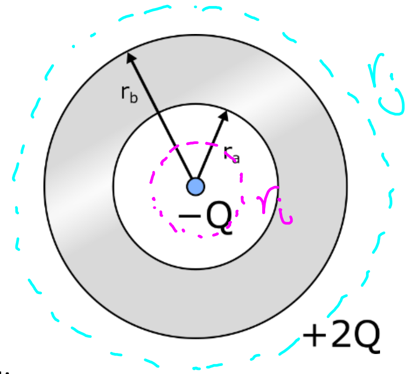
**Problem E3.1(☆☆):** A hollow metal sphere carrying charge +2Q encloses a point charge of –Q, as pictured right.




- **(a)** What is the electric field inside the hollow region of the sphere?
- **(b)** What is the electric field outside the sphere?
- **(c)** How much charge is gathered on the inside and outside surface of the hollow sphere?
- **(d)** Draw the magnitude of the electric field as a function of radius.

(a) We can apply Gauss' Law to a surface inside the hollow sphere:

$$\Phi_i = \oint \vec{E}(r_i) \cdot d\vec{A} = E(r_i) \oint r_i \, d\theta \, d\phi$$

$$= 4\pi r_i^2 E(r_i)$$

$$\Phi_i = \frac{q_{enc}}{\varepsilon_0} = \frac{-Q}{\varepsilon_0}$$

$$\therefore \vec{E}(r<r_a) = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

(b) Similarly, we can apply Gauss' Law:

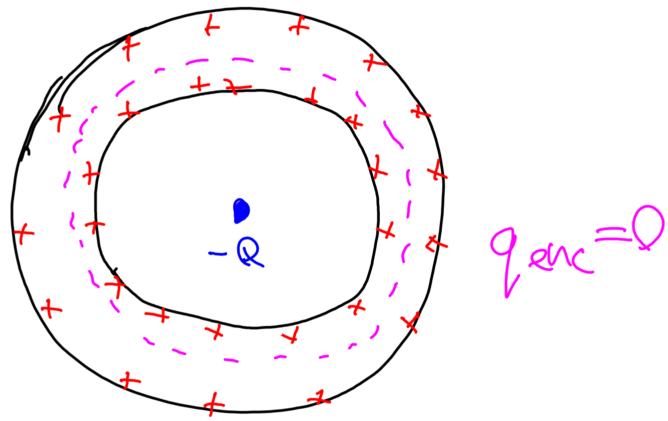
$$4\pi r_j^2 E(r_j) = \frac{(2Q-Q)}{\varepsilon_0}$$

$$\therefore \vec{E}(r>r_b) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

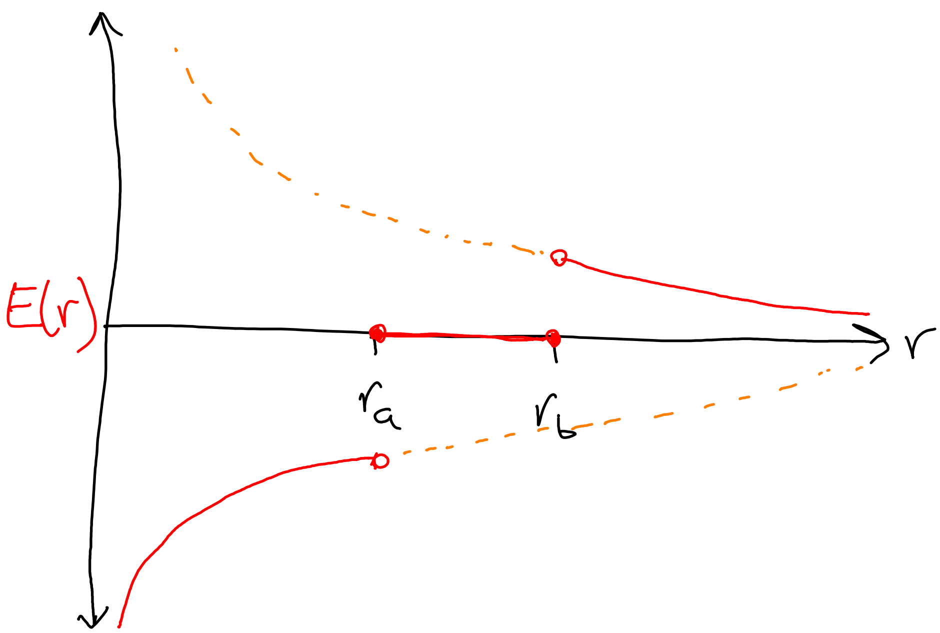
(c) E-field inside conductor must be zero. Hence charge enclosed in Gaussian surface inside conductor must be zero.

$\therefore Q_{in} = +Q$

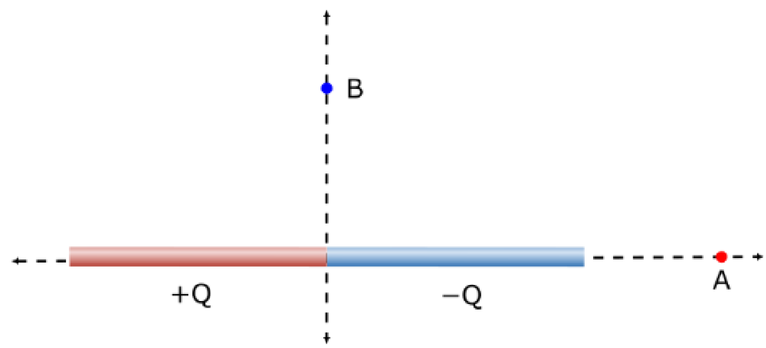
$Q_{out} = +Q$




(d)



**Problem E4.1(☆☆):** A thin rod of length $2a$ has charge $-Q$ uniformly distributed on its left half and charge $+Q$ on its right half.

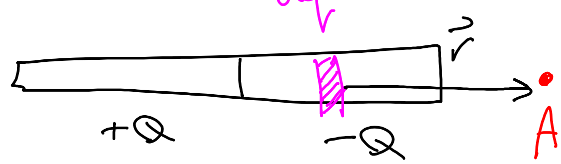



- **(a)** Find the potential $V$ at points A (x = d) and B (y = h) indicated in the figure.
- **(b)** From the potential $V$ find the electric field $E$, amplitude and direction, at point A. Compare that to the result of the direct calculation that we did previously.
- **(c)** Is it possible to obtain the field at point $B$ from the potential calculated in (a)? Discuss with your table.

Look up any integrals that are unfamiliar.

$$\lambda = \frac{Q}{a} = \frac{dq}{dx}$$

$$dq = \lambda dx$$




(a) To find the potential, we integrate for infinitesimal point charges:

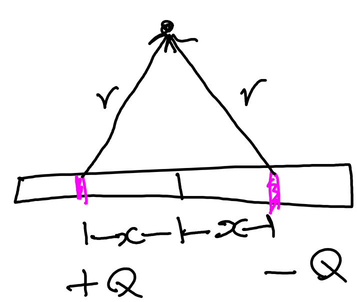
$$V_A = \int \frac{k\,dq}{r} dr = k\lambda \int_{d+a}^{d} \frac{dx}{x}\hat{x} + (-k\lambda)\int_{d}^{d-a} \frac{dx}{x}\hat{x}$$

$$\int \frac{dx}{x} = \ln(x)$$

$$\therefore V_A = k\lambda\left(\ln(d) - \ln(d+a)\right) - k\lambda\left(\ln(d-a) - \ln(d)\right)$$

$$= k\lambda\left(\ln\left(\frac{d}{d+a}\right) + \ln\left(\frac{d}{d-a}\right)\right) = \frac{2kQ}{a}\ln\left(\frac{d^2}{d^2-a^2}\right)$$

If we try for B, we find




$$dV_B = \frac{k\,dq}{r} + \frac{k(-dq)}{r} = 0$$

This is true for all pairs so $V_B = 0$.

(b) We get the E-field from $E_x = \frac{dV}{dx}$. Note that we can only get the component force in the direction we take the derivative.

For A, our solution holds for arbitrary $d$, so when we take the derivative w.r.t. $d$, we get $E_x$:

$$E_x = -\frac{dV_a}{dd} = -k\lambda \frac{d}{dd}\left(\ln(d^2) - \ln(d^2-a^2)\right)$$

$$= -k\lambda\left(2d \cdot \frac{-1}{d^2} - 2d \cdot \frac{-1}{d^2-a^2}\right)$$

$$= \frac{2kQ}{a}\left(\frac{1}{d} - \frac{d}{d^2-a^2}\right)$$

You can check that this is the same as the previous tutorial.

Note that we could also argue that $E_y(A)=0$ from symmetry.

(c) Since $V_B = 0$ for all B on the y-axis, we can say $E_y(B) = 0$.

However we would need to know how V varies with $x$ at B to get $E_x(B)$, which would break the symmetry and be a much more involved calculation.