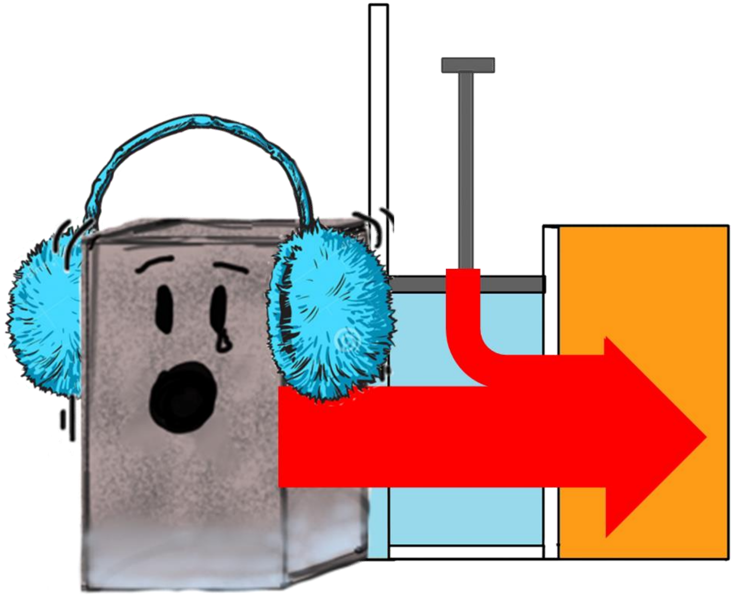
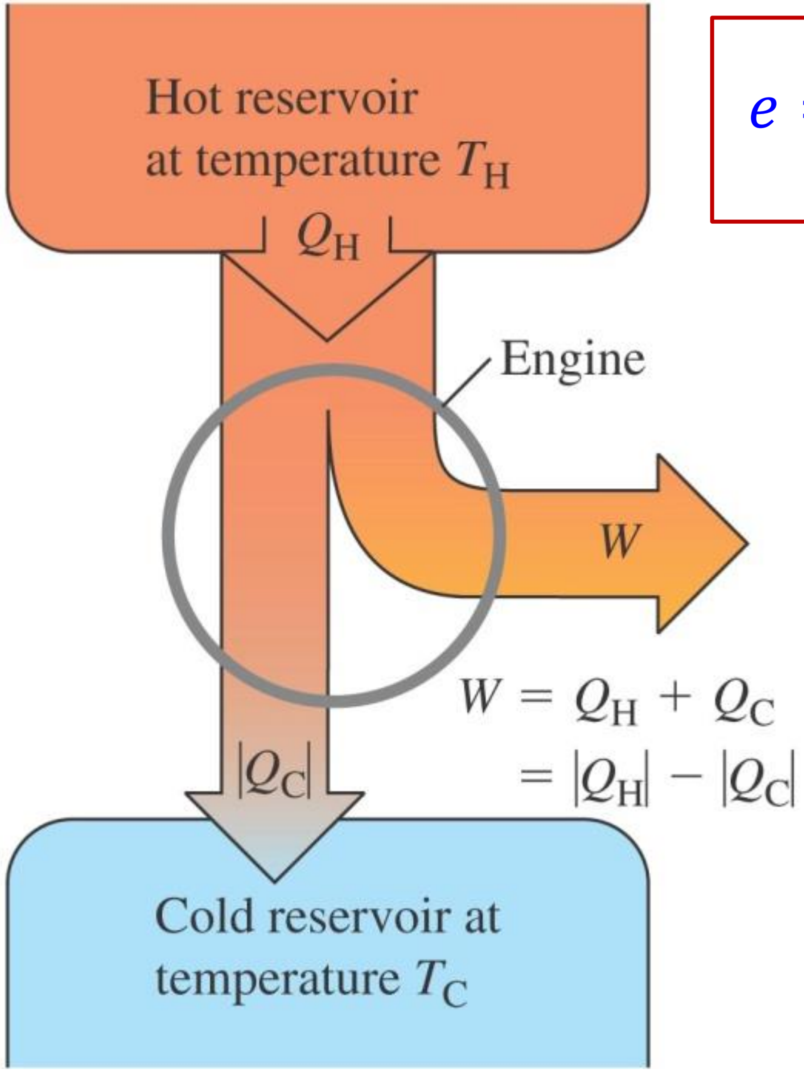
# Lecture 21.

# Refrigerators. Stirling engine.



what we got
―――――――――
what we spent

## Heat Engine

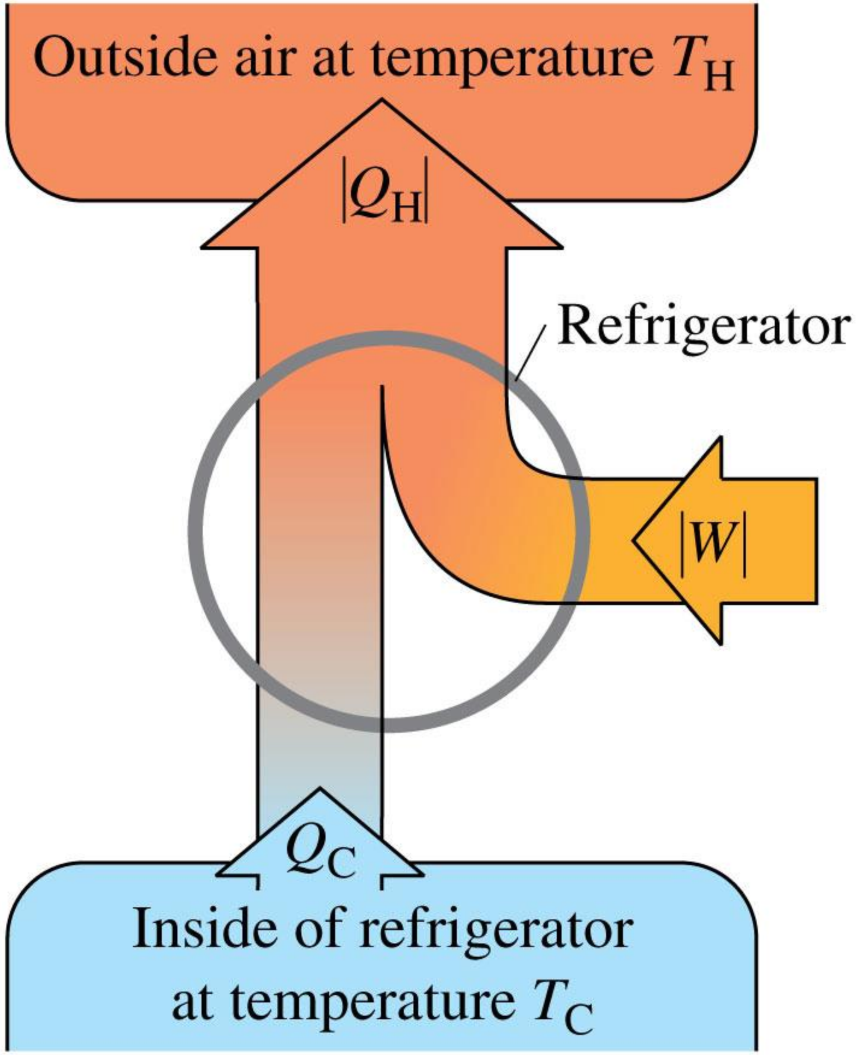



$$e = \frac{W_{net}}{Q_{in}}$$

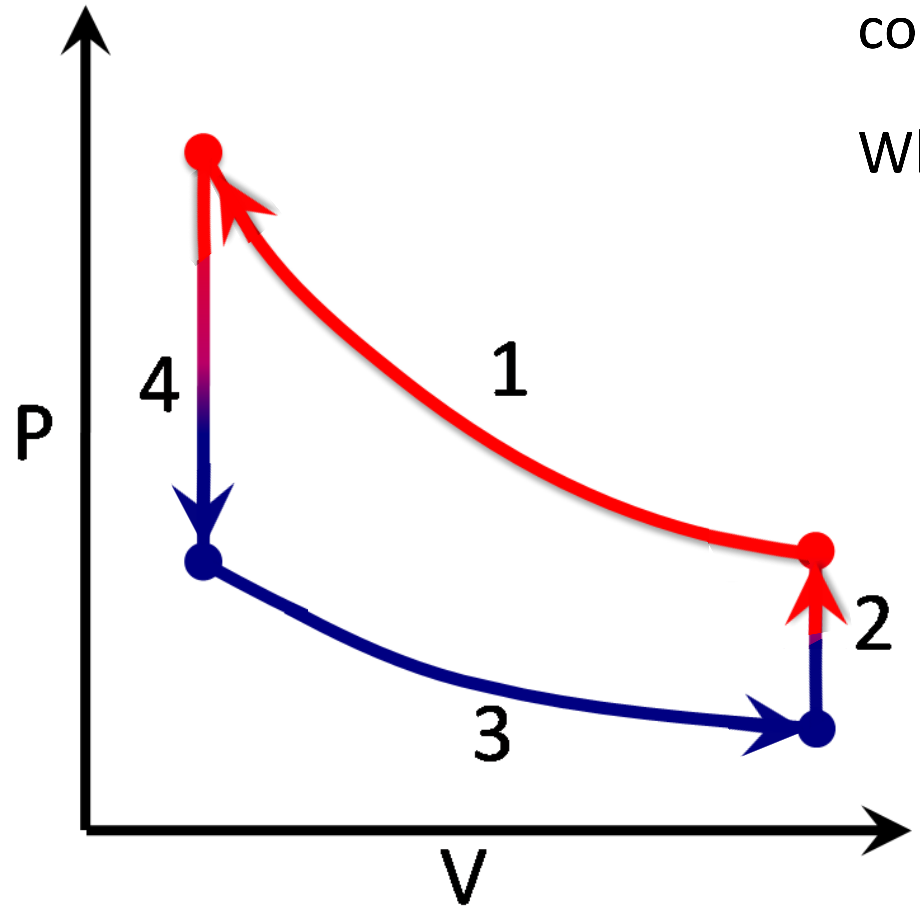
$$K = \frac{Q_C}{W_{net}}$$

# Last Time

## Refrigerator



# Last Time




Problem:

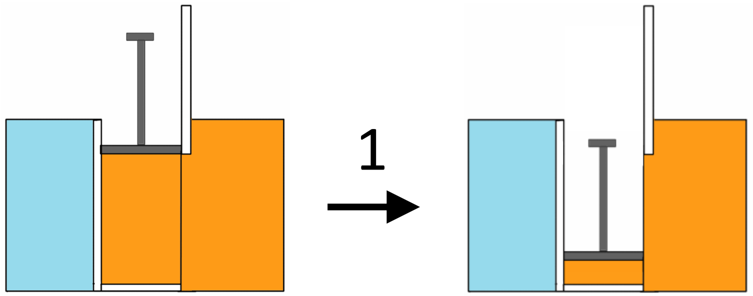
1 mole of nitrogen gas ($C_v = 5/2\ R$) is compressed at constant temperature $T = 20\ ^oC$ from 20 L to 5 L.

What is its COP?

Refrigerator

1 mole, $C_v = 5/2\, R$

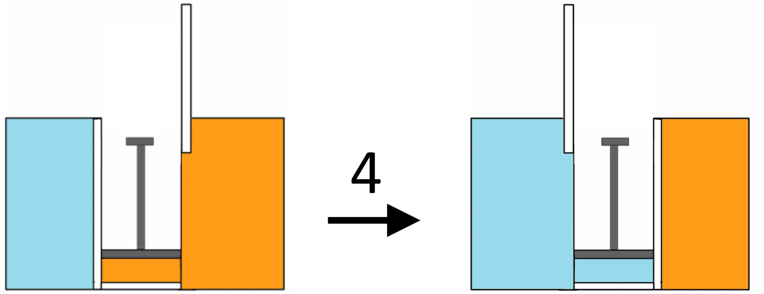
# Last Time




- Process 1: Q? W?

$\Delta U = 0 \rightarrow Q = W$

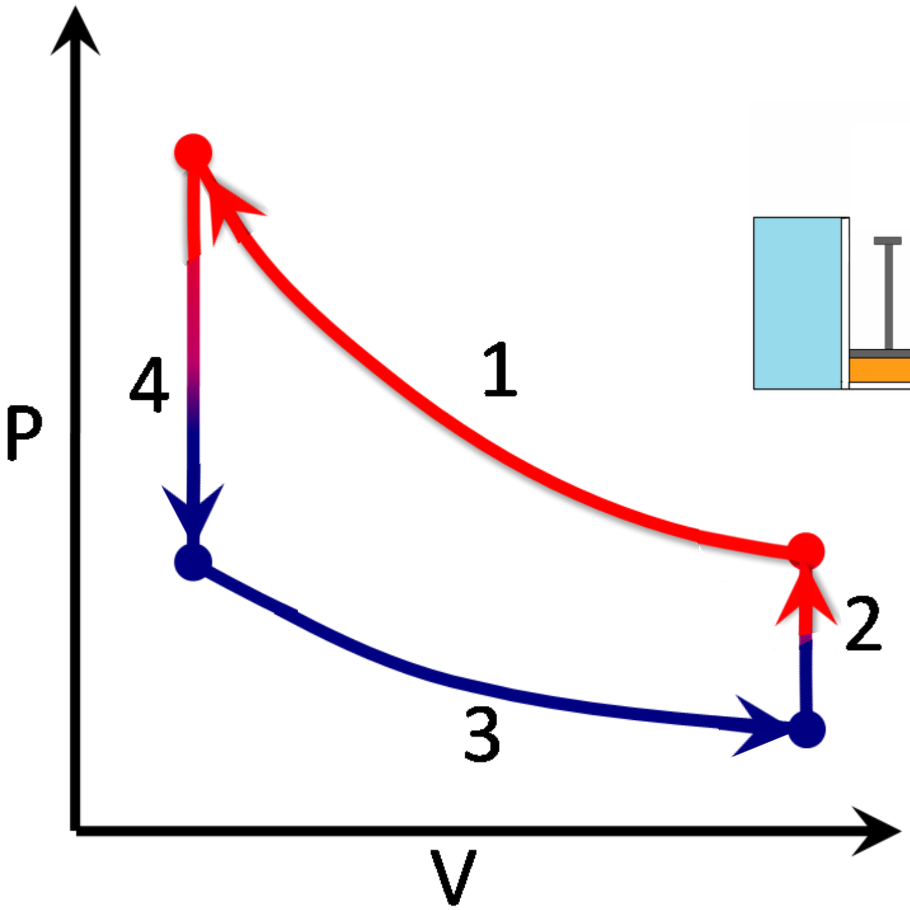
$W < 0 \rightarrow Q < 0$




- Process 4: Q? W? P?

$\Delta V = 0 \rightarrow W = 0$

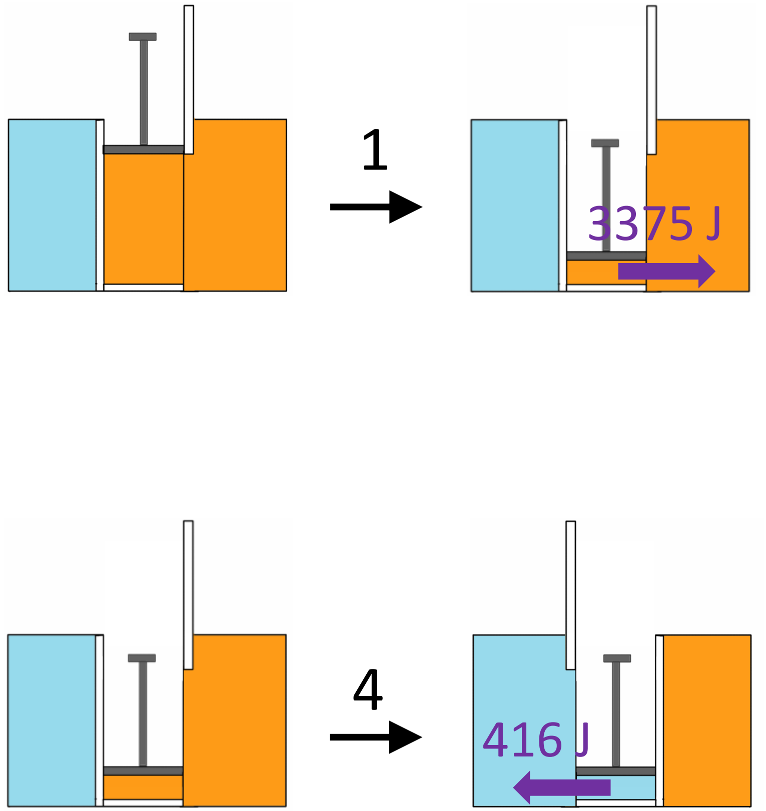
$Q = \Delta U = n C_v \underbrace{\Delta T}_{<0} < 0$



Refrigerator

# Last Time

1 mole, $C_v = 5/2\, R$




- Process 1: Q? W?

isothermal $\Longrightarrow \Delta U = 0$

$$Q = W = nRT ln\left(\frac{V_f}{V_i}\right) = -3375\, J$$

Heat goes out

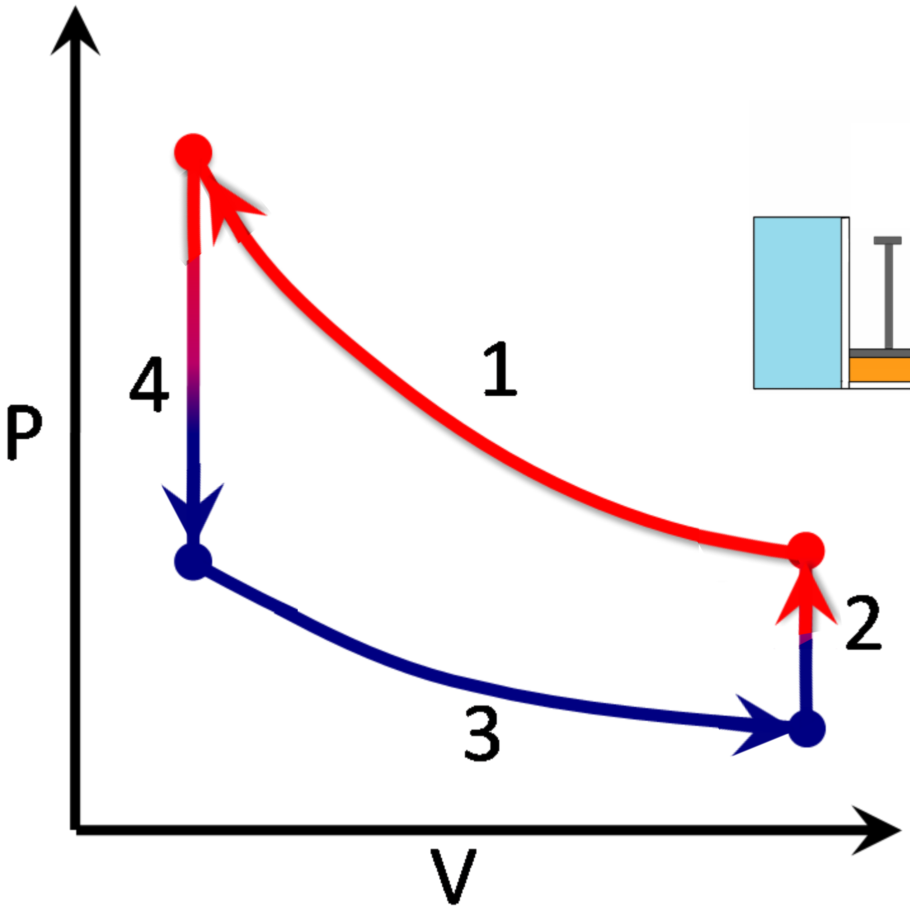
- Process 4: Q? W? P?

Constant $V \Longrightarrow W = 0$

$$Q = \Delta U = nC_v \Delta T = -416\, J$$

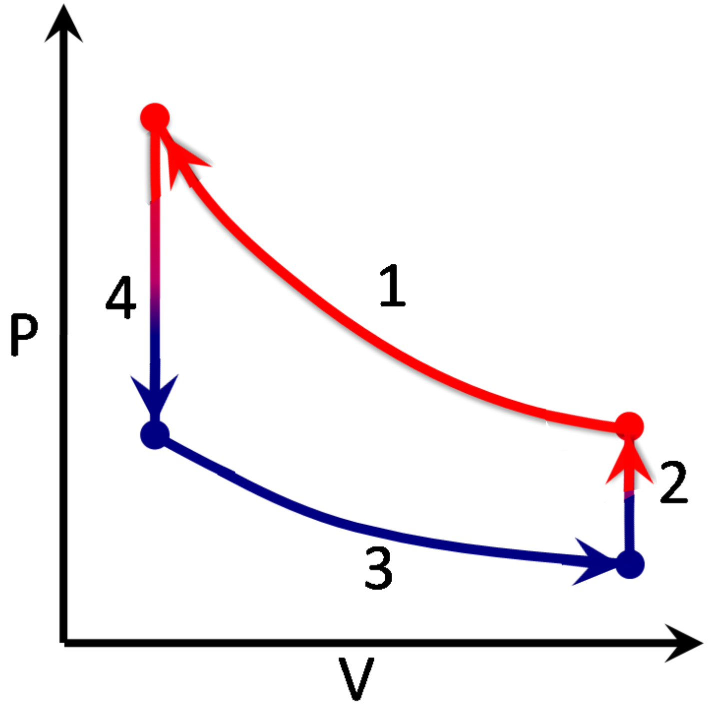
Heat goes out

Use $P = \frac{nRT}{V} \approx 454\, kPa$



1 mole, $C_v = 5/2\,R$

- Process 3 (5 L => 20 L, $T_c$ = const):
  - Q? W? P?
  - What happens with heat?
  - Contact with which reservoir?




$\Delta U = 0 \rightarrow Q = \underbrace{W}_{>0} = \underbrace{n c_v \ln \frac{V_f}{V_i}}_{+3145\,J}$

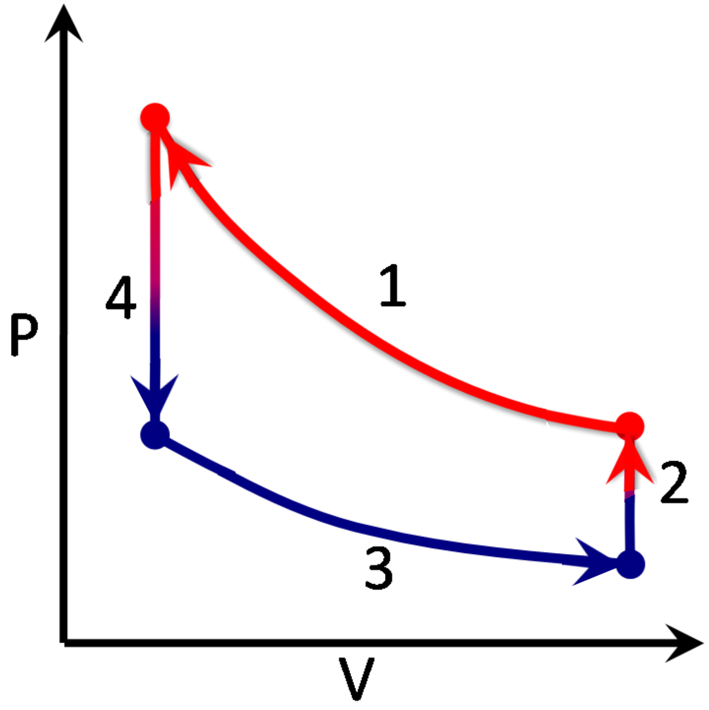
$Q > 0 \rightarrow$ into the gas (receives heat)

A. Cold

B. Hot

$1$ mole, $C_v = 5/2\,R$

- Process 3 (5 L => 20 L, T = const):
  - Q? W? P?
  - What happens with heat?
  - Contact with which reservoir?

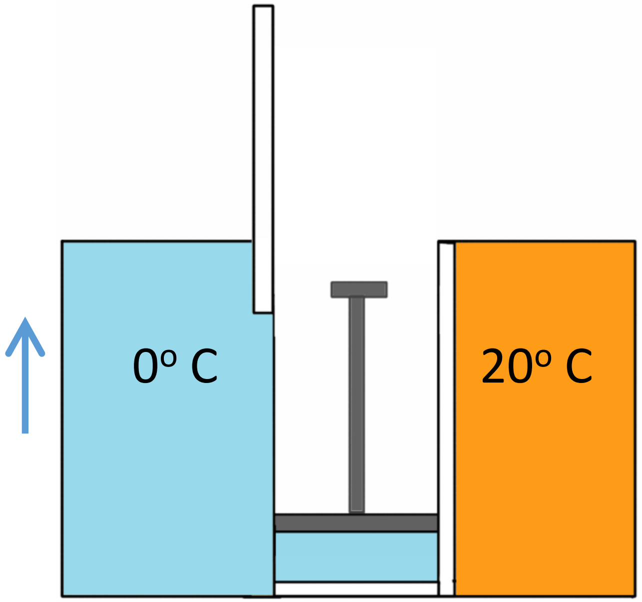


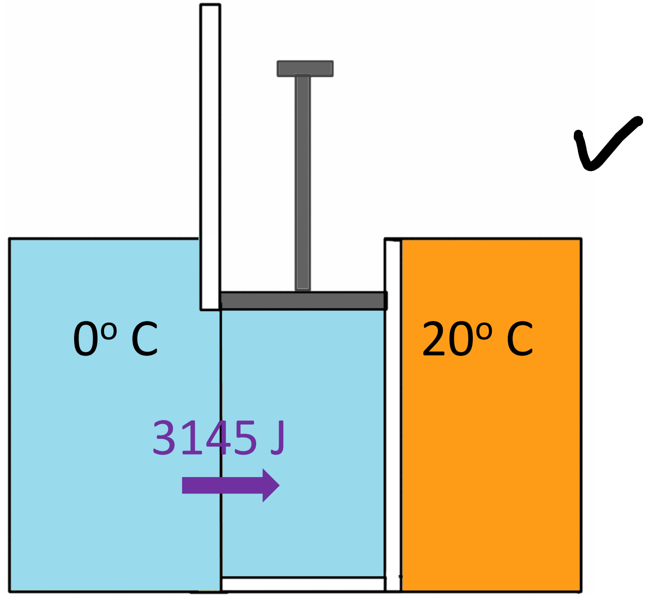

- Isothermal $\Longrightarrow \Delta U = 0$

  $$Q = W = nRTln\left(\frac{V_f}{V_i}\right) = +3145\,J$$
- Heat goes in
- Contact with cold reservoir

- $P_i V_i = P_f V_f \quad \Rightarrow \quad P_f =$

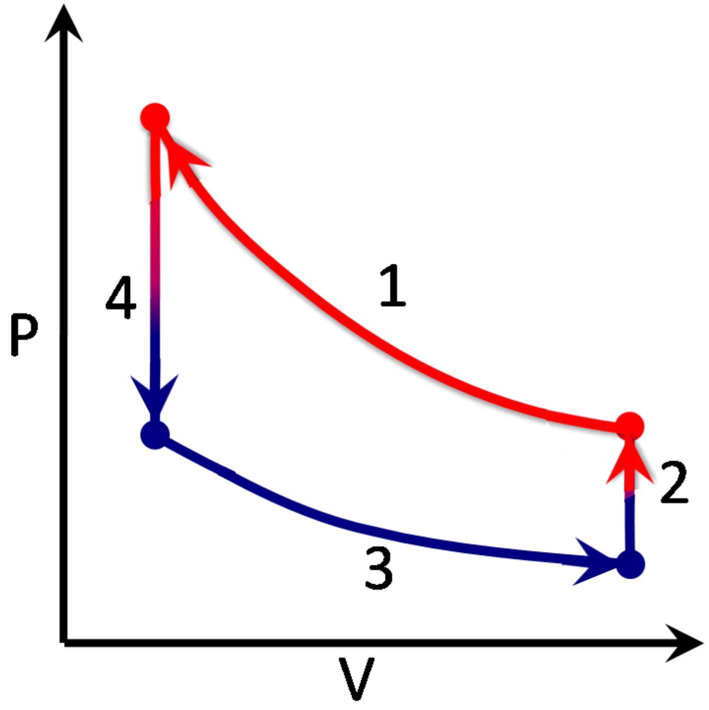
  $$(454\ kPa)\left(\frac{5}{20}\right) = 113\,kPa$$



$1$ mole, $C_v = 5/2\, R$

- Process 2 (0 C => 20 C, V = 20 L):
  - Q? W?
  - What happens with heat?
  - Contact with which reservoir?




$W = 0 \quad (\text{no } \Delta V)!$

$Q = \Delta U = \underbrace{n C_v \underbrace{\Delta T}_{+20}}_{+3145\text{ J}} > 0$

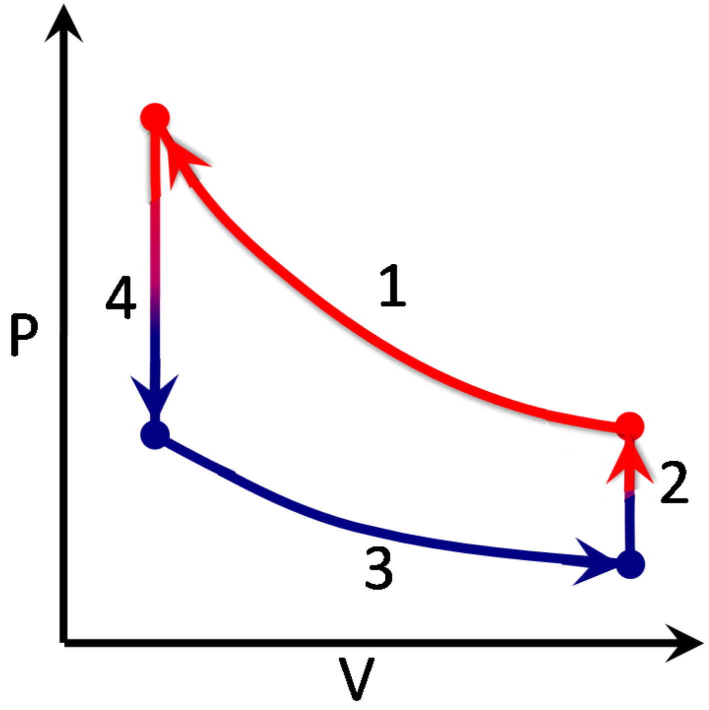
A. Cold

B. Hot

It receives heat from:

1 mole, $C_v = 5/2\, R$

- Process 2 (0 C => 20 C, V = 20 L):
  - Q? W?
  - What happens with heat?
  - Contact with which reservoir?

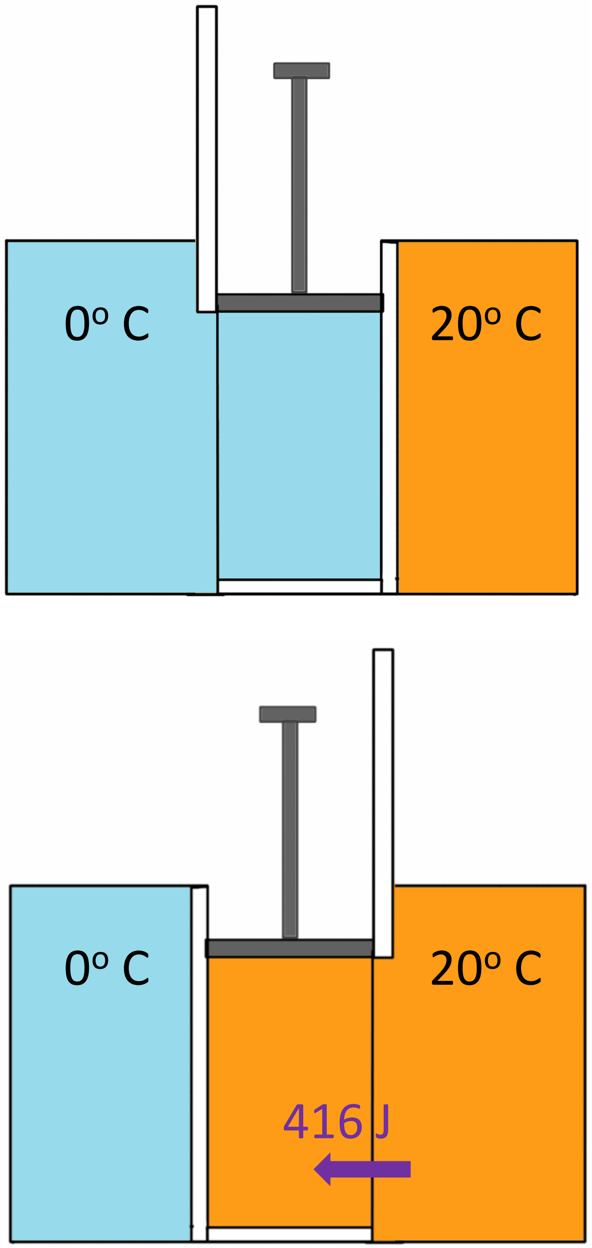


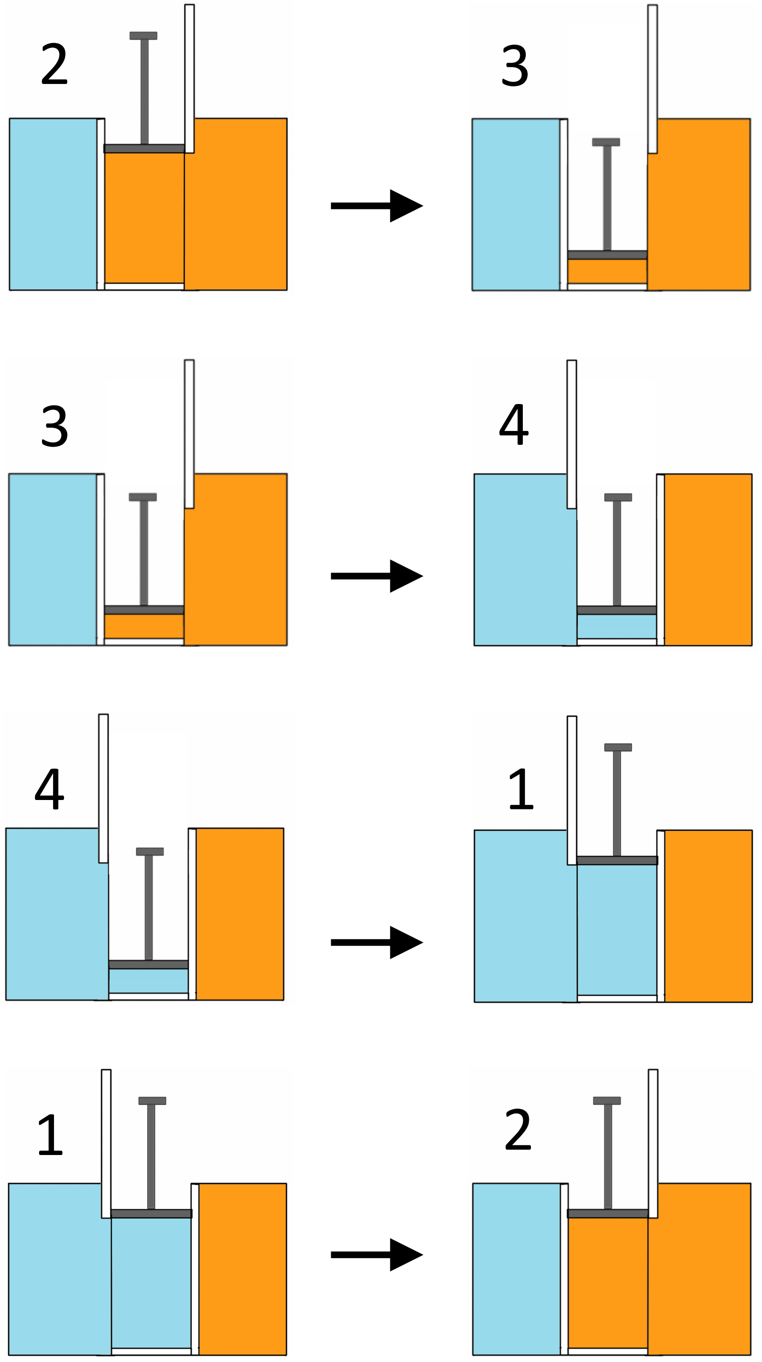

Constant $V \Longrightarrow W = 0$

$Q = \Delta U = nC_v\Delta T = +416\, J$

Heat goes in

Contact with hot reservoir (heating)



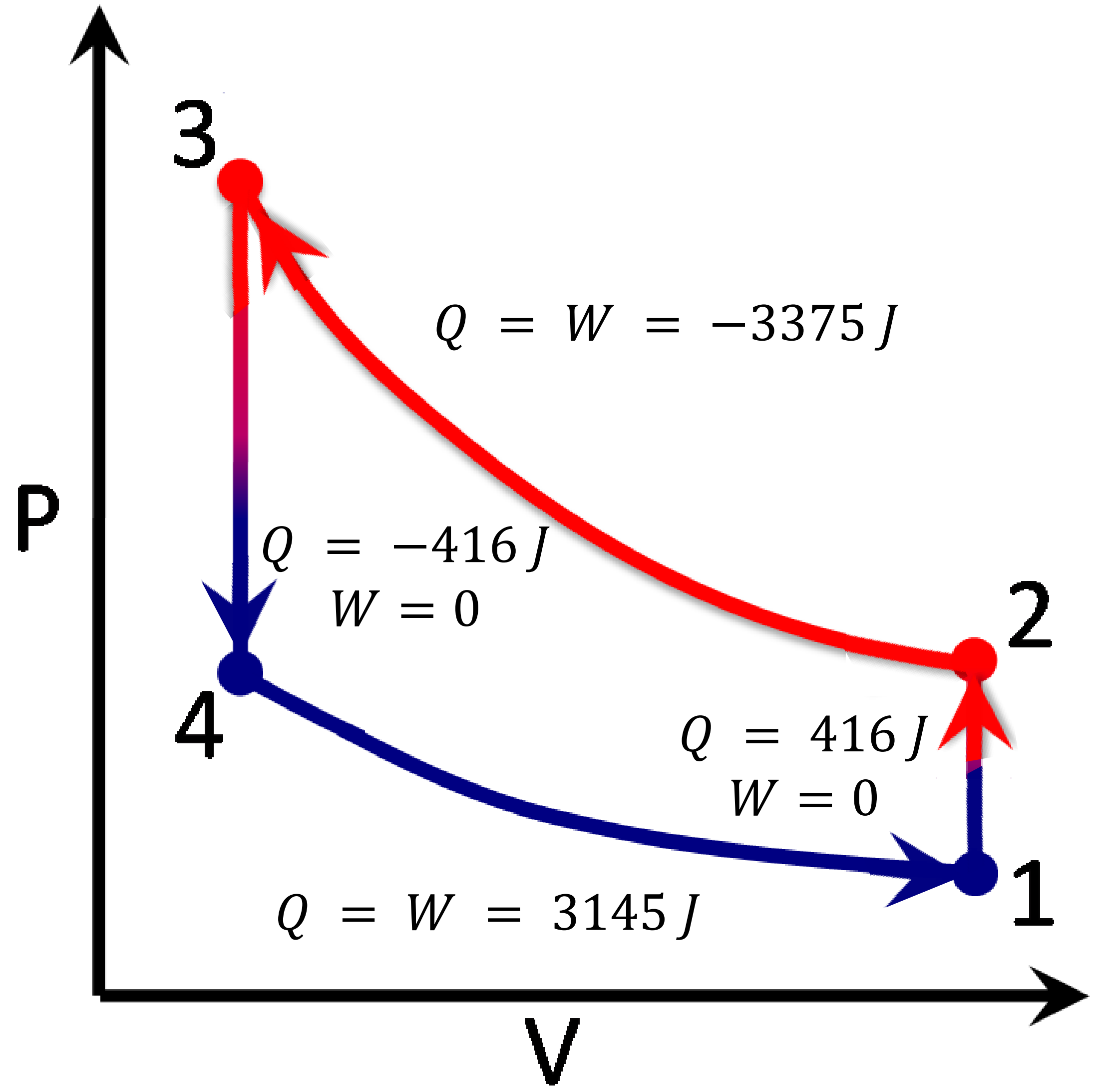



constant temperature compression

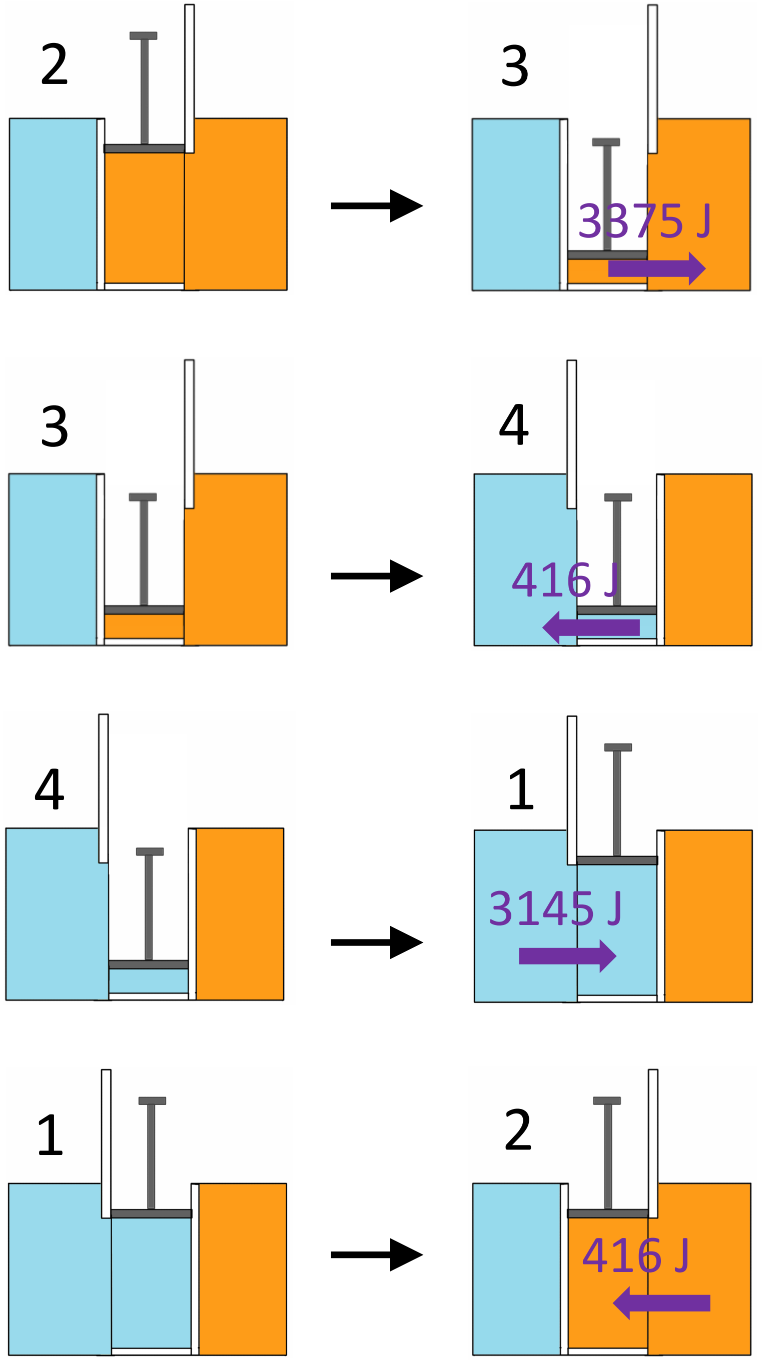
constant volume cooling

constant temperature expansion

constant volume heating




- Net $Q$ out of cold system:
- Net $W$ input:
- Net $Q$ into hot system:

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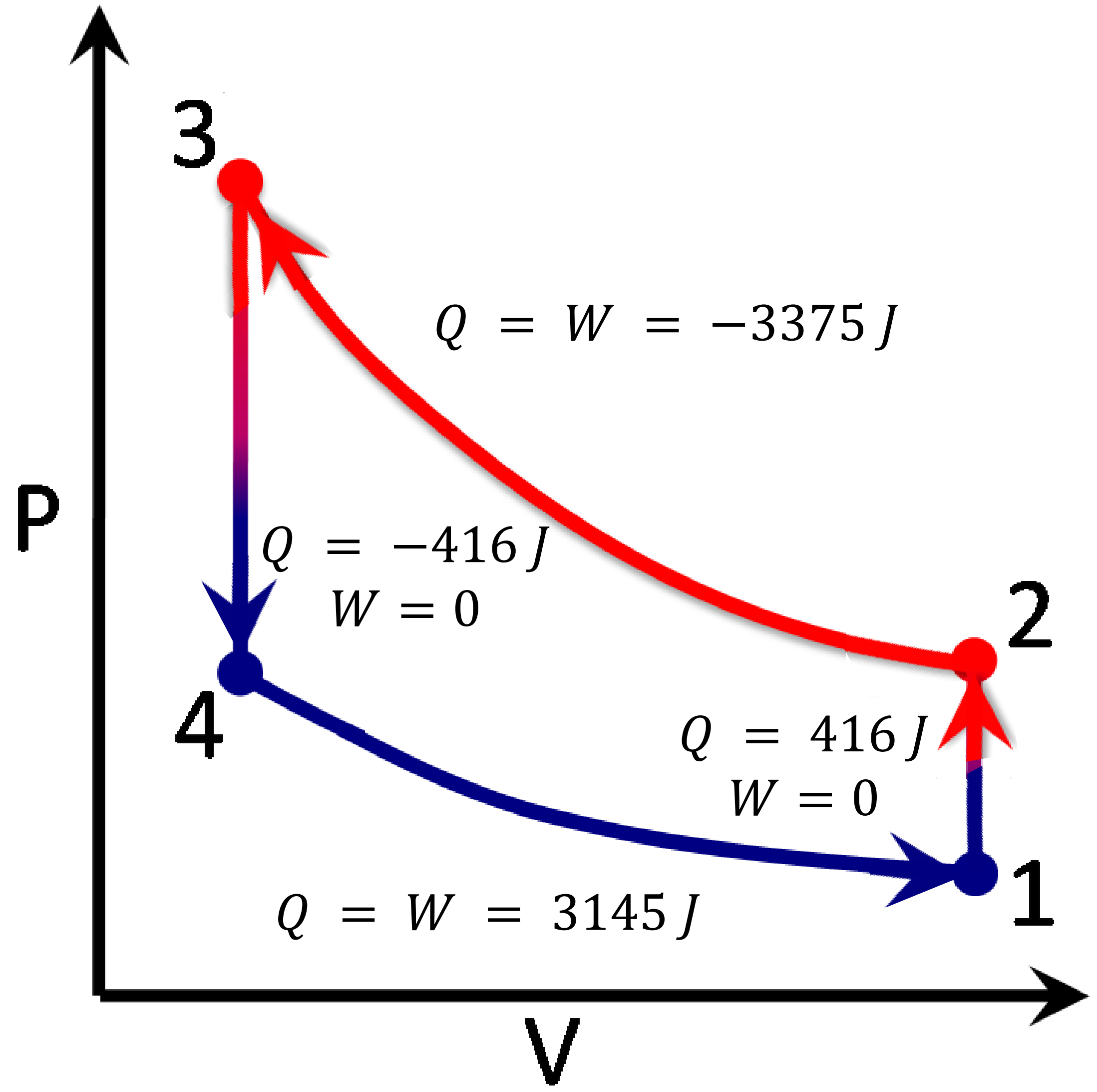
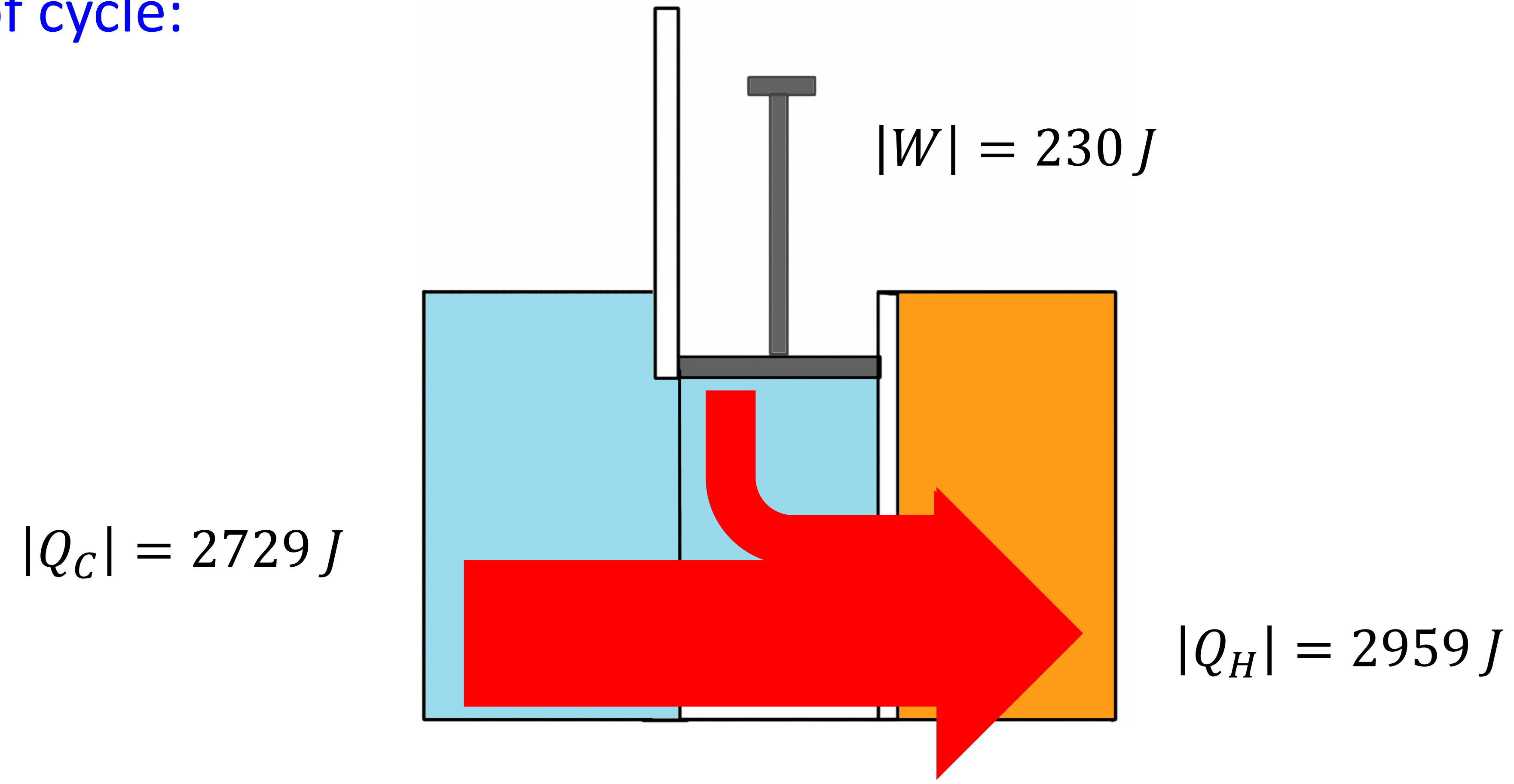


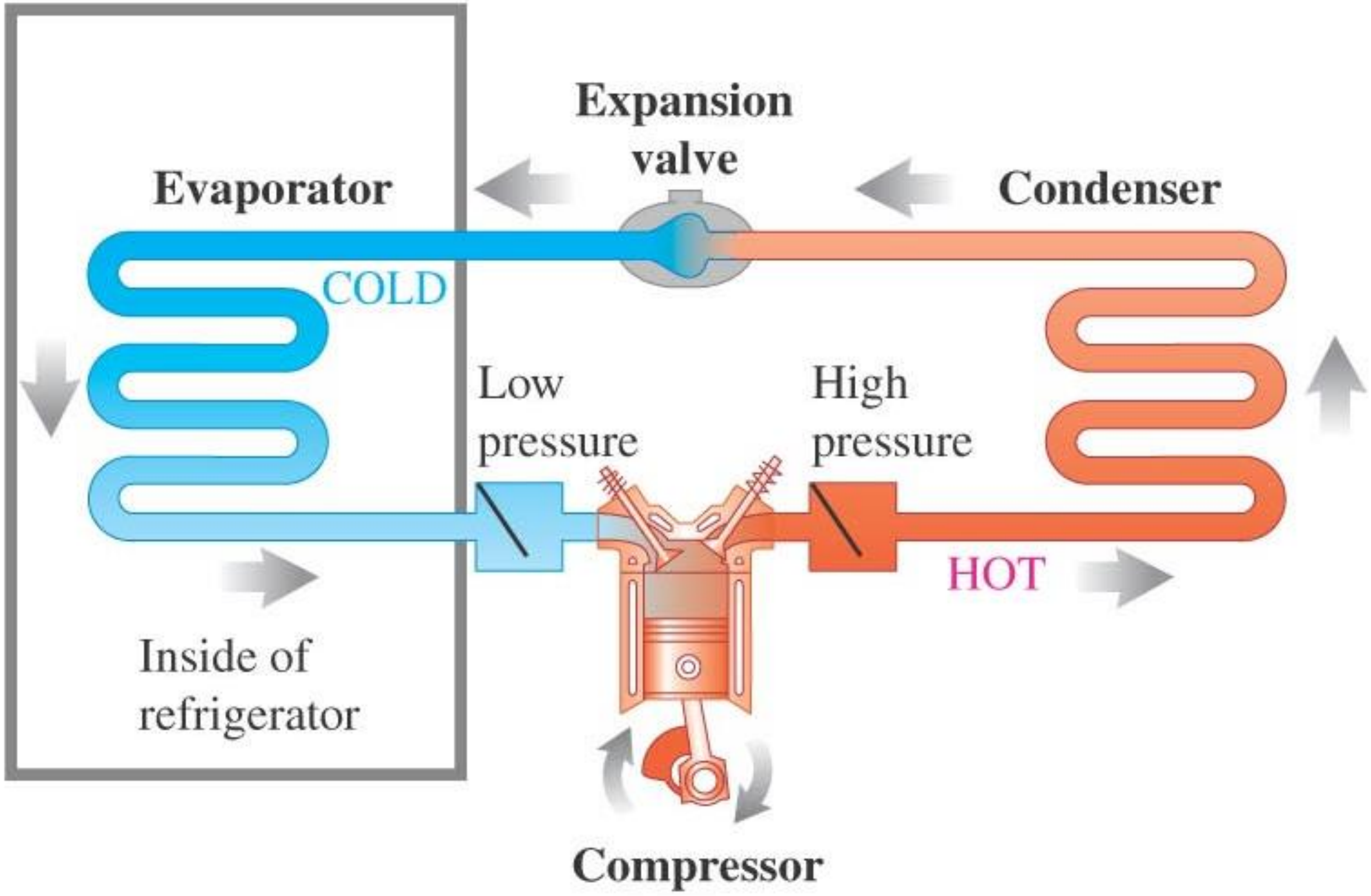

- Net $Q$ out of cold system: 2729 J
- Net $|W|$ input: 230 J
- Net $Q$ into hot system: 2959 J

# Net result of cycle:

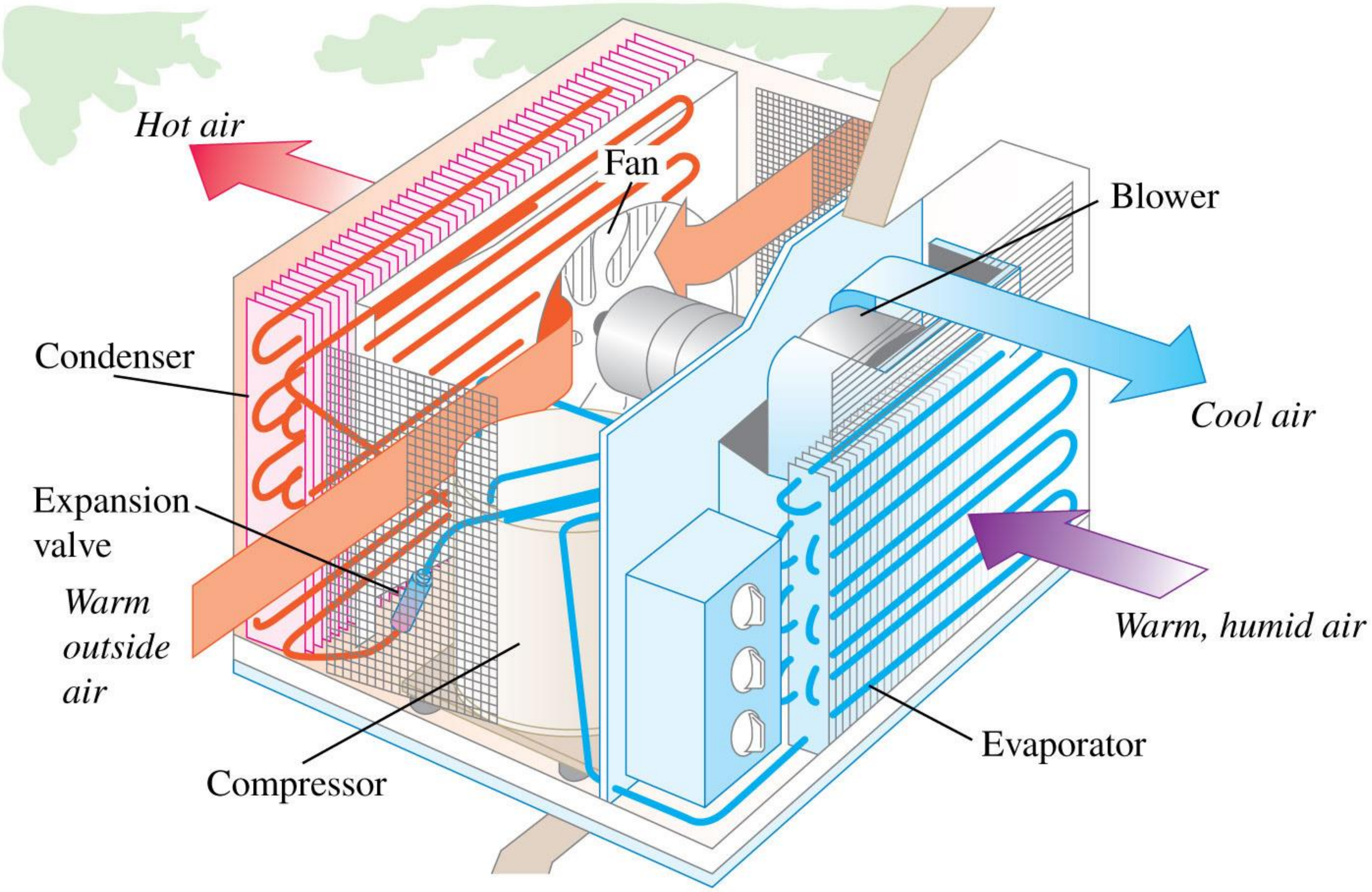



- Coefficient of performance, $K = \frac{|Q_C|}{|W|} = \frac{2729}{230} = 11.9$
- Typical real systems have COP ~ 3 to 4

# Typical home refrigerator:



# Typical home air conditioner:



Q: It’s a hot day and your house doesn’t have air conditioning. Your friend Sam suggests leaving the refrigerator door open in order to cool down the kitchen. What is an appropriate response here?

A. That’s a great idea, let’s do it!
B. Yes it will cool down the kitchen, but it’s a total waste of energy
C. That won’t have any effect at all on the temperature of the room, but the food will go bad
D. Hey Sam, that’s great that you’re thinking creatively, but it will actually make the room warmer than leaving the fridge door closed

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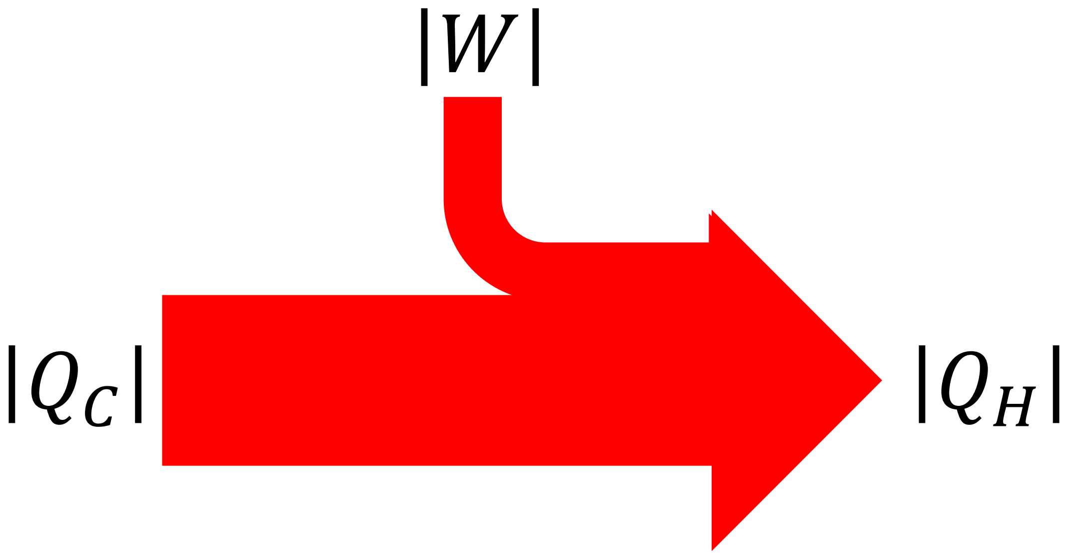



A. That’s a great idea, let’s do it!

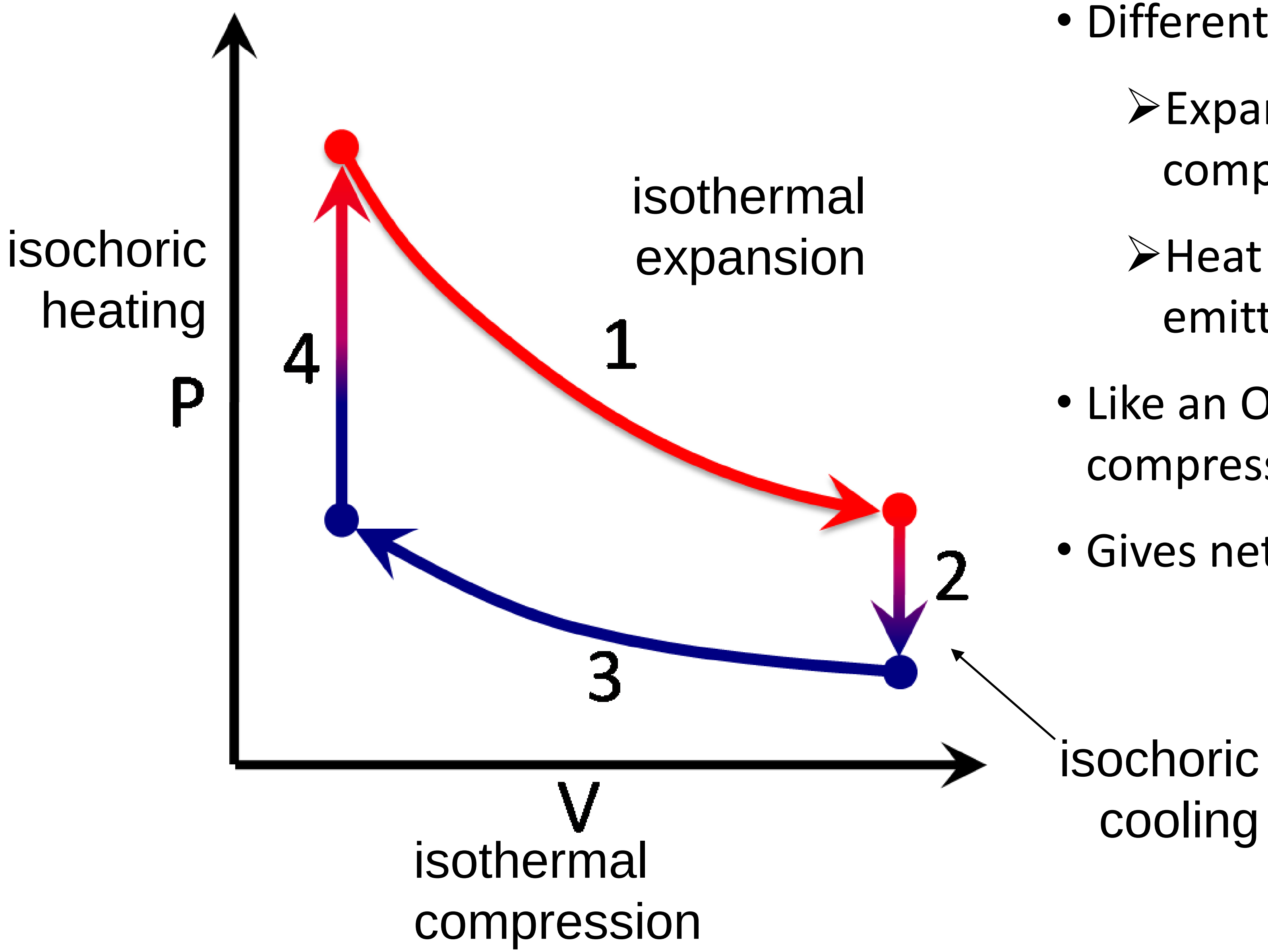
B. Yes it will cool down the kitchen, but it’s a total waste of energy

C. That won’t have any effect at all on the temperature of the room, but the food will go bad

D. Hey Sam, that’s great that you’re thinking creatively, but it will actually make the room warmer than leaving the fridge door closed

- expelled heat = heat taken in plus work done
- compressor runs more often when door is open, so more heat into room

# Stirling Engine: Heat supplied by external reservoir




- Different from refrigerator:
  - Expansion at high $T$ instead of compression at high $T$
  - Heat absorbed at high $T$ instead of emitted
- Like an Otto cycle engine, but isothermal compression/expansion instead of adiabatic
- Gives net work out

# Demo: Stirling Engine

# Stirling Engine Efficiency

- Analysis of $Q, W$ just like for refrigerator…
- …but all processes are reversed, so signs of $Q, W$ are reversed

230 J

Q: Find its efficiency with parameters from previous example:

$$e = \frac{W_{done}}{Q_{in}} =$$

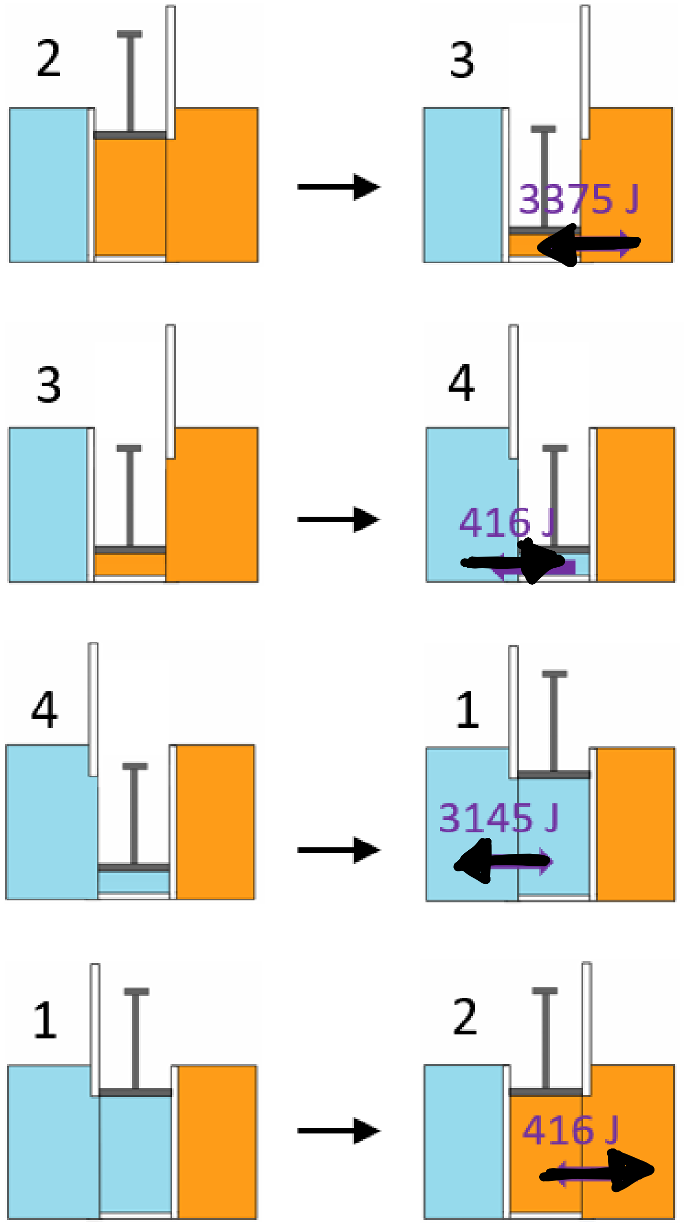
$$= \frac{230}{3375 + 416}$$

A. 6.5%

B. 6.1%

C. 7.8%

D. Something else




constant temperature compression

constant volume cooling

constant temperature expansion

constant volume heating

# Stirling Engine Efficiency

- Analysis of $Q, W$ just like for refrigerator…
- …but all processes are reversed, so signs of $Q, W$ are reversed

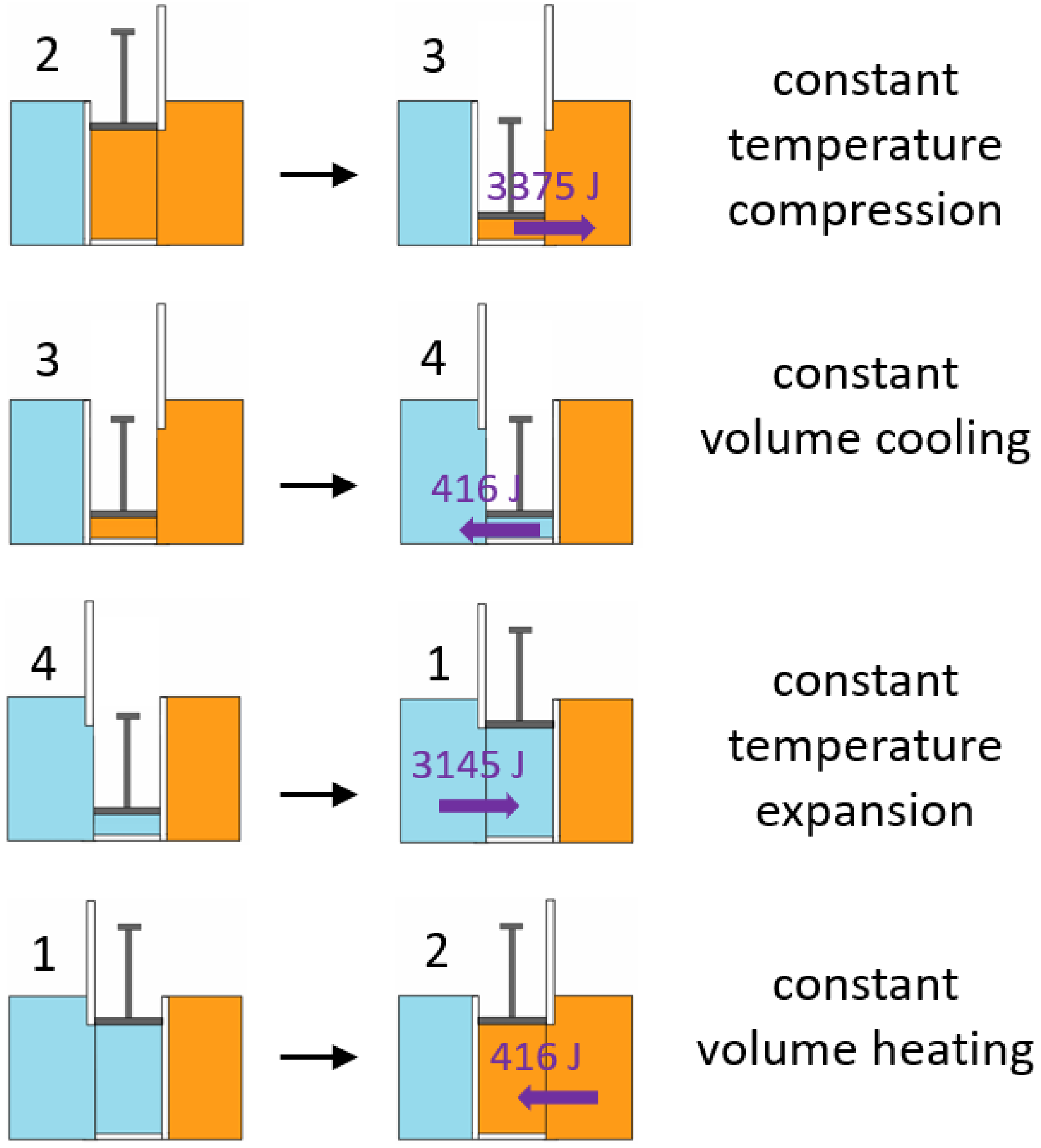
Q: Find its efficiency with parameters from previous example:

A. 6.5%

B. 6.1%

C. 7.8%

D. Something else



# Stirling Engine Efficiency

- $e = \frac{W_{net}}{Q_{in}}$
- $W_{net} = 230\ J$ as before (but now done by gas)
- $Q_{in} = 3375 + 416 = 3791$

Q: Find its efficiency with parameters from previous example:

A. 6.5%
B. 6.1%
C. 7.8%
D. Something else

# Our refrigerator