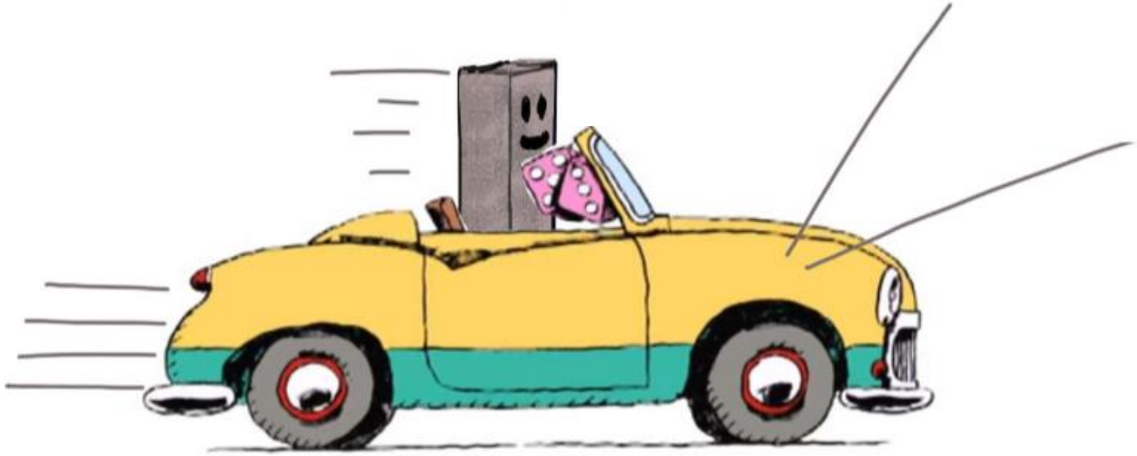
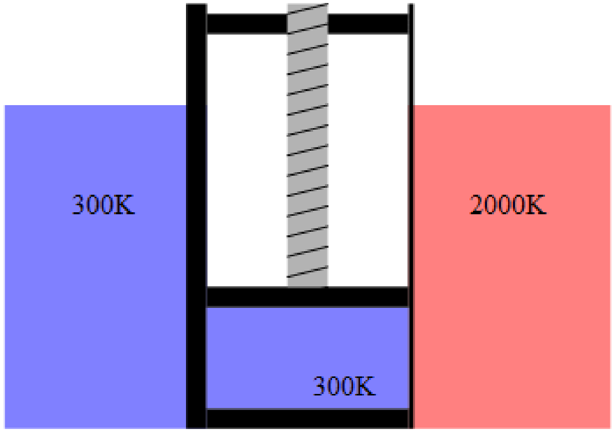
Lecture 19.

Otto cycle.



Heat engines

- (partially) convert heat to work via cyclic process



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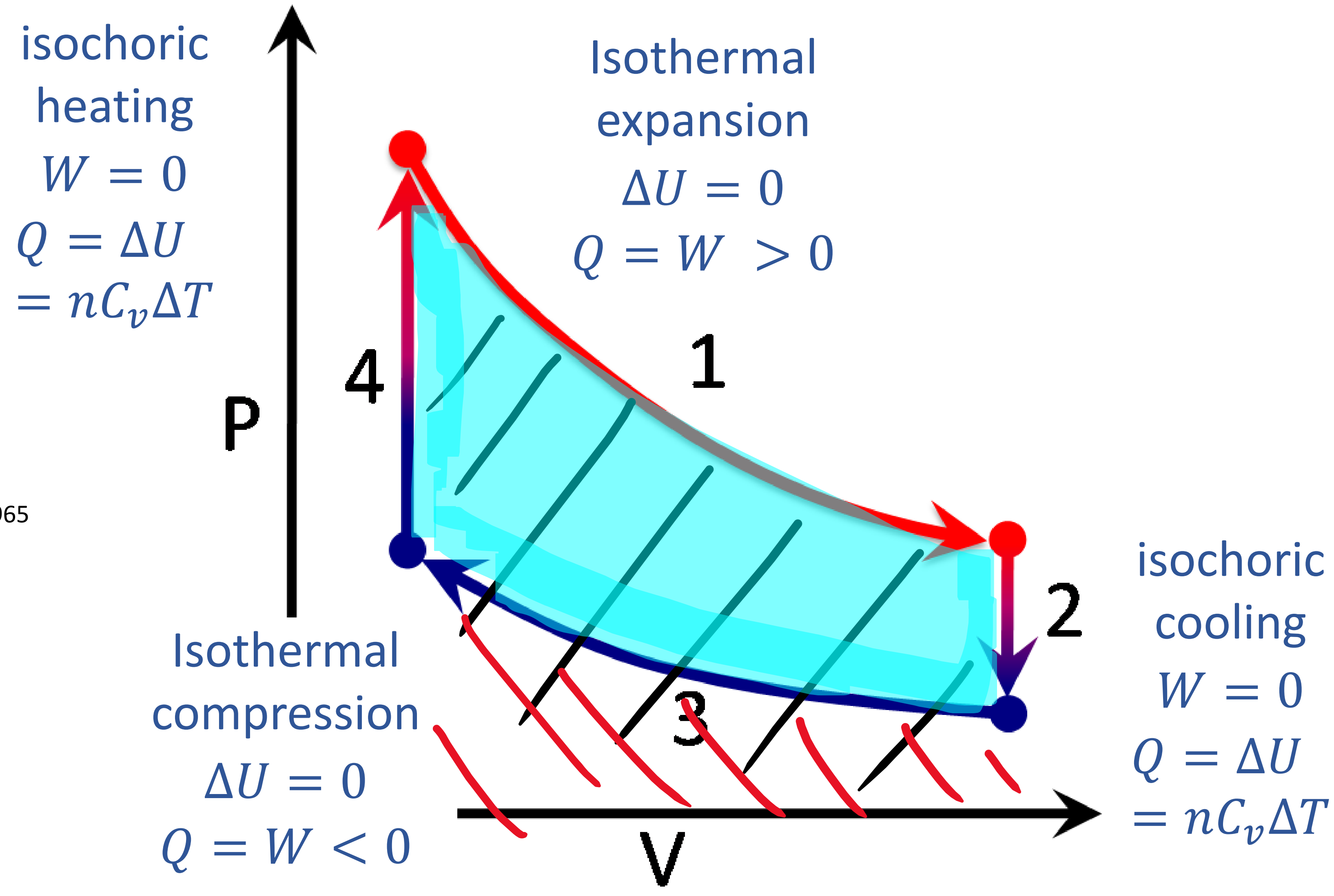
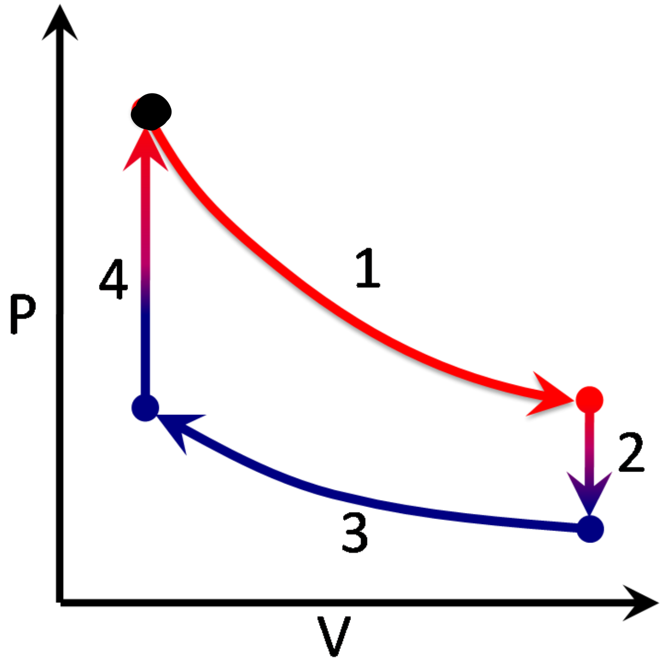
- example:

  "Stirling cycle"



Q: Around a full cycle, we can say that the net heat flow Q is



$$0 = \Delta U = Q_{in} - W_{out}$$

$$Q_{in} = W_{out}$$

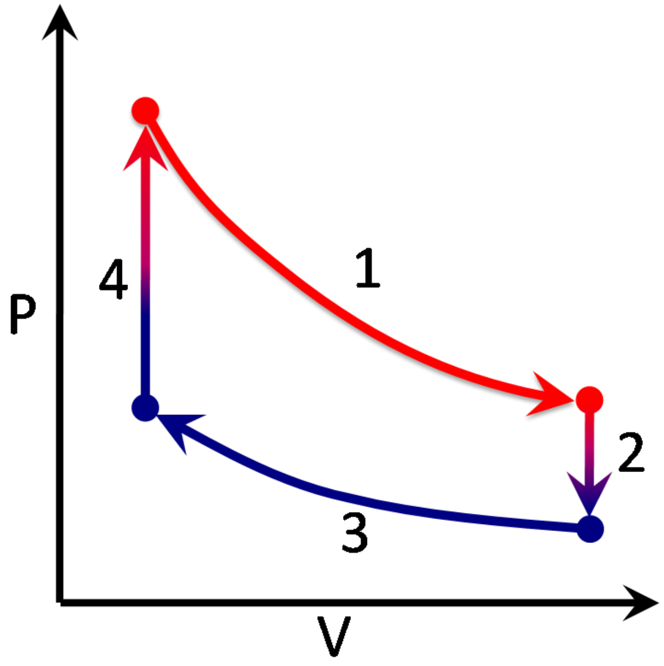
A. greater than the net work W

B. equal to the net work W

C. less than the net work W

D. Any of the above are possible, depending on the cycle

Q: Around a full cycle, we can say that the net heat flow Q is



$\Delta U = 0$ for full cycle

so $Q_{net} = W_{net}$

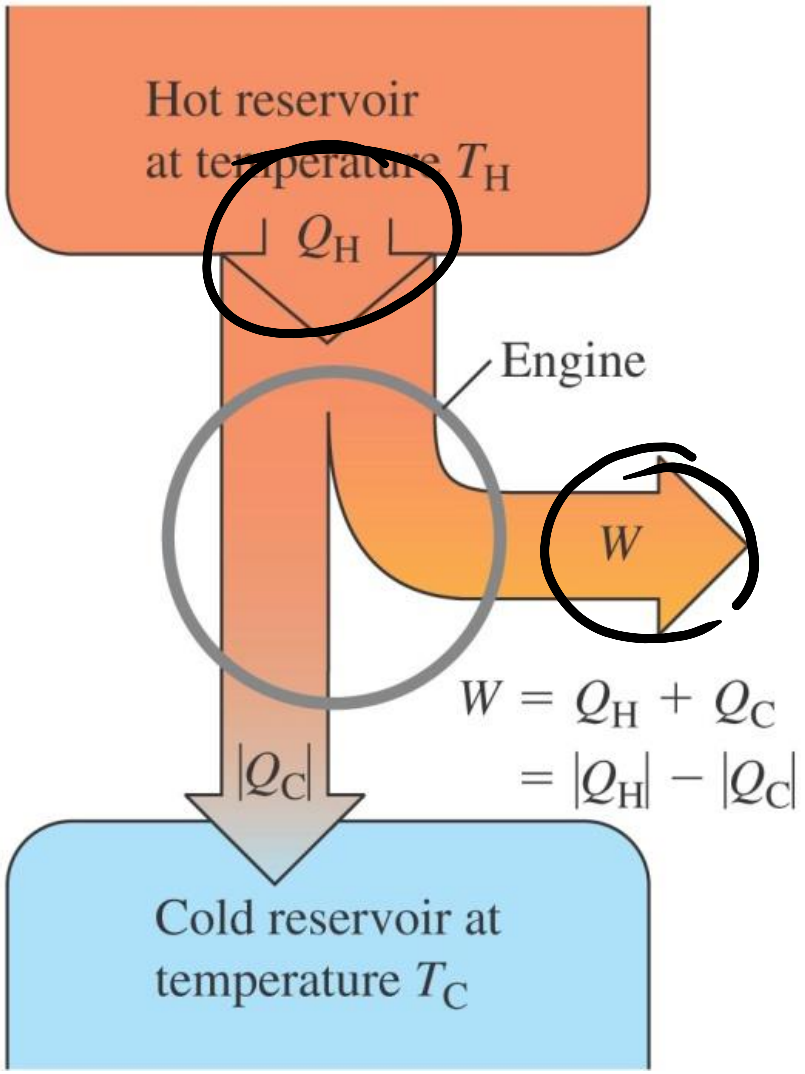
A. greater than the net work W

B. equal to the net work W ✓

C. less than the net work W

D. Any of the above are possible, depending on the cycle

Efficiency of an Engine:



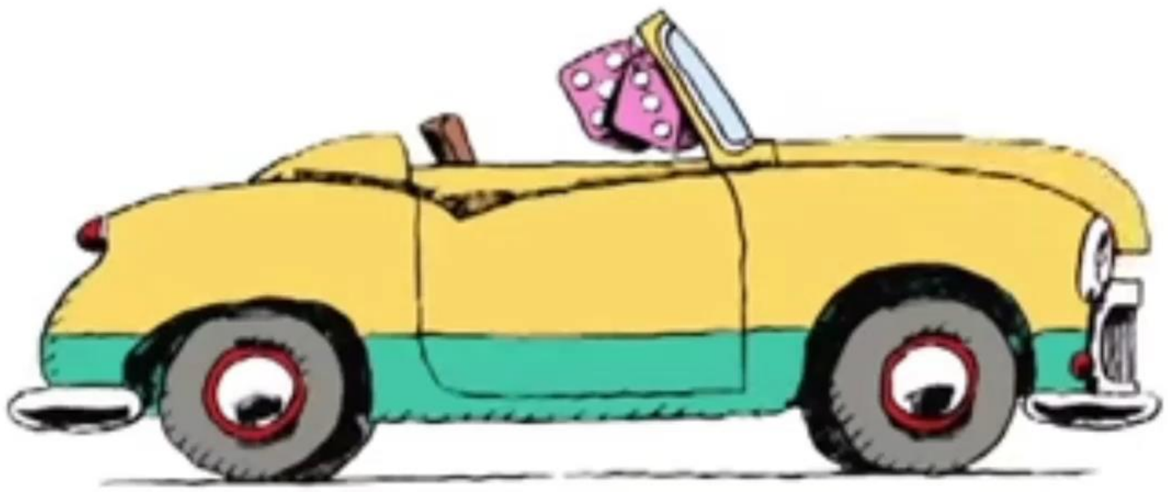
- Efficiency $(e) = \dfrac{\text{net work we get out}}{\text{heat we need to supply}}$

- Q_H: Heat absorbed by gas each cycle
- Q_C: Heat expelled by gas each cycle
- W: Net work done each cycle

$$\boxed{e = \frac{W}{Q_H} = 1 - \frac{|Q_C|}{|Q_H|}}$$

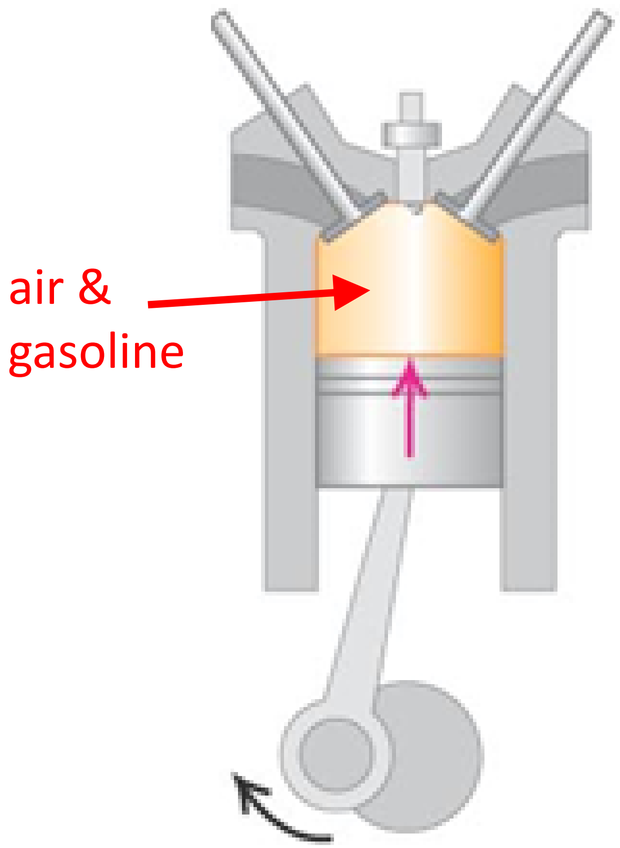
$0 < e < 1$

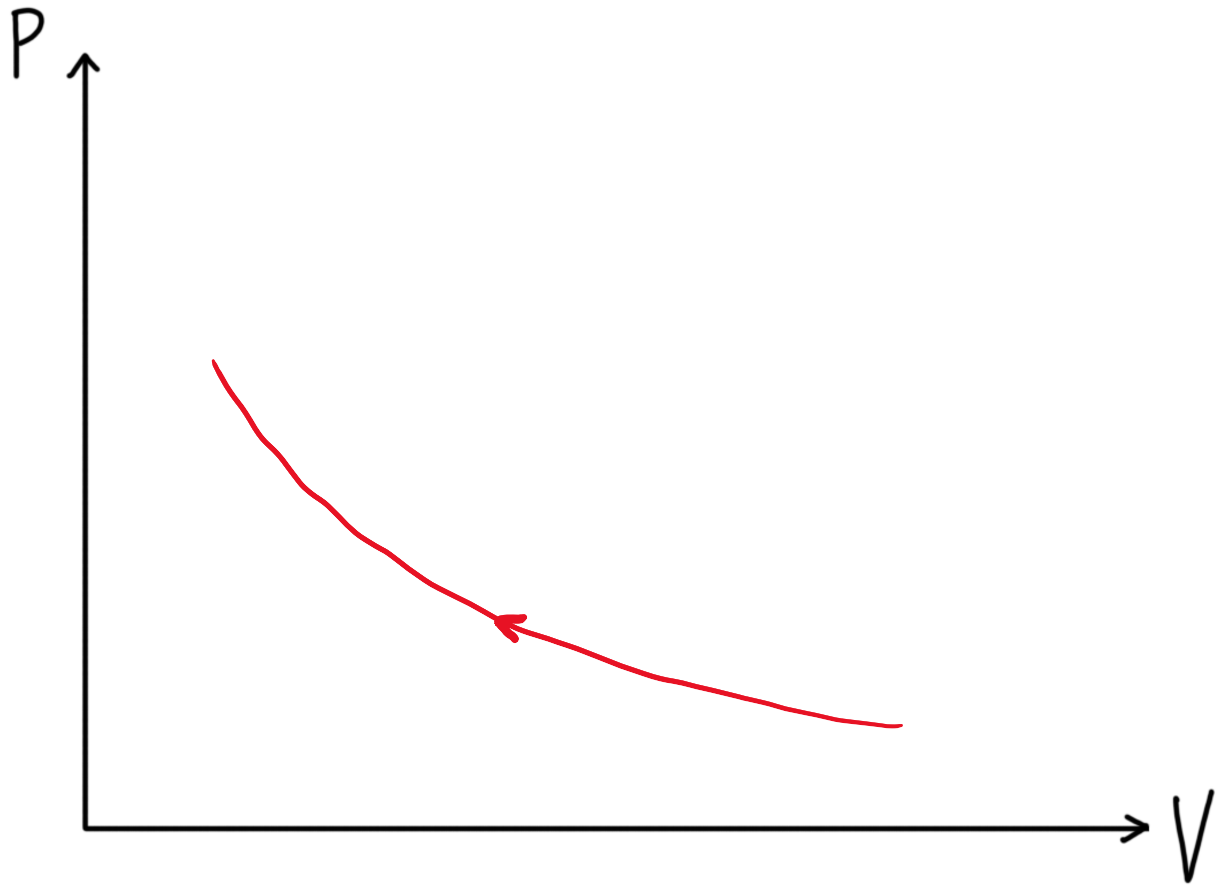
Internal combustion engine movie



https://www.youtube.com/watch?app=desktop&v=5tN6eynMMNw&feature=youtu.be&t=26

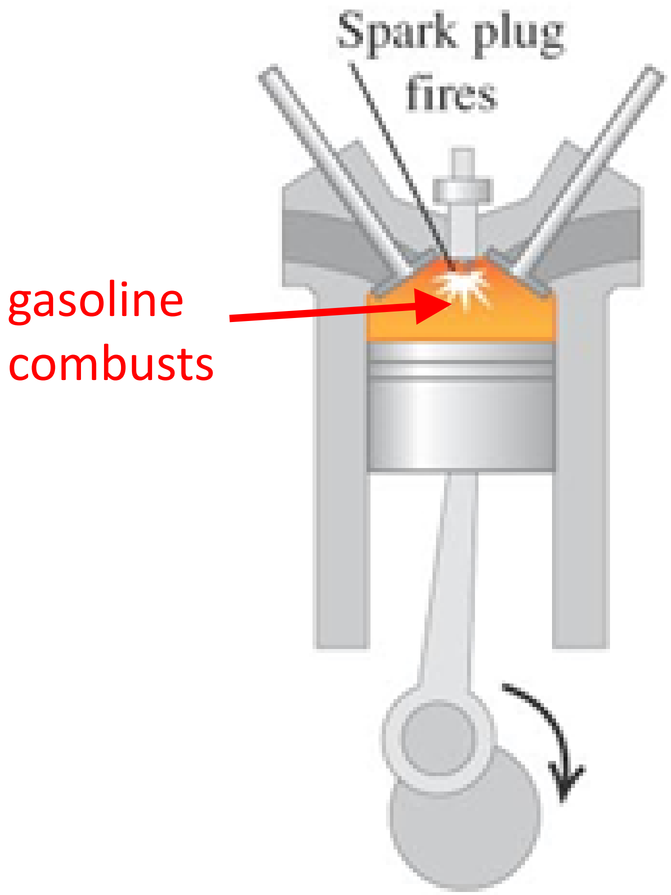
Step 1: adiabatic compression

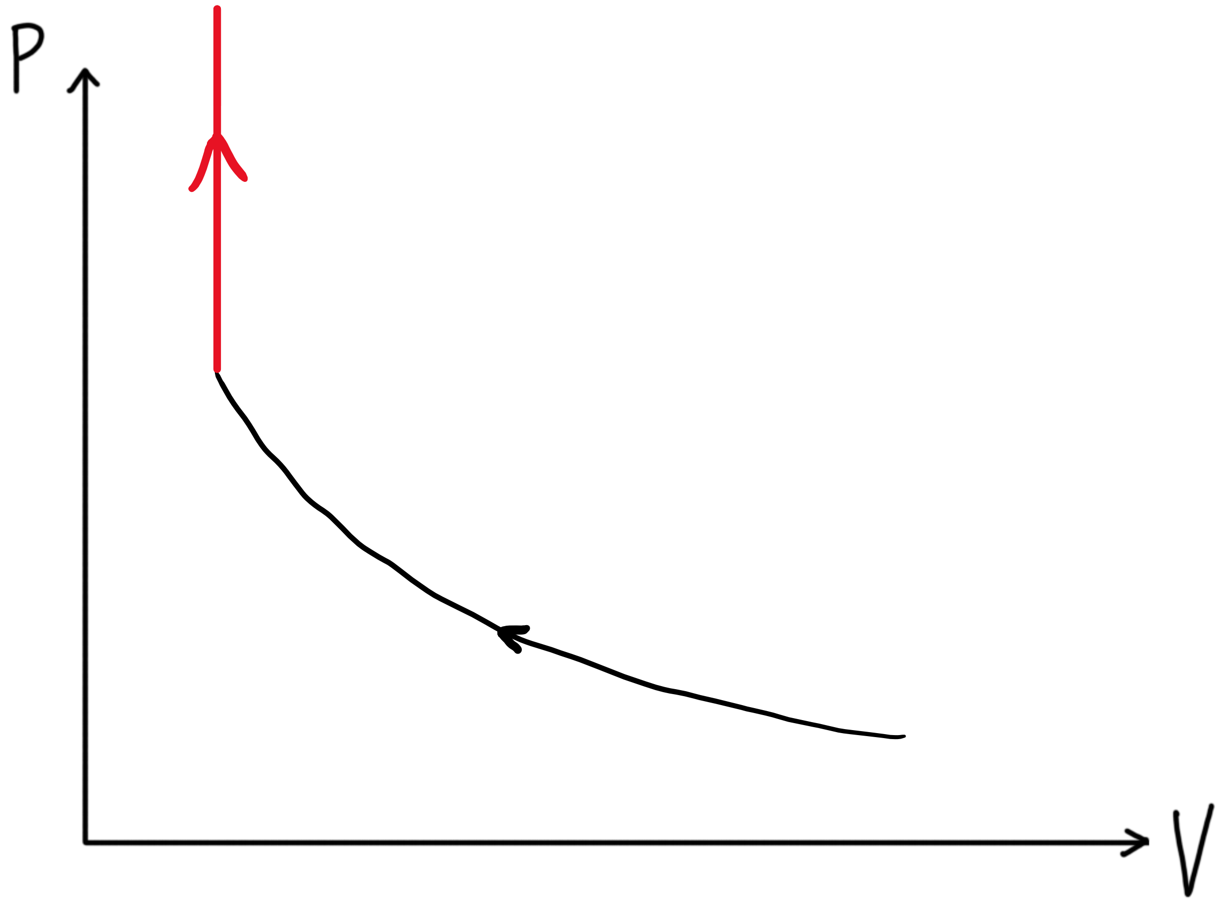




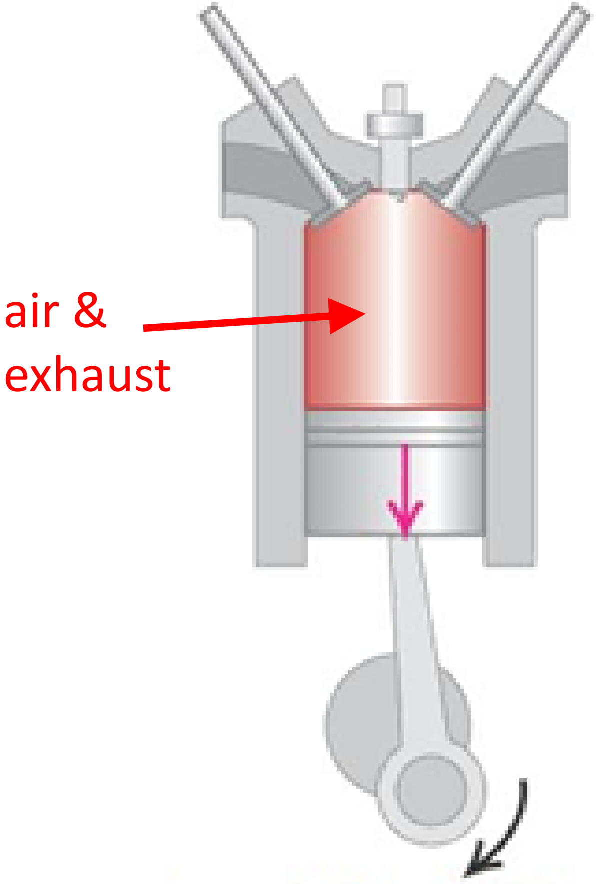
"compression stroke"

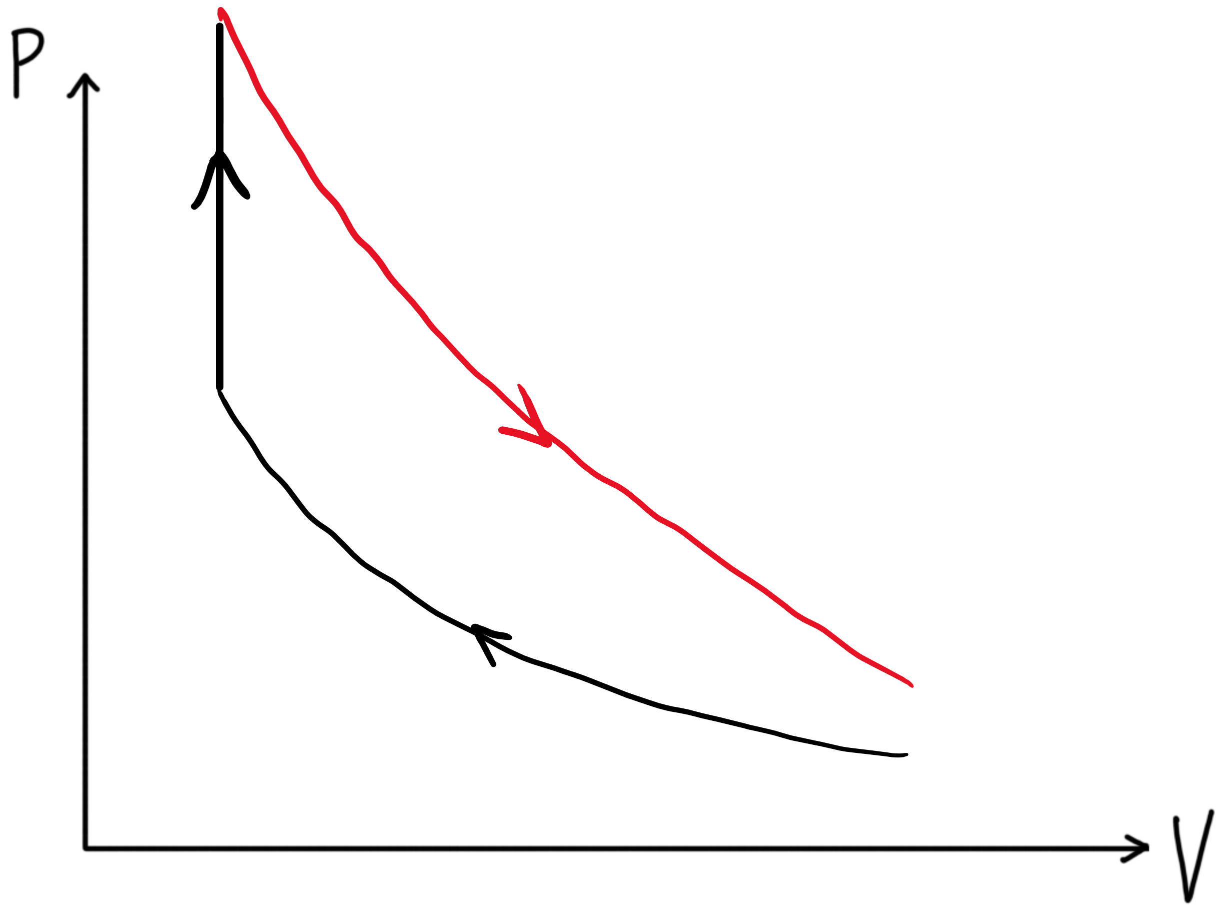
Step 2: combustion of gasoline (≈heating at a constant volume)





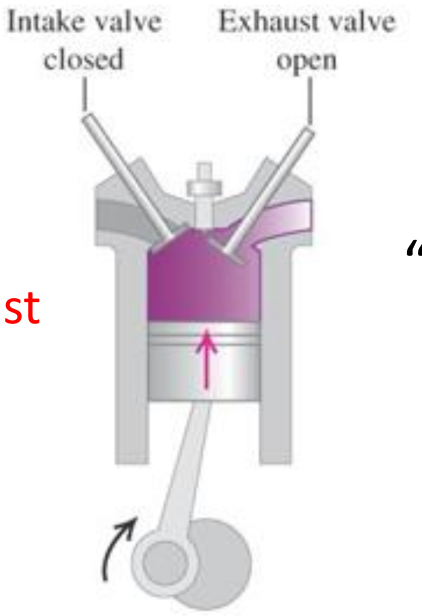
Step 3: adiabatic expansion





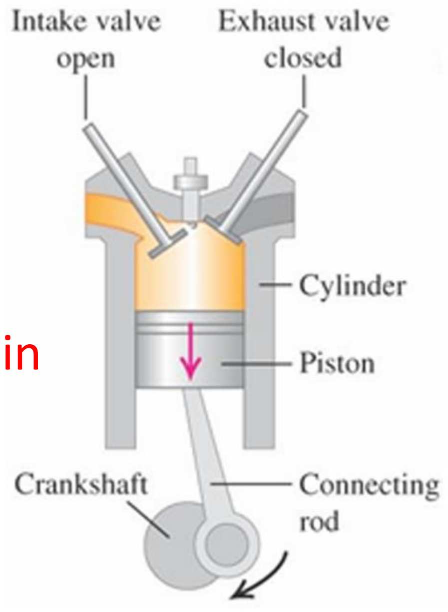
“power stroke”

Otto Cycle



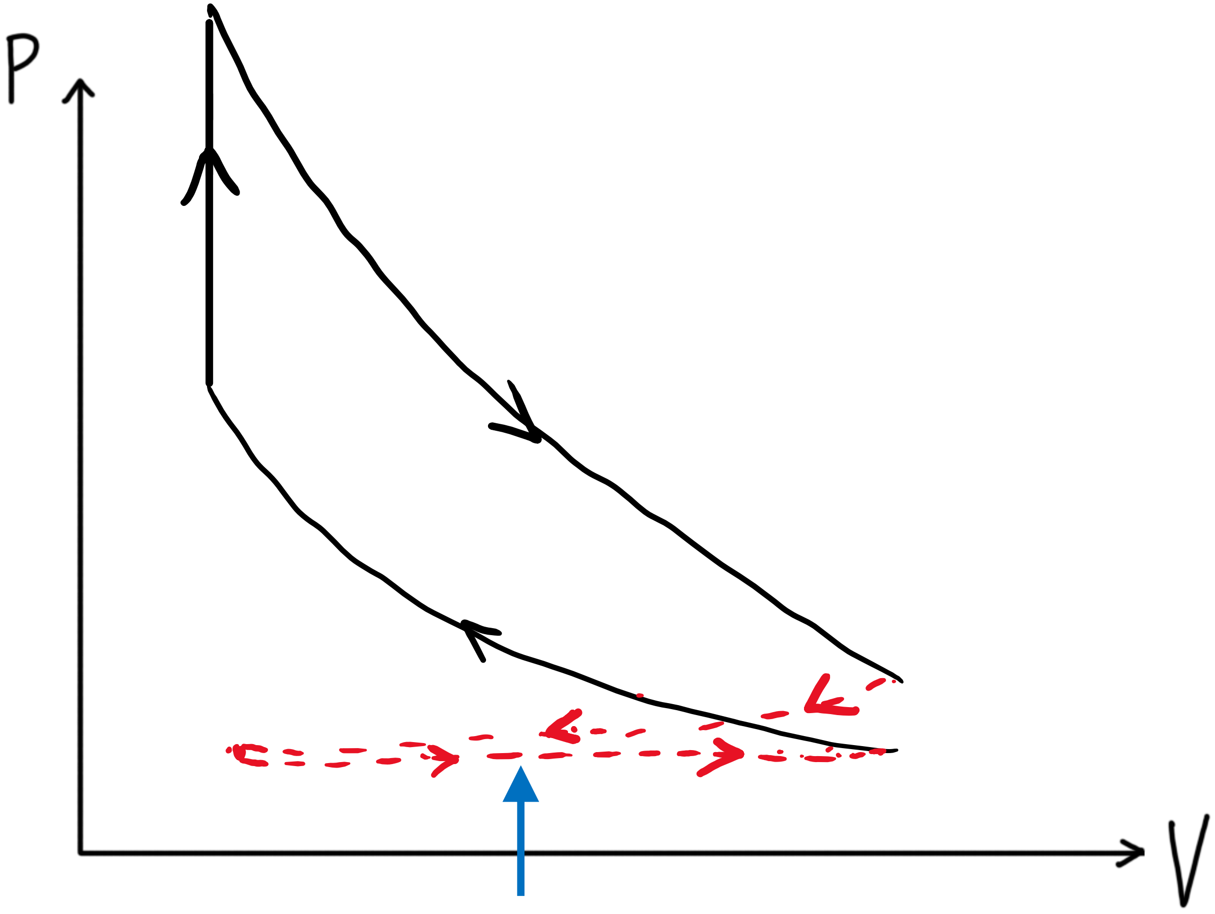
exhaust
out

“exhaust
stroke”



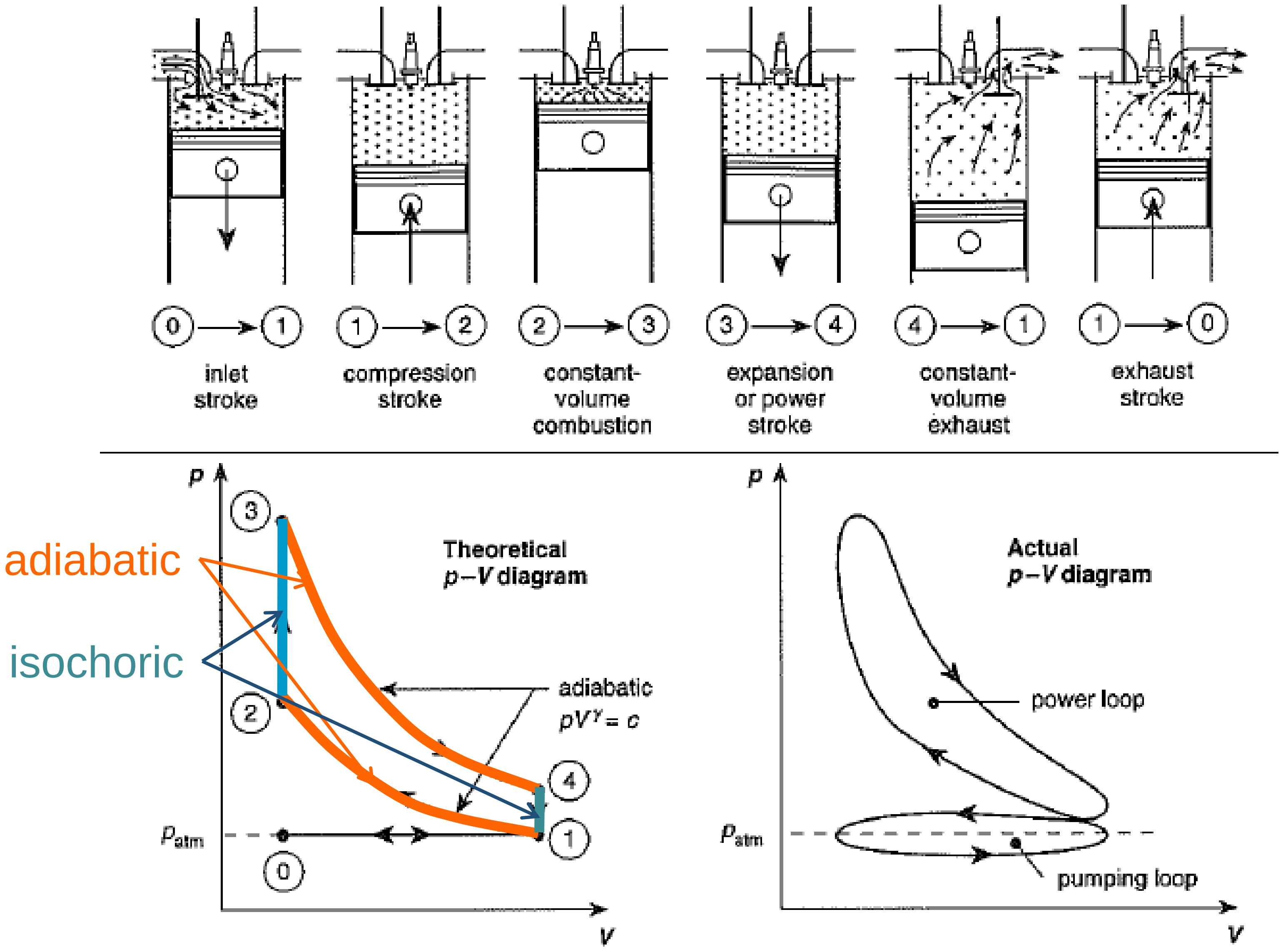
air &
gasoline in

“intake
stroke”



Not a significant amount of
net work, so model as
constant volume process

Otto cycle: model of internal combustion engine



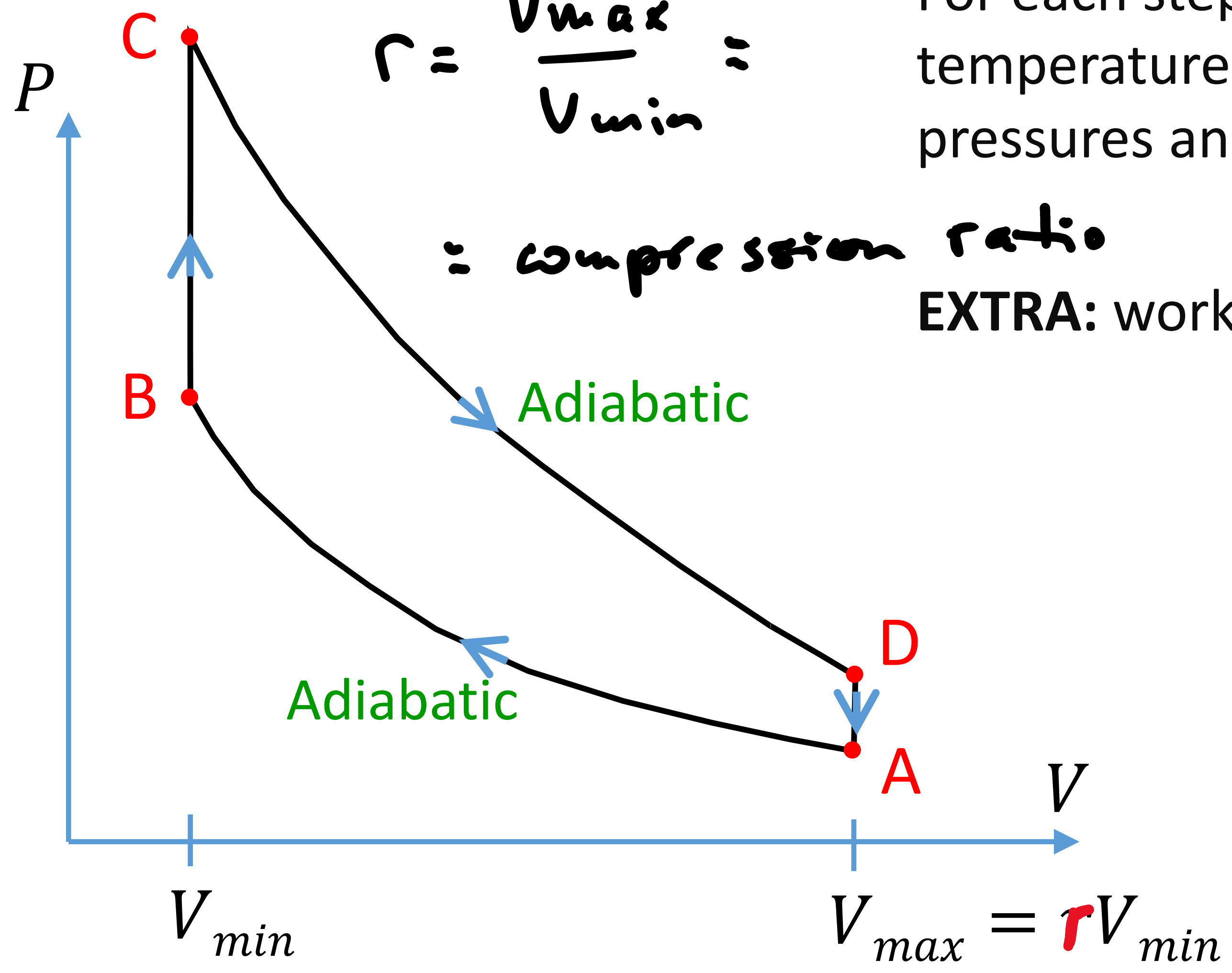
"Otto cycle"
(expansion & compression are done adiabatically)

Example problem

Q: Calculate the efficiency of the internal combustion engine operating via the cycle shown.

$r = \frac{V_{max}}{V_{min}} =$

$=$ compression ratio



For each step, write an equation that relates the final temperature to the initial temperature, assuming all pressures and volumes are known.

EXTRA: work out temperatures in term of T_A, r, and $\frac{P_C}{P_B}$

- Given: r
- Assume that we know T_A, V_{min}, V_{max}
- Let's figure out the rest

What are you trying to calculate?

- n : use $PV = nRT$ (always)

- The following equations are generally used for constant n:

  - T, V, or P : use $\frac{PV}{T} = \text{constant}$ (always)

    $\frac{P}{T} = \text{constant}$ (const V) $\frac{V}{T} = \text{constant}$ (const P) $PV = \text{constant}$ (const T)

    $PV^{\gamma} = \text{constant}$ (adiabatic) $TV^{\gamma-1} = \text{constant}$ (adiabatic)

  - ΔU : have $\Delta U = nC_v \Delta T$ (always)

  - W : have $W = \int_{V_i}^{V_f} P(V)\, dV$ (always)

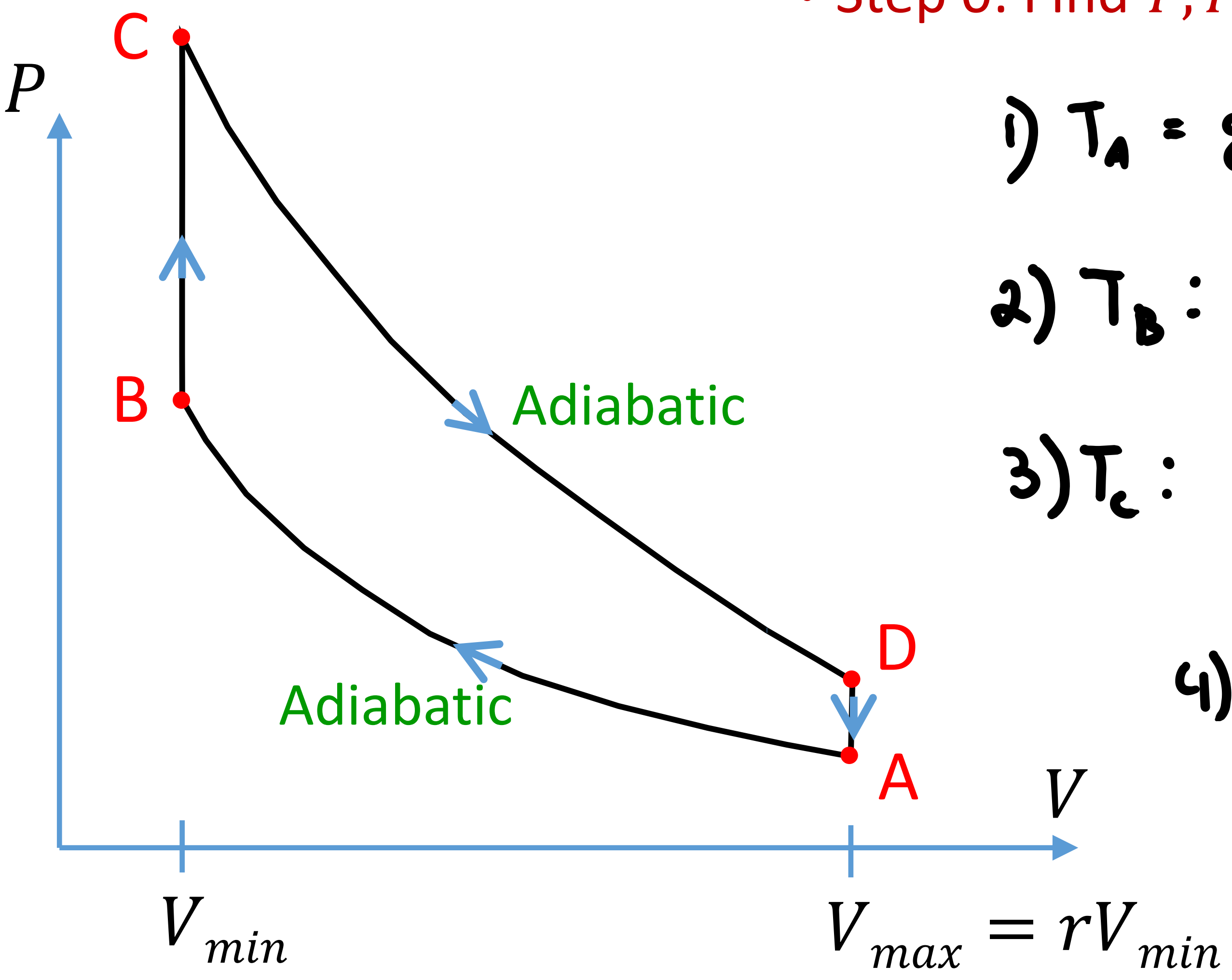
    $W = 0$ (const V) $W = P\Delta V$ (const P) $W = nRT \ln\left(\frac{V_f}{V_i}\right)$ (const T)

  - Q : use $Q = \Delta U + W$ (always)

    $Q = nC_v \Delta T$ (const V) $Q = nC_p \Delta T$ (const P) $Q = 0$ (adiabatic)

Example problem

Q: Calculate the efficiency of the internal combustion engine operating via the cycle shown.



- Step 0: Find T, P, V for the various points

1) T_A = given

2) T_B: $T_B V_{min}^{\gamma-1} = T_A V_{max}^{\gamma-1}$

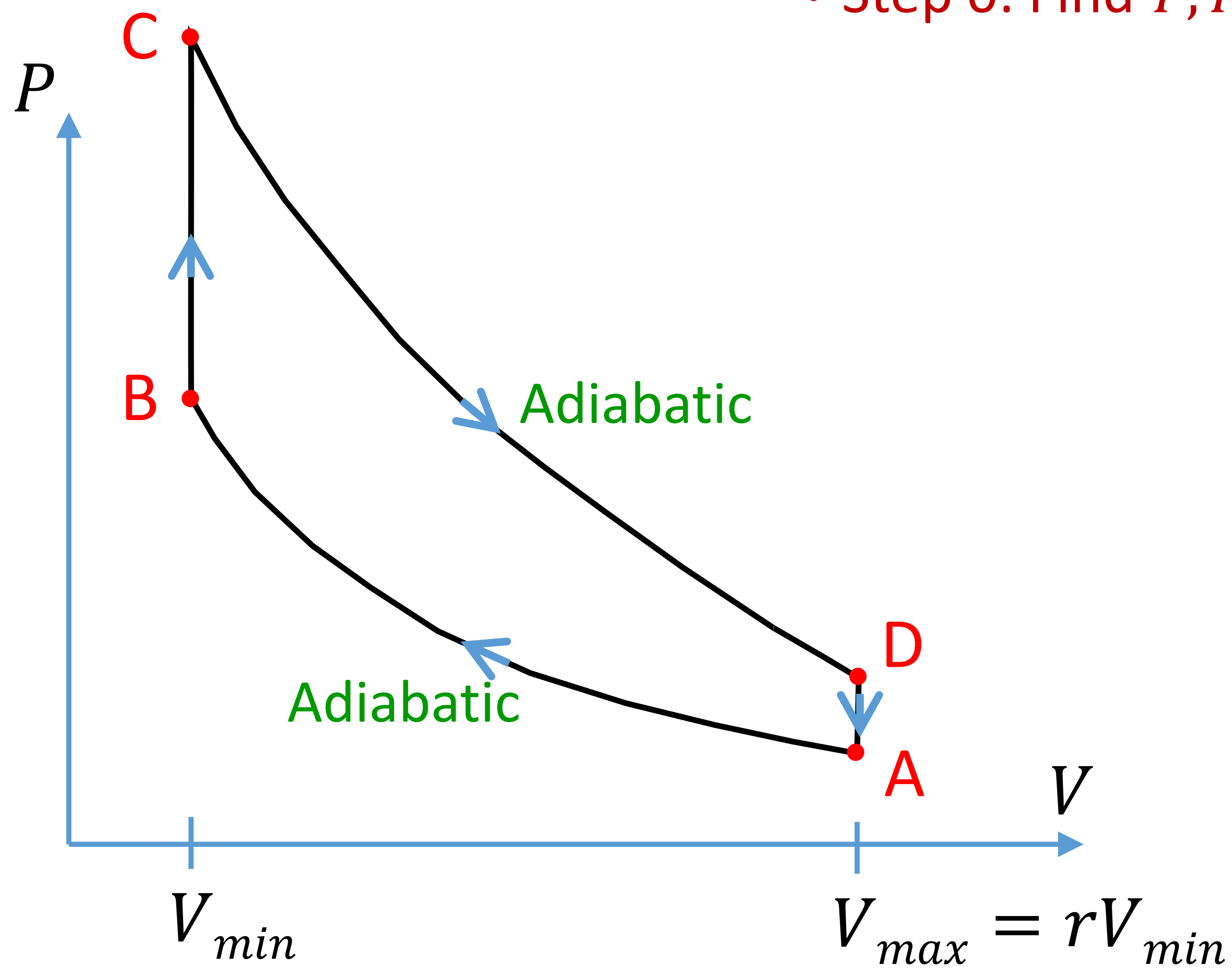
3) T_C: $\frac{P_B}{T_B} = \frac{P_C}{T_C} \rightarrow T_C P_B = T_B P_C$

4): T_D: $T_D V_{max}^{\gamma-1} = T_C V_{min}^{\gamma-1}$

Example problem

Q: Calculate the efficiency of the internal combustion engine operating via the cycle shown.

- Step 0: Find T, P, V for the various points

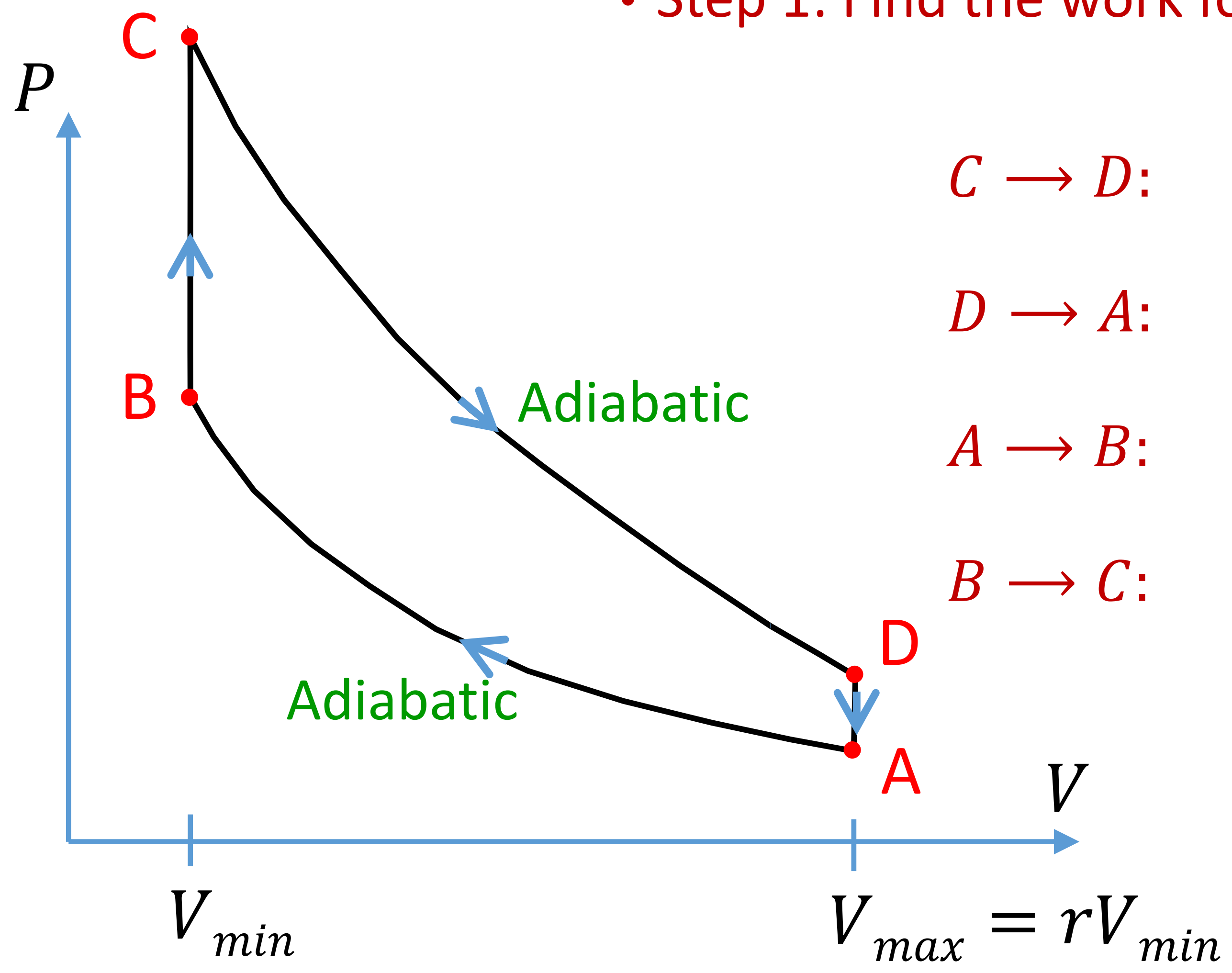


1) T_A: given.
2) $T_B V_{min}^{\gamma-1} = T_A V_{max}^{\gamma-1}$
3) $\frac{T_C}{P_C} = \frac{T_B}{P_B} \quad \Longrightarrow \quad T_C P_B = P_C T_B$
4) $T_C V_{min}^{\gamma-1} = T_D V_{max}^{\gamma-1}$

Example problem

$$\Delta U = \cancel{Q}^{\,0} - W \rightarrow W = -\Delta U$$

Q: Calculate the efficiency of the internal combustion engine operating via the cycle shown.



- Step 1: Find the work for each part, and add them up

$C \longrightarrow D$: $W_{C\to D} = -nC_v(T_D - T_C)$

$D \longrightarrow A$: $W = 0$

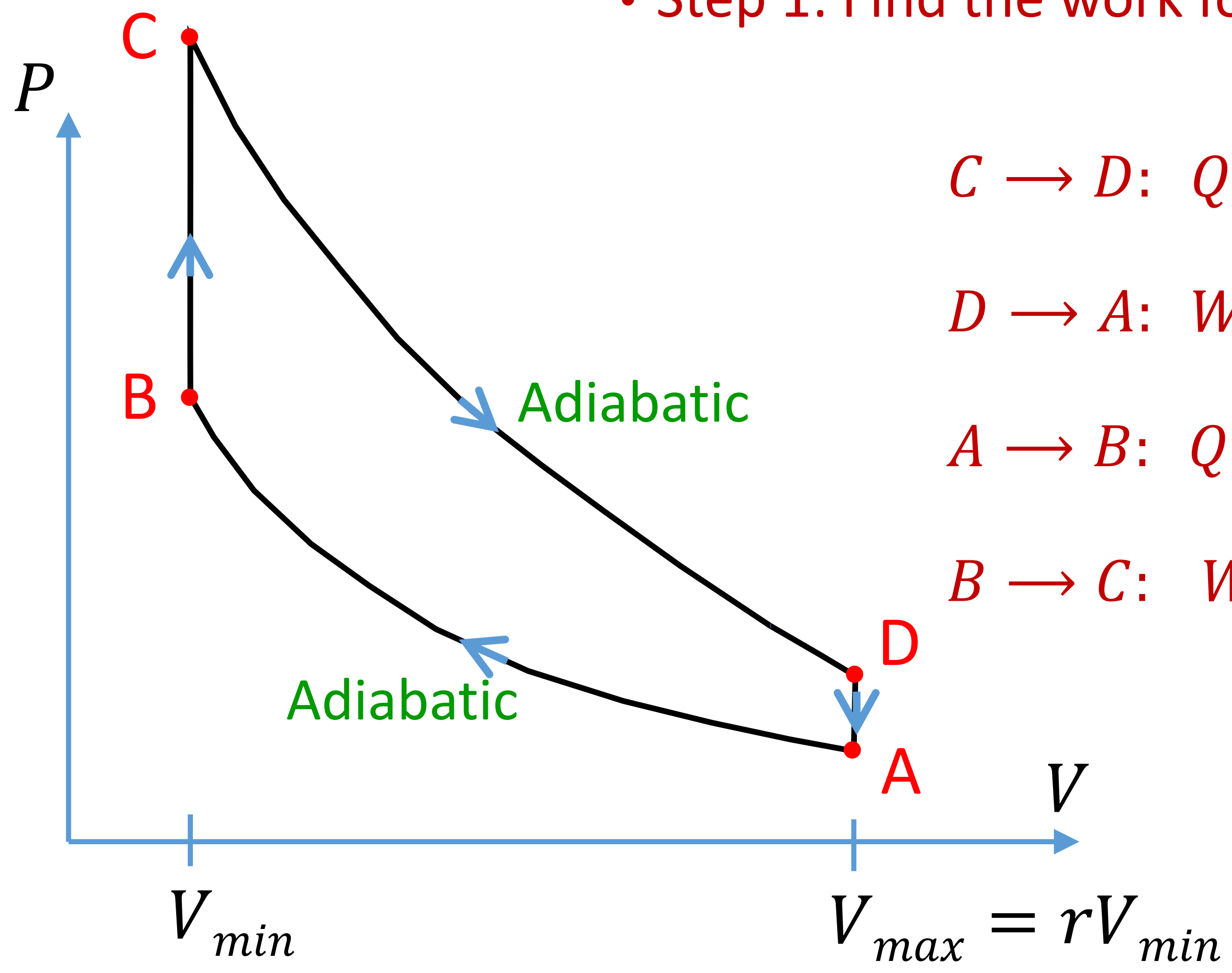
$A \longrightarrow B$: $W_{A\to B} = -nC_v(T_B - T_A)$

$B \longrightarrow C$: $W = 0$

$$W_{net} = nC_v(T_A - T_B + T_C - T_B)$$

Example problem

Q: Calculate the efficiency of the internal combustion engine operating via the cycle shown.



- Step 1: Find the work for each part, and add them up

$$C \longrightarrow D: \quad Q = 0 \text{ so } W_{C\longrightarrow D} = -\Delta U = -nC_v(T_D - T_C)$$

$$D \longrightarrow A: \quad W_{D\longrightarrow A} = 0 \text{ (const volume)}$$

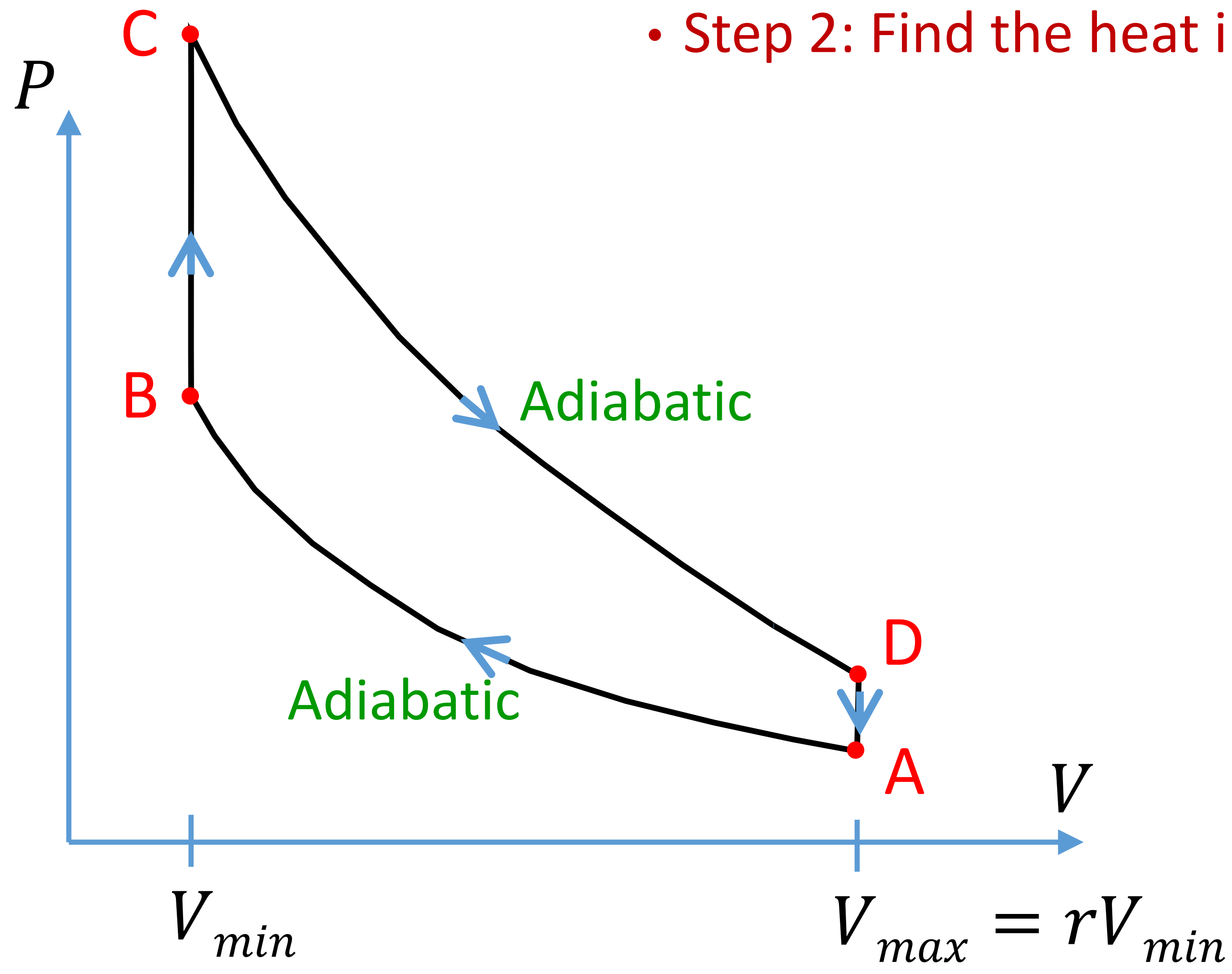
$$A \longrightarrow B: \quad Q = 0 \text{ so } W_{A\longrightarrow B} = -\Delta U = -nC_v(T_B - T_A)$$

$$B \longrightarrow C: \quad W_{B\longrightarrow C} = 0 \text{ (const volume)}$$

$$W_{net} = nC_v(T_C - T_D + T_A - T_B)$$

Example problem

Q: Calculate the efficiency of the internal combustion engine operating via the cycle shown.



- Step 2: Find the heat input with the steps with $Q > 0$

Q: Which processes have $Q > 0$?

A. A → B Q = 0 (adiabatic!)

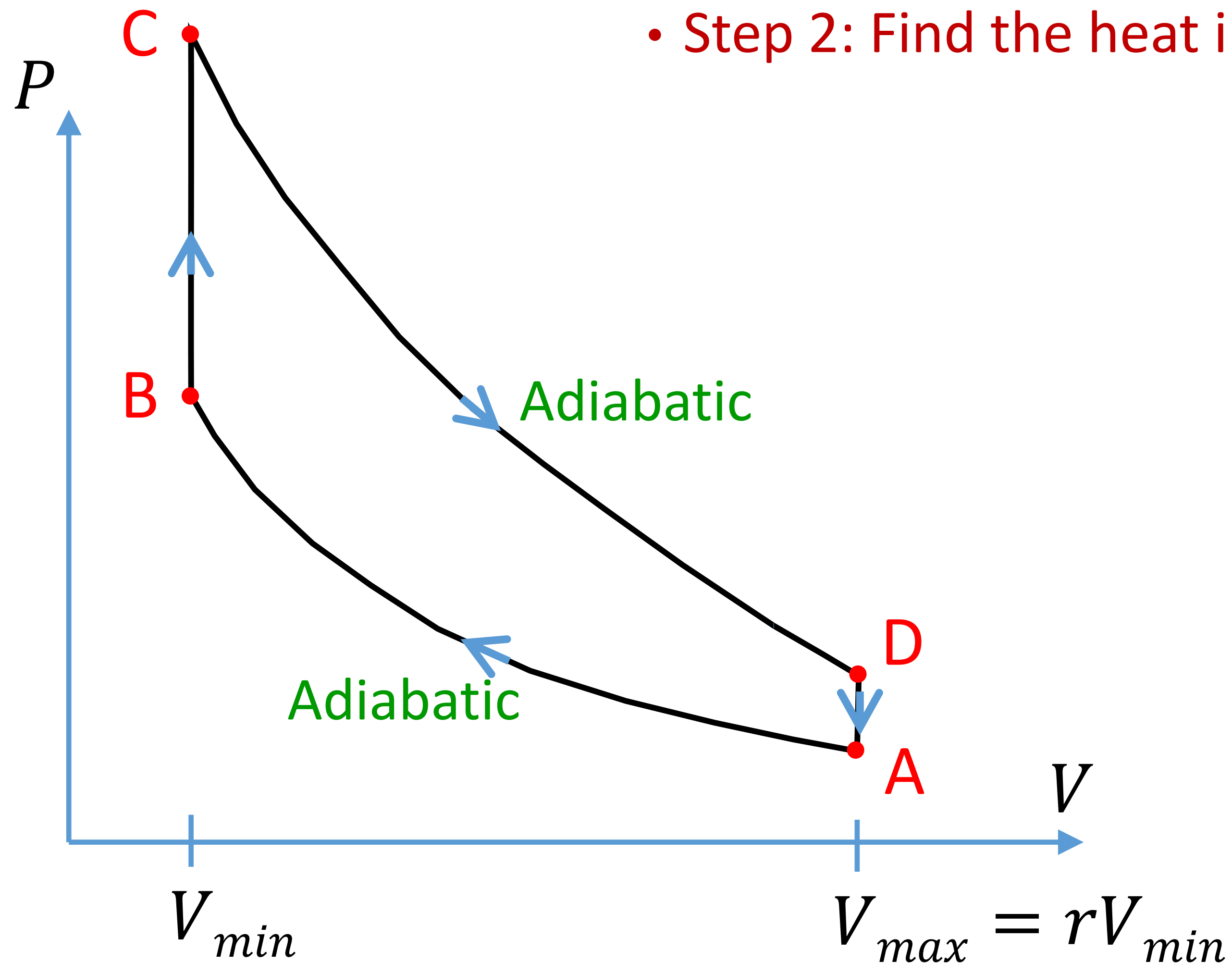
B. B → C Heating!

C. C → D Q = 0

D. D → A Cooling

Example problem

Q: Calculate the efficiency of the internal combustion engine operating via the cycle shown.



- Step 2: Find the heat input with the steps with $Q > 0$

Q: Which processes have $Q > 0$?

A. A → B $Q = 0$

B. B → C $Q > 0$ ✓

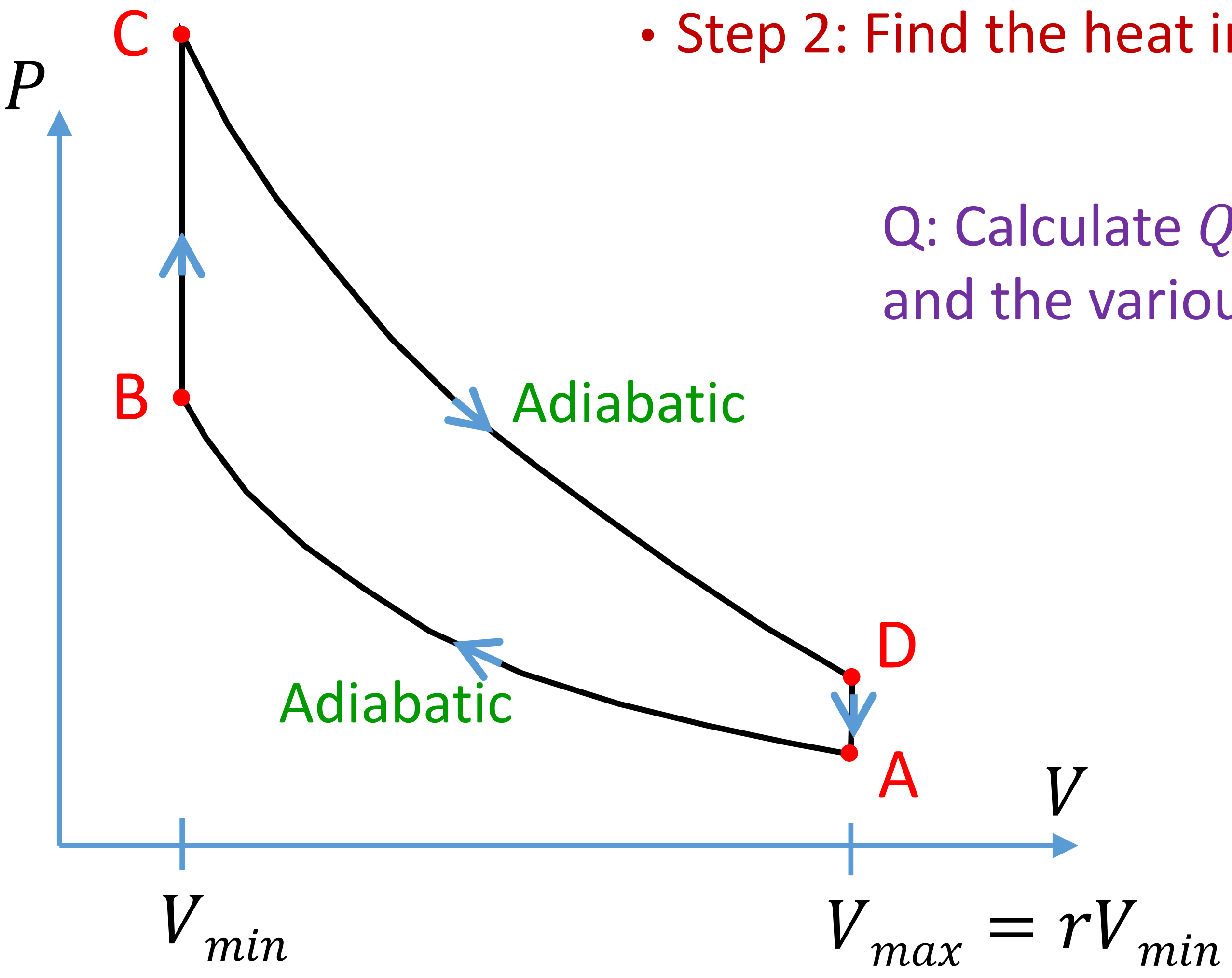
C. C → D $Q = 0$

D. D → A $Q < 0$

Example problem

$nC_v(T_f - T_i) = \Delta U = Q - \cancel{W}^{\,0} = Q$

Q: Calculate the efficiency of the internal combustion engine operating via the cycle shown.



- Step 2: Find the heat input with the steps with $Q > 0$

Q: Calculate Q for the process B→C in terms of $n, C_v,$ and the various temperatures, pressures, and volumes

A. $Q = 0$

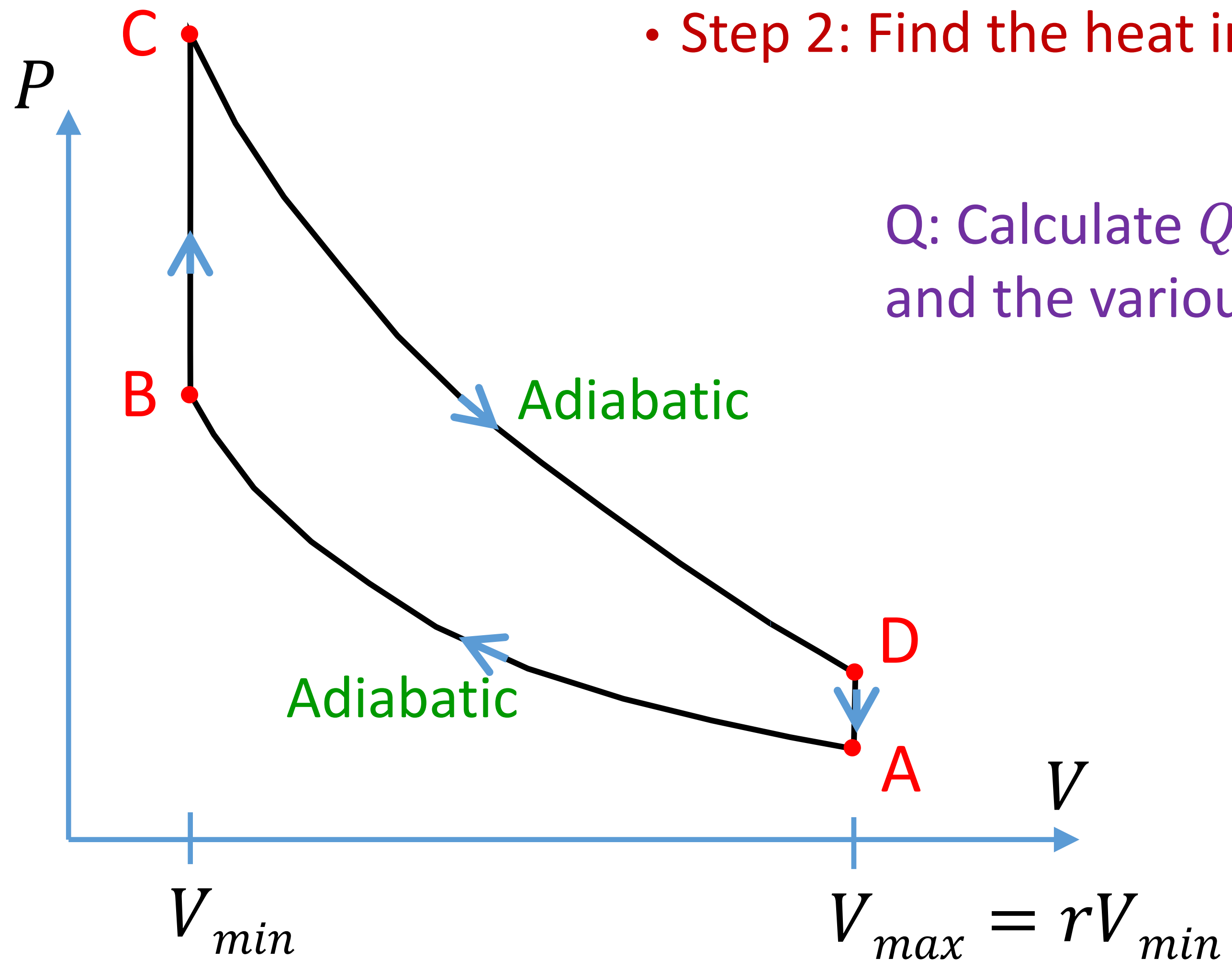
B. $Q = nC_v(T_C - T_B)$

C. $Q = V_B(P_C - P_B)$

D. $Q = nC_v(T_B - T_C)$

Example problem

Q: Calculate the efficiency of the internal combustion engine operating via the cycle shown.



- Step 2: Find the heat input with the steps with $Q > 0$

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A. $Q = 0$

B. $Q = nC_v(T_C - T_B)$ ✓

C. $Q = V_B(P_C - P_B)$

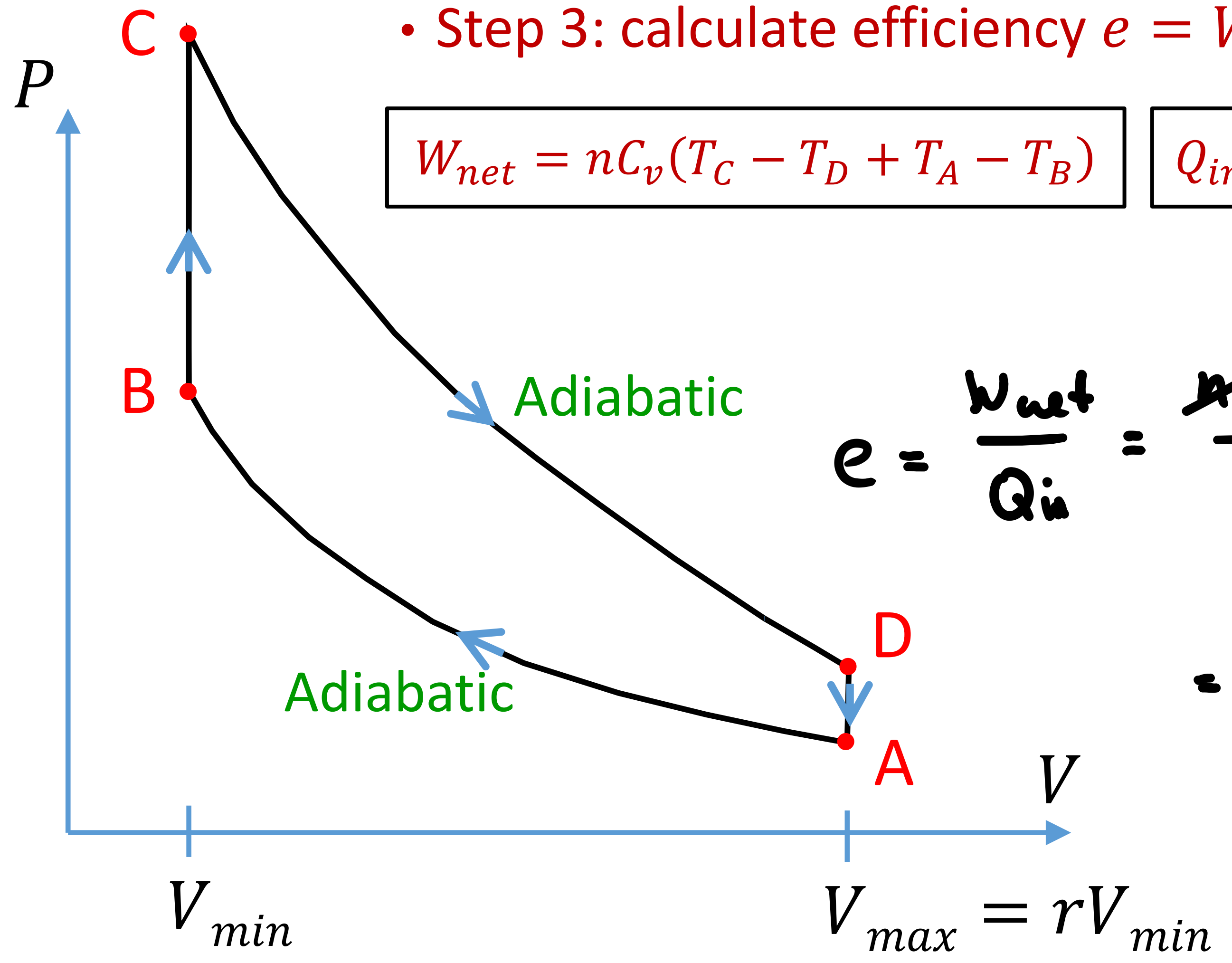
D. $Q = nC_v(T_B - T_C)$

Constant volume $\Longrightarrow W = 0$

$$Q = \Delta U = nC_v(T_C - T_B)$$

Example problem

Q: Calculate the efficiency of the internal combustion engine operating via the cycle shown.



- Step 3: calculate efficiency $e = W_{net}/Q_{in}$

$\boxed{W_{net} = nC_v(T_C - T_D + T_A - T_B)}$ $\boxed{Q_{in} = nC_v(T_C - T_B)}$

1) T_A: given.

2) $T_B V_{min}^{\gamma-1} = T_A V_{max}^{\gamma-1}$

3) $\frac{T_C}{P_C} = \frac{T_B}{P_B} \implies T_C P_B = P_C T_B$

4) $T_C V_{min}^{\gamma-1} = T_D V_{max}^{\gamma-1}$

$$e = \frac{W_{net}}{Q_{in}} = \frac{\cancel{nC_v}(T_C - T_D + T_A - T_B)}{\cancel{nC_v}(T_C - T_B)} =$$

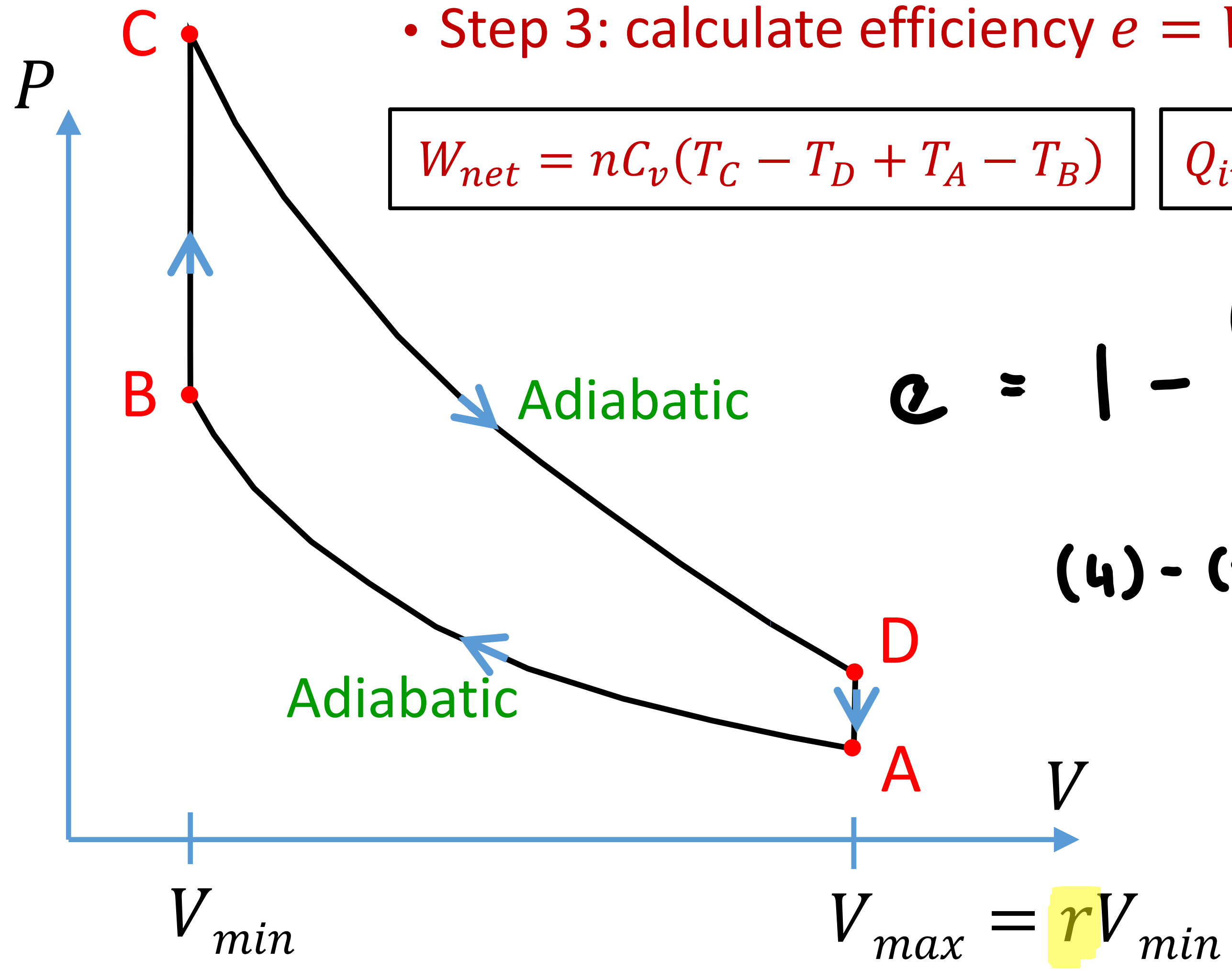
$$= \frac{T_C - T_D + T_A - T_B}{(T_C - T_B)} = \frac{(T_C - T_B) - (T_D - T_A)}{(T_C - T_B)}$$

$$= 1 - \frac{(T_D - T_A)}{(T_C - T_B)}$$

Example problem

Q: Calculate the efficiency of the internal combustion engine operating via the cycle shown.

$r = V_{max}/V_{min}$



- Step 3: calculate efficiency $e = W_{net}/Q_{in}$

$\boxed{W_{net} = nC_v(T_C - T_D + T_A - T_B)}$ $\boxed{Q_{in} = nC_v(T_C - T_B)}$

1) T_A: given.

2) $T_B V_{min}^{\gamma-1} = T_A V_{max}^{\gamma-1}$

3) $\frac{T_C}{P_C} = \frac{T_B}{P_B} \Longrightarrow T_C P_B = P_C T_B$

4) $T_C V_{min}^{\gamma-1} = T_D V_{max}^{\gamma-1}$

$$e = 1 - \frac{(T_D - T_A)}{(T_C - T_B)}$$

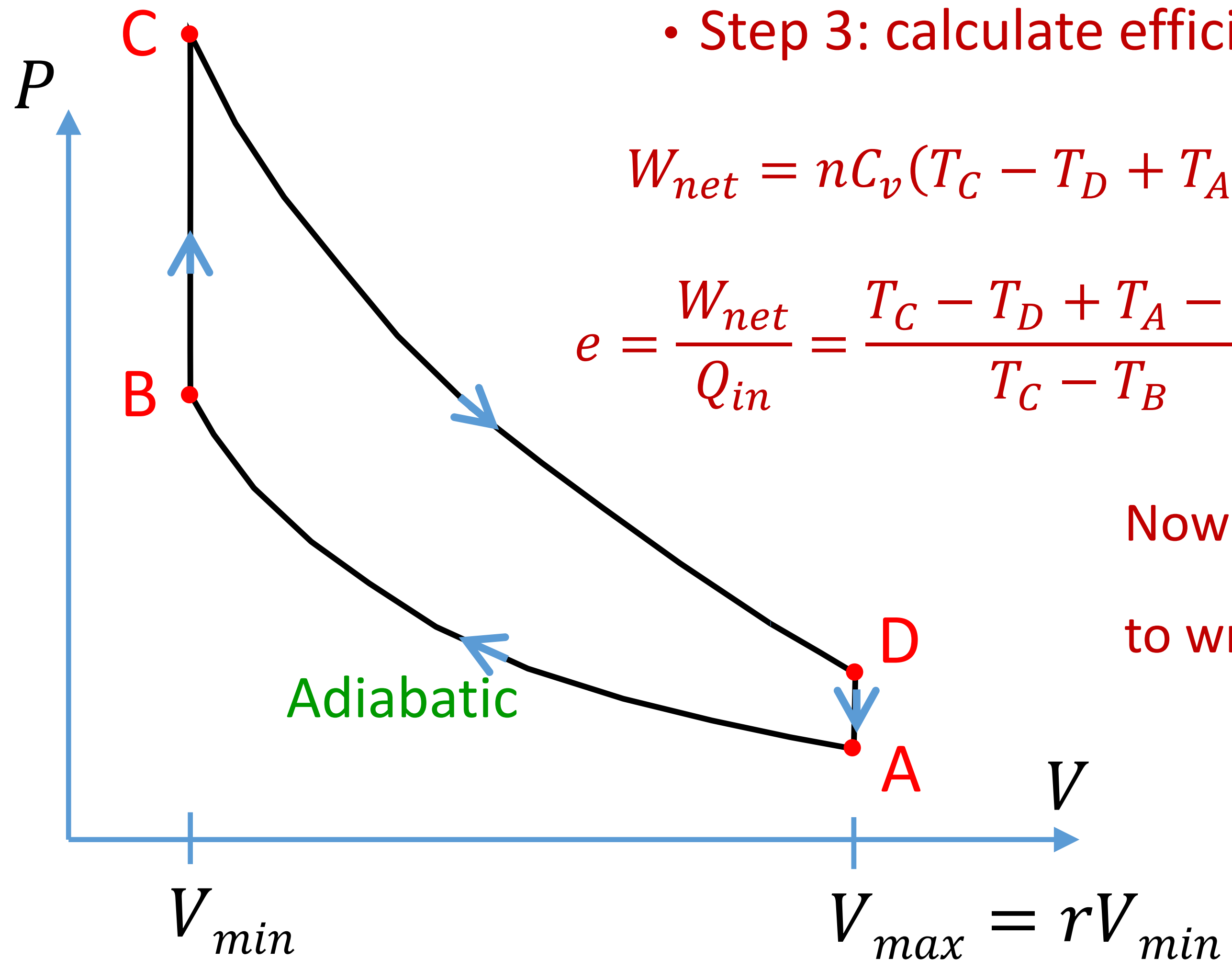
$$(4)-(2): \quad (T_C - T_B) V_{min}^{\gamma-1} = (T_D - T_A) V_{max}^{\gamma-1}$$

$$\frac{(T_D - T_A)}{(T_C - T_B)} = \frac{V_{min}^{\gamma-1}}{V_{max}^{\gamma-1}} = \frac{1}{r^{\gamma-1}}$$

$$\boxed{e = 1 - \frac{1}{r^{\gamma-1}}}$$

Example problem

Q: Calculate the efficiency of the internal combustion engine operating via the cycle shown.



- Step 3: calculate efficiency $e = W_{net}/Q_{in}$

$$W_{net} = nC_v(T_C - T_D + T_A - T_B), \qquad Q_{in} = \Delta U = nC_v(T_C - T_B)$$

$$e = \frac{W_{net}}{Q_{in}} = \frac{T_C - T_D + T_A - T_B}{T_C - T_B} = \frac{(T_C - T_B) - (T_D - T_A)}{(T_C - T_B)} = 1 - \frac{(T_D - T_A)}{(T_C - T_B)}$$

Now use $T_D V_{max}^{\gamma-1} = T_C V_{min}^{\gamma-1}$ and $T_A V_{max}^{\gamma-1} = T_B V_{min}^{\gamma-1}$

to write $(T_D - T_A)V_{max}^{\gamma-1} = (T_C - T_B)V_{min}^{\gamma-1}$ $\Longrightarrow$

$$\boxed{e = 1 - \frac{1}{r^{\gamma-1}}}$$

Otto cycle

- efficiency is $e = 1 - \frac{1}{r^{\gamma - 1}}$ (in theory)
- Higher efficiency for larger compression ratio $r = \frac{V_{max}}{V_{min}}$
- BUT: gasoline will spontaneously ignite if r is too large: “engine knocking”
- High octane fuel: higher ignition temp, so less knocking
- In real engines: $r \sim 8 - 10$ and $\gamma \sim 1.22$

$$\Rightarrow e \sim 38\%$$