TMUA Chapter 2 - Quiz 3:
Counting and Probabilities Supplements S02

Xie Tao TMUA Workbook 2024 (5th Edition)

Time Allowed: 90 minutes
Number of Questions: 20
Difficulty: ⋆ ⋆ ⋆○

Supplement Questions

SQ1

Anne, Bert, Clare, Derek and Emily are planning to play a game for which they need to divide themselves into three teams. Each team must have at least one member. The number of different ways they can do this is

(A) 10

(B) 15

(C) 25

(D) 30

SQ2

The faces of a cube are coloured red or blue. Exactly three are red and three are blue. The number of distinguishable cubes that can be produced (allowing the cube to be turned around) is?

(A) 2

(B) 4

(C) 6

(D) 20

SQ3

A grid of size 3 cm $\times$ 5 cm is drawn, ruled at 1 cm intervals. The number of squares that can be drawn using the grid is

(A) 15

(B) 18

(C) 26

(D) 37

SQ4

A cube painted black is cut into 125 identical cubes. How many of them are not painted at all?

(A) 21

(B) 25

(C) 27

(D) 30

SQ5

90 people enter a maze. At each junction a third will go left and two thirds will go right. After three such junctions, what is the most likely combination of turns people will have taken?

(A) Gone right three times

(B) Gone left three times

(C) Gone right twice and once left

(D) Gone twice left and once right

(E) It is impossible to tell

SQ6

A bag contains b blue balls and r red balls. If two balls are picked at random and removed from the bag, what is the probability P that they are different colours?

(A) $\frac{2br}{(b+r)(b+r-1)}$

(B) $\frac{br}{(b+r)(b+r-1)}$

(C) $\frac{br}{(b+r)^2}$

(D) $\frac{2br}{(b+r)^2}$

(E) $2br$

SQ7

We wish to represent integer numbers by using our ten fingers. A finger is assumed to be either stretched out or curled up. How many different integers can we represent with our fingers?

(A) 10

(B) 512

(C) 1000

(D) 20

(E) 1024

SQ8

Ten students need to complete their compulsory practicals for their high school examinations as detailed in the table below:

No. of students	No. of different practicals to complete
2	1
4	2
4	3

The school only has one laboratory in which several different experiments can be set up simultaneously. A maximum of six students are allowed in the school's laboratory for a lesson. Each practical takes one lesson. What is the minimum number of lessons required to complete all the practicals?

(A) 3

(B) 4

(C) 5

(D) 6

(E) 10

SQ9

To get to work, Sylvie first catches a bus and then catches a train.

- The probability that the bus is on time is 0.6.
- The probability that the bus is late is 0.4.
- If the bus is on time, then the probability that she will catch the train is 0.8.
- If the bus is late, then the probability that she will catch the train is 0.6.

Given that Sylvie catches the train, what is the probability that the bus was on time?

(A) $\frac{1}{3}$

(B) $\frac{12}{25}$

(C) $\frac{2}{5}$

(D) $\frac{3}{5}$

(E) $\frac{2}{3}$

(F) $\frac{18}{25}$

(G) $\frac{6}{7}$

SQ10

I have two six-sided dice, each with faces numbered from 1 to 6. One of the dice is fair, but the other is not; it will land on numbers 1 to 5 with equal probability, but lands on 6 with a different probability.

When I roll the dice the probability that I get a total of 12 is $\frac{1}{18}$.

What is the probability that I get a total of 2 when I roll the dice?

(A) $\frac{1}{72}$

(B) $\frac{1}{45}$

(C) $\frac{1}{36}$

(D) $\frac{1}{18}$

(E) $\frac{1}{9}$

SQ11

The ratio of the number of boys to the number of girls in a class is 1:3

The number of boys in the class is n.

Two students are chosen at random from the class.

The probability that both the students are boys is p.

Which one of the following is a correct expression for n, the number of boys in the class?

(A) $n = \frac{3p-1}{9p-1}$

(B) $n = \frac{3p+1}{9p-1}$

(C) $n = \frac{1}{1-9p}$

(D) $n = \frac{1}{9p-1}$

(E) $n = \frac{4p-1}{16p-1}$

(F) $n = \frac{4p+1}{16p-1}$

(G) $n = \frac{1}{1-16p}$

(H) $n = \frac{1}{16p-1}$

SQ12

A bag contains only n red balls and $2n$ green balls.

One ball is picked and its colour recorded. It is then put back in the bag, and an additional ball of the same colour is added to the bag.

A second ball is then picked.

What is the probability that the two balls picked are not the same colour?

(A) $\frac{2n}{3(3n-1)}$

(B) $\frac{4n}{3(3n-1)}$

(C) $\frac{5n}{3(3n-1)}$

(D) $\frac{5n-3}{3(3n-1)}$

(E) $\frac{2n}{3(3n+1)}$

(F) $\frac{4n}{3(3n+1)}$

(G) $\frac{5n}{3(3n+1)}$

(H) $\frac{5n+3}{3(3n+1)}$

SQ13

A bag only contains $2n$ blue balls and n red balls. All the balls are identical apart from colour.

One ball is randomly selected and not replaced. A second ball is then randomly selected.

What is the probability that at least one of the selected balls is red?

(A) $\frac{n-1}{3(3n-1)}$

(B) $\frac{3n-1}{3(3n-1)}$

(C) $\frac{4n-2}{3(3n-1)}$

(D) $\frac{4n}{3(3n-1)}$

(E) $\frac{5n-1}{3(3n-1)}$

(F) $\frac{5n-5}{3(3n-1)}$

SQ14

Two identical fair six-sided dice each have their faces numbered from 1 to 6, with one number on each face.

Both dice are thrown, and the number on each of the dice is recorded.

They are then both thrown again, and the number on each of the dice is recorded.

What is the probability that at least one of the four recorded numbers is even?

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{9}{16}$

(D) $\frac{3}{4}$

(E) $\frac{15}{16}$

SQ15

Eight people are sitting around a circular table, each holding a fair coin. All eight people flip their coins and those who flip heads stand while those who flip tails remain seated. What is the probability that no two adjacent people will stand?

(A) $\frac{47}{256}$

(B) $\frac{3}{16}$

(C) $\frac{49}{256}$

(D) $\frac{25}{128}$

(E) $\frac{51}{256}$

SQ16

A choir director must select a group of singers from among his 6 tenors and 8 basses. The only requirements are that the difference between the number of tenors and basses must be a multiple of 4, and the group must have at least one singer. Let N be the number of groups that can be selected. What is the remainder when N is divided by 100?

(A) 47

(B) 48

(C) 83

(D) 95

(E) 96

SQ17

How many 15-letter arrangements of 5 A's, 5 B's and 5 C's have no A's in the first 5 letters, no B's in the next 5 letters and no C's in the last 5 letters?

(A) $\sum_{k=0}^{5} \binom{5}{k}^3$

(B) $3^5 \cdot 2^5$

(C) 2^{15}

(D) $\frac{15!}{(5!)^3}$

(E) 3^{15}

SQ18

How many odd positive 3-digit integers are divisible by 3 but do not contain the digit 3?

(A) 96

(B) 97

(C) 98

(D) 102

(E) 120

SQ19

A pair of standard 6-sided fair dice is rolled once. The sum of the numbers rolled determines the diameter of a circle. What is the probability that the numerical value of the area of the circle is less than the numerical value of the circle's circumference?

(A) $\frac{1}{36}$

(B) $\frac{1}{12}$

(C) $\frac{1}{6}$

(D) $\frac{1}{4}$

(E) $\frac{5}{18}$

SQ20

Chlo chooses a real number uniformly at random from the interval [0, 2017]. Independently, Laurent chooses a real number uniformly at random from the interval [0, 4034]. What is the probability that Laurent's number is greater than Chlo's number?

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) $\frac{3}{4}$

(D) $\frac{5}{6}$

(E) $\frac{7}{8}$