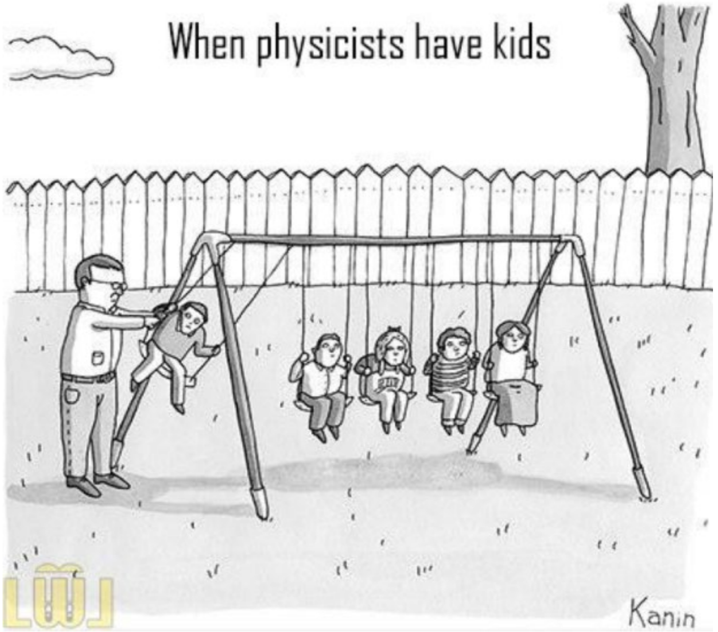
Linear Impulse and Momentum for a System of Particles



Text: 15.2-3

Content:

- Center of mass, its position and velocity
- Internal forces do not influence the momentum of a system of particles
- In the absence of net external force along some axis, the component of the momentum of the system along that axis conserves

Announcement

- Monday: Wrapping up:
  - Linear momentum

$$\boxed{\vec{L} = m\vec{v}}$$

  - Linear impulse

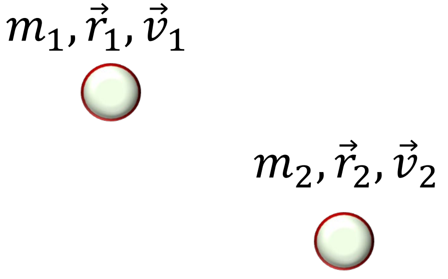
$$\boxed{\vec{I} = \int_{t_1}^{t_2} \vec{F}\, dt}$$

- Wednesday: Review
  - ➢ Thank you for submitting your suggestions!
    https://piazza.com/class/lq1h74hu27s4gl/post/910

$$\vec{L}_2 = \vec{L}_1 + \vec{I}_{1\to 2}$$

- Friday: No lecture

PRINCIPLE OF LINEAR IMPULSE & MOMENTUM: Two (or more) particles



- We have:

$$m_1 \vec{v}_{1,\mathrm{f}} = m_1 \vec{v}_{1,i} + \int_{t_i}^{t_\mathrm{f}} \left[\vec{F}_{ext,1} + \vec{f}_{int,1}\right] dt$$

$$+$$

$$m_2 \vec{v}_{2,\mathrm{f}} = m_2 \vec{v}_{2,i} + \int_{t_i}^{t_\mathrm{f}} \left[\vec{F}_{ext,2} + \vec{f}_{int,2}\right] dt$$

$\vec{F}_{ext,n}$ and $\vec{f}_{int,n}$ are **resultant** external and internal forces acting on n-th particle

- Sum over these two particles:

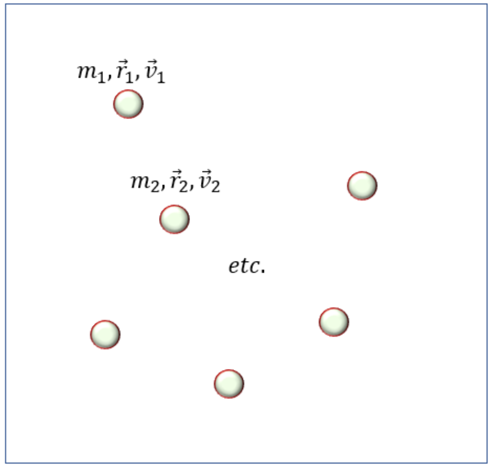
$$(m_1 \vec{v}_1 + m_2 \vec{v}_2)_\mathrm{f} = (m_1 \vec{v}_1 + m_2 \vec{v}_2)_\mathrm{i} + \int_{t_i}^{t_\mathrm{f}} [\vec{F}_{ext,1} + \vec{F}_{ext,2}] dt + \int_{t_i}^{t_\mathrm{f}} \left[\vec{f}_{int,1} + \vec{f}_{int,2}\right] dt$$

- Now: $\vec{f}_{int,1} + \vec{f}_{int,2} = \vec{f}_{2\mathrm{on}1} + \vec{f}_{1\mathrm{on}2} = 0$ by Newton's 3rd law!

- We get:

$$\underbrace{(m_1 \vec{v}_1 + m_2 \vec{v}_2)_\mathrm{f}}_{\text{Final momentum of the pair}} = \underbrace{(m_1 \vec{v}_1 + m_2 \vec{v}_2)_\mathrm{i}}_{\text{Initial momentum of the pair}} + \underbrace{\int_{t_i}^{t_\mathrm{f}} [\vec{F}_{ext,1} + \vec{F}_{ext,2}] dt}_{\text{Impulse imparted on the particles by external forces}}$$

PRINCIPLE OF LINEAR IMPULSE & MOMENTUM: Two (or more) particles



- For an arbitrary set of particles (more than two), we will get:

$$\sum_n m_n \vec{v}_{n,\mathrm{f}} = \sum_n m_n \vec{v}_{n,i} + \sum_i \left(\int_{t_i}^{t_\mathrm{f}} \vec{F}_{ext,n}\, dt \right)$$

here n is the # of the particle

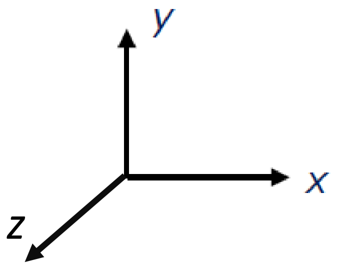
(internal forces cancel out due to 3rd Newton's law)

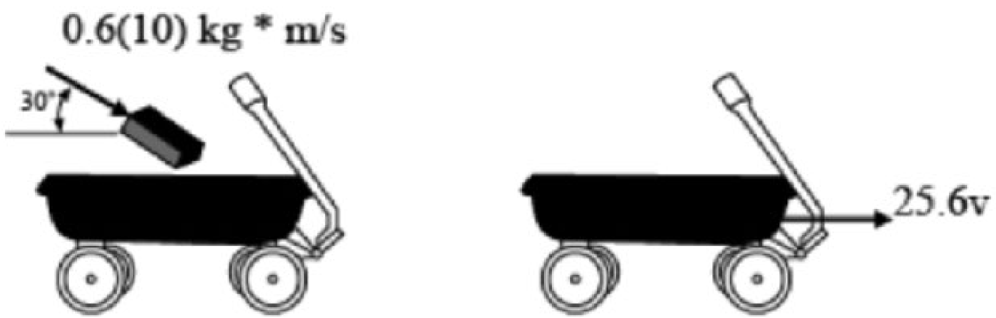
- Assume that there are no net external forces acting on the system (only inter-particle interactions)
  - ➢ …or that all the external forces are balanced, so that $\vec{F}_{R,ext} = 0$
- Then the total linear momentum of the system conserves:

$$\boxed{\sum_n m_n \vec{v}_{n,\mathrm{f}} = \sum_n m_n \vec{v}_{n,i}}$$

- Though it is difficult to imagine a system with no external forces acting on it, we can easily have a situation when there are no forces acting along one or two dimensions
  - ➢ …or with all the external forces balanced along one or two dimensions…
- Then linear momentum will conserve along these directions.

Q: A cart is moving to the right when a brick falls onto it as shown. What can you say about the momentum of the brick-cart system? Ignore air drag and rolling friction.





A. Only its z-component conserves
B. Its x- and z-components conserve
C. Its y- and z-components conserve
D. All its components conserve
E. None of its components conserve

W12-1. The free-rolling smooth ramp weighs 120 lb. The 80 lb crate slides 15 ft down the ramp to B from rest at A.

a) Determine the speed of the ramp when the crate reaches B

b) Determine the velocity of the crate when it reaches B. Express the velocity as a Cartesian vector in terms of the crate's speed and the angle the velocity makes with the horizontal

c) Determine the kinetic energies of the ramp and the crate when the crate reaches B.

Q: Which coordinate system?

A. Cartesian

B. (n,t)

C. Polar

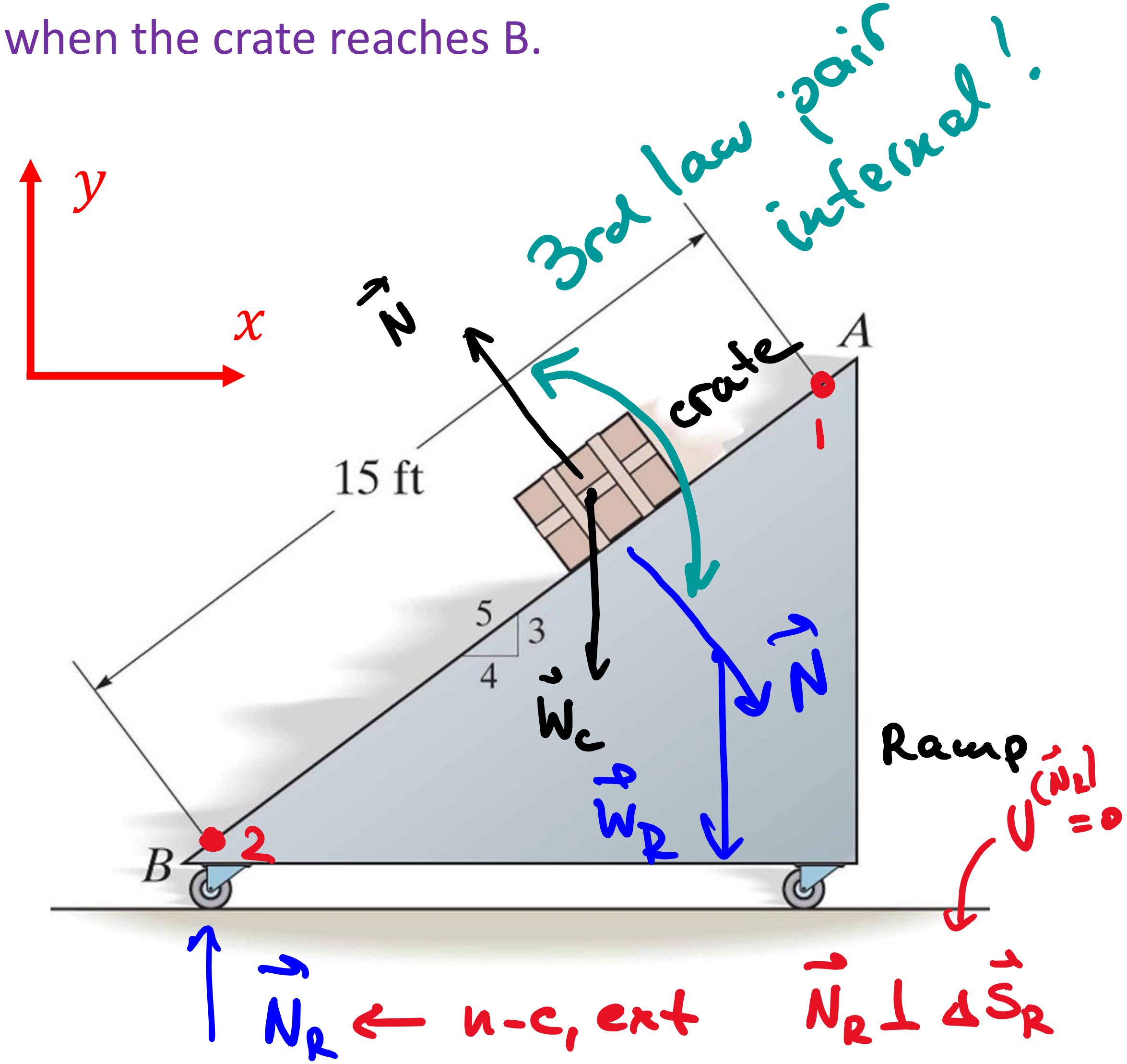
Q: Which system do you want to consider?

A. Crate

B. Ramp

C. Crate and Ramp, one at time

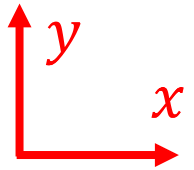
D. Combined system of Crate and Ramp



W12-1. The free-rolling smooth ramp weighs 120 lb. The 80 lb crate slides 15 ft down the ramp to B from rest at A. $v_{R2}=?$ $v_{c2}=?$

C + R

- Work-Energy principle:



1: $v_{c1}=0$, $v_{R1}=0$; $y_{c1}=15\sin\theta = 15\frac{3}{5}$, y_{R1} ;

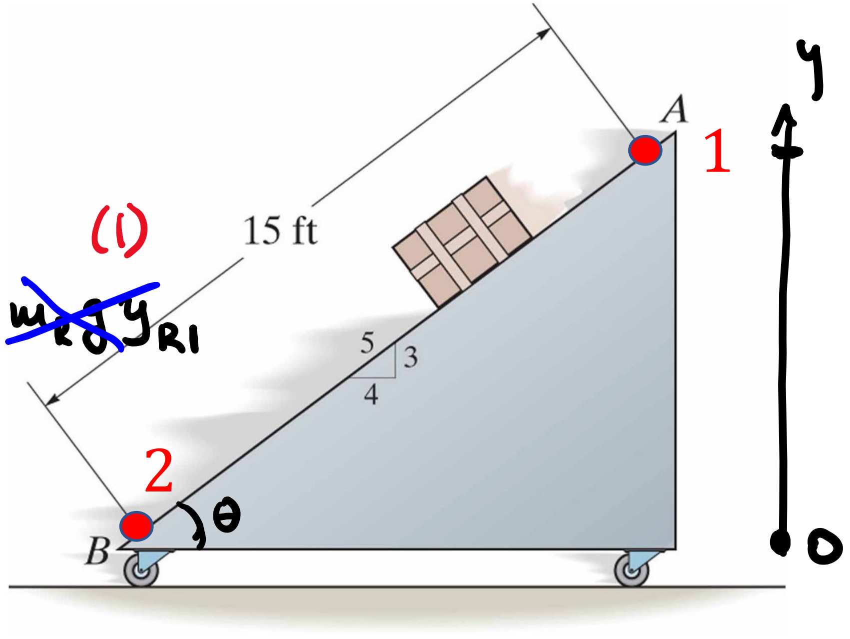
2: $v_{c2}=?$ $v_{R2}=?$; $y_{c2}=0$ (choice), y_{R2} ; no springs.

$$U_{1\to 2}^{(n-c)(\text{external})} = 0 \qquad (\vec{N}_R \perp \Delta\vec{s}_R)$$

$$ME_2 = ME_1 \qquad ME = T + V^{(g)}$$

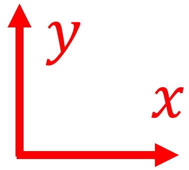
$$\frac{m_c v_{c2}^2}{2} + \frac{m_R v_{R2}^2}{2} + 0 + \cancel{m_R g y_{R2}} = 0 + 0 + m_c g\, 15\cdot\frac{3}{5} + \cancel{m_R g y_{R1}} \quad (1)$$

f i



W12-1. The free-rolling smooth ramp weighs 120 lb. The 80 lb crate slides 15 ft down the ramp to B from rest at A. $v_{R2}=?\ v_{c2}=?$

- Momentum conservation:



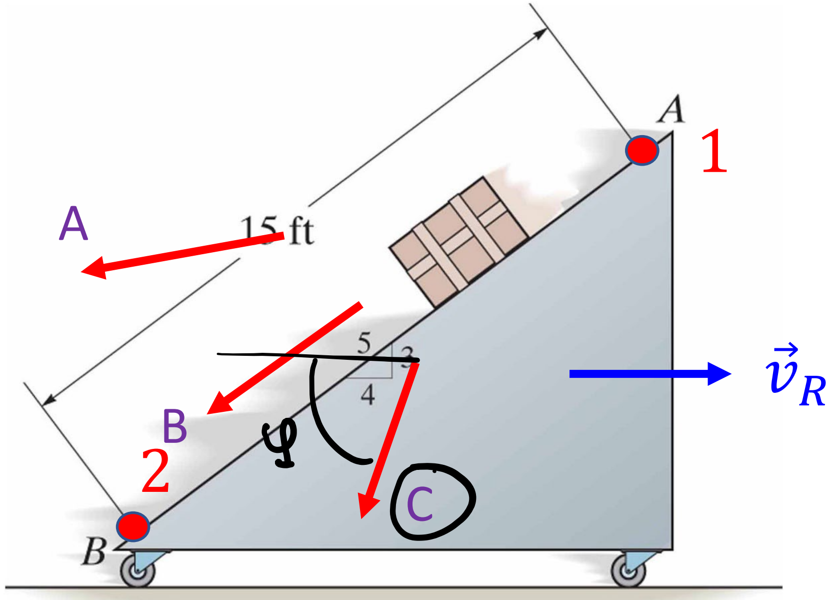
Q: Along which axes does the momentum conserve?

A. x
B. y
C. x and y
D. None

$$\boxed{L_{2x} = L_{1x}}$$

$$\vec{v}_R = \vec{i}\, v_R$$

$$\vec{v}_c = -\vec{i}\, v_c \cos\varphi - \vec{j}\, v_c \sin\varphi$$



Q: What is the direction of $\vec{v}_c$?

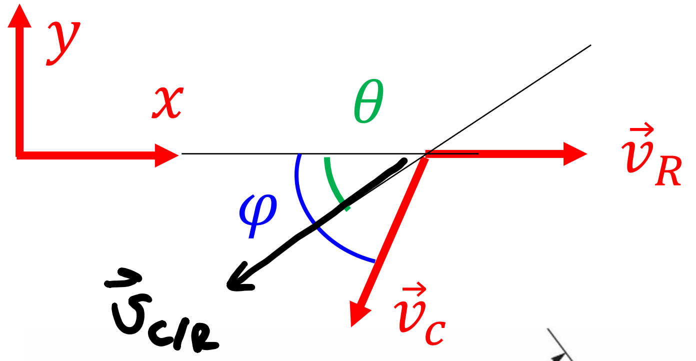
$$m_R v_{R2} - m_c v_{c2}\cos\varphi = m_R \cancel{v_{R1}}^{0} - m_c \cancel{v_{c1}}^{0}\cos\varphi = 0$$

$$m_R v_{R2} - m_c v_{c2}\cos\varphi = 0 \qquad (2)$$

D. None of the above

W12-1. The free-rolling smooth ramp weighs 120 lb. The 80 lb crate slides 15 ft down the ramp to B from rest at A.
$v_{R2} =?$ $v_{c2} =?$

- Relative motion: $\vec{v}_{C/R} = ?$

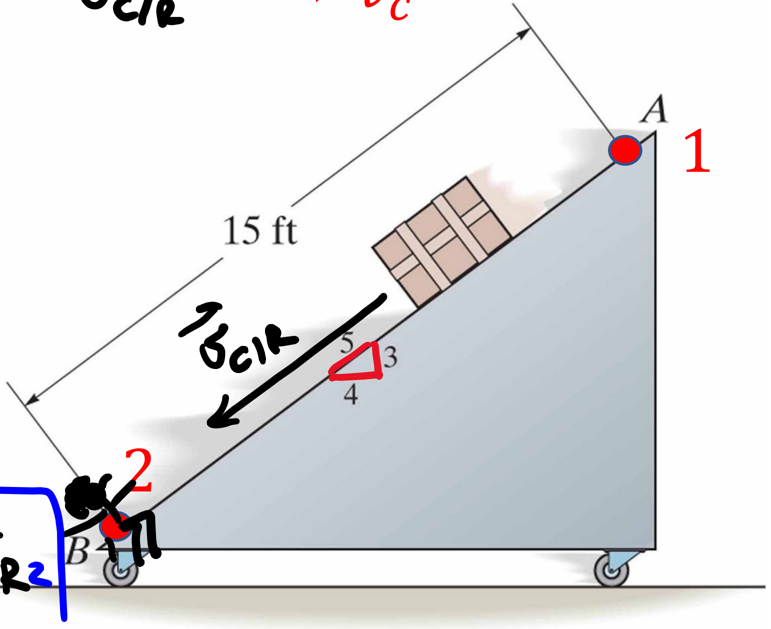


1) $\vec{v}_{C/R} = -\vec{i}\, v_{C/R}\cos\theta - \vec{j}\, v_{C/R}\sin\theta$

2) $\vec{v}_{C/R} = \vec{v}_C - \vec{v}_R$ ✓

$$\left\{\begin{array}{l} \vec{v}_C = -\vec{i}\, v_C\cos\varphi - \vec{j}\, v_C\sin\varphi \\ \vec{v}_R = \vec{i}\, v_R \end{array}\right.$$

$$\vec{v}_{C/R} = -\vec{i}\,(v_C\cos\varphi + v_R) - \vec{j}\, v_C\sin\varphi$$



$$v_{C/R}\overset{4/5}{\cos\theta} = v_C\cos\varphi + v_R \rightarrow \boxed{\frac{\cancel{5}}{3} v_{C2}\sin\theta \cdot \frac{4}{\cancel{5}} = v_{C2}\cos\varphi + v_{R2}}$$

$$v_{C/R}\overset{3/5}{\sin\theta} = v_C\sin\varphi \longrightarrow v_{C/R} = \frac{5}{3} v_C\sin\varphi$$

W12-1. The free-rolling smooth ramp weighs 120 lb. The 80 lb crate slides 15 ft down the ramp to B from rest at A.

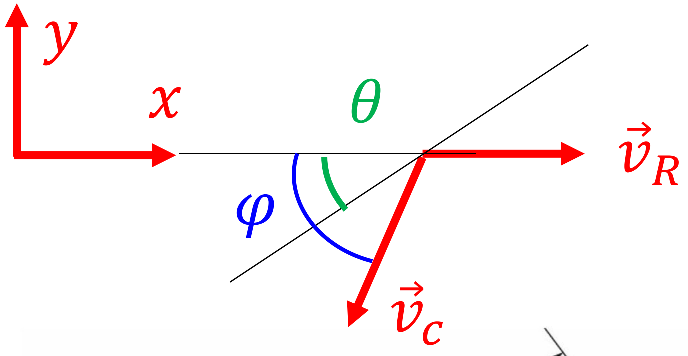
$v_{R2} =?\ v_{c2} =?$

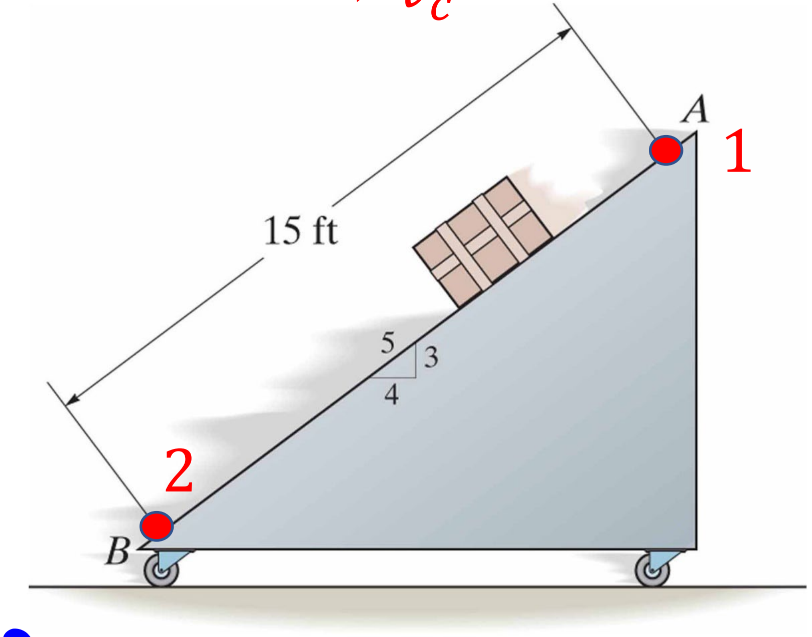
- Finalize:

$$m_C \frac{v_{C2}^2}{2} + m_R \frac{v_{R2}^2}{2} = \frac{3}{5} m_C g\, L$$

$$m_R v_{R2} - m_c v_{C2} \cos\varphi = 0$$

$$3 v_{C2} \cos\varphi + 3 v_{R2} = 4 v_{C2} \sin\varphi$$





$v_{R2} = 8.93\ \frac{ft}{s}$

$v_{C2} = 21.4\ \frac{ft}{s}$

$\varphi = 51.3°$

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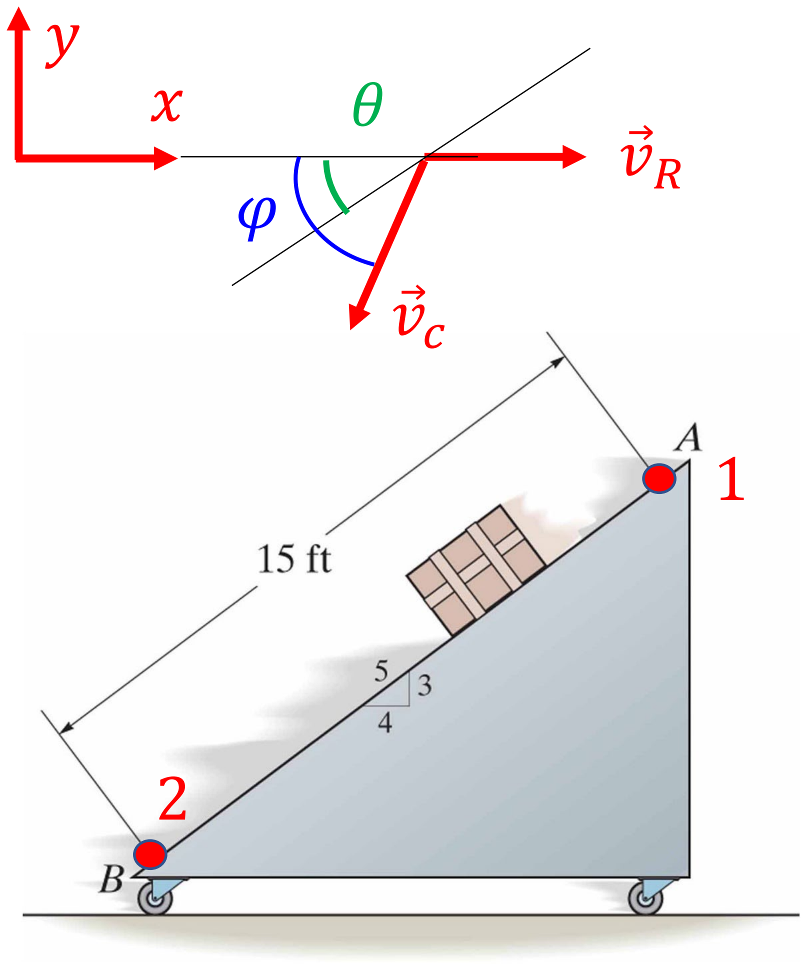
$v_{R2} = 8.93$ ft/s, $\quad v_{C2} = 21.4$ ft/s, $\quad \varphi = 51.3^o$

Please finish on your own!



PHYS 170 CONCEPTS (the list is not exhaustive)

- Vector equations / Scalar equations
- Forces: translations and rotations / Couple moments: rotations
- Translational (forces) and rotational (couple moments + force moments) equilibrium
- External forces and moments / Reaction forces and moments
- Competing scenarios of breaking equilibrium / impending motion eqs vs restrictions

Statics

Dynamics

- Coordinate systems: many of them! / Choice: convenience / Switching between them
- Acceleration: Can change magnitude (a_t) or direction (a_n) of the velocity ($\vec{v} = \text{vector}$)
- Dependent (constrained) motion
- Relative motion
- Newton's 2nd law: $\vec{F} \Leftrightarrow m\vec{a}$
- Energy ⇔ Work / Work-Energy principle / Momentum ⇔ Impulse / Impulse-Momentum principle

Please fill out the teaching evaluation survey now!

- Teaching evaluations are anonymous
- Who reads your comments?
  - I read them (to understand what worked well, and what didn't)
  - Our administration (to make their decisions about future appointments)

If you are not one of those 11 well-organized people – please fill it out now!

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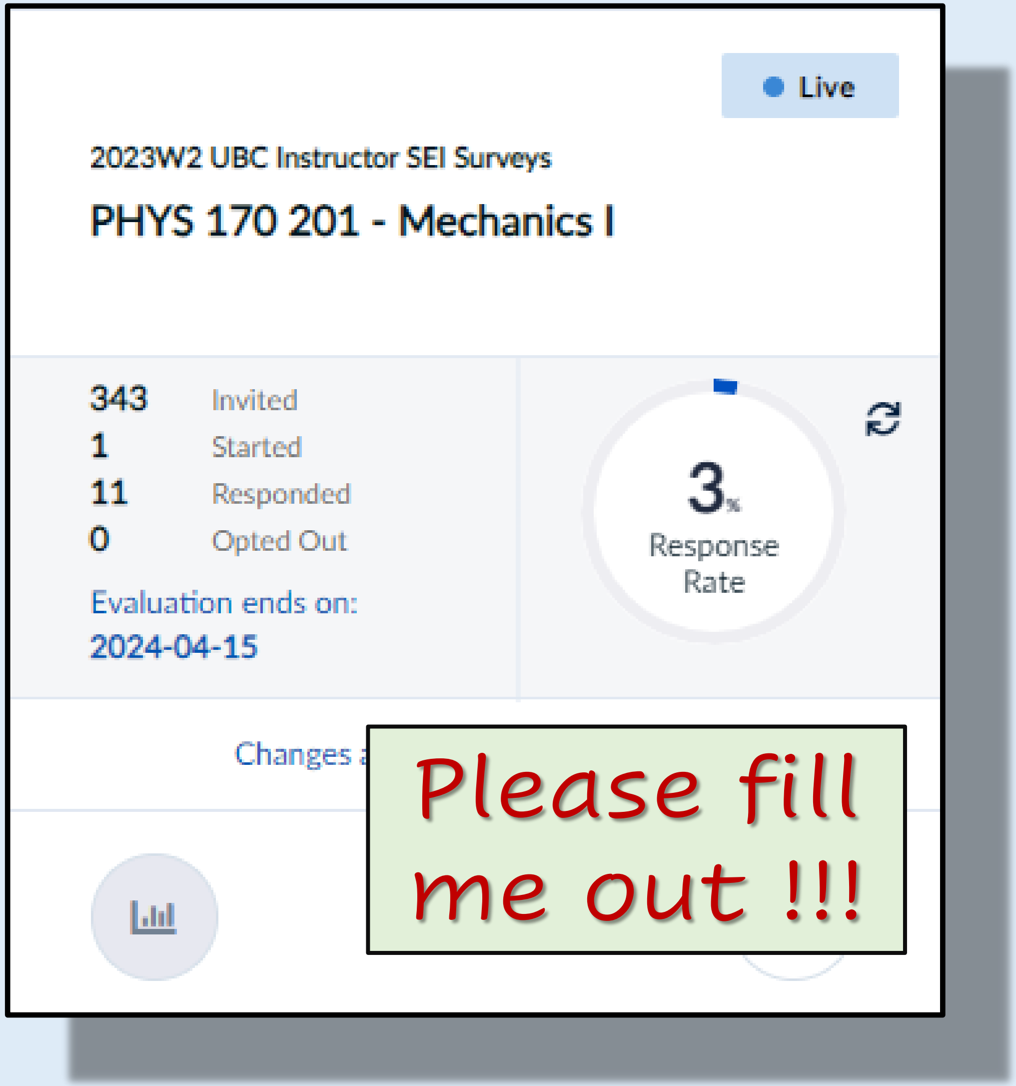

THE END !!!

- Happy final exam ! -