PC1101 Cheatsheet, by randomwish

https://github.com/randomwish/schoolNotes

1. Intro to light

Preliminaries

Electromagnetic field and Maxwell's equations

Nomenclature

- $\vec{E}$: direction along which a charged particle is pushed (polar vector). Perpendicular to direction of propagation.
- $\vec{D}$: electric displacement field.
- $\vec{H}$: magnetic field.
- $\vec{B}$: axis around which a charged particle will rotate (axial vector). Perpendicular to both $\vec{E}$ and direction of propagation.
- ∇: derivatives with respect to spatial coordinates.
- ρ: charge density.
- $\vec{j}$: current density.

Equations

- $\vec{\nabla} \times \vec{E}(\vec{x}, t) + \partial_t \vec{B}(\vec{x}, t) = 0$
  - $\vec{E}$ circulates around any region where $\vec{B}$ is changing with time.
- $\vec{\nabla} \cdot \vec{B}(\vec{x}, t) = 0$
  - No magnetic monopoles; net outflow of $\vec{B}$ is zero.
- $\vec{\nabla} \cdot \vec{D}(\vec{x}, t) = \rho(\vec{x}, t)$
  - The electric displacement field $\vec{D}$ relates to charge density.
- $\vec{\nabla} \times \vec{H}(\vec{x}, t) - \partial_t \vec{D}(\vec{x}, t) = \vec{j}(\vec{x}, t)$
  - Describes the circulation of $\vec{H}$ around a current density $\vec{j}$ and the time derivative of $\vec{D}$.

E-m field with matter

Forces on charged particle

In addition to factoring in Newton's equation with electric and Lorentz forces, include the Abraham-Lorentz force, which accounts for the energy radiated by an accelerated charge.

Interaction with a molecule

An electric dipole arises when positive and negative charges move in opposite directions, causing the centers of mass of both clouds to not overlap.
When an electric field oscillates, the dipole oscillates at the same frequency, emitting a wave at that frequency and scattering in all directions. Electrons are stable only in specific energy states. The energy difference between two states, $\Delta E_{jk} = E_j - E_k$, corresponds to a frequency via the relation

$$\Delta E_{jk} = h\nu_{jk} = \hbar\omega_{jk},$$

where h is Planck's constant.

Interaction with solids

In dielectric materials, such as glass (where electrons are bound to their atoms), light scattered by each atom remains in phase with the light scattered by other atoms. As a result, light passing through the material continues to propagate in a coherent direction.

Frequency

The dispersion relation is given by $\lambda v = c$, where c is the speed of light in vacuum (3×10^8 m/s). Humans can perceive light only when its wavelength lies within the visible spectrum.

2. Monochromatic Waves

Monochromatic: single color $\rightarrow$ wave has a single frequency.

- Frequency v: units in Hz.
- Phase φ: only matters if there are two or more waves (relative phase).
- Pulsation $\omega = 2\pi v$: units in rad/s.
- Time dependence: $\cos(2\pi vt + \varphi) \equiv \cos(\omega t + \varphi)$.
- Conversion: $\sin(\theta) = \cos(\theta - \frac{\pi}{2})$.

Space Dependence

1D Monochromatic Waves

We represent space as $x - ct$, where t represents time coordinates. Space and time dependence of a 1D monochromatic wave:

$$\cos(kx - \omega t + \varphi),$$

where k is the wave number:

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}.$$

The minus sign indicates that the wave moves to the right (positive direction).

3D Monochromatic Waves

- **Plane wave**: $\cos(\vec{k} \cdot \vec{x} - \omega t + \varphi)$.
  - Wavefronts are planes; propagates eternally from $-\infty$ to ∞ in the direction $\hat{e_k}$.
  - The space vibrates in unison in the perpendicular direction to $\hat{e_k}$.
- **Spherical wave**:

  $$\frac{1}{r(\vec{x})} \cos(kr(\vec{x}) - \omega t + \varphi),$$

  where k is the wave number and $r(\vec{x})$ is the distance of $\vec{x}$ from the center $\vec{x_0}$ where the wave originates.

  $$r(\vec{x}) = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}$$

- wave fronts are surfaces of constant r, or spheres centered at $\vec{x_0}$