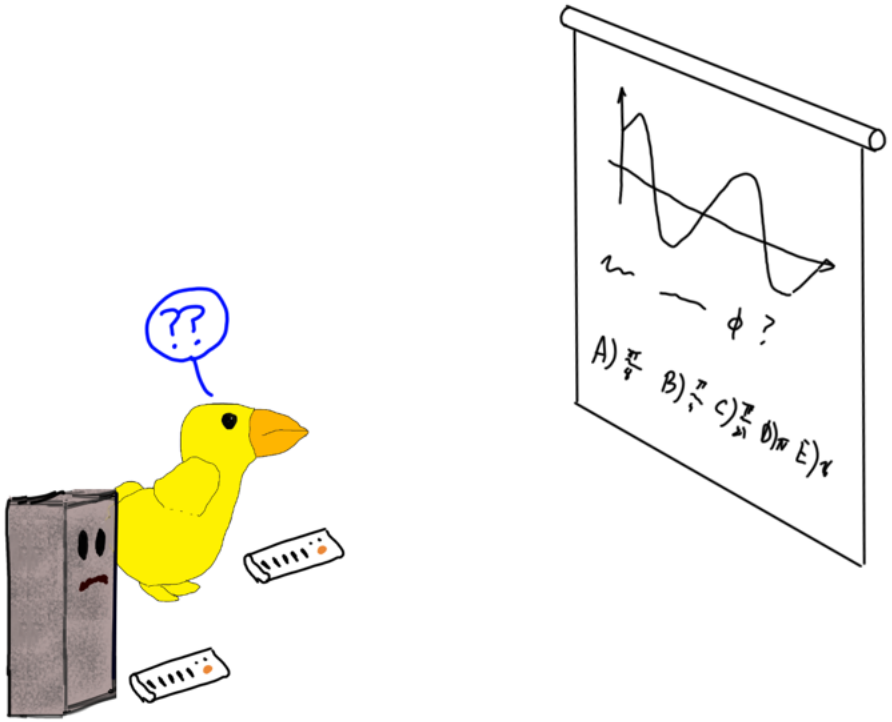
Lecture 28.

$x(t) \Leftrightarrow v(t) \Leftrightarrow a(t)$. Energy in SHM.



Midterm 2 Information

- 7:00-8:30 PM, Thursday, Nov 16th
- Location based on Tutorial section (and possibly first letter of last name):
  - Instructions in **Midterm 2 Details** posted on Canvas
- Format:
  - 6 Multiple choice conceptual questions + two written problems
  - 7:00-8:15 to work on exam; 8:15-8:30 to scan/upload exam to Canvas
- Content:
  - Material summarized in the **Midterm 2 Resource Guide** posted on Canvas
- Rules
  - Closed book but formula sheet will be provided (posted on Canvas)
  - Calculators allowed: any calculator without wireless capabilities
  - No communications or internet usage (except Canvas during upload period ONLY)

Office hours during the reading break:

- TBA, please check home page on Canvas

Last Time

$$x(t) = A\cos(\omega t + \phi)$$

- A = amplitude
- ω = angular frequency
- ϕ = phase

- Position:

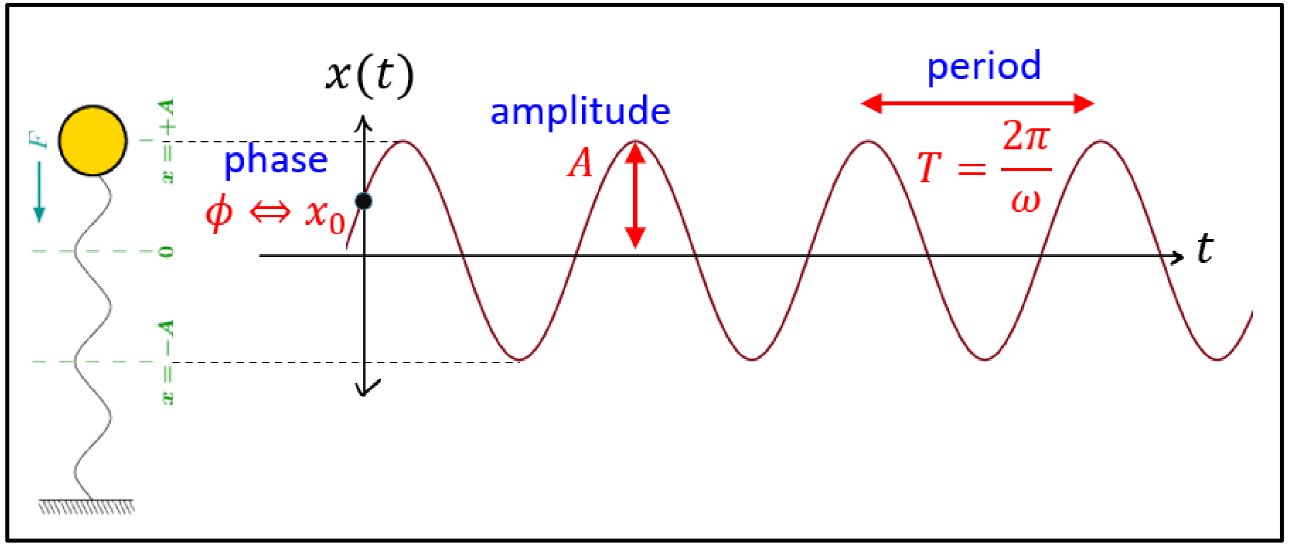
$$x(t) = A\cos(\omega t + \phi)$$

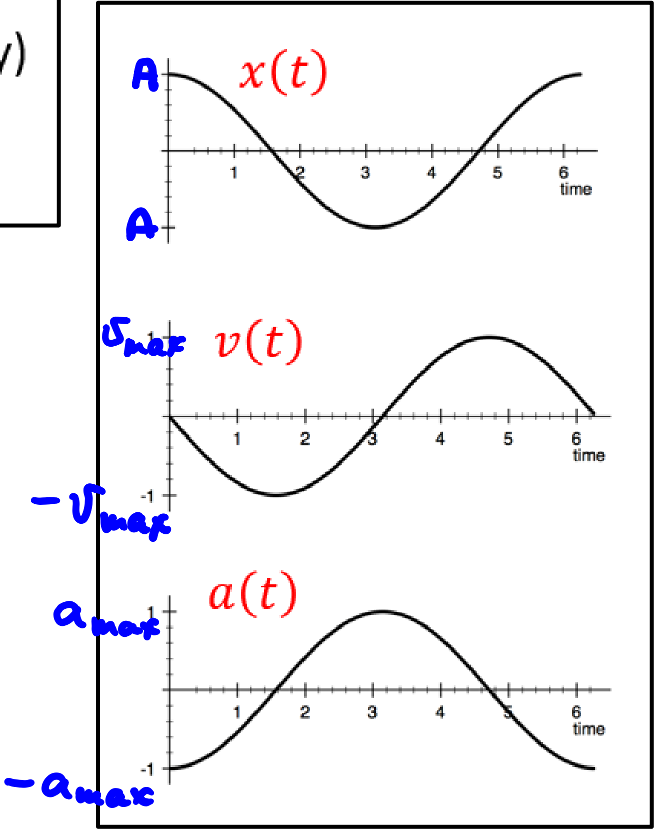
- Velocity (time derivative of position):

$$v(t) = -\underbrace{\omega A}_{v_{max}}\sin(\omega t + \phi)$$

- Acceleration (time derivative of velocity)

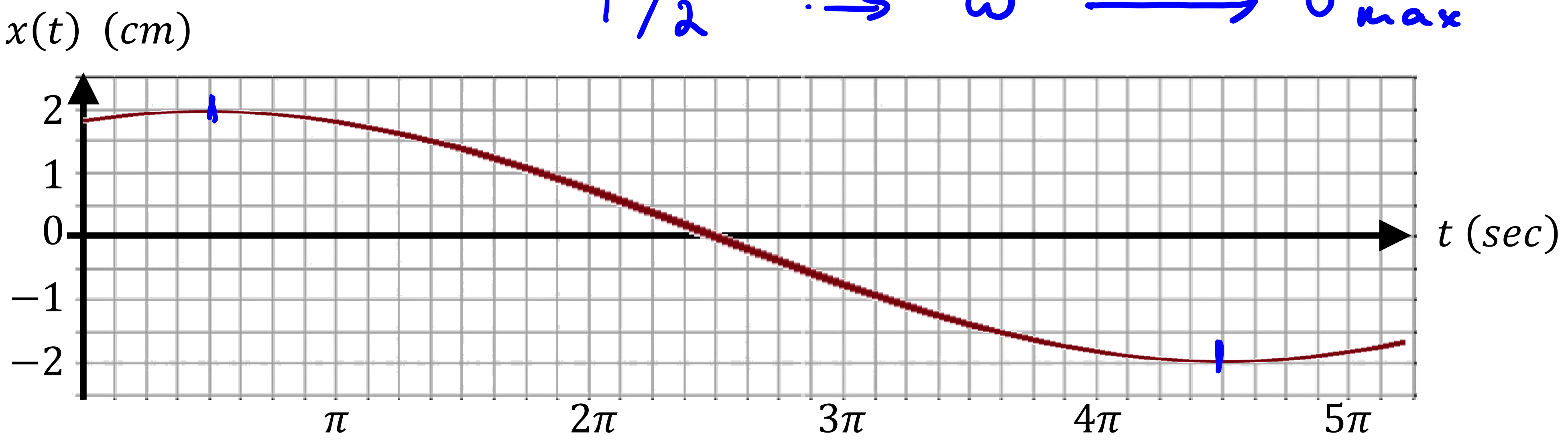
$$a(t) = -\underbrace{\omega^2 A}_{a_{max}}\cos(\omega t + \phi)$$





Q: For this displacement graph, what is the maximum magnitude of velocity, in cm/s?

$T/2 \rightarrow \omega \longrightarrow v_{max}$



A. 4

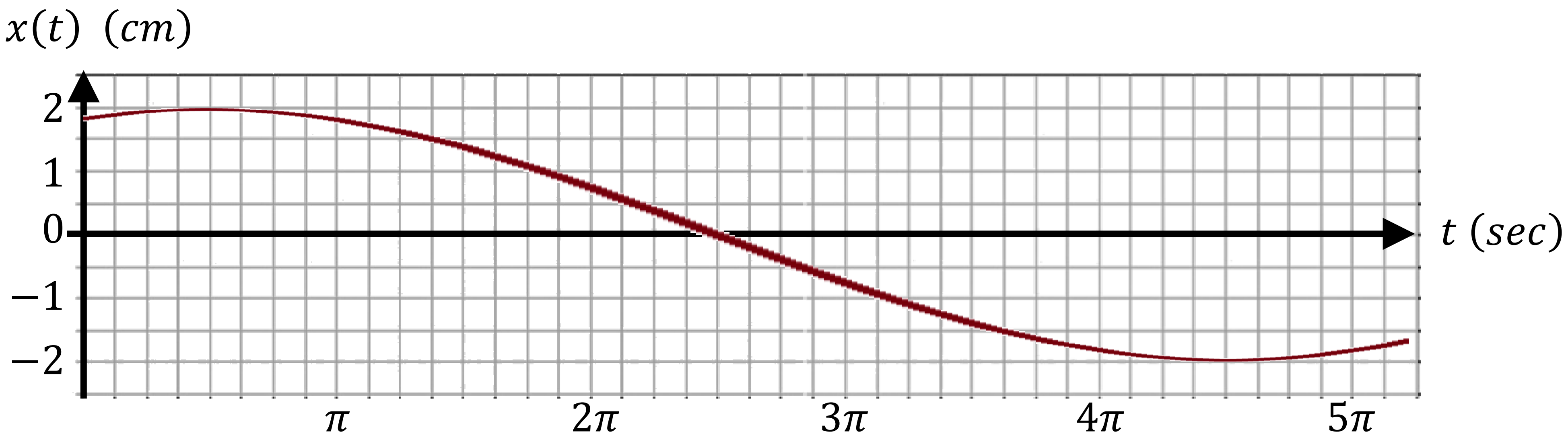
B. 2

C. 1

D. 1/2

E. 1/4

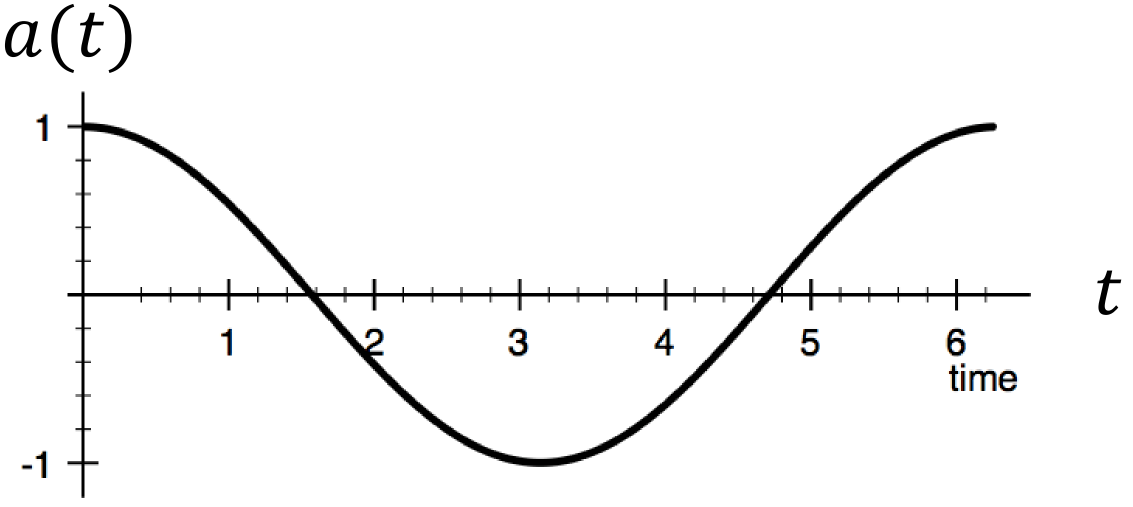
Q: For this displacement graph, what is the maximum magnitude of velocity, in cm/s?

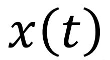


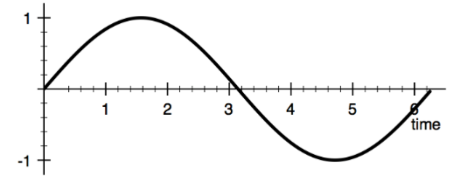
A. 4
B. 2
C. 1
D. 1/2 ✓
E. 1/4

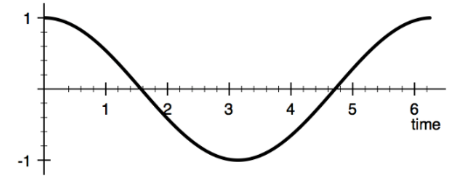
- $x(t) = A\cos(\omega t + \phi)$ $\qquad \omega = 2\pi/T$
- $v(t) = -A\,\omega\sin(\omega t + \phi)$ so maximum value is $A\,\omega$
- $A = 2\ cm \qquad \omega = \frac{2\pi}{T} = \frac{2\pi}{8\pi} = \frac{1}{4} \qquad \Longrightarrow v_{max} = A\,\omega = \frac{1}{2}$

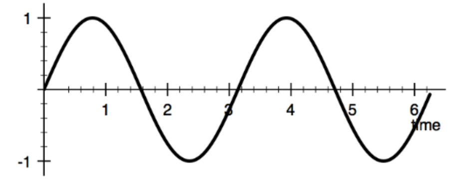
Q: A plot of upward **acceleration** (in cm/s^2) as a function of time (in s) is shown for a mass hanging from a spring. Which of the pictures to the right could represent $x(t)$?

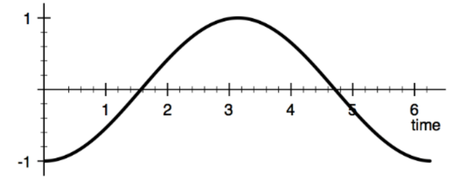


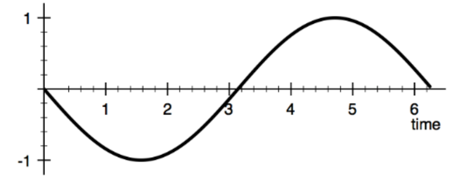


A.


B.


C.


D.


E.


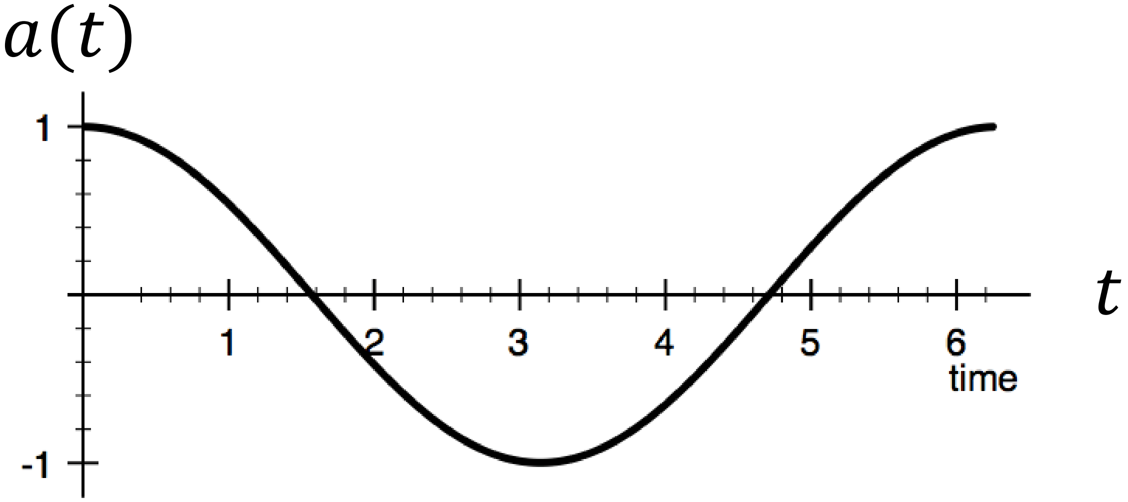
$$x(t) = A\cos(\omega t + \phi)$$

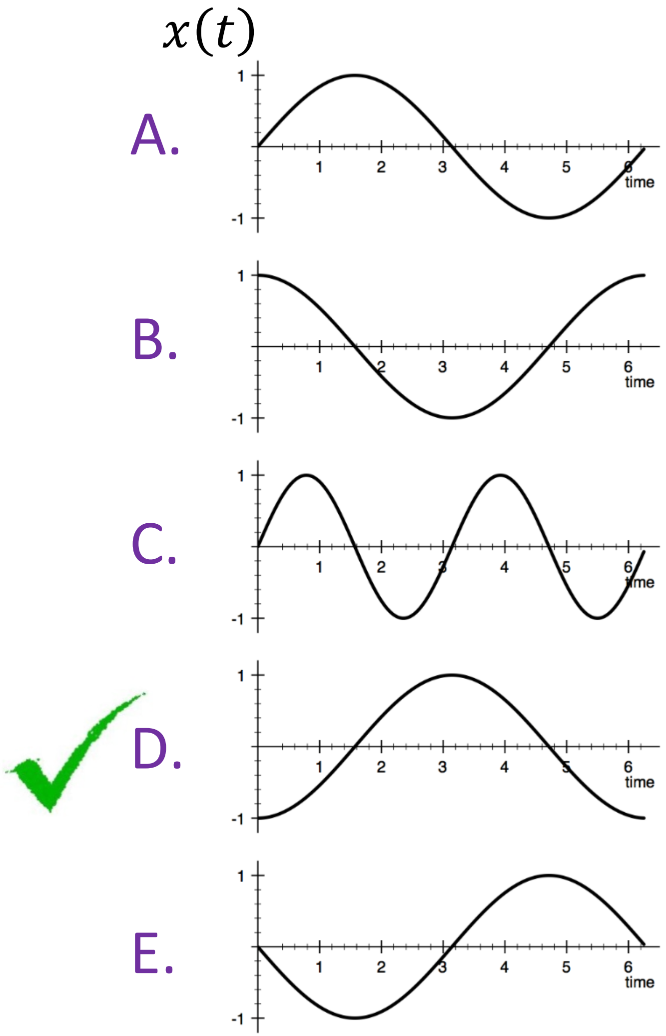
$$a(t) = -A\omega^2 \cos(\omega t + \phi)$$

SHM

$$\boxed{a(t) = -\omega^2 x(t)}$$

Q: A plot of upward **acceleration** (in cm/s^2) as a function of time (in s) is shown for a mass hanging from a spring. Which of the pictures to the right could represent $x(t)$?

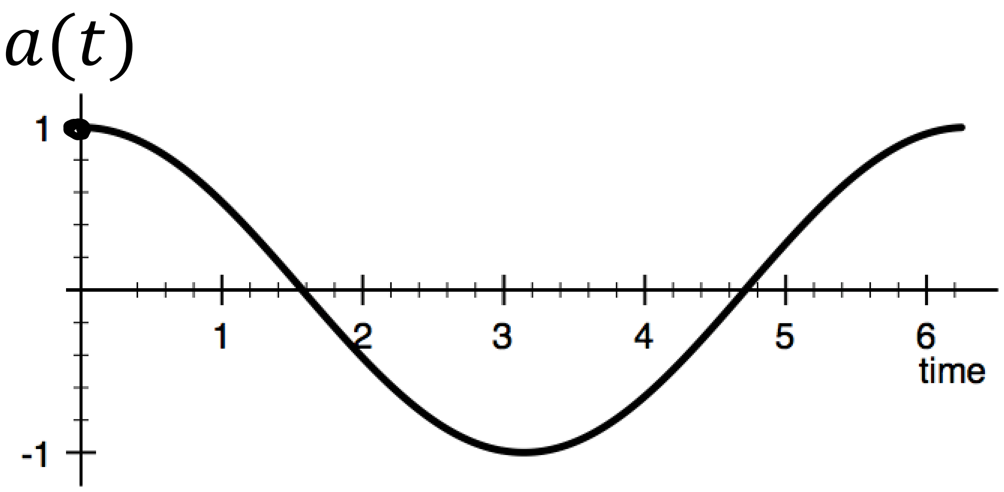


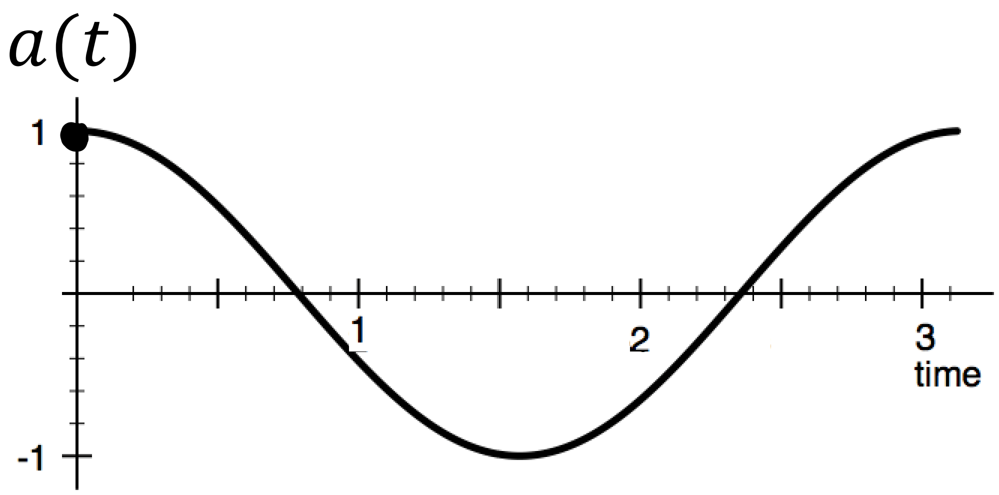


$$a(t) = -A\,\omega^2 \cos(\omega t + \phi) = -\omega^2 x(t)$$

So x is a maximum when a is a minimum

Q: The graphs show **acceleration** as a function of time for two different harmonic oscillators. The amplitude of the **displacement** in the first case is 1 cm. For the second oscillator, the amplitude of the **displacement** is:





A. 4 cm

B. 2 cm

C. 1 cm

D. 0.5 cm

E. 0.25 cm

$A = ?$

$A_1 = 1\,cm$

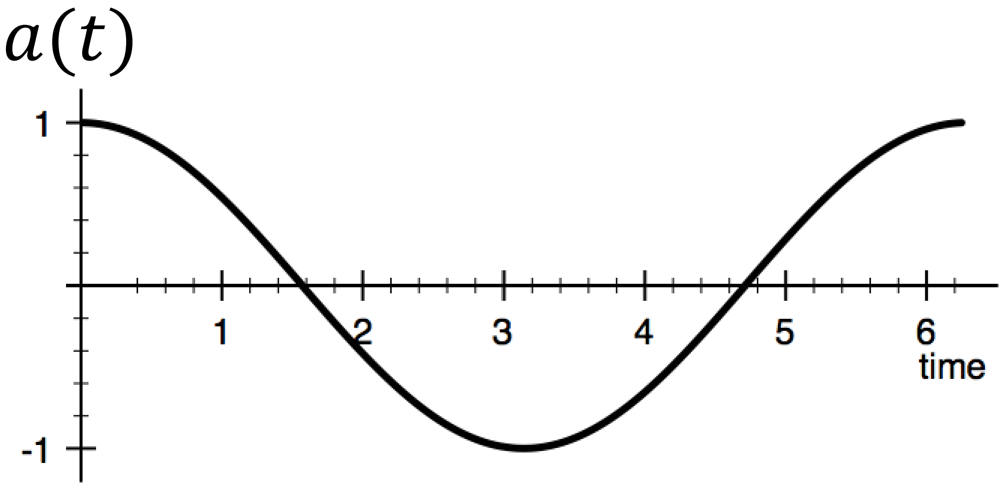
$\omega = \frac{2\pi}{T}$

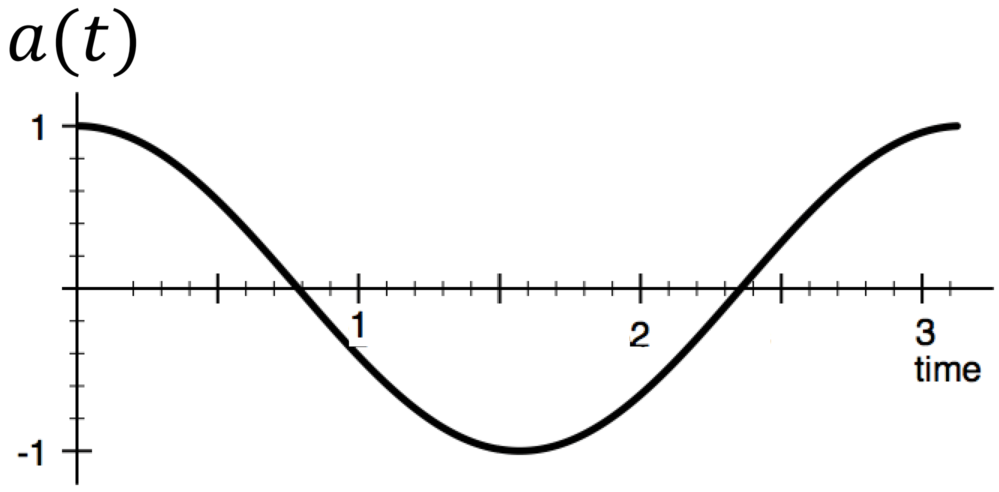
$a_{max,1} = a_{max,2}$

$A_1 \omega_1^2 = A_2 \omega_2^2$

$A_2 = A_1 \left(\omega_1 / \omega_2\right)^2 = A_1 \left(\frac{T_2}{T_1}\right)^2$

Q: The graphs show **acceleration** as a function of time for two different harmonic oscillators. The amplitude of the **displacement** in the first case is 1 cm. For the second oscillator, the amplitude of the **displacement** is:



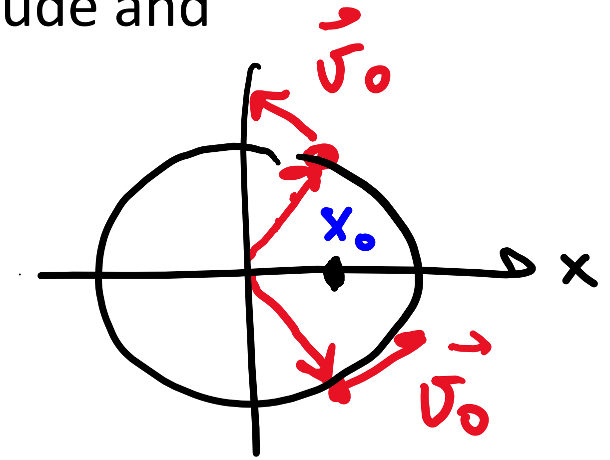


A. 4 cm
B. 2 cm
C. 1 cm
D. 0.5 cm
E. 0.25 cm ✓

- We have: $a = -\omega^2 x \Longrightarrow x = -\frac{a}{\omega^2}$
- T is half in 2nd case $\Longrightarrow \omega$ is double, so amplitude of x is ¼

Initial conditions (x_0, v_0) and the parameters of SHM (ϕ, A)

- If we are given the initial conditions x_0 and v_0, we can find the amplitude and the phase angle of the resulting SHM:



  - At $t = 0$, we know that:

$$x_0 = A\cos\phi \quad \text{and} \quad v_0 = -\omega A \sin\phi$$

  - Dividing the two gives us the phase angle:

$$\frac{v_0}{x_0} = -\omega \tan\phi \quad \Rightarrow \quad \boxed{\phi = \tan^{-1}\left(\frac{-v_0}{\omega x_0}\right)}$$

  - ...and we can use Pythagoras to get the amplitude:

$$x_0^2 + \left(\frac{v_0}{\omega}\right)^2 = A^2(\underbrace{\cos^2\phi + \sin^2\phi}_{1}) = A^2 \quad \Rightarrow \quad \boxed{A = \sqrt{x_0^2 + v_0^2/\omega^2}}$$

Exercise

A 0.500-kg mass on a spring has velocity function given by

$$v_x(t) = -\underbrace{\left(3.60\,\frac{\text{cm}}{\text{s}}\right)}_{v_{max} = A\omega} \sin\left[\left(\overset{\omega}{\underbrace{4.71}}\ \text{s}^{-1}\right)t - \frac{\pi}{2}\right].\ \text{Find:}$$

$\phi = -\frac{\pi}{2}$

a) the period $T = \frac{2\pi}{\omega}$

b) the amplitude $A = \frac{v_{max}}{\omega}$

c) the max acceleration $a_{max} = |-A\omega^2|$

d) the force constant of the spring $\omega = \sqrt{\frac{k}{m}} \longrightarrow k = m\omega^2$

Exercise

A 0.500-kg mass on a spring has velocity function given by

$$v_x(t) = -\left(3.60\,\frac{\mathrm{cm}}{\mathrm{s}}\right)\sin\left[(4.71\ \mathrm{s}^{-1})\mathrm{t} - \frac{\pi}{2}\right].\ \text{Find:}$$

$$v(t) = -A\,\omega\,\sin(\omega t + \phi),\quad \text{so } A\omega = 0.036\,m/s,\ \ \omega = 4.71\,s^{-1}, \phi = -{}^{\pi}/_{2}$$

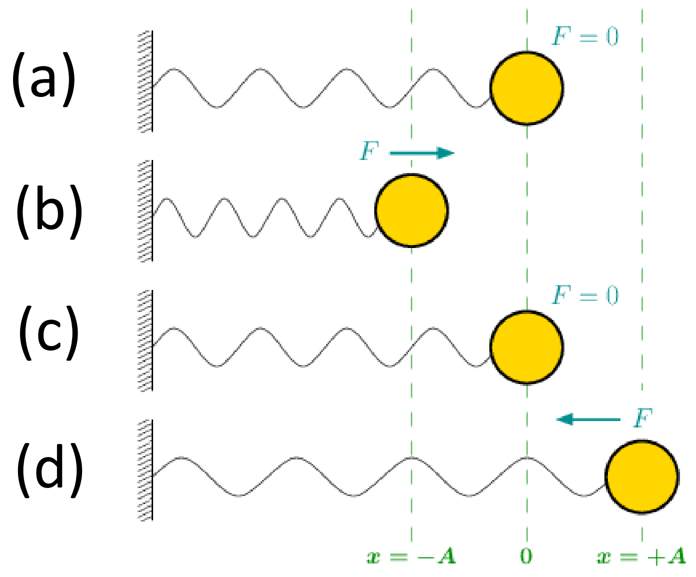
a) the period $\quad T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{4.71} = 1.33\,s$

b) the amplitude $\quad A = \dfrac{0.036}{4.71} = 0.0076\,m$

c) the max acceleration $\quad a_{max} = \omega^2 A = 0.17\ {}^{m}/_{s^2}$

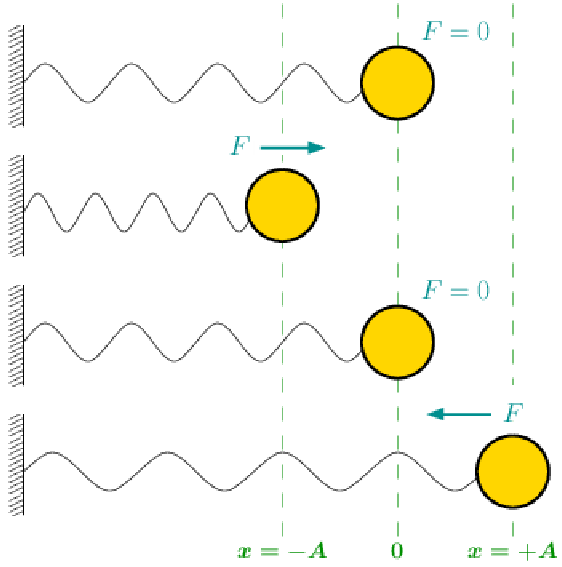
d) the force constant of the spring $\quad \omega = \sqrt{{}^{k}/_{m}} \Rightarrow k = \omega^2 m = 11.1\,{}^{N}/_{m}$

Q: The pictures show an object in simple harmonic motion at successive times. The kinetic energy of the system is largest at



A. (a)
B. (c)
C. Either (a) or (c)
D. Either (b) or (d)
E. The kinetic energy is the same at all times

Q: The pictures show an object in simple harmonic motion at successive times. The kinetic energy of the system is largest at



(a) $\leftarrow \vec{v}_{max} = A\omega$

(b) $\vec{v} = 0$

(c) $\vec{v}_{max} \rightarrow$

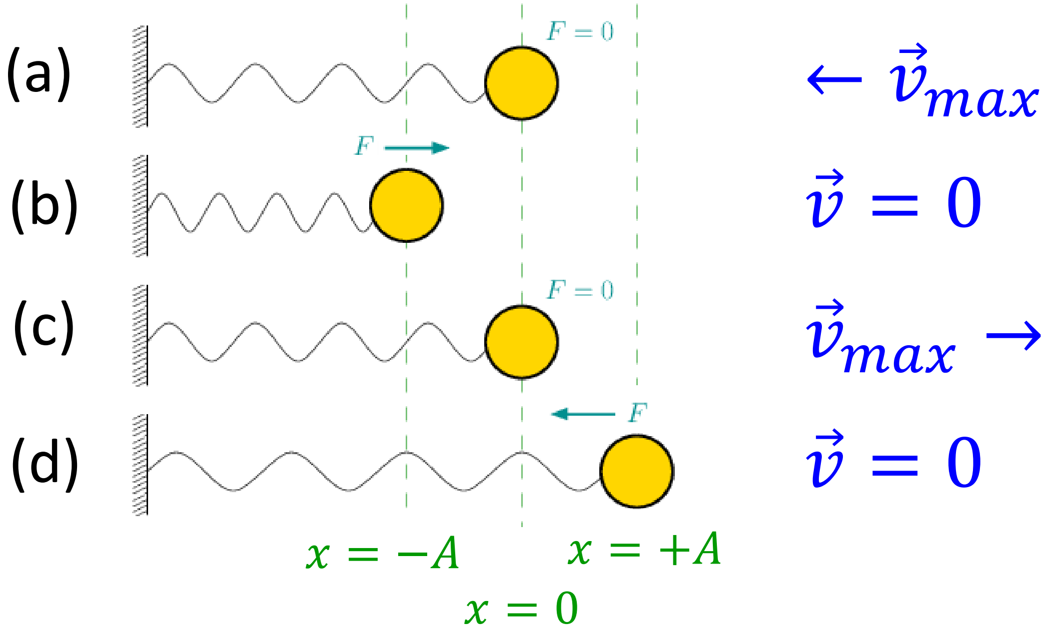
(d) $\vec{v} = 0$

A. (a)
B. (c)
C. Either (a) or (c)
D. Either (b) or (d)
E. The kinetic energy is the same at all times

Kinetic energy is $\mathrm{KE} = \frac{1}{2}mv^2$

Largest when speed is maximum $\Longrightarrow$ object is moving through equilibrium position

Q: The pictures show an object in simple harmonic motion at successive times. At which position(s) is the total energy of the system the largest?



A. (a) and (c)
B. (b) and (d)
C. (a), (b), (c), and (d)

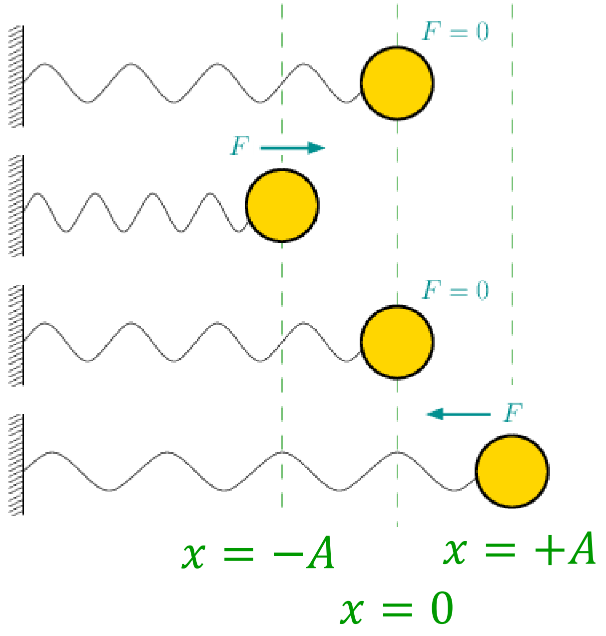
Q: The pictures show an object in simple harmonic motion at successive times. At which position(s) is the total energy of the system the largest?

(a) $\leftarrow \vec{v}_{max}$

(b) $\vec{v} = 0$

(c) $\vec{v}_{max} \rightarrow$

(d) $\vec{v} = 0$

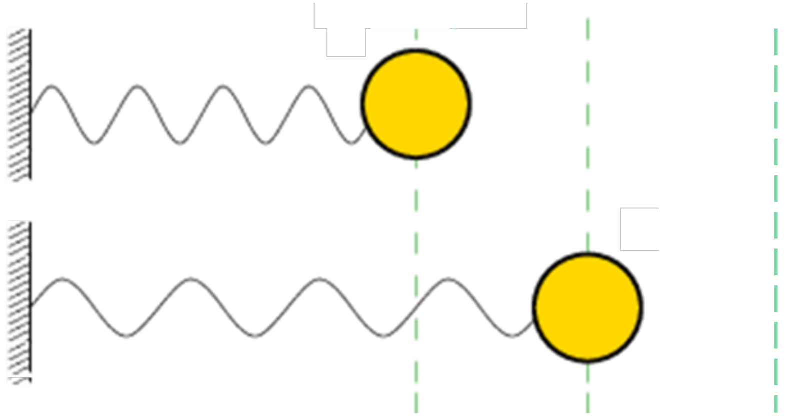


TOTAL energy is the same at all positions because of energy conservation

A. (a) and (c)
B. (b) and (d)
C. (a), (b), (c), and (d)

Kinetic energy in SHM

- An oscillating object with mass m in simple harmonic motion moving with speed v has kinetic energy $\mathrm{KE} = \frac{1}{2}mv^2$:



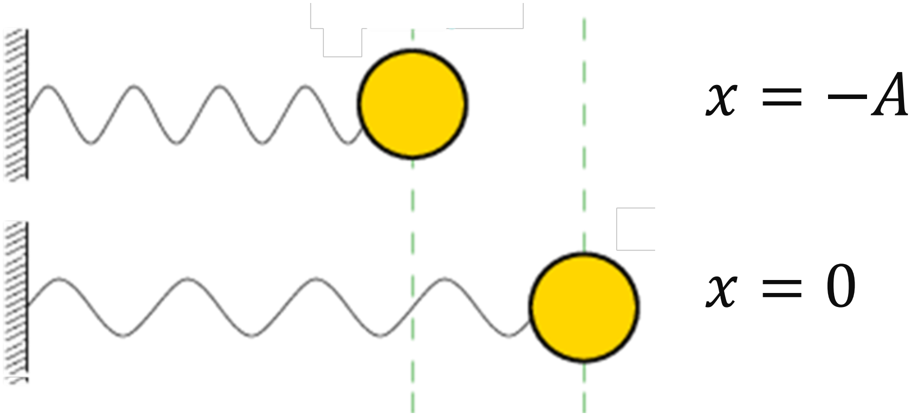
$x = -A \quad v = 0 \quad KE = 0$

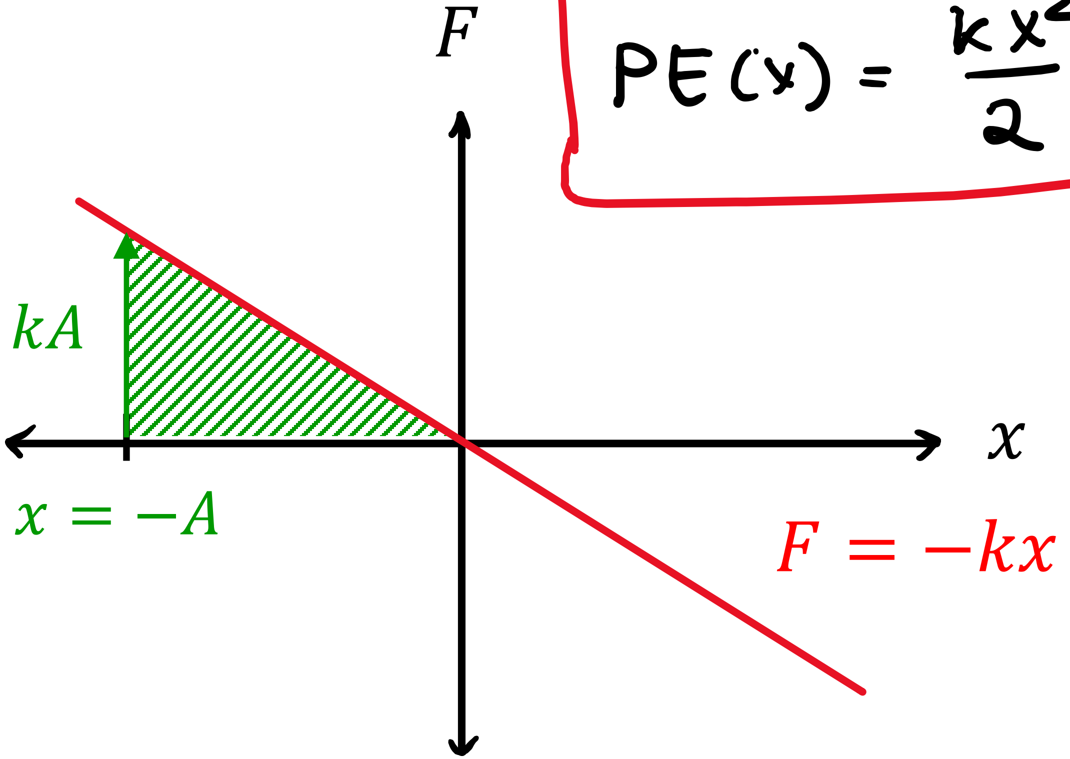
$x = 0 \quad v = v_{max} \quad KE = \frac{1}{2}mv^2$

Q: What is/are other form/forms of energy involved?

Work of the spring on the mass

- During the motion from $x = -A$ to $x = 0$ (the equilibrium position), what is the net work done by the spring on the particle?





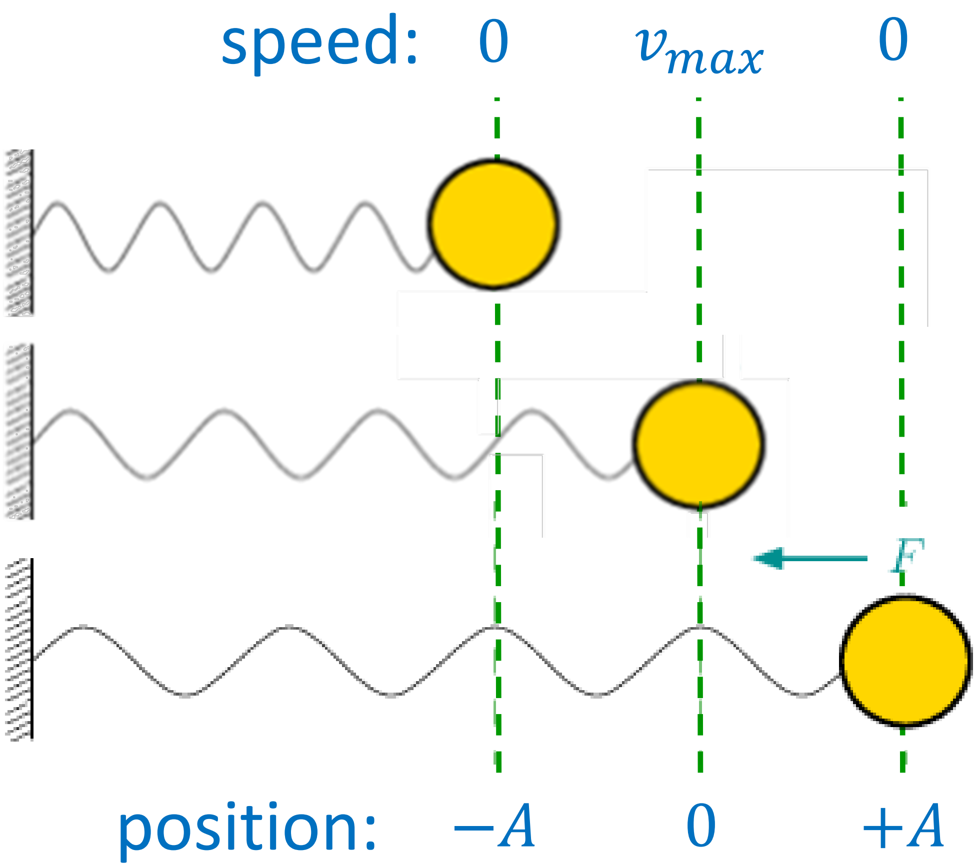
$$PE(x) = \frac{kx^2}{2}$$

- Work: $W = F \cdot \Delta x_{\parallel}$
- F & Δx in same direction, so W is positive
- F is changing, so $W = \sum F \Delta x =$ area under F vs x graph

$$\boxed{W = \frac{1}{2} A \cdot kA = \frac{1}{2} kA^2}$$

(Elastic) potential energy (PE) of the mass
(comes from the spring)

Energy in Simple Harmonic Motion



spring compressed: $PE = \frac{1}{2}kA^2 \quad KE = 0$

equilibrium position: $PE = 0 \quad KE = \frac{1}{2}mv_{max}^2$

spring stretched: $PE = \frac{1}{2}kA^2 \quad KE = 0$

- Is total energy conserved?

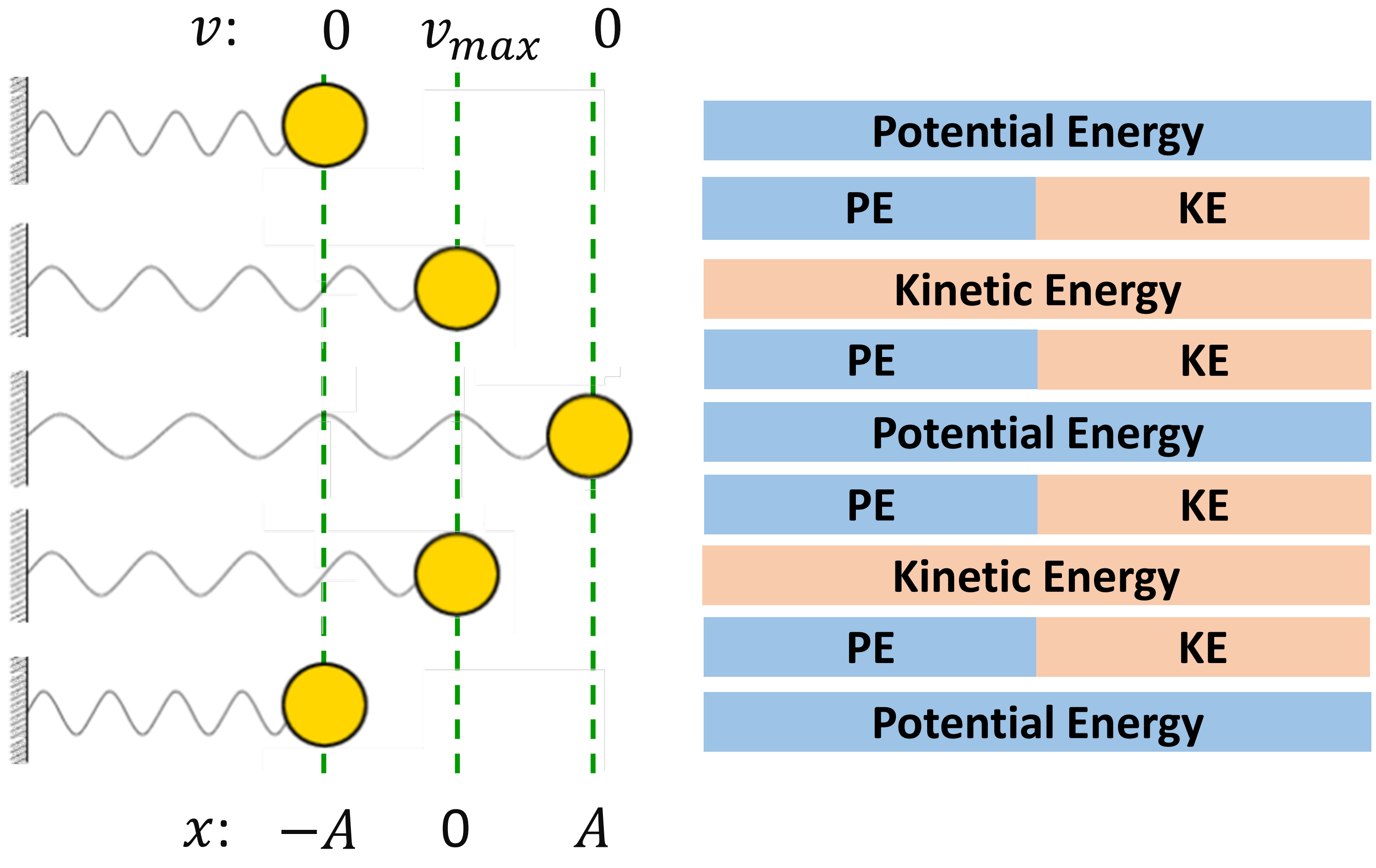
Recall that $v_{max} = A\omega$

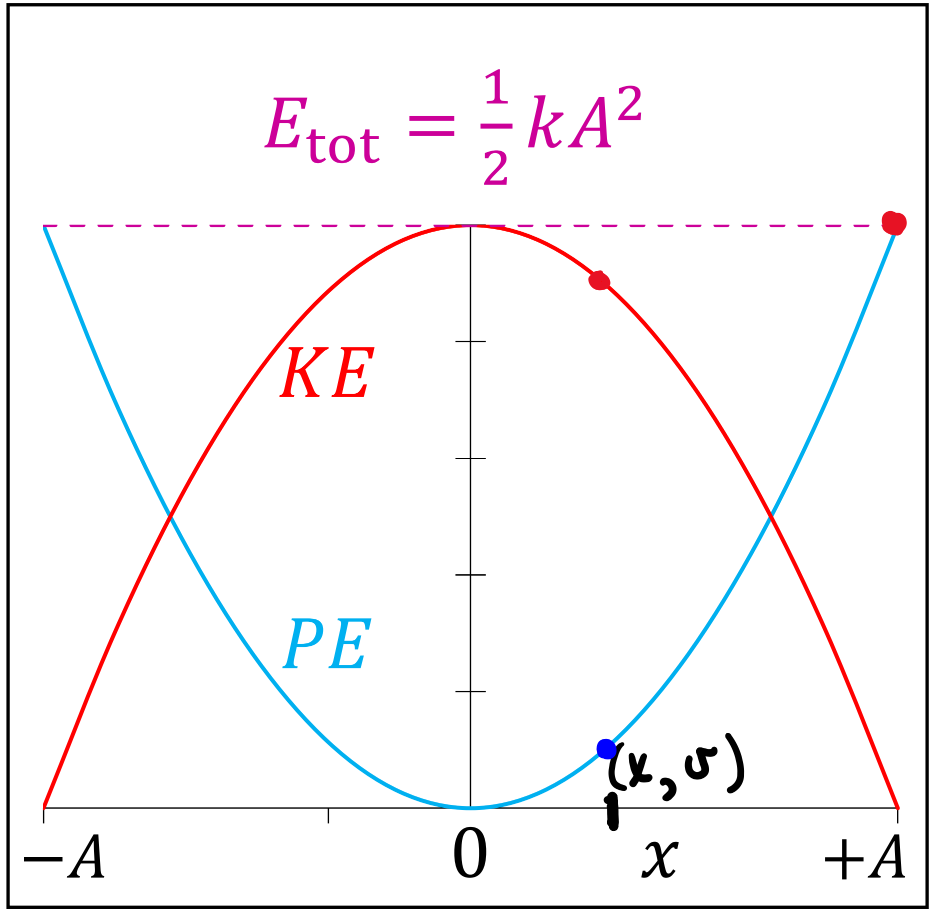
$$\Longrightarrow \frac{1}{2}mv_{max}^2 = \frac{1}{2}m(A\omega)^2 = \frac{1}{2}mA^2\frac{k}{m} = \frac{1}{2}kA^2$$

So energy is same at all three positions

Energy in SHM: Total energy is conserved

$$E_{\text{tot}} = KE + PE = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \text{constant} = \text{initial energy}$$





Energy in Simple Harmonic Motion

- Total mechanical (= KE + PE) energy of the system is conserved in SHM:

$$\boxed{E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2}$$ throughout the motion

- At the maximum displacement, $x = A$ and $v = 0$

$$\boxed{E = \frac{1}{2}m(0)^2 + \frac{1}{2}k(A)^2 = \frac{1}{2}kA^2}$$

E_{tot}

- Mass-on-a-spring is the classic example

- So at *any* other time:

$$\boxed{\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2}$$

$$mv^2 = k(A^2 - x^2) \quad \Rightarrow \quad \boxed{v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}}$$

Q: A mass oscillates on the end of a spring with SHM of amplitude A with equilibrium at $x = 0$. The kinetic energy of the mass will equal the potential energy of the spring when the position of the mass is:

$$PE(x) \stackrel{?}{=} KE(v)$$

$$PE + KE = E_{total}$$

$$2\,PE(x) = E_{total} = \frac{kA^2}{2}$$

$$2\,\frac{\cancel{k}x^2}{\cancel{2}} = \frac{\cancel{k}A^2}{\cancel{2}} \qquad x = \frac{A}{\sqrt{2}}$$

A. $x = 0$

B. $x = A/\sqrt{2}$

C. $x = A/2$

D. $x = A/4$

Q: A mass oscillates on the end of a spring with SHM of amplitude A with equilibrium at $x = 0$. The kinetic energy of the mass will equal the potential energy of the spring when the position of the mass is:

- Start with energy conservation:

$$PE + KE = E_{Total}$$

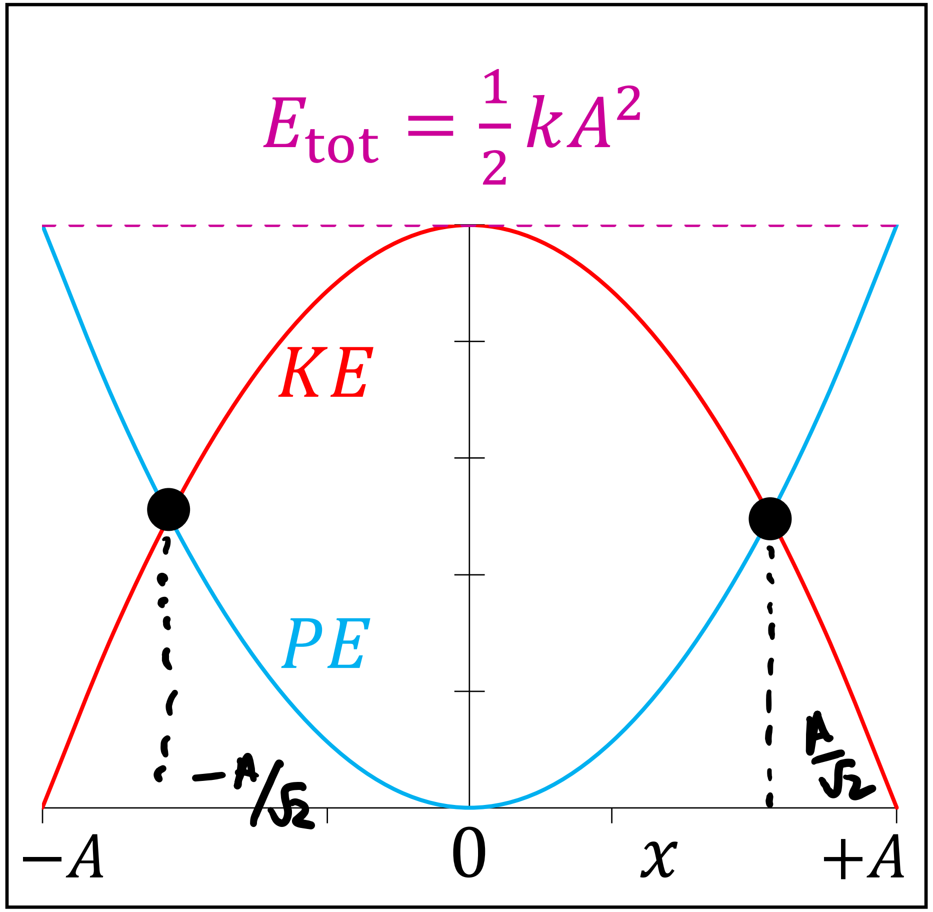
$$\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$$

- Want to know x when $PE = KE$

$$\Rightarrow 2PE = E_{\text{total}}$$

$$\Rightarrow 2\left(\frac{1}{2}kx^2\right) = \frac{1}{2}kA^2$$

- …which is true when $x = A/\sqrt{2}$



A. $x = 0$

B. $x = A/\sqrt{2}$ ✓

C. $x = A/2$

D. $x = A/4$

Q: A 0.5 kg mass is attached to a horizontal spring of spring constant 200 N/m. $= k$
If the spring is initially compressed by 0.1 m, and the mass is then released, what is the speed of the block when the spring is at its equilibrium length?

A

$$PE \longrightarrow KE$$

$$\frac{kA^2}{2} = \frac{mv^2}{2} \longrightarrow v = A\sqrt{\frac{k}{m}}$$

A. 1 m/s
B. 2 m/s
C. 3 m/s
D. 4 m/s
E. 5 m/s

Q: A 0.5 kg mass is attached to a horizontal spring of spring constant 200 N/m. If the spring is initially compressed by 0.1 m, and the mass is then released, what is the speed of the block when the spring is at its equilibrium length?

- Initially, $v = 0$ and $x = -A$:

$$\Longrightarrow E = PE = \frac{1}{2}kA^2$$

- At the equilibrium position, $v = v_{max}$ and $x = 0$:

$$\Longrightarrow E = KE = \frac{1}{2}mv_{max}^2$$

- Energy is conserved:

$$\Longrightarrow \frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2 \Longrightarrow v_{max} = \sqrt{\frac{k}{m}}A = 2\,m/s$$

A. 1 m/s

B. 2 m/s ✓

C. 3 m/s

D. 4 m/s

E. 5 m/s

Have a good reading break!