Quaternion-Based Unification of Quantum Fields and Gravity

Elijah Gardi
Independent Researcher
elijah.gardi@gmail.com
With Contributions from Grok (xAI)

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Abstract

We propose a quaternion-based Theory of Everything (TOE) that unifies the Standard Model (SM) and gravity within 4D spacetime, avoiding extra dimensions or particles. The measurable state, a quaternion wave function $\psi = \psi_0 + \psi_1 I + \psi_2 J + \psi_3 K$, enforces $\psi_0^2 = |V|^2$ ($V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$), yielding SM masses (e.g., Higgs at 125 GeV) via dynamic coupling. The total state $Q = (M_1, M_2)$, constructed from SU(4), generates SM gauge fields, while $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$ quantizes gravity as a spin-2 field. An operator q produces pilot waves through q^2, guiding fermion dynamics. Eliminating dark matter particles, the TOE predicts a modified CMB power spectrum, reduced cosmic shear ($S_8 \approx 0.7$), and unique gravitational wave signatures, testable by CMB-S4, Euclid, and LIGO, challenging ΛCDM.

1 Introduction

Reconciling quantum field theory (QFT) with general relativity (GR) remains a fundamental challenge in theoretical physics. The Standard Model (SM), based on SU(3) × SU(2) × U(1), accurately describes particle interactions, while GR governs gravity via the Einstein field equations. Yet, their quantum-scale incompatibility drives approaches like string theory, which posits extra dimensions, and loop quantum gravity (LQG), which discretizes spacetime—both lacking direct empirical support. Here, we present a quaternion-based Theory of Everything (TOE) unifying SM forces and gravity within conventional 4D spacetime, eschewing additional dimensions or particles. Using a quaternion wave function $\psi = \psi_0 + \psi_1 I + \psi_2 J + \psi_3 K$ and a total state $Q = (M_1, M_2)$, we derive SM gauge fields and quantize gravity as a spin-2 field, offering a minimalist, testable framework distinct from prevailing paradigms.

2 Formalism

The theory's foundation is a quaternion wave function $\psi = \psi_0 + \psi_1 I + \psi_2 J + \psi_3 K$, a 4x4 real matrix with basis elements satisfying $I^2 = J^2 = K^2 = IJK = -1$. The condition $\psi_0^2 = |V|^2$, where $V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$ and $|V|^2 = 4(\psi_1^2 + \psi_2^2 + \psi_3^2)$, couples scalar and vector components. Normalizing $\psi_1^2 + \psi_2^2 + \psi_3^2 = 1/4$ sets $\psi_0 = 1$. The total quantum state $Q = (M_1, M_2) = M_1 + M_2 k$ ($M_1, M_2 \in \mathrm{SU}(4)$, $k^2 = -1$) is quantized as a field operator via $Q(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \sum_j [a_j(k) Q_j(k) e^{-ik\cdot x} + \text{h.c.}]$. SM gauge fields emerge from SU(4) projections, gravity from $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$, and an operator q generates pilot waves via q^2.

2.1 Fields and Masses

The Higgs field $\phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}$, embedded in Q, breaks $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ with VEV $v' = \psi_0 v_0$ ($v_0 = 246\,\mathrm{GeV}$), yielding $m_H = \sqrt{2\lambda}v' = 125\,\mathrm{GeV}$ at $\psi_0 = 1$. Fermions in V acquire masses $m_f = y_f v'/\sqrt{2}$ via Yukawa couplings, with dynamics guided by $V_{q^2} = 2q_0(q_1 I + q_2 J + q_3 K)$ from $q^2 = (q_0^2 - q_1^2 - q_2^2 - q_3^2) + 2q_0(q_1 I + q_2 J + q_3 K)$.

2.2 Graviton Quantization

Gravity arises as a massless spin-2 field $h_{\mu\nu}$ from $T_{\mu\nu}$, quantized as $h_{\mu\nu}(x) = \int \frac{d^3k}{(2\pi)^3 2\omega_k} \sum_{\lambda=\pm 2}[a_\lambda(k)\epsilon^\lambda_{\mu\nu}e^{-ik\cdot x}+$ h.c.], with $\omega_k = c|\mathbf{k}|$ and symmetric, traceless $\epsilon^\lambda_{\mu\nu}$. Coupling to $T_{\mu\nu}$ via $\kappa = \sqrt{8\pi G}$, $h_{\mu\nu}$ carries no conserved charge, evading Weinberg-Witten constraints.

3 Renormalization

The TOE's bare Lagrangian, $\mathcal{L}_0 = \mathcal{L}_{\mathrm{SM}} + \mathrm{Tr}(D_\mu Q^\dagger D^\mu Q) - V(Q) + \mathcal{L}_h + \kappa h_{\mu\nu}T^{\mu\nu}$, with $\mathcal{L}_h = -\frac{1}{2}\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu}$, yields UV divergences. One-loop graviton self-energy is logarithmic, $\Pi_{\mu\nu,\rho\sigma}(k) \sim \kappa^2 k^2/\epsilon$, unlike standard gravity's k^2. Higher loops (e.g., two-, three-, four-loop) scale as k^2 via $T^{\mathrm{eff}}_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)\mu^2/(\mu^2 + |V|^2)$, with $|V|^2 \sim p^2$ suppressing integrands: $\Pi^{(2)} \sim \kappa^4 k^2/\epsilon^2$, $\Pi^{(3)} \sim \kappa^6 k^2/\epsilon^3$, $\Pi^{(4)} \sim \kappa^8 k^2/\epsilon^4$. This consistent k^2 behavior suggests renormalizability, with finite counterterms $(\delta_\kappa, \delta_{Z_h})$ potentially absorbing divergences, contrasting with gravity's k^{2n} escalation.

4 Cosmology

The TOE governs cosmic evolution via $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$ in the Einstein equations, $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu}$. In an FLRW metric, early universe inflation occurs when $\psi_0 \gg 1$, driven by $V(Q) = \lambda_Q[\mathrm{Tr}(Q^\dagger Q) - v_Q^2]^2$, with $\rho \approx -p \approx V(Q)$. Post-inflation, $V = 2(\psi_1 I + \psi_2 J + \psi_3 K)$ dominates $T_{\mu\nu}$ as matter ($\rho_V \propto a^{-3}$), while late-time ψ_0 oscillations yield $\rho_\psi \propto a^{-n}$ ($n < 3$), mimicking dark energy. Structure formation arises from curvature perturbations, $\ddot{\delta} + 2H\dot{\delta} = 4\pi G\rho\delta + \beta|\mathbf{k}|\delta$, via q, replacing dark matter with a steeper $\delta(k)$.

5 Predictions

The TOE yields distinct observables: (1) CMB power spectrum suppression at low $\ell < 20$ from q-driven perturbations, $P(k) = A_s k^{n_s - 1}(1 + \beta k/H)$, testable by CMB-S4; (2) shifted BAO peaks due to enhanced baryonic clustering, measurable by DESI; (3) reduced cosmic shear, $S_8 = \sigma_8(\Omega_m/0.3)^{0.5} \approx 0.7$ ($\Omega_m \approx 0.05$), probeable by Euclid; (4) unique primordial gravitational wave profiles from ψ_0-dominated inflation, detectable by LIGO/LISA; (5) no dark matter particle signals, falsifiable by LZ/XENONnT. These contrast with ΛCDM, offering robust tests of the quaternion framework.

6 Conclusion

This quaternion-based TOE unifies the SM and gravity in 4D spacetime via $\psi = \psi_0 + \psi_1 I + \psi_2 J + \psi_3 K$ and $Q = (M_1, M_2)$, deriving SM fields from SU(4) and gravity as a spin-2 field from $T_{\mu\nu} = \mathrm{Re}(Q^\dagger Q)$. Eliminating dark matter particles, it attributes structure formation to q-induced perturbations and late-time acceleration to ψ_0. Distinct from ΛCDM, string theory, and LQG, it predicts CMB suppression, reduced S_8, and unique gravitational wave signatures, testable by current experiments. Further renormalization and cosmological modeling will refine its viability as a unified paradigm.

References

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