Exercise 2A.1

In the example from the lecture, the following probability mass function was given for X, where X is the final grade in 6210:

Grade:	F	D	C	B	A
Value of X:	0	1	2	3	4
Probability:	0.05	0.05	0.1	0.6	0.2

What is $E(X)$?

ANSWER: 2.85

$$\begin{aligned}E(X) &= 0 \times P(X=0) + 1 \times P(X=1) + 2 \times P(X=2) + 3 \times P(X=3) + 4 \times P(X=4) \\ &= 0 \times 0.05 + 1 \times 0.05 + 2 \times 0.1 + 3 \times 0.6 + 4 \times 0.2 \\ &= 2.85\end{aligned}$$

Exercise 2A.2

Consider a game in which you (the contestant) and the banker flip a coin. Based on the result of the fair coin flip, the following occurs:

- Heads: The banker pays the contestant $10
- Tails: The contestant pays the banker $10

How much do you expect to win in the long run? Is this a fair game?

ANSWER: $0; the game is fair since you expect to "break even"

Let X *be the "winnings" on a single flip, so that the two options are:*
Heads $\rightarrow$ *you win $10* $\rightarrow X = +10$
Tails $\rightarrow$ *you lose $10* $\rightarrow X = -10$
Since the coin is fair, the probability of heads is the same as tails, so:
$P(X = +10) = P(X = -10) = 0.5$
Thus we can calculate the expected winnings, i.e., the expected value of X*:*
$E(X) = (+10) \times P(X = +10) + (-10) \times P(X = -10) = (+10) \times 0.5 + (-10) \times 0.5 = 0$

Exercise 2A.3

According to the U.S. Census Bureau, the number of people per household in the U.S. had the distributions below for the years 1900 and 2006:

	Number of People in the Household						
Year	1	2	3	4	5	6	7
1900	5.1%	15.0%	17.6%	16.9%	14.2%	10.9%	20.3%
2006	25.5%	33.1%	16.4%	14.6%	6.7%	2.3%	1.4%

(a) If one household was randomly selected in 1900, how many people do you expect to live there?

ANSWER: 4.34

Let X be the number of people in a randomly selected 1900 household

$$\begin{aligned}E(X) &= 1 \times P(X=1) + 2 \times P(X=2) + \cdots + 6 \times P(X=6) + 7 \times P(X=7) \\ &= 1 \times 0.051 + 2 \times 0.15 + 3 \times 0.176 + 4 \times 0.169 + 5 \times 0.142 + 6 \times 0.109 + 7 \times 0.203 \\ &= 4.34\end{aligned}$$

(b) If one household was randomly selected in 2006, how many people do you expect to live there?

ANSWER: 2.564

Let X be the number of people in a randomly selected 2006 household

$$\begin{aligned}E(X) &= 1 \times P(X=1) + 2 \times P(X=2) + \cdots + 6 \times P(X=6) + 7 \times P(X=7) \\ &= 1 \times 0.255 + 2 \times 0.331 + 3 \times 0.164 + 4 \times 0.146 + 5 \times 0.067 + 6 \times 0.023 + 7 \times 0.014 \\ &= 2.564\end{aligned}$$