De-Broglie matter waves as the basis for quantum gravity

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1 Abstract

The use of a quaternion algebra to represent the direction vector of a matter wave packet that travels at the speed of light allows one to obtain the regular format of solutions to the Schrodinger equation and also to display how two matter waves interact by matrix multiplication in a global coordinate system.

2 Interpretation

There are a number of interpretations of quantum mechanics that fail to maintain realism, nor explain quantum non-locality. The proposed interpretation aims to maintain realism, whilst explaining quantum non-locality.
The stochastic formulation of quantum mechanics requires intrinsic random forces acting to change the particle velocity direction, but doesn't explain the origin of these forces. The extension of brownian motion to the quantum scale can only be achieved with matter wave perturbations combining to create a wave packet (particle) contributed by the vacuum travelling at c. This means that the group velocity that we measure as the velocity of the particle, is the result of projecting the 3 dimensional velocity vector onto a one dimensional (orthogonal) measurement axis. This results in a velocity vector less than c as described mathematically below.

The complex number space of the Schrodinger equation can be used to construct the quaternion space representation of the 3D velocity vector. It is shown that because the waves exist in an orthogonal complex vector space and they interfere with the complex vector space of the Schrodinger equation, it is identical to a superposition in the complex number space. This is done by deriving the formula for the uncertainty principle. This solution can also be used to construct a stress energy tensor at the point of matter wave intersection which corresponds to a perfect fluid in the original stress energy tensor since it is made of light-matter waves travelling at $v = c$ (see $E = mc^2$ and $E = hc/\lambda$) $\implies m = h/\lambda c$).

Fundamentally, this interpretation proposes that the direction of a mass's velocity is stored as a complex number in the regular Schrodinger equation. This implies that the degrees of freedom in the system are equal to the dimension of the algebra used to store the direction vector. It also proposes that a quaternion algebra can be used to represent the direction vectors more generally, for 3D vacuum matter wave packets. A host of implications result from this including a definition for the pilot waves from quantum mechanics, the negative pressure associated with the expansion of the universe and the apparent "measurement" of gravity as a force.

If there were a wave function whose solutions were made of components contributed from the other dimensions, it might best be represented as a combination of complex numbers or of the basis vectors from the quaternion algebra.

$$\Psi = f(x, y, z, t) = Ae^{i(k_x x - w_x t)} e^{j(k_y y - w_y t)} e^{k(k_z z - w_z t)}$$

This concept is explored more thoroughly below and the equation derived.

3 Maths

3.1 Two dimensional wave function superposition

The formula for a normal wave function can be generated by neglecting two of the vacuum contributions to the quaternion wave function.

$$\Psi = f(x, t) = Ae^{i(k_x x - w_x t)}$$

This is a plane wave solution for the matter wave along the x-axis. It is not a general solution for all matter waves, as the other two dimensions contribute across an infinite range of values to the system. For one, this produces wave packets that are virtually instantaneously made of superpositions/interference of matter waves from the other two spatial dimensions. Over any length of time greater than the matter-wave length divided by c:

$$t > t_0 = \lambda/c = h/mc^2$$

The superposition of the other two dimensions' matter waves occurs (from vacuum pertubations). This is the time it takes light to move the distance λ and is understood to be the norm of the matter wave's vacuum perturbation velocity vector, which travels at c. $t_0 = h/m_e c^2 \approx 8 * 10^{-21} s$ for the electron. Maintaining the single axis representation of the wave packets velocity vector, the result of superimposing vacuum matter waves from the other two spatial dimensions can be written as:

$$\Psi = f(x, t) = (A_0 e^{i(k_0 x - w_0 t)}, A_1 e^{i(k_1 x - w_1 t)})$$

Where the second term corresponds to a wavefunction that exists in an orthogonal complex vector space and both constants normalise their wavefunction.
There are two ways of viewing the complex vector space that makes up a wave packet in the original Schrodinger equation. We can imagine the complex number as representing the angle that the instantaneous 4D quaternion algebra velocity vector projects onto a 2D complex plane perpendicular to the velocity vector of the wave packet (which is harder to think about) or we can think of the real part as being the amount (projection of) the direction vector is towards the 1D measurement axis (it is never the x axis because it is never $\theta = 0 \implies v = c$) and the imaginary component as the amount it is orthogonal to both the measurement axis and the velocity vector, which is the y-z plane if the velocity vector is the x-axis (1,0,0).
In this view, the superimposed matter wave perturbations from the y-z plane are orthogonal to the measurement axis and the x-axis of the wave groups velocity vector. This implies they belong to a different complex vector space, *which does not contain the measurement axis.*

When the wave functions are superimposed, the two orthogonal complex vector spaces form a two dimensional vector space over the complex numbers $\mathbb{C}^2$ which allows us to build the quaternion algebra $\mathbb{H}$. Let the superimposed matter wave functions' direction vectors belong to a two dimensional vector space over the complex numbers.

$$((a+bi),(c+di))$$

and choose a different set of complex basis vectors 1 and j, then:

$$(a+bi)1+(c+di)j = a+bi+cj+dij$$

Which, after defining $j^2 = -1$ and $ij = -ji = k$ results in the same multiplication rules as the quaternions.

For $a = A_0$, $b = (k_0x - w_0t)$, $c = A_1$ and $d = (k_1x - w_1t) \implies$

$$v = A_0 + (k_0x - w_0t)i + A_1j + (k_1x - w_1t)k$$

Since A_1 is an arbitrary constant we can choose this equal to $(k_3x - w_3t)$ giving

$$v = A_0 + (k_0x - w_0t)i + (k_3x - w_3t)j + (k_1x - w_1t)k$$

or

$$v = A_0 + (k_1x - w_1t)i + (k_2x - w_2t)j + (k_3x - w_3t)k$$

Normally, A_1 is the k vector of the electron (Compton wavelength). Using the quaternion representation we can obtain the complex representation by simply neglecting the other two spatial dimensions, noting that we lose information, (which is immeasurable anyway since it exists entirely within an orthogonal complex vector space).

$$\mathbb{H} \to \mathbb{C}$$

$$a + bi + cj + dij \to a + bi$$

The final formula for the amplitude of the wave packet is:

$$\Psi = f(x,t) = A_0 e^{i(k_0 x - w_0 t)} A_1 e^{j(k_1 x - w_1 t)} A_3 e^{k(k_2 x - w_2 t)}$$

$$\Psi = A e^{i(k_0 x - w_0 t)} e^{j(k_1 y - w_1 t)} e^{k(k_2 z - w_2 t)}$$

After combining constants and generalising to three dimensions. The use of this equation details the effects of the pilot wave/vacuum perturbations on a particle by setting a single dimensions' direction vector to the Compton wavelength of that particle and the others as stochastic functions of Δk where $k = k_0 - \Delta k$.

3.2 Wave group and phase velocities

3.2.1 Group velocity

The dot product of velocity (direction) vectors:

$$v \circ v = c(a,b,c,d) \circ c(a,b,c,d) = c^2(a^2 + b^2 + c^2 + d^2) = c^2$$

Which is the same as:

$$|v^2| = v^* v = c(a + ib + jc + kd)c(a - i - j - k) = c^2(a^2 + b^2 + c^2 + d^2) = c^2$$

So in the quaternion representation the phase and group velocities are equal to the speed of light. But in the original complex vector space, the speed is equal to:

$$|(v_g)^2| = v * v = c(a + ib)c(a - ib) = c^2(a^2 + b^2) < c^2$$

Since, $a^2 + b^2 + c^2 + d^2 = 1$ but discarding $c + d$ means $a^2 + b^2 < 1$.

The value for the group velocity is better defined as the dot product of the two direction vectors. This is zero when pointing directly away from the measurement axis. The maximum occurs when they are pointing directly towards the measurement axis giving a value of $v = c$. Neither of these will ever happen since the measurement axis is never perfectly collinear with the velocity vector.

The velocity vector in one of the two $\mathbb{C}$ is the projection onto the complex plane orthogonal to the z-axis of the velocity vector (changed from the x-axis used earlier for notation simplicity). This is the same as in the original Schrodinger equation and is equal to the speed of light times the projection of the Riemann sphere onto the surface $\zeta = x + iy$:

$$v_\Psi = c(cot(1/2\theta)e^{i\phi}) = c(\frac{x + iy}{1 - z}) = c(\frac{a + ib}{|v|/c - c_1})$$

for the positive velocity direction ($z > 0$) direction and onto the surface $\xi = x - iy$:

$$v_\Psi = c(tan(1/2\theta)e^{-i\phi}) = c(\frac{x - iy}{1 + z}) = c(\frac{a - ib}{|v|/c + c_1})$$

for the negative velocity direction ($z < 0$). Where $x^2 + y^2 + z^2 = 1$, is the local coordinate system of the wave packet. This is the direction vector in $\mathbb{C}$ divided by the magnitude of the quaternion velocity vector minus the amount it is in the complex component of the other $\mathbb{C}$.
These are conjugates of each other, owing to the measurement direction having two directions/conjugate. The wave packet's velocity vector is rotated by θ, ϕ w.r.t. the desired direction.

A good way to think about the vacuum matter wave contributions orthogonal to the velocity vector is to imagine the complex number as the angle the measurement vector makes with a plane perpendicular to its velocity. The additional terms in the quaternion are the vacuum contributions from this plane into the wave packet.

3.2.2 Phase velocity

The ratio of the resulting quaternion wavelength to the complex component wavelengths is:

$$\frac{\lambda_{\mathbb{H}}}{(\lambda_{\mathbb{C}} + \lambda_{\mathbb{C}})} = \frac{h/mc}{(h/mv + h/mv)} = \frac{c^2}{v_g} = v_p$$

This represents the overlap between the two constituent wavefunctions belonging to orthogonal $\mathbb{C}$ that combine to create the resulting quaternion at that point in space.
The velocity is the amount the velocity is pointing in the same direction between the two direction vectors $v = v_1 \circ v_2$. This makes the phase velocity superluminal, but we calculated $|v| = c$ in the quaternion representation, so for an infinitesimal point in space and time we could just as easily interpret the real part of the wavefunctions as travelling at $v_g = c$. If the instantaneous velocity is equal to the speed of light, that means the mass that contributes to the wave packet from each complex component wave changes.

$$\frac{\lambda_{\mathbb{H}}}{(\lambda_{\mathbb{C}} + \lambda_{\mathbb{C}})} = \frac{h/mc}{(h/m_1c + h/m_2c)} = \frac{p^2}{\sqrt{p_1p_2}} = \frac{m^2}{\sqrt{m_1m_2}} = m$$

This equation states the value for the measured mass is the same as the measured mass squared divided by the average of the two orthogonal $\mathbb{C}$ mass contributions. As such there is no way of knowing how much either $\mathbb{C}$ is contributing to the mass.

3.3 Uncertainty principle

Adding two orthogonal plane wave solutions to the Schrodinger equation each belonging to a separate $\mathbb{C}$ basis, creates a system indistinguishable from the superposition of two waves in the original basis. This is the same as adding two waves along different dimensions which combine and create a different wave

along the original dimension by interference. Since we can only interact with light that is pointing directly at us (interaction between our wavefunction and the particle belongs to the orthogonal 'real' axis), we can only measure along one dimension.
The superposition can be analysed as two waves that are added in the same dimension. At t=0, the superposition of any two plane wave solutions is:

$$\Psi(x,0) = A_0 e^{ikx} + A_1 e^{i((k+\Delta k)x}$$

$$\Psi^2 = A_0^2 + A_0 A_1 e^{-i\Delta kx} + A_0 A_1 e^{i\Delta kx} + A_1^2$$

Integrating from $0 - 2\pi$ deletes the middle part. Which means the probability to find the particle in the wave packet is $\Psi^2 = A_0^2 + A_1^2 = 2$. This will be normalised later though it doesn't explicitly require the quantisation of the field into discrete wave packets and the square of the wavefunction to represent a normalised probability.

The distinction between the integrating variable is ambiguous, we simply require:

$$\Delta kx = 2\pi = \frac{1}{\Delta\lambda}x =$$

$$\frac{\Delta p}{\hbar}x = 2\pi$$

$$\Delta px = 2\pi\hbar = h$$

This represents a formalism with an accurate location of the wave packet, because the integrating variable covers a single cycle $\Delta k = 0 \to 2\pi$, yet maintains its quantum uncertainty.

The same argument can be made for the energy time uncertainty, by setting the position to zero. $\Delta Et = h$. This provides further justification specifically in the context of quantum mechanics for the usage of a quaternion algebra interpretation of the wave packet velocity direction.
Defining the momentum (mass) of a particle using the measurement axis represents it as the limit of a Gaussian distribution i.e. Dirac delta function along that axis, which puts the orthogonal complex vector space which is the Fourier transform (position) at infinite bounds. The reverse of this has been shown. The position of the particle is definite, and the momentum (mass) is infinitely uncertain, though it also allows wave packets to repeat in space.
Defining the momentum of a particle is why the Fourier transform is used to switch between the Hilbert spaces of position and momentum, whereas in this theory there is no requirement, and the orthogonal complex vector space is the vacuum matter wave contributions from the other two dimensions. This will hopefully untie the strict bounds placed on the 1D measurement axis definition of the momentum (mass) of subatomic particles, allowing for a full spectrum of momenta.

To normalise the wave function we need to divide Ψ^2 by 2π and also 2 because we integrated over two wave functions:

$$\Psi(x,t) = \frac{1}{\sqrt{4\pi}}(A_0 e^{i(kx+\omega t)} + A_1 e^{i((k+\Delta k)x+(\omega+\Delta\omega)t)})$$

It can be seen that the relationship between the defined momenta (mass) uncertainty and the amplitude of the wave function of a defined position (wave packets) is directly related by a factor of 4π.

3.4 Stress energy tensor of two interacting wave packets

In the same way we used an orthogonal vector space to represent the overlapping waves from the other two spatial dimensions, we construct another orthogonal vector space to represent the global co-ordinate system.
Representing vacuum matter wave direction quaternions as a matrix, we can compute the interaction of two matter waves in our global coordinate system as their matrix product. Since the derivation of the matter waves were done at an infinitesimal point in space and time, it represents the derivatives of time and space (local coordinate system), whereas in general relativity the coordinate system aims to describe the global properties of space-time.

$$q = \begin{bmatrix} a_1 & ib_1 & jc_1 & kd_1 \end{bmatrix}$$

$$q^T = \begin{bmatrix} a_2 \\ ib_2 \\ jc_2 \\ kd_2 \end{bmatrix}$$

Then the stress energy tensor becomes:

$$T^{\mu\nu} = qq^T = \begin{bmatrix} a^2 & a_1 b_2 i & a_1 c_2 j & a_1 d_2 k \\ b_1 i a_2 & -b^2 & b_1 i c_2 j & b_1 i d_2 k \\ c_1 j a_2 & c_1 j b_2 i & -c^2 & c_1 j d_2 k \\ d_1 k a_2 & d_1 k b_2 i & d_1 k c_2 j & -d^2 \end{bmatrix}$$

The variables $a^2 + b^2 + c^2 + d^2 = 1$ don't require positive values, but $|a| < 1$ etc. This means each element corresponds to a rotation of the direction vector along each local axis. Let, l and k range from 1 to 3, then the interpretation for the components of this matrix is as follows:
The element $T^{0,0}$ is the energy density divided by the speed of light squared and the real/measurable value for the velocity vector squared at that point in space. Since the maximum value for this variable is the speed of light squared, the maximum for this element is 1. This is the real/measurable interaction that occurs between two matter waves and is interpreted to be directly related to the "force" of gravity, since the value is always positive.
The first row $T^{0,k}$ is the momentum/mass flux across the global x^k surface and

also the contribution from the left matter wave to the right matter wave along the resulting local complex axis.
The first column $T^{k,0}$ is similar, being the contribution of the right matter wave to the left matter wave along the resulting local complex axis and the momentum flux across the global x^k surface.
The elements $T^{k,l}$ are the pressure in the global directions resulting from this interaction, it is also the amount the matter waves are moving apart from each other. This is interpreted as the apparent negative pressure associated with the expansion of the universe.
The rest of the elements $T^{k,l}$ $k \neq l$, represent the global sheer stress and the amount each basis element contributes to the others' basis elements.

This matrix stores every possible rotation of two matter wave direction vectors while they interfere at a point in space and at $t = 0$. For $t > t_0$ the value for the new matter wave at the point is uncertain, just as the future of each $\mathbb{C}$ component direction vector is stochastic.

4 Implications for cosmology

This interpretation provides insight into the cause of the expansion of the universe, the origin of gravity as a force, the vacuum energy of empty space and the pilot wave/intrinsic random forces of stochastic mechanics. Its possible that an extension of this could be made into the octonions for descriptions of the future.

5 references

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