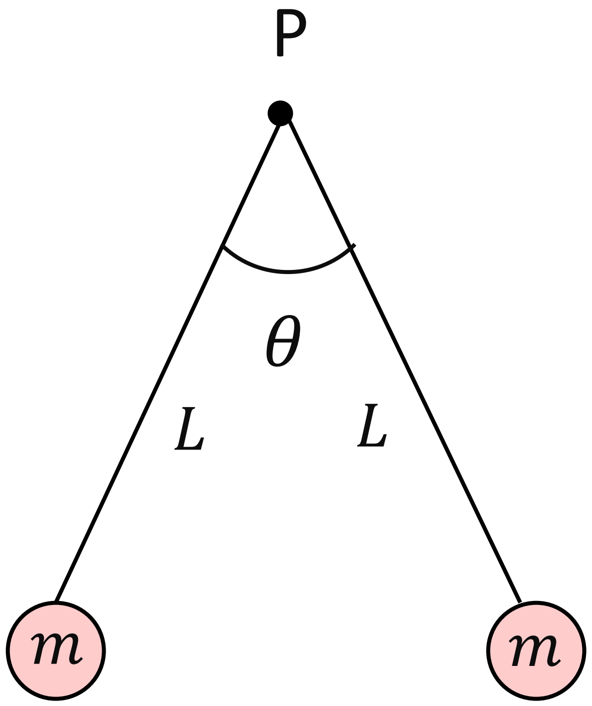
Lecture 14.

Coulomb force. Electric field.
Superposition principle.
E-field of a dipole.
E-field of continuous charge distribution

Q: Determine the charge on each identical balloon at equilibrium. The balloons are point masses m with equal charge Q. Calculate Q in terms of $m, L, k,$ and θ.

Hint: think about <u>all</u> the forces that act on one balloon



Q: Determine the charge on each identical balloon at equilibrium. The balloons are point masses m with equal charge Q. Calculate Q in terms of $m, L, k,$ and θ.

Hint: think about all the forces that act on one balloon

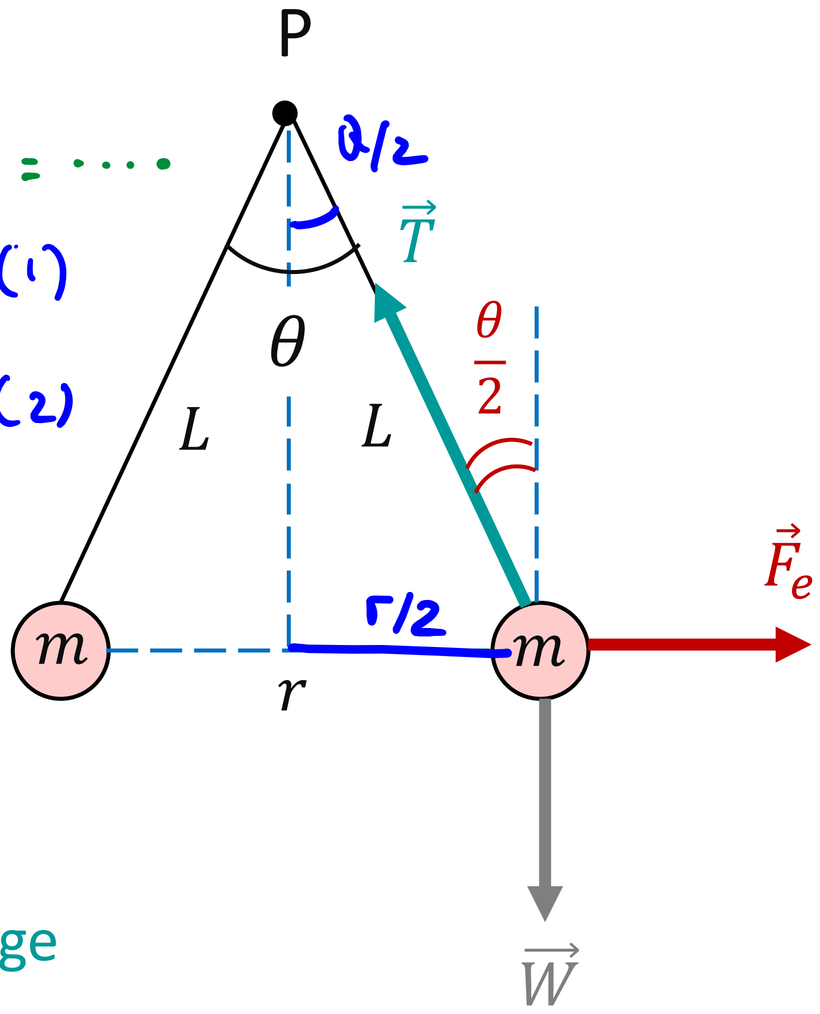
$\vec{T} + \vec{W} + \vec{F} = 0$

?

$Q = \cdots$

$T_x + \cancel{W_x} + F_x = 0 \quad (1)$

$T_y + W_y + \cancel{F_y} = 0 \quad (2)$



- Equilibrium equations:

x: $F_e = T\sin(\theta/2)$

y: $W = T\cos(\theta/2)$ where $W = mg$

- Eliminating T:

$F_e = mg\tan(\theta/2) = k\dfrac{QQ}{r}$

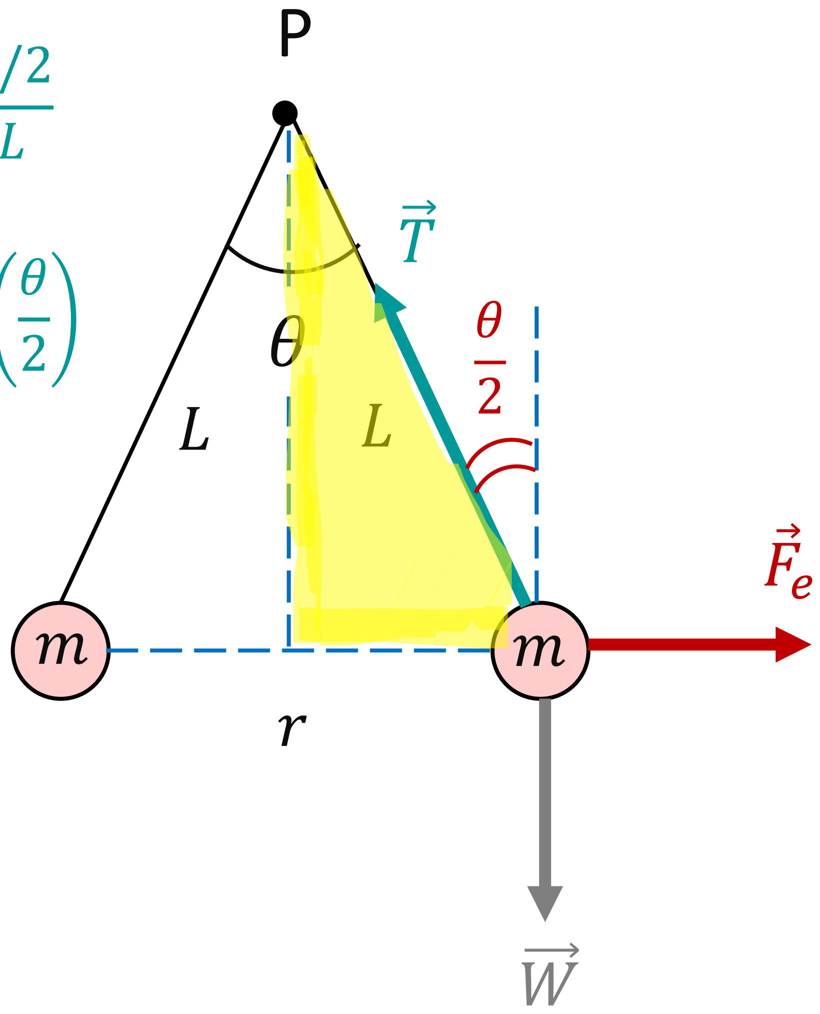
- Know force => find the charge

Q: Determine the charge on each identical balloon at equilibrium. The balloons are point masses m with equal charge Q. Calculate Q in terms of $m, L, k,$ and θ.

Hint: think about all the forces that act on one balloon

$$\sin\left(\frac{\theta}{2}\right) = \frac{r/2}{L}$$

$$r = 2L\sin\left(\frac{\theta}{2}\right)$$



$$F_e = mg\tan(\theta/2)$$

- Coulomb law:

$$F_e = mg\tan\left(\frac{\theta}{2}\right) = k\frac{Q^2}{r^2}$$

$$F_e = mg\tan\left(\frac{\theta}{2}\right) = k\frac{Q^2}{(2L)^2\sin^2\left(\frac{\theta}{2}\right)}$$

$$\boxed{Q^2 = \frac{4L^2}{k}\, mg\tan\left(\frac{\theta}{2}\right)\sin^2\left(\frac{\theta}{2}\right)}$$

What is E-field, and why do we need it?

Coulomb force:

- q_1 and q_2 are treated on the same footing: $\vec{\boldsymbol{F}} = k\dfrac{q_1 q_2}{r^2}\vec{u}_{\boldsymbol{r}} = k\dfrac{q_1 q_2}{r^2}\vec{u}_{\boldsymbol{r}} \equiv q_1\vec{\boldsymbol{E}}$

- Applicable only to point charges
- Looks simple only for charges at rest

Concept of a field:

Two-part analysis:

1) a charge (say, q_2) produces a field (as if the space is filled with “influence” of q_2);
2) another charge (q_1) is acted on by the field

Convenient notations:

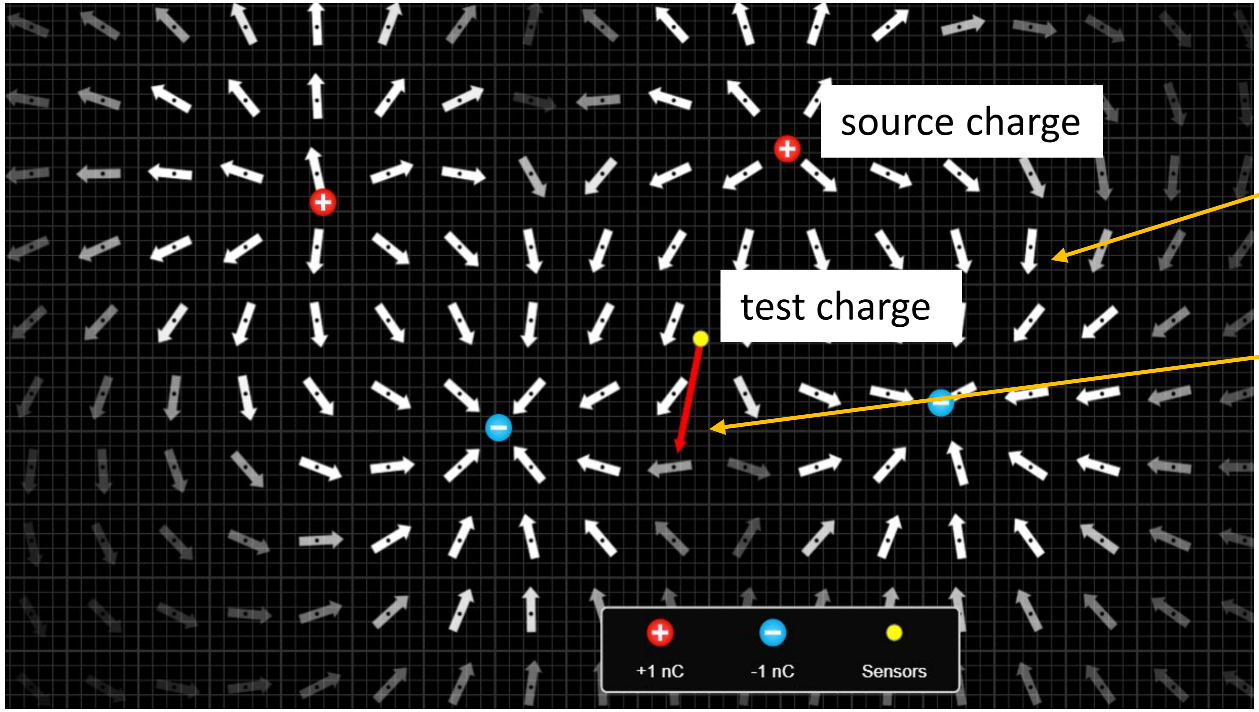
q_1 = ‘test’, or ‘probe’: t
q_2 = ‘source’: s

$$\boxed{\vec{F}_t = q_t\vec{E}_s}$$

- We are mentally splitting the Universe of electric charges into two distinct communities: one “community” creates electric field, the other “community” is acted by it!

E-field summary:

- Finite E-field = “somewhere there are charges, which we do not care about”. They modify the of space, and E-field is the property of that modified space.
- Accounts for combined effect of <u>all</u> the charges except for the one we are looking at



Electric field due to all source charges (white arrows show the direction, saturations show the magnitude of E-field at all points)

Electric force on the test charge (red arrow; its magnitude represents the magnitude of F)

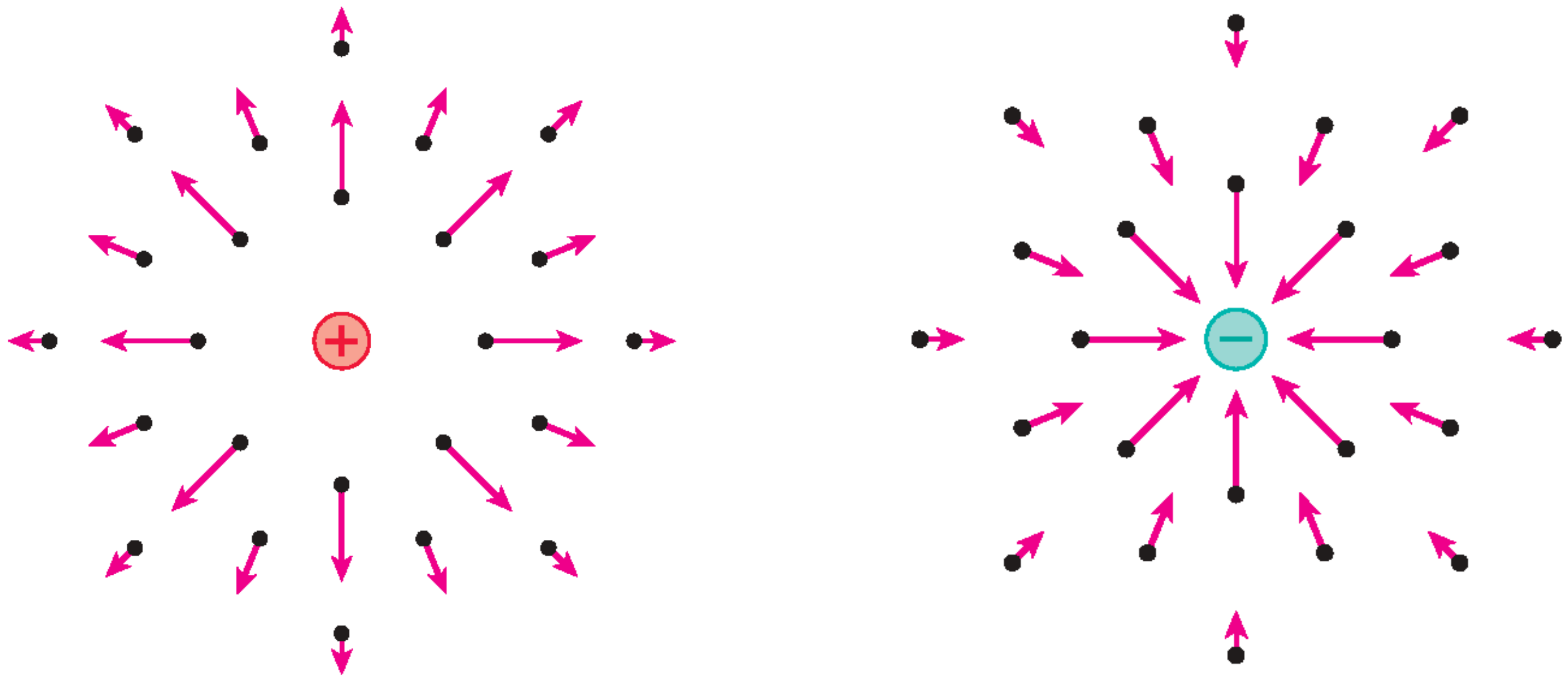
Electric field: DEMO

https://phet.colorado.edu/sims/html/charges-and-fields/latest/charges-and-fields_en.html

What is the field created by a point charge, $q = q_S$?

- Starting point: $\vec{F}_t = q_t \vec{E}_s$. Let's take a test charge of $q_t = +1C$.
- Then $\vec{E}$-vector will simply be $\vec{F}$, which we know how to calculate!

$$F_{12} = k\frac{|q_1||q_2|}{r^2}$$



Units: N/C

- Magnitude:

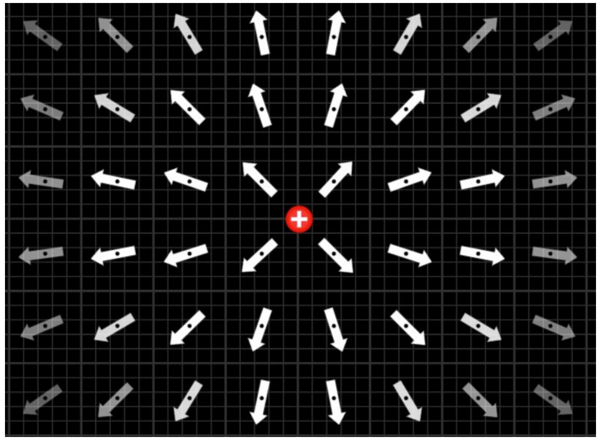
$$E = \frac{k|q_s|}{r^2}$$

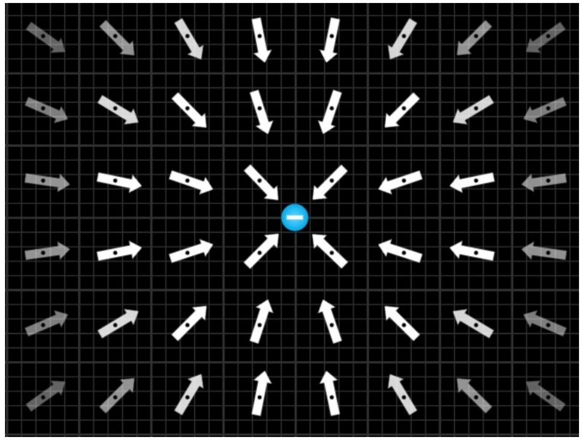
- Direction:
  - Outwards for q_+
  - Inwards for q_-

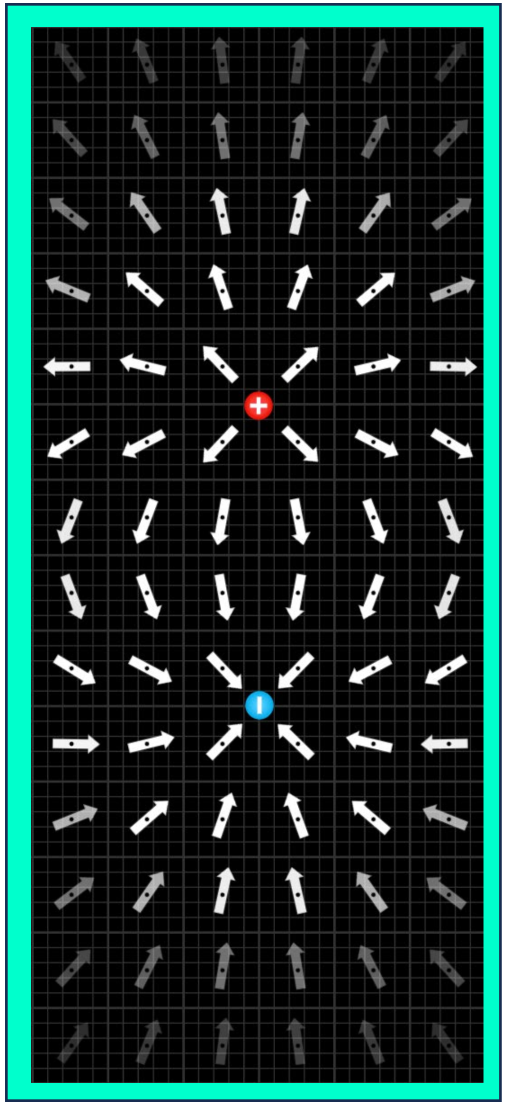
- Electric field = electric force per unit charge.

Superposition principle

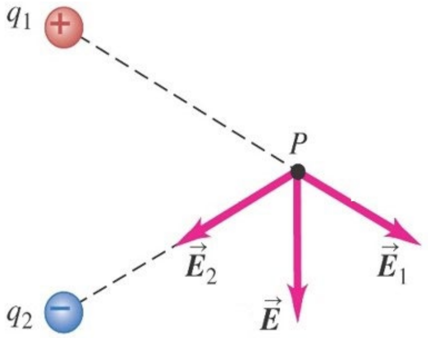
- Suppose that there are two or more charges.
- Each charge creates an electric field in space.







- Superposition principle:
  - ➢ The fields add at each point (as vectors !).

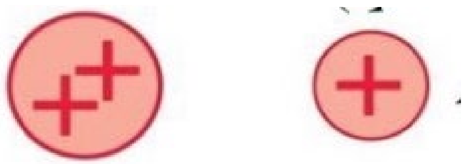


$\vec{E} = \vec{E}_1 + \vec{E}_2$ at all points

$E_x = E_{1,x} + E_{2,x}$ & $E_y = E_{1,y} + E_{2,y}$ & $E_z = E_{1,z} + E_{2,z}$

Electric field vs electric force

$2q$ q



Charge 1 Charge 2

Q: What is correct?

a)
A. $F_{1\ on\ 2} > F_{2\ on\ 1}$
B. $F_{1\ on\ 2} < F_{2\ on\ 1}$
C. $F_{1\ on\ 2} = F_{2\ on\ 1}$

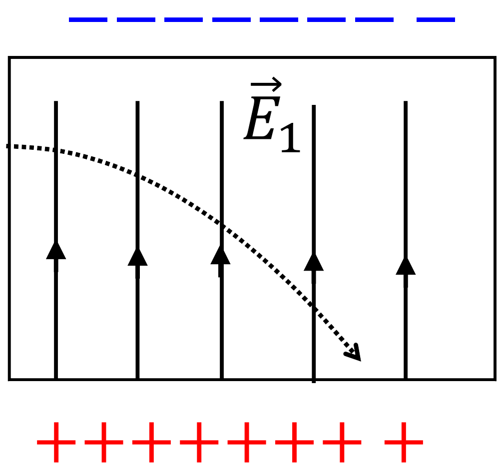
$$F_{12} = k\frac{|q_1||q_2|}{r^2}$$

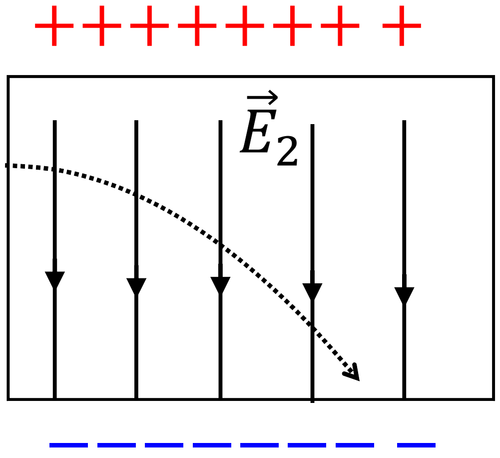
b)
A. $E_{1\ at\ 2} > E_{2\ at\ 1}$
B. $E_{1\ at\ 2} < E_{2\ at\ 1}$
C. $E_{1\ at\ 2} = E_{2\ at\ 1}$

$$E(r) = k\frac{q_s}{r^2}$$

Q: These pictures show uniform electric field created by two plates of charge of opposite signs, as shown.

An electron enters the E-field area from the left, and moves to the right with initial horizontal velocity v. Which of the two E-fields would produce the observed parabolic motion?





A. Only 1
B. Only 2
C. Both 1 and 2
D. Neither 1 nor 2
E. I have no idea

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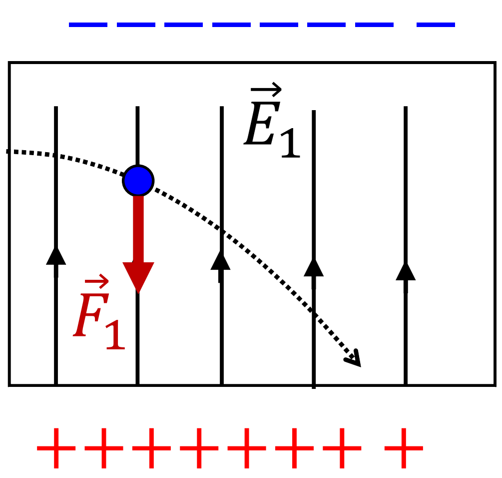
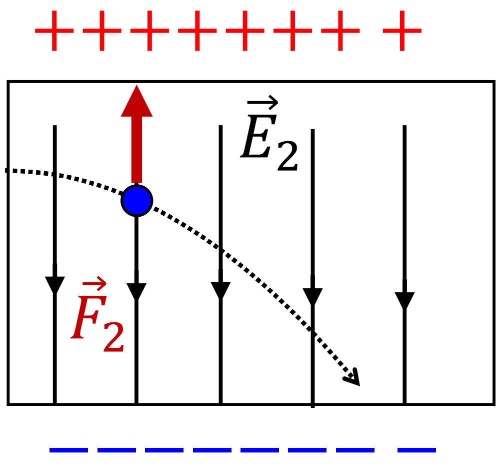


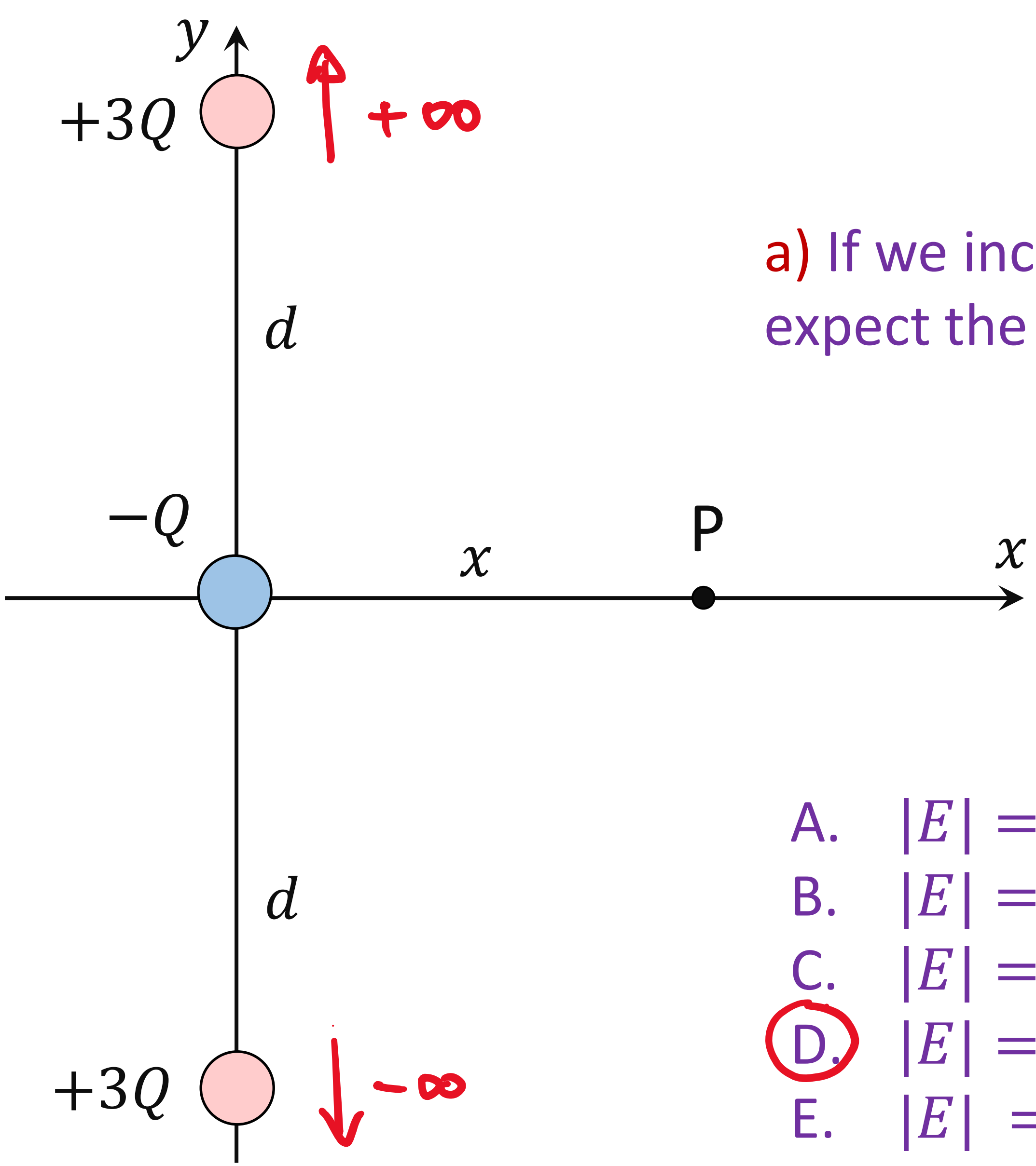


A. Only 1
B. Only 2
C. Both 1 and 2
D. Neither 1 nor 2
E. I have no idea

- Force on the electron $\vec{F} = e\vec{E} = -|e|\vec{E}$ is anti-parallel to the field

- Force $\vec{F}_1$ deflects the electron downwards. It keeps moving to the right but also deflects downwards => the result is a parabolic trajectory (similar to a parabolic trajectory of a projectile under the action of a downward gravity force)

Approximations



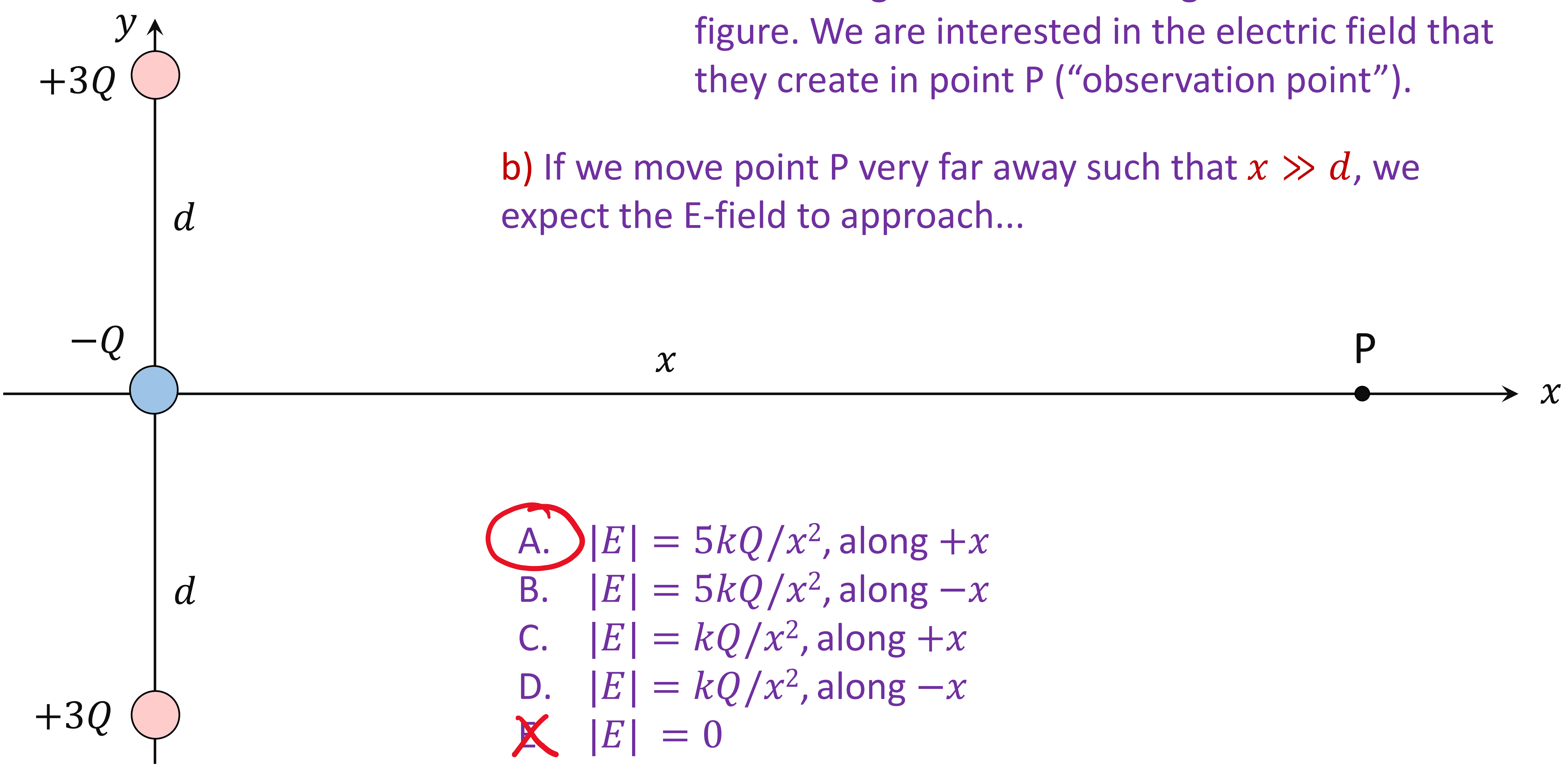
Three charges are in the configuration shown in the figure. We are interested in the electric field that they create in point P ("observation point").

a) If we increase the distance d such that $d \gg x$, we expect the E-field to approach...

A. $|E| = 5kQ/x^2$, along $+x$
B. $|E| = 5kQ/x^2$, along $-x$
C. $|E| = kQ/x^2$, along $+x$
D. $|E| = kQ/x^2$, along $-x$
E. $|E| = 0$

Approximations

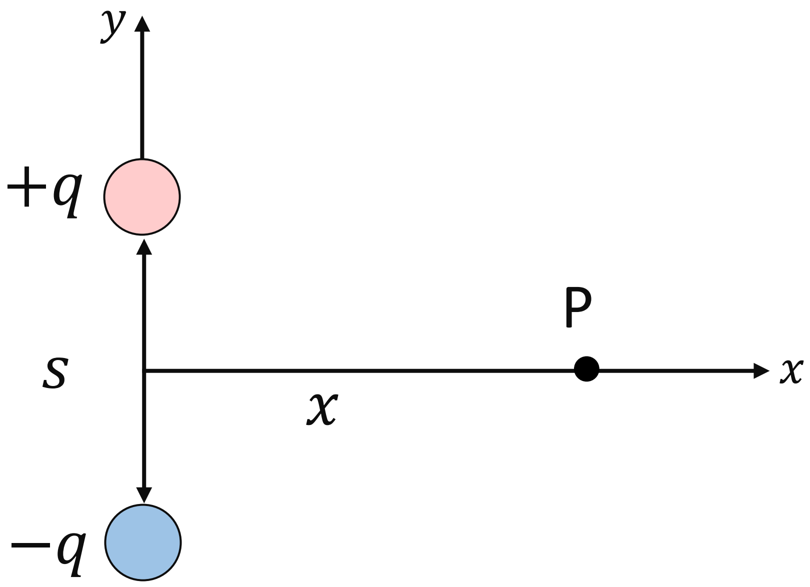
Three charges are in the configuration shown in the figure. We are interested in the electric field that they create in point P (“observation point”).



b) If we move point P very far away such that $x \gg d$, we expect the E-field to approach...

A. $|E| = 5kQ/x^2$, along $+x$
B. $|E| = 5kQ/x^2$, along $-x$
C. $|E| = kQ/x^2$, along $+x$
D. $|E| = kQ/x^2$, along $-x$
~~E.~~ $|E| = 0$

Electric field of a dipole



Q: Find electric field of a dipole at a point at a distance x from it, on the x-axis (see picture)

Point at y-axis: See 21.14 from textbook

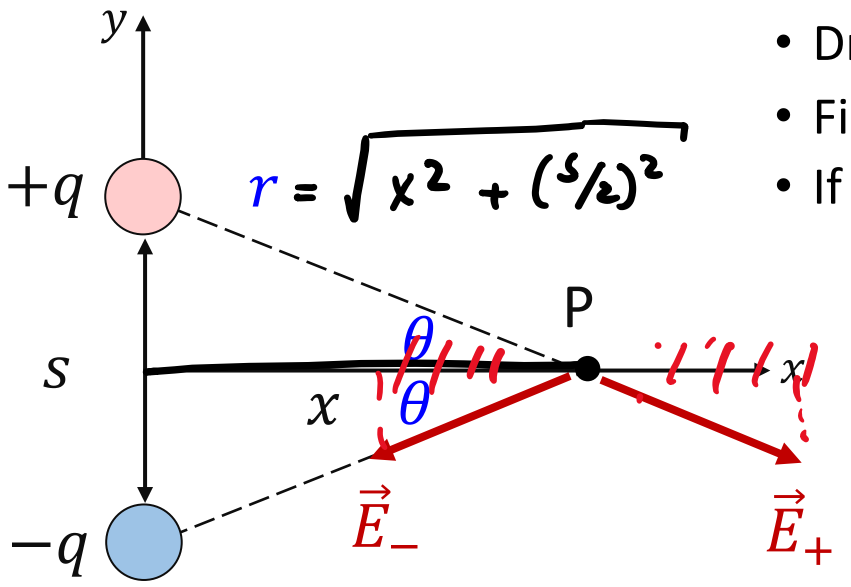
- Strategy: We will use superposition principle => will add two fields, $\vec{E}_+$ and $\vec{E}_-$, as vectors

- Draw $\vec{E}_+$ and $\vec{E}_-$ on the diagram. Find their components (write down their magnitudes, then project).
- Find the components of the net field: $E_x = E_{+x} + E_{-x}$ and $E_y = E_{+y} + E_{-y}$. (Anything special that you can observe due to symmetry?)
- Then $\vec{E} = \vec{u}_x E_x + \vec{u}_y E_y$. Done!
- If interested in its magnitude: $E^2 = E_x^2 + E_y^2$

$\cancel{E = E_+ + E_-}$

Electric field of a dipole

$\vec{E}_{dipole}(x) = ?$



$r = \sqrt{x^2 + (s/2)^2}$

- Draw $\vec{E}_+$ and $\vec{E}_-$. Find their components.
- Find $E_x = E_{+x} + E_{-x}$ and $E_y = E_{+y} + E_{-y}$. Then $\vec{E} = \vec{u}_x E_x + \vec{u}_y E_y$.
- If interested in its magnitude: $E^2 = E_x^2 + E_y^2$.

$$E_+ = E_- = k\frac{|q|}{r^2}$$

symmetry!

$$\vec{E}_+ = \vec{u}_x \cancel{E_+ \cos\theta} - \vec{u}_y E_+ \sin\theta$$

$$\vec{E}_- = \vec{u}_x \cancel{(-E_- \cos\theta)} - \vec{u}_y E_- \sin\theta$$

$$\sin\theta = \frac{s/2}{r}$$

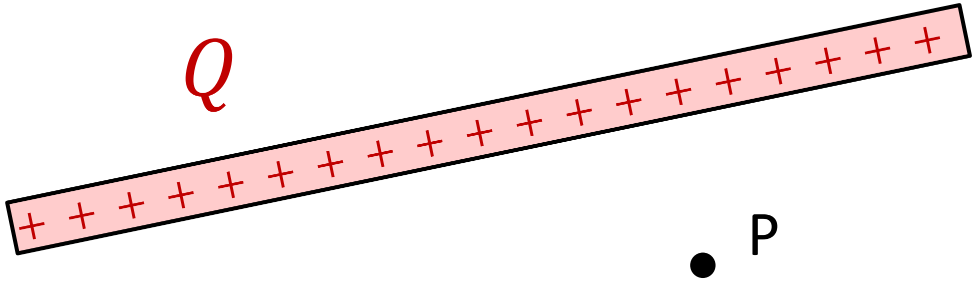
$$\vec{E}_+ + \vec{E}_- = \vec{E} = \vec{u}_x(0) - \vec{u}_y(E_+ \sin\theta + E_- \sin\theta)$$

$$\vec{E} = -\vec{u}_y E_y$$

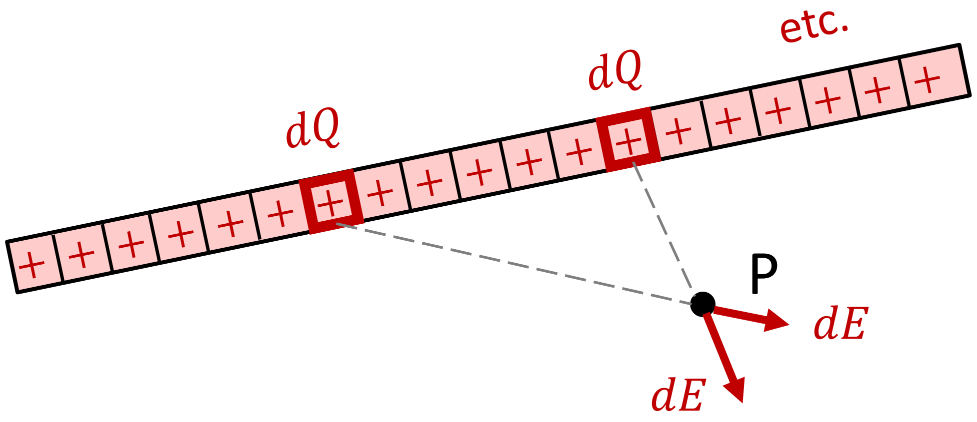
$$E_y = \frac{kq}{r^2}\frac{s/2}{r} = 2\frac{kq\, s/2}{r^3} = 2\frac{kq\, s/2}{(x^2 + (s/2)^2)^{3/2}} = \frac{kqs}{(x^2 + (s/2)^2)^{3/2}}$$

Electric field due to a continuous charge distribution

- What we know so far is the electric field produced by a point charge: $\vec{E} = \pm\frac{kq}{r^2}\,\vec{u}_r$
- How can we calculate the field produced by a continuous charge distribution?



➢ Say, we know that total charge Q is distributed evenly over this rod, and we want to know E-field at point P



➢ Big idea: let's cut the object into tiny almost-point-like charges dQ

➢ Then find the field created by each of them, and sum these fields up!