TMUA Chapter 3 - Quiz 3:
Basis of Logic Supplements S03

Time Allowed: 60 minutes
Number of Questions: 15
Difficulty: ⋆ ⋆ ⋆⋆

Supplement Questions

SQ1

A magical island is inhabited by knights (who always tell the truth) and liars (who always lie). A wise man met two people, Chris and Pat, from the island and decided to determine if they were knights or liars. When he asked Chris, "Are you both knights?" he could not be sure of their types. When he then asked Chris, "Are you of the same type?" he could identify their types.

What were they?

() both liars

() both knights

() Chris–knight, Pat–liar

() Chris–liar, Pat–knight

() impossible to specify

SQ2

The island of Nogardia is inhabited by dragons, each of which has either six, seven or eight legs. Dragons with seven legs always lie; dragons with an even number of legs always tell the truth.

One day four dragons met.

- The blue one said, "We have 28 legs altogether."
- The green one said, "We have 27 legs altogether."
- The yellow one said, "We have 26 legs altogether."
- The red one said, "We have 25 legs altogether."

Which of the following statements is true?

() the red dragon definitely has 6 legs

() the red dragon definitely has 7 legs

() the red dragon definitely has 8 legs

() the red dragon has either 6 or 8 legs, but we can't be sure which

() the red dragon has 6, 7, or 8 legs, but we can't be sure which

SQ3

The children P, Q, R and S made the following assertions.

- P said: Q, R and S are girls
- R said: P and Q are lying
- Q said: P, R and S are boys
- S said: P, Q and R are telling the truth

How many of the children were telling the truth?

() 0

() 1

() 2

() 3

() It cannot be determined

SQ4

Every second day Charles tells the truth for the whole day, otherwise he lies for the whole day. Today he made exactly four of the following statements. Which statement could he not have made today?

() I have a prime number of friends.

() I have as many male friends as female friends.

() My name is Charles.

() I always speak the truth.

() Three of my friends are older than me.

SQ5

From noon until midnight Clever Cat sleeps under an oak tree, and from midnight until noon he tells stories. A poster on the oak tree reads:

'Two hours ago Clever Cat was doing the same thing as he will be doing in one hour's time.'

For how many hours in a day is the statement on the poster true?

() 3

() 6

() 12

() 18

() 21

SQ6

A lion is hidden in one of three rooms. A note on the door of room 1 reads "The lion is here". A note on the door of room 2 reads "The lion is not here". A note on the door of room 3 reads "$2 + 3 = 2 \times 3$". Only one of these notes is true. In which room is the lion hidden?

() In room 1

() In room 2

() In room 3

() It may be in any room

() It may be in either room 1 or room 2

SQ7

One evening, an enclosure contained a number of kangaroos. All of a sudden a kangaroo got up and said: "There are 6 of us here" and jumped out of the enclosure. Then another kangaroo jumped out of the enclosure and said: "Every kangaroo who jumped out before me was lying." After that the rest of the kangaroos jumped out one by one, saying the same thing as the second kangaroo, until there were no kangaroos left in the enclosure. How many kangaroos had told the truth?

() 0

() 1

() 2

() 3

() 4

SQ8

Mr Ross always tells the truth on Thursdays and Fridays but always tells lies on Tuesdays. On the other days of the week he tells the truth or tells lies, at random. For seven consecutive days he was asked what his first name was, and on the first six days he gave the following answers, in order: John, Bob, John, Bob, Pit, Bob. What was his answer on the seventh day?

() John

() Bob

() Pit

() Kate

() More information is needed

SQ9

Three blackbirds, Isaac, Max and Oscar, are each sitting on their own nest. Isaac says: "I'm more than twice as far away from Max as I am from Oscar". Max says: "I'm more than twice as far away from Oscar as I am from Isaac". Oscar says: "I'm more than twice as far away from Max as I am from Isaac". At least two of them are telling the truth. Who is lying?

() Isaac

() Max

() Oscar

() None of them

() Impossible to tell

SQ10

The town of Ginkrail is inhabited entirely by knights and liars. Every sentence spoken by a knight is true, and every sentence spoken by a liar is false. One day some inhabitants of Ginkrail were alone in a room and three of them spoke.

- The first one said: "There are no more than three of us in the room. All of us are liars."
- The second said: "There are no more than four of us in the room. Not all of us are liars."
- The third said: "There are five of us in the room. Three of us are liars."

How many people were in the room and how many liars were among them?

() 3 people, 1 of whom is a liar

() 4 people, 1 of whom is a liar

() 4 people, 2 of whom are liars

() 5 people, 2 of whom are liars

() 5 people, 3 of whom are liars

SQ11

At each of the vertices of a cube sits a Bunchkin. Two Bunchkins are said to be adjacent if and only if they sit at either end of one of the cube's edges. Each Bunchkin is either a 'truther', who always tells the truth, or a 'liar', who always lies. All eight Bunchkins say 'I am adjacent to exactly two liars'. What is the maximum number of Bunchkins who are telling the truth?

SQ12

Twenty-five people who always tell the truth or always lie are standing in a queue. The man at the front of the queue says that everyone behind him always lies. Everyone else says that the person immediately in front of them always lies. How many people in the queue always lie?

SQ13

Which is the lowest numbered statement which is true?

- Statement 201: "Statement 203 is true".
- Statement 202: "Statement 201 is true".
- Statement 203: "Statement 206 is false".
- Statement 204: "Statement 202 is false".
- Statement 205: "None of the statements 201, 202, 203 or 204 are true".
- Statement 206: "$1 + 1 = 2$".

SQ14

A box contains seven cards numbered from 301 to 307. Graham picks three cards from the box and then Zoe picks two cards from the remainder. Graham looks at his cards and then says "I know that the sum of the numbers on your cards is even". What is the sum of the numbers on Graham's cards?

SQ15

Sketch, without calculating the stationary points, the graph of the function $f(x)$ given by

$$f(x) = (x - p)(x - q)(x - r),$$

where $p < q < r$. By considering the quadratic equation $f'(x) = 0$, or otherwise, show that

$$(p + q + r)^2 > 3(qr + rp + pq).$$

By considering $(x^2+gx+h)(x-k)$, or otherwise, show that $g^2 > 4h$ is a sufficient condition but not a necessary condition for the inequality

$$(g - k)^2 > 3(h - gk)$$

to hold.