# Hermitian Observables as Particles in a Quaternion-Based Theory of Everything

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**Abstract**

This document elucidates the particles of a quaternion-based Theory of Everything (TOE) as Hermitian observables derived from matrix structures. The total quantum state $Q = (M_1, M_2)$, with $M_1, M_2 \in \mathrm{SU}(4)$, and the operator $q$ define the field, with particles emerging as Hermitian operators projecting onto the real axis. We detail fermions (quarks and leptons), gauge bosons (gluons, photons, W/Z bosons), and the Higgs boson, leveraging the dynamic coupling $\psi_0^2 = 8(\psi_1^2 + \psi_2^2 + \psi_3^2)$ and orthogonality condition $E^2 = 0$. Quark energy transitions are explained through matrix dynamics, with the orthogonal 4x4 complex matrix projecting probability states onto the real axis via wave superposition. The 3x3 matrices' off-diagonal elements represent vacuum components, while diagonal elements are Casimir invariants, ensuring $E^2 = 0$.

# 1 Introduction

The quaternion-based Theory of Everything (TOE) unifies quantum mechanics and gravity within a 4D spacetime using a total quantum state $Q = (M_1, M_2)$, where $M_1, M_2 \in \mathrm{SU}(4)$, constructed via the Cayley-Dickson process. The measurable state is a quaternion wave function:

$$\psi = \psi_0 + \psi_1 I + \psi_2 J + \psi_3 K, \tag{1}$$

represented as a $4 \times 4$ real matrix, with $I, J, K$ satisfying $I^2 = J^2 = K^2 = IJK = -1$. A dynamic coupling governs the system:

$$\psi_0^2 = |V|^2 = 8(\psi_1^2 + \psi_2^2 + \psi_3^2), \tag{2}$$

where $V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$. The operator $q = q_0 + q_1 I + q_2 J + q_3 K$ acts via left multiplication, $Qq$, transforming field states. The energy-momentum relation imposes:

$$E^2 = E^\dagger E = 0, \tag{3}$$

ensuring orthogonality between momentum $p$ and mass components $p_m$.

In this TOE, particles are Hermitian observables, with real eigenvalues projecting the $\mathrm{SU}(4) \times \mathrm{SU}(4)$ matrix space onto the real axis. This document details fermions, gauge bosons, and the Higgs boson as Hermitian operators, and introduces quark energy transitions and the projection of probability states via an orthogonal 4x4 complex matrix, with 3x3 matrices' off-diagonal elements as vacuum components and diagonal elements as Casimir invariants.

# 2 Mathematical Framework

## 2.1 Quantum State and Observables

The wave function $\psi$ encodes the field state, with:

$$|V|^2 = 8(\psi_1^2 + \psi_2^2 + \psi_3^2), \tag{4}$$

and normalization $\psi_1^2 + \psi_2^2 + \psi_3^2 = \frac{1}{8}$, so $\psi_0 = 1$. The total state $Q$ projects Standard Model (SM) gauge fields via SU(4) subgroups, with $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$ sourcing gravity. Hermitian observables ($A^\dagger = A$) yield real eigenvalues, representing particle properties.

## 2.2 Energy-Momentum and Hermitian Matrices

The energy quaternion is:

$$E = mc^2 + (ip_x + jp_y + kp_z)c, \tag{5}$$

with $E^2 = m^2c^4 - p^2c^2$. Setting $E^2 = 0$ gives $m^2c^4 = -p^2c^2$, and mass is a vector quaternion:

$$p_m = \frac{|p|}{\sqrt{3}}(i + j + k), \quad p_m^2 = -\frac{p^2}{3}. \tag{6}$$

The norm is:

$$E^\dagger E = c^2 \left[ (p_x + p_{mx})^2 + (p_y + p_{my})^2 + (p_z + p_{mz})^2 \right], \tag{7}$$

forming a 3x3 Hermitian matrix when $p_m = -p$:

$$E^\dagger E = c^2 \begin{pmatrix} (p_x + p_{mx})^2 & (p_x + p_{mx})(p_y + p_{my}) & (p_x + p_{mx})(p_z + p_{mz}) \\ (p_y + p_{my})(p_x + p_{mx}) & (p_y + p_{my})^2 & (p_y + p_{my})(p_z + p_{mz}) \\ (p_z + p_{mz})(p_x + p_{mx}) & (p_z + p_{mz})(p_y + p_{my}) & (p_z + p_{mz})^2 \end{pmatrix}. \tag{8}$$

# 3 Particles as Hermitian Observables

## 3.1 Fermions: Quarks and Leptons

Fermions reside in $V$, with $\psi_1, \psi_2, \psi_3$ mapping to generational amplitudes. The Hermitian observable is:

$$V^\dagger V = 8(\psi_1^2 + \psi_2^2 + \psi_3^2) \cdot \mathbb{I}_{4\times 4}, \tag{9}$$

with masses:

$$m_f = y_f \psi_0 v_0 / \sqrt{2}, \quad v_0 = 246\,\mathrm{GeV}. \tag{10}$$

Quarks use SU(3):

$$E^\dagger E = \frac{c^2}{|E|} \sum_{a=1}^{8} \lambda^a (p^a + p_m^a)^2, \tag{11}$$

with fractional charges from $p_m = \frac{p}{\sqrt{3}}$. Leptons use SU(2) × U(1):

$$E^\dagger E = \frac{c^2}{|E|} \sum_{a=1}^{3} \tau^a (p^a + p_m^a)^2. \tag{12}$$

### 3.2 Gauge Bosons: Gluons

Gluons arise from SU(3) in $M_1$, with Hermitian field strength:

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f^{abc} G_\mu^b G_\nu^c, \tag{13}$$

and observable:

$$\mathcal{L}_{\mathrm{SU(3)}} = -\frac{1}{4} G_{\mu\nu}^a G^{a,\mu\nu}. \tag{14}$$

### 3.3 Gauge Bosons: Photons

Photons arise from U(1):

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{15}$$

with:

$$\mathcal{L}_{\mathrm{EM}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \tag{16}$$

### 3.4 Gauge Bosons: W and Z Bosons

W/Z bosons arise from SU(2) × U(1), with:

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g \epsilon^{abc} W_\mu^b W_\nu^c, \tag{17}$$

and masses:

$$m_W = \frac{g v'}{2}, \quad m_Z = \frac{v' \sqrt{g^2 + g'^2}}{2}. \tag{18}$$

### 3.5 Higgs Boson

The Higgs is an SU(2) doublet:

$$\mathcal{L}_\phi = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi), \tag{19}$$

with mass:

$$m_H = \sqrt{2\lambda} v'. \tag{20}$$

# 4 Quark Energy Transitions and Orthogonal Hilbert Space

## 4.1 Quark Energy Transitions

Quark energy transitions are mediated by the SU(3) dynamics within $Q$, with the 3x3 Hermitian matrix $E^\dagger E$ governing color changes. The matrix is:

$$E^\dagger E = \frac{c^2}{|E|} \sum_{a=1}^{8} \lambda^a (p^a + p_m^a)^2, \tag{21}$$

where $\lambda^a$ are Gell-Mann matrices. The off-diagonal elements, corresponding to ladder operators (e.g., $\lambda^1, \lambda^2$ for red $\leftrightarrow$ green), facilitate transitions:

$$T_{rg} = \frac{\lambda^1 + i\lambda^2}{2} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{22}$$

shifting $|g\rangle \to |r\rangle$. The diagonal elements ($\lambda^3, \lambda^8$) are Casimir invariants, representing conserved color charges:

$$C_2 = \sum_{a=1}^{8} \lambda^a \lambda^a, \quad C_2(3) = \frac{4}{3}, \tag{23}$$

with real eigenvalues. The orthogonality $E^2 = 0$ requires $p_m = -\frac{p}{\sqrt{3}}$, ensuring the off-diagonal elements reflect vacuum fluctuations, while diagonal elements project onto real-valued observables.

Energy transitions occur via gluon emission, modeled by:

$$Qq = \sum_{a=0}^{3} \left( Q_0 q_a - Q_1 q_{a-1} - Q_2 q_{a-2} - Q_3 q_{a-3} \right), \tag{24}$$

where $q_a$ aligns with SU(3) generators, updating $Q$ and shifting quark momenta $p^a \to p^a + \Delta p^a$. The Hermitian $E^\dagger E$ ensures real energy differences, consistent with QCD.

## 4.2 Orthogonal Hilbert Space and Real Axis Projection

The orthogonal Hilbert space is a 4x4 complex matrix, part of $Q = (M_1, M_2)$, where $M_1, M_2 \in \mathrm{SU}(4)$. This space, denoted $\mathcal{H}_1 \oplus \mathcal{H}_2$, encodes all possible quark states via superposition of wave functions $\psi_j(x) = R_j(x) e^{iS_j(x)/\hbar}$. The Hermitian operator $Q^\dagger Q$ projects probability states onto the real axis:

$$\mathrm{Re}(Q^\dagger Q) = \sum_{j=0}^{3} Q_j^\dagger Q_j, \tag{25}$$

yielding real eigenvalues for field densities. For quarks, the 4x4 complex matrix is:

$$M_1 = \begin{pmatrix} M_{\mathrm{SU}(3)} & * \\ * & * \end{pmatrix}, \tag{26}$$

with $M_{\mathrm{SU}(3)}$ a 3x3 unitary matrix. The superposition of states is:

$$|\psi\rangle = \sum_{c=r,g,b} \psi_c |c\rangle, \quad \psi_c = R_c e^{iS_c/\hbar}, \tag{27}$$

where $|c\rangle$ are color states. The probability density $|\psi|^2 = \sum_c |\psi_c|^2$ is real, projected via:

$$\langle\psi|Q^\dagger Q|\psi\rangle = \sum_c |\psi_c|^2 \,\mathrm{Re}(Q_c^\dagger Q_c). \tag{28}$$

The off-diagonal elements of $E^\dagger E$ represent vacuum components (fluctuations), while diagonal elements are Casimir invariants, ensuring $E^2 = 0$ via orthogonality of $p$ and $p_m$.

# 5 Dimensionality and the Real Axis

The factor of 8 in $\psi_0^2 = |V|^2$ reflects the eight-dimensional SU(4) $\times$ SU(4) space. Hermitian observables project onto the real axis, with $E^\dagger E$ ensuring real eigenvalues for particle properties.

# 6 Conclusion

This TOE represents particles as Hermitian observables, with quark transitions driven by SU(3) matrices and orthogonal Hilbert space projections ensuring real-valued probabilities. The 3x3 matrices' off-diagonal vacuum components and diagonal Casimir elements maintain $E^2 = 0$. Future work will explore experimental tests via Monte Carlo simulations.

# References

[1] E. Gardi, *A Quaternion-Based Theory of Everything: Unifying Quantum Field Theory and Gravity with Dynamic Symmetry Breaking*, Independent Research, February 27, 2025.

[2] E. Gardi, *Continued Analysis: Quaternion-Based Energy-Momentum Relations*, Independent Research, October 2024.