Exercise 2D.1

Suppose the distribution of height in feet in the U.S. is normally distributed with a mean of 5.5 feet and a standard deviation of 0.25 feet. What is the distribution of height in inches?

ANSWER: N(66, 9)

X = *height in feet*
$E(X) = 5.5$
$SD(X) = 0.25$
$V(X) = SD(X)^2 = 0.25^2 = 0.0625$
Y = *height in inches*
$Y = 12X$
$E(Y) = E(12X) = 12E(X) = 12 \times 5.5 = 66$ *inches*
$V(Y) = V(12X0 = 12^2V(X) = 144 \times 0.0625 = 9$ *inches*^{2}
Since X *is normally distributed,* Y *will be too*
$Y \sim N(66, 9)$

Exercise 2D.2

One kilogram is equal to 2.2 pounds. The weight of a randomly selected full term infant can be modeled as a random variable with mean 7.8 and standard deviation 1.77 pounds. Describe this distribution in kilograms.

ANSWER: mean = 3.55 kg; standard deviation = 0.805; variance = 0.647

L = *weight of a randomly selected full term infant in* <u>*pounds*</u> *(lbs)*
$E(L) = 7.8$ *lbs*
$SD(L) = 1.77$ *lbs*
$V(L) = SD(L)^2 = 1.77^2 = 3.1329$ *lbs*^{2}
K = *weight of a randomly selected full term infant in* <u>*kilograms*</u> *(kg)*
$K = L/2.2$ *(if 1 kg = 2.2 lbs, then you would take the number of lbs and divide by 2.2 to get kg)*
$E(K) = E(L/2.2) = E(L)/2.2 = 7.8/2.2 = 3.545$ *kg*
$V(K) = V(L/2.2) = (1/2.2)^2 V(L) = (1/2.2)^2 \times 3.1329 = 0.6473$ *kg*^{2}
$SD(K) = \sqrt{V(K)} = \sqrt{0.6473} = 0.8045$

Exercise 2D.3

If a random variable Z is standard normal, what is the distribution of $2Z - 10$?

ANSWER: $N(-10, 4)$

$Z \sim N(0, 1) \rightarrow E(Z) = 0, V(Z) = 1$
$E(2Z - 10) = 2E(Z) - 10 = 2 \times 0 - 10 = -10$
$V(2Z - 10) = 2^2 \times V(Z) = 4 \times 1 = 4$
Since Z *is normally distributed,* $2Z - 10$ *will be too*
Distribution is $N(-10, 4)$

Exercise 2D.4

If a random variable X is normally distributed with a mean of 25 and a standard deviation of 5, what will be the distribution of $\frac{X-25}{5}$?

ANSWER: $N(0,1)$

This is just a z-score transformation – subtracting the mean and then dividing by the standard deviation, thus the distribution will be standard normal: $N(0,1)$

Exercise 2D.5

The weight of jaguars is normally distributed with a mean of 168 pounds and a variance of 121 pounds^2.

(a) A single jaguar is captured and its weight is measured; it weighs 192 pounds. What is the z-score for this jaguar?

ANSWER: 2.18

$x = 192$ *(weight of the specific jaguar captured)*
$\mu = 168, \sigma^2 = 121 \rightarrow \sigma = \sqrt{121} = 11$
$$z = \frac{x - \mu}{\sigma} = \frac{192 - 168}{11} = 2.18$$

(b) What is the probability that a randomly captured jaguar weighs 192 pounds or more?

ANSWER: 0.0146

$X =$ *weight of a randomly captured jaguar*
$X \sim N(168, 121)$
$$P(X > 192) = P\left(Z > \frac{192 - 168}{11}\right) = P(Z > 2.18) = 0.0146 \text{ (Normal Table, Column B)}$$

(c) Another jaguar is captured and it weighs 152 pounds. What is the probability that a randomly captured jaguar weighs 152 pounds or less?

ANSWER: 0.0735

$$P(X < 152) = P\left(Z < \tfrac{152-168}{11}\right) = P(Z < -1.45) = P(Z > 1.45) = 0.0735 \text{ (Column B)}$$

Exercise 2D.6

Let X be the IQ scores for a certain population, and assume that X follows a normal distribution with $\mu = 100$ and $\sigma = 15$. Find x such that:

(a) $P(X > x) = 0.25$ (What percentile is this?)

ANSWER: 110.05 (75th percentile)

25% of values above $\rightarrow$ *75% of values below* $\rightarrow$ *75th percentile* $\rightarrow P(Z < a) = 0.75$
From Normal Table, Column A: $P(Z < 0.67) = 0.7486 \rightarrow a = 0.67$
$x = a \times \sigma + \mu = 0.67 \times 15 + 100 = 110.05$

(b) $P(X < x) = 0.95$ (What percentile is this?)

ANSWER: 124.75 (95th percentile)

95% of values below $\rightarrow$ *95th percentile* $\rightarrow P(Z < a) = 0.95$
From Normal Table, Column A: $P(Z < 1.65) = 0.9505 \rightarrow a = 1.65$
$x = a \times \sigma + \mu = 1.65 \times 15 + 100 = 124.75$

(c) $P(X > x) = 0.75$ (What percentile is this?)

ANSWER: 89.95 (25th percentile)

75% of values above $\rightarrow$ *25% of values below* $\rightarrow$ *25th percentile* $\rightarrow P(Z < a)$
Need trick of symmetry to find in table: $P(Z < a) = P(Z > -a) = 0.25$
From Normal Table, Column A: $P(Z > 0.67) = 0.2514 \rightarrow -a = 0.67 \rightarrow a = -0.67$
$x = a \times \sigma + \mu = -0.67 \times 15 + 100 = 89.95$

Note: could recognize that the 25th percentile would be as far below the mean as the 75th was above. . .
The 75th percentile is $110.05 = 100 + 10.05$, *thus the 25th percentile* $= 100 - 10.05 = 89.95$