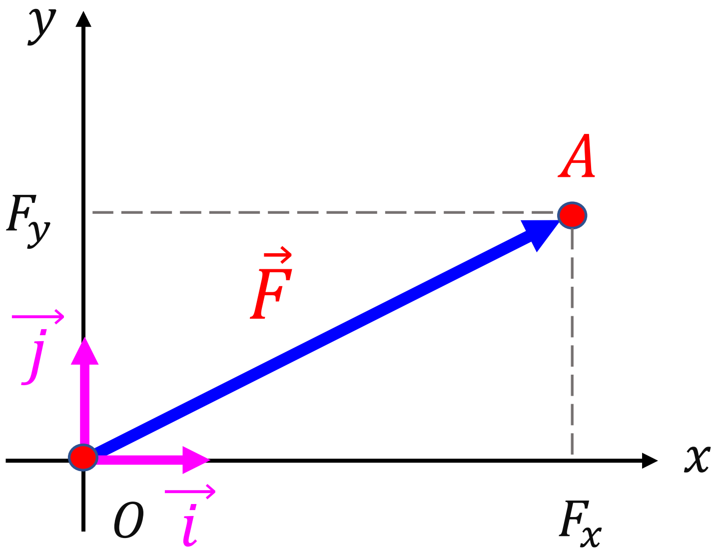
Announcements

- **Mastering Engineering** (Homework assignments)
  - ➢ Registration: please use your student number as your "student ID" (will help us to give your Mastering Engineering marks to <u>you</u>, and not to someone else)
  - ➢ First assignments (due Sunday, January 21, 10:00 pm):
    - ❖ Introductory assignment (worth marks!) – already available
    - ❖ First homework assignment – will be available Friday, 6:00 pm

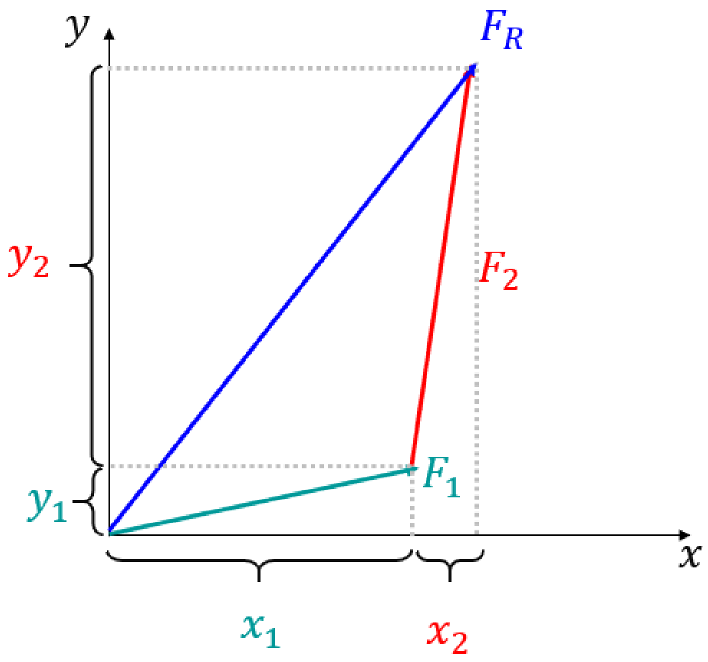
- **Tutorials** start next week
  - ➢ Paper, straightedge, calculator!!!

Last Time



$$\vec{F} = \vec{i}\ F_x + \vec{j}\ F_y$$

- Each vector can be expressed in Cartesian components



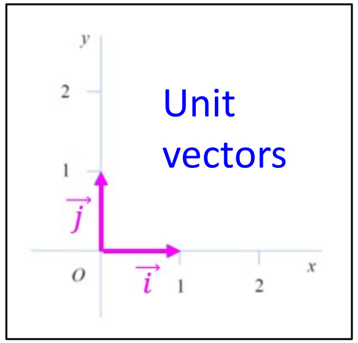
$$F_{R,x} = F_{1,x} + F_{2,x}$$

$$F_{R,y} = F_{1,y} + F_{2,y}$$

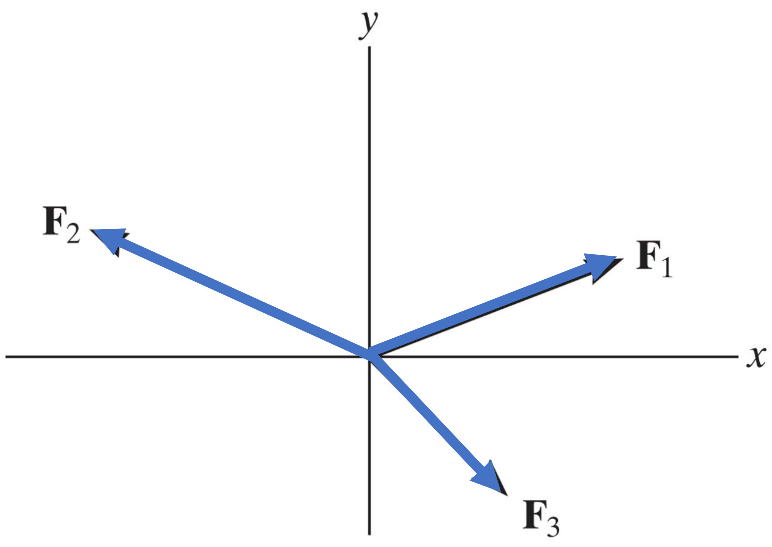
- Cartesian components are a perfect tool for adding vectors

VECTOR ADDITION: Analytical approach

$$\boxed{\vec{F} = \vec{i}\, F_x + \vec{j}\, F_y}$$



- What is so good about Cartesian representation of a vector?
- It gives us a super simple way to add up vectors: **vectors add up component-wise**



$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 =$$

$$= \left(\vec{i}\, F_{1x} + \vec{j}\, F_{1y}\right) + \left(\vec{i}\, F_{2x} + \vec{j}\, F_{2y}\right) + \left(\vec{i}\, F_{3x} + \vec{j}\, F_{3y}\right) =$$

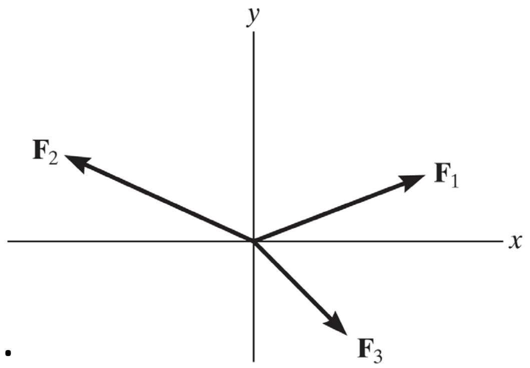
$$= \vec{i}\left(F_{1x} + F_{2x} + F_{3x}\right) + \vec{j}\left(F_{1y} + F_{2y} + F_{3y}\right) =$$

$$= \vec{i}\, F_{Rx} + \vec{j}\, F_{Ry}$$

positive or negative

- Q: Explain the recipe of finding the Cartesian components of the resultant vector.
  Now, how can you find its magnitude?

ADDING UP VECTORS IN CARTESIAN COORDINATES: Summary



- Vectors add up **component-wise !!!**

- To get x-component of the resultant, just add up x-components of all the vectors, etc.:

$$\boxed{F_{Rx} = \sum_i F_{ix} \quad \text{and} \quad F_{Ry} = \sum_i F_{iy}}$$

❖ Cartesian components of the resultant vector

- Knowing F_{Rx} and F_{Ry}, you can determine the magnitude and the direction of $\vec{F}_R$. For example:

❖ Magnitude of the resultant vector:

$$\boxed{F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}}$$

❖ Angle from positive-x direction:

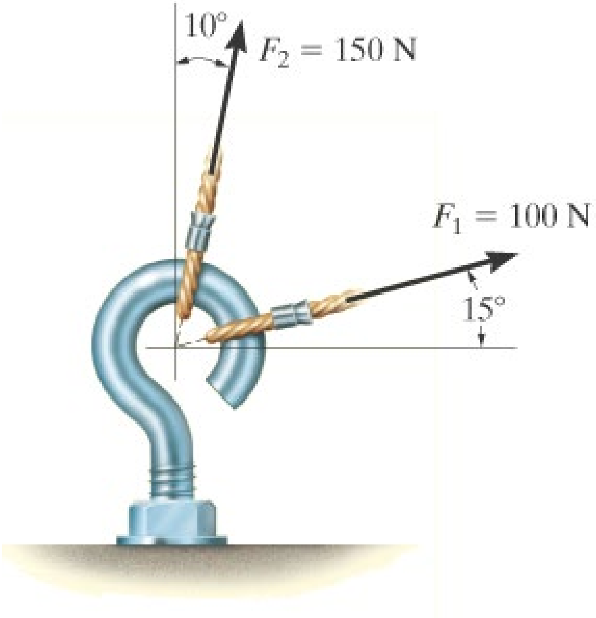
$$\boxed{\tan\theta_R = \frac{F_{Ry}}{F_{Rx}}}$$

(and remember to check whether the answer coming out of your calculator makes sense)

W1-1b: The screw eye is subject to two forces, $\vec{F}_1$ and $\vec{F}_2$. Determine the direction and the magnitude of the resultant force. Use vector addition in components.

Textbook:

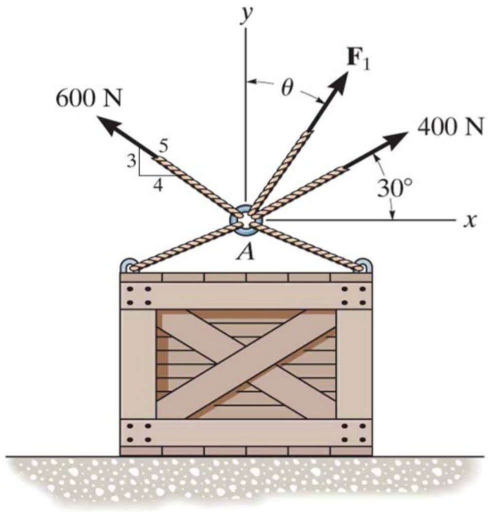
Example 2.1



On your own, please

$$F_1 = 100\ N$$
$$F_2 = 150\ N$$

W1-2: Determine the magnitude and direction measured counter-clockwise from the positive *x*-axis of the resultant force of the three forces acting on the ring *A*. Take F_1 = 500 N and θ = 20°.



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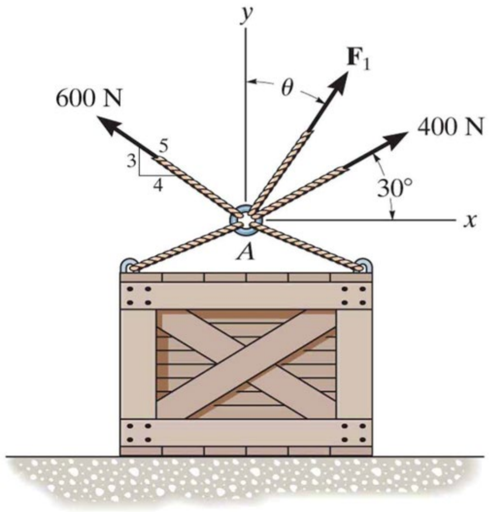
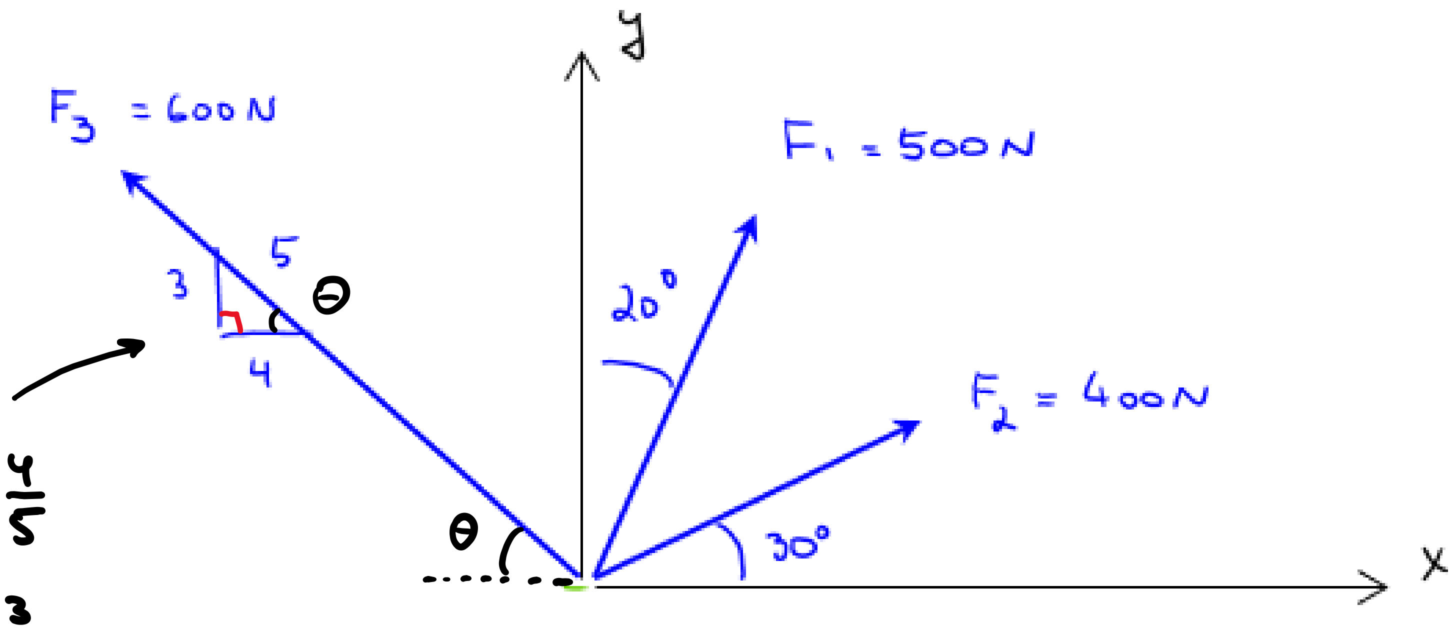


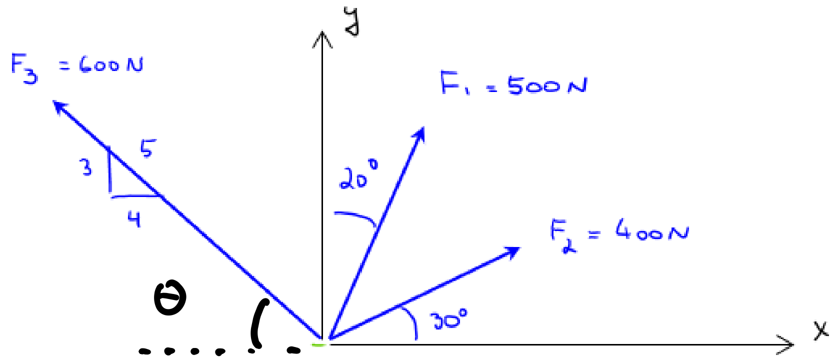
- Draw a picture using a straightedge, not something else!
- Label all givens!
- Think about what you are going to do



$\cos\theta = \frac{4}{5}$

$\sin\theta = \frac{3}{5}$

W1-2: Determine the magnitude and direction measured counter-clockwise from the positive *x*-axis of the resultant force of the three forces acting on the ring *A*. Take F_1 =500 N and θ = 20º.



$$F_{3x} = -F_3\cos\theta$$

$$F_{3y} = +F_3\sin\theta$$

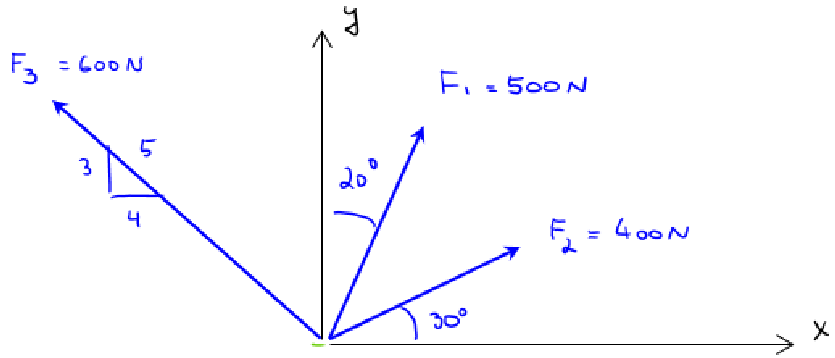
- Finding the resultant (vector):

$$
\begin{array}{l}
\vec{F}_1 = (500\sin 20^\circ)\,\vec{i} + (500\cos 20^\circ)\,\vec{j} \\
\qquad\quad + \qquad\qquad\qquad\quad + \\
\vec{F}_2 = (400\cos 30^\circ)\,\vec{i} + (400\sin 30^\circ)\,\vec{j} \\
\qquad\quad + \qquad\qquad\qquad\quad + \\
\vec{F}_3 = \left(-600\cdot\frac{4}{5}\right)\vec{i} + \left(600\cdot\frac{3}{5}\right)\vec{j} \\
\qquad\quad \| \qquad\qquad\qquad\quad \| \\
\qquad\quad F_{Rx} \qquad\qquad\quad\;\; F_{Ry}
\end{array}
$$

$$\vec{F}_R = (\;\ldots\;)\,\vec{i} + (\;\ldots\;)\,\vec{j}$$

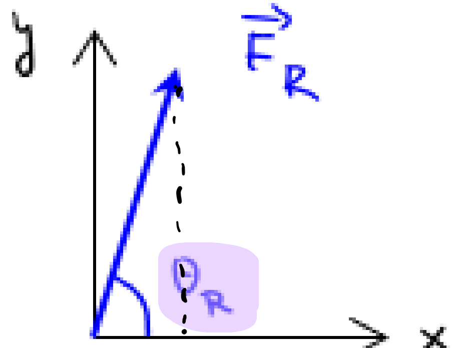
$$\vec{F}_R = \left(500\sin 20^\circ + 400\cos 30^\circ - 600\frac{4}{5}\right)\vec{i} + \left(500\cos 20^\circ + 400\sin 30^\circ + 600\frac{3}{5}\right)$$

W1-2: Determine the magnitude and direction measured counter-clockwise from the positive *x*-axis of the resultant force of the three forces acting on the ring *A*. Take F_1 =500 N and θ = 20º.



- Finding the resultant's magnitude and direction:

$$\vec{F}_R = \underbrace{37.42}_{F_{Rx}}\vec{i} + \underbrace{1030}_{F_{Ry}}\vec{j}$$

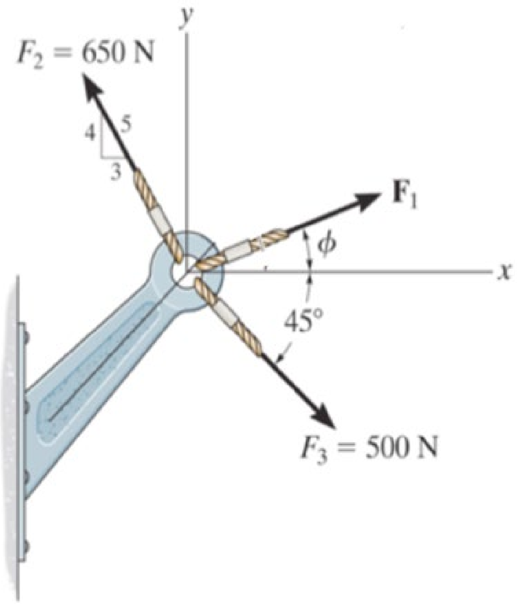


$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} =$$

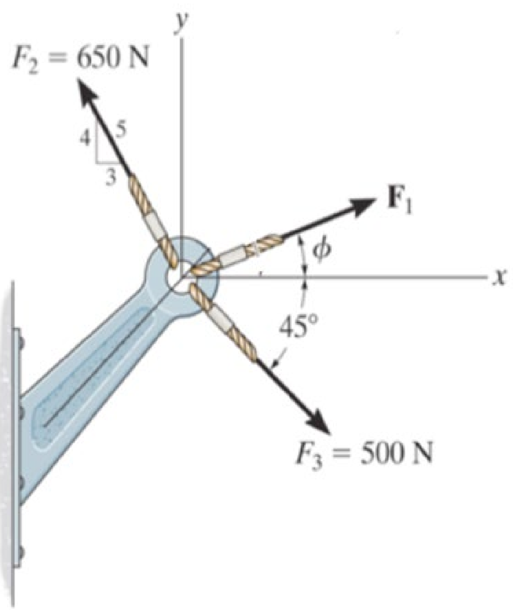
$$= 1031\text{ N} = 1031\text{ N} \rightarrow \boxed{F_R = 1.03\text{ kN}}$$

$$\tan\theta_R = \frac{F_{Ry}}{F_{Rx}} = \frac{1030}{37.42} \longrightarrow \boxed{\theta_R = 87.9^\circ}$$

W1-3: The magnitude of the resultant force acting on the bracket is 400 N. Determine the magnitude of $\vec{F}_1$. Take ϕ = 30º.



W1-3: The magnitude of the resultant force acting on the bracket is 400 N. Determine the magnitude of $\vec{F}_1$. Take ϕ = 30º.



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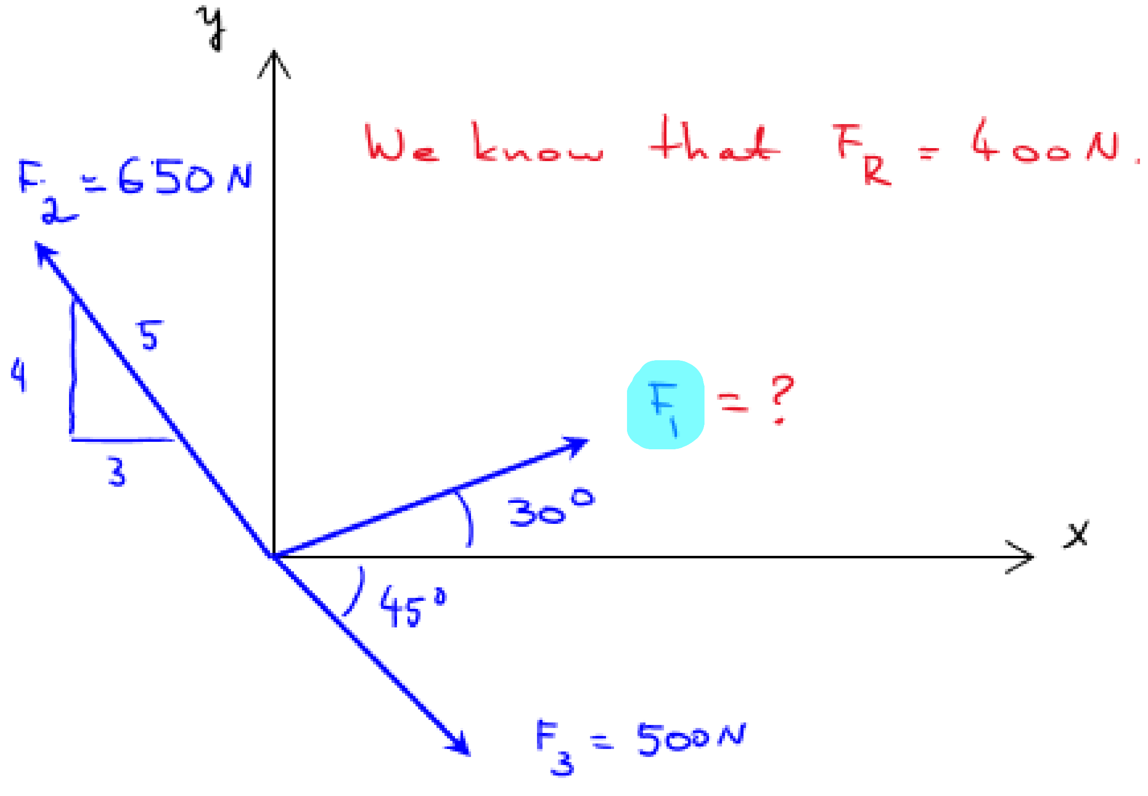
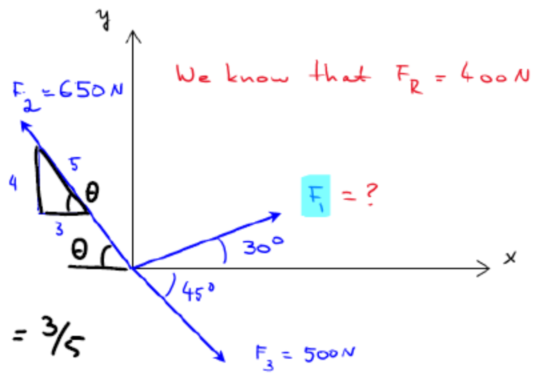


- $F_R = 400\ N$
- $\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$

$$F_{Rx}^2 + F_{Ry}^2 = 400^2$$

- Finding the resultant (vector):



$\cos\theta = 3/5$

$\sin\theta = 4/5$

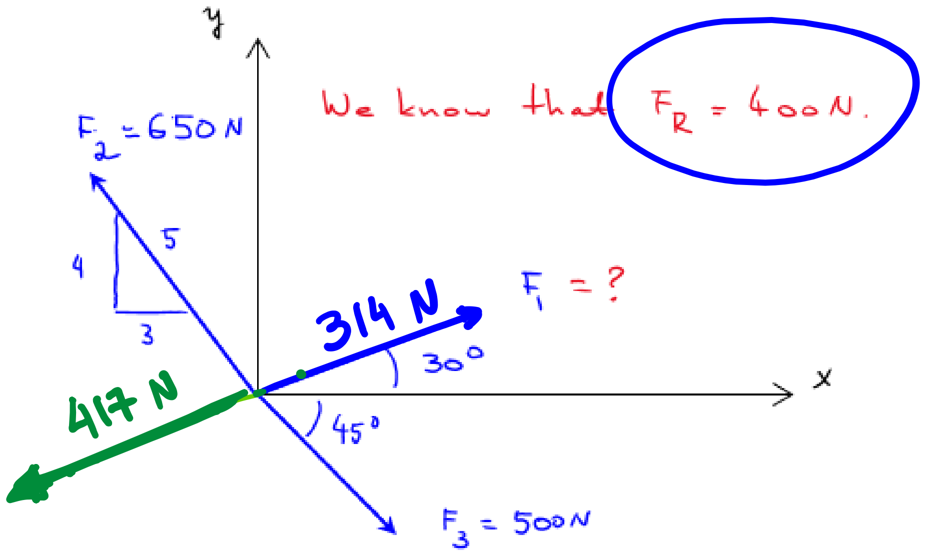
$$\begin{aligned}
\vec{F}_1 &= (F_1 \cos 30^\circ)\,\vec{i} + (F_1 \sin 30^\circ)\,\vec{j} \\
&\quad + \qquad\qquad\qquad + \\
\vec{F}_2 &= \left(-650\,\tfrac{3}{5}\right)\vec{i} + \left(650\,\tfrac{4}{5}\right)\vec{j} \\
&\quad + \qquad\qquad\qquad + \\
\vec{F}_3 &= (500 \cdot \cos 45^\circ)\,\vec{i} + (-500 \sin 45^\circ)\,\vec{j}
\end{aligned}$$

$$\vec{F}_R = (F_{Rx})\,\vec{i} + (F_{Ry})\,\vec{j}$$

$$\vec{F}_R = \left(\overbrace{F_1 \cos 30^\circ - 650\,\frac{3}{5} + 500\,\frac{1}{\sqrt{2}}}^{F_{Rx}}\right)\vec{i} + \left(\overbrace{F_1 \sin 30^\circ + 650\,\frac{4}{5} - 500\,\frac{1}{\sqrt{2}}}^{F_{Ry}}\right)\vec{j}$$

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

- Finding F_1:



$$\vec{F}_R = \left(F_1 \frac{\sqrt{3}}{2} - 36.45\right)\vec{i} + \left(F_1 \frac{1}{2} + 166.4\right)\vec{j}$$

$$400^2 = \left(F_1 \frac{\sqrt{3}}{2} - 36.45\right)^2 + \left(F_1 \frac{1}{2} + 166.4\right)^2$$

$$F_1^2 + 103.27 F_1 - 130966.4 = 0$$

$F_1 =$ 314 N (A.)

$F_1 =$ ~~-417 N~~ (B.)

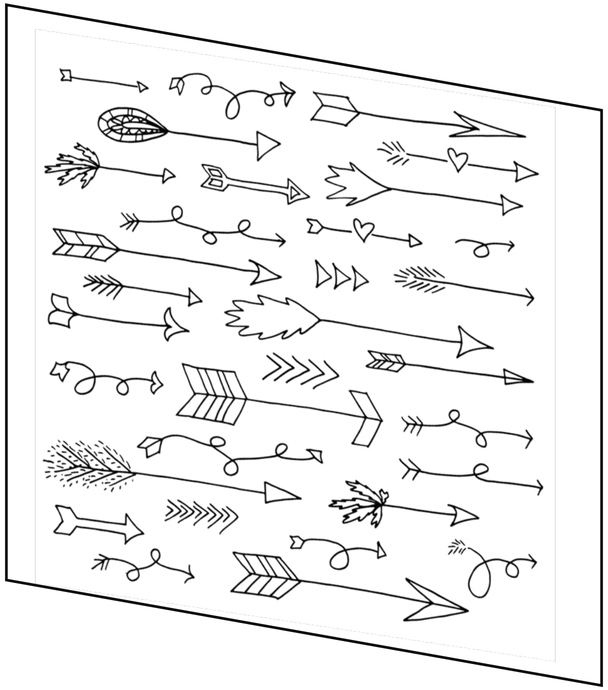
C.

REMARKS

- Things to be careful about when adding up vectors in Cartesian components:
  - Sin or Cos? Depends on how the angles are defined.
  - Sign of the projection!
  - Always draw a sketch, to understand where the vector is pointing => what should be the signs of its components. Don’t trust your calculator blindly!

- Mathematical & general stuff:
  - We will solve non-trivial multi-step problems. Try to make a plan, think about your strategy, get ready to go over the cycle “plan => try => reflect => adjust your plane => try => …”
  - Number of equations should always be equal to the number of unknowns => then you can find all your unknowns. Checking the number of equations and the number of unknowns may help you to understand whether you are on a right track.
  - Practice will help.

Force Vectors in 3D: Intro



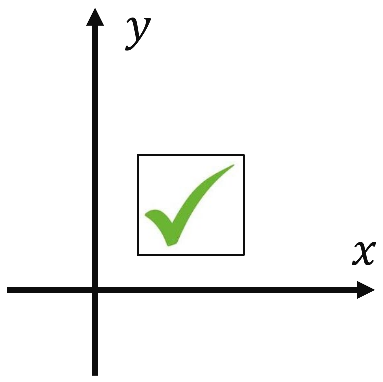
Text: 2.5-2.6

Content:

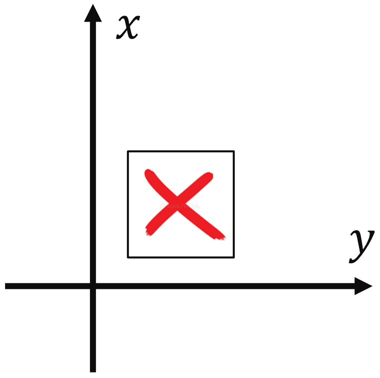
- Right-handed coordinate systems
- Description of Vectors in 3D:
  - Cartesian coordinates: Unit vectors, Projections of a vector
  - Representation through direction cosines
  - Conversion between these two descriptions
- Adding force vectors in 3D
- Practice (W2-1 – W2-3)

RIGHT-HANDED COORDINATE SYSTEMS

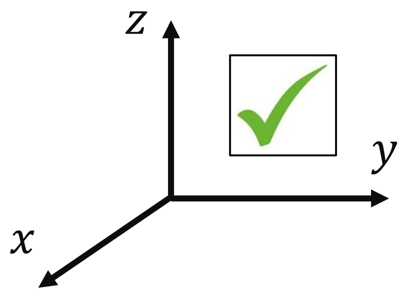
- Even if it goes unnoticed, we always draw a 2D coordinate system in a specific way:



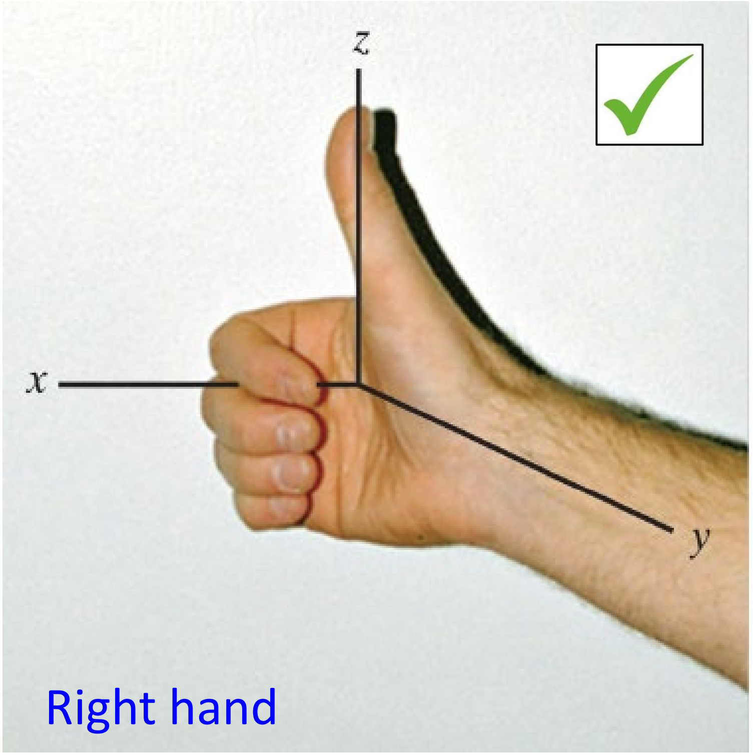
NOT



- Same applies for a 3D ("right-handed" coordinate system):

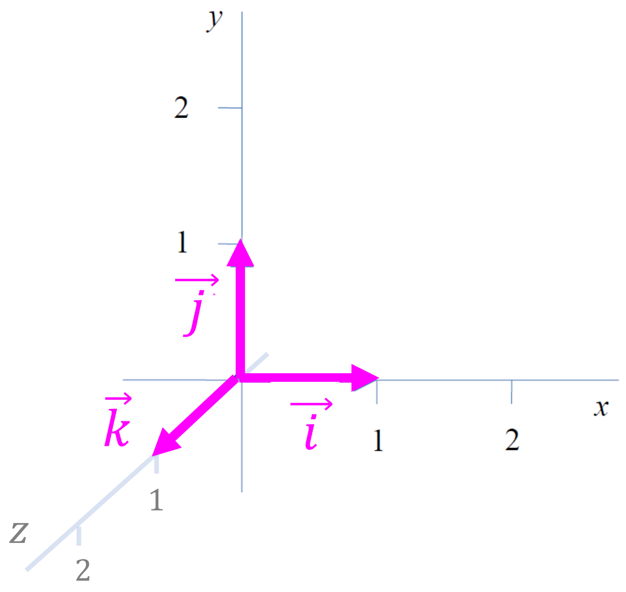


- You can rotate this object as you please, but you cannot interchange, e.g., x and y axes



VECTORS IN A CARTESIAN COORDINATE SYSTEM: Three Dimensions (3D)

- **Cartesian coordinate system in 3D**:



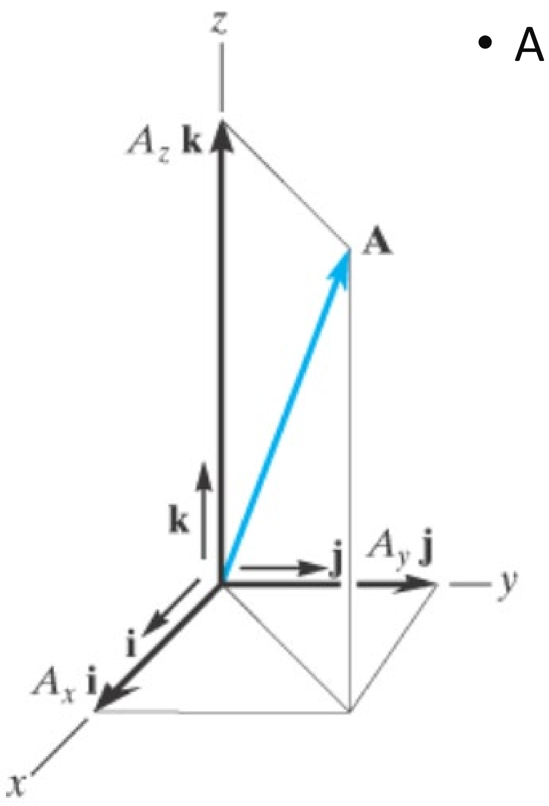
- Perpendicular axes: x, y and z (right-hand rue)
- **Unit vectors** (= “orts”, = “basis vectors”):
  - $\vec{i}$ -- vector of length 1 in the positive direction of x-axis
  - $\vec{j}$ -- vector of length 1 in the positive direction of y-axis
  - $\vec{k}$ -- vector of length 1 in the positive direction of z-axis

- Why they are called “unit vectors”:

$$i = |\vec{i}| = j = |\vec{j}| = k = |\vec{k}| = 1$$

VECTORS IN A CARTESIAN COORDINATE SYSTEM: Three Dimensions (3D)

- **Vector $\vec{A}$ in Cartesian coordinates in 3D**:



- Any vector $\vec{A}$ in 3D can be uniquely **resolved into three components** along x, y and z:

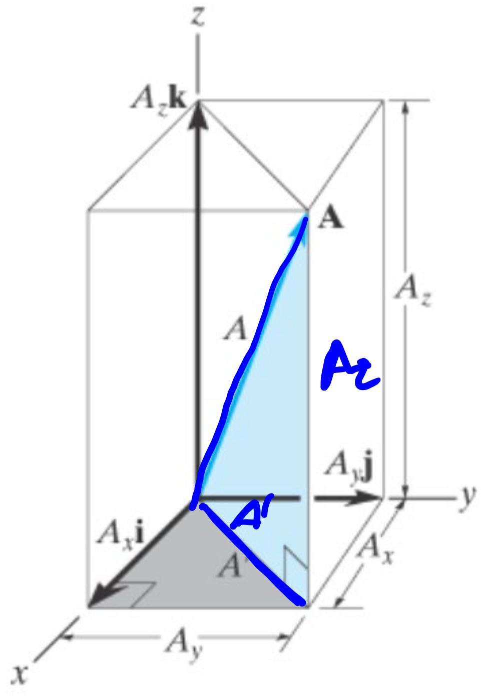
$$\boxed{\vec{A} = \vec{i}\ A_x + \vec{j}\ A_y + \vec{k}\ A_z}$$

$$A^2 = A_z^2 + (A')^2 \quad \text{(blue)}$$

$$(A')^2 = A_x^2 + A_y^2 \quad \text{(grey)}$$

- The magnitude of a 3D vector:

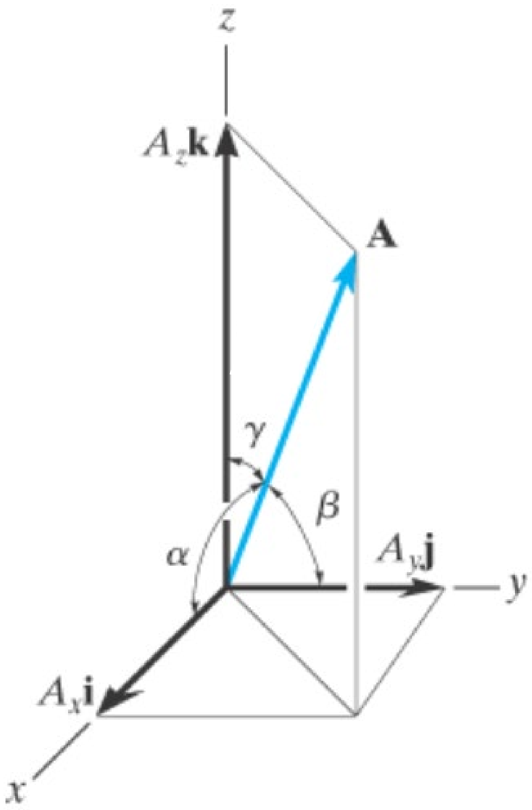
$$\boxed{A = |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}}$$



- Everything we discussed in 2D (positive / negative components, component-wise vector addition, etc.) applies also in 3D

DIRECTON ANGLES: Three Dimensions (3D)

- **Another description**:



- Let us define the angles between the vector $\vec{A}$ and the **positive** directions of x,y,z-axes: α, β, γ

$$A_x = A\cos\alpha \qquad A_y = A\cos\beta \qquad A_z = A\cos\gamma$$

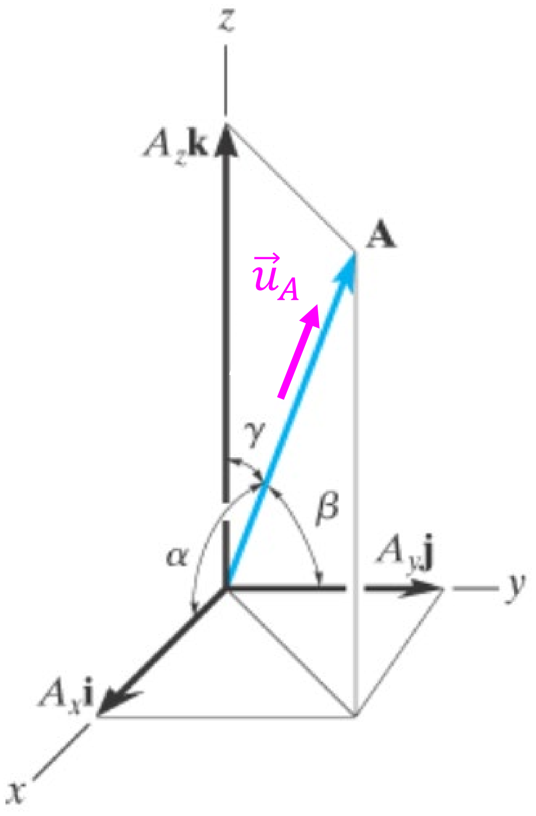
$$\cos\alpha = \frac{A_x}{A} \qquad \cos\beta = \frac{A_y}{A} \qquad \cos\gamma = \frac{A_z}{A}$$

- Writing a vector in terms of these **direction cosines**:

$$\boxed{\vec{A} = \vec{i}\,A_x + \vec{j}\,A_y + \vec{k}\,A_z = \vec{i}\,A\cos\alpha + \vec{j}\,A\cos\beta + \vec{k}\,A\cos\gamma}$$

UNIT VECTOR IN THE DIRECTION OF $\vec{A}$: Three Dimensions (3D)

- Hence, we have:



$$\vec{A} = \vec{i}\,A_x + \vec{j}\,A_y + \vec{k}\,A_z = \vec{i}\,A\cos\alpha + \vec{j}\,A\cos\beta + \vec{k}\,A\cos\gamma =$$

$$= \underbrace{A\left(\vec{i}\,\cos\alpha + \vec{j}\,\cos\beta + \vec{k}\,\cos\gamma\right)}_{\vec{u}_A} = A\,\vec{u}_A$$

Magnitude Direction

$\vec{u}_A$

unit vector (length 1) in the direction of $\vec{A}$.

- Hence, for any vector $\vec{A}$ with known direction angles α, β, γ we can define a unit vector $\vec{u}_A$, that carries the information about its direction, as follows:

$$\boxed{\vec{u}_A = \vec{i}\,\cos\alpha + \vec{j}\,\cos\beta + \vec{k}\,\cos\gamma = \vec{i}\,\frac{A_x}{A} + \vec{j}\,\frac{A_y}{A} + \vec{k}\,\frac{A_z}{A}}$$

- Since $\vec{u}_A$ is a unit vector, we have: $\boxed{\cos^2(\alpha) + \cos^2(\beta) + \cos^2(\gamma) = 1}$