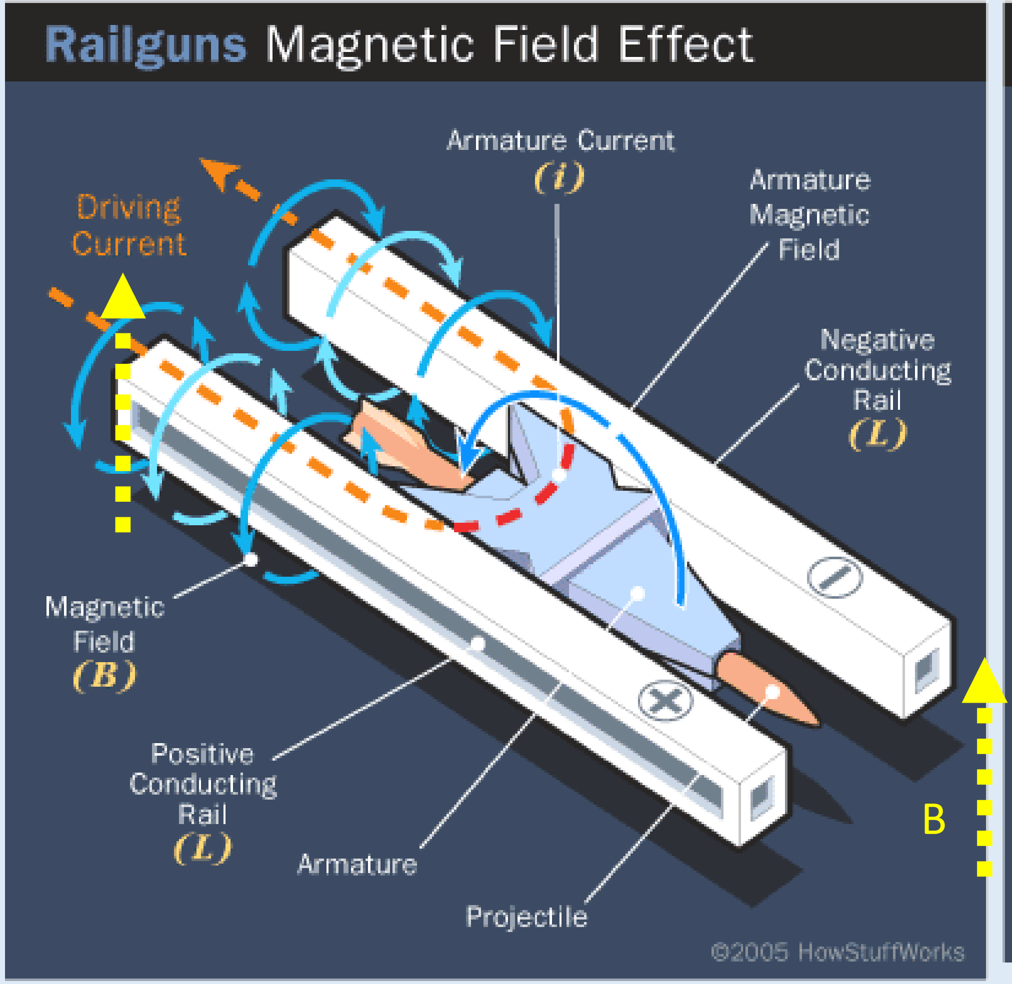
Lecture 28.

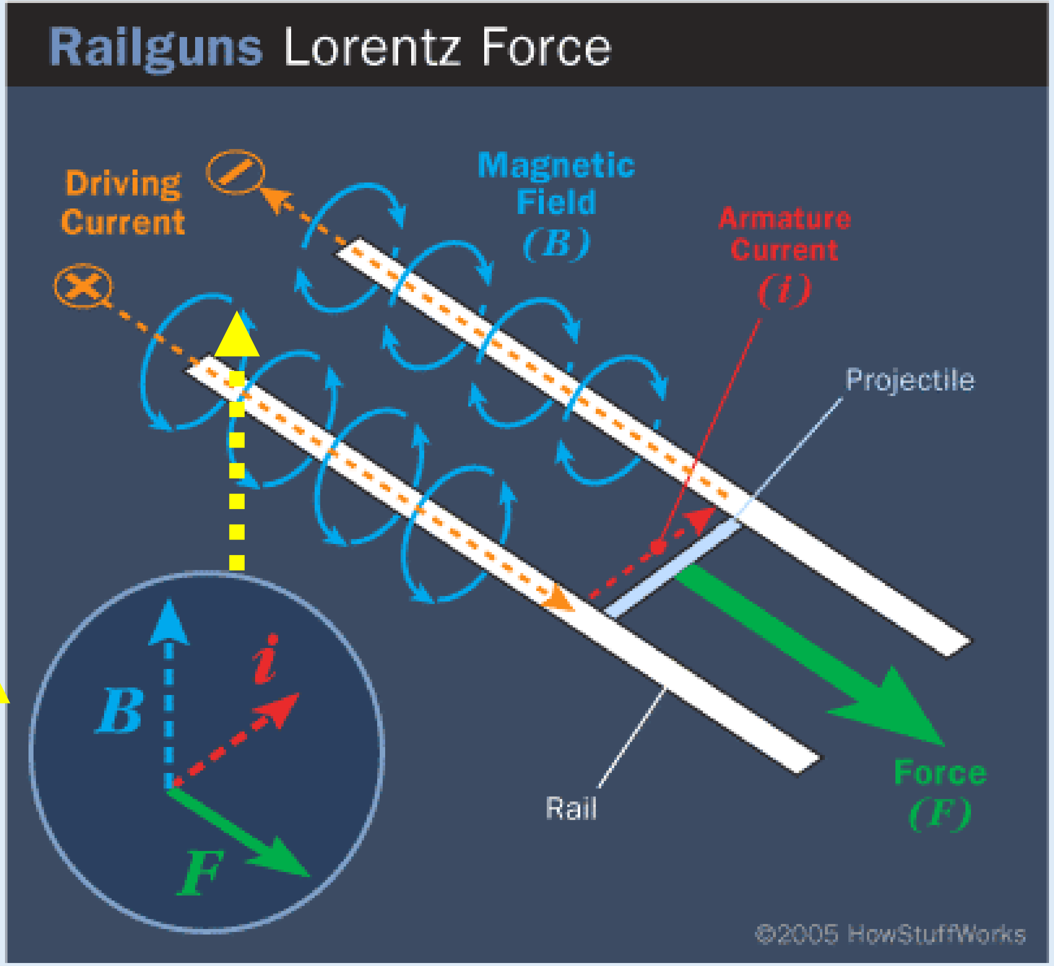
Sources of magnetic field.
Biot-Savart law.
B-field due to a ring.
B-field due to a wire (short segment and long wire)

Demo!

Rail

Gun

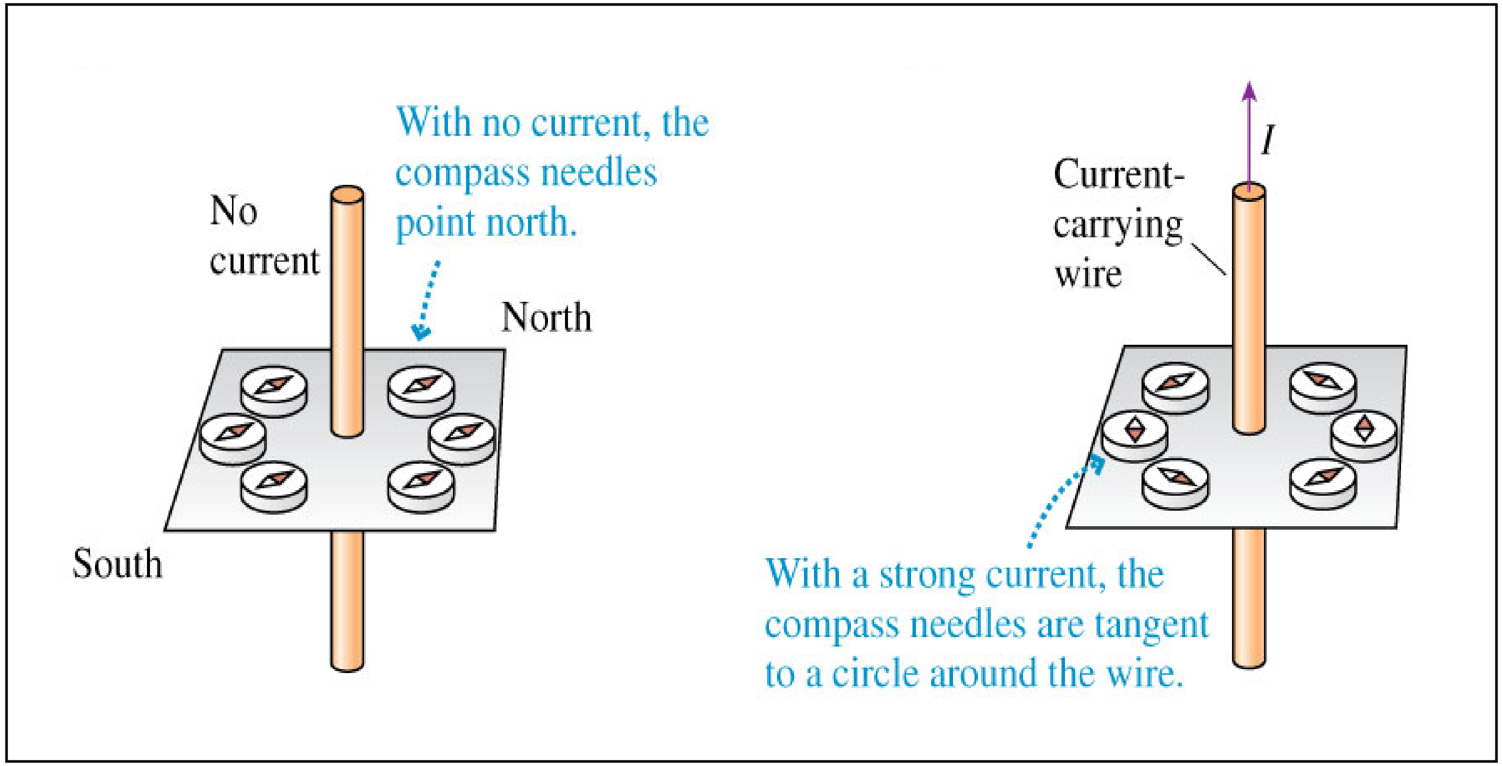




- The rails and the steel bar form a closed circuit. When the battery is on, a current starts flowing => it creates magnetic field => the magnetic field in the rails interact with the current in steel bar and exerts a Lorentz (magnetic) force on it => hence the push.

Sources of Magnetic Field

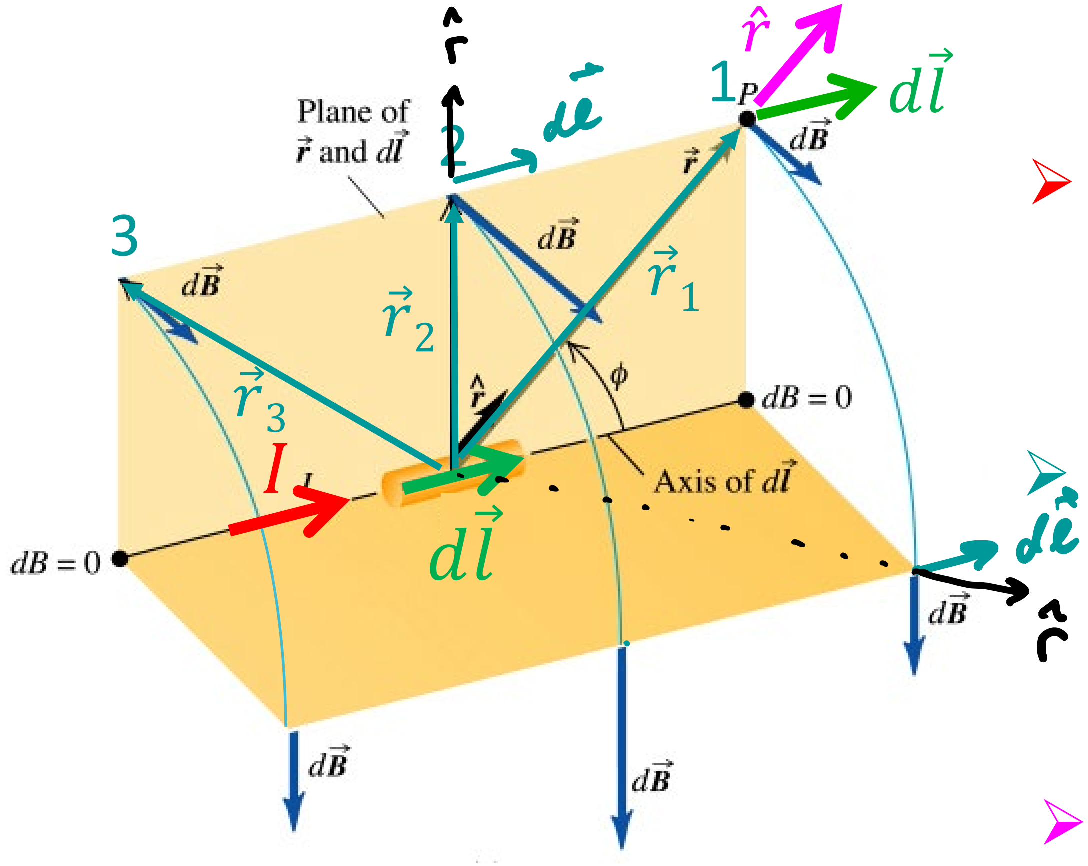
- In 1819 Hans Christian Oersted discovered that an electric current in a wire causes a compass to turn.



Electric current creates magnetic field !

Biot-Savart law

- allows us to calculate the magnetic field produced by any current distribution!



➤ Magnetic constant:

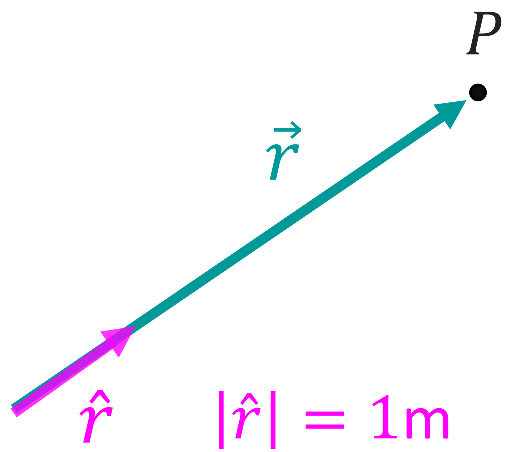
$$\mu_0 = 4\pi \times 10^{-7}\,\frac{Tm}{A}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

➤ $I d\vec{l}$: current flowing in the segment dl

$$d\vec{B} \propto \frac{1}{r^2}$$

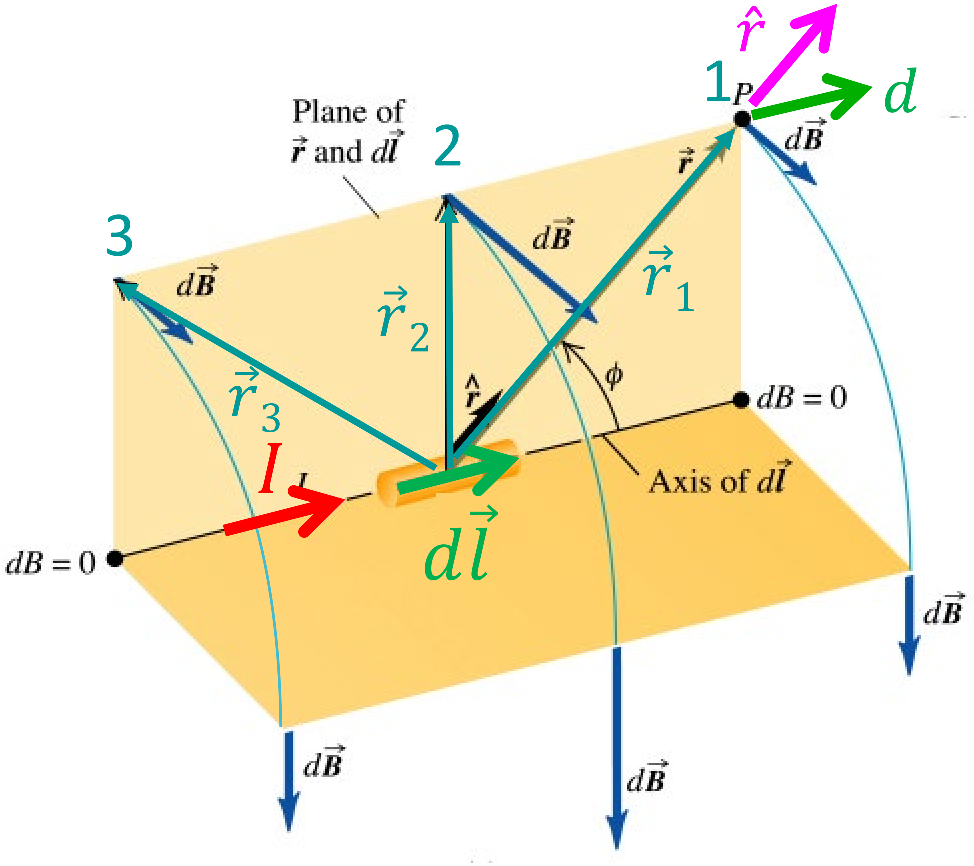
➤ $\vec{r}$: displacement vector from the current element to the observation point P



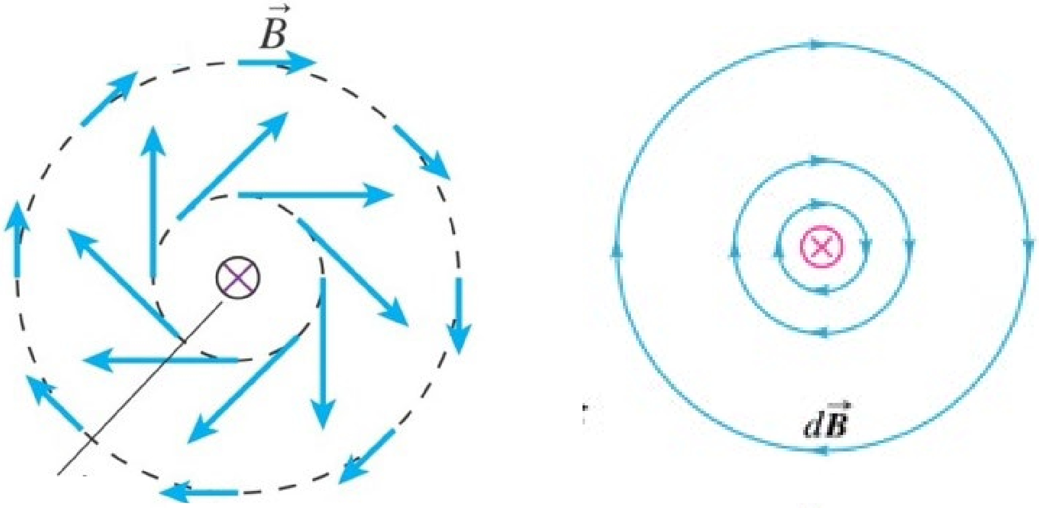
➤ $\hat{r}$: unit vector for $\vec{r}$

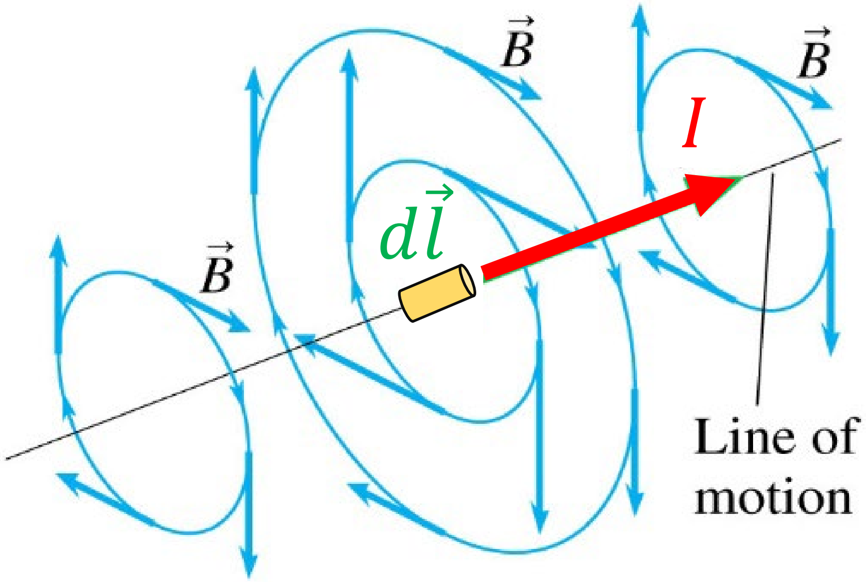
Biot-Savart law: Visualization

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}$$



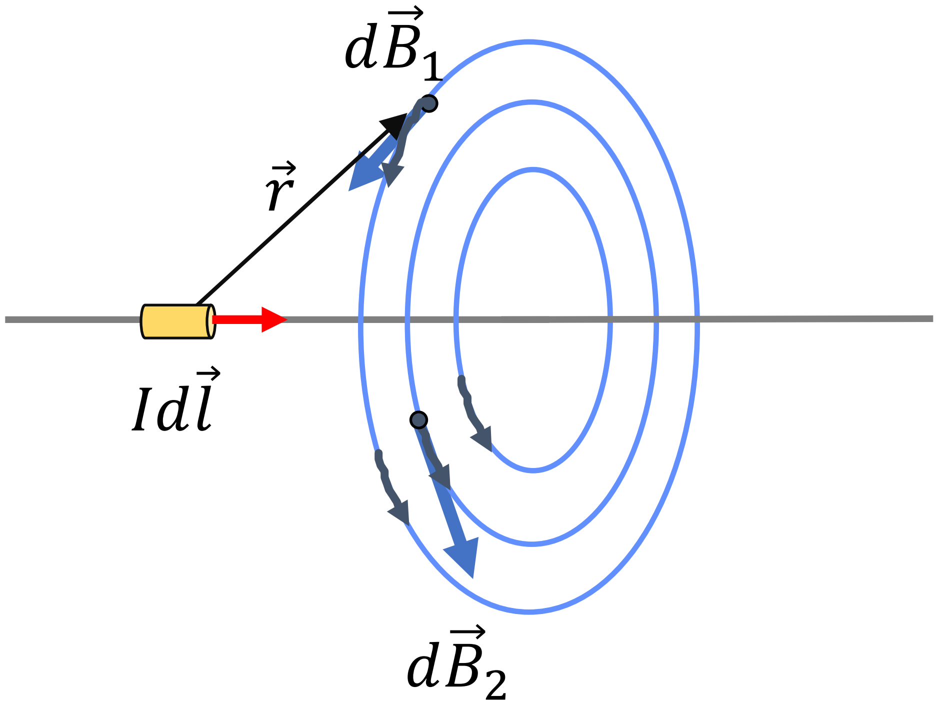
- View form behind





- Magnetic lines are always in form of closed loops.

Electric current as a source of magnetic field



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

Biot-Savart law

- Note the circular pattern of B-field for each little piece of the source current, with dB undefined along the line of that bit of current, i.e. at $r = 0$

- B-field decays as $1/r^2$ with the distance from the linear current (wire)

Compare E and B Fields: Summary

$$I d\vec{l} = \frac{dq}{dt}\, d\vec{l} = dq\, \frac{d\vec{l}}{dt} = dq\, \vec{v}$$

- Electric force:

$$\vec{F}_e = q_\pm\, \vec{E}$$

- Electric field:

charges produce E-field

$$d\vec{E} = \frac{k\, q_\pm}{r^2}\, \hat{r}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9\, \frac{Nm^2}{C^2}$$

- Magnetic force:

$$\vec{F}_m = q_\pm\, \vec{v} \times \vec{B} = \int I d\vec{l} \times \vec{B}$$

- Magnetic field:

currents produce B-field

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$$

$$\frac{\mu_0}{4\pi} = 10^{-7}\, \frac{T \cdot m}{A}$$

- Both fields for an elementary carrier (point charge dq & very short wire $I\, d\vec{l}$) vary as $1/r^2$

Magnetic field of a circular ring

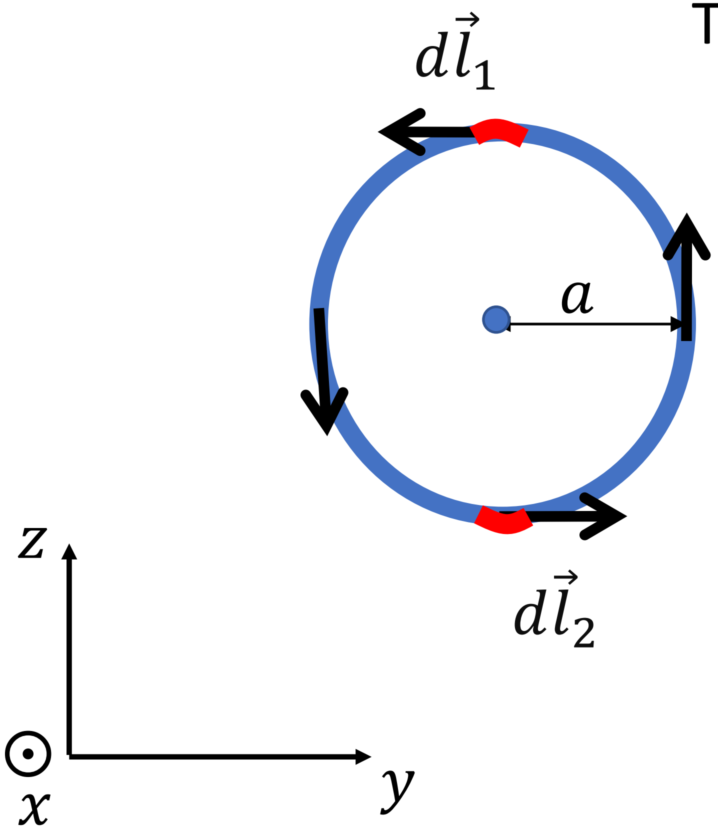
UHF Loop Antenna & part of a magnetic trap

Q: Use the Biot-Savart Law to compute magnetic field B created by a circular current loop ("ring of current") of radius a carrying a current I at:

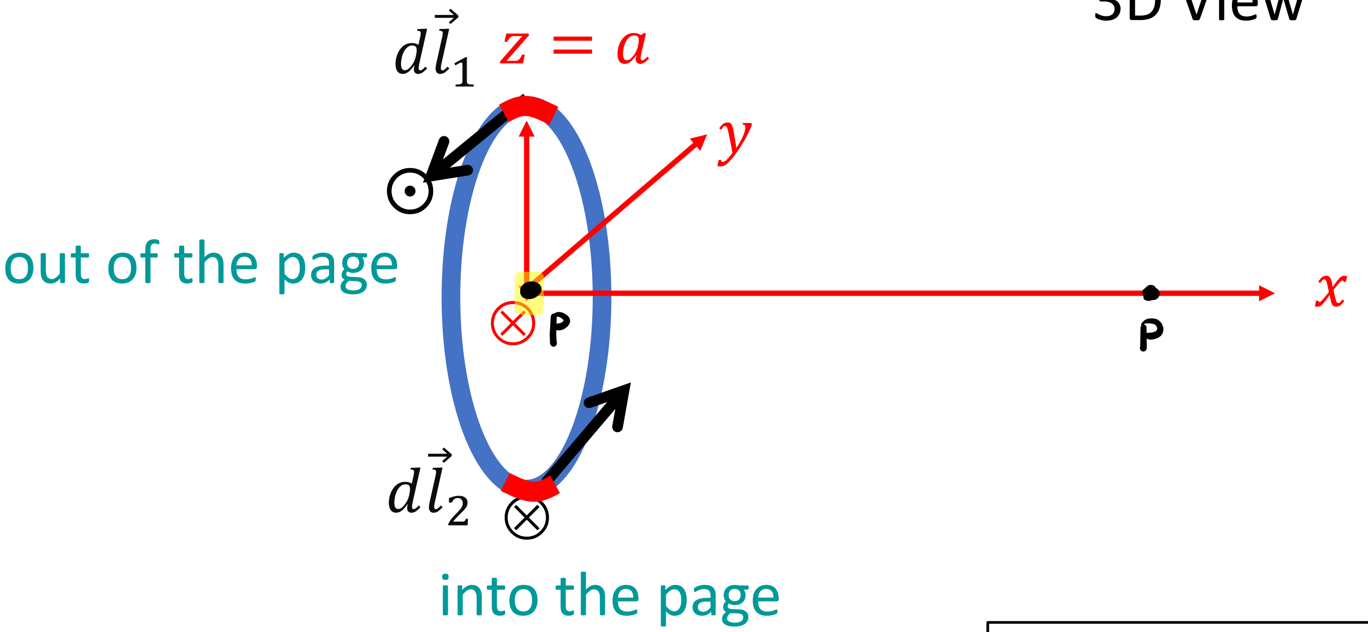
a) $x = 0$

b) $x > 0$

Top View



3D View



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

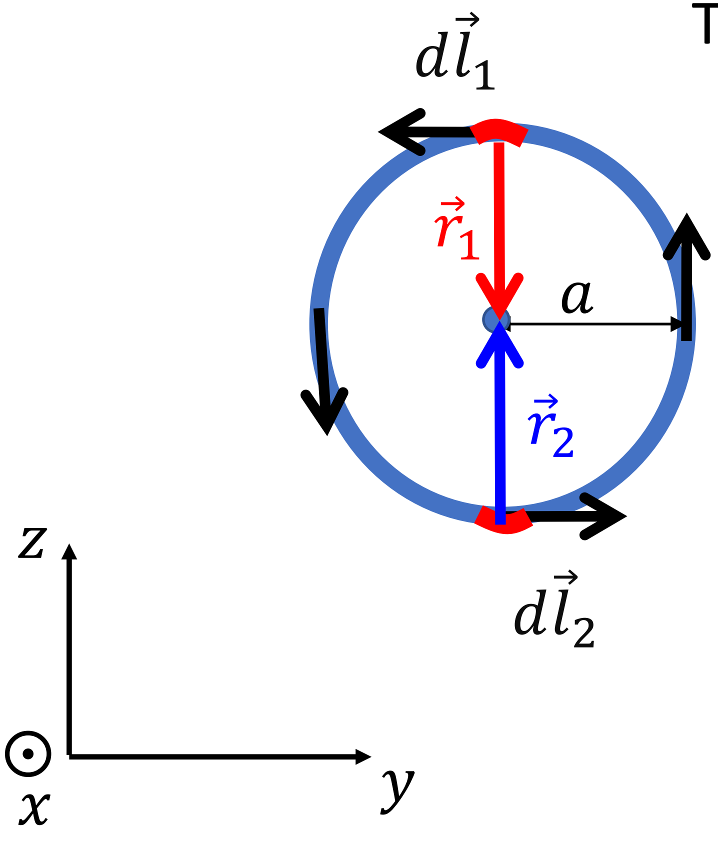
Magnetic field of a circular ring

Q: Use the Biot-Savart Law to compute magnetic field B created by a circular current loop ("ring of current") of radius a carrying a current I at:

a) $x = 0$

b) $x > 0$

$$d\vec{l}_1 \times \hat{r}_1 = \underline{dl_1} \cdot \underbrace{|\hat{r}_1|}_{1} \cdot \underbrace{\sin 90^\circ}_{1}$$



Top View

$$d\cancel{\vec{B}}_1 = \frac{\mu_0}{4\pi}\frac{I\,dl_1}{a^2} \quad \text{in x-direction}$$

$$d\cancel{\vec{B}}_2 = \frac{\mu_0}{4\pi}\frac{I\,dl_2}{a^2} \quad \text{in x-direction}$$

…

$$B_{tot} = \int dB = \frac{\mu_0}{4\pi}\int\frac{I\,dl}{a^2}$$

$$\vec{B}_{tot} = \frac{\mu_0}{\underset{2}{\cancel{4\pi}}}\frac{I\,(\cancel{2\pi}a)}{a^{\cancel{2}}} \quad \text{in x-direction}$$

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}}$$

Magnetic field of a circular ring

$|d\vec{\ell} \times \hat{r}| = d\ell$

Q: Use the Biot-Savart Law to compute magnetic field B created by a circular current loop ("ring of current") of radius a carrying a current I at:

a) $x = 0$

b) $x > 0$

- By symmetry: $\vec{B} = (B_x, 0, 0)$
- Assume this angle is θ.

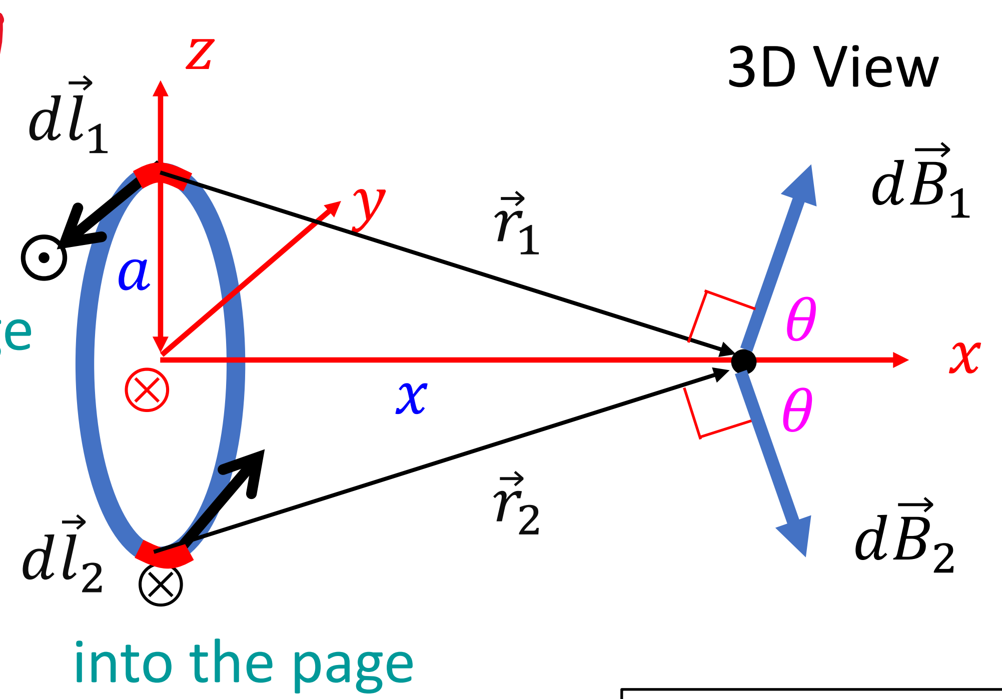
not $\vec{dB}$!!!

$$\int dB_x = \int dB \cos\theta$$

➢ Magnitude:

$$dB = \frac{\mu_0}{4\pi} \frac{I\, dl}{(x^2 + a^2)}$$

➢ Projection = ?



$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}}$$

Magnetic field of a circular ring

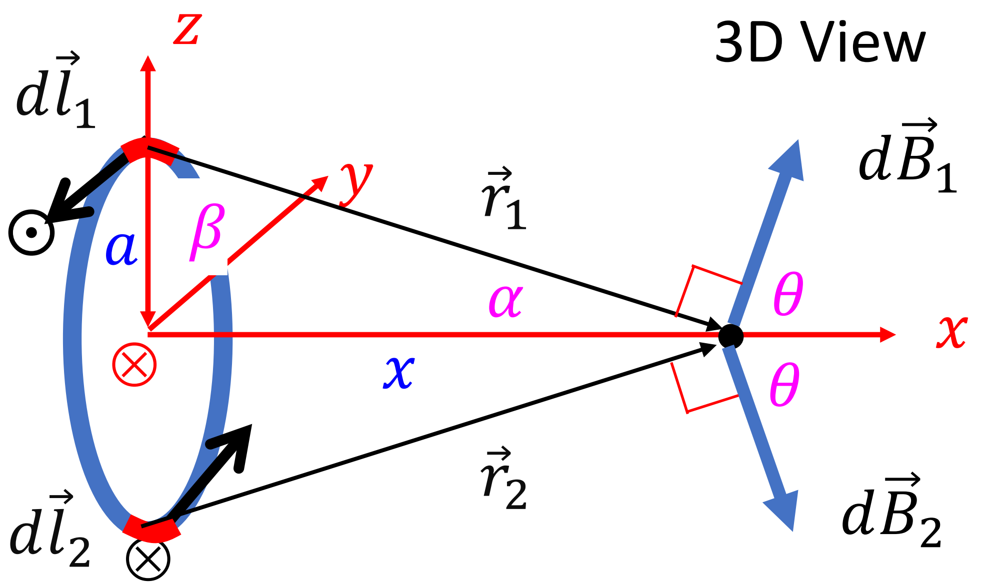
Q: Use the Biot-Savart Law to compute magnetic field B created by a circular current loop ("ring of current") of radius a carrying a current I at:

a) $x = 0$

b) $x > 0$

- By symmetry: $\vec{B} = (B_x, 0,0)$

$$dB_x = dB\cos\theta \qquad dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{(x^2+a^2)}$$



Which is true?

A. $dB_x = dB\frac{a}{\sqrt{x^2+a^2}}$

B. $dB_y = dB\frac{x}{\sqrt{x^2+a^2}}$

C. None is true

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

Magnetic field of a circular ring

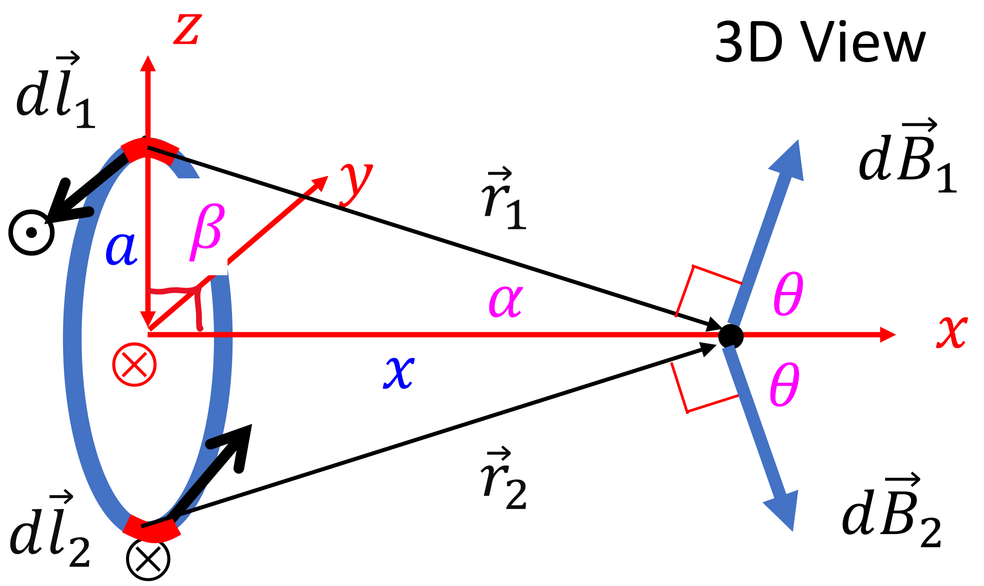
Projecting magnetic field is responsible for a huge amount of mark loss in E&M courses

$\alpha + \theta = 90^o$

$\alpha + \beta = 90^o$

Hence, $\theta = \beta$, <u>not</u> α!

$\cos\theta = a/r$, <u>not</u> x/r!



Which is true?

A. $dB_x = dB\frac{a}{\sqrt{x^2+a^2}}$

B. $dB_x = dB\frac{x}{\sqrt{x^2+a^2}}$

C. None is true

Magnetic field of a circular ring

Q: Use the Biot-Savart Law to compute magnetic field B created by a circular current loop ("ring of current") of radius a carrying a current I at:

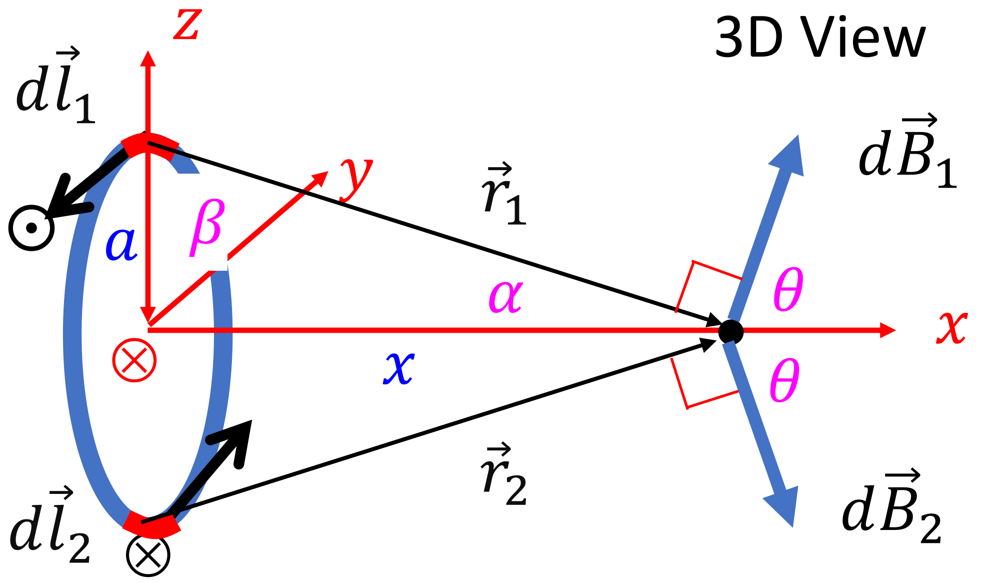
a) $x = 0$

b) $x > 0$

- By symmetry: $\vec{B} = (B_x, 0, 0)$

$dB_x = dB\cos\theta$ $\qquad$ $dB = \dfrac{\mu_0}{4\pi}\dfrac{I\,dl}{(x^2+a^2)}$

$\cos\theta = a/r$ $\qquad$ $\int dB: \quad dl \Rightarrow 2\pi a$



$$B_x = \frac{\mu_0}{4\pi}\frac{I\,(2\pi a)}{(x^2+a^2)}\frac{a}{\sqrt{x^2+a^2}} = \frac{\mu_0}{2}\frac{I\,a^2}{(x^2+a^2)^{3/2}}$$

$\propto \dfrac{1}{x^3}$ far away

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}}$$

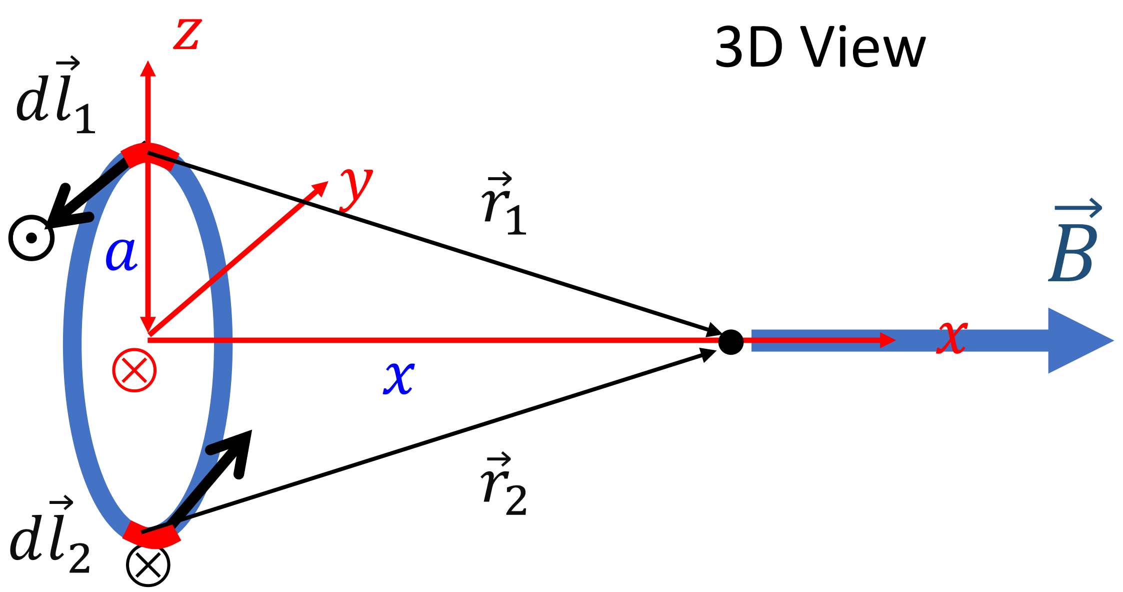
Magnetic field of a circular ring

Magnetic field B created by a circular current loop ("ring of current") of radius a carrying a current I at a distance x from its center is:

$x \to \infty: \quad \frac{\mu_0}{2\pi} \frac{I \cdot \pi a^2}{x^3}$ $IA = \mu$

- By symmetry: $\vec{B} = (B_x, 0, 0)$

$$B_x(x) = \frac{\mu_0}{2} \frac{I\, a^2}{(x^2 + a^2)^{3/2}}$$



- At $x = 0$:

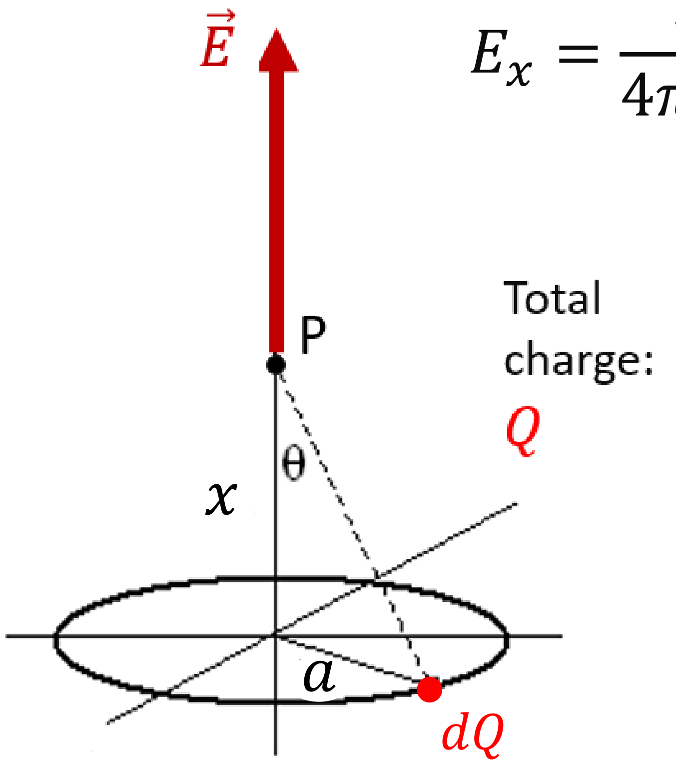
$$B_x(x = 0) = \frac{\mu_0 I}{2a} \quad \text{(our previous result)}$$

Compare:

Electric field $\vec{E}$ created by a "ring of current" of radius a carrying a charge $+Q$ at a distance z from its center is:

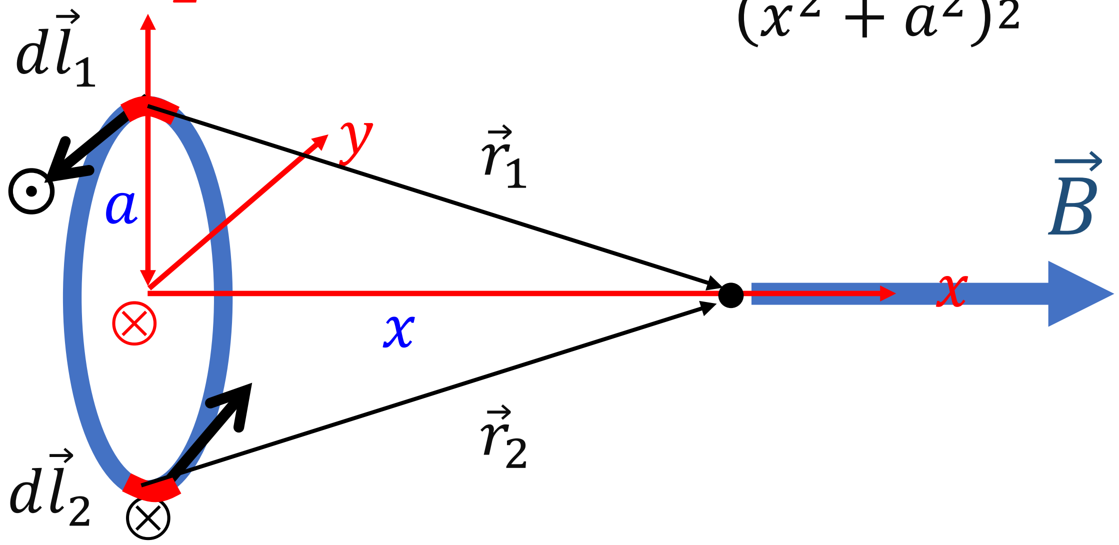
- By symmetry: $\vec{E} = (E_x, 0{,}0)$

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{x\,Q}{(x^2+a^2)^{\frac{3}{2}}}$$



Magnetic field $\vec{B}$ created by a "ring of current" of radius a carrying a current I at a distance x from its center is:

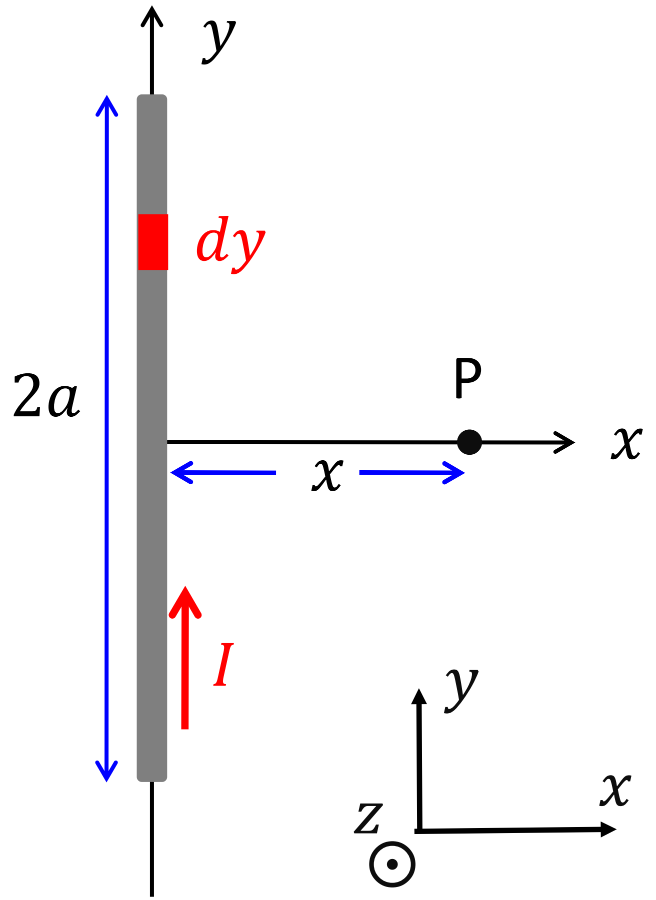
$$B_x = \frac{\mu_0}{2\pi}\frac{I(\pi a^2)}{(x^2+a^2)^{\frac{3}{2}}}$$



- By symmetry: $\vec{B} = (B_x, 0{,}0)$

Magnetic field of a short straight wire

Q: Use the Biot-Savart Law to compute magnetic field B created by a short current segment of length $2a$ with current I at the symmetry axis of the wire. Your answer should be a vector.



Exercise: Before you do the math, think about how to solve the problem and write a few sentences outlining your strategy.

- First consider a small wire segment dy => find the field $d\vec{B}$ produced by it at P => considering symmetry, integrate its components to get the resultant field at P

You might need:

$$\int \frac{x\,dy}{(x^2+y^2)^{3/2}} = \frac{y}{x\sqrt{x^2+y^2}}$$

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}}$$