Lecture 5.

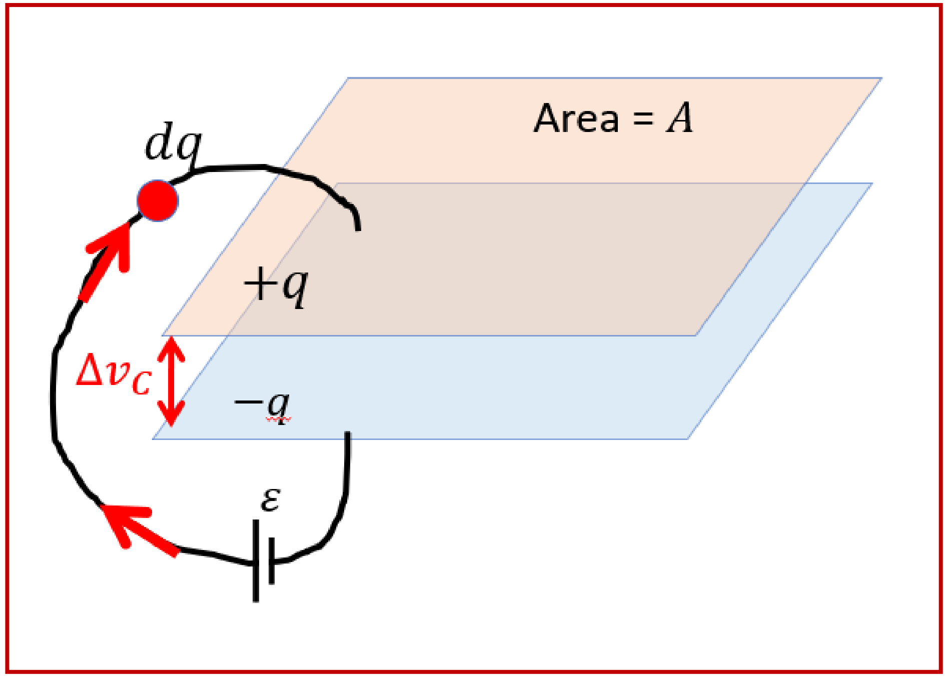
Energy stored in capacitors: practice.
Capacitors and switches.
RC-circuits (intro).

Last Time:

$$\boxed{Q = C\Delta V_C}$$

- C: capacitance

- Charging a capacitor



- Energy stored:

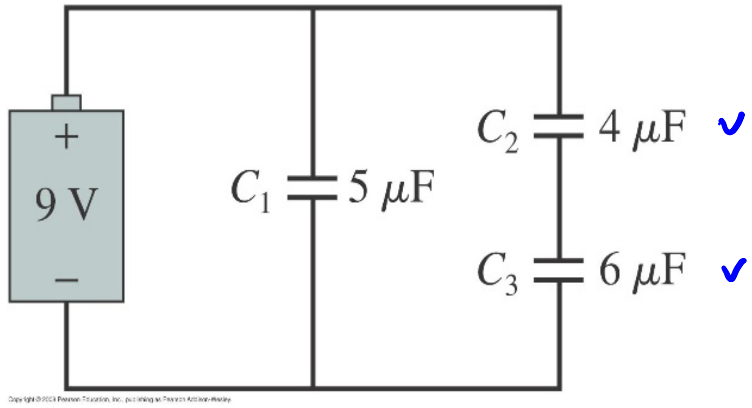
$$\boxed{U_c = \frac{Q_f^2}{2C} = \frac{C\Delta V_f^2}{2}}$$

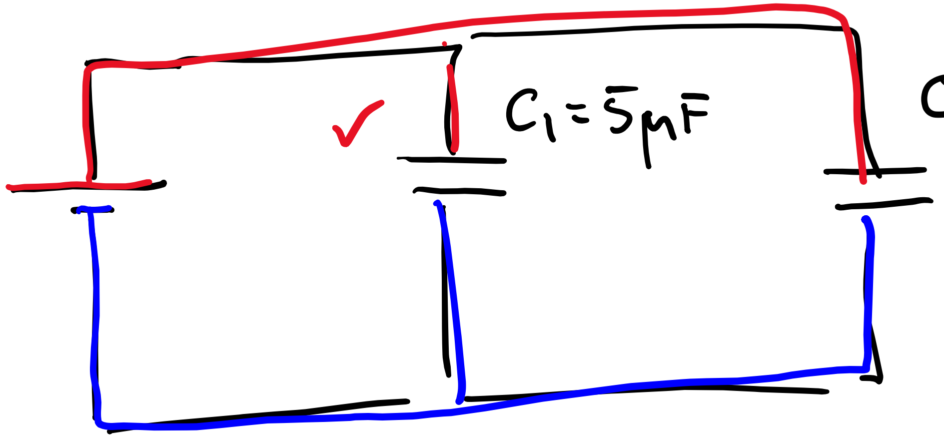
$Q_f = C\,\Delta V_f$

- Q_f: final charge at the plates
- V_f: final voltage across the plates

Q: Rank the three capacitors according to their stored energy.

$$U_c = \frac{Q^2}{2C} = \frac{C(\Delta V_C)^2}{2}$$





$C_{23} = \frac{6 \cdot 4}{6+4} = 2.4 \mu F$

A. $U_1 > U_2 > U_3$

B. $U_3 > U_1 > U_2$

C. $U_1 > U_2 = U_3$

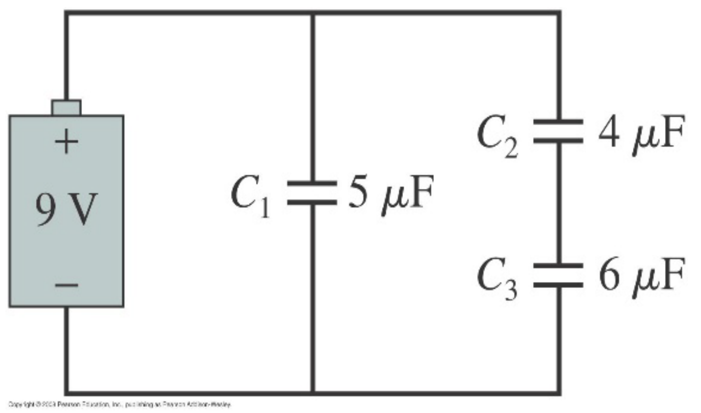
D. $U_2 > U_3 > U_1$

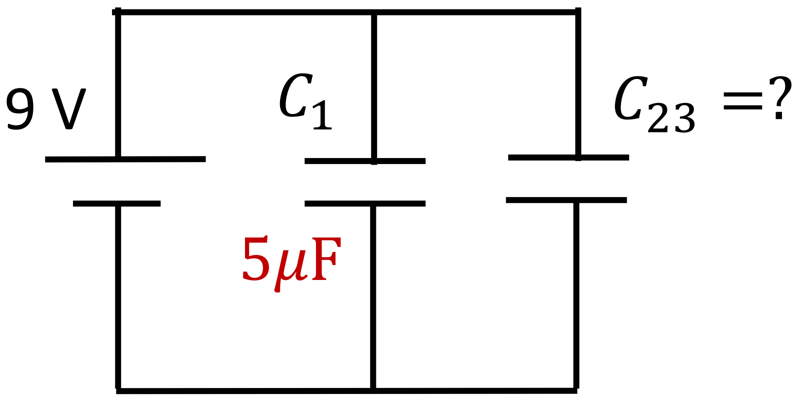
E. $U_1 > U_3 > U_2$

$$U_c = \frac{Q^2}{2C}$$

Q: Rank the three capacitors according to their stored energy.

$$U_c = \frac{Q^2}{2C} = \frac{C(\Delta V_C)^2}{2}$$





$$C_{23} = \frac{4 \cdot 6}{(4+6)} = 2.4\ \mu F$$

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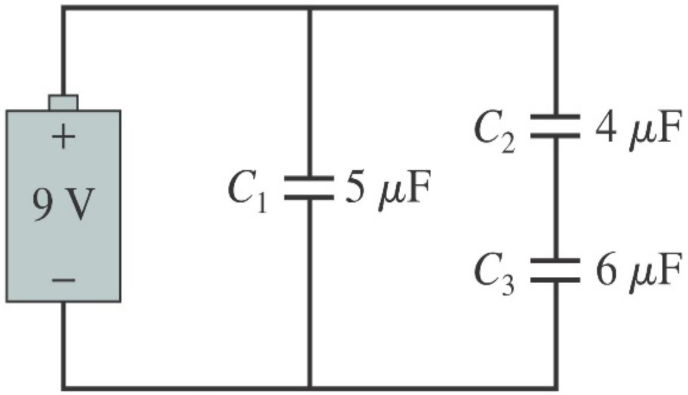
C. $U_1 > U_2 = U_3$
D. $U_2 > U_3 > U_1$
E. $U_1 > U_3 > U_2$

- $U_1 > U_{23}$, using $U_c = \frac{C(\Delta V_C)^2}{2}$ (since they have the same ΔV_C, and we only need to compare C_1 and C_{23})
- U_1 is the largest of all. How we can compare U_2 and U_3?
- $U_2 > U_3$, using $U_c = \frac{Q^2}{2C}$ (since they have the same Q and we only need to compare C_2 and C_3)

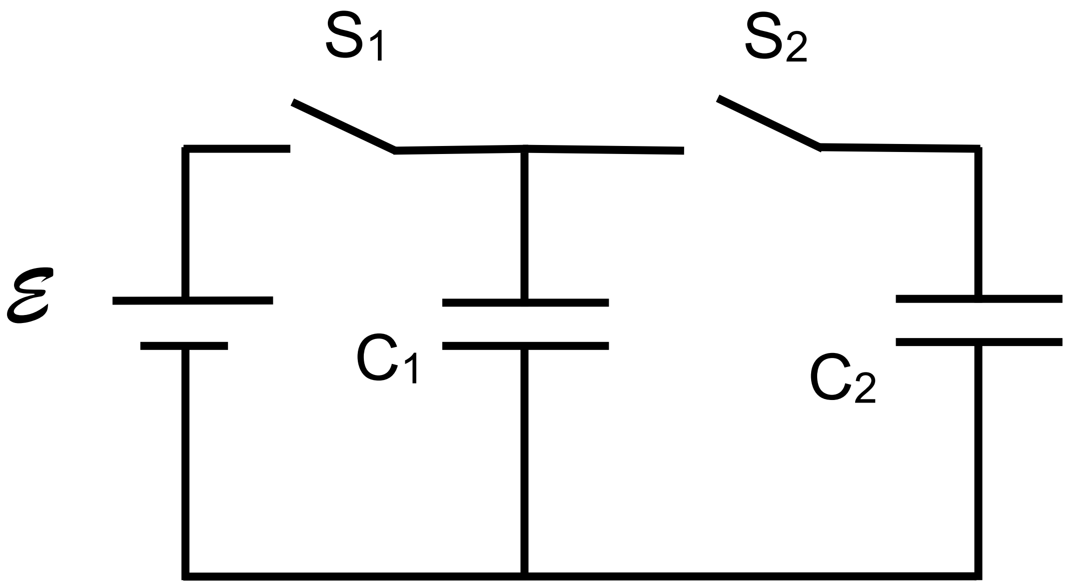
Q: Exercise, on your own: calculate the energies explicitly.

$$\boxed{U_c = \frac{Q^2}{2C} = \frac{C(\Delta V_C)^2}{2}}$$



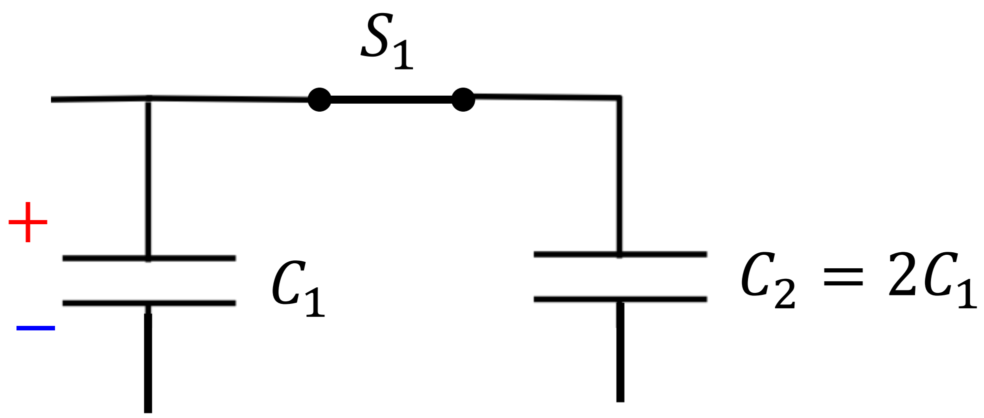
Answer: $U_1 = 203\ \mu J$; $U_2 = 58\ \mu J$; $U_3 = 39\ \mu J$

Circuits with switches



Switches change the characteristics of a circuit by providing or removing paths for current

Q: A Physics 159 student has wired the circuit below. Suppose C_1 has charge Q, and C_2 has charge = 0 before the switch S_1 is closed.



After S_1 is closed the final charges on C_1 and C_2 are:

A. $Q_1 = Q/2,\ \ Q_2 = Q/2$
B. $Q_1 = 0,\ \ Q_2 = +Q$
C. $Q_1 = Q/4,\ \ Q_2 = 3Q/4$
D. $Q_1 = Q,\ \ Q_2 = 0$
E. None of the above

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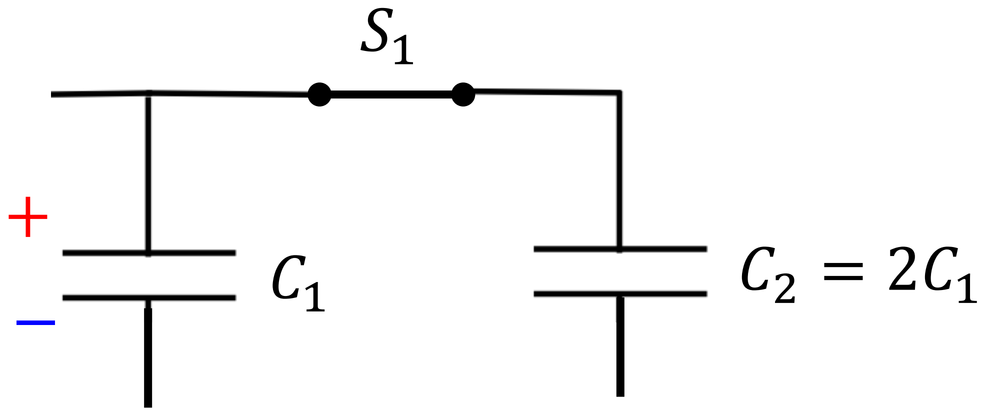
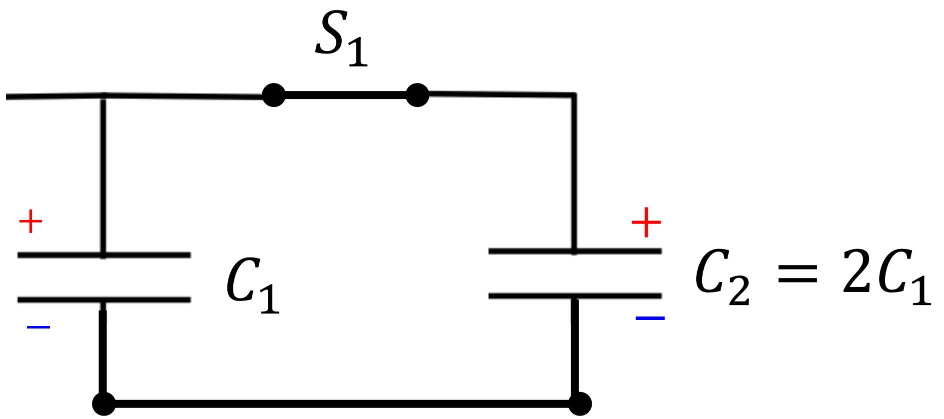


- The positive charges on the top plate repel each other and "want" to spread, but they are attracted by the negative charges at the bottom pate, which are "locked" at the lower plate if the bottom plates are not connected.

After S_1 is closed the final charges on C_1 and C_2 are:

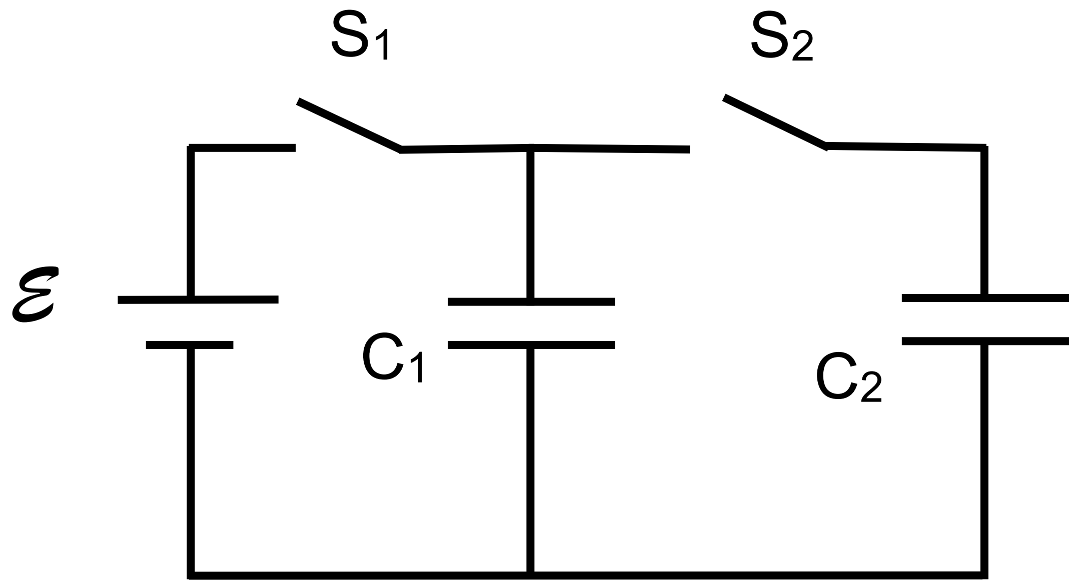
A. $Q_1 = Q/2, \quad Q_2 = Q/2$
B. $Q_1 = 0, \quad Q_2 = +Q$
C. $Q_1 = Q/4, \quad Q_2 = 3Q/4$
D. $Q_1 = Q, \quad Q_2 = 0$
E. None of the above

- What happens if we close the circuit by connecting bottom plates?
- Now both + and – can spread over the two capacitors => the second capacitor gets charged!



Q: Initially both capacitors are uncharged, switches are open. The following procedure is then performed:

1. Switch S_1 is closed for a long time.
2. Switch S_1 is opened.
3. Switch S_2 is closed.



a) What happens right after step 3, i.e. when switch S_2 is closed? Explain.

b) What is conserved in step 3? Explain.

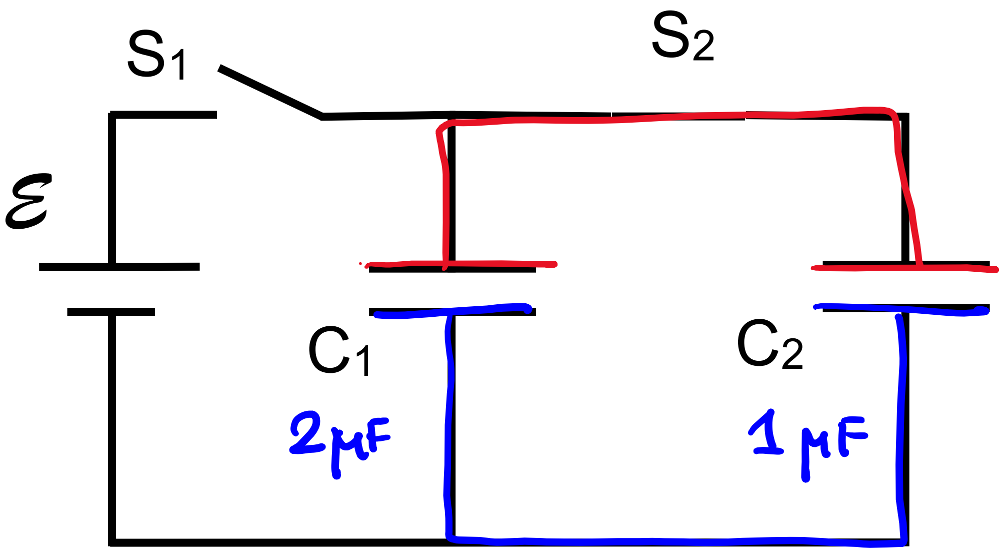
c) Calculate the final charges on both C_1. Use $\varepsilon = 24\ V,\ C_1 = 2\ \mu F$ and $C_2\ =\ 1\ \mu F$.

Q1: What is conserved in Step 3 (when S_2 is closed)?

A. Energy and Charge
B. Energy
C. Charge
D. Voltage
E. Voltage and Energy

Q2: After S_2 is closed and new equilibrium is reached, what is true about the two capacitors?

A. They must have the same charge across them
B. They must have the same energy
C. The voltage across them is the same ✓
D. More than one is true



Q1: What is conserved in Step 3 (when S_2 is closed)?

A. Energy and Charge
B. Energy
C. Charge
D. Voltage
E. Voltage and Energy

- Net charge is always conserved (law of Nature).
- Whether or not energy conserves is to be verified (we are changing the system by closing the switch!)

Q2: After S_2 is closed and new equilibrium is reached, what is true about the two capacitors?

A. They must have the same charge across them
B. They must have the same energy
C. The voltage across them is the same
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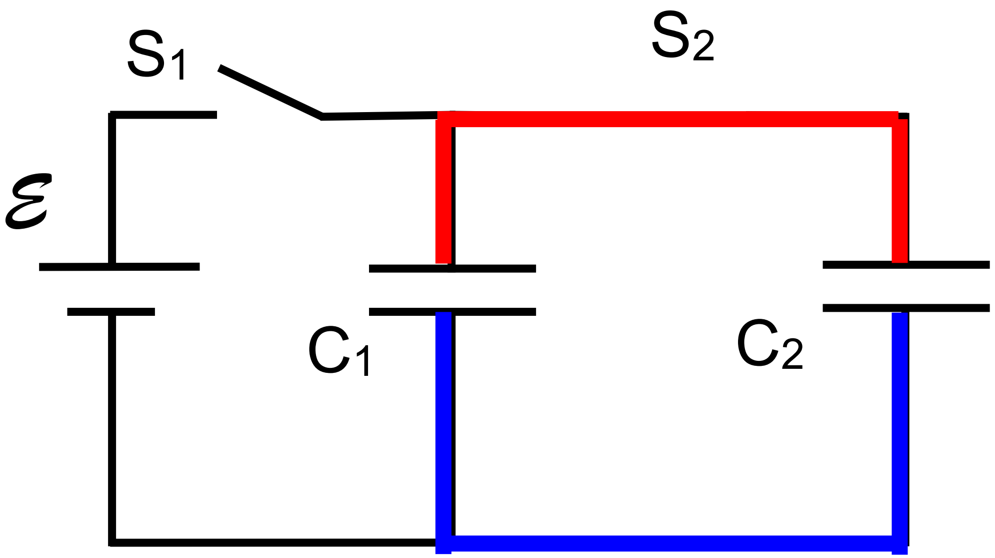


Q: The final charges Q_{1f} and Q_{2f} on C_1 and C_2 are:

A. 48 μC, 48 μC

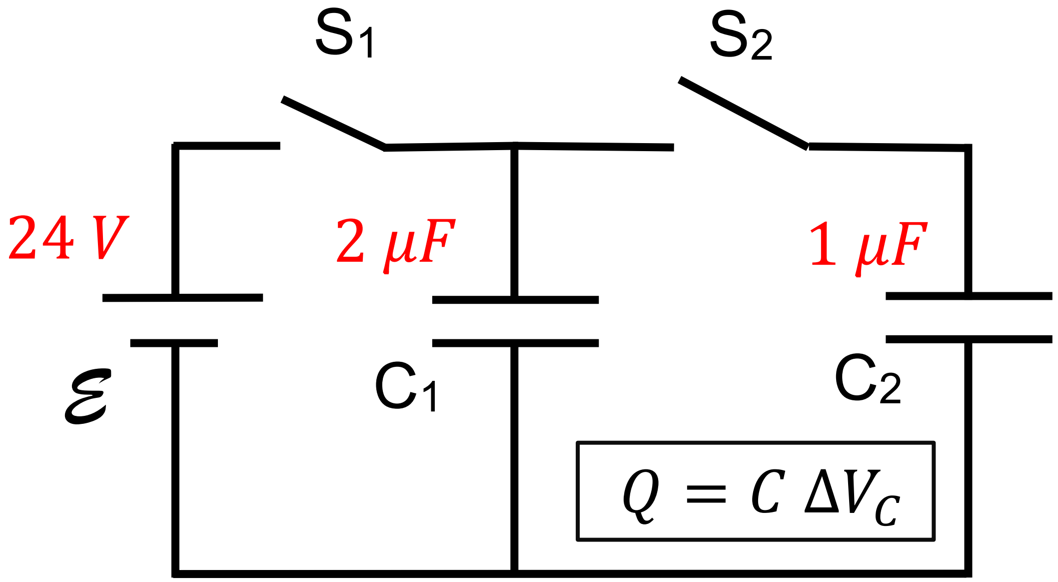
B. 24 μC, 24 μC

C. 40 μC, 20 μC

D. 48 μC, 24 μC

E. 32 μC, 16 μC

$Q_i = C_1 \mathcal{E} = 48\,\mu C$

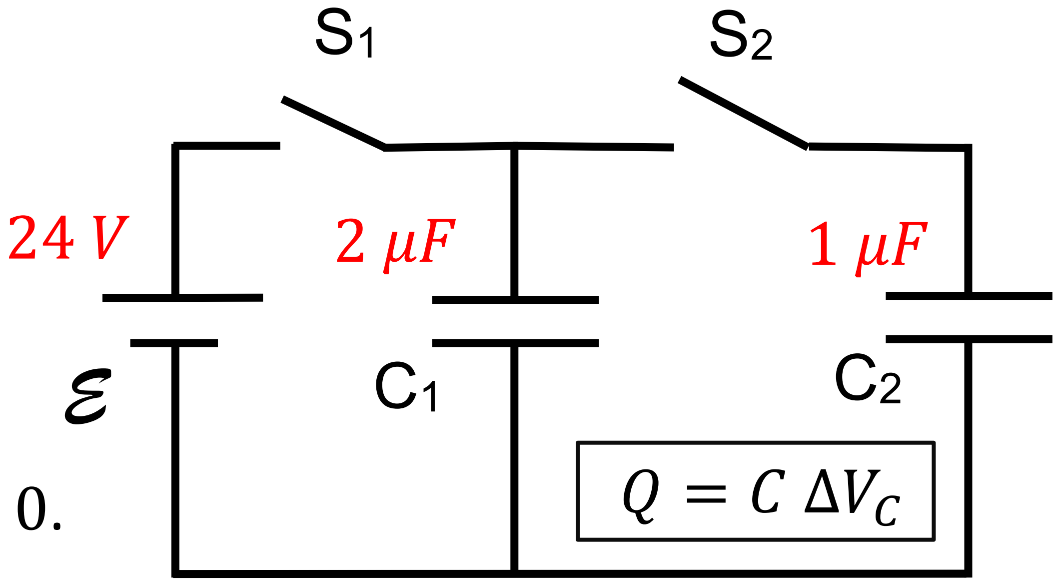


Q: The final charges Q_{1f} and Q_{2f} on C_1 and C_2 are:

A. 48 μC, 48 μC
B. 24 μC, 24 μC
C. 40 μC, 20 μC
D. 48 μC, 24 μC
E. 32 μC, 16 μC

- **Step 1:** C_1 is charged to:
$Q_{1i} = (2\,\mu F)(24V) = 48\,\mu C$,
and $Q_{2i} = 0$ (disconnected).

- **Hence:** $Q_{1i} = 48\,\mu C$ and $Q_{2i} = 0$.



$Q = C\,\Delta V_C$

- **Step 2:** Charge is always conserved (Total charge before) = (Total charge after)

$$Q_{1i} + Q_{2i} = \underbrace{Q_{1f} + Q_{2f} = 48\,\mu C}_{\text{eq (1)}}$$

$\Delta V_f = \frac{Q_{1f}}{C_1} = \frac{Q_{2f}}{C_2}$ eq (2)

Two unknowns: Q_{1f} and Q_{2f}

$Q_{1f} + Q_{2f} = 48\,\mu C$

$\frac{Q_{1f}}{C_1} = \frac{Q_{2f}}{C_2}$

$Q_1 = 32\,\mu C$

$Q_2 = 16\,\mu C$

- **Note:** the final voltage is the same on C_1 and C_2. If we call it ΔV_f:

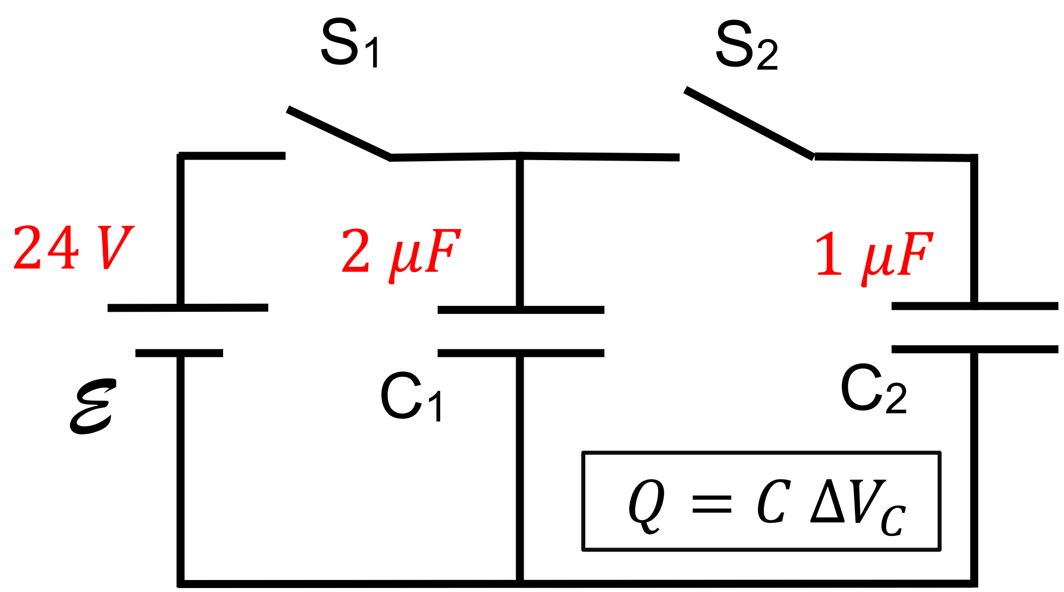
$$Q_{1f} + Q_{2f} = C_1\Delta V_f + C_2\Delta V_f = 48\,\mu C \quad \Rightarrow \quad \Delta V_f = (48\,\mu C)/(C_1 + C_2) = 16\,V$$

- **We get:** $Q_{1f} = C_1\Delta V_f = (2\,\mu F)(16\,V) = 32\,\mu C$ and $Q_{2f} = C_2\Delta V_f = (1\,\mu F)(16\,V) = 16\,\mu C$

alternative solution for Q_{1f} and Q_{2f}

What about the energy?

- So $\Delta V_i = \varepsilon = 24\ V$, and $\Delta V_f = \frac{48\ \mu C}{3\ \mu F} = 16\ V$
- Electrical Energy $= U_c = \frac{C \Delta V_C^2}{2}$



$$\frac{(48\mu C)^2}{2(2\mu F)}$$

$$=$$

$$U_i = \frac{C_1 \Delta V_{1i}^2}{2} = \frac{(2\ \mu\text{F})(24\ V)^2}{2} = 576\ \mu J$$ (initially there is no charge on C_2)

$$U_f = \frac{C_1 \Delta V_{1f}^2}{2} + \frac{C_2 \Delta V_{2f}^2}{2} = \frac{(2\ \mu\text{F} + 1\ \mu F)(16\ V)^2}{2} = 384\ \mu J$$ (parallel combination)

$$\frac{(32\mu C)^2}{2(2\mu F)} + \frac{(16\mu C)^2}{2(1\mu F)}$$

U_i is not equal to U_f!

Q: Where did the missing energy go?

Q: Where did the missing energy go?

Quoting from Wikipedia: “This problem has been discussed in electronics literature at least as far back as 1955.” In short:

- A model of a circuit with two capacitors and zero-resistance-wires is a bit “too ideal”. Though it gives us correct predictions about the distribution of charges, voltages and stored energies, it cannot explain where the energy goes. It also predicts infinite current when the switch is closed – unphysical!
- To remedy the situation, we need to account for small (but generally non-zero) resistance in wires. Then we can state that the missing energy was converted to heat and emitted as electromagnetic waves.
- This nicely brings us to the topic of RC-circuits (circuits with capacitors and resistors) which we are going to switch now.

$$\Delta V = IR \rightarrow I = \frac{\Delta V}{0} = \infty!$$

https://en.wikipedia.org/wiki/Two_capacitor_paradox

Time-dependent DC circuits

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Time-dependent DC circuits: RC-circuits

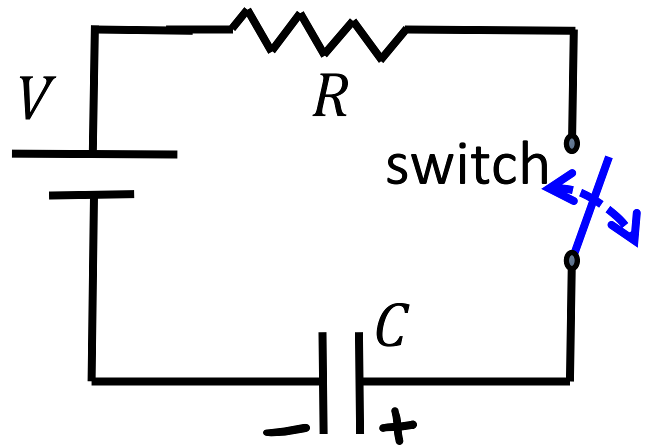
Text: 26.4

- Charging a capacitor
- Discharging a capacitor
- Time constant ($\tau = RC$)

Combining elements - ***RC* circuits + switch**

- Thus far we have considered only steady and continuous currents. However, many important circuit applications use combination of capacitors and resistors (and inductors) to produce time-dependent currents.
- Examples: Intermittent car wipers, filtering (or 'cleaning') wireless signals in a cordless phone, remote control, etc.

Simple RC Series Circuit

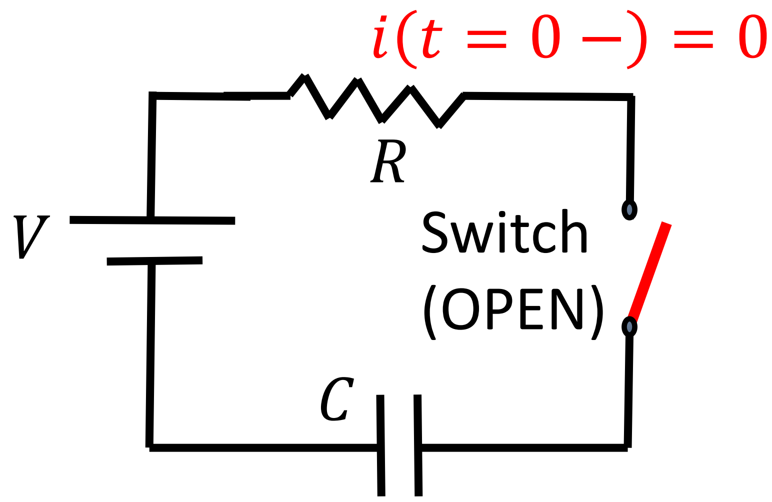


- When the switch is open:
  - No current can flow in the circuit.
  - The charge on the capacitor and the voltage across it both remain zero ($\Delta V_C = Q/C$).
- After the switch has been closed for a very long time:
  - Charge on the capacitor is $Q = CV$
  - Again, no current flows in the circuit (capacitor does not accept more charge).
- Intermediate times:
  - Dynamics! $I \Rightarrow i(t)$: current depends on time

CASE 1: Charging a capacitor - 1

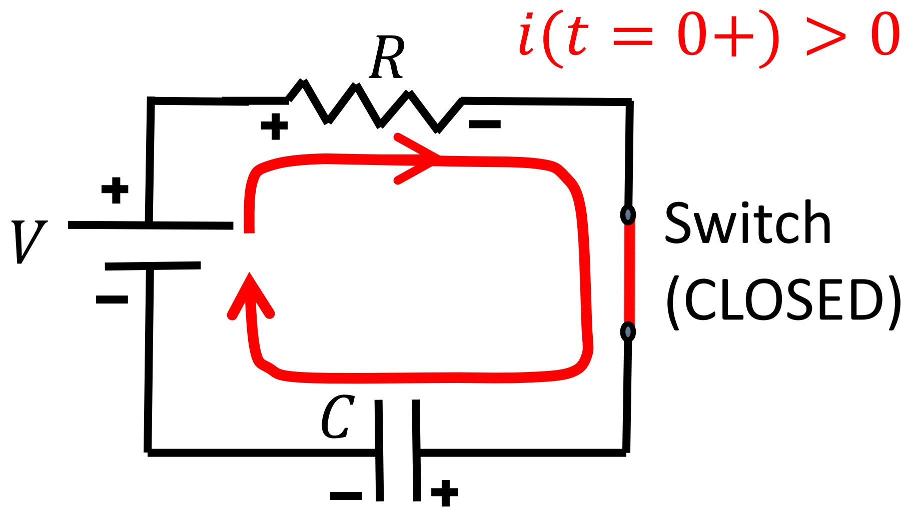
Switch is open until $t = 0$

Initial charge $q_c(t = 0) = 0$



Switch is suddenly

closed at $t = 0 +$



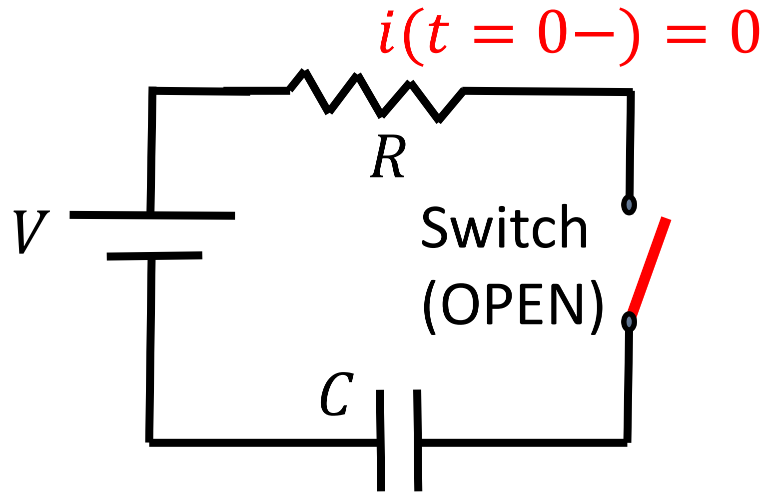
- Once the switch is closed, current starts flowing in the circuit.

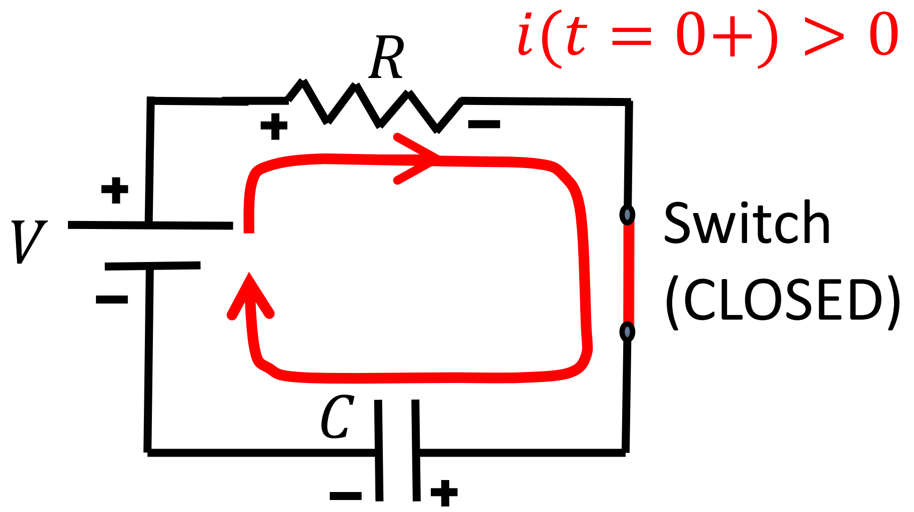
- Electric charges cannot propagate through the empty space between capacitor's plates. However, what happens is that the capacitor is charging: battery is delivering charges to one plate and removes them from the other as if the current is passing through the capacitor.

CASE 1: Charging a capacitor - 2

Switch is open until $t = 0$
Initial charge $q_c(t = 0) = 0$

Switch is suddenly closed at $t = 0+$





- Voltage drop across a capacitor:
  - Magnitude: $\Delta V_c = \frac{q}{C}$ with $q = q(t)$ => hence $\Delta V_c = \Delta V_c(t)$
  - Sign: voltage is higher at the plate that carries positive charge

- **Note that immediately after the switch is closed**, the capacitor acts like an ideal wire (short circuit). There is no charge on the capacitor so $\Delta V_C = 0$ and $i\ (0+) = V/R$

CASE 1: Charging a capacitor - 3

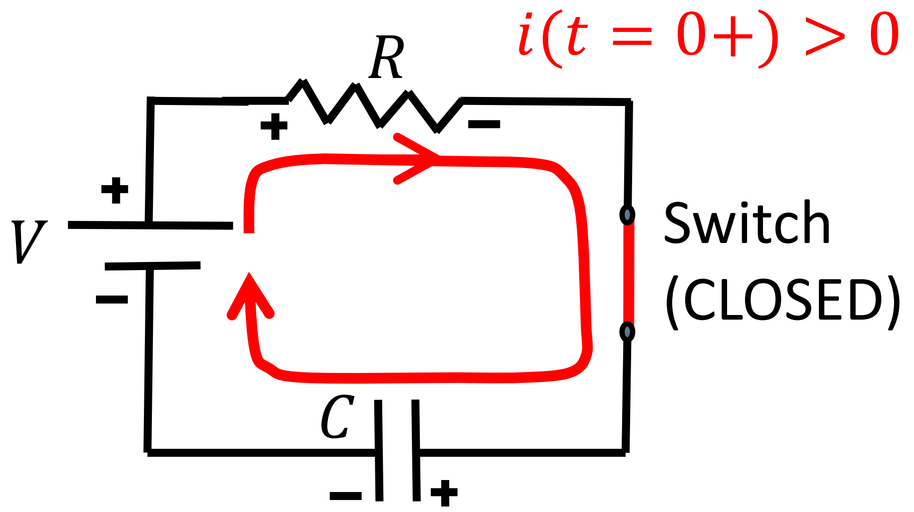
Initial charge $q_c(t=0) = 0$
Switch is suddenly
closed at $t = 0+$

- Kirchhoff loop law (travel CW):

$$V - iR - \frac{q}{C} = 0$$

- Let's take time derivative:

$$0 - R\frac{di}{dt} - \frac{1}{C}\frac{dq}{dt} = 0$$



- Note that: $\boxed{\frac{dq}{dt} = i,}$ hence: $\boxed{-R\frac{di}{dt} - \frac{i}{C} = 0.}$

- Here we take $\frac{dq}{dt} = +i$ since current i brings + charge to the positive plate => charge on the plates increases => $\frac{dq}{dt} > 0.$

- We have got a differential equation for $i(t)$, and a connection between $q(t)$ and $i(t)$