Review plan:

0. Course Summary

1. Coordinate systems
   - How to convert from one coordinate system to another
   - When to use which coordinate system
   - Problems that involve polar coordinates

2. Relative motion
   - General idea: Review

3. Rigid body equilibrium
   - Couple moments, moments, reactions -- OMG
   - Review of how to solve rigid body equilibrium problems

4. Wrench
   - Please review Lecture 12 and come to talk if you have questions!

Q: What are the elements required in a free body diagram for each type of question?

- **Reserve at least 5 mins for a good FBD.**
- **Use straightedge.** No exceptions. Draw the FBD **large** and **clean**.
- Show **axes** and **unit vectors:** $(\vec{i},\vec{j})$ and/or $(\vec{u}_n,\vec{u}_t)$ and/or $(\vec{u}_\rho,\vec{u}_\theta)$.
- **Object** (outlined shape)
- All **forces** and **moments** acting on the object (label them – give them names!)
- All relevant **points** (give their coordinates!)
- **Acceleration**: it's a must in your FBD if you are going to use Newton's second law, $\vec{F}_R = m\vec{a}$. If you are using Work and Energy principle only, then you can drop it (you won't be penalized for including it).

PHYS 170 CONCEPTS (the list is not exhaustive)

- Vector equations / Scalar equations
- Forces: translations and rotations / Couple moments: rotations
- Translational (forces) and rotational (couple moments + force moments) equilibrium
- External forces and moments / Reaction forces and moments
- Competing scenarios of breaking equilibrium / impending motion eqs vs restrictions

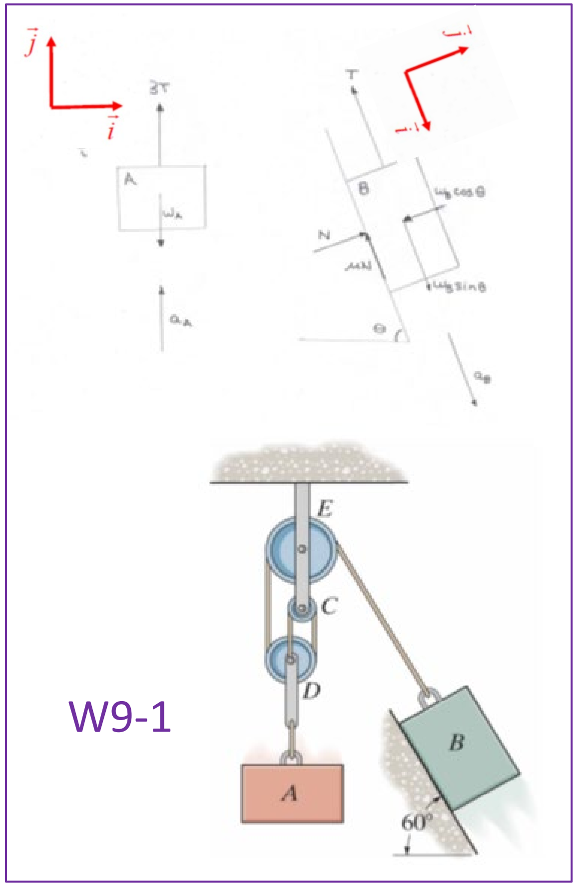
Statics

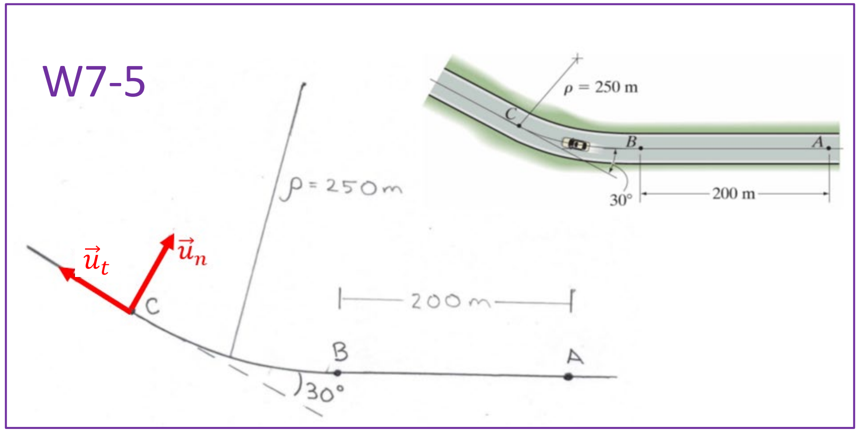
Dynamics

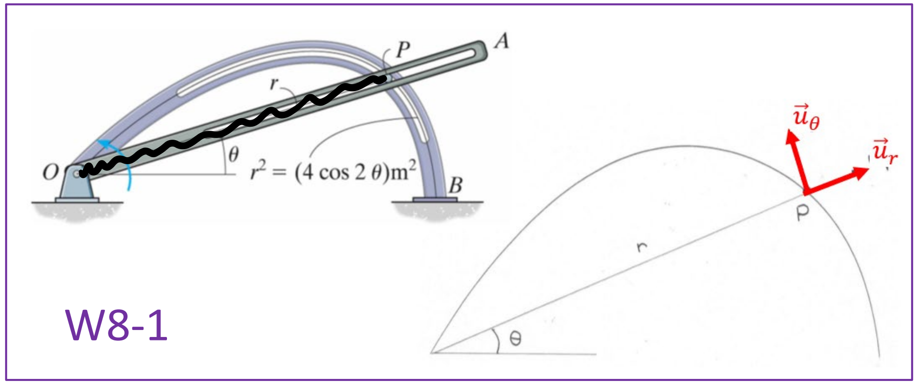
- Coordinate systems: many of them! / Choice: convenience / Switching between them
- Acceleration: Can change magnitude (a_t) or direction (a_n) of the velocity ($\vec{v} = \text{vector}$)
- Dependent (constrained) motion
- Relative motion
- Newton's 2nd law: $\vec{F} \Leftrightarrow m\vec{a}$
- Energy ⇔ Work / Work-Energy principle / Momentum ⇔ Impulse / Impulse-Momentum principle

1. Coordinate systems

- When to use which?







- Normal-Tangential:
  - "Portable" coordinate system
  - Curvilinear motion
  - Good if you deal with forces acting at normal or tangential directions to the trajectory of the particle

- Polar:
  - "Portable" coordinate system
  - Curvilinear motion
  - Good if you deal with forces acting from origin O to the particle, or in the direction perpendicular to it

- Cartesian:
  - Stationary coordinate system
  - Good for motion along straight lines

$$\sum F_i = ma_i\,, \quad i = x, y, z$$

$\vec{v} = v\vec{u}_t$ (tangential!)

$$\sum F_t = m\dot{v}$$

$$\sum F_n = m\frac{v^2}{\rho}$$

$$\vec{v} = \dot{\rho}\vec{u}_\rho + \rho\dot{\theta}\vec{u}_\theta$$

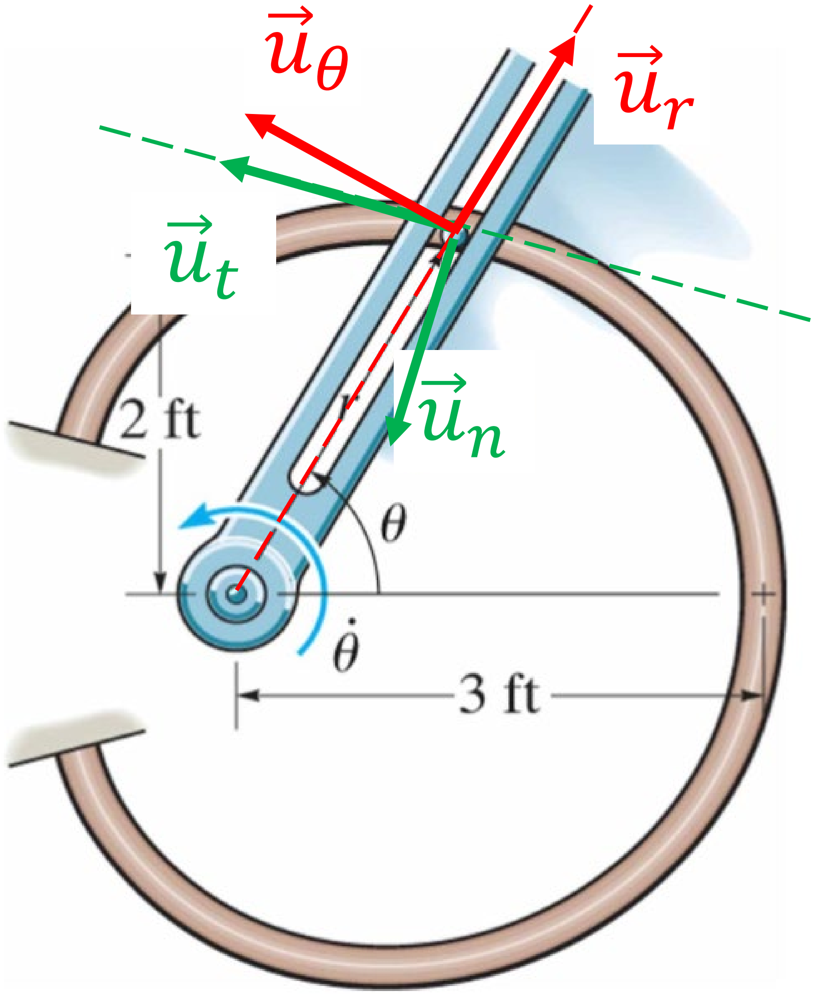
$$\sum F_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$\sum F_\theta = m(2\dot{\theta}\dot{r} + r\ddot{\theta})$$

1. Coordinate systems

- How to draw them?

Extra practice



- $\vec{u}_t, \vec{u}_n, \vec{u}_r$ and $\vec{u}_\theta$ are unit vectors (their length is equal to 1)

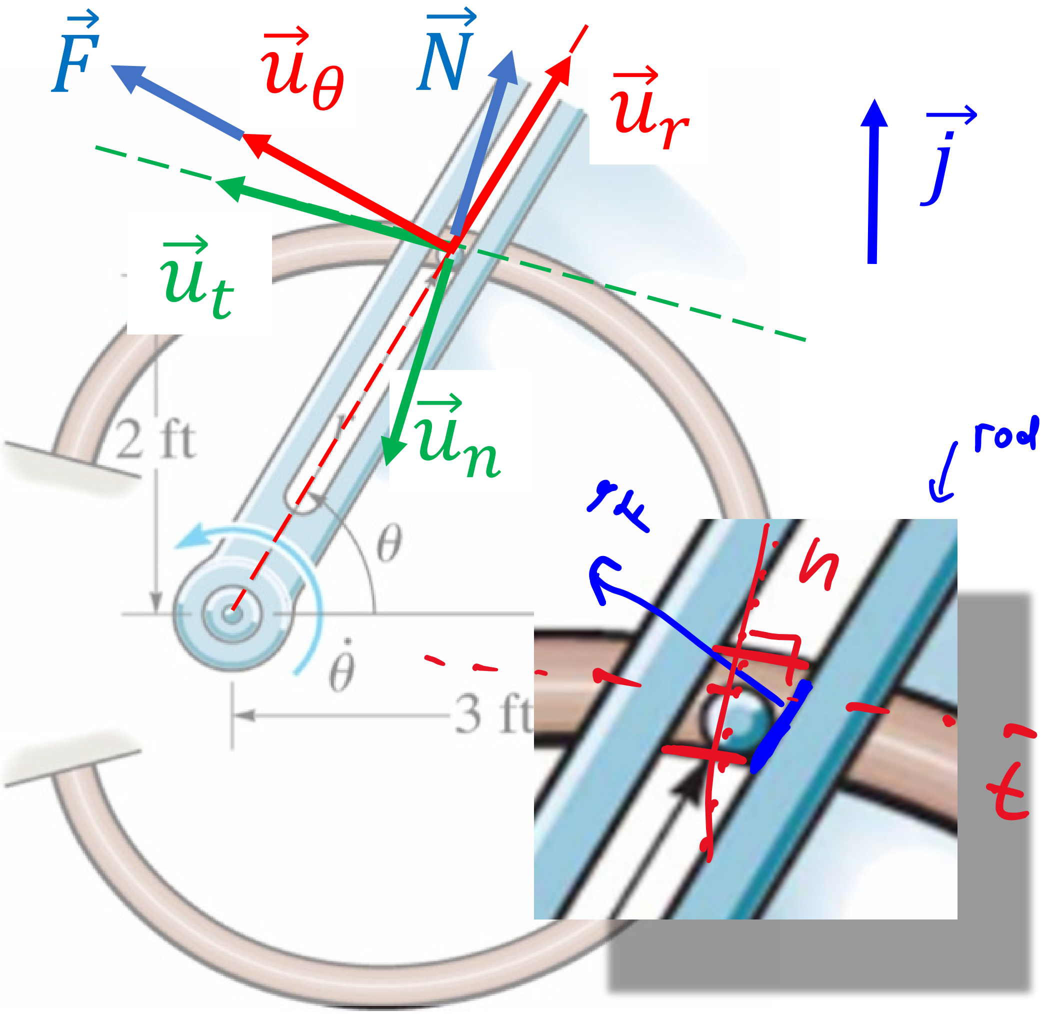
- (n,t):
  - Start with the tangent line => $\vec{u}_t$ (in the direction of motion)
  - $\vec{u}_n$ is perpendicular to $\vec{u}_t$, AND <u>into</u> the trajectory

- Polar:
  - Start with the radial line => $\vec{u}_r$ (away from the origin)
  - $\vec{u}_\theta$ is perpendicular to $\vec{u}_r$, AND in the direction of motion

1. Coordinate systems

• How to express involved forces?

Extra practice



• $\vec{u}_t, \vec{u}_n, \vec{u}_r$ and $\vec{u}_\theta$ are unit vectors (their length is equal to 1)

• Let's find natural unit vectors for involved forces:

➢ Force exerted by the rod, $\vec{F}$

○ It's a <u>normal</u> force exerted by the surface of the pushing rod => must be perpendicular to it => $\vec{u}_r$ along the rod & $\vec{u}_\theta \perp \vec{u}_r$ => $\vec{F}$ along $\vec{u}_\theta$

➢ Force exerted by the slot, $\vec{N}$

○ It's a <u>normal</u> force exerted by the slot, which is also particle's <u>trajectory</u> => must be perpendicular to the trajectory => $\vec{u}_t$ along the trajectory & $\vec{u}_n \perp \vec{u}_t$ => $\vec{N}$ along $\vec{u}_n$

○ Is $\vec{N} \uparrow\uparrow \vec{u}_n$ or $\vec{N} \uparrow\downarrow \vec{u}_n$? You can make a guess!

➢ Weight, $\vec{W}$

○ Always strictly downwards => $\vec{W}$ along $-\vec{j}$

➢ Likewise, analyse other forces if they are present

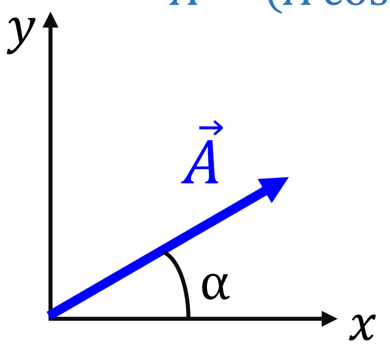
1. Coordinate systems

$\psi = 100°$ $\beta = 10°$

- How to translate between them?

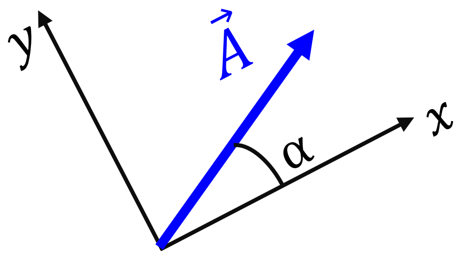
- "Regular vector" in a "regular" coordinate system:

$$\vec{A} = (A\cos\alpha)\vec{i} + (A\sin\alpha)\vec{j}$$

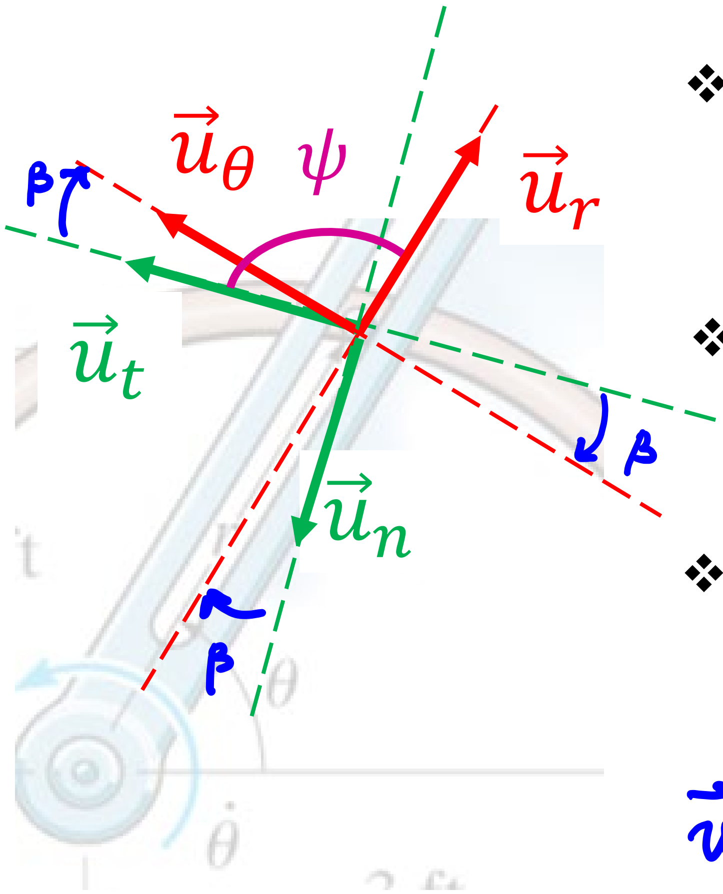


❖ Nothing changes if the coordinate system is tilted:

$$\vec{A} = (A\cos\alpha)\vec{i} + (A\sin\alpha)\vec{j}$$



- Example: (n,t)-coordinates ⇒ polar coordinates:



❖ Now polar coordinates are your basis vectors, and $\vec{u}_t$ and $\vec{u}_n$ are two vectors to be projected on them.

❖ Assume that you know the angle between $\vec{u}_t$ and $\vec{u}_t$, shown here as ψ

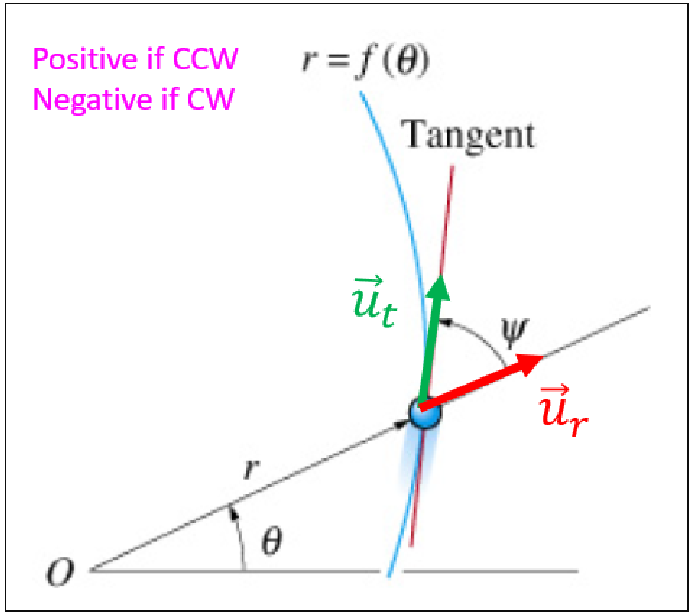
❖ You can project $\vec{u}_t$ and $\vec{u}_n$ onto the red axes using regular projection rules:

$$\vec{u}_t = (-\sin\beta)\vec{u}_r + (\cos\beta)\vec{u}_\theta$$

$$\vec{u}_n = (-\cos\beta)\vec{u}_r + (-\sin\beta)\vec{u}_\theta$$

- But where to get the angles from?

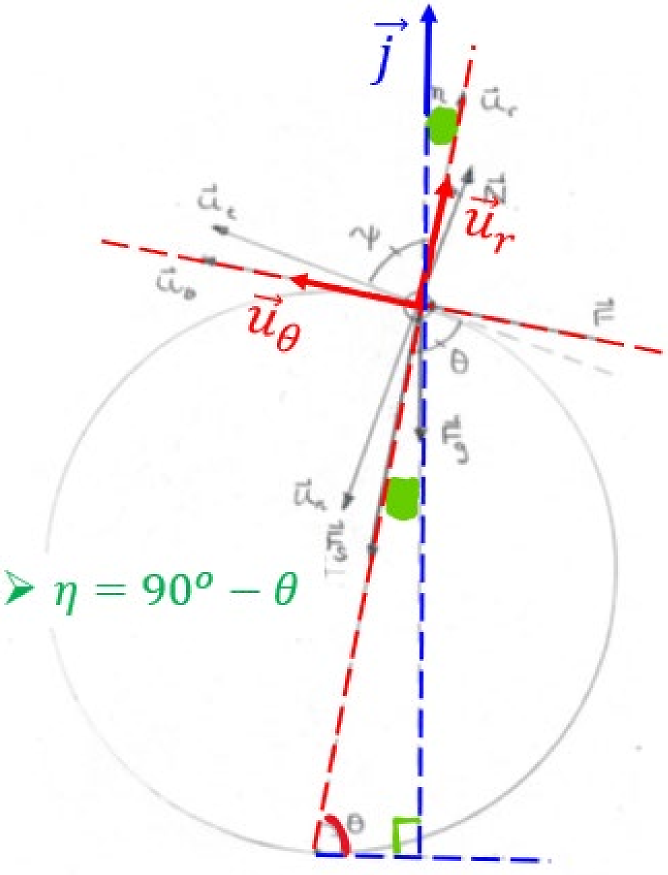
- (n,t) ⇔ polar converter: ψ-angle:



- ψ is the angle from $\vec{u}_r$ to $\vec{u}_t$, which can be calculated using this magic formula:

$$\tan\psi = \left(\frac{r}{dr/d\theta}\right)$$

- $\vec{j}$ ⇔ polar converter: (we need it for weight)
  - ➢ Use geometry!
  - ➢ Example (W10-2):

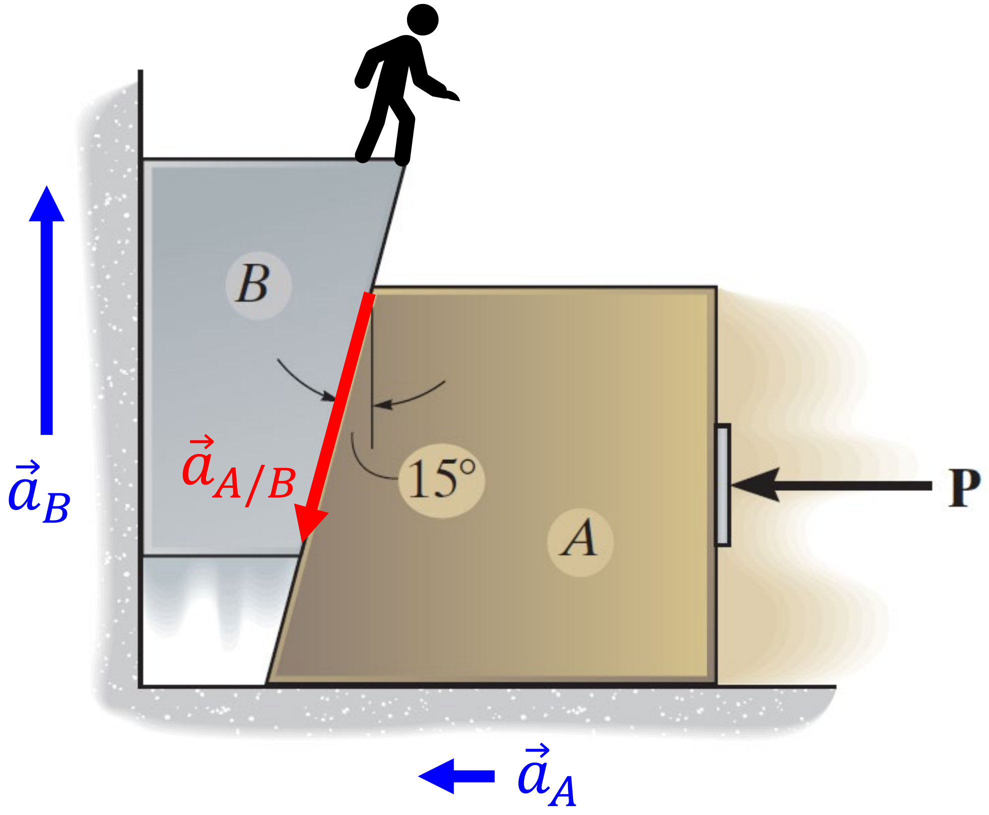


2. Relative motion

- What's the idea?

HW-9

Observer moving with B



- This observer sees all other objects moving "with respect to B" (since for him the object B is stationary)

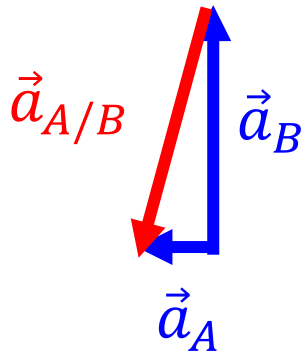
- To look at the world through his eyes, we introduce "acceleration of A with respect to B": $\vec{a}_{A/B}$

w.r.t.

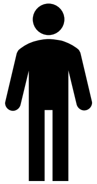
➢ At the lectures we have shown that these accelerations are connected through

$$\vec{a}_{A/B} = \vec{a}_A - \vec{a}_B$$

$$\vec{a}_B + \vec{a}_{A/B} = \vec{a}_A$$



Stationary observer



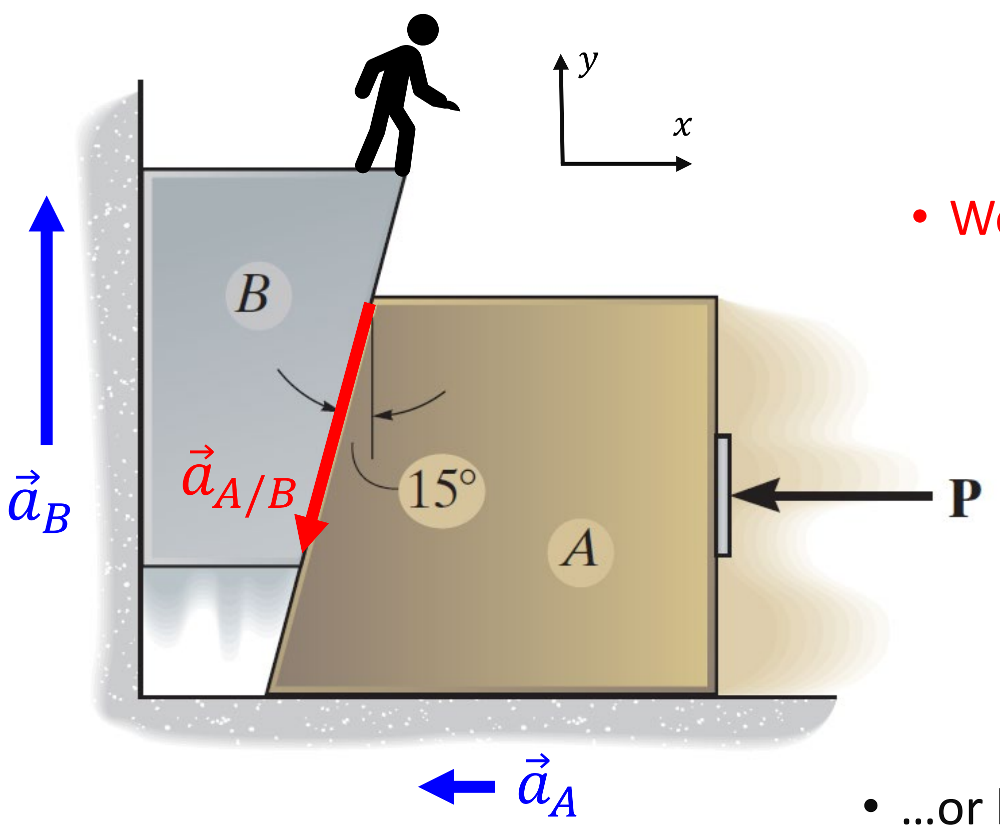
- This observer sees two blocks moving with accelerations $\vec{a}_A$ and $\vec{a}_B$.

2. Relative motion

- What's the idea?

HW-9

Observer moving with B



- We can express $\vec{a}_{A/B}$ through $\vec{a}_A$ and $\vec{a}_B$

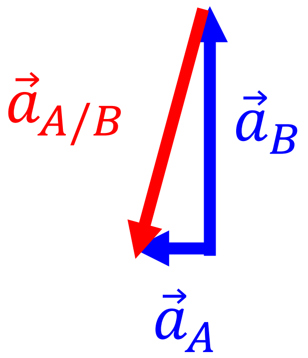
$$\vec{a}_{A/B} = \vec{a}_A - \vec{a}_B = (\overset{-a_A}{a_{Ax}} - \overset{0}{a_{Bx}})\,\vec{i} + (\overset{0}{a_{Ay}} - \overset{a_B}{a_{By}})\,\vec{j}$$

- We can also see from the picture that:

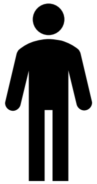
$$\vec{a}_{A/B} = (-a_{A/B}\sin 15^o)\,\vec{i} + (-a_{A/B}\cos 15^o)\,\vec{j}$$

- We have added <u>one unknown</u> ($a_{A/B}$) and <u>two equations</u> => gain !!!

- ...or here we can just look at this "acceleration triangle" and observe that $\tan 15^o = a_A/a_B$...



Stationary observer



- This observer sees two blocks moving with accelerations $\vec{a}_A$ and $\vec{a}_B$.

3. Rigid body equilibrium

- Neither translates nor rotates!

- There are two distinct "types" of motion
  - Translations: produced by forces
  - Rotations: produced by moments AND forces

- Accordingly, a body is in equilibrium if it neither translates nor rotates:
  - Translational equilibrium: $\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0$
  - Rotational equilibrium: $\sum M_{\underline{\text{pivot}},x} = 0, \quad \sum M_{\underline{\text{pivot}},y} = 0, \quad \sum M_{\underline{\text{pivot}},z} = 0$

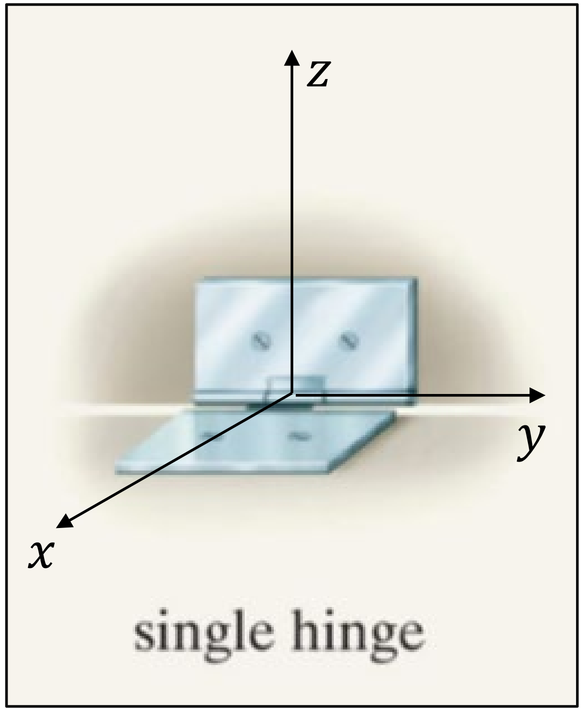
❖ Note that here we have to choose (and indicate) the pivot point;

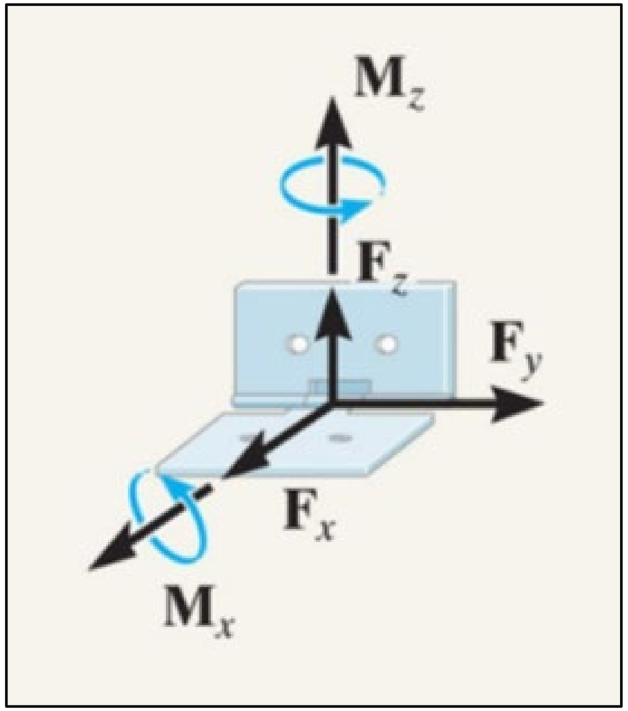
❖ While for translational equilibrium we simply add up and equate to zero all forces (<u>active</u> and <u>reactive</u>) acting on the object, for rotational equilibrium we add up the <u>external moments</u> ("couple moments"), <u>reactive moments</u>, as well as <u>the moments produced by forces</u> (active and reactive) acting on the object

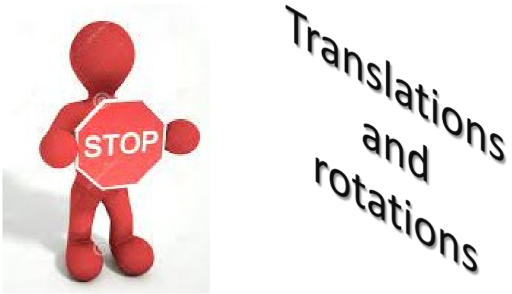
3. Rigid body equilibrium

- What "reactions" are?

- Let us recall what are reaction forces and reaction moments







❖ Reaction forces: <u>forces</u> created by a support in order to prevent <u>translation motion</u> of the attached member

❖ Reaction moments: <u>moments</u> created by a support in order to prevent <u>rotation</u> of the attached member

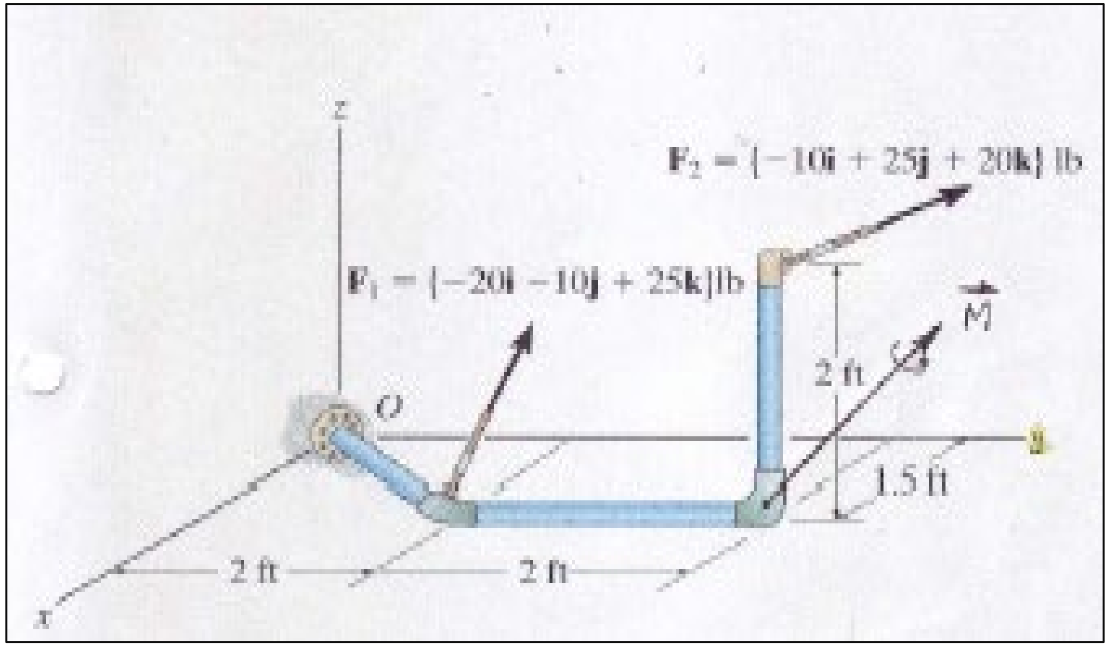
➢ Reaction forces: translation along which axes are forbidden? A_x, A_y, A_z

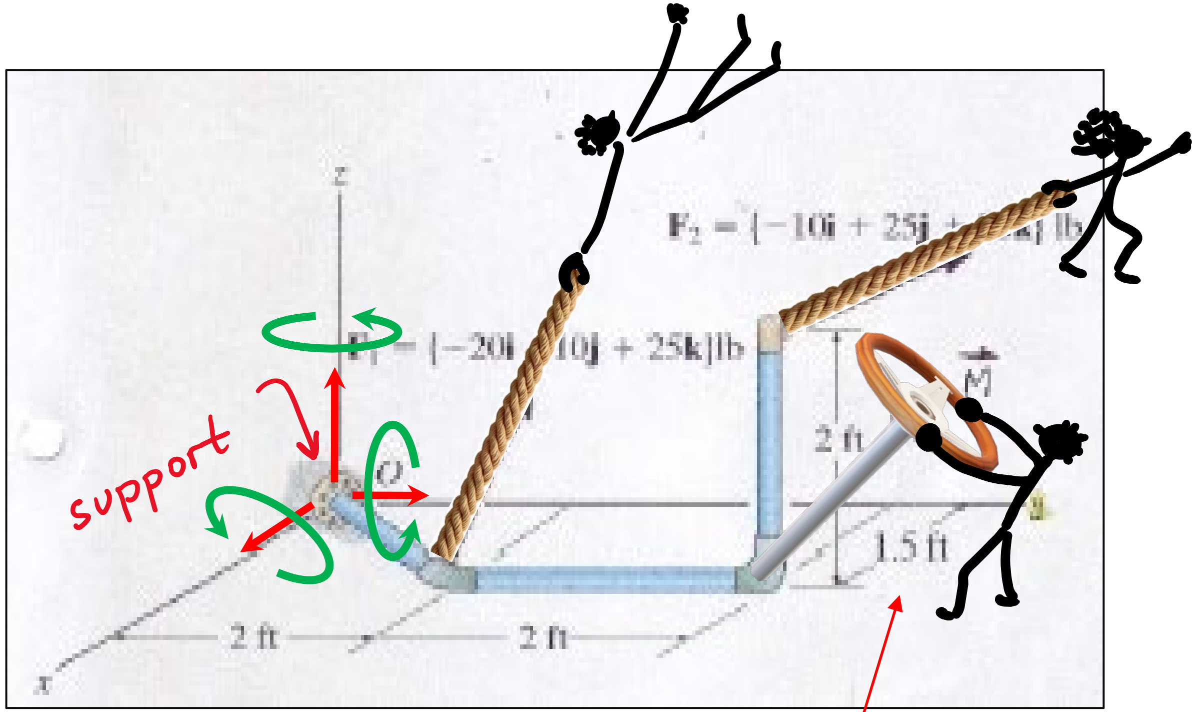
➢ Reaction moments: rotation about which axes are forbidden? M_x, M_z

➢ Reaction moment is NOT a moment of a reaction force!

- Piece of art

➢ Active vs Reactive forces and moments: creative view

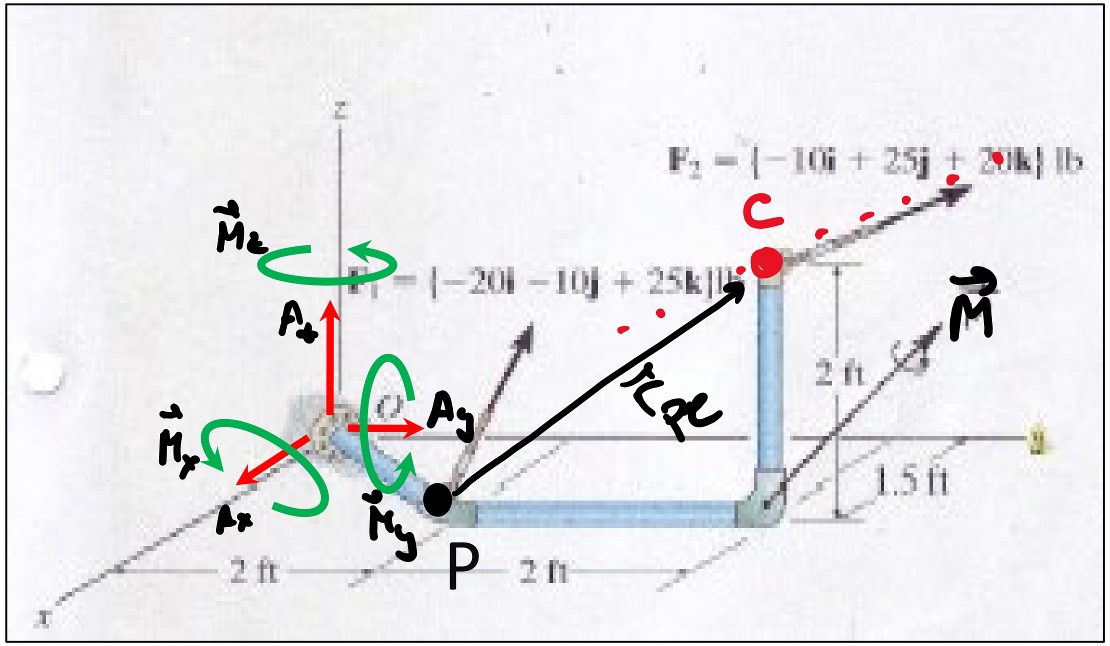




➢ Both of them are external forces.

❖ Couple moment

- Forces:
  - External forces? $\vec{F}_1 \quad \vec{F}_2$
  - Reaction forces? $(A_x, \; A_y, \; A_z) = \vec{A}$



- Moments:

❖ Pivot point: consider moments about point P (this choice is not the best, I took it to show more moments in our discussion)

  - External moments? $\vec{M}$
  - Reaction moments? $\vec{M}_x, \; \vec{M}_y, \; \vec{M}_z$
  - Moments produced by all the forces? $(\vec{M}_2)_P = \vec{r}_{PC} \times \vec{F}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ & & \\ & & \end{vmatrix}$

$\left\{ \begin{array}{l} (\vec{M}_A)_P = \\ \vec{r}_{PO} \times \vec{A} \end{array} \right.$

Study tips

- Study with someone, if possible. If you are studying alone, try to explain what you are doing out loud, pretending that you are teaching this material to someone. This way you can find gaps in your understanding that can go unnoticed otherwise.
- You should be comfortable with the content of worksheets, homework sets and tutorial problems. After that, you can do as much extra problems (posted on Canvas or from the text) as you can on the topics you feel less secure with.
- After some initial preparation, try posted final exams from previous years. Time yourself. Adjust your study plan using this type of feedback.
- Never ever read posted solutions! Use them only to check your work or if you feel stuck and need a jump-start. In the second case, close them after you have looked them up and do the problem on your own from scratch.
- Note that PHYS 170 is about accuracy and precision (check a typical mark split in posted exams).
- Fluency and confidence come with practice.
- You are not just an engineering student, first and foremost you are a human being. Take breaks, go out, walk and exercise, eat well and sleep enough. It will pay of.

Happy final exam!