Exercise 5B.1

A multimedia program designed to improve dietary behavior among low-income women was evaluated by randomly assigning women to intervention and control groups. One outcome was the score on a knowledge test taken about 2 months after the program. A summary of the data is below:

Group	n	$\bar{x}$	s
Intervention	15	5.08	1.15
Control	12	4.33	1.16

Is there evidence that there is a difference between intervention and control?

$n_1 = 15, \bar{x}_1 = 5.08, s_1 = 1.15$ *and* $n_2 = 12, \bar{x}_2 = 4.33, s_2 = 1.16$

$s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2} = \frac{(15-1)1.15^2+(12-1)1.16^2}{15+12-2} = 1.33 \rightarrow s_p = \sqrt{1.33} = 1.15$

$H_0 : \mu_1 = \mu_2$

$H_a : \mu_1 \neq \mu_2$

$t = \frac{\bar{x}_1-\bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{5.08-4.33}{1.15\sqrt{\frac{1}{15}+\frac{1}{12}}} = \frac{0.75}{0.4454} = 1.68$

Under $H_0, T \sim t_{n_1+n_2-2} = t_{25}$

Critical Value Method:

critical value from t_{25} *with 0.025 in the tail is* $t^* = 2.060$

Is $|t| > t^*$*? No,* $|1.68| < 2.060 \rightarrow$ *Fail to reject* H_0

P-value Method:

p-value $= 2 \times P(t_{25} > |1.68|) = 2 \times P(t_{25} > 1.68)$

$P(t_{25} > 1.708) = 0.05$ *and* $P(t_{25} > 1.316) = 0.1$*, so* $P(t_{25} > 1.68)$ *between 0.05 & 0.1*

Thus p-value = between 0.1 *and* 0.2 *(Exact p-value from Stata* $= 0.105$*)*

Is p-value $< \alpha$*? No,* $\rightarrow$ *Fail to reject* H_0

Exercise 5B.2

An entrepreneur wants to compare the daily sales for two different designs of web pages for her Internet business. She randomly assigns the next 60 days to either Design A or Design B, 30 days to each.

(a) Would you use a one-sided or two-sided significance test? Why?

two-sided; no reason to think A>B or vice-versa

(b) How many degrees of freedom will the test statistic have?

$n_1 = 30, n_2 = 30$
DF $= n_1 + n_2 - 2 = 30 + 30 - 2 = 58$

(c) The t statistic for comparing the mean sales is 2.06. Draw a conclusion for the entrepreneur.

$H_0 : \mu_A = \mu_B$ *where* μ_A *= mean sales under Design A,* μ_B *= mean sales under Design B*
$H_a : \mu_A \neq \mu_B$
$t = 2.06$ *(given)*
Under H_0, $T \sim t_{58}$ *DF=58 not on table, approximate with DF=50 (closest smaller value)*

Critical Value Method:
critical value from t_{50} *with 0.025 in the tail is* $t^* = 2.009$
Is $|t| > t^*$*? Yes,* $|2.06| > 2.009 \rightarrow$ *Reject* H_0

P-value Method:
p-value $\approx 2 \times P(t_{50} > |2.06|) = 2 \times P(t_{50} > 2.06)$
$P(t_{50} > 2.403) = 0.01$ *and* $P(t_{50} > 2.009) = 0.025$*, so* $P(t_{50} > 2.06)$ *between 0.01 & 0.025*
Thus p-value = between 0.02 *and* 0.05 *(Exact p-value from Stata* $= 0.044$*)*
Is p-value $< \alpha$*? Yes,* $\rightarrow$ *Reject* H_0

Exercise 5B.3

In the University of New Hampshire student survey data, information is collected on each student's gender and his/her GPA. This sample was used to test whether there is a difference in GPA between male and female students (at UNH in 1988). Use the Stata output below to answer the questions on the next page.

```
. ttest gpa, by(gender)

Two-sample t test with equal variances
------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
  Female |     119    2.882941    .0412487    .4499705    2.801257    2.964625
    Male |      99    2.718788    .0458589      .45629    2.627782    2.809793
---------+--------------------------------------------------------------------
combined |     218    2.808394    .0310989    .4591705      2.7471    2.869689
---------+--------------------------------------------------------------------
    diff |            .1641533    .0616013                .0427366      .28557
------------------------------------------------------------------------------
    diff = mean(Female) - mean(Male)                              t =   2.6648
Ho: diff = 0                                     degrees of freedom =      216

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.9959         Pr(|T| > |t|) = 0.0083          Pr(T > t) = 0.0041
```

(a) What is the estimate of the difference in GPA, females minus males?

0.1641533

(b) What is the 95% confidence interval for the mean difference in GPA between females and males?

(0.0427366, 0.28557)

(c) What is the null hypothesis being tested?

$H_0 : \mu_F = \mu_M$ *where* μ_F *= mean GPA for females,* μ_M *= mean GPA for males*

(d) What is the test statistic for the test?

t*=2.6648*

(e) What is the distribution of this test statistic under the null hypothesis?

$T \sim t_{216}$

(f) If my alternative hypothesis is that females and males have different GPAs, what is the p-value?

0.0083

(g) If my alternative hypothesis is that females have higher GPA than males, what is the p-value?

0.0041