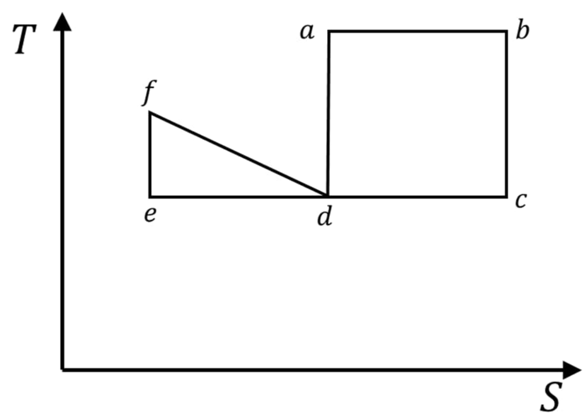
# Physics 157 Tutorial 7

**Name:**
**Student number:**
**Group member names:**

In this tutorial, you will get some practice with problems involving entropy.

**Question 1:** A thermodynamic system undergoes a cyclic processs as shown in the figure. The cycle consists of two closed loops. The cycle is the path d → a → b → c → d then d → e → f → d.

You are given the following parameters: $T_a = T_b = 400$ K, $T_c = T_d = T_e = 200$ K, $T_f = 300$ K, $S_b = S_c = 5$ J/K, $S_a = S_d = 3$ J/K, and $S_e = S_f = 1$ J/K.




a) Find the net work done by the system for one complete cycle, including both loops.

The net work done by the system is the area enclosed by the cycles, and is positive for clockwise cycles and negative for counterclockwise cycles. Here, both loops are clockwise:

$$W_{net} = W_{abcd} + W_{def} = (T_a - T_d) \cdot (S_b - S_a) + \tfrac{1}{2}(T_f - T_e) \cdot (S_d - S_e)$$
$$= 200 \text{ K} \cdot 2 \text{ J/K} + \tfrac{1}{2}\, 100 \text{ K} \cdot 2 \text{ J/K}$$
$$= 500 \text{ J}$$

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b) Find the net heat input to the system, $Q_{in}$, for one complete process, including both loops.

The heat input to the system is the area under the $\Delta S > 0$ parts, so

$$Q_{in} = Q_{ab} + Q_{fd} = T_a \cdot (S_b - S_a) + T_e \cdot (S_d - S_e) + \tfrac{1}{2}(T_f - T_e) \cdot (S_d - S_e)$$
$$= 400\text{ K} \cdot 2\text{ J/K} + 200\text{ K} \cdot 2\text{ J/K} + \tfrac{1}{2}\,100\text{ K} \cdot 2\text{ J/K}$$
$$= 1300\text{ J}$$

c) Is this thermodynamic cycle acting as an enginer or as a refrigerator?

The cycle is clockwise around each loop, so it is operating as an engine.

d) If your answer to part c was engine, how could you operate the cycle so that it acts as a refrigerator? If your answer to part c was refrigerator, how could you operate the cycle so that it acts as an engine?

Operating both cycles in reverse (i.e., counterclockwise) would convert if from an engine to a refrigerator.

e) Find the net work done by the system for one complete cycle, including both loops, when you operate it as described in part d.

The sign of the net work done by the system is reversed when the direction of the cycle is reversed:

$$W_{net} = -W_{abcd} - W_{def} = -(T_a - T_d) \cdot (S_b - S_a) - \tfrac{1}{2}(T_f - T_e) \cdot (S_d - S_e)$$
$$= -200\text{ K} \cdot 2\text{ J/K} - \tfrac{1}{2}\,100\text{ K} \cdot 2\text{ J/K}$$
$$= -500\text{ J}$$

f) Find the net heat input to the system, $Q_{in}$, for one complete process, including both loops, when you operate it as described in part d.

The heat input to the system is the area under the $\Delta S > 0$ parts. When the system is operated in reverse, this is the processs from e $\rightarrow$ d , and d $\rightarrow$ c.

$$
\begin{aligned}
Q_{in} &= Q_{ed} + Q_{dc} \\
&= T_e \cdot (S_d - S_e) + T_e \cdot (S_c - S_d) \\
&= 200\ \text{K} \cdot 2\ \text{J/K} + 200\ \text{K} \cdot 2\ \text{J/K} \\
&= 400\ \text{J}
\end{aligned}
$$

g) What is the efficiency of the system when operated as an engine?

From parts a and b:

$$e = W_{net} / Q_{in} = 500\ \text{J} / 1300\ \text{J} = 0.38$$

e) What is the coefficient of performance when operated as a refrigerator?

The coefficient of performance, $\kappa$, of a refrigerator is defined in terms of the work done ON the system, $W_{in}$, and the heat into the system. Since Win = – $W_{net}$, we have from part e,

$$W_{in} = -W_{net} = 500\ \text{J}$$

And from part f, the heat into the system is

$$Q_{in} = 400\ \text{J}.$$

The coefficient of performance of the system when operated as a regrigerator is thus

$$\kappa = Q_{in} / W_{in} = 400\ \text{J} / 500\ \text{J} = 0.8$$

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f) What would the efficiency of a carnot engine be if it were operated between the maximum and minimum temperatures in this cycle?

For this cycle, the hottest temperature is $T_H = 400$ K, and the coldest temperature is $T_C = 200$ K. The efficiency of a carnot engine operated between these values for $T_H$ and $T_C$ would be

$e = (T_H – T_C) / T_H = 1 – T_C / T_H = 1 – 200 / 400 = 0.5$

The efficiency of our cycle is less than that of the carnot cycle, as expected.

g) What would the coefficient of performance of a carnot refrigerator be if it were operated between the maximum and minimum temperators in this cycle?

For this cycle, the hottest temperature is $T_H = 400$ K, and the coldest temperature is $T_C = 200$ K. The coefficient of performance of a carnot refrigerator operated between these avlues for $T_H$ and TC would be

$\kappa = T_C / (T_H – T_C) = 200 / 200 = 1.0$

The coefficient of performance of our cycle is less than that of the carnot cycle, as expected.

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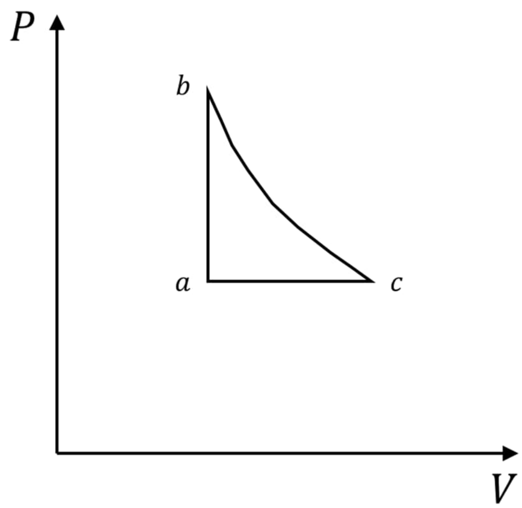
**Question 2:** A cylinder contains Nitrogen gas ($C_v = 5R/2$, $\gamma = 1.4$) at a pressure of 2.0 atm and a temperature of 300 K. Its initial volume is 2.0 liters. The Nitrogen gas is next carried through the following processes:

1) It is heated at constant volume to a temperature of 450 K.
2) It is expanded at constant temperature to a volume of 3.0 liters.
3) It is compressed at constant pressure until its volume is 2.0 liters, returning it to its initial state.

a) Draw the PV curve of this cycle. Be sure to label the each endpoint.




b) Determine the temperature, pressure, and volume at each endpoint.

We are given $P_a = 2$ atm, $V_a = V_b = 2.0$ L, $V_c = 3.0$ L, $T_a = 300$ K, $T_b = T_c = 450$ K. The remaining unknowns, $P_b$ and $P_c$ are given by:

$P_b = P_a \times T_b / T_a = 2 \text{ atm} \times 450 / 300 = 3 \text{ atm}$

$P_c = P_b\, V_b/V_c = 3 \text{ atm} \times 2 / 3 = 2 \text{ atm}$

and the compression ratio, $r = V_c / V_b = 1.5$

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c) What is the efficiency of this heat engine?

$e = W_{net} / Q_{in}$

$W_{net} = W_{bc} + W_{ca} = n\,R\,T_b \ln(V_c / V_b) + P_a\,(V_a – V_c)$

This can be rewritten using PV = nRT as

$W_{net} = n\,R\,T_b \ln(V_c / V_b) + n\,R\,(T_a – T_c)$

$Q_{in} = Q_{ab} + Q_{bc} = n\,C_v\,(T_b – T_a) + n\,R\,T_b \ln(V_c / V_b)$
$\quad = n\,R\,(5/2)\,(T_b – T_a) + n\,R\,T_b \ln(V_c / V_b)$

The efficiency is thus

$e = [\,T_b \ln(r) + (T_a – T_c)\,] / [\,(5/2)\,(T_b – T_a) + T_b \ln(r)\,] = 5.8\%$

d) What is the change in entropy for the second process?

The second process is isothermal, so dQ = dW = nRT dV / V.

The differential change in entropy is dS = dQ / T = nR dV / V

Integrating this gives

$S = nR \ln(V_c / V_b) = (P_b\,V_b / T_b) \ln(V_c / V_a) = (3 \times 2 / 450)\; 100 \ln(1.5)$

Where the factor of 100 converts form L-atm to J. Evaluating this gives

S = 0.54 J/K.