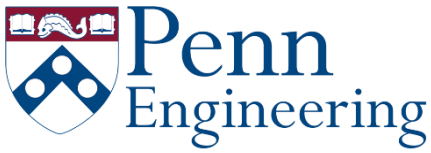
Robotics

Estimation and Learning
with Dan Lee

Week 1.
Gaussian Model Learning

1.2.1 1D Gaussian Distribution



Gaussian Distribution

Why Gaussian?

- The two parameters (mean and variance) are easy to compute and interpret.高斯分布是一种模型，只需要两个参数就可以控制其形状。
(类似三角函数 sine, 有周期、相位、大小，可以组成其他函数/傅立叶)

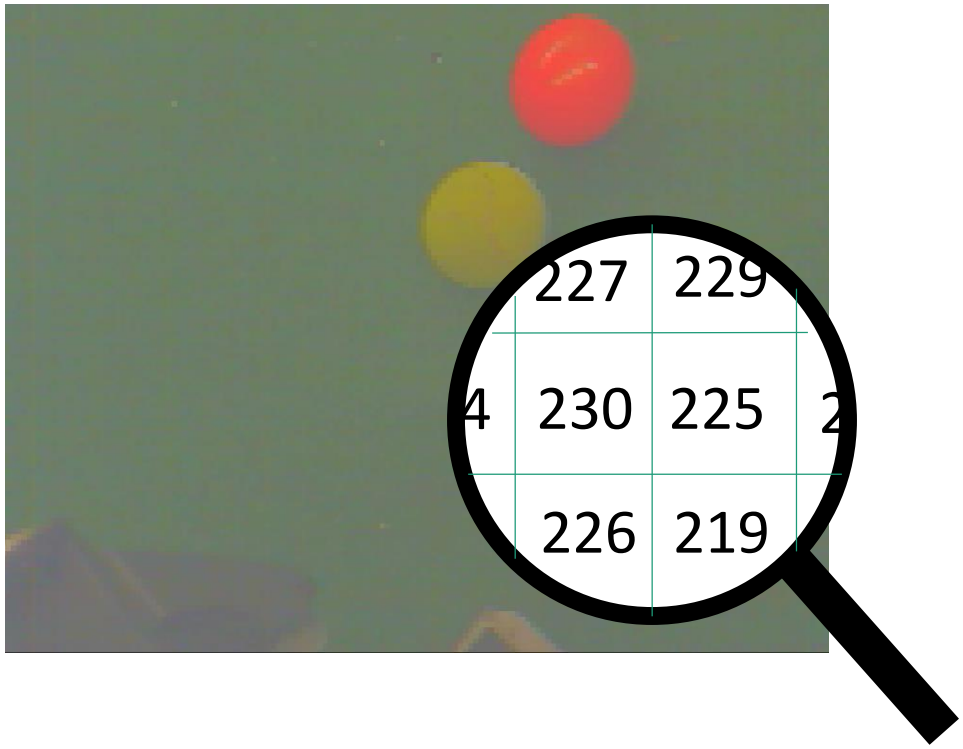
- Good mathematical properties:
e.g., product of Gaussian distributions forms Gaussian.
高斯分布满足闭包性(这意味着有很多数学理论都可以用)

- Central limit theorem:
Expectation of the mean of any random variables converges to Gaussian.
现实中很多事件属于高斯分布（神奇！）

Gaussian Distribution : Example

Ball color distribution

Color Image

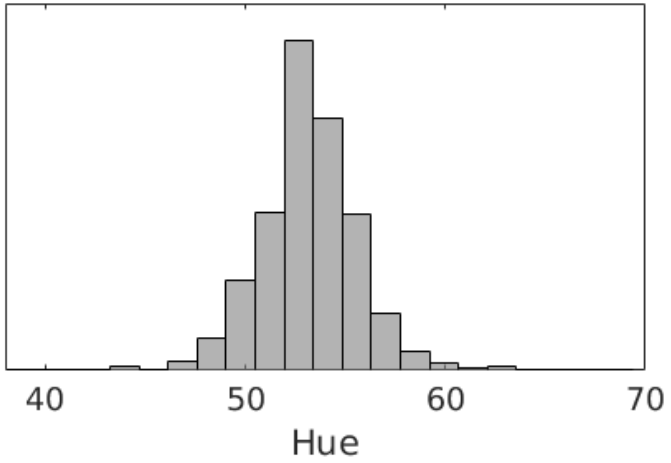


“Yellow”?
“Red”?

Gaussian Distribution : Example

Ball color distribution

Color Image



如果这是 3d 的话，你还能看到一座山呢。

Gaussian Distribution (1D)

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

这个不用担心，因为你必须查公示表的。

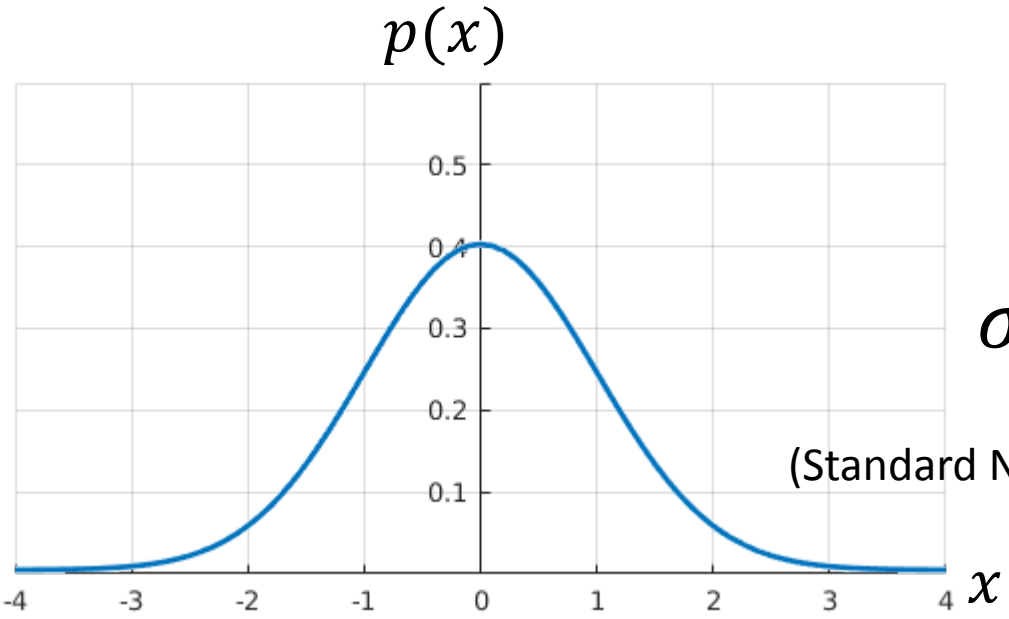
x	Variable 随机变量
μ	Mean 两个用来计算高斯模型的值
σ^2	Variance
σ	Standard deviation

(这种是可以逆运算的
知道高斯模型， 可算 mean, variance)
知道 mean 和 variance, 也可以反向描绘高斯分布(如果符合高斯分布的话，
这至少要用实验检验)

Gaussian Distribution (1D)

下面都是例子，了解一下即可

$$p(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$



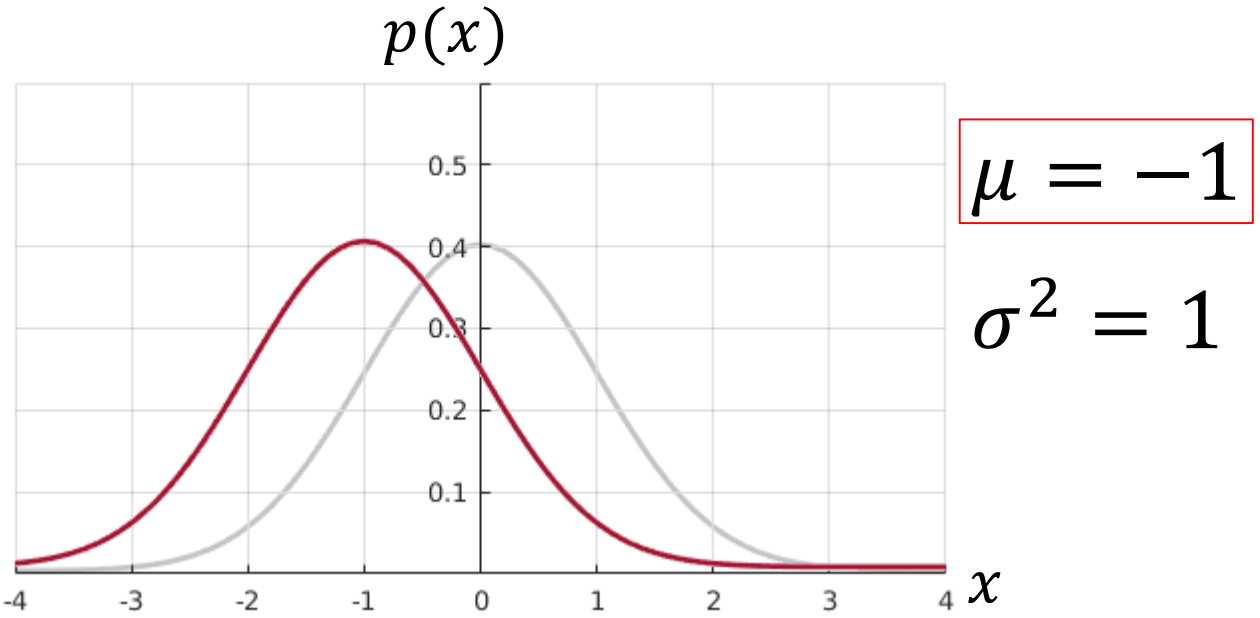
$$\mu = 0$$

$$\sigma^2 = 1$$

(Standard Normal Distribution)

Gaussian Distribution (1D)

$$p(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x+1)^2}{2}}$$

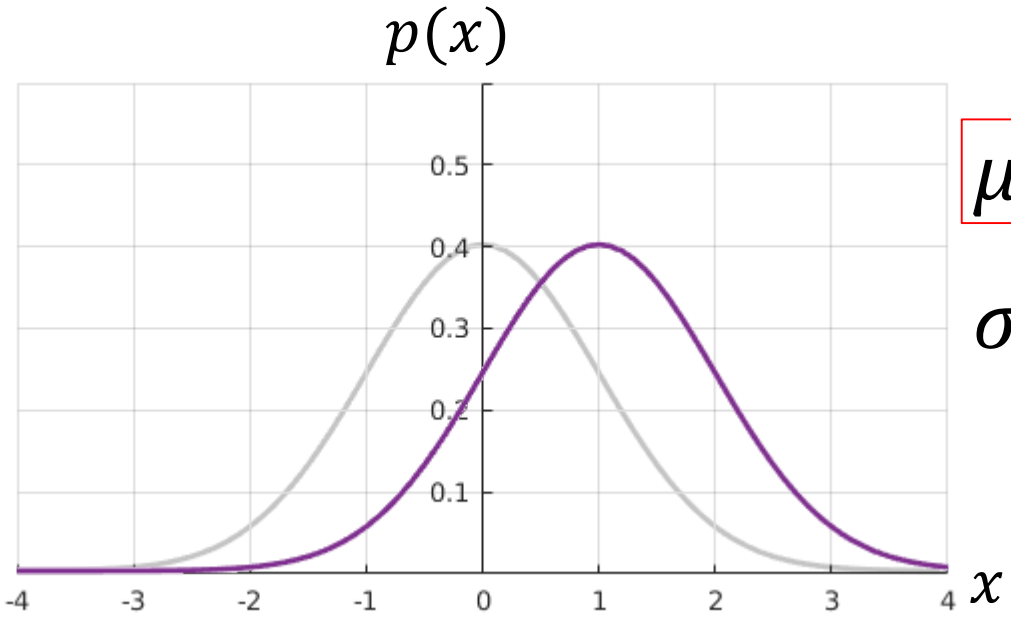


$\mu = -1$

$\sigma^2 = 1$

Gaussian Distribution (1D)

$$p(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-1)^2}{2}}$$

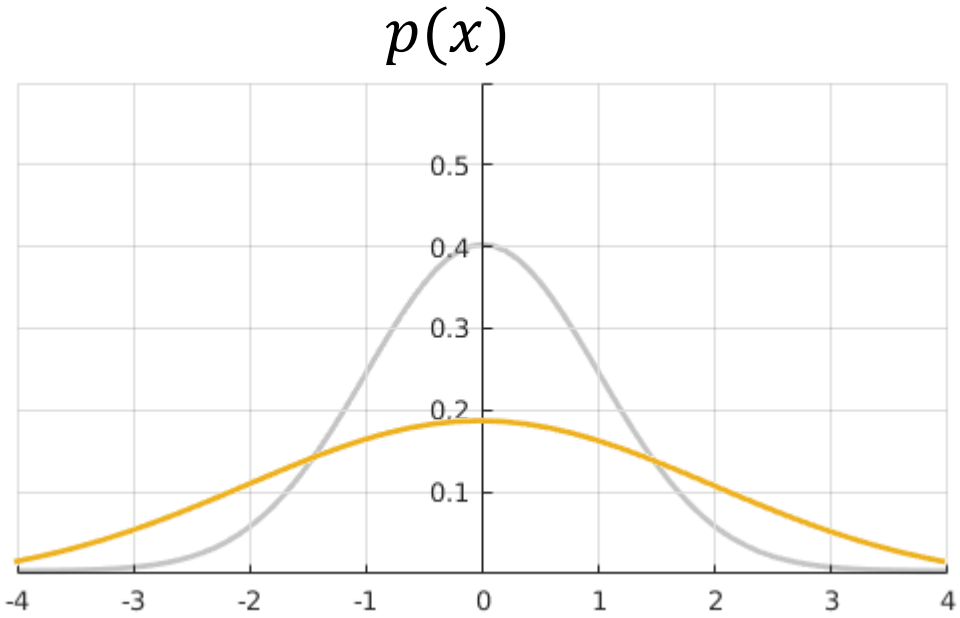


$\mu = 1$

$\sigma^2 = 1$

Gaussian Distribution (1D)

$$p(x) = \frac{1}{2\sqrt{\pi}} e^{-\frac{x^2}{4}}$$

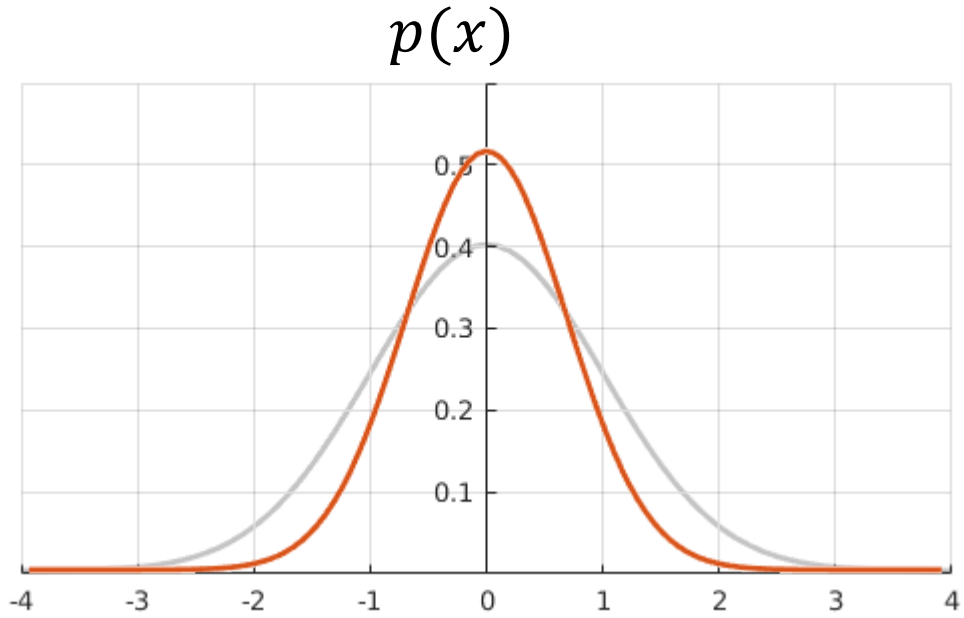


$\mu = 0$

$\sigma^2 = 2$

Gaussian Distribution (1D)

$$p(x) = \frac{1}{\sqrt{1.4\pi}} e^{-\frac{x^2}{1.4}}$$

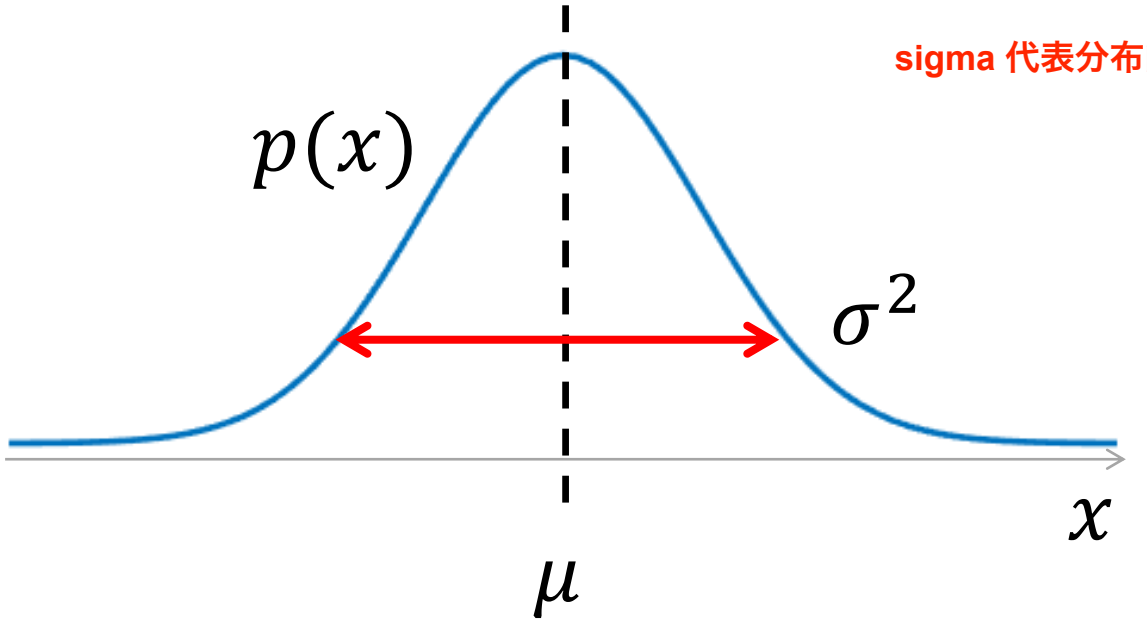


$\mu = 0$

$\sigma^2 = 0.7$

Gaussian Distribution (1D)

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

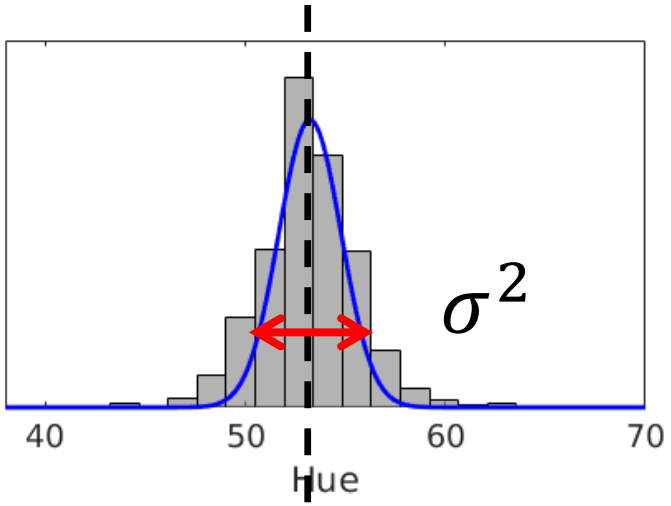


sigma 代表分布的宽度

Gaussian Distribution : Example

Ball color distribution

Color Image



μ

Acknowledgement

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