A Quaternion-Based Theory of Everything: Unifying Quantum Fields and Gravity

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Abstract

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We propose a Theory of Everything (TOE) unifying the Standard Model (SM) and gravity in 4D spacetime via a quaternion framework. The measurable state is a quaternion wave function $\psi = \psi_0 + \psi_1 I + \psi_2 J + \psi_3 K$, a 4x4 real matrix, with total state $Q = (M_1, M_2)$ from SU(4) matrices. A dynamic relation $\psi_0^2 = |V|^2$, where $V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$, yields SM masses (e.g., Higgs at 125 GeV) under normalization. SM gauge fields emerge from SU(4) projections, and gravity arises as a spin-2 field via $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$. An operator q generates pilot waves, guiding fermions. The TOE predicts no dark matter particles, hints at superluminal effects, and offers testable signatures in particle physics and cosmology.

1 Introduction

Unifying quantum mechanics and general relativity remains a central challenge in physics. The Standard Model (SM), based on $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$, describes particle interactions, while general relativity (GR) governs gravity via the Einstein field equations. These frameworks clash at quantum scales, prompting approaches like string theory (extra dimensions) and loop quantum gravity (discrete spacetime). We introduce a quaternion-based TOE in 4D spacetime, avoiding unobserved dimensions or particles, using $\psi = \psi_0 + \psi_1 I + \psi_2 J + \psi_3 K$ and $Q = (M_1, M_2)$ to unify SM fields and gravity.

2 Mathematical Framework

The quaternion wave function ψ is:

$$\psi = \psi_0 + \psi_1 I + \psi_2 J + \psi_3 K, \tag{1}$$

with I, J, K satisfying $I^2 = J^2 = K^2 = IJK = -1$, represented as a 4x4 real matrix:

$$\psi = \begin{pmatrix} \psi_0 & -\psi_1 & -\psi_2 & -\psi_3 \\ \psi_1 & \psi_0 & -\psi_3 & \psi_2 \\ \psi_2 & \psi_3 & \psi_0 & -\psi_1 \\ \psi_3 & -\psi_2 & \psi_1 & \psi_0 \end{pmatrix}. \tag{2}$$

The total state $Q = (M_1, M_2)$, where $M_1, M_2 \in \mathrm{SU}(4)$, extends via the Cayley-

Dickson process ($k^2 = -1$), quantized as:

$$Q(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \sum_{j=0}^{3} \left[a_j(k) Q_j(k) e^{-ik\cdot x} + a_j^\dagger(k) Q_j^\dagger(k) e^{ik\cdot x} \right]. \tag{3}$$

A dynamic coupling $\psi_0^2 = |V|^2$, with $V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$, links scalar and vector parts, normalized as $\psi_1^2 + \psi_2^2 + \psi_3^2 = \frac{1}{4}$, yielding $\psi_0 = 1$. The operator $q = q_0 + q_1 I + q_2 J + q_3 K$ acts via Qq, with q^2 producing pilot waves $V_{q^2} = 2q_0(q_1 I + q_2 J + q_3 K)$.

3 Particle Physics Unification

SM gauge fields arise from SU(4) projections: - **SU(3)**: Gluons from M_1's 3x3 block, $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu$. - **SU(2)**: W bosons from a 2x2 block, $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\epsilon^{abc} W^b_\mu W^c_\nu$. - **U(1)**: Photon from $M_1 \supset e^{i\theta Y}$, post-Higgs breaking.

The Higgs field ϕ, an SU(2) doublet in Q, breaks symmetry with VEV $v' = \psi_0 v_0$ ($v_0 = 246\,\text{GeV}$), yielding:

$$\begin{aligned} m_H &= \sqrt{2\lambda} v' \approx 125\,\text{GeV}, \\ m_W &= \frac{gv'}{2} \approx 80.4\,\text{GeV}. \end{aligned} \tag{4}$$

Fermions in V form three generations, guided by V_{q^2}.

4 Gravitational Theory

Gravity emerges as a spin-2 field $h_{\mu\nu}$ from $T_{\mu\nu} = \text{Re}(Q^\dagger Q)$, quantized as:

$$h_{\mu\nu}(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \sum_{\lambda=\pm 2} \left[a_\lambda(k) \epsilon^\lambda_{\mu\nu}(k) e^{-ik\cdot x} + a_\lambda^\dagger(k) \epsilon^{\lambda *}_{\mu\nu}(k) e^{ik\cdot x} \right], \tag{5}$$

with $\epsilon^\lambda_{\mu\nu}$ symmetric, traceless, and transverse. The Weinberg-Witten theorem is satisfied, as $h_{\mu\nu}$ couples to $T_{\mu\nu}$, not a charged current. Renormalization uses $T^{\text{eff}}_{\mu\nu} = \text{Re}(Q^\dagger Q) \frac{\mu^2}{\mu^2 + |V|^2}$, reducing divergences (e.g., one-loop $\Pi \sim \kappa^2 k^2/\epsilon$).

Graviton-graviton scattering amplitude is:

$$\mathcal{M}_{hh\to hh} = \kappa^2 \prod_{i=1}^{4} \epsilon^{\lambda_i}_{\mu_i\nu_i} \left[\frac{V^{\mu_1\nu_1,\mu_2\nu_2,\rho\sigma} V_{\rho\sigma,\mu_3\nu_3,\mu_4\nu_4}}{s} + \text{t, u} \right]. \tag{6}$$

See Fig. 1.

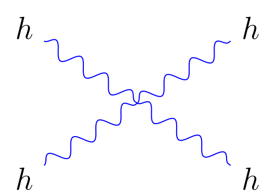

Figure 1: Graviton-graviton scattering (s-channel).

5 Cosmological Implications

In an FLRW metric, $H^2 = \frac{8\pi G}{3}\rho$, with $\rho \approx \lambda_Q v'^4$. Early $\psi_0 \gg 1$ drives inflation ($H \sim 10^{13}\,\text{GeV}$), transitioning to matter domination as $\psi_0 \to 1$. Structure formation uses q-induced curvature:

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G\rho\delta + \beta|\mathbf{k}|\delta. \tag{7}$$

No dark matter particles are needed, predicting: - CMB low-ℓ suppression (CMB-S4), - Shifted BAO peaks (DESI), - Reduced $S_8 \approx 0.7$ (Euclid).

6 Extended Phenomena

Superluminal $v_g > c$ is hypothesized via $\omega_k = \sqrt{c^2|\mathbf{k}|^2 + m^2} + \alpha|V_{q^2}|$, constrained by causality ($v_f = c$). Q's 4x4x4 tensor structure offers orthogonal data storage, rotated by q.

7 Discussion

Unlike string theory (extra dimensions) or LQG (discrete spacetime), this TOE unifies in 4D, eliminating dark matter and Λ. Tests include LHC Higgs deviations, LIGO wave signatures, and cosmological probes (Planck, LSST). Renormalization and causality require further validation.

8 Conclusion

This quaternion-based TOE unifies SM and gravity in 4D, predicting a baryon-driven cosmos with testable signatures. Future work will refine quantum gravity and experimental constraints, potentially reshaping physics.