WELCOME TO OUR INAUGURAL UNDERGRAD SCIENCE SLAM!

The Greatest Science Communication Competition...Ever!

PHAS Undergrad Slammers will explain complex science topics WITHOUT Academic slides or language....in 5 minutes! Can they do it????

★ Cheer on our Slammers!

Tuesday March 12th, 5:30-7:30pm in HENN 200

Email: outreach@phas.ubc.ca

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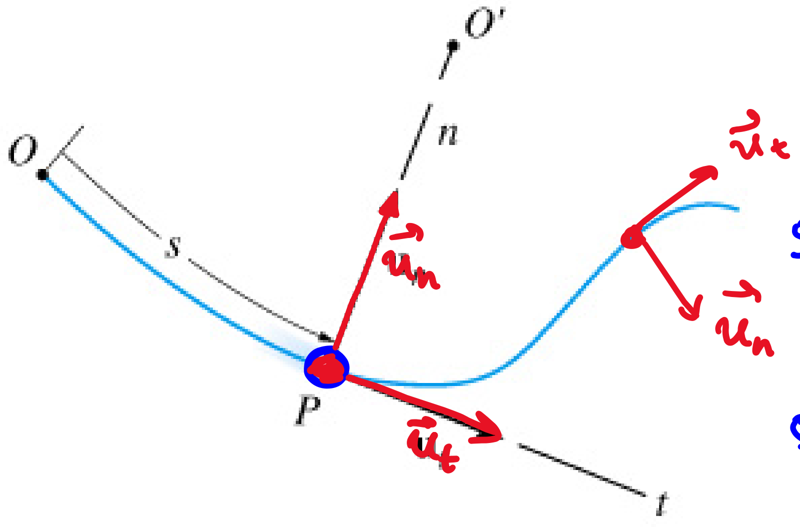
Science Slam

PHYSICS ASTRONOMY

UBC

ACCELERATION



Q: What can you say about acceleration in normal & tangential components? Consider a general situation.

50% A. I remember that acceleration always points inwards => it only has a normal component and no tangential component.

5% B. It only has a tangential component, since $\vec{a} = \frac{d\vec{v}}{dt}$, and $\vec{v}$ only has a tangential component.

41 % C. Acceleration has both normal and tangential components

D. One cannot define acceleration in this coordinate system

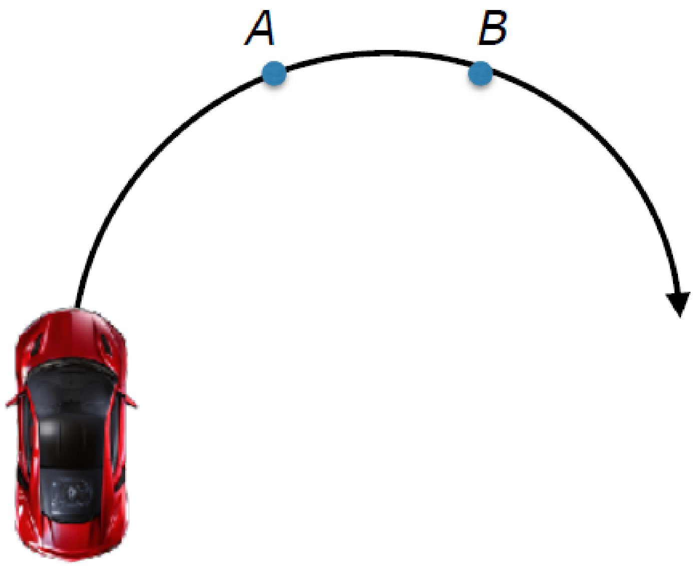
E. Not sure

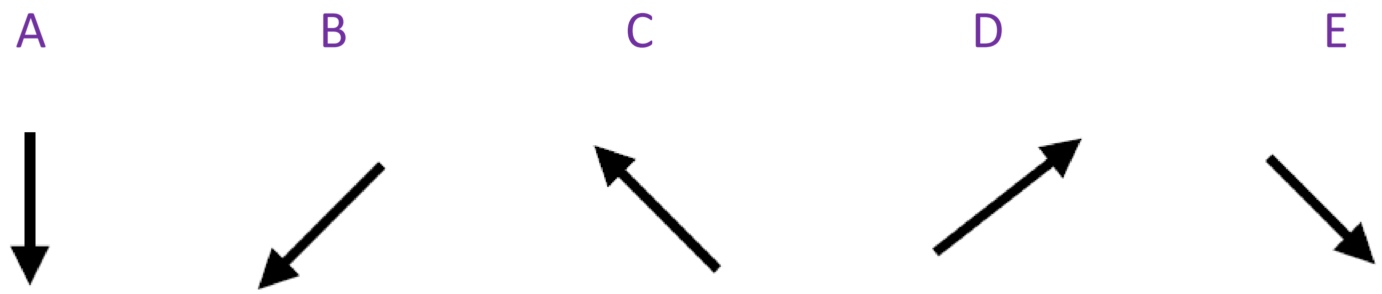
$$\boxed{\vec{v} = \frac{ds}{dt}\,\vec{u}_t}$$

ACCELERATION

Q: **a)** A car turns a corner keeping the same speed following a circular trajectory. Which vector best describes the average acceleration from point A to point B?

b) What if the car's speed increases from point A to point B?





ACCELERATION

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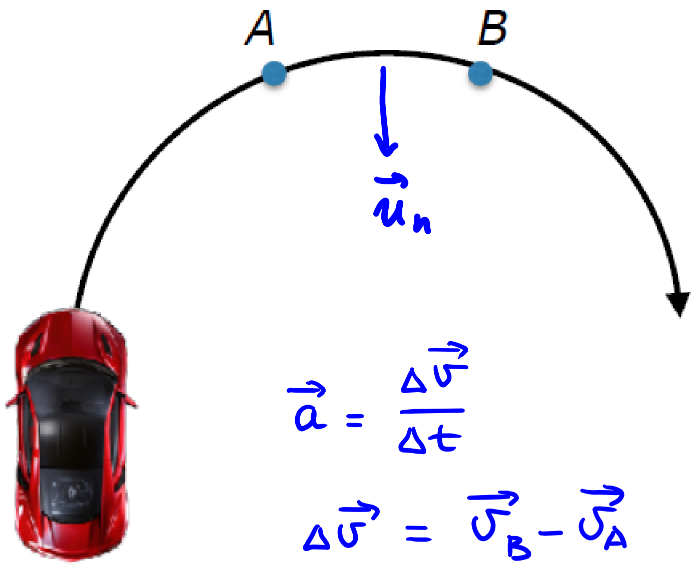
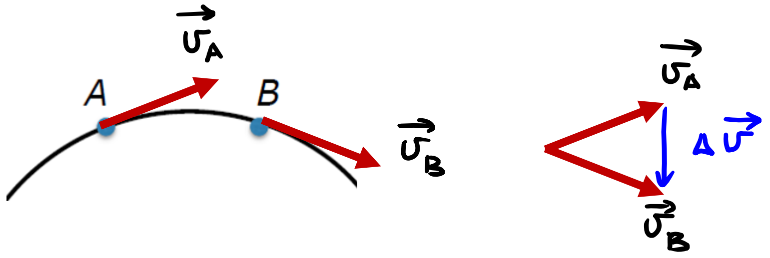


$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

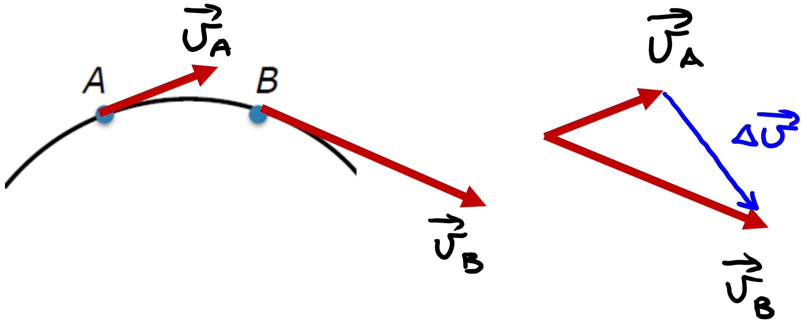
$$\Delta\vec{v} = \vec{v}_B - \vec{v}_A$$

a)				b)
A	B	C	D	E

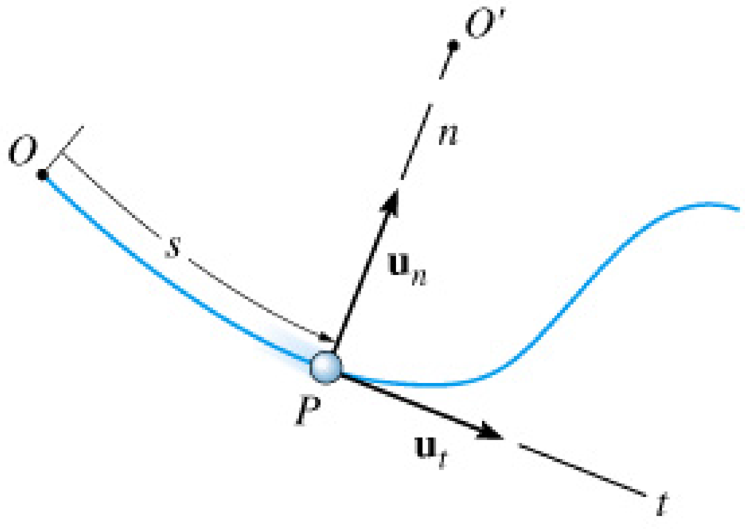
a)



b)



TANGENTIAL AND NORMAL ACCELERATION



Consider an object moving along a curved path with velocity

$$\vec{v} = v\,\vec{u}_t$$

Then the acceleration is:

speed

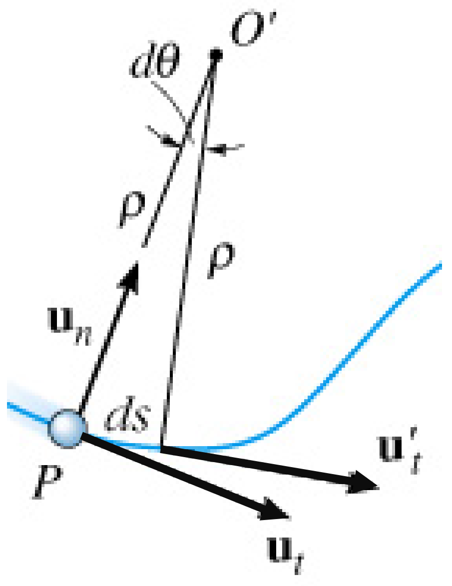
direction of motion

$$\boxed{\vec{a} = \frac{d\vec{v}}{dt} = \frac{d(v\,\vec{u}_t)}{dt} = \left(\frac{dv}{dt}\right)\vec{u}_t + v\left(\frac{d\vec{u}_t}{dt}\right)}$$

This term is acceleration
due to going faster

This term is acceleration
due to turning
(unit vector $\vec{u}_t$ does not change length, only direction)

TANGENTIAL AND NORMAL ACCELERATION: Derivation



$$\vec{a} = \frac{dv}{dt}\vec{u}_t + v\frac{d\vec{u}_t}{dt}, \qquad \frac{d\vec{u}_t}{dt} = ?$$

$\hookrightarrow \perp \vec{u}_t \rightarrow \parallel \vec{u}_n$

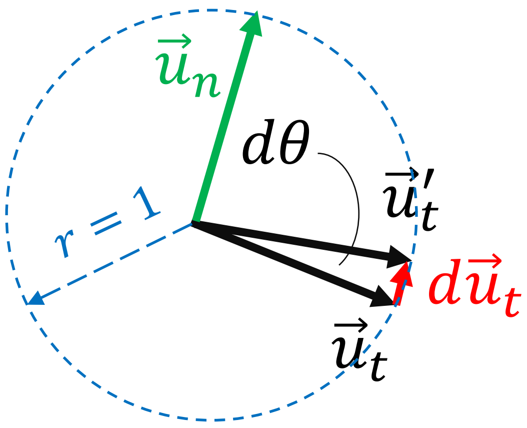
Geometry:

- The angle $d\theta$ between $\vec{u}_t$ and $\vec{u}_t'$ is the same as the angle $d\theta$ between two radius-vectors from the center of curvature O' drawn to these two locations

- Direction of $d\vec{u}_t$: parallel to $\vec{u}_n$.

- Magnitude: $du_t = 1 \cdot \sin d\theta = d\theta$

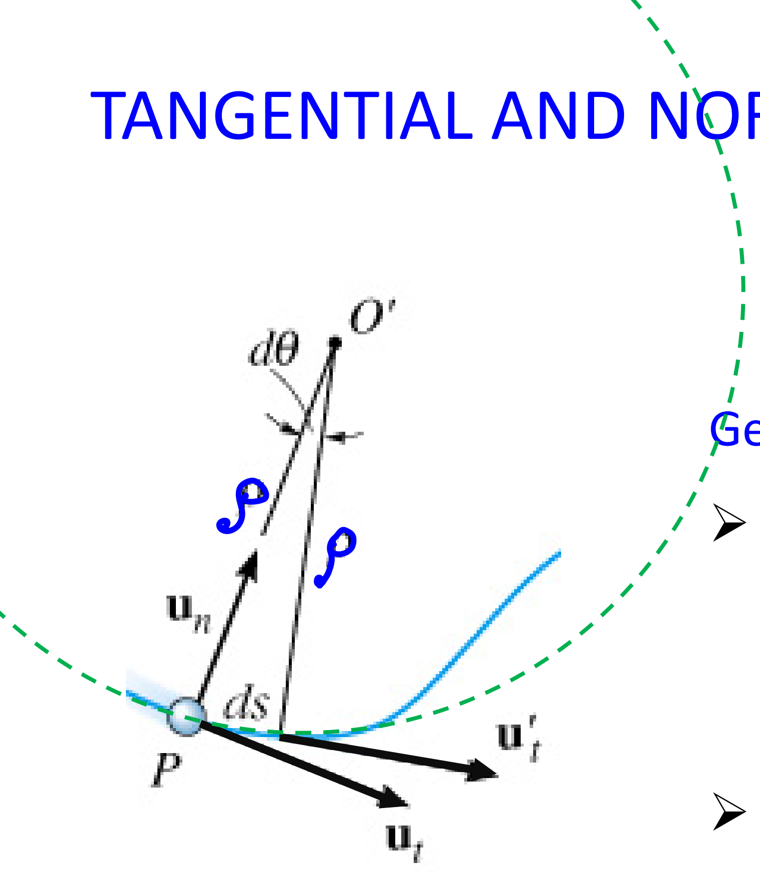
Then: $d\vec{u}_t = d\theta\, \vec{u}_n$



$$\boxed{\vec{a} = \frac{dv}{dt}\vec{u}_t + v\frac{d\vec{u}_t}{dt} = \overbrace{\left(\frac{dv}{dt}\right)}^{a_t}\vec{u}_t + \overbrace{\left(v\frac{d\theta}{dt}\right)}^{a_n}\vec{u}_n}$$

- We now need to relate $\dot{\theta}$ with v and ρ.

TANGENTIAL AND NORMAL ACCELERATION: Derivation



$$\vec{a} = \frac{dv}{dt}\vec{u}_t + v\frac{d\theta}{dt}\,\vec{u}_n, \qquad \frac{d\theta}{dt} = ?$$

Geometry:

- Connection between the arc ds, curvature radius ρ and the angle $d\theta$:

$$\frac{d\theta}{2\pi} = \frac{ds}{2\pi\rho} \qquad d\theta = \frac{ds}{\rho}$$

- Now:

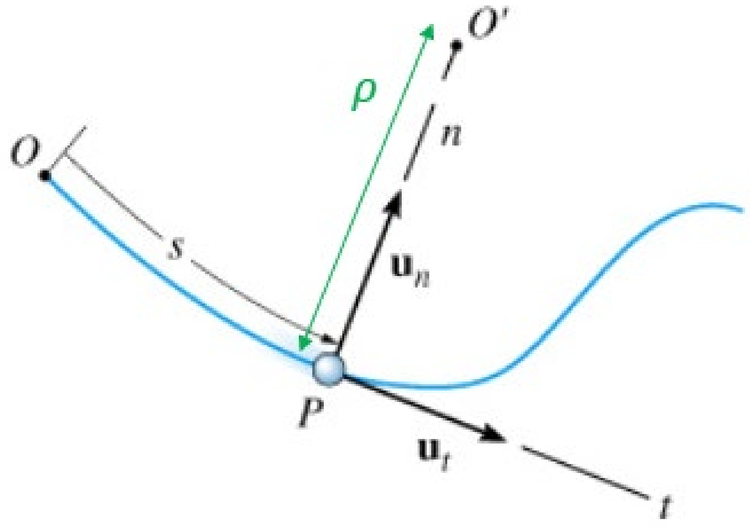
$$\frac{d\theta}{dt} = \frac{1}{\rho}\left(\frac{ds}{dt}\right) = \frac{v}{\rho}$$

- Finally:

$$\boxed{v\frac{d\theta}{dt} = \frac{v^2}{\rho} \equiv \ a_n}$$

(n, t)-coordinates: Position, Velocity, Acceleration (summary)

- Position:
  - There is no position vector (the object “carries” the coordinate system)
  - Instead, there is coordinate s (the distance travelled along the trajectory)



- Velocity:
  - Tangential component only

$$\vec{v} = \frac{ds}{dt}\,\vec{u}_t$$

- Acceleration:
  - Tangential and normal components

$$\vec{a} = a_t\,\vec{u}_t + a_n\,\vec{u}_n$$

$$a_t = \dot{v} = \ddot{s}, \qquad a_n = \frac{v^2}{\rho}$$

$$a = \sqrt{a_t^2 + a_n^2}$$

Limiting Cases:

$$\vec{a} = a_t\,\vec{u}_t + a_n\,\vec{u}_n$$

$$a_t = \dot{v} = \ddot{s}, \qquad a_n = \frac{v^2}{\rho}$$

- Linear Motion:
  - $\rho \to \infty$ (infinite radius of curvature)
  - Then $a_n \to 0$, and all the tangential components describe just a usual 1D motion, with $a = a_t = \dot{v} = \ddot{s}$

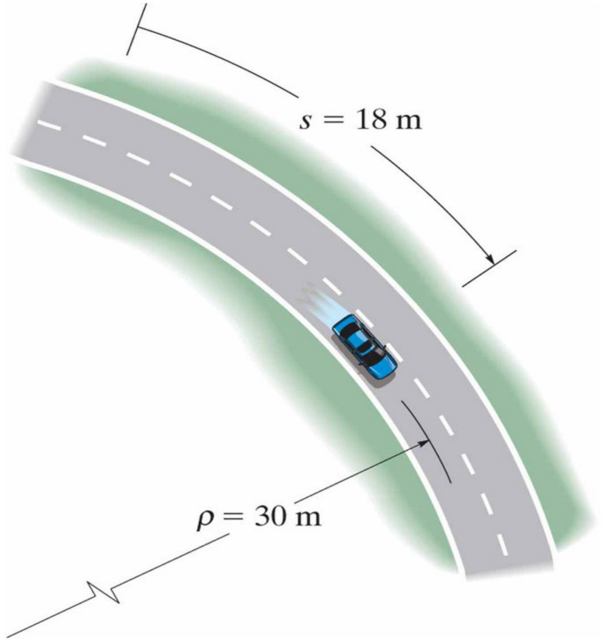
- Motion at constant speed:
  - $\dot{v} = 0$ (derivative of a const is zero)
  - Then $a_t \to 0$, and net acceleration becomes: $a = a_n = \frac{v^2}{\rho}$
  - Since it always acts towards the center of curvature, this component is sometimes called the *centripetal acceleration*

$a = a(t)$

W7-4. The car travels along the circular path. Starting from rest, its acceleration along the path is $\boxed{0.5\, e^t \text{ m/s}^2}$ where t is in seconds.

a) Determine how long it takes the car to travel 18 m.

b) Determine the car's speed and acceleration at this time.



W7-4. The car travels along the circular path. Starting from rest, its acceleration along the path is $0.5\ e^{t}$ m/s² where t is in seconds.

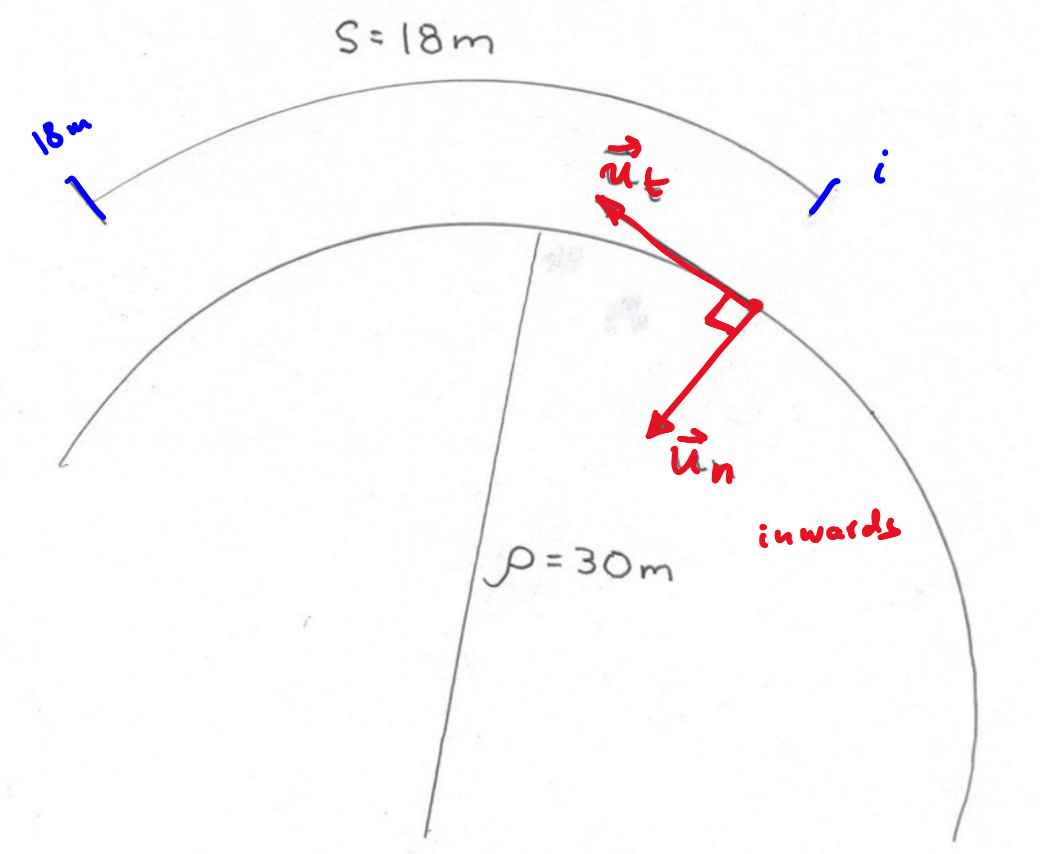
a) Determine how long it takes the car to travel 18 m.

b) Determine the car's speed and acceleration at this time.

- Plan?

$t: \quad s(t) = 18\,m$

$s(t) \xleftarrow{\int} v(t) \xleftarrow{\int} a(t)$



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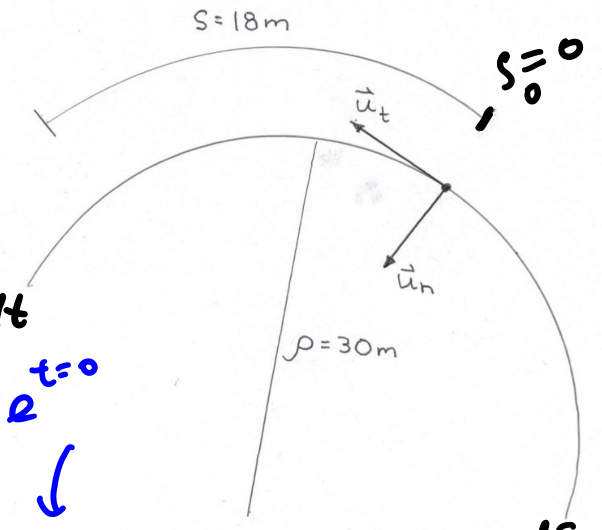


a) Determine how long it takes the car to travel 18 m.

b) Determine the car's speed and acceleration at this time.

$$a=\frac{dv}{dt} \longrightarrow dv = a\,dt$$

- Data: $a = a(t) = 0.5e^{t}$; $v_0 = 0$; t: $s(t) = 18\,m.$

$$v=\frac{ds}{dt} \qquad ds = v\,dt$$

$v(t)$: $$v(t) = \underset{0}{\underset{\shortparallel}{v_0}} + \int_{t=0}^{t=t} a(t)\,dt = \int_{t=0}^{t=t} 0.5e^{t}\,dt = 0.5\,e^{t}\Big|_{t=0}^{t=1} = 0.5\,[e^{t}-1]$$

$e^{t=0}$

$s(t)$: $$s(t) = \underset{0}{\underset{\shortparallel}{S_0}} + \int_{t=0}^{t=t} v(t)\,dt = \int_{t=0}^{t=t} 0.5\,[e^{t}-1]\,dt = 0.5\left[\int_0^t e^{t}dt - \int_0^t dt\right] =$$

$$= 0.5\left[e^{t}\Big|_0^t - t\Big|_0^t\right] = 0.5\,[e^{t}-1-t] = 18\,m$$

$$\boxed{t = 3.70538}$$

A. $t = 2.52$ s

B. $t = 3.71$ s

C. $t = 4.23$ s

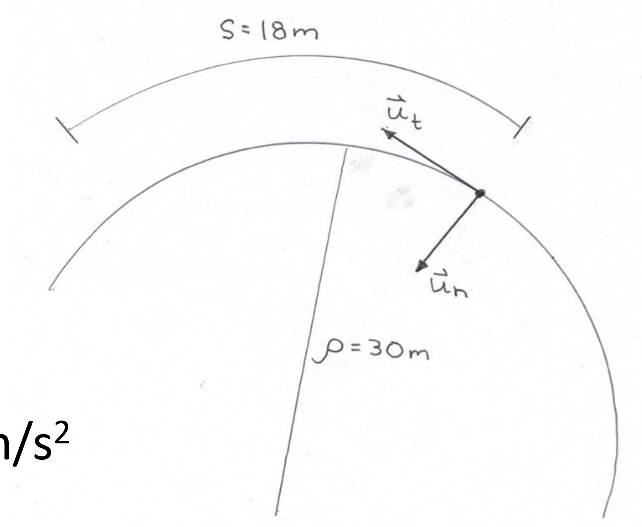
D. $t = 5.04$ s

E. I am lost

W7-4. The car travels along the circular path. Starting from rest, its acceleration along the path is $0.5\ e^t$ m/s² where t is in seconds.

a) Determine how long it takes the car to travel 18 m.

b) Determine the car's speed and acceleration at this time.



$s(t) = 0.5(e^t - t - 1)$ $\quad v(t) = 0.5(e^t - 1)$ m/s $\quad a_t(t) = 0.5\ e^t$ m/s²

3.70638

$t = \cancel{3.70638}$ s

$s(t) = 18m$

$v(t) = 19.9\ m/s$

$$a_t(t) = 0.5\ e^{3.70638} = 20.35\ \frac{m}{s^2}$$

$$a_n(t) = \frac{v^2}{\rho} = \frac{(19.9)^2}{30} = 13.14\ \frac{m}{s^2}$$

$$a = \sqrt{a_n^2 + a_t^2} = 24.2\ \frac{m}{s^2}$$

A. 13.2 m/s^2

B. $20.4\ m/s^2$

C. $24.2\ m/s^2$

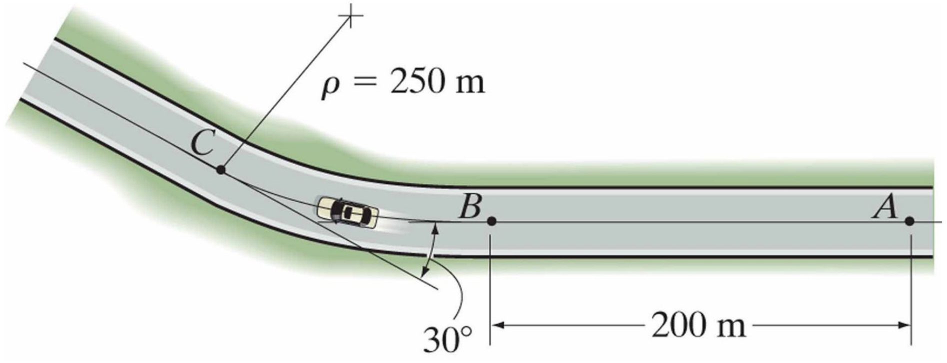
D. Something else

a_t

W7-5. The car is travelling at 25 m/s at A. The brakes are applied at A and its speed is reduced by $\boxed{t^{1/2}/4}$ m/s^2 where t is in seconds.

a) Determine how long it takes the car to travel from A to C.

b) Determine the car's speed and acceleration when it reaches C.



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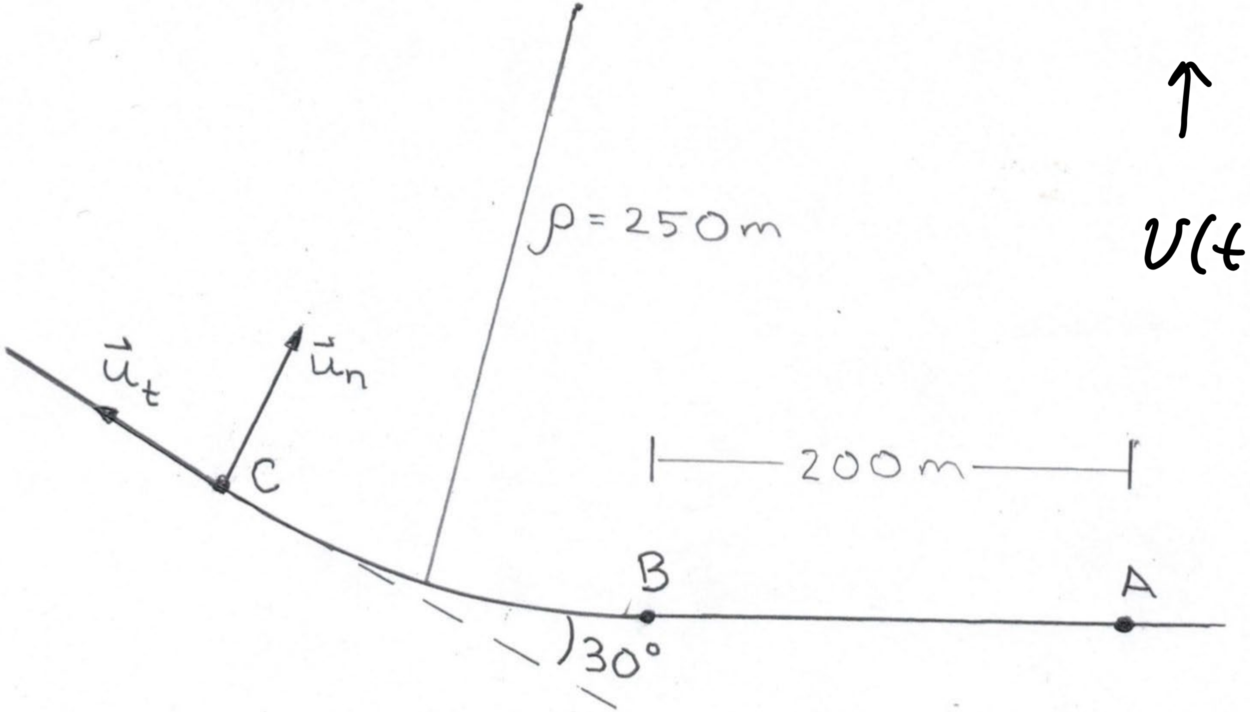
- Plan:



$$S(t) = \overset{200}{AB} + BC$$

$$\uparrow$$

$$v(t) \longleftarrow \underset{t}{a(t)}$$

W7-5. The car is travelling at 25 m/s at A. The brakes are applied at and its speed is reduced by $t^{1/2}/4$ m/s² where t is in seconds.

a) Determine how long it takes the car to travel from A to C.

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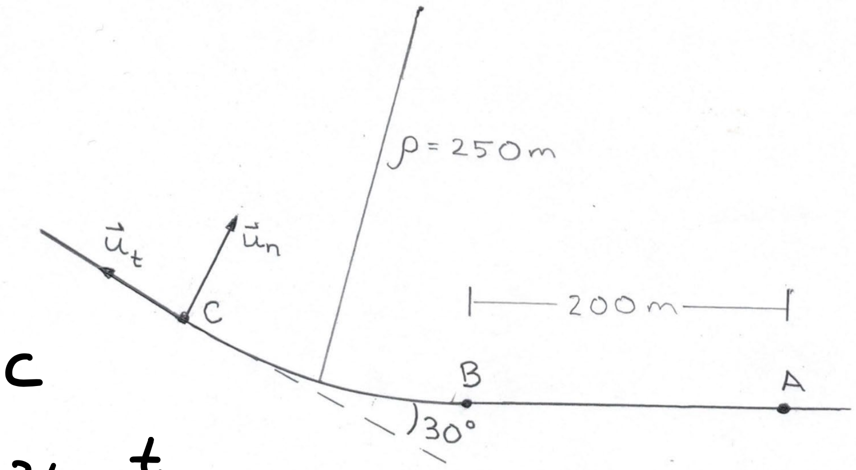


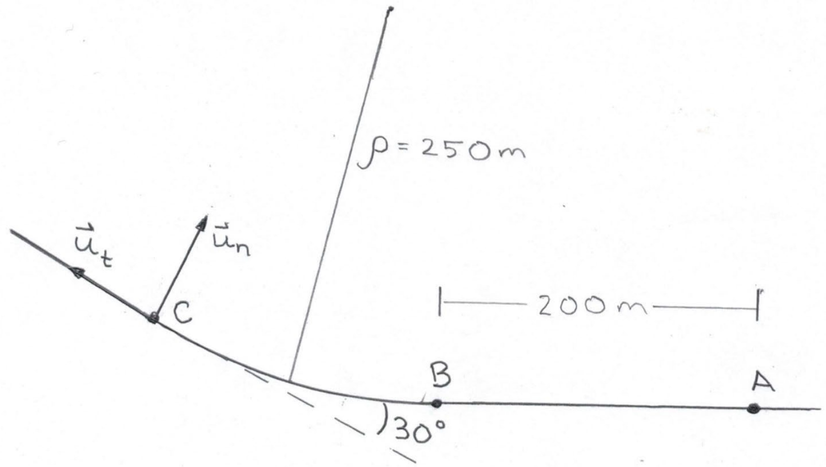
- Data: $v_A = 25\,\frac{m}{s}$ $\quad a_t = -\frac{1}{4}t^{1/2}$ $\quad t: \; s(t) = AB + BC$

$$v(t) = v_A + \int_{t=0}^{t} a(t)\,dt = 25 - \int_0^t \frac{1}{4}t^{1/2} = 25 - \frac{1}{4}\frac{t^{3/2}}{3/2}\Bigg|_0^t = 25 - \frac{1}{6}t^{3/2}$$

$$s(t) = \overset{0}{s_0} + \int_{t=0}^{t=t} v(t)\,dt = \int_0^t \left[25 - \frac{t^{3/2}}{6}\right]dt = 25\cdot t\Big|_0^t - \frac{1}{6}\frac{t^{5/2}}{5/2} = 25t - \frac{t^{5/2}}{15}$$

$$25t - \frac{t^{5/2}}{15} = AC + BC = 200 + \overset{?}{BC}$$

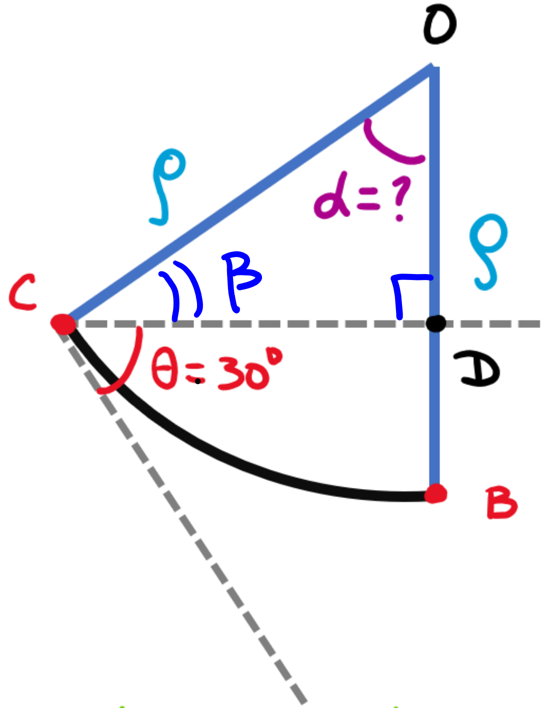
W7-5. The car is travelling at 25 m/s at A. The brakes are applied at and its speed is reduced by $t^{1/2}/4$ m/s^2 where t is in seconds.



a) Determine how long it takes the car to travel from A to C.

b) Determine the car's speed and acceleration when it reaches C.

$$s(t) = \left(25t - \frac{1}{15} * t^{\frac{5}{2}}\right)$$



$$\left.\begin{aligned} \beta + 30^\circ &= 90^\circ \\ \beta + \alpha &= 90^\circ \end{aligned}\right\}$$

$$\alpha = 30^\circ$$

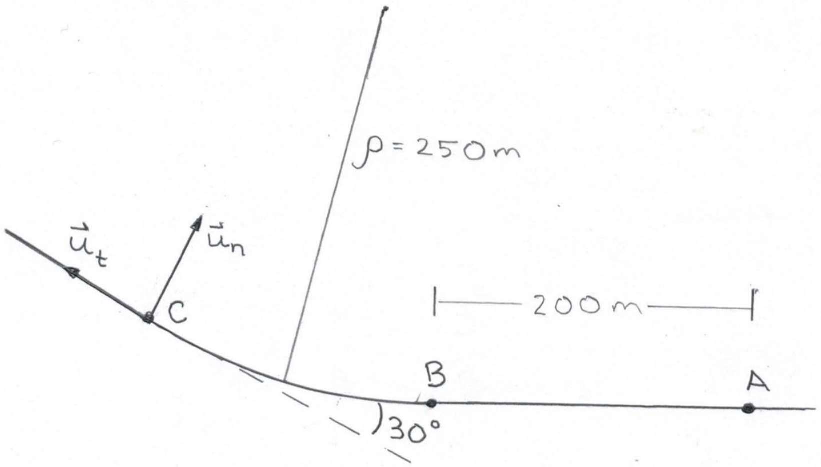
$$\frac{30^\circ}{360^\circ} = \frac{BC}{2\pi\rho} \rightarrow BC = \frac{\pi}{6}\rho =$$

130.9 m

W7-5. The car is travelling at 25 m/s at A. The brakes are applied at and its speed is reduced by $t^{1/2}/4$ m/s² where t is in seconds.

a) Determine how long it takes the car to travel from A to C.

b) Determine the car's speed and acceleration when it reaches C.



$$s(t) = \left(25t - \frac{1}{15} \cdot t^{\frac{5}{2}}\right) = 200 + 130.9 \text{ m}$$

$t = 15.942$ s

$t = 39.674$ s

$$v(t) = 25 - \frac{1}{6} t^{3/2}$$

$$a_t(t) = -\frac{1}{4}\sqrt{t}$$