Exercise 4D.1

A test of the null hypothesis $H_0 : \mu = \mu_0$ gives test statistic $z = -2.12$.

(a) What is the p-value if the alternative is $H_a : \mu \neq \mu_0$?

ANSWER: 0.0340

p-value $= P(Z > |-2.12|) + P(Z < -|-2.12|)$
$= P(Z > 2.12) + P(Z < -2.12)$
$= 2 \times P(Z > 2.12)$ *by symmetry*
$= 2 \times 0.0170 = 0.0340$

(b) What is the p-value if the alternative is $H_a : \mu < \mu_0$?

ANSWER: 0.0170

p-value $= P(Z < -2.12)$
$= P(Z > 2.12)$ *by symmetry*
$= 0.0170$

(c) What is the p-value if the alternative is $H_a : \mu > \mu_0$?

ANSWER: 0.983

p-value $= P(Z > -2.12)$
$= 1 - P(Z < -2.12)$ *by complement rule*
$= 1 - P(Z > 2.12)$ *by symmetry*
$= 1 - 0.0170 = 0.983$

Exercise 4D.2

The p-value for a two-sided significance test of $H_0 : \mu = 9$ is 0.082.

(a) Do you reject the null hypothesis at level $\alpha = 0.05$?

ANSWER: No, since 0.082 $>$ 0.05

(b) Do you reject the null hypothesis at level $\alpha = 0.1$?

ANSWER: Yes, since 0.082 $<$ 0.1

(c) Would a 95% confidence interval for μ contain 9? Why?

ANSWER: Yes. Since we fail to reject at $\alpha = 0.05$, the null hypothesis value of 9 will be in the 95% confidence interval.

(d) Would a 90% confidence interval for μ contain 9? Why?

ANSWER: No. Since we reject at the $\alpha = 0.1$ level, the null hypothesis value of 9 will not be in the 90% confidence interval.

Exercise 4D.3

For each of the following scenarios, state whether a type I error, a type II error, or no error will be made. Assume all tests are two-sided at the $\alpha = 0.05$ level.

(a) A test of $H_0 : \mu = 10$ produces a p-value = 0.31 and the true population mean is 14.

ANSWER: Type II error

The p-value 0.31 $>$ 0.05, so you fail to reject the null, but this is the wrong conclusion because the null hypothesis is not true (truth is $\mu = 14$). Failing to reject when you should reject is a Type II error.

(b) A test of $H_0 : \mu = 6$ produces a p-value = 0.02, and the true population mean is 10.

ANSWER: No error made.

The p-value 0.02 $<$ 0.05, so you reject the null, which is the correct conclusion because the null is not true (truth is $\mu = 10$).

(c) A test of $H_0 : \mu = 12$ produces a p-value = 0.005 and the true population mean is 12.

ANSWER: Type I error

The p-value 0.005 $<$ 0.05, so you reject the null, but this is the wrong conclusion because the null hypothesis is true (truth is $\mu = 12$). Rejecting when you shouldn't reject is a Type I error.