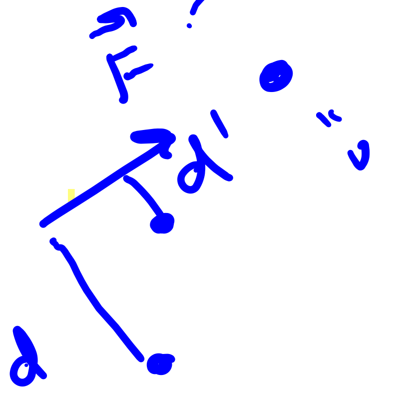
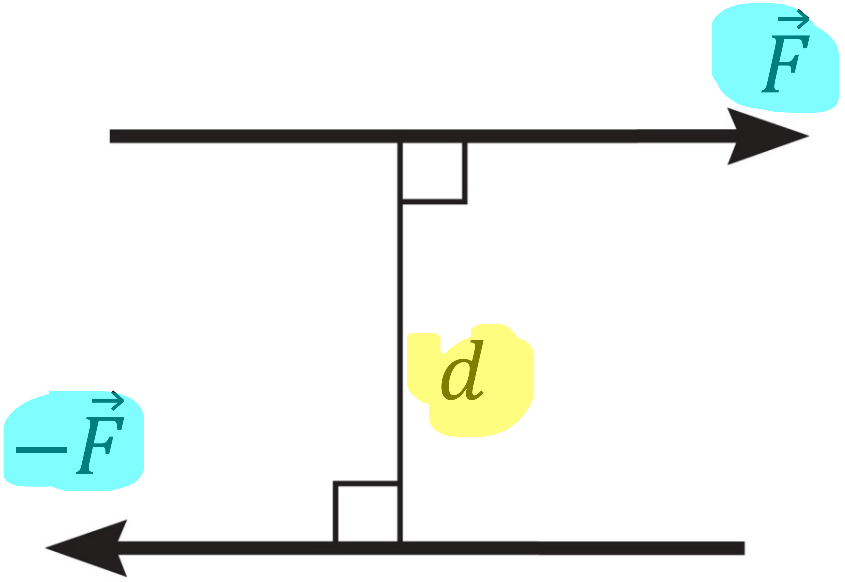
Last Time



sliding vector

- A moment of a couple

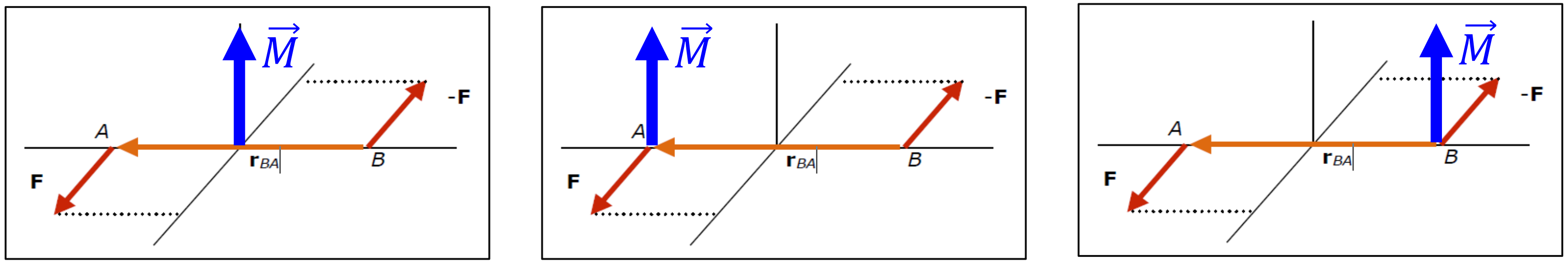


- A moment of a couple does NOT depend on the choice of the point about which it is calculated

↳ free vector

MOMENT OF A COUPLE: Free Vector

- We just have shown that a moment of a couple does not depend on the choice of the point about which we calculate it.
- We say that a moment of a couple is a free vector (you can apply it at any point):

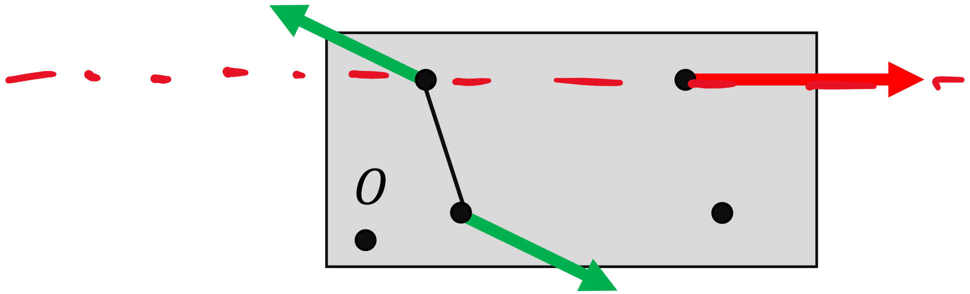


etc.

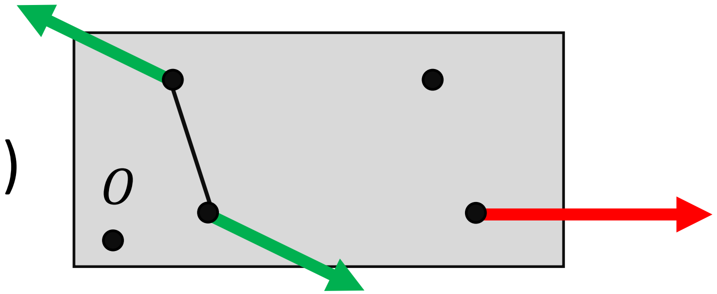
- Note that it is in a striking contrast with a <u>moment of one force about a point</u>, which is NOT a free vector!

- To not forget about which point the moment of one force was calculated, we always show it as a vector applied at that very point.
- In contrast, the moment of a couple can be applied at any point (we can shift it around without changing physical properties of the system).

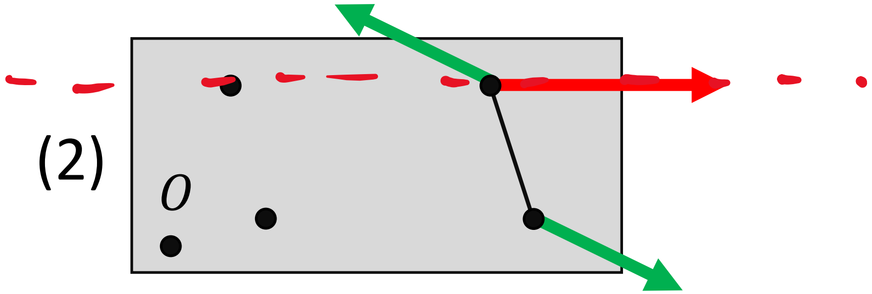
Q: Consider rotation of these 2D rectangles about point O. The green arrows show a couple, the red arrow is a force vector. Which of the rectangles (1)—(4) perform(s) the same rotation as this rectangle?



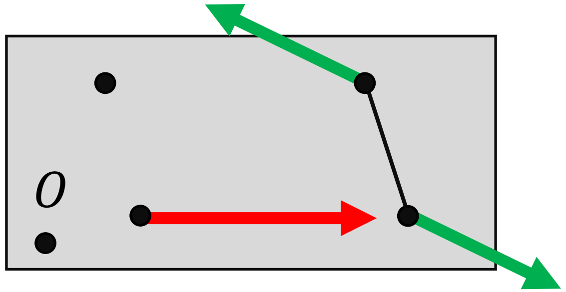
(1)



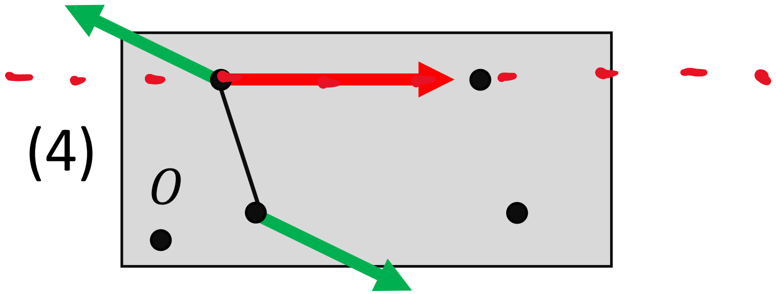
(2)



(3)

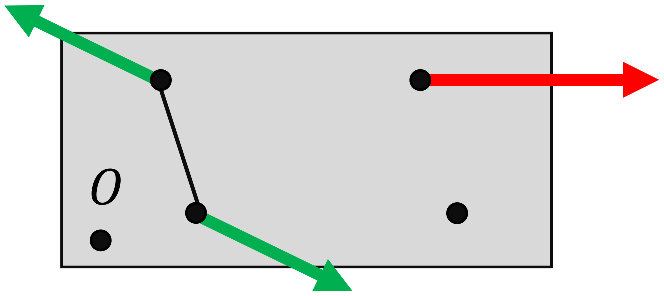


(4)



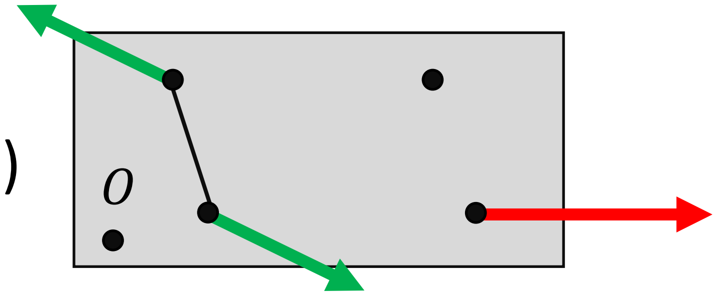
A. Only (1)
B. Only (2)
C. Only (3)
D. Only (4)
E. More than one (which?)

Q: Consider rotation of these 2D rectangles about point O. The green arrows show a couple, the red arrow is a force vector. Which of the rectangles (1)—(4) perform(s) the same rotation as this rectangle?

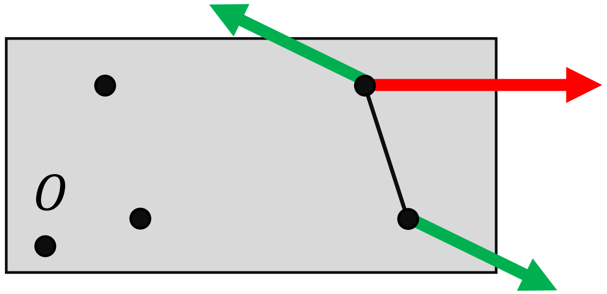


Force is a sliding vector (we can shift it along its line of action), and couple moment is a free vector (we can shift it to any point)

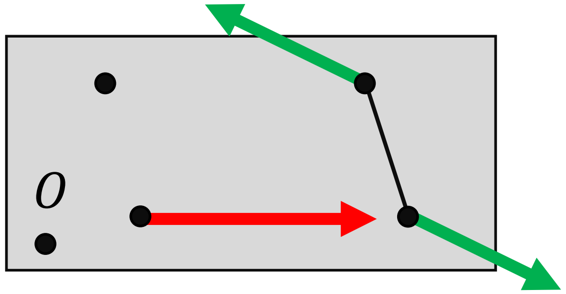
(1)



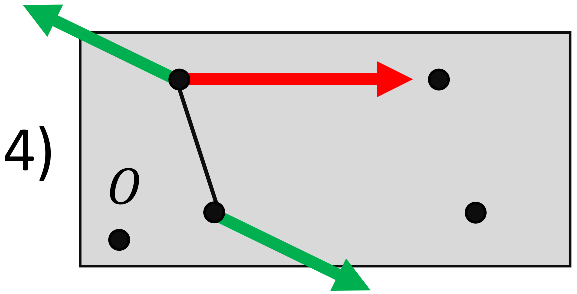
(2)



(3)



(4)

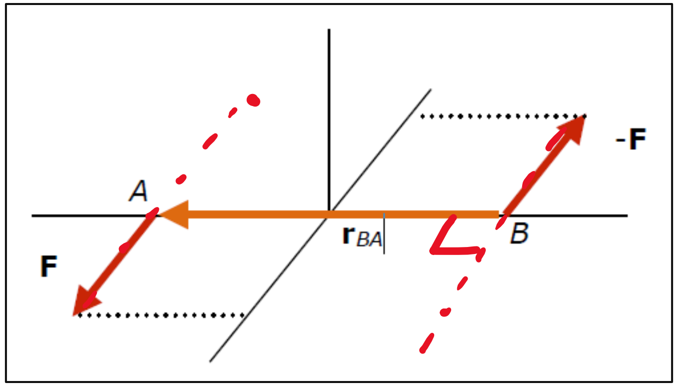


A. Only (1)
B. Only (2)
C. Only (3)
D. Only (4)
E. More than one (which?)

(2) and (4)

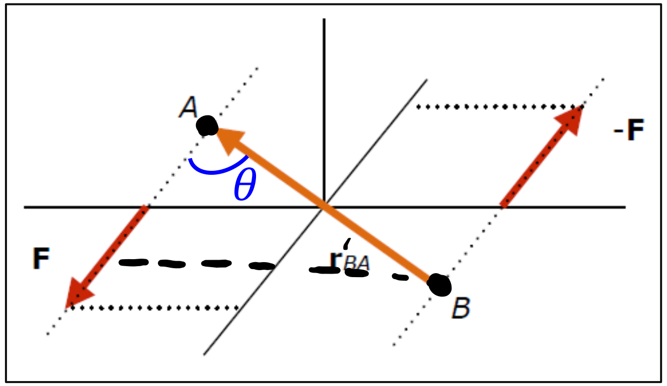
MOMENT OF A COUPLE: Two more properties

1. A moment of a couple is the same for ANY vector connecting ANY two points on the lines of actions of these forces:



$$\boxed{\vec{M} = \vec{r} \times \vec{F}}$$

$$|\vec{M}| = rF\sin\theta$$

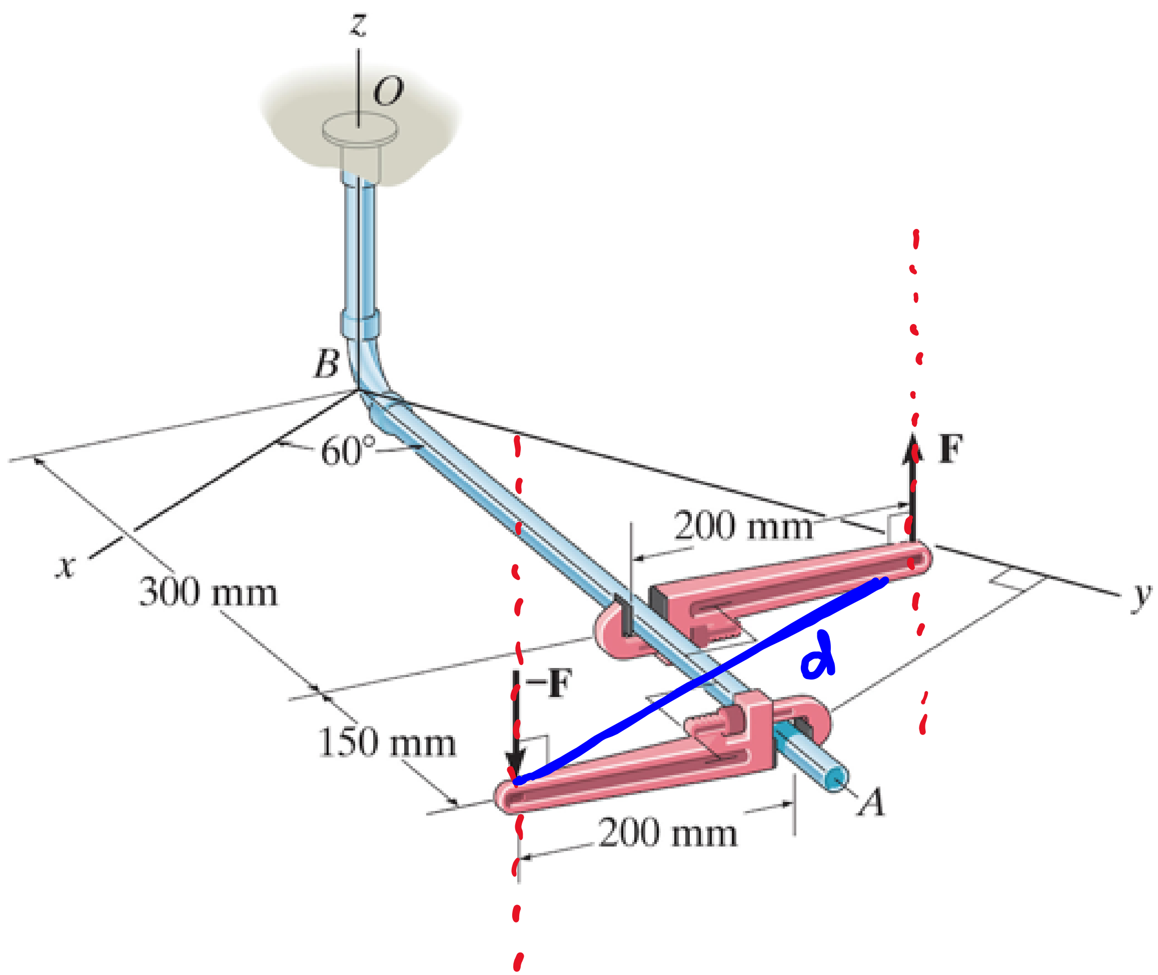


$M = F\, r_{BA} \sin 90^o = F\, r_{BA}$

$M = F\, r'_{BA} \sin\theta = F\, r_{BA}$

2. Since the couple moment can be calculated about any point, let's calculate it about point B, where B is a point on the line of action of $-\vec{F}$. The moment of $-\vec{F}$ about B is zero, the moment of $\vec{F}$ about B is $\vec{r}_{BA} \times \vec{F}$. Hence, $\boxed{\vec{M} = \vec{r}_{BA} \times \vec{F}}$

Q: If the magnitude of the force applied to the wrenches is $F = 35$ N, what is the magnitude of the resultant couple moment?

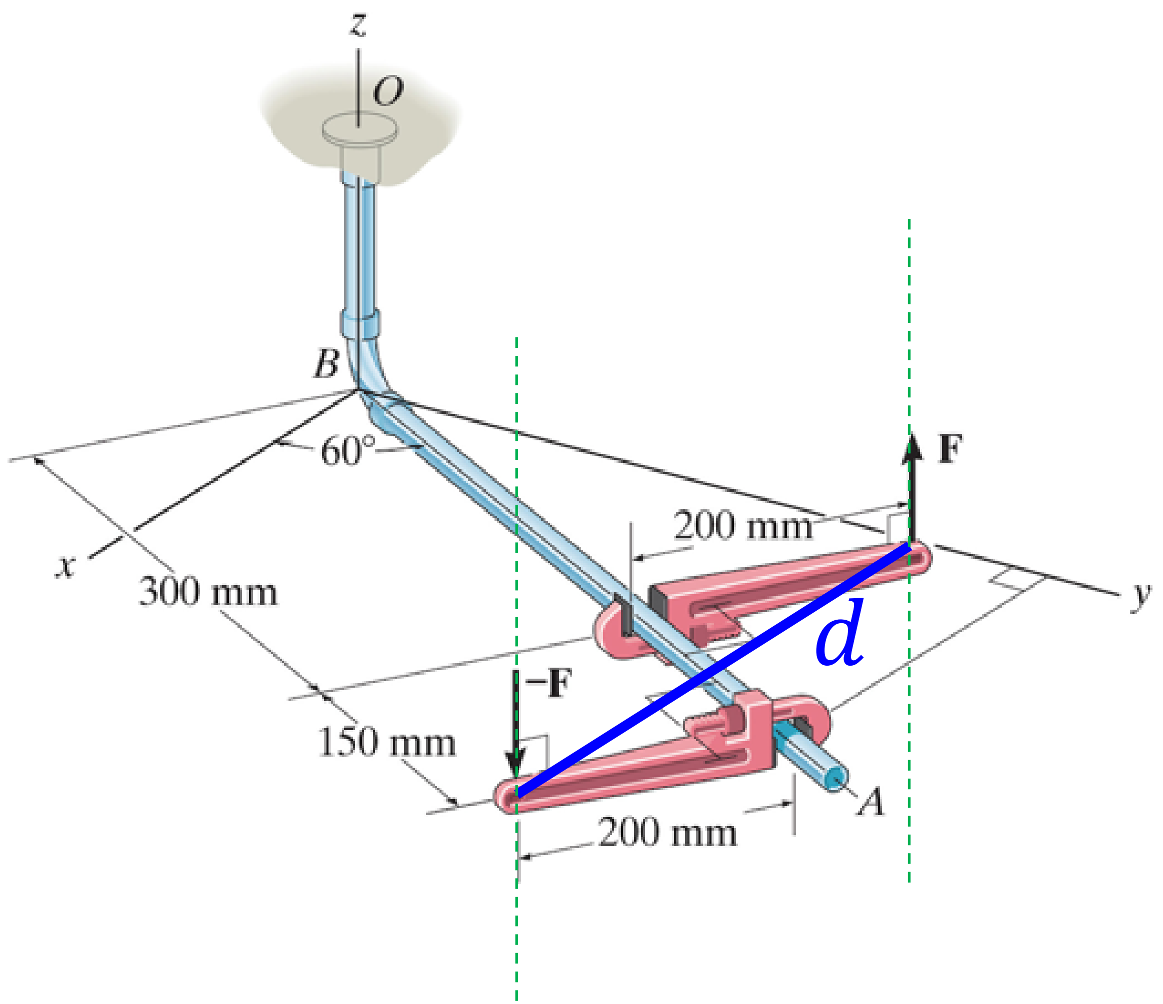


A. 35 N · (200+200) mm
B. 35 N · 150 mm
C. 35 N · 200 mm
D. 35 N · $\sqrt{200^2 + 150^2}$ mm
E. 35 N · $\sqrt{400^2 + 150^2}$ mm

arm

$M = F \cdot d$

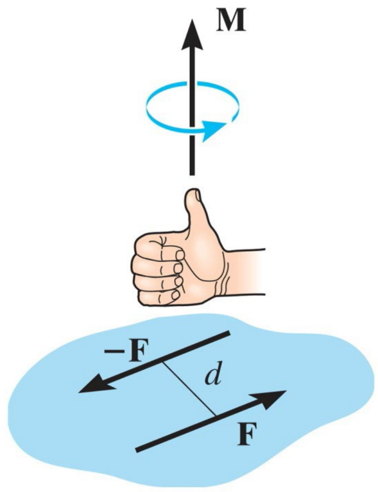
Q: If the magnitude of the force applied to the wrenches is $F = 35$ N, what is the magnitude of the resultant couple moment?



$M = Fd$, where d is the shortest distance between the lines of action of the two forces.

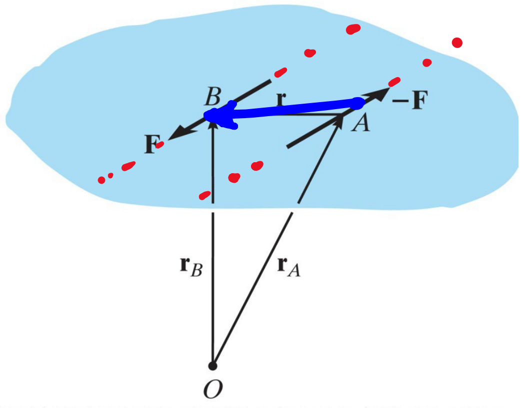
A. 35 N · (200+200) mm
B. 35 N · 150 mm
C. 35 N · 200 mm
D. 35 N · $\sqrt{200^2 + 150^2}$ mm
E. 35 N · $\sqrt{400^2 + 150^2}$ mm

MOMENT OF A COUPLE: summary



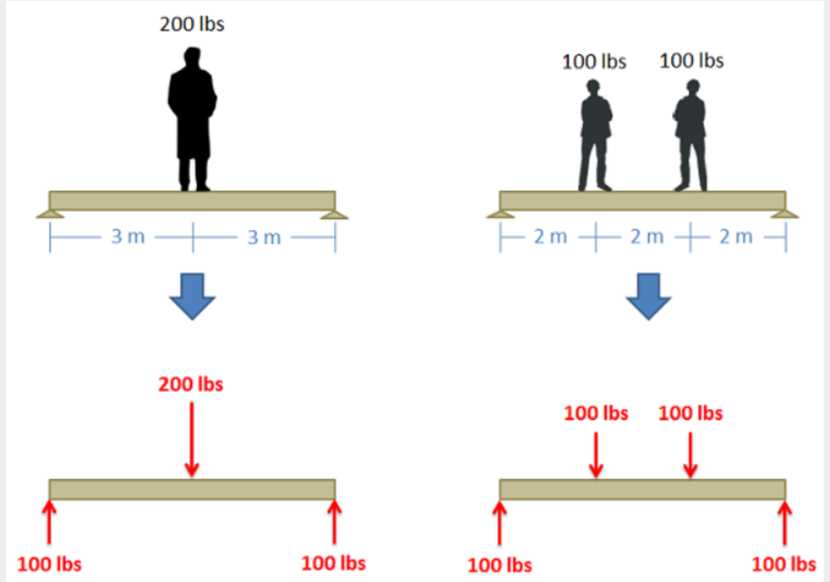
- Scalar formulation:
    - **Magnitude** of the couple moment is $M = Fd$, where d is the perpendicular distance
    - **Direction**: right-hand rule, fingers curl with the sense of rotation produced by the couple, thumb showing the direction of the moment

- Vector formulation:
    - $\vec{M} = \vec{r} \times \vec{F}$, where $\vec{r}$ is an **arbitrary** vector connecting the lines of actions of these two forces



- It is a free vector (can be applied at any point). We will use this property to calculate the net moment of a system of forces and couple moments acting on a body.

Equivalent Systems



A single 200 lb man standing at the center of a beam causes the same reaction forces as two 100 lb children standing evenly along the beam as shown above. The weight force of the man is therefore statically equivalent to the weight forces of the two children.

Text: 4.7

Content:

- Equivalent systems: same $\vec{F}_R$ and $\vec{M}_R$
- A force can be shifted along its line of action
- Moving a force away from its line of action adds couple moment

From: http://mechanicsmap.psu.edu/websites/4_statically_equivalent_systems/statically_equivalent_systems/staticallyequivalentsystems.html

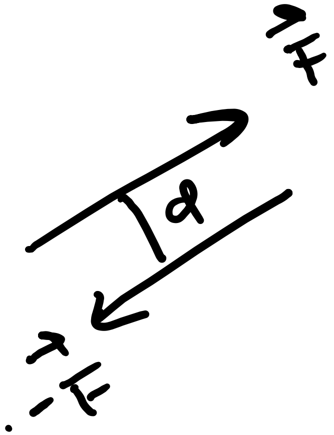
Q1: A couple moment applied to an object in general produces:

A. Translational effect

B. Rotational effect

C. Neither

D. Both



Pure rotation!

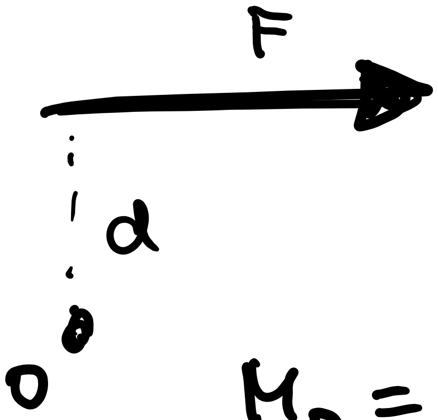
Q2: A force applied to an object in general produces:

A. Translational effect

B. Rotational effect

C. Neither

D. Both



$$M_O = F \cdot d$$

Q1: A couple moment applied to an object in general produces:

A. Translational effect
B. Rotational effect
C. Neither
D. Both

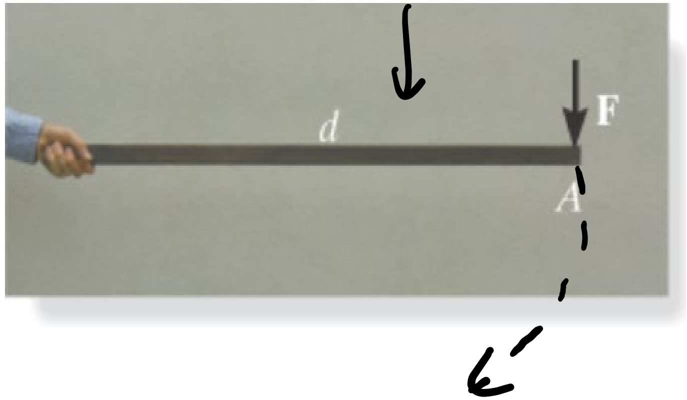
Q2: A force applied to an object in general produces:

A. Translational effect
B. Rotational effect
C. Neither
D. Both

EQUIVALENT SYSTEMS

- This is a story about simplifying things (not about making them more complex!)

- Two systems of forces and / or moments are **equivalent** if they produce the same effect on the system.

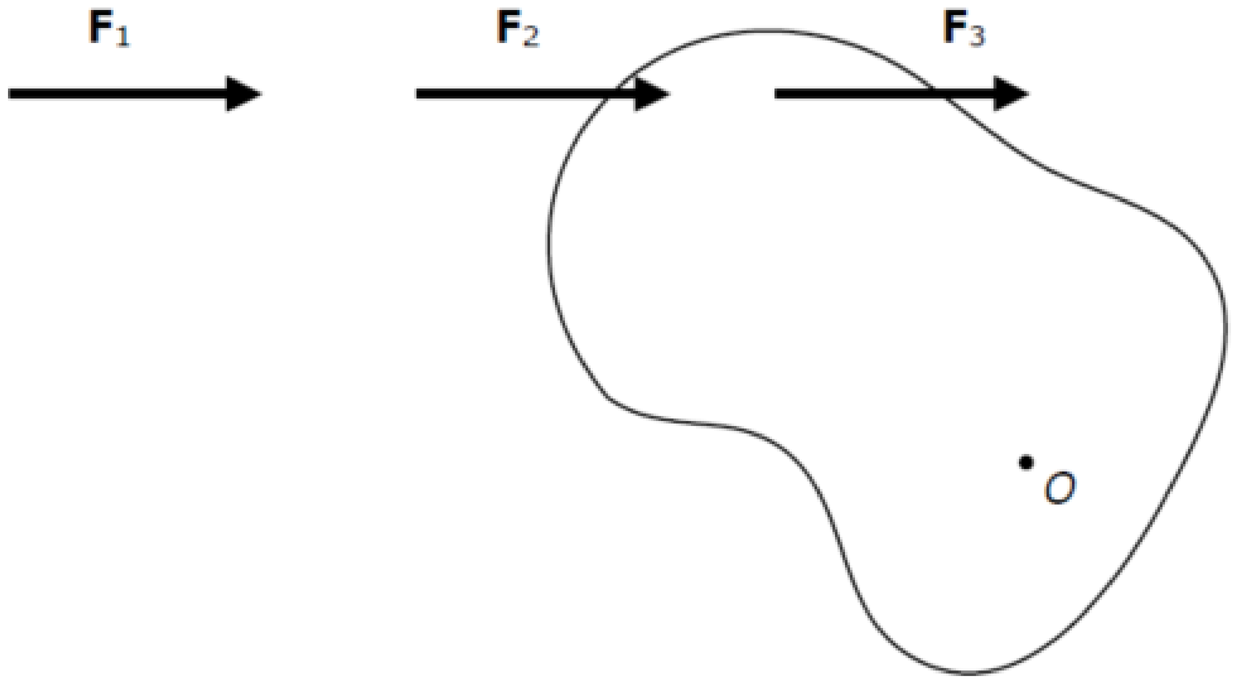


- Same said with the help of equations: Two systems are **equivalent** if:
  - The resultant forces $\vec{F}_R = \sum \vec{F}_i$ acting on them are equal, **<u>and</u>**
  - The resultant moments about some point O, $\vec{M}_R = \sum \vec{M}_{\text{couple}} + \sum \vec{M}_O$, are the same.

- Our goal will be to replace many forces and couple moments acting on the system by the resultant force $\vec{F}_R$ and the resultant moment $\vec{M}_R$ (about some chosen point) <u>maintaining the equivalency</u>.

MOVING FORCES

Q: Which of these forces results in the greatest moment about point O? Assume $F_1 = F_2 = F_3$.



A. Force $\vec{F}_1$

B. Force $\vec{F}_2$

C. Force $\vec{F}_3$

D. All of them produce the same moment

E. Not sure

MOVING FORCES

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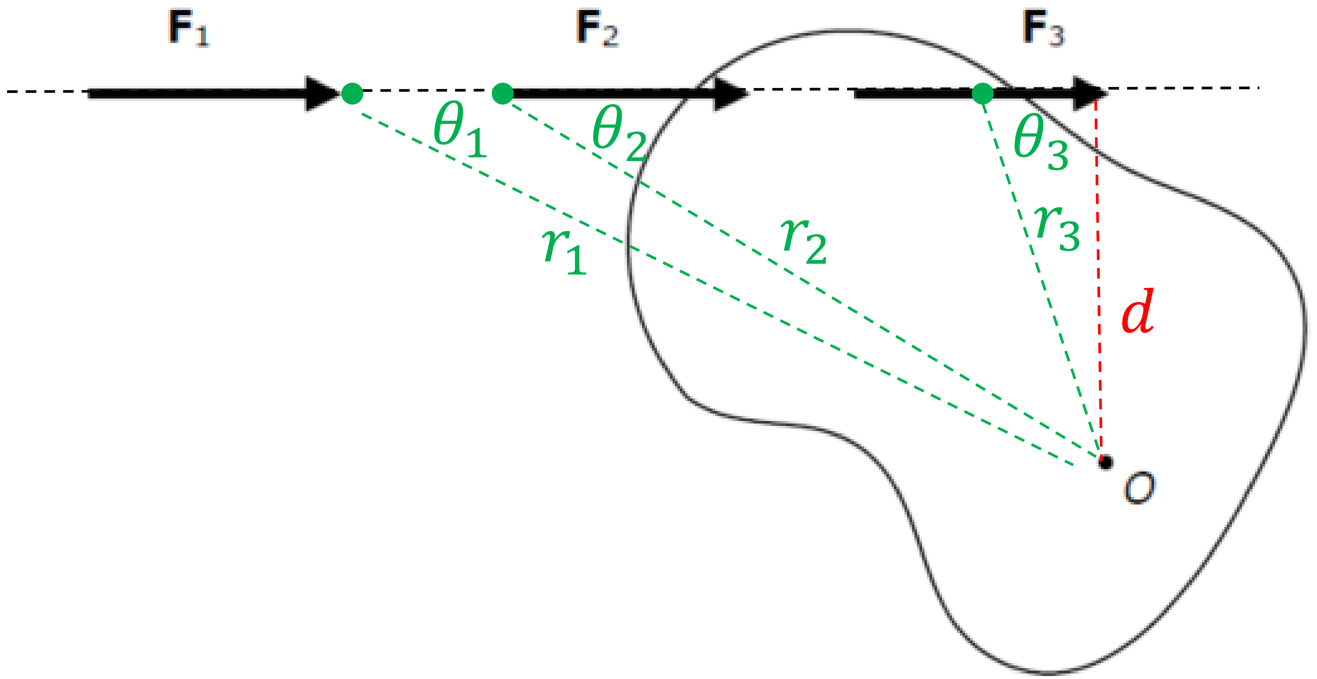
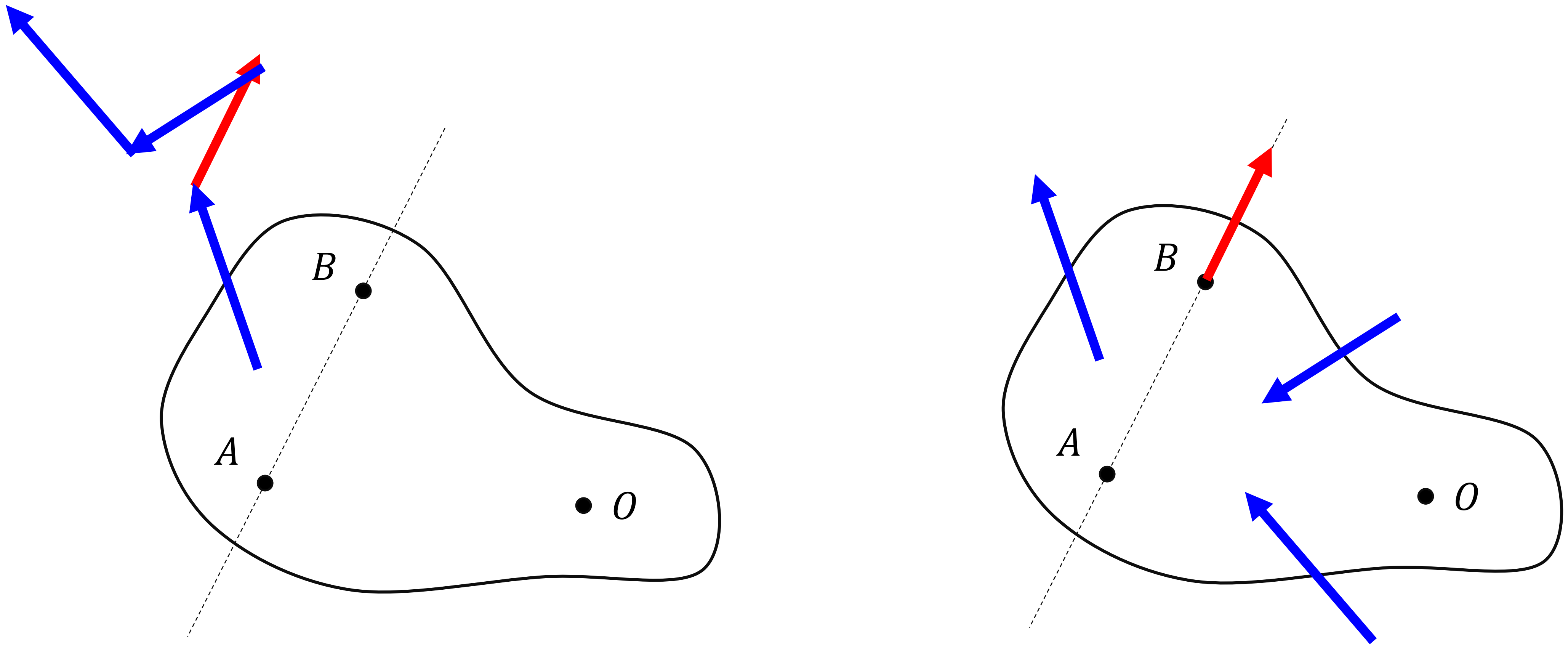


1. From scalar definition: $M = F\,d$, where d is the perpendicular distance from O to the line of action of the force => same moment, since they have the same line of action, and hence the same moment arm.
2. From vector definition: $M = F\,r\sin\theta$, where r is a vector connecting O with an arbitrary point on the line of action of the force. We see that for any point chosen on the axis we have: $r\sin\theta = d$ => $M = F\,d$ (same as with using the scalar definition), and d, again, is the same for all these forces.

MOVING FORCES

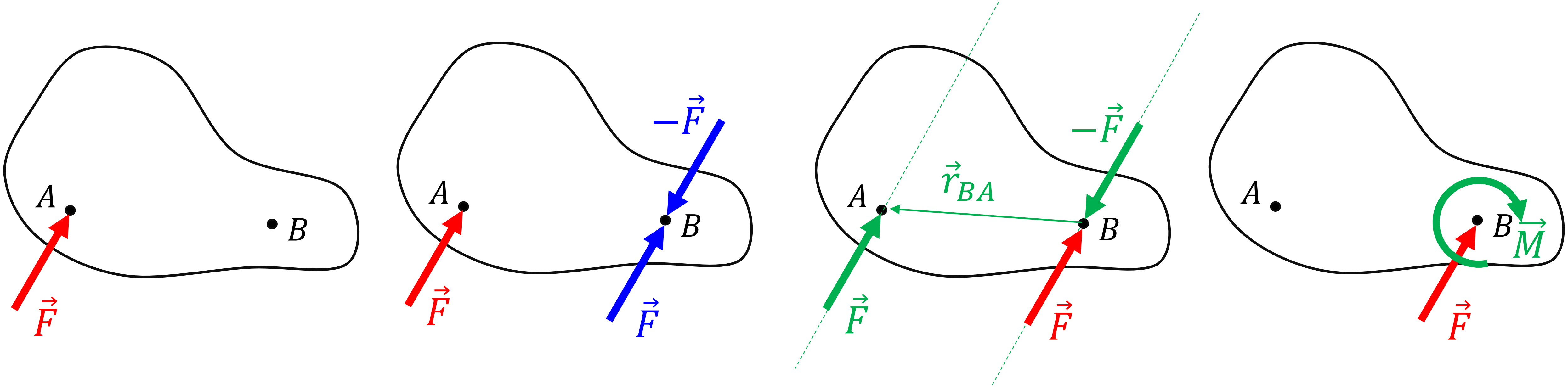
A force can be moved along its line of action.
This does not change anything.



- The force acting at *A* would produce the same resultant force and the same moment as when acting at *B*
- These two systems are **equivalent**

MOVING FORCES

- What happens if we move a force NOT along its line of action?



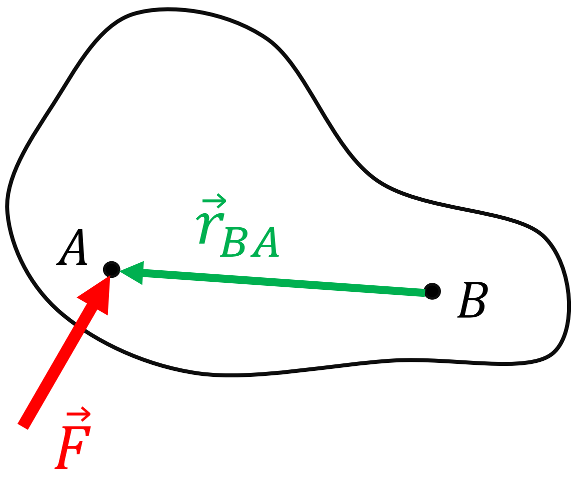
Force at A…

…shifted to point B…

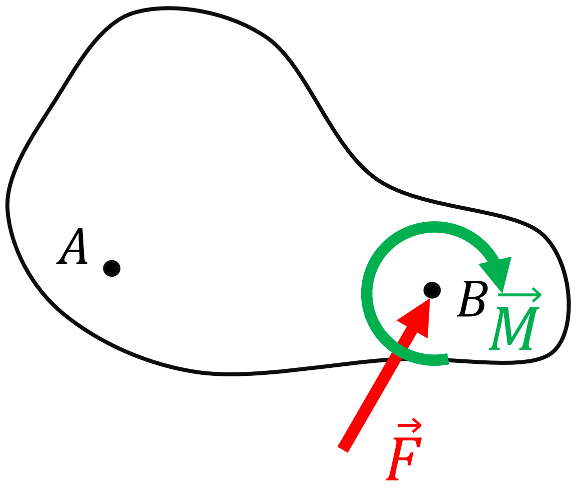
… is equivalent to a force acting at B and to added couple moment $\overrightarrow{M} = \vec{r}_{BA} \times \vec{F}$

MOVING FORCES

- What happens if we move a force NOT along its line of action?



The added couple moment is simply equal to the moment of the force placed at its initial location about the point we are going to shift it to.



Force at A…

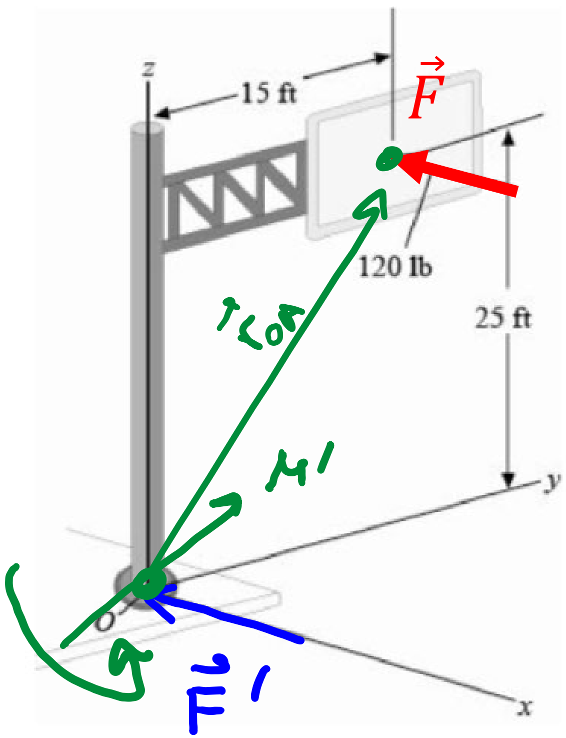
… is equivalent to a force acting at B and to added couple moment $\vec{M} = \vec{r}_{BA} \times \vec{F}$
"*compensating couple moment*"

NB: In general, a force applied to an object produces (i) *Translational effect*, and (ii) *Rotational effect*

• **Example**: The resultant force of a wind loading acts perpendicular to the face of the sign as shown. Replace this force by an equivalent force and couple moment acting at point *O*.

$\vec{F}' =$ $(-120)\vec{i}$ lb $= \vec{F}$

$\vec{M}' =$ $(-3000\vec{j} + 1800\vec{k})$ lb·ft



$\vec{M}' = \vec{r}_{OA} \times \vec{F} =$

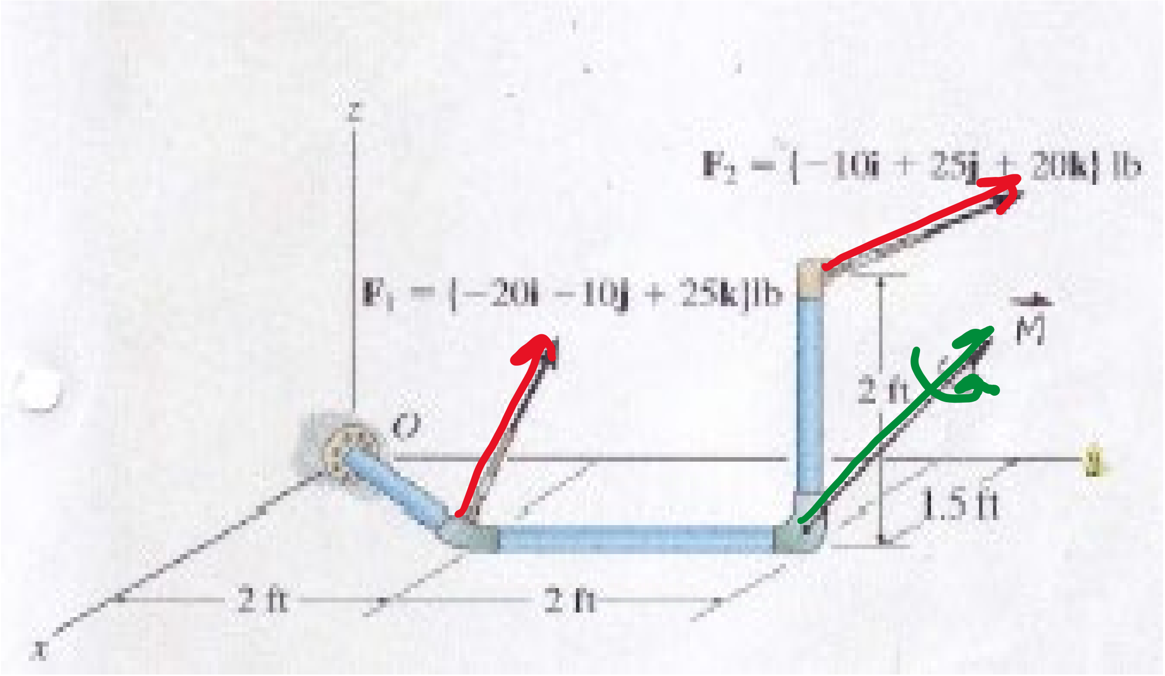
$\vec{r}_{OA} = (0)\vec{i} + (15)\vec{j} + (25)\vec{k}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 15 & 25 \\ -120 & 0 & 0 \end{vmatrix} = +\vec{i}\,(0) - \vec{j}\,(0-(25)(-120)) + \vec{k}\,(0-(15)(-120))$$

$= (-3000\vec{j} + 1800\vec{k})$ lb·ft

W4-2. The pipe assembly is acted on by forces $\vec{F}_1$ and $\vec{F}_2$ and by a couple moment $\vec{M} = (5\vec{i} + 6\vec{j} + 7\vec{k})$ lb·ft.

1) Determine the magnitude and coordinate direction angles of $\vec{M}$.
2) Determine the magnitude of each of the forces comprising the couple when the moment arm of the couple is 0.5 ft.
3) Replace the force-couple system by a resultant force and couple moment at O. Express the results in Cartesian vector form.



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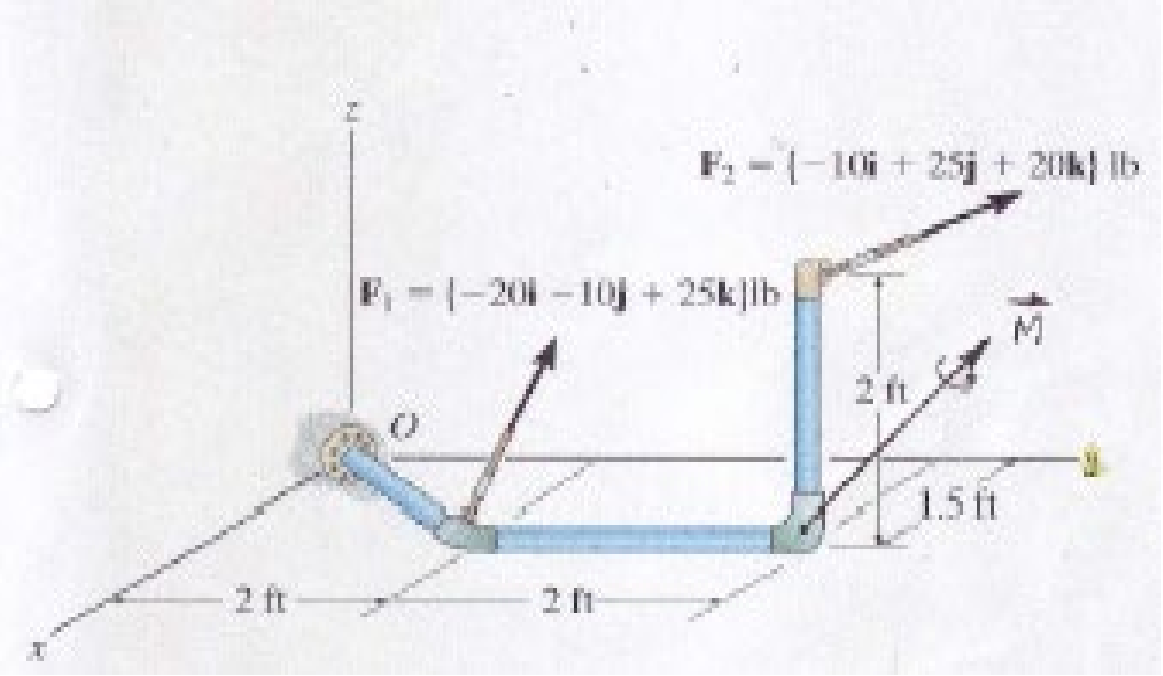

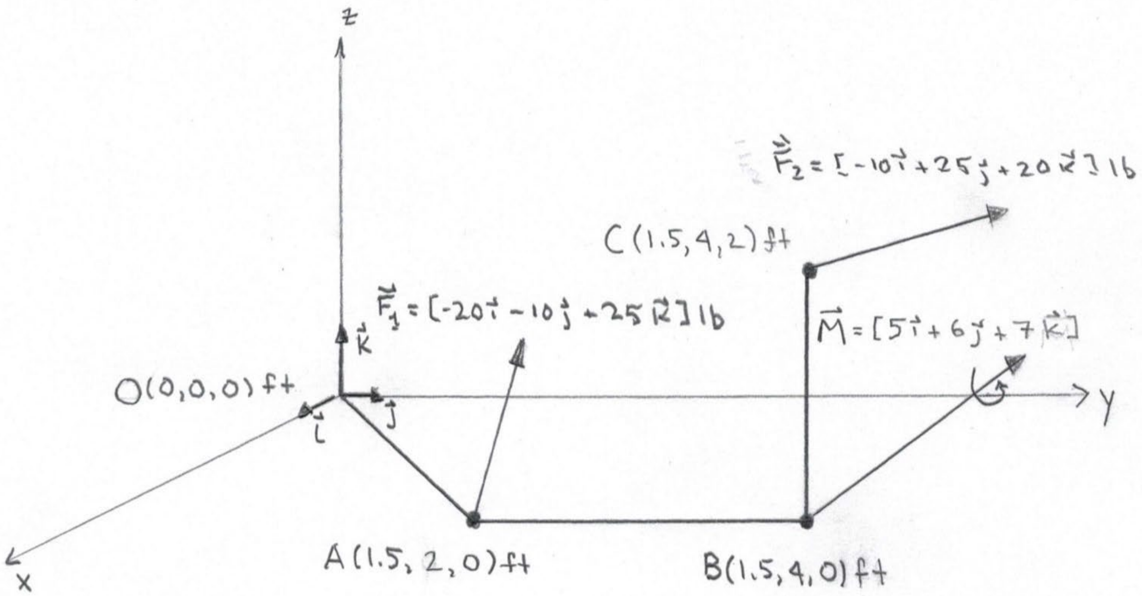

W4-2. The pipe assembly is acted on by forces $\vec{F}_1$ and $\vec{F}_2$ and by a couple moment $\vec{M} = \boxed{(5\vec{i} + 6\vec{j} + 7\vec{k})\text{ lb·ft.}}$

α, β, γ

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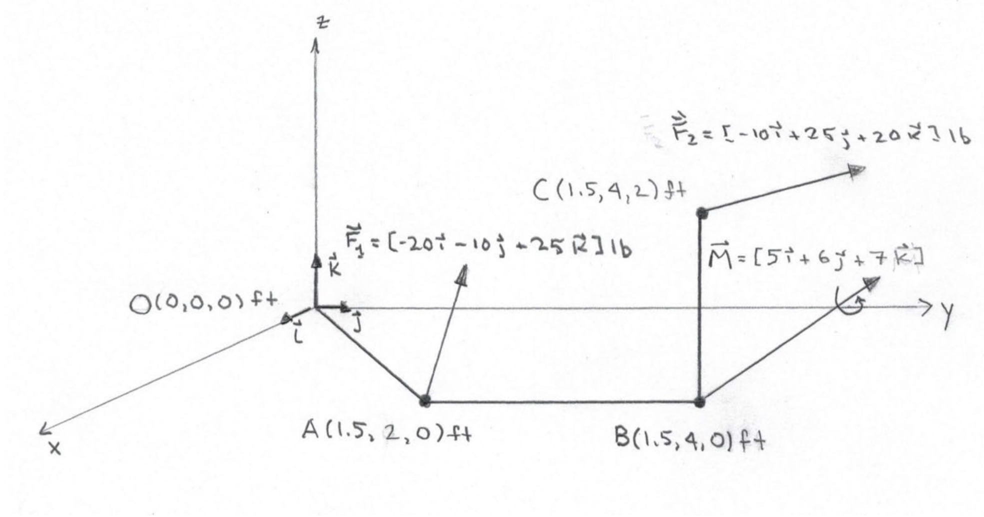

$$M = \sqrt{5^2 + 6^2 + 7^2} = \underline{\underline{10.49}}$$

$$\boxed{M = 10.5\ \text{lb·ft}}$$

$M_x = M\cos\alpha$ $M_y = M\cos\beta$ $M_z = M\cos\gamma$

$$\cos\alpha = \frac{M_x}{M} = \frac{5}{10.45} \rightarrow \alpha = 61.5^\circ$$

$\beta = 55.1^\circ$

$\gamma = 48.1^\circ$

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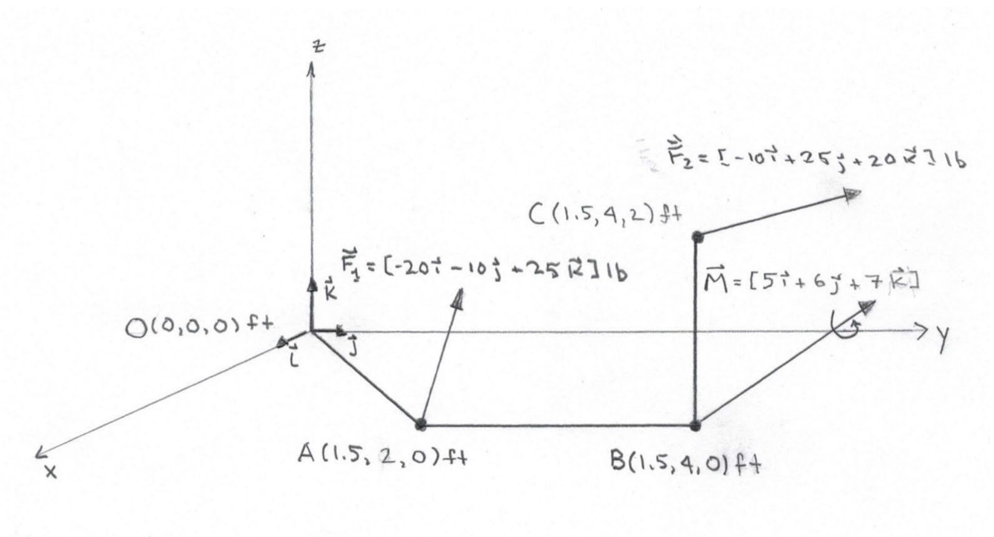
- $M = 10.5$ lb ft



$$M = F \cdot d$$

$$F = \frac{M}{d} = \frac{10.49}{0.5} = 21.0 \text{ lb}$$