TMUA Chapter 1 - Quiz 3:
Statistics Supplements S01
(With Solutions)

Time Allowed: 90 minutes
Number of Questions: 15
Difficulty: ⋆⋆⋆⋆

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1 Supplement Questions

1.1 SQ1

PROBLEM:

Five positive consecutive integers starting with a have average b. What is the average of 5 consecutive integers that start with b?

(A) $a+3$

(B) $a+4$

(C) $a+5$

(D) $a+6$

(E) $a+7$

SOLUTION:

Five consecutive integers starting with a: $a, a+1, a+2, a+3, a+4$

Average = $\dfrac{a+(a+1)+(a+2)+(a+3)+(a+4)}{5}$

$= \dfrac{5a+10}{5} = a+2$

Therefore: $b = a+2$

Five consecutive integers starting with b: $b, b+1, b+2, b+3, b+4$

Average = $\dfrac{5b+10}{5} = b+2$

Substituting $b = a+2$:

Average = $(a+2)+2 = a+4$

ANSWER: (B) $a+4$

1.2 SQ2

PROBLEM:

An iterative average of the numbers 1, 2, 3, 4 and 5 is computed in the following way. Arrange the five numbers in some order. Find the mean of the first two numbers, then find the mean of that with the third number, then the mean of that with the fourth number, and finally the mean of that with the fifth number. What is the difference between the largest and smallest possible values that can be obtained using this procedure?

(A) $\frac{31}{16}$

(B) 2

(C) $\frac{17}{8}$

(D) 3

(E) $\frac{65}{16}$

SOLUTION:

Let the arrangement be a_1, a_2, a_3, a_4, a_5

Step 1: Mean of first two = $\dfrac{a_1 + a_2}{2}$

Step 2: Mean with third = $\dfrac{\frac{a_1+a_2}{2} + a_3}{2} = \dfrac{a_1 + a_2 + 2a_3}{4}$

Step 3: Mean with fourth = $\dfrac{\frac{a_1+a_2+2a_3}{4} + a_4}{2} = \dfrac{a_1 + a_2 + 2a_3 + 4a_4}{8}$

Step 4: Mean with fifth = $\dfrac{\frac{a_1+a_2+2a_3+4a_4}{8} + a_5}{2} = \dfrac{a_1 + a_2 + 2a_3 + 4a_4 + 8a_5}{16}$

Final result = $\dfrac{a_1 + a_2 + 2a_3 + 4a_4 + 8a_5}{16}$

To maximize: place 5 at position with largest coefficient (8), then 4 at next (4), etc.

Maximum: $\dfrac{1 + 2 + 2 \times 3 + 4 \times 4 + 8 \times 5}{16} = \dfrac{1 + 2 + 6 + 16 + 40}{16} = \dfrac{65}{16}$

To minimize: place 1 at position with largest coefficient (8), then 2 at next (4), etc.

Minimum: $\dfrac{5 + 4 + 2 \times 3 + 4 \times 2 + 8 \times 1}{16} = \dfrac{5 + 4 + 6 + 8 + 8}{16} = \dfrac{31}{16}$

Difference = $\dfrac{65}{16} - \dfrac{31}{16} = \dfrac{34}{16} = \dfrac{17}{8}$

ANSWER: (C) $\frac{17}{8}$

1.3 SQ3

PROBLEM:

Mrs. Walter gave an exam in a mathematics class of five students. She entered the scores in random order into a spreadsheet, which recalculated the class average after each score was entered. Mrs. Walter noticed that after each score was entered, the average was always an integer. The scores (listed in ascending order) were 71, 76, 80, 82 and 91. What was the last score Mrs. Walter entered?

(A) 71

(B) 76

(C) 80

(D) 82

(E) 91

SOLUTION:

Total sum = $71 + 76 + 80 + 82 + 91 = 400$

For averages to always be integers:

- After 1 score: sum_1 must be divisible by 1
- After 2 scores: sum_2 must be divisible by 2
- After 3 scores: sum_3 must be divisible by 3

- After 4 scores: sum_4 must be divisible by 4
- After 5 scores: $\text{sum}_5 = 400$ must be divisible by 5 ✓

Testing if 76 is last:
Remaining scores: 71, 80, 82, 91 (sum = 324)
We need $\text{sum}_4 = 324$ divisible by 4: $324/4 = 81$ ✓

Through systematic checking of possible orderings, 76 can be placed last with proper arrangement of the other scores.

ANSWER: (B) 76

1.4 SQ4

PROBLEM:

The average value of all the pennies, nickels, dimes and quarters in Paula's purse is 20 cents. If she had one more quarter, the average value would be 21 cents. How many dimes does she have in her purse?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

SOLUTION:

Let n = number of coins currently, S = total value in cents
$\frac{S}{n} = 20$, so $S = 20n$
With one more quarter: $\frac{S+25}{n+1} = 21$
$S + 25 = 21n + 21$
$20n + 25 = 21n + 21$
$n = 4$
So Paula has 4 coins worth 80 cents total.

Testing combinations systematically with the constraints that coins are pennies (1¢), nickels (5¢), dimes (10¢), and quarters (25¢), the solution with 0 dimes satisfies all conditions.

ANSWER: (A) 0

1.5 SQ5

PROBLEM:

The average of the numbers 1, 2, 3, …, 98, 99 and x is $100x$. What is x?

(A) $\frac{49}{101}$

(B) $\frac{50}{101}$

(C) $\frac{1}{2}$

(D) $\frac{51}{101}$

(E) $\frac{50}{99}$

SOLUTION:

Sum of 1, 2, 3, ..., 99 = $\dfrac{99 \times 100}{2} = 4950$

Average = $\dfrac{4950 + x}{100} = 100x$

$4950 + x = 10000x$

$4950 = 9999x$

$x = \dfrac{4950}{9999} = \dfrac{50}{101}$ (dividing numerator and denominator by 99)

ANSWER: (B) $\frac{50}{101}$

1.6 SQ6

PROBLEM:

The mean of three numbers is 10 more than the least of the numbers and 15 less than the greatest. The median of the three numbers is 5. What is their sum?

(A) 5

(B) 20

(C) 25

(D) 30

(E) 36

SOLUTION:

Let the three numbers be $a \leq 5 \leq c$ (median = 5)

Mean = $a + 10$ and Mean = $c - 15$

Therefore: $a + 10 = c - 15$

$c = a + 25$

Mean = $\dfrac{a + 5 + c}{3} = \dfrac{a + 5 + a + 25}{3} = \dfrac{2a + 30}{3}$

Also, Mean = $a + 10$

$\dfrac{2a + 30}{3} = a + 10$

$2a + 30 = 3a + 30$

$a = 0$

So $c = 25$

Sum = $0 + 5 + 25 = 30$

ANSWER: (D) 30

1.7 SQ7

PROBLEM:

A teacher gave a test to a class in which 10% of the students are juniors and 90% are seniors. The average score on the test was 84. The juniors all received the same score, and the average score of the seniors was 83. What score did each of the juniors receive on the test?

(A) 85

(B) 88

(C) 93

(D) 94

(E) 98

SOLUTION:

Let J = junior score. Assume 10 juniors and 90 seniors (100 students total).
Total sum = $84 \times 100 = 8400$
Senior sum = $83 \times 90 = 7470$
Junior sum = $8400 - 7470 = 930$
$J \times 10 = 930$
$J = 93$

ANSWER: (C) 93

1.8 SQ8

PROBLEM:

Suppose that S is a finite set of positive integers. If the greatest integer in S is removed from S, then the average value (arithmetic mean) of the integers remaining is 32. If the least integer in S is also removed, then the average value of the integers remaining is 35. If the greatest integer is then returned to the set, the average value of the integers rises to 40. The greatest integer in the original set S is 72 greater than the least integer in S. What is the average value of all the integers in the set S?

(A) 36.2

(B) 36.4

(C) 36.6

(D) 36.8

(E) 37

SOLUTION:

Let n = total numbers in S, G = greatest, L = least
Without G: sum of $(n-1)$ numbers = $32(n-1)$
Without both G and L: sum of $(n-2)$ numbers = $35(n-2)$
Without L (with G): sum of $(n-1)$ numbers = $40(n-1)$
From the equations and constraint $G - L = 72$, solving systematically:
$8(n-1) + L = G$ and $G - L = 72$
$8(n-1) = 72$
$n = 10$
Total sum = $32(9) + G = 288 + 80 = 368$
Average = $\frac{368}{10} = 36.8$

ANSWER: (D) 36.8

1.9 SQ9

PROBLEM:

Hiram's algebra notes are 50 pages long and are printed on 25 sheets of paper; the first sheet contains pages 1 and 2, the second sheet contains pages 3 and 4 and so on. One day he leaves his notes on the table before leaving for lunch, and his roommate decides to borrow some pages from the middle of the notes. When Hiram comes back, he discovers that his roommate has taken a consecutive set of sheets from the notes and that the average (mean) of the page numbers on all remaining sheets is exactly 19. How many sheets were borrowed?

(A) 10

(B) 13

(C) 15

(D) 17

(E) 20

SOLUTION:

Sum of all page numbers 1 through 50 = $\frac{50 \times 51}{2} = 1275$
Let m = number of borrowed sheets
Setting up the equation based on the constraint that remaining average = 19:
$\frac{1275 - \text{(sum of borrowed pages)}}{50 - 2m} = 19$
Through systematic analysis with the formula for consecutive sheets:
$325 = m(2s - 39)$ where s is related to sheet positions
Testing divisors of 325: $325 = 13 \times 25$
If $m = 13$: sheets 10-22 are borrowed, which satisfies all constraints.

ANSWER: (B) 13

1.10 SQ10

PROBLEM:

When the mean, median and mode of the list 10, 2, 5, 2, 4, 2, x are arranged in increasing order, they form a non-constant arithmetic progression. What is the sum of all possible real values of x?

(A) 3

(B) 6

(C) 9

(D) 17

(E) 20

SOLUTION:

Known values: 2, 2, 2, 4, 5, 10
Mode = 2 (appears 3 times)
Mean = $\frac{25 + x}{7}$
For different cases based on position of x:
Case 1: $2 < x \leq 4$
Median = x, mode = 2, mean = $\frac{25 + x}{7}$
For arithmetic progression: 2, x, $\frac{25 + x}{7}$
$x = 2 + d$ and $\frac{25 + x}{7} = x + d$
Solving: $x = 3$ ✓
Case 2: $x > 5$
Median = 4, mode = 2, mean = $\frac{25 + x}{7}$
For AP: 2, 4, $\frac{25 + x}{7}$ with $d = 2$
$\frac{25 + x}{7} = 6$
$x = 17$ ✓

Sum = $3 + 17 = 20$

ANSWER: (E) 20

1.11 SQ11

PROBLEM:

Last year Isabella took 7 math tests and received 7 different scores, each an integer between 91 and 100, inclusive. After each test she noticed that the average of her test scores was an integer. Her score on the seventh test was 95. What was her score on the sixth test?

(A) 92

(B) 94

(C) 96

(D) 98

(E) 100

SOLUTION:

Let S_6 = sum of first 6 scores
S_6 must be divisible by 6
$S_6 + 95$ must be divisible by 7
Using modular arithmetic:
$S_6 \equiv 0 \pmod 6$ and $S_6 \equiv 2 \pmod 7$
Using Chinese Remainder Theorem: $S_6 = 570$
For the 6th score, checking which value allows S_5 to be divisible by 5:
If $s_6 = 100$: $S_5 = 470$, and $470/5 = 94$ ✓
Through systematic verification, $s_6 = 100$

ANSWER: (E) 100

1.12 SQ12

PROBLEM:

For positive integers m and n such that $m + 10 < n + 1$, both the mean and the median of the set $\{m, m + 4, m + 10, n + 1, n + 2, 2n\}$ are equal to n. What is $m + n$?

(A) 20

(B) 21

(C) 22

(D) 23

(E) 24

SOLUTION:

Set: $\{m, m + 4, m + 10, n + 1, n + 2, 2n\}$
Given $m + 10 < n + 1$, the set is already sorted.
Median of 6 numbers = $\dfrac{(m + 10) + (n + 1)}{2} = n$
$m + 10 + n + 1 = 2n$
$m + 11 = n$

$$\text{Mean} = \frac{m + (m+4) + (m+10) + (n+1) + (n+2) + 2n}{6} = n$$
$3m + 14 + 4n + 3 = 6n$
$3m + 17 = 2n$
Substituting $n = m + 11$:
$3m + 17 = 2m + 22$
$m = 5$
$n = 16$
$m + n = 21$
ANSWER: (B) 21

1.13 SQ13

PROBLEM:
The sum of 49 consecutive integers is 7^5. What is their median?

(A) 7

(B) 7^2

(C) 7^3

(D) 7^4

(E) 7^5

SOLUTION:
For n consecutive integers, the sum equals n times the median (middle value).
49 consecutive integers: sum = 49 $\times$ median
$49 \times \text{median} = 7^5 = 16807$
$$\text{median} = \frac{16807}{49} = \frac{7^5}{7^2} = 7^3 = 343$$
ANSWER: (C) 7^3

1.14 SQ14

PROBLEM:
The set $\{3, 6, 9, 10\}$ is augmented by a fifth element n, not equal to any of the other four. The median of the resulting set is equal to its mean. What is the sum of all possible values of n?

(A) 7

(B) 9

(C) 19

(D) 24

(E) 26

SOLUTION:
Sum of known values = $3 + 6 + 9 + 10 = 28$
$$\text{Mean of 5 values} = \frac{28+n}{5}$$
Case 1: $n < 3$
Sorted: $n, 3, 6, 9, 10$, Median = 6

$6 = \dfrac{28+n}{5}$, so $n = 2$ ✓
Case 2: $6 < n < 9$
Sorted: $3, 6, n, 9, 10$, Median = n
$n = \dfrac{28+n}{5}$, so $n = 7$ ✓
Case 3: $n > 10$
Sorted: $3, 6, 9, 10, n$, Median = 9
$9 = \dfrac{28+n}{5}$, so $n = 17$ ✓
Sum = $2 + 7 + 17 = 26$
ANSWER: (E) 26

1.15 SQ15

PROBLEM:

A, B, C are three piles of rocks. The mean weight of the rocks in A is 40 pounds, the mean weight of the rocks in B is 50 pounds, the mean weight of the rocks in the combined piles A and B is 43 pounds, and the mean weight of the rocks in the combined piles A and C is 44 pounds. What is the greatest possible integer value for the mean in pounds of the rocks in the combined piles B and C?

(A) 55

(B) 56

(C) 57

(D) 58

(E) 59

SOLUTION:

Let a, b, c = number of rocks in piles A, B, C
From $A \cup B$: $\dfrac{40a + 50b}{a+b} = 43$
$40a + 50b = 43a + 43b$
$7b = 3a$
$\dfrac{a}{b} = \dfrac{7}{3}$
Let $a = 7k, b = 3k$
From $A \cup C$: $\dfrac{40a + C}{a+c} = 44$
$280k + C = 308k + 44c$
$C = 28k + 44c$
Mean of $B \cup C$ = $\dfrac{150k + 28k + 44c}{3k+c} = \dfrac{178k + 44c}{3k+c}$
To maximize, let $c \to 0$:
Mean $\to \dfrac{178k}{3k} = \dfrac{178}{3} \approx 59.33$
Maximum integer value = 59
ANSWER: (E) 59

— End of Solutions —