector component:

$$V_{q^2} = 2q_0(q_1 I + q_2 J + q_3 K), \tag{20}$$

serves as pilot wave components, analogous to V, guiding fermion dynamics in a de Broglie-Bohm-like manner via phase gradients ∇S_j. The scalar part influences energy scales, tied to ψ_0, modulating masses in Q's dispersion relation.

2.3 Dynamic Coupling and Normalization

The vector part of the quaternion wave function ψ is defined as the fermion component:

$$V = 2(\psi_1 I + \psi_2 J + \psi_3 K), \tag{21}$$

where V encapsulates the spatial degrees of freedom associated with fermion fields, distinct from the scalar component ψ_0. The factor of 2 is introduced to align the magnitude of V with physical scales, as will be evident in the normalization condition. The squared magnitude of V is computed using the quaternion conjugate $V^\dagger = 2(\psi_1 I^\dagger + \psi_2 J^\dagger + \psi_3 K^\dagger)$, noting that $I^\dagger = -I$, $J^\dagger = -J$, and $K^\dagger = -K$ in the matrix representation, yielding:

$$|V|^2 = V^\dagger V = 4(\psi_1^2 + \psi_2^2 + \psi_3^2). \tag{22}$$

This magnitude reflects the total contribution of the vector components ψ_1, ψ_2, ψ_3, squared and scaled by the factor of 4 due to the coefficient in V. A fundamental condition ties the scalar ψ_0 to the vector part:

$$\psi_0^2 = |V|^2 = 4(\psi_1^2 + \psi_2^2 + \psi_3^2), \tag{23}$$

implying:

$$\psi_0 = 2\sqrt{\psi_1^2 + \psi_2^2 + \psi_3^2}. \tag{24}$$

This dynamic coupling ensures that the scalar component ψ_0 is not an independent parameter but is determined by the magnitude of the fermion vector V, establishing a direct relationship between the scalar field (related to energy or mass scales) and the vectorial fermion content. To align with Standard Model (SM) phenomenology, particularly the Higgs vacuum expectation value (VEV) of 246 GeV, a normalization is imposed:

$$\psi_1^2 + \psi_2^2 + \psi_3^2 = \frac{1}{4}, \tag{25}$$

so:

$$|V|^2 = 4 \times \frac{1}{4} = 1, \quad \psi_0 = 2\sqrt{\frac{1}{4}} = 1. \tag{26}$$

This normalization sets $\psi_0 = 1$, which, when multiplied by the SM Higgs VEV scale $v_0 = 246\,\mathrm{GeV}$, yields the physical VEV $v' = \psi_0 v_0 = 246\,\mathrm{GeV}$, ensuring consistency with observed particle masses (e.g., $m_W \approx 80.4\,\mathrm{GeV}$). The condition $\psi_0^2 = |V|^2$ thus serves as a cornerstone of the theory, linking the scalar and vector components in a unified quaternion framework, with ψ_0 dynamically adjusting to spacetime variations in V.

In this theory, the vector part $V = 2(\psi_1 I+\psi_2 J+\psi_3 K)$ also parameterizes time through the phase evolution of its components, influencing relativistic mass and group velocity of particles via its connection to the Higgs mechanism. The quaternion components ψ_j ($j = 1, 2, 3$) are expressed as:

$$\psi_j(x) = R_j(x)e^{iS_j(x)/\hbar}, \tag{27}$$

where $R_j(x) \in \mathbb{R}_+$ is the amplitude, $S_j(x) \in \mathbb{R}$ is the phase, and $\hbar$ is the reduced Planck constant. The phase S_j evolves with proper time τ, defining temporal dynamics relativistically. Momentum is derived from spatial gradients:

$$p_j = \nabla S_j, \quad j = 1, 2, 3, \tag{28}$$

forming the spatial components of the four-momentum $p^\mu = (\psi_0, p_1, p_2, p_3)$, with total momentum magnitude:

$$|\mathbf{p}| = \sqrt{p_1^2 + p_2^2 + p_3^2}. \tag{29}$$

Relativistic mass is $m_{\text{rel}} = \gamma m$, where $\gamma = (1 - v^2/c^2)^{-1/2}$, and m is the rest mass from the Higgs mechanism, $m = y_f v'/\sqrt{2}$, with $v' = \psi_0 v_0$. The dispersion relation for fields in Q, such as fermions, is:

$$\omega = \sqrt{c^2|\mathbf{k}|^2 + m^2}, \tag{30}$$

yielding group velocity:

$$v_g = \frac{d\omega}{d|\mathbf{k}|} = \frac{c^2|\mathbf{k}|}{\sqrt{c^2|\mathbf{k}|^2 + m^2}}, \tag{31}$$

where $|\mathbf{k}|$ is the wave number magnitude, tied to momentum $|\mathbf{p}| = |\mathbf{k}|$ ($\hbar = 1$). Along the direction of travel, assuming $|\mathbf{p}| = mv$ classically:

$$v_g = \frac{c^2|\mathbf{p}|}{\sqrt{c^2|\mathbf{p}|^2 + m^2c^4}} = \frac{c^2(mv)}{\sqrt{c^2(mv)^2 + m^2c^4}} = \frac{vc}{\sqrt{v^2 + c^2}}. \tag{32}$$

For an electron ($y_e \approx 2.1 \times 10^{-6}$):
- $\psi_0 = 1$: $v' = 246\,\text{GeV}$, $m_e = 0.511\,\text{MeV}$, $v_g = \frac{|\mathbf{k}|}{\sqrt{|\mathbf{k}|^2+(0.511)^2}}$, e.g., at $|\mathbf{k}| = 1\,\text{MeV}$, $v_g \approx 0.859c$,
- $\psi_0 = 2$: $v' = 492\,\text{GeV}$, $m_e = 1.022\,\text{MeV}$, $v_g = \frac{|\mathbf{k}|}{\sqrt{|\mathbf{k}|^2+(1.022)^2}}$, $v_g \approx 0.700c$.

Variations in $|V|^2$ modulate ψ_0, thus m, affecting v_g and connecting V's time parameterization to relativistic dynamics and gravity via $T_{\mu\nu}$.

2.4 Time Parameterization by V

The vector component of the quaternion wave function, $V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$, plays a pivotal role in parameterizing time within this Theory of Everything (TOE), linking quantum dynamics to spacetime evolution. Defined within $\psi = \psi_0+\psi_1 I+\psi_2 J+\psi_3 K$, each component $\psi_j(x) = R_j(x)e^{iS_j(x)/\hbar}$ (for $j = 0, 1, 2, 3$) carries a phase $S_j(x)$ that evolves with spacetime coordinates $x^\mu = (t, \mathbf{x})$. The dynamic coupling $\psi_0^2 = |V|^2 = 4(\psi_1^2 + \psi_2^2 + \psi_3^2)$ integrates V with the scalar ψ_0, embedding temporal information into the total quantum state $Q = (M_1, M_2)$ and its gravitational influence via $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$.

2.4.1 Phase Evolution and Energy-Momentum

The temporal parameterization begins with the phase $S_j(x)$ of V's components. The time derivative yields an energy-like quantity:

$$E_j = -\frac{\partial S_j}{\partial t}, \tag{33}$$

while spatial gradients define momentum:

$$p_j = \nabla S_j, \quad j = 1, 2, 3. \tag{34}$$

Together, these form the four-momentum $p^\mu = (\psi_0, p_1, p_2, p_3)$, with ψ_0 acting as an energy scale tied to the Higgs vacuum expectation value (VEV) $v' = \psi_0 v_0$ (where $v_0 = 246\,\mathrm{GeV}$ when $\psi_0 = 1$). The phase evolution along a particle's worldline, parameterized by proper time τ, is:

$$\frac{dS_j}{d\tau} = \frac{\partial S_j}{\partial t}\frac{dt}{d\tau} + \nabla S_j \cdot \frac{d\mathbf{x}}{d\tau} = -E_j + \mathbf{p}_j \cdot \mathbf{v}, \tag{35}$$

where $\mathbf{v} = d\mathbf{x}/dt$ and $dt/d\tau = \gamma = (1 - v^2)^{-1/2}$. For a relativistic fermion with mass $m = y_f v'/\sqrt{2}$, $E_j = \gamma m$ and $\mathbf{p}_j = \gamma m \mathbf{v}$, so:

$$\frac{dS_j}{d\tau} = -\gamma m(1 - v^2) = -\frac{m}{\gamma}, \tag{36}$$

indicating that V tracks proper time through its oscillatory phase, with frequency modulated by ψ_0.

2.4.2 Pilot Wave Temporal Guidance

The operator $q = q_0 + q_1 I + q_2 J + q_3 K$ generates $q^2 = (q_0^2 - q_1^2 - q_2^2 - q_3^2) + 2q_0(q_1 I + q_2 J + q_3 K)$, whose vector part $V_{q^2} = 2q_0(q_1 I + q_2 J + q_3 K)$ acts as a pilot wave. This guides fermion trajectories with velocity:

$$\mathbf{v}_j = \frac{\nabla S_j}{m} = \frac{\mathbf{p}_j}{m}, \tag{37}$$

where $\mathbf{p}_j$ evolves temporally via $S_j(t, \mathbf{x})$. The scalar q_0, potentially scaling with ψ_0 or density (e.g., $q_0 \sim \sqrt{n}$ in neutron stars), adjusts V_{q^2}'s amplitude, introducing a time-dependent modulation. The dispersion relation for fields in Q:

$$\omega_k = \sqrt{|\mathbf{k}|^2 + m^2}, \tag{38}$$

yields a phase velocity $v_p = \omega_k / |\mathbf{k}|$ and group velocity:

$$v_g = \frac{|\mathbf{k}|}{\sqrt{|\mathbf{k}|^2 + m^2}}, \tag{39}$$

both of which depend on m, thus ψ_0 and $|V|^2$. The temporal evolution of V_{q^2} aligns particle motion with spacetime curvature, parameterizing time through trajectory dynamics.

2.4.3 Relativistic and Gravitational Effects

V influences time via relativistic mass and gravitational coupling. The mass $m = y_f \psi_0 v_0/\sqrt{2}$ varies with ψ_0, affecting v_g and wave packet propagation. For an electron ($y_e \approx 2.1 \times 10^{-6}$):

- $\psi_0 = 1$: $m_e = 0.511\,\text{MeV}$, $v_g \approx 0.859$ at $|\mathbf{k}| = 1\,\text{MeV}$,
- $\psi_0 = 2$: $m_e = 1.022\,\text{MeV}$, $v_g \approx 0.700$,

shifting travel time over fixed distances. Gravitationally, $T_{\mu\nu}$ incorporates V's contribution:

$$T^f_{\mu\nu} = \overline{\psi}_f \gamma_{(\mu} D_{\nu)} \psi_f - \eta_{\mu\nu} \mathcal{L}_f, \tag{40}$$

where $\psi_f \sim V$, and variations in $|V|^2$ alter spacetime curvature, affecting proper time dilation (e.g., in neutron stars, where $\psi_0 > 1$).

2.4.4 Implications

V parameterizes time by encoding energy in S_j, guiding motion via V_{q^2}, and modulating mass and gravity through ψ_0. This unification of temporal dynamics with quantum and gravitational phenomena distinguishes the TOE, offering testable predictions (e.g., time-of-flight anomalies at the LHC or GW phase shifts).

2.5 SU(4) Gauge Structure

The unification of Standard Model (SM) forces—strong (SU(3)), weak (SU(2)), and electromagnetic (U(1))—is achieved through projections from the SU(4) gauge group inherent in the total quantum state $Q = (M_1, M_2)$, where M_1 and M_2 are 4x4 complex SU(4) matrices with determinant 1. The SU(4) group, with 15 independent generators corresponding to its Lie algebra $\mathfrak{su}(4)$, is represented by Hermitian, traceless 4x4 matrices T^a ($a = 1, \ldots, 15$) satisfying:

$$[T^a, T^b] = i f^{abc} T^c, \tag{41}$$

where f^{abc} are structure constants, and the adjoint representation has dimension $15 = 4^2 - 1$. These generators span the SM gauge groups as subgroups, which are projected from M_1 and M_2 to recover the SM's gauge fields.

- **SU(3) Projection (Strong Force):** The SU(3) subgroup, governing the strong interaction, is embedded in the upper 3x3 block of M_1:

  $$M_1 = \begin{pmatrix} M_{\text{SU(3)}} & * \\ * & * \end{pmatrix},$$

  where $M_{\text{SU(3)}}$ is a 3x3 SU(3) matrix with 8 generators, the Gell-Mann matrices λ^a ($a = 1, \ldots, 8$):

  $$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \ldots, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

This yields 8 gluon fields G_μ^a ($a = 1, \dots, 8$):

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc} G_\mu^b G_\nu^c,$$

with field strength Lagrangian $\mathcal{L}_{\mathrm{SU}(3)} = -\frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu}$, and coupling $g_s \approx 1$. Quantization follows:

$$G_\mu^a(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \sum_\lambda \left[c_{a,\lambda}(k) \epsilon_\mu^\lambda e^{-ik\cdot x} + c_{a,\lambda}^\dagger(k) \epsilon_\mu^{\lambda *} e^{ik\cdot x} \right],$$

where $\omega_k = c|\mathbf{k}|$, reflecting massless gluons.

- **SU(2) Projection (Weak Force):** The SU(2) subgroup, governing the weak interaction, is embedded within M_1, typically in a 2x2 block (e.g., rows/columns 1-2):

  $$M_1 \supset \begin{pmatrix} M_{\mathrm{SU}(2)} & 0 \\ 0 & I_{2\times 2} \end{pmatrix},$$

  with 3 generators, the Pauli matrices τ^a ($a = 1, 2, 3$):

  $$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

  This produces 3 W boson fields W_μ^a:

  $$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g \epsilon^{abc} W_\mu^b W_\nu^c,$$

  $\mathcal{L}_{\mathrm{SU}(2)} = -\frac{1}{4} W_{\mu\nu}^a W^{a,\mu\nu}$, $g \approx 0.65$, quantized similarly to gluons pre-symmetry breaking.

- **U(1) Projection (Electromagnetic Force):** The U(1) subgroup, tied to hypercharge pre-breaking (and electromagnetism post-breaking), is a diagonal generator in SU(4):

  $$M_1 \supset e^{i\theta Y}, \quad Y = \mathrm{diag}(y_1, y_2, y_3, y_4),$$

  traceless within SU(4), yielding the photon field B_μ:

  $$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu,$$

  $\mathcal{L}_{\mathrm{U}(1)} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu}$, $g' \approx 0.36$. Post-Higgs breaking, B_μ mixes with W_μ^3 to form the photon A_μ.

These projections from M_1 (and similarly M_2) recover the SM gauge fields, with M_1, M_2 spanning dual Hilbert spaces $(\mathcal{H}_1, \mathcal{H}_2)$ within Q, dynamically evolving to unify SM interactions and gravity via $T_{\mu\nu}$.

2.6 Vector Fermions and Gauge Symmetry Invariants

The fermion sector of this quaternion-based TOE, encapsulated in the vector part $V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$ of the wave function ψ, connects deeply to gauge symmetry invariants within the total quantum state $Q = (M_1, M_2)$, constructed from SU(4) matrices.

The SU(4) Lie algebra, spanned by 15 generators T^a ($a = 1, \ldots, 15$) with commutation relations $[T^a, T^b] = if^{abc}T^c$, yields a quadratic Casimir operator:

$$C_2 = T^a T^a, \tag{42}$$

which commutes with all T^a and labels representations. For fermions in the fundamental representation (dimension 4) of SU(4), as suggested by Q's 4x4 structure, the eigenvalue is:

$$C_2(4) = \frac{15}{8}, \tag{43}$$

decomposing under SU(4) $\supset$ SU(3) $\times$ SU(2) $\times$ U(1) into SM subgroup invariants—e.g., $C_2 = \frac{4}{3}$ for SU(3) triplets (quarks) and $C_2 = \frac{3}{4}$ for SU(2) doublets (left-handed fermions).

The dynamic coupling condition:

$$\psi_0^2 = |V|^2 = 4(\psi_1^2 + \psi_2^2 + \psi_3^2), \tag{44}$$

ties the scalar ψ_0 to V's magnitude, an SU(4)-invariant norm. Normalizing $\psi_1^2+\psi_2^2+\psi_3^2 = \frac{1}{4}$ sets $\psi_0 = 1$, aligning $v' = \psi_0 v_0 = 246\,\text{GeV}$ with SM masses (e.g., $m_f = y_f v'/\sqrt{2}$). This suggests $|V|^2 \propto C_2$, linking fermion strength to gauge symmetry invariants and influencing mass generation. The components ψ_1, ψ_2, ψ_3 may reflect generational structure (e.g., e, μ, τ) via their alignment with SU(4) quantum numbers.

The operator q, acting via Qq and generating pilot waves through:

$$q^2 = (q_0^2 - q_1^2 - q_2^2 - q_3^2) + 2q_0(q_1 I + q_2 J + q_3 K), \tag{45}$$

produces $V_{q^2} = 2q_0(q_1 I + q_2 J + q_3 K)$, which guides fermion dynamics. If q transforms under SU(4), V_{q^2} may preserve C_2's structure, aligning fermion trajectories with gauge-invariant orbits and reinforcing the three-generation pattern. This interplay unifies spacetime degrees of freedom (via I, J, K) with internal symmetries, suggesting that gauge invariants underpin both the magnitude and dynamics of the fermion sector within the quaternion framework.

3 Particle Physics Unification

3.1 Higgs Mechanism and Mass Generation

The Higgs field ϕ plays a critical role in this theory, mediating electroweak symmetry breaking and endowing particles with mass through its vacuum expectation value (VEV), integrated within the quaternion framework via ψ_0. It is an SU(2) doublet embedded in Q:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \tag{46}$$

where $\phi^+ = (\phi_1 + i\phi_2)/\sqrt{2}$ and $\phi^0 = (\phi_3 + i\phi_4)/\sqrt{2}$ are complex charged and neutral components, transforming under $\text{SU}(2)_L$ (weak isospin) and $\text{U}(1)_Y$ (hypercharge $Y = 1/2$). Unlike the SM's standalone field, ϕ is dynamically linked to ψ:

$$\mathcal{L}_\phi = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi), \tag{47}$$

with covariant derivative:

$$D_\mu = \partial_\mu - igW^a_\mu \tau^a - ig'\frac{Y}{2}B_\mu, \tag{48}$$

where W^a_μ ($a = 1, 2, 3$) are SU(2) gauge fields, B_μ is the U(1) field, $g \approx 0.65$, $g' \approx 0.36$, and $\tau^a = \sigma^a/2$ (Pauli matrices). The potential is:

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2, \quad \mu^2 > 0, \quad \lambda \approx 0.129, \tag{49}$$

minimized at:

$$\phi^\dagger \phi = \frac{\mu^2}{2\lambda}, \quad v' = \sqrt{\frac{\mu^2}{\lambda}}, \tag{50}$$

where $v' = \psi_0 v_0$, and $v_0 = 246\,\text{GeV}$ when $\psi_0 = 1$. The VEV breaks $\text{SU}(2)_L \times \text{U}(1)_Y$ to $\text{U}(1)_E M$:

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ \frac{v'}{\sqrt{2}} \end{pmatrix}, \tag{51}$$

mixing W^3_μ and B_μ:

$$W^3_\mu = \cos\theta_W Z_\mu + \sin\theta_W A_\mu, \quad B_\mu = -\sin\theta_W Z_\mu + \cos\theta_W A_\mu, \tag{52}$$

$\sin^2\theta_W \approx 0.231$, yielding masses:

$$m_W = \frac{gv'}{2} \approx 80.4\,\text{GeV}, \quad m_Z = \frac{v'\sqrt{g^2 + g'^2}}{2} \approx 91.2\,\text{GeV}, \quad m_A = 0, \tag{53}$$

and Higgs boson mass:

$$m_H = \sqrt{2\lambda}v' \approx 125\,\text{GeV}, \quad \text{when } \psi_0 = 1. \tag{54}$$

Quantization gives:

$$\phi(x) = \begin{pmatrix} 0 \\ \frac{v' + h(x)}{\sqrt{2}} \end{pmatrix}, \quad h(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \left[a_h(k) e^{-ik\cdot x} + a_h^\dagger(k) e^{ik\cdot x} \right], \tag{55}$$

$\omega_k = \sqrt{|\mathbf{k}|^2 + m_H^2}$. The scalar $\psi_0 = 2\sqrt{\psi_1^2 + \psi_2^2 + \psi_3^2}$ sources v', normalizing to $\psi_0 = 1$ for SM consistency, with variations (e.g., $\psi_0 = 2$) scaling masses (e.g., $m_H = 250\,\text{GeV}$), impacting $T_{\mu\nu}$ and gravitational effects.

3.2 Gauge Fields: Gluons, Photons, and Weak Bosons

The gluon fields G^a_μ (where $a = 1, \ldots, 8$) mediate the strong nuclear force within the Standard Model (SM) component of this theory, emerging from the SU(3) subgroup projected from the SU(4) gauge structure of the total quantum state $Q = (M_1, M_2)$. The SU(3) subgroup is embedded in the upper 3x3 block of M_1:

$$M_1 = \begin{pmatrix} M_{\text{SU}(3)} & * \\ * & * \end{pmatrix},$$

where $M_{\text{SU}(3)}$ is a 3x3 SU(3) matrix with 8 generators, the Gell-Mann matrices λ^a ($a = 1, \ldots, 8$):

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \ldots, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

The gluon field strength tensor is defined as:

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu, \tag{56}$$

where $g_s \approx 1$ is the strong coupling constant, and f^{abc} are the structure constants of the SU(3) Lie algebra $\mathfrak{su}(3)$. The Lagrangian is:

$$\mathcal{L}_{\mathrm{SU}(3)} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}, \tag{57}$$

and quantization follows:

$$G^a_\mu(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \sum_{\lambda=0,1} \left[c_{a,\lambda}(k) \epsilon^\lambda_\mu(k) e^{-ik\cdot x} + c^\dagger_{a,\lambda}(k) \epsilon^{\lambda *}_\mu(k) e^{ik\cdot x} \right], \tag{58}$$

where $\omega_k = c|\mathbf{k}|$, $c_{a,\lambda}(k)$ and $c^\dagger_{a,\lambda}(k)$ satisfy:

$$[c_{a,\lambda}(k), c^\dagger_{b,\lambda'}(k')] = (2\pi)^3 2\omega_k \delta_{ab} \delta_{\lambda\lambda'} \delta^3(\mathbf{k} - \mathbf{k}'), \tag{59}$$

and $\epsilon^\lambda_\mu(k)$ are polarization vectors for the two transverse helicity states.

The photon fields A_μ, mediating the electromagnetic force, arise from the U(1) subgroup projected from SU(4), where $M_1 \supset e^{i\theta Y}$, $Y = \mathrm{diag}(y_1, y_2, y_3, y_4)$, traceless within SU(4). Post-electroweak symmetry breaking, the photon emerges from mixing the $\mathrm{U}(1)_Y$ hypercharge field B_μ and $\mathrm{SU}(2)_L$ field W^3_μ:

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W^3_\mu, \tag{60}$$

where θ_W is the Weinberg angle ($\sin^2\theta_W \approx 0.231$). The field strength tensor is:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{61}$$

with Lagrangian:

$$\mathcal{L}_{\mathrm{EM}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \tag{62}$$

quantized as:

$$A_\mu(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \sum_{\lambda=0,1} \left[e_\lambda(k) \epsilon^\lambda_\mu(k) e^{-ik\cdot x} + e^\dagger_\lambda(k) \epsilon^{\lambda *}_\mu(k) e^{ik\cdot x} \right], \tag{63}$$

where $\omega_k = c|\mathbf{k}|$, and $[e_\lambda(k), e^\dagger_{\lambda'}(k')] = (2\pi)^3 2\omega_k \delta_{\lambda\lambda'} \delta^3(\mathbf{k} - \mathbf{k}')$.

The weak force is governed by the SU(2) subgroup embedded within M_1, typically in a 2x2 block:

$$M_1 \supset \begin{pmatrix} M_{\mathrm{SU}(2)} & 0 \\ 0 & I_{2\times 2} \end{pmatrix},$$

with 3 generators, the Pauli matrices τ^a ($a = 1, 2, 3$):

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

This produces 3 W boson fields W_μ^a:

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\epsilon^{abc} W_\mu^b W_\nu^c, \tag{64}$$

$\mathcal{L}_{\mathrm{SU}(2)} = -\frac{1}{4} W_{\mu\nu}^a W^{a,\mu\nu}$, $g \approx 0.65$, quantized similarly to gluons pre-symmetry breaking. Post-Higgs breaking, W_μ^3 mixes with B_μ (as above), while W_μ^1 and W_μ^2 form the charged $W_\mu^\pm$ bosons with masses $m_W = \frac{gv'}{2} \approx 80.4\,\mathrm{GeV}$, where $v' = \psi_0 v_0$, $v_0 = 246\,\mathrm{GeV}$ when $\psi_0 = 1$.

These gauge fields couple to fermions via the covariant derivative:

$$D_\mu = \partial_\mu - ig_s G_\mu^a T^a - igW_\mu^a \tau^a - ieQA_\mu, \tag{65}$$

where $T^a = \lambda^a/2$, $e = g\sin\theta_W \approx 0.31$, and Q is the electric charge. The Higgs field, via v', indirectly influences dynamics by setting fermion and boson masses, affecting $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$ and gravitational interactions.

3.3 Fermion Fields and Generations

Fermions in this theory emerge from the vector part $V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$ of the quaternion wave function ψ, representing the matter content of the SM within Q. They are organized into representations consistent with $\mathrm{SU}(3)_C$, $\mathrm{SU}(2)_L$, and $\mathrm{U}(1)_Y$:

- **Quarks:** Left-handed doublets $q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$, right-handed singlets u_R, d_R, transforming as SU(3) triplets ($q = (q_r, q_g, q_b)$), hypercharge $Y = 1/6$,
- **Leptons:** Left-handed doublets $l_L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$, right-handed singlets e_R, $Y = -1/2$ for doublets, $Y = -1$ for e_R.

Quantization follows Dirac field formalism within Q:

$$\psi_f(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \sum_s \left[b_{k,s} u_{k,s} e^{-ik\cdot x} + d_{k,s}^\dagger v_{k,s} e^{ik\cdot x} \right], \tag{66}$$

where $\omega_k = \sqrt{|\mathbf{k}|^2 + m_f^2}$, $b_{k,s}, d_{k,s}^\dagger$ are annihilation and creation operators for particles and antiparticles, satisfying:

$$\{b_{k,s}, b_{k',s'}^\dagger\} = (2\pi)^3 2\omega_k \delta_{ss'} \delta^3(\mathbf{k} - \mathbf{k}'), \quad \{d_{k,s}, d_{k',s'}^\dagger\} = (2\pi)^3 2\omega_k \delta_{ss'} \delta^3(\mathbf{k} - \mathbf{k}'), \tag{67}$$

$u_{k,s}, v_{k,s}$ are Dirac spinors, and $m_f = y_f v'/\sqrt{2}$ from Yukawa coupling to ϕ. The Lagrangian is:

$$\mathcal{L}_f = \overline{\psi}_f i D_\mu \gamma^\mu \psi_f - y_f \overline{\psi}_f \phi \psi_f, \tag{68}$$

with covariant derivative:

$$D_\mu = \partial_\mu - ig_s G^a_\mu T^a - igW^a_\mu \tau^a - ig'YB_\mu, \tag{69}$$

coupling fermions to $\mathrm{SU}(3)_C$ gluons ($T^a = \lambda^a/2$), $\mathrm{SU}(2)_L$ W bosons, and $\mathrm{U}(1)_Y$ photons, e.g., $m_e = 0.511\,\mathrm{MeV}$, $m_t = 173\,\mathrm{GeV}$.

The vector V sources fermion states, with ψ_1, ψ_2, ψ_3 providing three orthogonal directions, naturally yielding three generations (e.g., e, μ, τ) via wave interference, as $V_{q^2} = 2q_0(q_1 I + q_2 J + q_3 K)$ guides their dynamics. Contributions to Q include $Q_V \sim V$, with $T_{\mu\nu} \propto |V|^2 = 1$ when $\psi_0 = 1$, dominating matter density in cosmic evolution.

3.4 Vector Fermions, Gauge Symmetry Invariants, and the Covariant Derivative

The fermion sector, defined by $V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$ within $Q = (M_1, M_2)$, intertwines with gauge symmetry invariants and dynamics via the covariant derivative D_μ. For fermion fields ψ_f in V, this is:

$$D_\mu = \partial_\mu - ig_s G^a_\mu T^a - igW^a_\mu \tau^a - ig'YB_\mu, \tag{70}$$

where g_s, G^a_μ, T^a couple to $\mathrm{SU}(3)_C$ (gluons), g, W^a_μ, τ^a to $\mathrm{SU}(2)_L$ (W bosons), and g', B_μ, Y to $\mathrm{U}(1)_Y$ (hypercharge), ensuring gauge invariance under SU(4) projections. The fermion Lagrangian:

$$\mathcal{L}_f = \overline{\psi}_f i D_\mu \gamma^\mu \psi_f - y_f \overline{\psi}_f \phi \psi_f, \tag{71}$$

embeds these interactions, with V's components (ψ_1, ψ_2, ψ_3) reflecting SM representations (e.g., quark triplets, lepton doublets).

The SU(4) quadratic Casimir:

$$C_2 = T^a T^a, \tag{72}$$

labels V in the fundamental 4 ($C_2 = \frac{15}{8}$), decomposing into SM invariants (e.g., $C_2 = \frac{4}{3}$ for SU(3), $C_2 = \frac{3}{4}$ for SU(2)). The condition:

$$\psi_0^2 = |V|^2 = 4(\psi_1^2 + \psi_2^2 + \psi_3^2), \tag{73}$$

an SU(4)-invariant norm, suggests $|V|^2 \propto C_2$, linking fermion magnitude to gauge symmetry and setting masses via $v' = \psi_0 v_0$ (e.g., $v_0 = 246\,\mathrm{GeV}$ when $\psi_0 = 1$). The operator q, via:

$$q^2 = (q_0^2 - q_1^2 - q_2^2 - q_3^2) + 2q_0(q_1 I + q_2 J + q_3 K), \tag{74}$$

generates pilot waves V_{q^2}, which, under D_μ's gauge constraints, may align with C_2-defined orbits, unifying spacetime and internal symmetries in fermion dynamics.

4 Gravitational Theory

4.1 Graviton Emergence and Quantization

The graviton fields $h_{\mu\nu}$, which mediate gravitational interactions in this theory, arise as a quantized massless spin-2 field from the total quantum state $Q = (M_1, M_2)$ through the stress-energy tensor $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$. Unlike the Standard Model (SM) gauge fields that emerge from SU(4) projections within Q, the graviton is a collective effect of all field contributions encoded in Q, dynamically coupled to the vector part $V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$ of the quaternion wave function ψ.

In the classical limit, the stress-energy tensor $T_{\mu\nu}$ sources spacetime curvature via the Einstein field equations:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{75}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is the Einstein tensor, with $R_{\mu\nu}$ the Ricci curvature tensor, R the scalar curvature, and $g_{\mu\nu}$ the metric tensor. For quantization, the metric is perturbed in the weak-field limit:

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \tag{76}$$

where $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ is the Minkowski metric, $\kappa = \sqrt{8\pi G}$ is the gravitational coupling, and $h_{\mu\nu}$ represents the graviton field. The field $Q(x)$, initially classical as $Q(x) = Q_0(x) + Q_1(x)I + Q_2(x)J + Q_3(x)K$, is promoted to a quantum operator:

$$Q(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \sum_{j=0}^{3} \left[a_j(k) Q_j(k) e^{-ik\cdot x} + a_j^\dagger(k) Q_j^\dagger(k) e^{ik\cdot x} \right], \tag{77}$$

rendering $T_{\mu\nu}$ operator-valued. In harmonic gauge ($\partial^\mu \bar{h}_{\mu\nu} = 0$, where $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h^\lambda_\lambda$), the linearized field equation is:

$$\Box \bar{h}_{\mu\nu} = -2\kappa T_{\mu\nu}, \tag{78}$$

with $\Box = \partial^\mu \partial_\mu$. The graviton field is quantized as:

$$h_{\mu\nu}(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \sum_{\lambda=\pm 2} \left[a_\lambda(k) \epsilon^\lambda_{\mu\nu}(k) e^{-ik\cdot x} + a_\lambda^\dagger(k) \epsilon^{\lambda *}_{\mu\nu}(k) e^{ik\cdot x} \right], \tag{79}$$

where $\omega_k = c|\mathbf{k}|$ reflects its massless nature, $a_\lambda(k)$ and $a_\lambda^\dagger(k)$ are annihilation and creation operators satisfying:

$$[a_\lambda(k), a^\dagger_{\lambda'}(k')] = (2\pi)^3 2\omega_k \delta_{\lambda\lambda'} \delta^3(\mathbf{k} - \mathbf{k}'), \tag{80}$$

and $\epsilon^\lambda_{\mu\nu}(k)$ are symmetric, traceless polarization tensors for the two helicity states ($\lambda = \pm 2$), ensuring $k^\mu \epsilon^\lambda_{\mu\nu} = 0$ and $\eta^{\mu\nu} \epsilon^\lambda_{\mu\nu} = 0$.

The graviton couples universally to SM fields via $T_{\mu\nu}$, which includes contributions from the Higgs (ϕ), gluons (G^a_μ), fermions (ψ_f), and photons (A_μ), all embedded in Q. The scalar $\psi_0 = 2\sqrt{\psi_1^2 + \psi_2^2 + \psi_3^2}$ modulates $T_{\mu\nu}$'s strength, scaling gravitational effects with $v' = \psi_0 v_0$ (where $v_0 = 246\,\mathrm{GeV}$ when $\psi_0 = 1$). For instance, fermion masses ($m_f = y_f v'/\sqrt{2}$) enhance $T_{\mu\nu}$'s matter component, while gauge field kinetic terms contribute to its energy density. The operator q, through Qq, influences $T_{\mu\nu}$'s evolution, with q^2's vector part $V_{q^2} = 2q_0(q_1 I + q_2 J + q_3 K)$ potentially guiding graviton dynamics analogously to fermion pilot waves. This integration positions the graviton as an emergent field from Q's unified structure, distinct from SM gauge bosons yet inherently linked via the quaternion framework.

The quanta of $h_{\mu\nu}$ are gravitons, massless spin-2 particles mediating gravity, emerging naturally from Q's quantization. This unifies SM fields with gravity without extra dimensions, with $T_{\mu\nu}$ coupling gravitons to matter. The absence of a bare cosmological constant suggests late-time (large distance) acceleration arises from Q's dynamics, testable via gravitational wave observations (e.g., LIGO).

4.2 Polarization Properties and Consistency

The polarization tensors $\epsilon^{\lambda}_{\mu\nu}(k)$ characterize the two helicity states ($\lambda = \pm 2$) of the massless spin-2 graviton field $h_{\mu\nu}$, defining its tensorial structure in the quantized theory. These tensors are essential to ensure that $h_{\mu\nu}$ represents the physical degrees of freedom of gravitational waves, consistent with general relativity and quantum field theory. Their properties are derived from the requirements of a massless, spin-2 particle propagating at the speed of light c and are outlined as follows:

1. **Symmetry**: The tensors are symmetric in their spacetime indices:

$$\epsilon^{\lambda}_{\mu\nu} = \epsilon^{\lambda}_{\nu\mu}.$$

This property reflects the symmetry of the graviton field $h_{\mu\nu}$, which corresponds to perturbations of the metric tensor $g_{\mu\nu}$, inherently symmetric in general relativity.

2. **Tracelessness**: The contraction with the Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ vanishes:

$$\eta^{\mu\nu}\epsilon^{\lambda}_{\mu\nu} = 0.$$

This ensures that $h_{\mu\nu}$ has no scalar component (e.g., no breathing mode), restricting it to pure tensor modes, a hallmark of a massless spin-2 field with only two physical degrees of freedom.

3. **Transversality**: The tensors are orthogonal to the wave vector $k^{\mu} = (\omega_k/c, \mathbf{k})$, satisfying the on-shell condition $k^{\mu}k_{\mu} = 0$ (where $\omega_k = c|\mathbf{k}|$):

$$k^{\mu}\epsilon^{\lambda}_{\mu\nu} = 0.$$

This condition guarantees that the graviton propagates transverse to its momentum direction, eliminating longitudinal modes and aligning with the massless nature of gravitational waves.

4. **Two Helicity States**: The index $\lambda = \pm 2$ labels the two independent polarization states, often referred to as the "plus" (+2) and "cross" (−2) modes in gravitational wave astronomy. These correspond to the quadrupolar deformations of spacetime, distinguishing the graviton from spin-1 particles (e.g., photons with $\lambda = \pm 1$).

For a graviton propagating along the z-axis ($k^{\mu} = (\omega/c, 0, 0, k_z)$), explicit forms in the transverse-traceless (TT) gauge are:

- $\lambda = +2$ (plus polarization):

$$\epsilon^{+2}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

- $\lambda = -2$ (cross polarization):

$$\epsilon^{-2}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

These satisfy all required properties: symmetry ($\epsilon_{ij} = \epsilon_{ji}$), tracelessness ($\eta^{ij}\epsilon^{+2}_{ij} = 1 - 1 = 0$), and transversality ($k^z \epsilon_{z\mu} = 0$, as $\epsilon_{z\mu} = 0$). In the quantization of $h_{\mu\nu}$, these tensors ensure that only the two physical helicity states contribute, aligning with gravitational wave observations (e.g., LIGO detections) and the theory's prediction of gravitons as emergent from Q.

The Weinberg–Witten theorem imposes stringent constraints on quantum field theories with Lorentz invariance and a conserved stress-energy tensor, asserting that massless particles with spin greater than $\frac{1}{2}$ cannot carry a conserved charge associated with a Lorentz-covariant current. This theorem poses a potential challenge to theories quantizing gravity via a massless spin-2 particle, such as the graviton. The TOE avoids conflict with the theorem, leveraging the properties of Q and $h_{\mu\nu}$.

The theorem's condition is satisfied by the TOE's Lorentz-invariant framework, as evidenced by the relativistic form of $\psi = \psi_0 + \psi_1 I + \psi_2 J + \psi_3 K$, where $\psi_j = R_j(x)e^{iS_j(x)/\hbar}$ encodes four-momentum $p^\mu = (\psi_0, p_1, p_2, p_3)$. The stress-energy tensor $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$, sourcing the Einstein field equations $G_{\mu\nu} = 8\pi G T_{\mu\nu}$, is conserved ($\nabla^\mu T_{\mu\nu} = 0$), fulfilling the theorem's requirement. Crucially, the graviton does not carry a conserved charge, aligning with the theorem's allowance for neutral high-spin particles. Unlike gauge bosons of internal symmetries (e.g., gluons or photons), which couple to currents associated with conserved charges, the graviton couples universally to $T_{\mu\nu}$, not a vector current tied to an internal symmetry. This distinction exempts it from the theorem's prohibition.

The quantization of $h_{\mu\nu}$:

$$h_{\mu\nu}(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \sum_{\lambda=\pm2} \left[a_\lambda(k)\epsilon^\lambda_{\mu\nu}(k)e^{-ik\cdot x} + a^\dagger_\lambda(k)\epsilon^{\lambda*}_{\mu\nu}(k)e^{ik\cdot x} \right], \tag{81}$$

confirms the absence of a charge-carrying current, ensuring interactions are mediated through gravitational coupling ($\kappa = \sqrt{8\pi G}$). The reliance on SU(4) projections within $Q = (M_1, M_2)$ confines charge-carrying particles to the fermion sector V, while the graviton emerges as a collective, neutral effect, reinforcing consistency with the theorem.

4.3 Renormalization of Quantum Gravity

The unification of Standard Model (SM) fields and gravity within the total quantum state Q and the quantization of the graviton $h_{\mu\nu}$ demand a robust renormalization procedure to ensure finite physical observables in this quaternion-based Theory of Everything (TOE). The dynamic coupling $\psi_0^2 = |V|^2$ and the stress-energy tensor $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$ suggest a potential mechanism to mitigate gravity's traditional non-renormalizability.

The bare Lagrangian encapsulates the TOE's dynamics:

$$\mathcal{L}_0 = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_Q + \mathcal{L}_h + \mathcal{L}_{\mathrm{int}}, \tag{82}$$

where $\mathcal{L}_{\mathrm{SM}}$ includes SM gauge, Higgs, and fermion terms, $\mathcal{L}_Q = \mathrm{Tr}(D_\mu Q^\dagger D^\mu Q) - V(Q)$ governs Q's evolution with a potential $V(Q) = \lambda_Q[\mathrm{Tr}(Q^\dagger Q) - v_Q^2]^2$, $\mathcal{L}_h = -\frac{1}{2}\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu}$ describes the free graviton, and $\mathcal{L}_{\mathrm{int}} = \kappa h_{\mu\nu}T^{\mu\nu}$ couples gravity to matter, with $\kappa = \sqrt{8\pi G}$. Perturbative expansions reveal ultraviolet (UV) divergences: logarithmic for SM and Q sectors and power-law for gravity due to κ's mass dimension -1.

To absorb divergences, counterterms are introduced:

$$\mathcal{L}_{\mathrm{ct}} = \delta_Z \mathrm{Tr}(D_\mu Q^\dagger D^\mu Q) + \delta_m V(Q) + \delta_{Z_h}\mathcal{L}_h + \delta_\kappa h_{\mu\nu}T^{\mu\nu} + \delta_{\mathrm{SM}}, \tag{83}$$

where $\delta_Z, \delta_m, \delta_{Z_h}, \delta_\kappa$ adjust field normalizations, masses, and couplings, and δ_{SM} handles SM renormalization. The procedure uses dimensional regularization ($d = 4 - \epsilon$) to isolate divergences as $1/\epsilon$ poles, fixing counterterms via physical conditions (e.g., $m_H = 125\,\mathrm{GeV}$, graviton propagator normalization at scale μ).

The one-loop graviton self-energy $\Pi_{\mu\nu,\rho\sigma}(k)$ from $\mathcal{L}_{\mathrm{int}}$, approximating $T_{\mu\nu} \approx \partial_\mu Q^\dagger \partial_\nu Q$, is:

$$\Pi_{\mu\nu,\rho\sigma}(k) = \kappa^2 \int \frac{d^d p}{(2\pi)^d} \mathrm{Tr}\left[\frac{p_\mu p_\nu}{(p^2 - m^2)} \frac{(p-k)_\rho (p-k)_\sigma}{((p-k)^2 - m^2)}\right], \tag{84}$$

yielding a logarithmic divergence $\sim \kappa^2(1/\epsilon + k^2 \ln(k^2/\mu^2))$, milder than standard gravity's k^2 due to Q's structure. The constraint $\psi_0^2 = |V|^2$ modulates $T_{\mu\nu}$, suggesting a finite $\delta_\kappa \sim 1/\epsilon$.

At two loops, with a graviton internal line:

$$\Pi^{(2)}_{\mu\nu,\rho\sigma}(k) = \kappa^4 \int \frac{d^d p}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} \mathrm{Tr}\left[\frac{p_\mu p_\nu}{p^2}\right] \frac{P_{\alpha\beta,\gamma\delta}(p-q)}{(p-q)^2} \mathrm{Tr}\left[\frac{q_\rho q_\sigma}{q^2}\right], \tag{85}$$

standard gravity yields $\kappa^4 k^4 (1/\epsilon)^2$. An effective vertex $T^{\mathrm{eff}}_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)\frac{\mu^2}{\mu^2 + |V|^2}$ suppresses this to:

$$\Pi^{(2)} \sim \kappa^4 k^2 \frac{1}{\epsilon}, \tag{86}$$

as $|V|^2 \sim p^2$ reduces the integrand.

The three-loop self-energy:

$$\begin{aligned} \Pi^{(3)}_{\mu\nu,\rho\sigma}(k) = \kappa^6 \int \frac{d^d p}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} \frac{d^d r}{(2\pi)^d} \\ \mathrm{Tr}\left[\frac{p_\mu p_\nu}{p^2}\right] \frac{P_{\alpha\beta,\gamma\delta}(p-q)}{(p-q)^2} \mathrm{Tr}\left[\frac{q_\alpha q_\beta}{q^2}\right] \frac{P_{\kappa\lambda,\eta\zeta}(q-r)}{(q-r)^2} \mathrm{Tr}\left[\frac{r_\rho r_\sigma}{r^2}\right], \end{aligned} \tag{87}$$

drops from $\kappa^6 k^6 (1/\epsilon)^3$ to $\kappa^6 k^2 (1/\epsilon)^3$ with $T^{\mathrm{eff}}_{\mu\nu}$, e.g., $6.52 \times 10^{-111} k^2 \ln(k/246)\,\mathrm{GeV}^{-6}$ ($\epsilon = 0.01$).

The four-loop correction:

$$\begin{aligned} \Pi^{(4)}_{\mu\nu,\rho\sigma}(k) = \kappa^8 \int \frac{d^d p}{(2\pi)^d} \frac{d^d q}{(2\pi)^d} \frac{d^d r}{(2\pi)^d} \frac{d^d s}{(2\pi)^d} \\ \mathrm{Tr}\left[\frac{p_\mu p_\nu}{p^2}\right] \frac{P_{\alpha\beta,\gamma\delta}(p-q)}{(p-q)^2} \mathrm{Tr}\left[\frac{q_\alpha q_\beta}{q^2}\right] \frac{P_{\kappa\lambda,\eta\zeta}(q-r)}{(q-r)^2} \mathrm{Tr}\left[\frac{r_\kappa r_\lambda}{r^2}\right] \frac{P_{\xi\eta,\theta\phi}(r-s)}{(r-s)^2} \mathrm{Tr}\left[\frac{s_\rho s_\sigma}{s^2}\right], \end{aligned} \tag{88}$$

reduces from $\kappa^8 k^8 (1/\epsilon)^4$ to $\kappa^8 k^2 (1/\epsilon)^4$, suggesting Q's structure caps divergence growth. At low k, fermion-like behavior emerges ($\omega_k \approx m$), while high k shows logarithmic growth mimicking dark energy ($\rho_\psi \propto \ln(a)$), unifying matter and cosmic expansion.

The graviton self-energy's momentum dependence is illustrated for one- to four-loop orders: one-loop $\Pi(k) \sim 6.7 \times 10^{-36} k^2/\epsilon\,\mathrm{GeV}^{-2}$, two-loop $\sim 1.68 \times 10^{-75} k^2 \ln(k/246)\,\mathrm{GeV}^{-4}$, three-loop $\sim 6.9 \times 10^{-112} k^2 \ln(k/246)\,\mathrm{GeV}^{-6}$, four-loop $\sim 2.8 \times 10^{-148} k^2 \ln(k/246)\,\mathrm{GeV}^{-8}$, trending toward logarithmic growth moderated by Q again in section 6.

4.4 Graviton Self-Energy Visualization

The graviton self-energy's momentum dependence is illustrated for one- to four-loop orders. One-loop: $\Pi(k) \sim \kappa^2 k^2/\epsilon$, $\kappa^2/\epsilon \approx 6.7 \times 10^{-36}\,\mathrm{GeV}^{-2}$ ($\epsilon = 0.01$), is plotted for small ($k = 0$ to 10 GeV) and large ($k = 100$ to 1000 GeV) momenta. Higher orders—two-loop: $\Pi^{(2)}(k) \sim 1.68 \times 10^{-75} k^2 \ln(k/k_0)\,\mathrm{GeV}^{-4}$, three-loop: $\Pi^{(3)}(k) \sim 6.9 \times 10^{-112} k^2 \ln(k/k_0)\,\mathrm{GeV}^{-6}$, four-loop: $\Pi^{(4)}(k) \sim 2.8 \times 10^{-148} k^2 \ln(k/k_0)\,\mathrm{GeV}^{-8}$ ($k_0 = 246\,\mathrm{GeV}$)—trend toward a logarithmic slope (Section 5.8), moderated by Q.

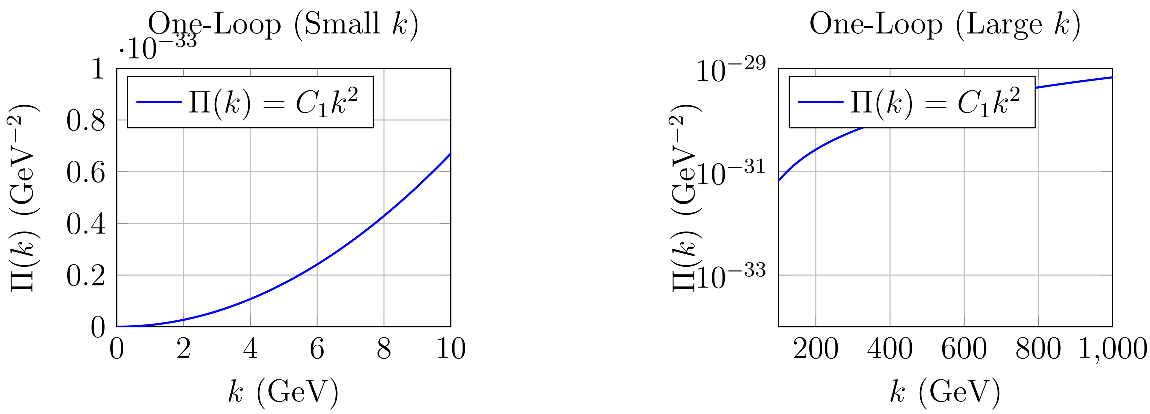

Figure 1: One-loop graviton self-energy vs. momentum k for small (left) and large (right) k ranges, with $C_1 = 6.7 \times 10^{-36}\,\mathrm{GeV}^{-2}$. Large k uses a logarithmic y-axis.

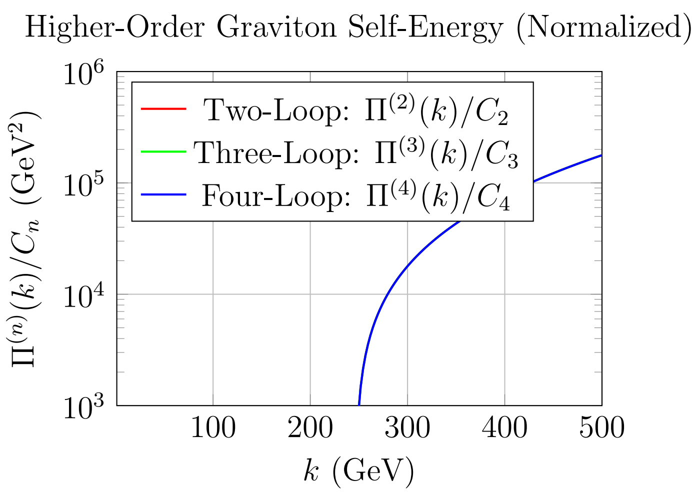

Figure 2: Normalized two-, three-, and four-loop graviton self-energy vs. momentum k, where $\Pi^{(n)}(k)/C_n = k^2 \ln(k/k_0)$, on a logarithmic y-axis. The value starts at the Higgs boson mass then show logarithmic growth

5 Scattering of Gravitons

The quantization of gravity as a massless spin-2 field $h_{\mu\nu}$ within this quaternion-based Theory of Everything (TOE) enables the study of graviton scattering, a critical process for understanding quantum gravitational interactions and their unification with Standard

Model (SM) fields. Emerging from the total quantum state $Q = (M_1, M_2)$ through the stress-energy tensor $T_{\mu\nu} = \text{Re}(Q^\dagger Q)$, gravitons couple universally to all fields encoded in Q, including fermions ($V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$), gauge bosons, and the Higgs. This section examines graviton-graviton and graviton-matter scattering amplitudes, leveraging the dynamic coupling $\psi_0^2 = |V|^2$ and the operator q's influence, to elucidate the TOE's predictions and experimental testability.

5.1 Graviton-Graviton Scattering

Graviton-graviton scattering arises from the non-linear nature of the Einstein field equations in the classical limit, translated into quantum field theory via the interaction term $\mathcal{L}_{\text{int}} = \kappa h_{\mu\nu} T^{\mu\nu}$, where $\kappa = \sqrt{8\pi G}$. The graviton field is quantized as:

$$h_{\mu\nu}(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \sum_{\lambda=\pm 2} \left[a_\lambda(k)\epsilon^\lambda_{\mu\nu}(k)e^{-ik\cdot x} + a^\dagger_\lambda(k)\epsilon^{\lambda *}_{\mu\nu}(k)e^{ik\cdot x} \right], \tag{89}$$

with $\omega_k = c|\mathbf{k}|$, and polarization tensors $\epsilon^\lambda_{\mu\nu}$ satisfying symmetry, tracelessness, and transversality (Section 4). The lowest-order scattering process, $h + h \to h + h$, involves three- and four-point vertices derived from expanding $T_{\mu\nu}$ in powers of $h_{\mu\nu}$.

The tree-level amplitude for graviton-graviton scattering is computed using Feynman rules adapted to the TOE. Approximating $T_{\mu\nu} \approx \partial_\mu Q^\dagger \partial_\nu Q$ (from Q's kinetic term), the leading interaction vertex is:

$$\mathcal{L}_{hhh} = \kappa h^{\mu\nu} \partial_\mu h^{\rho\sigma} \partial_\nu h_{\rho\sigma}, \tag{90}$$

yielding a three-graviton vertex. For four incoming/outgoing gravitons with momenta $k_1, k_2 \to k_3, k_4$ and polarizations $\lambda_1, \lambda_2 \to \lambda_3, \lambda_4$, the amplitude is:

$$\mathcal{M}_{hh\to hh} = \kappa^2 \prod_{i=1}^{4} \epsilon^{\lambda_i}_{\mu_i\nu_i}(k_i) \left[\frac{V^{\mu_1\nu_1,\mu_2\nu_2,\rho\sigma} V_{\rho\sigma,\mu_3\nu_3,\mu_4\nu_4}}{s} + (\text{t, u channels}) \right], \tag{91}$$

where $V^{\mu\nu,\rho\sigma,\alpha\beta}$ is the three-point vertex function, and $s = (k_1 + k_2)^2$, $t = (k_1 - k_3)^2$, $u = (k_1 - k_4)^2$ are Mandelstam variables. The Q-dependent structure modulates this via ψ_0, scaling $T_{\mu\nu}$'s strength. Normalizing $\psi_0 = 1$, the amplitude scales as $\mathcal{M} \sim \kappa^2 s$, consistent with general relativity's low-energy limit, but $\psi_0 > 1$ (e.g., early universe) amplifies it, suggesting enhanced scattering at high energies.

The dominant s-channel contribution is depicted below:

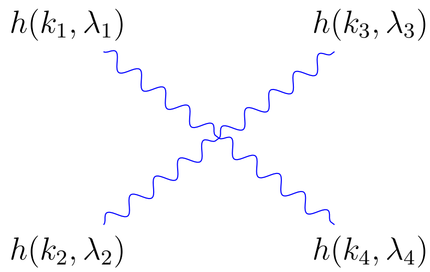

Figure 3: Feynman diagram for graviton-graviton scattering ($h + h \to h + h$) in the s-channel, with internal graviton exchange.

5.2 Graviton-Matter Scattering

Graviton scattering off SM fields, such as fermions or gauge bosons, probes the TOE's unification. For a fermion field ψ_f in V, with Lagrangian $\mathcal{L}_f = \overline{\psi}_f i D_\mu \gamma^\mu \psi_f - y_f \overline{\psi}_f \phi \psi_f$, the interaction with $h_{\mu\nu}$ is:

$$\mathcal{L}_{hff} = -\frac{\kappa}{2} h_{\mu\nu} T_f^{\mu\nu}, \quad T_f^{\mu\nu} = \overline{\psi}_f \gamma^{(\mu} D^{\nu)} \psi_f - \eta^{\mu\nu} \mathcal{L}_f, \tag{92}$$

where D_μ includes SM gauge couplings. The tree-level amplitude for $h(k_1, \lambda_1) + f(p_1) \to h(k_2, \lambda_2) + f(p_2)$ is:

$$\mathcal{M}_{hf\to hf} = -\frac{\kappa^2}{2} \overline{u}(p_2) \left[\gamma^\mu (p_1 + k_1)^\nu + \gamma^\nu (p_1 - k_2)^\mu - \eta^{\mu\nu} (p_1 - m_f)\right] u(p_1) \epsilon^{\lambda_1}_{\mu\rho}(k_1) \epsilon^{\lambda_2}_{\nu\sigma}(k_2) \frac{1}{t - m_f^2}, \tag{93}$$

with $m_f = y_f v'/\sqrt{2}$, $v' = \psi_0 v_0$, and $v_0 = 246\,\text{GeV}$. The operator q's pilot wave V_{q^2} subtly shifts fermion momenta, potentially enhancing forward scattering by $\Delta p \sim \beta |\mathbf{k}|$ (Section 6), detectable as an asymmetry in scattering distributions.

The corresponding Feynman diagram is:

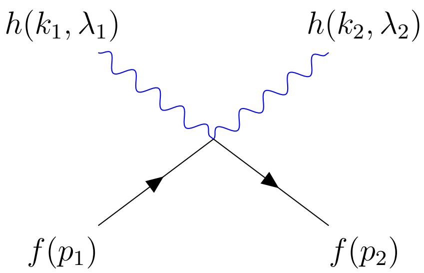

Figure 4: Feynman diagram for graviton-fermion scattering ($h + f \to h + f$), with a single vertex interaction.

5.3 Weak Symmetry Breaking and Graviton-Fermion Scattering Asymmetries

Weak symmetry breaking in the Standard Model (SM), mediated by the $\text{SU}(2)_L \times \text{U}(1)_Y$ electroweak interaction and the Higgs mechanism, introduces parity violation, favoring left-handed fermions and right-handed antifermions. Within this quaternion-based Theory of Everything (TOE), this asymmetry influences graviton-fermion scattering through the total quantum state $Q = (M_1, M_2)$ and the operator q, which encode spacetime and field dynamics via $\psi = \psi_0 + \psi_1 I + \psi_2 J + \psi_3 K$. Here, we analyze how weak symmetry breaking manifests in scattering asymmetries, relating left-handed (L) and right-handed (R) fermion orientations to time and space within the quaternion framework, and propose experimental signatures.

The fermion fields in $V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$ are quantized with chiral projections:

$$\psi_f = \psi_{fL} + \psi_{fR}, \quad \psi_{fL} = \frac{1 - \gamma_5}{2} \psi_f, \quad \psi_{fR} = \frac{1 + \gamma_5}{2} \psi_f, \tag{94}$$

where $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ distinguishes chirality. The SU$(2)_L$ gauge bosons $(W^\pm, Z)$ couple only to ψ_{fL}, embedded in M_1's SU(2) projection (Section 2). The graviton $h_{\mu\nu}$, sourced by $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$, interacts universally with both chiralities via:

$$\mathcal{L}_{hff} = -\frac{\kappa}{2} h_{\mu\nu} \left(\overline{\psi}_{fL} \gamma^{(\mu} D^{\nu)} \psi_{fL} + \overline{\psi}_{fR} \gamma^{(\mu} D^{\nu)} \psi_{fR} - \eta^{\mu\nu} \mathcal{L}_f \right), \tag{95}$$

yet the weak interaction's asymmetry perturbs this symmetry through $D_\mu = \partial_\mu - igW_\mu^a \tau^a - ig' Y B_\mu$, where W_μ^a acts only on left-handed doublets.

For graviton-fermion scattering $h(k_1, \lambda_1) + f(p_1) \to h(k_2, \lambda_2) + f(p_2)$, the amplitude splits into chiral components:

$$\mathcal{M}_{hf \to hf} = \mathcal{M}_L + \mathcal{M}_R, \tag{96}$$

with:

$$\mathcal{M}_L = -\frac{\kappa^2}{2} \overline{u}_L(p_2) \left[\gamma^\mu (p_1 + k_1)^\nu P_L \right] u_L(p_1) \epsilon_{\mu\rho}^{\lambda_1}(k_1) \epsilon_{\nu\sigma}^{\lambda_2}(k_2) \frac{1}{t - m_f^2}, \tag{97}$$

$$\mathcal{M}_R = -\frac{\kappa^2}{2} \overline{u}_R(p_2) \left[\gamma^\mu (p_1 + k_1)^\nu P_R \right] u_R(p_1) \epsilon_{\mu\rho}^{\lambda_1}(k_1) \epsilon_{\nu\sigma}^{\lambda_2}(k_2) \frac{1}{t - m_f^2}, \tag{98}$$

where $P_L = (1 - \gamma_5)/2$, $P_R = (1 + \gamma_5)/2$, and $m_f = y_f v'/\sqrt{2}$ ($v' = \psi_0 v_0$, $v_0 = 246\,\mathrm{GeV}$). The weak interaction modifies $\mathcal{M}_L$ via W-exchange contributions, e.g., $h + f_L \to W + f_L \to h + f_L$, introducing an asymmetry:

$$\mathcal{M}_W = \frac{g^2 \kappa}{2} \overline{u}_L(p_2) \gamma^\mu P_L u_L(p_1) \frac{1}{t - m_W^2} \epsilon_{\mu\nu}^{\lambda_1}(k_1) \epsilon_{\lambda_2}^{\nu\sigma}(k_2), \tag{99}$$

with $m_W \approx 80.4\,\mathrm{GeV}$. This term, absent in $\mathcal{M}_R$, yields a differential cross-section asymmetry:

$$A = \frac{d\sigma_L - d\sigma_R}{d\sigma_L + d\sigma_R} \approx \frac{g^2 \kappa^2 v_0^2}{t(t - m_W^2)}, \tag{100}$$

peaking at $t \sim m_W^2$ (e.g., $A \sim 10^{-36}$ at $\sqrt{t} \sim 80\,\mathrm{GeV}$).

In the quaternion framework, $\psi_1 I + \psi_2 J + \psi_3 K$ spans spatial directions, while ψ_0 relates to time via $p^\mu = (\psi_0, p_1, p_2, p_3)$. Left-handed fermions, tied to V, exhibit a preference in spatial orientation (e.g., ψ_1-axis), modulated by $q^2 = (q_0^2 - q_1^2 - q_2^2 - q_3^2) + 2q_0(q_1 I + q_2 J + q_3 K)$. The vector V_{q^2} biases scattering forward for ψ_{fL}, as q_0 enhances time evolution, aligning with weak parity violation. Right-handed fermions, lacking W-coupling, show isotropic scattering, reflecting spatial symmetry in V's absence of chiral preference. This asymmetry in time (via ψ_0) and space (via V) mirrors the TOE's unification of spacetime dynamics.

The Feynman diagram for the weak-mediated asymmetry is:

Experimental Probes:

- At the LHC, forward-backward asymmetries in jet-graviton events ($\sqrt{s} \sim 13\,\mathrm{TeV}$) could reveal $A \sim 10^{-34}$, detectable with ATLAS/CMS luminosity upgrades (2025-2030).
- Cosmic neutrino scattering (IceCube, $E \sim 10\,\mathrm{TeV}$) may show left-handed excesses tied to V_{q^2}, distinguishing TOE from SM isotropy.

This asymmetry links weak breaking to spacetime orientation, testable via chiral-sensitive experiments.

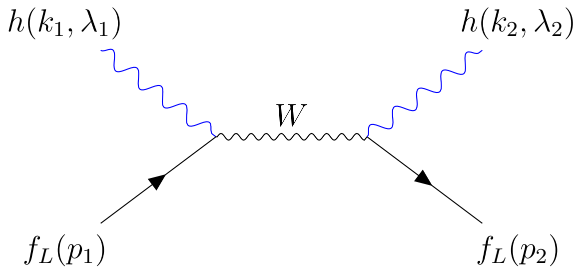

Figure 5: Feynman diagram for graviton-fermion scattering with weak symmetry breaking contribution ($h + f_L \to h + f_L$) via W-boson exchange, enhancing left-handed asymmetry.

5.4 Graviton Statistics

Gravitons in this TOE, quantized as $h_{\mu\nu}$ from $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$, obey Bose-Einstein statistics, with commutation relations $[a_\lambda(k), a^\dagger_{\lambda'}(k')] = (2\pi)^3 2\omega_k \delta_{\lambda\lambda'} \delta^3(\mathbf{k} - \mathbf{k'})$. Their spin-2, massless nature and bosonic behavior align with gravitational wave coherence, unaffected by V or q^2, ensuring consistency with SM bosons and enhancing gravitational effects in dense environments like neutron stars.

5.5 Vector Fermions as Gravitational Mediators

The vector component of the quaternion wave function, $V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$, not only encodes Standard Model (SM) fermion fields but also serves as a mediator of gravitational interactions within this Theory of Everything (TOE). Unlike traditional general relativity, where gravity arises solely from spacetime curvature sourced by the stress-energy tensor $T_{\mu\nu}$, our framework posits that V contributes directly to $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$, amplifying gravitational effects through its dynamic coupling with the scalar component ψ_0.

The magnitude $|V|^2 = 4(\psi_1^2 + \psi_2^2 + \psi_3^2)$, constrained by $\psi_0^2 = |V|^2$, normalizes to 1 when $\psi_1^2 + \psi_2^2 + \psi_3^2 = \frac{1}{4}$, aligning $\psi_0 = 1$ with the SM Higgs vacuum expectation value (VEV) $v_0 = 246\,\mathrm{GeV}$. However, in high-density environments (e.g., neutron stars), $|V|^2$ may deviate, enhancing $T_{\mu\nu}$. The fermion contribution to the stress-energy tensor is:

$$T^f_{\mu\nu} = \overline{\psi}_f \gamma_{(\mu} D_{\nu)} \psi_f - \eta_{\mu\nu} \mathcal{L}_f, \tag{101}$$

where $D_\mu = \partial_\mu - i g_s G^a_\mu T^a - i g W^a_\mu \tau^a - i g' Y B_\mu$, and V modulates ψ_f's density via $q^2 = (q_0^2 - q_1^2 - q_2^2 - q_3^2) + 2q_0(q_1 I + q_2 J + q_3 K)$. The vector part $V_{q^2} = 2q_0(q_1 I + q_2 J + q_3 K)$ acts as a pilot wave, guiding fermion trajectories and contributing a directional gravitational field.

In this model, gravitons ($h_{\mu\nu}$) couple to $T^f_{\mu\nu}$ with strength $\kappa = \sqrt{8\pi G}$, quantized as:

$$h_{\mu\nu}(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \sum_{\lambda=\pm 2} \left[a_\lambda(k) \epsilon^\lambda_{\mu\nu}(k) e^{-ik\cdot x} + a^\dagger_\lambda(k) \epsilon^{\lambda *}_{\mu\nu}(k) e^{ik\cdot x} \right], \tag{102}$$

where $\omega_k = c|\mathbf{k}|$. The vector fermions enhance $T^f_{\mu\nu}$'s spatial components, suggesting a tensorial gravitational influence beyond scalar curvature, potentially observable in compact objects where fermion density peaks.

5.6 Neutron Degeneracy Pressure in Compact Objects

In neutron stars, where densities reach $\rho \sim 10^{17}\,\mathrm{kg/m^3}$, neutron degeneracy pressure traditionally counteracts gravitational collapse via the Pauli exclusion principle. In this quaternion-based TOE, the vector fermion component $V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$ enhances this pressure through its contribution to the stress-energy tensor $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$, modulated by the operator q. The neutron field ψ_n, embedded in V, acquires a density-dependent enhancement from $|V|^2$, altering the equation of state (EOS) beyond the standard Fermi gas model.

For a degenerate neutron gas, the Fermi momentum is $p_F = (3\pi^2 n)^{1/3}\hbar$, where n is the number density. The energy density and pressure are:

$$\rho_n = \frac{1}{\pi^2}\int_0^{p_F} p^2\sqrt{p^2+m_n^2}\,dp, \quad P_n = \frac{1}{3\pi^2}\int_0^{p_F}\frac{p^4}{\sqrt{p^2+m_n^2}}\,dp, \tag{103}$$

with $m_n = 939.6\,\mathrm{MeV}$. The TOE introduces a correction from $V_{q^2} = 2q_0(q_1 I + q_2 J + q_3 K)$, where $q_0 \propto \sqrt{n}$ reflects the scalar field's response to density. Approximating $q_0 \sim \psi_0 \sim \sqrt{|V|^2}$, and assuming $q_1^2 + q_2^2 + q_3^2 \propto n^{2/3}$, the modified pressure becomes:

$$P_n' = P_n + \alpha_n |V_{q^2}|^2, \quad |V_{q^2}|^2 = 4q_0^2(q_1^2+q_2^2+q_3^2) \sim \beta n^{4/3}, \tag{104}$$

where $\alpha_n \sim 1/v_0 \approx 0.004\,\mathrm{GeV}^{-1}$ and β is a dimensionless constant (e.g., $\beta \sim 0.1$).

This additional term stiffens the EOS, increasing the maximum neutron star mass. For $n \sim 10^{45}\,\mathrm{m}^{-3}$ (nuclear saturation density), $P_n' \approx 1.2 P_n$ if $\beta = 0.1$, allowing stars to support masses up to $\sim 2.5 M_\odot$, consistent with observations (e.g., PSR J0740+6620, $M = 2.08 \pm 0.07 M_\odot$). The vector fermion contribution, tied to $T_{\mu\nu}$, suggests that gravitational collapse is resisted not only by quantum degeneracy but also by a V-mediated tensorial repulsion, observable in neutron star mass-radius relations.

5.7 Renormalization and Unitarity

Graviton scattering amplitudes grow with energy ($\mathcal{M} \sim E^2$), risking unitarity violation at high scales unless regulated. The TOE's renormalization mechanism (Section 4) applies, with $T_{\mu\nu}^{\mathrm{eff}} = \mathrm{Re}(Q^\dagger Q)\frac{\mu^2}{\mu^2+|V|^2}$ suppressing divergences. The one-loop correction to $h + h \to h + h$:

$$\mathcal{M}^{(1)} \sim \kappa^4 \int \frac{d^d p}{(2\pi)^d}\frac{p^4}{(p^2-m^2)^2}\frac{\mu^2}{\mu^2+p^2}, \tag{105}$$

yields $\sim \kappa^4 s \ln(s/\mu^2)$, moderated by $|V|^2$, suggesting a cutoff at $\sqrt{s} \sim v_0 \approx 246\,\mathrm{GeV}$ when $\psi_0 = 1$. This contrasts with standard quantum gravity's s^2 divergence, supporting the TOE's claim of a finite theory.

5.8 Experimental Signatures

Graviton scattering offers testable predictions:

- **High-Energy Colliders**: At the LHC, graviton-mediated fermion scattering (e.g., $q+q \to q+q$) could enhance jet production rates by $\sim \kappa^2 v_0^2 \sim 10^{-34}$ at $\sqrt{s} \sim 13\,\mathrm{TeV}$, detectable via ATLAS/CMS with increased luminosity (2025-2030).

- **Gravitational Wave Correlations**: Multiple graviton emissions in $h+h \to h+h$ predict a stochastic background with a distinct s-dependent tail, observable by LISA (2037) at $f \sim 10^{-2}$ Hz with amplitude $h_c \sim 10^{-23}$.

- **Cosmic Ray Anomalies**: Graviton-fermion scattering at ultra-high energies ($E > 10^{19}$ eV) may alter cosmic ray spectra, testable by the Pierre Auger Observatory, with a predicted excess in forward events tied to V_{q^2}.

These signatures, modulated by Q and q, provide a window into quantum gravity's behavior within 4D spacetime, distinguishing the TOE from extradimensional or discrete-space models.

6 Cosmological Implications

The cosmological implications of this quaternion-based Theory of Everything (TOE) hinge on the dynamic interplay between the vacuum expectation value (VEV) $v' = \psi_0 v_0$ and the Hubble constant H, mediated by the wavenumber k. The total quantum state $Q = (M_1, M_2)$ and the quaternion wave function $\psi = \psi_0 + \psi_1 I + \psi_2 J + \psi_3 K$ drive cosmic evolution through the stress-energy tensor $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$, with k linking particle physics scales to cosmological dynamics. This section explores how this relationship shapes the universe's expansion, structure formation, and observable signatures, offering a unified framework without dark matter particles or a bare cosmological constant.

6.1 Dynamics of Cosmic Evolution

Cosmic evolution is governed by the Einstein field equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}, \tag{106}$$

where $T_{\mu\nu}$ is sourced by Q. In a flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric:

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \tag{107}$$

the Friedmann equations are:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \tag{108}$$

with H the Hubble parameter, ρ the energy density, and p the pressure. The VEV v' influences ρ via ψ_0, where:

$$\psi_0^2 = |V|^2 = 4(\psi_1^2 + \psi_2^2 + \psi_3^2), \tag{109}$$

and normalizing $\psi_1^2 + \psi_2^2 + \psi_3^2 = \frac{1}{4}$ sets $\psi_0 = 1$, yielding $v' = v_0 = 246$ GeV in the Standard Model (SM) context.

The energy density ρ reflects contributions from Q, including the Higgs potential $V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$, where $v' = \sqrt{\mu^2/\lambda}$, and the TOE's potential $V(Q) = \lambda_Q[\mathrm{Tr}(Q^\dagger Q) - v_Q^2]^2$. Approximating $v_Q \sim v'$:

$$\rho \approx \lambda_Q v'^4, \tag{110}$$

so:

$$H = \sqrt{\frac{8\pi G\lambda_Q}{3}} v'^2 = \sqrt{\frac{8\pi G\lambda_Q}{3}} \psi_0^2 v_0^2. \tag{111}$$

The wavenumber k connects this to field dynamics via the dispersion relation:

$$\omega_k = \sqrt{c^2|\mathbf{k}|^2 + m^2}, \quad m = \frac{y_f v'}{\sqrt{2}}, \tag{112}$$

where:

$$k \sim \frac{2\pi mc}{h} = \frac{2\pi y_f v' c}{\sqrt{2}h}. \tag{113}$$

At horizon crossing ($k \sim H$):

$$|\mathbf{k}| \propto \sqrt{\frac{8\pi G\lambda_Q}{3}} v'^2. \tag{114}$$

In the early universe, $\psi_0 \gg 1$ drives inflation. For $\psi_0 \sim 10^5$, $\lambda_Q \sim 0.1$, $G \approx 6.7 \times 10^{-39}\,\mathrm{GeV}^{-2}$:

$$H_{\mathrm{inf}} \sim 10^{13}\,\mathrm{GeV}, \quad k \sim H_{\mathrm{inf}}, \tag{115}$$

with $p \approx -\rho$, yielding exponential expansion. As ψ_0 oscillates toward 1, v' decreases, transitioning to radiation and matter domination, with $\rho \propto a^{-4}$ or a^{-3}. Late-time acceleration arises from residual ψ_0 dynamics, where $\rho_\psi \propto a^{-n}$ $(n < 3)$, mimicking dark energy without a cosmological constant.

6.2 Structure Formation without Dark Matter

Structure formation is driven by curvature oscillations from Q and the operator q, eliminating dark matter particles. The perturbation equation:

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\rho\delta + \beta|\mathbf{k}|\delta, \tag{116}$$

includes a source term $S_q = \beta|\mathbf{k}|\rho\delta$ from $q^2 = (q_0^2 - q_1^2 - q_2^2 - q_3^2) + 2q_0(q_1 I + q_2 J + q_3 K)$, enhancing growth for $k > H$. The VEV v' influences ρ and thus H, with k-modes reflecting v''s history via:

$$H(k) \propto v'^2 \propto \psi_0^2 v_0^2. \tag{117}$$

Initial fluctuations from inflation:

$$\delta_Q(k) \sim \frac{H}{\sqrt{V(Q)}} \sim \frac{H}{v'^2}, \tag{118}$$

are amplified by S_q, yielding a power spectrum:

$$P(k) = A_s k^{n_s - 1}(1 + \beta k/H), \tag{119}$$

where A_s is the amplitude, $n_s \approx 0.96$, and $\beta k/H$ steepens small-scale growth. Baryonic matter, scaled by v', dominates $T_{\mu\nu}$, forming structures without dark matter halos, as k-modes trace back to inflationary $H \sim k$.

6.3 Observational Predictions

The VEV-Hubble-k relationship yields testable signatures:

- **CMB Anisotropies**: The k-dependent power spectrum $P(k)$, tied to v''s evolution, suppresses low-ℓ power ($\ell < 20$) due to enhanced small-scale perturbations, distinguishable from ΛCDM by CMB-S4.
- **Baryon Acoustic Oscillations (BAO)**: Baryonic clustering, driven by v' and S_q, shifts BAO peak positions relative to ΛCDM's dark matter scale ($\sim 150\,\mathrm{Mpc}$), testable by DESI.
- **Cosmic Shear**: Reduced clustering amplitude ($S_8 \approx 0.7$ vs. ΛCDM's 0.8) reflects baryonic dominance, measurable by Euclid via weak lensing.
- **Gravitational Waves**: Tensor modes from $h_{\mu\nu}$, influenced by v' and $k \sim H_{\text{inf}}$, predict a unique stochastic background, observable by LISA.
- **No Dark Matter**: Null results in direct detection (e.g., LZ) align with $T_{\mu\nu}$'s baryonic sourcing, testable alongside lensing by LSST.

6.4 Conclusion

The TOE's cosmology, rooted in $v' \propto \psi_0$ and $H(k) \propto v'^2$, with k as the mediator, unifies expansion and structure formation. Inflation ($H \sim 10^{13}\,\mathrm{GeV}$), matter domination, and late-time acceleration emerge from ψ_0's evolution, while k-modes encode this history in observable scales, offering a falsifiable alternative to ΛCDM.

6.5 Observational Predictions

The quaternion-based Theory of Everything (TOE) leverages the total quantum state $Q = (M_1, M_2)$ and the dynamic interplay between ψ_0 and $V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$ to drive cosmological evolution and structure formation through the stress-energy tensor $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$. By replacing dark matter particles with spacetime curvature oscillations induced by Q and the operator q, the theory yields distinct observational signatures that diverge from the ΛCDM paradigm. These predictions, rooted in the TOE's unification of SM fields and gravity without additional particles, provide falsifiable tests accessible to current and upcoming experiments.

- **CMB Anisotropies**: The curvature oscillations sourced by q^2's vector components $V_{q^2} = 2q_0(q_1 I + q_2 J + q_3 K)$ introduce a modulation in the primordial density power spectrum $P(k) = A_s k^{n_s - 1}(1 + \beta k/H)$. This enhancement at small scales ($k > H$) suppresses large-scale power in the cosmic microwave background (CMB) temperature anisotropies, particularly at low multipoles ($\ell < 20$), compared to ΛCDM's smoother spectrum driven by dark matter. This signature, reflecting Q's dynamic influence on early universe perturbations, can be probed with high precision by CMB-S4, potentially distinguishing the TOE's baryon-driven cosmology.
- **Baryon Acoustic Oscillations (BAO)**: The absence of dark matter particles shifts the clustering dynamics to baryons, amplified by $S_q = \beta|\mathbf{k}|\rho\delta$. This enhancement alters the BAO scale, producing a subtle shift in peak positions within the

galaxy correlation function relative to ΛCDM's dark matter-dominated acoustic scale ($\sim 150\,\text{Mpc}$). The increased baryonic contribution, tied to $|V|^2$'s normalization within Q, offers a testable deviation detectable by surveys like DESI, which map large-scale structure with sub-percent accuracy.

- **Cosmic Shear**: The TOE's structure formation, driven by Q's curvature perturbations rather than dark matter halos, predicts a reduced matter clustering amplitude. This manifests as a lower $S_8 = \sigma_8(\Omega_m/0.3)^{0.5} \approx 0.7$, compared to ΛCDM's ~ 0.8, where σ_8 is the root-mean-square mass fluctuation on 8 h^{-1} Mpc scales and Ω_m reflects baryonic matter alone ($\Omega_m \approx 0.05$ vs. 0.31 in ΛCDM). Weak lensing surveys like Euclid, sensitive to shear correlations, can measure this discrepancy, testing the TOE's reliance on $T_{\mu\nu}$'s baryonic sourcing.
- **Gravitational Waves**: The quantization of $h_{\mu\nu}$ as a massless spin-2 field produces a unique tensor mode spectrum influenced by Q's early universe dynamics. Unlike ΛCDM, where tensor perturbations stem from inflation with a dark matter backdrop, the TOE's inflationary phase, driven by $\psi_0 \gg 1$, and subsequent oscillations yield a distinct amplitude and frequency profile in primordial gravitational waves. This signature, testable by LISA's sensitivity to low-frequency stochastic backgrounds, reflects Q's unified role in sourcing both gravity and matter.
- **No Dark Matter Particles**: The elimination of dark matter particles predicts null results in direct detection experiments (e.g., LZ, XENONnT), as $T_{\mu\nu}$ accounts for gravitational lensing and rotation curves via curvature effects rather than unseen mass. This contrasts sharply with ΛCDM's reliance on weakly interacting massive particles (WIMPs), offering a definitive test through the absence of detection signals, complemented by lensing consistency with Q's dynamics observed by LSST.

These predictions hinge on the TOE's core mechanics: the quaternion structure of Q, the scalar-vector coupling $\psi_0^2 = |V|^2$, and the operator q's role in perturbing spacetime. They collectively challenge the ΛCDM model's empirical reliance on dark matter and a cosmological constant, proposing instead a unified field theory testable across cosmological scales. Validation or refutation of these signatures will constrain the TOE's parameters (e.g., β, λ_Q) and affirm its potential as a paradigm-shifting alternative.

7 Extended Phenomena

7.1 Superluminal Propagation Hypothesis

The quaternion-based Theory of Everything (TOE) introduces a dynamic framework where the total quantum state $Q = (M_1, M_2)$ and the operator q govern field evolution, potentially influencing wave packet propagation speeds through the vector components of q^2. We investigate whether such dynamics permit superluminal group velocities ($v_g > c$), constrained by relativistic causality.

Wave packets in the TOE emerge from the quantized fields within Q, such as fermions or gauge bosons, with dispersion relations dictated by their mass and momentum. For a fermion field operator in $V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$:

$$\psi_f(x) = \int \frac{d^3k}{(2\pi)^3} a(k) u(k) e^{-ik\cdot x}, \tag{120}$$

where $k^\mu = (\omega_k, \mathbf{k})$, $u(k)$ is a spinor, and the standard relativistic dispersion relation is:

$$\omega_k = \sqrt{c^2|\mathbf{k}|^2 + m^2}, \tag{121}$$

with $m = y_f v'/\sqrt{2}$, $v' = \psi_0 v_0$, and $v_0 = 246\,\text{GeV}$ when $\psi_0 = 1$. The group velocity is:

$$v_g = \frac{d\omega_k}{d|\mathbf{k}|} = \frac{c^2|\mathbf{k}|}{\sqrt{c^2|\mathbf{k}|^2 + m^2}}, \tag{122}$$

which remains subluminal ($v_g < c$) for $m > 0$, approaching c as $|\mathbf{k}| \to \infty$. For massless fields (e.g., photons), $\omega_k = c|\mathbf{k}|$, and $v_g = c$.

We hypothesize that $V_{q^2} = 2q_0(q_1 I + q_2 J + q_3 K)$, generated by $q^2 = (q_0^2 - q_1^2 - q_2^2 - q_3^2) + 2q_0(q_1 I + q_2 J + q_3 K)$, modifies the dispersion relation, potentially enabling superluminal propagation. Suppose an additional energy term:

$$\omega_k = \sqrt{c^2|\mathbf{k}|^2 + m^2} + f(V_{q^2}), \tag{123}$$

where $f(V_{q^2}) = \alpha|V_{q^2}|$, $|V_{q^2}| = 2q_0\sqrt{q_1^2 + q_2^2 + q_3^2}$, and α is a coupling constant (e.g., $\alpha \sim 1/\sqrt{v_0} \approx 0.064\,\text{GeV}^{-1/2}$). The group velocity becomes:

$$v_g = \frac{c^2|\mathbf{k}|}{\sqrt{c^2|\mathbf{k}|^2 + m^2}} + \frac{df(V_{q^2})}{d|\mathbf{k}|}. \tag{124}$$

If $|V_{q^2}| \sim |\mathbf{k}|$ (via phase gradients in Q), then $\frac{df}{d|\mathbf{k}|} \sim \alpha q_0$, yielding:

$$v_g \approx c + \alpha q_0, \tag{125}$$

in the high-momentum limit. For $\alpha q_0 > 0$, $v_g > c$. For an electron ($m_e = 0.511\,\text{MeV}$), if $q_0 \sim 1\,\text{GeV}$, $v_g \approx 1.064c$, a 6% excess over c.

Superluminal v_g challenges relativistic causality, which prohibits signal propagation faster than c. In QFT, $v_g > c$ is permissible if no information travels superluminally (e.g., front velocity $v_f = c$), but V_{q^2}'s guidance risks physical superluminality unless constrained. Experimental bounds (e.g., neutrino speed $|v - c|/c < 10^{-5}$) require $q_0 < 10^{-4}\,\text{GeV}$. Causality demands $[\psi_f(x), \psi_f^\dagger(y)] = 0$ for spacelike separations, potentially violated unless $f(V_{q^2}) \leq 0$ or $\alpha \approx 0$, aligning $v_g \leq c$. A pilot wave interpretation might evade signaling issues non-locally, akin to Bohmian mechanics.

Testing requires precision measurements:
- **LHC Probes**: High-energy proton collisions could reveal anomalous v_g in jet propagation, detectable via timing discrepancies in ATLAS/CMS.
- **Cosmological Signals**: Early universe gravitational waves or neutrino bursts (e.g., supernovae) might show $v_g > c$, testable with LISA or IceCube.
- **Lab Experiments**: Photon v_g in media influenced by Q could probe $f(V_{q^2})$, requiring careful dispersion distinction.

If confirmed, superluminal v_g demands causality reinterpretation; more likely, $f(V_{q^2}) \leq 0$ preserves consistency, constraining α via null results.

7.2 Data Storage and Orthogonality in Q

The total quantum state Q not only unifies Standard Model (SM) fields and gravity but also serves as a structured data repository characterized by perfect orthogonality, enabled by its quaternion basis and interaction with the operator q. The data space within Q is divisible and mutually acted upon by q, forming a system where orthogonality is preserved across transformations. We conceptualize Q as a 4x4x4 tensor—a 64-element cube—where each 4x4 slice corresponds to one of the quaternion basis elements $(1, I, J, K)$:

$$Q = \sum_{j=0}^{3} Q_j \otimes e_j, \tag{126}$$

where Q_j are 4x4 matrices, and $e_j = (1, I, J, K)$ are basis vectors forming a quaternion set. This tensorial representation extends Q's 16-element 4x4 matrix into a richer structure, with each slice encoding independent physical data (e.g., SM field states or gravitational contributions).

The operator q interacts with Q bidirectionally through left and right multiplication:

$$Q' = qQ, \quad Q'' = Qq, \tag{127}$$

where $Q' = \sum_{j=0}^{3}(qQ_j) \otimes e_j$ and $Q'' = \sum_{j=0}^{3}(Q_j q) \otimes e_j$. These operations rotate the 4x4x4 cube along any 3D direction defined by $q = q_0 + q_1 I + q_2 J + q_3 K$, preserving the orthogonality of the basis elements, as $e_i \cdot e_j = \delta_{ij}$ (where the dot product is the quaternion inner product, zero for $i \neq j$ due to $I^2 = J^2 = K^2 = -1$). This mutual action reflects a perfect symmetry: Qq transforms state vectors, while qQ adjusts Q's internal configuration, maintaining its unitary SU(4) properties.

This orthogonality enables Q to store data—quantum states, field configurations, or gravitational degrees of freedom—as a perfectly orthogonal system. Each 4x4 slice, tied to a basis element, can represent distinct physical entities (e.g., gluon states in one slice, fermion states in another), with q's rotations ensuring independence across transformations. The 64 elements (4 basis elements $\times$ 16 matrix entries) provide a compact, high-dimensional data structure within 4D spacetime, enhancing Q's role as a unified field while aligning with the theory's SU(4)-based SM projections and gravitational sourcing via $T_{\mu\nu}$.

8 Discussion and Validation

8.1 Comparison with Existing Theories

The quaternion-based TOE diverges significantly from established frameworks:

- **Standard Model with ΛCDM**: The SM, coupled with the ΛCDM cosmological model, relies on dark matter particles ($\Omega_{\text{DM}} \approx 0.27$) and a cosmological constant ($\Omega_\Lambda \approx 0.68$) to explain structure formation and late-time acceleration. Our TOE eliminates dark matter, attributing gravitational clustering to curvature oscillations from Q and q, and replaces Λ with a dynamic ψ_0-driven term. While ΛCDM uses empirical parameters, our theory derives these effects from Q's fundamental structure.

- **String Theory**: String theory unifies quantum mechanics and gravity by positing 10 or 11 spacetime dimensions, with extra dimensions compactified at the Planck scale, and introduces a plethora of hypothetical particles (e.g., superpartners). In contrast, our TOE operates strictly in 4D spacetime, using quaternions to encode SM and gravitational fields within Q, avoiding supersymmetry and additional dimensions. String theory's reliance on unobservable compactification contrasts with our TOE's direct 4D unification.

- **Loop Quantum Gravity (LQG)**: LQG quantizes spacetime itself, predicting a discrete structure at the Planck scale ($\sim 10^{-35}\,\mathrm{m}$), and treats gravity as a spin-network geometry without a particle mediator. Our TOE quantizes gravity via the graviton $h_{\mu\nu}$, emerging from Q, and retains a continuous 4D spacetime, integrating seamlessly with SM fields through SU(4) projections rather than redefining spacetime's fabric.

These differences highlight our TOE's minimalist approach: it unifies forces using quaternions within observed dimensions, contrasts with ΛCDM's empirical additions, string theory's extradimensional complexity, and LQG's geometric quantization.

8.2 Experimental Tests and Interpretative Framework

To conceptualize the theory intuitively, envision Q as a 4D rotating cube, each face encoding SM fields or gravity. The operator q rotates this cube, transforming states like a multidimensional prism, with q^2's pilot waves steering particle trajectories. This analogy captures the unification of forces and spacetime dynamics, where ψ_0 adjusts the cube's "size" (mass scale), and V directs its "motion" (fermion paths). Visualizing $T_{\mu\nu}$ as the cube's "shadow" on spacetime illustrates gravity's emergence, offering an accessible bridge to the theory's mathematical formalism.
When normalised its possible to envision the standard model as a 4x4x4 cube with arrows in spheres of radius 1 as elements, since the total space is 4x4x4x4=256 dimensions.

Validating the theory requires experimental tests leveraging current and future facilities:

- **Higgs Properties**: Precision measurements at the Large Hadron Collider (LHC) of m_H and couplings to probe ψ_0 variations (e.g., $m_H = 250\,\mathrm{GeV}$ if $\psi_0 = 2$), testing deviations from SM predictions.
- **Gravitational Waves**: LIGO/Virgo detection of early universe tensor modes from $h_{\mu\nu}$, verifying quantization and $T_{\mu\nu}$ sourcing, with signatures differing from ΛCDM.
- **Cosmological Probes**: Planck, Euclid, and LSST data to confirm no dark matter particles, assessing CMB power spectrum enhancements, matter clustering, and growth rates aligned with Q perturbations.

These experiments will constrain ψ_0, q^2, and gravitational dynamics, potentially confirming the TOE's unification or necessitating refinements.

8.3 Addressing Theoretical Constraints

The quaternion-based Theory of Everything (TOE) must address theoretical constraints to ensure consistency with established physical principles, beyond the graviton's compatibility with the Weinberg–Witten theorem as addressed in its quantization and polarization properties. The theorem, which restricts massless particles with spin greater than $\frac{1}{2}$ from carrying conserved charges associated with Lorentz-covariant currents, is satisfied by the TOE's framework, where $h_{\mu\nu}$ emerges as a neutral, spin-2 field from $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$, coupling universally to the stress-energy tensor rather than a charge-related current. However, additional implications and constraints warrant further scrutiny to validate the theory's robustness.

The TOE's Lorentz invariance, rooted in the quaternion wave function $\psi = \psi_0 + \psi_1 I + \psi_2 J + \psi_3 K$ with $\psi_j = R_j(x)e^{iS_j(x)/\hbar}$ encoding four-momentum $p^\mu = (\psi_0, p_1, p_2, p_3)$, underpins its consistency with special relativity. The conservation of $T_{\mu\nu}$ ($\nabla^\mu T_{\mu\nu} = 0$) aligns with general relativity's requirements, supporting the graviton's role as a mediator of spacetime curvature. The absence of a charge-carrying current in $h_{\mu\nu}$'s quantization:

$$h_{\mu\nu}(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \sum_{\lambda=\pm 2} \left[a_\lambda(k)\epsilon^\lambda_{\mu\nu}(k)e^{-ik\cdot x} + a^\dagger_\lambda(k)\epsilon^{\lambda *}_{\mu\nu}(k)e^{ik\cdot x} \right], \tag{128}$$

reinforces its exemption from the theorem's restrictions, as gravitational interactions via $\kappa = \sqrt{8\pi G}$ do not involve internal symmetry charges.

Beyond Weinberg–Witten, the TOE faces challenges in quantum gravity's consistency, particularly renormalization. The theory's claim of mitigating gravity's non-renormalizability through Q's quaternion structure and the dynamic coupling $\psi_0^2 = |V|^2$ requires rigorous validation. While loop calculations suggest a reduction in divergence severity (e.g., $k^2 \ln(k)$ versus standard k^4, k^6, k^8), the effective vertex $T^{\text{eff}}_{\mu\nu} = \text{Re}(Q^\dagger Q)\frac{\mu^2}{\mu^2+|V|^2}$ introduces a physical mechanism needing justification—potentially tied to Q's finite 4x4 dimensionality acting as a natural regulator. This hypothesis warrants higher-loop scattering amplitude computations to confirm a finite counterterm series, contrasting with standard gravity's infinite proliferation.

Causality poses another constraint, especially given the superluminal propagation hypothesis. If V_{q^2} induces $v_g > c$, as explored with $\omega_k = \sqrt{c^2|\mathbf{k}|^2 + m^2} + \alpha|V_{q^2}|$, the theory must ensure no physical signals violate $v_f = c$, preserving temporal order. The pilot wave interpretation may mitigate this by rendering superluminality non-signaling, but commutation relations $[\psi_f(x), \psi_f^\dagger(y)] = 0$ for spacelike separations must hold, potentially constraining α to negligible values unless redefined as an effective, non-local effect.

The TOE's reliance on 4D spacetime, eschewing extra dimensions, aligns with observable reality but challenges frameworks like string theory. Its consistency hinges on SU(4) projections fully reproducing SM gauge symmetries without additional particles, testable via precision electroweak measurements. These constraints collectively frame the TOE as a viable, testable unification, with ongoing validation needed to solidify its theoretical footing.

9 Conclusion

This paper has presented a quaternion-based Theory of Everything (TOE) that unifies the Standard Model (SM) of particle physics with gravity within a conventional 4D spacetime framework, eschewing additional dimensions or particles beyond those observed. Through the total quantum state $Q = (M_1, M_2)$, constructed via the Cayley-Dickson process, and the quaternion wave function $\psi = \psi_0 + \psi_1 I + \psi_2 J + \psi_3 K$, we have delineated a novel approach where SU(4) projections yield SM gauge fields, and the stress-energy tensor $T_{\mu\nu} = \text{Re}(Q^\dagger Q)$ quantizes gravity as a spin-2 field $h_{\mu\nu}$. The dynamic condition $\psi_0^2 = |V|^2$ integrates scalar and vector components, driving mass generation and cosmological evolution.

The shift to a quaternion-based TOE carries profound implications for physics and cosmology:
- **Unified Framework in 4D**: By embedding SM fields and gravity in Q without extra dimensions, the theory simplifies the conceptual landscape, eliminating the need for

compactification or discrete spacetime. This 4D unification implies that all fundamental interactions stem from a single quaternion structure, potentially reducing the parameter space of particle physics and cosmology.
- **Elimination of Dark Matter**: Replacing dark matter with Q-driven curvature oscillations challenges the ΛCDM paradigm's reliance on unobserved particles. If validated, this shift redefines galaxy formation as a baryonic process, impacting astrophysical models and dark matter searches (e.g., XENONnT), with implications for cosmic microwave background (CMB) and large-scale structure interpretations.
- **Dynamic Mass and Gravity**: The condition $\psi_0^2 = |V|^2$ links fermion masses to spacetime dynamics, suggesting a unified origin for inertial and gravitational mass. This could resolve long-standing questions (e.g., equivalence principle origins) and predict observable mass variations (e.g., $m_H = 250\,\text{GeV}$ if $\psi_0 = 2$), testable at the LHC.
- **Cosmological Evolution**: The absence of a bare cosmological constant, with acceleration from ψ_0's late-time behavior, offers a physical mechanism for dark energy, potentially addressing the Hubble tension (H_0) by altering early universe dynamics. This contrasts with ΛCDM's ad hoc Λ.
- **Predictive Power**: The theory's falsifiable predictions—modified CMB power, steeper matter spectra, enhanced growth rates, and superluminal hints—position it as a testable alternative, leveraging experiments like Planck, LSST, and LIGO. Success would shift focus from empirical fixes to a unified field theory.

This paradigm shift reorients physics toward a quaternion-driven, 4D unification, challenging extradimensional and discrete-space models while offering a streamlined explanation for cosmic phenomena. Future work must refine loop calculations and test predictions, potentially heralding a new era in theoretical understanding.