Exercise 2B.1

Assume that everyone in Ohio Appalachia has a probability of developing lung cancer within the next year of 0.001 and each resident is independent of one another. The population of Ohio Appalachia is 1.5 million. Let X be the number of people in Ohio Appalachia who will develop lung cancer within the next year.

(a) Define the distribution of X.

ANSWER: $X \sim Binomial(1500000, 0.001)$

(b) How many people do we expect to develop lung cancer in this population within the next year?

ANSWER: 1500 people

$E(X) = np = 1500000 \times 0.001 = 1500$

(c) Calculate the standard deviation of X (be explicit about the units).

ANSWER: 38.7 people

$V(X) = np(1-p) = 1500000 \times 0.001 \times (1 - 0.001) = 1498.5$ *people*^{2}
$SD(X) = \sqrt{V(X)} = \sqrt{1498.5} = 38.71$ *people*

Exercise 2B.2

Births in a hospital occur randomly at an average rate of 1.8 births per hour.

(a) What is the probability of observing 4 births in a given hour at the hospital?

ANSWER: 0.0723

X = *number of births in one hour*, $X \sim Poisson(1.8)$

$P(X = 4) = \frac{e^{-1.8} 1.8^4}{4!} = 0.0723$

(b) What is the probability of observing no births in a given hour at the hospital?

ANSWER: 0.1653

$$P(X = 0) = \frac{e^{-1.8} 1.8^0}{0!} = 0.1653$$

(c) What is the probability of observing 2 or more births in a given hour at the hospital?

ANSWER: 0.5372

$$\begin{aligned} P(X \geq 2) &= 1 - P(X < 2) = 1 - [P(X = 0) + P(X = 1)] = 1 - \left[\frac{e^{-1.8} 1.8^0}{0!} + \frac{e^{-1.8} 1.8^1}{1!}\right] \\ &= 1 - [0.1653 + 0.2975] = 1 - 0.4628 = 0.5372 \end{aligned}$$

(d) How many births do we expect to see in a given hour at the hospital?

ANSWER: 1.8

$$E(X) = \lambda = 1.8$$

Exercise 2B.3
What is the probability that if you throw a (fair, 6-sided) die 10 times it will land on "6" exactly 4 times?

ANSWER: 0.0543

$X =$ *number of times the die lands on 6*
$X \sim Binomial(10, 1/6)$ *since there are* $n = 10$ *rolls and* $p = P($*land on a 6*$) = 1/6$

$$P(X = 4) = \binom{10}{4} 0.1667^4 (1 - 0.1667)^{10-4} = 210 \times 0.0007716 \times 0.3349 = 0.0543$$

Note: if you tried to solve this via the counting method, you would find that there are $6^{10} = 60466176$ *different outcomes of the 10 rolls! It sure would take you a long time to write out all those outcomes and figure out which ones had exactly 4 sixes. . . .*

Exercise 2B.4
What is the probability that if you throw a (fair, 6-sided) die 10 times it will land on an even number on exactly half the rolls?

ANSWER: 0.2461

$X =$ *number of times the die lands on an even number*
$X \sim Binomial(10, 0.5)$ *since there are* $n = 10$ *rolls and* $p = P($*land on even*$) = 3/6 = 0.5$
"exactly half the rolls" $=$ *5 rolls*

$$P(X = 5) = \binom{10}{5} 0.5^5 (1 - 0.5)^{10-5} = 252 \times 0.03125 \times 0.03125 = 0.2461$$

Note: could also find this probability in the Binomial table