
ADDITIONAL MATHEMATICS

Paper 1

MARK SCHEME

Maximum Mark: 80

0606/11

October/November 2019

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

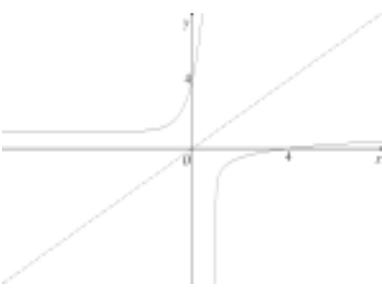
- M Method marks, awarded for a valid method applied to the problem.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘dep’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

| | |
|------|----------------------------|
| awrt | answers which round to |
| cao | correct answer only |
| dep | dependent |
| FT | follow through after error |
| isw | ignore subsequent working |
| nfww | not from wrong working |
| oe | or equivalent |
| rot | rounded or truncated |
| SC | Special Case |
| soi | seen or implied |

| Question | Answer | Marks | Guidance |
|----------|--|-------|--|
| 1 | $A' \cap B$ oe | B1 | |
| | $(X \cap Y) \cup (X \cap Z)$ or $X \cap (Y \cup Z)$ | B1 | |
| 2 | $2x^2 + 3x + k = kx - 3$ | M1 | For an attempt to equate and simplify to a 3 term quadratic equation, allow an error in one term |
| | $2x^2 + (3-k)x + (k+3) = 0$ | A1 | |
| | $(3-k)^2 - 4 \times 2 \times (k+3)$ | M1 | For attempt to use the discriminant, allow previous error, leading to a quadratic equation in terms of k |
| | $k^2 - 14k - 15 = 0$ giving critical values of -1 and 15 | A1 | For critical values |
| | $-1 < k < 15$ | A1 | |
| 3 | Either $7^x \times 7^{2y}$ or $49^{\frac{x}{2}} \times 49^y$ or $5^{5x} \times 5^{2y}$ or $25^{\frac{5x}{2}} \times 25^y$ | M1 | For expressing the terms on the left hand side of either one of the 2 equations in terms of powers of 7, 49, 5 or 25 |
| | $7^x \times 7^{2y} = 7^0$ or $49^{\frac{x}{2}} \times 49^y = 49^0$ | A1 | |
| | $5^{5x} \times 5^{2y} = 5^{-2}$ or $25^{\frac{5x}{2}} \times 25^y = 25^{-1}$ | A1 | |
| | leading to $x + 2y = 0$ and $5x + 2y = -2$ | M1 | For attempt to solve two linear equations, with integer coefficients and constants, in terms of x and y |
| | $x = -\frac{1}{2}$, $y = \frac{1}{4}$ | A1 | |
| 4(i) | $\frac{d}{dx} (\ln(4x^2 + 1)) = \frac{8x}{4x^2 + 1}$ | B1 | |
| | $\frac{dy}{dx} = \frac{(2x-3)\frac{8x}{(4x^2+1)} - 2\ln(4x^2+1)}{(2x-3)^2}$ | M1 | For attempt to differentiate a quotient |
| | | A1 | For all other terms, not including $\frac{8x}{4x^2+1}$, correct |
| 4(ii) | When $x = 2$, $\frac{dy}{dx} = \frac{16}{17} - 2\ln 17$ $= -4.73$ | M1 | For attempt to find value of $\frac{dy}{dx}$ when $x = 2$ and multiply by h |
| | Change in $y = -4.73h$ | A1 | |

| Question | Answer | Marks | Guidance |
|----------|---|-----------|---|
| 5(i) | $f > 1$ | B1 | Must be using correct notation |
| | $g \in \mathbb{R}$ | B1 | Must be using correct notation |
| 5(ii) | $g(0) = 1, g(1) = 2$ and attempt at $f(2)$ | M1 | For attempt at g^2 and correct order |
| | $f(2) = 164.8$ awrt 165 | A1 | |
| 5(iii) |  | B3 | B1 for correct f and $(0, 4)$, must be in first and second quadrant B1 for correct f^1 and $(4, 0)$, must be in first and fourth quadrant B1 for $y = x$ and/or symmetry implied, by ‘matching intercepts’. No intersection. |
| 6 | $\frac{dy}{dx} = k(8x + 5)^{-\frac{1}{2}}$ | M1 | For attempt to differentiate, must be in the form $k(8x + 5)^{-\frac{1}{2}}$ |
| | $\frac{dy}{dx} = 4(8x + 5)^{-\frac{1}{2}}$ | A1 | |
| | When $x = \frac{1}{2}, y = 3$ | B1 | |
| | Normal: $y - 3 = -\frac{3}{4}\left(x - \frac{1}{2}\right)$ | M1 | For attempt at the normal when $x = \frac{1}{2}$, using correct process for <i>their</i> $\frac{dy}{dx}$ and <i>their</i> y . |
| | $6x + 8y - 27 = 0$ | A1 | |

| Question | Answer | Marks | Guidance |
|----------|---|-----------|---|
| 7(i) | $\lg y = \lg A + x \lg b$ | B1 | For statement, may be implied by subsequent work |
| | Either $6 = \lg A + 3.4 \lg b$ or $3.6 = \lg A + 2.2 \lg b$ | M1 | For one correct equation |
| | | M1 | For another correct equation and attempt to solve simultaneously |
| | $\lg b = 2, b = 100$ | A1 | |
| | $\lg A = -0.8, A = 10^{-0.8}$ or 0.158 | A1 | |
| | Or Gradient = $\lg b = 2$ | M1 | equating gradient to $\lg b$ and attempt to evaluate |
| | $b = 100$ | A1 | Must be identified as b |
| | $6 = \lg A + 3.4 \lg b$ or $3.6 = \lg A + 2.2 \lg b$ | M1 | For a correct equation and attempt to find $\lg A$ |
| | $\lg A = -0.8, A = 10^{-0.8}$ or 0.158 | A1 | Must be identified as A |
| 7(ii) | $\lg 900 = -0.8 + 2x$ oe | M1 | For correct use of $y = 900$ |
| | $x = 1.88$ | A1 | |
| 8(i) | $\begin{aligned} BC^2 &= (7 + \sqrt{5})^2 + (7 - \sqrt{5})^2 \\ &= 49 + 14\sqrt{5} + 5 + 49 - 14\sqrt{5} + 5 \\ &= 108 \end{aligned}$ | M1 | For use of Pythagoras' theorem and attempt to expand and simplify |
| | $BC = 6\sqrt{3}$ | A1 | |
| | Perimeter = $22 + 6\sqrt{5} + 6\sqrt{3}$ | A1 | |

| Question | Answer | Marks | Guidance |
|----------|--|-----------|--|
| 8(ii) | Either $\frac{1}{2}(4+3\sqrt{5}+11+2\sqrt{5})(7+\sqrt{5})$ $= \frac{1}{2}(15+5\sqrt{5})(7+\sqrt{5})$ $= \frac{1}{2}(105+35\sqrt{5}+15\sqrt{5}+25)$ | M1 | Either For a valid method and attempt to expand out and simplify |
| | Or $(4+3\sqrt{5})(7+\sqrt{5}) + \frac{1}{2}(7+\sqrt{5})(7-\sqrt{5})$ $= 28 + 21\sqrt{5} + 4\sqrt{5} + 15 + \frac{1}{2}(49 - 5)$ | M1 | Or For a valid method and attempt to expand out and simplify |
| | Area = $65 + 25\sqrt{5}$ | A2 | A1 for each term |
| 9(i) | Either $15^2 = 10^2 + 10^2 - 200 \cos AOB$ $\cos AOB = -0.125$ | M1 | For use of cosine rule |
| | $AOB = 1.696$ so 1.70 to 2 dp | A1 | Must have justification to 2 dp |
| | Or $\sin\left(\frac{AOB}{2}\right) = \frac{7.5}{10}$ $\frac{AOB}{2} = 0.8481$ | M1 | For use of basic trig |
| | $AOB = 1.696$ so 1.70 to 2 dp | A1 | |

| Question | Answer | Marks | Guidance |
|----------|---|-----------|---|
| 9(ii) | Angle $DOC = \frac{\pi}{3}$ | B1 | |
| | Either $AOD = BOC = 0.5\left(2\pi - \frac{\pi}{3} - 1.696\right)$ $AOD = BOC = 1.77$ | M1 | For attempt to get AOD or BOC |
| | Arc lengths = 17.7 | M1 | For attempt at arc length using their previous answer |
| | Perimeter = $15 + 10 + (2 \times 17.7) = 60.4$ | A1 | |
| | Or $\text{Arc } AB = 17$ or $\text{Arc } CD = \frac{10\pi}{3}$ | M1 | For either arc length |
| | $(20\pi - \text{arc } AB - \text{arc } CD)$ | M1 | |
| | Perimeter = 60.4 | A1 | |
| 9(iii) | Either Area of each sector = $\frac{1}{2}10^2(1.770)$ | M1 | For area of sector using their BOC |
| | Area of triangles = $\left(\frac{1}{2} \times 100 \times \sin \frac{\pi}{3}\right) + \left(\frac{1}{2} \times 100 \sin 1.70\right)$ | M1 | For area of one triangle using the sine rule oe |
| | Total area = $177 + 43.3 + 49.6$ | M1 | For plan |
| | Area = awrt 270 | A1 | |
| | Or Area of upper segment = $\frac{1}{2}10^2(1.696 - \sin 1.696)$ | M1 | For area of a sector or area of a triangle using the sine rule oe |
| | Area of lower segment = $\frac{1}{2}10^2\left(\frac{\pi}{3} - \sin \frac{\pi}{3}\right)$ | M1 | For whichever has not been obtained in previous part |
| | Shaded area = $100\pi - \text{are of the 2 segments}$ Area = $314.2 - 35.2 - 9.06$ | M1 | For plan |
| | Area = awrt 270 | A1 | |

| Question | Answer | Marks | Guidance |
|----------|--|-----------|--|
| 10 | $1.5 = 2 + \cos 3x$ $\cos 3x = -0.5$ | M1 | For correct attempt to find points of intersection |
| | $3x = \frac{2\pi}{3}, \frac{4\pi}{3}$ | M1 | For dealing with $3x$ correctly |
| | $x = \frac{2\pi}{9}$ or 40° | A1 | |
| | $x = \frac{4\pi}{9}$ or 80° | A1 | |
| | Either $\int_{\frac{2\pi}{9}}^{\frac{4\pi}{9}} 1.5 - (2 + \cos 3x) \, dx$ | M1 | For subtraction method – condone omission of or incorrect limits |
| | $[-0.5x - k \sin 3x]_{\frac{2\pi}{9}}^{\frac{4\pi}{9}}$ | M1 | For attempt to integrate – condone omission of or incorrect limits |
| | $\left[-0.5x - \frac{1}{3} \sin 3x \right]_{\frac{2\pi}{9}}^{\frac{4\pi}{9}}$ | A1 | All correct – condone omission of or incorrect limits |
| | $\left(-\frac{2\pi}{9} + \frac{\sqrt{3}}{6} \right) - \left(-\frac{\pi}{9} - \frac{\sqrt{3}}{6} \right)$ | M1 | Dep for application of limits, must be in radians |
| | $\text{Area} = \frac{\sqrt{3}}{3} - \frac{\pi}{9}$ | A1 | |
| | Or $\left(1.5 \times \frac{2\pi}{9} \right)$ | M1 | For attempt at rectangle (must include subtraction subsequently) |
| | $[2x + k \sin 3x]_{\frac{2\pi}{9}}^{\frac{4\pi}{9}}$ | M1 | For attempt to integrate – condone omission of or incorrect limits |
| | $\left[2x + \frac{1}{3} \sin 3x \right]_{\frac{2\pi}{9}}^{\frac{4\pi}{9}}$ | A1 | All correct – condone omission of or incorrect limits |
| | $\left(\left(\frac{8\pi}{9} - \frac{\sqrt{3}}{6} \right) - \left(\frac{4\pi}{9} + \frac{\sqrt{3}}{6} \right) \right)$ | M1 | Dep for application of limits, must be in radians |
| | $\text{Area} = \frac{\sqrt{3}}{3} - \frac{\pi}{9}$ | A1 | |

| Question | Answer | Marks | Guidance |
|------------|---|-----------|--|
| 11(a)(i) | 362 880 | B1 | |
| 11(a)(ii) | 7! × 2 | B1 | For 7! |
| | 10 080 | B1 | For 7! × 2 leading to 10080 |
| 11(a)(iii) | Total = 4! × 4! × 3! = 3456 | B3 | B1 for treating as 4 separate units 4! B1 for either number of ways of arranging the maths books amongst themselves 4! or the number of ways of arranging the physics books amongst themselves 3! |
| 11(b)(i) | 18 564 | B1 | |
| 11(b)(ii) | Total 3738 | B4 | B1 4 boys 3150 B1 5 boys 560 B1 6 boys 28 |
| 12 | $\frac{dy}{dx} = k \cos\left(x + \frac{\pi}{3}\right) + c$ | M1 | For attempt to integrate |
| | $\frac{dy}{dx} = -2 \cos\left(x + \frac{\pi}{3}\right) + c$ | A1 | All correct, condone omission of +c |
| | $5 = -2 \cos\frac{2\pi}{3} + c$ | M1 | Dep for attempt to find c |
| | $\frac{dy}{dx} = -2 \cos\left(x + \frac{\pi}{3}\right) + 4$ | A1 | |
| | $y = p \sin\left(x + \frac{\pi}{3}\right) (+qx + d)$ | M1 | attempt to integrate a second time to obtain $y = p \sin\left(x + \frac{\pi}{3}\right)$ |
| | $y = -2 \sin\left(x + \frac{\pi}{3}\right) + 4x + d$ | A1 | All correct, condone omission of +d |
| | $\frac{5\pi}{3} = -2 \sin\frac{2\pi}{3} + \frac{4\pi}{3} + d$ | M1 | Dep for attempt to find a second arbitrary constant |
| | $y = -2 \sin\left(x + \frac{\pi}{3}\right) + 4x + \frac{\pi}{3} + \sqrt{3}$ or $y = -2 \sin\left(x + \frac{\pi}{3}\right) + 4x + 2.78$ | A1 | |