Lecture 2.

Ohm’s law (continued).
Equivalent resistance: Series & Parallel.
Kirchhoff’s laws (loop & junction).

Q: What's your preference for TA-led homework help sessions (5-6 pm, 3 per week)?

A. One online, two in-person 46%

B. ~~Two~~ in-person, ~~one~~ online 54%
One two

Last Time:

Resistance – intrinsic property of any conductor

Origin: scattering of charge carriers on thermal fluctuations of atoms of the resistor)

Ohm's law

$$\boxed{\Delta V_R = IR}$$

Note: No current – no voltage drop!

$$\boxed{\Delta V_R = IR}$$

Ohm's law

$R = 0\ \Omega$

Q: What is the voltage drop across an ideal ($R = 0$) wire?

A. Zero (it is all under the same voltage)
B. Depends on the current running through it
C. Not sure

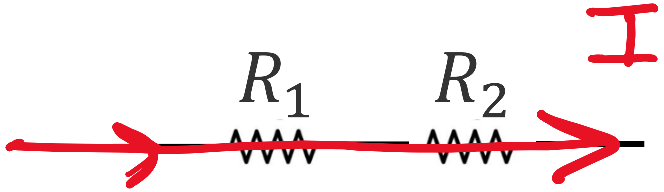
Ohm's law and Equivalent resistance

$$\boxed{\Delta V_R = IR}$$

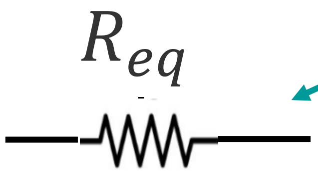
Ohm's law is applicable to one resistor at a time: voltage drop across a resistor is equal to current through this resistor times its resistance.

- Q: What to do if a circuit has more than one resistor?
- A: You can calculate an "equivalent resistance" for a combination of resistors. By doing this, you mentally replace many resistors by one "equivalent" resistor => you can apply Ohm's law to this one "equivalent" resistor

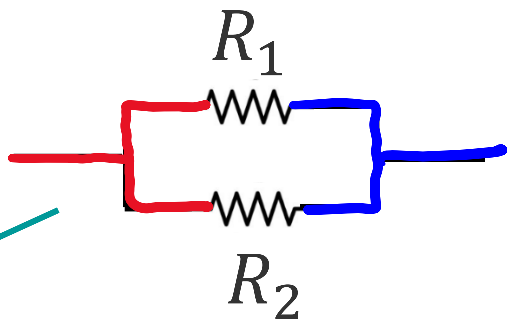
Resistors "in series":



$$\boxed{R_{eq,s} = R_1 + R_2}$$



Resistors "in parallel":



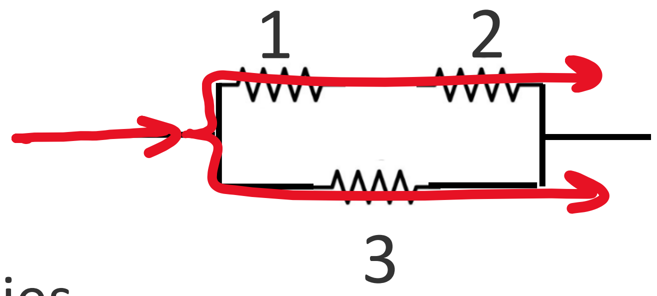
$$R_{eq,p} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\boxed{\frac{1}{R_{eq,p}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

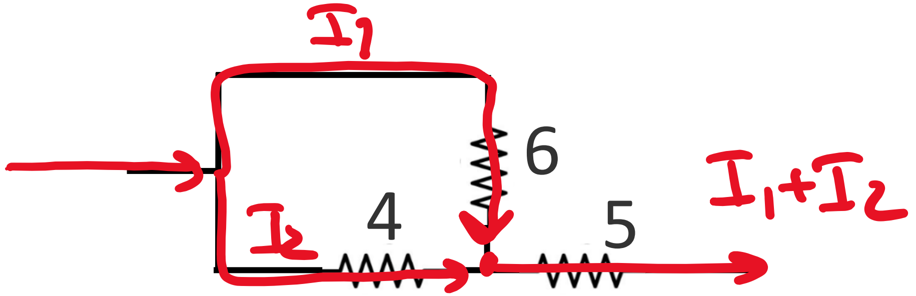
To calculate the equivalent resistance of complex circuit correctly, you need to understand well which resistors are in series, which are in parallel, and which are neither in series nor in parallel.

Resistors "in series":

" X and Y are in series":
same current through them.

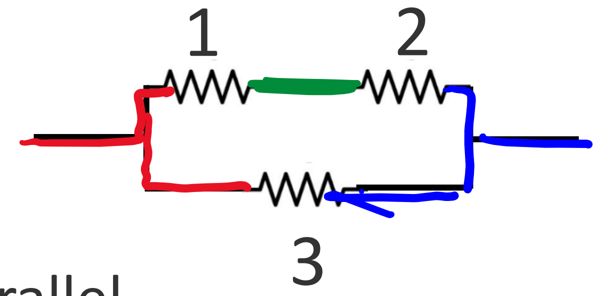


- 1 and 2 are in series
- 4 and 5 are not in series

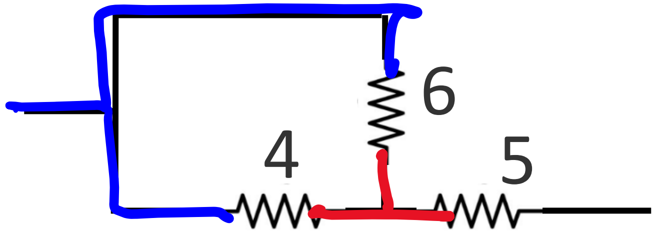


Resistors "in parallel":

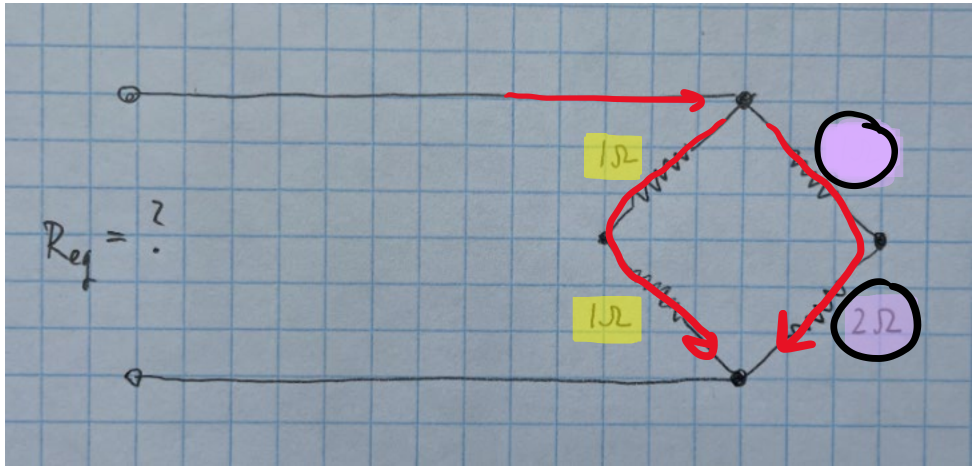
" X and Y are in parallel":
same voltage across them.

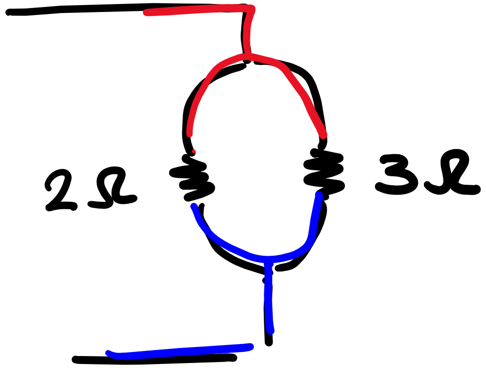


- 1 and 3 are not in parallel
- 4 and 6 are in parallel



Q: What is the equivalent resistance of this circuit?





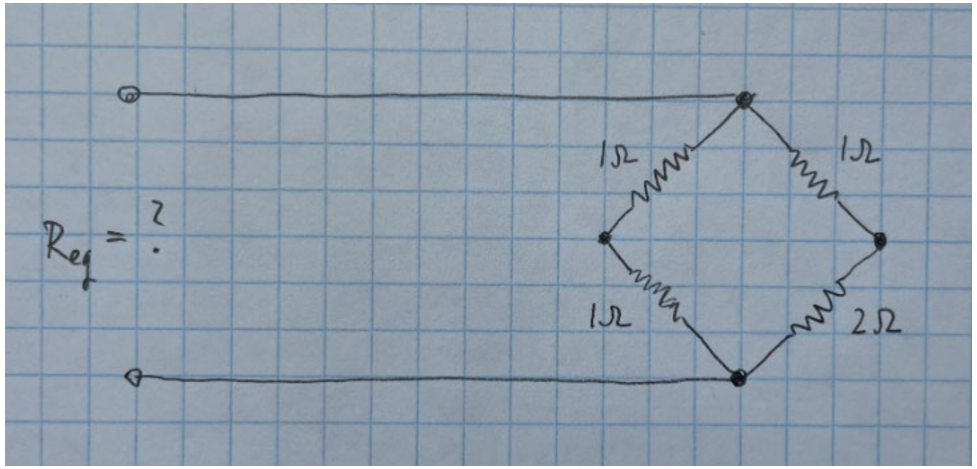
$\rightarrow R_{eq} = \frac{2\cdot 3}{2+3}$

A. 5/6 Ω
B. 6/5 Ω
C. 6/11 Ω
D. 13/6 Ω
E. 13/11 Ω

$\rightarrow$ $\boxed{R_{eq,s} = R_1 + R_2}$

$$\boxed{\frac{1}{R_{eq,p}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

Q: What is the equivalent resistance of this circuit?



Here we have two combinations connected in parallel, and each of these combinations consists of two resistors connected in series.

$$\frac{1}{R_{eq}} = \frac{1}{(1\Omega + 1\Omega)} + \frac{1}{(1\Omega + 2\Omega)} = \frac{5}{6}$$

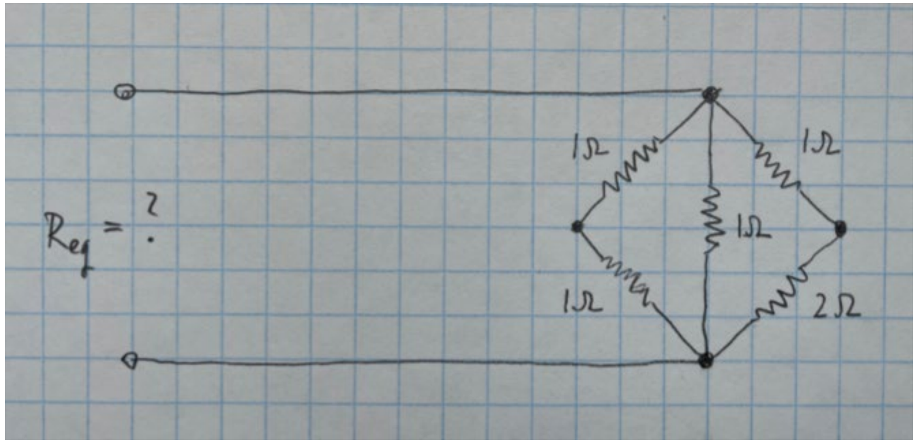
A. 5/6 Ω
B. 6/5 Ω
C. 6/11 Ω
D. 13/6 Ω
E. 13/11 Ω

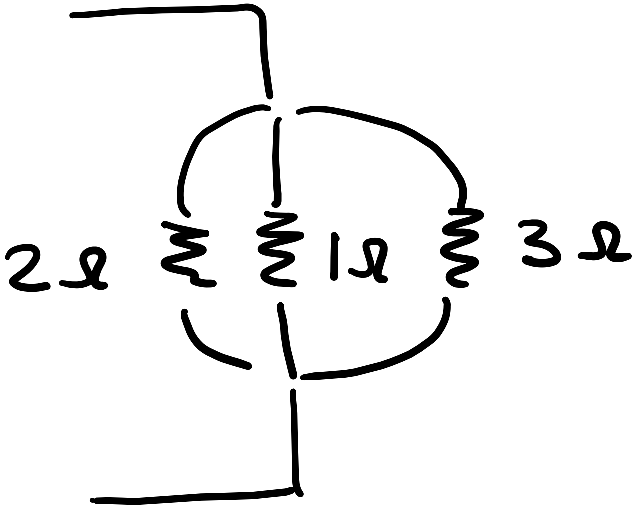
$$R_{eq} = \frac{6}{5}$$

$$\boxed{R_{eq,s} = R_1 + R_2}$$

$$\boxed{\frac{1}{R_{eq,p}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

Q: What is the equivalent resistance of this circuit?





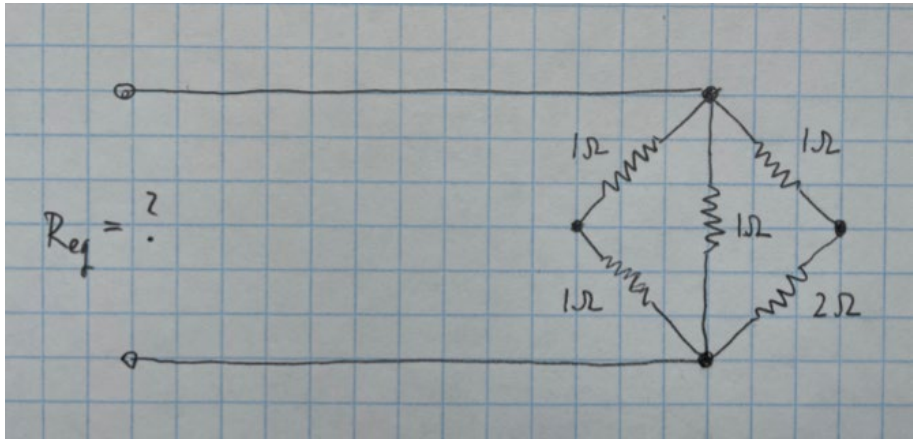
$$\frac{1}{R_{eq,p}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

A. 5/6 Ω
B. 6/5 Ω
C. 6/11 Ω
D. 13/6 Ω
E. 13/11 Ω

$$\boxed{R_{eq,s} = R_1 + R_2}$$

$$\boxed{\frac{1}{R_{eq,p}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

Q: What is the equivalent resistance of this circuit?



Here we have <u>three</u> units connected in parallel:

- a 1Ω resistor
- a combinations of two resistors (1Ω and 1Ω) connected in series
- a combinations of two resistors (1Ω and 2Ω) connected in series

$$\frac{1}{R_{eq}} = \frac{1}{(1\Omega + 1\Omega)} + \frac{1}{(1\Omega)} + \frac{1}{(1\Omega + 2\Omega)} = \frac{11}{6}$$

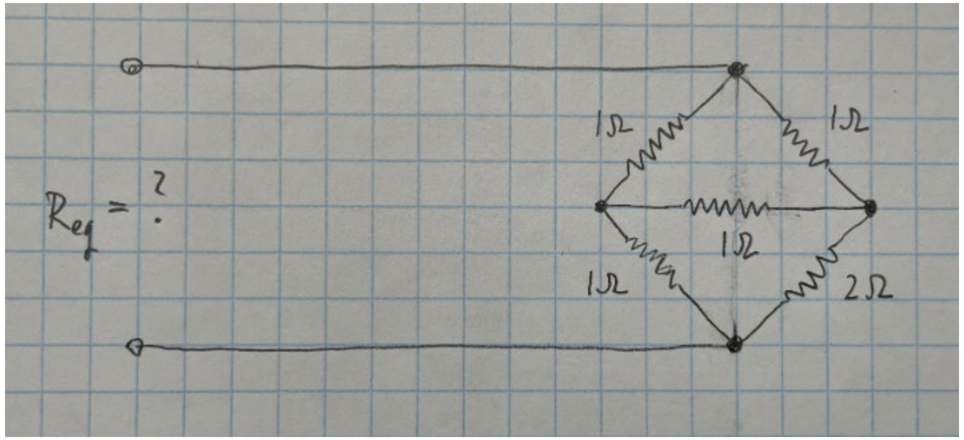
A. 5/6 Ω
B. 6/5 Ω
C. 6/11 Ω
D. 13/6 Ω
E. 13/11 Ω

$$R_{eq} = \frac{6}{11}$$

$$\boxed{R_{eq,s} = R_1 + R_2}$$

$$\boxed{\frac{1}{R_{eq,p}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

Q: What is the equivalent resistance of this circuit?

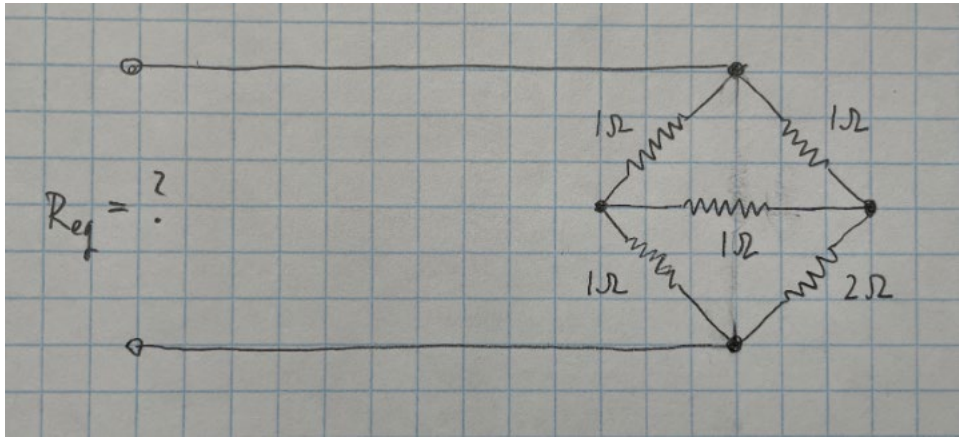


A. 5/6 Ω
B. 6/5 Ω
C. 6/11 Ω
D. 13/6 Ω
E. 13/11 Ω

$$\boxed{R_{eq,s} = R_1 + R_2}$$

$$\boxed{\frac{1}{R_{eq,p}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

Q: What is the equivalent resistance of this circuit?



This is an example of a circuit where simple series/parallel rules are not useful…

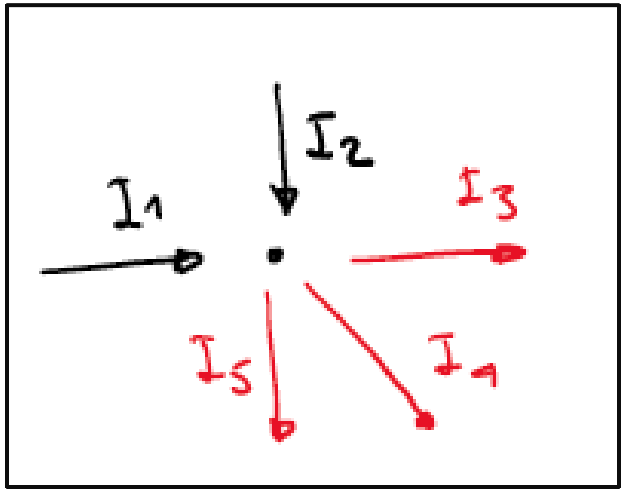
A. 5/6 Ω
B. 6/5 Ω
C. 6/11 Ω
D. 13/6 Ω
E. 13/11 Ω ←

We will see using the Kirchhoff laws that this is the right answer!

$$\boxed{R_{eq,s} = R_1 + R_2}$$

$$\boxed{\frac{1}{R_{eq,p}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

- Kirchhoff’s junction law (K1)

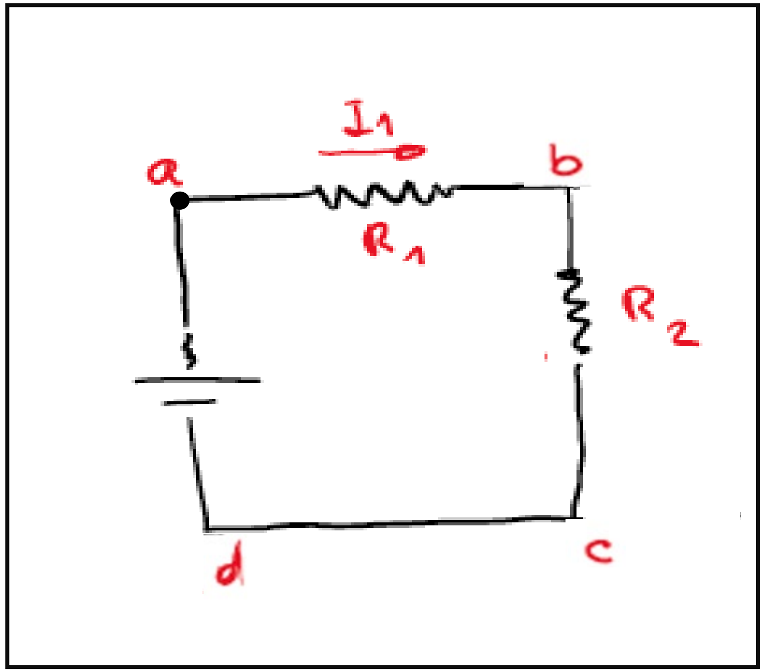


Related to conservation of charge

$$\boxed{\sum I_{in} = \sum I_{out}}$$

- Kirchhoff’s loop law (K2)

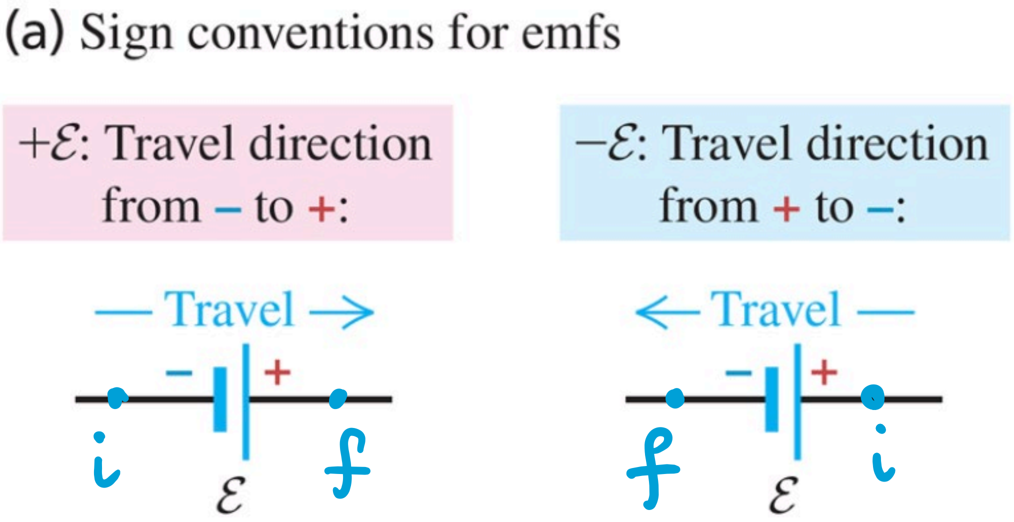
Going over a loop, you come back to the same voltage you have started with

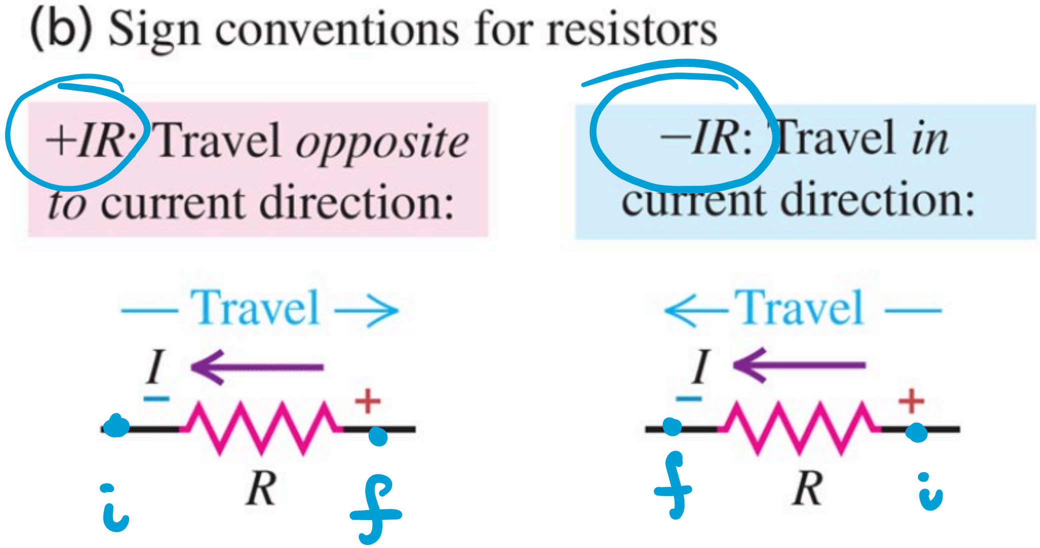


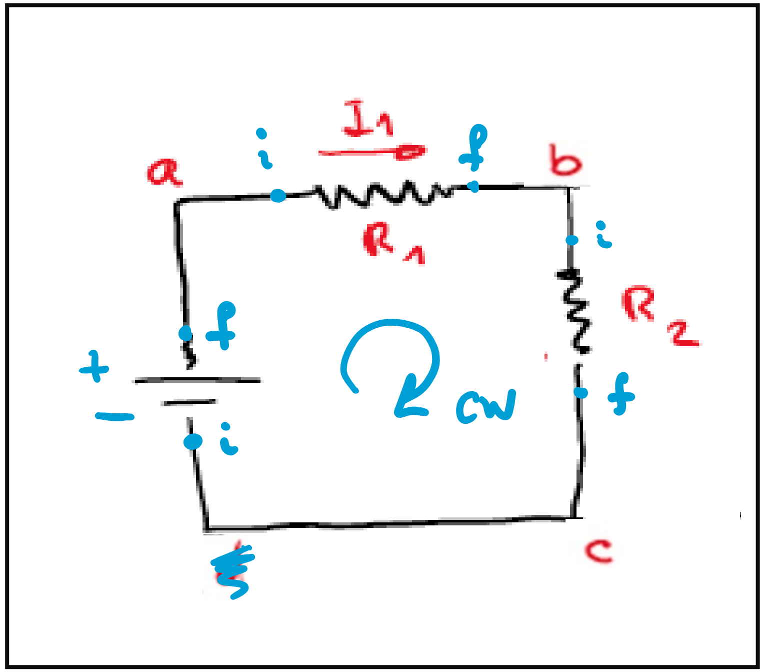
$$\boxed{\sum_{\text{loop}} \Delta V = 0}$$

$$\sum_{\text{loop}} \Delta V = \Delta V_{ab} + \Delta V_{bc} + \overbrace{\Delta V_{cd}}^{=0 \text{ (wire!)}} + \Delta V_{da} = 0$$

- Kirchhoff's loop law (K2): Sign convention







$$\sum_{\text{loop}} \Delta V = \Delta V_{ab} + \Delta V_{bc} + \Delta V_{ca} = 0$$

$$-IR_1 \; - \; IR_2 \; + \; \varepsilon \; = 0$$

(traveling clockwise)

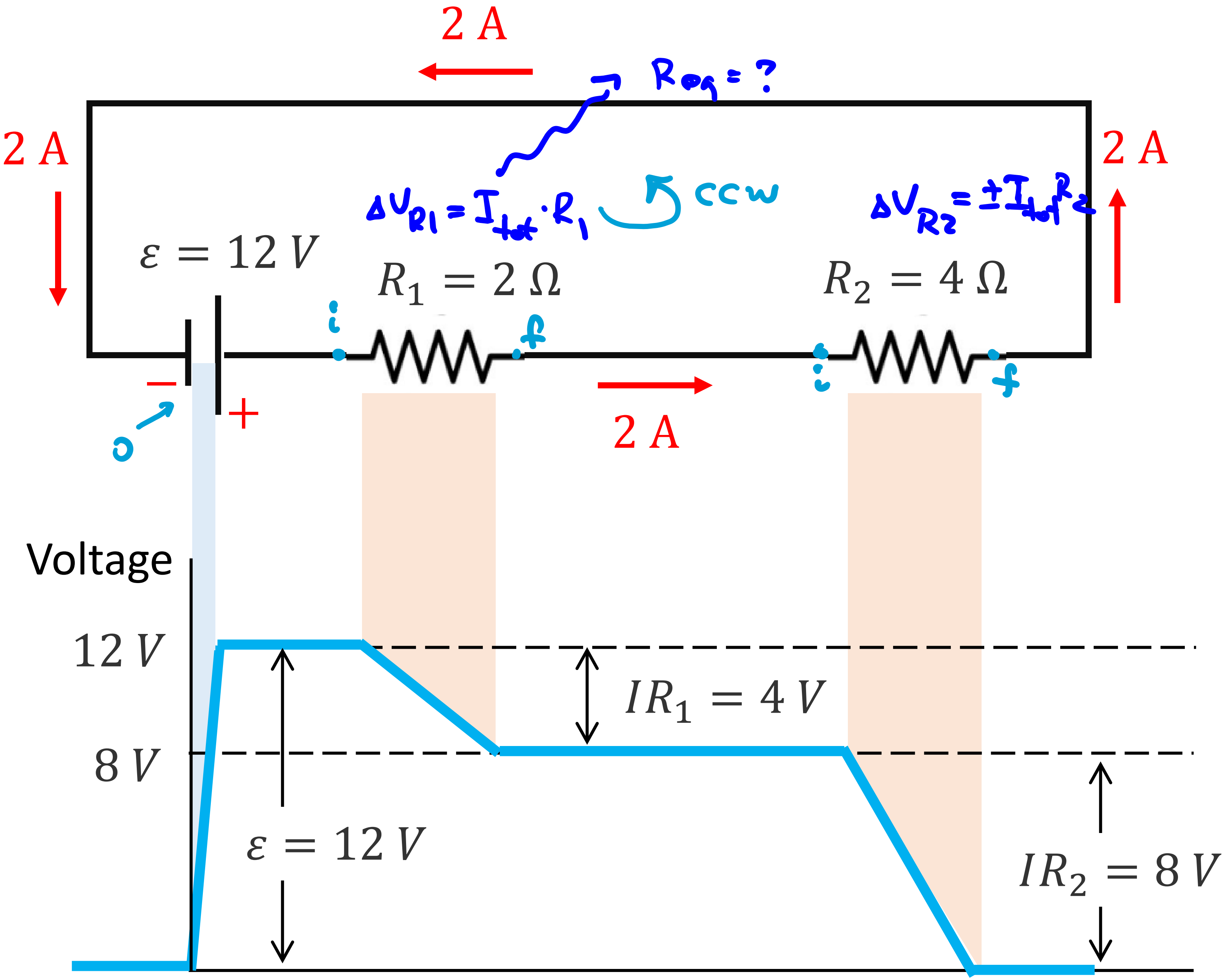
K2: Voltage drops across a loop

$$R_{eq} = R_1 + R_2 = 6\ \Omega$$

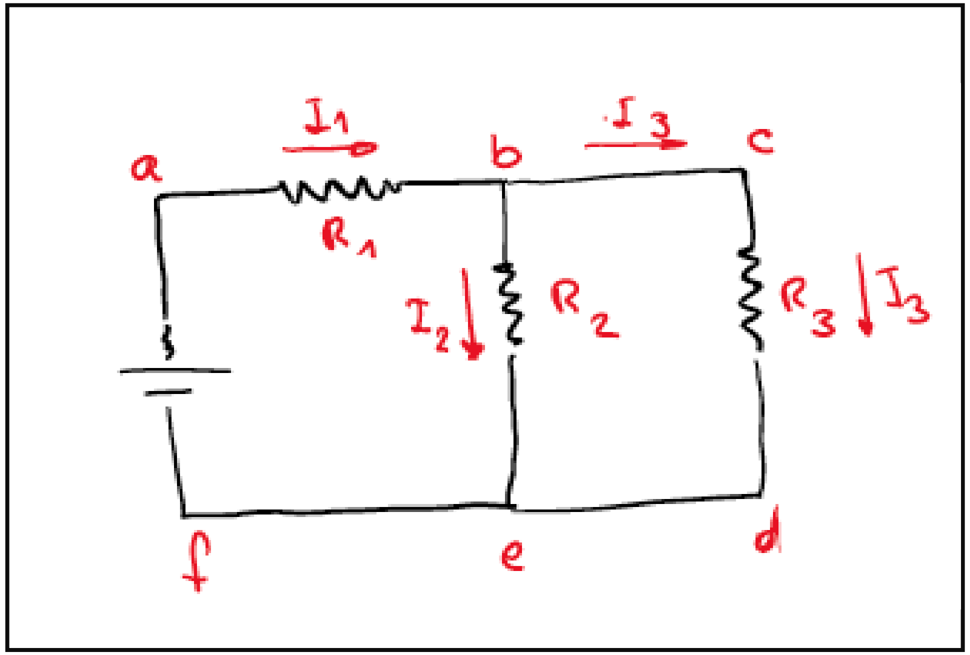
$$I = \frac{\varepsilon}{R_{eq}} = \frac{12\ V}{6\ \Omega} = 2\ A$$

$$\Delta V_{R1} = \pm I R_1 = -4\ V$$

$$\Delta V_{R2} = \pm I R_2 = -8\ V$$



- In circuits with more than one loop, we can combine K1 and K2



Q: write down Kirchhoff's laws to solve for I_1, I_2, I_3

K1: $I_1 = I_2 + I_3$ (1)

K2: $\sum \Delta V_{loop} = 0$

$$\sum I_{in} = \sum I_{out}$$

$$\sum_{\text{loop}} \Delta V = 0$$

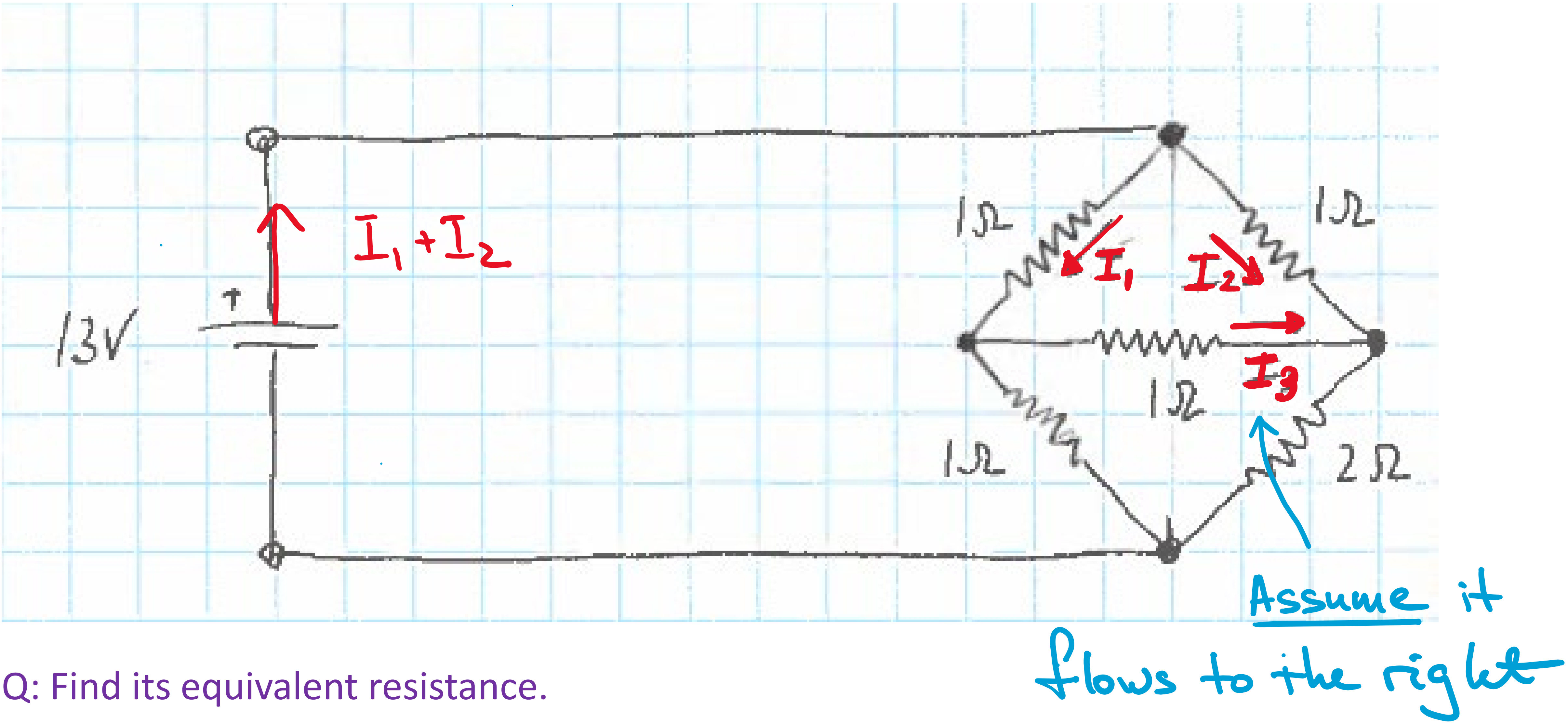
K2, right loop: $\sum_{b,c,d,e,b} \Delta V = -I_3 R_3 + I_2 R_2 = 0$ (2)

K2, left loop: $\sum_{f,a,b,e,f} = \varepsilon - I_1 R_1 - I_2 R_2 = 0$ (3)

K2, big loop: $\sum_{f,a,b,c,d,e,f} = \varepsilon - I_1 R_1 - I_3 R_3 = 0$

Practice

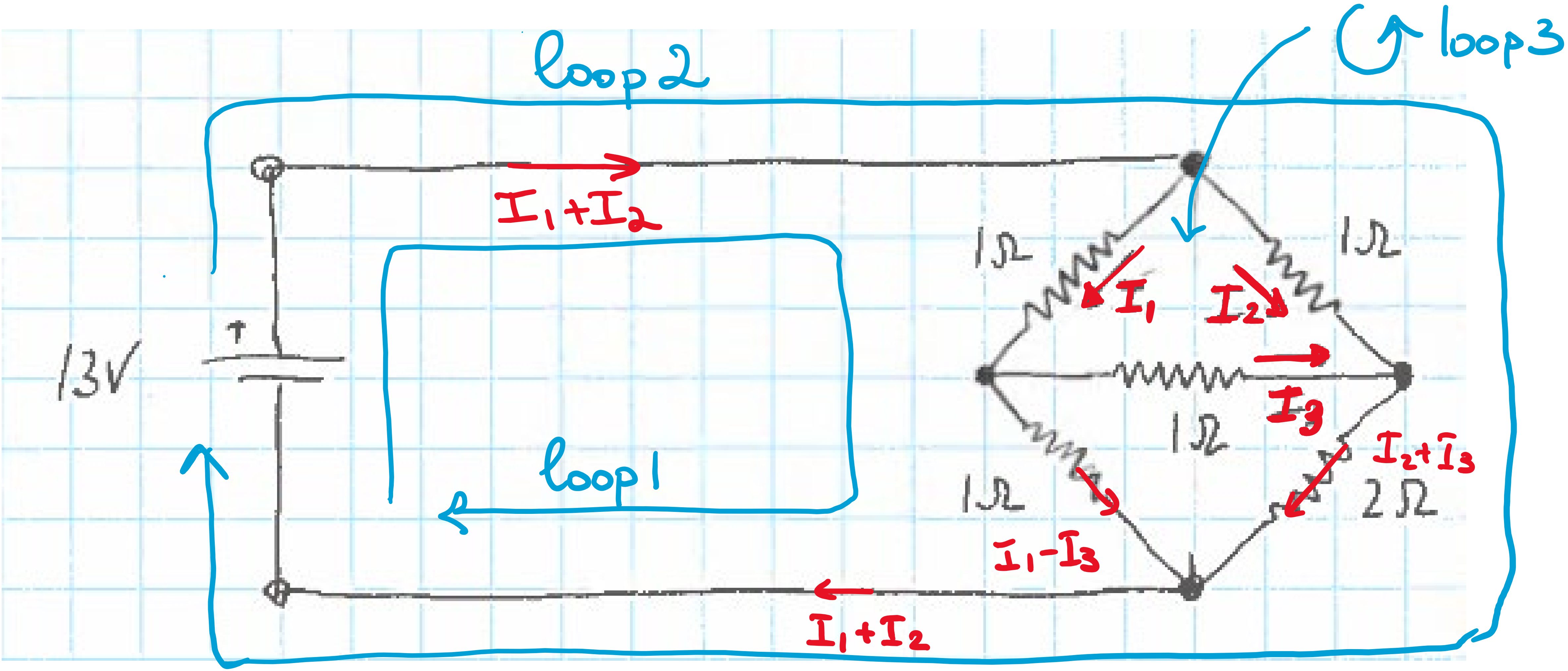
Q: Write down Kirchhoff's loop equations to solve for currents in all parts of the circuit



Q: Find its equivalent resistance.

Practice

Q: Write down Kirchhoff’s loop equations to solve for currents in all parts of the circuit



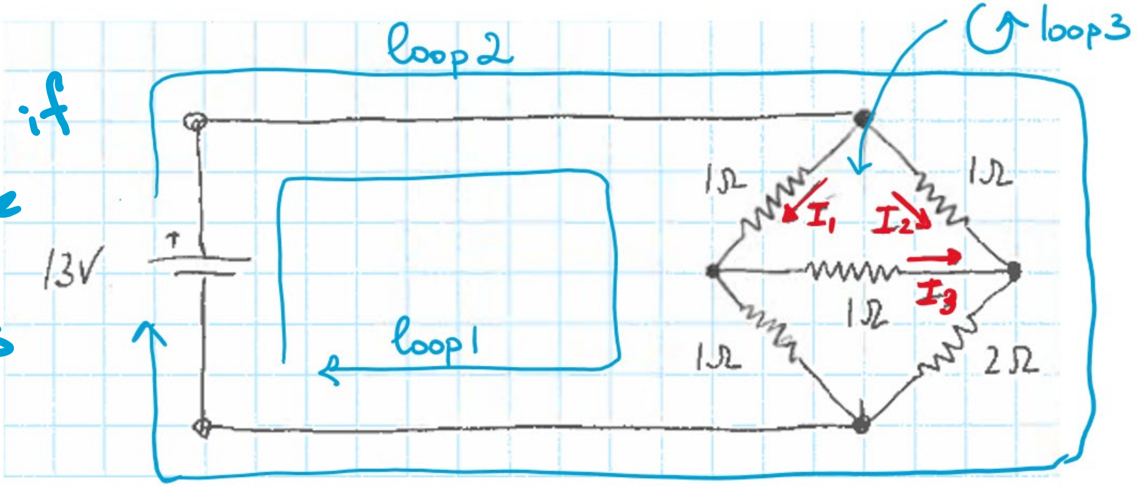
Q: Find its equivalent resistance.

Practice

Q: Write down Kirchhoff's loop equations to solve for I_1, I_2, I_3

Q: Find equivalent resistance.

Check text example 26.6 if you are not sure about how to solve this system of equations



- Loop 1: $13\,V - I_1(1\Omega) - (I_1 - I_3)(1\Omega) = 0$
- Loop 2: $13\,V - I_2(1\Omega) - (I_2 + I_3)(2\Omega) = 0$
- Loop 3: $-I_1(1\Omega) - I_3(1\Omega) + I_2(1\Omega) = 0$

- Solution:

$I_1 = 6\,A$

$I_2 = 5\,A$

$I_3 = -1\,A.$

$I_{tot} = I_1 + I_2 = 11\,A$

1A to the left! (Initial assumption was wrong, but it's okay)

Q: Does it make sense or not?

- Equivalent resistance: $R_{eq} = \frac{\varepsilon}{I_{tot}} = \frac{\varepsilon}{I_1 + I_2} = \frac{13\,V}{11\,A} = \frac{13}{11}\,\Omega$