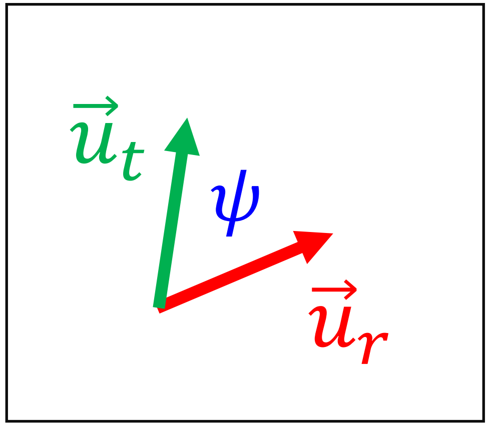
PHYS 170

Week 10: Kinetics: Force and Acceleration-2

Section 201 (Mon Wed Fri 12:00 – 13:00)

Equation of motion: Polar coordinates



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Content:

- Second Newton's law, $\vec{F}_R = m\vec{a}$, as a mean to find force knowing acceleration
- Coordinate systems for curvilinear motion: Recap
- Practice

PLANS FOR THIS WEEK

- Last week: “know forces => find acceleration”
- We used: dependent motion, relative motion, …
- This week we will go in the opposite direction:

“know acceleration => find forces”

Week 9 $\longrightarrow$

$$\vec{F}_R = m\vec{a}$$

$\longleftarrow$ this week

1) Kinematic characteristics (velocity, position) => can find acceleration

- Cartesian coordinates
- Normal-tangential coordinates
- Polar coordinates

2) Knowing acceleration, we can find the net force on the object using 2nd Newton’s law

NEWTON'S SECOND LAW AND COORDINATE SYSTEMS: Summary

- In Cartesian coordinates:

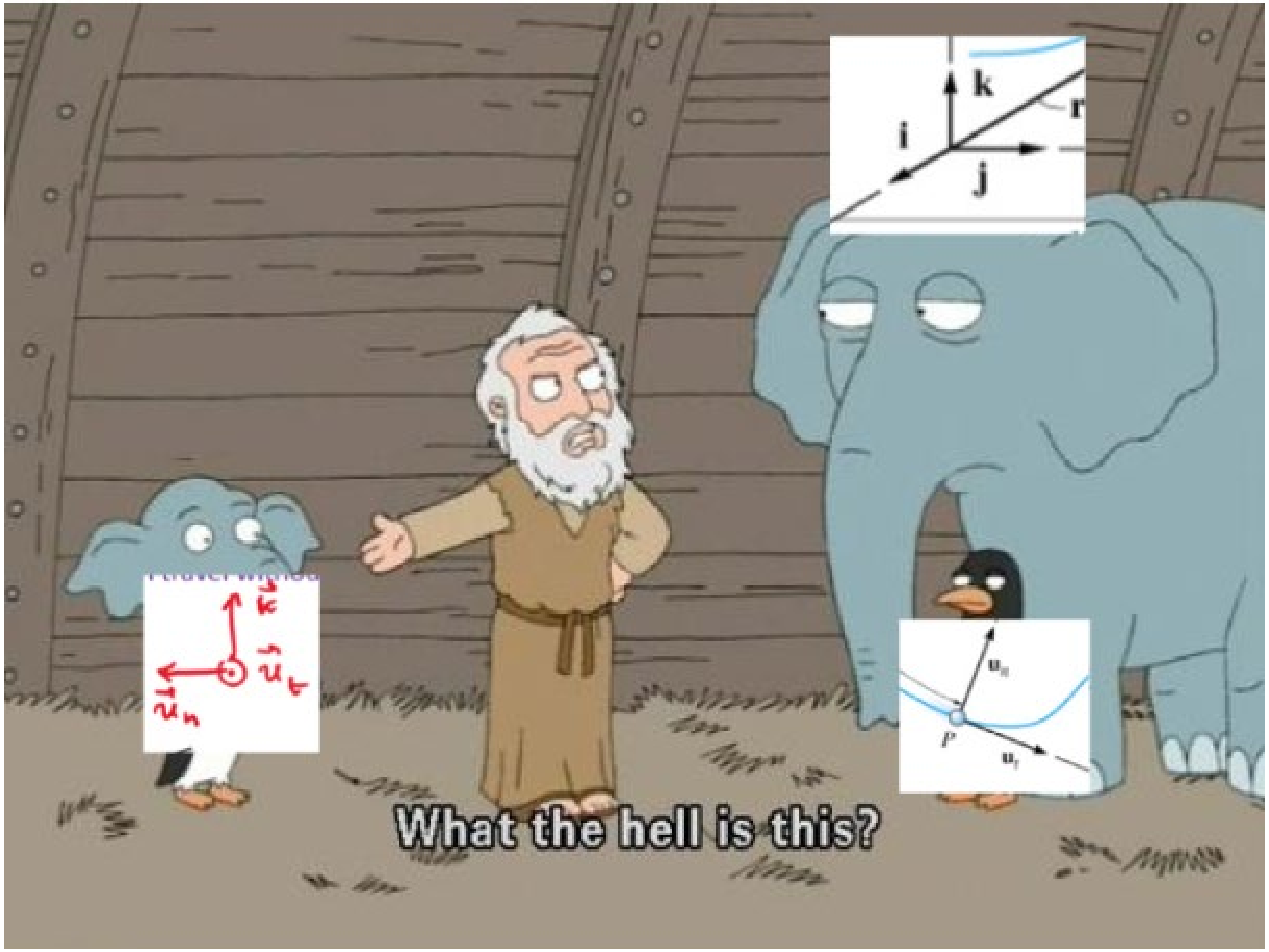
$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z$$

- In normal-tangential coordinates (2D) / add z-axis (3D):

$$\sum F_t = m\underbrace{\dot{v}}_{a_t} \qquad \sum F_n = m\underbrace{\frac{v^2}{\rho}}_{a_n} \qquad \sum F_z = m\ddot{z}$$

- In polar (2D) / cylindrical (3D) coordinates:

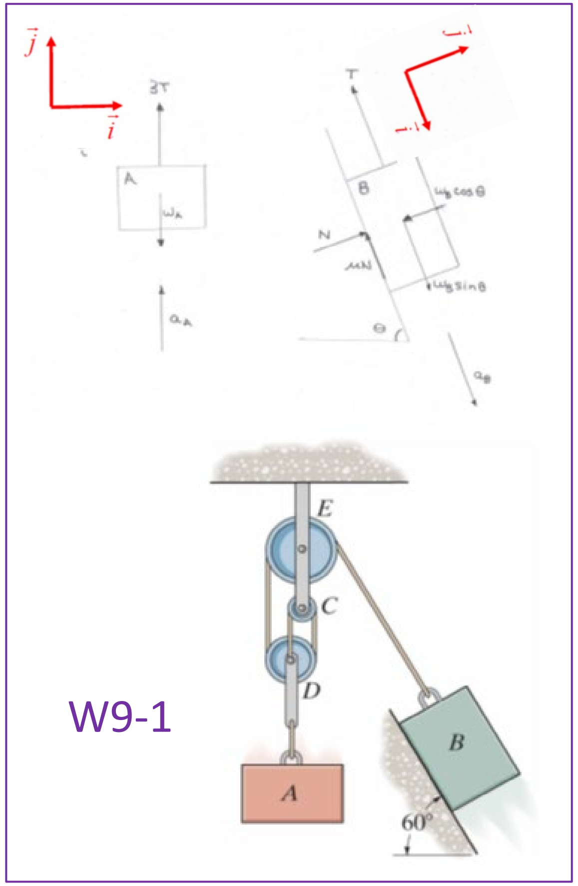
$$\sum F_r = m(\underbrace{\ddot{r} - r\dot{\theta}^2}_{a_r}) \qquad \sum F_\theta = m(\underbrace{2\dot{\theta}\dot{r} + r\ddot{\theta}}_{a_\theta}) \qquad \sum F_z = m\ddot{z}$$

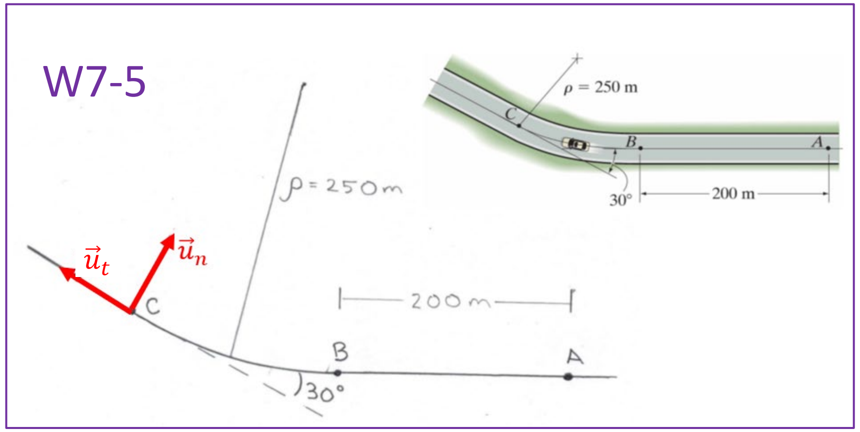


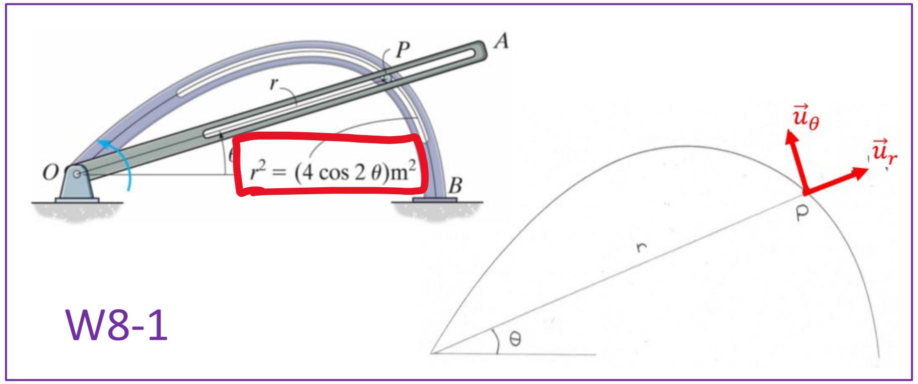
Credits: Anon. Poet

COORDINATE SYSTEMS

Q: What determines the choice of the coordinate system?







- Cartesian:
  - Stationary coordinate system
  - Good for motion along straight lines

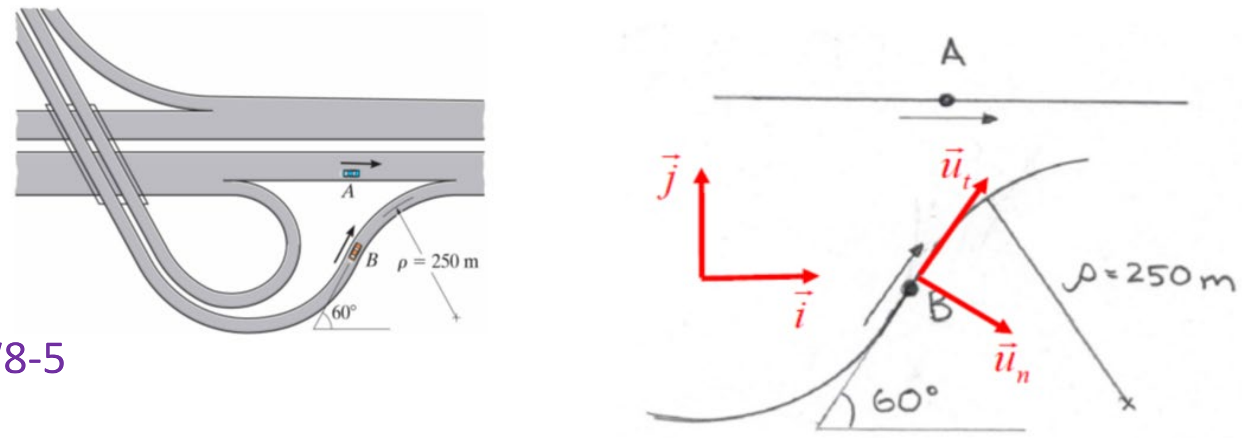
- Normal-Tangential:
  - "Portable" coordinate system
  - Curvilinear motion
  - Good when motion is naturally split into normal and tangential components (Example: car on a track)

$v = 50\,\frac{km}{h}$ $\quad \vec{v} = 50\,\vec{u}_t$

- Polar:
  - "Portable" coordinate system
  - Curvilinear motion
  - Good when the trajectory is given in terms of position vector and angle:
    - $r = r(t), \theta = \theta(t)$
    - $r = r(\theta), \theta = \theta(t)$

A: Convenience!

COORDINATE SYSTEMS



W8-5

- We can switch between coordinate systems on the fly!

- Example:
  - Describe car A in Cartesian coordinate system (because it moves along a straight line)
  - Describe car B in normal-tangential components (because it undergoes curvilinear motion)
  - Express (n, t)-component in Cartesian components:

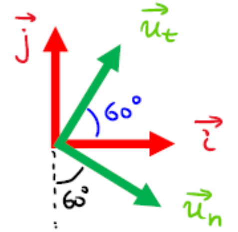
- This step is done using the geometry of your system
  - look at how you "guest" unit vectors (here: $\vec{u}_t$ and $\vec{u}_n$) are oriented with respect to your "host" unit vectors (here: $\vec{i}$ and $\vec{j}$)
  - Find projections of $\vec{u}_t$ on $\vec{i}$ and $\vec{j}$
  - Now you can express all $(\vec{u}_t, \vec{u}_n)$-dependent vectors through $\vec{i}$ and $\vec{j}$.

$\vec{v}_B = \vec{u}_t \cdot 15;$ $\quad \vec{a}_B = \vec{u}_t(-0.8) + \vec{u}_n\left(\frac{15^2}{250}\right)$

Let us express unit vectors $\vec{u}_t, \vec{u}_n$ in Cartesian coordinantes $\vec{i}, \vec{j}$:

$\vec{u}_t = \vec{i}(\cos 60°) + \vec{j}(\sin 60°)$

$\vec{u}_n = \vec{i}(\sin 60°) - \vec{j}(\cos 60°)$



Then:

$\vec{v}_B = 15\cos 60° \cdot \vec{i} + 15\sin 60° \cdot \vec{j}$

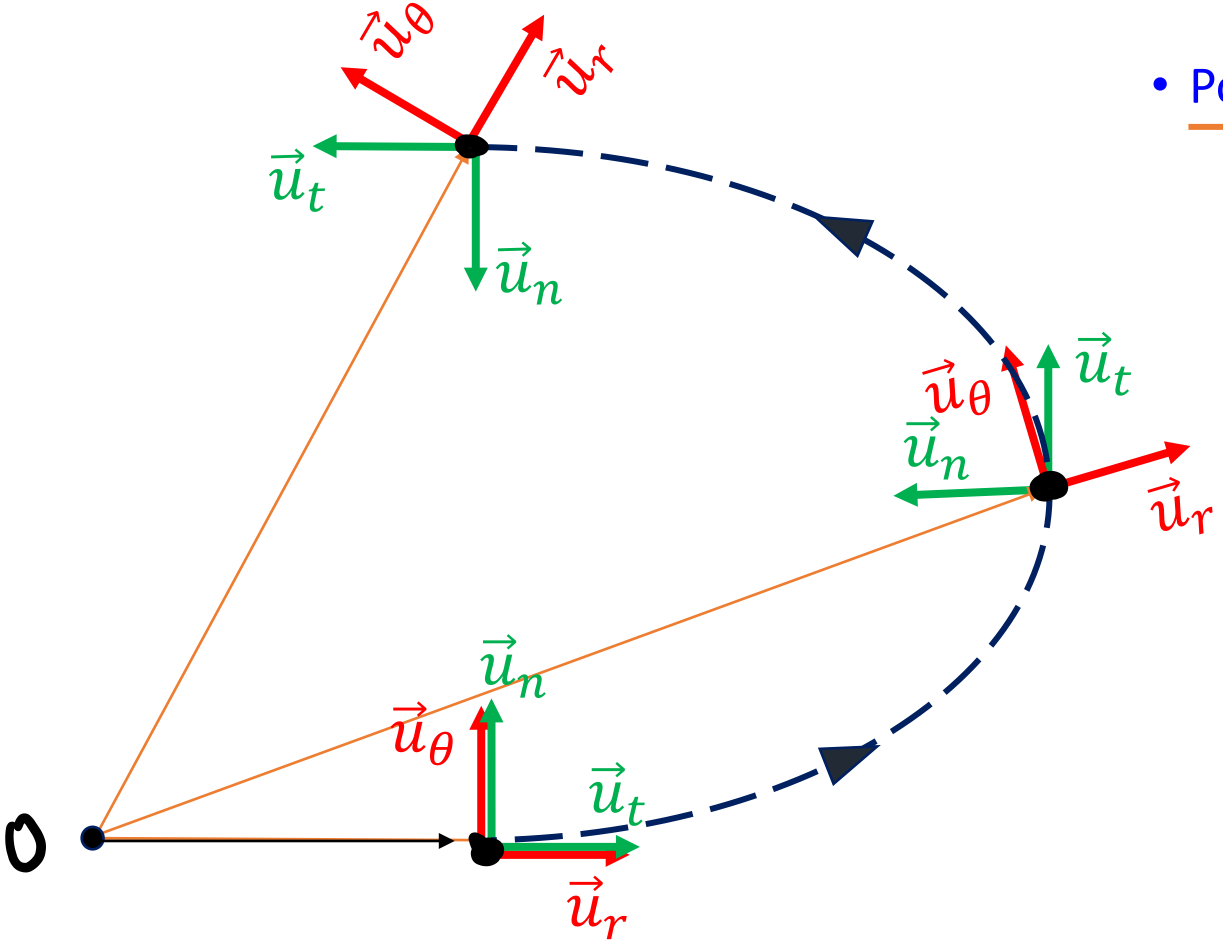
$\vec{a}_B = (-0.8)[\vec{i}\cos 60° + \vec{j}\sin 60°] + 0.9[\vec{i}\sin 60° - \vec{j}\cos 60°] =$

$= \vec{i}(0.9\sin 60° - 0.8\cos 60°) - \vec{j}(0.8\sin 60° + 0.9\cos 60°) =$

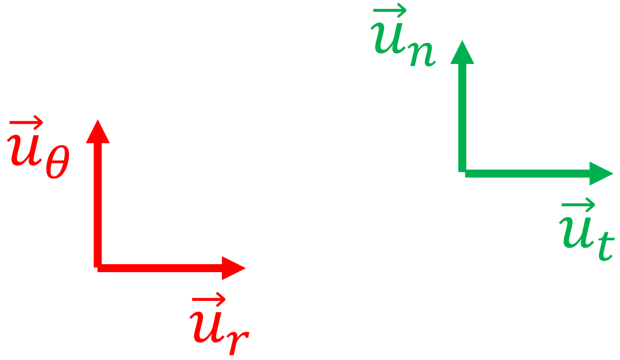
COORDINATE SYSTEMS: Polar (r, θ) and (n, t)

- They both are "portable" (travel with the particle).

But:



- Polar and (n, t)-coordinate systems are very different!



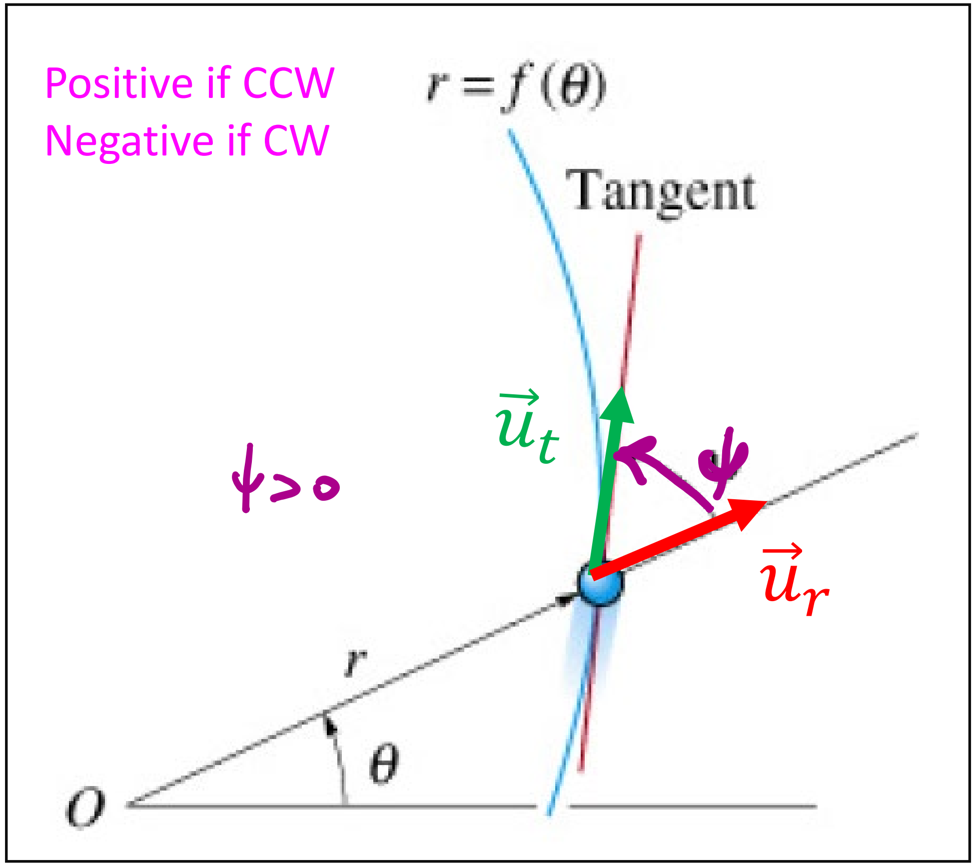
Q: Place these coordinate systems at a few locations on this trajectory.

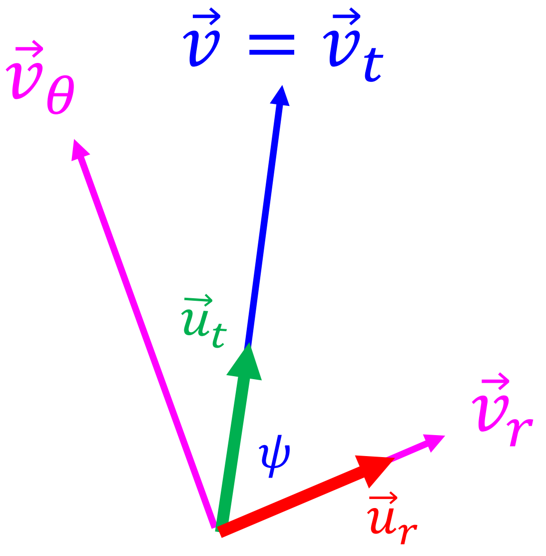
Polar ⇔ (n, t) converter: ψ −angle (“psi”)

ψ −angle definition:
Angle between r- and t-axes

- Let us prove that:

$$\boxed{\tan\psi = \frac{r}{dr/d\theta}}$$



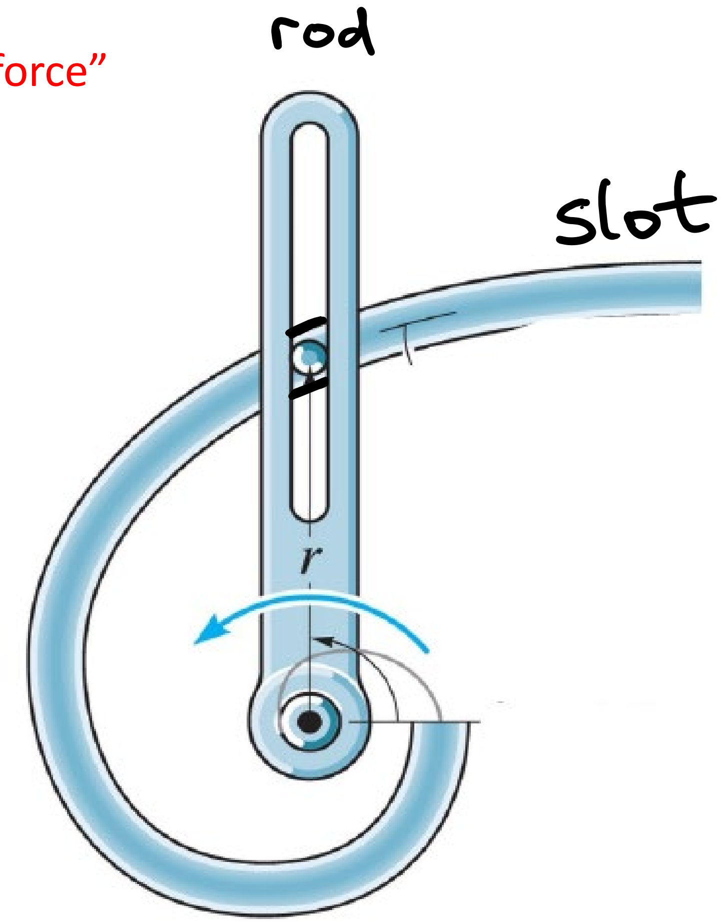


- We know that $\vec{v}$ is always parallel to t-axis:
  $\vec{v} = \vec{v}_t$
- But we also can express it in polar coordinates:
  $\vec{v} = \vec{u}_r v_r + \vec{u}_\theta v_\theta$
- Geometry:
  $\tan\psi = \frac{v_\theta}{v_r}$
- Now: $\tan\psi = \frac{v_\theta}{v_r} = \frac{r\,\dot{\theta}}{\dot{r}} = \frac{r\,(d\theta/\cancel{dt})}{(dr/\cancel{dt})} = \frac{r}{(dr/d\theta)}$

W10-1. The slotted rod moves the 4 kg particle around the curved slot whose shape is given by $r = 0.6/\theta$ m, where θ is in radians. The motion is in the horizontal plane. Determine the force that the rod exerts on the particle and the force of the slot on the particle when $\theta = 90^o$, $\dot{\theta} = 0.5$ rad/s, and $\ddot{\theta} = 0.6$ rad/s^2. Neglect friction.

$\vec{W} \perp$ to this picture

= "ignore gravity force"



Which coordinate system is most natural to describe the acceleration of this particle?

A. Cartesian

B. (n, t)

C. Polar

D. All of them

E. None of them…

$a_r = \ddot{r} - r\dot{\theta}^2$

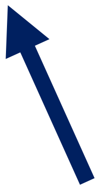
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$\dot{r}, \ddot{r}$ – easy!

$r(\theta) = \frac{0.6}{\theta}$

Q: What is the direction of the force exerted **by the rod** on the particle at the moment shown in the figure?

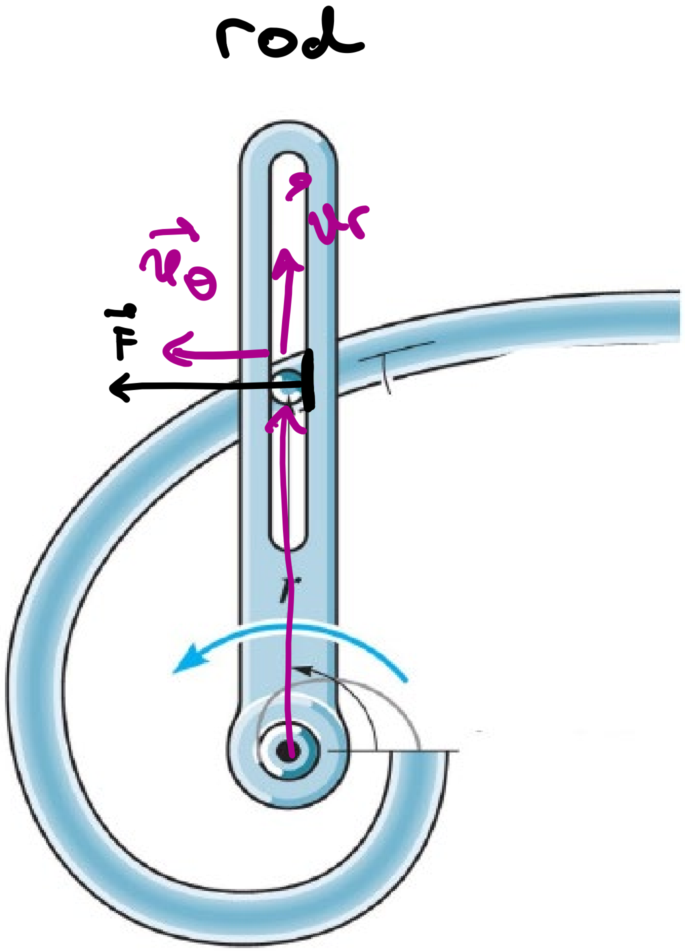
A


B


C


D


E. None of these



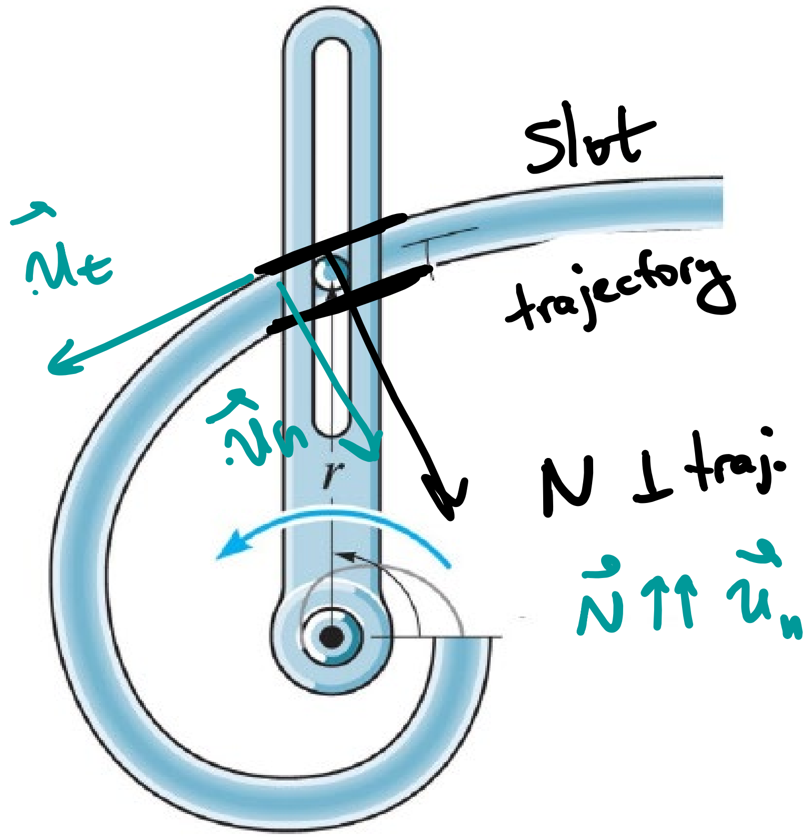
- Which coordinate system is most natural for this force?

A. (n,t)
B. polar
~~C.~~ Cartesian

Q: What is the direction of the force exerted **by the slot** on the particle at the moment shown in the figure?

A B C D

E. None of these



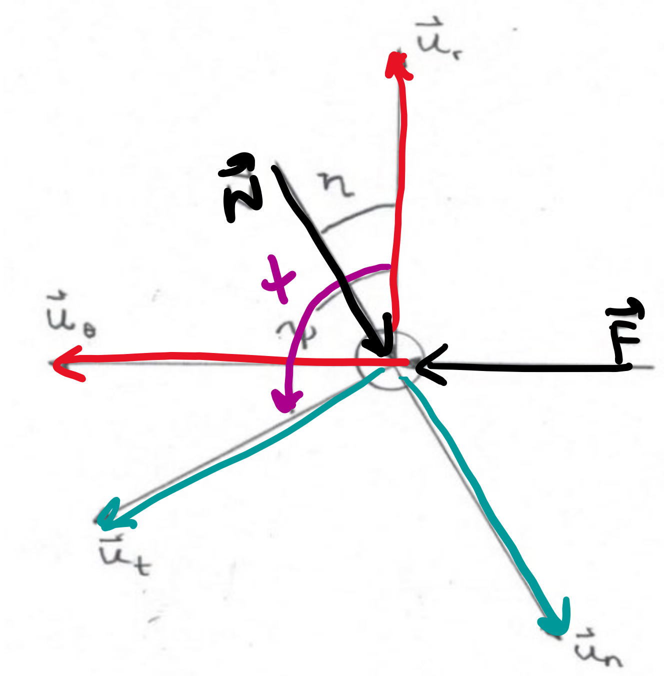
A. (u,t)
B. Polar
C. Cartesian

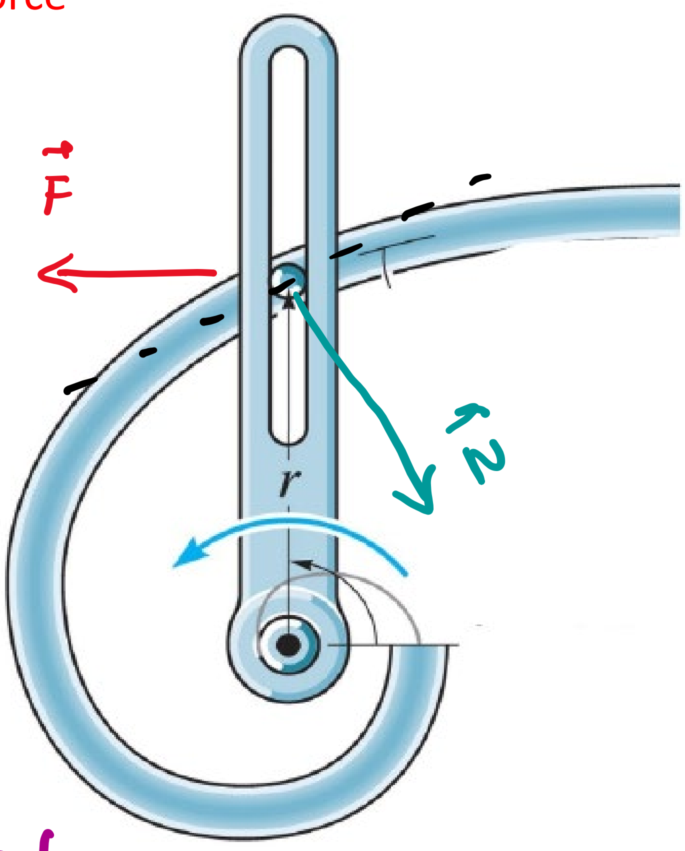
- Which coordinate system is most natural for this force?

W10-1. The slotted rod moves the 4 kg particle around the curved slot whose shape is given by $r = 0.6/\theta$ m, where θ is in radians. The motion is in the horizontal plane. Determine the force that the rod exerts on the particle and the force of the slot on the particle when $\theta = 90^o$, $\dot{\theta} = 0.5$ rad/s, and $\ddot{\theta} = 0.6$ rad/s². Neglect friction.

= "ignore gravity force"

0. Diagram:

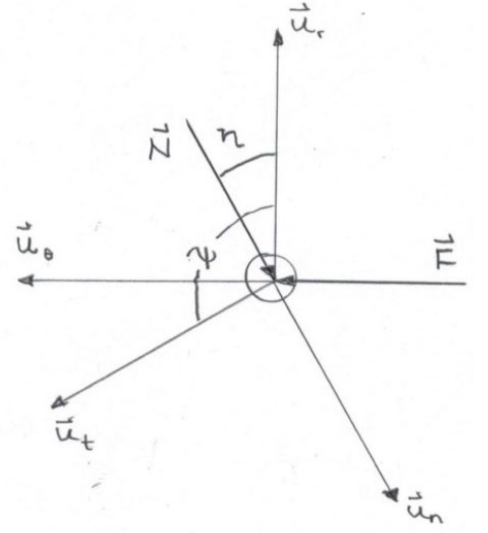




$$\vec{F}_R = \underbrace{\vec{F}}_{\text{polar}} + \underbrace{\vec{N}}_{(n,t)} = m\underbrace{\vec{a}}_{\text{polar}}$$

W10-1. The slotted rod moves the 4 kg particle around the curved slot whose shape is given by $\boxed{r = 0.6/\theta \text{ m},}$ where θ is in radians. The motion is in the horizontal plane. Determine the force that the rod exerts on the particle and the force of the slot on the particle when $\theta = 90^o$, $\dot\theta = 0.5$ rad/s, and $\ddot\theta = 0.6$ rad/s². Neglect friction.

↳ $\pi/2$



1. Find acceleration in polar coordinates:

$$a_r = \ddot r - r\dot\theta^2, \qquad a_\theta = r\ddot\theta + 2\dot r\dot\theta$$

$$\frac{d}{dt}\frac{f}{g} = \frac{\dot f g - \dot g f}{g^2}$$

We are interested in the moment when:

- $\theta = \frac{\pi}{2}$; $\quad \dot\theta = 0.5\,\frac{\text{rad}}{s}$; $\quad \ddot\theta = 0.6\,\frac{\text{rad}}{s^2}$

$$r = \frac{0.6}{\theta} \xrightarrow{\text{at } \theta = \pi/2} 0.3820 \text{ m}$$

$$\dot r = \frac{d}{dt}\frac{0.6}{\theta} = -\frac{0.6}{\theta^2}\cdot\dot\theta \xrightarrow{\text{at } \theta = \frac{\pi}{2},\ \dot\theta = 0.5} -0.1216\,\frac{m}{s}$$

$$\ddot r = \frac{d}{dt}\left(-\frac{0.6}{\theta^2}\dot\theta\right) = -0.6\,\frac{\ddot\theta\theta^2 - 2\theta\dot\theta^2}{\theta^4} = -0.6\,\frac{\ddot\theta\theta - 2\dot\theta^2}{\theta^3} \xrightarrow{\text{at } \theta = \frac{\pi}{2} \text{ etc}} -0.0685\,\frac{m}{s^2}$$

$$a_r\left(\theta = \tfrac{\pi}{2}\right) = -0.1640\,\frac{m}{s^2} \qquad a_\theta = 0.1076\,\frac{m}{s^2}$$