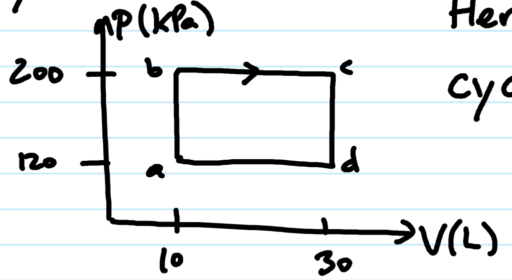
Homework 6 Written Solution.

We will analyze each cycle in turn.

Cycle A:



Here, the net work is the area inside the cycle, $W = 80\,\text{kPa} \cdot 20\,\text{L} = 1600\,\text{J}$.

Heat is added in step $a \to b$ and $b \to c$.

For $a \to b$ $W = 0$ so $Q = \Delta U = nC_v(T_b - T_a)$

We have $\frac{T_b}{T_a} = \frac{P_b}{P_a} = \frac{5}{3}$, so $T_b = 500\,\text{K}$.

Also, $nR = \frac{P_a V_a}{T_a} = \frac{120 \cdot 10}{300} \frac{\text{J}}{\text{K}} = 4 \frac{\text{J}}{\text{K}}$, so $nC_v = \frac{5}{2} nR = 10 \frac{\text{J}}{\text{K}}$

Thus $Q_{a \to b} = 10\,\text{J/K} \cdot (200\,\text{K}) = 2000\,\text{J}$.

Next, $Q_{b \to c} = nC_p \Delta T$ (constant pressure)

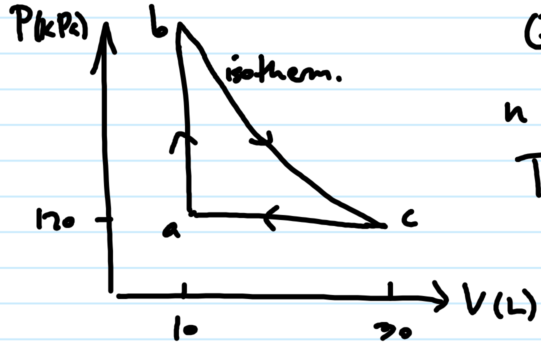
$= \frac{7}{2} nR (T_c - T_b)$

Also: $\frac{T_c}{T_b} = \frac{V_c}{V_b} = 3$ so $T_c = 1500\,\text{K}$.

Then $Q_{b \to c} = \frac{7}{2} \cdot 4 \cdot 1000 = 14{,}000\,\text{J}$

Finally, the efficiency is $\frac{W}{Q_{b \to c} + Q_{a \to b}} = \frac{1600}{2000 + 14000} = 0.1$

Cycle B:



Here, for $a \to b$, we have $W = 0$ and

$Q = \Delta U = nC_v(T_b - T_a)$

n is the same as before since P_a, V_a, and T_a are all the same. We have $T_b = T_c$,

and $\frac{T_c}{T_a} = \frac{V_c}{V_a} = 3$ so $T_b = T_c = 900\,\text{K}$

Thus $Q_{a \to b} = 10 \frac{\text{J}}{\text{K}} \cdot 600\,\text{K} = 6000\,\text{J}$

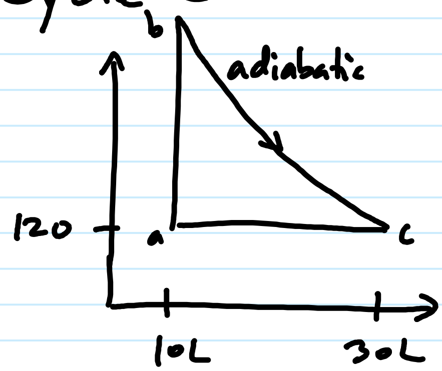
For $b \to c$, we have $\Delta U = 0$ so $Q = W = nRT \ln\left(\frac{V_f}{V_i}\right)$

This gives $Q = W = 4\,\frac{J}{K} \cdot 900K \cdot \ln(3) = 3955J$

Finally, for $c \to a$, we have $Q < 0$ and $W = P\Delta V = -2400J$

Overall, we get $W = 1555J$ and $Q_{in} = 9955J$, so the efficiency is $e = \frac{W}{Q_{in}} = \frac{1555}{9955} = 0.156$. Better.

Cycle C:



Here, T_c is 900K as above. To find T_b, we use that $b \to c$ is adiabatic. We have that $T_b V_b^{\gamma-1} = T_c V_c^{\gamma-1}$ so:

$$\frac{T_b}{T_c} = \left(\frac{V_c}{V_b}\right)^{\gamma-1} = 3^{0.4} = 1.552,$$

so $T_b = 1397K$

Now, for $a \to b$, $W = 0$ and $Q = \Delta U = nC_v(T_b - T_a)$

$$= 10\,\tfrac{J}{K} \cdot 1097 = 10970J$$

For $b \to c$, $Q = 0$ and $W = -\Delta U = nC_v(T_b - T_c)$

$$= 10\,\tfrac{J}{K} \cdot 497K = 4970J$$

Finally, for $c \to a$, $W = P\Delta V = -2400J$ and $Q = \Delta U + W < 0$.

So $W_{net} = 2570J$ and $Q_{in} = 10970J$ giving

$e = 2570/10970 = \underline{\underline{0.234}}$ Winner!

For the winning cycle C, if we want 10 cycles per minute, this is 600 cycles/hour. We need 10970J per cycle of heat added, so this is 6.58×10^6 J/hour. Burning gasoline gives us 35MJ/L, so we need

$\frac{6.58}{35} = 0.188$ L/hour. That's only ~30¢/hour to keep your baby happy!