This event is one where students come in to provide feedback or "Beef" about their current classes and receive free pizza in return for their participation. "Feedback" can include a variety of things extending from lecture pacing, to test material, to office hours and tutorial help.

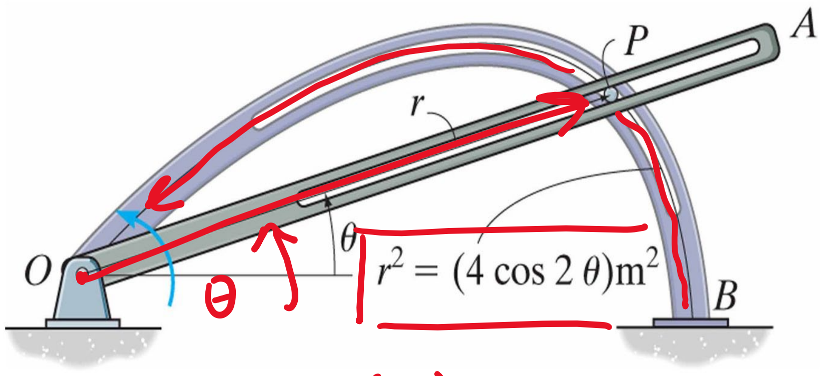
W8-1. $\boxed{\dot{\theta} = 3t^{3/2}}$ rad/s where t is in seconds, and $\theta = 0$ when $t = 0$. Also, $r^2 = (4\cos 2\theta)m^2$.

a) Determine the time when $\theta = 30^\circ$.

b) Determine the radial and transverse components of the ball's velocity and acceleration when $\theta = 30^\circ$.

$$\frac{d\theta}{dt} = 3t^{3/2} \ \frac{\text{rad}}{\text{s}} \longrightarrow \int_{\theta=0}^{\theta} d\theta = \int_{t=0}^{t} 3t^{3/2}\,dt$$

$$\theta(t) = \underset{\substack{\parallel \\ 0}}{\theta_0} + \int_0^t 3t^{3/2}\,dt = 3\left.\frac{t^{5/2}}{5/2}\right|_{t=0}^{t=t} = \frac{6}{5}t^{5/2}$$



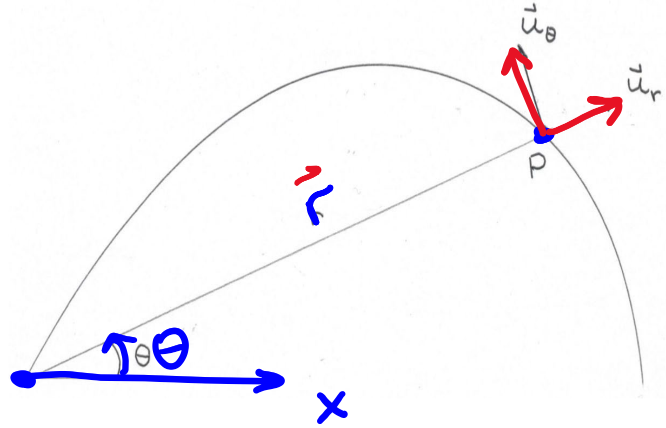
$r = r(\theta)$

Last Time

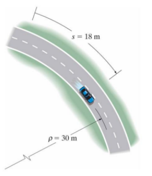
t: $\theta(t) = \frac{6}{5}t^{5/2} = \frac{\pi}{6}$

$$t^{5/2} = \frac{5\pi}{36}$$

$$\boxed{t = \left(\frac{5\pi}{36}\right)^{2/5} = 0.7177 \text{ s}}$$



W7-4. The car travels along the circular path. Starting from rest, its acceleration along the path is $0.5\,e^t$ m/s² where t is in seconds.
a) Determine how long it takes the car to travel 18 m.
b) Determine the car's speed and acceleration at this time.



W8-1. $\dot{\theta} = 3t^{3/2}$ rad/s where t is in seconds, and $\theta = 0$ when $t = 0$. Also, $\boxed{r^2 = (4\cos 2\theta)m^2.}$ $\vec{u}_r, \vec{u}_\theta$

a) Determine the time when $\theta = 30^o$ **=>** $\boldsymbol{t = 0.7177}$ **s**

b) Determine the radial and transverse components of the ball's velocity and acceleration when $\theta = 30^o$.

$\theta = \frac{\pi}{6}$

$v_r = \dot{r}$,	$v_\theta = r\dot{\theta}$
$a_r = \ddot{r} - r\dot{\theta}^2$,	$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

- Find $\theta, \dot{\theta}, \ddot{\theta}$.

(1) $\theta = \pi/6$

(2) $\dot{\theta} = 3t^{3/2} \xrightarrow{t=0.7177} \dot{\theta} = 1.8240 \ \frac{rad}{s}$

(3) $\ddot{\theta} = \frac{d}{dt}\dot{\theta} = \frac{d}{dt}(3t^{3/2}) = 3 \cdot \frac{3}{2} t^{1/2} \xrightarrow{t=0.7177} 3.8123 \ \frac{rad}{s^2}$

$r = \sqrt{4 \cdot \cos(2\theta)} \xrightarrow{\theta = \pi/6} \dots$

$\frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} = \frac{dr}{d\theta} \cdot \dot{\theta}$

W8-1. $\dot{\theta} = 3t^{3/2}$ rad/s where t is in seconds, and $\theta = 0$ when $t = 0$. Also, $r^2 = (4\cos 2\theta)m^2$.

a) Determine the time when $\theta = 30^o$ **=>** $\boldsymbol{t_* = 0.7177}$ **s**

b) Determine the radial and transverse components of the ball's velocity and acceleration when $\theta = 30^o$.

$$v_r = \dot{r}, \qquad v_\theta = r\dot{\theta}$$

$$a_r = \ddot{r} - r\dot{\theta}^2, \qquad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

- Find $r, \dot{r}, \ddot{r}$:

$$r^2 = (4\cos 2\theta)$$

✓ (1) $\theta = \frac{\pi}{6}$

✓ (2) $\dot{\theta}(t) = 3\,t^{3/2} = 1.8240\,\frac{\text{rad}}{s}$

(3) $\ddot{\theta} = \frac{9}{2}\,t^{\frac{1}{2}} = 3.8123\,\frac{\text{rad}}{s^2}$

(4) $r(t = 0.7177) = r(\theta = \pi/6)$

$r^2 = 4 \cdot 1/2 = 2$ ✓ $r(t = \dots) = \sqrt{2}$.

$\dot{r}$: $\frac{d}{dt}\left[r^2 = 4\cos 2\theta\right]$

$2r\dot{r} = 4 \cdot (-\sin 2\theta)\, 2 \cdot \dot{\theta}$

$\rightarrow r\dot{r} = -4\dot{\theta}\sin 2\theta$

$$\dot{r} = -\frac{4\dot{\theta}\sin 2\theta}{r} \xrightarrow{t = 0.7177} -4.4679\,\frac{m}{s}$$

$\ddot{r}$: $\frac{d}{dt}\left[r\dot{r} = -4\dot{\theta}\sin 2\theta\right]$

$\dot{r}\dot{r} + r\ddot{r} = -4\left[\ddot{\theta}\sin 2\theta + \dot{\theta}\cos 2\theta \cdot 2\dot{\theta}\right]$

$\dot{r}^2 + r\ddot{r} = -4\left[\ddot{\theta}\sin 2\theta + 2\dot{\theta}^2\cos 2\theta\right]$

$$\ddot{r} = \frac{-4\left[\ddot{\theta}\sin 2\theta + 2\dot{\theta}^2\cos 2\theta\right] - \dot{r}^2}{r} \xrightarrow{t = 0.7177} -32.864\,\frac{m}{s^2}$$

W8-1. $\dot{\theta} = 3t^{3/2}$ rad/s where t is in seconds, and $\theta = 0$ when $t = 0$. Also, $r^2 = (4\cos 2\theta)m^2$.

a) Determine the time when $\theta = 30^o$ **=>** $\boldsymbol{t_* = 0.7177}$ **s**

$t = 0.7177\ s$

b) Determine the radial and transverse components of the ball's velocity and acceleration when $\theta = 30^o$.

r $\quad$ θ

$v_r = \dot{r}, \qquad v_\theta = r\dot{\theta}$
$a_r = \ddot{r} - r\dot{\theta}^2, \qquad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

(1): $\theta = \frac{\pi}{6}$

(2): $\dot{\theta}(t) = 3\,t^{3/2} = 1.8240\,\frac{\text{rad}}{s}$

(3): $\ddot{\theta} = \frac{9}{2}\,t^{\frac{1}{2}} = 3.8123\,\frac{\text{rad}}{s^2}$

(4): $r = 2\sqrt{\cos 2\theta} = \sqrt{2}\,m$

(5): $\dot{r} = -\frac{4\dot{\theta}\sin(2\theta)}{r} = -4.4679\,\frac{m}{s}$

(6): $\ddot{r} = \frac{-4[\ddot{\theta}\sin 2\theta + 2\dot{\theta}^2\cos 2\theta] - \dot{r}^2}{r} = -32.864\,\frac{m}{s^2}$

$v_r = -4.47\ m/s$

$v_\theta = 2.58\ m/s$

$v = \sqrt{v_r^2 + v_\theta^2} =$

$= \dots$

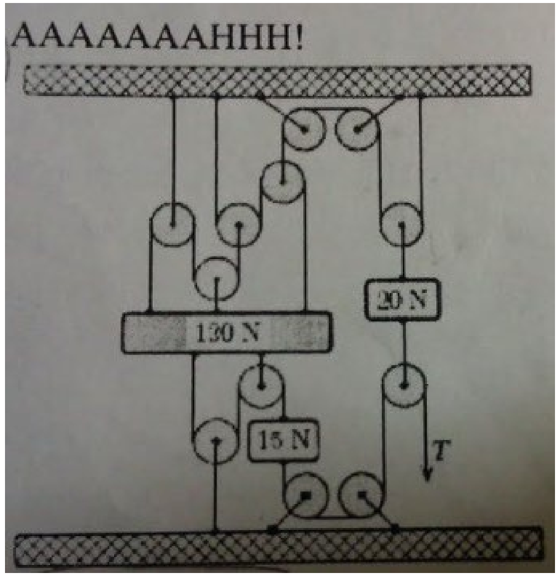
$a_r = -37.6\ \frac{m}{s^2}$

$a_\theta = -10.9\ \frac{m}{s^2}$

$a = \sqrt{a_r^2 + a_\theta^2} =$

$= \dots$

Absolute Dependent Motion: Pulleys and Ropes

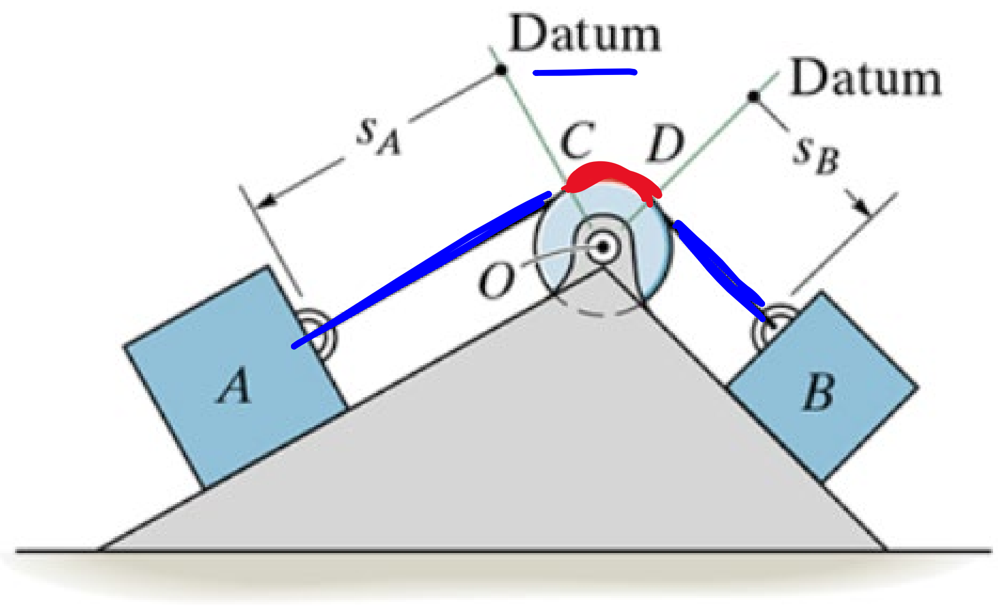


Text: 12.9

Content:

- Rope equation and paths equations
- Solving scary systems of blocks and pulleys

CONSTRAINED MOTION



- Big idea:

The motion of A and B are subject to the constraint dictated by the length of the rope:

$$\frac{d}{dt}\left[s_A + s_B + CD = const = L_{rope} \right]$$

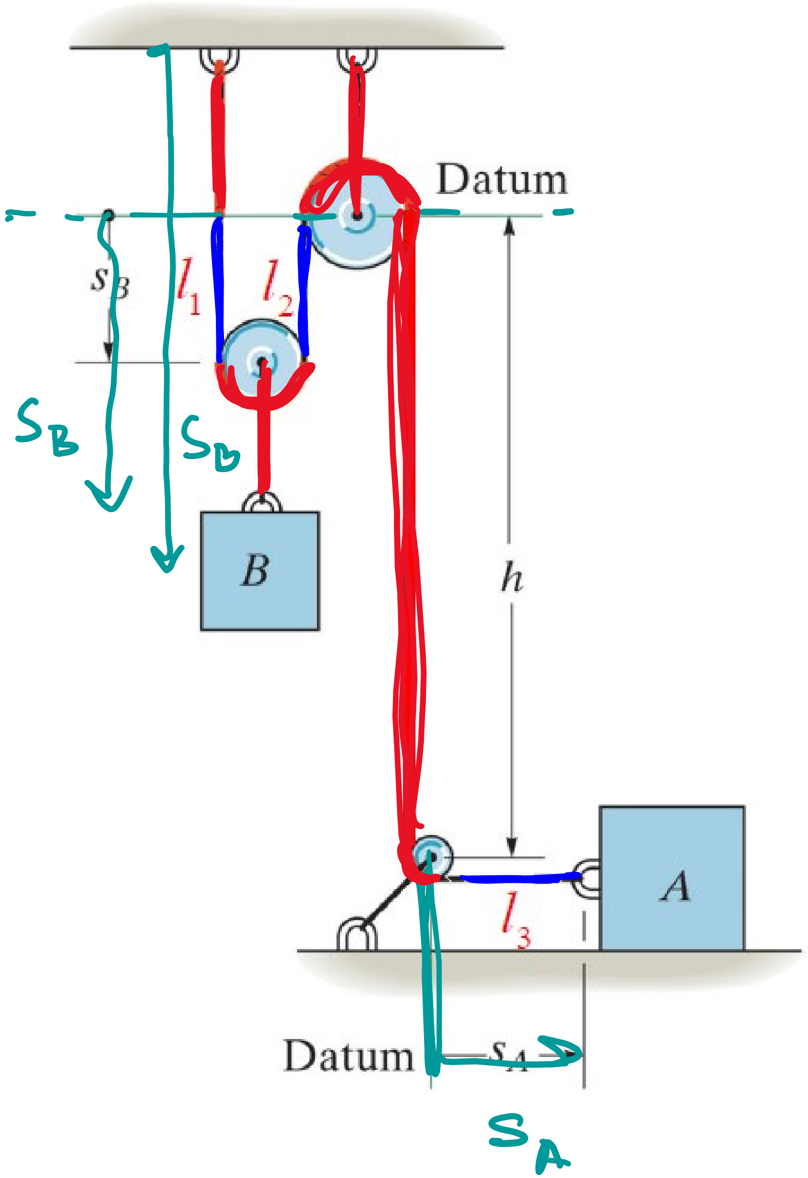
The derivative of this equation tells us that

$$\dot{s}_A = -\dot{s}_B$$

That means that these two blocks:

- Have the same speeds!
- When block A moves parallel to s_A, block B moves anti-parallel to s_B => know the direction of motion!

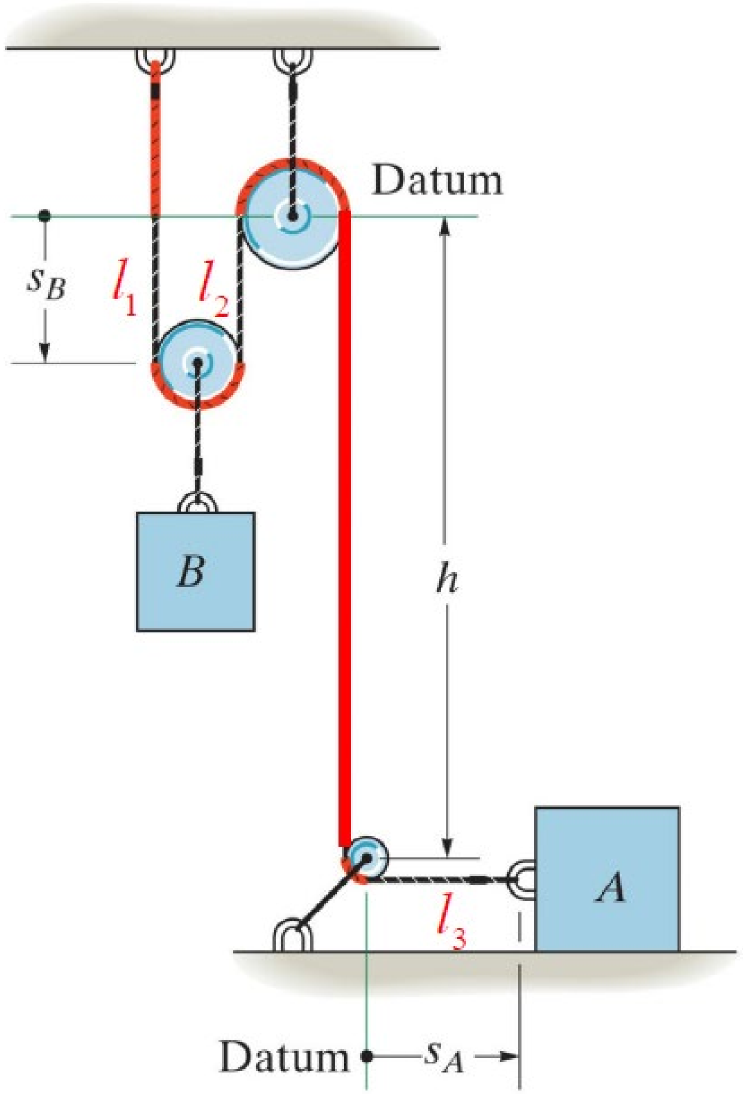
ROPE EQUATION & PATH EQUATION



Q: Determine the relationship between v_A and v_B

- Paths: s_A and s_B
  - Distances from a fixed point ("datum") to the object
  - Paths are directional
- Rope segments: l_1, l_2 and l_3
  - All <u>non-constant</u> segments of the rope
  - They are just magnitudes
- Highlighted in red are all <u>constant</u> segments of the rope

ROPE EQUATION & PATH EQUATION



All red segments

L_{rope} - const

- Rope equation(s):

(1) ➢ $L_{rope} = l_1 + l_2 + l_3 + const$ => $\boxed{l_1 + l_2 + l_3 = const'}$

- Path equation(s):

(a) + (b) + (c) :

(a) ➢ $l_1 = s_B$ (b) ➢ $l_2 = s_B$ (c) ➢ $l_3 = s_A$

- Combine path equations to get a const :

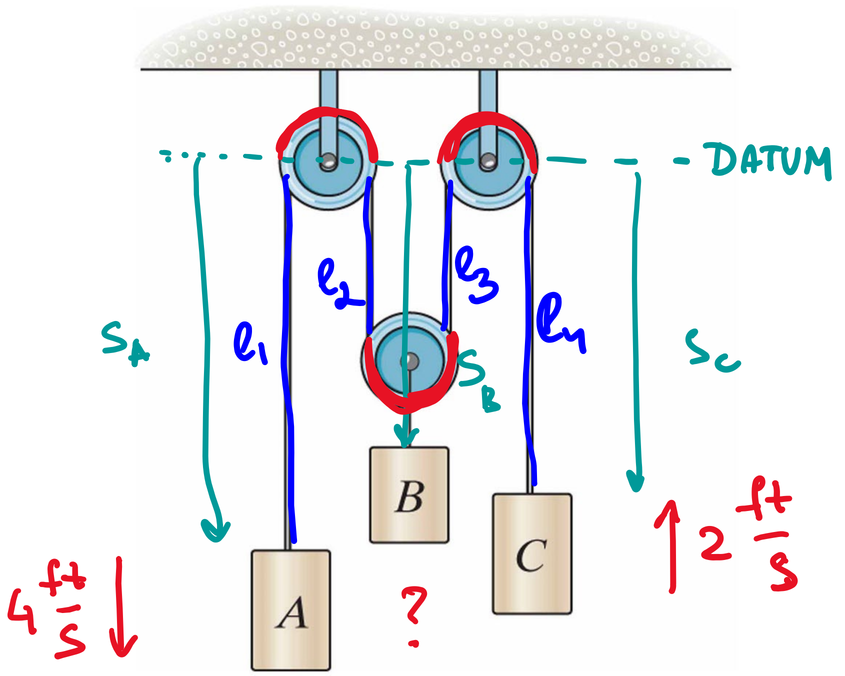
➢ $\boxed{l_1 + l_2 + l_3}$ $\frac{d}{dt}\left[s_A + 2s_B = const\right]$

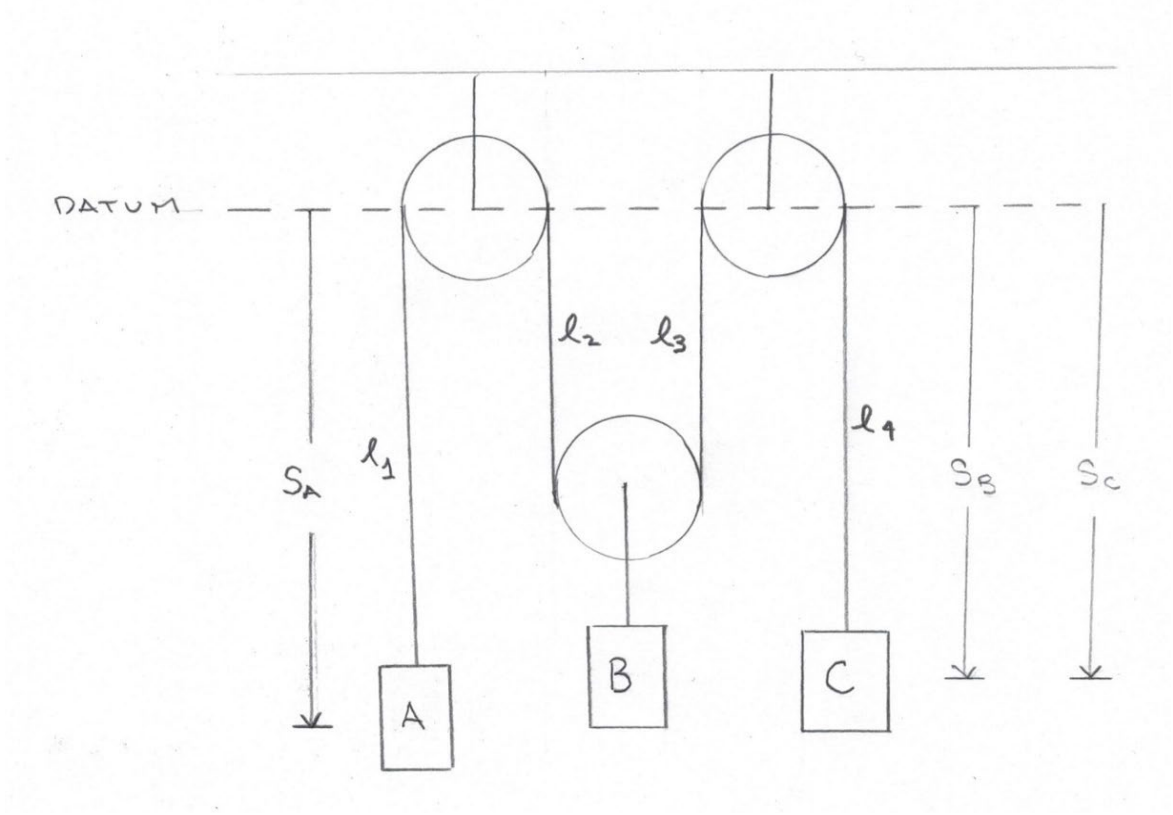
➢ $0 = \dot{s}_A + 2\dot{s}_B$

➢ $v_A = -2v_B$

➢ "−" tells us about their mutual directions
(if B moves along s_B, then A moves against s_A)

W8-2. Block A is moving down at 4 ft/s. Block C is moving up at 2 ft/s. Determine the speed and direction of motion of block B.





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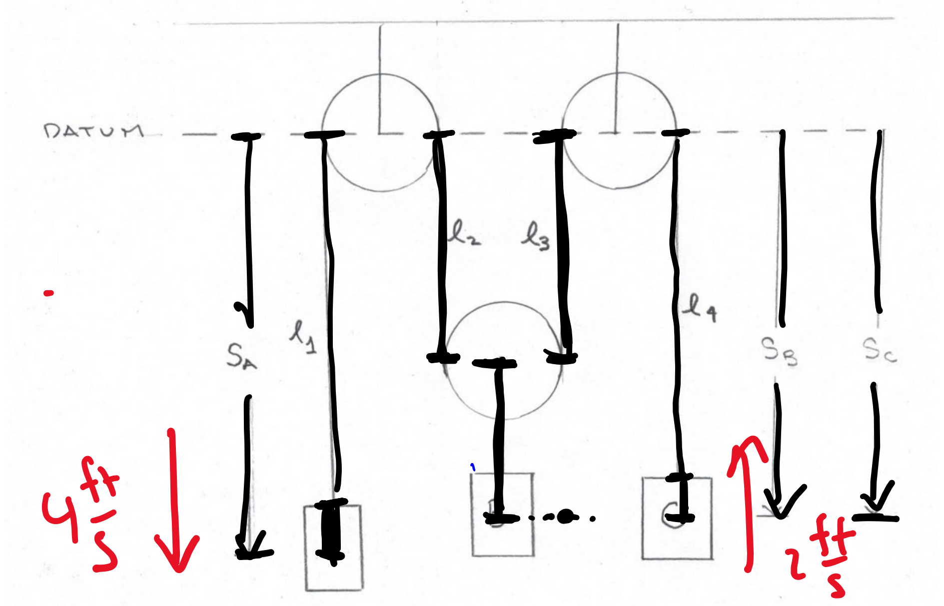

Q: v_B is:

A. 1 ft/s, up

B. 1 ft/s, down

C. 2 ft/s, up

D. 3 ft/s, down

E. Something else

$\dot{S}_A = +4\,\frac{ft}{s}$

$\dot{S}_C = -2\,\frac{ft}{s}$

$\boxed{v_B = 1\,\frac{ft}{s},\ \text{up}}$

Rope equation:

$l_1 + l_2 + l_3 + l_4 + \text{const} = L_{rope}$

(1) $\boxed{l_1 + l_2 + l_3 + l_4 = \text{const}}$

Path eqs:

(a) $l_1 + \text{const} = S_A$

(b) $l_2 + \text{const} = S_B$

(c) $l_3 + \text{const} = S_B$

(d) $l_4 + \text{const} = S_C$

(a) + (b) + (c) + (d):

$l_1 + \text{const} + l_2 + \text{const} + l_3 + \text{const} + l_4 + \text{const} = S_A + 2S_B + S_C$

$\underbrace{\boxed{l_1 + l_2 + l_3 + l_4}}_{\text{const}} + \text{const} = S_A + 2S_B + S_C$

$\frac{d}{dt}\left[S_A + 2S_B + S_C = \cancel{\text{const}}^{\,0}\right]$

$\dot{S}_A + 2\dot{S}_B + \dot{S}_C = 0$

$\dot{S}_B = -\frac{1}{2}(\dot{S}_A + \dot{S}_C) = -1\,\frac{ft}{s}$