Exercise 1C.1

What is the probability of two die rolls summing to 5, given that (at least) one of the rolls is a 2?

ANSWER: 2/11

Let A = at least one roll is a 2, B = rolls sum to 5

We want $P(B|A) = \dfrac{P(A \cap B)}{P(A)}$

To find P(A), let X = 2 on first roll, Y = 2 on second roll

$P(A) = P(X \cup Y) = P(X) + P(Y) - P(X \cap Y) = 1/6 + 1/6 - 1/36 = 11/36$

To find $P(A \cap B)$, note that when rolling the die twice you have:

$k = 6$ *outcomes per roll, repeat experiment (roll die)* $n = 2$ *times* $\rightarrow k^n = 6^2 = 36$ *total outcomes*

Of those, there are only 2 ways you can sum to 5 with one die a 2: {2,3}, {3,2}

Therefore, $P(A \cap B) = 2/36$

Thus,

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{2/36}{11/36} = 2/11$$

Note: another way to do this would be to write out all the possible ways you can roll 2 die and have at least one of them be a 2 (event A) – that's subsetting the sample space. Then determine what fraction of these have die that sum to 5 (event B). So you would have:

Reduced sample space = {2,1},{2,2},{2,3},{2,4},{2,5},{2,6},{1,2},{3,2},{4,2},{5,2},{6,2}

= 11 outcomes

Of these, only 2 of them sum to 5: {2,3},{3,2}

Thus the desired probability is 2/11

Exercise 1C.2

Twenty-three brain tumor patients went through a radiotherapy. Time to recurrence of brain metastasis after the treatment was recorded. The proportion of subjects metastasis-free for four weeks was 0.783. The proportion of subjects metastasis-free for eight weeks was 0.566. What is the chance that a patient surviving four weeks would be metastasis-free for another four weeks?

Hint: being met-free for 8 weeks means being met-free for the first four weeks **and** *the next four weeks.*

ANSWER: 0.723

Let A = *met-free for 1st 4 weeks,* B = *met-free for 2nd 4 weeks*
Given in problem:
$P(\text{met-free for 1st 4 weeks}) = P(A) = 0.783$
$P(\text{met-free for 8 weeks}) = P(\text{met-free for 1st 4 weeks AND met-free for 2nd 4 weeks})$
$= P(A \cap B) = 0.566$
Want to know: $P(\text{met-free for 2nd 4 weeks GIVEN met-free for 1st 4 weeks}) = P(B|A)$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.566}{0.783} = 0.723$$

Exercise 1C.3

Suppose I have a bag containing two white balls and one black ball. Let:

$A =$ white ball on the first draw from the bag
$B =$ black ball on the second draw

(a) If I put balls back into the bag whenever I draw one, are A and B independent?

ANSWER: Yes

$P(B) = 1/3$ regardless of whether the first draw is white (A) or not white (A^C), since the ball that is drawn is replaced. This means that $P(B|A) = P(B|A^C) = P(B)$, so A and B are independent.

(b) What about if I keep balls out of the bag when I draw them?

ANSWER: No

The probability of B changes depending on whether A or A^C happens.
$P(B|A) = 1/2$ *($A =$ white ball drawn and removed on 1st draw, so 1 white and 1 black ball left)*
$P(B|A^C) = 0/2$ *($A^C =$ black ball drawn and removed on 1st draw, so 2 white balls left)*
$P(B|A) \neq P(B|A^C) \rightarrow A$ *and* B *are not independent*