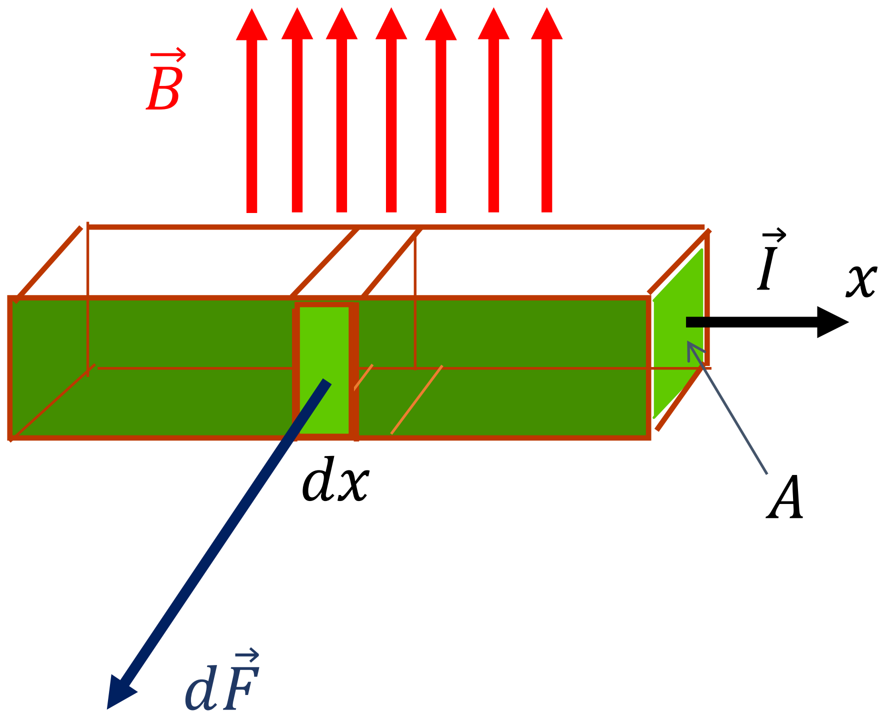
Lecture 27.

Force on a current-carrying wire and on a loop.
Magnetic torque on a loop.

Last Time:



- Force on a wire segment dx:

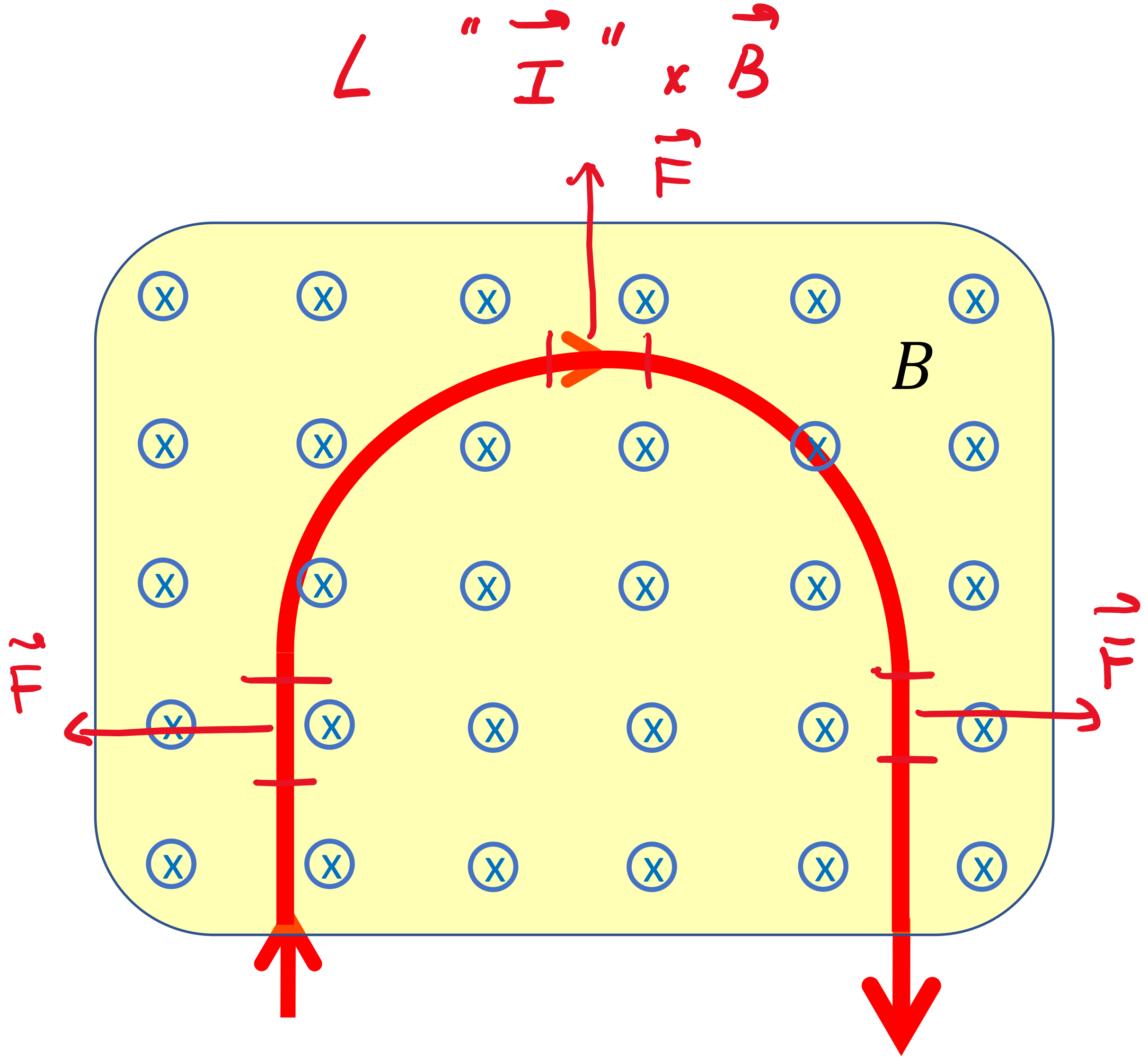
$$d\vec{F} = I\, d\vec{x} \times \vec{B}$$

- Force on a straight piece of wire in a unform field:

$$\vec{F} = I\, \vec{L} \times \vec{B}$$

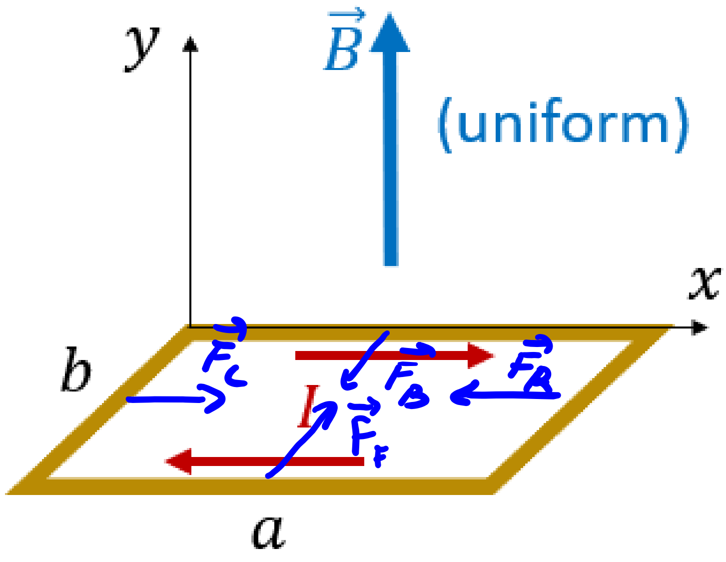
Magnitude: $F = ILB \sin\theta_{\vec{B},\, dir\, I}$

Q: A current carrying wire passes through a region with a uniform B-field. Will there be a force on this wire? If so, in which direction?



A. Upward
B. Downward
C. Right
D. Left
E. No net force (symmetry)

Q: This rectangular loop carries a clockwise current. A uniform magnetic field is perpendicular to the plane of the loop. What is the net magnetic force and the torque on the loop?



$$F = L \cdot I \cdot B \cdot \underbrace{\sin 90^\circ}_{1}$$

$$\vec{F}_{net} = 0$$

$$\tau \overset{?}{=} 0$$

(unstable equil.)

A. $F_{net} = 0, \tau = 0$

B. $F_{net} \neq 0, \tau = 0$

C. $F_{net} = 0, \tau \neq 0$

D. $F_{net} \neq 0, \tau \neq 0$

$$\boxed{\vec{F}_{wire} = L\,\vec{I} \times \vec{B}}$$

Q: Now the uniform magnetic field is at 60° with respect to the plane of the loop. What are the magnetic force and the torque now?

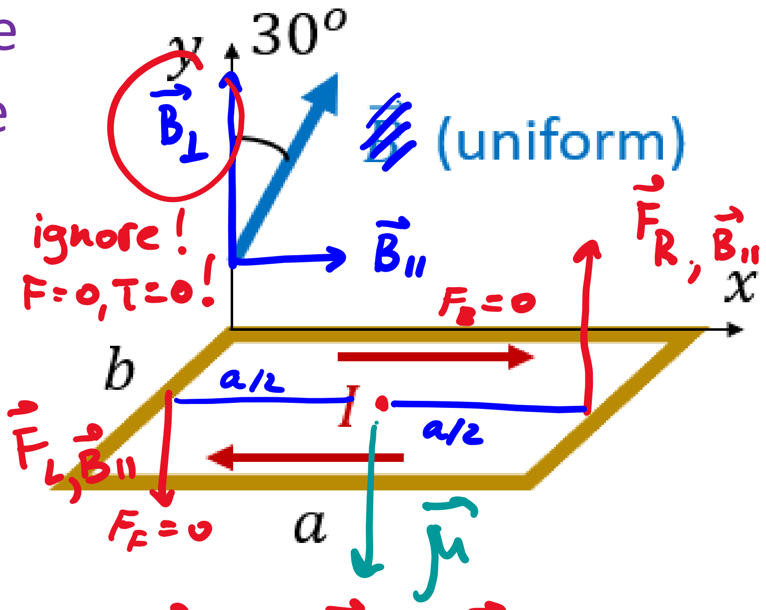
$\vec{F}_{net} = 0$ (B = same)

$F_L = F_R = b\cdot I\cdot B_{\parallel} = bIB\sin 30^\circ$

$\tau = F_L \frac{a}{2} + F_R \frac{a}{2} = 2\cdot\frac{a}{2}\cdot bIB\sin 30^\circ$

$\tau = (ab)I\,B\sin 30^\circ$

$\tau = \underbrace{A\cdot I}_{\mu}\cdot B\cdot\sin 30^\circ$, CCW (see fig)



$\vec{F} = L\,\text{"}\vec{I}\text{"} \times \vec{B}_{\parallel}$

A. $F_{net} = 0, \tau = 0$

B. $F_{net} \neq 0, \tau = 0$

C. $F_{net} = 0, \tau \neq 0$

D. $F_{net} \neq 0, \tau \neq 0$

$\vec{F}_{wire} = L\vec{I} \times \vec{B}$

Magnetic moment

- Magnetic dipole moment of a current-carrying loop is a vector, called $\vec{\mu}$:

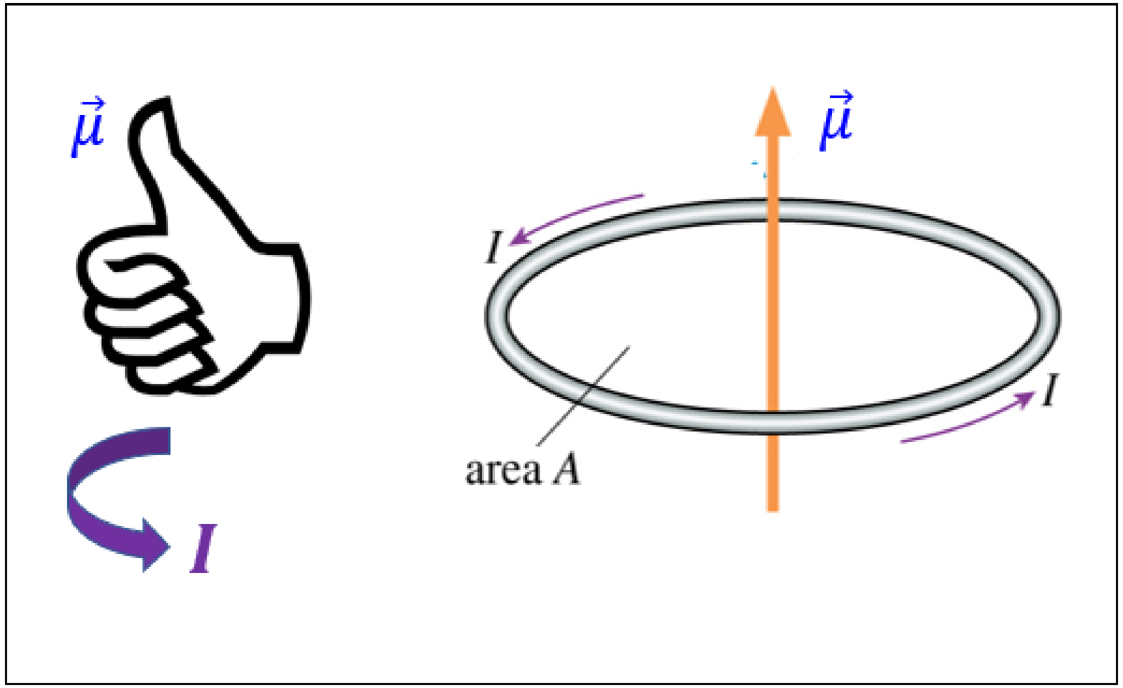
➢ Magnitude: $\mu = IA$

❖ I = current in the loop

❖ A = its area

➢ Direction: curled-fingers right-hand rule

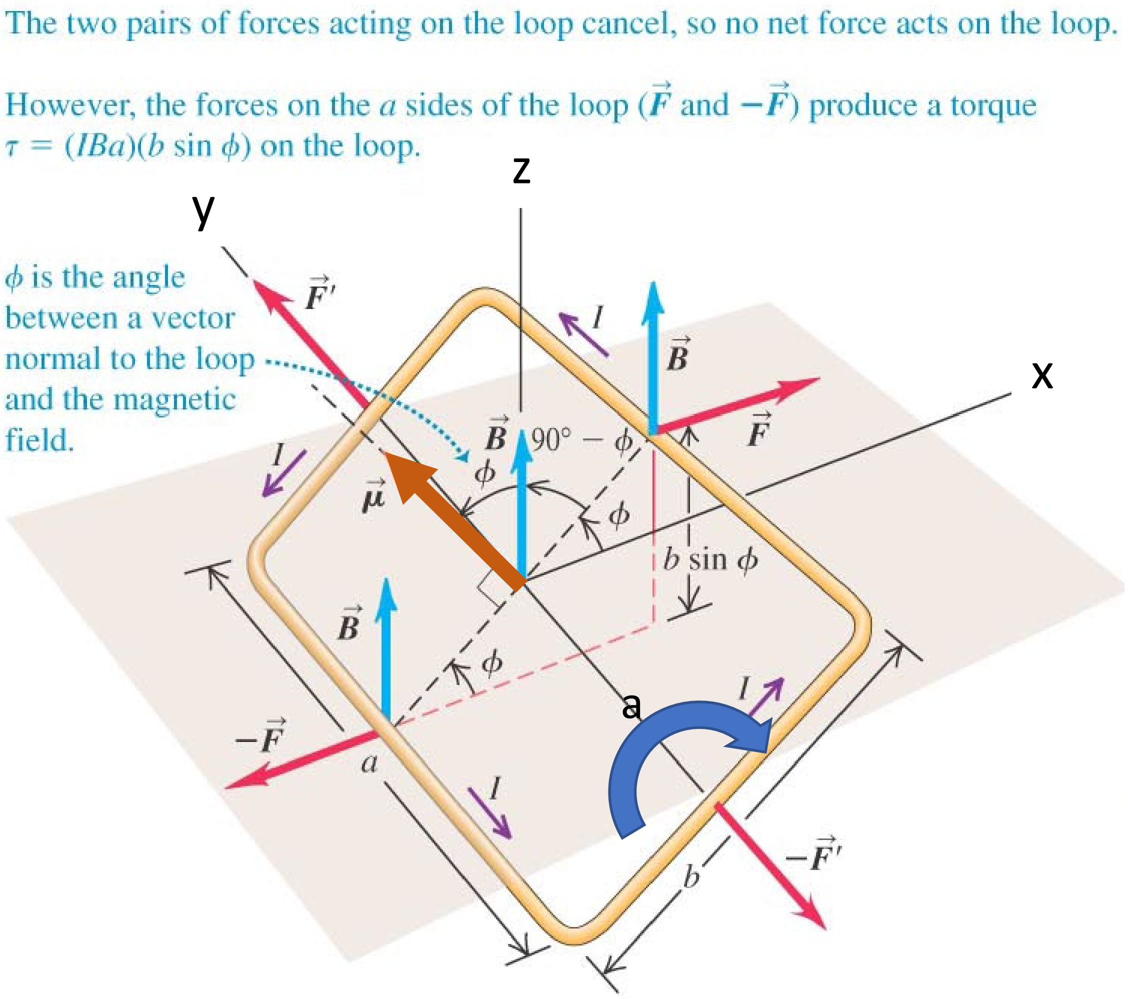
❖ Curled fingers along the current, right thumb shows $\vec{\mu}$

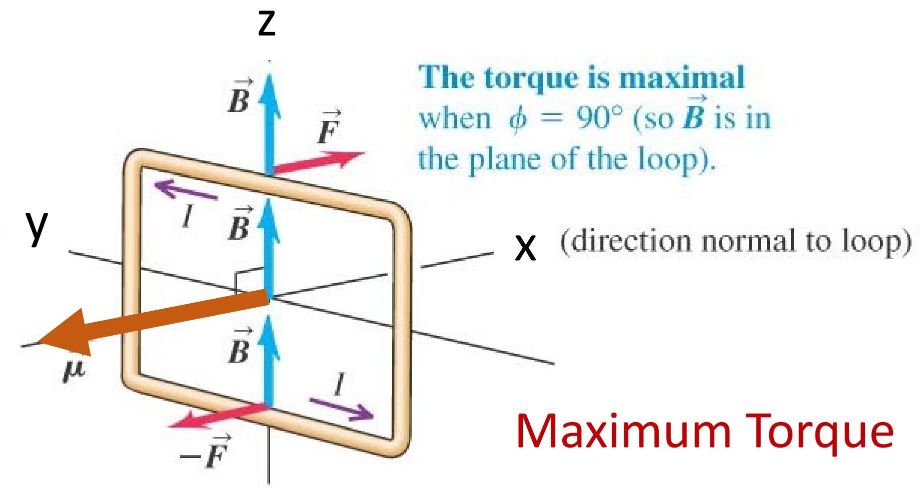


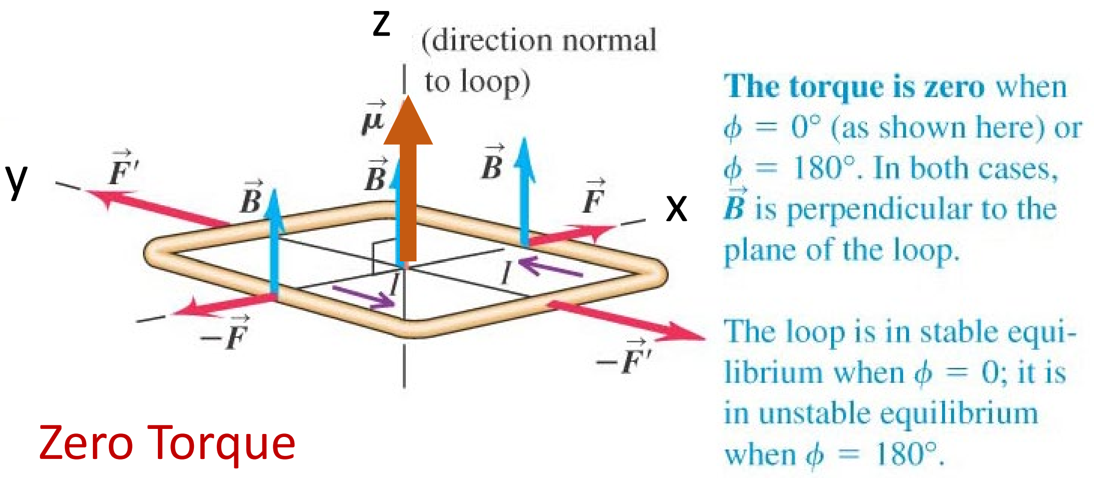
Magnetic torque

$$\tau = Fb\sin\phi$$
$$= I(ab)B\sin\phi = \mu B\sin\phi$$

ϕ = angle between $\vec{B}$ and $\vec{\mu}$







Y&F 27.31

Magnetic Moment and Magnetic Torque

- Magnetic torque vector: $\boxed{\vec{\tau}_B = \vec{\mu} \times \vec{B}}$

- For comparison: $\vec{\tau}_E = \vec{p} \times \vec{E}$

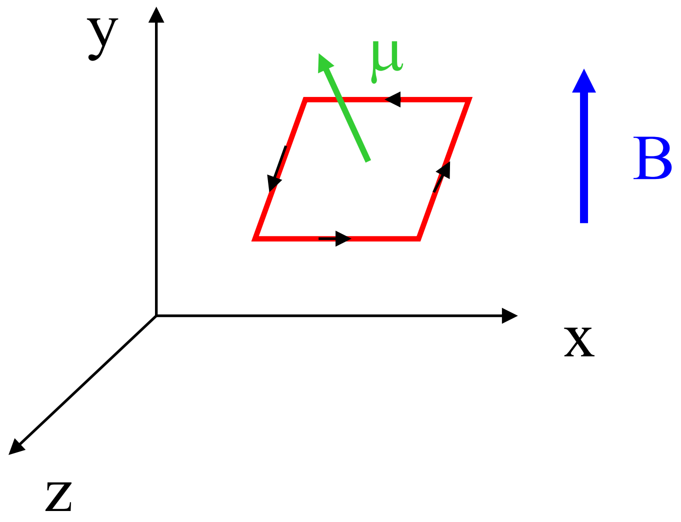
- Magnitude of the torque:

$$\tau = \mu\, B \sin\theta$$

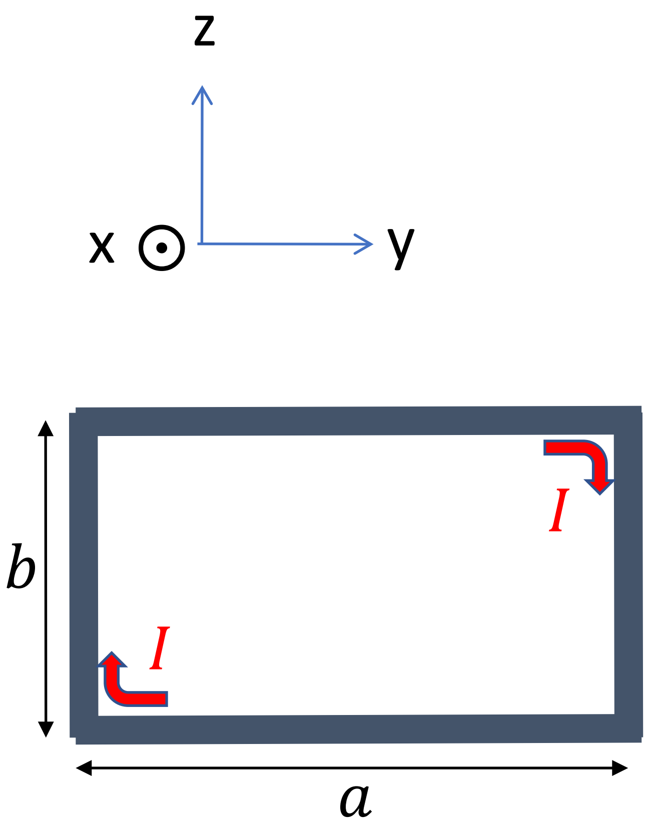
with $\mu = IA$ and θ being the angle between the B-field and the normal to the loop (= direction of $\vec{\mu}$).

- Note that the torque is non-zero if $\vec{\mu}$ is not aligned with $\vec{B}$.

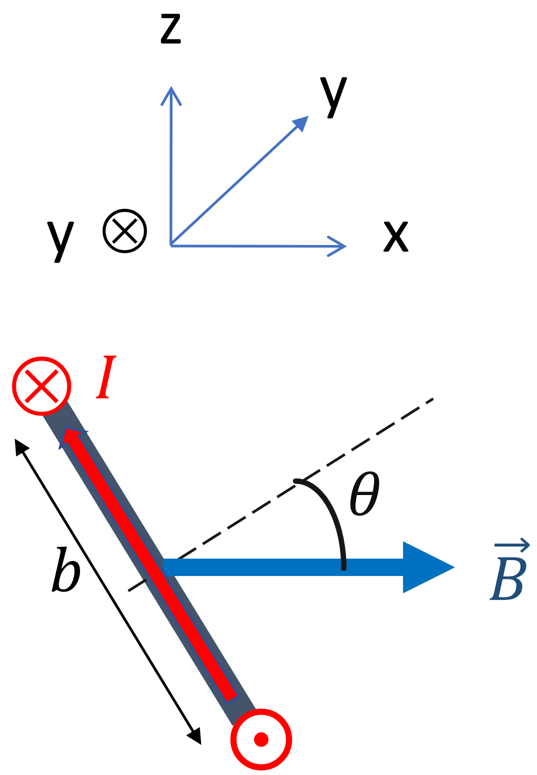
- In other words: magnetic torque aligns $\vec{\mu}$ with $\vec{B}$!



Q: A current-carrying wire loop is immersed in an external B-field as shown below. The B-field is at an angle θ with respect to the normal to the loop. What is the direction for the torque on the wire loop ?



Top view



Side view

A. CW about the x axis
B. CW about the y axis
C. CW about the z axis
D. CCW about the x axis
E. CCW about the y axis

Q: A current-carrying wire loop is immersed in an external B-field as shown below. The B-field is at an angle θ with respect to the normal to the loop. What is the direction for the torque on the wire loop ?

- Torque tends to align the magnetic moment $\vec{\mu}$ with the external field $\vec{B}$

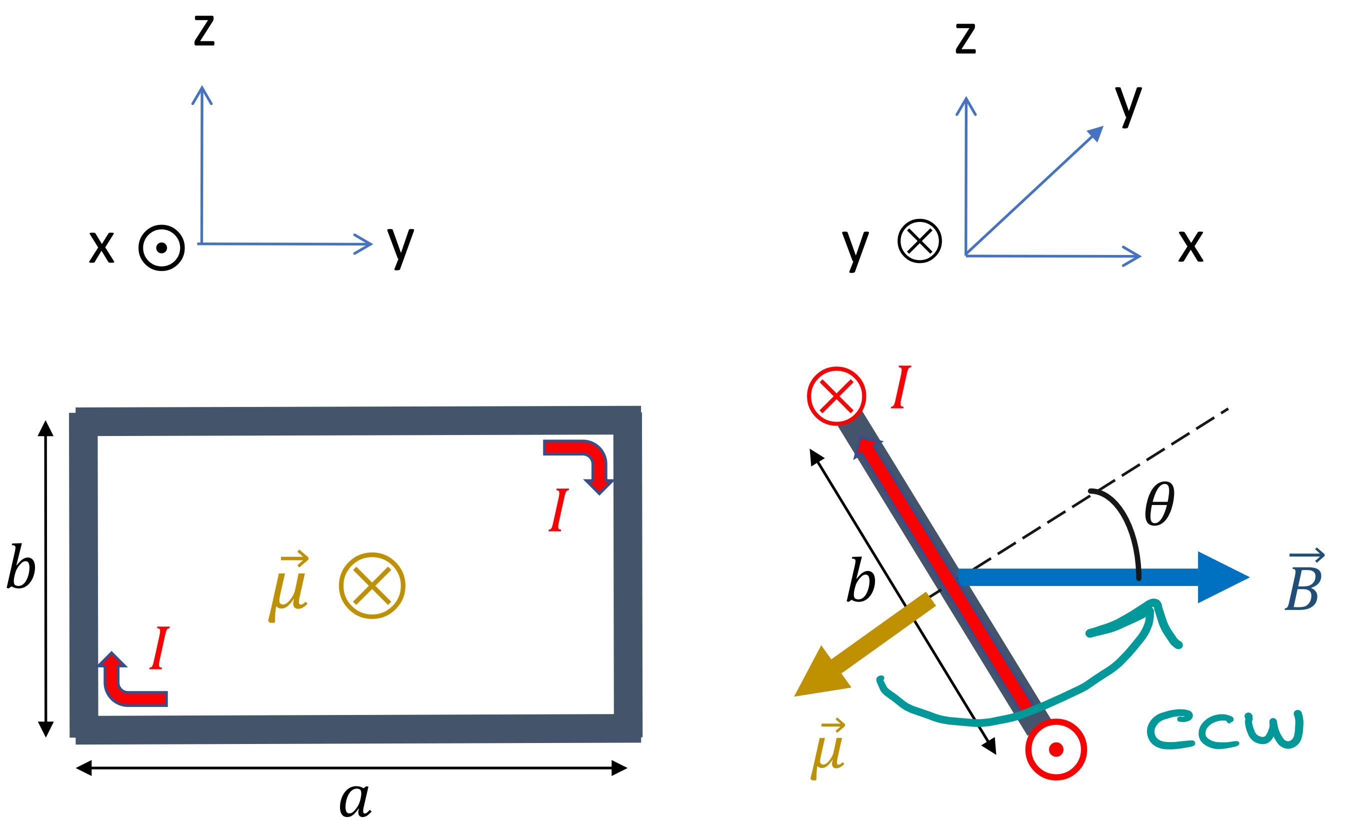
A. CW about the x axis

B. CW about the y axis

C. CW about the z axis

D. CCW about the x axis

E. CCW about the y axis

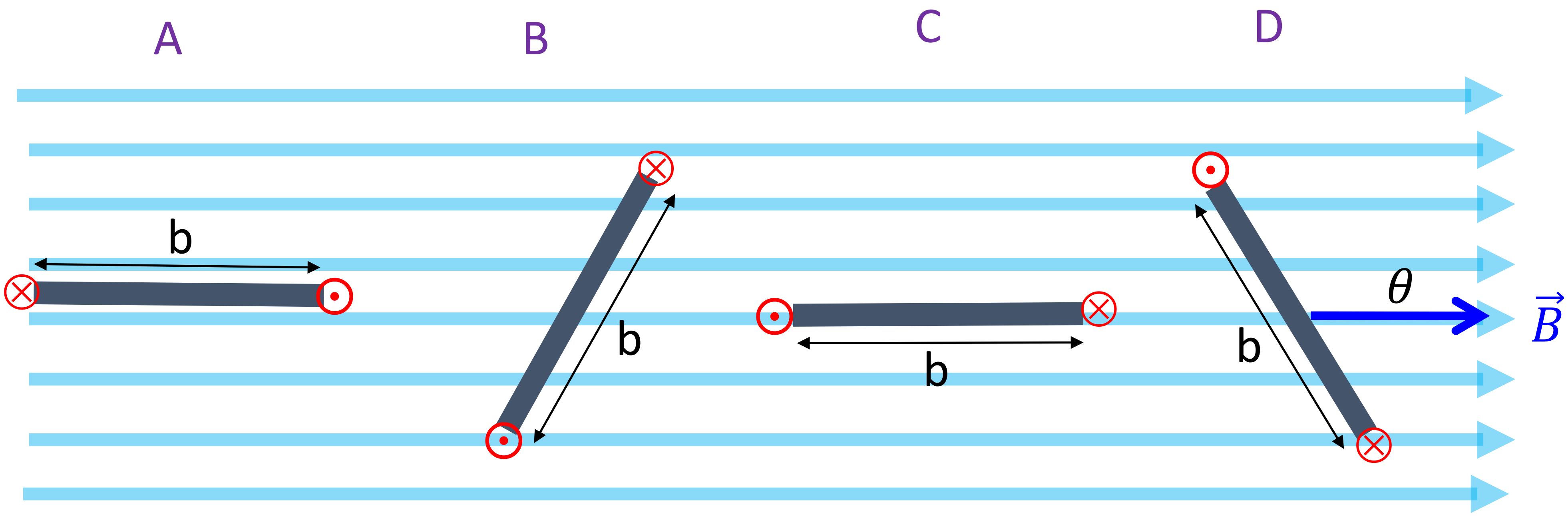


Top view

Side view

Because of the torque, potential energy can be stored in a current carrying wire loop when it is immersed in an external B-field.

Which orientation below corresponds to the highest value of the stored potential energy?

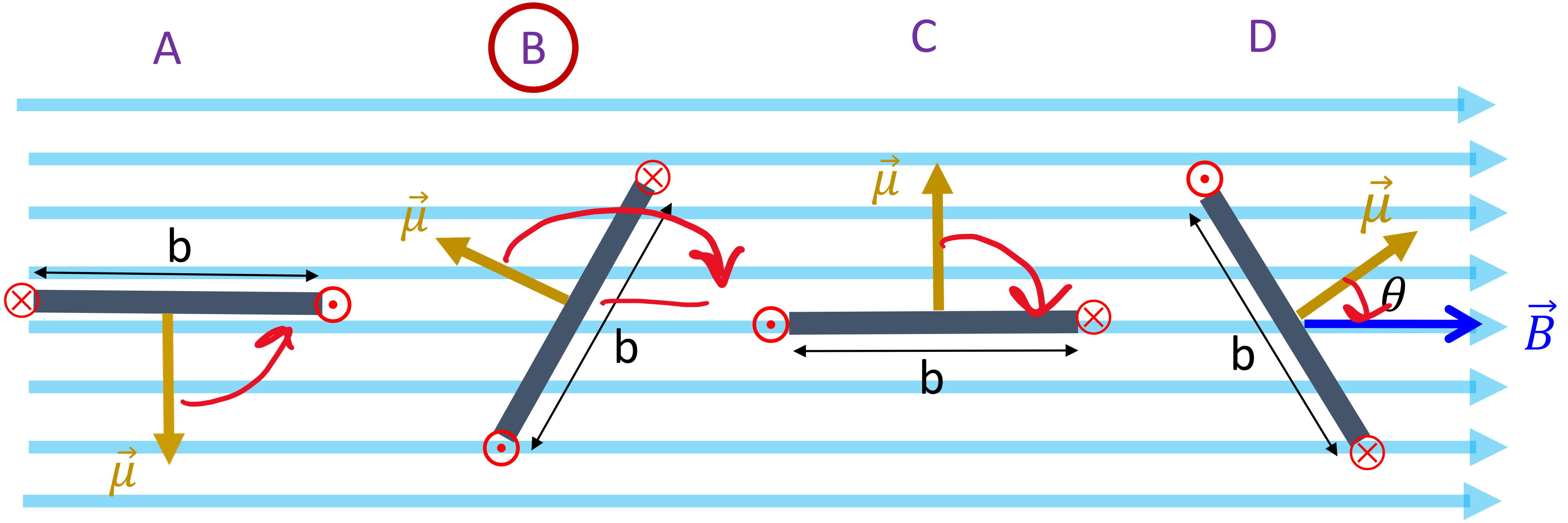


Because of the torque, potential energy can be stored in a current carrying wire loop when it is immersed in an external B-field.

Which orientation below corresponds to the highest value of the stored potential energy?

- The torque on the magnetic dipole moment $\vec{\mu}$ always tries to align it with the external B- field (make it parallel to the B-field)

- The potential energy is: $\boxed{U_B = -\vec{\mu}\cdot\vec{B}}$ (compare: $U_E = -\vec{p}\cdot\vec{E}$)

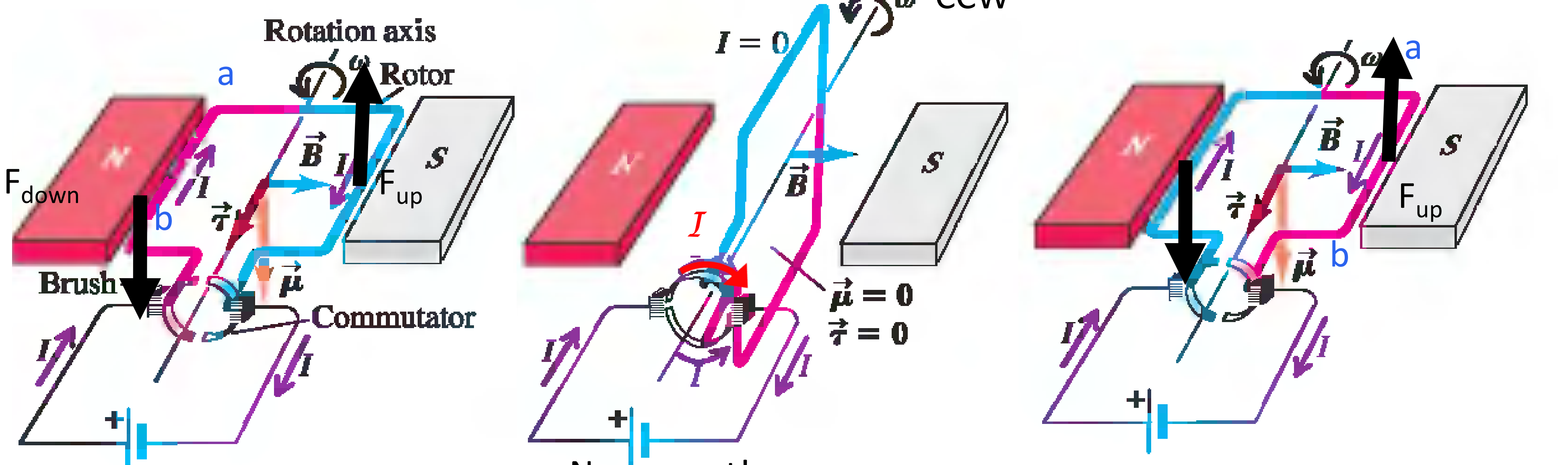


Direct Current electric motor

Y&F 27.39

The rotor is a wire loop free to rotate about an axis

coil is horizontal | coil is vertical | coil is horizontal



No current!

Current into on the red side and out of on the blue side

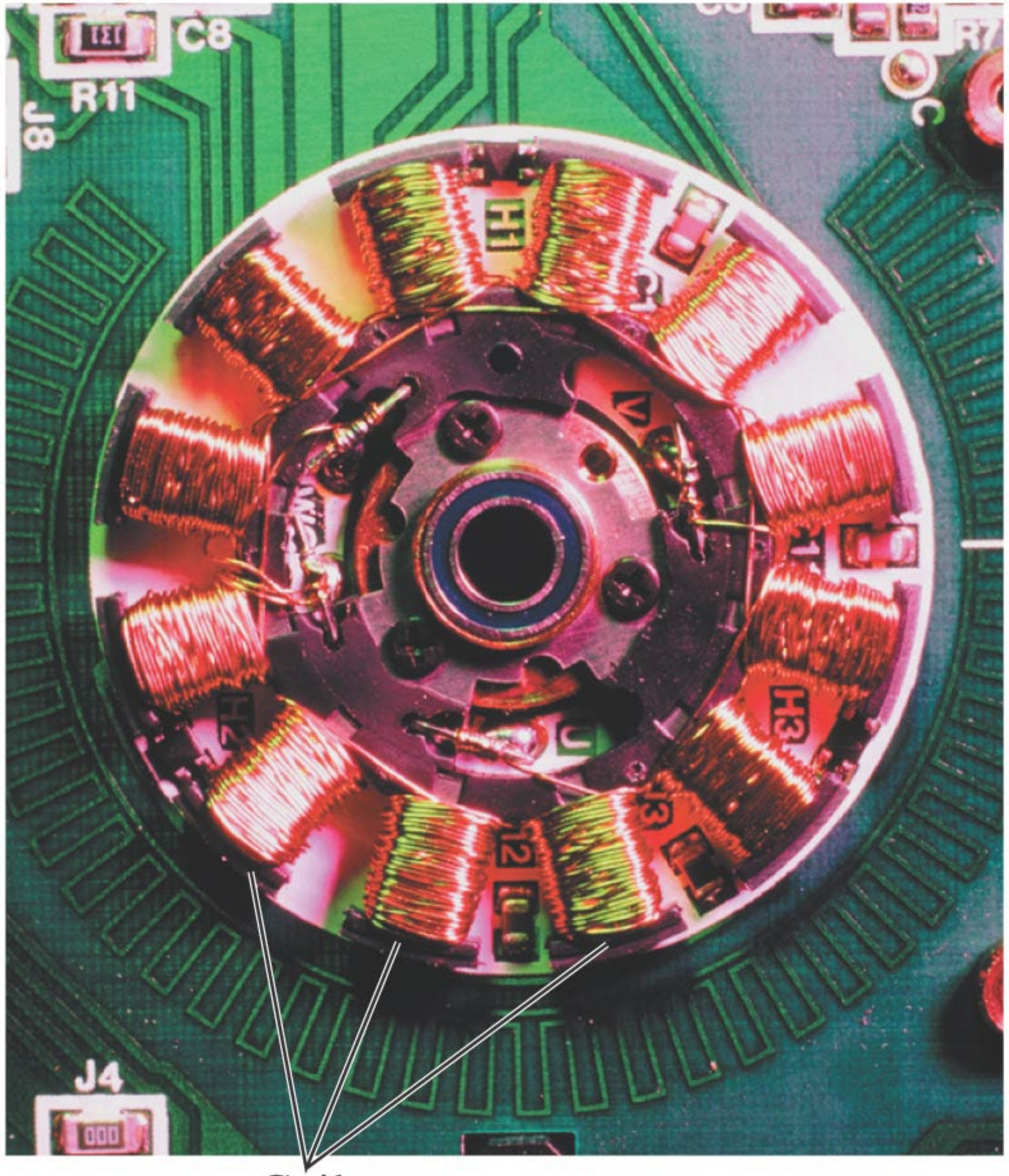
Current into on the blue side and out of on the red side

Key idea: switching the current direction (using a split ring)

Torque is counterclockwise in both orientations

Disk Drive DC Motor – 12 coils

Y&F 27.40



Coils

The permanent magnets can also be located on the rotating turntable. By using several fixed coils a constant speed can be maintained

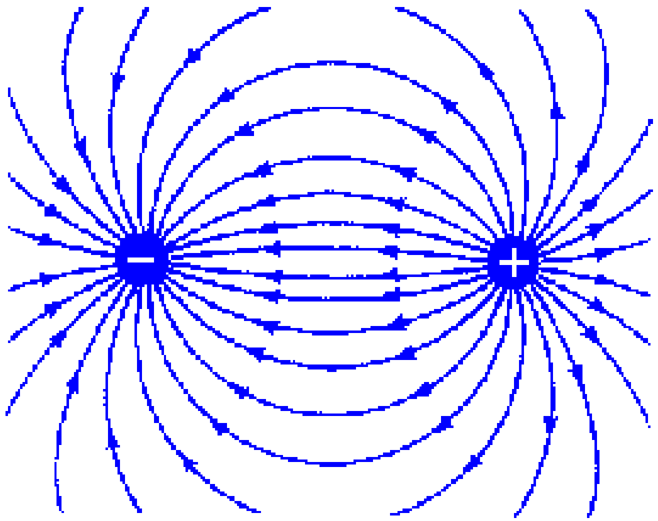
https://www.youtube.com/watch?v=ZAY5JInyHXY&t=24s

https://www.youtube.com/watch?v=bCEiOnuODac

Magnetic Field of a Magnetic Dipole (aka Magnet)

Electric field from an electric dipole

$\vec{E}$

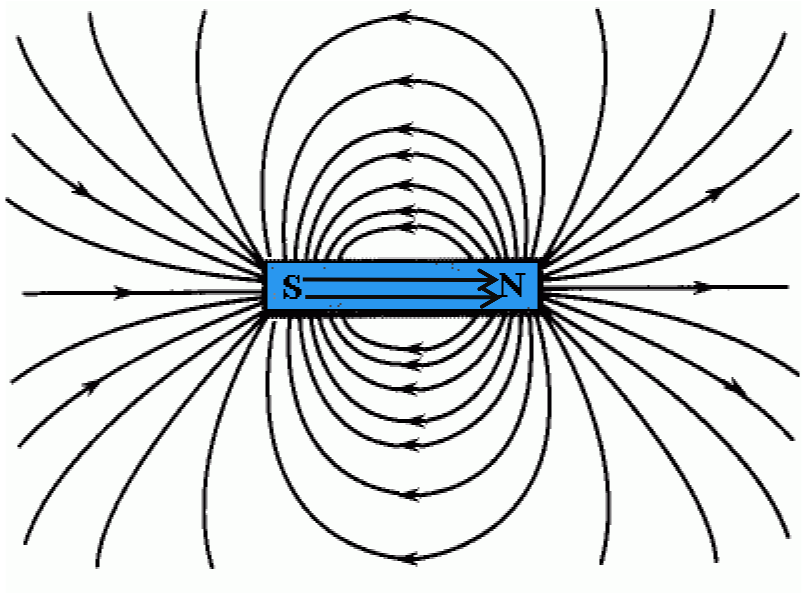


$\vec{B}$

Magnetic field from a magnetic dipole (magnet).

Note that the magnetic field lines are continuous – they do NOT stop at the poles!!

They are always in the shape of a loop.

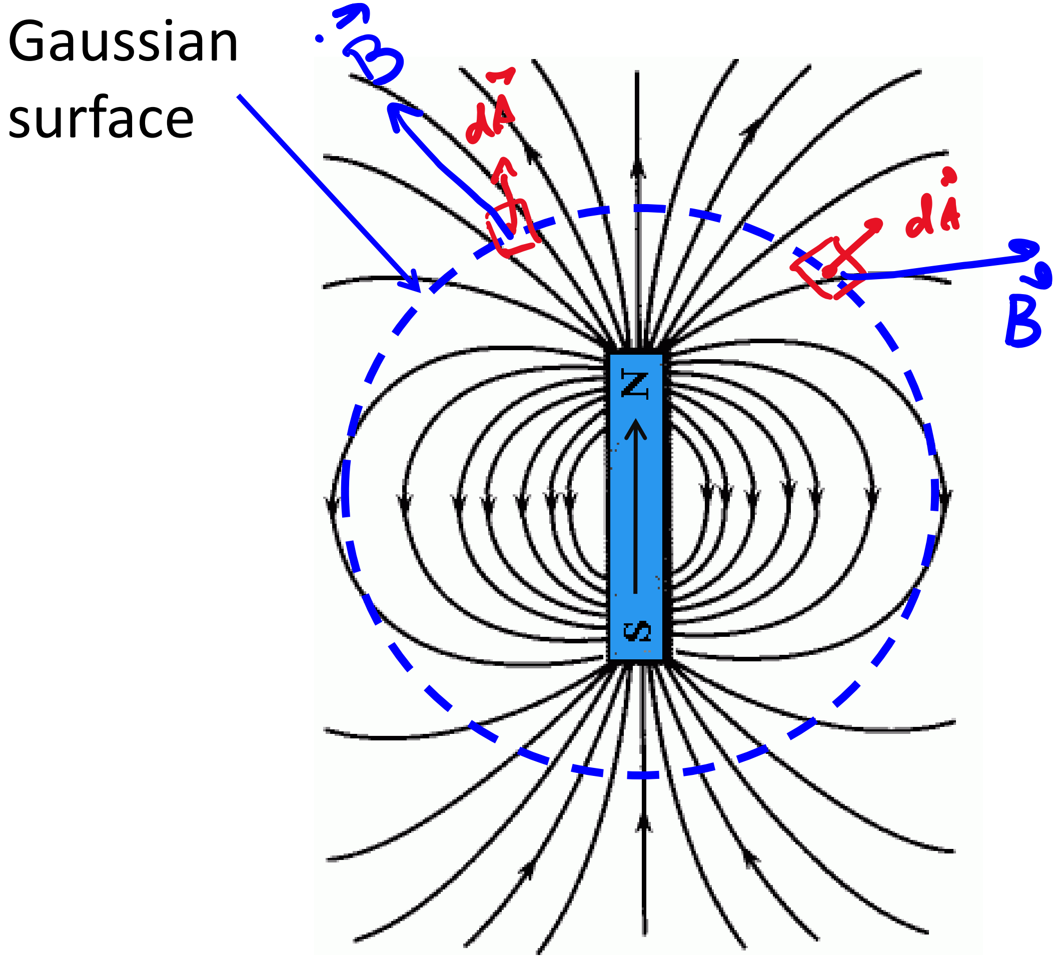


- Both fields have the same shape !!

The magnetic field lines from a magnet point away from the North pole and point into the South pole.

What can you say about the magnetic flux passing through this Gaussian surface?

$$\Phi_B = \oint \vec{B} \cdot d\vec{A}$$



A. $\Phi_B = 0$

B. $\Phi_B > 0$

C. $\Phi_B < 0$

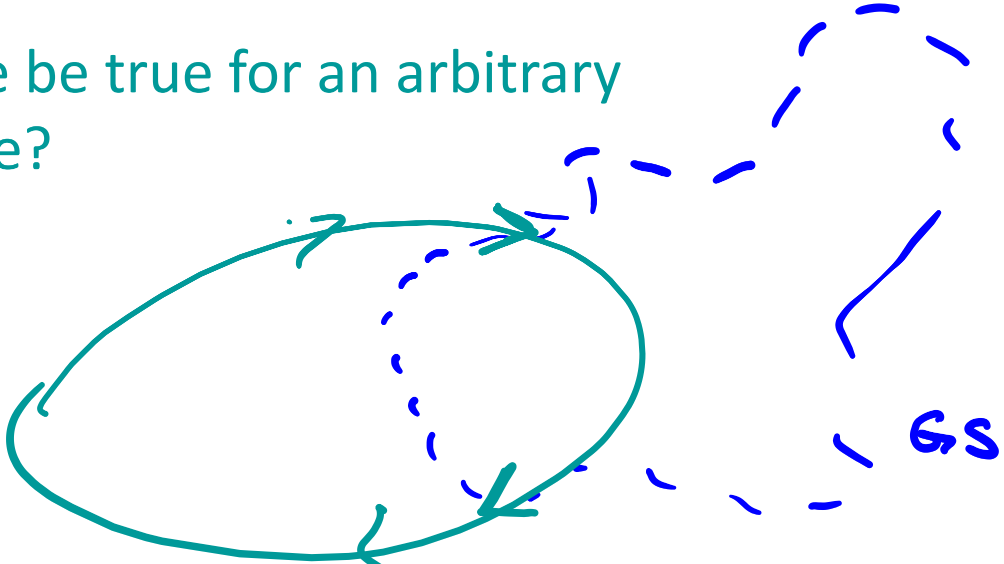
D. Can't tell without evaluating the integral

The magnetic field lines from a magnet point away from the North pole and point into the South pole.

What can you say about the magnetic flux passing through this Gaussian surface?

$$\Phi_B = \oint \vec{B} \cdot d\vec{A}$$

- By symmetry, the same number of flux lines enter and leave the spherical Gaussian surface
- Would the same be true for an arbitrary Gaussian surface?



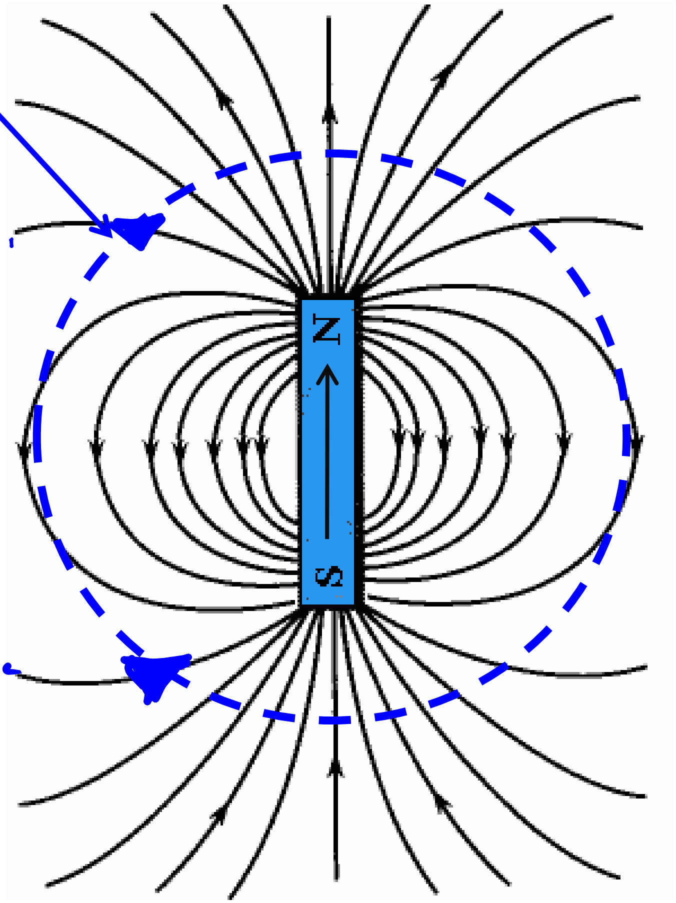
A. $\Phi_B = 0$

B. $\Phi_B > 0$

C. $\Phi_B < 0$

D. Can't tell without evaluating the integral

Gaussian surface



Gauss's law for Magnetism

- The magnetic flux through any closed surface is ALWAYS zero:

$$\boxed{\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0}$$

no enclosed single magnetic pole

- There is no way to isolate a North or South magnetic pole => magnetic lines are always in the shape of a loop

- The simplest E-field is from a point charge, while the simplest B-field is from a magnetic dipole (e.g. Bar Magnet)

Maxwell's equations

- Essentially all of Electricity & Magnetism can be described by a set of 4 equations, referred to as Maxwell's equations.
- We now have two of them!

Gauss's law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\varepsilon_0} \qquad \Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

Faradays' law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} + \frac{1}{c^2}\frac{d\Phi_E}{dt}$$

- We will learn about these other two Maxwell equations shortly