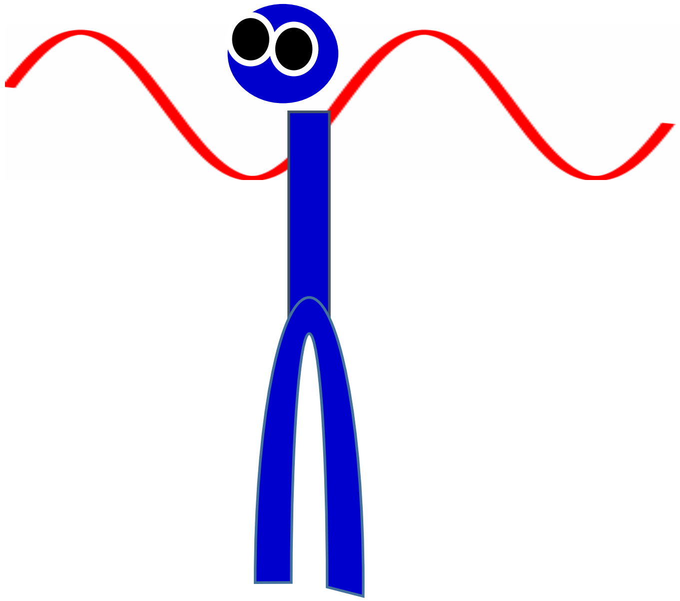
# Lecture 33.

# Superposition principle. Standing waves.
# Speed of a wave.



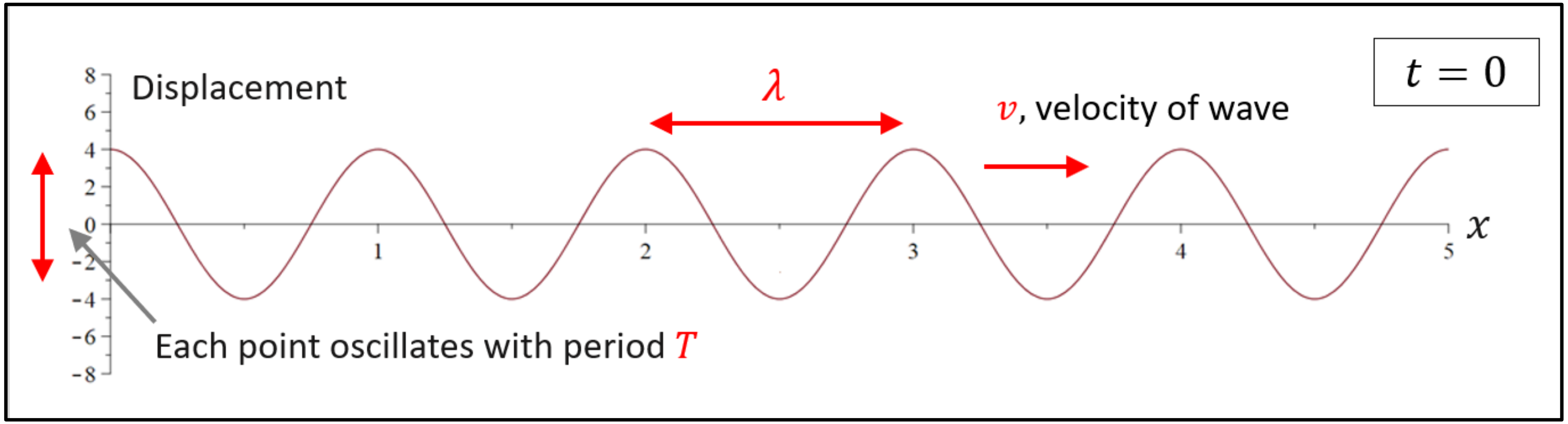
# Announcement

## Student Experience of Instruction (SEI) Survey

- You should have received an email asking you to complete a course evaluation survey on Canvas
- Your feedback is important to us and to our administration
- Survey is anonymous
- Please take a moment to complete the survey

# Last Time

## Travelling harmonic waves




$$T = \frac{2\pi}{\omega}$$

$$\lambda = \frac{2\pi}{k}$$

- Right-moving wave: $D(x,t) = A \cdot \cos(kx - \omega t + \phi_0)$
- Left-moving wave: $D(x,t) = A \cdot \cos(kx + \omega t + \phi_0)$

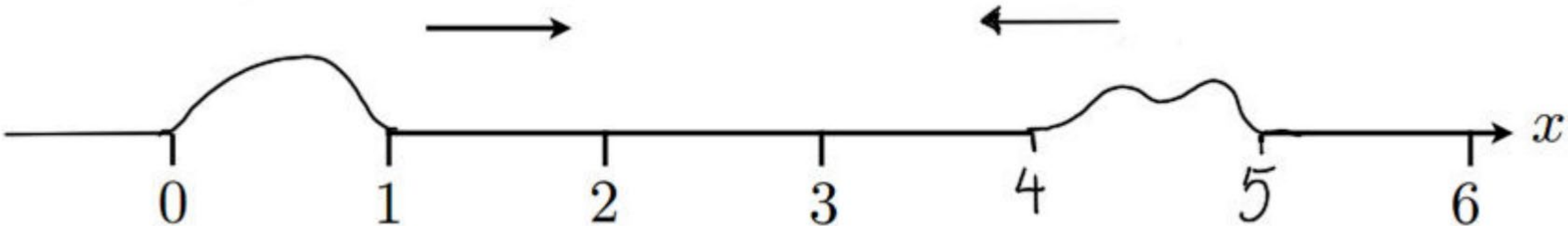
$$\lambda = v \cdot T$$

$$v = \lambda f$$

$$v = \omega / k$$

Q: Two waves are travelling towards each other as shown. When they meet, they will:

# Last Time

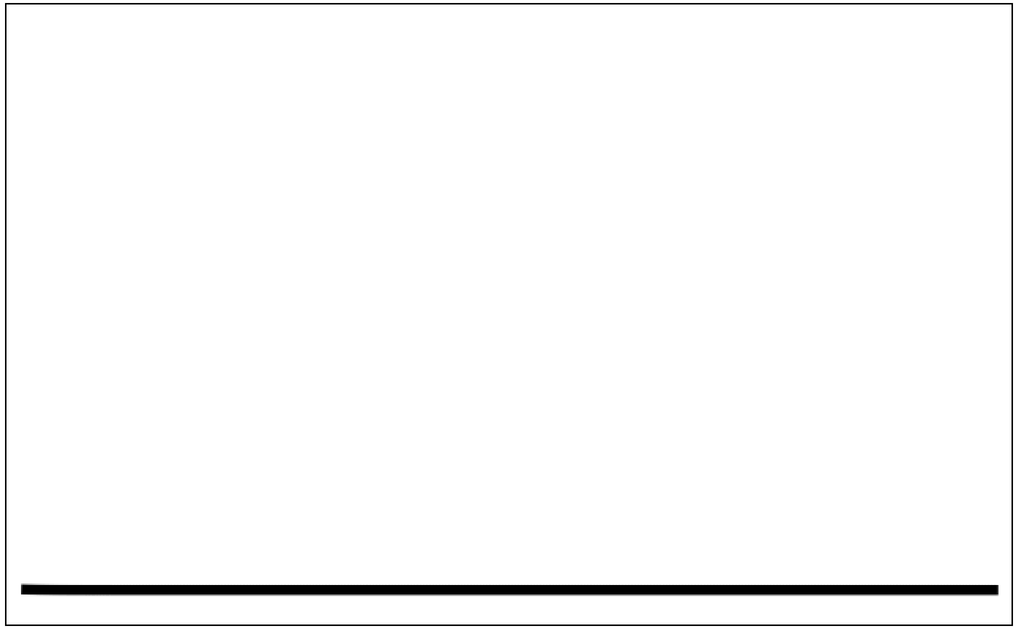



The presence of one wave does not change the equation of motion for the other wave. They each behave like the other were not there, so they pass right through each other

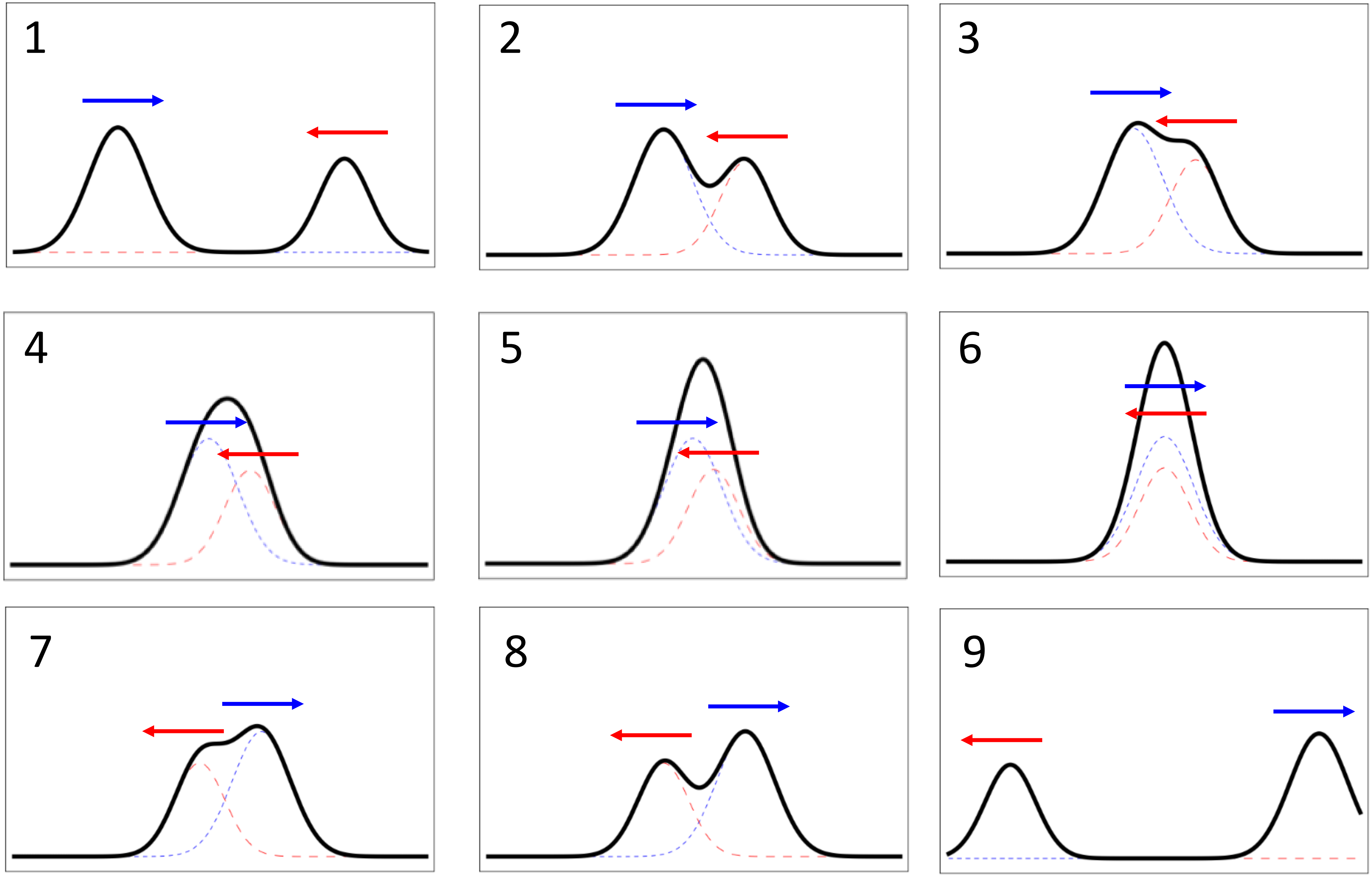
A. Bounce off each other and reflect backwards
B. Destroy each other, leaving a few random ripples going in either direction
C. Pass right through each other ✓

# The principle of linear superposition

- If $D_1(x,t)$ is an allowed wave and $D_2(x,t)$ is an allowed wave, then $D_1(x,t) + D_2(x,t)$ is an allowed wave
- When two or more waves overlap, the total displacement at that point is the sum of the displacements due to each individual wave
    - ➢ Waves add without disturbing each other!
- This phenomena is called "interference"
    - Animation: two pulses travelling in opposite directions

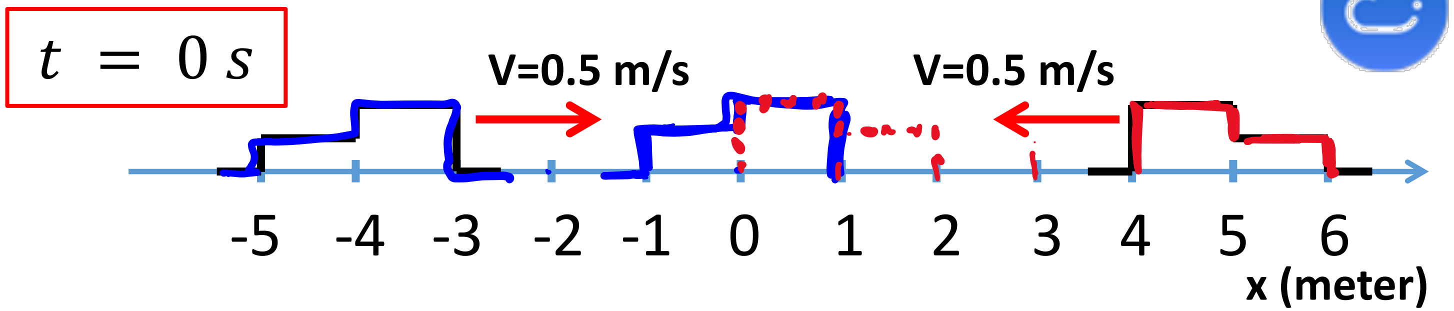


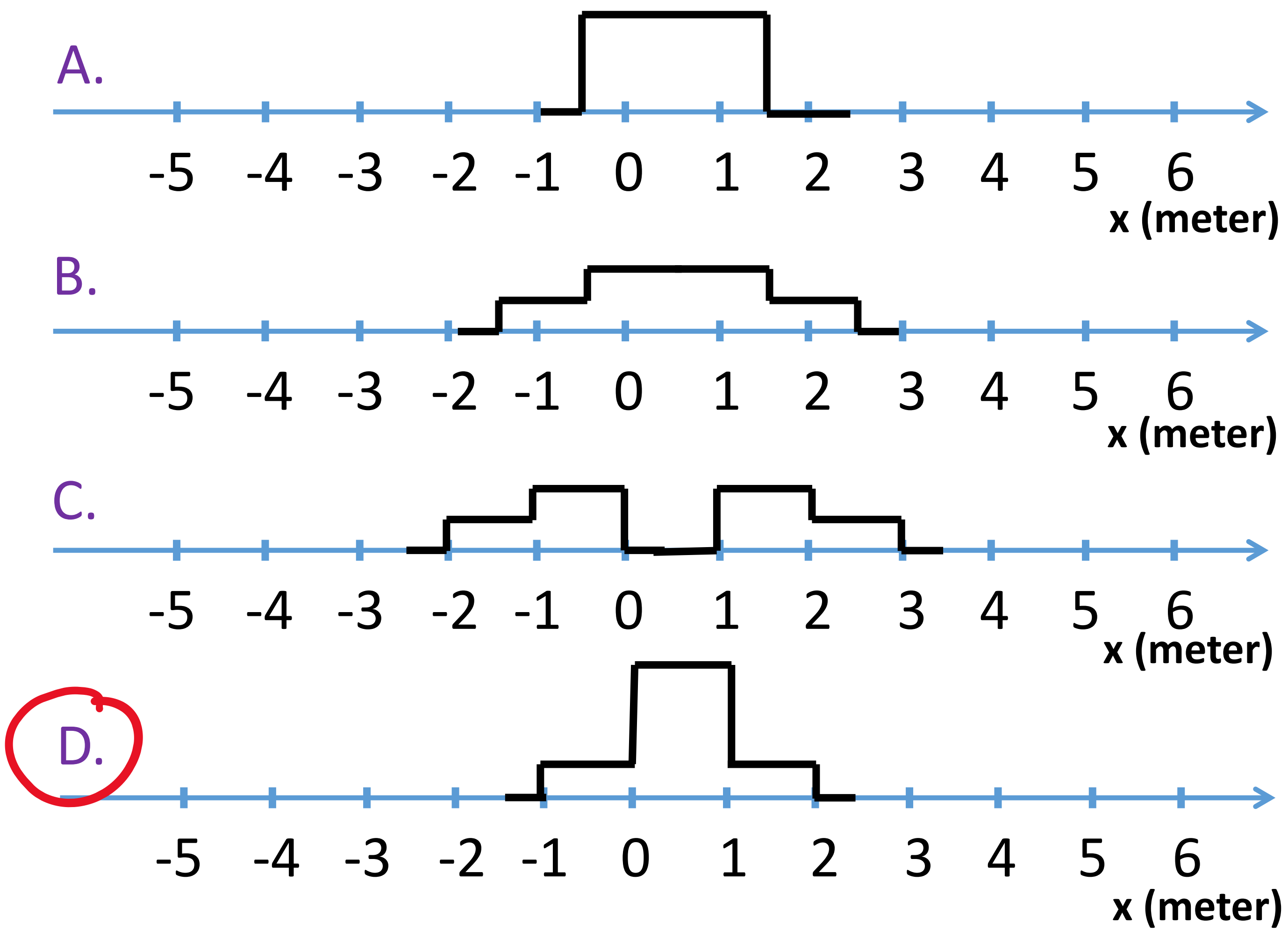
https://www.acs.psu.edu/drussell/Demos/superposition/superposition.html



1
2
3
4
5
6
7
8
9

Q: Two pulses are traveling toward each other in opposite directions. What is the shape of the net disturbance at $t = 8\ s$ ?



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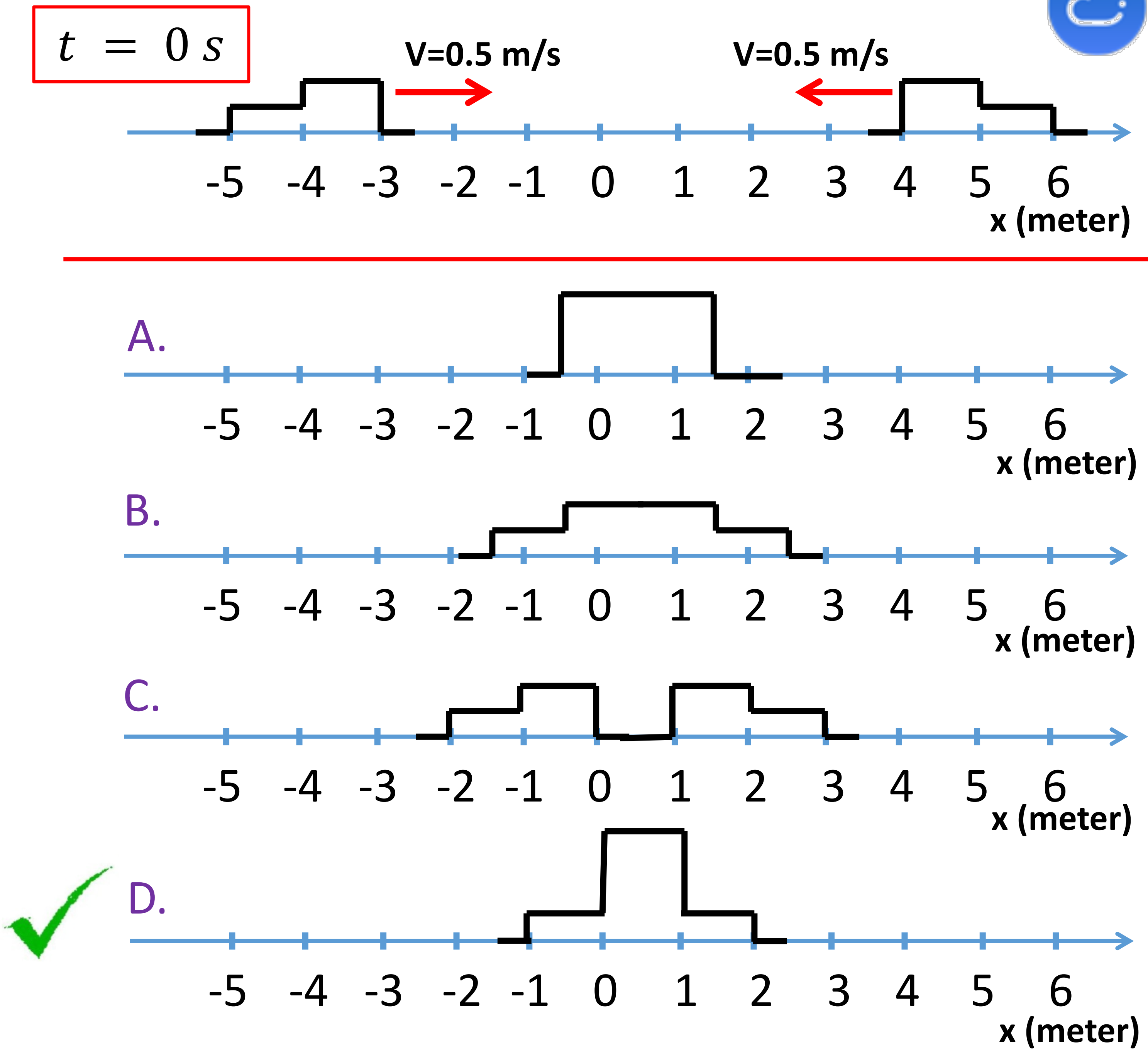

Wine glass shattered by sound waves:

Once again!

- Resonance: exaggerated response (= large amplitude of oscillations) if the driving at the natural frequency, $\omega_{nat}$

Q: What determines $\omega_{nat}$ for this glass?

A: Geometry of the glass.

sponds to two wavelengths of the bending wave:

$$2\lambda_B = 2\pi R. \quad (2)$$

Substituting $\lambda_B = \pi R$ into Eq. (1) and solving for $f$, the

## "Whispering" waves in a wineglass

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(Received 23 July 1984; accepted for publication 6 December 1984)

Surface waves in a wineglass do not whisper. But they do illuminate a special class of wave phenomena and, therefore, are instructive to study. The wine is an added benefit.

### I. INTRODUCTION

There has been a long history of fruitful research involving waves in axisymmetrical geometries. This research has been even more enjoyable when the axisymmetric geometry has been a wineglass. It is said that Gallileo proved that the musical consonance associated with what we call an octave was truly a doubling of frequency. He did this by observing twice as many waves per unit length on the surface of water in a glass[1] when the pitch was changed from the lower pitch to the higher consonant one. I would not be surprised if his glass were actually filled with wine, not

tension to liquid density. The former problem has been treated ni... the basic ... an oversi... dences of ... and thick... A bend... nite, two-... given by[6] $V_B = ($... where $V$...

## *In Vino Veritas*: A study of wineglass acoustics

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(Received 16 August 1982; accepted for publication 28 September 1982)

This paper describes an investigation of the natural resonant frequencies of vessels such as wineglasses. Measurements on a number of glasses are interpreted with the help of theoretical predictions based on the analysis of vibrating systems by means of the energy method. Results and analysis are given for empty glasses and for glasses containing different amounts of liquid. Evidence for vibrational modes above the lowest is presented.

### I. INTRODUCTION

The pure tones emitted when one rubs a moistened finger around the rim of a wineglass must have been a subject of informal experiment at countless dinner tables. The quan-

right angles to the first [Fig. 1(a)]. All other horizontal sections of the wineglass will go through similar motions, but with amplitudes that decrease as one goes down from the level of the rim.

On a simplified view, one might represent the wineglass

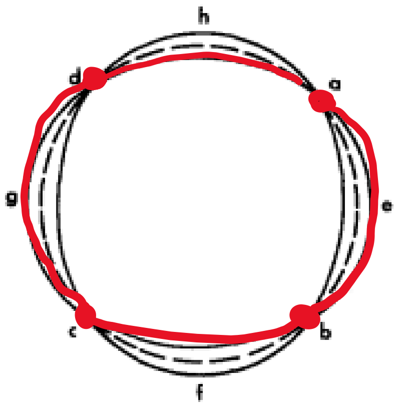
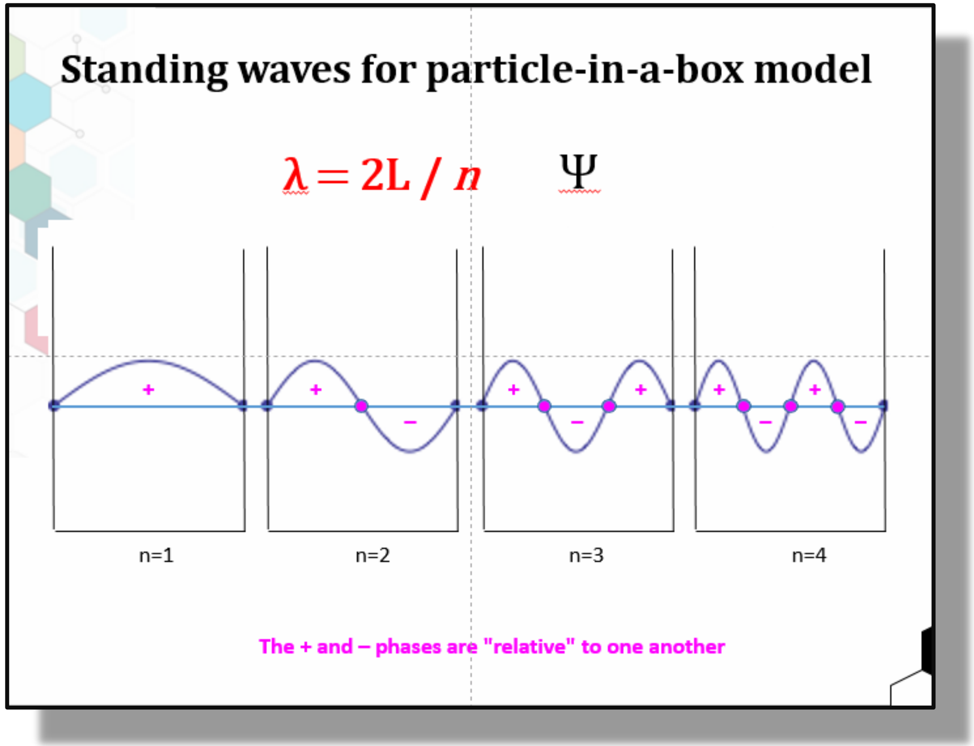
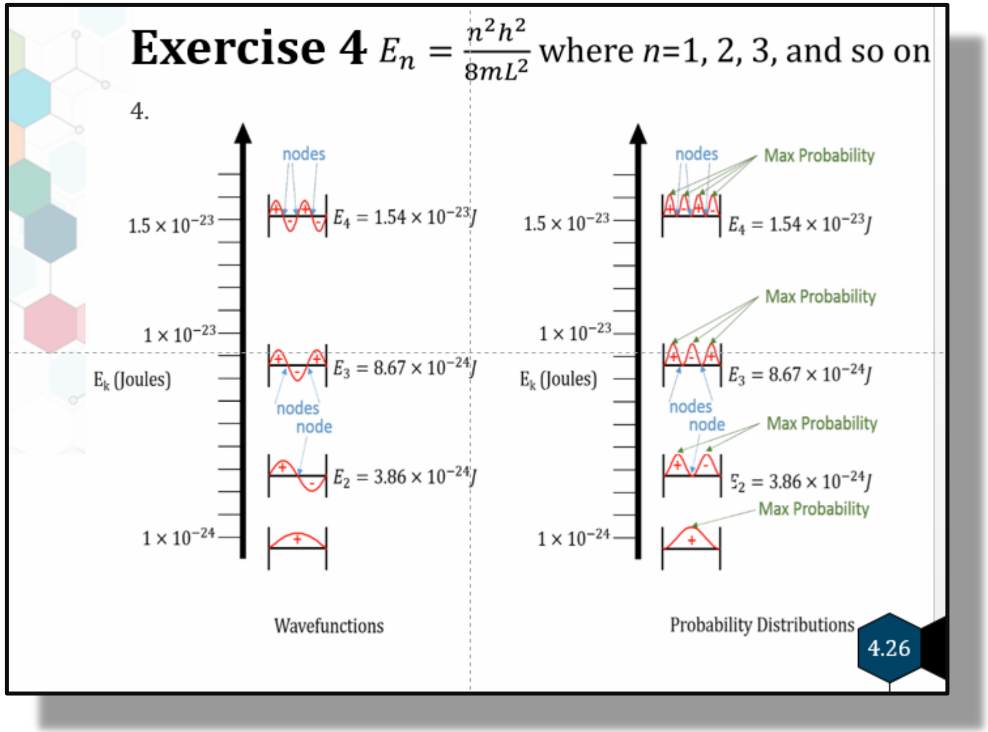



Fig. 2. The quadrupole mode of vibration of a cylindrical vessel.



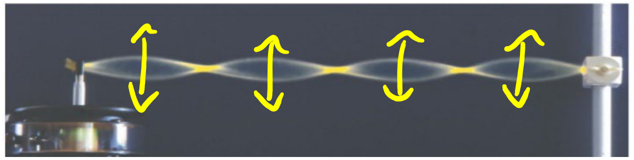

- Chem 121 stuff:

"Particle-in-a-box model"




- Applying particle-in-the-box idea to the circumference of a ~~wine glass~~ electron orbit in an atom brought Niels Bohr to the first quantum model of the atom!

# Standing Wave: Special Kind of wave



- Key idea: the geometry of the wave should match the geometry of the object that supports this wave

- Geometry of the wave:
  is linked to its wavelength, $\lambda$

- Geometry of the object:
  - String, or tube, or 1D box: length $L$
  - Glass: circumference $2\pi r$
  - …

- If the wavelength is 'just right', the wave will form a stationary pattern.

- Typically there is more than one 'just right' wavelength. $\begin{pmatrix}\lambda_1\\ f_1\end{pmatrix}, \begin{pmatrix}\lambda_2\\ f_2\end{pmatrix}, \begin{pmatrix}\lambda_3\\ f_3\end{pmatrix}, \dots$

- There are specific frequencies corresponding to these wavelengths: remember, $f = v/\lambda$.
  They are called resonant frequencies of the object (string, open or closed tube, box...).

## Standing waves

Stanging wave

$$D(x,t) = \underbrace{\left[A\sin(kx)\right]}_{A(x)}\sin(\omega t)$$

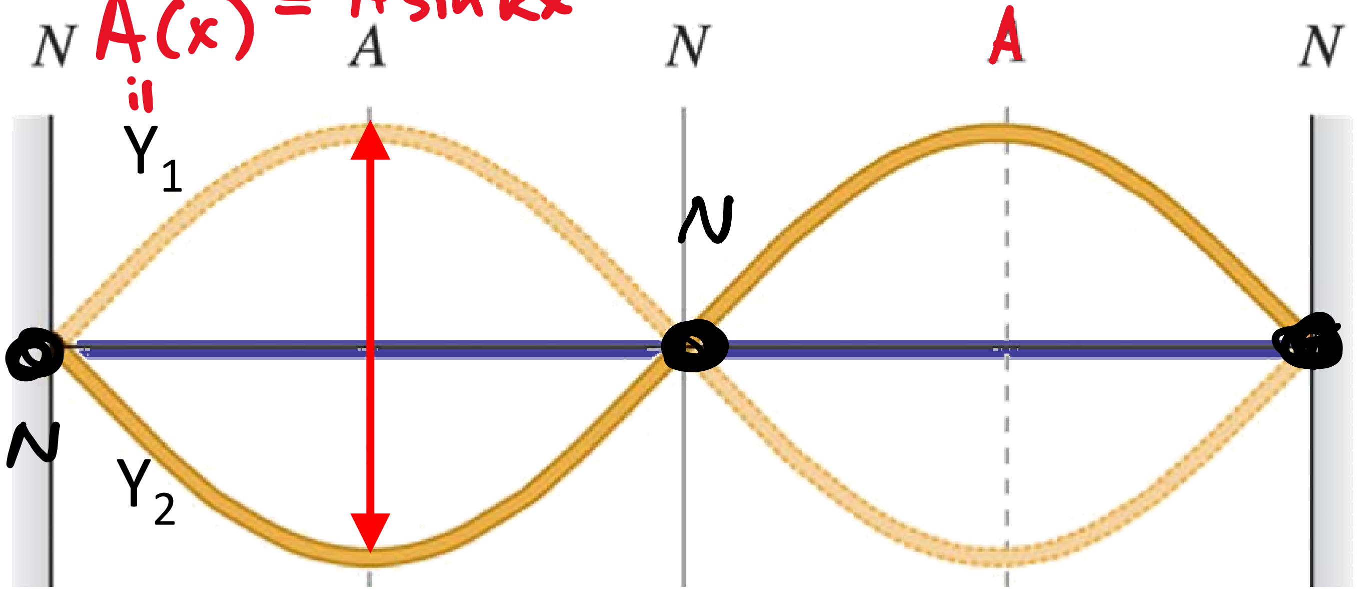
- Often the displacement must be zero on both ends (example: guitar string). It's called "fixed boundary conditions", or "fixed ends".

Travelling wave:

$\vec{D}(x,t) = A\cdot\cos(kx-\omega t)$

compare!

$A(x) = A\sin kx$




sin kx = 0

sin kx = ±1

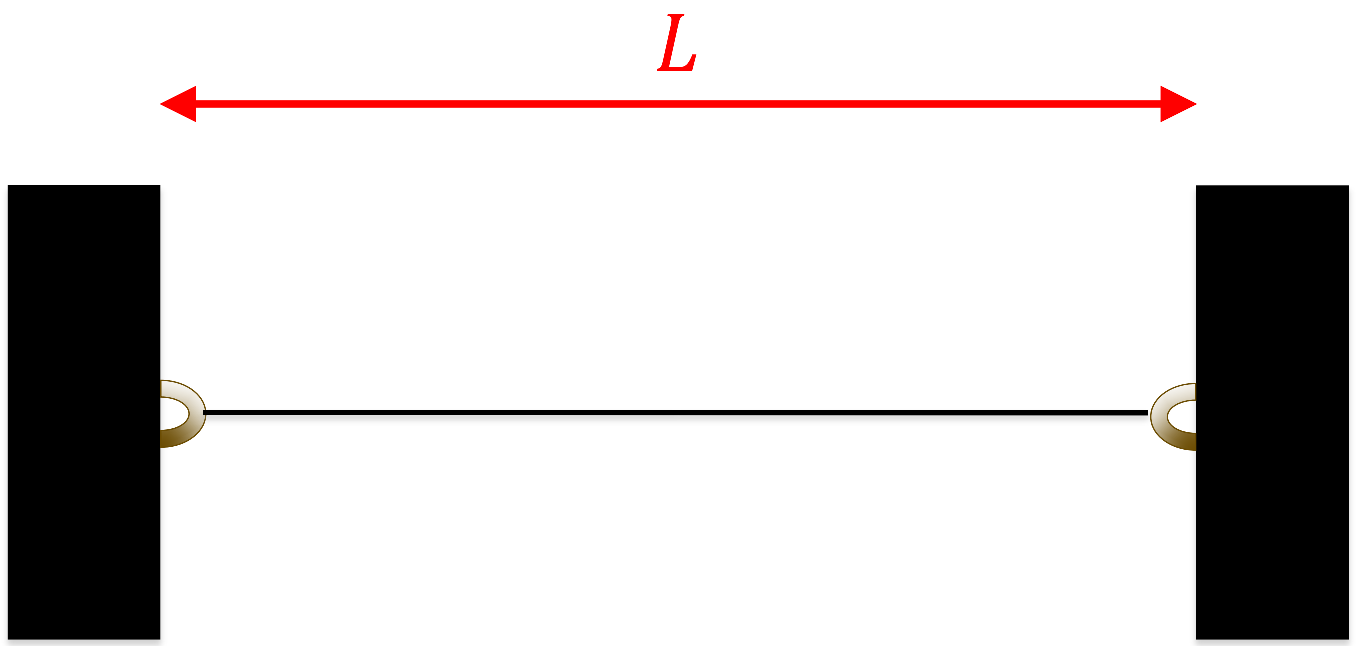
N = nodes: points at which the displacement is always zero

A = antinodes: points at which the amplitude of displacement is greatest

$Y_1$ and $Y_2$: the shape of the string at two different times

# Example: Guitar string – fixed at both ends

What oscillations will fit ??



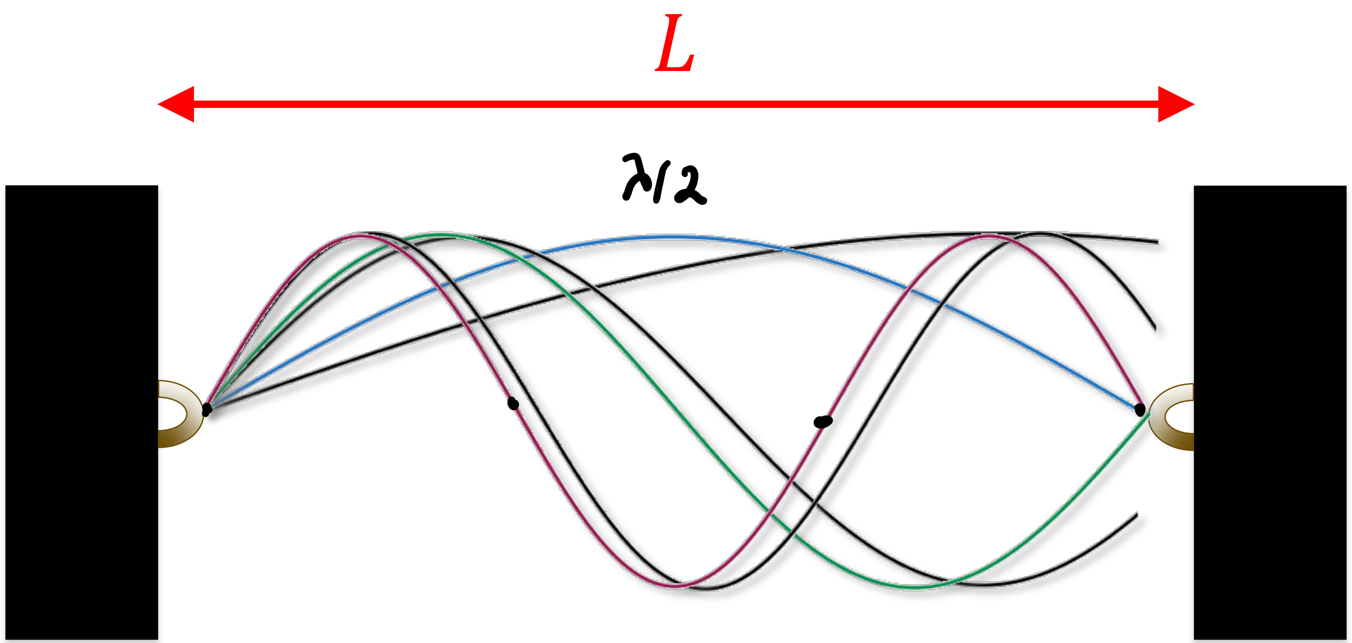
# Example: Guitar string – fixed at both ends

What oscillations will fit ??

$\lambda = 2L \rightarrow$

$3 \cdot \frac{\lambda}{2} = L \rightarrow$

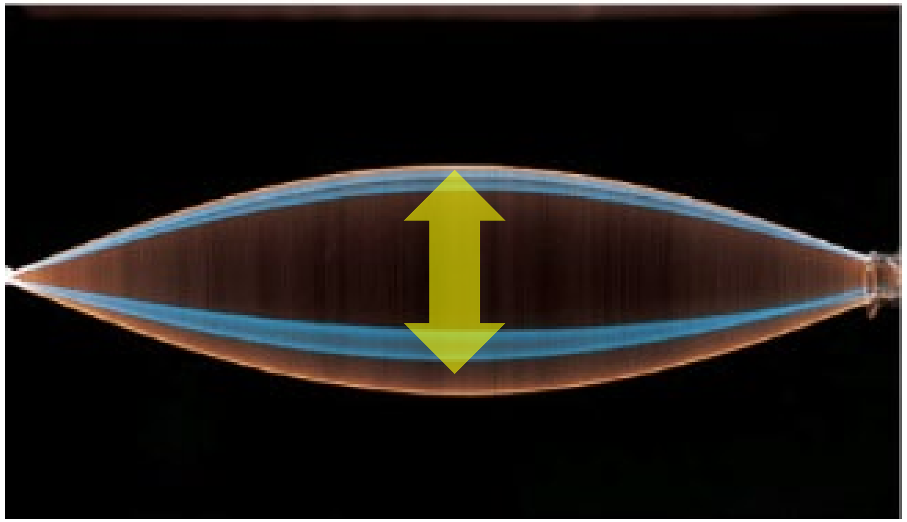
$\lambda = \frac{2L}{3}$




$$\lambda = 2L, \frac{2L}{2}, \frac{2L}{3} \ldots \frac{2L}{n}$$

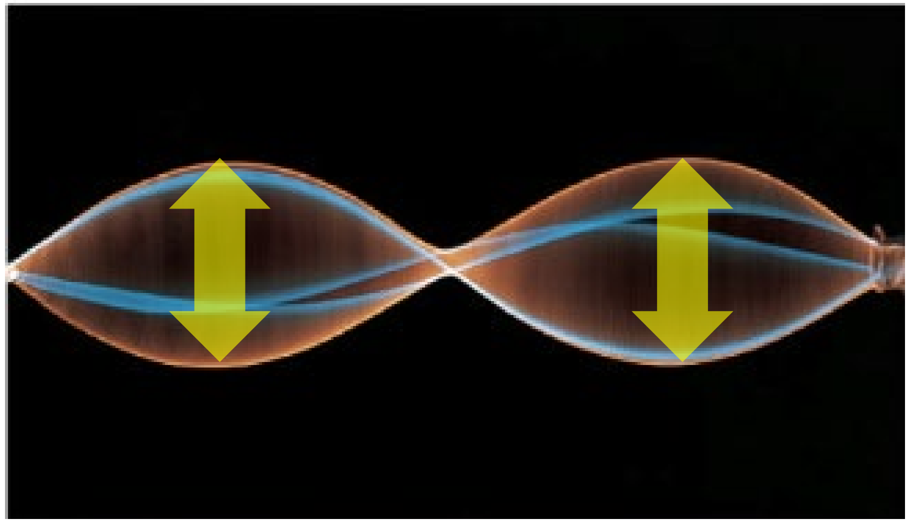
# Transverse standing waves on a string

(a) String is one-half wavelength long



$$\lambda = \frac{2L}{1}$$
$$= 2L$$

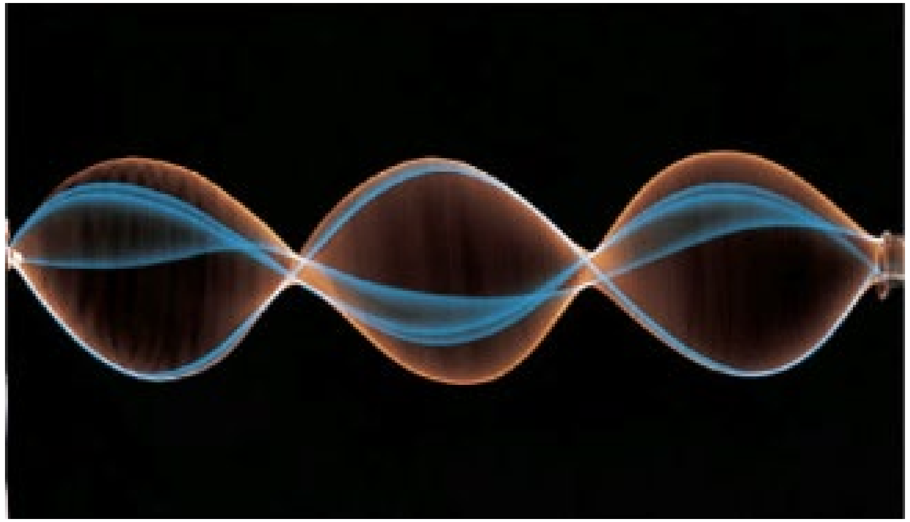
(b) String is one wavelength long



$$\lambda = \frac{2L}{2}$$
$$= L$$

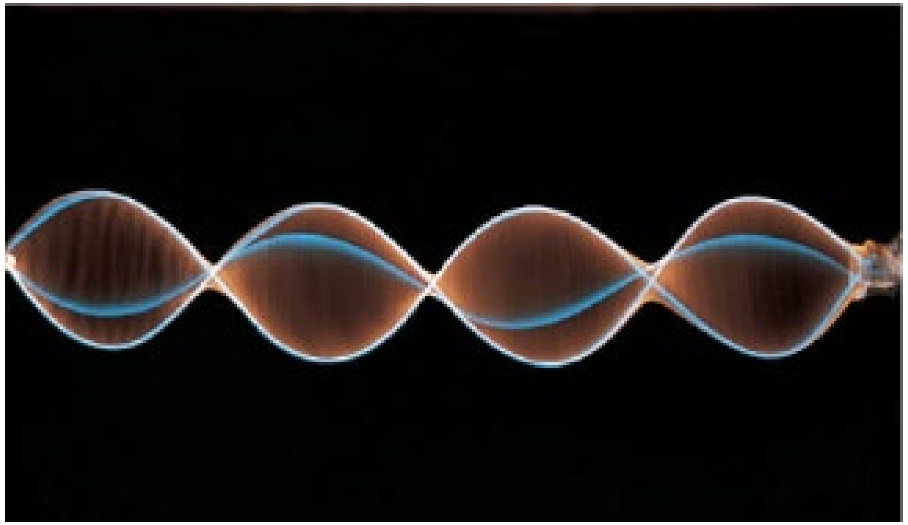
$$L = n \cdot \frac{\lambda}{2}$$
$$\lambda = \frac{2L}{n}$$

(c) String is one and a half wavelengths long



$$\lambda = \frac{2L}{3}$$
$$= \frac{2}{3}L$$

(d) String is two wavelengths long

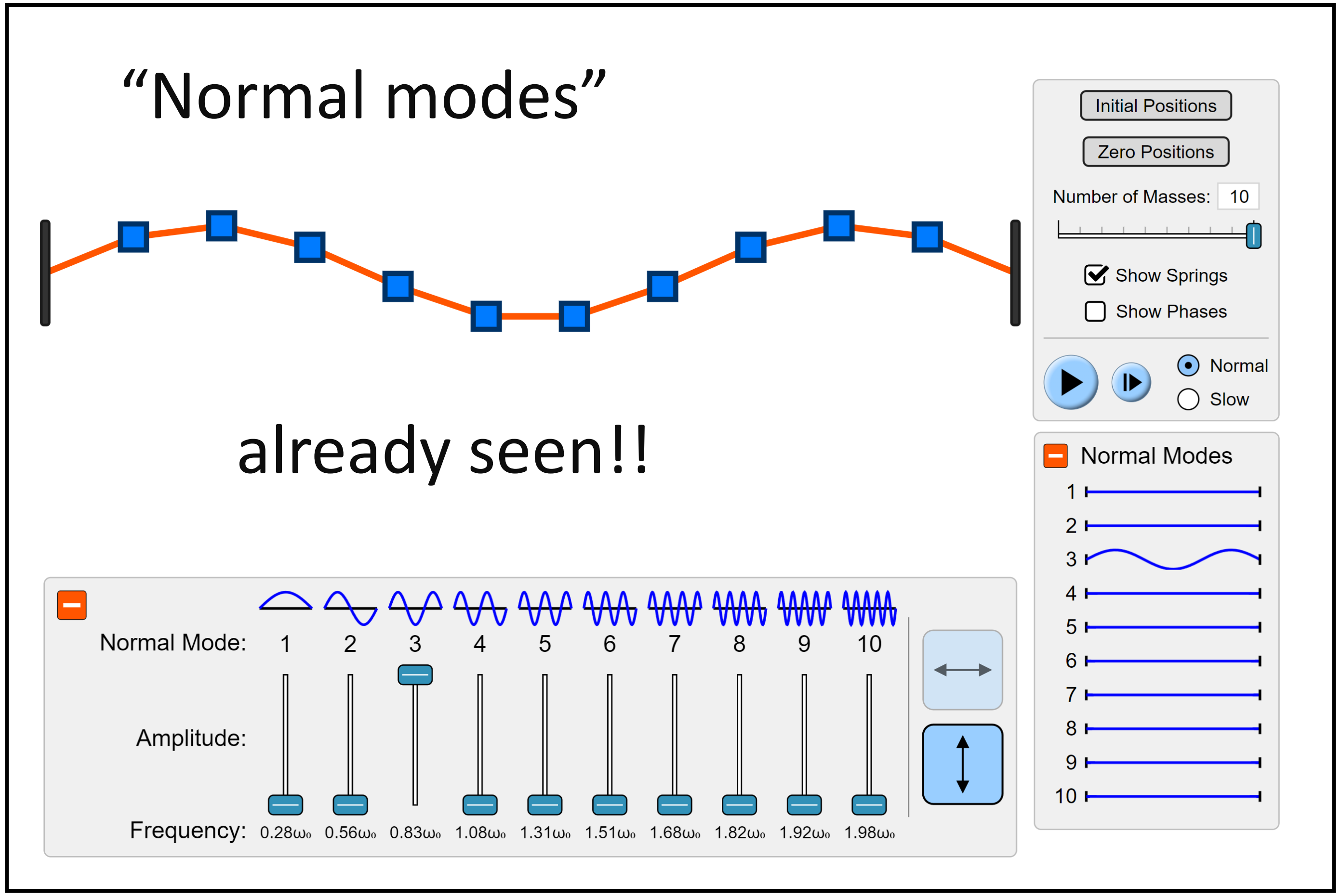


$$\lambda = \frac{2L}{4}$$
$$= \frac{1}{2}L$$

https://www.acs.psu.edu/drussell/Demos/StandingWaves/StandingWaves.html

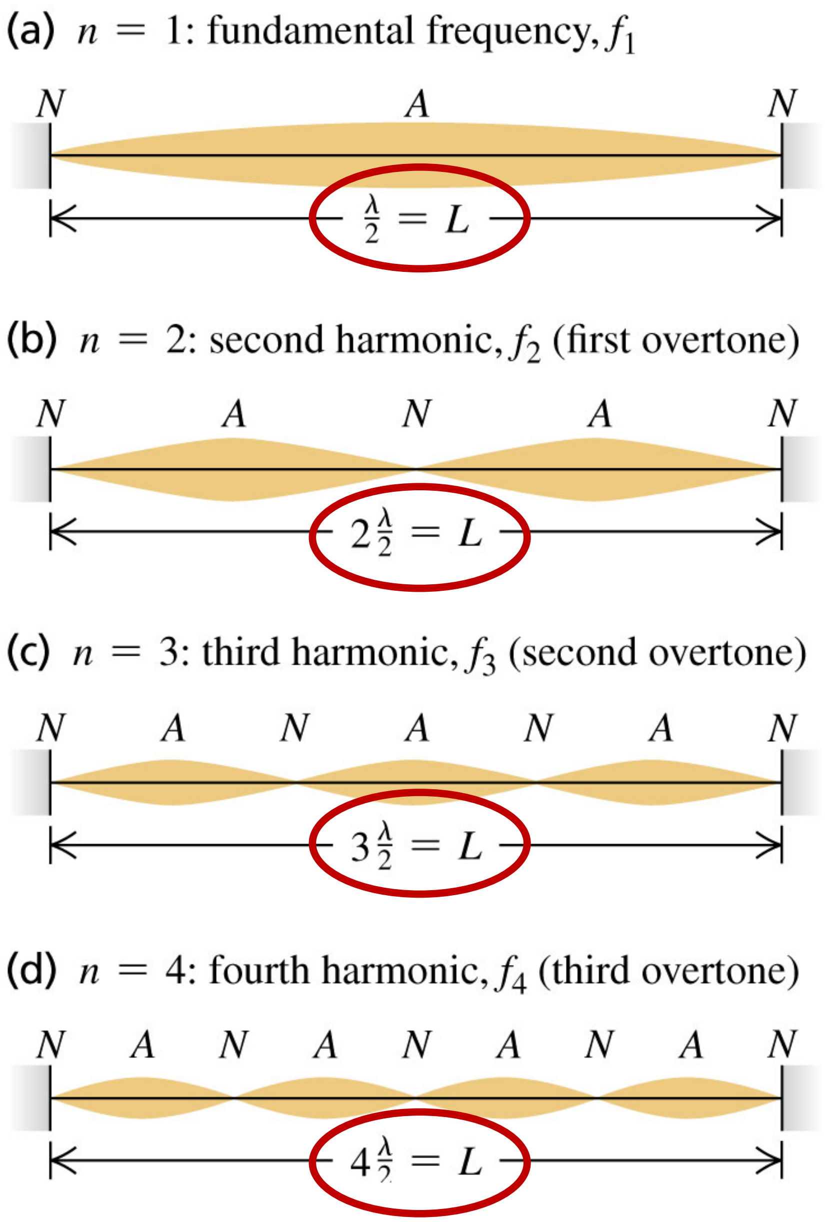
# Example: Guitar string – fixed at both ends

“Normal modes”




already seen!!

https://phet.colorado.edu/sims/html/normal-modes/latest/normal-modes_all.html




First 4 harmonics
(also called Normal Modes)

# Example: Guitar string – fixed at both ends

- For a taut string fixed at both ends, the displacement must be zero at ends

$f_n \lambda_n = v$

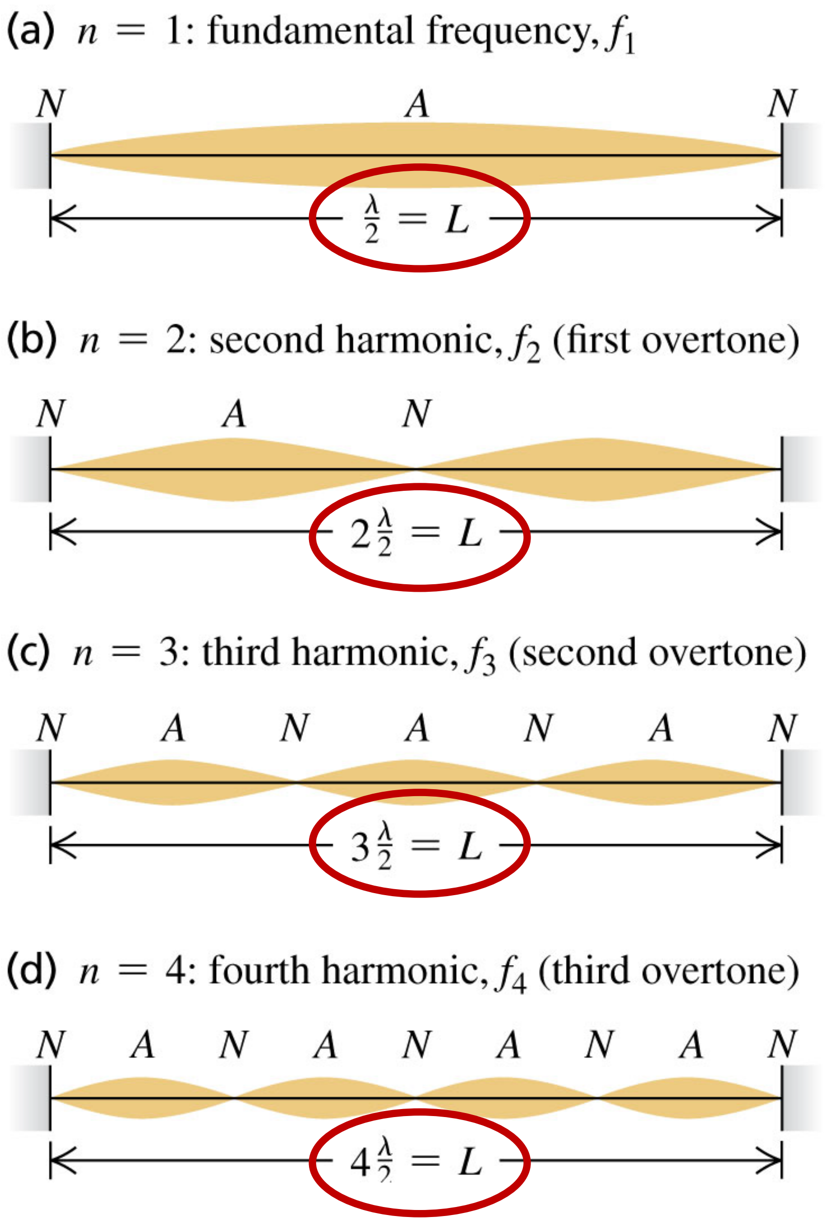
- Possible wavelengths are: $\lambda_n = 2L/n$ =>

- Possible frequencies are $f_n = \frac{v}{\lambda_n} = \frac{nv}{2L} = nf_1$, where $n = 1, 2, 3 \ldots$
  - $f_1 = \frac{v}{2L}$ is the fundamental frequency
  - $f_2 = \frac{2v}{2L}$ is the second harmonic (1st overtone)
  - $f_3 = \frac{3v}{2L}$ is the third harmonic, etc. (2nd overtone)

⋮

Q: Is the speed, $v$, same for all modes??

A: Yes!!! $v$ is determined by properties of the system.



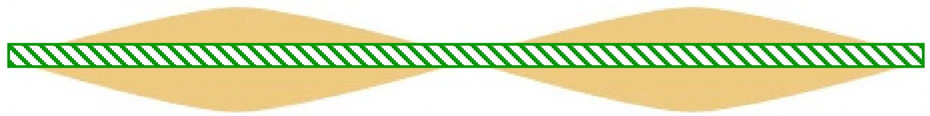

First 4 harmonics
(also called Normal Modes)

# Speed of waves on a string: Dimensional Analysis

(detailed derivation in 15.4)

- Important: $v$ only depends on properties of the string, not $\lambda$ or $f$ (though the frequency and the wavelength match so that $v = f\lambda$)

- What determines $v$?



  - depends on tension $T$ units: $N = \frac{kg \cdot m}{s^2}$
  - depends on mass per unit length $\mu = m_{\text{string}}/L$ units: $\frac{kg}{m}$

- What combination of these has units of velocity?

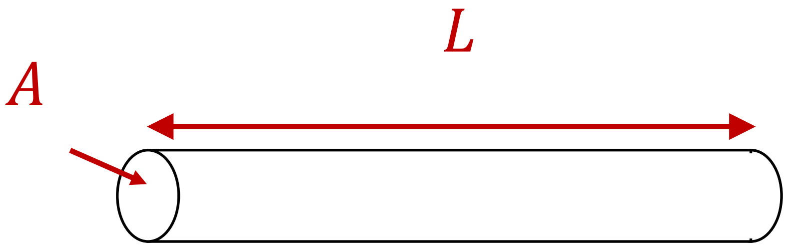
- Combination with the right units: (see 15.4 for derivation, if you wish)

$$\boxed{v = \sqrt{\frac{T}{\mu}}}$$

Q: The four strings of a musical instrument are all made of the same material and are under the same tension, but have different thicknesses. Waves travel:

A. fastest on the thickest string
B. fastest on the thinnest string
C. at the same speed on all strings
D. either A or B is possible
E. any of A, B, or C is possible

Q: The four strings of a musical instrument are all made of the same material and are under the same tension, but have different thicknesses. Waves travel:




$$m = V\rho = AL\rho$$

$$v = \sqrt{T/\mu}$$

- String mass per unit length: $\mu = \frac{m}{L} = \frac{AL\rho}{L} = A\rho$
- Thinnest string has smallest $\mu$ and fastest $v$

A. fastest on the thickest string
B. fastest on the thinnest string
C. at the same speed on all strings
D. either A or B is possible
E. any of A, B, or C is possible

## Speed of travelling waves in a material

- The speed of waves in a material depends on the material's characteristics
- Gases and liquids only support longitudinal waves
  - ➢ In an ideal gas, the speed of longitudinal waves is:

    $v = \sqrt{\gamma RT/M}$ $\gamma$: $C_P/C_V$ $R$: Gas constant $T$: temperature $M$: Molar mass

  - ➢ In a liquid, the speed of longitudinal waves is:

    $v = \sqrt{B/\rho}$ $B$: Bulk Modulus $\rho$: Density

- Solids support both longitudinal and transverse waves
  - ➢ In a solid, the speed of longitudinal waves is:

    $v = \sqrt{Y/\rho}$ $Y$: Young's Modulus $\rho$: Density

  - ➢ In a solid, the speed of transverse waves is:

    $v = \sqrt{S/\rho}$ $S$: Shear Modulus $\rho$: Density

# Speed of sound of various materials

| Material | Speed of Longitudinal Sound (m/s) | Speed of Transverse Sound (m/s) |
|---|---|---|
| **Gases** | | |
| Air (20 degrees Celsius) | 344 | |
| Helium (20 degrees Celsius) | 999 | |
| Hydrogen (20 degrees Celsius) | 1330 | |
| **Liquids** | | |
| Liquid helium (4 K) | 211 | |
| Mercury (20 degrees Celsius) | 1451 | |
| Water (0 degrees Celsius) | 1402 | |
| Water (20 degrees Celsius) | 1482 | |
| Water (100 degrees Celsius) | 1543 | |
| **Solids** | | |
| Aluminum | 6420 | 3040 |
| Lead | 1960 | 690 |
| Steel | 5941 | 3220 |

Q1: How does a wave "know" to fit a string?

Q2: Okay, we know that on a string $v = \sqrt{T/\mu}$. But we are talking about a <u>standing wave</u>, where nothing moves in the horizontal direction.

What for the speed is that???

- On Wednesday!