Continued analysis

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1 Introduction to orthogonality

In the standard approach to the energy momentum relation, $E = pc$ and $E = mc^2$ where one side of the triangle corresponds to a photons energy (momentum gained) and the other side the rest mass respectively. This could only both be true if they were orthogonal to each other. It is proposed that this orthogonality should be extended into four dimensions. The Pythagoras theorem being used thus represents the connection between the variables in the four dimensional space.

In this paper we decompose the rest mass of a particle in terms of overlapping light-matter waves from each of the 3 dimensions. This allows one to represent both massive and massless particles from the four dimensional space in a 3 dimensional basis. This is done by setting the energy and angle to the fourth dimension to zero, representing a conservation equation for the energy-momentum relation.

Let the rest energy be a real quaternion and momentum be a vector quaternion i, j, k.

$$|E|^2 = m^2c^4 + p^2c^2 =$$

$$E^\dagger E = m^2c^4 + c(ip_x + jp_y + kp_z)^\dagger c(ip_x + jp_y + kp_z)$$

This means the energy is the square root of a complex number.

$$E = \sqrt{m^2c^4 + p^2c^2} = \sqrt{|m^2c^4 + p^2c^2|}e^{\frac{i\phi}{2}}$$

$$= \pm m^2c^4 \mp p^2c^2$$

Where ϕ is the angle between the 3D momentum vector and the fourth dimension of mass. This can be further decomposed into angles along each coordinate axis as in the quaternion to Euler conversion.

$$E_m = |E|cos(\phi/2)$$

$$E_x = |E|sin(\phi/2)cos(\beta_x)$$

$$E_y = |E|sin(\phi/2)cos(\beta_y)$$

$$E_z = |E|sin(\phi/2)cos(\beta_z)$$

Where the β is the angle to the corresponding coordinate axis. This is understood as the photon components angle and the half ϕ is the angle to the massive component. Typically, our co-ordinate choices are photon trajectories in flat space, so massive particles behave as spinors.

$$E^2 = 0 = |m^2c^4 + p^2c^2|e^{i\phi} - >$$

$$m^2c^4e^{i\phi} = -p^2c^2e^{i\phi} - >$$

$$m^2c^2e^{i\phi} = p^2e^{i(\phi+\pi)} - >$$

$$mc = pe^{\frac{i\pi}{2}} = ip$$

This complex number is the angle between the mass and all photon components, or the real part to the vector quaternion part. A similar result is possible with different algebraic manipulations. Let $\phi = 0$

$$E^2 = 0 = m^2c^4 + p^2c^2 - >$$

$$mc = \sqrt{-p^2} = p_m$$

$$p_m = (ip_{mx} + jp_{my} + kp_{mz}) = \frac{1}{\sqrt{3}}(ip_x + jp_y + kp_z)$$

This result presents the problem as a three dimensional one. This equation represents three imaginary numbers as the angle of the mass to each photon component separately. Notice that the mass and photon components are equal by a factor of $\sqrt{3}$ (this result will be used in the strong force section).

The main takeaway is that by setting the mass and photon components equal, for the energy to be non-zero we require multiplication by a conjugate energy quaternion.

$$E^2 = m^2c^4 - p^2c^2 = 0$$

$$E^{\dagger}E = m_1^{\dagger}m_2c^4 + p_1^{\dagger}p_2c^2$$

The quaternion based decomposition represents the colours of the strong force as the basis elements of the quaternion group for the momenta matrices. This reproduces all unitary 3x3 matrices in an eight element basis. Interestingly, the real component appears to be regular photons, or perhaps neutrinos.

The same is done with the massive component of the energy momentum relation, representing the quarks and leptons as being combinations of the basis elements of the vector quaternion group. Since quaternions can be used to represent a rotation, these are further shown to have half integer spin, by their parameterisation into Euler angles.

Taking both components into account and not expanding into quaternion basis elements, presents a matrix with anti-commutation relations between the momentum and rest energy, potentially explaining the parity inversion of the weak force.

Interference patterns observed can be explained by an orthogonal Hilbert space of equal dimension, this is akin to the superfluid vacuum hypothesis. Overlapping light-matter waves perpendicular to the measurement and time dimensions. Since we can only measure along a single axis, we can't detect the orthogonal light-matter waves, only their effects.
This theory predicts that only hermitian operators are observable.

2 Energy momentum relation

Proof of the four to three dimensional mapping. Let the mass be a vector quaternion:

$$E = (ip_x + jp_y + kp_z)c + (ip_{mx} + jp_{my} + kp_{mz})c$$

$$E^\dagger E = c^2(i(p_x+p_{mx})+j(p_y+p_{my})+k(p_z+p_{mz}))^\dagger(i(p_x+p_{mx})+j(p_y+p_{my})+k(p_z+p_{mz}))$$

$$E^\dagger E = c^2((p_x + p_{mx})^2 + (p_y + p_{my})^2 + (p_z + p_{mz})^2)$$

$$= c^2(p^2 + p_m^2 + pp_m + p_m p) = 0$$

Only when p and p_m are equal and opposite magnitude.

This means the only way to go from the four dimensional space into the Euclidean $3D$ space is to make the space contain a conjugate vector quaternion valued mass. This ensures it reduces to the regular energy momentum relation by having the mass component perpendicular.

$$E^- = (p_x - ip_{mx}, p_y - jp_{my}, p_z - kp_{mz})c^2 - >$$

$$E^2 = E^- E^+ = E^\dagger E = (p_x - ip_{mx}, p_y - jp_{my}, p_z - kp_{mz})(p_x + ip_{mx}, p_y + jp_{my}, p_z + kp_{mz})c^2$$

$$E^2 = (p^2 + p_m^2)c^2 = p^2c^2 + m^2c^4$$

If we don't conjugate the energy, we must maintain agreement between $E^2 = E^\dagger E$, thus we require $E = 0$.

If this is not the case something with momentum and mass components, will show a parity inversion. The order of operations for the photon/momentum and mass components impact their interactions.
Something with only mass components, since it belongs to the local coordinate

system or intrinsic vector quaternion space, will have a parameterization into Euler angles aka "half spin".
Something with only light/momentum components doesn't show half spin, nor parity inversion or non-commutation.

If you represent everything in the 3D space, the mass must be opposite magnitude to the momentum. If using a complex valued mass, you can represent the uncertainty from vacuum contributions as a perpendicular contribution (j,k components). If you use a quaternion valued mass, you can represent the vacuum fluctuations as the conjugate required to multiply to have positive energy. Also you can use the fact that the max of two/three random numbers, is the same as the square/cube root of that number.

If momentum is allowed to take any value, then the corresponding mass must be opposite in value to conserve total energy-momentum. If this were not the case, computing the energy squared without conjugation results in a negative value for energy, which is nonphysical.

$$E^2 = -E^\dagger E = -c^2((p_x + p_{mx})^2 + (p_y + p_{my})^2 + (p_z + p_{mz})^2)$$

$E^2 = E^\dagger E$ only when $E^2 = E^\dagger E = 0$. This requires $p_m = -p$.

Note, we repeat this logic multiple times during this paper.

3 SU(2) algebra and the weak force

$$E = [0, p_x + p_{mx}, p_y + p_{my}, p_z + p_{mz}]$$

Convoluting this equation with itself (matrix multiplication) effectively produces a Jacobian matrix.

Expanding out to find the relationship between massive and massless particles.

$$\Psi = \frac{c^2}{|E|}$$

$$\begin{bmatrix} (p_x + p_{mx})^2 & (p_x + p_{mx})(p_y + p_{my}) & (p_x + p_{mx})(p_z + p_{mz}) \\ (p_y + p_{my})(p_x + p_{mx}) & (p_y + p_{my})^2 & (p_y + p_{my})(p_z + p_{mz}) \\ (p_z + p_{mz})(p_x + p_{mx}) & (p_z + p_{mz})(p_y + p_{my}) & (p_z + p_{mz})^2 \end{bmatrix} =$$

$$= \frac{c^2}{|E|} \begin{bmatrix} p_x^2 & p_x p_y & p_x p_z \\ p_y p_x & p_y^2 & p_y p_z \\ p_z p_x & p_z p_y & p_z^2 \end{bmatrix} + \frac{c^2}{|E|} \begin{bmatrix} p_{mx}^2 & p_{mx} p_{my} & p_{mx} p_{mz} \\ p_{my} p_{mx} & p_{my}^2 & p_{my} p_{mz} \\ p_{mz} p_{mx} & p_{mz} p_{my} & p_{mz}^2 \end{bmatrix} +$$

$$\frac{c^2}{|E|}\begin{bmatrix} p_x p_{mx} & p_x p_{my} & p_x p_{mz} \\ p_y p_{mx} & p_y p_{my} & p_y p_{mz} \\ p_z p_{mx} & p_z p_{my} & p_z p_{mz} \end{bmatrix} + \frac{c^2}{|E|}\begin{bmatrix} p_{mx} p_x & p_{mx} p_y & p_{mx} p_z \\ p_{my} p_x & p_{my} p_y & p_{my} p_z \\ p_{mz} p_x & p_{mz} p_y & p_{mz} p_z \end{bmatrix} = 0$$

Computing this element by element we see the argument is similar to earlier. We require, $p_m^2 = -p^2$ so that $E^2 = E^\dagger E = 0$. However, we also require $p_{my} p_x = -p_y p_{mx}$. This is essentially saying the axes are interchangeable, but that the mass-photon components are non-commutative.
This could be the cause of the weak force's parity inversion. For the weak force, flipping the sign of the coordinates only changes the order of operations, not the mass component. This means if the mass component changes in the order of operations, it becomes the negative of its prior value, indicating that this symmetry breaking is a requirement for the conservation of energy.

This approach can be considered identical to the approach used earlier. The equation for energy momentum can be solved with the same conditions, (it just reveals more to show $p + p_m = 0$ in the simpler equation).

The requirement for light to overlap with matter waves potentially explains the extremely short range and low energy of the weak nuclear force.

Rewriting after the anti-commutation and Z^0 boson mass negation:

$$= \frac{c^2}{|E|}\begin{bmatrix} p_x^2 & p_x p_y & p_x p_z \\ p_y p_x & p_y^2 & p_y p_z \\ p_z p_x & p_z p_y & p_z^2 \end{bmatrix} + \frac{c^2}{|E|}\begin{bmatrix} -p_{mx}^2 & -p_{mx} p_{my} & -p_{mx} p_{mz} \\ -p_{my} p_{mx} & -p_{my}^2 & -p_{my} p_{mz} \\ -p_{mz} p_{mx} & -p_{mz} p_{my} & -p_{mz}^2 \end{bmatrix} +$$

$$\frac{c^2}{|E|}\begin{bmatrix} p_x p_{mx} & p_x p_{my} & p_x p_{mz} \\ p_y p_{mx} & p_y p_{my} & p_y p_{mz} \\ p_z p_{mx} & p_z p_{my} & p_z p_{mz} \end{bmatrix} + \frac{c^2}{|E|}\begin{bmatrix} -p_x p_{mx} & -p_x p_{my} & -p_x p_{mz} \\ -p_y p_{mx} & -p_y p_{my} & -p_y p_{mz} \\ -p_z p_{mx} & -p_z p_{my} & -p_z p_{mz} \end{bmatrix} = 0$$

W^+ bosons are the antiparticle of W^-. This is seen by adding the third and fourth matrix together. The Z^0 and γ bosons are their own antiparticles. This is seen by recognising that they are hermitian as they are no longer of complex nature. They also have no polarity as $p^2 - p_m^2$ is equivalent to $p_m^2 - p^2$, i.e. $E^\dagger E = E E^\dagger = E^2 = 0$.

4 Quarks and leptons

Let $p^2 c^2 = 0$, this implies $E \neq 0$, but this is not the case. All energy must be borrowed from the orthogonal components.

The matrix can be decomposed into each quaternion component. This section exhibits redundancy and is longer than what is required by this theory.

$$E = [0, p_{mx}, p_{my}, p_{mz}]$$

$$E^\dagger E = c^2 \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix}$$

The left side corresponds to the massive particles and the right side their anti-particles.

Expanding out to see each component:

$$E^\dagger E = c^2 \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} = 0 =$$

$$\begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} - \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} +$$

$$i \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} - i \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} +$$

$$j \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} - j \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} +$$

$$k \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} - k \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix}$$

The electron and positron are postulated to be the first two components. Quarks are postulated to be combinations of the vector quaternion parts. All have half integer spin as their parameterisation into Euler angles, but only the vector quaternion parts have fractional charge as detailed by their equivalence to photon components.

$$p_m = (ip_{mx} + jp_{my} + kp_{mz}) = \frac{1}{\sqrt{3}}(ip_x + jp_y + kp_z) - >$$

$$p_m^\dagger p_m = \frac{1}{3} p^\dagger p$$

This set of matrices can be further decomposed into the angle made by the mass and the Euclidean coordinate system, as the quaternion represents a rotation. This represents the spin as the factor in the sin, relationship between

the vector quaternion mass and the photon momentum as in the quaternion to Euler angle conversion.
The quarks that make up the standard model appear to exhibit redundancy in their charge, ($\mp\frac{1}{3}$ and $\pm\frac{2}{3}$). This theory potentially explains the origin of this apparent redundancy and also the mechanism by which the strong force changes the "colours" of quarks and the mechanism behind electron capture.

An up quark can be represented by:

$$UpQuark = i \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} + j \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} =$$

$$\frac{1}{3}(i \begin{bmatrix} p_x^2 & p_xp_y & p_xp_z \\ p_yp_x & p_y^2 & p_yp_z \\ p_zp_x & p_zp_y & p_z^2 \end{bmatrix} + j \begin{bmatrix} p_x^2 & p_xp_y & p_xp_z \\ p_yp_x & p_y^2 & p_yp_z \\ p_zp_x & p_zp_y & p_z^2 \end{bmatrix})$$

The factor $\frac{1}{3}$ is the same factor used earlier when equating the mass to the photon components. This is directly related to the number of co-ordinate axes.

The larger quarks contain other terms, which are equivalent to but smaller than gluons (perhaps travelling apart from each other as they are $\pm$ in orientation/charge). As such an anti-bottom quark can be represented by:

$$AntiBottom = \frac{1}{3}(k \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} +$$

$$-i \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} + j \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix}$$

$$-j \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} + i \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix})$$

Notice the electron and positron charges have not been used, though it is easy to see how they could be incorporated and explain electron capture.

$$UpQuark + electron = downQuark + neutrino =$$

$$i \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} + j \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} +$$

$$- \begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} =$$

$$i\begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} + j\begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix}$$

$$-(i+j+k)\begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} =$$

$$-k\begin{bmatrix} p_{mx}^2 & p_{mx}p_{my} & p_{mx}p_{mz} \\ p_{my}p_{mx} & p_{my}^2 & p_{my}p_{mz} \\ p_{mz}p_{mx} & p_{mz}p_{my} & p_{mz}^2 \end{bmatrix} = -\frac{1}{3}k\begin{bmatrix} p_x^2 & p_xp_y & p_xp_z \\ p_yp_x & p_y^2 & p_yp_z \\ p_zp_x & p_zp_y & p_z^2 \end{bmatrix}$$

5 SU(3) algebra and the strong nuclear force

Let $m^2c^4 = 0$. This implies $E^2 \neq 0$, but this is not the case. All energy must therefore be taken from the vacuum/orthogonal components, which requires overlap of orthogonal light waves, potentially explaining why the strong force only happens in a small volume. This borrowing of energy potentially explains the paired nature of gluons when mediating the strong force.

$$E = c[p_x, p_y, p_z]$$

Convoluting this equation with itself (matrix multiplication) effectively produces a Jacobian matrix.

$$E^\dagger E = c^2\begin{bmatrix} p_x^2 & p_xp_y & p_xp_z \\ p_ypx & p_y^2 & p_yp_z \\ p_zp_x & p_zp_y & p_z^2 \end{bmatrix}$$

The only way this matrix can still be zero, is if it can be decomposed into orthogonal components. Assuming each element of the matrix can be separated into the basis elements of the vector quaternion, this produces 3 colours and 3 anti-colours. The sum of all colours is zero.

$$E^\dagger E = qE^\dagger E - qE^\dagger E = 0$$

Where q denotes the unit quaternion $a + i + j + k$, $|q| = 1$. Dividing any independent matrix by the magnitude of it's independent energy-momentum four vector produces a unitary matrix, having a determinant of 1. Effectively a normalized probability function of the orthogonal variables, though it originated from the energy-momentum four vector.

6 SU(2)xU(1) and the Yang-Mills field

Assuming the previous matrix can be decomposed into complex components we obtain:

$$e^{i\phi/2}$$