PHYS 170

Week 9: Kinetics: Force and Acceleration

Section 201 (Mon Wed Fri 12:00 – 13:00)

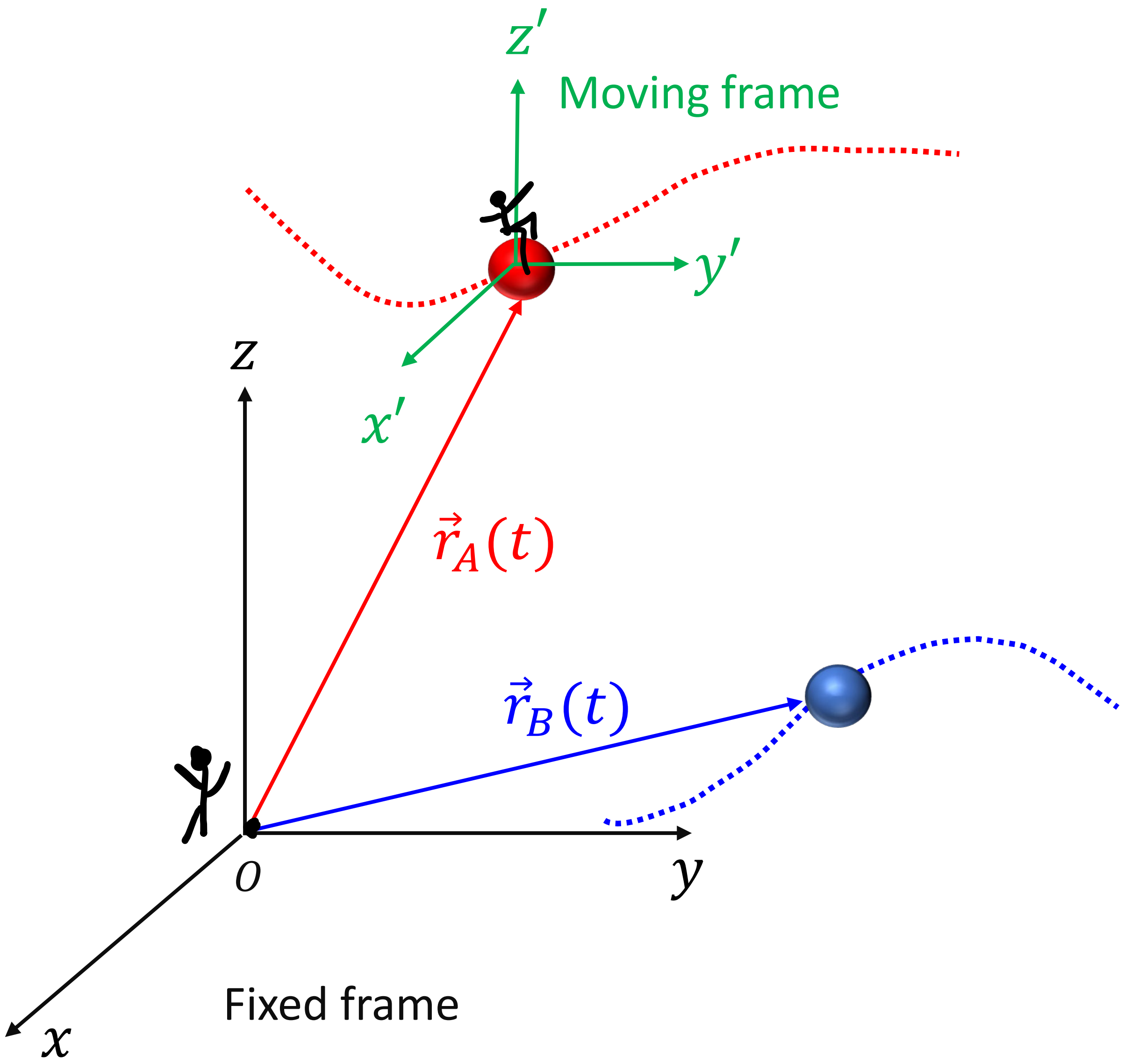
Relative Motion

Text: 12.10

Content:

- Translating (but not rotating!) moving coordinate systems
- Relative velocity, relative acceleration

RELATIVE MOTION



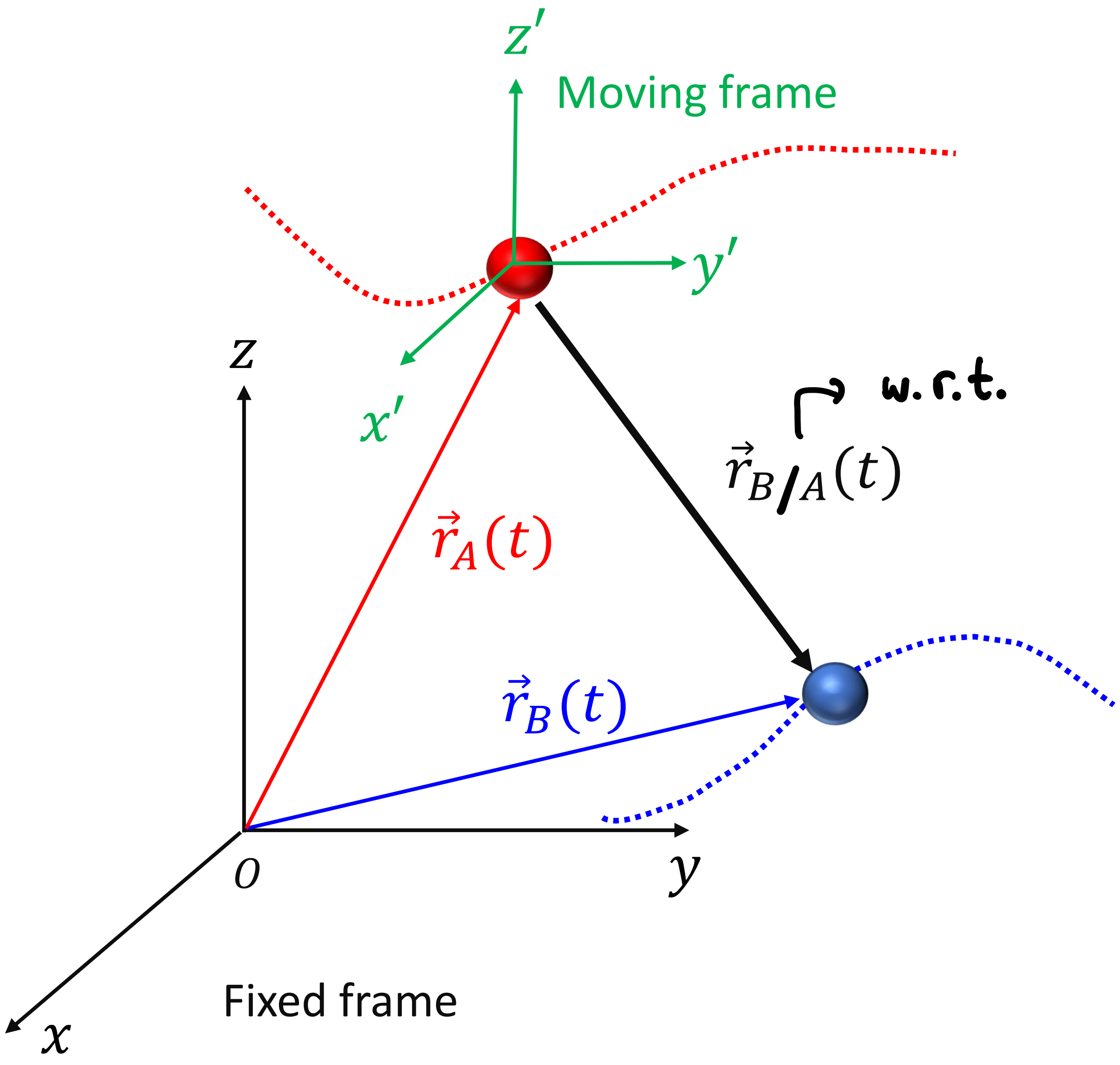
- Velocity and acceleration of both particles in the fixed frame, O:

$$\vec{v}_A = \dot{\vec{r}}_A \qquad \vec{v}_B = \dot{\vec{r}}_B$$

$$\vec{a}_A = \dot{\vec{v}}_A \qquad \vec{a}_B = \dot{\vec{v}}_B$$

- How can we describe motion of particle B in the frame moving together with particle A (which, in the fixed frame, moves with velocity $\vec{v}_A$ and acceleration $\vec{a}_A$)?

RELATIVE MOTION



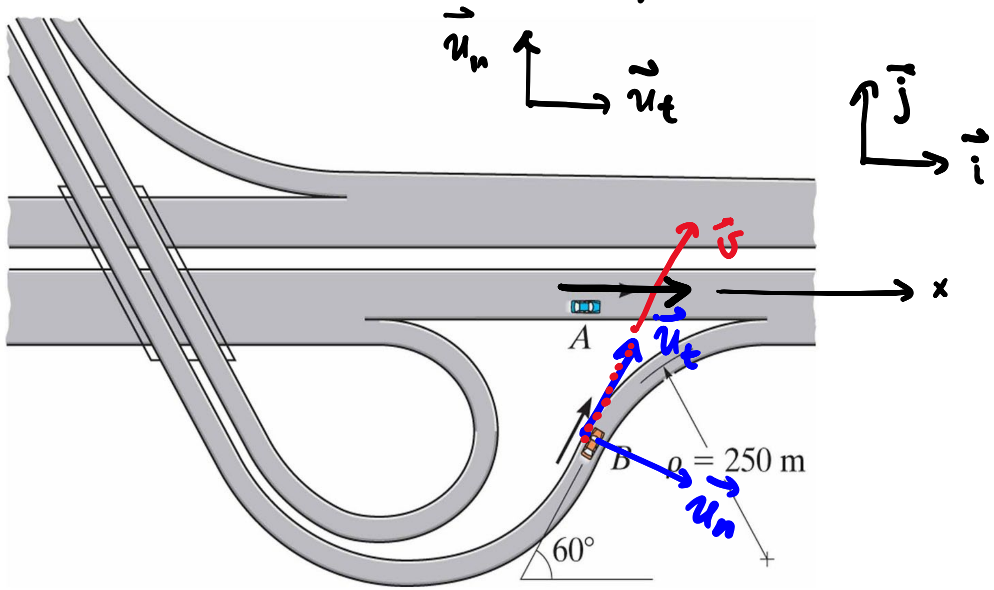
- Position of B relative to A is described by the position vector $\vec{r}_{B/A}(t)$
- Then: $\vec{v}_{B/A} = \dot{\vec{r}}_{B/A}$, $\vec{a}_{B/A} = \dot{\vec{v}}_{B/A}$
- Note that: $\vec{r}_{B/A} = \vec{r}_B - \vec{r}_A$
- Differentiating this equality with respect to time we find:

$$\boxed{\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A}$$

$$\boxed{\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A}$$

W8-5. At the instant shown, car A travels east along the highway at 30 m/s and accelerates at 2 m/s². At the same instant, car B travels on the interchange curve at 15 m/s and decelerates at 0.8 m/s². Determine the velocity and acceleration of B relative to A at this instant.

$\vec{v}_{B/A}$, $\vec{a}_{B/A}$



Which coordinate system will you use for this problem? Explain.

A. Cartesian for both cars.

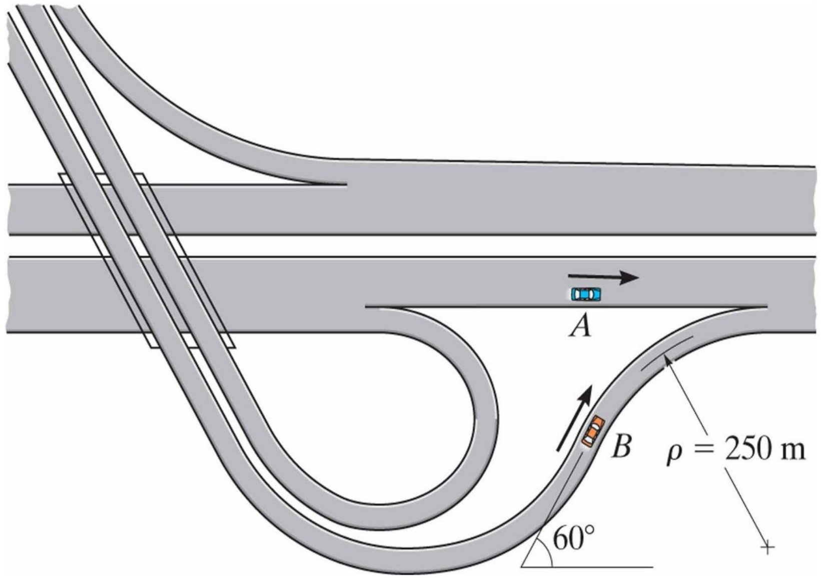
→ B. (n,t)-coordinates for both cars.

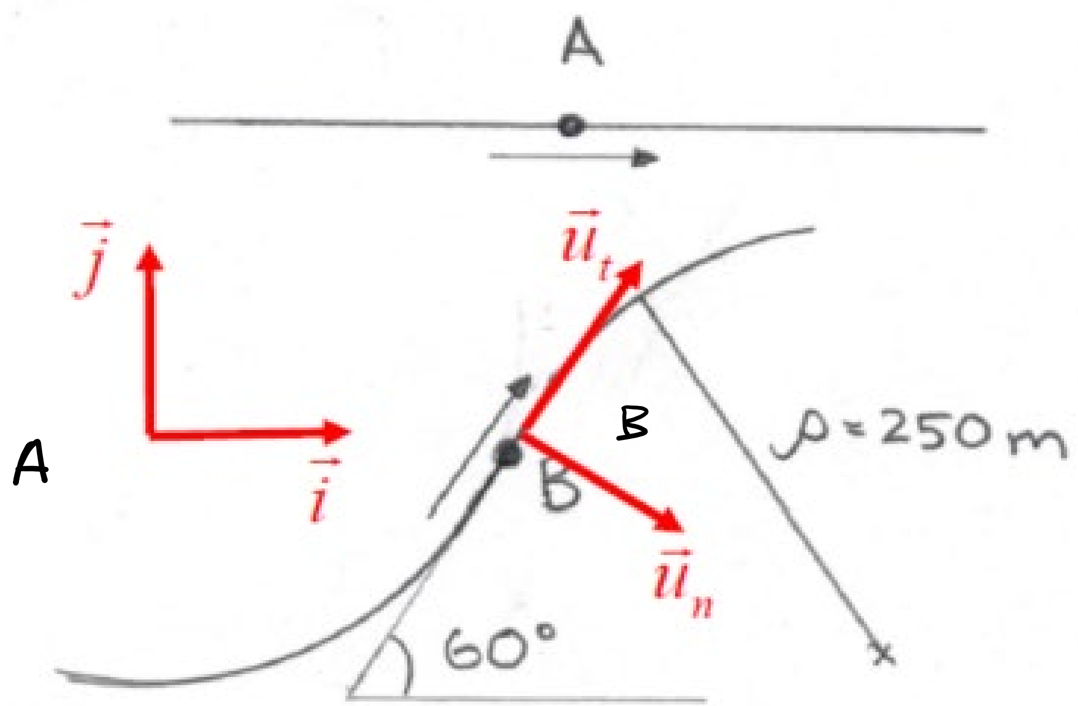
~~C.~~ Polar for both cars.

→ D. A mixture of two of the above.

E. It actually does not matter.

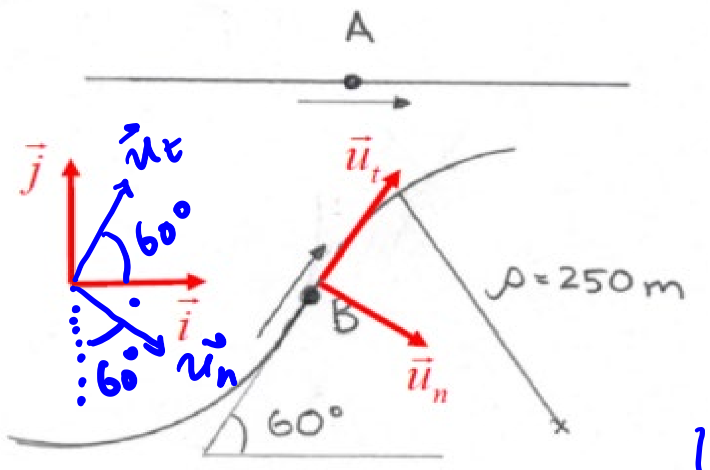
W8-5. At the instant shown, car A travels east along the highway at 30 m/s and accelerates at 2 m/s^2. At the same instant, car B travels on the interchange curve at 15 m/s and decelerates at 0.8 m/s^2. Determine the velocity and acceleration of B relative to A at this instant.





W8-5. At the instant shown, car A travels east along the highway at 30 m/s and accelerates at 2 m/s². At the same instant, car B travels on the interchange curve at 15 m/s and decelerates at 0.8 m/s². Determine the velocity and acceleration of B relative to A at this instant.

$\boxed{\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A}$ $\boxed{\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A}$



- $\vec{v}_A, \vec{a}_A$: $\vec{v}_A = (30)\vec{i}$ $\qquad \vec{a}_A = (+2)\vec{i}$

speedin/slowing $\qquad$ turning $\quad \frac{v^2}{\rho}$

- $\vec{v}_B, \vec{a}_B$: $\vec{v}_B = (15)\vec{u}_t$ $\qquad \vec{a}_B = (-0.8)\vec{u}_t + \left(\frac{15^2}{250}\right)\vec{u}_n$

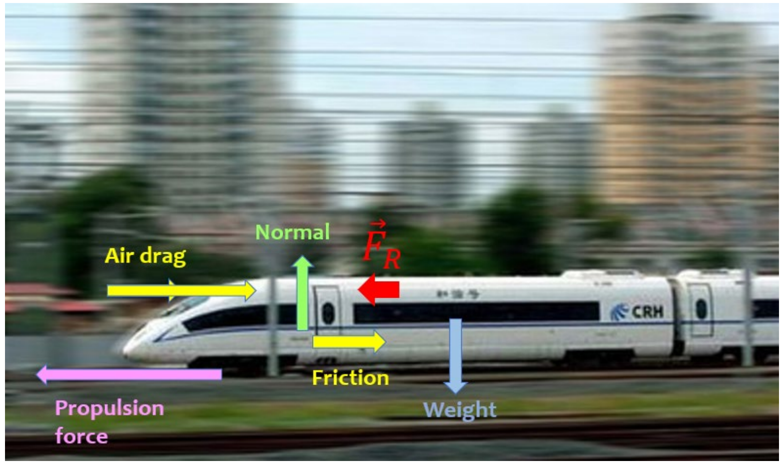
$$\vec{u}_t = (u_t \cos 60°)\vec{i} + (u_t \sin 60°)\vec{j} = \vec{i}\cos 60° + \vec{j}\sin 60°$$

$$\vec{u}_n = (u_n \sin 60°)\vec{i} - (u_n \cos 60°)\vec{j} = \vec{i}\sin 60° - \vec{j}\cos 60°$$

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A = (15)\left[\vec{i}\cos 60° + \vec{j}\sin 60°\right] - (30)\vec{i} = (-22.5)\vec{i} + (13.0)\vec{j} \text{ m/s}$$

$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A = \left\{(-0.8)\left[\vec{i}\cos 60° + \vec{j}\sin 60°\right] + \left(\frac{15^2}{250}\right)\left[\vec{i}\sin 60° - \vec{j}\cos 60°\right]\right\} - (+2)\vec{i} = \left[-1.62\,\vec{i} - 1.14\,\vec{j}\right] \text{ m/s}^2$$

Kinetics: Intro



Text: 13.1-13.3

Content:

- Mass / weight
- Inertia
- Inertial coordinate systems
- Second Newton's law: $\vec{F}_R = m\vec{a}$

FUNDAMENTAL LAWS

- Newton's 1st law:

A particle which is originally at rest or is moving in a straight line with a constant velocity, will remain in this state provided the particle is not subjected to unbalanced forces (or *motion with a constant velocity along a straight line is a natural state and it does NOT require a constant force to maintain this velocity* – very counter-intuitive, since we live in the world where we always cause motion by applying a force to compensate for friction/drag)

✓

- Newton's 2nd law:

A particle acted upon by an unbalanced force experiences an acceleration in the same direction as the net force with a magnitude proportional to the force (or $\boxed{\vec{F}_R = m\vec{a}}$)

✓

- Newton's 3rd law:

The mutual forces of "action" and "reaction" between particles are equal and opposite (or "all forces appear in pairs", or *you cannot touch without being touched*).

✓

- Newton's law of gravitation:

Gravity force due to two masses (magnitude): $F_G = G\,\frac{m_1 m_2}{r^2}$ ⇒ on the Earth: $\boxed{W = mg}$

Please read:

- Section 13.1

Mass (m) vs Weight (W):

- Mass:
  - Internal property of each object
  - Units: kg (SI) / slugs (FPS)

- Weight:
  - Gravity **force** acting on the object
  - Units: N (SI) / lb (FPS)

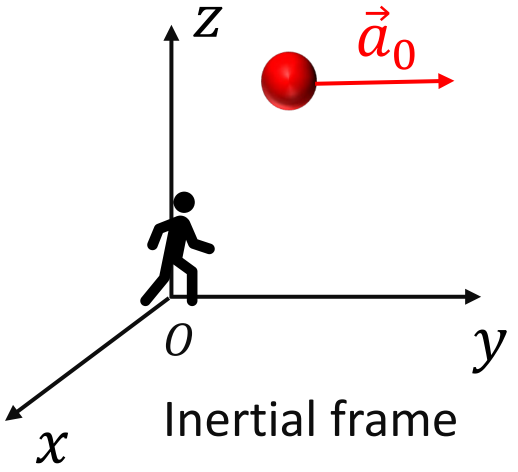
$$\boxed{m = \frac{W}{g}}$$

$$\boxed{W = mg}$$

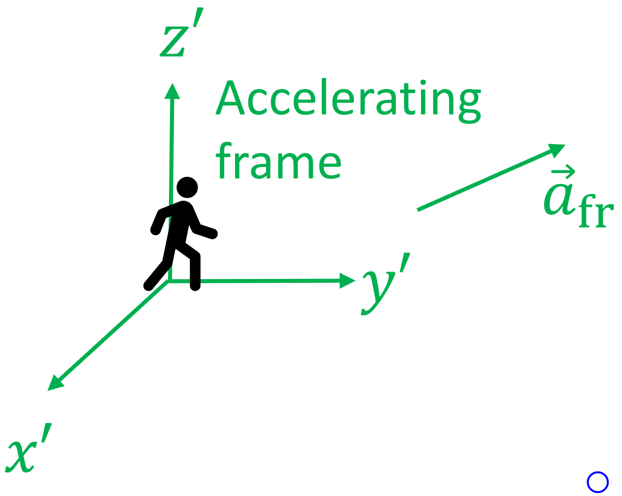
- g = 9.81 m/s^2 = 32.2 ft/s^2
- Units: kg (SI) / slugs (FPS)
- 1 slug $= \dfrac{\text{1 lb}}{\text{32.2 ft/s}^2}$

Please have a look:

- Section 13.2
    - Inertia: tendency of a massive object to resist change in its velocity
    - Inertial reference frame (= coordinate system): It is not accelerating (which also means that it is not rotating)
        - If a particle's acceleration in some inertial frame is $\vec{a}_0$, and another system moves with acceleration $\vec{a}_{\mathrm{fr}}$ with respect to that system, the particle's acceleration in the second system will be $\vec{a}_{0/\mathrm{fr}} = \vec{a}_0 - \vec{a}_{\mathrm{fr}}$
        - Internal coordinate systems: observers will agree on acceleration
        - Observer in non-inertial coordinate system: will measure a *different* acceleration



Inertial frame



- The equation of motion $\vec{F}_R = m\vec{a}$ will look different for these two observers!

Optional:

- Section 13.3
  - ➢ Equation of motion for a system of particles
  - ➢ You can skip it since we did not discuss the concept of center of mass
  - ➢ …but it is nice and is worth reading if you have time

FUTURE PLANS

particles!

- Kinematic characteristics (velocity, position)

$$\vec{F}_R = m\vec{a}$$

- Free-body diagrams
- 3rd Newton's law pairs
- Kinetic friction
- .....

- Cartesian coordinates!
- Tangential-normal coordinates!
- Polar coordinates!

- Dependent motion
- Relative motion
- .....

Equation of motion: Examples

Text: 13.4-13.5

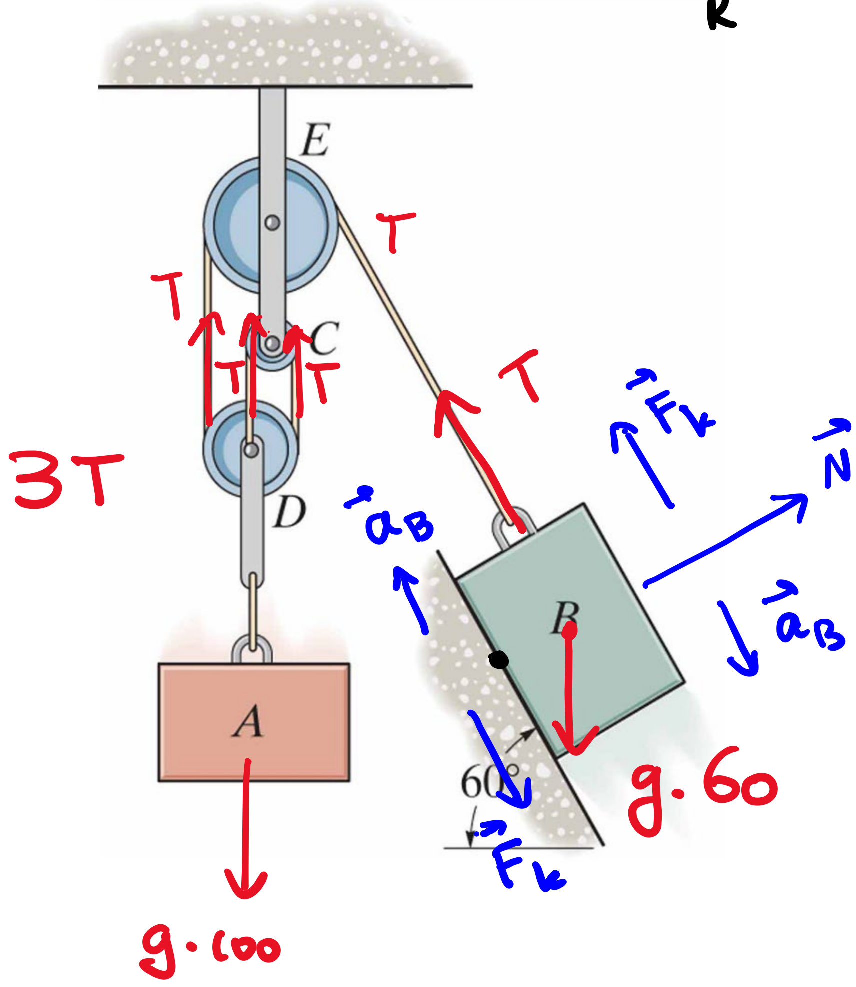
Content:

- Equation of motion in Cartesian components
- Equation of motion in tangential-normal components

W9-1. The mass of block A is 100 kg. The mass of block B is 60 kg. The coefficient of kinetic friction between block B and the inclined plane is 0.4. A and B are released from rest. Determine the acceleration of block A and the tension in the cord. Neglect the mass of the pulleys and the cord.

$$\vec{F}_R = m\vec{a}$$

$$\boxed{F_k = \mu_k N}$$



Direction of accelerations?

A. a_A up, a_B up

B. a_A down, a_B down

C. a_A up, a_B down

? D. a_A down, a_B up

~~E.~~ It does not really matter. Let's chose them somehow and correct after we know the sign of the answer

$F_k < 0 \longrightarrow N < 0$!

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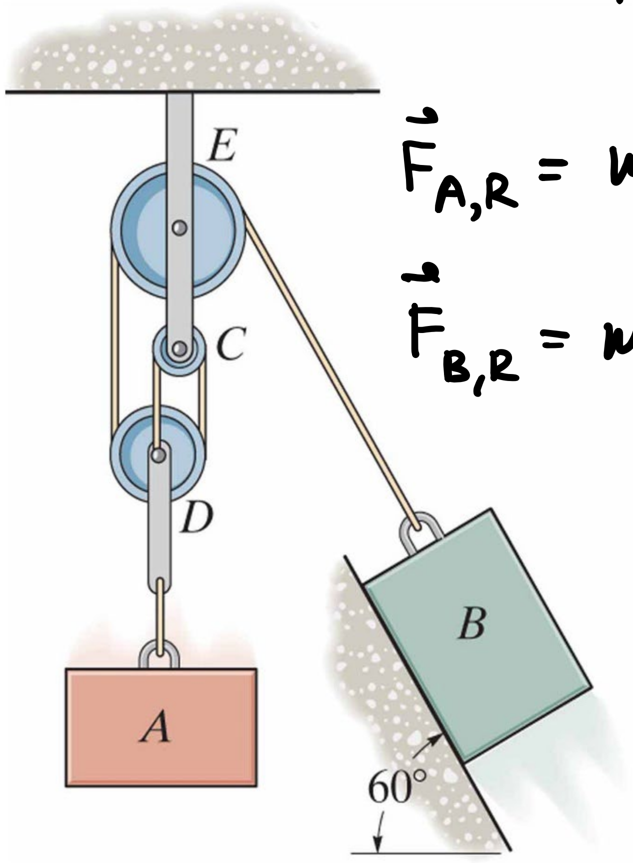
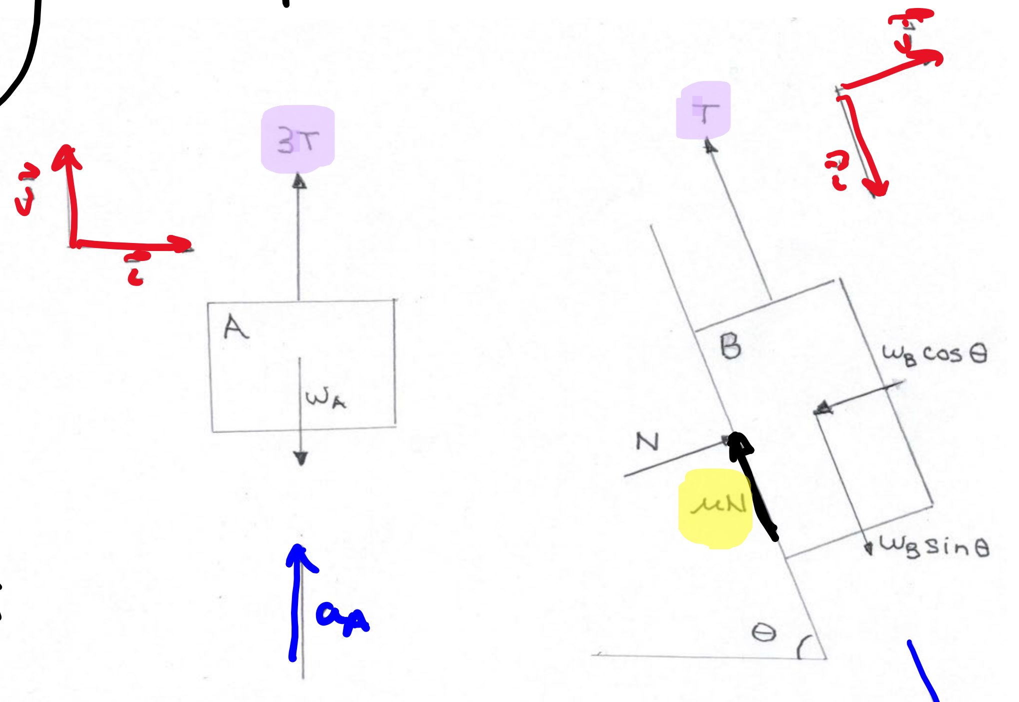
$$\vec{F}_R = m\vec{a} \quad \rightarrow \quad 2\text{ eq}$$



$$\vec{F}_{A,R} = m_A \vec{a}_A$$

$$\vec{F}_{B,R} = m_B \vec{a}_B$$

2 eq



dependent motion : a_A, a_B