SDS384-7 Project — Bayesian Signal Segmentation

Yuyang Xie (yx2979)

May 4, 2016

Introduction

Signal segmentation is an important step before feature extraction when analyzing signals like audio, video, biomedical signals, etc. Many different methods based on statistical analysis have been proposed to address this problem [3]. Figure 1 gives an example of audio signal segmentation using typical detection threshold method.

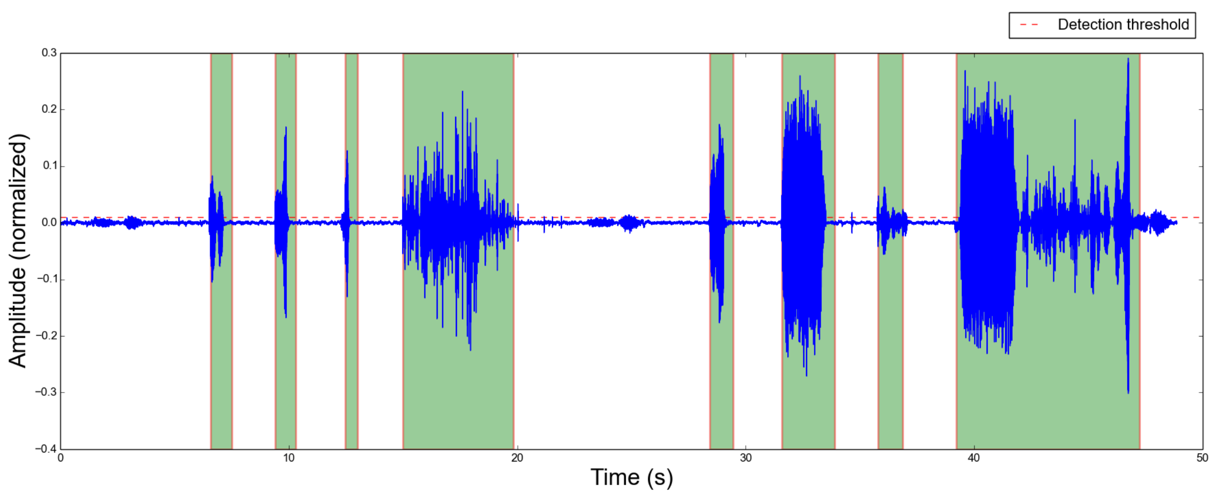

Figure 1: Example of Audio Signal Segmentation

However, there are few literature solving this problem from a Bayesian viewpoint, except a few notable publications [1, 2, 4]. In this project, the Bayesian signal segmentation algorithm will be studied and applied to segment both simulation signals and biomedical signals.

Methodology

The proposed signal segmentation algorithm slightly modifies the algorithms presented in [4]. In the project, the number of segment is assumed to be known and denoted as N_s, while the change point for the i^{th} segment is denoted as t_i, and the length of i^{th} segment is denoted as T_i. The noise level is assumed to be the same over all the segments, and having standard deviation σ_N.

Time Series Model

We need a time series model to describe signals in each segment. For the i^{th} segment of signal from t_i to t_{i+1}, the time series model is assumed to be an autoregressive (AR) model of order p and given as:

$$y_{t_i:t_{i+1}} = X_{t_i:t_{i+1}}\beta_i + \beta_{0_i} + \epsilon_{t_i:t_{i+1}}$$

where,

$$y_{t_i:t_{i+1}} = [y_{t_i}, y_{t_i}+1, \ldots, y_{t_{i+1}}]^T$$

$$X_{t_i:t_{i+1}} = \begin{bmatrix} y_{t_i-1} & \cdots & y_{t_i-p} \\ \vdots & \ddots & \vdots \\ y_{t_{i+1}-1} & \cdots & y_{t_{i+1}-p} \end{bmatrix}$$

$$\beta_i = [\beta_i^1, \beta_i^2, \ldots, \beta_i^p]^T$$

$$\epsilon_{t_i:t_{i+1}} \sim N(0, \sigma_N^2 I)$$

Parameter Inference and Prior Selection

In this project, we need to estimate model parameters (β_i and β_{0_i}), noise level σ_N and the change time (t_i). Denote L as length of data, and assume the number of segments are given. The Following priors are chosen for inference. Please be noted the signals are normalized to [-1,1]. Therefore, the mean level β_{0_i} are in the range of [-1,1].

$$\beta_i \sim N(\mu_i, 2)$$

$$\beta_{0_i} \sim N(0, 2)$$

$$e_t \sim N(0, \sigma_N^2)$$

$$\mu_i \sim N(0, 2)$$

$$\frac{1}{\sigma_N} \sim Gamma(0.001, 0.001)$$

$$\frac{T_i}{L} \sim Unif(0, 1)$$

A graph representation of the Bayesian signal segmentation model

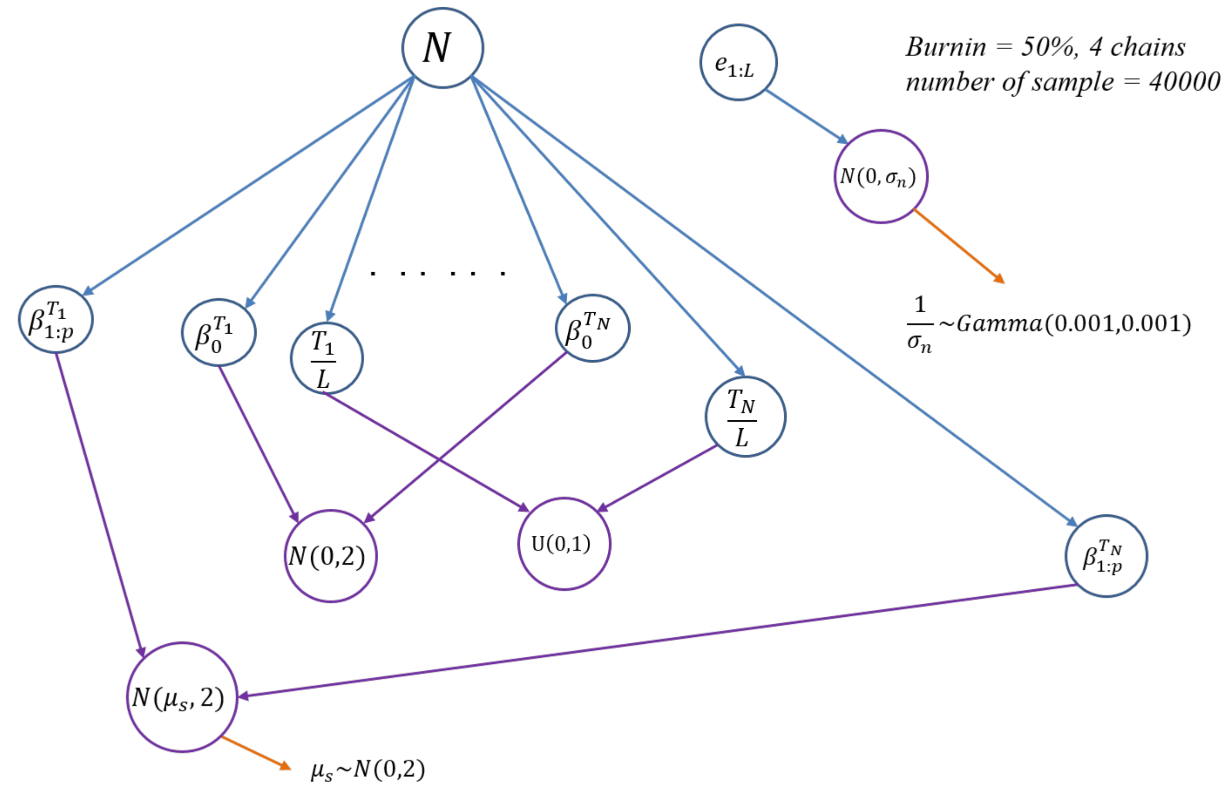

Figure 2: Graph Representation of Bayesian Signal Segmentation Model

Result and Discussion

(a) Simulation Data: Random Walk + Noise Process

Fig. 3 shows the result for the first case. The simulation data comes from the following equations:

$$y_s = \begin{cases} y_{s-1} + e_s, & s \in [1, t_1) \\ 0.2y_{s-1} + e_s, & s \in [t_1, t_2) \\ y_{s-1} + e_s, & s \in [t_2, t_3) \\ 0.2y_{s-1} + e_s, & s \in [t_3, L] \end{cases} \tag{1}$$

where, $t_1 =$ The 1st and 3rd segments are random walk while the 2nd and 4th segments are noise with damping. We can see that the algorithm performs as expected and segment the signal into four segments correctly.

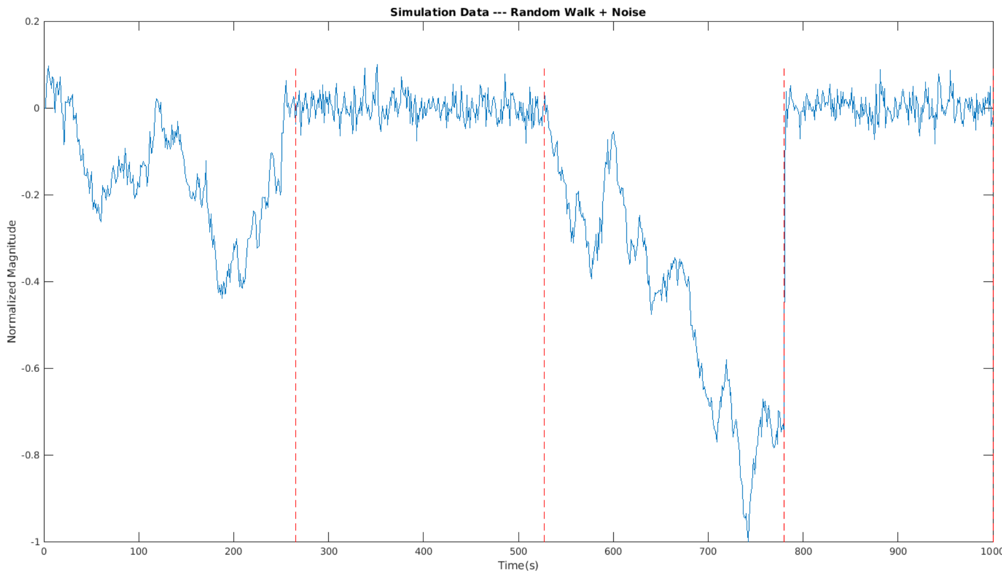

Figure 3: Random Walk + Noise Process

Model Diagnostics

In this case, the parameter β_1 is of our interest, since only β_1 is changed in the dynamic equations. Denote $\hat{\beta}_1$ as the posterior mean of β_1. Figure.4 shows the trace and posterior density of β_1 at different segments, with detailed statistics shown in the following table.

Segment	β_1	95% C.I. of $\hat{\beta}_1$	$\hat{\beta}_1$	Rhat
1	1	(0.79352,1.0246)	0.9068	1.0416
2	0.2	(0.07016,0.3218)	0.1946	1.5144
3	1	(0.76599,1.0334)	0.9063	1.0113
4	0.2	(0.11285,0.2928)	0.2021	1.000

Table 1: Statistics for β_1

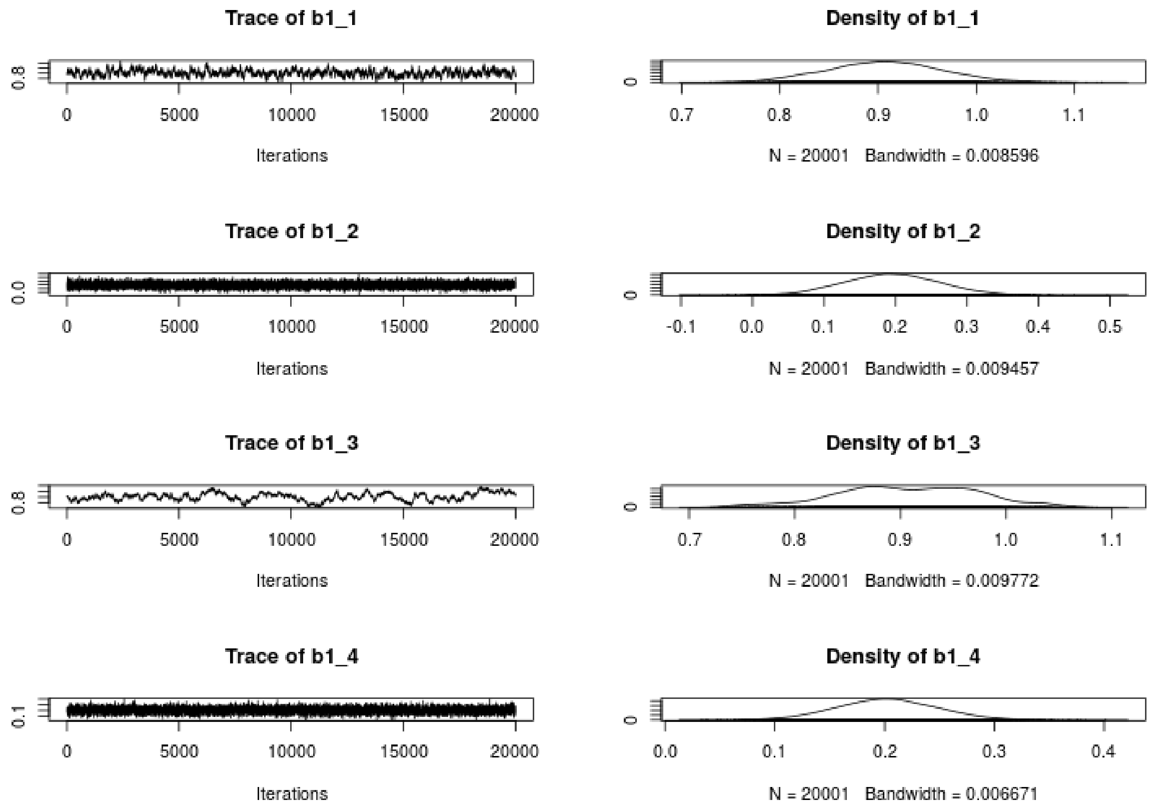

Figure 4: β_1 at Different Segments

There are two major observations. Firstly, all the β_1 in different segments fall into 95% confidence interval. It means that Bayesian Signal Segmentation algorithm perfectly segmented the signal. The second observation is that Rhat for the 2nd segment is away from 1. But it still correctly finds the parameters.

(b) Simulation Data: Changing Mean Levels

Fig. 5 shows the result for the second case. The simulation data comes from the following equations:

$$y_s = \begin{cases} 0.2y_{s-1} + e_s, \quad s \in [1, t_1) \\ 1 + 0.2y_{s-1} + e_s, \quad s \in [t_1, t_2) \\ y_{s-1} + e_s, \quad s \in [t_2, t_3) \\ -1 + 0.2y_{s-1} + e_s, \quad s \in [t_3, L] \end{cases} \tag{2}$$

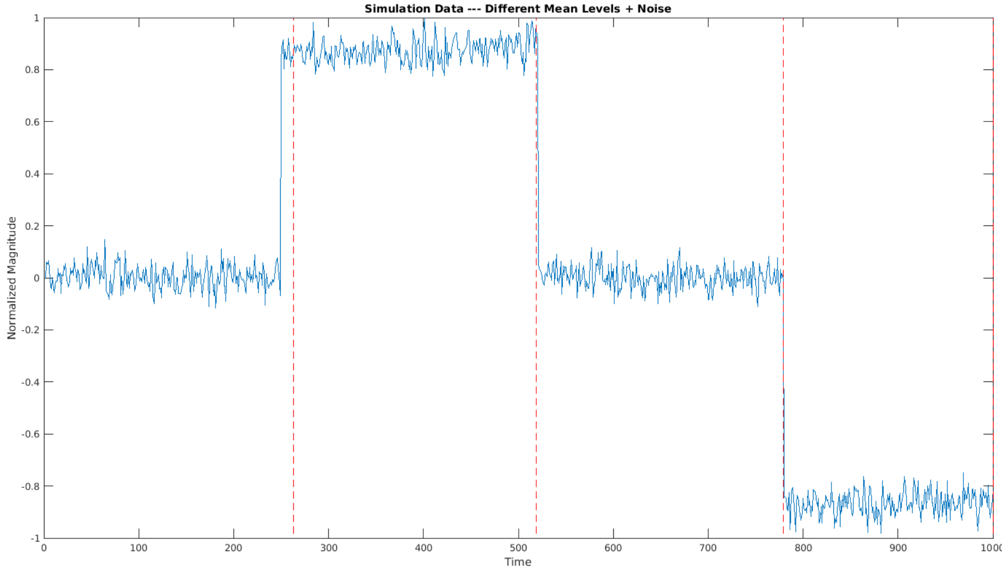

Figure 5: Noise Process with Changing Mean Levels

Model Diagnostics

In this case, the parameter β_0 is of our interest, since only β_0 is changed in the dynamic equations. Denote $\hat{\beta}_0$ as the posterior mean of β_0. Figure.6 shows the trace and posterior density of β_0 at different segments, with detailed statistics shown in the following table.

Segment	β_0	95% C.I. of $\hat{\beta}_0$	$\hat{\beta}_1$	Rhat	$\hat{\beta}_1$	$\hat{\beta}_2$
1	0	(-0.001631,0.011994)	0.005194	1.0001	0.8109	0.1437
2	1	(0.420019,0.733174)	0.575038	1.0017	0.4070	-0.0667
3	0	(-0.005389,0.007764)	0.001099	1.0012	0.2566	0.0072
4	-1	(-0.885713,-0.718859)	-0.797546	1.0057	0.1002	-0.0116

Table 2: Statistics for β_1

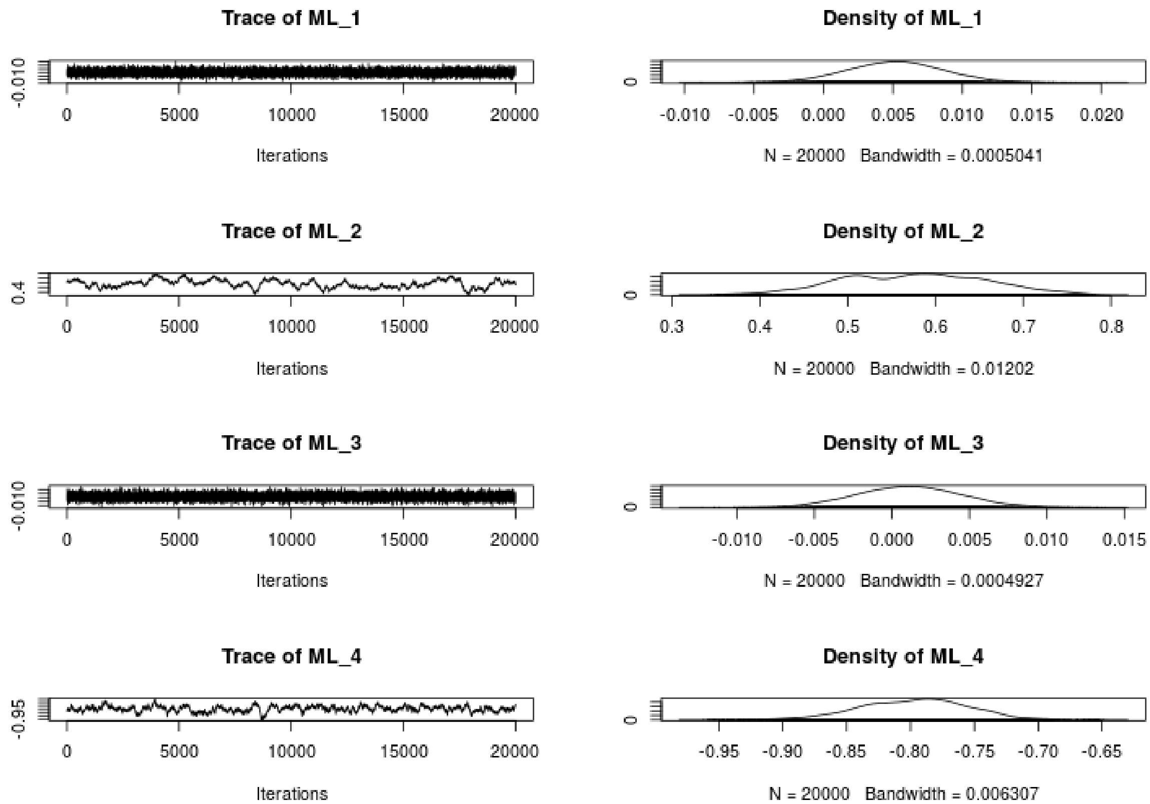

Figure 6: β_0 At Different Segments

There are two major observations. Firstly, the β_0 in the segments 2 and 4 did not fall into 95% confidence interval. It means that Bayesian Signal Segmentation algorithm did not capture the dynamics in segments 2 and 4. The second observation is that Rhat for all the segment are close to 1. It means that convergence of MCMC does not necessarily mean getting the "true" parameters.

(c) sEMG Signal from Temporalis Muscle

In this section, I would like to see how this segmentation algorithm work on real data, as given in Figure.7. This segment of signal comes from surface electromyographic (sEMG) signal of a Temporalis muscle, during mouth opening-closing motion. The two stronger portions of signal represents the mouth opening and mouth closing process.

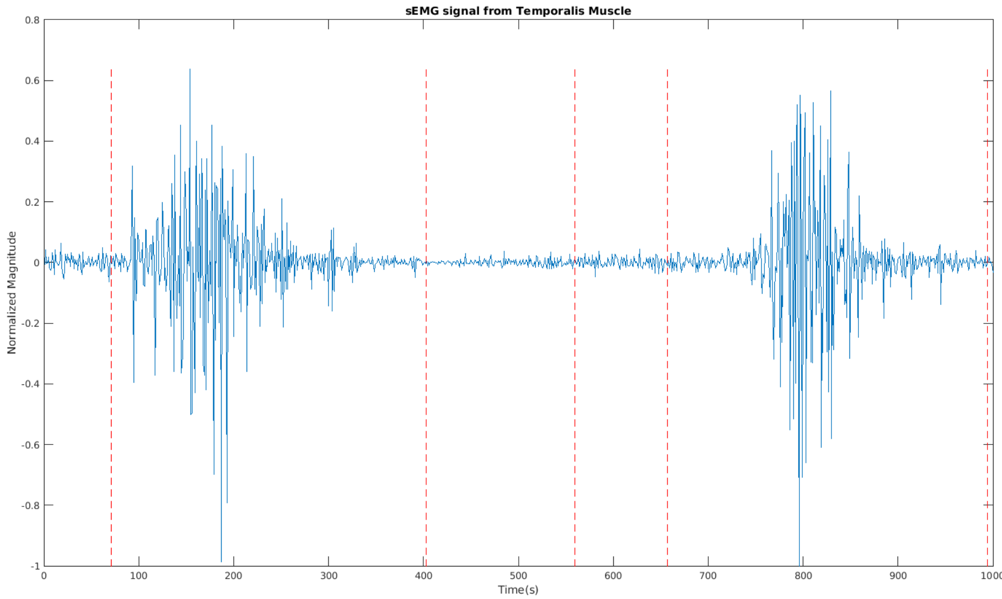

Figure 7: sEMG Signal Collected from Temporalis Muscle at sampling rate of 500Hz

It is noticed that the Bayesian approach has successfully segmented the signal into five segments, with the two strong portions in two separated segments. However, the segmentation is not as perfect as expected, since there are some extra ”noise” portions. It may be due to the selection of prior model structure.

Conclusion and Future Work

To conclude, the Bayesian signal segmentation algorithm successfully segments both simulation signal and real biomedical signal. However, the convergence rate is much slower than the traditional methods. The segmentation result will be sensitive to prior selection and model structure. Therefore, potential future works can be further concentrated on the following two directions. Firstly, improve convergence rate of MCMC by incorporating Hamiltonian dynamics, as the current MCMC is a random walk on graph. Secondly,

Appendix

JAGS code for Bayesian Signal Segmentation Model

```
model{
   idx[1] <- 1
    ncp[1] <- 1
    cp[1] <- round(lambda[1])
   for (seg in 2:(Nseg-1))
   {
        tmp[seg-1] <- cp[seg-1] + round(lambda[seg])
        cp[seg] <- ifelse(tmp[seg-1]>length(y),length(y),tmp[seg-1])
    }
    cp[Nseg] <- length(y)

   for (i in 1:Ndata)
   {
      y[i] ~ dnorm(mu[i], tau)
        mu[i] <- inprod(b[idx[i],1:ARorder], X[i,1:ARorder]) + MeanLevel[idx[i]]
        idx[i+1] <- ifelse(i==cp[ncp[i]],idx[i]+1,idx[i])
        ncp[i+1] <- ifelse(i==cp[ncp[i]],ncp[i]+1,ncp[i])
   }

   # set time segment
   for (seg in 1:(Nseg-1))
   {
        lambda[seg] ~ dunif(1,length(y))
   }

   # set mean level
   for (seg in 1:(Nseg))
   {
        MeanLevel[seg] ~ dnorm(0,1)

   }

    # set regression coefficient
   for (seg in 1:Nseg)
   {
      for (AR_i in 1:ARorder)
      {
         b[seg,AR_i] ~ dnorm(mu_b[seg,AR_i],2)
```

```
    }
  }

   # hyper-parameter for mu_b
  for (seg in 1:Nseg)
  {
      for (AR_i in 1:ARorder)
    {
         mu_b[seg,AR_i] ~ dnorm(0,2)
      }
  }

  tau ~ dgamma(0.001,0.001)
  sig <- 1/tau
}
```

References

[1] Paul Fearnhead. Exact bayesian curve fitting and signal segmentation. *Signal Processing, IEEE Transactions on*, 53(6):2160–2166, 2005.

[2] Paul Fearnhead. Exact and efficient bayesian inference for multiple changepoint problems. *Statistics and computing*, 16(2):203–213, 2006.

[3] Geoffroy Peeters, Amaury La Burthe, and Xavier Rodet. Toward automatic music audio summary generation from signal analysis. In *ISMIR*, pages 1–1, 2002.

[4] Elena Punskaya, Christophe Andrieu, Arnaud Doucet, and William J Fitzgerald. Bayesian curve fitting using mcmc with applications to signal segmentation. *Signal Processing, IEEE Transactions on*, 50(3):747–758, 2002.