Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Wednesday 6 October 2021 – Afternoon

Time 2 hours

Paper reference **9MA0/01**

Mathematics

Advanced
PAPER 1: Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question*.

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

1. $$f(x) = ax^3 + 10x^2 - 3ax - 4$$

Given that $(x - 1)$ is a factor of $f(x)$, find the value of the constant a.

You must make your method clear.

(3)

If $(x-1)$ is a factor of $f(x)$, $f(1)=0$

$f(1) = a(1)^3 + 10(1)^2 - 3a(1) - 4 = 0$ 1

$a + 10 - 3a - 4 = 0$

$-2a + 6 = 0$ 1

$2a = 6$

$a = 3$ 1

Question 1 continued

(Total for Question 1 is 3 marks)

2. Given that

$$f(x) = x^2 - 4x + 5 \qquad x \in \mathbb{R}$$

(a) express f(x) in the form $(x + a)^2 + b$ where a and b are integers to be found.

(2)

The curve with equation $y = f(x)$

- meets the y-axis at the point P
- has a minimum turning point at the point Q

(b) Write down

(i) the coordinates of P

(ii) the coordinates of Q

(2)

a) $f(x) = x^2 - 4x + 5$
$= (x-2)^2 - 4 + 5$ 1
$= (x-2)^2 + 1$ 1

b) meets y-axis when $x = 0$
$f(0) = (0-2)^2 + 1 = 4 + 1 = 5$

(i) $P(0,5)$ 1

minimum turning point:
min $f(x) = (x-2)^2 + 1$
happens when $(x-2)^2 = 0 \Rightarrow x = 2$
$f(2) = 1$

(ii) $Q(2,1)$ 1

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Question 2 continued

(Total for Question 2 is 4 marks)

3. The sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = k - \frac{24}{u_n} \qquad u_1 = 2$$

where k is an integer.

Given that $u_1 + 2u_2 + u_3 = 0$

(a) show that

$$3k^2 - 58k + 240 = 0$$

(3)

(b) Find the value of k, giving a reason for your answer.

(2)

(c) Find the value of u_3

(1)

a) $u_1 = 2 \qquad u_2 = k - \frac{24}{u_1} \qquad u_3 = k - \frac{24}{u_2}$

$u_2 = k - \frac{24}{2} \qquad u_3 = k - \frac{24}{k-12}$ 1

$u_2 = k - 12$

$u_1 + 2u_2 + u_3 = 0$

sub in known values of u_1, u_2, u_3

$2 + 2(k-12) + k - \frac{24}{k-12} = 0$ 1

$3k - 22 - \frac{24}{k-12} = 0$

$\times (k-12)$

$(k-12)(3k-22) - 24 = 0$

$3k^2 - 22k - 36k + 264 - 24 = 0$

$3k^2 - 58k + 240 = 0$ 1 as required

P 6 8 7 3 1 A 0 6 5 2

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Question 3 continued

b) we have $3k^2 - 58k + 240 = 0$

$(3k - 40)(k - 6) = 0$

$k = \frac{40}{3}$ or $k = 6$ (1)

choose $k=6$ because k is an integer (1)

c) $U_3 = k - \frac{24}{k-12}$

$U_3 = 6 - \frac{24}{6-12}$

$U_3 = 6 + 4$

$U_3 = 10$ (1)

(Total for Question 3 is 6 marks)

4. The curve with equation $y = f(x)$ where

$$f(x) = x^2 + \ln(2x^2 - 4x + 5)$$

has a single turning point at $x = \alpha$

(a) Show that α is a solution of the equation

$$2x^3 - 4x^2 + 7x - 2 = 0$$

(4)

The iterative formula

$$x_{n+1} = \frac{1}{7}(2 + 4x_n^2 - 2x_n^3)$$

is used to find an approximate value for α.

Starting with $x_1 = 0.3$

(b) calculate, giving each answer to 4 decimal places,

(i) the value of x_2

(ii) the value of x_4

(3)

Using a suitable interval and a suitable function that should be stated,

(c) show that α is 0.341 to 3 decimal places.

(2)

a) $f(x) = x^2 + \ln(2x^2 - 4x + 5)$

$f'(x) = 2x + \dfrac{4x-4}{2x^2-4x+5}$ (2)

turning point has $f'(x) = 0$

$2x + \dfrac{4x-4}{2x^2-4x+5} = 0$ $\times(2x^2-4x+5)$

$2x(2x^2-4x+5) + 4x - 4 = 0$ (1)

$4x^3 - 8x^2 + 14x - 4 = 0$

$2x^3 - 4x^2 + 7x - 2 = 0$ (1)

Question 4 continued

b) $x_{n+1} = \frac{1}{7}(2 + 4x_n^2 - 2x_n^3)$

$x_1 = 0.3$

$x_2 = \frac{1}{7}(2 + 4(0.3)^2 - 2(0.3)^3)$ 1

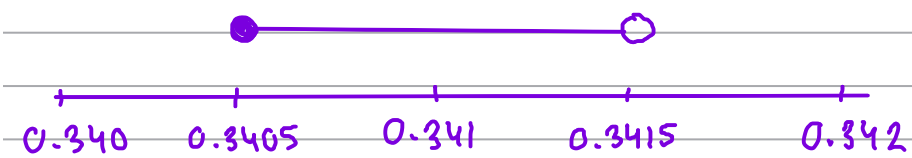
(i) $x_2 = 0.329428...$
$= 0.3294$ (4dp) 1

(ii) $x_4 = 0.339823...$
$= 0.3398$ (4dp) 1

c) let $h(x) = 2x^3 - 4x^2 + 7x - 2$
$h(x) = 0$ represents a turning point of $f(x)$.
want to show that $\alpha = 0.341$ to 3dp.

$h(0.3415) = 0.00366... > 0$
$h(0.3405) = -0.00130... < 0$ 1

- since there is a change in sign
- and $f'(x)$ is a continuous function
- $\alpha = 0.341$ to 3dp 1



Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A company made a profit of £20 000 in its first year of trading, Year 1

A model for future trading predicts that the yearly profit will increase by 8% each year, so that the yearly profits will form a geometric sequence.

According to the model,

(a) show that the profit for Year 3 will be £23 328

(1)

(b) find the first year when the yearly profit will exceed £65 000

(3)

(c) find the total profit for the first 20 years of trading, giving your answer to the nearest £1000

(2)

$a = 20{,}000 \quad r = 1.08 \quad u_n = ar^{n-1}$

a) Year 3 $= u_3$

$u_3 = 20{,}000 \times (1.08)^2 = £23{,}328$ 1

b) find n such that $u_n > 65{,}000$

$20{,}000(1.08)^{n-1} > 65{,}000$ 1

$(1.08)^{n-1} > \frac{13}{4}$

$n-1 > \frac{\log(13/4)}{\log(1.08)}$ 1

$n > 16.31\ldots$

n is an integer so $n = 17$.

Profits first exceed £65,000 in Year 17 1

c) $S_{20} = \frac{20{,}000[1-(1.08)^{20}]}{1-1.08}$ 1 $= £915{,}000$ 1

(nearest 1000)

Question 5 continued

(Total for Question 5 is 6 marks)

6.

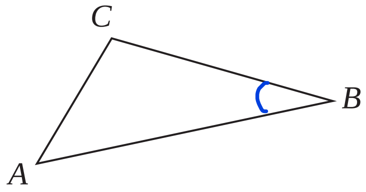


Figure 1

Figure 1 shows a sketch of triangle ABC.

Given that

- $\overrightarrow{AB} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$
- $\overrightarrow{BC} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$

(a) find $\overrightarrow{AC}$

(2)

(b) show that $\cos ABC = \dfrac{9}{10}$

(3)

a) $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \begin{pmatrix}-3\\-4\\-5\end{pmatrix} + \begin{pmatrix}1\\1\\4\end{pmatrix} = \begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ 1

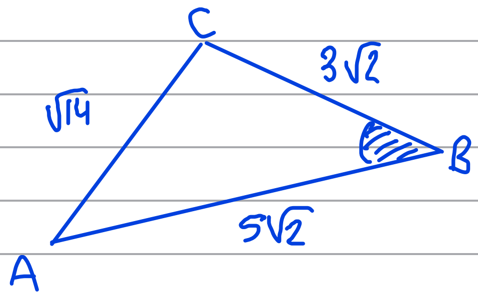
$\overrightarrow{AC} = -2\underline{i} - 3\underline{j} - \underline{k}$ 1

b) $|AC| = \sqrt{(-2)^2 + (-3)^2 + (-1)^2} = \sqrt{14}$

cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

$|AB| = \sqrt{(-3)^2 + (-4)^2 + (-5)^2} = 5\sqrt{2}$

$|BC| = \sqrt{1^2 + 1^2 + 4^2} = 3\sqrt{2}$ 1



$\cos ABC = \dfrac{(5\sqrt{2})^2 + (3\sqrt{2})^2 - (\sqrt{14})^2}{2 \times 5\sqrt{2} \times 3\sqrt{2}}$ 1

$\cos ABC = \dfrac{9}{10}$ 1

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 5 marks)

7. The circle C has equation

$$x^2 + y^2 - 10x + 4y + 11 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C, giving your answer as a simplified surd.

(4)

The line l has equation $y = 3x + k$ where k is a constant.

Given that l is a tangent to C,

(b) find the possible values of k, giving your answers as simplified surds.

(5)

a) $C: x^2+y^2-10x+4y+11=0$ complete the square on x and y

$(x-5)^2-25+(y+2)^2-4+11=0$ 1

$(x-5)^2+(y+2)^2=18$ 1

centre: $(5,-2)$ 1 radius: $\sqrt{18}=3\sqrt{2}$ 1 simplified surd

b) $L: y=3x+k$

L is tangent to C, so they have only 1 point of intersection.
Method: sub $y=3x+k$ into C to form a quadratic in x, then set the discriminant equal to 0.

sub L into C: $x^2+(3x+k)^2-10x+4(3x+k)+11=0$ 1

$x^2+9x^2+6xk+k^2-10x+12x+4k+11=0$

$10x^2+(6k+2)x+(k^2+4k+11)=0$ 1

Question 7 continued

$10x^2 + (6k+2)x + (k^2 + 4k + 11) = 0$

This quadratic has only 1 solution, so discriminant = 0.

"$b^2 - 4ac$" $= 0$

$(6k+2)^2 - 4 \times 10 \times (k^2 + 4k + 11) = 0$ 1

$36k^2 + 24k + 4 - 40k^2 - 160k - 440 = 0$

$-4k^2 - 136k - 436 = 0$

$4k^2 + 136k + 436 = 0$ 1

using calculator

$k = -17 \pm 6\sqrt{5}$ 1

Question 7 continued

Question 7 continued

(Total for Question 7 is 9 marks)

8. A scientist is studying the growth of two different populations of bacteria.

The number of bacteria, N, in the **first** population is modelled by the equation

$$N = Ae^{kt} \qquad t \geqslant 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 1000 bacteria in this population at the start of the study
- it took exactly 5 hours from the start of the study for this population to double

(a) find a complete equation for the model.

(4)

(b) Hence find the rate of increase in the number of bacteria in this population exactly 8 hours from the start of the study. Give your answer to 2 significant figures.

(2)

The number of bacteria, M, in the **second** population is modelled by the equation

$$M = 500e^{1.4kt} \qquad t \geqslant 0$$

where k has the value found in part (a) and t is the time in hours from the start of the study.

Given that T hours after the start of the study, the number of bacteria in the two different populations was the same,

(c) find the value of T.

(3)

a) $N = Ae^{kt}$

given: at $t=0$, $N=1{,}000$
at $t=5$, $N=2{,}000$

sub in $t=0$, $N=1{,}000$ to find A:

$1000 = Ae^0 \qquad e^0 = 1$
$A = 1000$ (1)

sub in $t=5$, $N=2{,}000$ to find k

$2{,}000 = 1000e^{5k}$
$e^{5k} = 2$ (1)
$5k = \ln 2$
$k = \frac{1}{5}\ln 2$ (1)

$A = 1000$
$k = \frac{1}{5}\ln 2$

so $N = 1{,}000e^{\frac{t}{5}\ln 2}$ (1)

P 6 8 7 3 1 A 0 2 2 5 2

Question 8 continued

b) $N = 1000e^{\frac{t}{5}\ln 2}$

differentiate N to find rate of increase:

$$\frac{dN}{dt} = \left(\frac{1}{5}\ln 2\right) \times 1000e^{\frac{t}{5}\ln 2}$$

$$\frac{dN}{dt} = 200\ln 2 \times e^{\frac{t}{5}\ln 2}$$

$$\left.\frac{dN}{dt}\right|_{t=8} = 200\ln 2 \times e^{\frac{8}{5}\ln 2} \quad 1 \quad = 420.245\ldots$$
$$= 420 \text{ (2sf)} \quad 1$$

c) $M = 500e^{1.4kt}$ $\quad N = 1000e^{kt}$ $\quad \left(k = \frac{1}{5}\ln 2\right)$

Populations same when $t = T$

$$500e^{1.4kT} = 1000e^{kT} \quad 1$$

$$\frac{e^{1.4kT}}{e^{kT}} = \frac{1000}{500}$$

$$e^{0.4kT} = 2$$

$$0.4kT = \ln 2 \quad 1$$

$$T = \frac{\ln 2}{0.4k} = \frac{25}{2} = 12.5 \text{ hours} \quad 1$$

Question 8 continued

Question 8 continued

(Total for Question 8 is 9 marks)

9.

$$f(x) = \frac{50x^2 + 38x + 9}{(5x+2)^2(1-2x)} \qquad x \neq -\frac{2}{5} \quad x \neq \frac{1}{2}$$

Given that f(x) can be expressed in the form

$$\frac{A}{5x+2} + \frac{B}{(5x+2)^2} + \frac{C}{1-2x}$$

where A, B and C are constants

(a) (i) find the value of B and the value of C

(ii) show that $A = 0$

(4)

(b) (i) Use binomial expansions to show that, in ascending powers of x

$$f(x) = p + qx + rx^2 + \dots$$

where p, q and r are simplified fractions to be found.

(ii) Find the range of values of x for which this expansion is valid.

(7)

a) $f(x) = \dfrac{50x^2+38x+9}{(5x+2)^2(1-2x)}$

i) $\dfrac{50x^2+38x+9}{(5x+2)^2(1-2x)} = \dfrac{A}{5x+2} + \dfrac{B}{(5x+2)^2} + \dfrac{C}{1-2x}$

$50x^2+38x+9 = A(5x+2)(1-2x) + B(1-2x) + C(5x+2)^2$

multiplied by $(5x+2)^2(1-2x)$ ①

sub in $x = \frac{1}{2}$:

$50\left(\frac{1}{2}\right)^2 + 38\left(\frac{1}{2}\right) + 9 = C\left(5\left(\frac{1}{2}\right)+2\right)^2$

$\frac{81}{2} = \frac{81}{4}C$

$C = 2$

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Question 9 continued

$50x^2+38x+9 = A(5x+2)(1-2x)+B(1-2x)+C(5x+2)^2$

sub in $x = -\frac{2}{5}$:

$50\left(-\frac{2}{5}\right)^2+38\left(-\frac{2}{5}\right)+9 = B\left(1-2\left(-\frac{2}{5}\right)\right)$

$\frac{9}{5} = \frac{9}{5}B$

$B = 1$ (1)

ii) sub in $x=0$: $9 = 2A+B+4C$ (1)

$9 = 2A+1+8$

$2A = 0$

$A = 0$ (1)

$f(x) = \frac{1}{(5x+2)^2} + \frac{2}{1-2x}$

b i) $f(x) = (5x+2)^{-2} + 2(1-2x)^{-1}$

$(5x+2)^{-2} = \left(2\left[\frac{5x}{2}+1\right]\right)^{-2} = 2^{-2}\left(1+\frac{5}{2}x\right)^{-2}$ (1)

$\left(1+\frac{5}{2}x\right)^{-2} = 1-2\left(\frac{5}{2}x\right)+\frac{(-2)(-3)}{2!}\left(\frac{5}{2}x\right)^2+\dots$ (1)

$= 1-5x+\frac{75}{4}x^2+\dots$

$\therefore 2^{-2}\left(1+\frac{5}{2}x\right)^{-2} = \frac{1}{4}-\frac{5}{4}x+\frac{75}{16}x^2+\dots$ (1)

Question 9 continued

$$(1-2x)^{-1} = 1 + 2x + \frac{(-1)(-2)}{2!}(-2x)^2 + \ldots \quad (1)$$

$$= 1 + 2x + 4x^2 + \ldots$$

$$\therefore\ 2(1-2x)^{-1} = 2 + 4x + 8x^2 + \ldots$$

$$f(x) = \frac{1}{(5x+2)^2} + \frac{2}{1-2x}$$

$$= \left[\frac{1}{4} - \frac{5}{4}x + \frac{75}{16}x^2 + \ldots\right] + \left[2 + 4x + 8x^2 + \ldots\right] \quad (1)$$

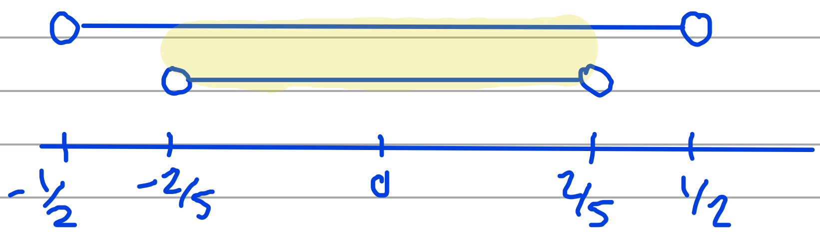
$$= \frac{9}{4} + \frac{11}{4}x + \frac{203}{16}x^2 + \ldots \quad (1)$$

b ii) $\left(1+\frac{5}{2}x\right)^{-2}$ is valid for $\left|\frac{5x}{2}\right| < 1$ $\quad \times \frac{2}{5}$

$|x| < \frac{2}{5}$

$(1-2x)^{-1}$ is valid for $|-2x| < 1$

$|x| < \frac{1}{2}$



want the overlap region, so

$|x| < \frac{2}{5}$ (1)

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Question 9 continued

(Total for Question 9 is 11 marks)

10. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that $1 + \cos 2\theta + \sin 2\theta \neq 0$ prove that

$$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} \equiv \tan\theta$$

(4)

(b) Hence solve, for $0 < x < 180°$

$$\frac{1 - \cos 4x + \sin 4x}{1 + \cos 4x + \sin 4x} = 3\sin 2x$$

giving your answers to one decimal place where appropriate.

(4)

using the identity for $\cos 2\theta$ which will eliminate the 1s

a) $\dfrac{1-\cos 2\theta + \sin 2\theta}{1+\cos 2\theta + \sin 2\theta} = \dfrac{\boxed{1} - (1 - 2\sin^2\theta) + 2\sin\theta\cos\theta}{\boxed{1} + (2\cos^2\theta - 1) + 2\sin\theta\cos\theta}$ ① ①

$= \dfrac{2\sin^2\theta + 2\sin\theta\cos\theta}{2\cos^2\theta + 2\sin\theta\cos\theta} = \dfrac{2\sin\theta\,\cancel{(\sin\theta + \cos\theta)}}{2\cos\theta\,\cancel{(\cos\theta + \sin\theta)}}$ ①

$= \tan\theta$ as required ①

b) using (i), LHS $= \tan 2x$

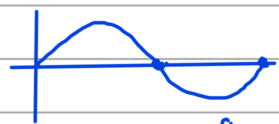
range: $0 < x < 180°$
$0 < 2x < 360°$

$\tan 2x = 3\sin 2x$ ①
$\sin 2x = 3\sin 2x \cos 2x$ $\big) \times \cos 2x$
$0 = 3\sin 2x\cos 2x - \sin 2x$
$0 = \sin 2x(3\cos 2x - 1)$ ①

$\therefore$ either $\sin 2x = 0$ or $3\cos 2x - 1 = 0$

$\Rightarrow \cos 2x = \dfrac{1}{3}$

$2x = \cos^{-1}\left(\frac{1}{3}\right)$
or $2x = 360 - \cos^{-1}\left(\frac{1}{3}\right)$



out of range

$2x = 180°, 360°$
$x = 90°$ ①

$x = 35.3°, 144.7°$ ①

$x = \{35.3°, 90°, 144.7°\}$

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Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 8 marks)

11.

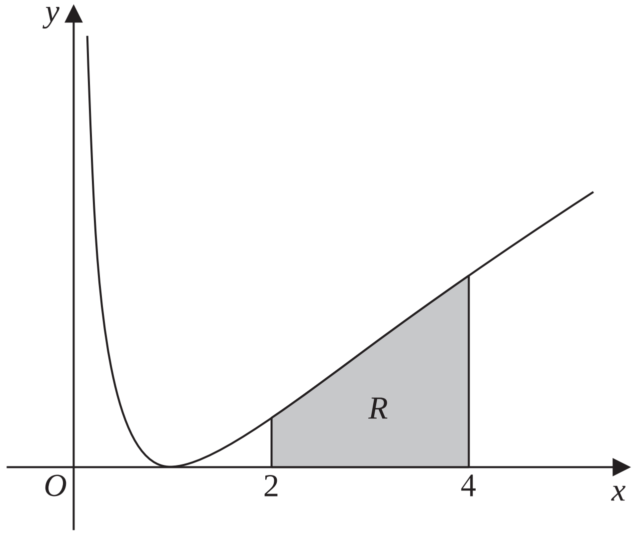

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = (\ln x)^2 \qquad x > 0$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

The table below shows corresponding values of x and y, with the values of y given to 4 decimal places.

$4-3.5=0.5$ $\quad h=0.5$ 1

x	2	2.5	3	3.5	4
y	0.4805	0.8396	1.2069	1.5694	1.9218

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of R, giving your answer to 3 significant figures.

(3)

(b) Use algebraic integration to find the exact area of R, giving your answer in the form

$$y = a(\ln 2)^2 + b\ln 2 + c$$

where a, b and c are integers to be found.

(5)

a) $R = \int_2^4 (\ln x)^2 dx \approx \frac{0.5}{2}\left[0.4805+1.9218+2(0.8396+1.2069+1.5694)\right]$ 1

$= 2.41$ (3sf) 1

Question 11 continued

b) $R = \int_2^4 (\ln x)^2 \, dx$

by parts: $\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx$

find $\int (\ln x)^2 \, dx$

let $u = (\ln x)^2$ $\quad \frac{dv}{dx} = 1$

$\frac{du}{dx} = \frac{2\ln x}{x}$ $\quad v = x$ $\quad$ 1

$\int (\ln x)^2 \, dx = x(\ln x)^2 - 2\int \ln x \, dx$ $\quad$ 1

$\int \ln x \, dx$

let $u = \ln x$ $\quad \frac{dv}{dx} = 1$

$\frac{du}{dx} = \frac{1}{x}$ $\quad v = x$

$\int \ln x \, dx = x\ln x - \int 1 \, dx$

$= x\ln x - x$

$\int (\ln x)^2 \, dx = x(\ln x)^2 - 2(x\ln x - x)$

$= x(\ln x)^2 - 2x\ln x + 2x$ $\quad$ 1

$R = \int_2^4 (\ln x)^2 \, dx = \left[x(\ln x)^2 - 2x\ln x + 2x\right]_2^4$

$= 4[\ln 4]^2 - 8\ln 4 + 8 - 2(\ln 2)^2 + 4\ln 2 - 4$

$= 4[2\ln 2]^2 - 16\ln 2 - 2[\ln 2]^2 + 4\ln 2 + 4$ $\quad$ 1

$= 16(\ln 2)^2 - 12\ln 2 - 2(\ln 2)^2 + 4$

$= 14(\ln 2)^2 - 12\ln 2 + 4$ $\quad$ 1

$\ln 4 = \ln 2^2 = 2\ln 2$

Question 11 continued

Question 11 continued

(Total for Question 11 is 8 marks)

12.

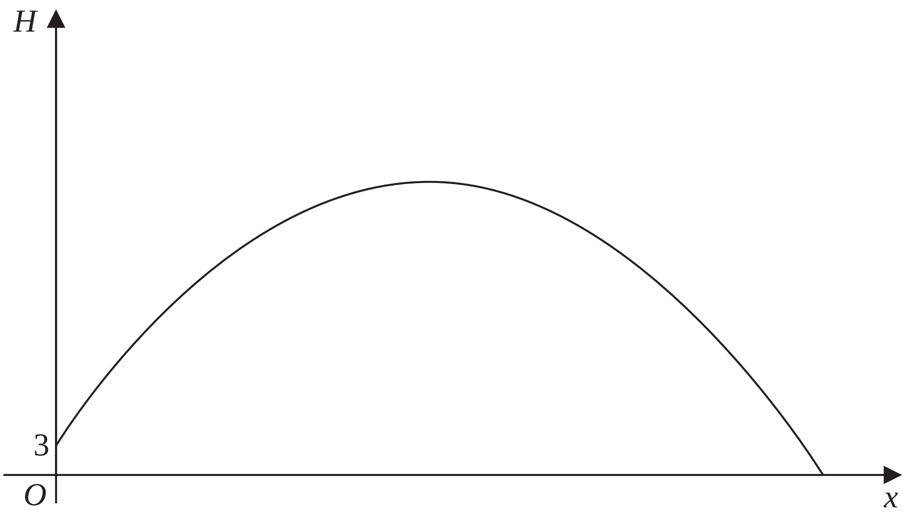

Figure 3

Figure 3 is a graph of the trajectory of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, H metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

Given that the ball

- is hit from a point on the top of a platform of vertical height 3 m above the ground
- reaches its maximum vertical height after travelling a horizontal distance of 90 m
- is at a vertical height of 27 m above the ground after travelling a horizontal distance of 120 m

Given also that H is modelled as a **quadratic** function in x

(a) find H in terms of x

(5)

(b) Hence find, according to the model,

(i) the maximum vertical height of the ball above the ground,

(ii) the horizontal distance travelled by the ball, from when it was hit to when it first hits the ground, giving your answer to the nearest metre.

(3)

(c) The possible effects of wind or air resistance are two limitations of the model. Give one other limitation of this model.

(1)

a) info given: 1. contains $(0, 3) \Rightarrow$ H-intercept 3
2. turning point at $x = 90$
3. contains $(120, 27)$

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Question 12 continued

let $H = ax^2 + bx + c$

1. contains $(0, 3)$

$3 = a(0)^2 + b(0) + c$
$c = 3$ 1

2. turning point at $x = 90$

$H = ax^2 + bx + 3$

$\frac{dH}{dx} = 2ax + b$ 1

$2ax + b = 0$ when $x = 90$

$180a + b = 0$ (1) 1

3. contains $(120, 27)$

$27 = a(120)^2 + b(120) + 3$
$24 = 14400a + 120b$ (2) 1

solve (1) and (2) simultaneously using calculator:

$a = -\frac{1}{300}$, $b = \frac{3}{5}$

$H = -\frac{1}{300}x^2 + \frac{3}{5}x + 3$ 1

b) (i) maximum vertical height is at $x = 90$

$H = -\frac{1}{300}(90)^2 + \frac{3}{5}(90) + 3$

$= 30\text{m}$ 1

(ii) find roots of H, i.e. when $H = 0$

$-0.03x^2 + 0.6x + 3 = 0$ 1

$x = -4.868\ldots$, $x = 184.868\ldots$

horizontal distance is 185m (nearest metre) 1

c) The ball's dimensions are not considered. 1

Question 12 continued

Question 12 continued

(Total for Question 12 is 9 marks)

13. A curve C has parametric equations

$$x = \frac{t^2+5}{t^2+1} \qquad y = \frac{4t}{t^2+1} \qquad t \in \mathbb{R}$$

Show that all points on C satisfy

$$(x-3)^2 + y^2 = 4$$

(3)

Rearrange for t^2:

$$x = \frac{t^2+5}{t^2+1} \qquad y = \frac{4t}{t^2+1}$$

$$y^2 = \frac{16t^2}{(t^2+1)^2}$$

eliminate t

$$x(t^2+1) = t^2+5$$

$$xt^2 + x = t^2+5$$

$$xt^2 - t^2 = 5 - x$$

$$t^2(x-1) = 5-x$$

$$t^2 = \frac{5-x}{x-1}$$

$$y^2 = \frac{16\left(\frac{5-x}{x-1}\right)}{\left(\frac{5-x}{x-1}+1\right)^2} \quad 1$$

$$\frac{5-x}{x-1} + 1 = \frac{5-x}{x-1} + \frac{x-1}{x-1} = \frac{4}{x-1}$$

$$y^2 = \frac{16(5-x)}{x-1} \times \left[\frac{(x-1)}{4}\right]^2$$

$$= (5-x)(x-1) \quad 1$$

$$= 5x - 5 - x^2 + x$$

$$y^2 = -x^2 + 6x - 5$$

$$y^2 = -x^2 + 6x - 5$$

$$y^2 + x^2 - 6x + 5 = 0$$

complete square

$$y^2 + (x-3)^2 - 9 + 5 = 0$$

$$y^2 + (x-3)^2 = 4 \quad 1$$

as required

P 6 8 7 3 1 A 0 4 2 5 2

Question 13 continued

(Total for Question 13 is 3 marks)

14. Given that

$$y = \frac{x-4}{2+\sqrt{x}} \qquad x > 0$$

show that

$$\frac{dy}{dx} = \frac{1}{A\sqrt{x}} \qquad x > 0$$

where A is a constant to be found.

(4)

$$y = \frac{x-4}{2+x^{1/2}} \qquad \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$\frac{dy}{dx} = \frac{(2+x^{1/2}) - \frac{1}{2}x^{-1/2}(x-4)}{(2+x^{1/2})^2} \; 1 \; 1 = \frac{2+x^{1/2} - \frac{1}{2}x^{1/2} + 2x^{-1/2}}{(2+x^{1/2})^2}$$

$$= \frac{2+\frac{1}{2}x^{1/2} + 2x^{-1/2}}{(2+x^{1/2})^2} \; 1 \qquad \times 2\sqrt{x}$$

$$= \frac{4x^{1/2} + x + 4}{2x^{1/2}(2+x^{1/2})^2} = \frac{\cancel{(2+x^{1/2})^2}}{2\sqrt{x}\cancel{(2+x^{1/2})^2}} = \frac{1}{2\sqrt{x}} \; 1$$

Alternative: $y = \frac{x-4}{2+\sqrt{x}} = \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{2+\sqrt{x}} = \sqrt{x} - 2$

$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

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Question 14 continued

Question 14 continued

Question 14 continued

(Total for Question 14 is 4 marks)

15. (i) Use proof by exhaustion to show that for $n \in \mathbb{N}, n \leqslant 4$

$$(n+1)^3 > 3^n$$

(2)

(ii) Given that $m^3 + 5$ is odd, use proof by contradiction to show, using algebra, that m is even.

(4)

i) $n \in \mathbb{N}, n \leq 4$ so $n = 1, 2, 3$ or 4 $(0 \notin \mathbb{N})$

$n=1$, $2^3 = 8$, $3^1 = 3$ $8 > 3$

$n=2$, $3^3 = 27$, $3^2 = 9$ $27 > 9$

$n=3$, $4^3 = 64$, $3^3 = 27$ $64 > 27$

$n=4$, $5^3 = 125$, $3^4 = 81$ $125 > 81$ ①

so if $n \in \mathbb{N}, n \leq 4$, then $(n+1)^3 > 3^n$ ①

ii) Assume there exists an m which is odd such that m^3+5 is odd. ①

Set $m = 2k+1$, $k \in \mathbb{Z}$

①

$m^3+5 = (2k+1)^3 + 5 = 8k^3 + 12k^2 + 6k + 1 + 5$

$= 8k^3 + 12k^2 + 6k + 6$

$= 2(4k^3 + 6k^2 + 3k + 3)$ which is even. ①

This is a contradiction.

So if m^3+5 is odd then m must be even. ①

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Question 15 continued

Question 15 continued

Question 15 continued

Question 15 continued

(Total for Question 15 is 6 marks)

TOTAL FOR PAPER IS 100 MARKS