Lecture 20.

Work and Potential Energy.
Electric potential.

Work and Electric Potential Energy. Electric potential.

Text: 23.1

- Ch 23.1: Work and electric potential energy
- Ch 23.2-3: Electric potential
- Ch 23.4: Equipotential surfaces
- Ch 23.5: Electric field gradient

Energy at a glance

Another way to say the same:

An object possesses potential energy when it is placed in a field (created by another object)

- There are two well-known forms of energy:

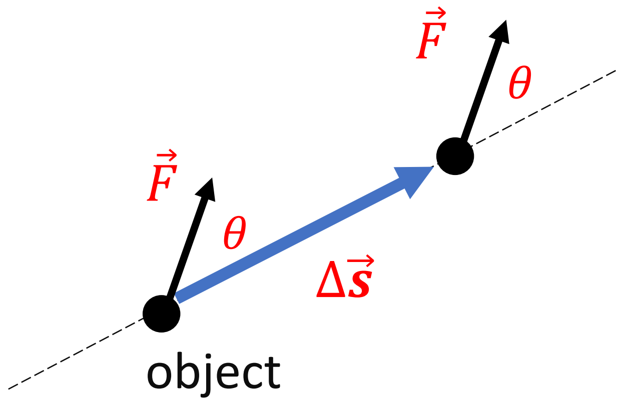
  - ➢Kinetic energy (K) is due to motion: $K = mv^2/2$
  - ➢Potential energy (U) is due to interaction between objects
    - ❖ Gravitational energy: $U_g = mgy$
      (attraction between Earth and object ⇔ gravitational field)
    - ❖ Electric energy of two charges: $U_e = \ldots$ (coming soon)
      (attraction/repulsion between two charges ⇔ electric field)

- These two forms of energy can transform into each other.
- Knowing object's energy is an important task that allows one to make a lot of predictions about its behaviour.

- Potential energy is related to the concept of work (W), which we are going to start with

Work of a force

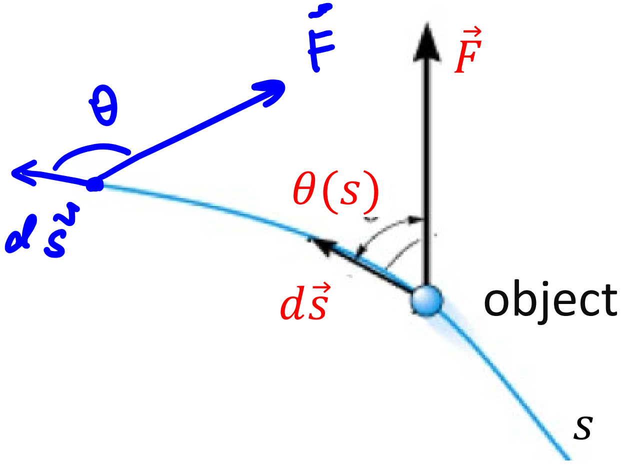
- Work of a constant force $\vec{F}$ acting on the object while it is being displaced by $\Delta\vec{s}$:



$$W = \vec{F}\cdot\Delta\vec{s} = F\,\Delta s\cos\theta$$

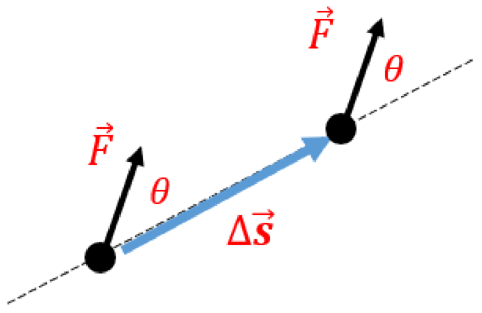
θ is the angle between $\vec{F}$ and $\Delta\vec{s}$

- Work of a non-uniform force $\vec{F}$ acting on the object along some path:



$$dW = \vec{F}\cdot d\vec{s} = F(s)\,ds\cos\theta(s) \quad \Rightarrow \quad \boxed{W = \int_{\text{path}} \vec{F}\cdot d\vec{s}}$$

Properties of work:



- Work is a scalar quantity

$$\boxed{W = \vec{F} \cdot \Delta\vec{s} = F\,\Delta s \cos\theta}$$

- Sign of W depends on mutual orientation of the force $\vec{F}$ and the displacement $d\vec{s}$:

  - $0 \leq \theta < 90^o$: $\cos\theta > 0$ => $\vec{F}$ and $d\vec{r}$ have the same sense => Work is positive
  - $90^o < \theta \leq 180^o$: $\cos\theta < 0$ => $\vec{F}$ and $d\vec{r}$ have opposite sense => Work is negative
  - $\theta = 90^o$: $\cos\theta = 0$ => $\vec{F}$ and $d\vec{r}$ are orthogonal => Work is zero

- SI Units: Joules (J) 1 J = 1 N · 1 m

Potential energy of an object

- Why did we start talking about a work of a force?
- Because it is linked to the potential energy of the object the force acts on.

The change of the potential energy of an object is equal to negative work of a (*conservative*) force acting on it:

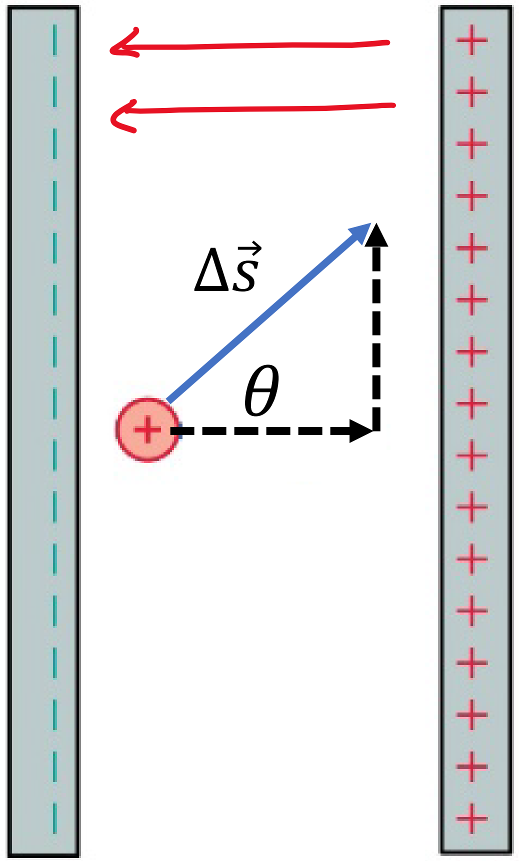
$$\Delta U_e = U_f - U_i = -W_e$$

- Hence, if there is an electric force acting on a charge when it gets displaced, its electric potential energy will change.
- FYI: In PHYS 170 we will talk about work and energy in more detail. In particular, we will learn about gravitational potential energy ($U_g = mgy$) and elastic potential energy ($U_s = kx^2/2$). They are linked, respectively, to the work of the gravity force and of the elastic force.

Q: Let the electric field inside the capacitor be E. A proton is shifted from one location two another as shown by the blue arrow.

a) What is the final potential energy of the proton?
b) What is its final energy if it is shifted along the path shown by black dashed arrows?
c) Does the proton gain or lose electric potential energy?
d) What the answer would be if we replace the proton with an electron, will it lose or gain potential energy?

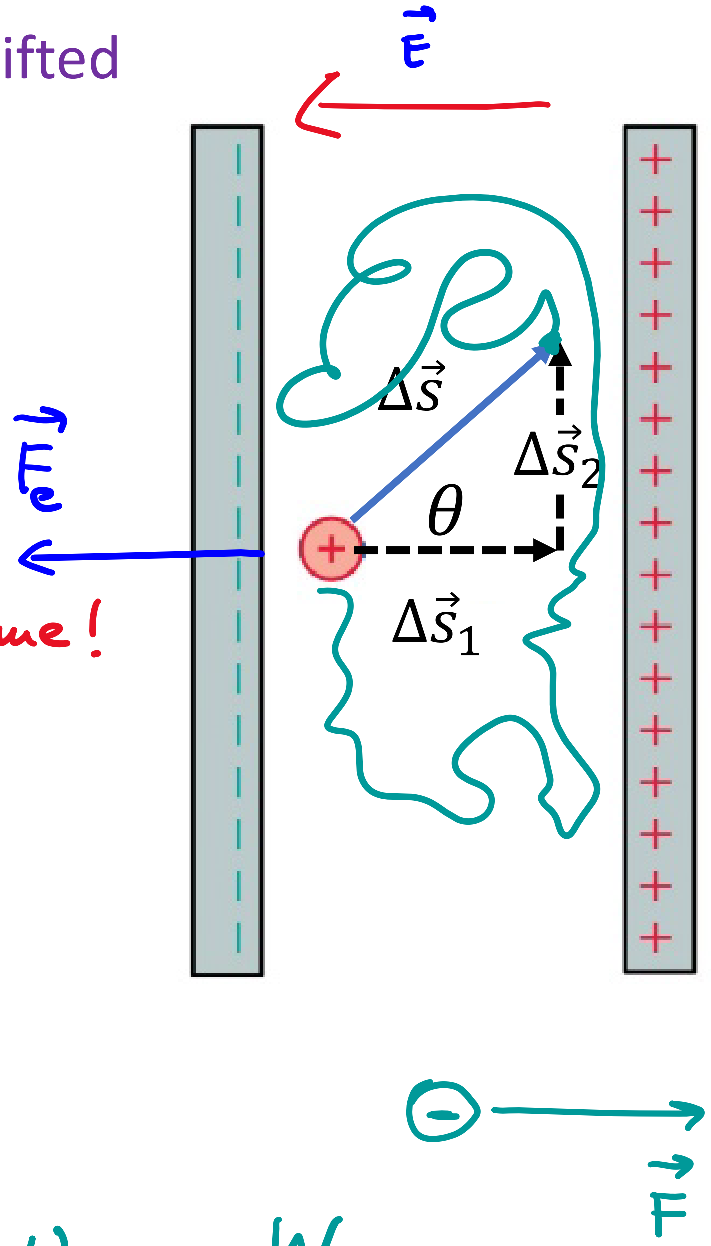
$E = \text{const} = \frac{\sigma}{\varepsilon_0}$



Q: Let the electric field inside the capacitor be E. A proton is shifted from one location two another as shown by the blue arrow.

A. It gains potential energy
B. It loses potential energy
C. Its energy remains the same

$\boxed{\vec{F}_e \text{ is conservative!}}$



Δs_1

$$W_{\Delta\vec{s}} = \vec{F}_e \cdot \Delta\vec{s} = F\Delta s\cos(\pi-\theta) = -F(\Delta s\cos\theta)$$

same!

$$W_{\Delta\vec{s}_1} = \vec{F}_e \cdot \Delta\vec{s}_1 = F\Delta s_1\cos(180^\circ) = -F\Delta s_1$$

$$W_{\Delta\vec{s}_2} = \vec{F}\cdot\Delta\vec{s}_2 = F\Delta s_2\cos(90^\circ) = 0$$

$$\Delta U_e = -W_e$$

Proton => electron: will it lose or gain potential energy?

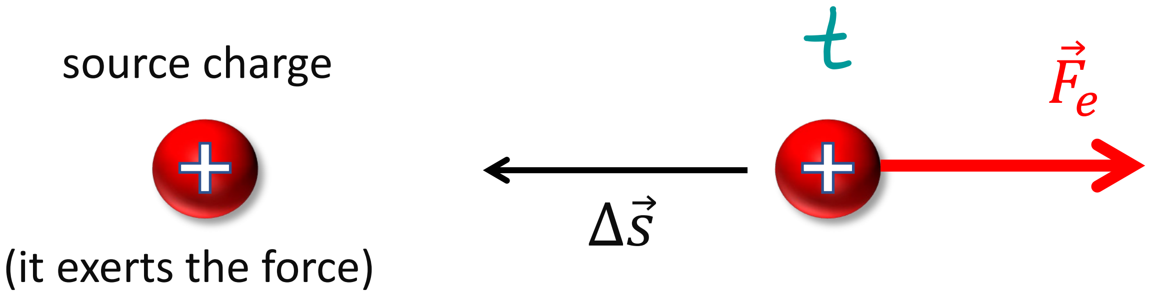
Warning

- Let us look carefully what are the variables in this equation: $\Delta U_e = -W_e =$

$= -\vec{F}_e \cdot \Delta\vec{s}$

➢ $\Delta U = U_f - U_i$ is the change in the electric potential energy of the object

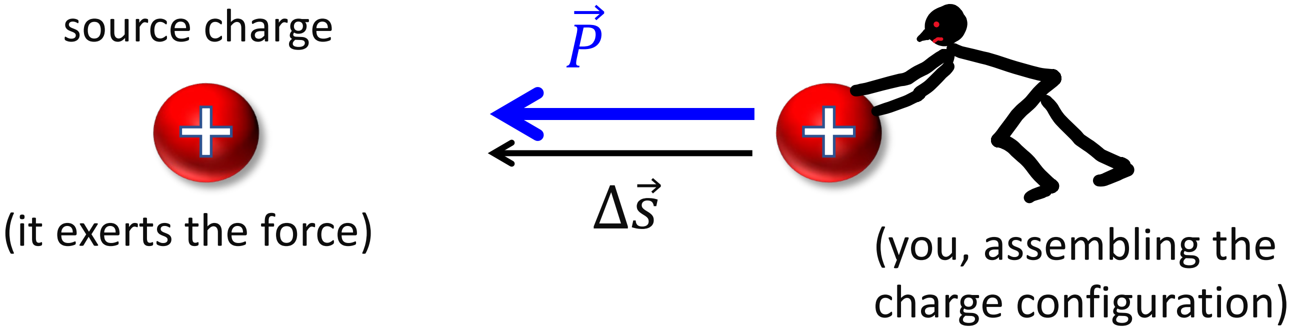
➢ Hence, W is the work of the electric force acting on the displaced charge: $W = \vec{F}_e \cdot \Delta\vec{s}$



$W_e = \vec{F}_e \cdot \Delta\vec{s} = -F_e\Delta s$

(negative, in this example)

➢ You can be asked instead "how much work you have to do to assemble a certain configuration of charges". In this case, you are asked specifically about the work of the "pushing" force $\vec{P}$ that <u>you</u> have to apply <u>against</u> the electric force:



$W_{you} = \vec{P} \cdot \Delta\vec{s} = P\Delta s$

(positive, in this example)

$\Delta U_e = -W_e = W_{you}$

Electric potential energy of two positive charges

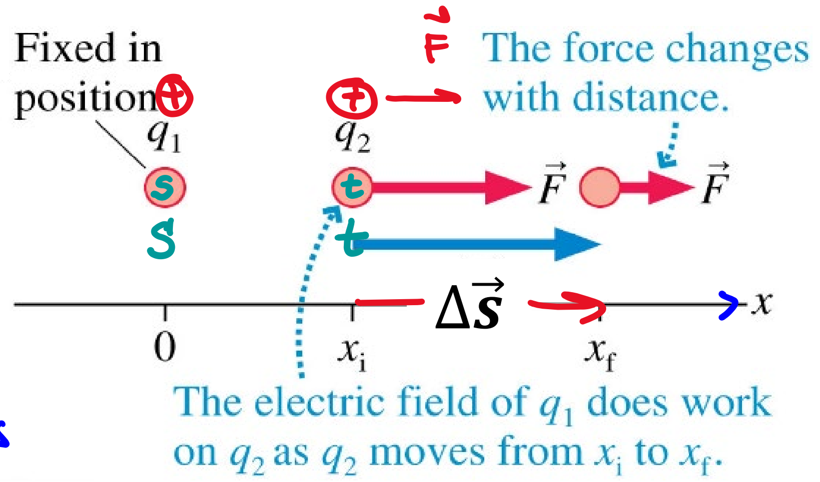
Q: Calculate:

$ds = dx$

a) the work done by the electric force of q_1 when it pushes q_2 from x_i to x_f,

b) the change in the potential energy of this system

magnitudes



Tip: integrate outwards, then $ds = dx$

$$W = \int_{x_i}^{x_f} \vec{F}_{1\,on\,2}\cdot d\vec{s} = \int_{x_i}^{x_f} F_{1\,on\,2}\cdot dx = \int_{x_i}^{x_f} \frac{kq_1q_2}{x^2}\,dx =$$

$$= kq_1q_2\int_{x_i}^{x_f}\frac{dx}{x^2} = kq_1q_2\left(-\frac{1}{x}\right)\Big|_{x_i}^{x_f} = -\Delta U = -(U_f - U_i)$$

$$= -\underbrace{\boxed{\frac{kq_1q_2}{x_f}}}_{U_f} + \underbrace{\boxed{\frac{kq_1q_2}{x_i}}}_{U_i} = -U_f + U_i$$

$$\boxed{U_e(q_1,q_2) = \frac{kq_1q_2}{r}}$$

Hint: $\int x^n dx = \frac{x^{n+1}}{n+1}$

Electric potential energy for two point charges

$$\boxed{U = k\frac{q_1^{(\pm)} q_2^{(\pm)}}{r}}$$

is the energy of the system of two point charges q_1 and q_2.

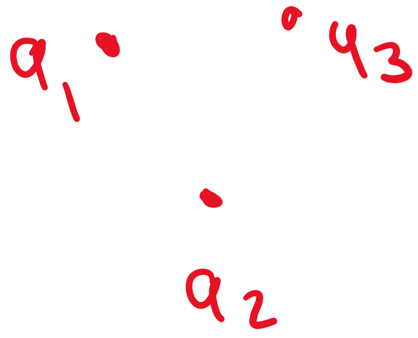
! We have to use the signs of positive and negative charges explicitly in the equation !

(explain why!)

- This formula implies that when the charges are at the infinite distance from each other, their potential energy is zero.

Three (and more) point charges

Consider all interacting particles in pairs:



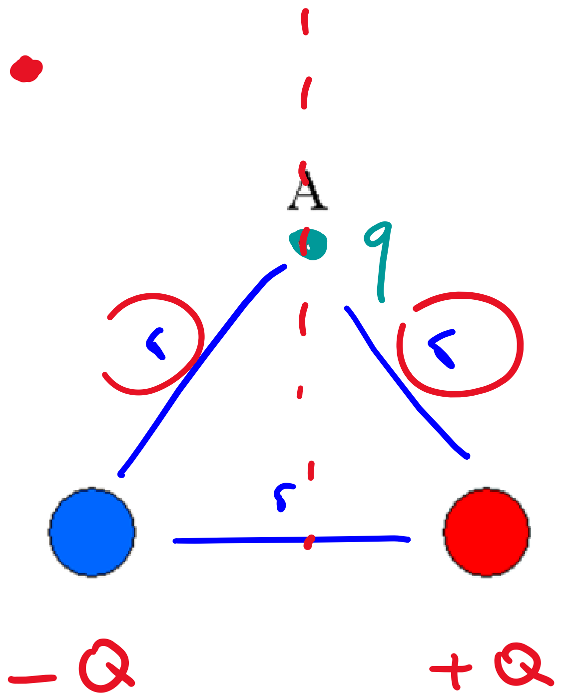
$$U = k\frac{q_1^{(\pm)}q_2^{(\pm)}}{r_{12}} + k\frac{q_1^{(\pm)}q_3^{(\pm)}}{r_{13}} + k\frac{q_2^{(\pm)}q_3^{(\pm)}}{r_{23}}$$ etc.

- This is the electric potential energy stored in a given configuration of charges
- Note that it is equal to the work that must be done by an ‘external agent’ to assemble the system, bringing each charge in from infinity to its final position.

Q: Two charges which are equal in magnitude, but opposite in sign, are placed at equal distances from point A as shown. If a third charge is added to the system and placed at point A, how does the electric potential energy of the charge collection change?

$$U_i = \frac{k(Q)(-Q)}{r}$$

$$U_f = \frac{k(Q)(-Q)}{r} + \cancel{k\frac{(-Q)(-q)}{r}} + \cancel{k\frac{(+Q)(-q)}{r}}$$

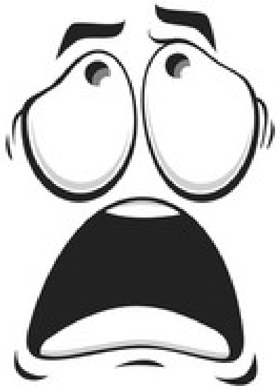


A. Potential energy increases

B. Potential energy decreases

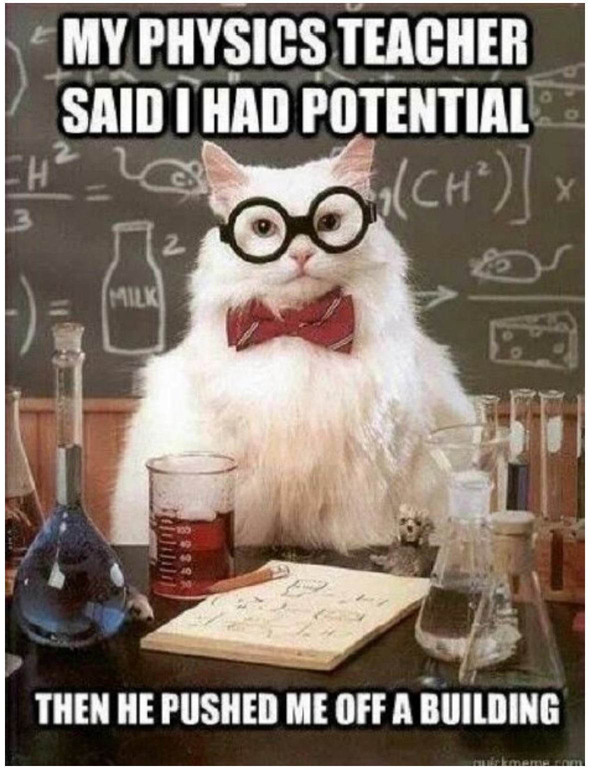
C. Potential energy does not change

D. The answer depends on the sign of the third charge



Electric potential

- Definition
- Electric potential vs Electric potential energy
- How positive / negative test particles behave in electric potential
- Electric potential of a set of point charges, and other simple charge distributions



Electric potential vs Electric potential energy

Never ever mix them up:

(sometimes called "electric potential energy", to distinguish it from e.g. gravitational potential energy – it adds to confusion!)

$U =$ Potential energy $[J]$

adjective *noun*

$V =$ Electric potential (V)

adjective *noun*

- Potential energy of a pair of point charges separated by a distance r is (assuming $U = 0$ when they are infinitely far apart):

$$U = \frac{k\, Q_1\, Q_2}{r}$$

- We can define the electric potential due to Q_1 at distance r from it as

$$V = \frac{U}{Q_2} = \frac{k\, Q_1}{r}$$

Hence, electric potential simply is electric potential energy per unit charge.