MINDFULNESS MATTERS

Dr. Ishan Shivanand

Reframe your mindset & build a positive outlook

Manage high-pressure situations & anxiety

Learn how to become mentally resilient!

MARCH 27th, 2024

5:00-7:00 PM

Chemical and Biological Engineering Building (CHBE) - 101

Food & refreshments will be provided.

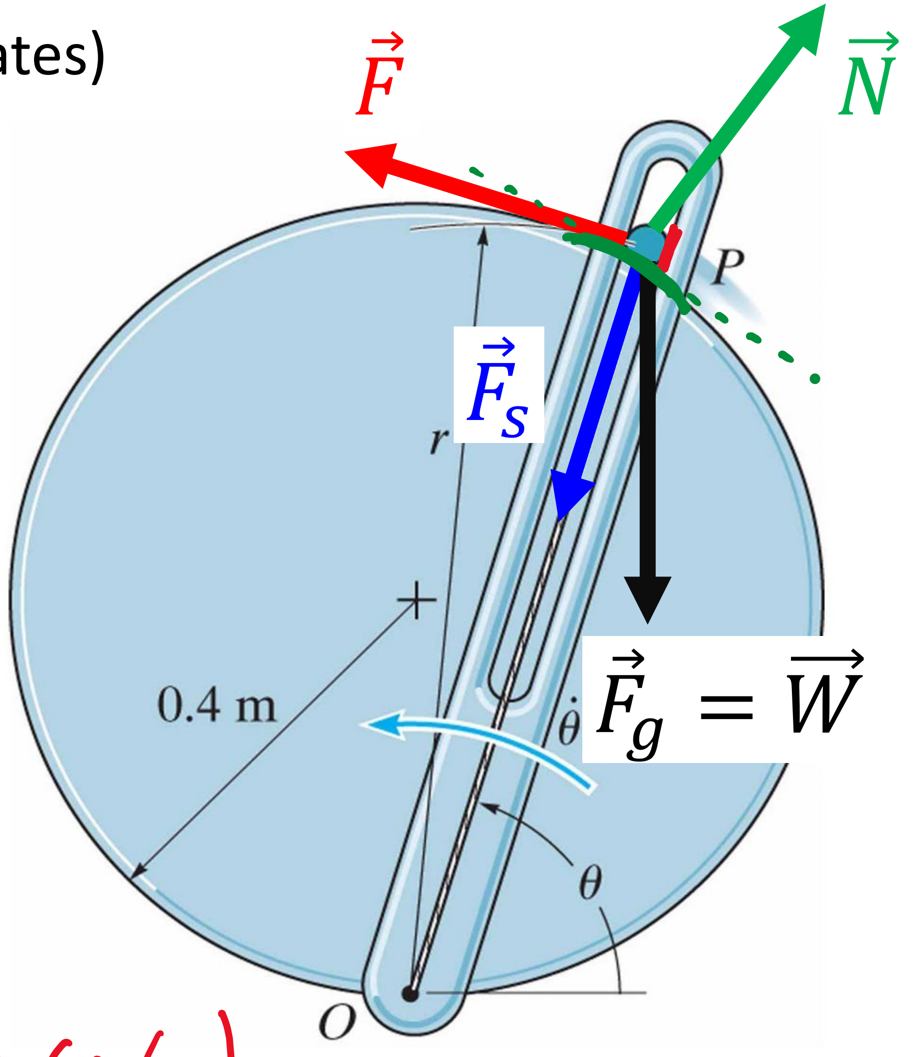
DE-STRESS WITH US!

RSVP!

Last time:

Finding forces from acceleration (written in polar coordinates)

- Acceleration -- in polar coordinates
- Forces in convenient coordinates
- Translate forces into polar coordinates
- Apply 2nd Newton's law



$a_r = -3704 \frac{m}{s^2}$ $\quad a_\theta = 15.18 \frac{m}{s^2}$ (polar)

$\vec{F}_s = -F_s \vec{u}_r$ (polar) $\quad \vec{F}_g = -mg\,\vec{j}$ (cart)

$\vec{F} = F\,\vec{u}_\theta$ (polar) $\quad \vec{N} = -N\,\vec{u}_n$ (n,t)

(polar)

a : $\vec{u}_r, \vec{u}_\theta$ (polar)

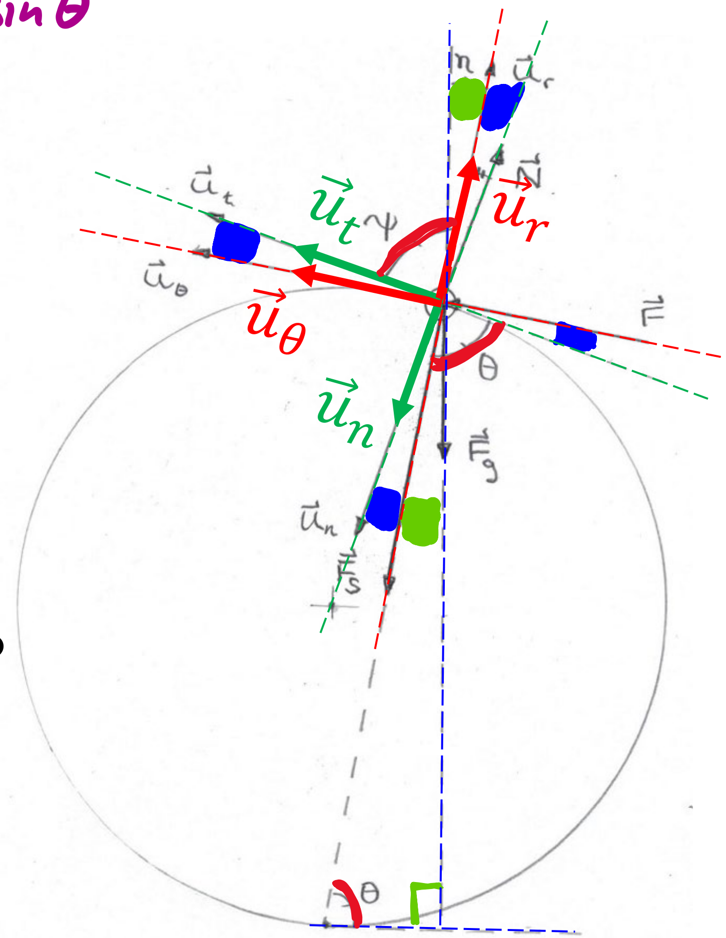
W10-2. The slotted guide moves the 150 g particle P around the 0.4 m radius circular disk. Motion is in the vertical plane. Attached to P is an elastic cord extending from O. The cord has stiffness 30 N/m and unstretched length 0.25 m. Friction may be neglected. Determine the force of the guide on P and the normal force of the disk on P when $\theta = 70^o$, $\dot{\theta} = 5$ rad/s, and $\ddot{\theta} = 2$ rad/s².

$r(\theta) = 2R \sin\theta$

3. Reduce to the same coordinate system: $\vec{u}_n$ and $\vec{j}$ to $(\vec{u}_r, \vec{u}_\theta)$-components

- For projections, we need to know angles. $\tan\psi = \dfrac{r}{dr/d\theta}$
- Let us highlight the three coordinate systems: (n,t) & polar & z-axis
- ψ (angle between $\vec{u}_r$ and $\vec{u}_t$) $= \theta$ (vertical angles) $= 70°$
- η (angle between $\vec{u}_r$ and $\vec{j}$) $= 90^o - \theta$ (vertical right triangle) $= 20°$
- β (angle between $\vec{u}_t$ and $\vec{u}_\theta$) $= 90^o - \psi$ (from 90° between $\vec{u}_\theta$ and $\vec{u}_r$) $= 20°$

- We get:
  - ➢ $\psi = \theta = 70^o$
  - ➢ $\eta = \beta = 20^o$



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3. Reduce to the same coordinate system: $\vec{u}_n$ and $\vec{j}$ to $(\vec{u}_r, \vec{u}_\theta)$-components

- $\vec{N} = -N\,\vec{u}_n$

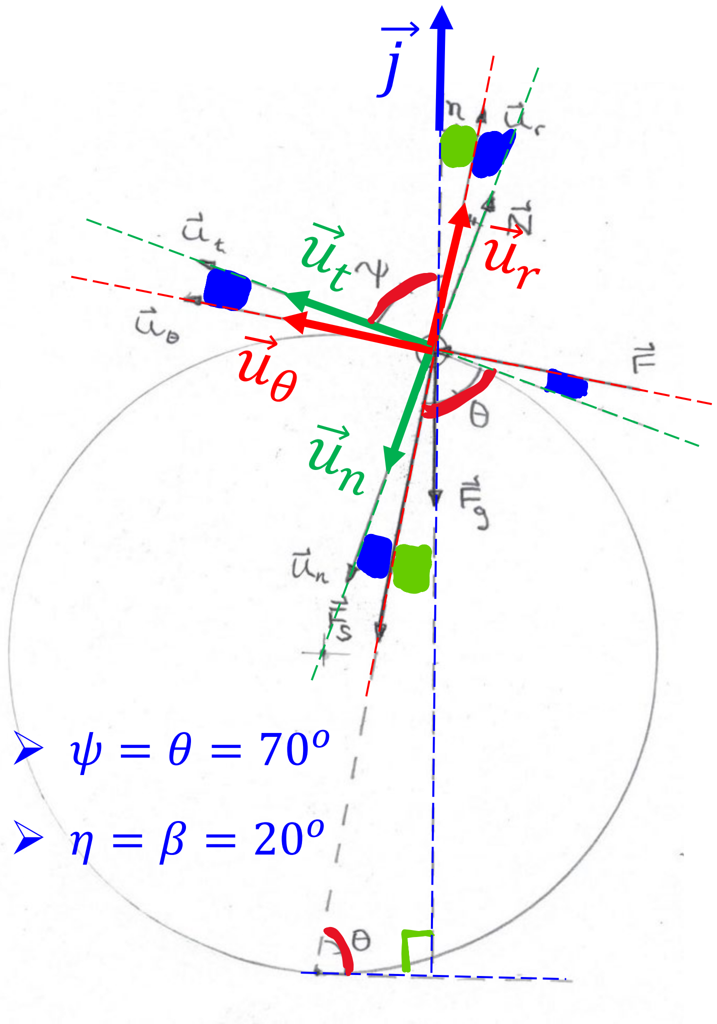
$$\vec{u}_n = -\cos 20^\circ\,\vec{u}_r + \sin 20^\circ\,\vec{u}_\theta$$

$$\vec{N} = N\cos 20^\circ\,\vec{u}_r - N\sin 20^\circ\,\vec{u}_\theta$$

- $\vec{F}_g = -mg\,\vec{j}$

$$\vec{j} = \cos 20^\circ\,\vec{u}_r + \sin 20^\circ\,\vec{u}_\theta$$

$$\vec{F}_g = -\cos 20^\circ\,\vec{u}_r - \sin 20^\circ\,\vec{u}_\theta$$



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$|\vec{F}_s| = k\,\Delta x = 30\,(r(\theta) - r_0) = 15.05\ N$

4. Apply 2nd Newton's law:

$\vec{F} + \vec{N} + \vec{F}_s + \vec{F}_g = m\vec{a}$

$\vec{F} = F\vec{u}_\theta$;

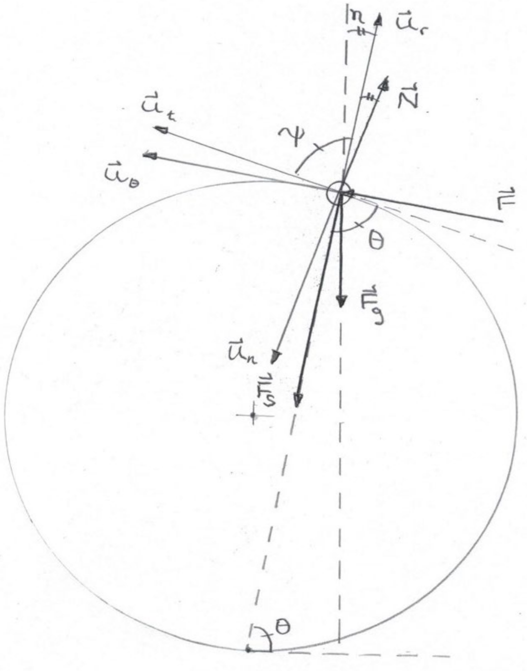
$\vec{F}_s = -\underbrace{\boxed{k(r - r_0)}}_{15.05}\vec{u}_r$;

$\vec{N} = N\cos 20^o\,\vec{u}_r - N\sin 20^o\,\vec{u}_\theta$;

$\vec{W} = \vec{F}_g = -mg\cos 20^o\,\vec{u}_r - mg\sin 20^o\,\vec{u}_\theta$

$a_r = -37.04\,\frac{m}{s^2}$, $a_\theta = 15.184\,\frac{m}{s^2}$

$r(\theta) = 2R\sin\theta \Rightarrow r(70^\circ) = 2 \cdot 0.4 \cdot \sin(70^\circ)$



a_r

$r:\ 0 + N\cos 20^\circ - 15.05 - mg\cos 20^\circ = (0.150)(-37.04)$

$\theta:\ F - N\sin 20^\circ + 0 - mg\sin 20^\circ = (0.150)(15.184)$

a_θ

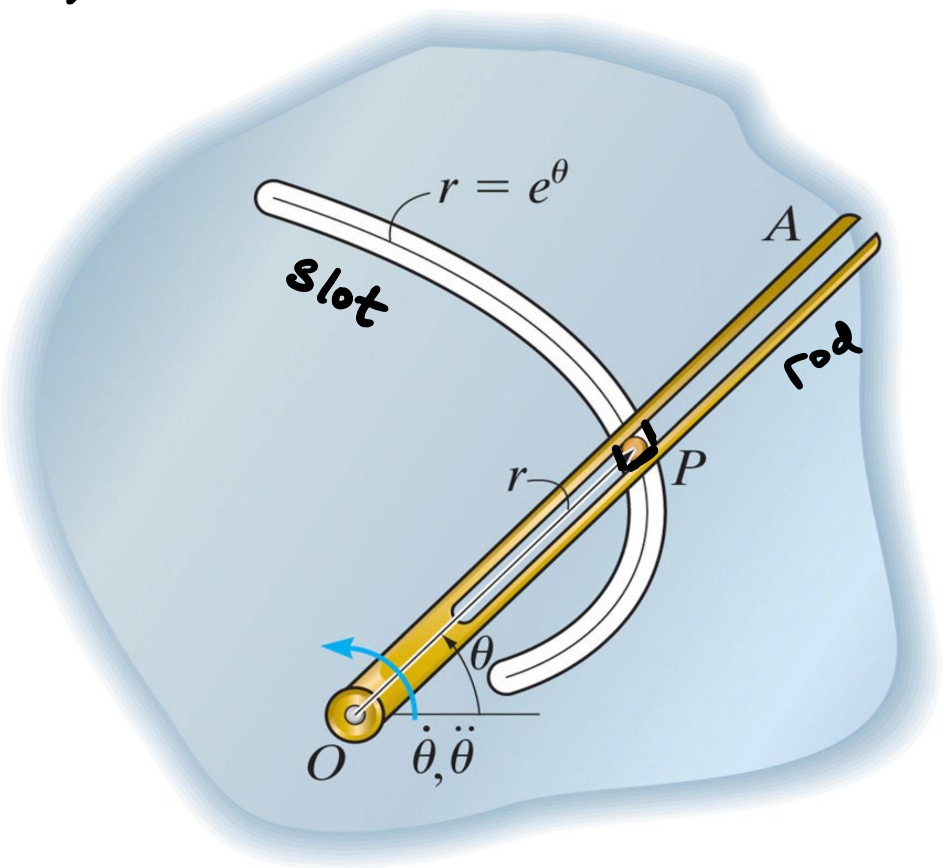
$r:\ \boxed{N = 11.6\ N}$

$\boxed{F = 6.74\ N}$

W10-3. The forked rod OA moves the 1.6 kg particle P around the curved slot whose shape is given by $r = 0.6\,e^{\theta}$ m where θ is in radians. Motion is in the vertical plane. A force $\vec{S}$ acts on the particle in the direction opposite to its velocity. The magnitude of $\vec{S}$ is 8.4 N. Determine the force that the rod exerts on the particle and the force of the slot on the particle when $\theta = 50^o$, $\dot{\theta} = 1.3$ rad/s, and $\ddot{\theta} = 1.4$ rad/s².

$\dot{r}, \ddot{r}$

- include / ignore gravity force?



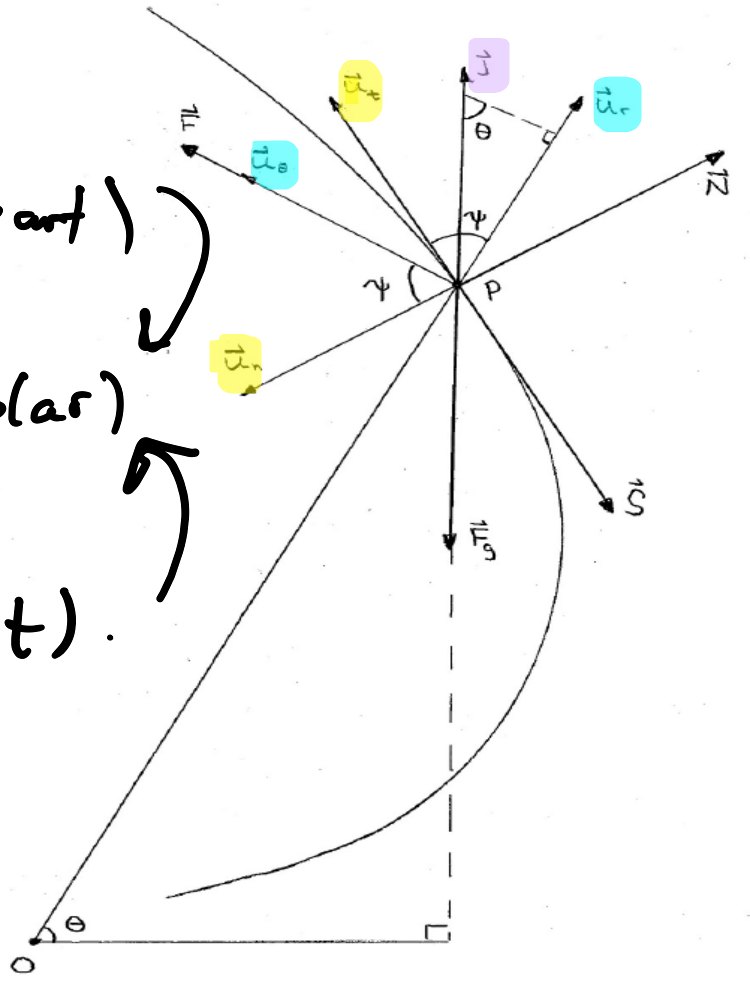
$\vec{F}_g \uparrow\downarrow \vec{j}$ (cart)

$\vec{F} \uparrow\uparrow \vec{u}_\theta$ (polar)

$\vec{N} \uparrow\downarrow \vec{u}_n$ (n,t).

$\vec{S} \uparrow\downarrow \vec{u}_t$

$\vec{a}$ (polar)



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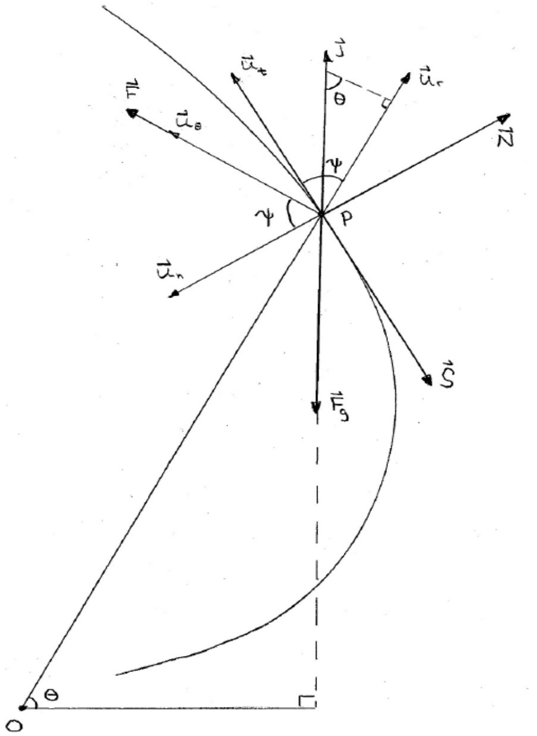
1. Find acceleration in polar coordinates:



We are interested in the moment when:

- $\theta = \frac{50\,\pi}{180}$; $\quad \dot{\theta} = 1.3\,\frac{\text{rad}}{s}$; $\quad \ddot{\theta} = 1.4\,\frac{\text{rad}}{s^2}$

$$r = 0.6\, e^{\theta} \xrightarrow{@\,\theta = \dots} 1.436 \text{ m}$$

$$\dot{r} = 0.6\, e^{\theta} \cdot \dot{\theta} \xrightarrow{@\,\theta = \dots\ \dot{\theta} = \dots} 1.867\ \frac{\text{m}}{\text{s}}$$

$$\ddot{r} = 0.6 \frac{d}{dt}\left(e^{\theta}\dot{\theta}\right) = 0.6\left[e^{\theta}\cdot\dot{\theta}^2 + e^{\theta}\ddot{\theta}\right] \xrightarrow{\theta = \dots\ \dot{\theta} = \dots\ \ddot{\theta} = \dots} 4.437\ \frac{\text{m}}{\text{s}^2}$$

SIG. FIGS. !!!

$a_r = 2.010$ m/s^2 $\qquad a_\theta = 6.864\ \frac{\text{m}}{\text{s}^2}$

$$a_r = \ddot{r} - r\dot{\theta}^2, \qquad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

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2. Write down all the forces:

$\vec{F} = F\vec{u}_\theta$

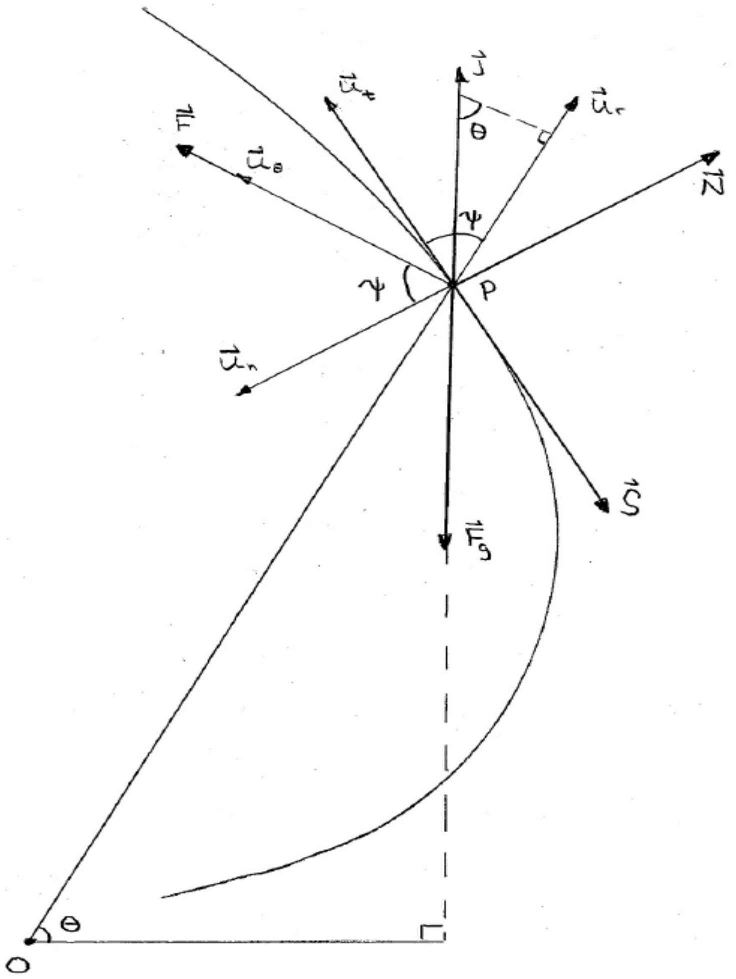
$\vec{N} = -N\vec{u}_n$

polar

$\vec{S} = -S\vec{u}_t$, $S = 8.4$ N

$\vec{F}_g = -mg\vec{j}$, $mg = (1.6)(9.81)$

$\vec{a} =$ polar



W10-3. The forked rod OA moves the 1.6 kg particle P around the curved slot whose shape is given by $r = 0.6\ e^{\theta}$ m where θ is in radians. Motion is in the vertical plane. A force $\vec{S}$ acts on the particle in the direction opposite to its velocity. The magnitude of $\vec{S}$ is 8.4 N. Determine the force that the rod exerts on the particle and the force of the slot on the particle when $\theta = 50^o$, $\dot{\theta} = 1.3$ rad/s, and $\ddot{\theta} = 1.4$ rad/s^2.

3. Translate between coordinate systems:

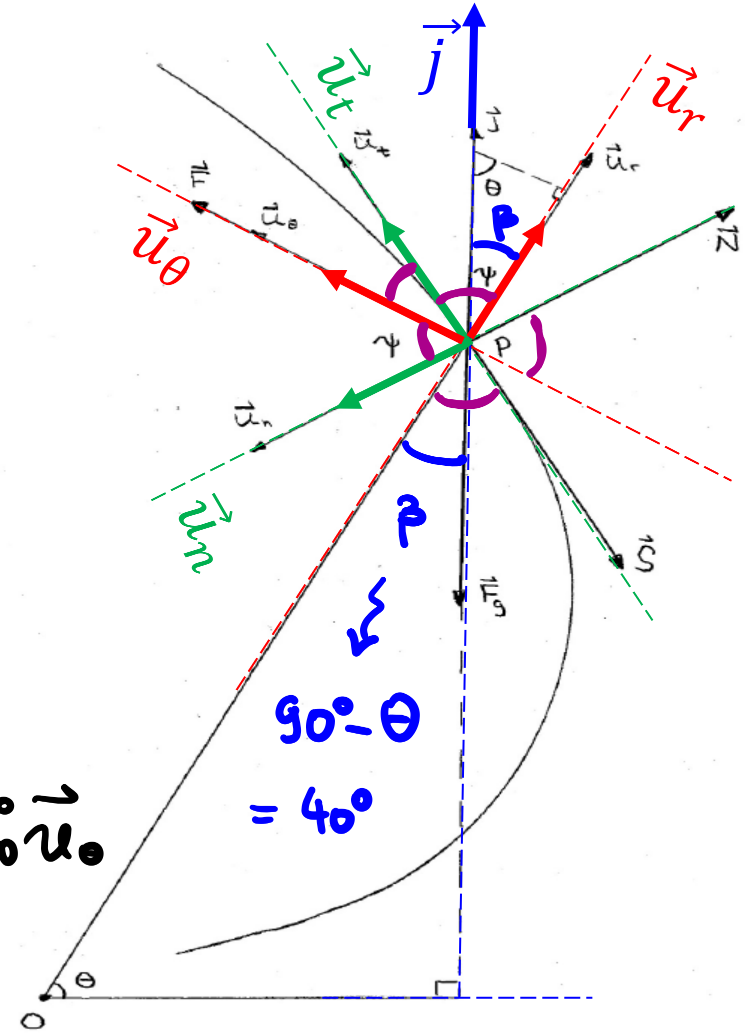
$$\vec{F} = F\vec{u}_{\theta}; \quad \vec{N} = -N\vec{u}_n; \quad \vec{S} = -S\vec{u}_t; \quad \vec{W} = \vec{F}_g = -mg\ \vec{j}$$

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{\cancel{0.6}e^{\theta}}{\cancel{0.6}e^{\theta}} = 1 \qquad \boxed{\psi = 45^\circ}$$

$$\vec{u}_t = \frac{\vec{u}_r}{\sqrt{2}} + \frac{\vec{u}_\theta}{\sqrt{2}} \longrightarrow \vec{S} = -\frac{S}{\sqrt{2}}\vec{u}_r - \frac{S}{\sqrt{2}}\vec{u}_\theta$$

$$\vec{u}_n = -\frac{\vec{u}_r}{\sqrt{2}} + \frac{\vec{u}_\theta}{\sqrt{2}} \longrightarrow \vec{N} = +\frac{N}{\sqrt{2}}\vec{u}_r - \frac{N}{\sqrt{2}}\vec{u}_\theta$$

$$\vec{j} = \cos 40^\circ\,\vec{u}_r + \sin 40^\circ\,\vec{u}_\theta \longrightarrow -mg\cos 40^\circ\,\vec{u}_r - mg\sin 40^\circ\,\vec{u}_\theta$$



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4. Apply 2nd Newton's law:

$$\vec{F} = F\vec{u}_\theta; \qquad \vec{N} = \frac{N}{\sqrt{2}}\vec{u}_r - \frac{N}{\sqrt{2}}\vec{u}_\theta \qquad \eta = 90^o - \theta$$

$$\vec{S} = -\frac{S}{\sqrt{2}}\vec{u}_r - \frac{S}{\sqrt{2}}\vec{u}_\theta; \qquad \vec{W} = \vec{F}_g = -mg\cos 40^o\,\vec{u}_r - mg\sin 40^o\,\vec{u}_\theta$$

$$\vec{F} + \vec{N} + \vec{S} + m\vec{g} = m\vec{a} \qquad a_r = 2.010\frac{m}{s^2}, \qquad a_\theta = 6.864\frac{m}{s^2}$$

$$r: \quad 0 + \frac{N}{\sqrt{2}} - \frac{S}{\sqrt{2}} - mg\cos 40^\circ = (1.6)(2.010)$$

$$\theta: \quad F - \frac{N}{\sqrt{2}} - \frac{S}{\sqrt{2}} - mg\sin 40^\circ = (1.6)(6.864)$$

$$\boxed{N = 30.0\ N} \qquad \boxed{F = 48.2\ N}$$