Announcement

From: Engineering Undergraduate Society

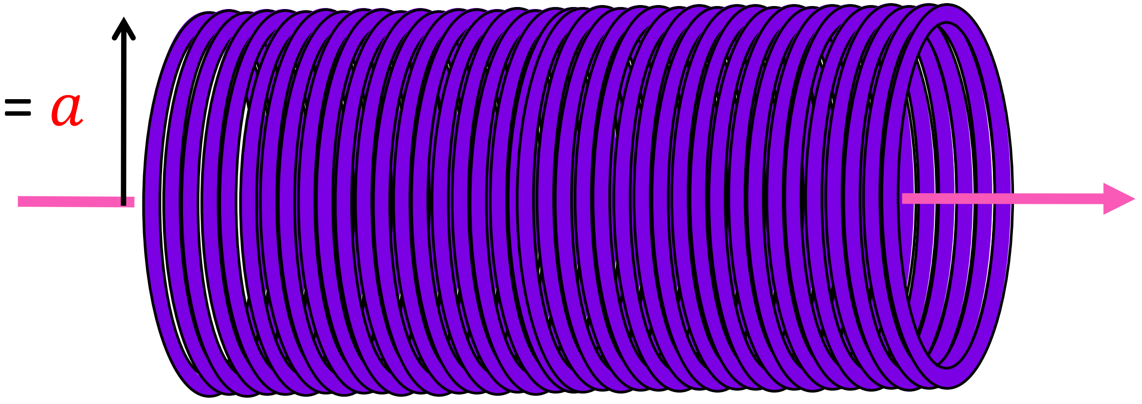
To celebrate the end of midterms, we are putting on one more event called "PP semi" which is essentially a semi formal dance for first year engineering students and their plus ones. The event will take place on the 27th of March from 7:30-10:00pm.

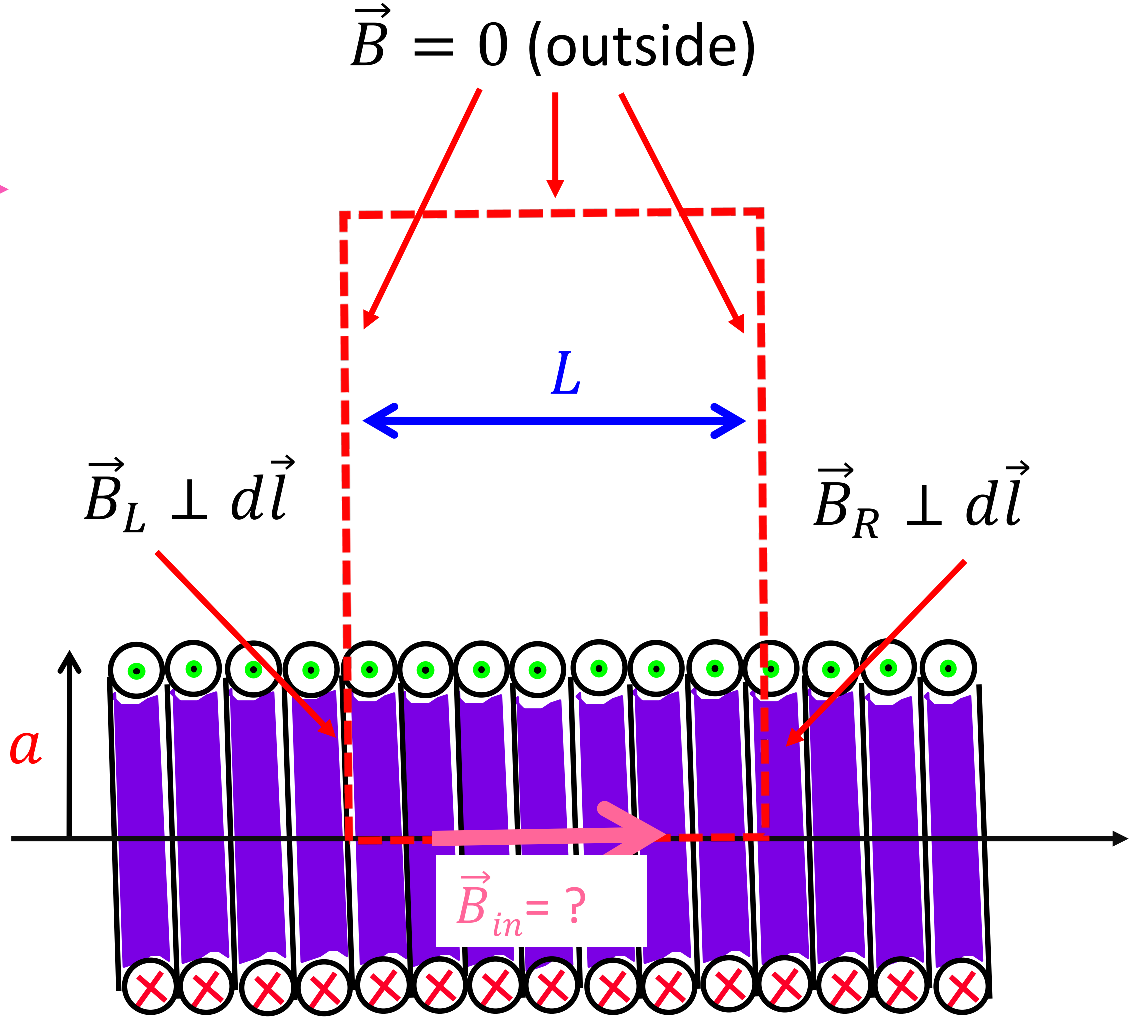
Lecture 31.

Real-life solenoids.
Faraday’s law & Lenz’s law: making sense

Ideal solenoid

Last Time:





- Ampere's law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$

$$\oint \vec{B} \cdot d\vec{l} = B_{in} L$$

$$I_{\text{encl}} = InL$$

$$\boxed{B_{in} = \mu_0 In}$$

Finite Length Solenoid

Unfortunately, Engineers have to deal with finite length solenoids!

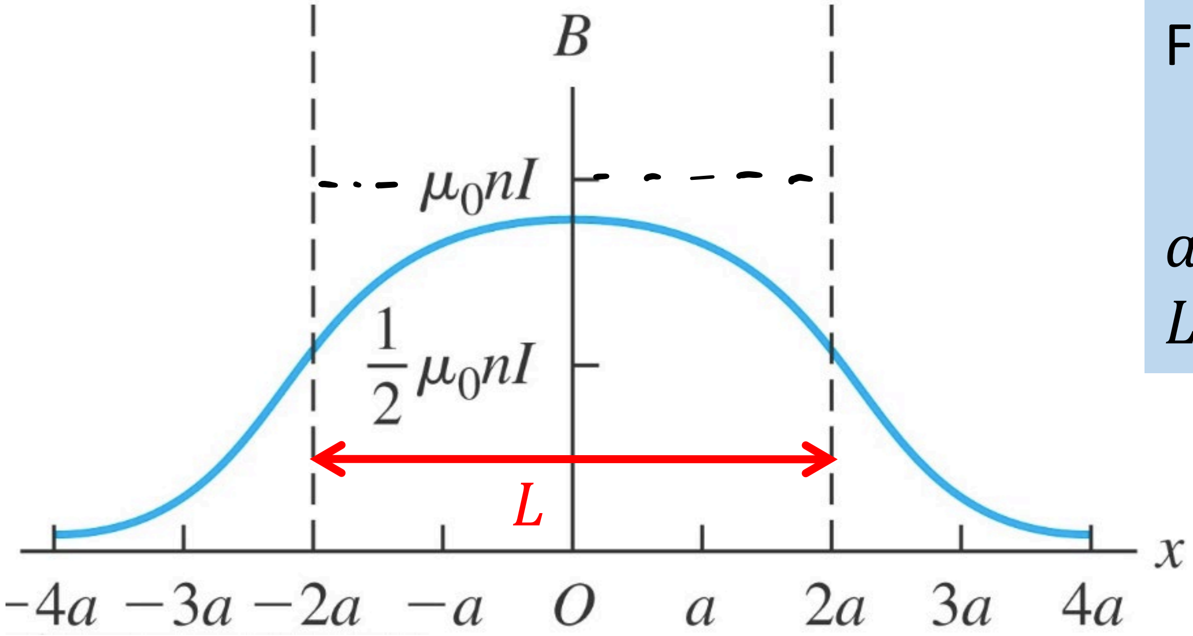


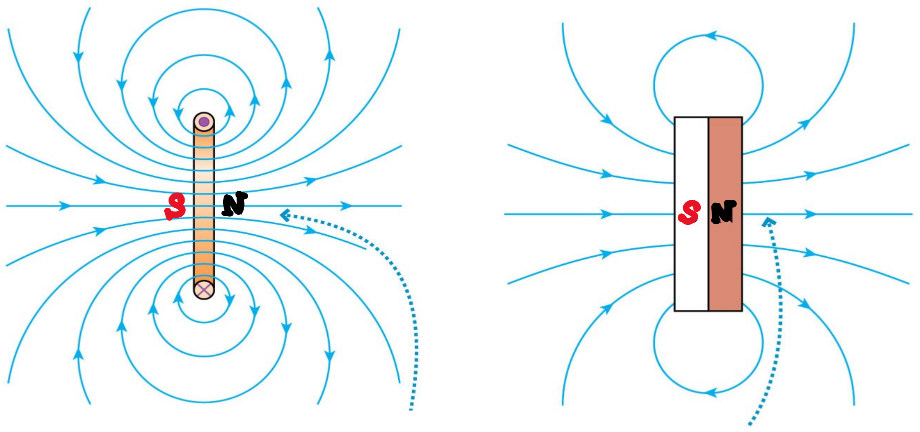
Fig 28.24
$L = 4a$

a = radius
L = length

Look at website -- http://www.falstad.com/vector3dm

Fringe effects

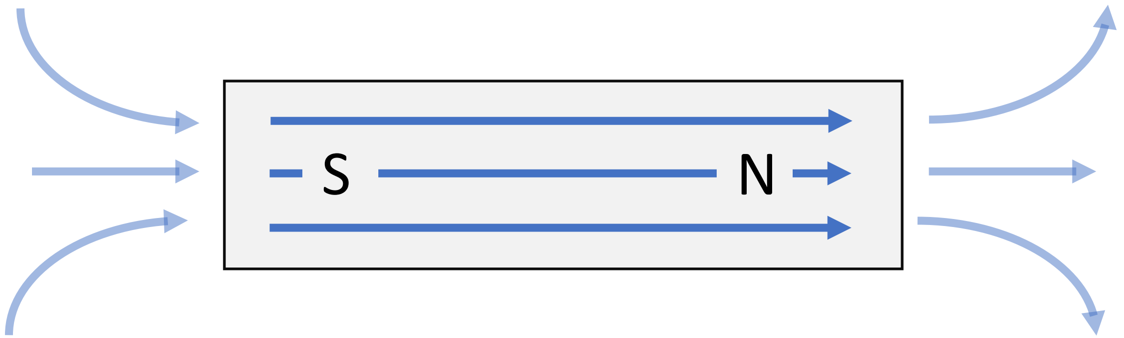
To better understand fringe effects, let's look at the B-field of one ring



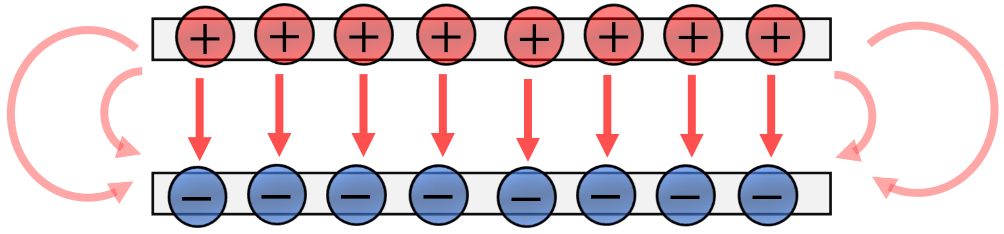
- If we look at the field of a current-carrying ring, we see that it is equivalent to the field of a tiny magnet!

- You can even define North and South poles for such a loop…

- Putting many loops together (solenoid / magnet):

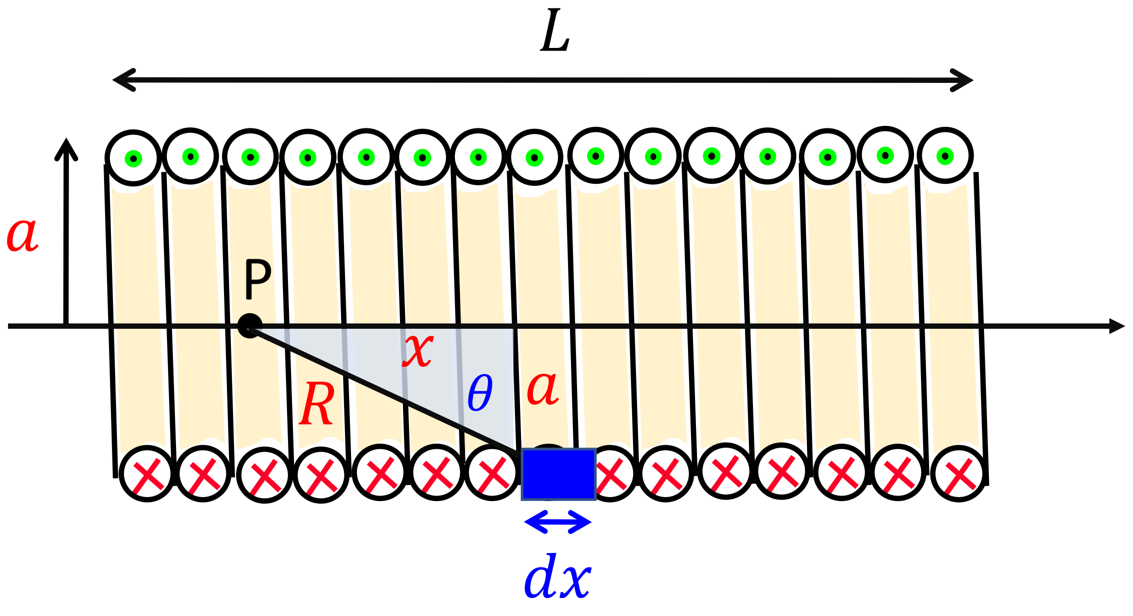


- Doesn't it remind a capacitor?



- Store magnetic / electric energy, uniform field…

Finite Length Solenoid



- Recall B_x on the axis of a circular loop:

$$B_x(x) = \frac{\mu_0}{2}\frac{I\,a^2}{(x^2+a^2)^{3/2}}$$

- Consider magnetic field created at the observation point P by a tiny segment of solenoid (thickness dx):

$$dB_x(x) = \frac{\mu_0}{2}\frac{N dx}{L}\frac{I a^2}{(x^2+a^2)^{3/2}}$$

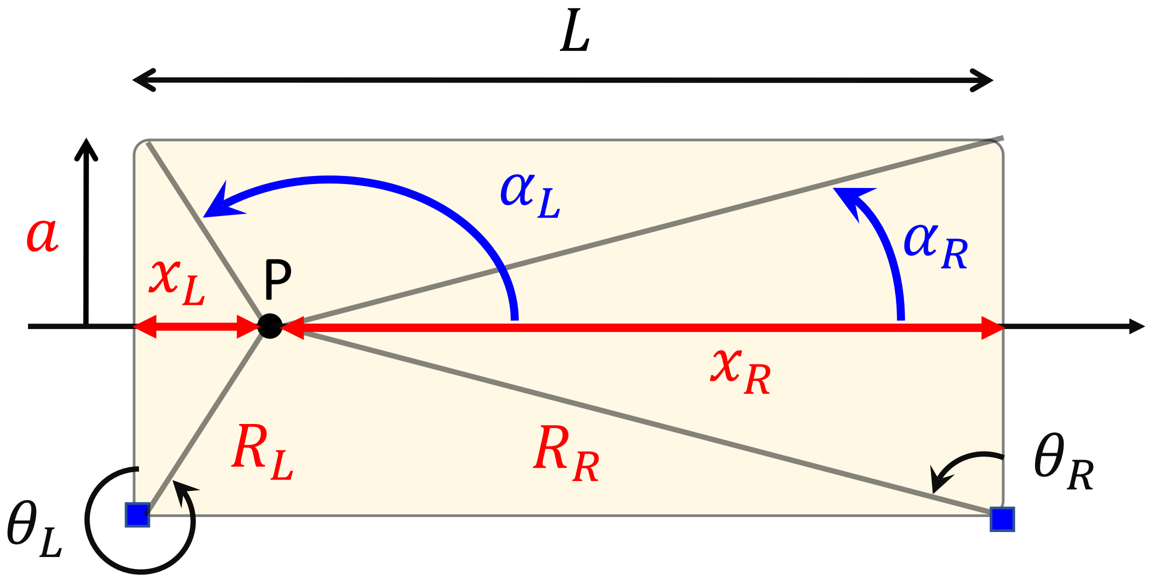
- Same way as with a finite wire: introduce θ

$$x = a\tan\theta \quad \Rightarrow \quad dx = \frac{a\,d\theta}{\cos^2\theta}, \quad \text{and} \quad R\cos(\theta) = a \quad \Rightarrow \quad dB_x = \frac{\mu_0}{2}\frac{N}{L} I\cos(\theta)\,d\theta$$

check it!

$$\boxed{B_{\text{real solenoid}}(P) = \int dB_x = \frac{\mu_0}{2}\frac{N}{L} I \left\{\sin(\theta) = \pm\frac{x}{\sqrt{x^2+a^2}}\right\}\Bigg|_{x_L}^{x_R}}$$

Finite Length Solenoid & Trigonometry



$$B_{\text{real solenoid}}(@P) = \frac{\mu_0}{2}\frac{N}{L} I(\sin\theta_R - \sin\theta_L)$$

- Let's brush it up:

$$\sin\theta_L = -\sin(2\pi - \theta_L) = -\frac{x_L}{R_L} = \cos\alpha_R$$

$$\sin\theta_R = \frac{x_R}{R_R} = \cos\alpha_R$$

- We get: $$\boxed{B_{\text{real solenoid}}(@P) = \frac{\mu_0}{2} n\, I(\cos\alpha_R - \cos\alpha_L)}$$

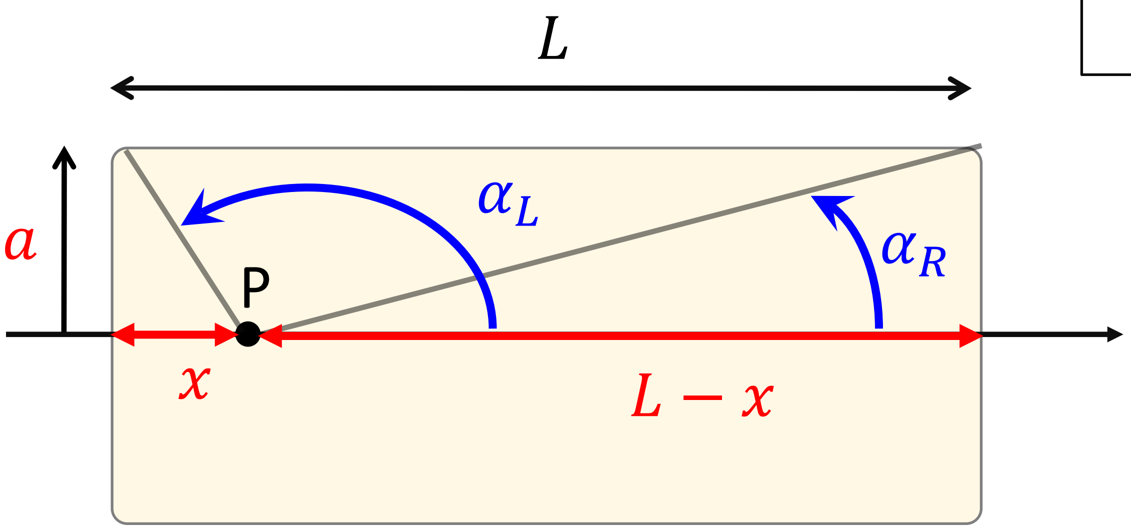
- $n = N/L$ is the density of coils
- α_L and α_R are the angles at which you see the edges of the solenoid from point P.

- For a long solenoid: $\alpha_R \to 0 \quad \& \ \alpha_L \to \pi \qquad \Rightarrow \qquad \boxed{B_{\text{ideal solenoid}} = \mu_0\, n\, I = \text{const}}$

Finite Length Solenoid & Trigonometry

$$B_{\text{real solenoid}}(@P) = \frac{\mu_0}{2} n\, I (\cos\alpha_R - \cos\alpha_L)$$

$$B_{\text{ideal solenoid}} = \mu_0\, n\, I = \text{const}$$



$$\boxed{B_{\text{real solenoid}}(x) = B_{\text{ideal}} \times \frac{1}{2}\left(\frac{L-x}{\sqrt{a^2+(L-x)^2}} + \frac{x}{\sqrt{a^2+x^2}}\right)}$$

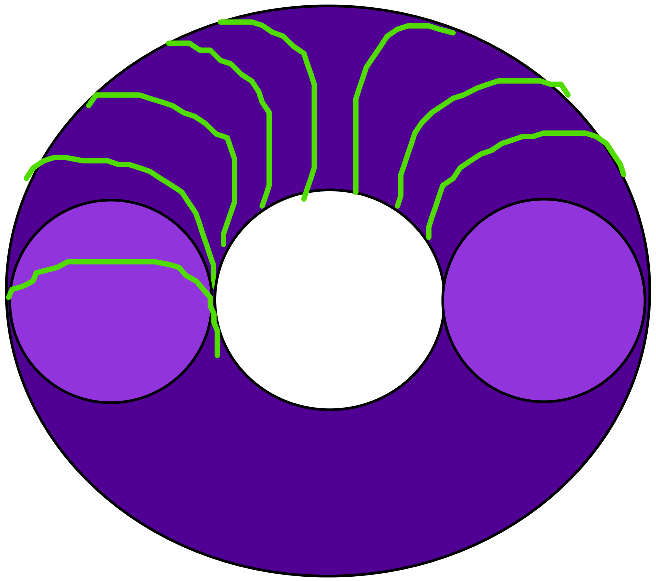
- Show that the approximation of ideal solenoid works well when $L \gg a$

$L \sim 4a$ – fringe effects

https://www.desmos.com/calculator/maau7qidjc

Magnetic Applications – Toroid Solenoid

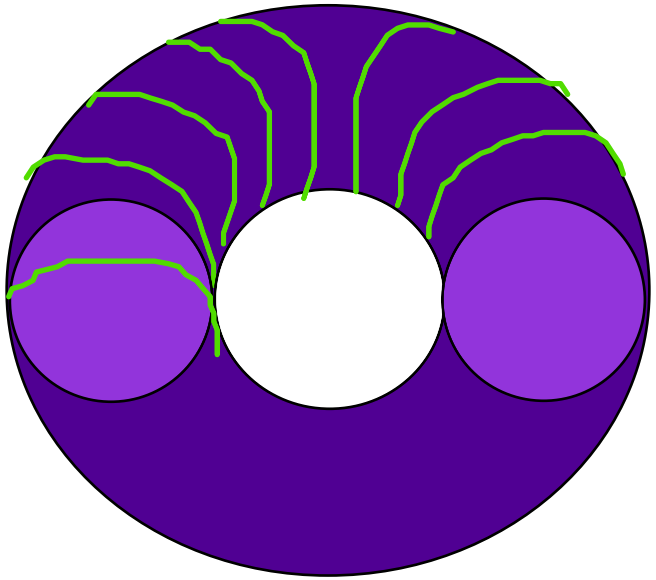
- Let's get rid of fringes at the ends – by getting rid of the ends themselves!



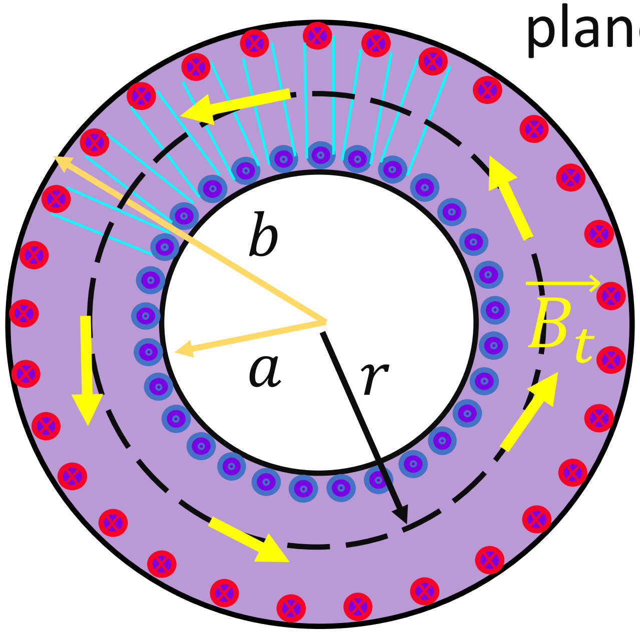
- Here we show a toroid with a circular cross-section, i.e. a "doughnut" wrapped with N "turns" of wire.
- The wire carries current I
- This is a solenoid twisted into a circle.

- What the magnetic field in this structure is?

Magnetic Applications – Toroid Solenoid



plane view



$\vec{B}_t$ = tangential B-field

$$\oint \vec{B} \cdot d\vec{l} = 2\pi r\, B_t = \mu_0 I_{encl} = \mu_0 IN$$

$$\boxed{B_t = \frac{\mu_0 IN}{2\pi r}}$$

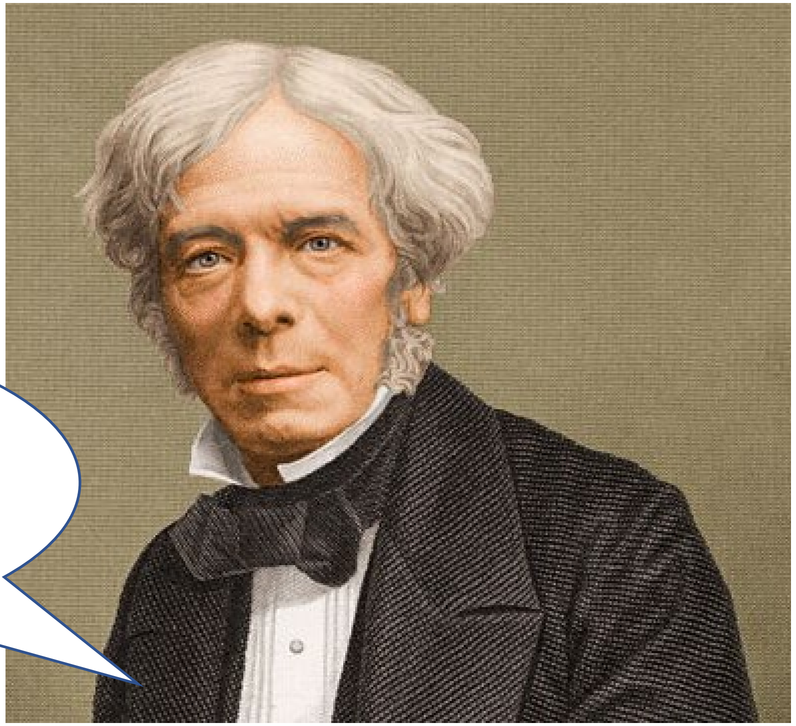
Note -- $B \cong 0$ for $r < a$ or $r > b$

Fig 28.25

Lenz’s law

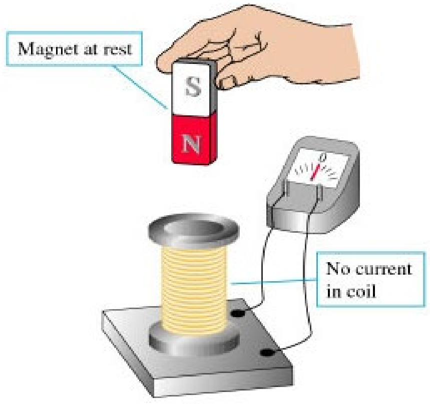
$$\varepsilon = -\frac{d\Phi_B}{dt}$$

Faraday’s law

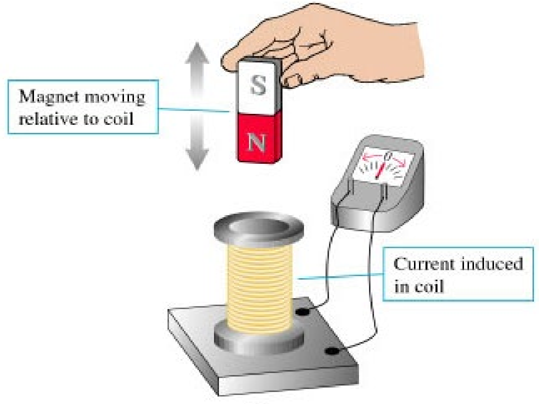


$$\varepsilon = \left|\frac{d\Phi_B}{dt}\right|$$

Experiments on induced EMF

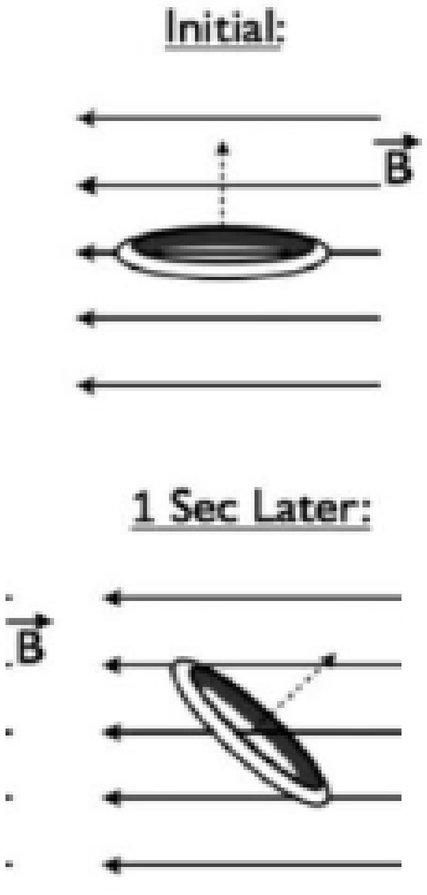


(a)



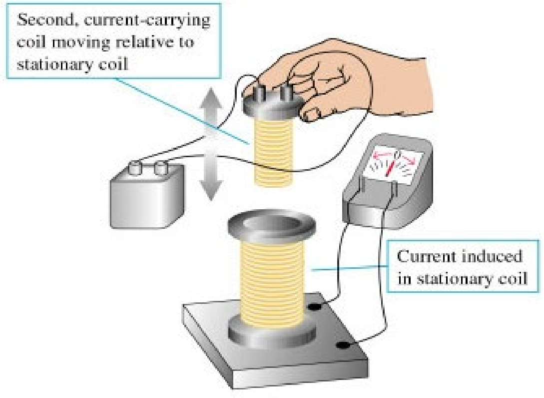
(b)

- Moving a magnet around a coil creates current in the coil



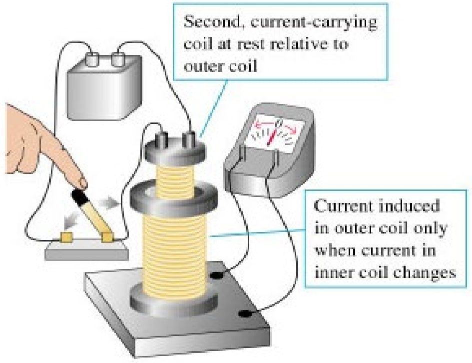
- Rotating a loop in an external magnetic field creates current in the loop

- Moving a current-carrying solenoid around a coil creates current in the solenoid



(c)

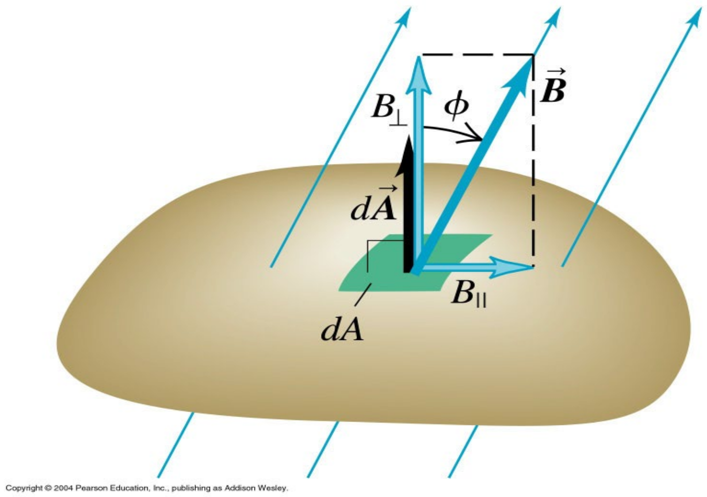
- Switching a current on in the inner coil momentarily creates current in the outer coil



(d)

- This experiment works for any closed (so that it can support electric current) loop

Induced EMF



$$\Phi_B = \oint_{\text{area}} \vec{B} \cdot d\vec{A}$$

- Common denominator:
  - In these experiments, there in no battery in the loop. The current is excited literally by changing the magnetic flux through the loop!

- Okay, we have current => there must be EMF

- We can define the so-called “motional emf”: (emf = electromotive force)

$$\boxed{\varepsilon = \left|\frac{d\Phi_B}{dt}\right|}$$

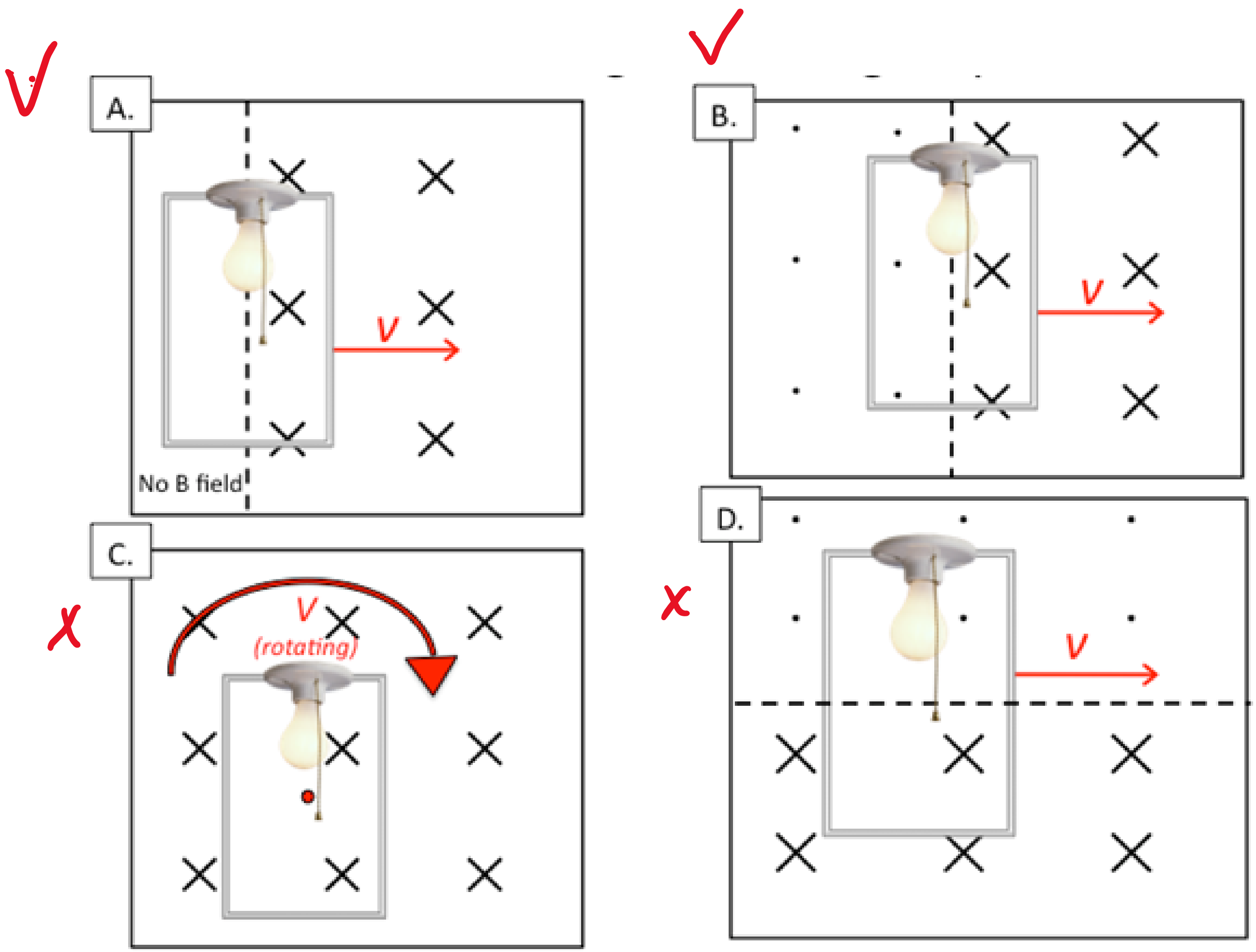
an emf induced in a conducting loop if the magnetic flux Φ_B through it changes ⇒ “emf responsible for that current”

Q: Which of the following could light up the bulb?

$\mathcal{E} = max$

A. A only
B. A and B
C. A, B and C
D. A and C
E. All of them

$\frac{d\Phi}{dt} = max$



Q: Which of these will light the bulb most strongly?

A. A
B. B
C. Same

Direction of Induced EMF

- So $\varepsilon = \left|\frac{d\Phi_B}{dt}\right|$ is the magnitude of induced EMF. We need to know its direction, CW or CCW

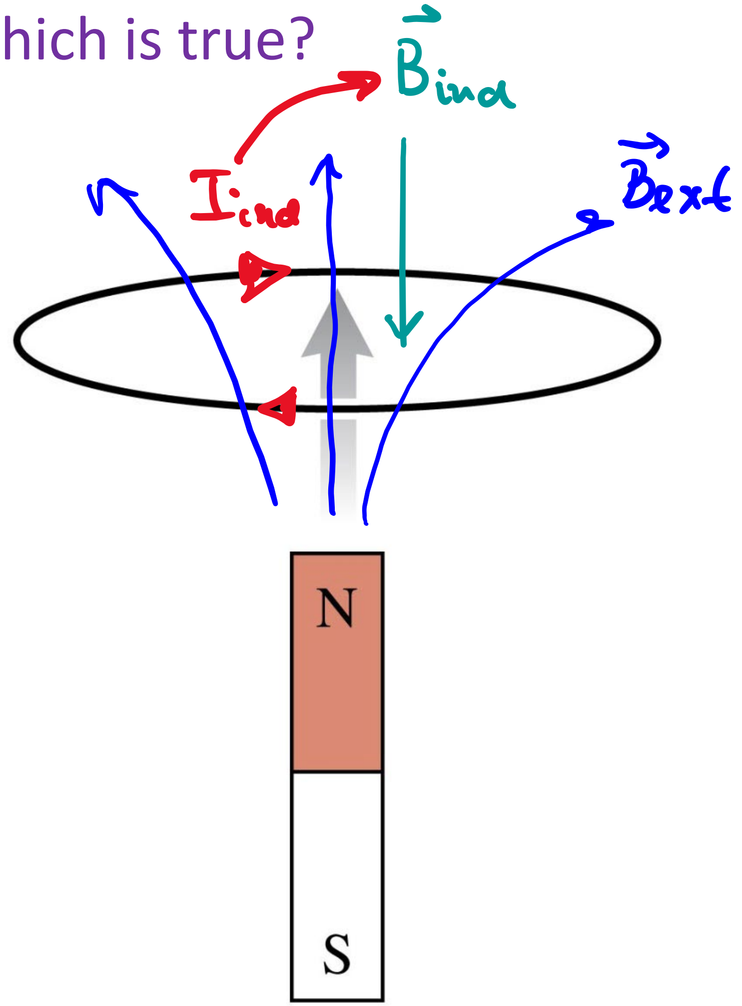
- Induced current is a current, and hence it creates (induces) magnetic field!
- Their directions are related by the RHR.

- The direction of the induced magnetic field (and, therefore, the direction of the induced current!) is determined by the Lenz law:

The direction of the induced current is such that the induced magnetic field opposes the change in the flux.

$$\varepsilon = -\frac{d\Phi_m}{dt}$$

Q: The bar magnet is pushed toward the center of a wire loop. Which is true?



A. There is a clockwise induced current in the loop.

B. There is a counterclockwise induced current in the loop.

C. There is no induced current in the loop.

D. Not sure

$\vec{B}_{ext} \uparrow$, increasing $\quad \vec{B}_{ind} \downarrow$

$\vec{B}_{ind} \uparrow\downarrow \vec{B}_{ext}$

(assume we look at the loop from above)

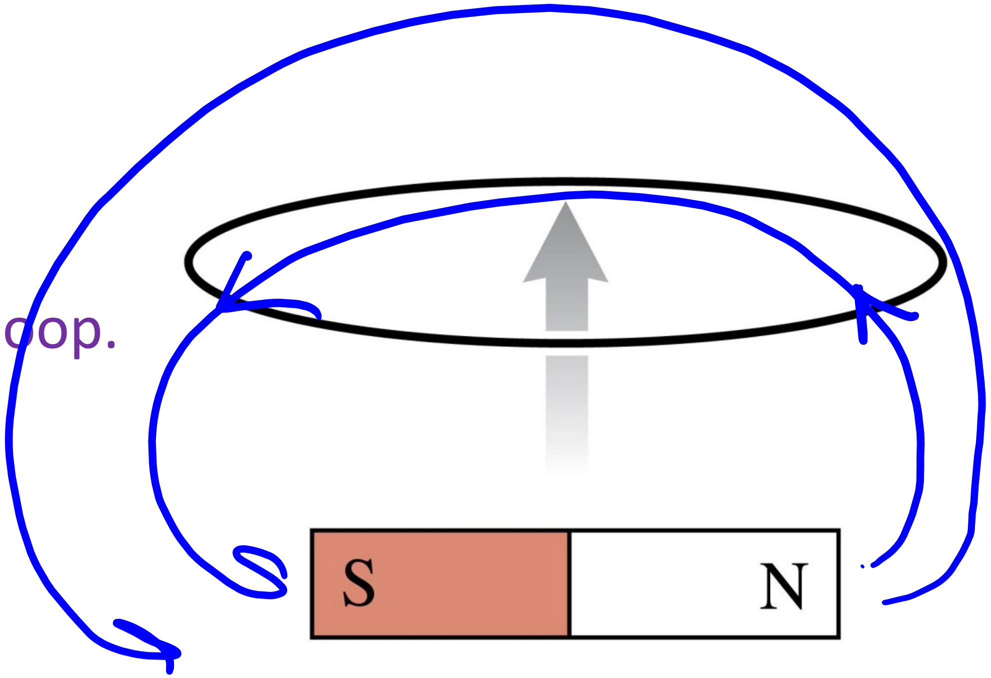
Q: The bar magnet is pushed toward the center of a wire loop. Which is true?

A. There is a clockwise induced current in the loop.

B. There is a counterclockwise induced current in the loop.

C. There is no induced current in the loop.

D. Not sure



(assume we look at the loop from above)