$F_k = 0$

W11-3. The 1.5 kg collar is released from rest at A and travels along the smooth vertical guide. The unstretched length of the spring is 0.1 m. The spring constant is 100 N/m. Determine the following:

a) The normal force exerted on the collar at A

b) The acceleration of the collar at A

c) The speed of the collar at B

d) The tangential and normal components of the acceleration at B

e) The normal force exerted on the collar at B

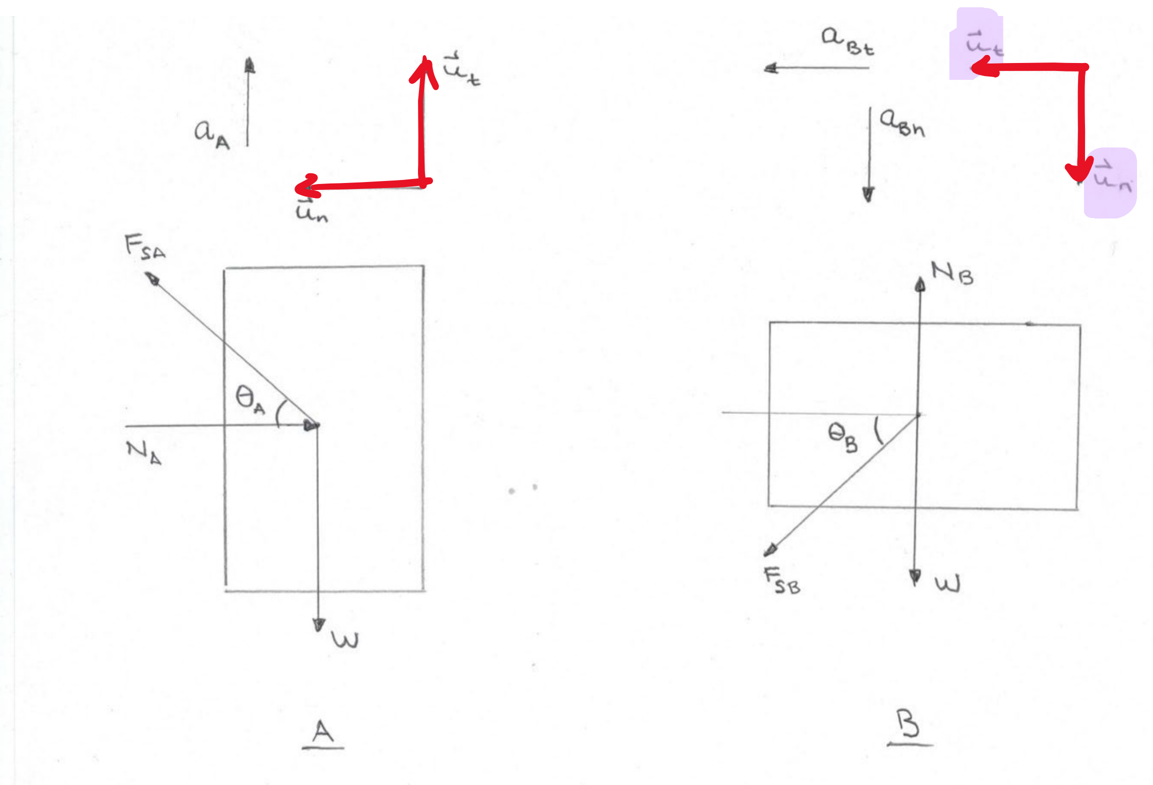
$\vec{F}_R = m\vec{a}$

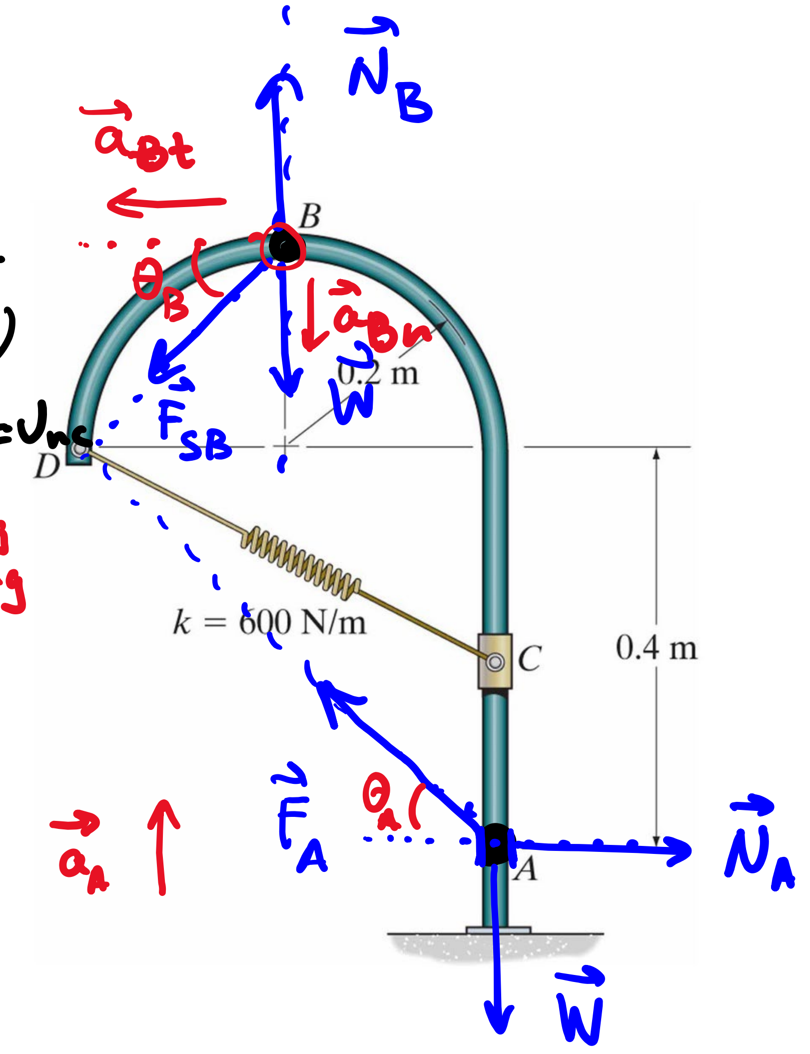
$N \quad a$

$v \rightarrow T \rightarrow \Delta ME = U_{nc}$

a_t — speeding up

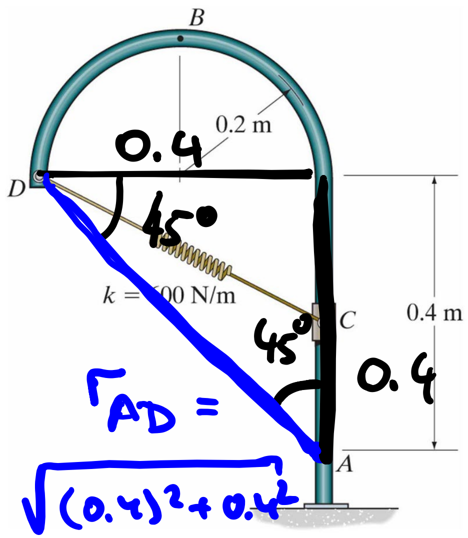
a_n — turning around





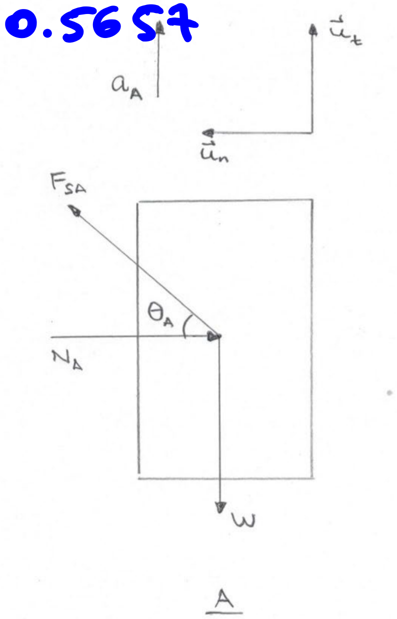
W11-3. The 1.5 kg collar is released from rest at A and travels along the smooth vertical guide. The unstretched length of the spring is 0.1 m. The spring constant is 100 N/m. $N_A, a_A, v_B, a_{B,t}, a_{B,n}, N_B$ =?

ℓ_0



$r_{AD} = \sqrt{(0.4)^2 + 0.4^2}$

$= 0.5657$



$\vec F_R = \vec W + \vec N_A + \vec F_{SA} = m\vec a_A$ N. 2nd Law

n: $F_{SA} \cos\theta_A - N_A = m a_n = 0$

t: $F_{SA} \sin\theta_A - mg = m a_t = m a_A$

$|F_{SA}| = k \Delta x_A = k(r_{AD} - \ell_0) =$

$= 100(0.5657 - 0.1) \Rightarrow F_{SA} = 46.57\ N$

Elastic Force

$F_s = k\Delta x$

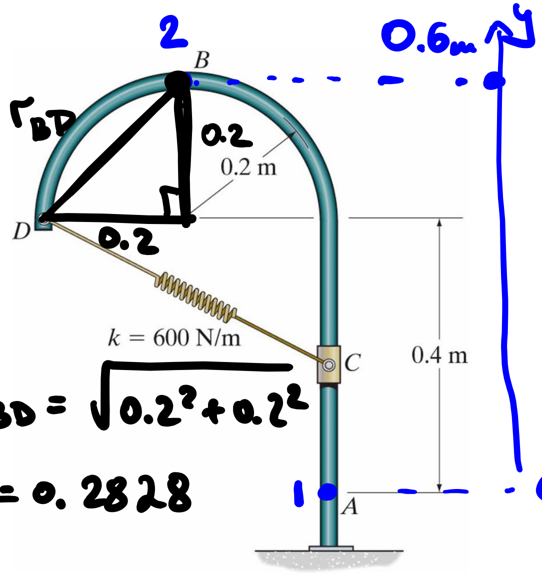
$V_s^{(e)} = \frac{k \Delta x^2}{2}$

Elastic potential energy

$\boxed{a_A = 12.1\ m/s^2}$

$\boxed{N_A = 32.9\ N}$ ← positive direction correct

W11-3. The 1.5 kg collar is released from rest at A and travels along the smooth vertical guide. The unstretched length of the spring is 0.1 m. The spring constant is 100 N/m. $N_A, a_A, v_B, a_{B,t}, a_{B,n}, N_B = ?$



$r_{BD} = \sqrt{0.2^2 + 0.2^2}$

$= 0.2828$

$$ME_2 = ME_1 + U_{1\to 2}^{(n-c)}$$

$$T_2 + V_2^{(g)} + V_2^{(s)} = T_1 + V_1^{(g)} + V_1^{(s)} + \cancel{U_{1\to 2}^{(n-c)}}$$

B:

$\vec{N}_B \perp d\vec{e}$

$\vec{F}_s$

$\vec{W}$

$U^{(N)} = 0$

$1 = A: \quad v_A = 0, \quad y_A = 0 \text{ (choice)}, \quad \Delta x_A = (r_{AD} - l_0) = 0.4657$

$2 = B: \quad v_B = ?, \quad y_B = 0.6\,m, \quad \Delta x_B = (r_{BD} - l_0) = 0.1828$

$U_{1\to 2}^{n-c} = 0 \; !$

$$\frac{m v_B^2}{2} + mg \cdot 0.6 + \frac{k \Delta x_B^2}{2} = 0 + 0 + \frac{k \Delta x_A^2}{2}$$

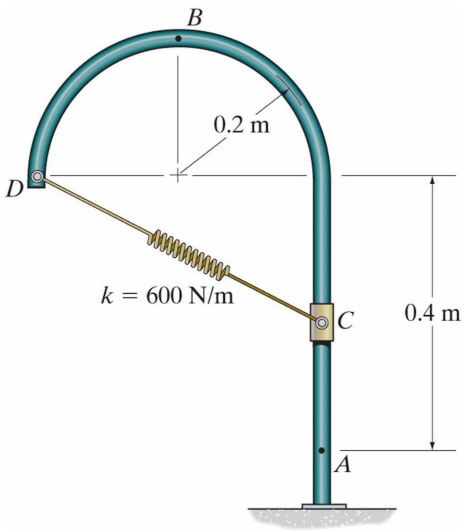
$$\boxed{v_B = 0.676 \,\frac{m}{s}}$$

W11-3. The 1.5 kg collar is released from rest at A and travels along the smooth vertical guide. The unstretched length of the spring is 0.1 m. The spring constant is 100 N/m. $N_A, a_A, v_B, a_{B,t}, a_{B,n}, N_B = ?$

$v_B = 0.676$ m/s

$\Delta x_A = 0.4657$

$\Delta x_B = 0.1828$

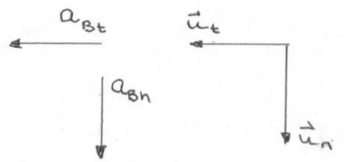


$$\vec{W} + \vec{N}_B + \vec{F}_B = m\vec{a}$$

$$n: \quad mg + F_{SB}\sin\theta_B - N_B = m a_{Bn} = m\frac{v_B^2}{0.2}$$

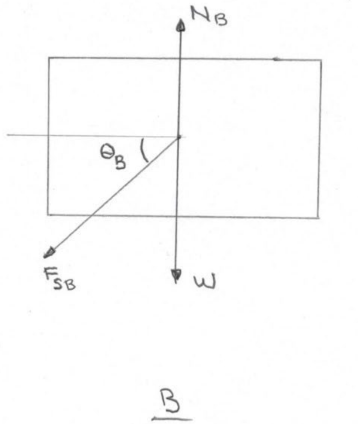
$$t: \quad F_{SB}\cos\theta_B = m a_{Bt}$$

$$a_{Bn} = \frac{v_B^2}{0.2} = 2.29\ \frac{m}{s^2}$$



$$\boxed{N_B = 24.2\ N}$$

$$a_{Bt} = 8.62\ \frac{m}{s^2}$$



$$a_B = \sqrt{a_{Bt}^2 + a_{Bn}^2} = 8.92\ \frac{m}{s^2}$$

A. $a_B = 2.29\ m/s^2$
B. $a_B = 8.62\ m/s^2$
C. $a_B = 8.92\ m/s^2$
D. Help!!!

PHYS 170

Week 12: Momentum and Impulse

Section 201 (Mon Wed Fri 12:00 – 13:00)

Linear Impulse and Momentum

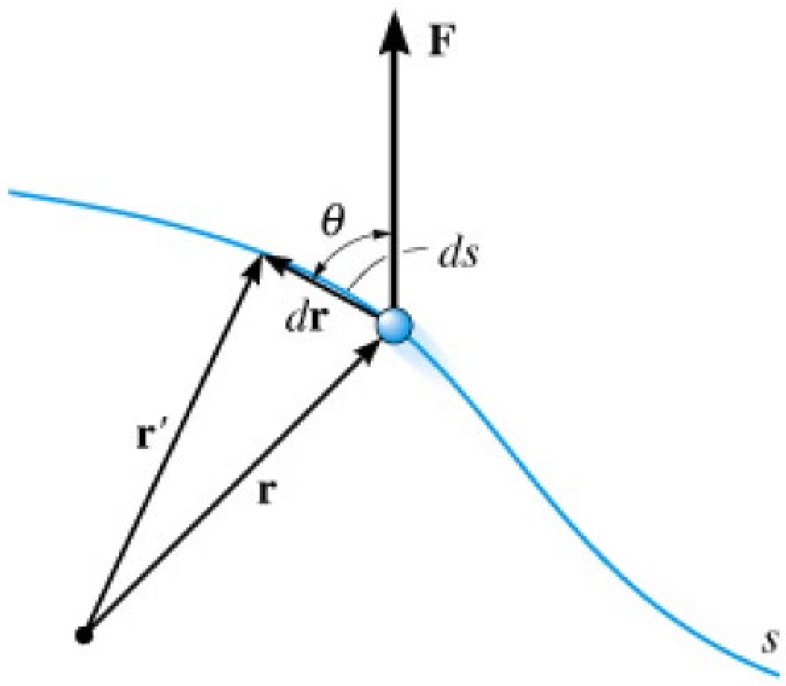
Text: 15.1

Content:

- Definitions of momentum and impulse
- Vectorial nature of these quantities
- Principle of linear impulse and momentum
- This principle applies to <u>components</u> of the momentum and impulse!

DEFINITIONS

A particle with mass m moves on a certain trajectory and is acted by a (resultant) force $\vec{F}$. Its speed is $\vec{v}(t)$.



- Linear momentum

$$\vec{L} = m\vec{v}$$

- Units:
  - SI: kg · m/s
  - FPS: slug · ft/s

- Linear impulse

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}\, dt$$

- Units:
  - SI: N · s
  - FPS: lb · s

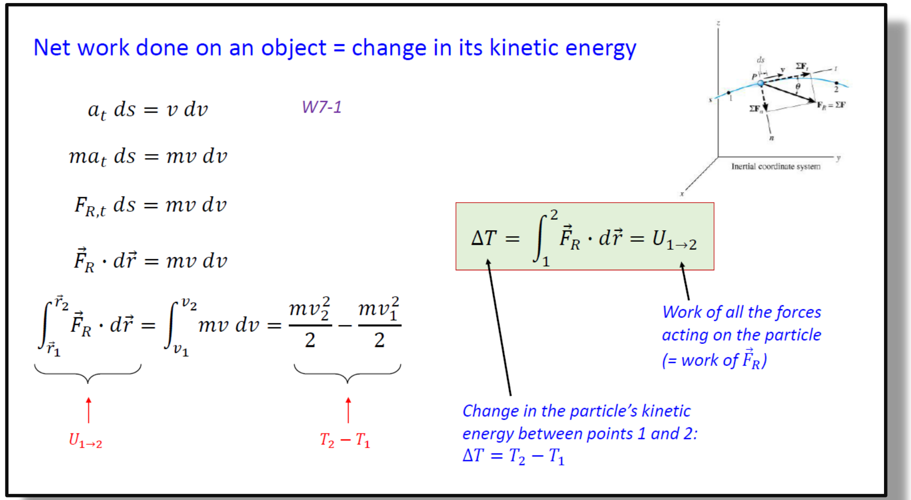
$F = ma$

- Exercise: find connection between the units of linear momentum and linear impulse.

$$N \cdot s = kg \cdot \frac{m}{s^2} \cdot s = kg \cdot \frac{m}{s}$$

Impulse = change in object's momentum

- We have shown earlier that integrating the resultant force over coordinate gives us the Principle of Work and Energy:
- Now let us show that integrating it over time gives us the Principle of Linear Impulse and Momentum:



$$\vec{F}_R = m\vec{a} = m\frac{d\vec{v}}{dt}$$

- As usual, $\vec{F}_R = \sum \vec{F}$ is the sum of all forces acting on the particle

$$\vec{F}_R\,dt = m\,d\vec{v}$$

$$\underbrace{\int_{t_1}^{t_2} \vec{F}_R\,dt}_{\vec{I}_{1\to 2}} = m\int_{\vec{v}_1}^{\vec{v}_2} d\vec{v} = \underbrace{m\vec{v}_2 - m\vec{v}_1}_{\vec{L}_2 - \vec{L}_1}$$

$$\boxed{\Delta\vec{L} = \int_{t_1}^{t_2} \vec{F}_R\,dt = \vec{I}_{1\to 2}}$$

Change in the particle's linear momentum between points 1 and 2: $\Delta\vec{L} = \vec{L}_2 - \vec{L}_1$

Impulse acting on the particle

PRINCIPLE OF LINEAR IMPULSE & MOMENTUM

$$\vec{L} = m\vec{v}$$

$$\vec{I}_{1\to 2} = \int_{t_1}^{t_2} \vec{F}_R \, dt$$

- We can rewrite this relationship as:

$$\boxed{\vec{L}_2 = \vec{L}_1 + \vec{I}_{1\to 2}}$$

- **In words**: The final momentum of the particle is equal to the initial momentum of a particle plus the total impulse acting on the particle.

- If a force does not change with time its impulse is very easy to calculate:

$$\int_{t_1}^{t_2} \vec{F} dt = \vec{F}(t_2 - t_1)$$

- Note that in the absence of forces (i.e. if $\vec{F}_R \equiv 0$), particle's momentum conserves: $\vec{L}_2 = \vec{L}_1$

PRINCIPLE OF LINEAR IMPULSE & MOMENTUM: In components !

$$\boxed{m\vec{v}_2 = m\vec{v}_1 + \sum\left(\int_{t_1}^{t_2} \vec{F}dt\right)}$$

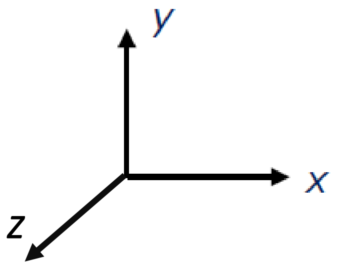
- One vector equation = Up to three scalar equation!

- We can rewrite this relationship in Cartesian components as:

$$\begin{cases} mv_{2,x} = mv_{1,x} + \sum\left(\int_{t_1}^{t_2} F_x\, dt\right) \\ mv_{2,y} = mv_{1,y} + \sum\left(\int_{t_1}^{t_2} F_y\, dt\right) \\ mv_{2,z} = mv_{1,z} + \sum\left(\int_{t_1}^{t_2} F_z\, dt\right) \end{cases}$$

- So if there are no force components in one of the directions:
  - ➢ The momentum in that direction will conserve
  - ➢ In other directions the change in the momentum will be determined by the impulse of the relevant force components

Q: A cannon shoots a cannonball at the angle of 45° to the horizon. In which direction the momentum of the cannonball conserves <u>throughout its flight</u>? Ignore air resistance.



A. x and y
B. x and z
C. y and z
D. x, y, and z
E. it isn't conserved in any of these directions