Exercise 1B.1

While visiting the Three Bears' house, Goldilocks finds three bowls of porridge. Upon tasting a bowl she will declare it either Too Hot, Too Cold, or Just Right. How many possible outcomes are there?

ANSWER: 27

$n = 3$ *bowls tasted*
$k = 3$ *possible outcomes for each bowl.*
Total number of outcomes $= k^n = 3^3 = 27$

Exercise 1B.2

We have 4 imaging modalities available for diagnosing a brain tumor: CT, MRI, MRS, PET.

(a) If all four modalities have to be used, in how many different orders can the physician request them?

ANSWER: 24

$n = 4$ *modalities*
\# *of ways to order* $= n! = 4! = 24$

(b) Suppose that only three modalities are used and if they all agree, the patient will be considered positive for a brain tumor. How many different combinations of the three chosen modalities can the physician use?

ANSWER: 4

$n = 4$ *modalities*
$k = 3$ *chosen without regard to order*

$$\# \text{ of ways} = {}_nC_k = \binom{n}{k} = \binom{4}{3} = \frac{4!}{3!(4-3)!} = 4$$

(c) How many different sequences (orderings) of three modalities could the physician order?

ANSWER: 24

$n = 4$ *modalities*
$k = 3$ *chosen **with** regard to order*

$$\# \text{ of ways} = {}_nP_k = {}_4P_3 = \frac{4!}{(4-3)!} = 24$$

Exercise 1B.3

A pair of (standard, six-sided) dice are rolled. Let A be the event that the first die is a 6 and B be the event that the second die is a 6.

(a) Calculate $P(A \cap B)$.

ANSWER: 1/36

$A \cap B$ = *6 on first roll and 6 on second roll*
Each roll has $k = 6$ *outcomes and we are repeating the experiment* $n = 2$ *times*
Total number of outcomes = $k^n = 6^2 = 36$
Total number of outcomes associated with the event of interest = 1 ({6,6})
$P(A \cap B) = 1/36$

(b) Calculate $P(A \cup B)$.

ANSWER: 11/36

$P(A) = P($*6 on first roll*$) = 1/6$
$P(B) = P($*6 on second roll*$) = 1/6$
$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 1/6 + 1/6 - 1/36 = 11/36$

Exercise 1B.4

A social scientist is recruiting adults who have either diabetes or asthma to participate in a pilot study investigating strategies for managing chronic disease. She will cold-call adults in Columbus, Ohio to recruit study subjects. In Ohio, 6.8% of adults have asthma, 10.5% of adults have diabetes, and 0.8% of adults have both.

(a) What is the probability that a randomly called adult will be eligible for her study?

ANSWER: 0.165

A = *subject has asthma,* $P(A) = 0.068$
B = *subject has diabetes,* $P(B) = 0.105$
$A \cap B$ = *subject has asthma and diabetes,* $P(A \cap B) = 0.008$
"eligible" = *has asthma OR diabetes* = $P(A \cup B)$
$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.068 + 0.105 - 0.008 = 0.165$

(b) What is the probability that a randomly called adult will not be eligible for her study?

ANSWER: 0.835

"not eligible" = *complement of "eligible"* = $(A \cup B)^C$
$P((A \cup B)^C) = 1 - P(A \cup B) = 1 - 0.165 = 0.835$

Exercise 1B.5

Suppose there are two tests for a disease, test G and test H. In the whole population, test G gives a positive result 20% of the time and test H gives a positive result 30% of the time. Test G and test H both give a positive result 10% of the time.

(a) What is the probability of either test G or test H being positive?

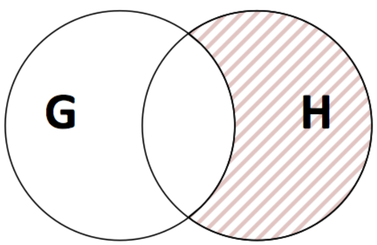
ANSWER: 0.4

$P(G) = 0.2$
$P(H) = 0.3$
$P(G \cap H) = 0.1$
$P(G \cup H) = P(G) + P(H) - P(G \cap H) = 0.2 + 0.3 - 0.1 = 0.4$

(b) What is the probability of test G being negative and test H being positive? (A Venn Diagram may be useful. . .)

ANSWER: 0.2

We want "H and not G", which in the Venn diagram is shaded:



$P(H \cap G^C) = P(H) - P(G \cap H)$ *(all of H, minus the intersection with G)*
$P(H \cap G^C) = 0.3 - 0.1 = 0.2$

Exercise 1B.6

CHALLENGE: In Texas Hold 'Em poker, each player makes the strongest hand he can with 5 cards. Strength of the hand is based on the probability of that specific combination of 5 cards. How likely is four of a kind? ("four of a kind" = 4 cards all the same #, plus 1 other card) Note that there are 52 cards in a standard deck.

ANSWER: 0.00024

Number of possible hands of 5 cards $= \binom{52}{5} = 2598960$
Number of ways to have a hand with 4 of same #, plus one other card
$=$ *(# of ways to have 4 of same #)* $\times$ *(# of ways to select the other card, i.e., not one of those 4)*
$= 13 \times (52 - 4) = 13 \times 48 = 624$
$P($*4 of a kind*$) = 624/2598960 = 0.00024$