Lecture 18.

Applications of Gauss's law (continued).

E-field of:

- infinite line of charge
- charged sphere

Practice with Gauss's law.

Announcement

- Studying for the midterm: you need to understand the ideas and the procedures (what exactly you should do to calculate some physical quantity)

- You understand a concept if you can explain it to someone else in simple words

- You understand an equation if you can explain in simple words what each letter in it means, what this equation describes, and in which situations you might need it.

- Resources:
  - Practice exams &&& clicker questions and problems that we solved in class, HW, tutorial problems
  - Help sessions (MTW on the exam week), Piazza…

Announcement

- Study guides (@ Weekly pages on canvas):
  - Topics
  - Practice problems
  - Written by previous students

Homework

HW-5-MP due Sunday, Feb 18 at 11:59pm

Study Guides

2-Capacitors and Inductors Study Guide (Chapters 24.1-24.6 + 30.2).pdf

3-RC Circuits Study Guide (Chapter 26.4-26.5).pdf

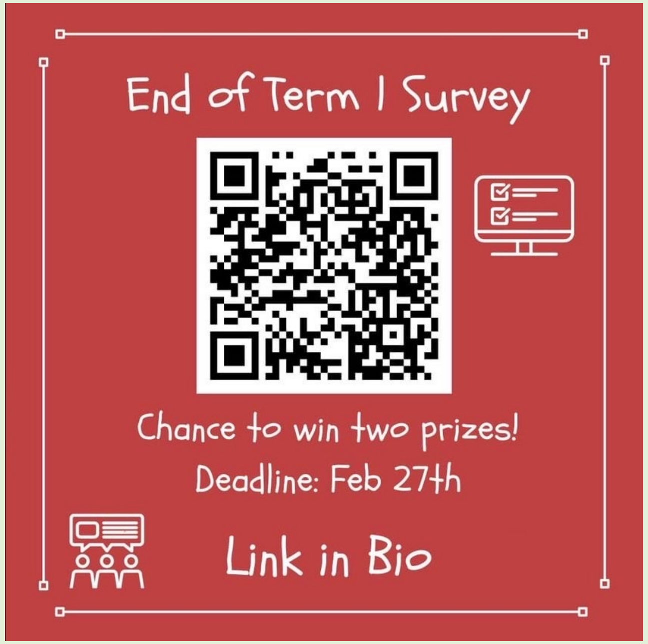
4-RLC Circuits Study Guide (Chapters 30.6 + 31.3).pdf

- Weekly lecture notes: (Modules / Posted Notes)

Posted Notes

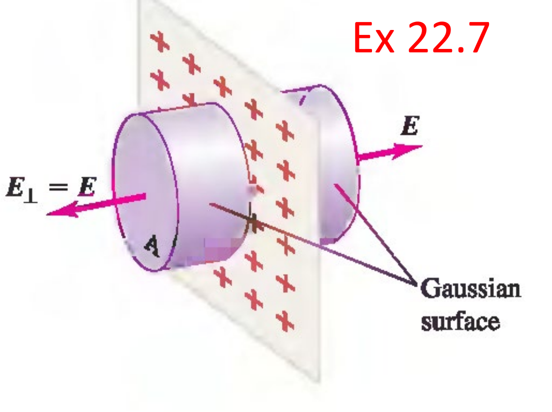
- Selected Pre-Lecture Slides-202
- Lecture Slides-202
- Lecture Slides-201
- HW-Solutions

- Survey (Engineering)



Four steps for everybody who wants to use the Gauss law to relate charges and fields

Ex 22.7



1. Choose Gaussian surface with a symmetry matching that of the charge distribution (if such a surface exists…) and passing through your observation point.

2. Calculate the flux through it: $\Phi_e = \oint \vec{E} \cdot d\vec{A} = E\, A_{\mathrm{GS}}$

3. Find the charge inside the Gaussian surface, Q_{inside}

4. Apply the Gauss law: $E\, A_{\mathrm{GS}} = \dfrac{Q_{\mathrm{inside}}}{\varepsilon_0}$

➢ Bingo! You obtained a relation between the charge Q_{inside} and the field E created by it at the observation point.

$$\boxed{E(d) = \frac{\sigma}{2\varepsilon_0}}$$

$\sigma = \dfrac{Q}{A_{\mathrm{plane}}}$ is surface charge density

$$\varepsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\, \frac{C^2}{Nm^2}$$

Using Gauss's law (example 3)

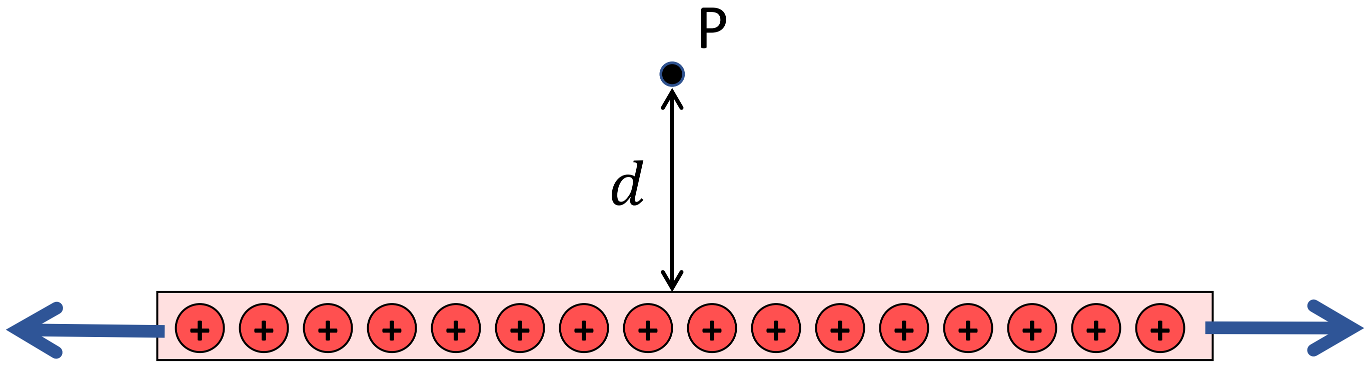
Q: Find electric field of an infinite charged rod at a vertical distance d from it. Assume uniform linear charge density $\lambda = \frac{Q}{L}$ (C/m)

1. Matching GS?

2. Flux: $\Phi_e = \oint \vec{E} \cdot d\vec{A} = ?$

3. Enclosed charge: $Q_{in} = ?$

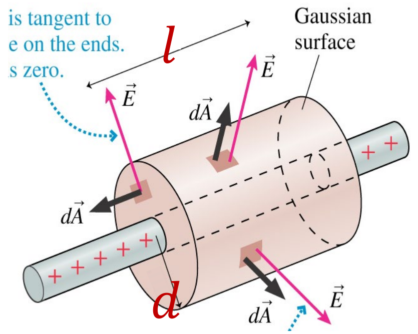
4. Gauss's law: $\Phi_e = Q_{in}/\varepsilon_0$



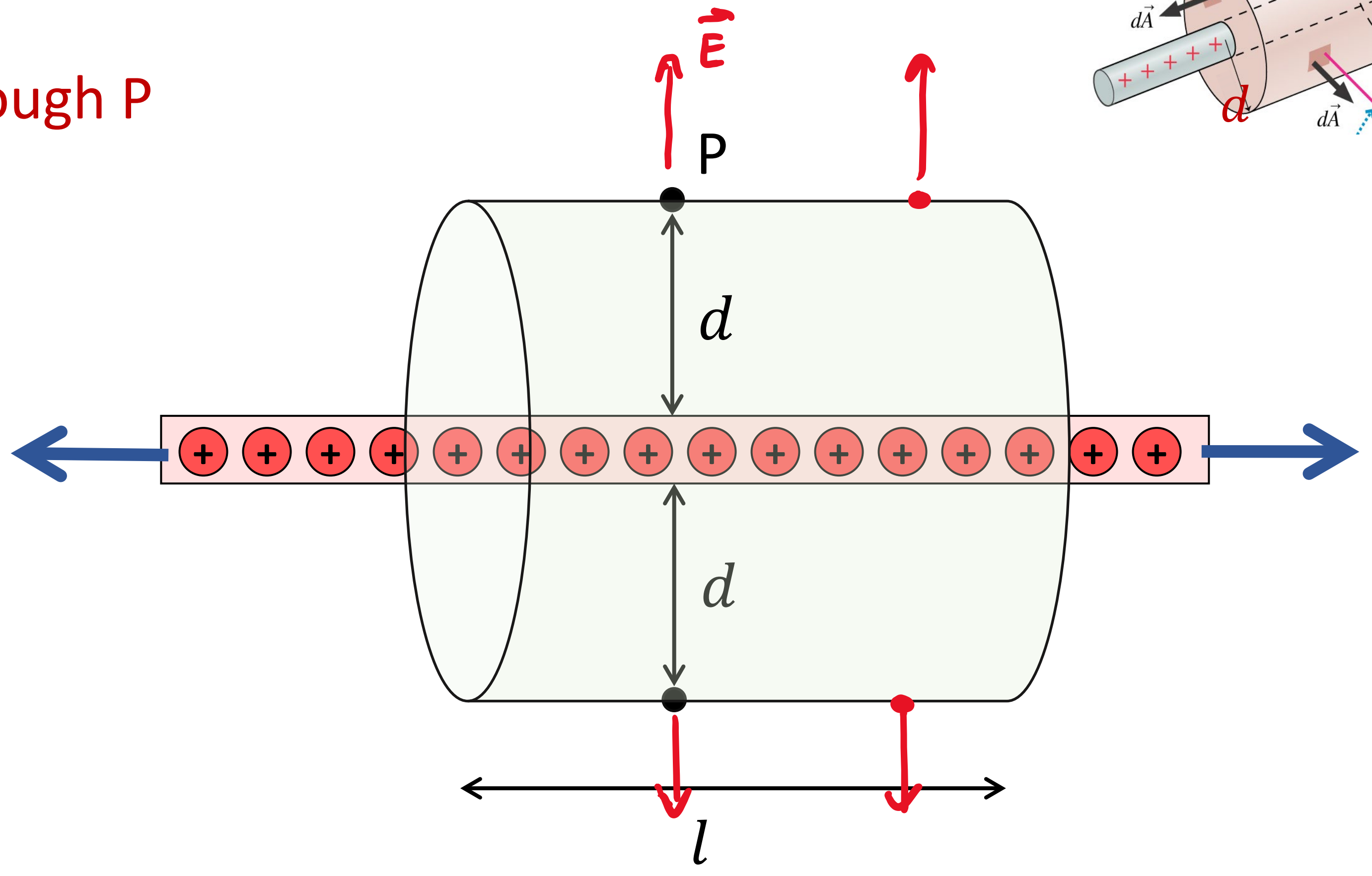
Using Gauss's law (example 3)

Q: Find electric field of an infinite charged rod at a vertical distance d from it. Assume uniform linear charge density $\lambda = \frac{Q}{L}$ (C/m)

1. Matching GS?



- Cylinder, with the side passing through P



- By symmetry, the electric field lines should be perpendicular to the rod => no flux is lost through the top & bottom of the cylinder => all flux goes through its side

Using Gauss's law (example 3)

Q: Find electric field of an infinite charged rod at a vertical distance d from it. Assume uniform linear charge density $\boxed{\lambda = \frac{Q}{L}}$ (C/m)

1. Matching GS?

Cylinder, with the side passing through P (only S = side)

2. Flux: $\Phi_e = \oint \vec{E} \cdot d\vec{A} = ?$ $\int_S E\, dA$

$$\Phi_e = \int_S E\, dA \cos 0^0 = E \int_S dA = E\, A_{\text{side}} = E \cdot l \cdot 2\pi d$$

3. Enclosed charge: $Q_{in} = ?$

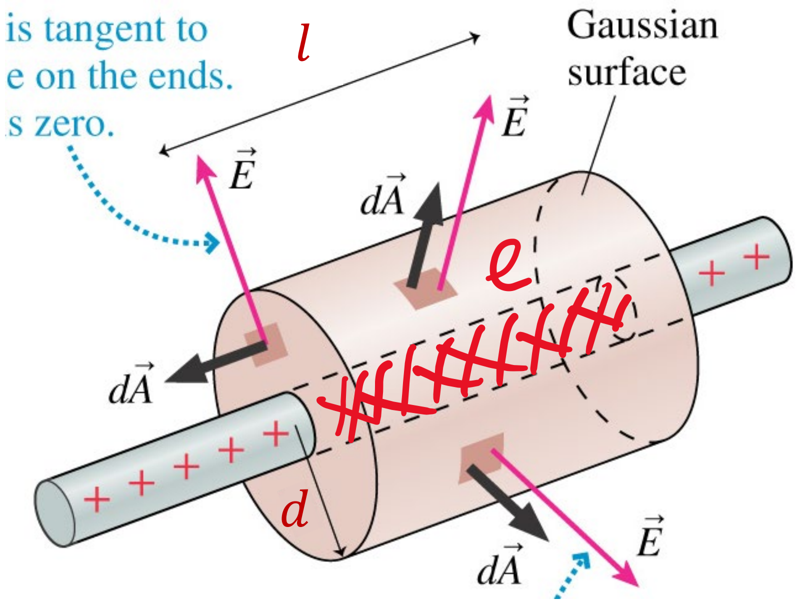
$\frac{1}{4\pi\varepsilon_0} = k$

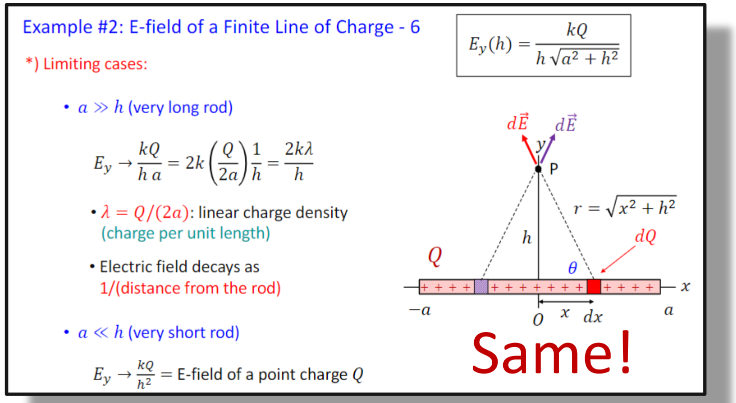
Charge sitting on the segment of length l: $Q_{in} = \lambda l$

4. Gauss's law: $\Phi_e = Q_{in}/\varepsilon_0$

$$E \cdot \cancel{l} \cdot 2\pi d = \lambda \cancel{l}/\varepsilon_0$$

$$\boxed{E(d) = \frac{\lambda}{2\pi\varepsilon_0\, d} = \frac{2k\lambda}{d}}$$





Using Gauss's law (example 4)

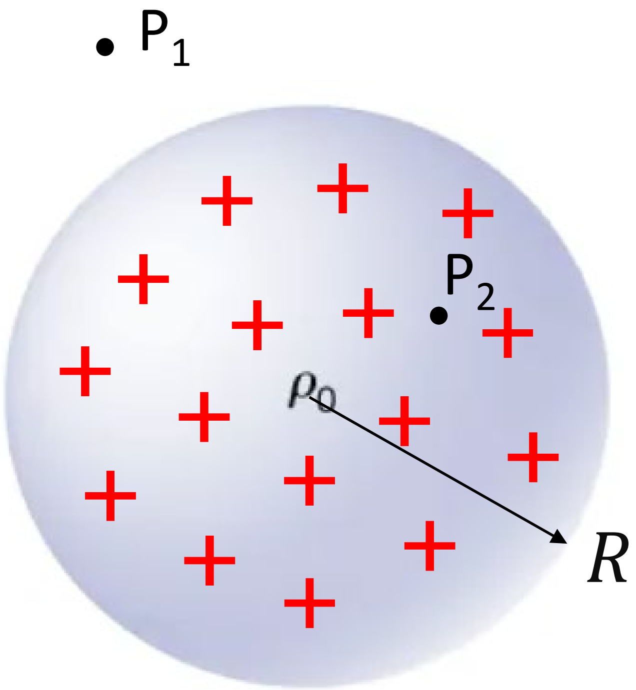
Q: Find electric field of a uniformly charged insulating sphere, both inside and outside. Assume uniform volume charge density $\rho = \frac{Q}{V_{\text{spere}}}$ (C/m^3)

1. Symmetry? GS?

2. Flux: $\Phi_e = \oint \vec{E} \cdot d\vec{A} = ?$

3. Enclosed charge: $Q_{in} = ?$

4. Gauss's law: $\Phi_e = Q_{in}/\varepsilon_0$



Using Gauss's law (example 4, outside)

1. GS: Sphere of radius $\underline{\ r\ }$

2. Flux: $\Phi_e = \oint \vec{E} \cdot d\vec{A}$?

$= \oint E\,dA = E \int_{GS\ (sphere)} dA$

$= E \cdot A_{sphere} = E \cdot 4\pi r^2$

A. $\Phi_e = E\, 4\pi\, r^2$

B. $\Phi_e = E\, 4\pi\, R^2$

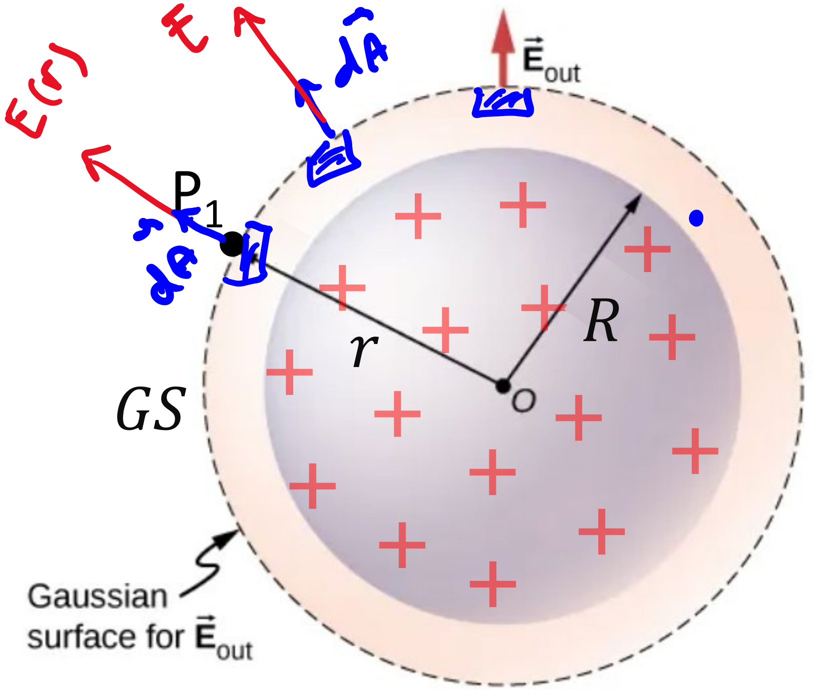
C. $\Phi_e = E\, \frac{4}{3}\pi\, R^2$

D. $\Phi_e = E\, \frac{4}{3}\pi\, r^3$

3. Enclosed charge: $Q_{in} = Q$

4. Gauss's law: $\Phi_e = Q_{in}/\varepsilon_0$

$E(r) \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$ $\qquad E(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$



R: radius of the sphere

r: distance from the center to the observation point

$\boxed{Q\text{: total charge}}$

$\rho = \frac{Q}{V_{sphere}}$

- $E_{sphere}(r > R) = \frac{kQ}{r^2}$

Using Gauss's law (example 4, inside)

1. GS: Sphere of radius r

2. Flux: $\Phi_e = \oint \vec{E} \cdot d\vec{A}$?

$\sqrt{\int_{GS} E \cdot dA} = E A_{GS}$

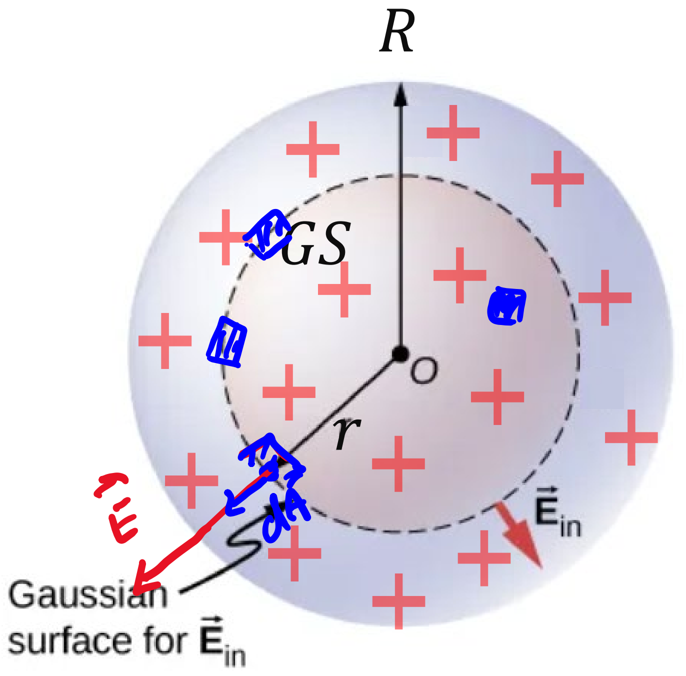
$= E \cdot 4\pi r^2$

A. $\Phi_e = E\,4\pi\,r^2$

B. $\Phi_e = E\,4\pi\,R^2$

C. $\Phi_e = E\,\frac{4}{3}\pi\,R^2$

D. $\Phi_e = E\,\frac{4}{3}\pi\,r^3$



R: radius of the sphere

r: distance from the center to the observation point

Q: total charge

3. Enclosed charge: $Q_{in} = ?$

D. Q E: else

A. $Q_{in} = Q\,\frac{r^3}{R^3}$

B. $Q_{in} = Q\,\frac{r^2}{R^3}$

C. $Q_{in} = Q\,\frac{r^2}{\frac{4}{3}\pi\,R^3}$

4. Gauss's law: $\Phi_e = Q_{in}/\varepsilon_0$

$E_{in}(r) \cdot 4\pi \cancel{r^2} = \frac{Q}{\varepsilon_0}\frac{r^{\cancel{3}}}{R^3}$

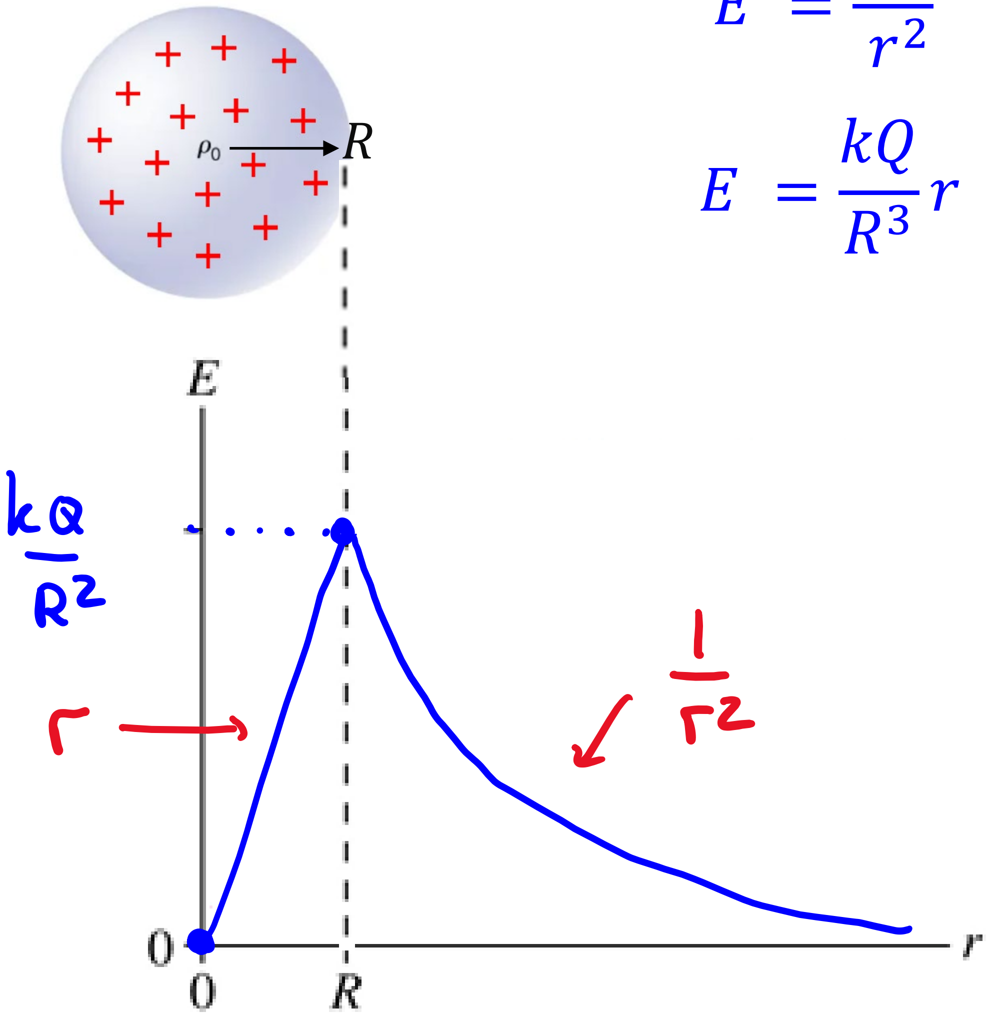
$E_{in}(r) = \frac{Q}{4\pi\varepsilon_0}\frac{r}{R^3}$

- $E_{\text{sphere}}(r < R) = \frac{kQr}{R^3}$

- Charge inside the Gaussian surface: Ratio arguments

$$\frac{Q_{in}}{Q} = \frac{V_{in}}{V_{sphere}} = \frac{\cancel{\frac{4\pi}{3}} r^3}{\cancel{\frac{4\pi}{3}} R^3} = \frac{r^3}{R^3}$$

Using Gauss's law (example 4, combined)



$$E = \frac{kQ}{r^2}$$ - outside (case 1)

$$E = \frac{kQ}{R^3} r$$ - inside (case 2)

Extra:

Note: a similar situation happens with the weight of an object in a deep pit. The gravity constant appears to be determined by the part of the Earth below the object:

$$g_{\text{eff}}(r) = \frac{GM_{\text{Earth}}}{R^3_{\text{Earth}}} r = g \frac{r}{R_{\text{Earth}}}$$

See, e.g. http://hyperphysics.phy-astr.gsu.edu/hbase/Mechanics/earthole.html

Summary:

$$A_{sphere} = 4\pi R^2 \qquad V_{sphere} = \frac{4}{3}\pi R^3$$

- Electric field created by highly symmetric (= “infinite”, with no edge effects) objects

$$E_{plane}(r) = \frac{\sigma}{2\varepsilon_0}$$

- Does not depend on distance from the plane! Same everywhere!

$$\sigma = \frac{Q}{A}$$

$$E_{rod}(r) = \frac{2k\lambda}{r}$$

- Decays as $1/r$

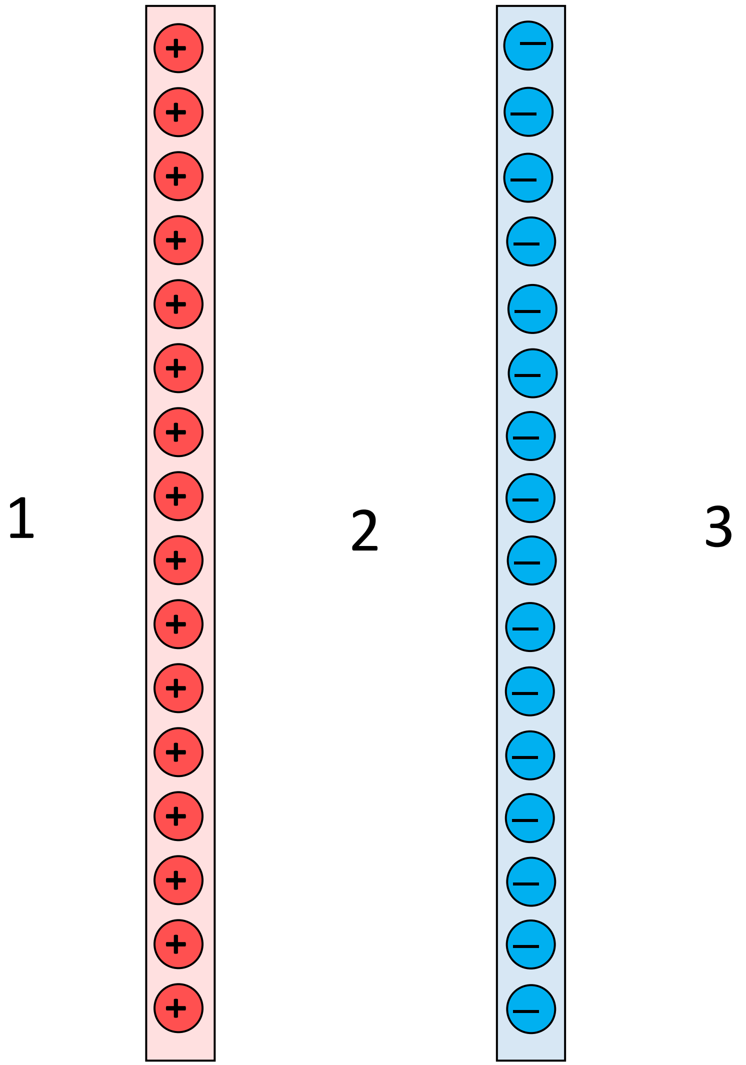
$$\lambda = \frac{Q}{L}$$

$$E_{sphere}(r > R) = \frac{kQ}{r^2}$$

- Decays as $1/r^2$
- True outside of any spherical shape carrying total charge Q (including point charge)

$$\rho = \frac{Q}{V}$$

Q: Two infinite planes are uniformly charged with the same charge per unit area, σ.
If one plane only were present, the magnitude of its electric field would be E.



a) What is the magnitude of the electric field in region 2?

b) What is the magnitude of the electric field in region 3?

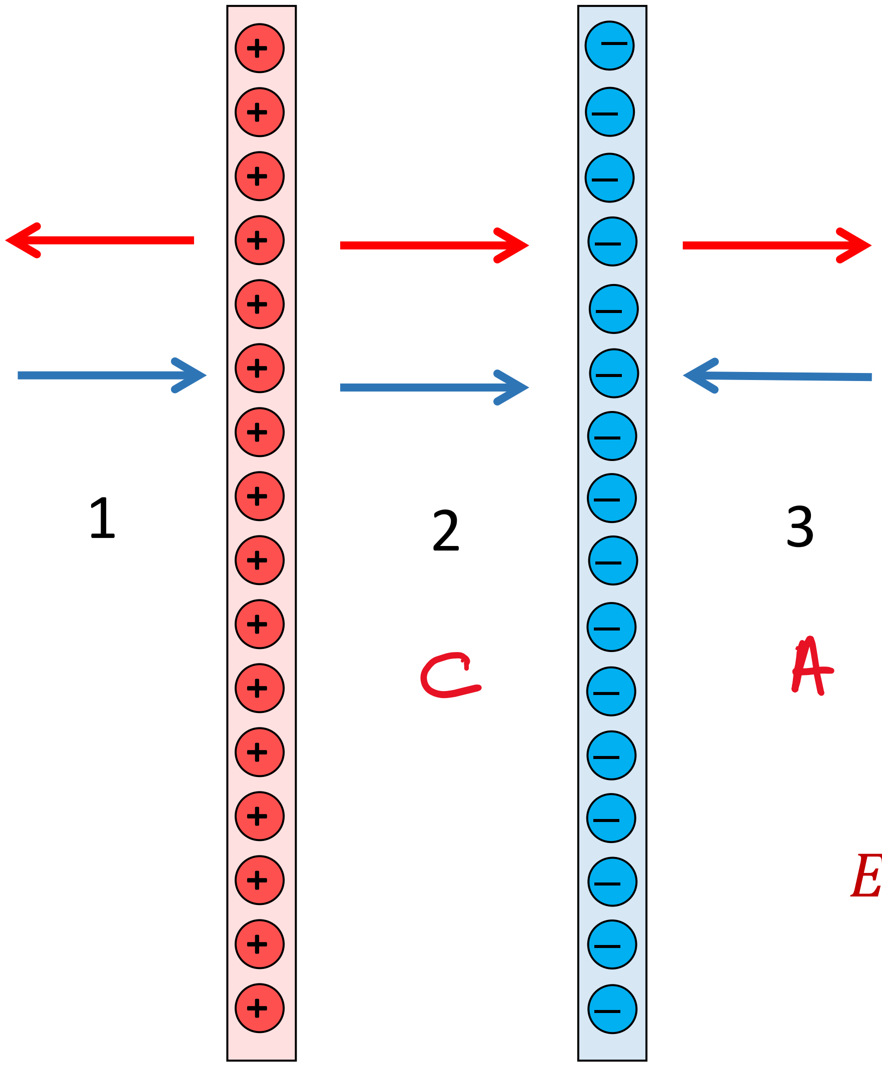
A. Zero

B. E

C. $2E$

D. Depends on exact location

Q: Two infinite planes are uniformly charged with the same charge per unit area, σ. If one plane only were present, the magnitude of its electric field would be E.



a) What is the magnitude of the electric field in region 2?

b) What is the magnitude of the electric field in region 3?

@2: E-fields add => $\frac{\sigma}{\varepsilon_0}$ everywhere inside

@1&3: E-fields cancel => 0 everywhere outside

$$E_{plane}(r) = \frac{\sigma}{2\varepsilon_0}$$

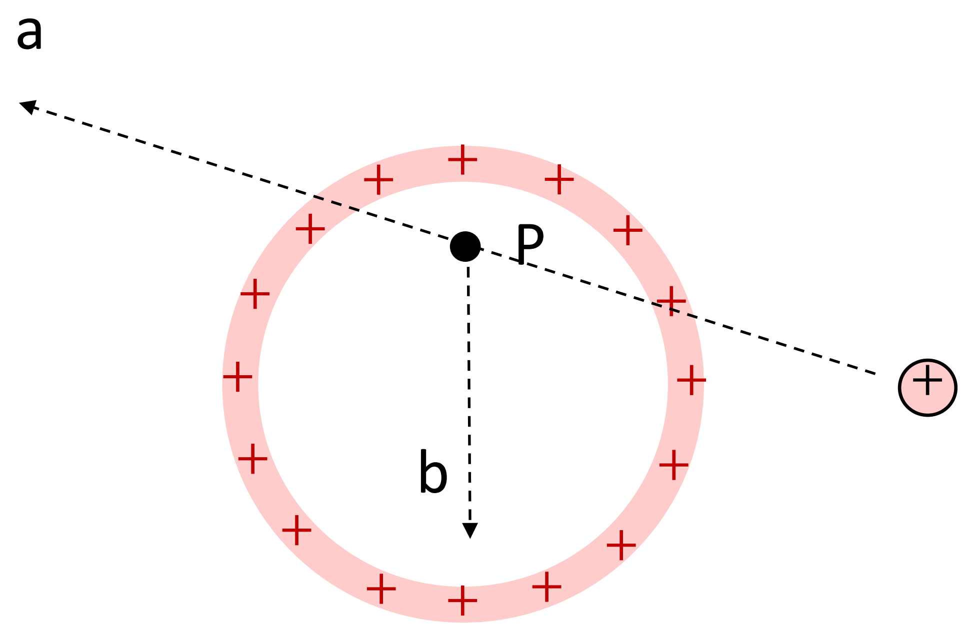
A. Zero

B. E

C. $2E$

D. Depends on exact location

Q: A spherical shell with a uniform positive charge density on its surface is neat a positive charge Q. This is an insulating shell, and the charges on its surface are not pushed away by the point charge. What can you say about the electric field at the point P ?



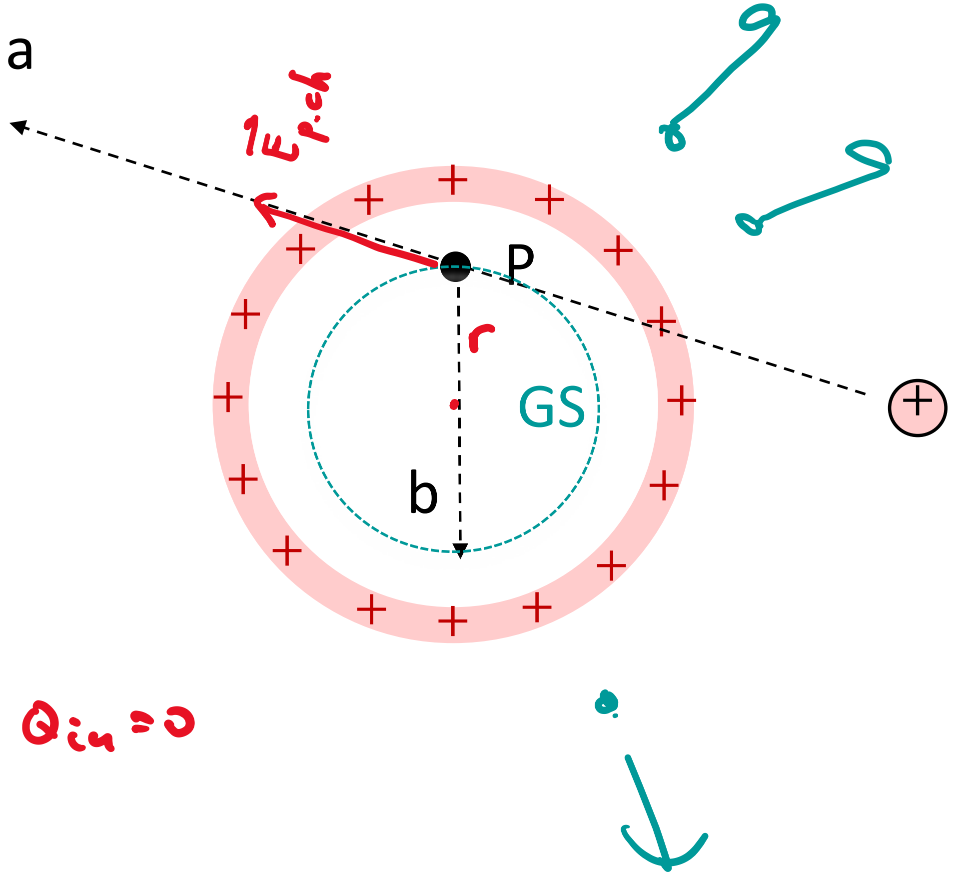
A. It is zero
B. It is directed along (a)
C. It is directed along (b)
D. It is directed between (a) and (b)
E. Something else

Q: A spherical shell with a uniform positive charge density on its surface is neat a positive charge Q. This is an insulating shell, and the charges on its surface are not pushed away by the point charge. What can you say about the electric field at the point P ?

Superposition: $\vec{E}_{\text{tot}} = \vec{E}_{\text{p.ch.}} + \vec{E}_{\text{shell}}$

$\vec{E}_{\text{shell}}(P) = 0$ by Gauss's law

$\vec{E}_{\text{tot}}(P) = \vec{E}_{\text{p.ch.}}(P)$



$E(r) \cdot 4\pi r^2 = 0$

$Q_{in} = 0$

A. It is zero
B. It is directed along (a)
C. It is directed along (b)
D. It is directed between (a) and (b)
E. Something else