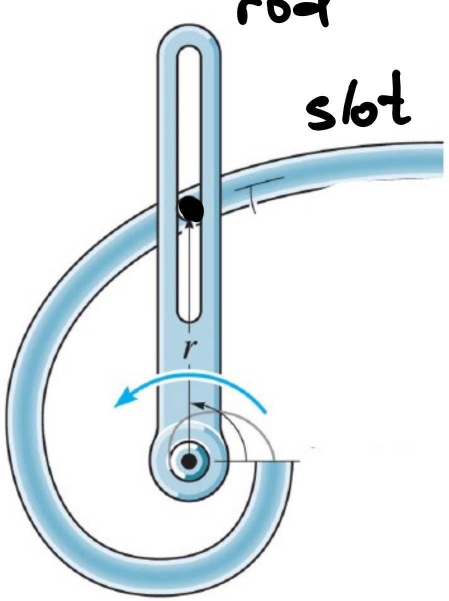
W10-1. The slotted rod moves the 4 kg particle around the curved slot whose shape is given by $\boxed{r = 0.6/\theta \text{ m}}$, where θ is in radians. The motion is in the horizontal plane. Determine the force that the rod exerts on the particle and the force of the slot on the particle when $\theta = 90^o$, $\dot{\theta} = 0.5$ rad/s, and $\ddot{\theta} = 0.6$ rad/s^2. Neglect friction.

$\hookrightarrow \pi/2$



1. Find acceleration in polar coordinates:

$$\boxed{a_r = \ddot{r} - r\dot{\theta}^2, \qquad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}}$$

We are interested in the moment when:

- $\theta = \frac{\pi}{2}$; $\quad \dot{\theta} = 0.5\,\frac{\text{rad}}{s}$; $\quad \ddot{\theta} = 0.6\,\frac{\text{rad}}{s^2}$

$$r = \frac{0.6}{\theta} \xrightarrow{\text{at } \theta = \pi/2} 0.3820 \text{ m}$$

$$\dot{r} = \frac{d}{dt}\frac{0.6}{\theta} = -\frac{0.6}{\theta^2}\cdot\dot{\theta} \xrightarrow{\text{at } \theta = \frac{\pi}{2},\ \dot{\theta} = 0.5} -0.1216\,\frac{m}{s}$$

$$\ddot{r} = \frac{d}{dt}\left(-\frac{0.6}{\theta^2}\dot{\theta}\right) = -0.6\,\frac{\ddot{\theta}\theta^2 - 2\theta\dot{\theta}^2}{\theta^4} = -0.6\,\frac{\ddot{\theta}\theta - 2\dot{\theta}^2}{\theta^3} \xrightarrow{\text{at } \theta = \frac{\pi}{2} \text{ etc}} -0.0685\,\frac{m}{s^2}$$

Last Time

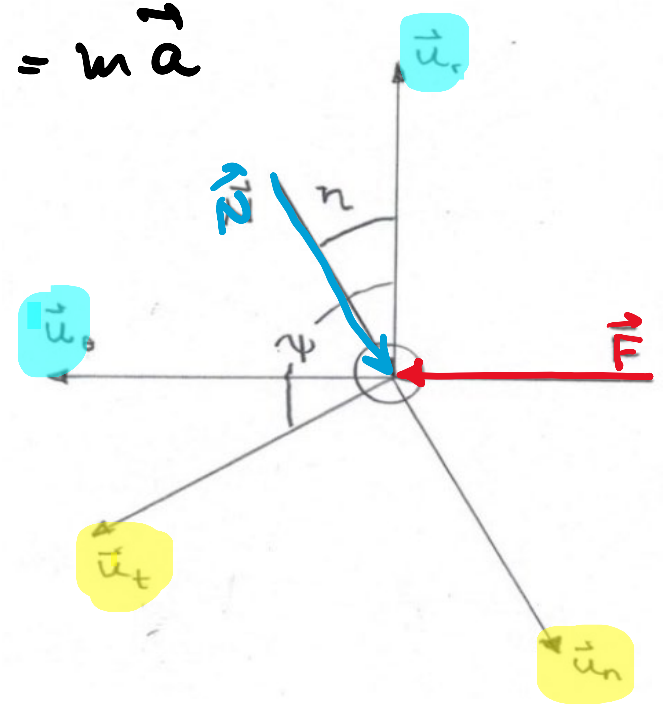
$a_r(\theta = \frac{\pi}{2}) = -0.1640\,\frac{m}{s^2}$ $\qquad a_\theta = 0.1076\,\frac{m}{s^2}$

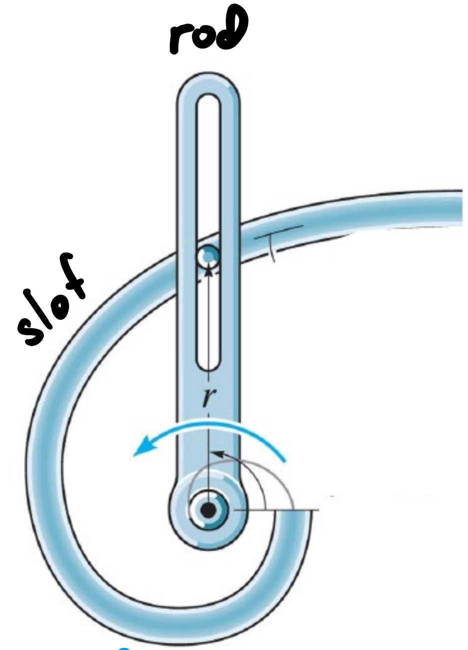
W10-1. The slotted rod moves the 4 kg particle around the curved slot whose shape is given by $r = 0.6/\theta$ m, where θ is in radians. The motion is in the horizontal plane. Determine the force that the rod exerts on the particle and the force of the slot on the particle when $\theta = 90^o$, $\dot{\theta} = 0.5$ rad/s, and $\ddot{\theta} = 0.6$ rad/s². Neglect friction.

→ neglect $\vec{W}$

2. Write down all the forces:

$$\vec{F}_R = \vec{F} + \vec{N} = m\vec{a}$$





$\vec{F} = F\vec{u}_\theta$ (polar)

$\vec{N} = N\vec{u}_n$ (n,t)

$\vec{a}$ (polar)

→ (polar) : $\vec{u}_n = (\dots)\vec{u}_r + (\dots)\vec{u}_\theta$

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$90° < \psi < 180°$

3. Translate between coordinate systems: $\vec{F} = F\,\vec{u}_\theta$; $\vec{N} = N\,\vec{u}_n$

$$\vec{u}_n = -\cos\eta\,\vec{u}_r - \sin\eta\,\vec{u}_\theta$$

$$\frac{dr}{d\theta} = -\frac{0.6}{\theta^2}$$

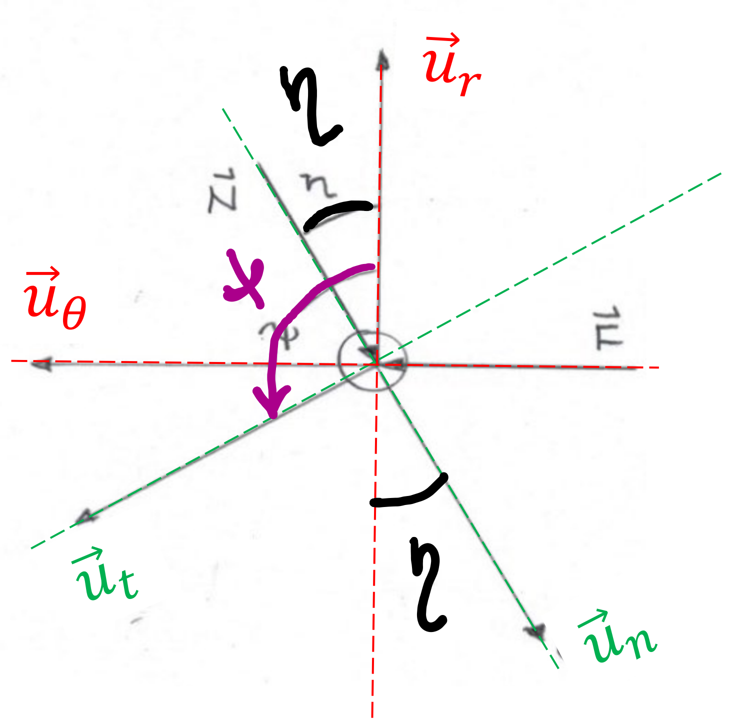
$$\eta = \psi - 90°$$

$$\psi: \boxed{\tan\psi = \frac{r}{dr/d\theta}} = \frac{0.6/\theta}{-0.6/\theta^2} = -\theta = -\frac{\pi}{2}$$

$$\psi = \tan^{-1}(\pi/2) = -57.518° + 180° = \underline{122.48°}$$

$$\eta = \psi - 90° = 32.48°$$

$$\vec{N} = -N\cos\eta\,\vec{u}_r - N\sin\eta\,\vec{u}_\theta$$



$$\tan\psi = \frac{r}{dr/d\theta}$$

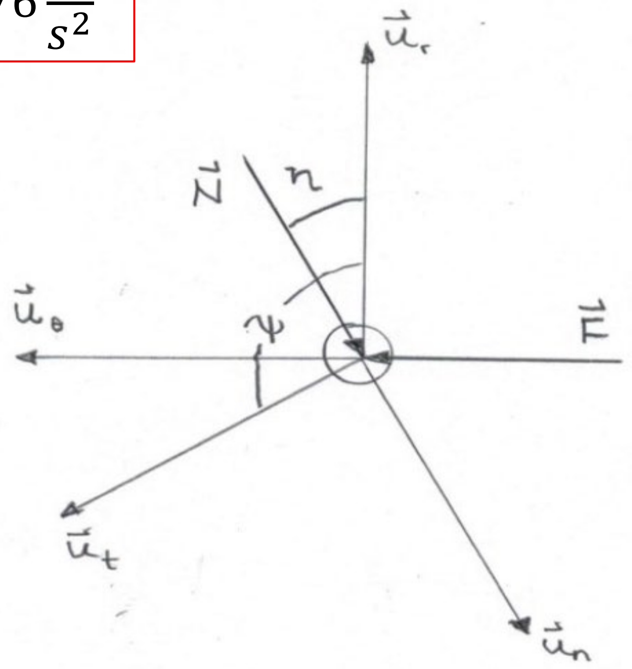
W10-1. The slotted rod moves the 4 kg particle around the curved slot whose shape is given by $r = 0.6/\theta$ m, where θ is in radians. The motion is in the horizontal plane. Determine the force that the rod exerts on the particle and the force of the slot on the particle when $\theta = 90^o$, $\dot{\theta} = 0.5$ rad/s, and $\ddot{\theta} = 0.6$ rad/s². Neglect friction.

$$\boxed{\vec{F} = F\,\vec{u}_\theta; \qquad \vec{N} = -N\cos\eta\,\vec{u}_r - N\sin\eta\,\vec{u}_\theta; \qquad \eta = 32.48^o}$$

4. Apply 2nd Newton's law:

$$\boxed{a_r\left(\theta = \frac{\pi}{2}\right) = -0.1640\,\frac{m}{s^2}, \qquad a_\theta\left(\theta = \frac{\pi}{2}\right) = 0.1076\,\frac{m}{s^2}}$$

$$\vec{F}_R = \vec{F} + \vec{N} = m\vec{a}$$



$r:\quad 0 - N\cos\eta = m a_r = (4)(-0.1640) \qquad (1)$

$\theta:\quad F - N\sin\eta = m a_\theta = (4)(0.1076) \qquad (2)$

$(1) \Rightarrow \quad N = 0.778\ N$

$(2) \Rightarrow \quad F = 0.848\ N$

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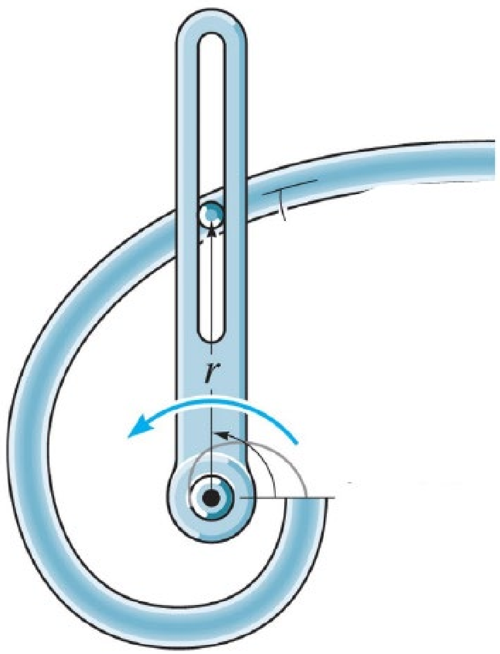
Let's summarize what we did (strategy):



0. Analyse the forces => Diagram (forces, axes)

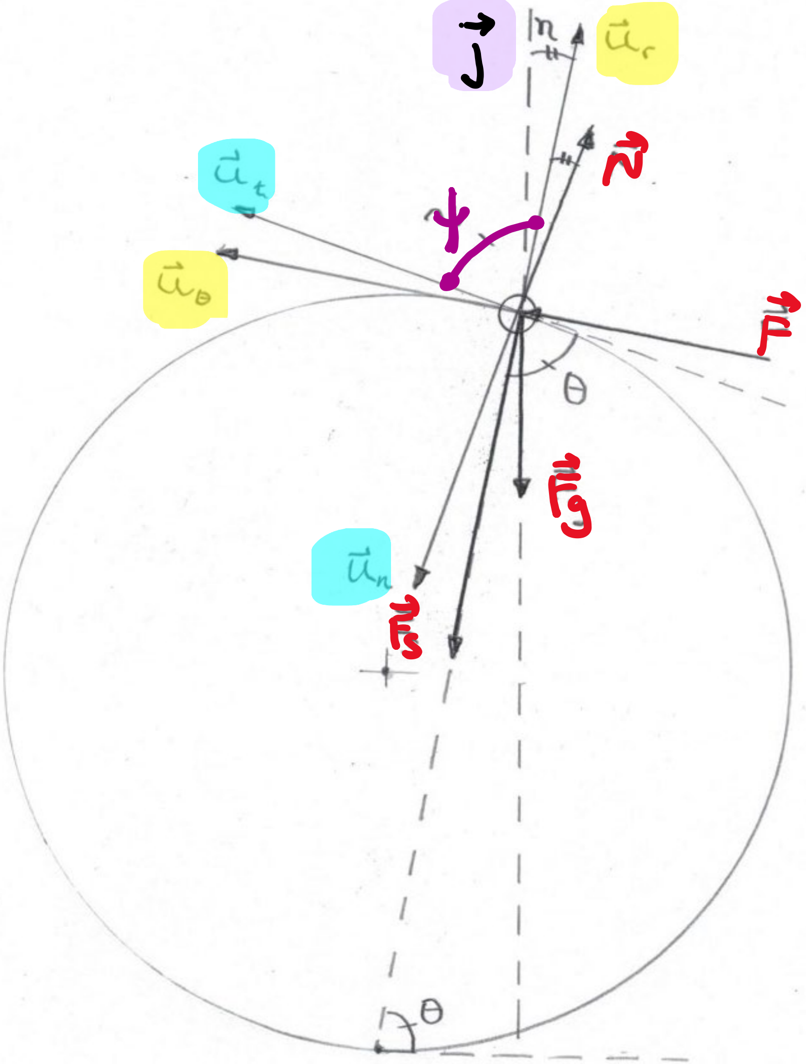
1. Found acceleration, in polar coordinates

2. Wrote down all the forces, in polar and (n,t)-coordinates

3. Translated between coordinate systems => reduced all the forces to the same coordinate system

4. Applied Newton's 2nd law to find the unknowns

W10-2. The slotted guide moves the 150 g particle P around the 0.4 m radius circular disk. Motion is in the vertical plane. Attached to P is an elastic cord extending from O. The cord has stiffness 30 N/m and unstretched length 0.25 m. Friction may be neglected. Determine the force of the guide on P and the normal force of the disk on P when $\theta = 70^o$, $\dot{\theta} = 5$ rad/s, and $\ddot{\theta} = 2$ rad/s².



$$\vec{F}_R = \vec{F} + \vec{N} + \vec{F}_g + \vec{F}_s = m\vec{a}$$

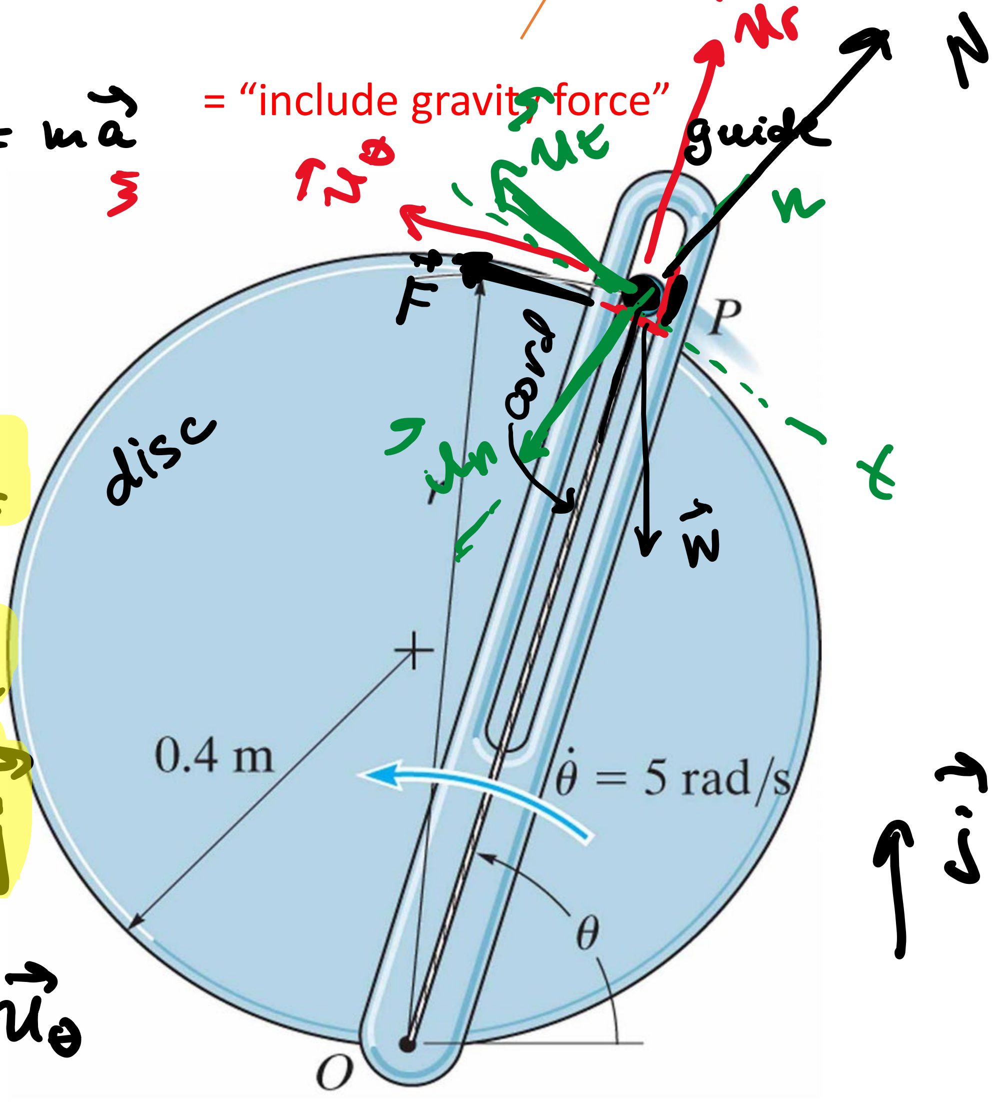
= "include gravity force"

Cord: $\vec{F}_s \uparrow\downarrow \vec{u}_r$

Disk: $\vec{N} \uparrow\downarrow \vec{u}_n$

Gravity: $\vec{F}_g \uparrow\downarrow \vec{j}$

Guide: $\vec{F} \uparrow\uparrow \vec{u}_\theta$



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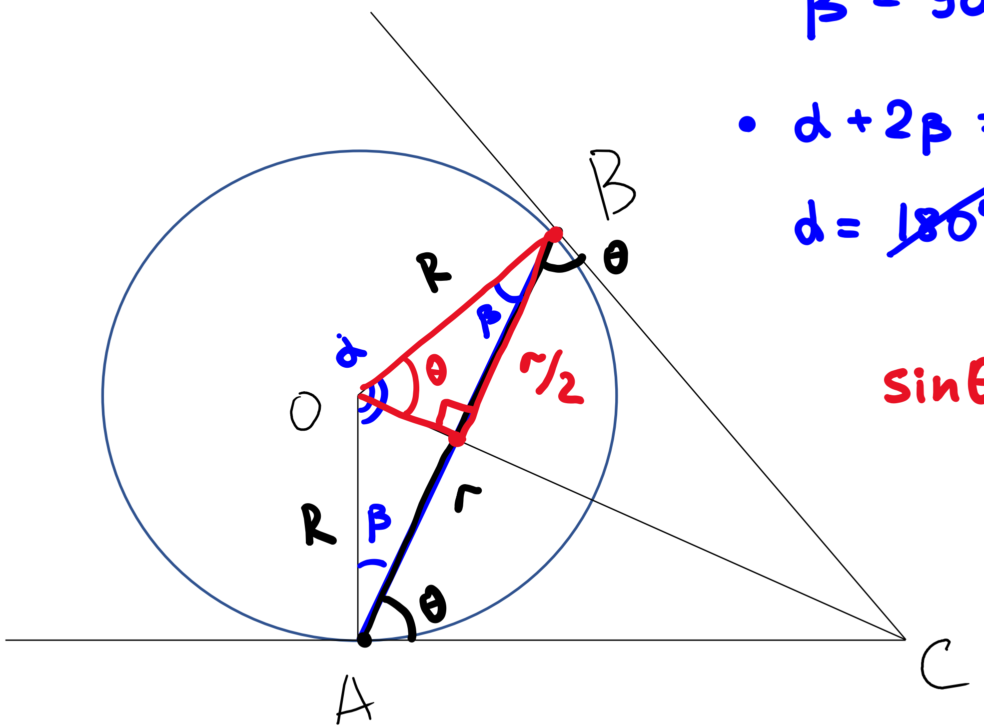
$\hookrightarrow \frac{70^\circ}{180^\circ}\pi$

1. Find acceleration in polar coordinates:

➢ What is $r(\theta)$??



- $\beta + \theta = 90^\circ$

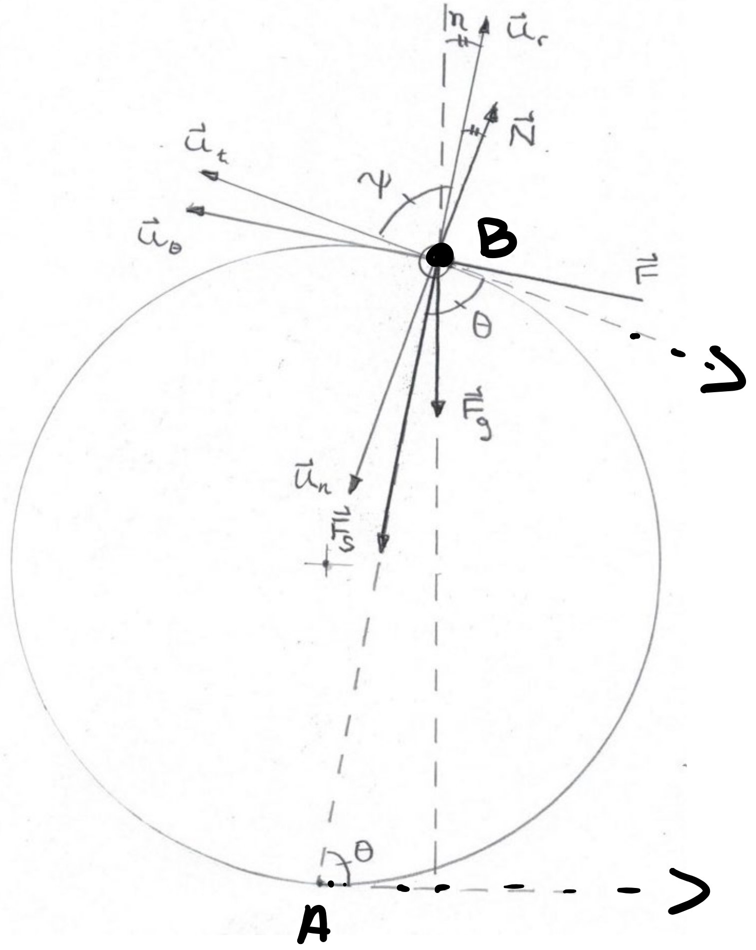
  $\beta = 90^\circ - \theta$

- $\alpha + 2\beta = 180^\circ$

  $\alpha = \cancel{180^\circ} - 2(90^\circ - \theta) = 2\theta$

$\sin\theta = \frac{r/2}{R}$

$$\boxed{r(\theta) = 2R\sin\theta}$$



$$a_r = \ddot{r} - r\dot{\theta}^2, \qquad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

W10-2. The slotted guide moves the 150 g particle P around the 0.4 m radius circular disk. Motion is in the vertical plane. Attached to P is an elastic cord extending from O. The cord has stiffness 30 N/m and unstretched length 0.25 m. Friction may be neglected. Determine the force of the guide on P and the normal force of the disk on P when $\theta = 70^o$, $\dot{\theta} = 5$ rad/s, and $\ddot{\theta} = 2$ rad/s².

1. Find acceleration in polar coordinates:

$$\boxed{r(\theta) = 2R\sin\theta}$$

We are interested in the moment when:

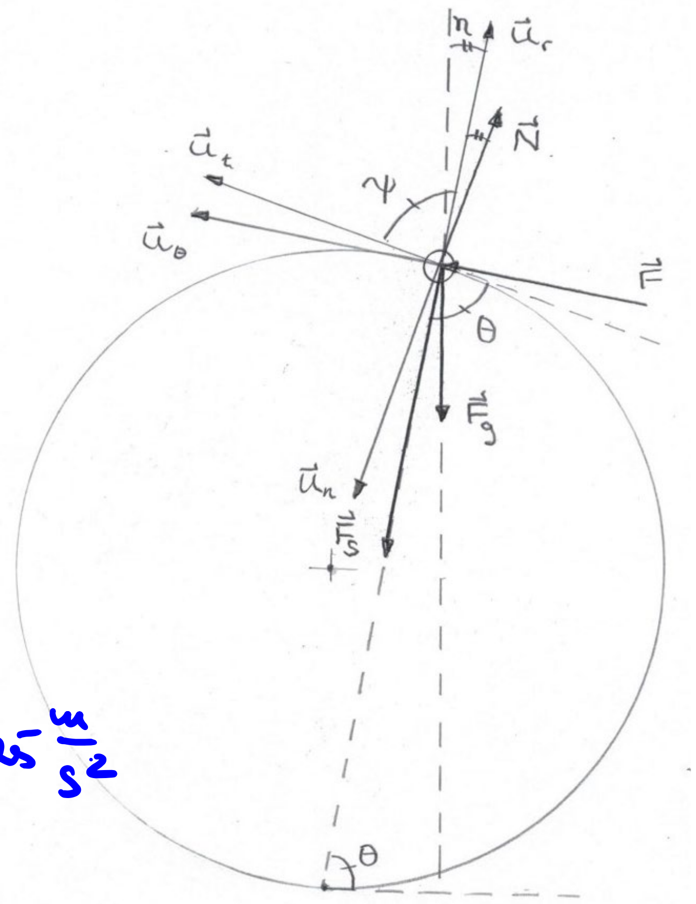
- $\theta = \frac{70\,\pi}{180}$; $\quad \dot{\theta} = 5\,\frac{\text{rad}}{s}$; $\quad \ddot{\theta} = 2\,\frac{\text{rad}}{s^2}$

$$r(\theta) = 2R\sin\theta \xrightarrow{\theta = \frac{70\pi}{180}} 0.7518\ \text{m}$$

$$\dot{r} = 2R\cos\theta\cdot\dot{\theta} \xrightarrow{\theta = \dots\ \dot{\theta} =} 1.368\ \frac{\text{m}}{\text{s}}$$

$$\ddot{r} = 2R\frac{d}{dt}\left(\dot{\theta}\cos\theta\right) = 2R\left(\ddot{\theta}\cos\theta - \sin\theta\cdot\dot{\theta}^2\right) \longrightarrow -18.25\ \frac{\text{m}}{\text{s}^2}$$

$$a_r = -3704\ \frac{\text{m}}{\text{s}^2} \qquad a_\theta = 15.18\ \frac{\text{m}}{\text{s}^2} \qquad (\text{polar})$$



$$\boxed{a_r = \ddot{r} - r\dot{\theta}^2, \qquad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}}$$

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2. Write down all the forces:

Guide (rod, slot) $\quad \vec{F} = F \cdot \vec{u}_\theta$ (polar)

Disk: $\quad \vec{N} = -N \vec{u}_n$ (n, t)

Cord: $\quad \vec{F}_s = -F_s \vec{u}_r$ (polar)

Weight: $\quad \vec{F}_g = -mg \vec{j}$ (Cart)

$\vec{a}$ (polar)