Exercise 5D.1

In the general population, the gestational length of uncomplicated pregnancies varies according to a normal distribution with mean 39 weeks and (known) standard deviation 2 weeks. An investigator wishes to test whether gestational ages of African American women have a different mean length, and believes that the true mean for these women is 38.5 weeks. How large a sample would be needed to detect a difference in gestational lengths of this size with 80% power for a two-sided test with level $\alpha = 0.05$?

$\mu_0 = 39, \mu_a = 38.5$
$\sigma = 2$
type I error $= \alpha = 0.05$, *two-sided test,* $\rightarrow z^*_{\alpha/2} = 1.96$
power $=$ *80%* $\rightarrow 1 - \beta = 0.8 \rightarrow \beta = 0.2 \rightarrow z^*_\beta = 0.84$

$$n = \frac{\sigma^2 \left(z^*_{\alpha/2} + z^*_\beta\right)^2}{(\mu_a - \mu_0)^2} = \frac{2^2(1.96 + 0.84)^2}{(38.5 - 39)^2} = 125.44$$

Round up: Need 126 women in the study.

Stata code to confirm answer:

```
power onemean 39 38.5, power(0.8) alpha(0.05) sd(2) knownsd
```

Exercise 5D.2

How large a sample is needed for a one-sided z-test with 90% power and $\alpha = 0.05$, if the null hypothesis assumes $\mu = 170$, we want to detect a difference of $+20$, and the standard deviation is known to be 40?

$\mu_a - \mu_0 = 20$
$\sigma = 40$
type I error $= \alpha = 0.05$, *one-sided test,* $\rightarrow z^*_\alpha = 1.645$
power = *90%* $\rightarrow 1 - \beta = 0.9 \rightarrow \beta = 0.1 \rightarrow z^*_\beta = 1.28$

$$n = \frac{\sigma^2 \left(z^*_\alpha + z^*_\beta\right)^2}{(\mu_a - \mu_0)^2} = \frac{40^2(1.645 + 1.28)^2}{20^2} = 34.2$$

Round up: Need sample size of 35.

Stata code to confirm answer:

```
power onemean 170 190, power(0.9) alpha(0.05) sd(40) knownsd onesided
```

Exercise 5D.3

The Ohio Department of Education is planning a study of SAT math scores of Ohio high school seniors. They will randomly give the exam to 500 students. Assume that the population standard deviation is known to be $\sigma = 100$. The hypotheses are:
$H_0 : \mu = 450$
$H_a : \mu > 450$
Is this test sufficiently powerful to detect an increase of 12 points in the population mean SAT score? Answer this question by calculating the power of the Z-test at $\alpha = 0.01$ against the alternative $\mu = 462$.

(1) Hypotheses stated in problem $(\mu_0 = 450)$
(2) Specific alternative to detect: $\mu_a = 462$
(3) Find rejection region:
one-sided z-test with $\alpha = 0.01 \rightarrow$ *all 0.01 in upper tail* $\rightarrow$ *critical value is* $z^* = 2.33$
reject H_0 *if* $z > z*$ *(matches direction of alternative hypothesis)*

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{\bar{x} - 450}{100/\sqrt{500}} > 2.33 \rightarrow \bar{x} > 450 + 2.33 \times \frac{100}{\sqrt{500}} = 460.41$$

reject H_0 *if* $\bar{x} > 460.41$
(4) Power = Probability of being in rejection region given H_a *is true*

$$P(\overline{X} > 460.41 | \mu_a = 462) = P\left(Z > \frac{460.41 - 462}{100/\sqrt{500}}\right) = P(Z > -0.36) = 0.6406$$

Study has 64.1% power

Stata code to confirm answer:

```
power onemean 450 462, n(500) alpha(0.01) sd(100) knownsd onesided
```