Robotics
Estimation and Learning
with Dan Lee

Week 2.
Kalman Filter

2.2 System and Measurement Models

Penn Engineering
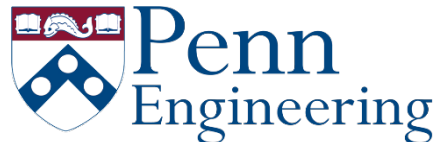
Bayesian Kalman Filtering

- Modeling motion and noise
- Mathematical underpinnings of Kalman filters
- Position tracking example

Linear Modeling

- Discrete Linear dynamical system of motion

  - $x_{t+1} = A\,x_t + B\,u_t$ $z_t = C\,x_t$

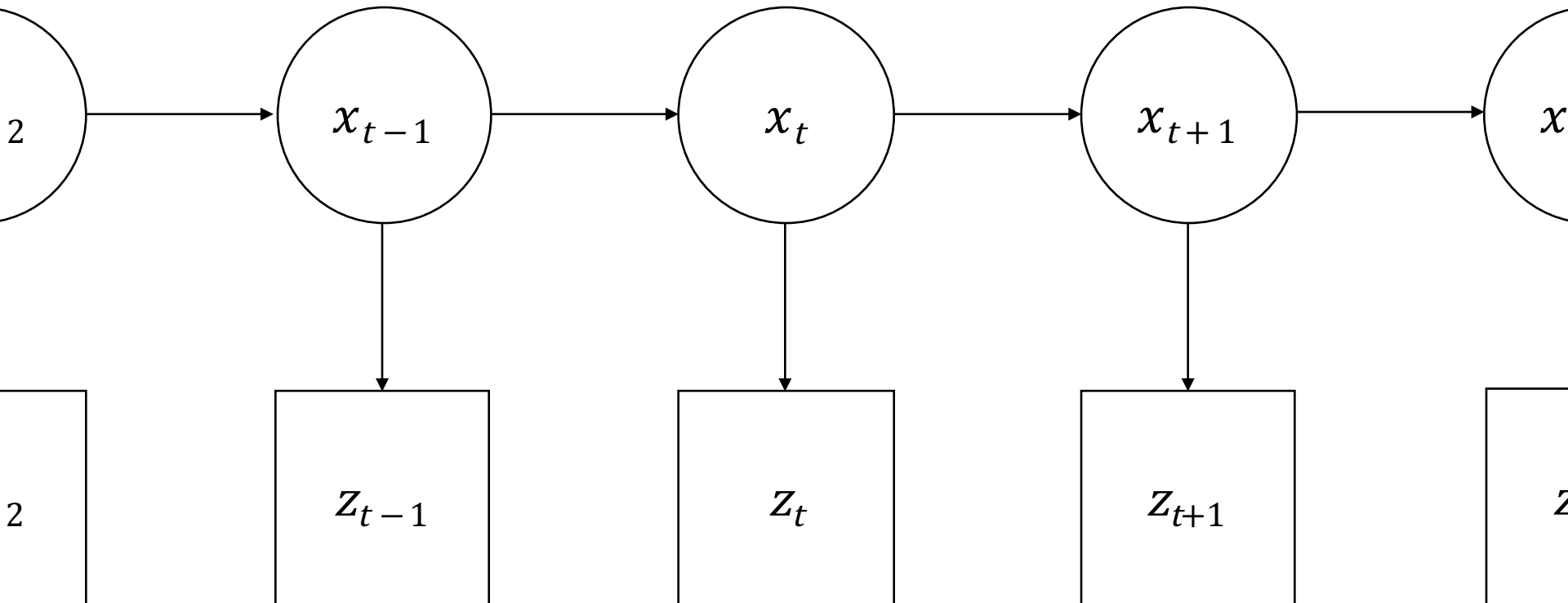   **假设物体的运动模型 (linear object), 其中 u 代表 external inputs (但此处为零)**

- Simple state vector, x, is position and velocity
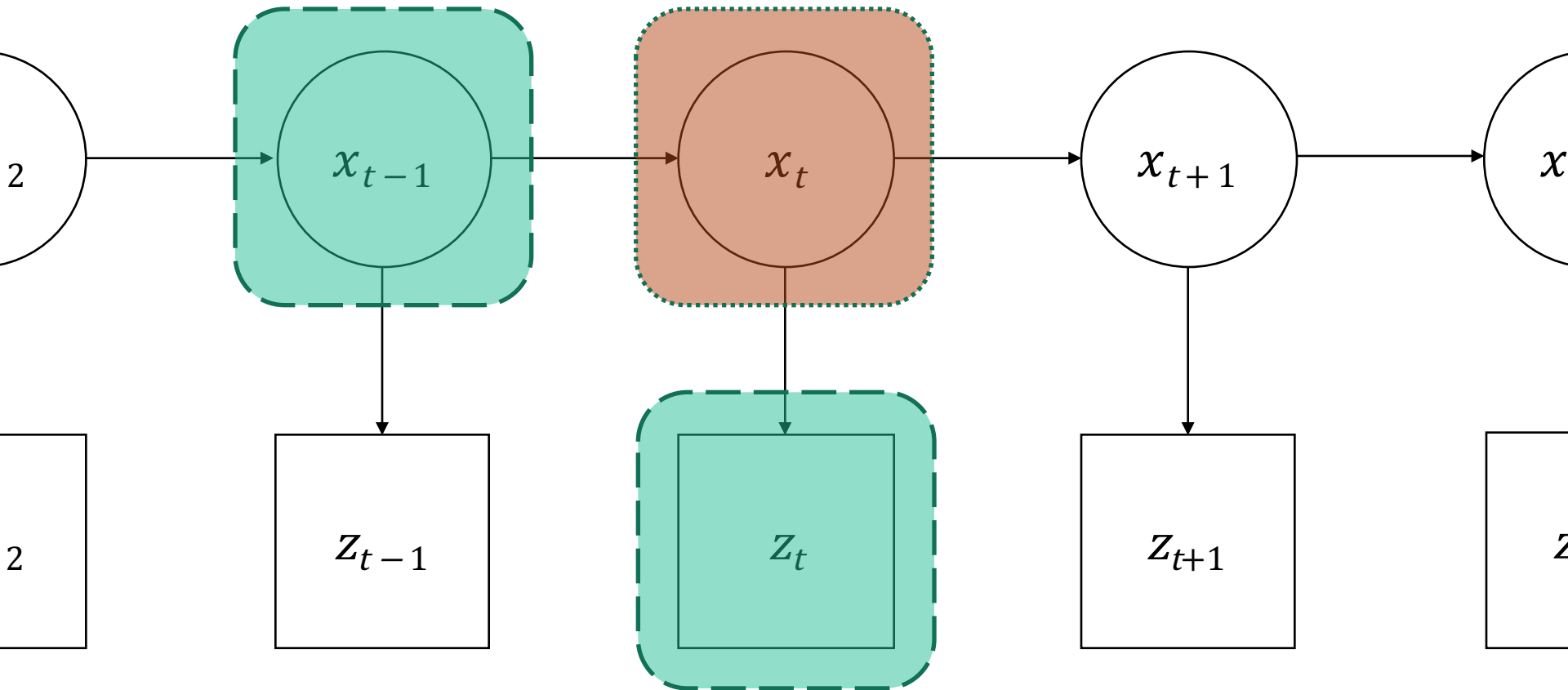
  - $x_{t+1} := [\,v \quad {}^{dv}/_{dt}\,]$

- Description of Dynamics

  - $A = \begin{bmatrix} 1 & dt \\ 0 & 1 \end{bmatrix}$

KF: Bayesian filtering

KF: Bayesian filtering

- Find x_t
- Know x_{t-1}
- Know z_t

Bayesian modeling

如何解决噪音问题?
- 从上一个状态到现在的状态应该符合物理规律（不会大起大落）
- 所以是概率性的

- Prediction using state dynamics model

$$p(x_{t+1}|x_t)$$

- Inference from noisy measurements

$$p(z_t|x_t)$$

- Model x_t with a Gaussian (mean and covariance)

$$p(x_t) = \mathcal{N}(x_t, P_t)$$

Bayesian filtering

- Apply linear dynamics

$$p(x_{t+1}|x_t) = Ap(x_t)$$

$$p(z_t|x_t) = Cp(x_t)$$

- Add noise for motion and observations

$$p(x_{t+1}|x_t) = Ap(x_t) + v_m$$

$$p(z_t|x_t) = Cp(x_t) + v_o$$

- Introduce Gaussian model of x_t

$$p(x_{t+1}|x_t) = A\mathcal{N}(x_t, P_t) + \mathcal{N}(0, \Sigma_m)$$

$$p(z_t|x_t) = C\mathcal{N}(x_t, P_t) + \mathcal{N}(0, \Sigma_o)$$

Bayesian filtering

- Consolidate expression using special properties

$$p(x_{t+1}|x_t) = A\mathcal{N}(x_t, P_t) + \mathcal{N}(0, \Sigma_m)$$

$$p(z_t|x_t) = C\mathcal{N}(x_t, P_t) + \mathcal{N}(0, \Sigma_o)$$

- Apply *linear* transform to Gaussian distributions

$$p(x_{t+1}|x_t) = \mathcal{N}(Ax_t, AP_tA^T) + \mathcal{N}(0, \Sigma_m)$$

$$p(z_t|x_t) = \mathcal{N}(Cx_t, CP_tC^T) + \mathcal{N}(0, \Sigma_o)$$

- Apply summation

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