

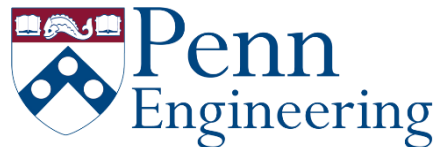
Robotics

Estimation and Learning
with Dan Lee

Week 1.

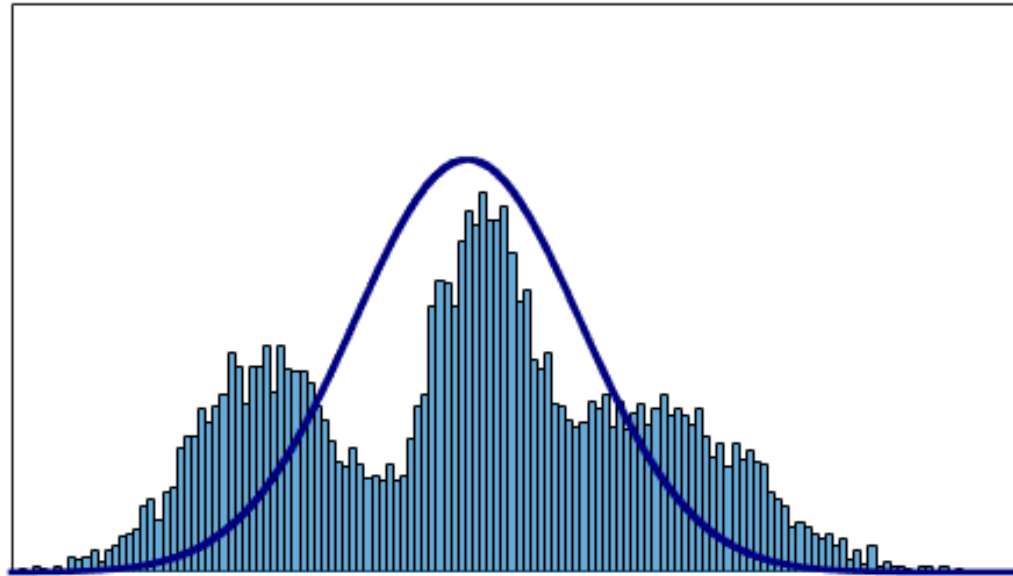
Gaussian Model Learning

1.4.1 Gaussian Mixture Model



Limitations of Single Gaussian

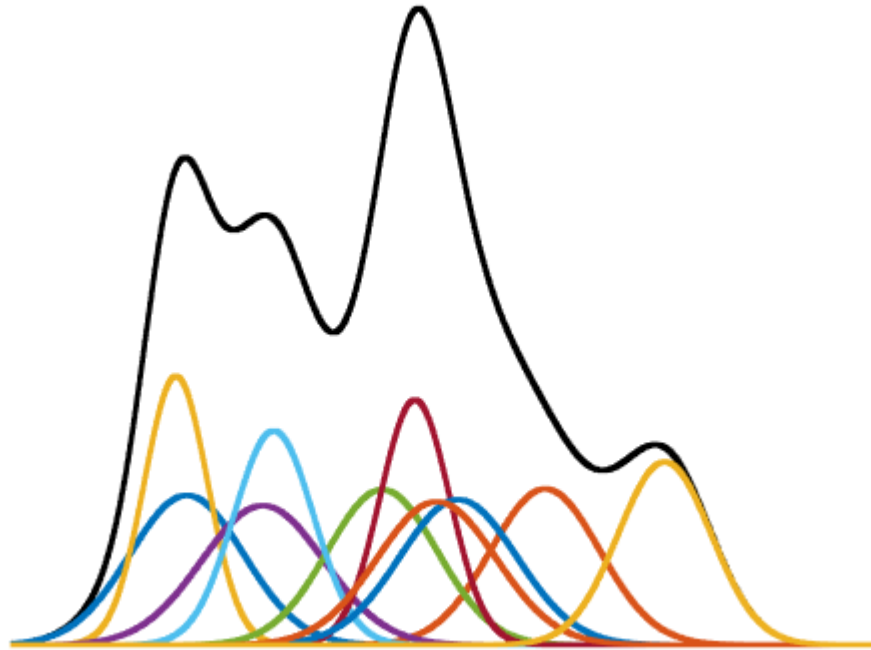
- Single Mode
- Symmetric



Gaussian Mixture Model

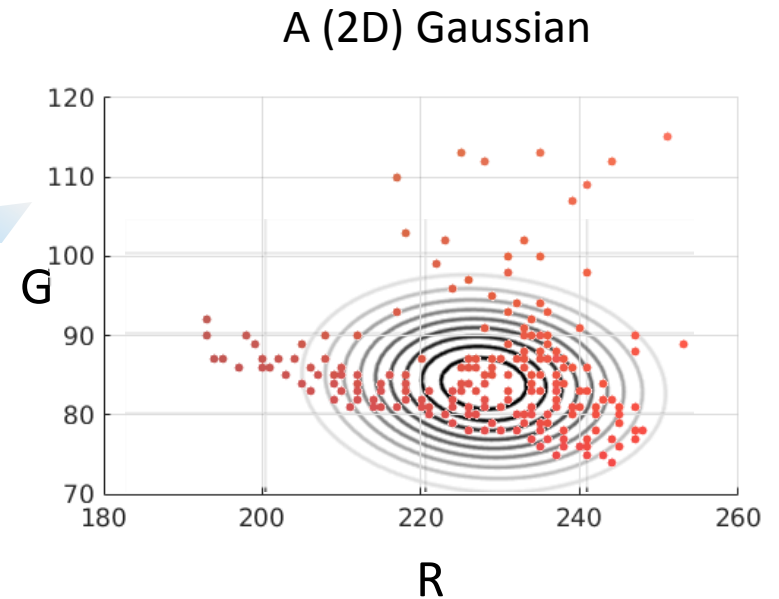
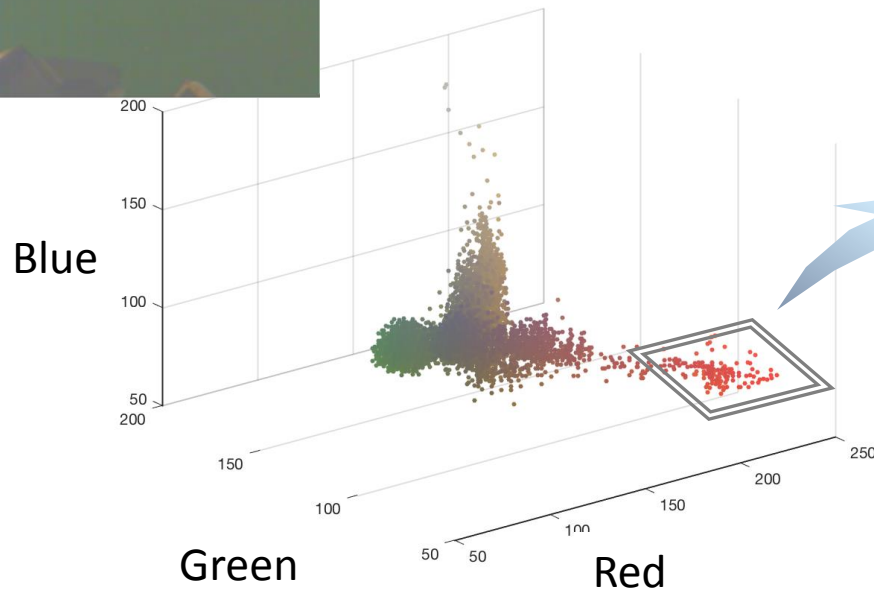
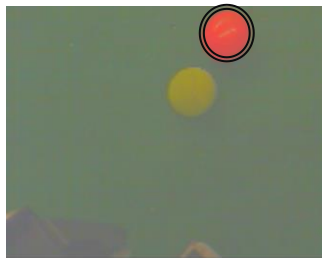
- Mixture (=Sum) of Gaussians

傅立叶！



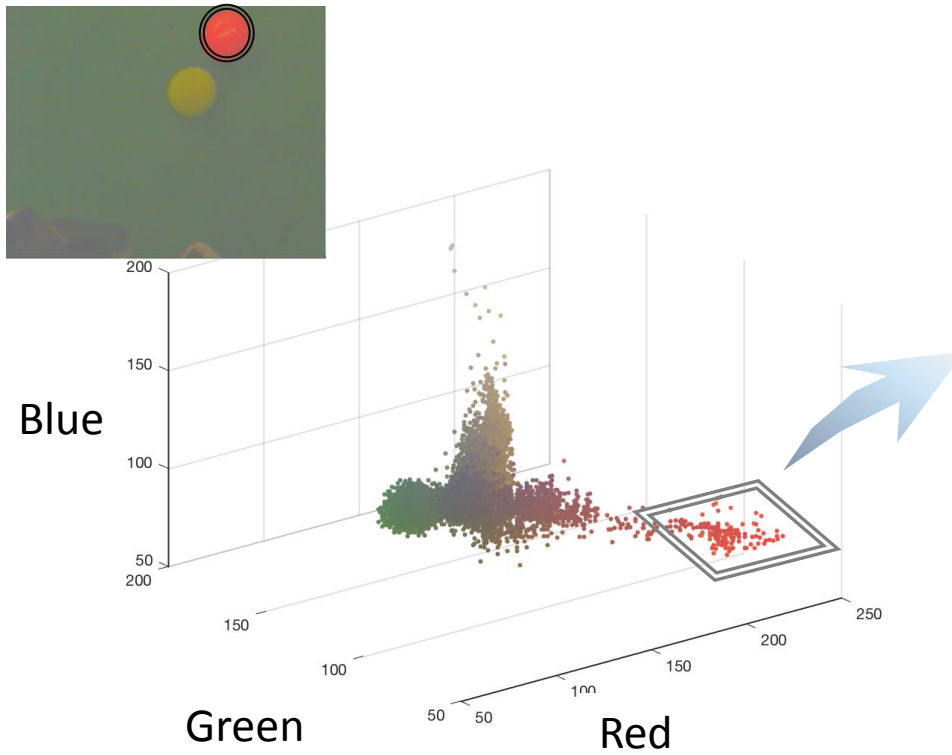
Multi-dimension Distribution: Example

- Ball color in multi-channels

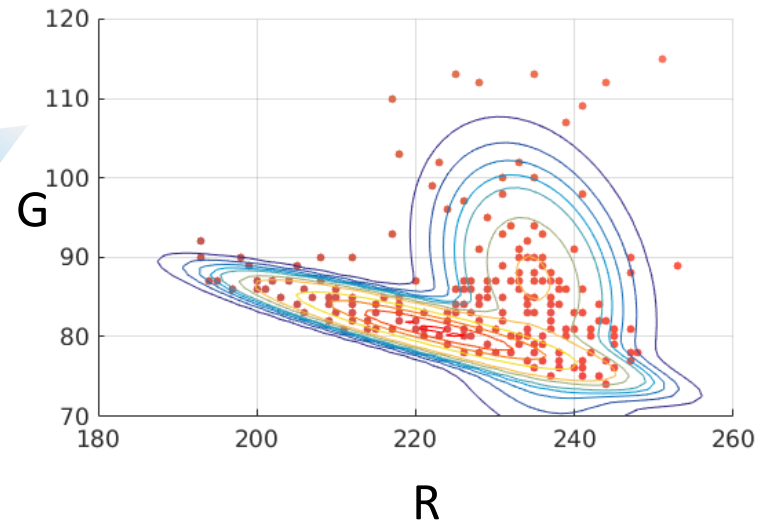


Multi-dimension Distribution: Example

- Ball color in multi-channels



A Mixture of Two Gaussians



Gaussian Mixture Model

- Mixture of Gaussians

$$p(\mathbf{x}) = \sum_{k=1}^K w_k g_k(\mathbf{x} | \boldsymbol{\mu}_k, \Sigma_k)$$

g_k : Gaussian with $\boldsymbol{\mu}_k$ and Σ_k

w_k : mixing coefficient (weight, a prior) $w_k > 0, \sum_{k=1}^K w_k = 1$

Using GMM



Flexibility



Parameters \uparrow



- No analytic solution
- Overfitting

$$\boldsymbol{\mu} = \{\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \dots, \boldsymbol{\mu}_K\}$$

$$\boldsymbol{\Sigma} = \{\boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2, \dots, \boldsymbol{\Sigma}_K\}$$

$$\boldsymbol{w} = \{w_1, w_2, \dots, w_K\}$$

K : Number of Components

Using GMM



Flexibility



Parameters \uparrow



- No analytic solution
- Overfitting

$$\boldsymbol{\mu} = \{\boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \dots, \boldsymbol{\mu}_K\}$$

$$\Sigma = \{\Sigma_1, \Sigma_2, \dots, \Sigma_K\}$$

$$w = w = 1/K$$

K : given number of Components