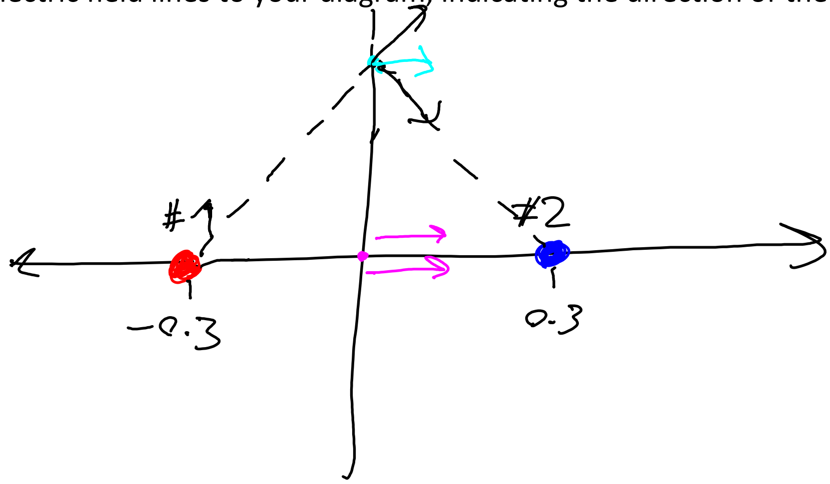
Problem E1.1 (☆): consider a point charge of $+5\ nC$ on the x-axis at $x = -0.3\ m$, and another point charge of $-5\ nC$ on the x-axis at $x = +0.3\ m$

(a) Draw a diagram of the two charges.
(b) What is the force experienced by the positive charge from the negative one?
(c) What is the electric field at the origin?
NB: electric field is a **vector ield**, so don't forget vector notation!
(d) What is the electric field at a point on the y-axis at $y = +0.4\ m$?
(e) Add electric field lines to your diagram, indicating the direction of the electric field.

(a)



(b) Coulomb's law: $\vec{F} = \frac{k|q_1 q_2|}{r^2}\hat{r}$

$$\vec{F}_{2\,on\,1} = k\frac{|(-5nC)(5nC)|}{(0.6)^2}\hat{x} = \underline{6.25\times10^{-7}\,N}\,\hat{x}$$

(c) E-field: $\vec{E} = \frac{kq}{r^2}\hat{r}$

$$\vec{E}_{tot} = \vec{E}_1 + \vec{E}_2 = k\left(\frac{5nC}{(0.3m)^2}\hat{x} + \frac{-5nC}{(0.3m)^2}(-\hat{x})\right)$$

$$= 2k\frac{5nC}{0.3m}\hat{x} = \underline{500\ N/C}$$

(d) The vertical component from #1 is cancelled by the vertical component of #2 due to symmetry.

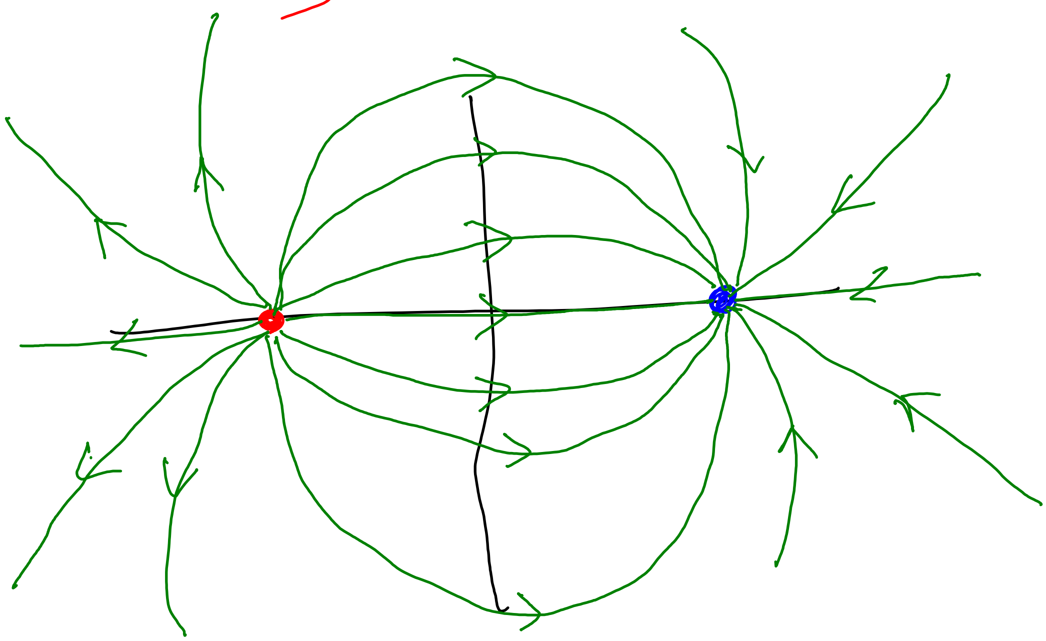
The horizontal components add.

$$\therefore \vec{E}_{tot} = 2k\,\frac{5nC}{r^2}\cos\theta\,\hat{x}$$

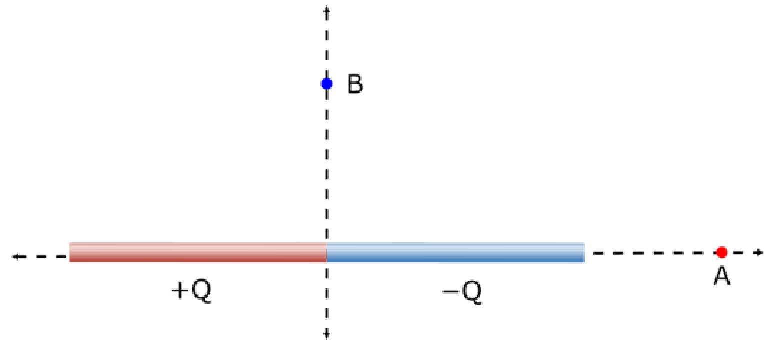
$$r = \sqrt{0.3^2 + 0.4^2} = 0.5 \qquad \cos\theta = \frac{0.3}{r} = \frac{3}{5}$$

$$\therefore \vec{E}_{tot} = 216\ N/C$$

(e)



Problem E1.3(☆☆☆): A thin rod of length $2a$ has charge Q uniformly distributed on its left half and $+Q$ on its right half.

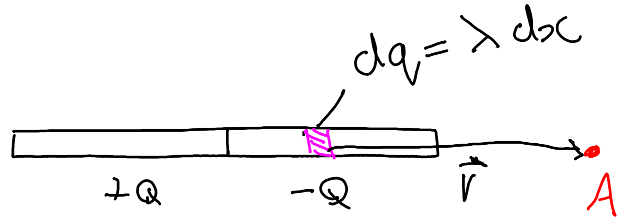


(a) Find the electric field E, amplitude and direction, at point A distance d from the rod center.

(b) Find the electric field E, amplitude and direction, at point B distance h above the rod center.

Look up any integrals that are unfamiliar.

(a) E-field for continuous distributions is:



$$\vec{E} = \int \frac{k\,dq}{r^2}\hat{r}$$

In our case, uniform density says $\lambda = \frac{dq}{dx} = \frac{Q}{a}$
so $dq = \lambda\, dx$.

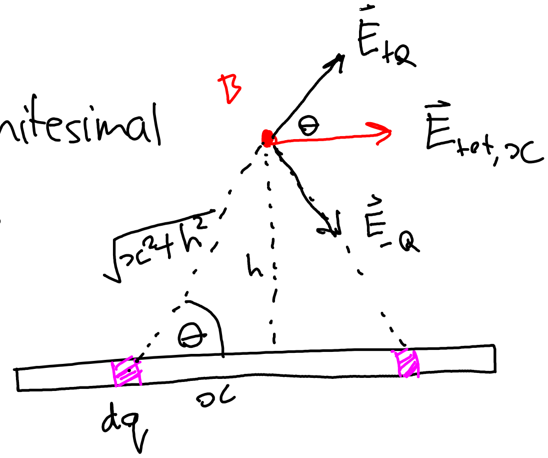
We will treat the opposite charged segments as 2 different bars:

$$\vec{E} = \underbrace{k\lambda \int_{-d-a}^{-d} \frac{dx}{x^2}\hat{x}}_{+Q \text{ half}} - \underbrace{k\lambda \int_{-d}^{-d+a} \frac{dx}{x^2}\hat{x}}_{-Q \text{ half}}$$

$$= k\lambda\hat{x}\left[-\frac{1}{x}\right]_{-d-a}^{-d} - k\lambda\hat{x}\left[-\frac{1}{x}\right]_{-d}^{-d+a}$$

$$= k\lambda\hat{x}\left(\frac{1}{d} - \frac{1}{d+a}\right) - k\lambda\hat{x}\left(\frac{1}{d-a} - \frac{1}{d}\right) = \frac{2kQ}{a}\left(\frac{1}{d} - \frac{d}{d^2-a^2}\right)\hat{x}$$

(b) For B, we can realise that the vertical components of two infinitesimal charge blocks on opposite sides cancel, whilst the horizontal components add. Thus



$$d\vec{E}_x = 2\cos\theta \frac{k\,dq}{\left(\sqrt{x^2+h^2}\right)^2}\hat{x}$$

θ and dq with x, so we must express them in terms of x.

$$d\vec{E} = 2k\frac{x}{\sqrt{x^2+h^2}}\frac{\lambda\,dx}{x^2+h^2}\hat{x} \qquad \cos\theta = \frac{x}{\sqrt{x^2+h^2}}$$

$$dq = \lambda\,dx$$

$$\therefore \vec{E} = 2k\lambda\int_0^a \frac{x\,dx}{(x^2+h^2)^{3/2}}\hat{x}$$

This integral is a common one in emag, so you can look up the result or put it on your cheat sheet. We can also solve it by making the substitution

$$u = x^2+h^2 \quad \therefore du = 2x\,dx \quad u(0) = h^2 \quad u(a) = a^2+h^2$$

$$\vec{E} = k\lambda\int_{h^2}^{a^2+h^2}\frac{du}{u^{3/2}}\hat{x} = k\lambda\left[-2u^{-\frac{1}{2}}\right]_{h^2}^{a^2+h^2}\hat{x}$$

$$= \frac{2kQ}{a}\left(\frac{1}{h} - \frac{1}{\sqrt{a^2+h^2}}\right)\hat{x}$$

NB: the unit vector has tracked the direction of the field throughout the calculation.