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Create your own study plan, pick up some helpful study tips, and learn essential time management strategies.

April 2 - 12

Lecture 33.

Mutual inductance & Self-inductance.
Uniform and non-uniform magnetic field.
Faraday's law applications: current generator.
Eddy currents.
Are B-field and E-field connected?

Week 10: External current $\Rightarrow$ External magnetic field 

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\ d\vec{l} \times \hat{r}}{r^2} \qquad B_{wire} = \frac{\mu_0 I}{2\pi r}, \text{ RHR}$$

Week 11:

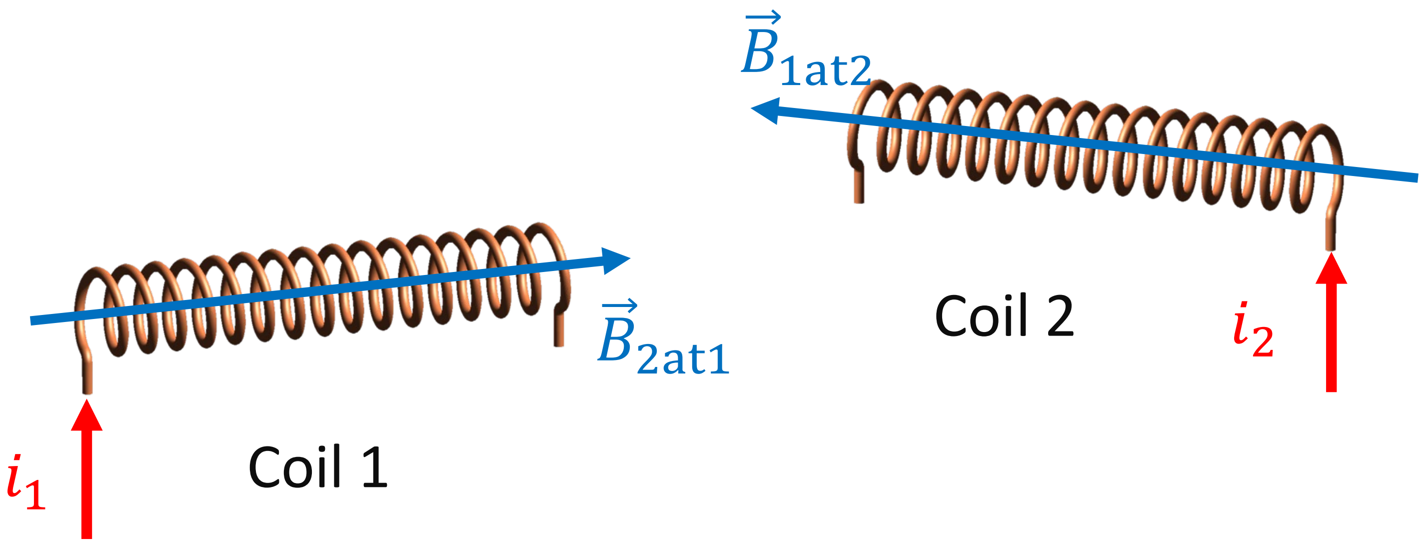
Changing external current $\Rightarrow$ Changing external magnetic field

$\Downarrow$ Lenz's law

Induced magnetic field $\Leftarrow$ Induced current (need a loop)

Inductance: Mutual inductance

- Current i_1 creates magnetic field, $B_{1\mathrm{at}2}$, at the location of Coil 2
- Current i_2 creates magnetic field, $B_{2\mathrm{at}1}$, at the location of Coil 1



- Now assume that $i_1 = i_1(t)$

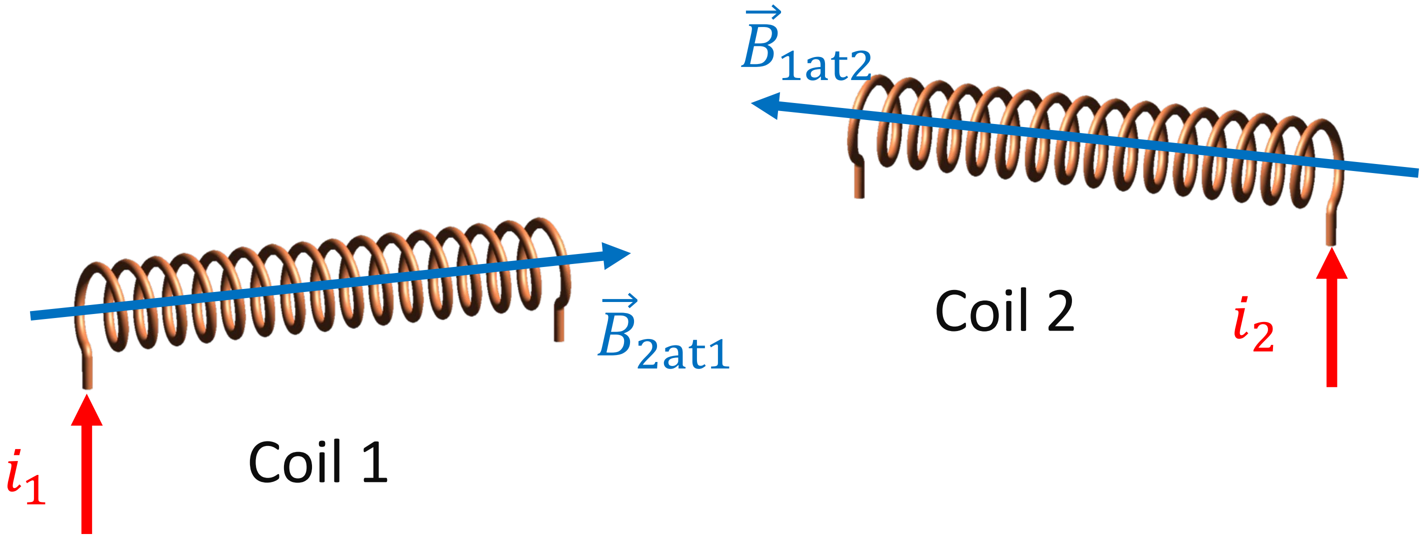
  - We have: i_1 changes => $B_{1\mathrm{at}2}$ changes => Φ_2 changes => $\varepsilon_2 = -\frac{d\Phi_2}{dt}$ appears.
  - Likewise: i_2 changes => $B_{2\mathrm{at}1}$ changes => Φ_1 changes => $\varepsilon_1 = -\frac{d\Phi_1}{dt}$ appears.

- This is called mutual inductance (change of current in one coil induces emf in another coil)

Inductance: Mutual inductance

- EMF induced in Coil 2:

$$\varepsilon_2 = -\frac{d\Phi_2}{dt}$$



$$\Phi_2 \propto B_{1at2} \propto i_1(t)$$

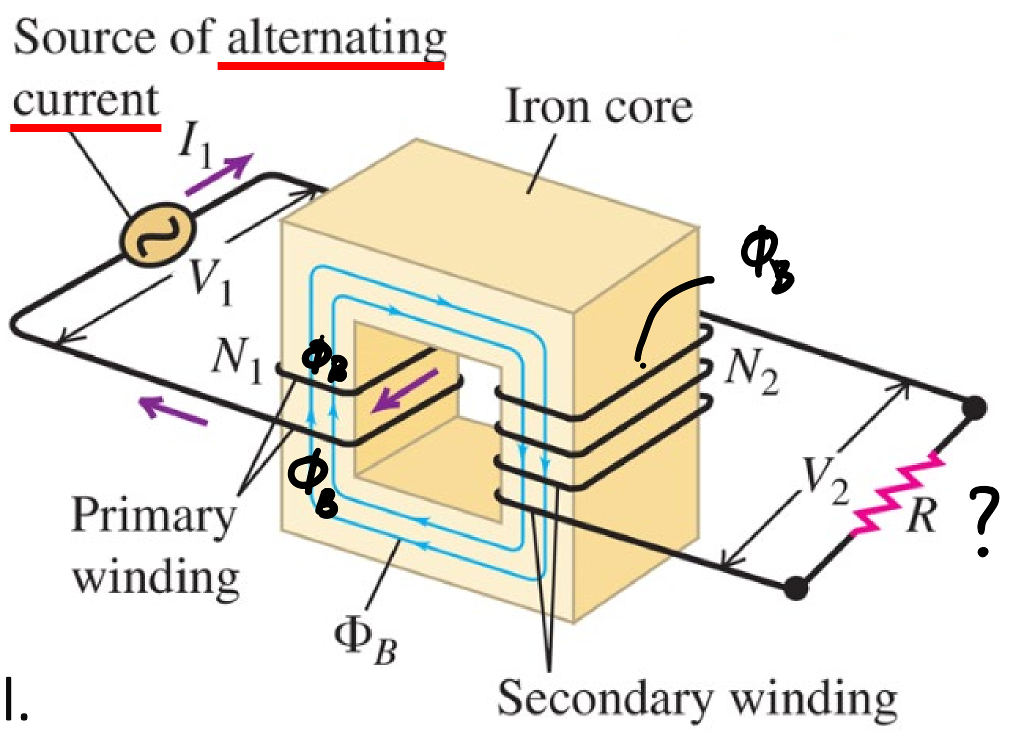
$\Phi_2 = M_{1at2}\, i_1(t)$, where M_{1at2} is some constant. Hence, $\boxed{\varepsilon_2 = -M_{1at2}\frac{di_1}{dt}}$

- Likewise, if i_2 changes, EMF is induced in coil 1: $\boxed{\varepsilon_1 = -M_{2at1}\frac{di_2}{dt}}$

- Interestingly, it is always true that: $M_{1at2} = M_{2at1} \equiv M$

Application: Transformers

$V_{max} \cos(\omega t) \rightarrow$



- Two windings over the same core, one connected to AC source, the other to a resistor
- The magnetic field lines due to a current in one winding are kept almost completely within the iron core (high magnetic permeability).
- Let Φ_B be the magnetic flux through one turn of a coil.

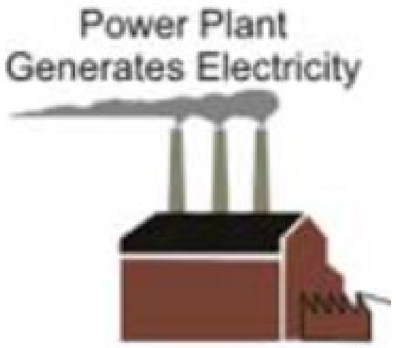
$$\varepsilon_1 = -\frac{d\Phi_{B1}}{dt} = -N_1 \frac{d\Phi_B}{dt}$$

$$\varepsilon_2 = -\frac{d\Phi_{B2}}{dt} = -N_2 \frac{d\Phi_B}{dt}$$

Hence, $$\boxed{\frac{\varepsilon_1}{\varepsilon_2} = \frac{N_1}{N_2}}$$

- By changing the ratio of the number of turns in the two coils, we can up- or down-convert the voltage of the AC source

Application: Transformers



- High voltage transmission (prevents losses): ~500 kV

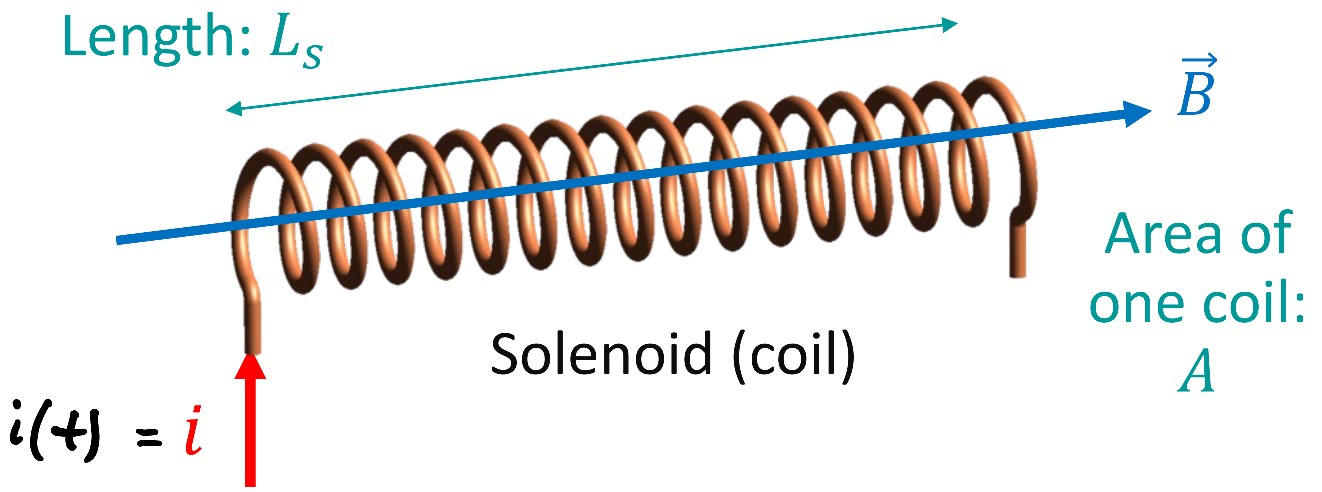
- Neighborhood transmission: ~25 kV

$$\boxed{\frac{\varepsilon_1}{\varepsilon_2} = \frac{N_1}{N_2}}$$

- Household: 120 V

- By changing the ratio of the number of turns in the two coils, we can up- or down-convert the voltage of the AC source

Inductance: Self-inductance



- Assume that the current through this solenoid changes.

  What happens?

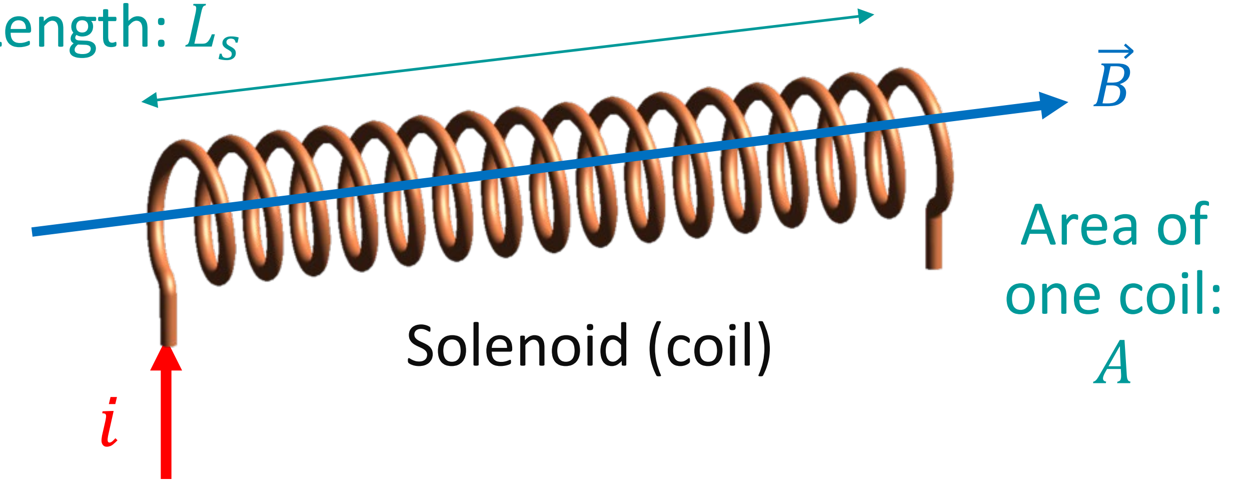
- We have: i changes => B changes => Φ_B changes => $\varepsilon = -\dfrac{d\Phi_B}{dt}$ appears.

Q: How does the induced EMF depend on the number of turns in the solenoid, N?

A. It does not depend on N
B. Proportional to N
C. Proportional to N^2
D. Something else

$$\boxed{B_{solenoid} = \mu_0 n I}$$

Inductance: Self-inductance



- Assume that the current through this solenoid changes.

  What happens?

- We have: i changes => B changes => Φ_B changes => $\varepsilon = -\frac{d\Phi_B}{dt}$ appears.

Q: How does the induced EMF depend on the number of turns in the solenoid, N?

A. It does not depend on N
B. Proportional to N
C. Proportional to N^2
D. Something else

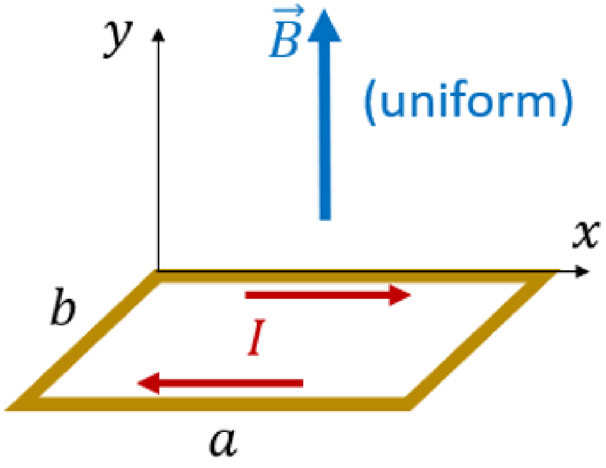
$$\varepsilon = -\frac{d\Phi_B}{dt} \qquad \Phi_B = N \cdot B(t)\, A \qquad B(t) = \mu_0 \frac{N}{L_s} I(t)$$

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 A N^2}{L_s}\frac{di}{dt} = -L\frac{di}{dt} = \Delta V_L$$

- Here L is what we know as inductance!!

$$\boxed{B_{solenoid} = \mu_0 n I}$$

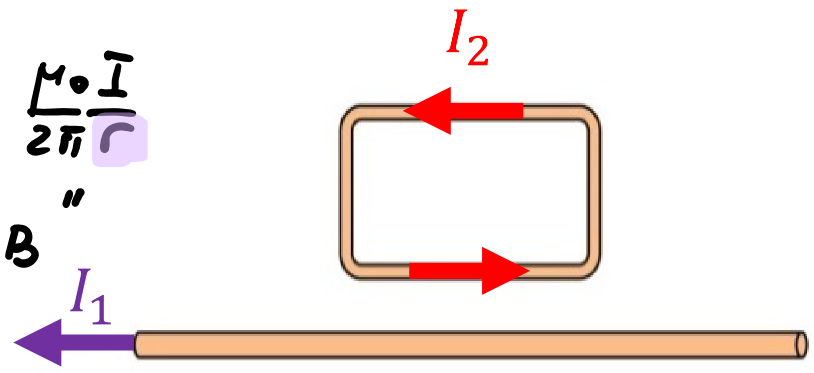
Uniform vs non-uniform magnetic field



$F_{wire} = I \cdot L \cdot B \cdot \sin\theta \ldots$

$\frac{\mu_0 I}{2\pi r}$

B "



Week 10: find net force

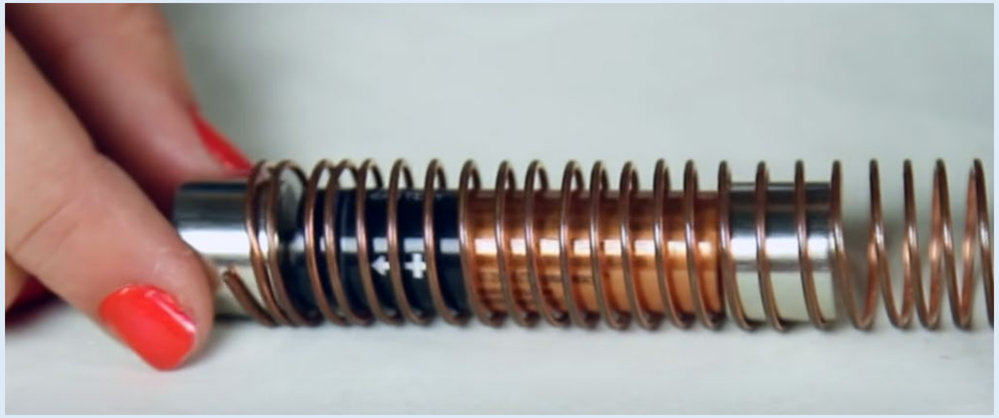
- Uniform magnetic field
- Same force on the pairs of sides of the loop
- Zero net force

HW 10: find net force

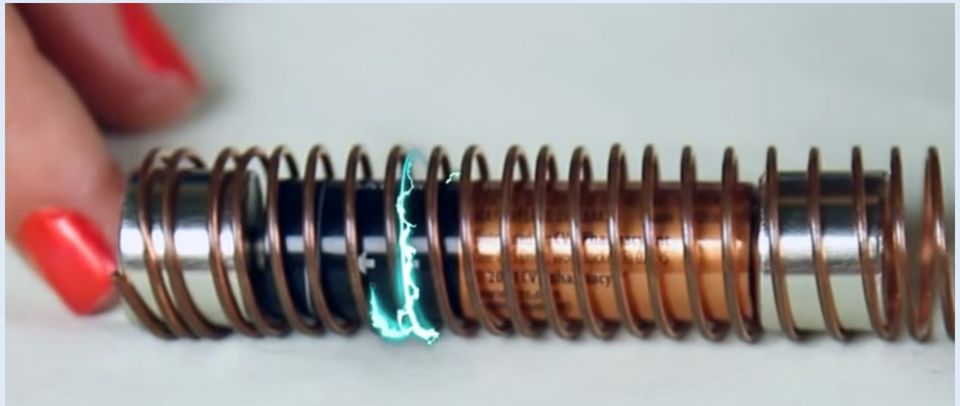
- Non-uniform magnetic field
- Different forces on the horizontal sides of the loop
- Non-zero net force

DEMO!

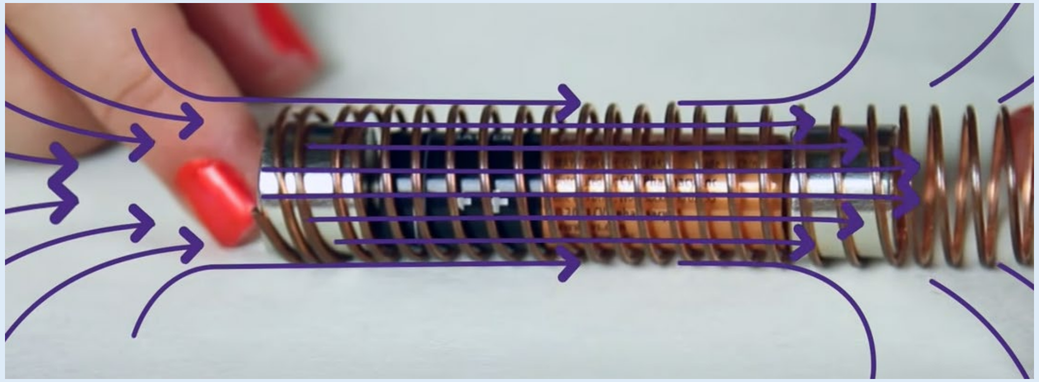
[World's Easiest DIY Electric Train]



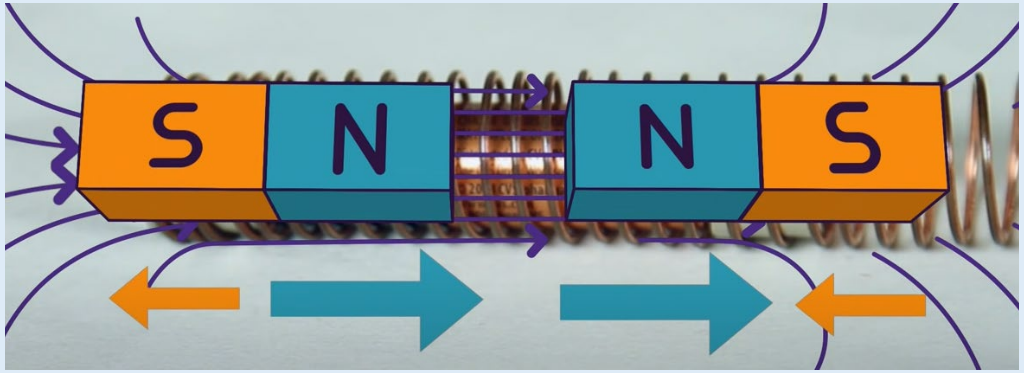
A battery with two attached magnets streams like crazy through a cooper coil



All parts are conducting => it's a closed circuit



...and B-field builds up inside this coil.



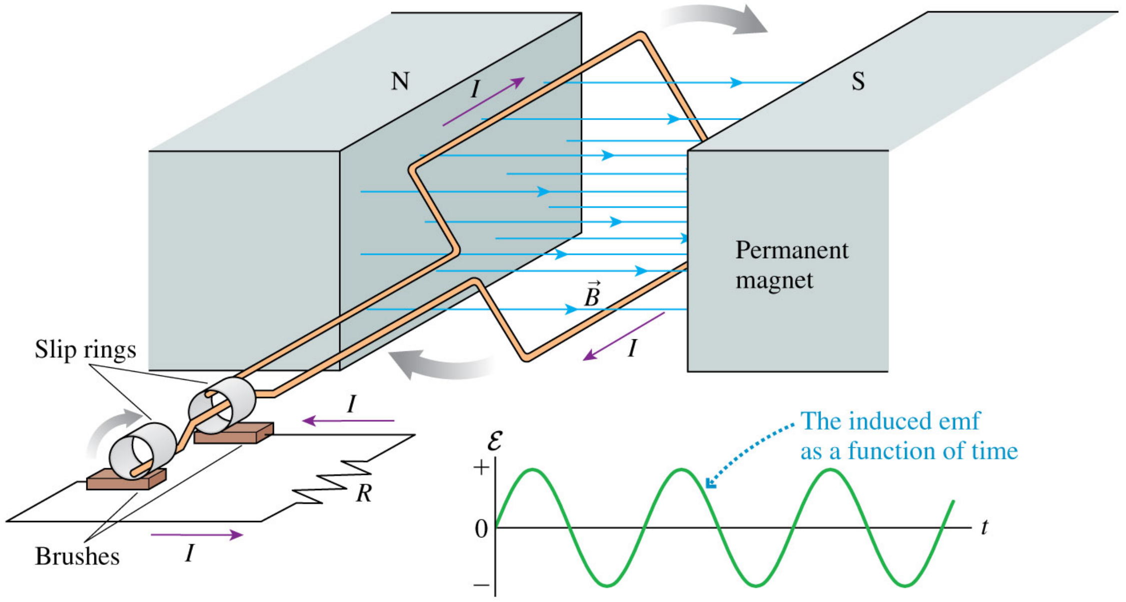
It's non-uniform => pushes S and N poles differently => net force on the "train"!

check: E-dipols in E-field (uniform vs non-uniform).

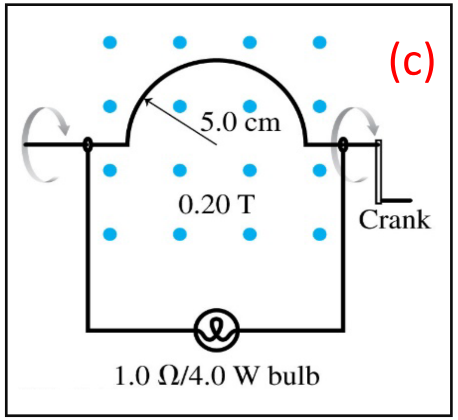
• Application of Faraday's law: Generator

A device that produces alternating current from mechanical rotation

- Loop rotates => effective area exposed to the magnetic field changes with time (or $\theta = \theta(t)$)
- Cycle: min => increasing => max => decreasing => min => …
- As a result, an alternating current is produced



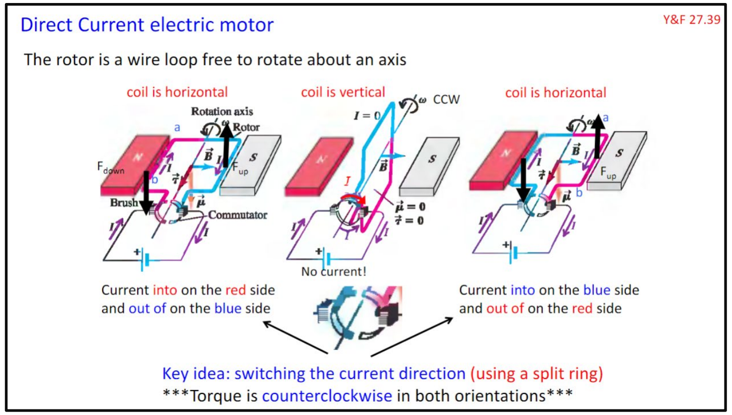
https://phet.colorado.edu/en/simulation/legacy/generator



HW-10

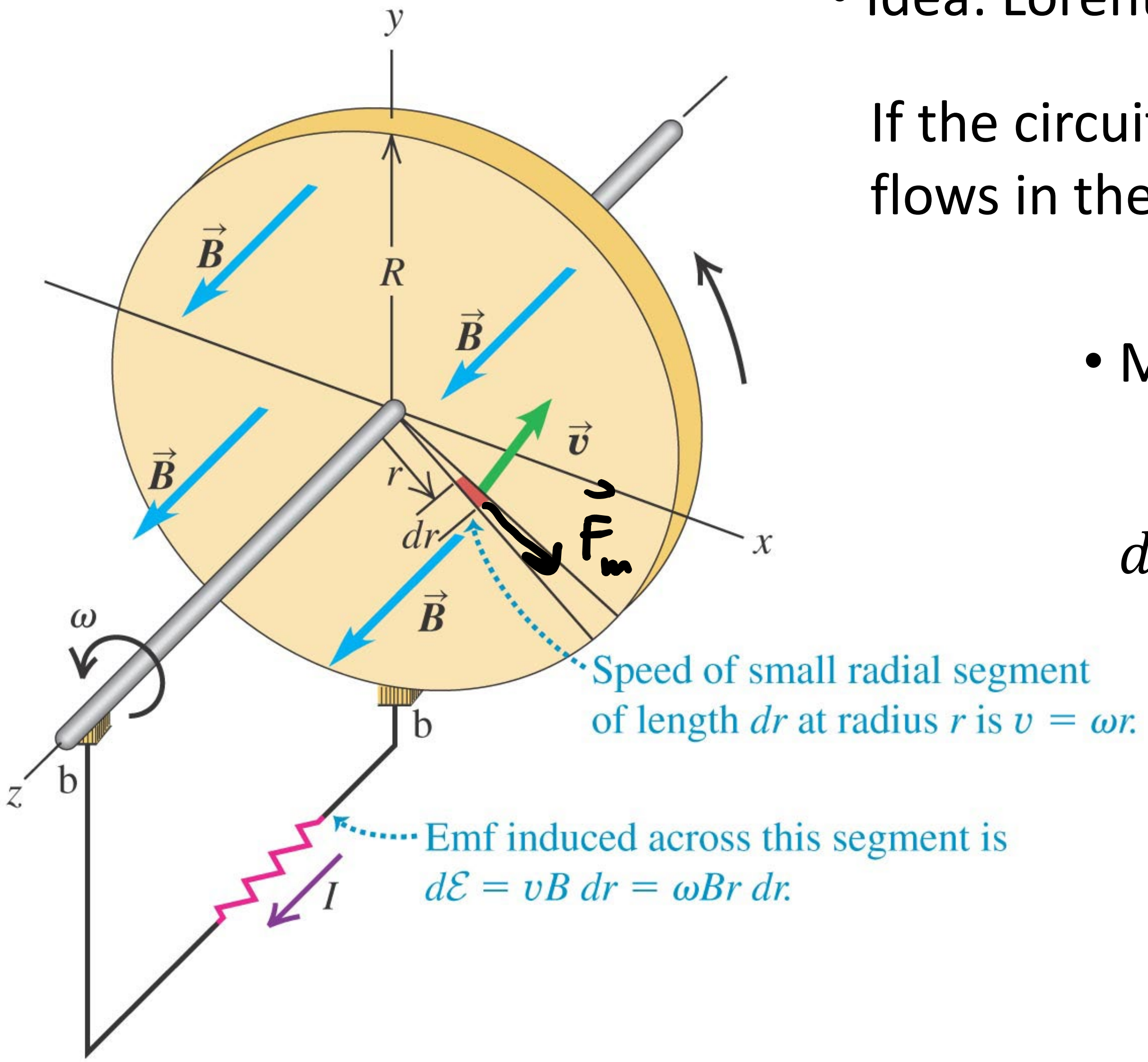
Cf: Week 9, electric motor

(current => mechanical rotation)



Example 29.10

- Another current generator: Faraday's disk dynamo



- Idea: Lorentz's force on electrons towards the center =>

  If the circuit is closed, conventional current flows in the "from the center" direction

- Motional EMF:

$$d\varepsilon = \vec{E}\cdot d\vec{r} = \frac{\vec{F}}{q}d\vec{r} = (\vec{v}\times\vec{B})\cdot d\vec{r} = vBdr$$

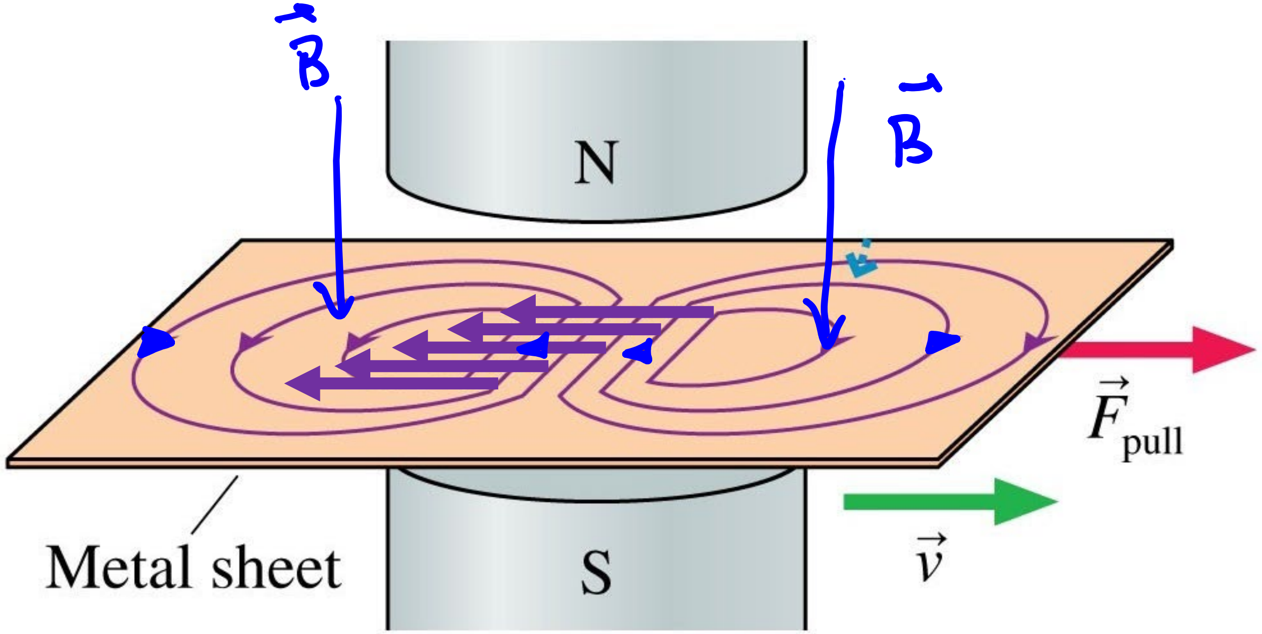
$$\varepsilon = \int_0^R d\varepsilon = \int_0^R vBdr \quad \text{and} \quad \boxed{v = \omega r}$$

$$\varepsilon = B\int_0^R (\omega r)\, dr = \frac{\omega B R^2}{2}$$

Everything is going so well so far…

- But: why changing magnetic flux creates electric current??
- What exactly moves the charges around?

- Does the induced emf appear only in a loop? Can it appear in a piece of metal?
- Yes. Example: Eddy currents.



> "Whirlpools" of electron motion; no wires to define the path of electrons.
> Heats the sheet
> Force in the direction opposite to $\vec{F}_{pull}$ $\Rightarrow$ train brakes

- Can induced emf appear in air? In vacuum?
- Yes, it can.