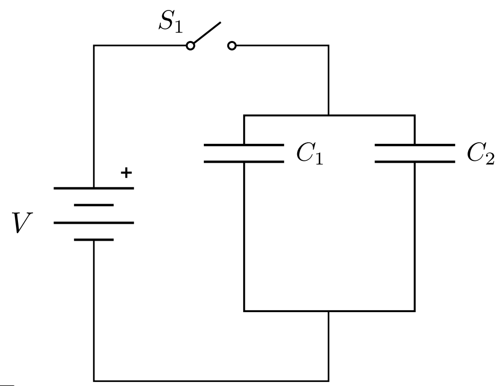


Problem C2.1 (☆☆): The capacitors shown left are both initially uncharged. The battery has $V = 210\ V$ and the capacitances are $C_1 = 3\ \mu F$ and $C_2 = 6\ \mu F$.

(a) How much charge is drawn from the battery when the switch S_1 is closed?

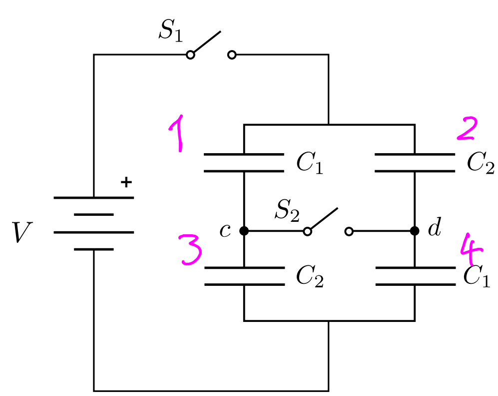
Equations we may need:

- Series capacitors: $C_{eq} = \sum_k \frac{1}{C_k}$
- Parallel capacitors: $C_{eq} = \sum_k C_k$
- Capacitor eqn: $q = CV$

Units: $C = F \cdot V$

(a) $C_{12} = (3\mu F + 6\mu F) = 9\mu F$

$q_{12} = C_{12} V = (9\mu F)(210V) = \underline{1.89\ mC}$ ✓



Now an additional pair of capacitors is added to give the circuit diagram shown left.

(b) How much charge is drawn from the battery now when the switch S_1 is closed?

(c) What is the potential difference across points c and d?

(b) $\frac{1}{C_{13}} = \frac{1}{C_{24}} = \frac{1}{C_1} + \frac{1}{C_3}$

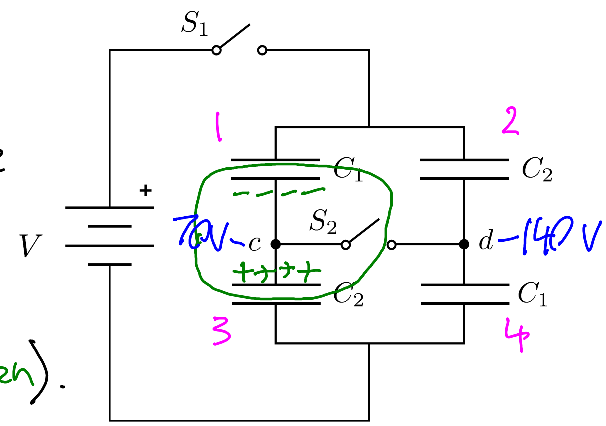
$\therefore C_{13} = 1/\left(\frac{1}{3\mu F} + \frac{1}{6\mu F}\right) = 2\mu F$

whilst S_2 is open:

$q_{tot} = q_{13} + q_{24} = 2q_{13} = 2C_{13}V = 2(2\mu F)(210V) = 840\mu C$

It is interesting to note that by adding two capacitors, we actually reduced the total charge drawn from the battery.

(c) Capacitors in series have the same charge because the 2 plates connecting them must be neutral (see green).



$$\therefore q_1 = q_3 = q_{13} = 420\,\mu C$$

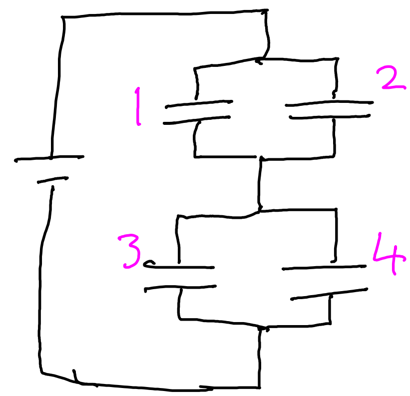
$$q = CV \Rightarrow V_1 = \frac{q_1}{C_1} = \frac{420\,\mu C}{3\,\mu F} = 140\,V = V_4$$

$$V_2 = \frac{q_2}{C_2} = \frac{420\,\mu C}{6\,\mu F} = 70\,V = V_3$$

$$\therefore \Delta V_{cd} = V_c - V_d = \underline{-70\,V}$$

Problem C2.1 (cont.): Now the second switch S_2 is closed.

- **(d)** What is the potential difference across each capacitor?
- **(e)** How much charge flowed across S_2 when it was closed?



(d) Now S_2 has closed we can redraw our circuit as left.

Our equivalent capacitors are now the same as in part (a).

$$C_{13} = C_{24} = C_1 + C_3 = 9\,\mu F$$

Because the equivalent capacitors are identical, we conclude that from symmetry:

$$V_{13} = V_{24} = V/2 = \underline{105\,V}$$

(e) When S_2 was open C_1 & C_3 had the same charge:

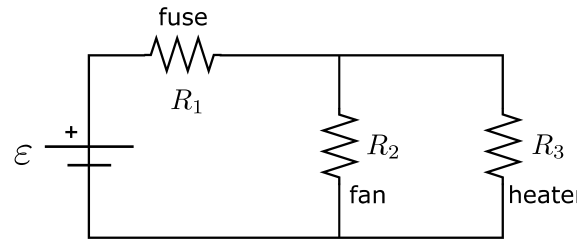
$$\Delta q = q_1 - q_3 = 0$$

when S_2 was closed C_1 & C_3 have the same potential:

$$\Delta q = C_1 V_1 - C_3 V_3$$
$$= (3\mu F)(105\,V) - (6\mu F)(105\,V)$$
$$= \underline{-315\,\mu C}$$

So $315\,\mu C$ must have flowed across S_2.

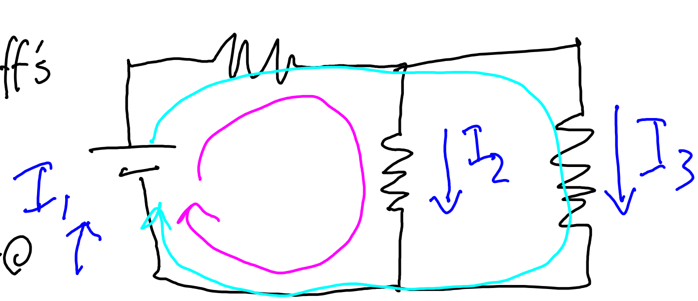
Problem C2.3 (☆☆): Power sources used to feature a fuse*, which is a small resistor (typically $10\ m\Omega$) that melts when the current drawn from the circuit becomes dangerous, thus breaking the circuit. Consider a standard $120\ V$ power socket, which is being used to power a hairdryer. The hairdryer uses a $50\ W$ fan and a $1\ kW$ heating element.



(a) What is the resistance of the fan and the heating element, given their power output?

(b) If the fuse melts after $5\ J$ of energy accumulation, how long would it take to melt with twice the operating current? Assume the fuse can dissipate $1\ W$ of heat.

(a) We will need Kirchoff's laws:



KLR#1 $\varepsilon - I_1R_1 - I_2R_2 = 0$

KLR#2 $\varepsilon - I_1R_1 - I_3R_3 = 0$

KJR: $I_1 = I_2 + I_3$

Whilst we don't know R_2 or R_3, we do know their power. since $P = VI = RI^2$:

$$R_2 = \frac{P_2}{I_2^2} \quad \& \quad R_3 = \frac{P_3}{I_3^2}$$

Solving Kirchoff's laws:

$$\varepsilon - I_1R_1 - I_2\frac{P_2}{I_2^2} = 0 \Rightarrow I_2 = \frac{P_2}{\varepsilon - I_1R_1}$$

*Modern circuit breakers use electromagnetics to open a switch when the current becomes too high and are thus reusable. We will learn how electromagnets can do this later in the course.

(a) Similarly $I_3 = \dfrac{P_3}{\mathcal{E} - I_1 R_1}$

$$\therefore I_1 = \frac{P_2 + P_3}{\mathcal{E} - I_1 R_1} \Rightarrow R_1 I_1^2 - \mathcal{E} I_1 + (P_2 + P_3) = 0$$

This quadratic has 2 solutions:

$$I_1 = 8.756 \text{ A or } 11.991 \text{ A}$$

The second solution is unphysical and blows up to ∞ as $R_1 \to 0$. Hence we discard it.

$$\therefore I_2 = 0.417 \text{ A} \Rightarrow \underline{R_2 = 288\ \Omega}$$

$$I_3 = 8.340 \text{ A} \Rightarrow \underline{R_3 = 14.4\ \Omega}$$

(b) Operating current is $I_1 = 8.76$A. So heat produced at $2I_{op}$:

$$P_{melt} = R_1 (2I_{op})^2 = 3.067 \text{ W}$$

But fuse can dissipate 1 W. So net heat buildup is:

$$P_{net} = P_{melt} - 1 = \frac{\Delta E}{\Delta t}$$

$$\therefore \Delta t_{melt} = \frac{\Delta E}{P_{net}} = \frac{5\text{J}}{2.067\text{ W}} \frac{\cancel{J}}{\cancel{J}/s} = \underline{2.419\ s}$$