Please check the examination details below before entering your candidate information

Candidate surname | Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Tuesday 6 June 2023

Afternoon (Time: 2 hours) | **Paper reference** **9MA0/01**

Mathematics
Advanced
PAPER 1: Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

1. Find

$$\int \frac{x^{\frac{1}{2}}(2x-5)}{3} \, dx$$

writing each term in simplest form.

(4)

$$x^{1/2}(2x-5) = 2x^{3/2} - 5x^{1/2} \quad ① \quad \leftarrow x^a \times x^b = x^{a+b}$$

$$\int \frac{2}{3}x^{3/2} - \frac{5}{3}x^{1/2} \, dx \quad ①$$

$$= \frac{2}{5} \times \frac{2}{3}x^{5/2} - \frac{2}{3} \times \frac{5}{3}x^{3/2} + C \quad ①$$

$$= \frac{4}{15}x^{5/2} - \frac{10}{9}x^{3/2} + C \quad ①$$

Question 1 continued

(Total for Question 1 is 4 marks)

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 4x^3 + 5x^2 - 10x + 4a \qquad x \in \mathbb{R}$$

where *a* is a positive constant.

Given $(x - a)$ is a factor of $f(x)$,

(a) show that

$$a(4a^2 + 5a - 6) = 0$$

(2)

(b) Hence

   (i) find the value of *a*

   (ii) use algebra to find the exact solutions of the equation

$$f(x) = 3$$

(4)

a) If $(x-a)$ is a factor of $f(x)$, $f(a) = 0$

$f(a) = 4a^3 + 5a^2 - 10a + 4a = 0$ ①

$\qquad 4a^3 + 5a^2 - 6a = 0$

$\qquad a(4a^2 + 5a - 6) = 0$ ①

b)i) $a = 0 \qquad$ or $\qquad 4a^2 + 5a - 6 = 0 \quad \leftarrow$ to make $f(a) = 0$

$\qquad\qquad\qquad (4a - 3)(a + 2) = 0$

$\qquad\qquad\qquad a = \dfrac{3}{4}$ or $a = -2$

$\qquad a > 0$ so $a = \dfrac{3}{4}$ ①

ii)

$\qquad\qquad$ set $= 3$ as the question wants $f(x) = 3$

$f(x) = 4x^3 + 5x^2 - 10x + 4\left(\dfrac{3}{4}\right) = 3$

$\qquad 4x^3 + 5x^2 - 10x = 0$ ①

$\qquad x(4x^2 + 5x - 10) = 0$

Question 2 continued

$x = 0$ ① or $4x^2 + 5x - 10 = 0$

$$x = \frac{-5 \pm \sqrt{185}}{8}$$ ①

(Total for Question 2 is 6 marks)

P 7 2 8 0 4 A 0 5 4 4

3. Relative to a fixed origin O

- the point A has position vector $5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$

- the point B has position vector $2\mathbf{i} + 4\mathbf{j} + a\mathbf{k}$

where a is a positive integer.

(a) Show that $|\overrightarrow{OA}| = \sqrt{38}$

(1)

(b) Find the smallest value of a for which

$$|\overrightarrow{OB}| > |\overrightarrow{OA}|$$

(2)

a) $|\overrightarrow{OA}| = \sqrt{5^2 + 3^2 + 2^2} = \sqrt{38}$ ①

b) $|\overrightarrow{OB}| = \sqrt{2^2 + 4^2 + a^2} = \sqrt{20 + a^2}$

when $a = 4$, $|\overrightarrow{OB}| = \sqrt{36} < \sqrt{38}$

when $a = 5$, $|\overrightarrow{OB}| = \sqrt{45} > \sqrt{38}$ ①

$\therefore a = 5$ ①

Question 3 continued

(Total for Question 3 is 3 marks)

4. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = f(x)$ where $x \in \mathbb{R}$

Given that

- $f'(x) = 2x + \dfrac{1}{2}\cos x$

- the curve has a stationary point with x coordinate α

- α is small

(a) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)

The point $P(0, 3)$ lies on C

(b) Find the equation of the tangent to the curve at P, giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(2)

a) $f'(\alpha) = 0$

↙ small angle approximation for $\cos\alpha$

$\therefore \ 2\alpha + \dfrac{1}{2}\left(1 - \dfrac{\alpha^2}{2}\right) = 0$ ①

$\left.\begin{array}{c} \\ \end{array}\right\rangle \times 2$

$4\alpha + 1 - \dfrac{\alpha^2}{2} = 0$

$\left.\begin{array}{c} \\ \end{array}\right\rangle \times -2$

$\alpha^2 - 8\alpha - 2 = 0$ ①

$\alpha = 8.243$ or $\alpha = -0.243$

choose $\alpha = -0.243$ as α is small. ①

b) $P(0, 3)$

$f'(0) = \dfrac{1}{2}\cos 0 = \dfrac{1}{2}$ ← gradient (m) when $x = 0$.

$y - 3 = \dfrac{1}{2}(x - 0)$ ① ← uses $y - y_1 = m(x - x_1)$ with point P.

$y = \dfrac{1}{2}x + 3$ ①

P 7 2 8 0 4 A 0 8 4 4

Question 4 continued

(Total for Question 4 is 5 marks)

5. A continuous curve has equation $y = \text{f}(x)$.

The table shows corresponding values of x and y for this curve, where a and b are constants.

$h = 4 - 3.8$
$= 0.2$ ①

x	3	3.2	3.4	3.6	3.8	4
y	a	16.8	b	20.2	18.7	13.5

The trapezium rule is used, with all the y values in the table, to find an approximate area under the curve between $x = 3$ and $x = 4$

Given that this area is 17.59

(a) show that $a + 2b = 51$

$$\int_0^{\wedge} \text{f}(x)\,dx = \tfrac{1}{2}h\left[(y_0 + y_n) + 2(y_1 + \ldots + y_{n-1})\right]$$
where h = interval size

(3)

Given also that the sum of all the y values in the table is 97.2

(b) find the value of a and the value of b

(3)

a) $\dfrac{0.2}{2}\left(a + 13.5 + 2[16.8 + b + 20.2 + 18.7]\right) = 17.59$ ①

$\quad a + 13.5 + 2(b + 55.7) = 175.9$

$\quad a + 2b + 111.4 + 13.5 = 175.9$

$\qquad a + 2b = 51$ ① ①

b) $a + 16.8 + b + 20.2 + 18.7 + 13.5 = 97.2$ ①
$\qquad a + b = 28$ ②

solve ① and ② simultaneously using calculator

$a = 5$ ① $b = 23$ ①

handwritten method:
① $a + 2b = 51$
② $a + b = 28$
② − ① ⟹ $b = 23$
$a + b = 28 \Rightarrow a = 28 - b$
$\qquad\qquad\qquad = 28 - 23$
$\qquad\qquad\qquad = 5$

Question 5 continued

(Total for Question 5 is 6 marks)

Turn over ▶

6. $\qquad a = \log_2 x \qquad b = \log_2(x + 8)$

Express in terms of a and/or b

(a) $\log_2 \sqrt{x}$

(1)

(b) $\log_2(x^2 + 8x)$

(2)

(c) $\log_2\left(8 + \dfrac{64}{x}\right)$

Give your answer in simplest form.

(3)

$\log a^b = b \log a$

a) $\log_2 \sqrt{x} = \log_2 x^{1/2} = \frac{1}{2}\log_2 x = \frac{1}{2}a$ ①

b) $\log_2(x^2 + 8x) = \log_2(x(x+8)) = \log_2 x + \log_2(x+8)$ ①
$\qquad\qquad\qquad\qquad\qquad = a + b$ ①

c) $8 + \dfrac{64}{x} = \dfrac{8}{x}(x+8)$ ①

$\log(ab) = \log a + \log b$

$\log_2\left(8 + \dfrac{64}{x}\right) = \log_2\left(\dfrac{8}{x}(x+8)\right)$

$\qquad\qquad = \log_2 8 + \log_2(x+8) - \log_2 x$ ①

$\qquad\qquad = 3 + b - a$ ①

$\log\dfrac{a}{b} = \log a - \log b$

Question 6 continued

(Total for Question 6 is 6 marks)

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7. The function f is defined by

$$f(x) = 3 + \sqrt{x - 2} \qquad x \in \mathbb{R} \quad x > 2$$

(a) State the range of f

(1)

(b) Find f^{-1}

(3)

The function g is defined by

$$g(x) = \frac{15}{x - 3} \qquad x \in \mathbb{R} \quad x \neq 3$$

(c) Find gf(6)

(2)

(d) Find the exact value of the constant a for which

$$f(a^2 + 2) = g(a)$$

(2)

a) $\sqrt{x-2} > 0 \quad \forall x > 2 \quad \therefore \quad f(x) > 3$ ① $\forall$ means "for all"

b) $f(x) = 3 + \sqrt{x-2}$ } swap x and y,
rearrange for y

$x = 3 + \sqrt{y-2}$ ①

$x - 3 = \sqrt{y-2}$

$y - 2 = (x-3)^2$

$y = (x-3)^2 + 2$

range of f is the
domain of f^{-1}

$f^{-1}(x) = (x-3)^2 + 2$ ① $, x > 3$ ①

c) $f(6) = 3 + \sqrt{6-2} = 3 + 2 = 5$ ①

$g(5) = \frac{15}{5-3} = \frac{15}{2}$ ①

Question 7 continued

d) $f(a^2 + 2) = g(a)$

$$3 + \sqrt{a^2 + 2 - 2} = \frac{15}{a - 3}$$

$$3 + a = \frac{15}{a - 3}$$

$$(a + 3)(a - 3) = 15$$

$$a^2 - 9 = 15 \quad ①$$

$$a^2 = 24$$

$$a = \pm 2\sqrt{6} \qquad a = -2\sqrt{6} \text{ doesn't work}$$
$$\qquad\qquad\qquad\qquad \text{in original equation}$$
$$a = 2\sqrt{6} \quad ①$$

P 7 2 8 0 4 A 0 1 5 4 4

Question 7 continued

P7 2804 A0 1 6 44

Question 7 continued

(Total for Question 7 is 8 marks)

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8.

FRONT OF STAGE
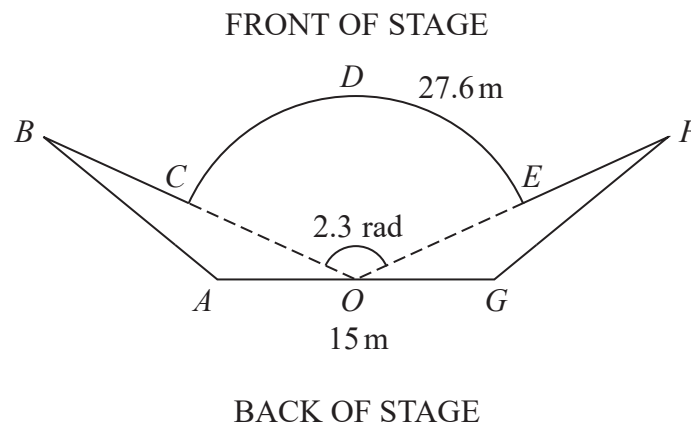

Diagram **NOT**
accurately drawn

BACK OF STAGE

Figure 1

Figure 1 shows the plan view of a stage.

The plan view shows two congruent triangles *ABO* and *GFO* joined to a sector *OCDEO* of a circle, centre *O*, where

- angle *COE* = 2.3 radians

- arc length *CDE* = 27.6 m

- *AOG* is a straight line of length 15 m

(a) Show that *OC* = 12 m.

(2)

(b) Show that the size of angle *AOB* is 0.421 radians correct to 3 decimal places.

(2)

Given that the total length of the front of the stage, *BCDEF*, is 35 m,

(c) find the total area of the stage, giving your answer to the nearest square metre.

(6)

a) arc length $CDE = r\theta = 27.6$ ①

   $\theta = 2.3$ ∴ $r = \dfrac{27.6}{2.3} = 12$ ①

   $r = OC$ ∴ $OC = 12$

b) $\angle AOG = \pi$ and $\angle AOB = \angle GOF$ so

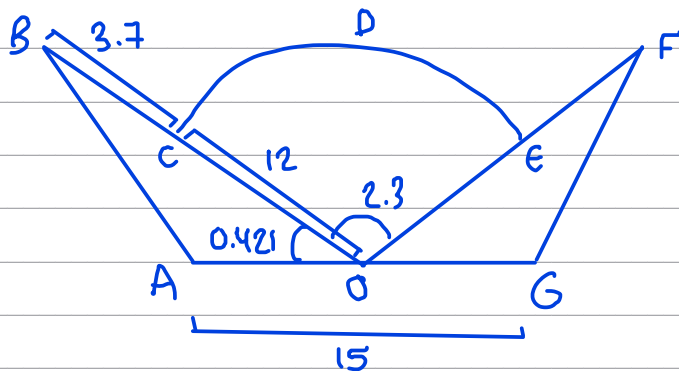 $\angle AOB = \dfrac{\pi - 2.3}{2}$ ① $= 0.421$ rad (3dp) ①

c) $BC = EF$

Question 8 continued

$\therefore 2BC + CDE = 35$

$2BC + 27.6 = 35$

$BC = \dfrac{35 - 27.6}{2} = 3.7 \quad \therefore OB = 3.7 + 12 = 15.7$ ①

Area $\triangle AOB = \dfrac{1}{2} \times \dfrac{15}{2} \times (3.7 + 12) \times \sin 0.421$ ① 　 using $\dfrac{1}{2}ab\sin C$

$= 24.04\ldots$

$= 24.1 \ (3sf) \ m^2$

Area $OCDE = \dfrac{1}{2} \times 12^2 \times 2.3 = 165.6 \ m^2$ ① ① 　 using $\dfrac{1}{2}r^2\theta$

Total area $= 165.6 + 2 \times 24.1$ ①

$= 213.69\ldots$

$= 214 \ m^2$ (nearest square metre) ①

P 7 2 8 0 4 A 0 1 9 4 4

Question 8 continued

P72804A02044

Question 8 continued

(Total for Question 8 is 10 marks)

Turn over ▶

9. The first three terms of a geometric sequence are

$$3k + 4 \qquad 12 - 3k \qquad k + 16$$

where k is a constant.

(a) Show that k satisfies the equation

$$3k^2 - 62k + 40 = 0$$

(2)

Given that the sequence converges,

(b) (i) find the value of k, giving a reason for your answer,

(ii) find the value of S_∞

(5)

a) $\dfrac{12-3k}{3k+4} = \dfrac{k+16}{12-3k}$ ① ← because scale between two terms should be the same.

$(12-3k)(12-3k) = (k+16)(3k+4)$

$144 - 72k + 9k^2 = 3k^2 + 52k + 64$

$6k^2 - 124k + 80 = 0$

$3k^2 - 62k + 40 = 0$ ①

b) $(k-20)(3k-2) = 0$
∴ $k = 20$ or $k = \dfrac{2}{3}$ ①

If $k = 20$, $r = \dfrac{12-3(20)}{3(20)+4} = -\dfrac{3}{4}$ $\left| -\dfrac{3}{4} \right| < 1$ ✓

If $k = \dfrac{2}{3}$, $r = \dfrac{12-3\left(\frac{2}{3}\right)}{3\left(\frac{2}{3}\right)+4} = \dfrac{5}{3}$

$\left| \dfrac{5}{3} \right| > 1$, so if $k = \dfrac{2}{3}$ the series cannot converge.

∴ $k = 20$ ①

Question 9 continued

$k = 20$ $\therefore a = 3(20) + 4 = 64$ ①

$r = -\dfrac{3}{4}$

$S_\infty = \dfrac{a}{1-r} = \dfrac{64}{1+\frac{3}{4}}$ ① $= \dfrac{256}{7}$ ①

(Total for Question 9 is 7 marks)

Turn over ▶

10. A circle C has equation

$$x^2 + y^2 + 6kx - 2ky + 7 = 0$$

where k is a constant.

(a) Find in terms of k,

    (i) the coordinates of the centre of C

    (ii) the radius of C

(3)

The line with equation $y = 2x - 1$ intersects C at 2 distinct points.

(b) Find the range of possible values of k.

(6)

a) $\quad x^2 + y^2 + 6kx - 2ky + 7 = 0$

$\quad\quad (x+3k)^2 - 9k^2 + (y-k)^2 - k^2 + 7 = 0$

$\quad\quad (x+3k)^2 + (y-k)^2 = 10k^2 - 7$ ①

(i) Centre $(-3k, k)$ ①

(ii) radius $\sqrt{10k^2 - 7}$ ①

b) sub $y = 2x - 1$ into C, then use $b^2 - 4ac > 0$

$\quad x^2 + (2x-1)^2 + 6kx - 2k(2x-1) + 7 = 0$

$\quad x^2 + 4x^2 - 4x + 1 + 6kx - 4kx + 2k + 7 = 0$ ①

$\quad 5x^2 + (2k-4)x + (2k+8) = 0$ ①

two distinct solutions, so "$b^2 - 4ac$" > 0

$\quad (2k-4)^2 - 4(5)(2k+8) > 0$ ①

$\quad 4k^2 - 16k + 16 - 40k - 160 > 0$

$\quad 4k^2 - 56k - 144 > 0$

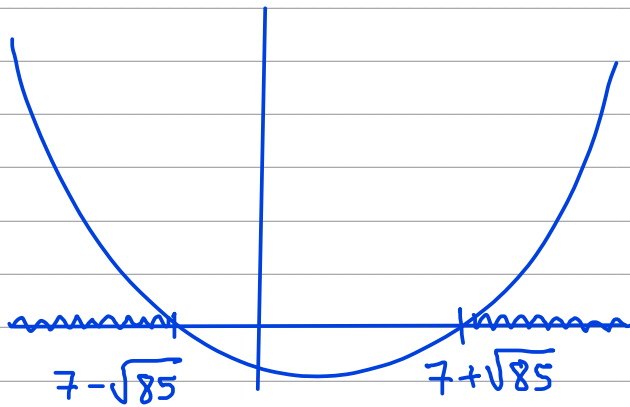**Question 10 continued**

$k^2 - 14k - 36 > 0$

critical values are solutions to $k^2 - 14k - 36 = 0$

$$k = 7 \pm \sqrt{85} \quad ①$$

mmm = want
this region

$7 - \sqrt{85} \qquad 7 + \sqrt{85}$

$k < 7 - \sqrt{85}$ or $k > 7 + \sqrt{85}$

$①$ $①$

Turn over ▶

Question 10 continued

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Question 10 continued

(Total for Question 10 is 9 marks)
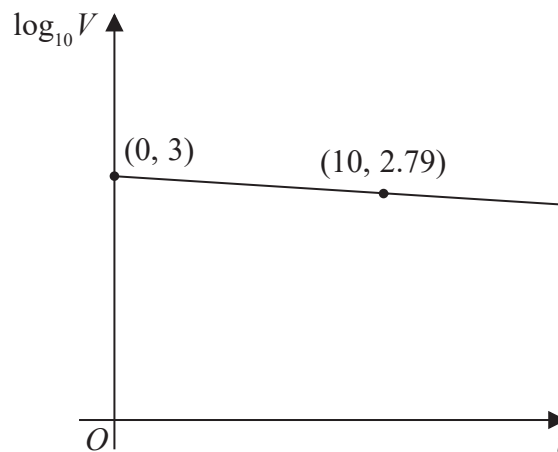
11.

Figure 2

The value, V pounds, of a mobile phone, t months after it was bought, is modelled by

$$V = ab^t$$

where a and b are constants.

Figure 2 shows the linear relationship between $\log_{10} V$ and t.

The line passes through the points $(0, 3)$ and $(10, 2.79)$ as shown.

Using these points,

(a) find the initial value of the phone,

(2)

(b) find a complete equation for V in terms of t, giving the exact value of a and giving the value of b to 3 significant figures.

(3)

Exactly 2 years after it was bought, the value of the phone was £320

(c) Use this information to evaluate the reliability of the model.

(2)

a) when $t=0$, $\log_{10} V = 3$ ①

$\therefore V = 10^3 = £1000$ ①

b) $V = ab^t$

$\log_{10} V = \log_{10} ab^t$

$= \log_{10} a + \log_{10} b^t$

$= \log_{10} a + t \log_{10} b$

Question 11 continued

$\therefore$ gradient $= \log_{10} b$ ①

$\log_{10} V$ intercept $= \log_{10} a$

$\log_{10} b = \dfrac{2.79 - 3}{10 - 0} = -0.021$ ①

$\therefore \log_{10} a = 3$

$a = 1000$

$b = 10^{-0.021} = 0.953 \ (3sf)$

$V = 1000(0.953)^t$ ①

c) 2 years = 24 months so sub in $t = 24$

$V = 1000(0.953)^{24} = £315 \ (3sf)$ ①

£315 is close to £320 so the model is suitable. ①

P 7 2 8 0 4 A 0 2 9 4 4

Question 11 continued

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Question 11 continued

(Total for Question 11 is 7 marks)

Turn over ▶

12.
$$y = \sin x$$

where x is measured in radians.

Use differentiation from first principles to show that

$$\frac{dy}{dx} = \cos x$$

You may

- use without proof the formula for $\sin(A \pm B)$

- assume that as $h \to 0$, $\dfrac{\sin h}{h} \to 1$ and $\dfrac{\cos h - 1}{h} \to 0$

(5)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

let $f(x) = \sin x$

$$f'(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h} \quad \text{①}$$

$$= \lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h} \quad \text{①} \quad \text{①}$$

$$= \lim_{h \to 0} \left[\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \right]$$

using $\dfrac{\sin h}{h} \to 1$ and $\dfrac{\cos h - 1}{h} \to 0$ as $h \to 0$

$$= \sin x (0) + \cos x (1) \quad \text{①}$$

$$= \cos x$$

so $\dfrac{dy}{dx} = \cos x \quad \text{①}$

Question 12 continued

(Total for Question 12 is 5 marks)

P72804A03344

13. On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 60 m

- a carriage starts a circuit at a vertical height of 2 m above the ground

- the ground is horizontal

The vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 20)^2$$

where a and b are positive constants.

(a) Find a complete equation for the model.

(3)

(b) Use the model to determine the height of the carriage above the ground when $t = 40$

(1)

In an alternative model, the vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is given by

$$H = 29\cos(9t + \alpha)° + \beta \qquad 0 \leqslant \alpha < 360°$$

where α and β are constants.

(c) Find a complete equation for the alternative model.

(2)

Given that the carriage moves continuously for 2 minutes,

(d) give a reason why the alternative model would be more appropriate.

(1)

a) $H = a - b(t-20)^2$

given:
  1. turning point is when $H=60$
  2. when $t=0$, $H=2$

turning point has $H=60$ and $t=20$ ∴ $60 = a + b(20-20)$
$a = 60$ ①

sub in $t=0$, $h=2$:   ①

$2 = 60 - b(0-20)^2 ⇒ 2 = 60 - 400b$

Question 13 continued

$$400b = 58$$
$$b = 0.145$$

$$H = 60 - 0.145(t-20)^2 \quad ①$$

b) sub in $t=40$

$$H = 60 - 0.145(40-20)^2$$
$$= 2m \quad ①$$

c) $H = 29\cos(9t + \alpha) + \beta$

$$\frac{dH}{dt} = -261\sin(9t+\alpha) = 0 \text{ when } t=20$$

$$\therefore \sin(180+\alpha) = 0$$

$$180+\alpha = 0 \Rightarrow \alpha = -180°, \text{ out of range}$$
$$180+\alpha = 360 \Rightarrow \alpha = 180°, \text{ in range} \quad ①$$

$$H = 29\cos(9t+180) + \beta \qquad (0 \le \alpha < 360)$$

sub in $t=0, H=2$

$$2 = 29\cos 180 + \beta$$
$$2 = -29 + \beta$$
$$\beta = 31$$

$$H = 29\cos(9t+180) + 31 \quad ①$$

d) The alternative model allows for more than one circuit ①

DO NOT WRITE IN THIS AREA

Question 13 continued

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Question 13 continued

(Total for Question 13 is 7 marks)

P 7 2 8 0 4 A 0 3 7 4 4

14. Prove, using algebra, that

$$(n + 1)^3 - n^3$$

is odd for all $n \in \mathbb{N}$

(4)

case 1: n is even
let n=2k , k ∈ ℤ

$(2k+1)^3 - (2k)^3 = 8k^3 + 12k^2 + 6k + 1 - 8k^3$ ①
$= 12k^2 + 6k + 1$
$= 2(6k^2 + 3k) + 1$
which is odd ① ← because 2n is even by definition, so 2n+1 is odd

case 2: n is odd
n = 2k+1 , k ∈ ℤ

$(2k+2)^3 - (2k+1)^3 = 8k^3 + 24k^2 + 24k + 8 - (8k^3 + 12k^2 + 6k + 1)$ ①
$= 12k^2 + 18k + 7$
$= 2(6k^2 + 9k + 3) + 1$

which is odd

Hence odd for all n ∈ ℕ ①

Question 14 continued

(Total for Question 14 is 4 marks)

P72804A03944

15. A curve has equation $y = \text{f}(x)$, where

$$\text{f}(x) = \frac{7xe^x}{\sqrt{e^{3x} - 2}} \qquad x > \ln\sqrt[3]{2}$$

(a) Show that

$$\text{f}'(x) = \frac{7e^x(e^{3x}(2 - x) + Ax + B)}{2(e^{3x} - 2)^{\frac{3}{2}}}$$

where A and B are constants to be found.

(5)

(b) Hence show that the x coordinates of the turning points of the curve are solutions of the equation

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4}$$

(2)

The equation $x = \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ has two positive roots α and β where $\beta > \alpha$

A student uses the iteration formula

$$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

in an attempt to find approximations for α and β

Diagram 1 shows a plot of part of the curve with equation $y = \dfrac{2e^{3x} - 4}{e^{3x} + 4}$ and part of the line with equation $y = x$

Using Diagram 1 on page 42

(c) draw a staircase diagram to show that the iteration formula starting with $x_1 = 1$ can be used to find an approximation for β

(1)

Use the iteration formula with $x_1 = 1$, to find, to 3 decimal places,

(d) (i) the value of x_2

    (ii) the value of β

(3)

Using a suitable interval and a suitable function that should be stated

(e) show that $\alpha = 0.432$ to 3 decimal places.

(2)

Question 15 continued

a) $f(x) = \dfrac{7xe^x}{(e^{3x}-2)^{1/2}}$

$\dfrac{d}{dx}(7xe^x):$ let $u = 7x \qquad v = e^x$

$\qquad\qquad\qquad \dfrac{du}{dx} = 7 \qquad \dfrac{dv}{dx} = e^x$

$\dfrac{du}{dx}v + \dfrac{dv}{dx}u = 7e^x + 7xe^x$

$\dfrac{d}{dx}\left((e^{3x}-2)^{1/2}\right) = \dfrac{1}{2} \times 3e^{3x} \times (e^{3x}-2)^{-1/2}$

$\qquad\qquad\qquad\qquad = \dfrac{3}{2}e^{3x}(e^{3x}-2)^{-1/2}$

$f(x) = \dfrac{7xe^x}{(e^{3x}-2)^{1/2}}$ using quotient rule

$f'(x) = \dfrac{(e^{3x}-2)^{1/2}(7e^x + 7xe^x) - 7xe^x\left(\frac{3}{2}e^{3x}(e^{3x}-2)^{-1/2}\right)}{e^{3x}-2}$

$\qquad = \dfrac{7(e^{3x}-2)^{-1/2}\left[e^x(e^{3x}-2)(1+x) - \frac{3}{2}xe^xe^{3x}\right]}{e^{3x}-2}$

factoring out $7(e^{3x}-2)^{-1/2}$

$\qquad = \dfrac{7e^x\left[(e^{3x}-2)(1+x) - \frac{3}{2}xe^{3x}\right]}{(e^{3x}-2)^{3/2}}$

factoring out e^x

moving $(e^{3x}-2)^{-1/2}$ to the denominator

Turn over ▶

Question 15 continued

Only use the copy of Diagram 1 if you need to redraw your answer to part (c).
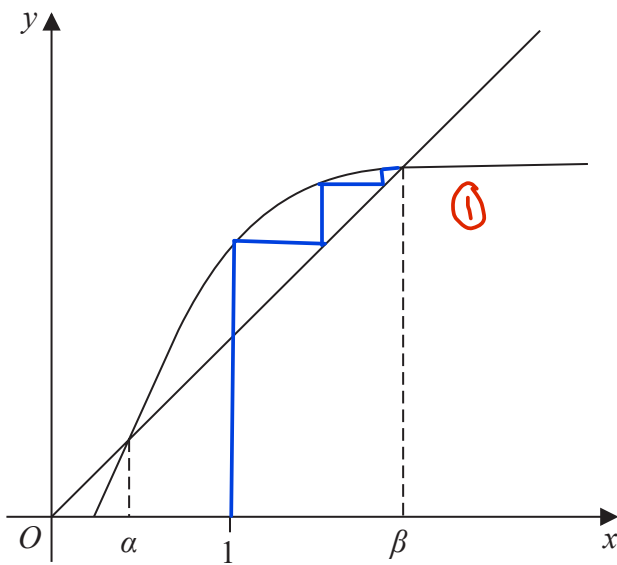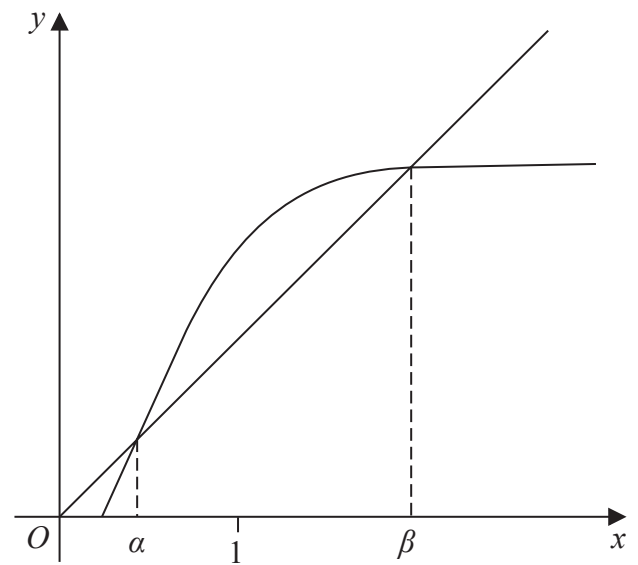

Diagram 1   **copy of Diagram 1**

$$f'(x) = \frac{7e^x\left[e^{3x} + xe^{3x} - 2 - 2x - \frac{3}{2}xe^{3x}\right]}{(e^{3x} - 2)^{3/2}}$$

expanding bracket

$$= \frac{7e^x\left[e^{3x} - \frac{1}{2}xe^{3x} - 2x - 2\right]}{(e^{3x} - 2)^{3/2}}$$

collecting like terms

$$= \frac{7e^x\left(2e^{3x} - xe^{3x} - 4x - 4\right)}{2(e^{3x} - 2)^{3/2}}$$

multiplying top and bottom by 2

$$= \frac{7e^x\left(e^{3x}(2 - x) - 4x - 4\right)}{2(e^{3x} - 2)^{3/2}}$$ ①   *as required.*

P 7 2 8 0 4 A 0 4 2 4 4

Question 15 continued

b) turning points have $f'(x) = 0$

$$\Rightarrow \frac{7e^x \left(e^{3x}(2-x) - 4x - 4\right)}{2(e^{3x}-2)^{3/2}} = 0$$

$e^x \neq 0$,
multiply by
$2\left(e^{3x}-2\right)^{3/2}$

$$e^{3x}(2-x) - 4x - 4 = 0$$

$$2e^{3x} - xe^{3x} - 4x - 4 = 0$$

$$x(e^{3x} + 4) = 2e^{3x} - 4 \quad ①$$

$$x = \frac{2e^{3x} - 4}{e^{3x} + 4} \quad ①$$

c) drawn on diagram

d) $$x_{n+1} = \frac{2e^{3x_n} - 4}{e^{3x_n} + 4}$$

(i) $$x_2 = \frac{2e^3 - 4}{e^3 + 4} = 1.50177\ldots \quad ①$$
$$= 1.502 \ (3dp) \quad ①$$

(ii) $\beta = 1.96757\ldots$
$\quad = 1.968 \ (3dp) \quad ①$

e) α is a solution of $x = \dfrac{2e^{3x} - 4}{e^{3x} + 4}$

$\therefore$ a solution of $\dfrac{2e^{3x} - 4}{e^{3x} + 4} - x = 0$

Turn over ▶

Question 15 continued

so define $h(x) = \dfrac{2e^{3x} - 4}{e^{3x} + 4} - x$, so $h(\alpha) = 0$.

$h(0.4315) = -0.000297\ldots < 0$

$h(0.4325) = 0.000947 > 0$ ①

- since there is a change of sign
- and $h(x)$ is continuous
- $\alpha = 0.432$ (to 3dp) ①

(Total for Question 15 is 13 marks)

TOTAL FOR PAPER IS 100 MARKS