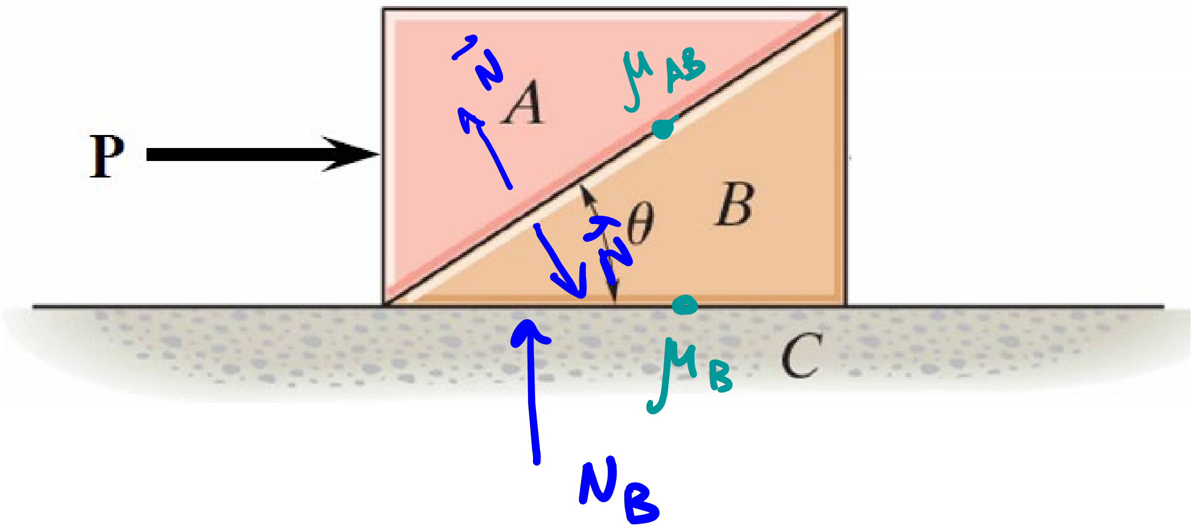
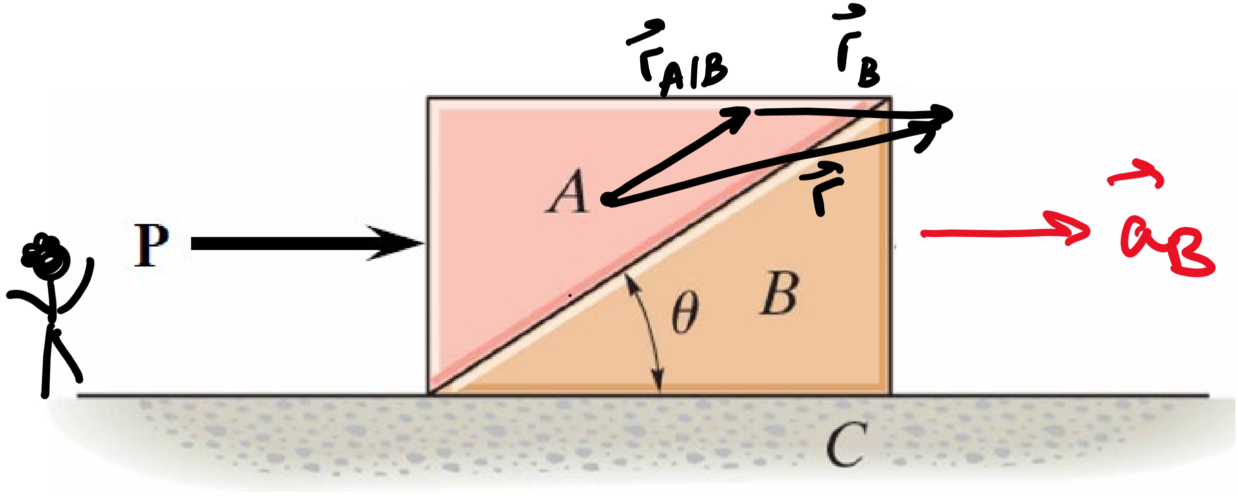
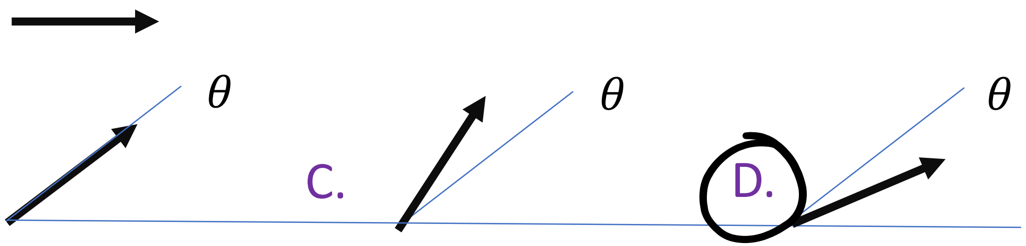
W9-3. The diagram shows two triangular blocks A and B each with mass 2 kg. B is on a horizontal surface. The sloped surface of B makes an angle θ=40° with the horizontal surface. The coefficient of kinetic friction between B and the horizontal surface is 0.5. The coefficient of kinetic friction between the two blocks is 0.2. A horizontal force $P = 50$ N acts to the right on A. Determine the acceleration of each block and the normal forces.



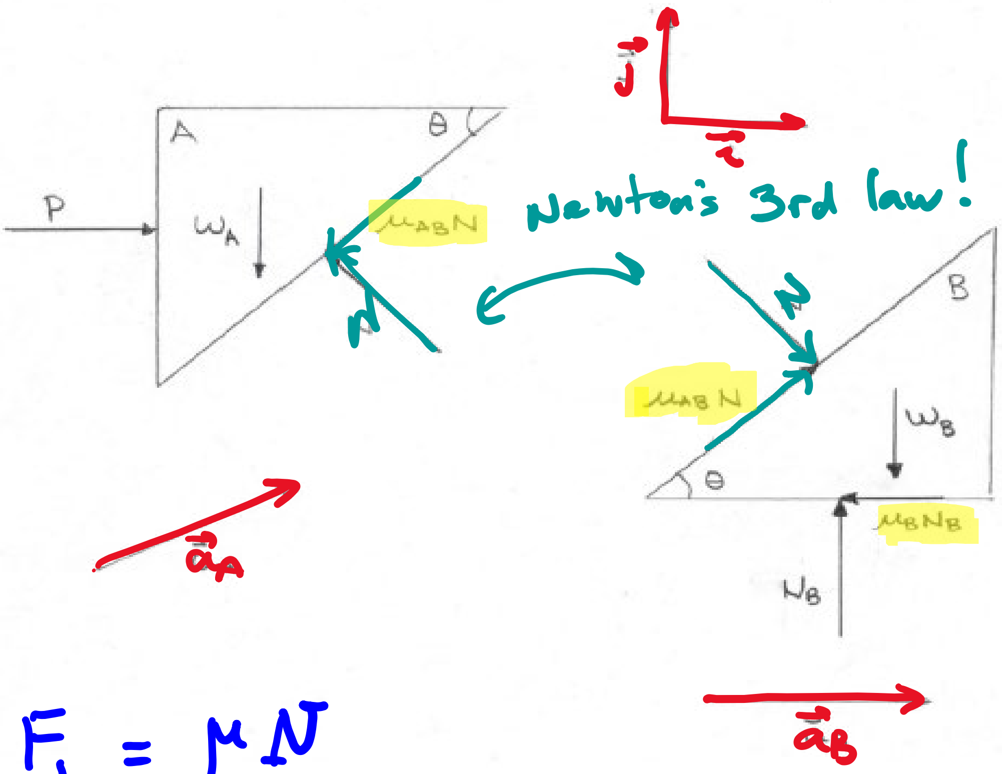
W9-3. The diagram shows two triangular blocks A and B each with mass 2 kg. B is on a horizontal surface. The sloped surface of B makes an angle θ=40° with the horizontal surface. The coefficient of kinetic friction between B and the horizontal surface is 0.5. The coefficient of kinetic friction between the two blocks is 0.2. A horizontal force $P = 50$ N acts to the right on A. Determine the acceleration of each block and the normal forces.



Okay, a_B is to the right. What is the direction of a_A?



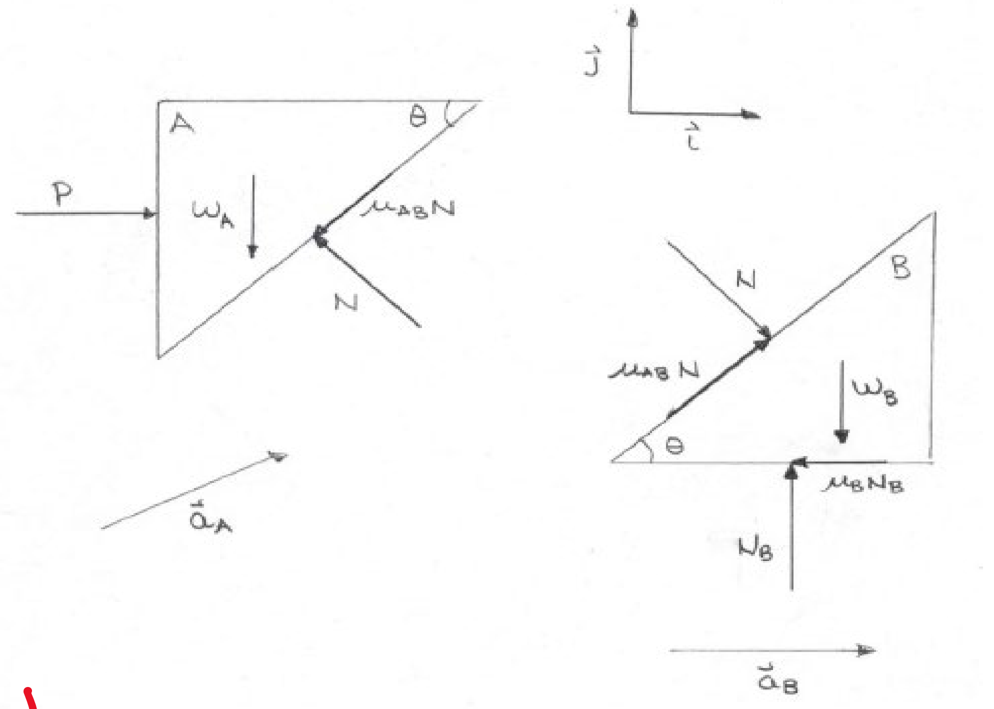
E. Other



$F_k = \mu N$

W9-3. $m_A = m_B = 2$ kg, θ=40°, $\mu_B = 0.5$, $\mu_{AB} = 0.2$, $P = 50$ N.

$a_A, a_B, N_B, N = ?$



$$A: \quad \vec{F}_{R,A} = m_A \vec{a}_A$$

$$x: \quad P - \mu_{AB} N \cos\theta - N \sin\theta = m_A a_{Ax} \quad (1)$$

$$y: \quad N\cos\theta - \mu_{AB} N \sin\theta - W_A = m_A a_{Ay} \quad (2)$$

$$B: \quad \vec{F}_{RB} = m_B \vec{a}_B$$

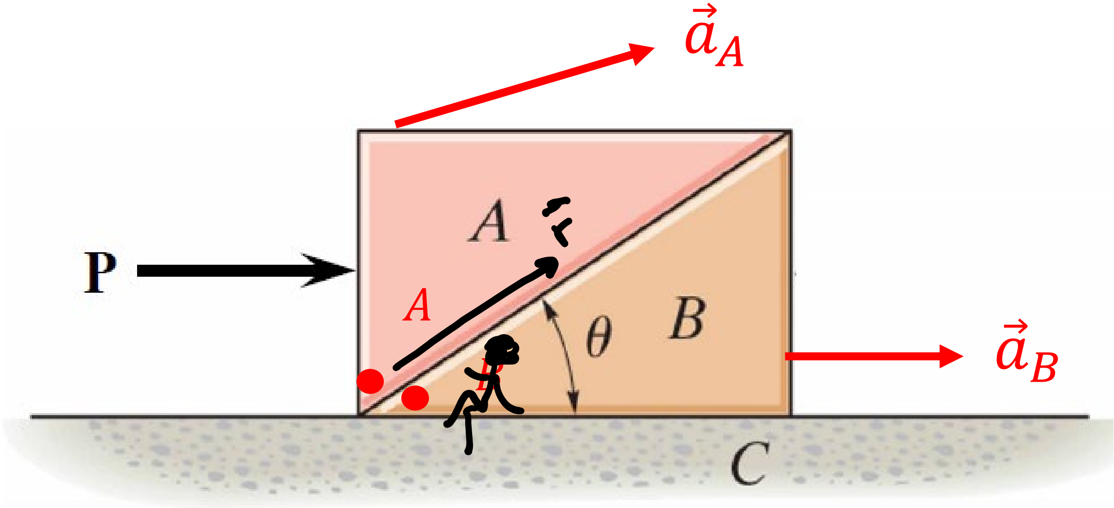
Relative motion!

$$x: \quad \mu_{AB} N\cos\theta + N\sin\theta - \mu_B N_B = m_B a_B \quad (3)$$

$$y: \quad N_B + \mu_{AB} N \sin\theta - N\cos\theta - W_B = 0 \quad (4) \quad (a_{By} = 0)$$

W9-3. $m_A = m_B = 2$ kg, θ=40º, $\mu_B = 0.5$, $\mu_{AB} = 0.2$, $P = 50$ N.

$a_A, a_B, N_B, N = ?$



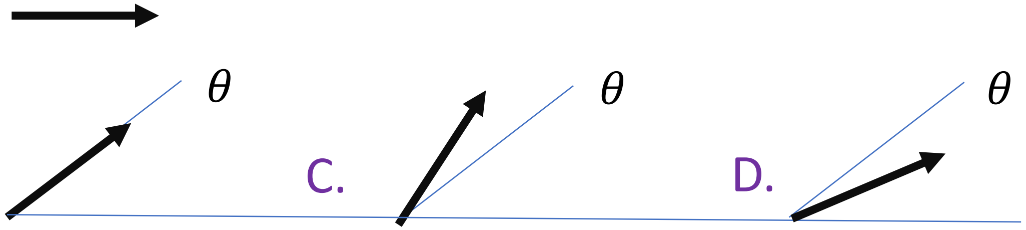
- Assume you are sitting in point B of block B and look at the point A of block A that is mowing away from you. What is $\vec{a}_{A/B}$?

A.

B.

C.

D.



E. Other

- First representation (visualization):

$$\vec{a}_{A/B} = \vec{i}\,(a_{A/B}\cos\theta) + \vec{j}\,(a_{A/B}\sin\theta)$$

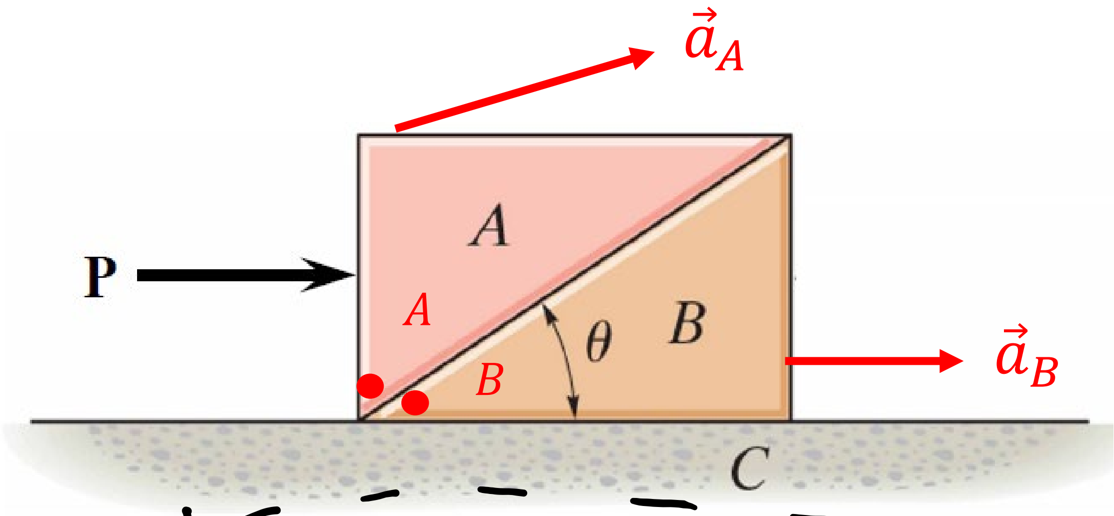
$a_{A/B}$

- Second representation: $\vec{a}_{A/B} = \vec{a}_A - \vec{a}_B$

$$\vec{a}_{A/B} = (\vec{i}\,a_{Ax} + \vec{j}\,a_{Ay}) - (\vec{i}\,a_B) = \vec{i}\,(a_{Ax} - a_B) + \vec{j}\,(a_{Ay})$$

W9-3. $m_A = m_B = 2$ kg, θ=40º, $\mu_B = 0.5$, $\mu_{AB} = 0.2$, $P = 50$ N.

$a_A, a_B, N_B, N = ?$



- First representation (visualization):

$$a_{A/B,x} = \underline{a_{A/B}\cos\theta}, \qquad a_{A/B,y} = \underline{a_{A/B}\sin\theta}$$

- Second representation: $\vec{a}_{A/B} = \vec{a}_A - \vec{a}_B$

$$a_{A/B,x} = \underline{a_{A,x} - a_B}, \qquad \underline{a_{A/B,y} = a_{A,y}}$$

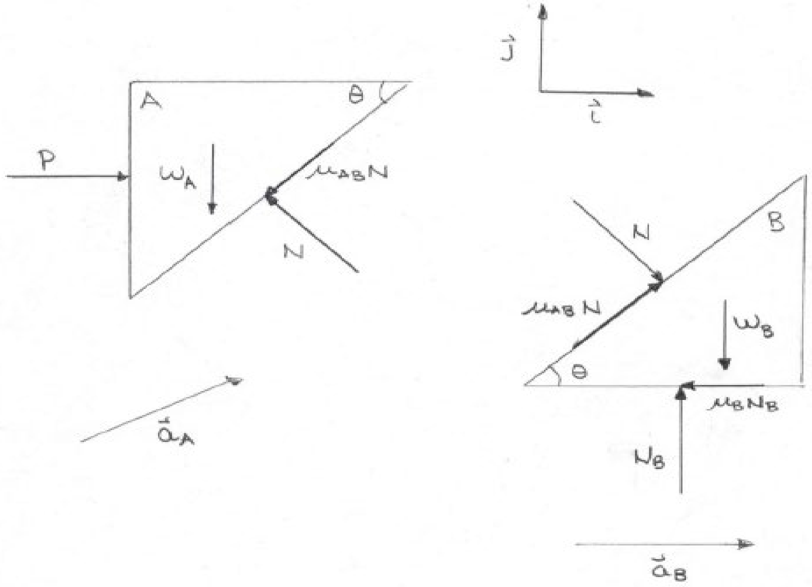
$$a_{A/B}\cos\theta = a_{Ax} - a_B$$

$$a_{A/B}\sin\theta = a_{Ay} \longrightarrow a_{A/B} = \frac{a_{Ay}}{\sin\theta}$$

$$\frac{a_{Ay}}{\sin\theta}\cos\theta = a_{Ax} - a_B \longrightarrow \boxed{a_{Ay} = \tan\theta\,(a_{Ax} - a_B)}$$

W9-3. $m_A = m_B = 2$ kg, θ=40°, $\mu_B = 0.5$, $\mu_{AB} = 0.2$, $P = 50$ N.

Acceleration of each block? Normal forces?



$$P - N\sin\theta - \mu_{AB}N\cos\theta = m_A a_{A,x}$$

$$N\cos\theta - \mu_{AB}N\sin\theta - W_A = m_A a_{A,y}$$

$$N\sin\theta + \mu_{AB}N\cos\theta - \mu_B N_B = m_B a_B$$

$$N_B + \mu_{AB}N\sin\theta - N\cos\theta - W_B = 0$$

$$a_{A,y} = \tan\theta\,(a_{A,x} - a_B)$$

Solver:

$$\left\{\begin{array}{c} \mathrm{aAx} \to 8.71336501465564, \\ \mathrm{aAy} \to 3.233420184957929, \\ \mathrm{aB} \to 4.859924892865397, \\ N \to 40.9213734480392, \\ \mathrm{NB} \to 45.70684036991585 \end{array}\right\}$$

Let's summarize:

To solve a kinetic problem (forces acting on a particle $\Rightarrow$ find its acceleration), we:

- Use Newton's second law, $\vec{F}_R = m\,\vec{a}$, for each particle
  - Draw FBD for each particle to find $\vec{F}_R$
- Project it onto the axes of our choice
  - For that we need to specify the direction of motion, and therefore the direction of acceleration
- Project these vector equations onto the coordinate axes;
- If number of unknowns is less than the number of equations:
  - Supplement these equations by a relationship between the accelerations of the particles
  - Relevant concepts: absolute dependent motion, relative motion
- Solve for the unknowns

W9-4. A 1700 kg sports car travels around a 100 m radius horizontal circle on a 20° banked track. The coefficient of static friction between the tires and the road is 0.20.

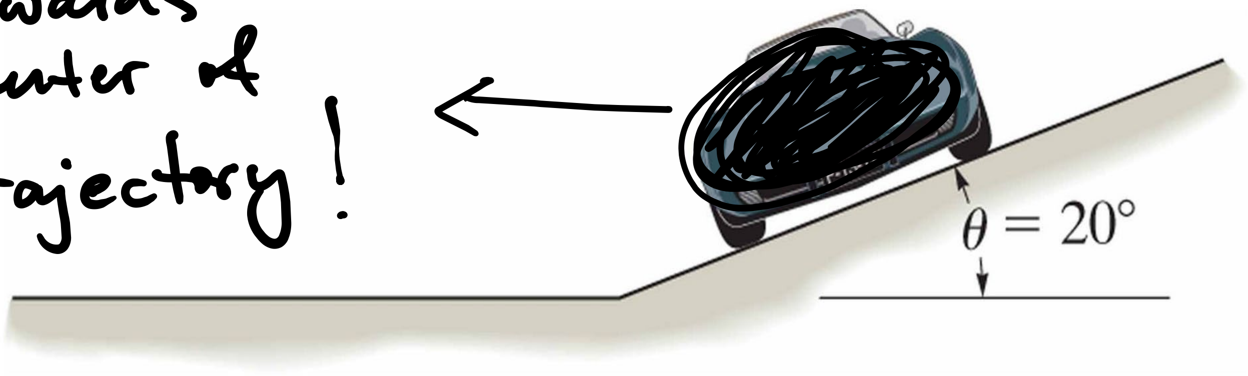
a) Determine the maximum constant speed the car can travel without slipping up the slope.

b) Determine the minimum constant speed the car can travel without slipping down the slope.

μ_s

$\vec{a}$

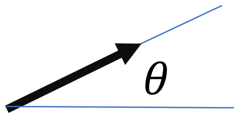
towards center of trajectory!

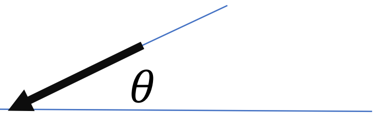


Q: What is the direction of normal acceleration at the instant shown?

A. →

B. ←

C. 

D. 

E. Other

W9-4. A 1700 kg sports car travels around a 100 m radius horizontal circle on a 20° banked track. The coefficient of static friction between the tires and the road is 0.20.

a) Determine the maximum constant speed the car can travel without slipping up the slope.

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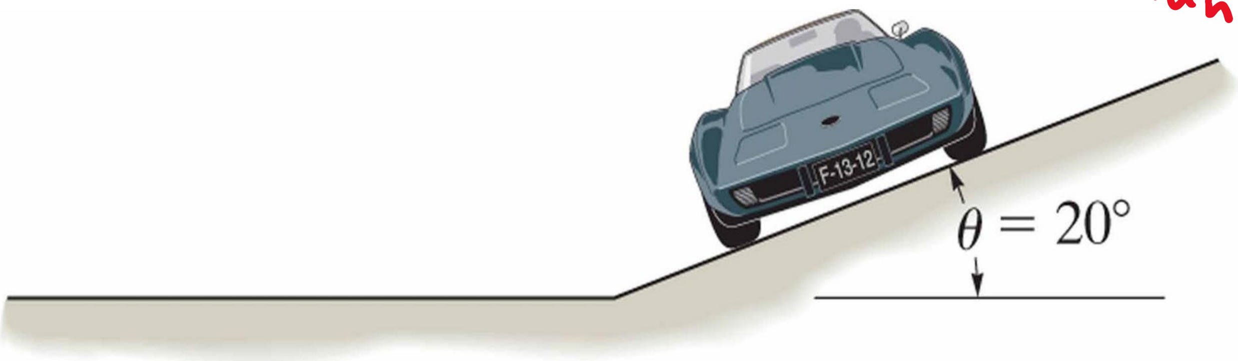
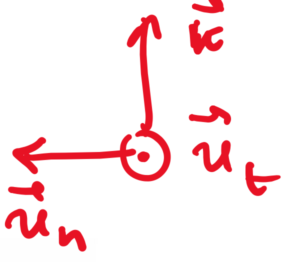
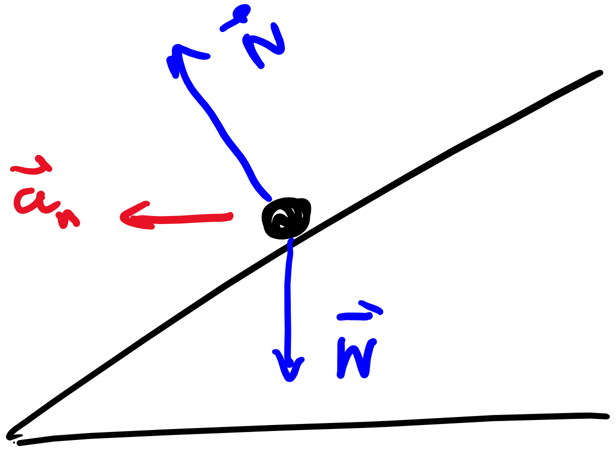






$$\vec{F}_R = \vec{W} + \vec{N} = m\vec{a}$$

$$n: \quad N\sin\theta = m a_n = m\frac{v^2}{R}$$

$$z: \quad N\cos\theta - mg = 0 \longrightarrow N = \frac{mg}{\cos\theta}$$

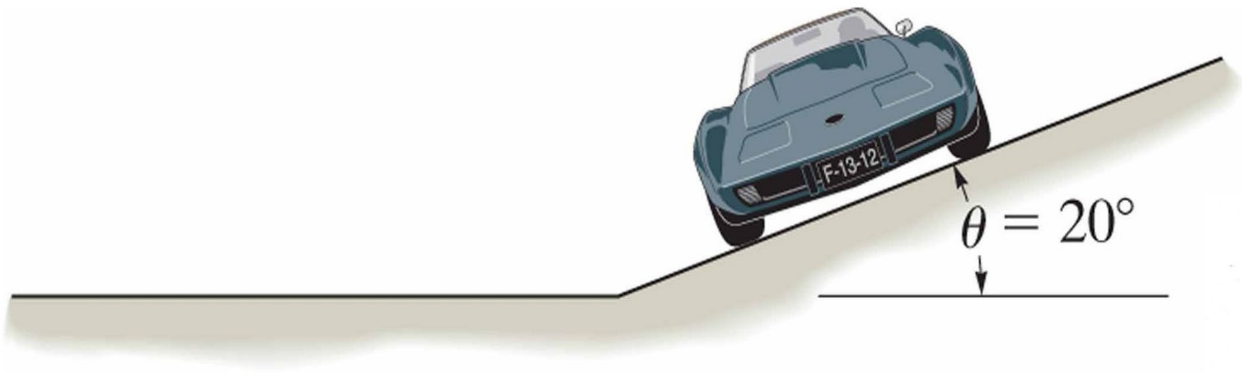
$$\cancel{m}g\tan\theta = \cancel{m}\frac{v^2}{R}$$

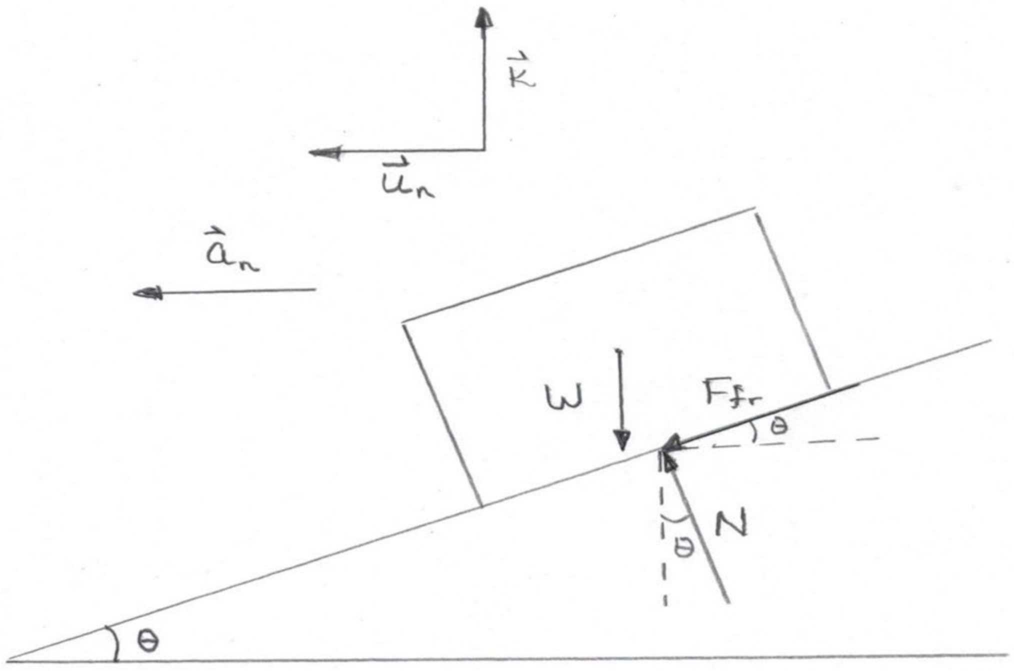
$$v^2 = Rg\tan\theta$$

$$v_* = \sqrt{Rg\tan\theta}$$

W9-4. A 1700 kg sports car travels around a 100 m radius horizontal circle on a 20º banked track. The coefficient of static friction between the tires and the road is 0.20.

a) Determine the maximum constant speed the car can travel without slipping up the slope.





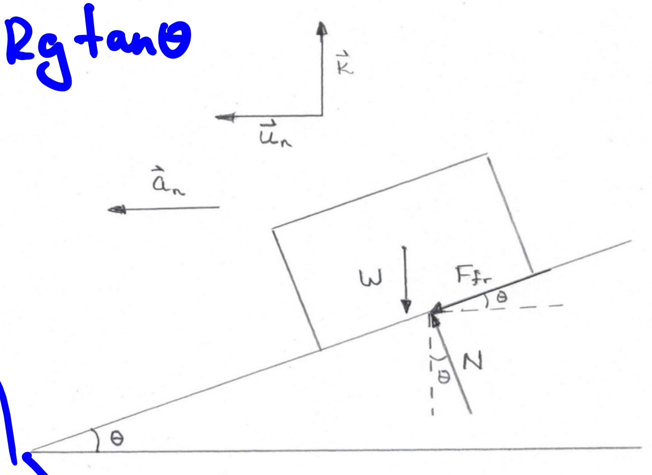
W9-4. A 1700 kg sports car travels around a 100 m radius horizontal circle on a 20° banked track. The coefficient of static friction between the tires and the road is 0.20.

R

a) Determine the maximum constant speed the car can travel without slipping up the slope.

$$\vec{F}_R = \vec{W} + \vec{N} + \vec{F}_s = m\vec{a}$$

$\mu = 0$: $\quad v = \sqrt{Rg\tan\theta}$



$\mu_s N$

n: $\quad N\sin\theta + F_s\cos\theta = m a_n = m\dfrac{v^2}{R}$ $\quad$ (1)

z: $\quad N\cos\theta - F_s\sin\theta - W = 0$ $\quad$ (2)

@: $\quad F_s = \mu_s N$ $\quad$ (3) Impending motion eq!

$$\boxed{v = \sqrt{Rg\,\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}}}$$

$$N = \frac{mg}{\cos\theta - \mu\sin\theta}$$

$$mg\,\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} = m\frac{v^2}{R}$$

24.4 m/s

↓

87.9 $\frac{km}{h}$

W9-4. A 1700 kg sports car travels around a 100 m radius horizontal circle on a 20º banked track. The coefficient of static friction between the tires and the road is 0.20.

b) Determine the minimum constant speed the car can travel without slipping down the slope.

From part a):

$$N \sin\theta - F_{\mathrm{fr}} \cos\theta = \frac{mv^2}{\rho}$$

$$N \cos\theta + F_{\mathrm{fr}} \sin\theta - mg = 0$$

$$F_{\mathrm{fr}} = \mu_s N$$

$$v = \sqrt{Rg\,\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}}$$

$12.2\ \frac{m}{s}$ $\quad$ $43.9\ \frac{km}{h}$

@ $\mu = 0$: $\quad v = v_* = \sqrt{Rg\tan\theta}$