Robotics

Estimation and Learning
with Dan Lee

Week 1.
Gaussian Model Learning

1.2.2 Maximum Likelihood Estimate

Penn Engineering

Maximum Likelihood Estimate of Gaussian Model Parameters

- Objective   如何从观察的数据中，计算 mean 和 variance

  Estimate the mean and the variance given observed data

Likelihood: $p(\{x_i\}|\mu, \sigma)$

Observed data   Unknown parameters

Likelihood is the probability of the observed data given model parameters.
- a function of model parameters give observed data.
- to maximize the likelihood, we tune the model parameters to fit the observed data.

Maximum Likelihood Estimate of Gaussian Model Parameters

- Objective

$$\hat{\mu}, \hat{\sigma} = \arg\max_{\mu,\sigma} p(\{x_i\}|\mu, \sigma)$$

Find the best model parameters to make the likehoold as large as possible.

其实很容易理解，在给定参数的情况下，想让模型更加贴合实际数据 P({xi} | u, q)

Maximum Likelihood Estimate of Gaussian Model Parameters

- Objective

$$\hat{\mu}, \hat{\sigma} = \arg\max_{\mu,\sigma} p(\{x_i\}|\mu, \sigma)$$

Assuming independence of observations,

$$p(\{x_i\}|\mu, \sigma) = \prod_{i=1}^{N} p(x_i|\mu, \sigma)$$

$$\boxed{\hat{\mu}, \hat{\sigma} = \arg\max_{\mu,\sigma} \prod_{i=1}^{N} p(x_i|\mu, \sigma)}$$

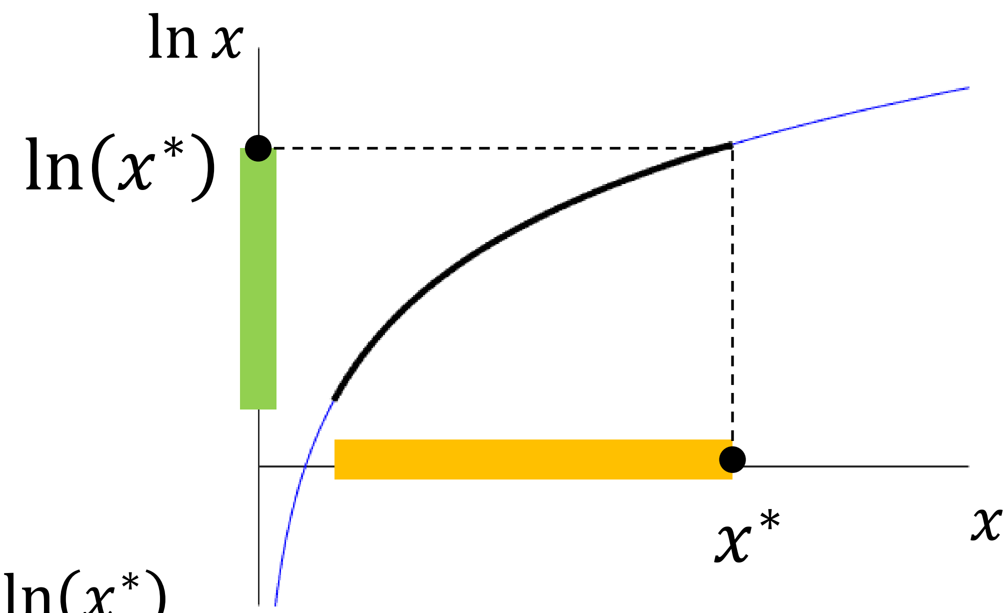
Maximum Likelihood Estimate of Gaussian Model Parameters

$$\hat{\mu}, \hat{\sigma} = \arg\max_{\mu,\sigma} \prod_{i=1}^{N} p(x_i|\mu, \sigma)$$

(1) Take the log!

Maximum Likelihood Estimate of Gaussian Model Parameters

$$\hat{\mu}, \hat{\sigma} = \arg\max_{\mu,\sigma} \prod_{i=1}^{N} p(x_i | \mu, \sigma)$$



NOTE 1:

$$x \leq x^* \leftrightarrow \ln(x) \leq \ln(x^*)$$

Maximum Likelihood Estimate of Gaussian Model Parameters

$$\hat{\mu}, \hat{\sigma} = \arg\max_{\mu,\sigma} \prod_{i=1}^{N} p(x_i|\mu,\sigma)$$

(1) $$\arg\max_{\mu,\sigma} \prod_{i=1}^{N} p(x_i|\mu,\sigma) = \arg\max_{\mu,\sigma} \ln\left\{\prod_{i=1}^{N} p(x_i|\mu,\sigma)\right\}$$

NOTE 1:

$$x \leq x^* \leftrightarrow \ln(x) \leq \ln(x^*)$$

Maximum Likelihood Estimate of Gaussian Model Parameters

$$\hat{\mu}, \hat{\sigma} = \arg\max_{\mu,\sigma} \prod_{i=1}^{N} p(x_i | \mu, \sigma)$$

(1) $$\arg\max_{\mu,\sigma} \prod_{i=1}^{N} p(x_i | \mu, \sigma) = \arg\max_{\mu,\sigma} \ln \left\{ \prod_{i=1}^{N} p(x_i | \mu, \sigma) \right\}$$

$$= \arg\max_{\mu,\sigma} \boxed{\sum_{i=1}^{N} \ln p(x_i | \mu, \sigma)}$$

NOTE 2:

$$\log(x_1 \times x_2 \times \cdots \times x_k) = \log(x_1) + \log(x_2) + \cdots + \log(x_k)$$

Maximum Likelihood Estimate of Gaussian Model Parameters

$$\hat{\mu}, \hat{\sigma} = \arg\max_{\mu,\sigma} \sum_{i=1}^{N} \ln \underline{p(x_i|\mu,\sigma)}$$

(2) Gaussian!

$$\frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x_i-\mu)^2}{2\sigma^2}\right\}$$

Maximum Likelihood Estimate of Gaussian Model Parameters

$$\hat{\mu}, \hat{\sigma} = \arg\max_{\mu,\sigma} \sum_{i=1}^{N} \underline{\ln p(x_i|\mu,\sigma)}$$

$$\ln \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x_i-\mu)^2}{2\sigma^2}\right\}$$

$$= \left\{-\frac{(x_i-\mu)^2}{2\sigma^2} - \ln\sigma - \ln\sqrt{2\pi}\right\}$$

Maximum Likelihood Estimate of Gaussian Model Parameters

$$\hat{\mu}, \hat{\sigma} = \arg\max_{\mu,\sigma} \sum_{i=1}^{N} \left\{ -\frac{(x_i - \mu)^2}{2\sigma^2} - \ln \sigma \right\}$$

$$\Updownarrow$$

$$\boxed{\hat{\mu}, \hat{\sigma} = \arg\min_{\mu,\sigma} \sum_{i=1}^{N} \left\{ \frac{(x_i - \mu)^2}{2\sigma^2} + \ln \sigma \right\}}$$

Maximum Likelihood Estimate of Gaussian Model Parameters

$$\hat{\mu}, \hat{\sigma} = \arg\min_{\mu,\sigma} \underbrace{\sum_{i=1}^{N} \left\{ \frac{(x_i - \mu)^2}{2\sigma^2} + \ln\sigma \right\}}_{J(\mu,\sigma)}$$

- At optimum,

$$\boxed{\frac{\partial J}{\partial \mu} = 0 \longrightarrow \hat{\mu}} \qquad \boxed{\frac{\partial J(\hat{\mu},\sigma)}{\partial \sigma} = 0 \longrightarrow \hat{\sigma}}$$

Maximum Likelihood Estimate of Gaussian Model Parameters

- The MLE Solution:

$$\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\hat{\sigma}^2 = \frac{1}{N}\sum_{i=1}^{N} (x_i - \hat{\mu})^2$$

MLE Estimate: Example

Ball color distribution

$$\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\hat{\sigma}^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \hat{\mu})^2$$

Segmented Ball Image