Lecture 7.

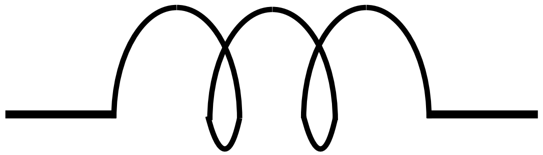
Inductors.
Circuits with inductors.

Time-dependent DC circuits with inductors

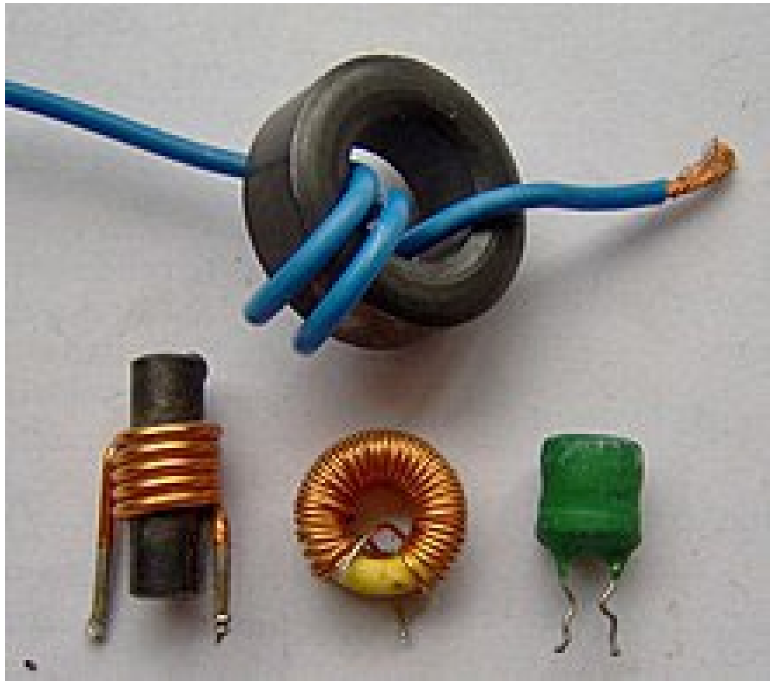
Text: 30.2-6

- Ch 30.2 – Inductors
- Ch 30.3 – Energy stored in inductors
- Ch 30.4 – RL circuits
- Ch 30.5 – LC circuits
- Ch 25.3 – RLC circuits

Inductors



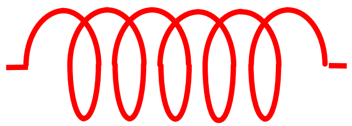
- An inductor is a passive electrical component that can store energy in the magnetic field inside the coil.
- Inductor acts as a stabilizer: an inductor opposes any change of current that flows through it (by creating “induced current”)



- Energy in a Capacitor is stored in the Electric Field
- Energy in an Inductor is stored in the Magnetic Field.

  - Later we will learn about a connection between electric current and magnetic field. Then we will understand that when the current changes, the magnetic flux through the inductor changes, too, and an induced current appears in response to that (Lenz’s law + Faraday’s law)

Inductance



- An inductor is a coil. Simply a coil of ideal wire.

- When the current flowing through it is stable (does not change), this coil is a "normal" piece of ideal wire. The voltage drop across it is equal to zero:

$$\Delta V_L \equiv 0$$

- However, when the current in an inductor changes, it produces a voltage drop across it:

$$\Delta V_L = \varepsilon_L = -L\, di/dt \quad \text{(back emf)}$$

- Here L is called inductance. It depends on the geometry of the inductor (cf. capacitance of a capacitor)

[L] = Henry

- NOTE -- Inductors can act like a BATTERY or like a WIRE –

but not at the same time, of course

Inductors: How it works?

- Let's scrutinize this equation: $\Delta V_L = -L\, di/dt.$

  Assume that we go in the direction of the current.

  - If the current is increasing, $\frac{di}{dt} > 0$, and the voltage drop produced by inductor, ΔV_L, is negative. In this case the inductor acts as a battery producing current in the direction opposite to the direction of the increasing current.

  - If the current is decreasing, $\frac{di}{dt} < 0$, and the voltage drop produced by inductor, ΔV_L, is positive. In this case the inductor acts as a battery producing current in the same direction as the decreasing current.

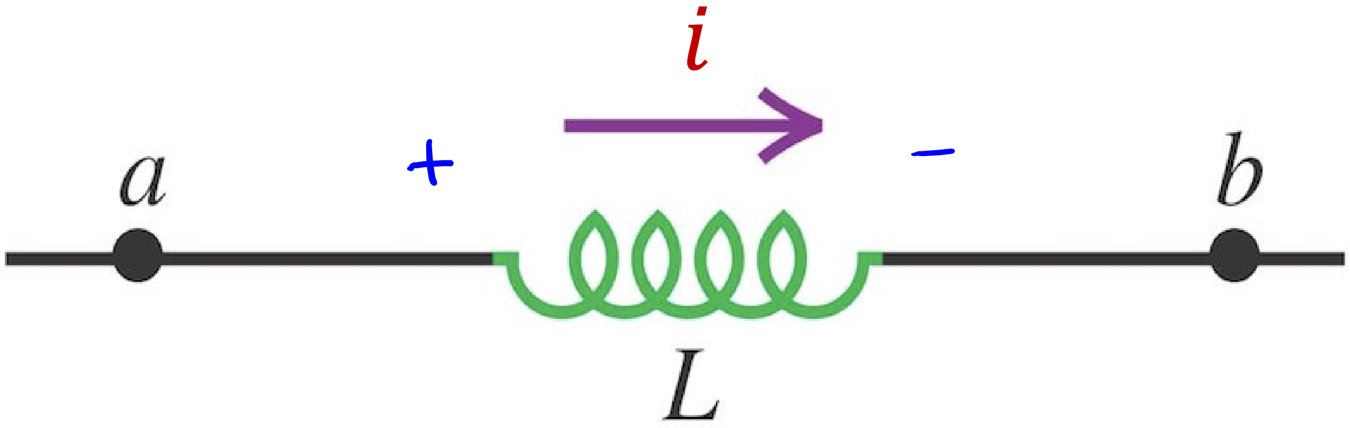
- **This can be rephrased as if the inductor induces EMF** which <u>tries to oppose the change</u>

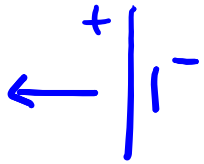
Inductors – try to preserve the "STATUS QUO"

Inductors: How it works?

Q: Compare V_a and V_b when…

a) the current i through the inductor is increasing





A. $V_b > V_a$

B. $V_a > V_b$

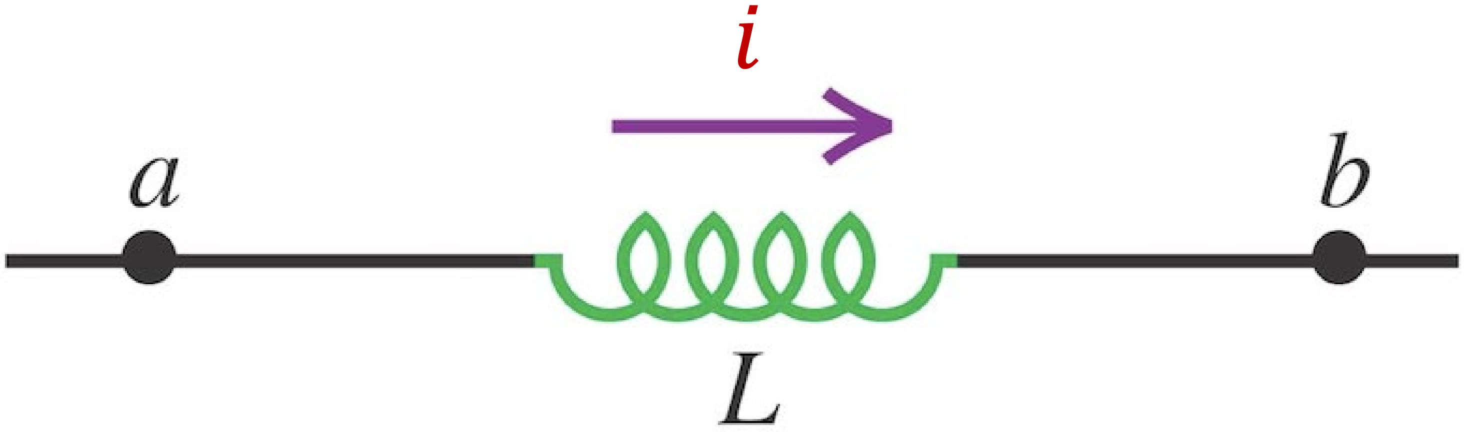
C. $V_a = V_b$

D. No idea

Inductors: How it works?

Q: Compare V_a and V_b when…

b) the current i through the inductor is decreasing



$\Delta V_{a\to b} > 0$

A. $V_b > V_a$

B. $V_a > V_b$

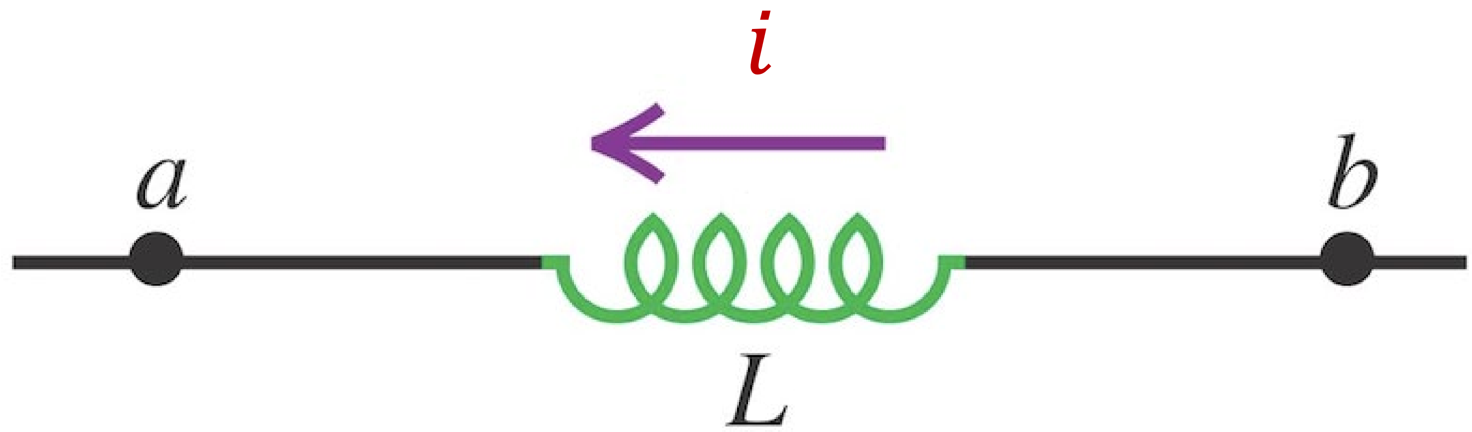
C. $V_a = V_b$

D. No idea

Inductors: How it works?

Q: Compare V_a and V_b when…

c) the current i through the inductor is decreasing



A. $V_b > V_a$

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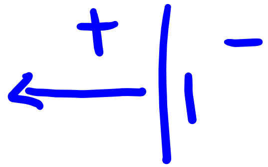
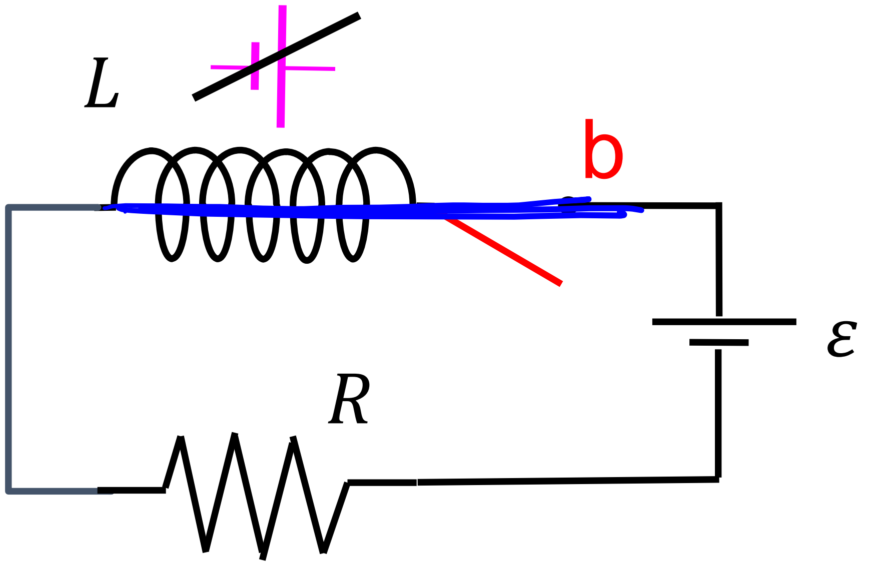


$\Delta V_{b \to a} > 0$

$\Delta V_{a \to b} < 0$

R-L Circuit

- What happens if the switch is moved to position b, to initiate the current flow?



- Current is now free to flow in the circuit, but there is a back EMF induced in L which opposes the change in current!

  - So at $t = 0+$, $i(0+) = 0$ Inductor acts like a BATTERY
  - After a long time, $i\,(\infty) = \varepsilon/R$ Inductor acts like a WIRE (inductor gives up)

- For comparison: in an R circuit, $I = \varepsilon/R$ at all times after you close the switch.

Q: The switch in the series circuit below is closed at $t = 0$.

a) What is the current i at this circuit right after the switch is closed (time = 0+)? $\rightarrow i = 0$

b) What is initial rate of change of current, di/dt, immediately after the switch is closed?

A. 0 A/s

B. 0.5 A/s

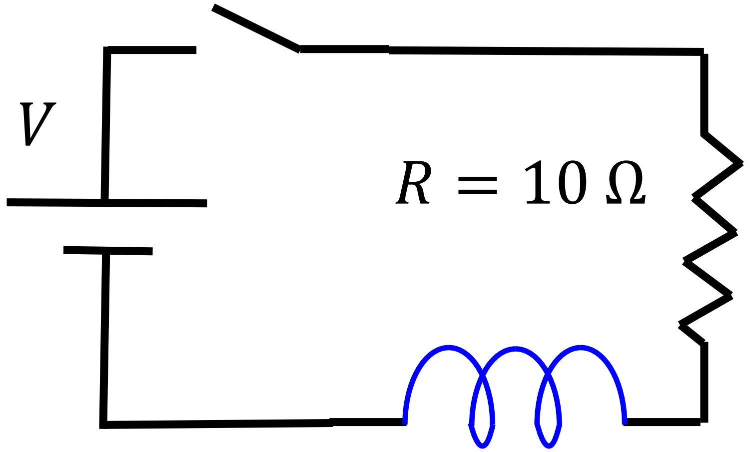
C. 1.0 A/s

D. 10 A/s

E. None of these.

$i \uparrow$

$\varepsilon_L = -L \frac{di}{dt}$



$\Delta V_R + \Delta V_L - \varepsilon = 0$

$\varepsilon = |\Delta V_R| + |\Delta V_L|$

$IR = 0$, $= 0$

$L \cdot \frac{di}{dt}$

$\frac{di}{dt} = \frac{\varepsilon}{L} = \frac{10\ V}{10\ L} = 1\ \frac{A}{S}$

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C. 1.0 A/s

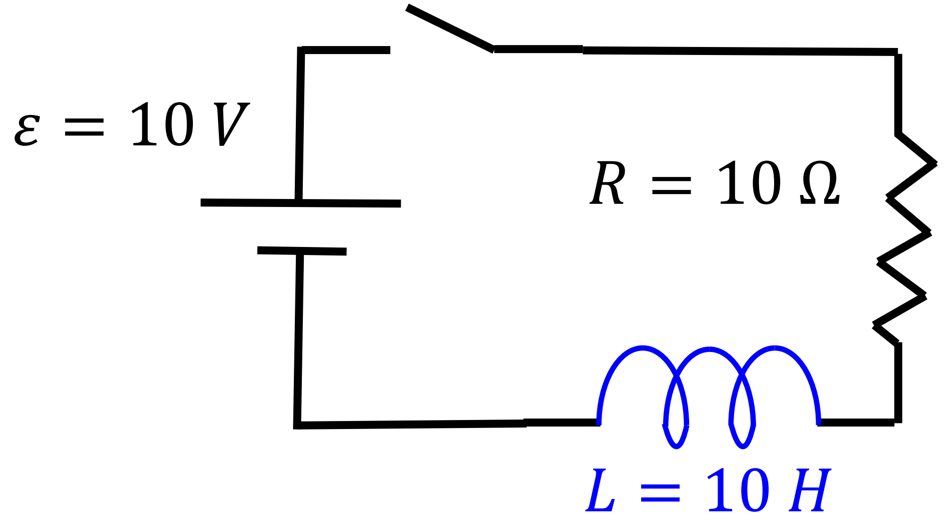
D. 10 A/s

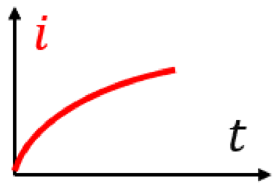
E. None of these.

Part a)

- Inductor prevents flow of current

  => $i(0+) = 0$





- $i(0+) = 0$
- $di/dt \neq 0$

Part b)

- We know: $|\Delta V_L| = L\,|di/dt|$. What is the voltage drop across the inductor at $t = 0+$?
- Kirchhoff's loop law: $\varepsilon + \Delta V_R + \Delta V_L = 0$
- At $t = 0$ we have: $i = 0$, so $\Delta V_R = iR = 0$, hence:

$$\Delta V_L = \varepsilon = 10\ V = L\frac{di}{dt} \Rightarrow \frac{di}{dt} = \frac{10\ V}{10\ H} = 1\ A/s$$

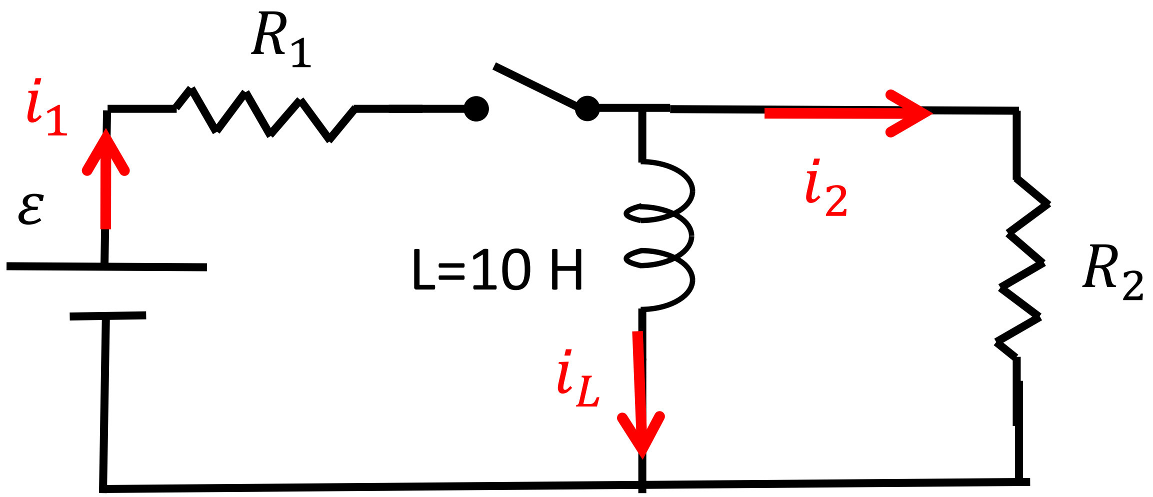
Q: A parallel two-loop LR circuit is shown below. Initially the switch is open.

Assume $R_1 = R_2 = 10\ \Omega,\ \ \varepsilon = 10\ V$.

a) Find all the currents (i_1, i_2, i_L) immediately after the switch is closed ($t = 0+$).

b) Now the switch is closed for a very long time. Find the currents i_1, i_2, i_L.

c) At $t = T_1$ the switch S is opened. Calculate all the currents right after $t = T_1$.



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- What is i_L? What is i_1? What is i_2?

$i_L(t=0+) = 0 = i_L(t=0-)$

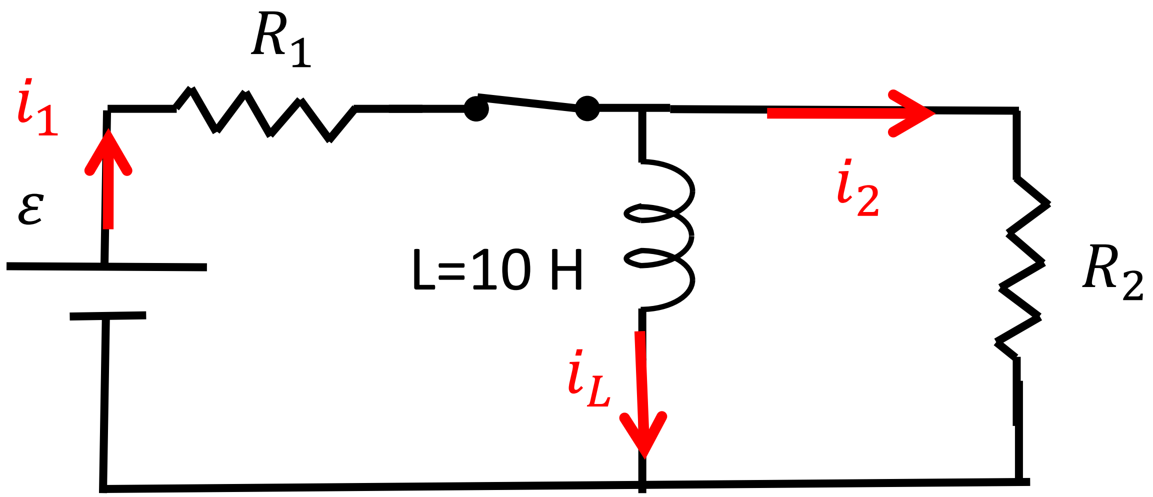
A. 0 A

B. 1 A

C. ε/R_1

D. $\varepsilon/(R_1 + R_2)$

E. None of these.



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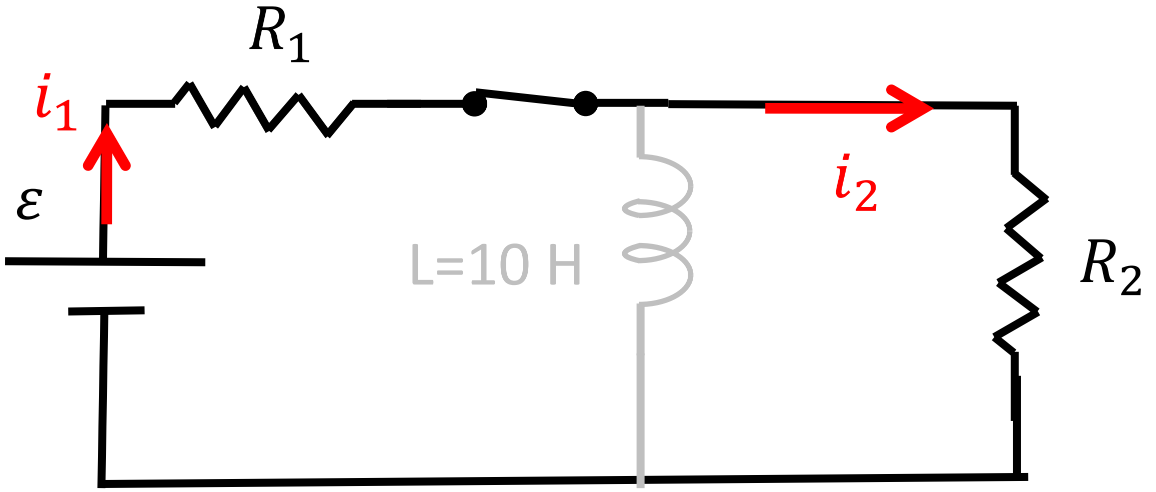
- What is i_L? What is i_1? What is i_2?

A. 0 A
B. 1 A
C. ε/R_1
D. ε/(R_1 + R_2)
E. None of these.

- At the very first moment, inductor acts like a battery to oppose the current flow. This prevents current flow through the inductor =>

$$i_L = 0$$



- No current in the inductor => at t = 0+, it's a circuit with two resistors in series.
- $R_{eq} = R_1 + R_2 = 20\ \Omega$
- $i_1 = i_2 = \frac{\varepsilon}{R_{eq}} = \frac{\varepsilon}{R_1 + R_2} = 0.5\ A$

Q: A parallel two-loop LR circuit is shown below. Initially the switch is open.

Assume $R_1 = R_2 = 10\ \Omega,\ \varepsilon = 10\ V$.

At $t = 0$: $i_1 = i_2 = \frac{\varepsilon}{R_1+R_2}$, $i_3 = 0$

b) Now the switch is closed for a very long time. Find the currents i_1, i_2, i_L.

- What are i_1 and i_2?

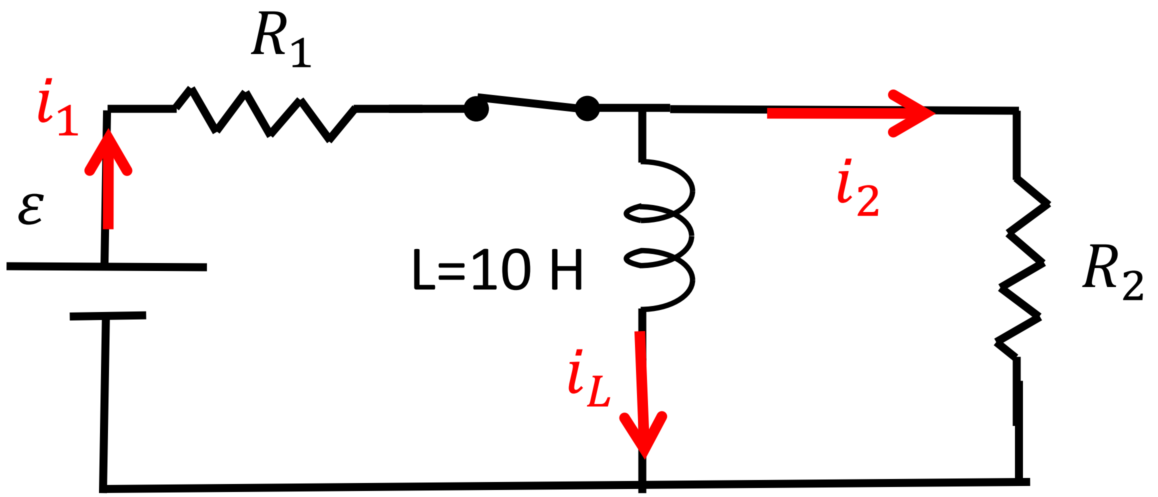
A. Both 0

B. ε/R_1 and 0

C. 0 and ε/R_2

D. Both $\varepsilon/(R_1 + R_2)$

E. None of these.



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A. Both 0

B. ε/R_1 and 0

C. 0 and ε/R_2

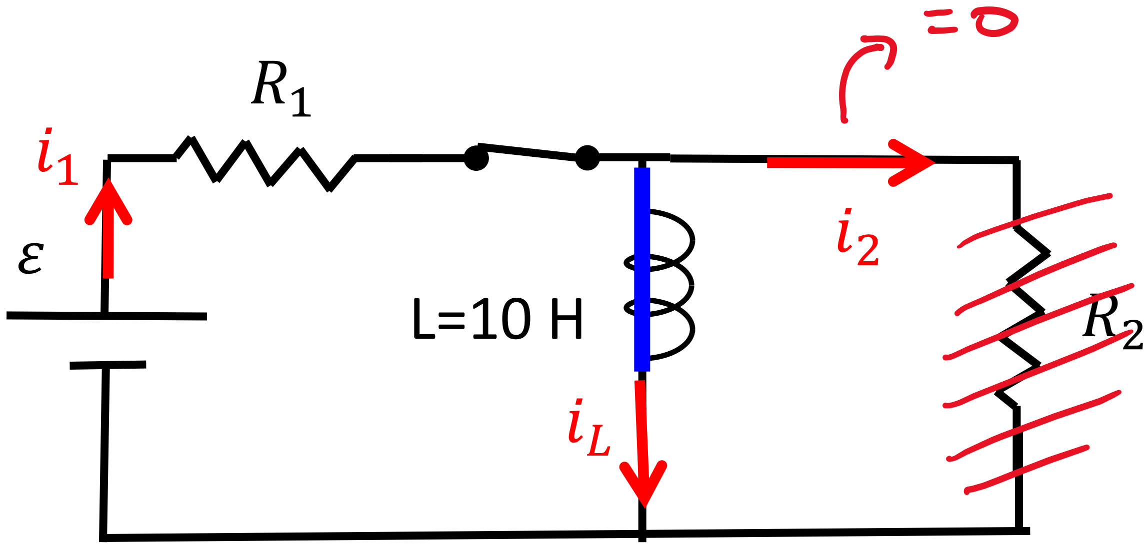
D. Both $\varepsilon/(R_1 + R_2)$

E. None of these.

- After a very long time, the current stabilizes =>

$\frac{di}{dt} = 0$ => $\Delta V_L = -L\frac{di}{dt} = 0$ =>

Inductor acts as ideal wire



- There is no voltage drop across R_2 => $i_2 = 0$
- It's a circuit with one resistor, R_1 => $i_1 = \frac{\varepsilon}{R_1}$

Q: A parallel two-loop LR circuit is shown below. Initially the switch is open.

Assume $R_1 = R_2 = 10\ \Omega,\ \varepsilon = 10\ V$.

At $t = \infty$: $i_2 = 0,\ i_1 = i_L = \frac{\varepsilon}{R_1}$

c) At $t = T_1$ the switch S is opened. Calculate all the currents right after $t = T_1$.

$i_L(\text{before}) = i_L(\text{after})$

- What are i_1 and i_2?

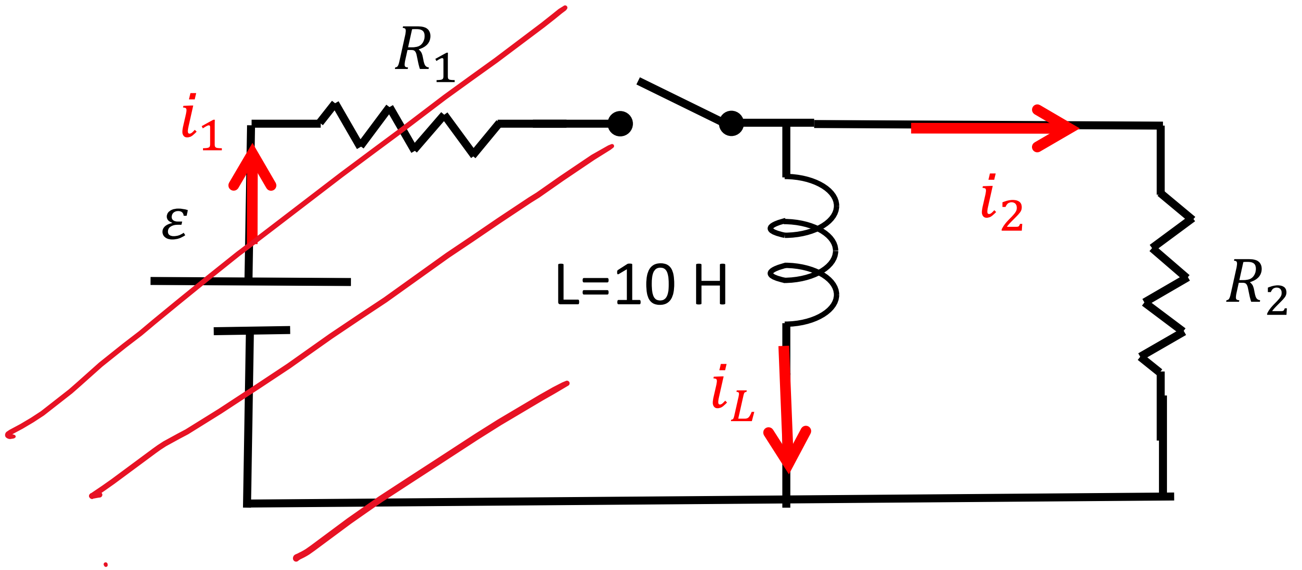
A. both 0

B. ε/R_1 and 0

C. both $\varepsilon/(R_1 + R_1)$

D. 0 and ε/R_1

E. 0 and ε/R_2



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Assume $R_1 = R_2 = 10\ \Omega,\ \varepsilon = 10\ V$.

At $t = \infty$: $i_2 = 0,\ i_1 = i_L = \frac{\varepsilon}{R_1}$

c) At $t = T_1$ the switch S is opened. Calculate all the currents right after $t = T_1$.

- No current through R_1 (open circuit), Hence, $i_1 = 0$.
- i_L through the inductor cannot change immediately. Hence, i_L(after) = i_L(before)
- Charge is conserved so we must have: i_2(after) = i_L(after) =i_L(before) = i_1(before)
  => i_2 flips direction!

- What are i_1 and i_2?

A. both 0

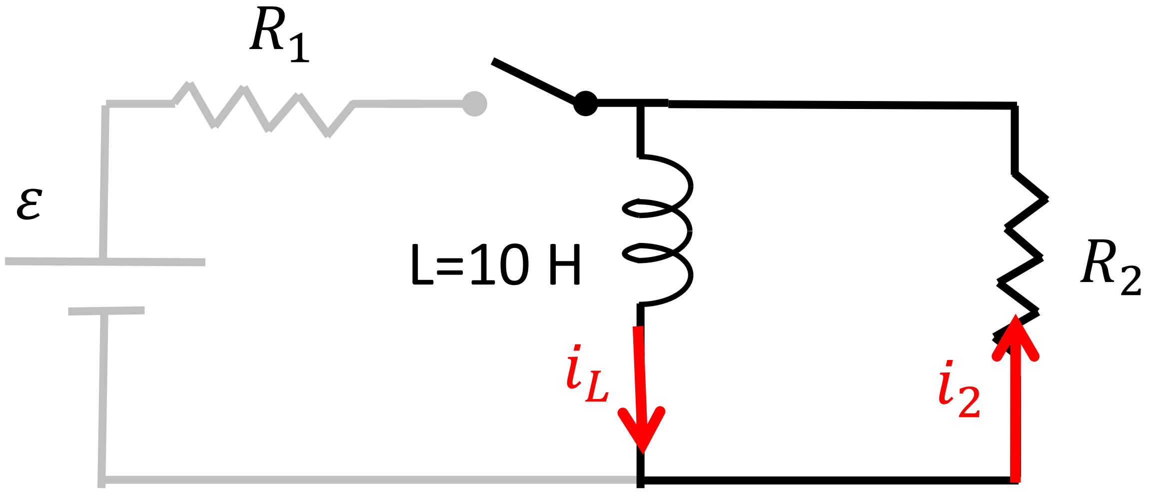
B. ε/R_1 and 0

C. both $\varepsilon/(R_1 + R_1)$

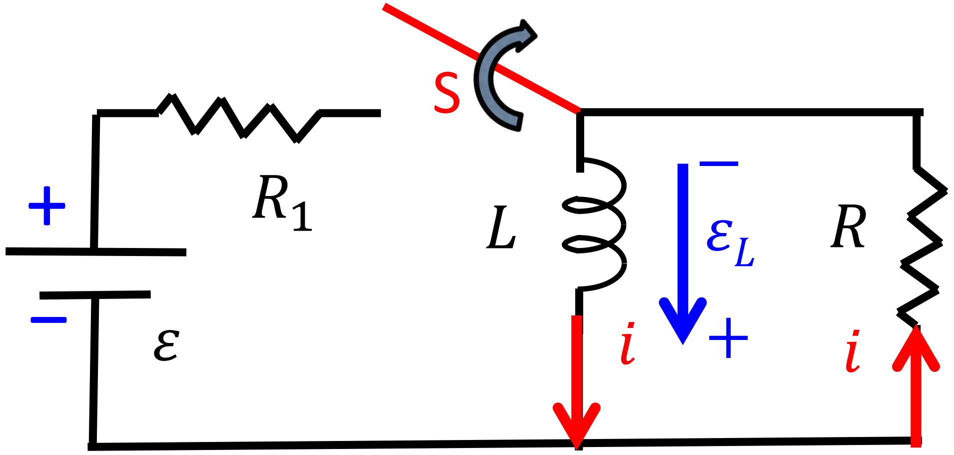
D. 0 and ε/R_1

E. 0 and ε/R_2

- Note that at that instant the inductor acts as a battery that drives the current in the right loop.



RL circuits -- 1



Suppose all components have all been connected for a very long time so that the currents through R_1 and R are ε/R_1 and 0.

At $t = 0+$ the switch S is opened, but the current will continue to flow through the Inductor (inductors always resist change => tries to maintain the current)

- Using the loop rules: $\Delta V_L - iR = 0$
- Voltage drop across an inductor: $\Delta V_L = \varepsilon_L = -L\dfrac{di}{dt}$

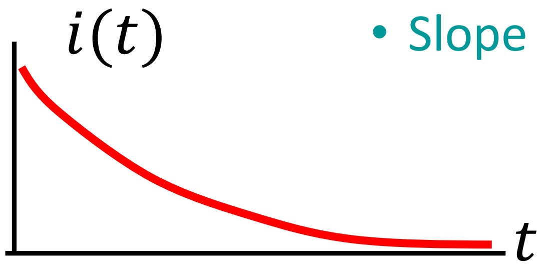
$$\boxed{-L\frac{di}{dt} - iR = 0}$$

$e^{-t/\tau_{RL}}$ $\tau_{RL} = L/R$

We can solve this equations using the method we presented for a discharging capacitor.
Result: exponential decay

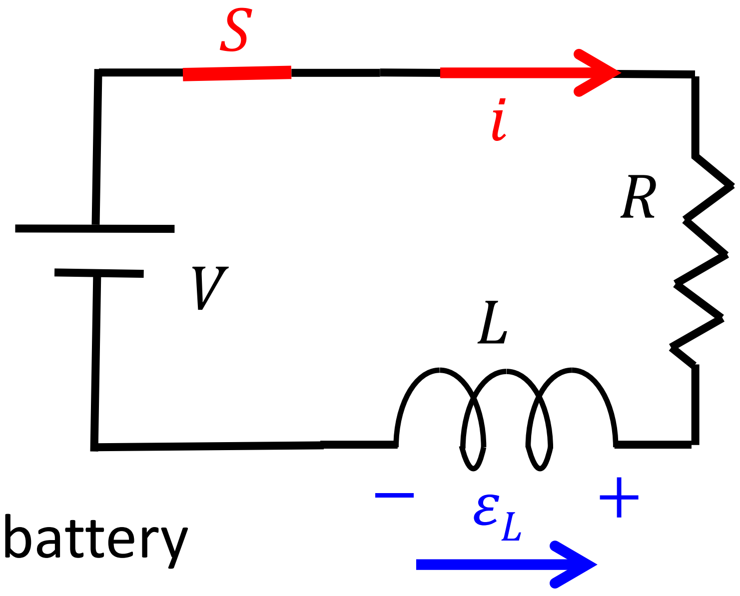
$$\boxed{i(t) = I_0\, e^{-(Rt/L)}}$$

$I_0 = i(t = 0+)$



- Note: $\frac{di}{dt} < 0$
- Slope < 0

RL circuits -- 2



Now consider the RL series case where the switch S is closed at $t = 0$.

Again, inductor tries to maintain the current by creating current in the direction opposite to the direction of the current from the battery

- Using the loop rules:

$$\boxed{V - iR - L\frac{di}{dt} = 0}$$

$I_{max} = \frac{\varepsilon}{R}$

depends !

Basically, it is equal to the current that will flow in the circuit if you replace the inductor by an ideal wire.

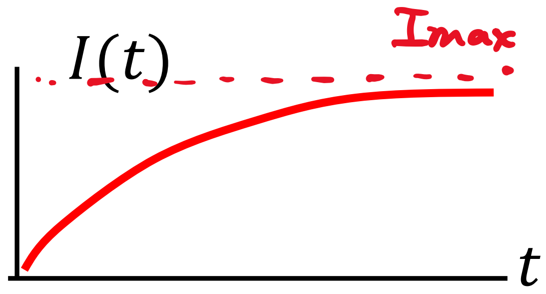
- Note: $\frac{dI}{dt} > 0$
- Slope > 0

We can solve this equations using the method we presented for a charging capacitor.

$$\boxed{i(t) = I_{\max}\left(1 - e^{-(Rt/L)}\right)}$$

$\downarrow$

$e^{-t/\tau_{RL}}$



$\tau_{RL} = L/R$

Result: exponential increase from 0 to $I_{\max}$