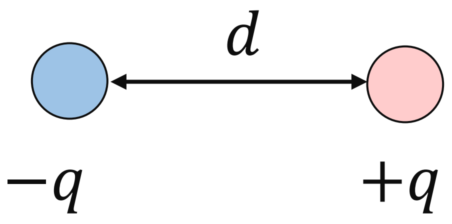
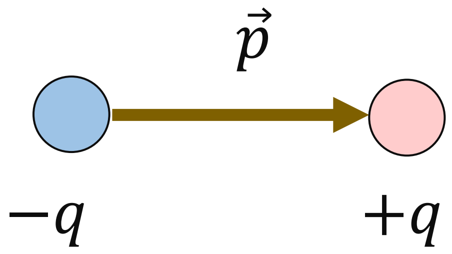
Lecture 16.

Dipoles in E-field.
Flux. Gauss's law.

Electric dipole

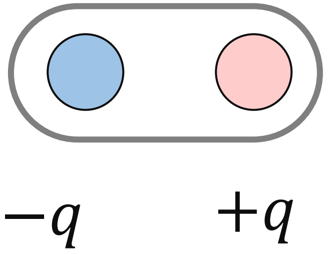




$|\vec{p}| = qd$

Dipole: Two charges of the same magnitude but opposite charge at a small distance, d

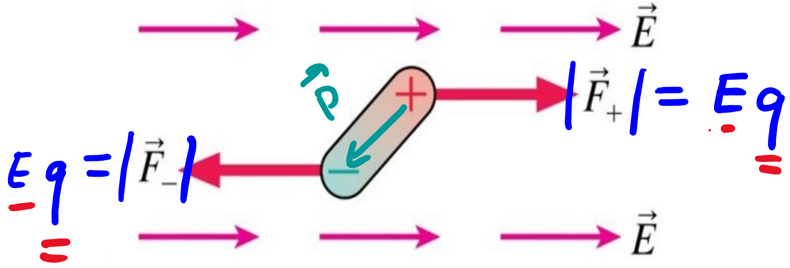
Electric dipole moment: points from negative to positive charge



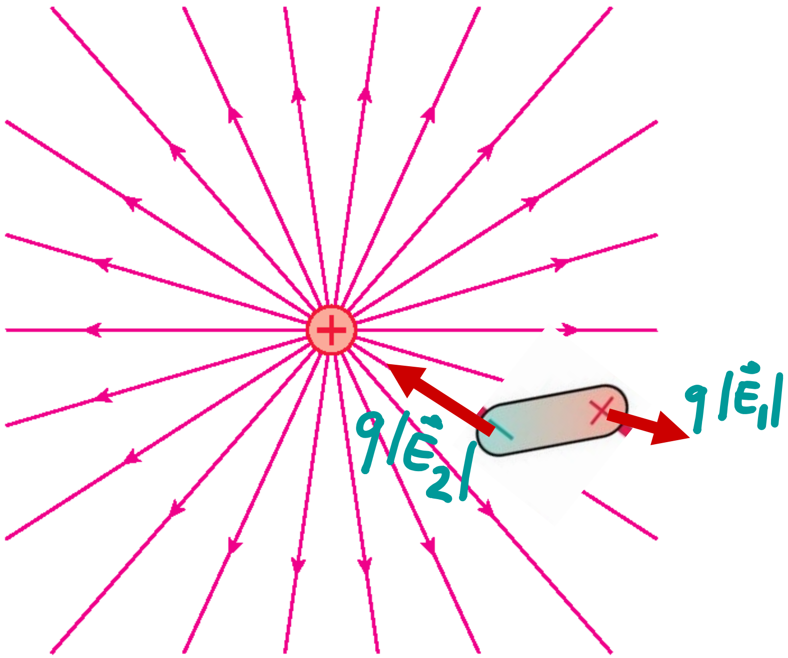
- You can visualize a dipole as a rigid unit that moves in electric field as a tiny rigid body under action of electric forces that act on its two charges

Dipole in electric field (qualitative description)

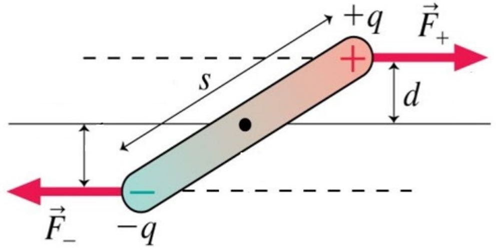
- In a uniform field:
  - Net force on the dipole is zero.
  - Field tends to align the dipole => may exert torque on the dipole, which will cause it to *rotate.*



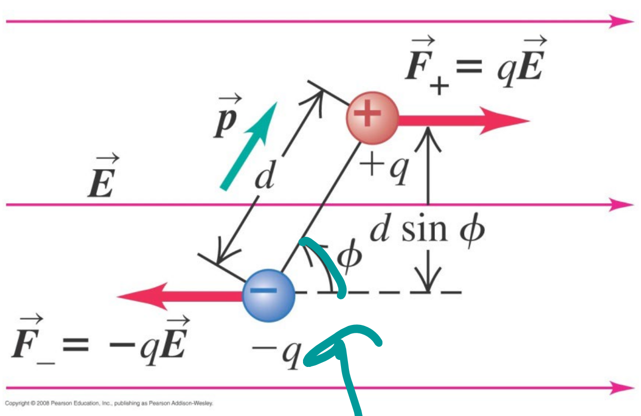
- In a non-uniform field:
  - Non-zero net force.
  - Torque: depends on the orientation.



Q: When torque on a dipole in a uniform field is equal to zero?



Torque on a dipole (quantitative description)



- When a dipole is immersed in an external E-field, we need to hold on to the dipole to keep it from rotating. In other words, the dipole experiences a torque.

- In a uniform electric field, the two forces acting on a dipole are a couple (PHYS 170).

- The moment of this coupe is: $\boxed{\vec{\tau} = \vec{p} \times \vec{F}}$

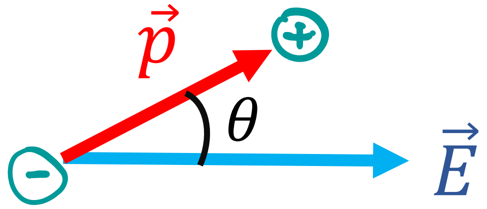
$\vec{F} = q\vec{E}$

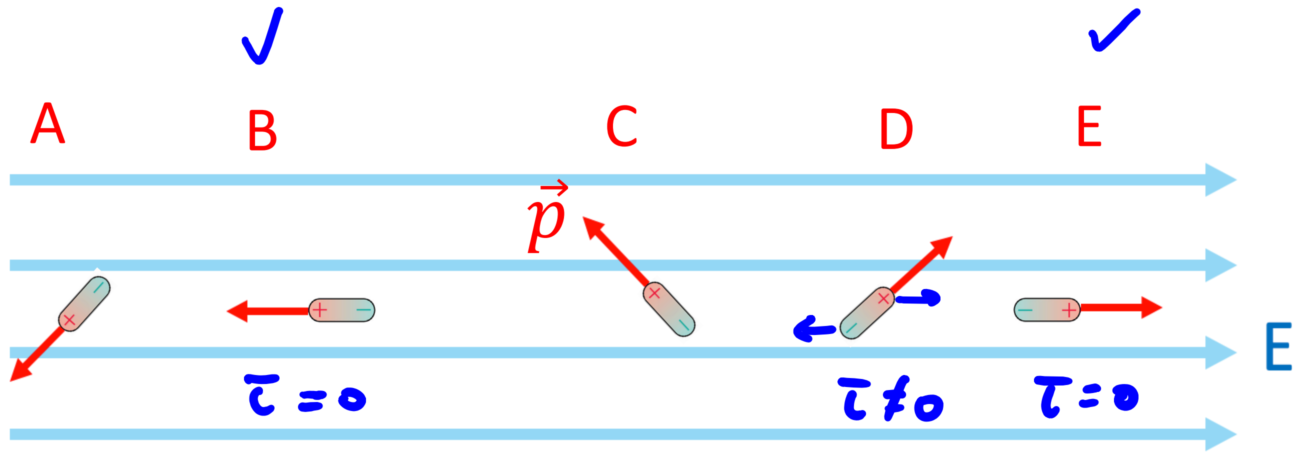
- The electric-field torque does work on a dipole to change its direction in an electric field, which corresponds to a change in its potential energy (coming soon).

- Potential energy of a dipole in E-field: $\boxed{U_e = -\vec{p} \cdot \vec{E}}$ $= -p \cdot E \cdot \cos\phi$ $\quad p = q \cdot d$

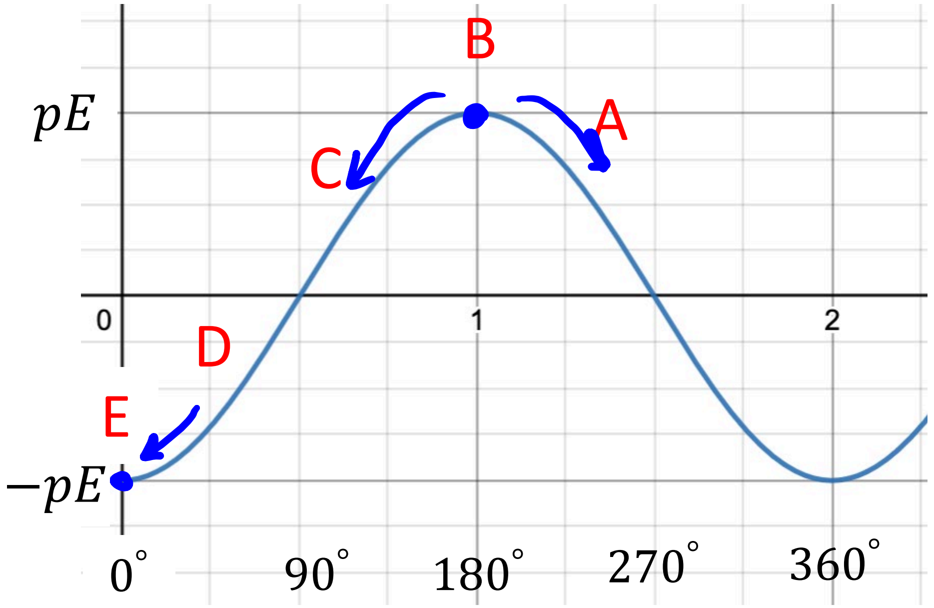
Electric Dipole in a Uniform Electric Field

$$\boxed{U_e = -\vec{p}\cdot\vec{E} = -pE\cos\theta}$$





$$U_e(\theta) = -pE\cos(\theta)$$



- Stable Equilibrium: $U_{\min} = -\,p\,E$: when $\vec{p}$ is parallel to $\vec{E}$ E
- Unstable Equilibrium: $U_{\max} = +\,p\,E$: when $\vec{p}$ is anti-parallel to $\vec{E}$ B

a) Which dipole experiences no net force in the electric field? E !

b) Which dipole experiences no net torque in the electric field? C

A. Dipole A.

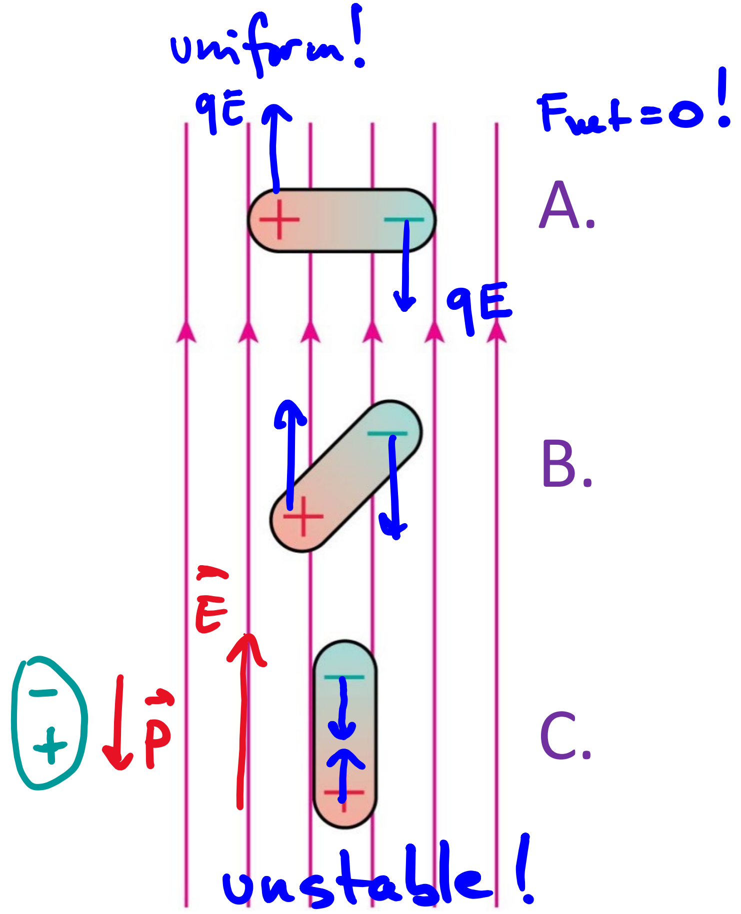
B. Dipole B.

C. Dipole C.

D. Both dipoles A and C.

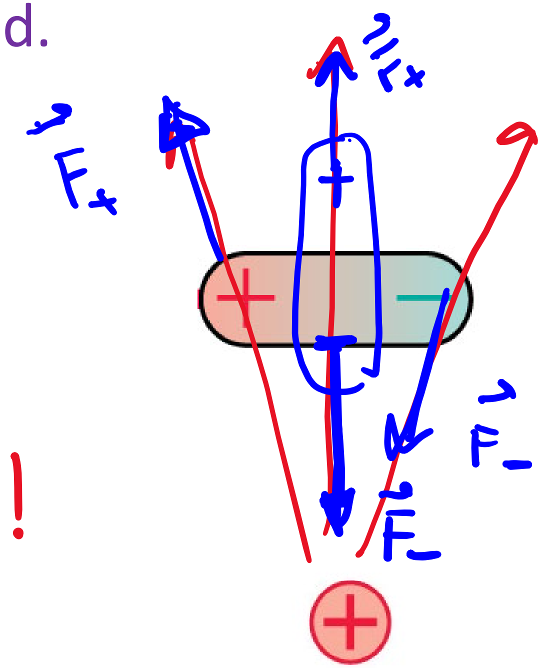
E. All three dipoles.

$$\vec{F} = q_{\pm}\vec{E}$$



Q: Suppose a dipole is in the electric field of a positive point charge as shown. What will happen?

A. The dipole will rotate counter-clockwise and then will be attracted.
B. The dipole will rotate counter-clockwise and then will be repelled.
C. The dipole will rotate clockwise and then will be attracted.
D. The dipole will rotate clockwise and then will be repelled.
E. The dipole will rotate clockwise.
F. The dipole will rotate counter-clockwise.



non-uniform!

cw

*) If F is your choice, don't submit your answer, make a wish instead.
It will come true if it is the correct answer.

Gauss's Law

Text: 22.1-4

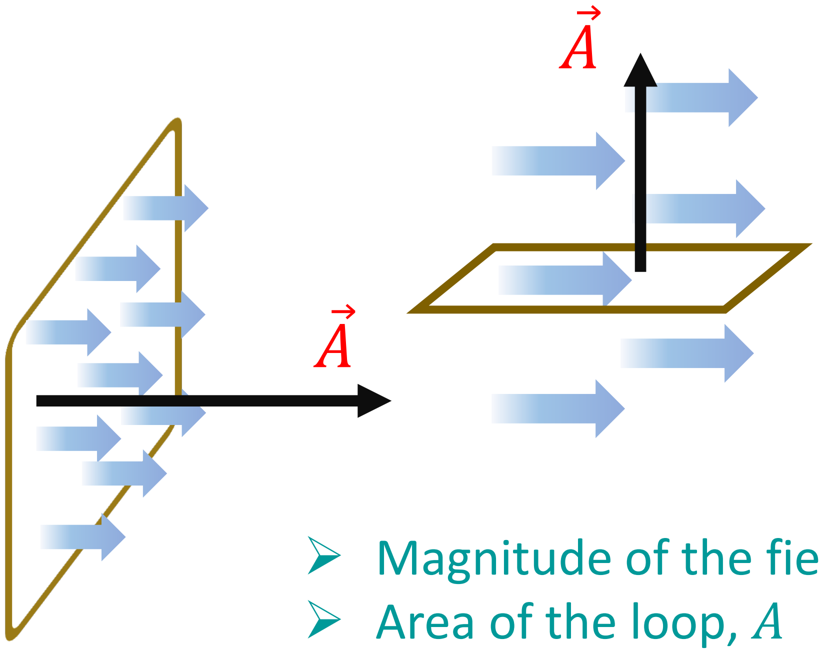
- Ch 22.1: Electric flux
- Ch 22.2: Calculating electric flux
- Ch 22.3: Gauss's law
- Ch 22.4: Applications: electric field of high-symmetry objects

Electric flux

- Definition: Flux = The amount of electric field passing through a surface

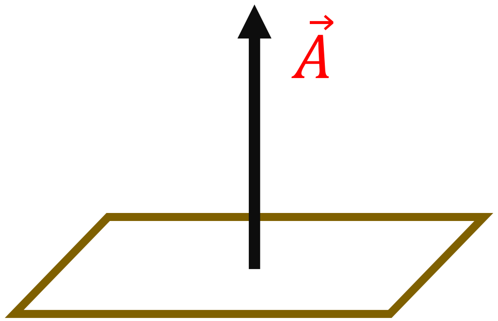
("number of electric field lines poking through the surface")

- What determines the flux?



- Magnitude of the field, E
- Area of the loop, A
- Angle between the loop and $\vec{E}$

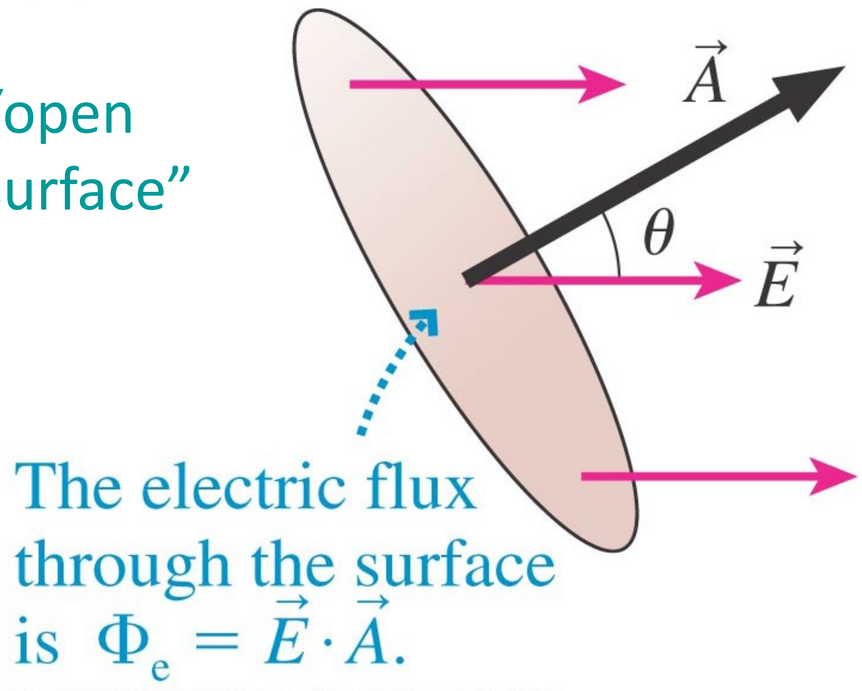
- Mathematical trick to account for orientation: let's make the area a vector quantity:
  - Magnitude: Area A
  - Direction: normal to the area



Electric flux through a plane surface with area A

- Electric flux for a uniform electric field and flat surface:

- “open surface”



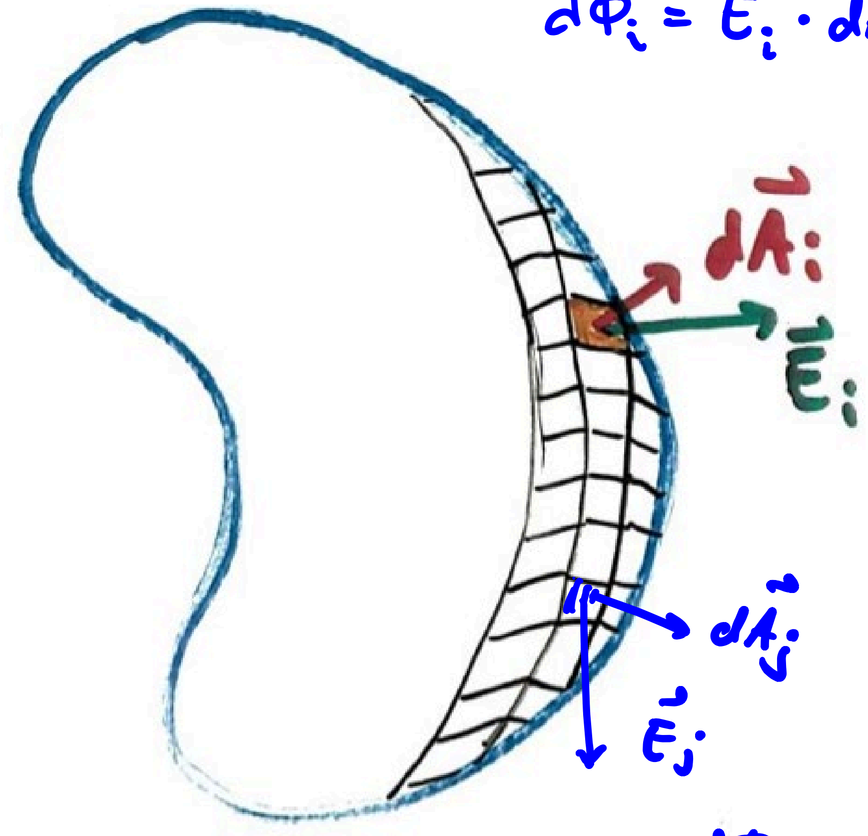
$$\boxed{\Phi_e = \vec{E} \cdot \vec{A} = EA\cos\theta}$$

- θ is the angle between $\vec{E}$ and $\vec{A}$
- $\vec{A}$ is the area vector (normal to the surface)

Electric flux through a curved surface and/or non-uniform electric field:

- Life is not always kind to us: The surface can be curved (and the vector $\vec{A}$ will change direction from point to point), and the electric field can be non-uniform (and the magnitude and direction of vector $\vec{E}$ will change from point to point)

$d\Phi_i = \vec{E}_i \cdot d\vec{A}_i$



$d\Phi_j = \vec{E}_j \cdot d\vec{A}_j$

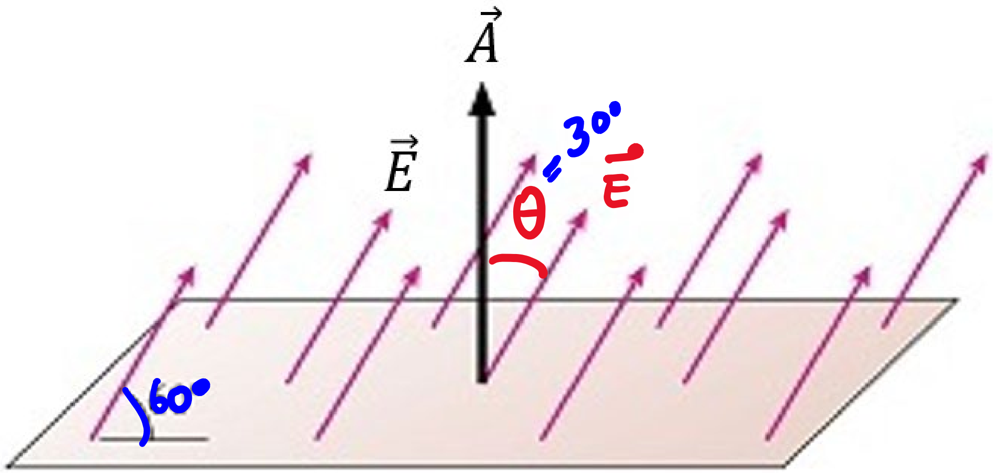
$$\Phi_e = \sum_i d\Phi_i = \sum_i \vec{E}_i \cdot d\vec{A}$$

$$\Phi_e = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

@ various points of the surface

➢ Note that in this integral $\vec{E}$ changes from point to point!

Q: A uniform electric field is passing through a flat rectangular sheet that is lying in the horizontal plane. What is the flux through the sheet of area A?



A. E A
B. E A cos(30)
C. E A tan(60)
D. E A cos(60)
E. Other

Q: A uniform electric field is passing through a flat rectangular sheet that is lying in the horizontal plane. What is the flux through the sheet of area A?

- D is a common mistake. Correct answer is B:
  - If the angle between the field and the surface is 60º, the angle between the field and the area vector is 30º!

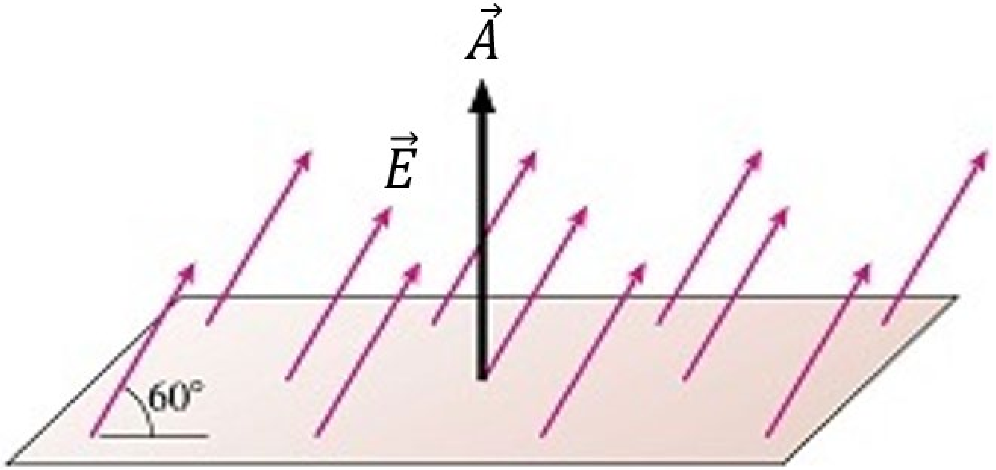
A. E A

B. E A cos(30)

C. E A tan(60)

D. E A cos(60)

E. Other



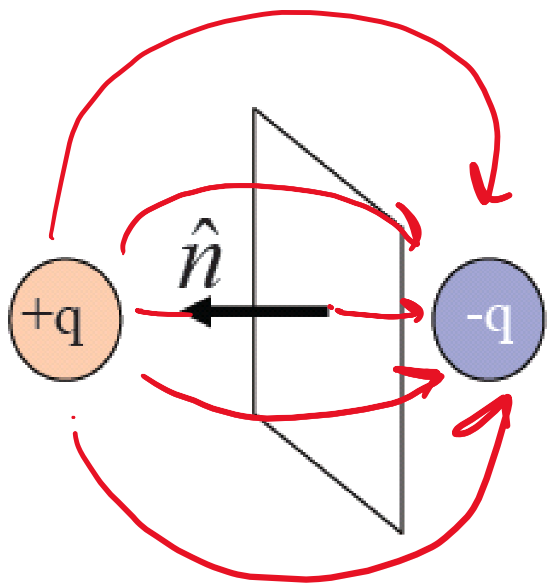
Q: A uniform electric field is passing through a flat rectangular sheet that is lying in the horizontal plane. What can you say about the flux through this sheet? Assume that the area vector is directed along the normal vector $\hat{n}$.

$90^\circ < \theta < 180^\circ$

$\cos\theta < 0$

$\Phi_e = \sum_i |E_i||dA_i| \underline{\underline{\cos\theta_i}}$

negative



A: The flux is positive.

B: The flux is negative.

C: The flux is zero.

D: I don't know

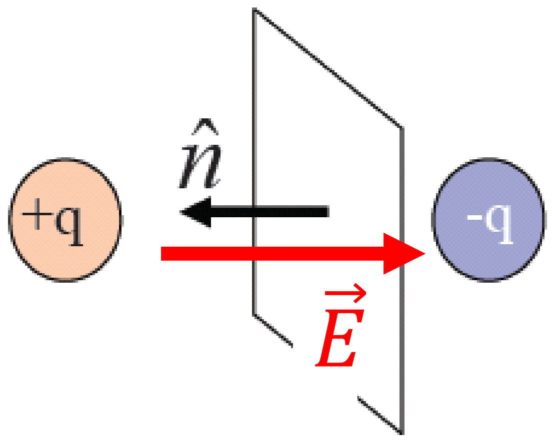
$\vec{E} \uparrow\uparrow \vec{A}$: $\Phi_e > 0$

$\vec{E} \uparrow\downarrow \vec{A}$: $\Phi_e < 0$

Q: A uniform electric field is passing through a flat rectangular sheet that is lying in the horizontal plane. What can you say about the flux through this sheet? Assume that the area vector is directed along the normal vector $\hat{n}$.

$$\boxed{\Phi_e = \vec{E} \cdot \vec{A} = EA\cos\theta}$$

- $\vec{E}$ always points from $+$ to $-$
- $\vec{E} \uparrow\downarrow \hat{n} \quad \Rightarrow \quad \cos\theta < 0 \quad \Rightarrow \quad$ flux is negative



A. The flux is positive.

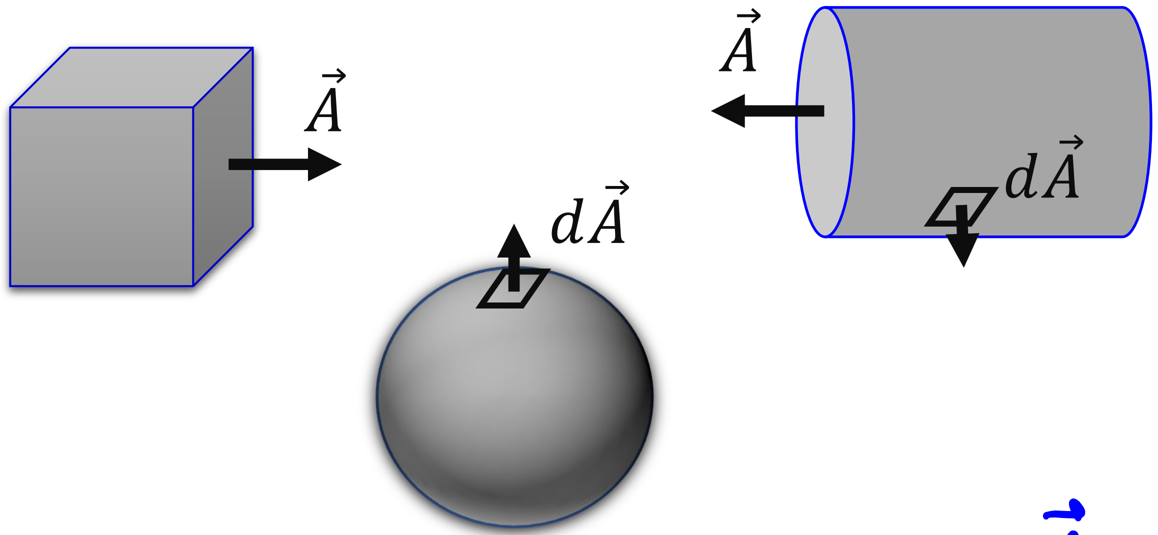
B. The flux is negative.

C. The flux is zero.

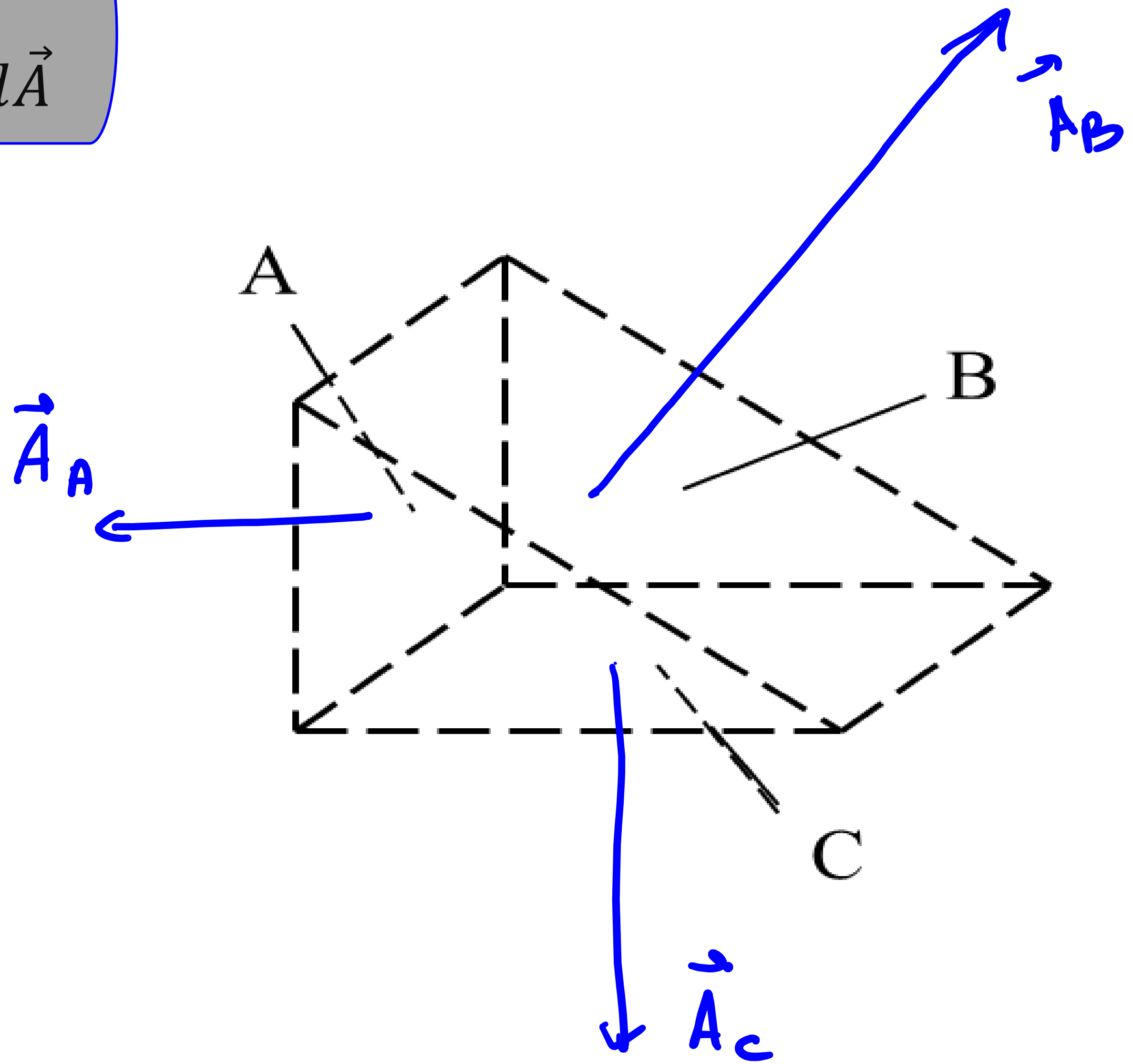
D. I don't know

- In fact, $\Phi_e = \vec{E} \cdot \vec{A}$ is not applicable here since the electric field is non-uniform, but the logic still works

- Convention: For a closed surface, we always take the area vector pointing outwards.



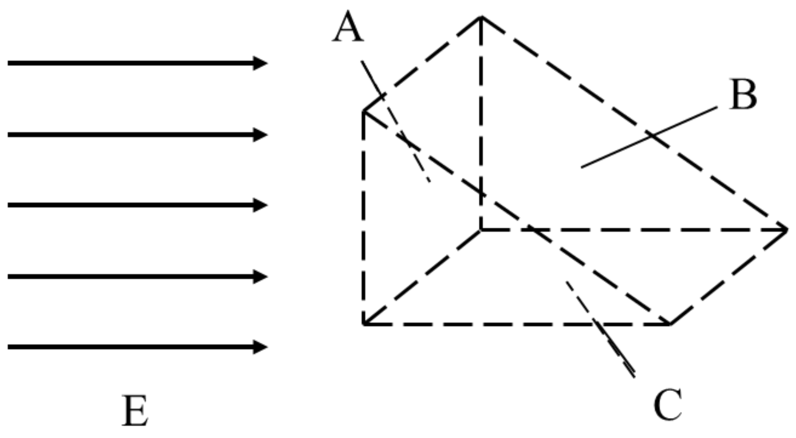
Q: Draw the area vectors for sides A, B, C of the prism shown in the figure. Your drawing should reflect both the directions and the magnitudes of the area vectors.



Q: A prism-shaped closed surface is in a constant, uniform E-field. The three rectangular faces of the prism are labeled A, B, and C. Face A is perpendicular to the E-field. The bottom face C is parallel to the field. Face B is the leaning face. There are no charges inside the prism.

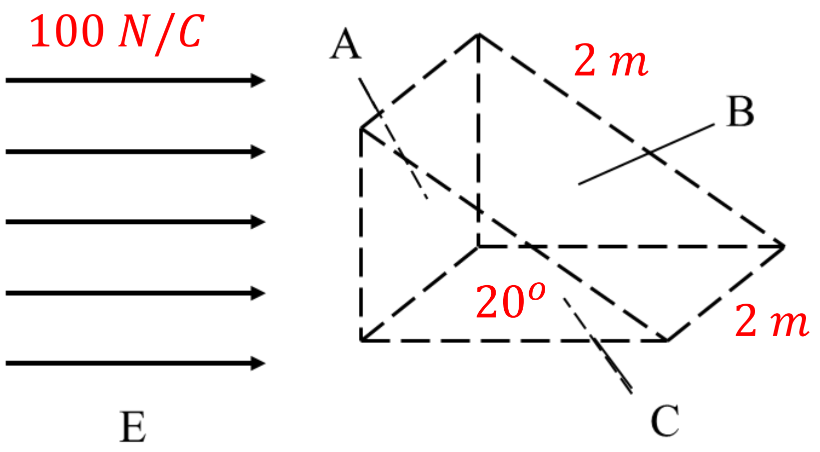
Which face has the largest magnitude of electric flux through it?

A. A
B. B
C. C
D. A and B have the same magnitude of flux
E. Other



Let’s calculate:

$$\boxed{\Phi_e = \vec{E} \cdot \vec{A}}$$



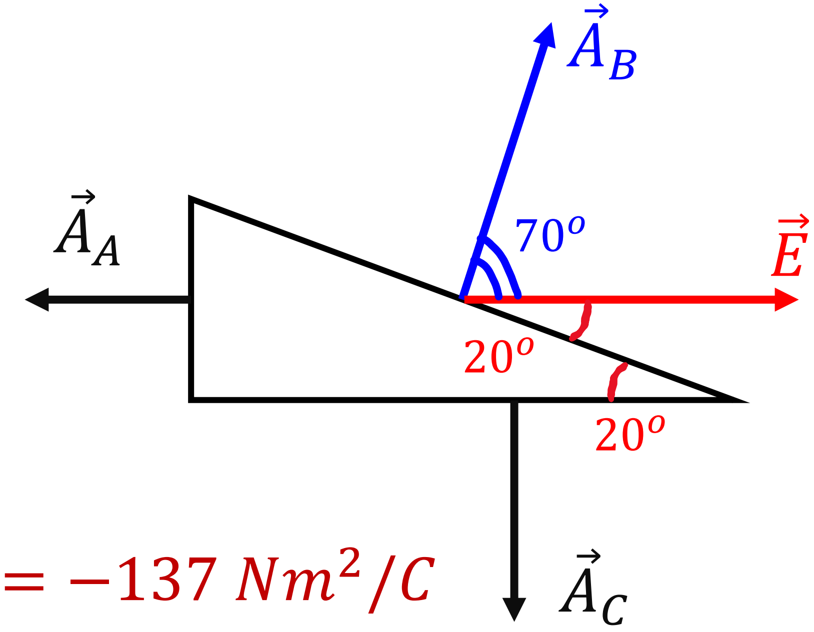
- The flux through the side C is …

$$\Phi_C = EA_C \cos 90^0 = 0$$

- The flux through the side B is …

$$\Phi_B = EA_B \cos 70^0 = \left(100\frac{N}{C}\right)(2 \times 2\ m^2)(\cos 70)$$

$$= \underline{137\ Nm^2/C}$$



- The flux through the side A is …

$$\Phi_A = EA_A \cos 180^0 = \left(100\frac{N}{C}\right)(2 \times 2 \sin 20\ \ m^2)\ (-1) = \underline{-137\ Nm^2/C}$$

- Is it a coincidence??

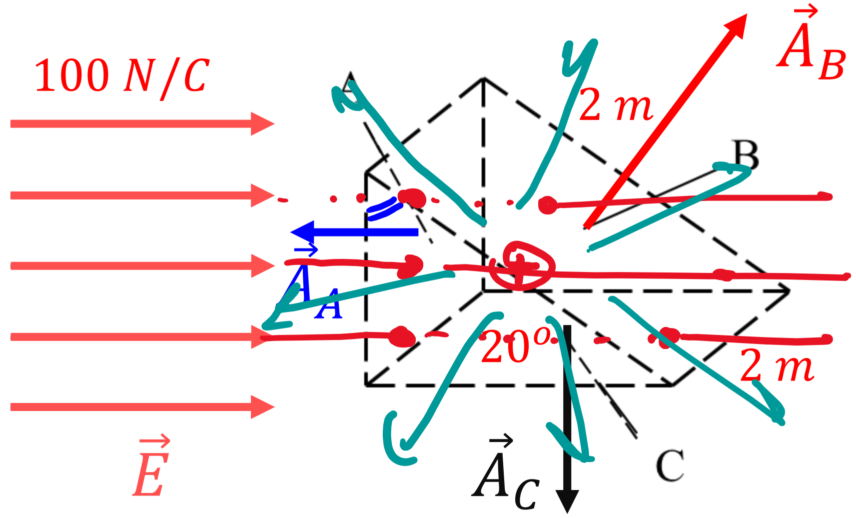
- We found: $\Phi_C = 0$ $\Phi_A = -\Phi_B$

$$\boxed{\Phi_e = EA\cos\theta}$$

$A_B \cos 70° = A_A$

- Is it a coincidence? Can we explain it in simple words?

The slanted face B has larger surface area, but the $\cos(\theta)$ keeps the flux equal to the flux through the perpendicular face A; okay. But why are $|\Phi_A|$ and $|\Phi_B|$ exactly equal?



- Note that the number of field lines entering the prism through the side A is equal to the number of field lines leaving it through side B => $|\Phi_A| = |\Phi_B|$!

- ...and Φ_B is outwards => must be positive, while Φ_A is inwards => must be negative!

Q: Would this nice and beautiful picture change if we put a charge inside the prism?

A: Yes, it will – since this new charge will create additional electric field, and this will break the balance between the external incoming and outgoing lines that we have now

Gauss’s Law

- This brings us to Gauss’s law – a law that relates the electric flux through a surface with the charge that this surface encloses.
- We can also put it slightly differently: Since the electric flux is a measure of electric field, Gauss’s law relates the charges with the electric fields that they produce.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0}$$

Purpose

Gauss's law establishes a connection between a charge and the field produced by that charge, in the integral form

- Why do we need Gauss's law if we already know how to apply Coulomb's law to calculate the electric field of any charge distribution?

  - Gauss's law is extremely useful for calculating the electric fields due to objects that are highly symmetric. It is aesthetically appealing and easy!
  - In conductors, it allows to predict charge distribution easily.
  - Coulomb's law: only electrostatics; Gauss's law: always valid!
  - Will be used (much) later to analyze the shape of electromagnetic waves.

- We do not throw Coulomb's law away. If the charge distribution is not symmetric enough, there is no other way to go as to apply Coulomb's law.

Gauss's law

$$\frac{1}{4\pi\varepsilon_0} = k$$

- Gauss's Law is a fundamental law in physics. It states:

  - Net electric flux through a closed surface = charge inside that surface/ε_0

$$\boxed{\Phi_e = \frac{Q_{\text{in}}}{\varepsilon_0}}$$

- Now, let's recall the definition of the flux: $\Phi_e = \oint \vec{E} \cdot d\vec{A}$. We get:

  - Q_{in}: total (net) charge inside a closed surface.
  - The $\oint \ldots d\vec{A}$ notation: integrate the flux over a closed surface (enclosing the charge Q_{in})

$$\boxed{\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0}}$$

We can say that Gauss's law relates electric field with charges that create this field

- Gauss's law is another way of saying "electric field is produced by charges".