



Oxford Cambridge and RSA

Wednesday 5 June 2019 – Morning

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

Time allowed: 2 hours



You must have:

- Printed Answer Booklet

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\text{ ms}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets [].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **8** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_r a^{n-r} b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \frac{u}{v}, \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

Integration by parts $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

Small angle approximations

$$\sin \theta \approx \theta, \cos \theta \approx 1 - \frac{1}{2}\theta^2, \tan \theta \approx \theta \text{ where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Sample variance

$$s^2 = \frac{1}{n-1} S_{xx} \text{ where } S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^n C_r p^r q^{n-r}$ where $q = 1 - p$

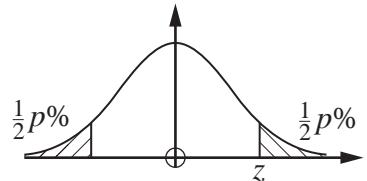
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u+v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Answer **all** the questions.

Section A (25 marks)

1 In this question you must show detailed reasoning.

Show that $\int_4^9 (2x + \sqrt{x}) dx = \frac{233}{3}$. [3]

- 2** Show that the line which passes through the points $(2, -4)$ and $(-1, 5)$ does not intersect the line $3x + y = 10$. [3]

- 3** The function $f(x)$ is given by $f(x) = (1 - ax)^{-3}$, where a is a non-zero constant. In the binomial expansion of $f(x)$, the coefficients of x and x^2 are equal.

(a) Find the value of a . [3]

(b) Using this value for a ,

(i) state the set of values of x for which the binomial expansion is valid, [1]

(ii) write down the quadratic function which approximates $f(x)$ when x is small. [1]

- 4** Fig. 4 shows a uniform beam of mass 4 kg and length 2.4 m resting on two supports P and Q. P is at one end of the beam and Q is 0.3 m from the other end.

Determine whether a person of mass 50 kg can tip the beam by standing on it. [3]

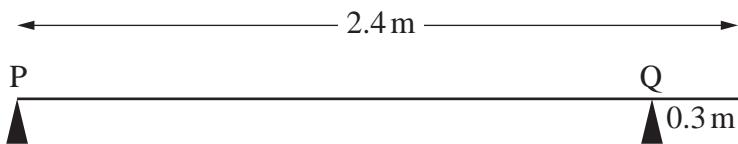


Fig. 4

- 5** A car of mass 1200 kg travels from rest along a straight horizontal road. The driving force is 4000 N and the total of all resistances to motion is 800 N.
Calculate the velocity of the car after 9 seconds. [4]

- 6 (a)** Prove that $\frac{\sin \theta}{1 - \cos \theta} - \frac{1}{\sin \theta} = \cot \theta$. [4]

- (b)** Hence find the exact roots of the equation $\frac{\sin \theta}{1 - \cos \theta} - \frac{1}{\sin \theta} = 3 \tan \theta$ in the interval $0 \leq \theta \leq \pi$. [3]

Answer **all** the questions.

Section B (75 marks)

- 7 The velocity $v \text{ ms}^{-1}$ of a particle at time $t \text{ s}$ is given by

$$v = 0.5t(7-t).$$

Determine whether the **speed** of the particle is increasing or decreasing when $t = 8$. [4]

- 8 An arithmetic series has first term 9300 and 10th term 3900.

(a) Show that the 20th term of the series is negative. [3]

(b) The sum of the first n terms is denoted by S . Find the greatest value of S as n varies. [4]

- 9 A cannonball is fired from a point on horizontal ground at 100 ms^{-1} at an angle of 25° above the horizontal. Ignoring air resistance, calculate

(a) the greatest height the cannonball reaches, [3]

(b) the range of the cannonball. [4]

- 10 (a) Express $7\cos x - 2\sin x$ in the form $R\cos(x+\alpha)$ where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact value of R and the value of α correct to 3 significant figures. [4]

(b) Give details of a sequence of two transformations which maps the curve $y = \sec x$ onto the curve $y = \frac{1}{7\cos x - 2\sin x}$. [3]

- 11 In this question, the unit vector **i** is horizontal and the unit vector **j** is vertically upwards.

A particle of mass 0.8 kg moves under the action of its weight and two forces given by $(k\mathbf{i} + 5\mathbf{j}) \text{ N}$ and $(4\mathbf{i} + 3\mathbf{j}) \text{ N}$. The acceleration of the particle is vertically upwards.

(a) Write down the value of k . [1]

Initially the velocity of the particle is $(4\mathbf{i} + 7\mathbf{j}) \text{ ms}^{-1}$.

(b) Find the velocity of the particle 10 seconds later. [4]

- 12 Fig. 12 shows a curve C with parametric equations $x = 4t^2$, $y = 4t$. The point P, with parameter t , is a general point on the curve. Q is the point on the line $x + 4 = 0$ such that PQ is parallel to the x -axis. R is the point $(4, 0)$.

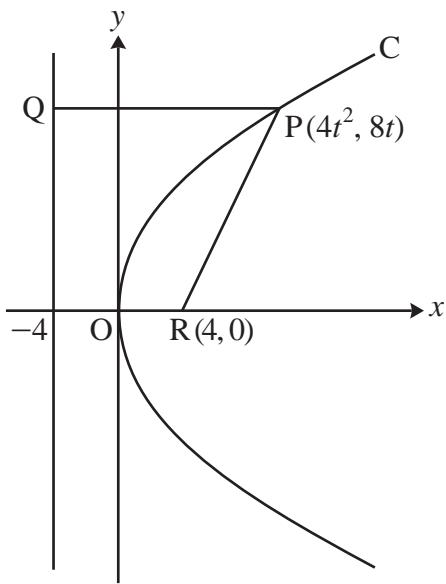


Fig. 12

- (a) Show algebraically that P is equidistant from Q and R. [4]
- (b) Find a cartesian equation of C. [2]
- 13 A 15 kg box is suspended in the air by a rope which makes an angle of 30° with the vertical. The box is held in place by a string which is horizontal.
- (a) Draw a diagram showing the forces acting on the box. [1]
- (b) Calculate the tension in the rope. [2]
- (c) Calculate the tension in the string. [2]

- 14 Fig. 14 shows a circle with centre O and radius $r\text{cm}$. The chord AB is such that angle $\text{AOB} = x$ radians. The area of the shaded segment formed by AB is 5% of the area of the circle.

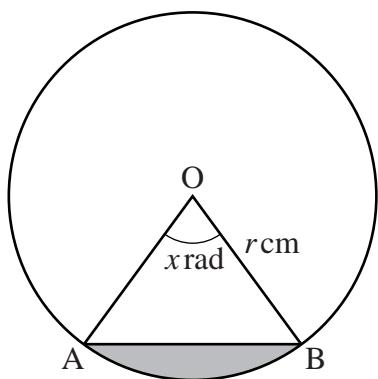


Fig. 14

- (a) Show that $x - \sin x - \frac{1}{10}\pi = 0$.

[4]

The Newton-Raphson method is to be used to find x .

- (b) Write down the iterative formula to be used for the equation in part (a).

[1]

- (c) Use three iterations of the Newton-Raphson method with $x_0 = 1.2$ to find the value of x to a suitable degree of accuracy.

[3]

- 15 A model for the motion of a small object falling through a thick fluid can be expressed using the differential equation

$$\frac{dv}{dt} = 9.8 - kv,$$

where $v \text{ m s}^{-1}$ is the velocity after t s and k is a positive constant.

- (a) Given that $v = 0$ when $t = 0$, solve the differential equation to find v in terms of t and k .

[7]

- (b) Sketch the graph of v against t .

[2]

Experiments show that for large values of t , the velocity tends to 7 m s^{-1} .

- (c) Find the value of k .

[2]

- (d) Find the value of t for which $v = 3.5$.

[1]

- 16** A particle of mass 2 kg slides down a plane inclined at 20° to the horizontal. The particle has an initial velocity of 1.4 m s^{-1} down the plane. Two models for the particle's motion are proposed.

In model A the plane is taken to be smooth.

(a) Calculate the time that model A predicts for the particle to slide the first 0.7 m. [5]

(b) Explain why model A is likely to underestimate the time taken. [1]

In model B the plane is taken to be rough, with a constant coefficient of friction between the particle and the plane.

(c) Calculate the acceleration of the particle predicted by model B given that it takes 0.8 s to slide the first 0.7 m. [2]

(d) Find the coefficient of friction predicted by model B, giving your answer correct to 3 significant figures. [6]

END OF QUESTION PAPER



Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.