- What is your personal preference in case of a longer Translink strike?

A. Strongly prefer in-person lectures to continue 34%

B. Simply can't get to the campus & need zoom lectures 34%

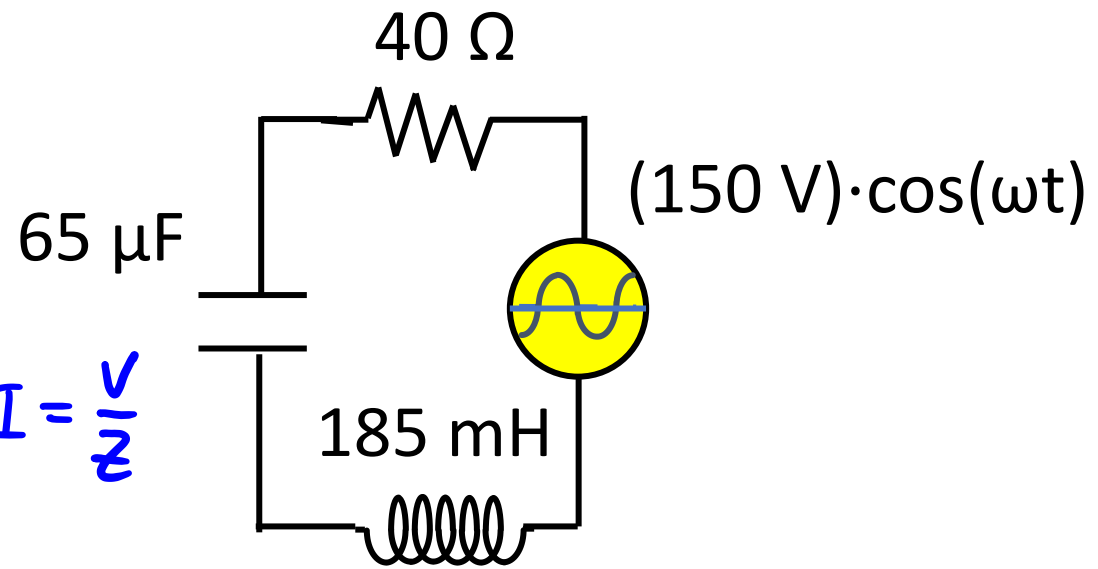
C. No strong preference, both ways work for me. 32%

❖ We will try to come up with a solution that addresses everybody's needs, and it's good to know how your preferences split.

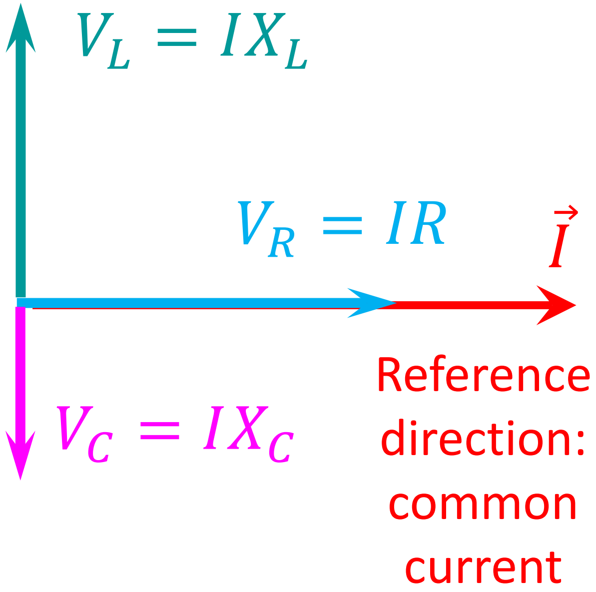
Q: An AC circuit with $V_{peak} = 150\text{ V}$ and $f = 50\text{ Hz}$ drives this RLC circuit.

a) What is the peak voltage across the resistor?

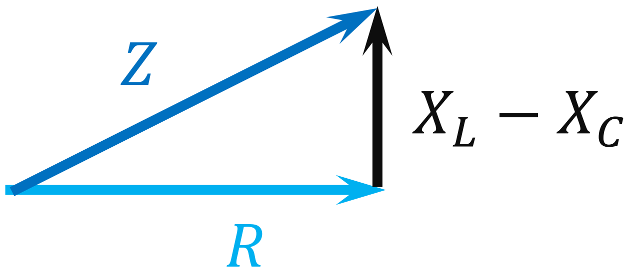
b) What is the peak voltage across the inductor?



$I = \frac{V}{Z}$



$V = IZ$



$X_L = \omega L = 58\ \Omega$

$X_C = \frac{1}{\omega C} = 49\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = 40.8\ \Omega$

A. 0 V

B. 4 V

C. 120 V

D. 145 V

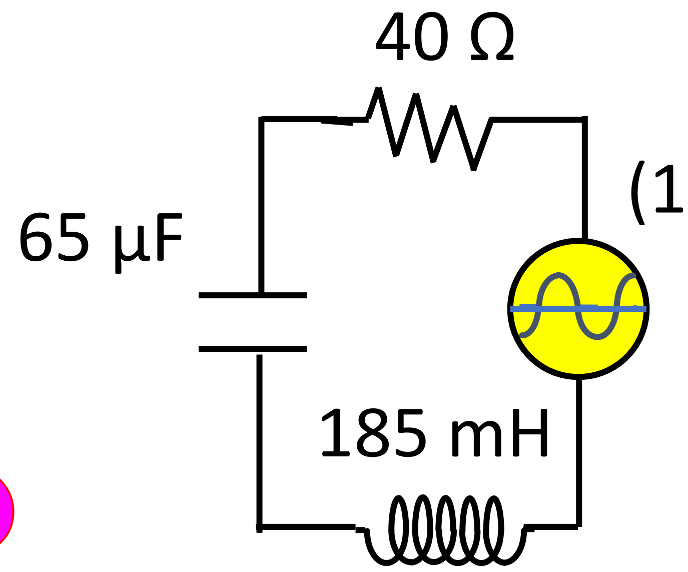
E. 215 V

$$\boxed{V_R = I_{max}R = V_{max}\frac{R}{Z} = 147\ V, \qquad V_L = I_{max}L = V_{max}\frac{X_L}{Z} = 214\ V}$$

Q: An AC circuit with $V_{peak} = 150\text{ V}$ and $f = 50\text{ Hz}$ drives this RLC circuit.

a) What is the peak voltage across the resistor?

b) What is the peak voltage across the inductor?



$V_R = 147\ \Omega$

$V_L = 214\ \Omega$

! Wait!! -- How can the peak voltage across the inductor be larger than the source voltage ?? ?

ANSWER: Kirchhoff's voltage law ALWAYS applies. In this case, the addition of instantaneous voltages involves both the amplitude and the phase of the instantaneous voltages.

$V_C \cos(\omega t - \frac{\pi}{2})$

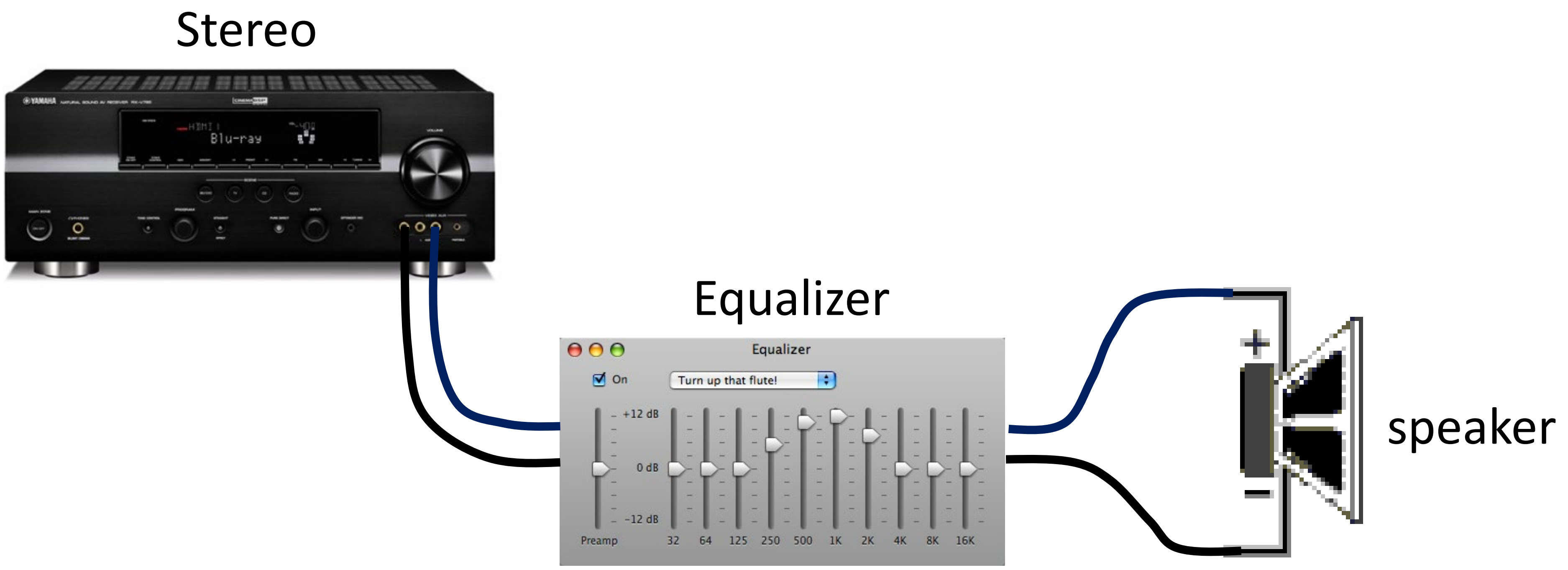
$$v_{\text{source}}(t) = v_R(t) + v_L(t) + v_C(t)$$

$V_{max} \cos \omega t$ $V_R \cos \omega t$ $V_L \cos(\omega t + \pi/2)$

When the voltage across the inductor is at max, the phases of the R and C are negative, and the three voltages add up to the source voltage, as it should be.

Application: electronic filters

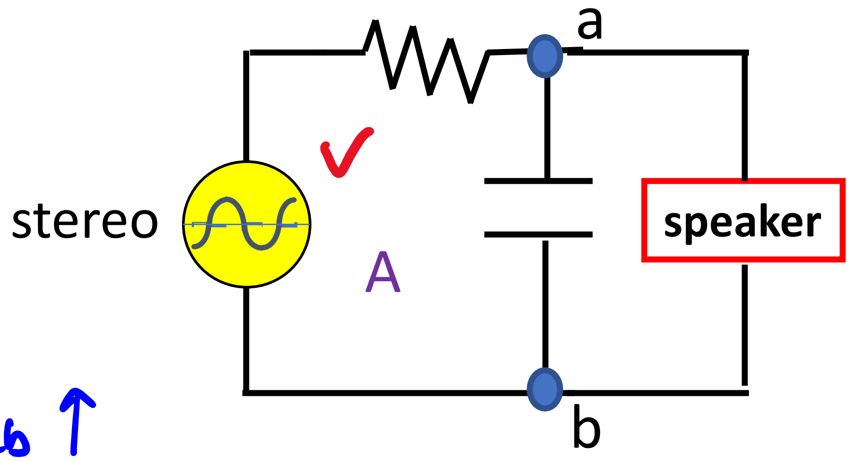
- What is the equalizer doing to the electrical signal coming out of the stereo?
- Specifically, if you want more bass in your music (i.e. higher voltage for low frequencies ω), what does the equalizer do to the output signal from the stereo ?

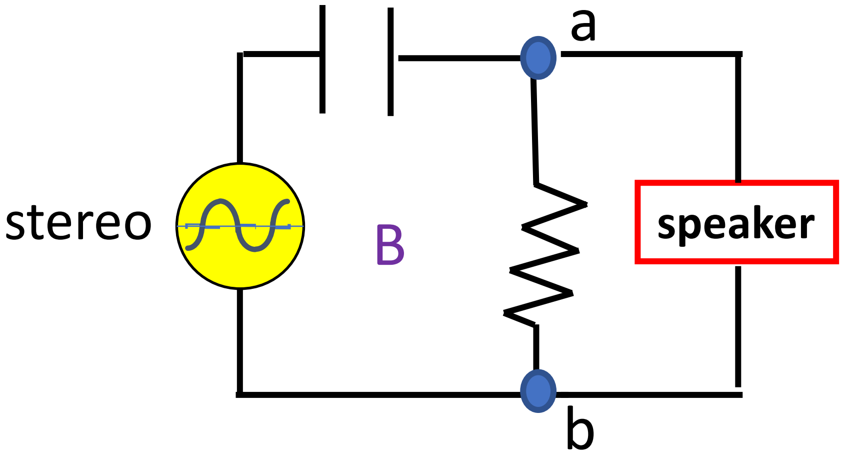


Q: Which of the electrical circuits below could be used as a low pass filter? $\omega \downarrow$ $\cancel{\omega \uparrow}$

"Low pass filter" : attenuate the higher frequencies = higher voltage for lower frequencies

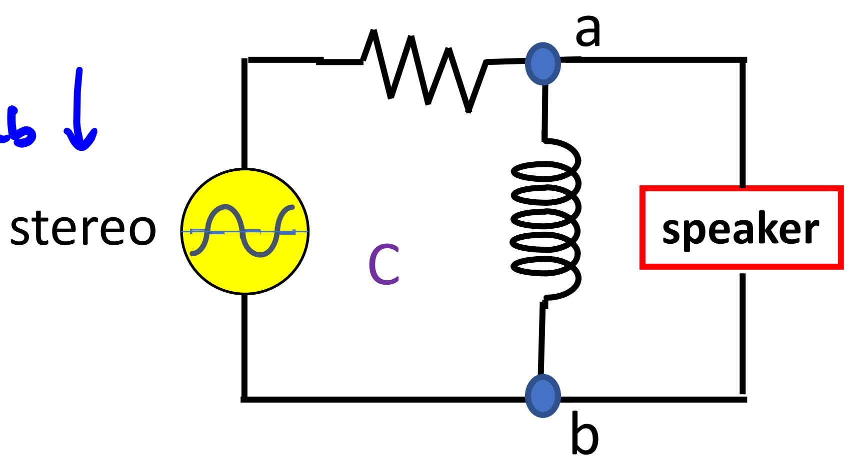
$\Delta V_{ab} = ?$

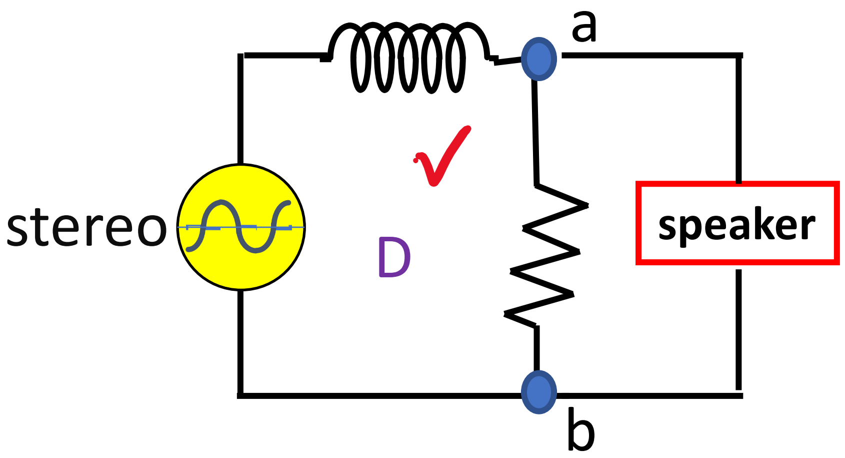




$\omega \downarrow$ $\Delta V_{ab} \uparrow$

$\cancel{\omega \uparrow}$ $\Delta V_{ab} \downarrow$





$X_L = L\omega$

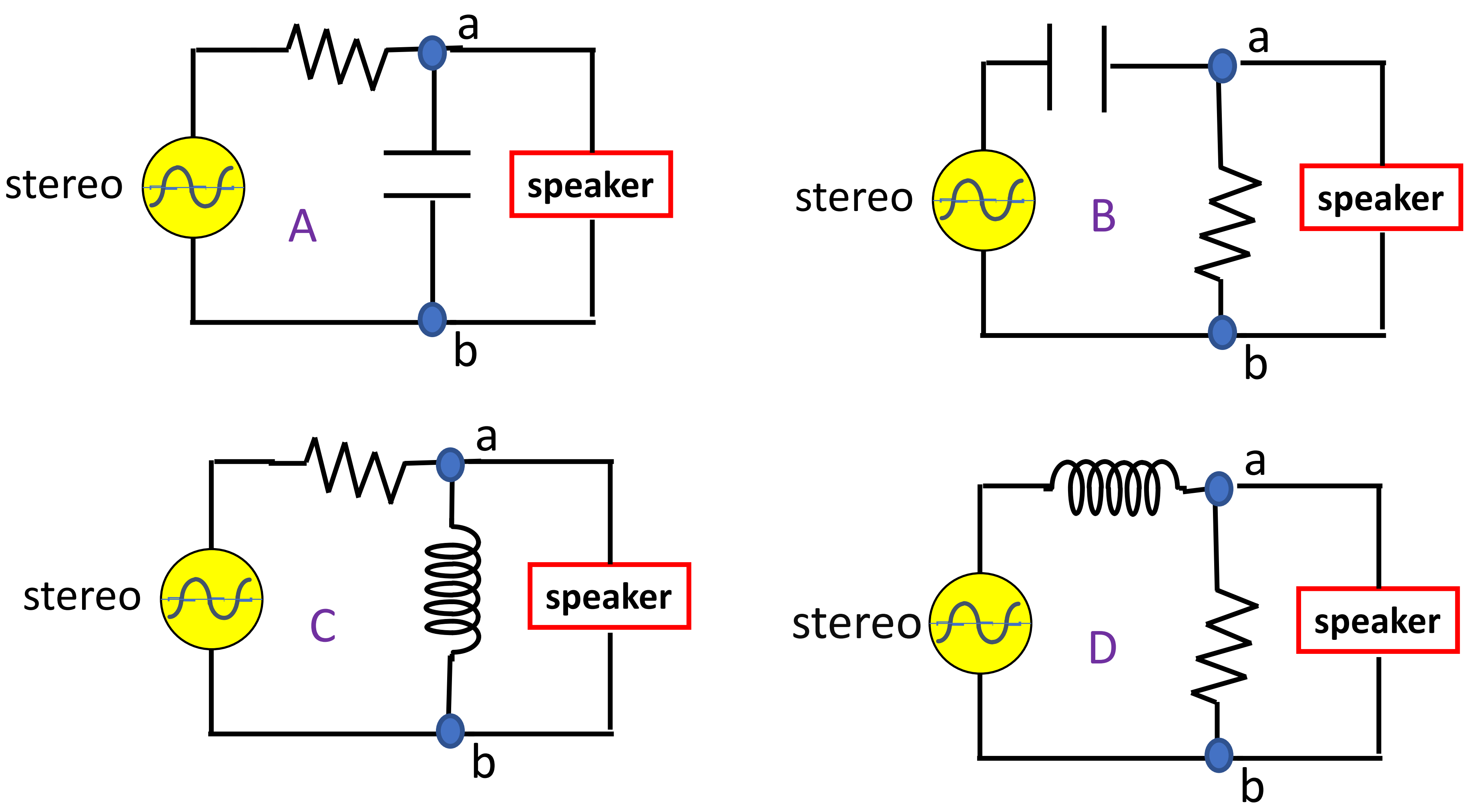
$X_C = \frac{1}{C\omega}$

E. More than one could work

$\Delta V_{R,L,C} = I X_{R,L,C}$

Q: Which of the electrical circuits below could be used as a low pass filter?

”Low pass filter” : attenuate the higher frequencies = higher voltage for lower frequencies

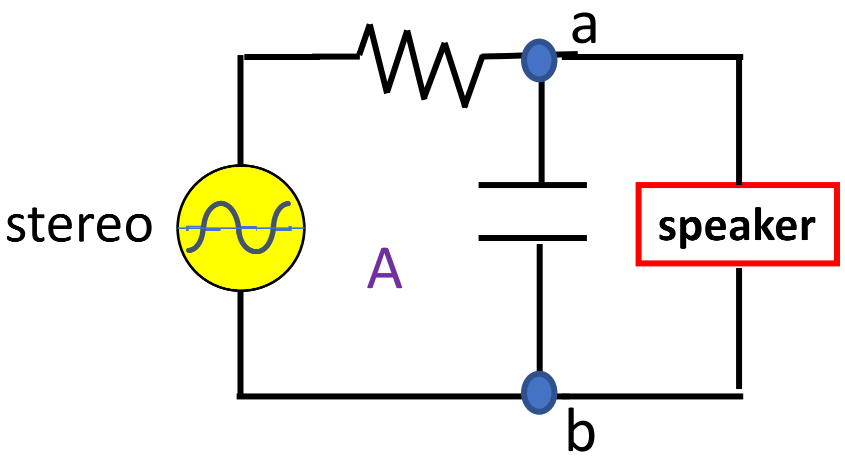


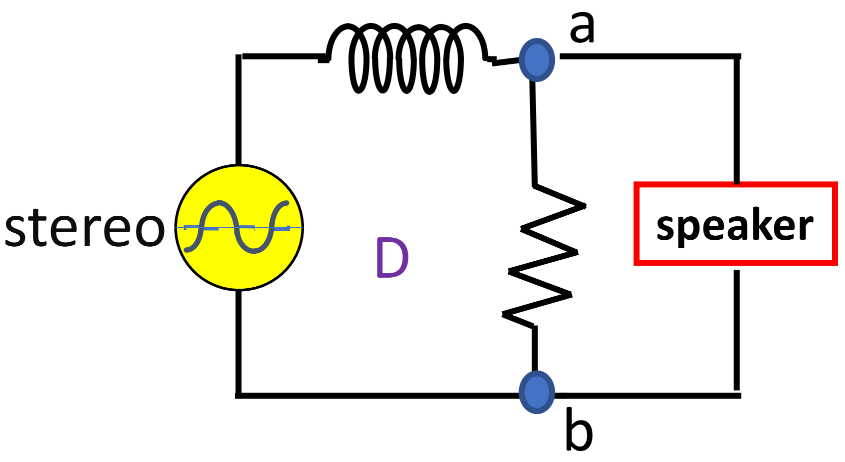
E. More than one could work

A & D

Q: Which of the electrical circuits below could be used as a low pass filter?

"Low pass filter" : attenuate the higher frequencies = higher voltage for lower frequencies





- Voltage drops across the two circuit elements in the left loop always add up to the source voltage => it's the question about how the voltage drops will be distributed among these two elements;
- $V_{\text{speaker}} = V_{ab}$

$$X_C = \frac{1}{\omega C} \to 0 \text{ as } \omega \to \infty$$

$$X_C = \frac{1}{\omega C} \to \infty \text{ as } \omega \to 0$$

- Thus, V_{ab} is larger for low frequencies

$$X_L = \omega L \to \infty \text{ as } \omega \to \infty$$

$$X_C = \frac{1}{\omega C} \to 0 \text{ as } \omega \to 0$$

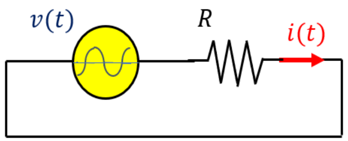
- Thus, V_{ab} is larger for low frequencies

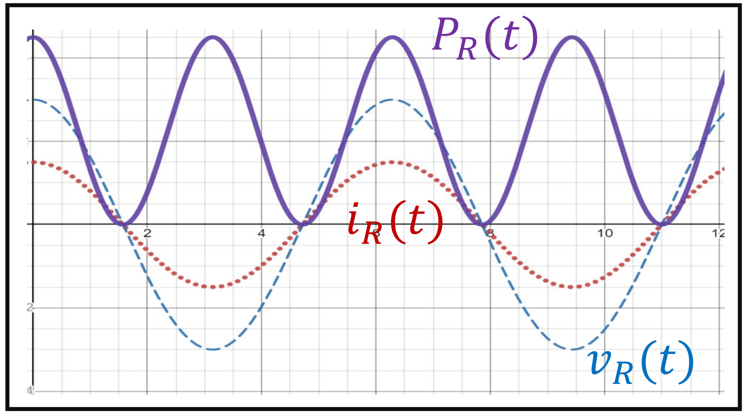
Power dissipation in AC circuits (Ch 31.4)

$\Delta V_R = IR$

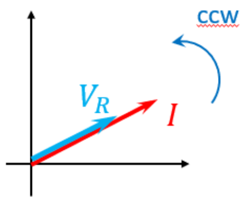
- From DC circuits: $P_R = IV = I^2 R = \dfrac{V^2}{R}$
- For AC circuits: $\langle P \rangle = \langle i(t)v(t) \rangle_T$ (average over a period)

pure R



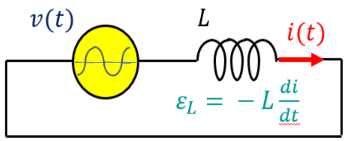


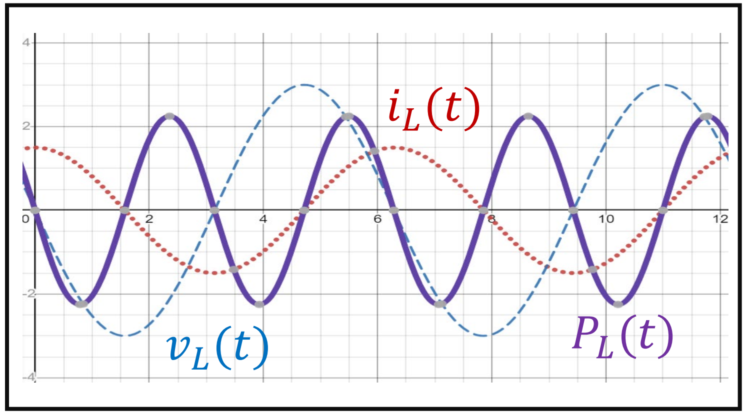
- Averages to something!



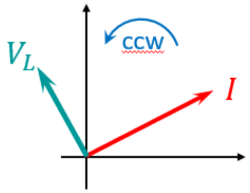
$$\langle P_R \rangle \propto \frac{1}{T}\int_0^T \cos(x)\cos(x)dx$$

pure L



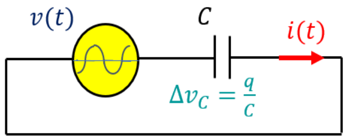


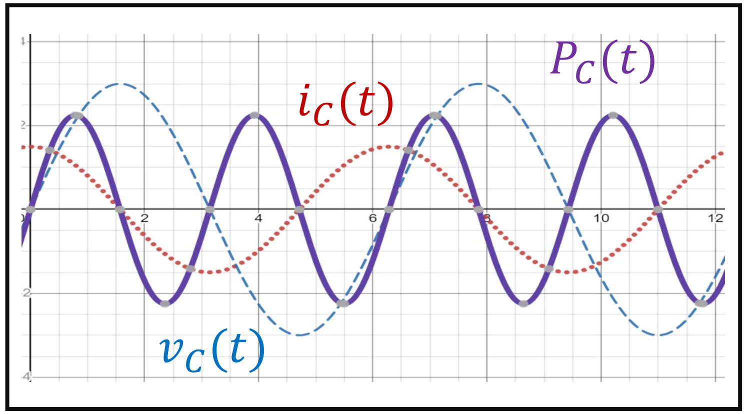
- Averages to zero!



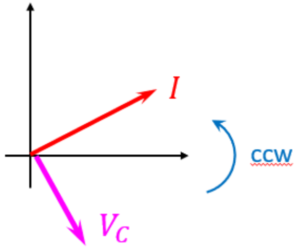
$$\langle P_L \rangle \propto \frac{1}{T}\int_0^T \cos(x)\cos(x-\frac{\pi}{2})\,dx$$

pure C





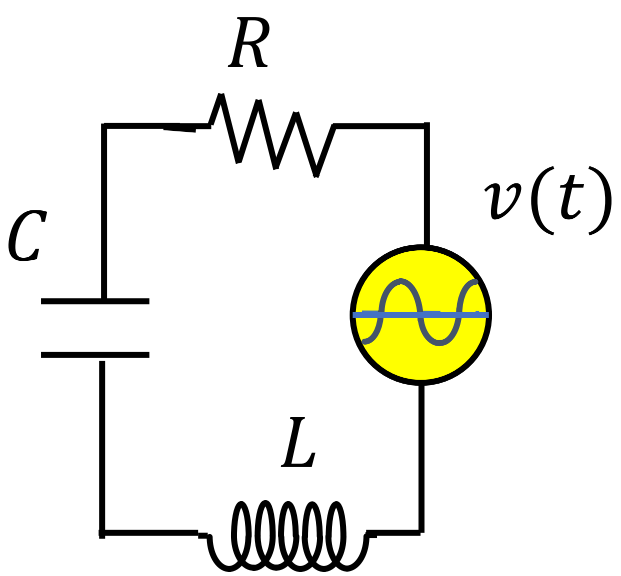
- Averages to zero!



$$\langle P_C \rangle \propto \frac{1}{T}\int_0^T \cos(x)\cos(x+\frac{\pi}{2})\,dx$$

Power dissipation in AC Series circuits

- Q: In which of these elements can the power be dissipated?

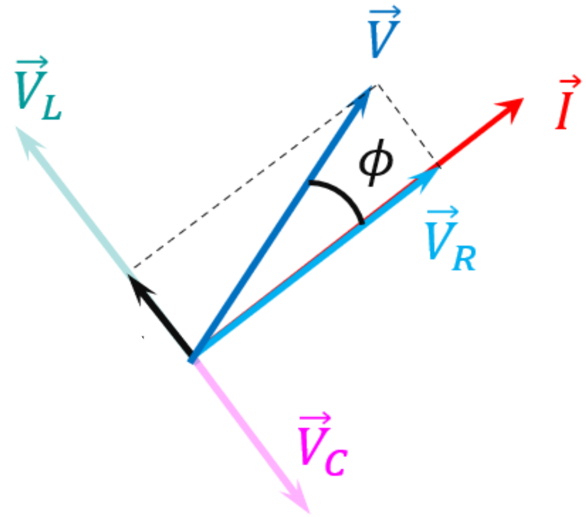


- A: Only in the resistor! The voltage phasors for R, L and C are in phase with / $\frac{\pi}{2}$ ahead / $\frac{\pi}{2}$ behind the current => the average over the period will give something / zero / zero.

$$\langle P_{RLC}\rangle = \frac{1}{T}\int_0^T v(t)\, i(t)\, dt \qquad v(t) = V_{peak}\cos(\omega t) \qquad i(t) = I_{peak}\cos(\omega t - \phi)$$

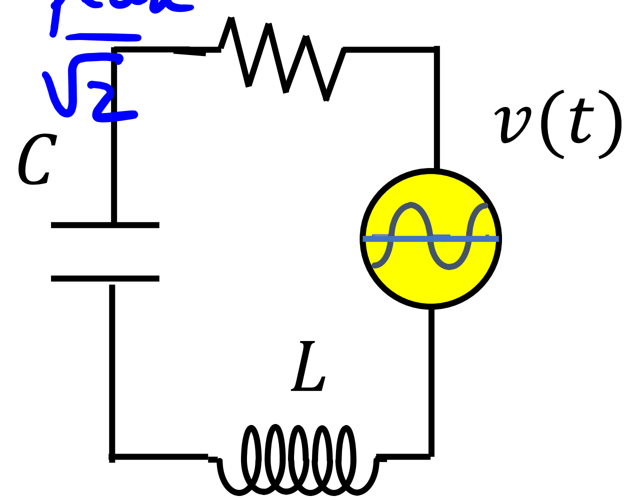
$$\langle P_{RLC}\rangle = \frac{V_{peak} I_{peak}}{T}\int_0^T \cos(\omega t)\cos(\omega t - \phi)\, dt =$$

$$= \frac{V_{peak} I_{peak}}{T}\cdot\frac{1}{2}\int_0^T (\underbrace{\cos(2\omega t - \phi)}_{0} + \underbrace{\cos\phi}_{\text{const}})dt = \frac{V_{peak}\, I_{peak}}{2}\cos\phi$$



Power dissipation in AC Series circuits

$V_{rms} = \frac{V_{peak}}{\sqrt{2}}$



$$\langle P_{RLC} \rangle = \frac{V_{peak}\, I_{peak}}{2} \cos\phi = V_{rms} I_{rms} \cos\phi$$

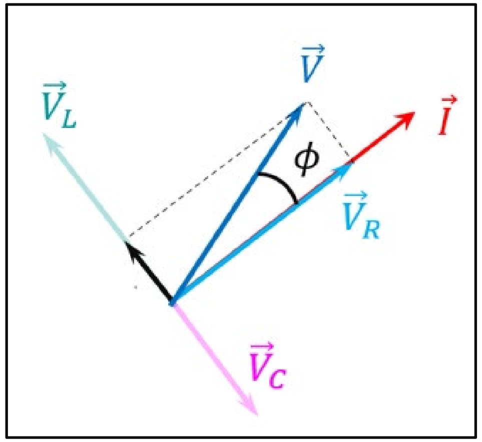
- $\cos\phi$: the power factor

- Note that $V_{peak} \cos\phi = V_R$

$V_R = IR$

$$\langle P_{RLC} \rangle = \frac{V_R\, I_{peak}}{2} = \frac{I_{peak}^2 R}{2}$$

- ½ comes from averaging over the cycle, $V_R = I_{peak} R$ from the fact that all this power dissipates in the resistor

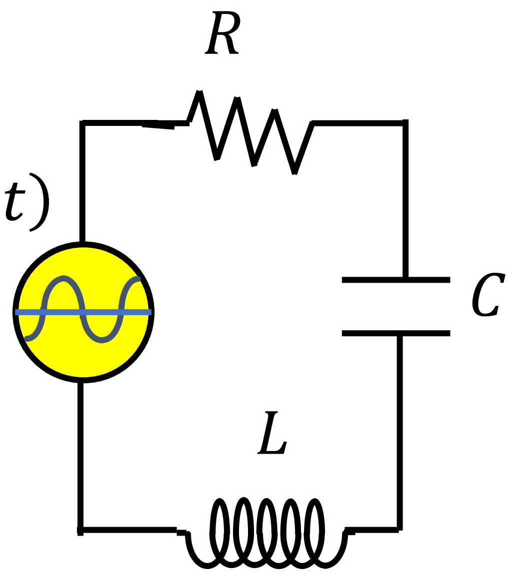


- L and C: modify voltage drop across R

Resonance in AC circuits - 1

- Imagine that the frequency ω of the voltage source V_{max} is changed smoothly.

$$v(t) = V_{max}\cos(\omega t)$$



- What happens to the current in an RLC circuit?

$$I = \frac{V_{source}}{Z} = \frac{V_{source}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

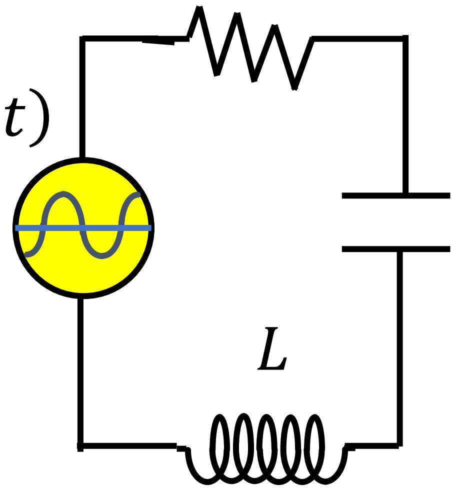
- Current as a function of ω:

$$I(\omega) = \frac{V_{source}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

- First of all, let's look at the limiting cases, $\omega \to 0$ and $\omega \to \infty$.

Resonance in AC circuits - 2

$$v(t) = V_{max}\cos(\omega t)$$



- Current as a function of ω:

$$I(\omega) = \frac{V_{max}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

- $\omega \to 0$: $I(\omega) \to 0$
- $\omega \to \infty$: $I(\omega) \to 0$

- Since $I = 0$ at both limits, it suggests that it must have a maximum somewhere in between!
- Maximum: $(X_L = \omega L) = (X_C = 1/\omega C)$ (the bracket in the denominator disappears)
- Hence, the maximum is reached at:

$$\boxed{\omega = \omega_0 = \frac{1}{\sqrt{LC}}}$$

- Resonance frequency

- At maximum:

$$\boxed{I(\omega) = I_{max} = \frac{V_{max}}{R}}$$

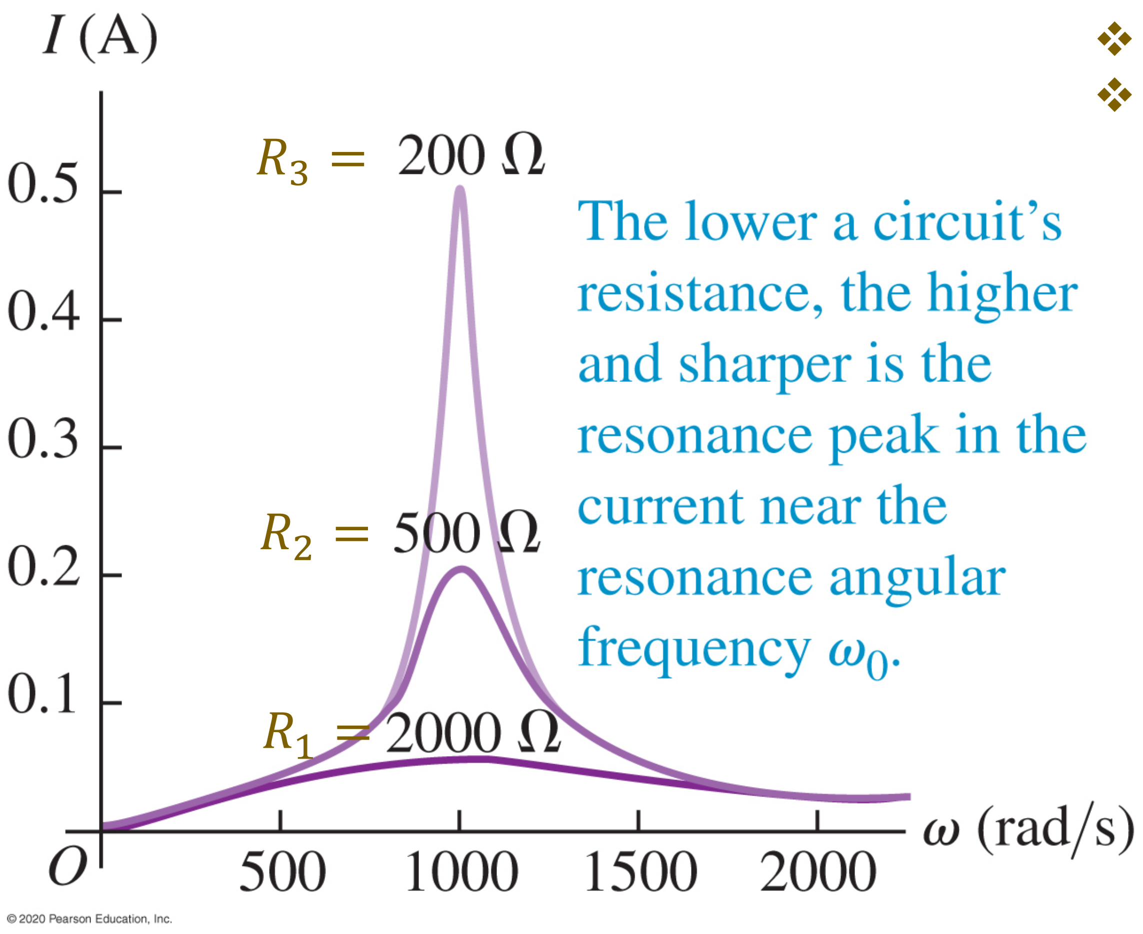
Resonance in AC circuits - 3

- $V_{max} = 100\ V$
- $L = 2\ H$
- $C = 0.5\ \mu F$

$$I(\omega) = \frac{V_{max}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$I_{max} = \frac{V_{max}}{R}$$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$



The lower a circuit's resistance, the higher and sharper is the resonance peak in the current near the resonance angular frequency ω_0.

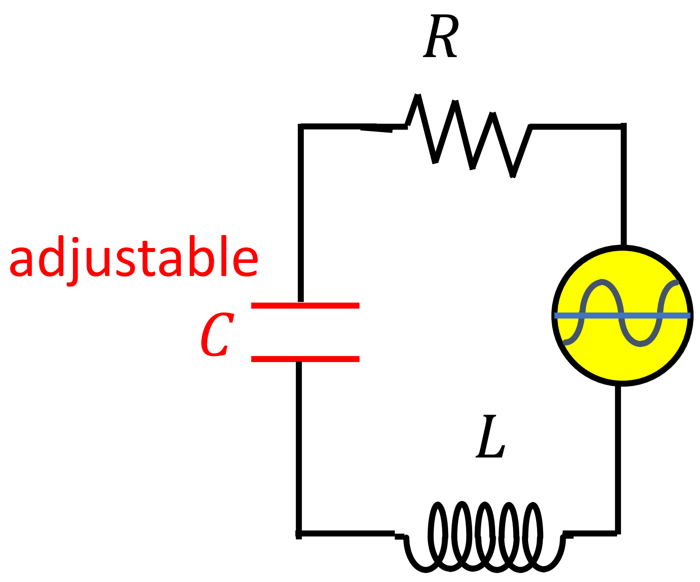
- Q: What does the expression for ω_0 remind you of?
- A: It's the natural frequency of oscillations in an LC-circuit!

Application of resonance in AC circuits

f

Q: An AM radio station picks up a 1 MHz signal with a peak voltage of 5 mV. The tuning circuit consists of a 60 μH inductor in series with a variable capacitor C. The circuit has a small resistance of 0.25Ω.

To what value should the capacitor be tuned to listen to this AM radio station? Pick the closest answer.



A. 1.5×10^{-2} F
B. 2.7×10^{-3} F
C. 5.0×10^{-6} F
D. 1.7×10^{-8} F
E. 4.0×10^{-10} F

$$2\pi f = \omega_0 = 1/\sqrt{LC}$$

Application of resonance in AC circuits

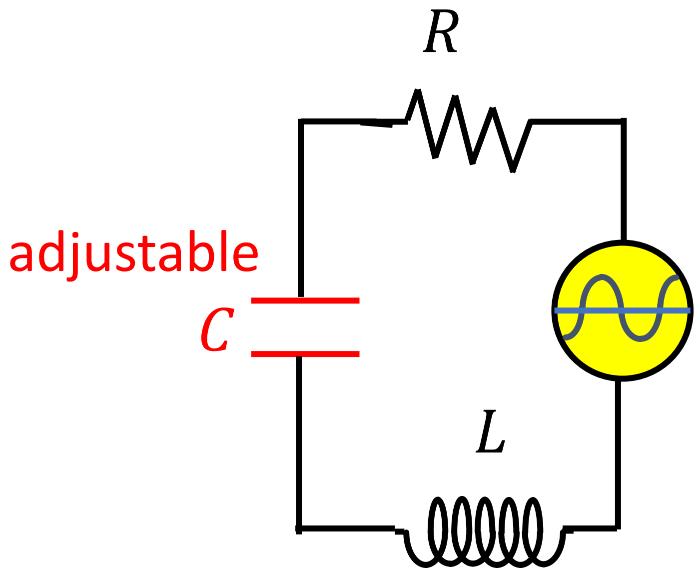
Q: An AM radio station picks up a 1 MHz signal with a peak voltage of 5 mV. The tuning circuit consists of a 60 μH inductor in series with a variable capacitor C. The circuit has a small resistance of 0.25Ω.

To what value should the capacitor be tuned to listen to this AM radio station? Pick the closest answer.

- $\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi\, f_{\text{station}}$

- $C = \frac{1}{(2\pi f_{\text{station}})^2\, L} = \frac{1}{(2\pi \cdot 10^6)^2\,(60 \cdot 10^{-6})}$

$= 4.2 \cdot 10^{-10} = 420\, pF$



A. 1.5×10^{-2} F
B. 2.7×10^{-3} F
C. 5.0×10^{-6} F
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E. 4.0×10^{-10} F

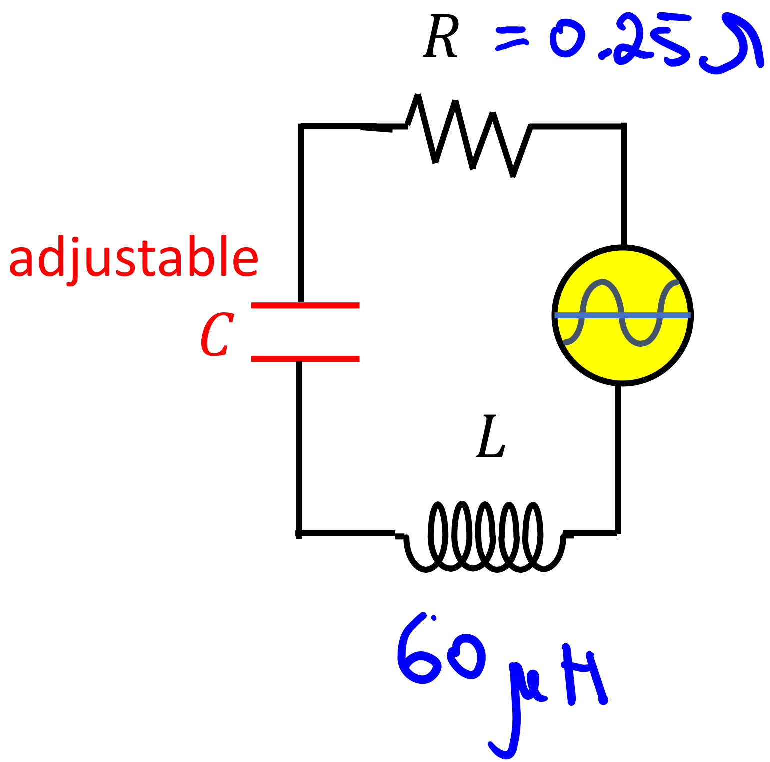
Application of resonance in AC circuits

Q: An AM radio station picks up a 1 MHz signal with a peak voltage of 5 mV. The tuning circuit consists of a 60 μH inductor in series with a variable capacitor C. The circuit has a small resistance of 0.25Ω.

What is the peak current through this circuit on resonance? Pick the closest answer.

$C = 420\ pF$

@ Res: $I_{peak} = \frac{V_{peak}}{R}$



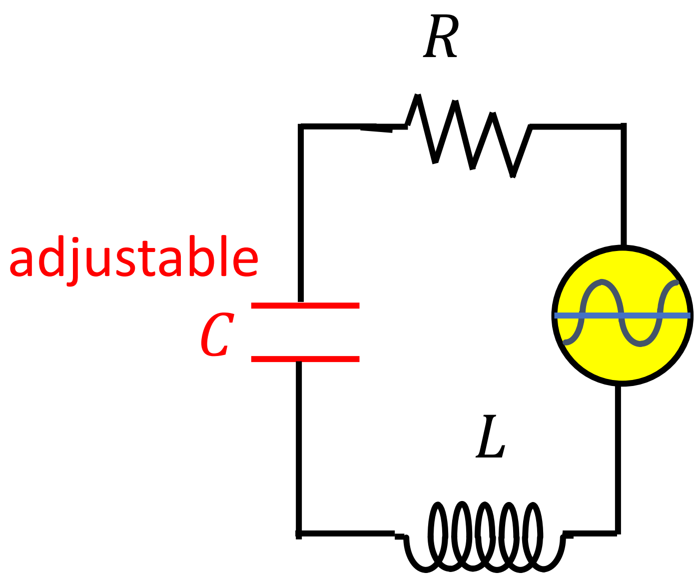
A. 1 mA
B. 20 mA
C. 50 mA
D. 100 mA
E. 350 mA

Application of resonance in AC circuits

Q: An AM radio station picks up a 1 MHz signal with a peak voltage of 5 mV. The tuning circuit consists of a 60 μH inductor in series with a variable capacitor C. The circuit has a small resistance of 0.25Ω.

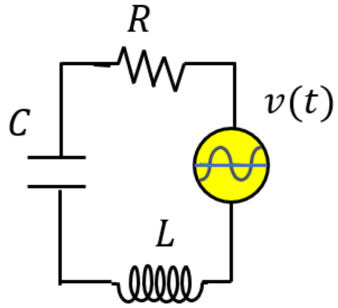
What is the peak current through this circuit on resonance? Pick the closest answer.

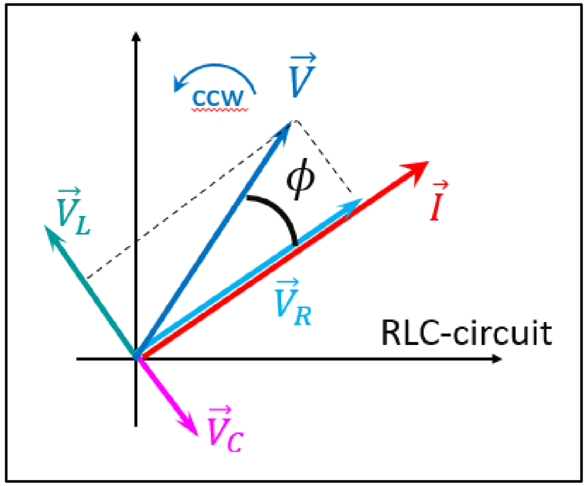
- $I_{max} = \frac{V_{max}}{R} = \frac{5\ mV}{0.25\ \Omega} = 0.02\ A = 20\ mA$



A. 1 mA
B. 20 mA
C. 50 mA
D. 100 mA
E. 350 mA

AC RLC series circuit: Summary





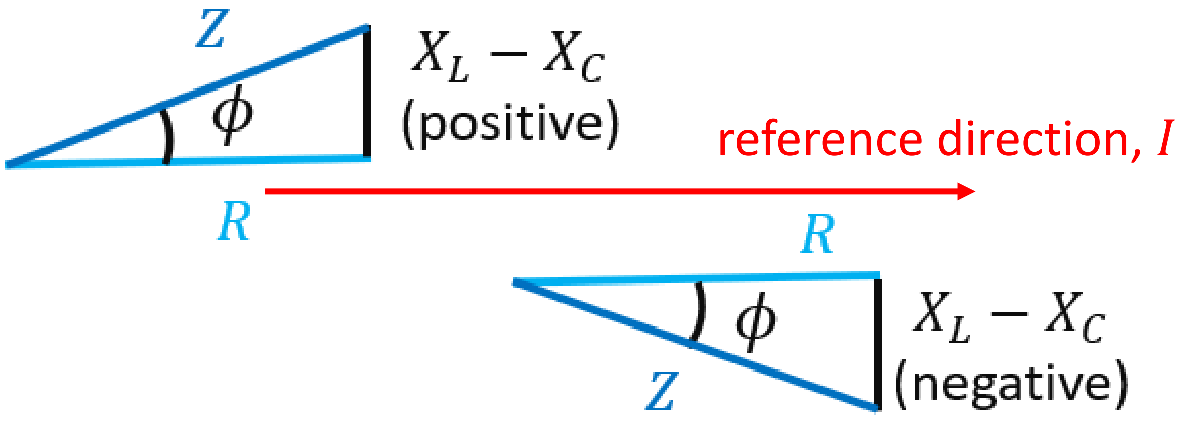
$$X_R = R$$

$$X_L = \omega L$$

$$X_C = 1/\omega C$$

$$V_{max} = I_{max}\, Z$$

- Compute X_L, X_C
- Set up impedance triangle
  - Find the impedance Z
  - Find the phase shift ϕ between V and



- Relations between voltages and current:
  - $V_{max} = I_{max} Z$
  - $V_R = I_{max}\, R = V_{max}\, \frac{R}{Z}$
  - $V_L = I_{max}\, X_L = V_{max}\, \frac{X_L}{Z}$
  - $V_C = I_{max}\, X_C = V_{max}\, \frac{X_C}{Z}$

- Note that the phasor diagram looks this way because the current I is common for all three elements (series)

- What if a circuit is parallel ??

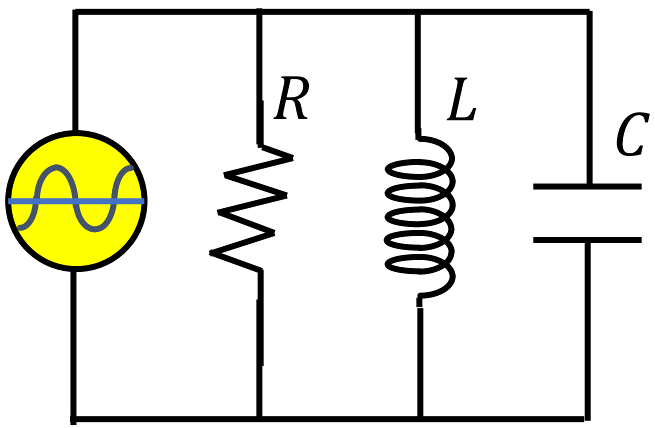
AC RLC parallel circuit

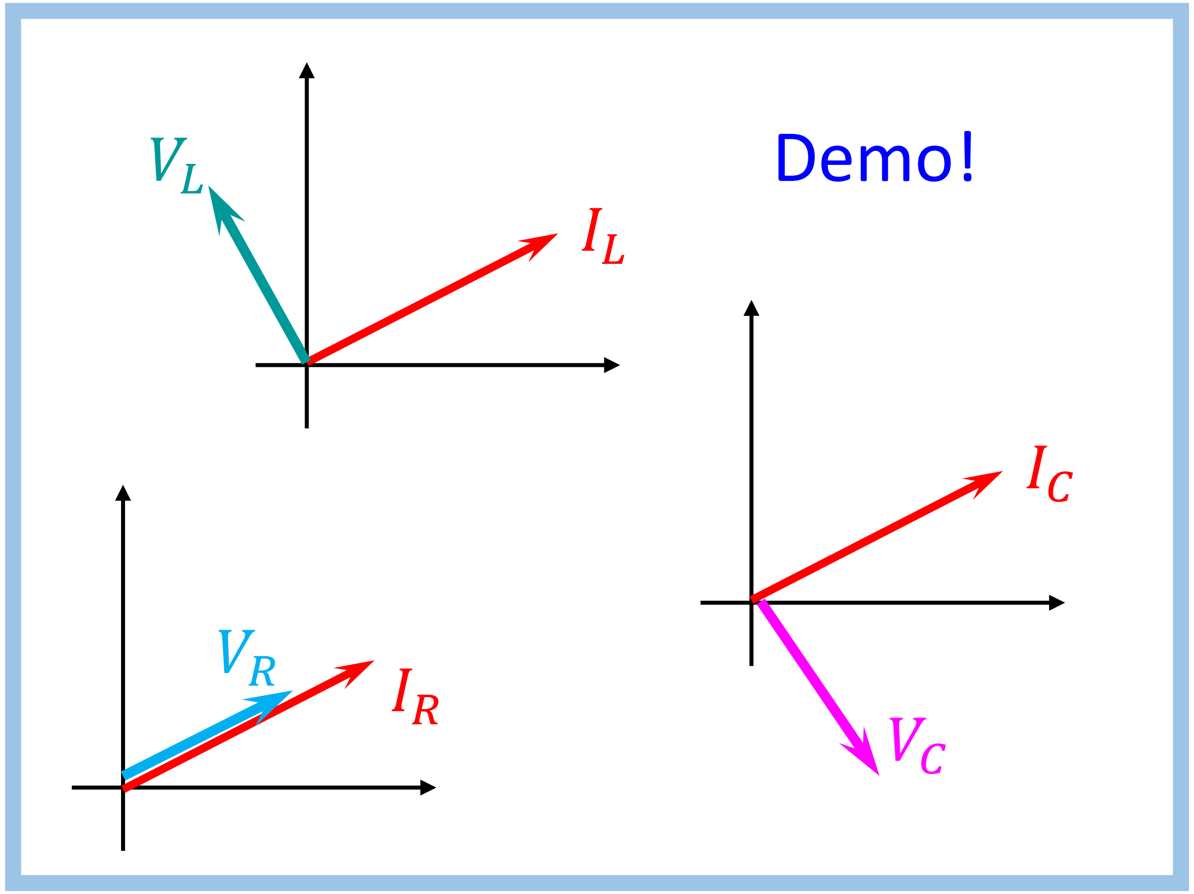
- Now voltage across each parallel element is the same!

- Series: $v(t) = v_R(t) + v_L(t) + v_C(t)$ – hence we were adding up voltages phase shifted with respect to common current.

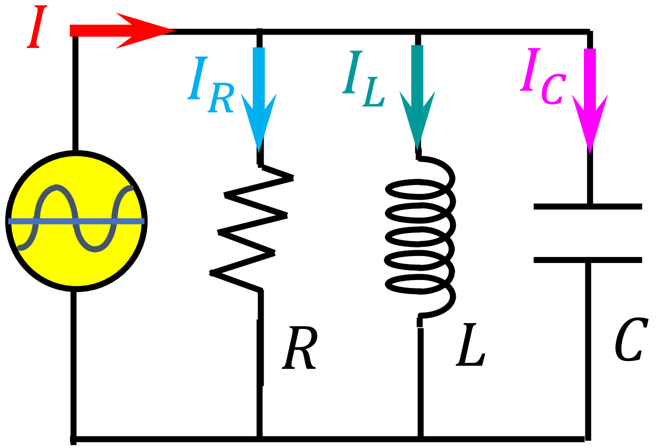
- Parallel: $i(t) = i_R(t) + i_L(t) + i_C(t)$ – hence we will be adding up currents phase shifted with respect to common voltage!

- Q: What the outcome would be?

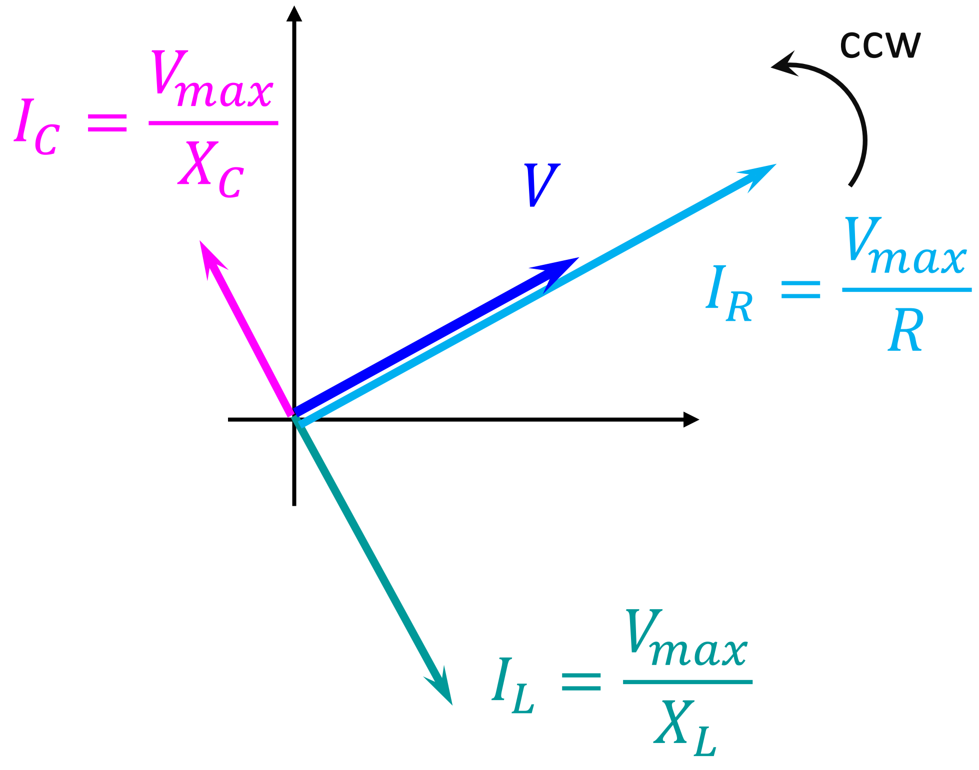




AC RLC parallel circuit

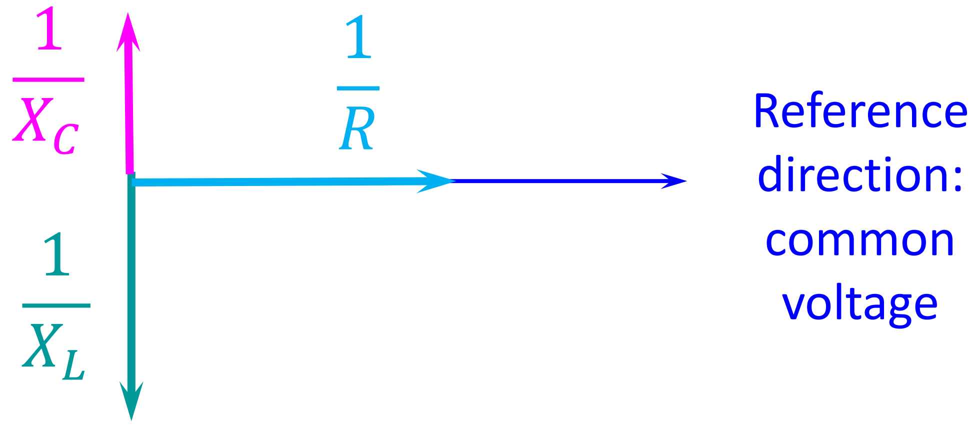


- Now let's combine the elementary phasor pairs, taking into account that they have common voltage (same magnitude and direction of $\vec{V}$):



$$\vec{I} = \vec{I}_R + \vec{I}_L + \vec{I}_C$$

- Rescale (divide by common V_{max}), rotate:



- Build the impedance triangle from reciprocal resistances!