TMUA Chapter 2 - Quiz 1:
Counting and Probabilities Exercises E02

Time Allowed: No limit
Number of Questions: 25
Difficulty: ⋆ ⋆ ⋆○

Pre-Quiz Questions

Quiz Pre-1

Three dice, each showing numbers 1 to 6, are coloured red, blue and yellow respectively. Each of the dice is rolled once. The total of the numbers rolled is 10. In how many different ways can this happen?

(A) 36

(B) 30

(C) 27

(D) 24

(E) 21

Quiz Pre-2

There are 10 girls in a mixed class. If two pupils from the class are selected at random to represent the class on the School Council, then the probability that both are girls is 0.15. How many boys are in the class?

(A) 10

(B) 12

(C) 15

(D) 18

(E) 20

Quiz Pre-3

A bag contains m blue and n yellow marbles. One marble is selected at random from the bag and its colour is noted. It is then returned to the bag along with k other marbles of the same colour. A second marble is now selected at random from the bag. What is the probability that the second marble is blue?

(A) $\frac{m}{m+n}$

(B) $\frac{n}{m+n}$

(C) $\frac{m}{m+n+k}$

(D) $\frac{m+k}{m+n+k}$

(E) $\frac{m+n}{m+n+k}$

Exercise Questions

Ex. 1

There are 6 gentlemen, A, B, C, D, E, F, and 4 ladies, X, Y, Z, W.

(i) Find the number of different ways when they stand in a line if

(a) there are no restrictions,

(b) all men stand next to each other,

(c) no lady stands next to another,

(d) A must stand next to B,

(e) A must stand next to B, and X must stand next to Y,

(f) A must stand next to B, and X must not stand next to Y,

(g) A must stand next to B, and X must not stand next to B.

(ii) In this part, the 6 gentlemen and 4 ladies are divided into 2 groups of five. Find the number of different ways when

(a) there are no other restrictions,

(b) all ladies cannot be in the same group,

(c) numbers of men and ladies are the same in each group,

(d) numbers of men and ladies are the same in each group, A and X must be in the same group, B and Y also must be in the same group,

(e) numbers of men and ladies are the same in each group, A and X must be in the same group, whereas B and Y must not be in the same group.

(iii) If they are divided into 3 groups, find the number of different ways when

(a) no group has a number of persons less than 2,

(b) there is at least one lady and two gentlemen in each group.

(iv) In this part, these 10 people sit down for dinner where they may order one of three types of meals, or order nothing.

(a) How many ways of their orders are possible?

(b) If A orders nothing, three people order the pork meal, three order the chicken meal, and three order the beef meal. A passes the nine meals to the other 9 people in random order. Find the number of ways in which A could pass the meal types to them such that exactly one person receives the type of meal ordered by that person.

Ex. 2

You go into a supermarket to buy two packets of biscuits, which may or may not be of the same variety. The supermarket has 20 different varieties of biscuits and at least two packets of each variety. In how many ways can you choose your two packets?

(A) 400

(B) 210

(C) 200

(D) 190

Ex. 3

Given an unlimited supply of 50p, £1 and £2 coins, in how many different ways is it possible to make a sum of £100?

(A) 1326

(B) 2500

(C) 2601

(D) 5050

(E) 10 000

Ex. 4

In a college there are 100 students taking A level French, German or Spanish. Of these students, 64 are female and the rest are male. There are 50 French students of whom 40 are female and 30 German students of whom 10 are female.

Find the probability that a randomly chosen student

(i) is taking Spanish,

(ii) is male, given that the student is taking Spanish.

College records indicate that 70% of the French students, 80% of the German students and 60% of the Spanish students have applied for University. A student is chosen at random.

(iii) Find the probability that this student has applied for University.

(iv) Given that the student had applied to University, find the probability that the student is studying French.

Ex. 5

Most students in a large college study Mathematics. A teacher chooses three different students at random, one after the other.

Consider these three probabilities:

- $R = P(\text{At least one of the students chosen studies Mathematics})$
- $S = P(\text{The second student chosen studies Mathematics})$
- $T = P(\text{All three of the students chosen study Mathematics})$

Which of the following is true?

(A) $R < S < T$

(B) $R < T < S$

(C) $S < R < T$

(D) $S < T < R$

(E) $T < R < S$

(F) $T < S < R$

Ex. 6

There are only red balls and green balls in a bag.

When I pick a ball from the bag, the probability of picking a red ball is p and the probability of picking a green ball is q, where $q \geq p$.

I pick a ball from the bag and record its colour. I then replace the ball in the bag.

I repeat this procedure once.

Given that

$$P(\text{the balls are of the same colour}) - P(\text{the balls are of different colours}) = \frac{1}{4}$$

find the value of

$$\frac{q}{p} - \frac{p}{q}$$

(A) 0

(B) $\frac{3}{2}$

(C) $\frac{5}{6}$

(D) $\frac{8}{3}$

(E) $\frac{247}{48}$

Ex. 7

Two players take turns to throw a fair six-sided die until one of them scores a six. What is the probability that the first player to throw the die is the first to score a six?

(A) $\frac{5}{9}$

(B) $\frac{3}{5}$

(C) $\frac{6}{11}$

(D) $\frac{7}{12}$

Ex. 8

Billie has a die with the numbers 1, 2, 3, 4, 5 and 6 on its six faces.

Niles has a die which has the numbers 4, 4, 4, 5, 5 and 5 on its six faces.

When Billie and Niles roll their dice the one with the larger number wins. If the two numbers are equal it is a draw.

The probability that Niles wins, when written as a fraction in its lowest terms, is $\frac{p}{q}$. What is the value of $7p + 11q$?

Ex. 9

Tom and Geri have a competition. Initially, each player has one attempt at hitting a target. If one player hits the target and the other does not then the successful player wins. If both players hit the target, or if both players miss the target, then each has another attempt, with the same rules applying. If the probability of Tom hitting the target is always $\frac{4}{5}$ and the probability of Geri hitting the target is always $\frac{2}{3}$, what is the probability that Tom wins the competition?

(A) $\frac{4}{15}$

(B) $\frac{8}{15}$

(C) $\frac{2}{3}$

(D) $\frac{4}{5}$

(E) $\frac{13}{15}$

Ex. 10

Each face of a cube is painted either red or blue, each with probability $\frac{1}{2}$. The color of each face is determined independently. What is the probability that the painted cube can be placed on a horizontal surface so that the four vertical faces are all the same color?

(A) $\frac{1}{4}$

(B) $\frac{5}{16}$

(C) $\frac{3}{8}$

(D) $\frac{7}{16}$

(E) $\frac{1}{2}$

Ex. 11

Suppose a and b are single-digit positive integers chosen independently and at random. What is the probability that the point (a, b) lies above the parabola $y = ax^2 - bx$?

(A) $\frac{11}{81}$

(B) $\frac{13}{81}$

(C) $\frac{5}{27}$

(D) $\frac{17}{81}$

(E) $\frac{19}{81}$

Ex. 12

A fair 6-sided die is repeatedly rolled until an odd number appears. What is the probability that every even number appears at least once before the first occurrence of an odd number?

(A) $\frac{1}{120}$

(B) $\frac{1}{32}$

(C) $\frac{1}{20}$

(D) $\frac{3}{20}$

(E) $\frac{1}{6}$

Quiz Questions

Quiz 1

A fancy bed and breakfast inn has 5 rooms, each with a distinctive color-coded decor. One day 5 friends arrive to spend the night. There are no other guests that night. The friends can room in any combination they wish, but with no more than 2 friends per room. In how many ways can the innkeeper assign the guests to the rooms?

(A) 2100

(B) 2220

(C) 3000

(D) 3120

(E) 3125

Quiz 2

A hockey team consists of 1 goalkeeper, 4 defenders, 4 midfielders and 2 forwards. There are 4 substitutes: 1 goalkeeper, 1 defender, 1 midfielder and 1 forward. A substitute may only replace a player of the same category eg: midfielder for midfielder. Given that a maximum of 3 substitutes may be used and that there are still 11 players on the pitch at the end, how many different teams could finish the game?

(A) 110

(B) 118

(C) 121

(D) 125

(E) 132

Quiz 3

As a special treat, Sammy is allowed to eat five sweets from his very large jar which contains many sweets of each of three flavours — Lemon, Orange and Strawberry. He wants to eat his five sweets in such a way that no two consecutive sweets have the same flavour. In how many ways can he do this?

(A) 32

(B) 48

(C) 72

(D) 108

(E) 162

Quiz 4

In a population, $\frac{3}{5}$ of the adults are overweight. The probability of an overweight adult having Type 2 diabetes is $\frac{9}{50}$; this probability is 6 times the probability of an adult who is not overweight having the disease. An adult is chosen at random from the population.

What is the probability the chosen adult has Type 2 diabetes?

(A) $\frac{27}{250}$

(B) $\frac{3}{25}$

(C) $\frac{63}{500}$

(D) $\frac{37}{250}$

(E) $\frac{39}{50}$

(F) $\frac{21}{100}$

Quiz 5

A bag contains 6 red and 6 green sweets. The sweets are identical apart from their colour. A child takes a sweet at random from the bag. If the sweet is red, the child stops taking sweets. If the sweet is green, it is not replaced and the child takes another sweet. This continues until a red sweet is taken at which point the child stops taking sweets.

What is the probability that the child takes more green sweets than red sweets?

(A) $\frac{3}{22}$

(B) $\frac{5}{22}$

(C) $\frac{3}{11}$

(D) $\frac{1}{2}$

(E) $\frac{8}{11}$

(F) $\frac{17}{22}$

Quiz 6

Ivana has two identical dice and on the faces of each are the numbers -3, -2, -1, 0, 1, 2. If she throws her dice and multiplies the results, what is the probability that their product is negative?

(A) $\frac{1}{4}$

(B) $\frac{11}{36}$

(C) $\frac{1}{3}$

(D) $\frac{13}{36}$

(E) $\frac{1}{2}$

Quiz 7

A train arriving at Edinburgh has 12 passengers.

The passengers got on the train at three different stations:

- 5 at Peterborough
- 4 at Newark
- 3 at York

The passengers leave the train one at a time in a random order.

What is the probability that the first three to leave did not all get on the train at the same station?

(A) $\frac{3}{11}$

(B) $\frac{41}{44}$

(C) $\frac{103}{110}$

(D) $\frac{19}{20}$

(E) $\frac{21}{22}$

(F) $\frac{43}{44}$

(G) $\frac{54}{55}$

(H) $\frac{219}{220}$

Quiz 8

Jerry starts at 0 on the real number line. He tosses a fair coin 8 times. When he gets heads, he moves 1 unit in the positive direction; when he gets tails, he moves 1 unit in the negative direction. The probability that he reaches 4 at some time during this process is $\frac{a}{b}$, where a and b are relatively prime positive integers. What is $a+b$? (For example, he succeeds if his sequence of tosses is HTHHHHHH.)

(A) 69

(B) 151

(C) 257

(D) 293

(E) 313

Quiz 9

Alice, Bob, and Carol play a game in which each of them chooses a real number between 0 and 1. The winner of the game is the one whose number is between the numbers chosen by the other two players. Alice announces that she will choose her number uniformly at random from all the numbers between 0 and 1, and Bob announces that he will choose his number uniformly at random from all the numbers between $\frac{1}{2}$ and $\frac{2}{3}$. Armed with this information, what number should Carol choose to maximize her chance of winning?

(A) $\frac{1}{2}$

(B) $\frac{13}{24}$

(C) $\frac{7}{12}$

(D) $\frac{5}{8}$

(E) $\frac{2}{3}$

Ex. 13

Find the sum of those numbers between 1000 and 6000 every one of whose digits is one of the numbers 0, 2, 5 or 7, giving your answer as a product of primes.