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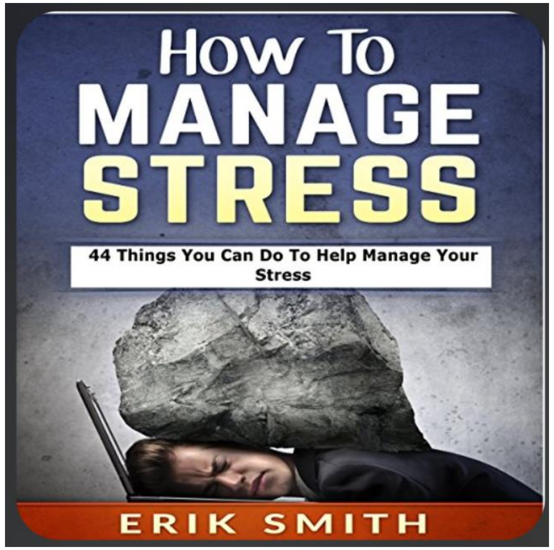
Lecture 6.
Stress and strain



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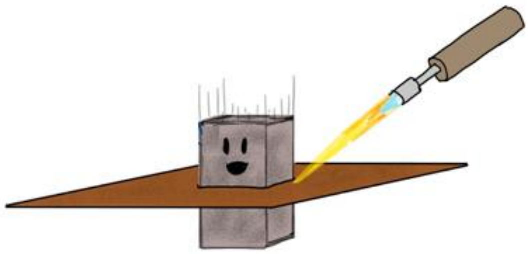
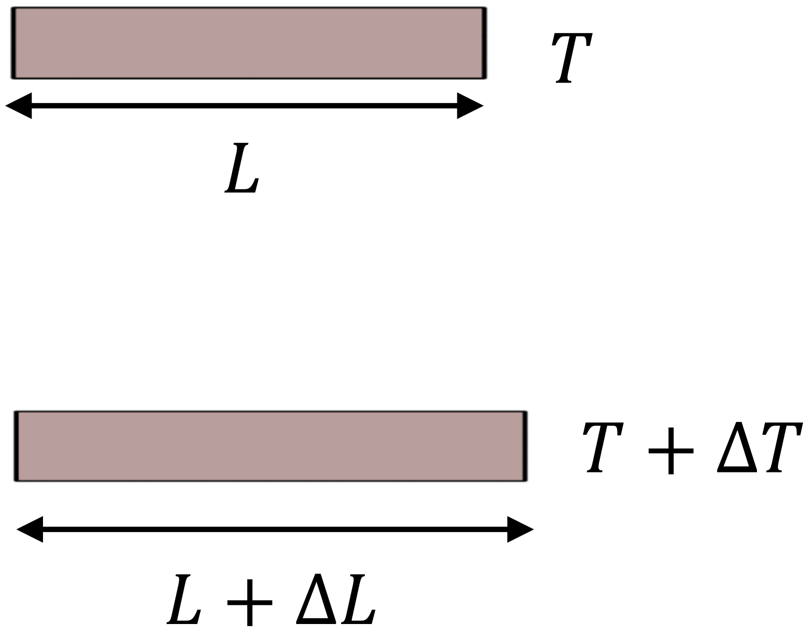
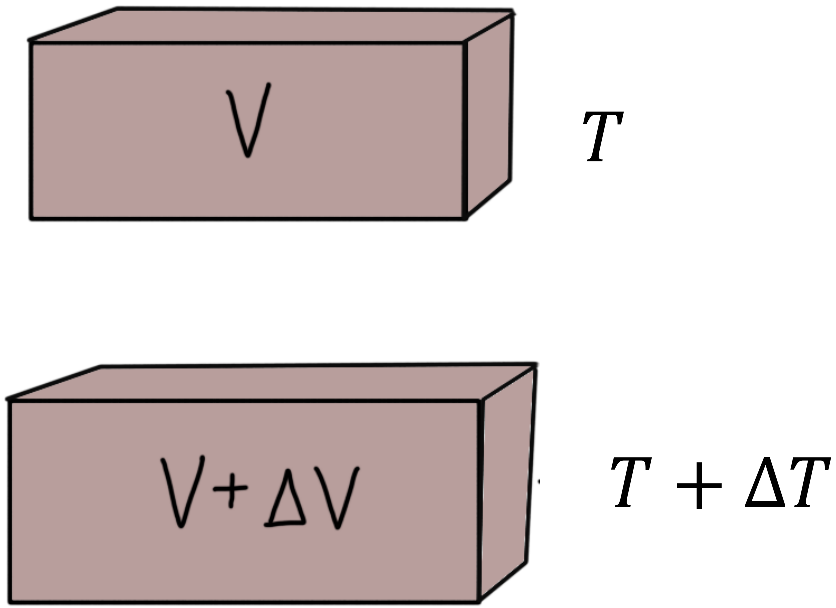






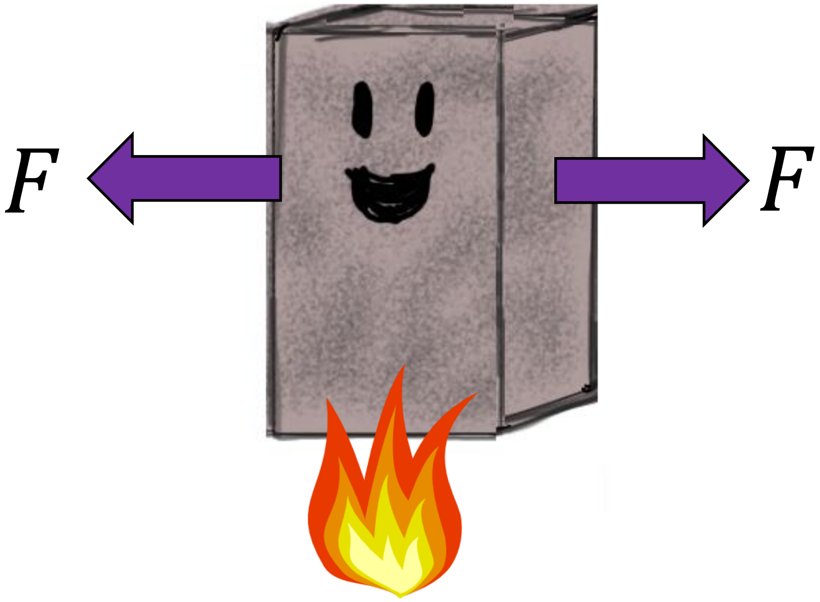
$\boxed{\Delta L = \alpha L_0 \Delta T}$

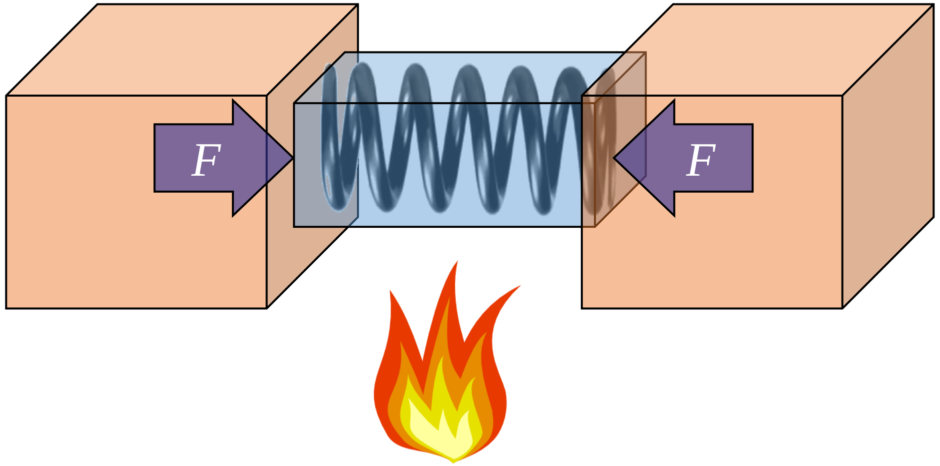
$\boxed{\Delta V = \beta V_0 \Delta T}$

$\boxed{\beta = 3\alpha}$

Stress, Strain, and Young's Modulus

- Often, we need to understand the combined effects of thermal expansion together with expansion/compression due to external mechanical forces





- Let us learn to quantify expansion/compression due to external forces!

Stress

- One kind of stress is overheating of your brain due to learning too much about thermodynamics 
- But that’s not the kind of stress that we will be talking about…

- Tensile stress occurs when an object is stretched by forces acting at the ends, such as a guitar string
- Compressive stress is similar, just the opposite sign (compression instead of extension)

Stress… and Strain

- When you pinch your nose, the force per area that you apply to your nose is called **stress.**

$$\text{stress} = \frac{F}{A} \qquad [Pa] = \frac{[N]}{[m^2]}$$

- Stress has units of pressure

- The fractional change in the size of your nose is called **strain:**

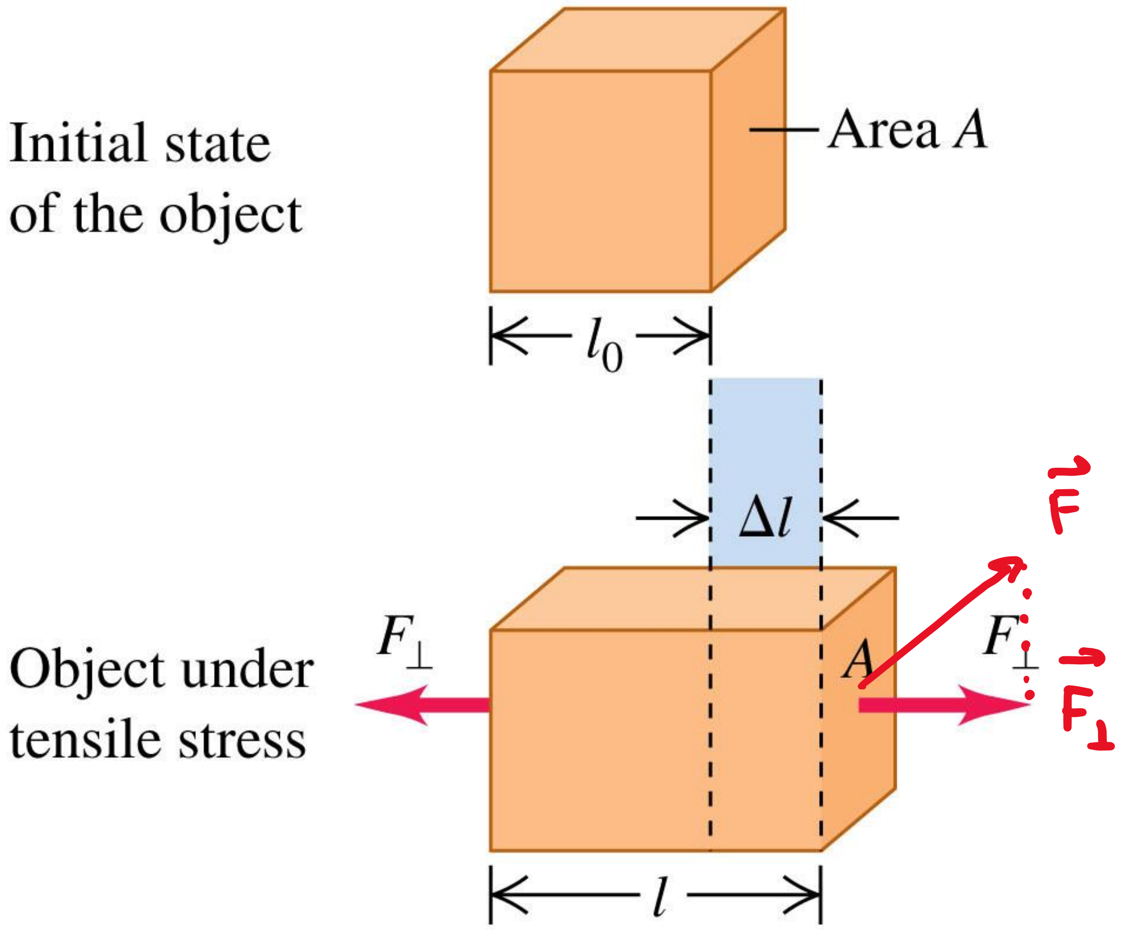
$$\text{strain} = \frac{\Delta L}{L_0}$$

- Strain is dimensionless

- The deformation is **elastic** if your nose springs back to its initial size when you stop pinching (and **plastic** if it does not…)

Tensile stress and strain

- An object in tension
- The net force on the object is zero, but the object deforms
- The tensile stress produces a tensile strain



Tensile stress = $\frac{F_\perp}{A}$ Tensile strain = $\frac{\Delta L}{L_0}$

Q: A copper wire of varying thickness is pulled at each end with a force of 40N. Which of the following statements is true?

$T = \text{const}$

$\text{stress} = \frac{T}{A} = \text{varies}$

A. The tension is a constant 80N all along the wire, and the stress is also constant along the wire.

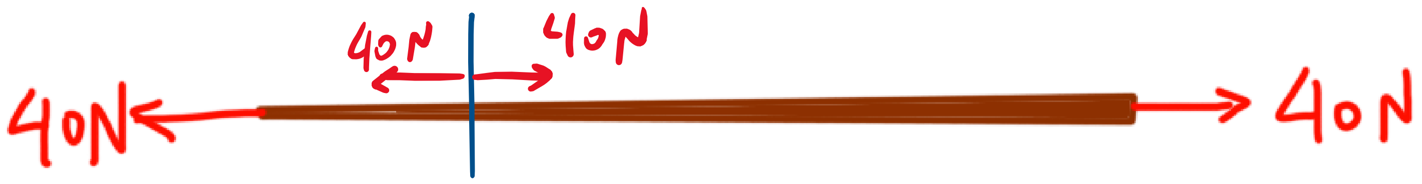
B. The tension is a constant 40N all along the wire, and the stress is also constant along the wire.

C. The stress is constant throughout the wire but the tension varies.

D. The tension is a constant 80N all along the wire, but the stress varies along the wire.

E. The tension is a constant 40N all along the wire, but the stress varies along the wire.

Q: A copper wire of varying thickness is pulled at each end with a force of 40N. Which of the following statements is true?



- At any point: force from each half on other is 40 N, since net force on each side is zero
- Stress = F/A varies since A varies

A. The tension is a constant 80N all along the wire, and the stress is also constant along the wire.

B. The tension is a constant 40N all along the wire, and the stress is also constant along the wire.

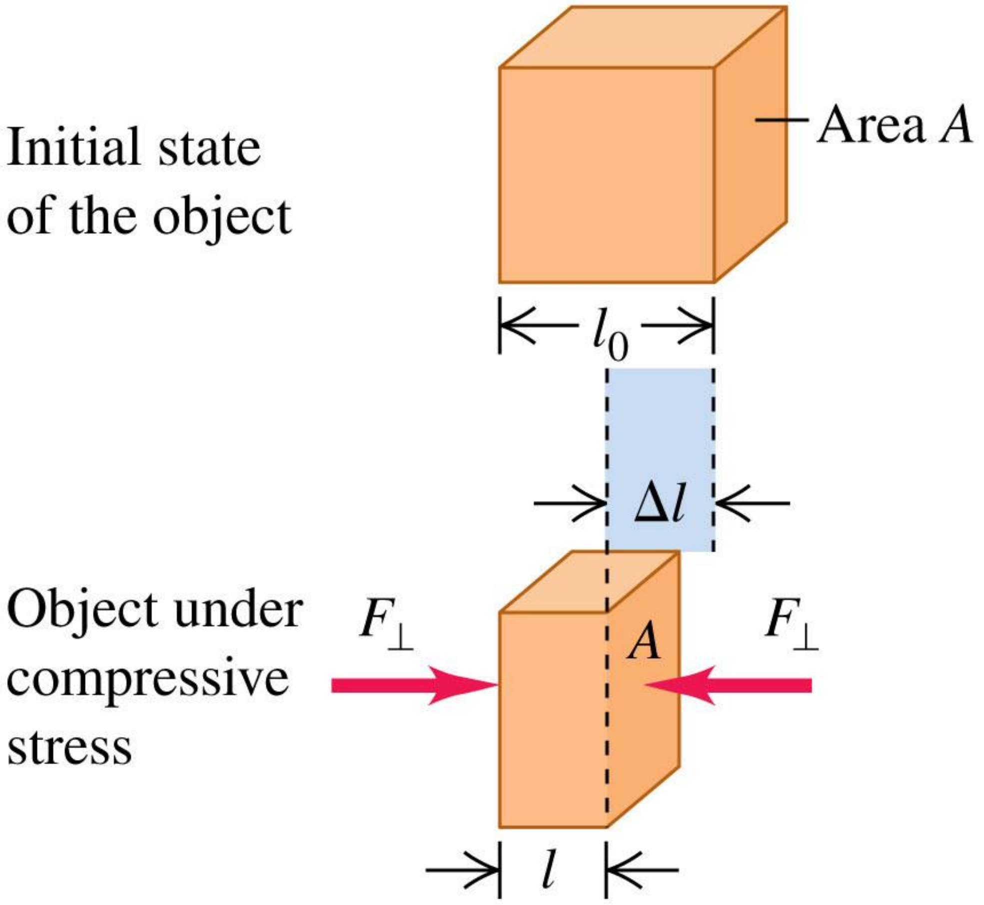
C. The stress is constant throughout the wire but the tension varies.

D. The tension is a constant 80N all along the wire, but the stress varies along the wire.

E. The tension is a constant 40N all along the wire, but the stress varies along the wire. ✓

Compressive stress and strain

- An object in compression
- The net force on the object is zero, but the object deforms
- The compressive stress produces a compressive strain. They are defined in the same way as tensile stress and strain, except that ΔL now denotes the distance that the object contracts



Compressive stress = $\frac{F_\perp}{A}$

Compressive strain = $\frac{\Delta L}{L_0}$

Young's modulus

- Experiment shows that for a sufficiently small tensile or compressive stress, stress and strain are linearly proportional

- The corresponding proportionality coefficient ("*elastic modulus*") is called Young's modulus, Y

$$\underbrace{\frac{F}{A}}_{\text{stress}} = Y \underbrace{\frac{\Delta L}{L_0}}_{\text{strain}}$$

- Y expresses resistance to stretching or compressing

stiffness!

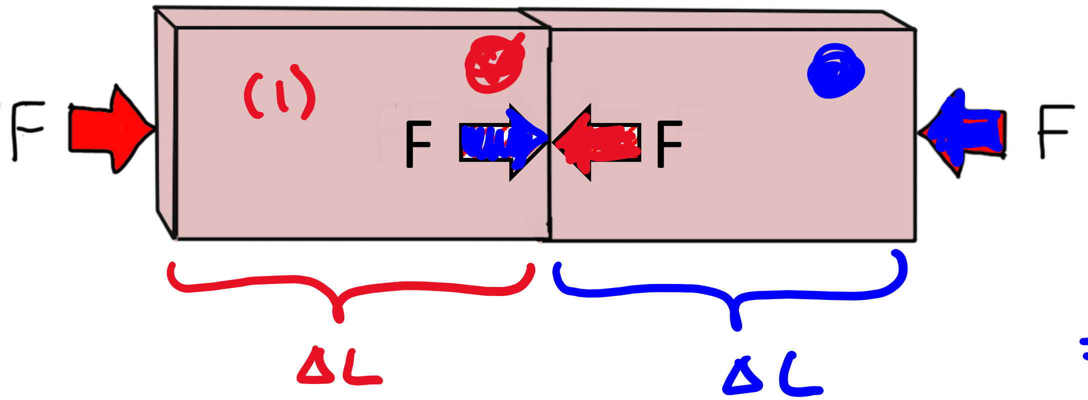
$$Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta L/L_0}$$

- How does this equation appear?

❖ Varying length of the sample, L_0

Q: In the picture, the force on the right brick from the left brick has magnitude

A. 0
B. F
C. 2F



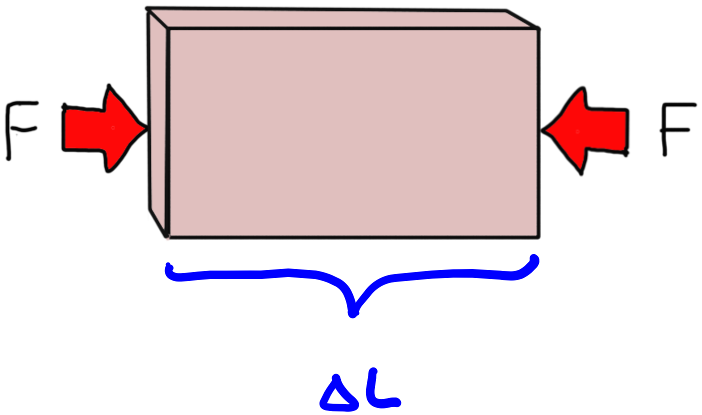
ΔL?

$= 2\Delta L = \Delta L_{tot}$

Q: How much is the brick in the picture above is compressed compared to this brick?

A. More
B. Less
C. Same

ΔL?



❖ Varying length of the sample, L_0

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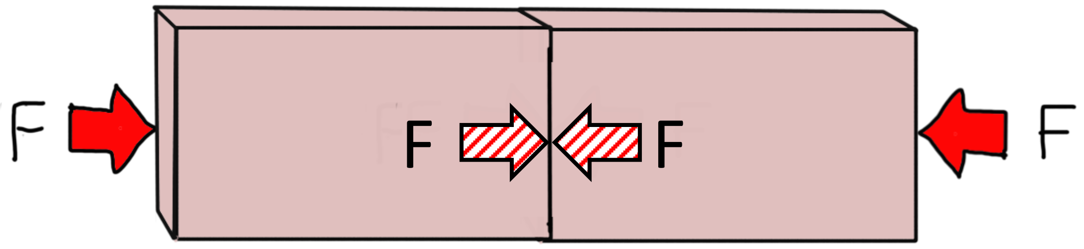
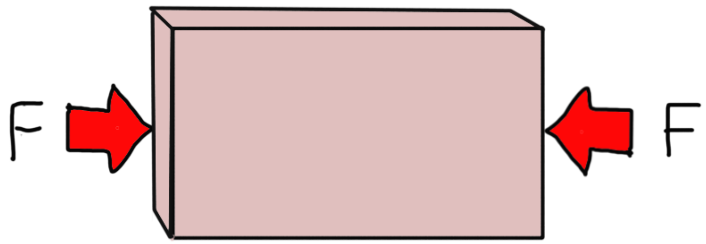


Bricks not moving so net force on right brick must be zero.

So force of left brick on right exactly opposes force from right.

Q: How much is the brick in the picture above is compressed compared to this brick?

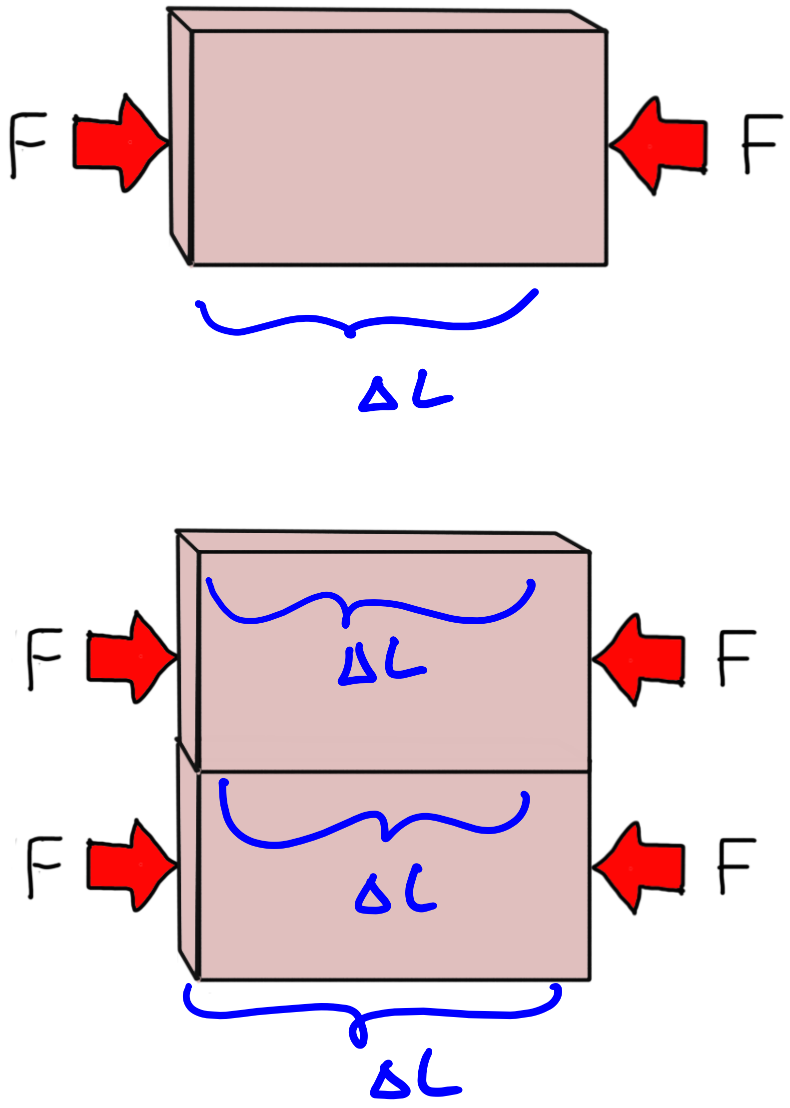
A. More
B. Less
C. Same



Each brick in the picture above is identical to this one brick => if the latter compresses by ΔL, the brick above compresses by $2\Delta L$

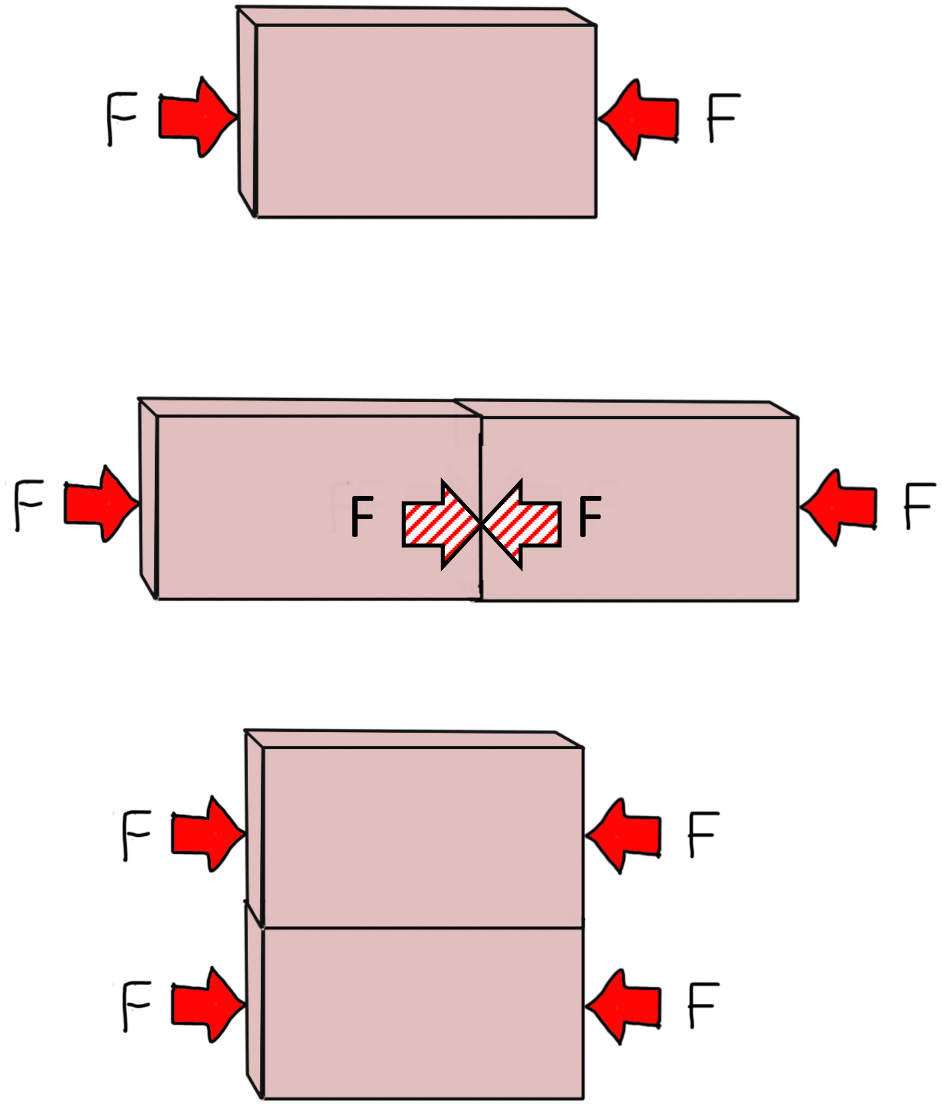
❖ Double L_0 $\Rightarrow$ ΔL doubles

❖ Varying the applied force F and the area A



- Assume that we double the area and double the applied force

- All these three blocks are equally compressed => ΔL does not change if we double F and A at the same time



Young's modulus

$$Y = \frac{F/A}{\Delta L/L_0}$$

- Start with linear relationship:
  - $F = \text{const} \cdot \Delta L$ const ($\underline{\underline{L_0}}$, A, material)
- Same F, A, double L_0 => ΔL doubles
  - $F = \text{const}' \cdot \dfrac{\Delta L}{L_0}$ const' (A, material)
- Same L_0, double $F\&A$ => ΔL remains the same
  - $F = \text{const}'' \cdot \dfrac{\Delta L}{L_0} A = Y \dfrac{\Delta L}{L_0} A$ const''(material)

Y depends only on the material (substance), not on geometry!

Young's Modulus of Various Materials

Material	Young's Modulus, Y (Pa)
Aluminum	7.0×10^{10}
Brass	9.0×10^{10}
Copper	11×10^{10}
Iron	21×10^{10}
Lead	1.6×10^{10}
Nickel	21×10^{10}
Silicone rubber	0.001×10^{10}
Steel	20×10^{10}
Tendon (typical)	0.12×10^{10}

HW

- Y has units of pressure (Pa)
- It's a measure of how much pressure is required to produce a significant fractional change in length
- For example, a pressure of 0.01% of Y on ends will give 0.01% compression

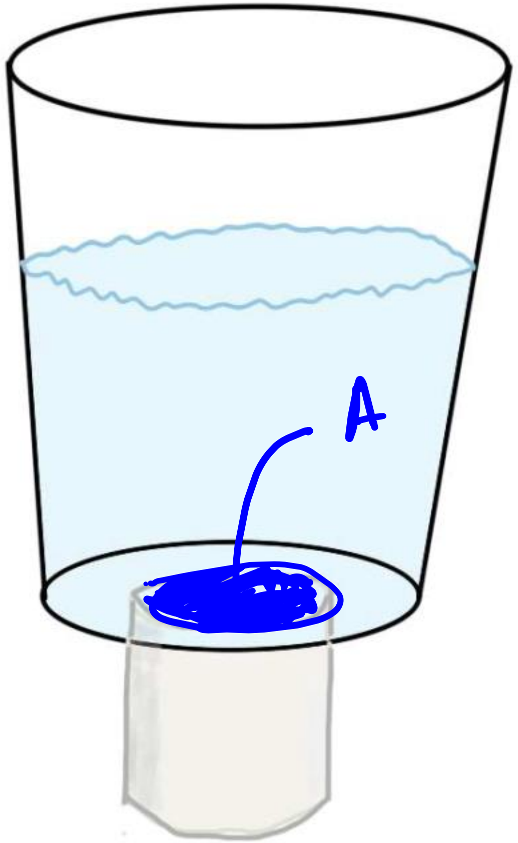
Y = material const

$$\frac{F}{A} = Y \frac{\Delta L}{L_0}$$

How to calculate Young's modulus, or

Young's modulus of a marshmallow

$$\frac{F}{A} = Y\,\frac{\Delta L}{L_0}$$



- $\frac{\Delta L}{L_0} \approx 0.1 - 0.2$
- $F \approx 1N$
- $A \approx 5\,cm^2 = 5 \times (10^{-2}m)^2$

- This gives: $Y \approx 10^4 \frac{N}{m^2}$

$Y_{\text{steel, copper}} \sim 10^{10}\,Pa$

Q: Suppose you repeated the measurements of Y for a mini-marshmallow.

In this case, we would expect a value of Y that is:

A. Significantly higher

B. Significantly lower

C. About the same

Y_{mini} ?

$Y = Y(\text{material})$!

Q: Suppose you repeated the measurements of Y for a mini-marshmallow.

In this case, we would expect a value of Y that is:

A. Significantly higher

B. Significantly lower

C. About the same ✓

Y is a property of the materials and doesn't depend on the size

Q: Do you expect that the Young's modulus you measure for a marshmallow is higher or lower than for steel?

A. Higher

B. Lower

C. Could be higher or lower depending on the relative dimensions of the steel and marshmallow

Y = resistance to compression / expansion

$Y_{marsh.m} \approx 10^4$ Pa

$Y_{steel} \approx 10^{10}$ Pa

Q: Do you expect that the Young's modulus you measure for a marshmallow is higher or lower than for steel?

A. Higher
B. Lower ✓
C. Could be higher or lower depending on the relative dimensions of the steel and marshmallow

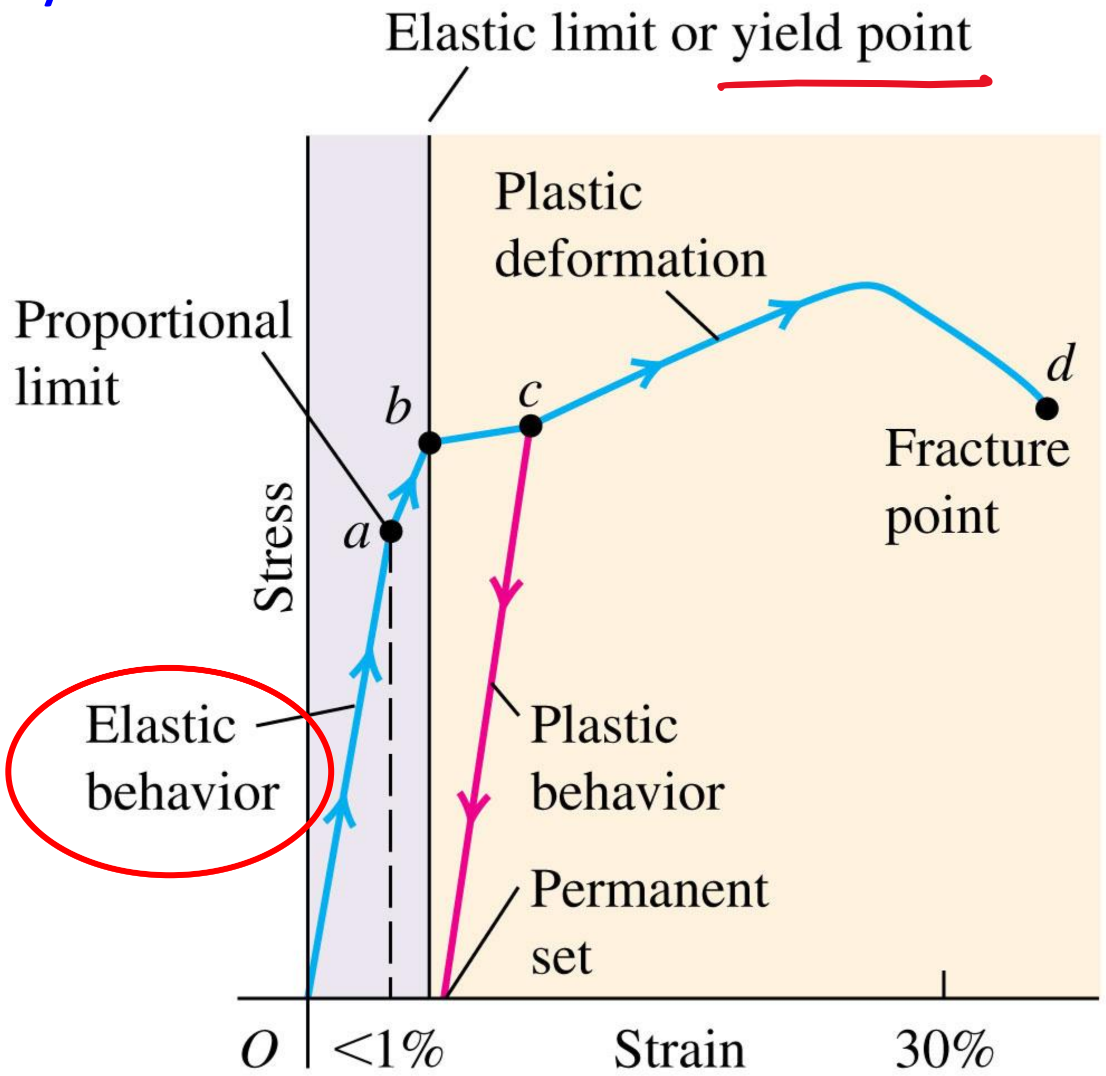
1) Y only depends on what the object is made of, not its size

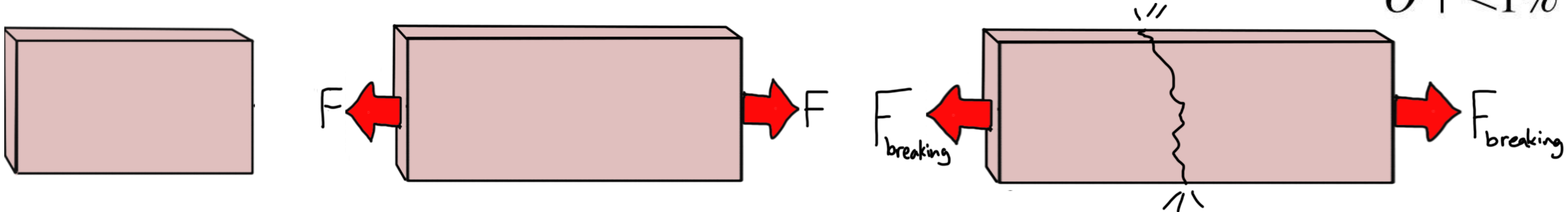
2) $\frac{F}{A} = Y\,\frac{\Delta L}{L_o}$ so Y lower if it takes less stress to give same change in L

Elasticity and plasticity

- Hooke's law—the proportionality of stress and strain in elastic deformations—has a limited range of validity
- Here is a typical stress-strain diagram for a **ductile** (=with large plastic region) **material**, such as copper or soft iron, under tension
- If fracture point close to yield point: **brittle material** (steel)

$$\frac{F}{A} = Y\,\frac{\Delta L}{L_0}$$ valid here





Q: If 0.2 mm diameter nylon fishing line is good for catching fish up to 2 kg, what thickness of line would you need to catch a 50 kg fish?

A. 0.5 mm
B. 1 mm
C. 2 mm
D. 5 mm
E. 300 mm

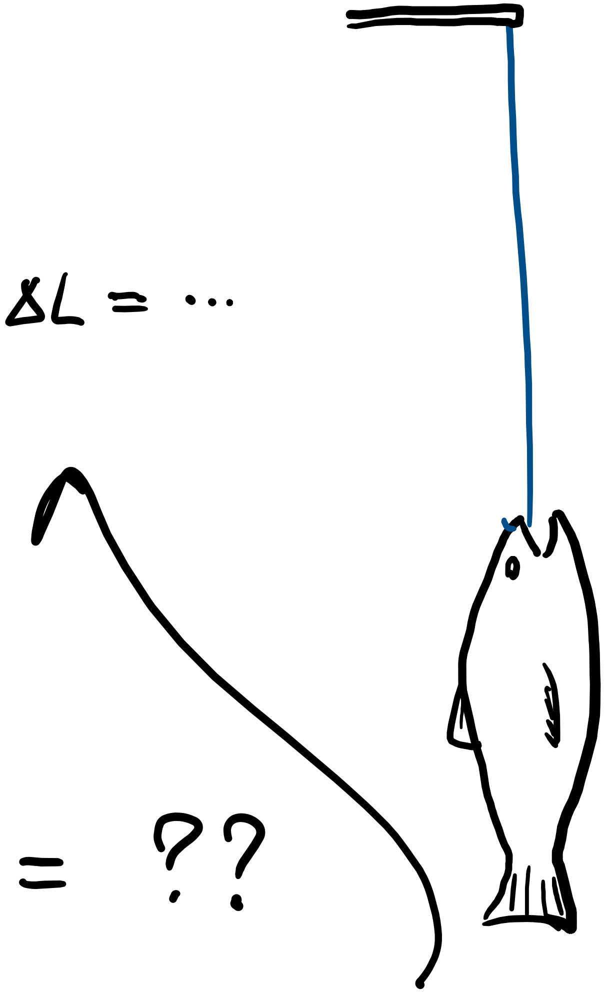
$F' = 25\ F$

$A' = 25\ A$

$D' = 5\ D$

$\hookrightarrow 0.2 \times ? = ??$

$\Delta L = \cdots$



Extra: By roughly how much would 1 m of 0.2 mm diameter line be stretched by a 2 kg fish? ($Y_{\text{nylon}} = 3$ GPa)

$$\boxed{\uparrow \frac{F}{A} = Y \frac{\Delta L}{L_0}}$$

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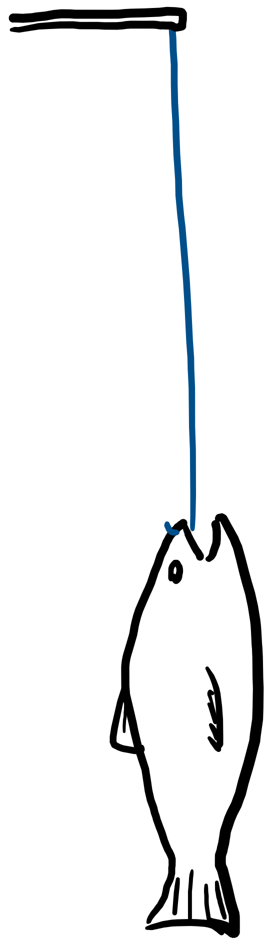
A. 0.5 mm
B. 1 mm
C. 2 mm
D. 5 mm
E. 300 mm

F is 25x greater. So to get equivalent “safe” stretching, need A to be 25x bigger and thus diameter must be 5x bigger

$$\Delta L = \frac{L_0}{Y}\frac{F}{A} \approx \frac{1\ m}{3 \times 10^9}\ \frac{20\ N}{\pi(10^{-4}\ m)^2} \approx 0.2\ m$$



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$$\boxed{\frac{F}{A} = Y\,\frac{\Delta L}{L_0}}$$