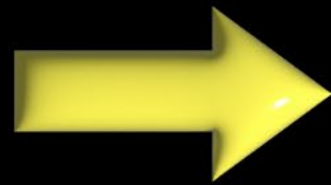


**FYSRE DEADLINE: JAN 31st!**



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**“FYSRE”**  
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Year  
Summer  
Research  
Experience  
Award



## **Vogt First Year Summer Research Experience - paid full-time summer research positions**

[The Erich Vogt First Year Summer Research Experience](#) (FYSRE, pronounced phyzzze) is a program offering summer research experiences to budding academic stars after their First Year Physics courses. Awardees will receive an opportunity for paid work experience in Physics or Astronomy research in [UBC Physics and Astronomy](#) or at [TRIUMF](#).

This award honors Dr. Erich Vogt (1929-2014) one of the most distinguished Canadian nuclear physicists of his generation. For over thirty-five years, Erich Vogt taught bright and eager First Year Physics students here at UBC.

The FYSRE awards give outstanding first year students an opportunity to gain work experience in paid summer research positions. Other such research opportunities usually give preference to second and higher year students.

### **Eligibility**

As a requirement for the program, you should have completed 2 semesters of first year physics including the lab, by the time you start the summer position, and plan to enter a Physics or Astronomy (or any combined Majors or Honors degree including Physics or Astronomy) or Engineering Physics program in your second year of study. You must obtain a cumulative average of at least A- over the first year of university study. This is a full-time paid research position. The internship selection process includes an interview.

Applicants must have first-class standing (an average of at least 80%) -although to date every recipient has had an average of at least 85%,

For full info, including application procedure, forms and past recipients and their work, see:

<https://www.phas.ubc.ca/erich-vogt-first-year-summer-research-experience-fysre>

Application deadline is Jan 31, 2024

## Lecture 4.

Capacitors. Capacitance.

Capacitors in Series and in Parallel & Effective capacitance.

Energy stored in a capacitor (if time permits).

## Time-independent DC Circuits with capacitors

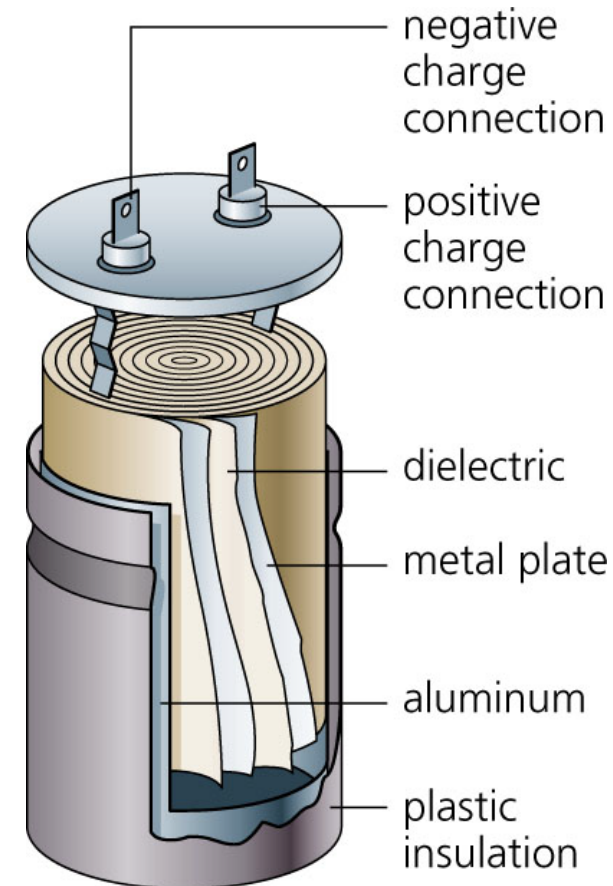
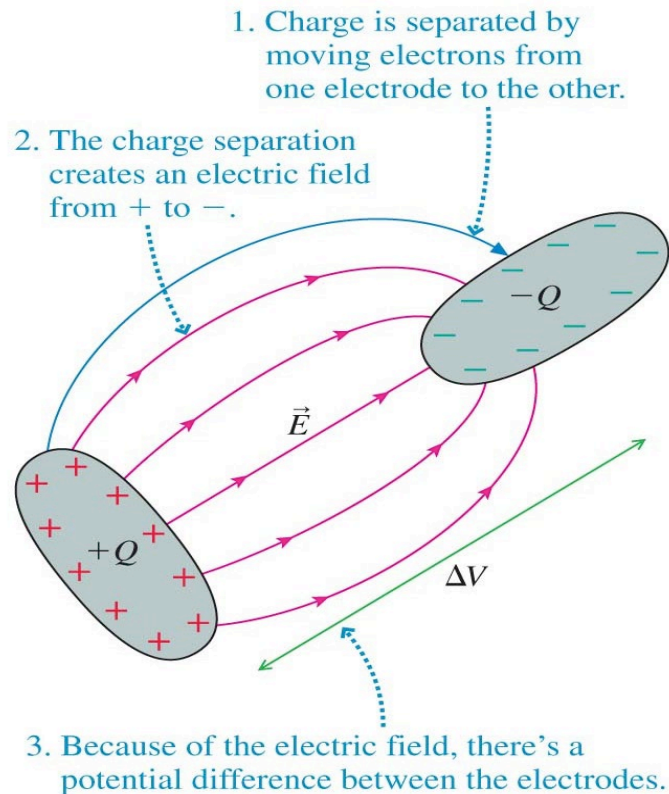
Text: 24.1-3

- Ch 24.1 – Capacitance (  $C = \frac{\Delta V}{\Delta Q}$  ) depends on geometry of a capacitor
- Ch 24.2 – Capacitors in series and in parallel, effective capacitance
- Ch 24.3 – Energy storage in capacitors

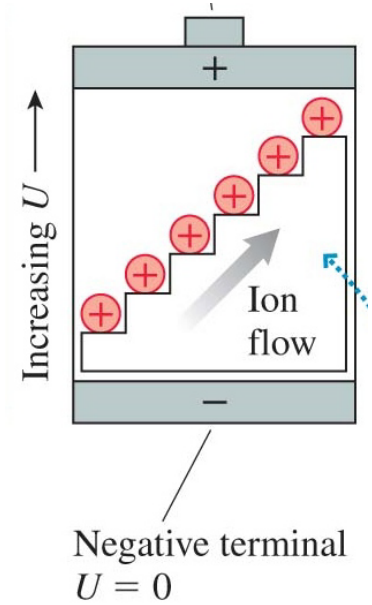


# Capacitor

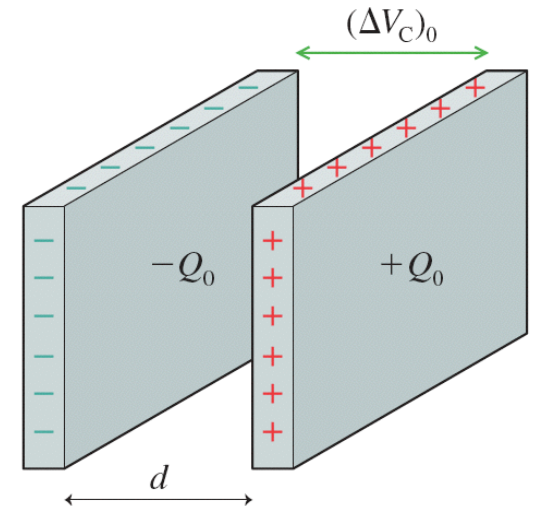
- A **capacitor** is any collection of conductors which stores electric charge and therefore electric energy.
- Example of capacitor with 2 conductors.



- **Batteries** are sources of voltage ( $V$ ):
  - Constant potential difference created by separating charges in electrochemical reactions.
  - Limited to low voltages. For higher voltages we use multiple batteries in series.
  - Internal resistance leads to a maximum current that a battery can deliver.



- **Capacitors** need to be charged:
  - Not a source of **constant voltage** ( $V$ )
  - Voltage changes during charge / discharge
  - Very high voltages possible
  - No internal resistance  $\rightarrow$  fast discharge



When we say a capacitor is charged with a charge  $Q$ , this implies that there is:

A.  $+Q/2$  on one plate, and  $-Q/2$  on the other plate  
(resulting in total charge  $Q$ )

B.  $+Q/2$  on one plate, no charge on the other plate

☒ C.  $+Q$  on one plate, and  $-Q$  on the other plate

D. Both answers A and B are correct

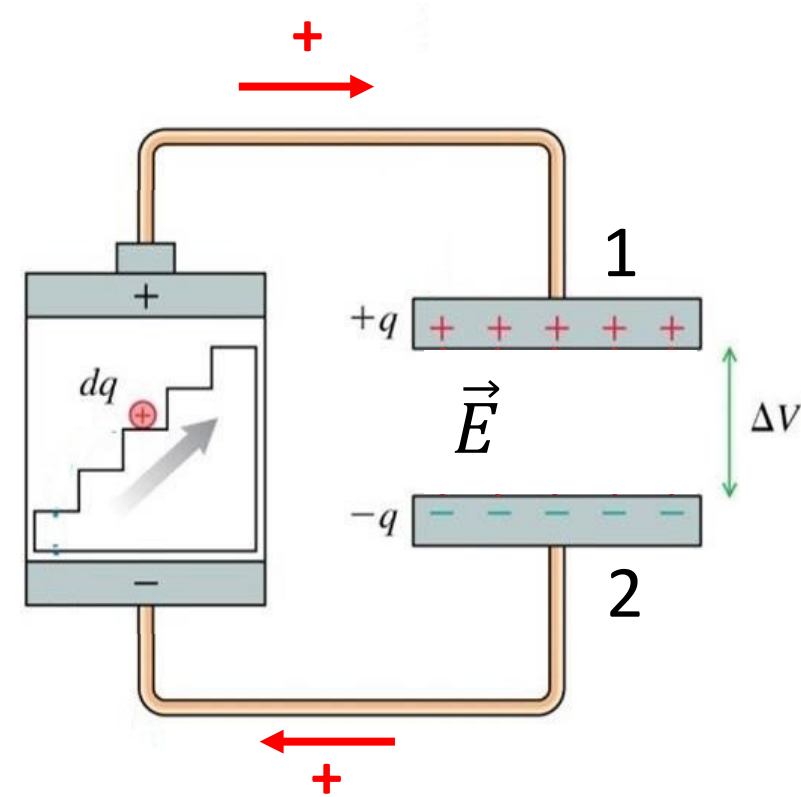
- Battery:

- Charge separation due to work done by a chemical reaction
- $\Delta V_{\text{bat}}$  = voltage drop across the terminals of the battery

- Charging capacitor:

- We attach a capacitor to a battery
- In the conventional current picture, charges are moved from the positive terminal of the battery to plate 1, and from plate 2 to the negative terminal of the battery
- Separated charges create a voltage drop,  $\Delta V_C$ , across the capacitor, and electric field  $\vec{E}$  inside the capacitor (more later)
- Note that **no charges travel through the capacitor!**  
Current flows in the wires, but not inside the capacitor.

## Charging a capacitor



- The capacitor is fully charged when  $\Delta V_{\text{bat}} = \Delta V_C$



# Capacitance

- In a charged capacitor the voltage created across its plates and the charge on its plates are proportional to each other:

$$Q = C \Delta V_C$$

- Capacitance:  $C = Q / \Delta V_C$

(meaning: how much charge we can store at a given voltage)

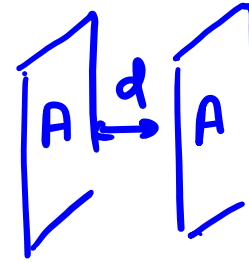
- Unit: Farad (1 F = 1 C / 1 V).
- Usually measured in: pF ( $10^{-12}$ ), nF ( $10^{-9}$ ),  $\mu$ F ( $10^{-6}$ ), mF ( $10^{-3}$ )

Q: A capacitor is charged with a 3 V battery. What will happen with its capacitance if we charge it with a 9 V battery?

- A. The capacitance will decrease.
- B. The capacitance will increase.
- ☒ C. The capacitance will stay the same.
- D. Not sure

Q: A capacitor is charged with a 3 V battery. What will happen with its capacitance if we charge it with a 9 V battery?

- A. The capacitance will decrease.
- B. The capacitance will increase.
- ☒ C. The capacitance will stay the same.
- D. Not sure



$$\epsilon_0 = \text{const}$$

$C$  depends only on the geometry of the capacitor

$$C = \frac{Q}{\Delta V_C} = \frac{A\epsilon_0}{d}$$

- $C = \frac{Q}{\Delta V_C}$  just means that doubling the charge, we will get twice larger voltage across the plates!

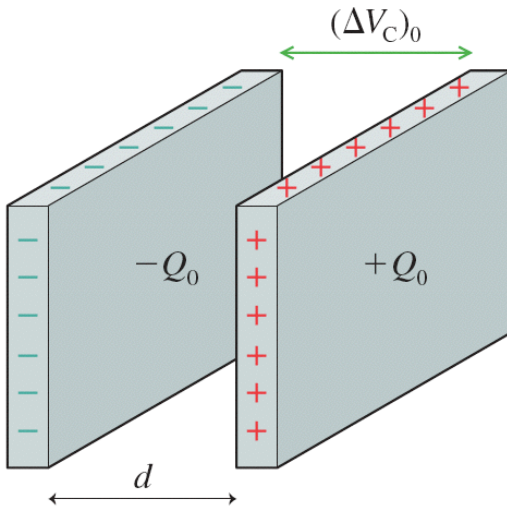
# Voltage drop across a capacitor

- Magnitude:

- Using definition of capacitance:  $C = Q/\Delta V_C \Rightarrow$

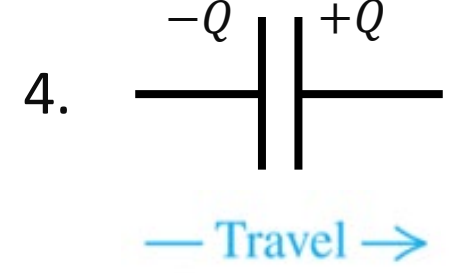
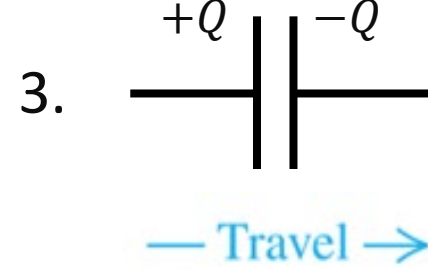
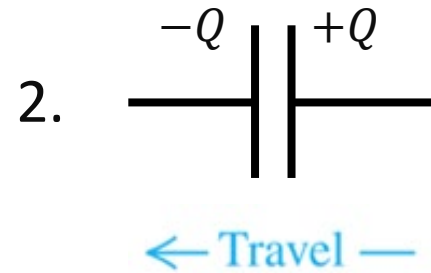
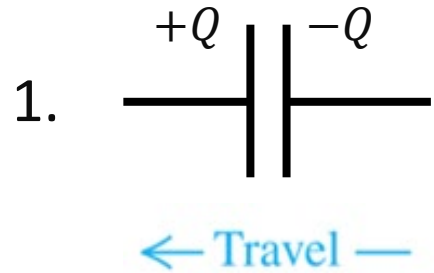
$$\Delta V_C = \frac{Q}{C}$$

- Sign convention:



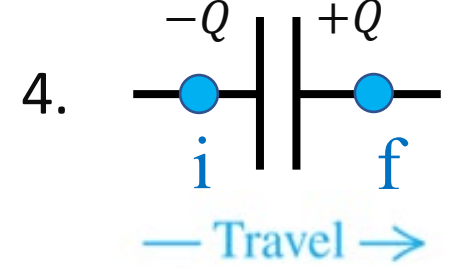
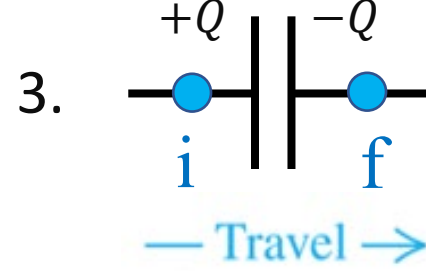
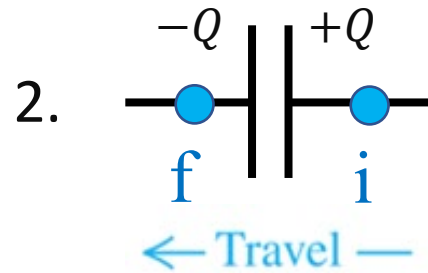
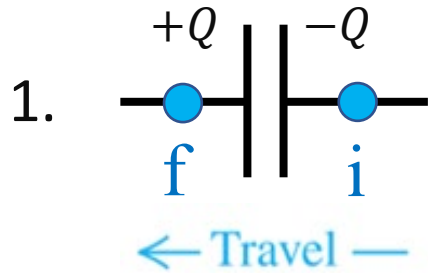
- Positive plate of a capacitor always is at a higher voltage than its negative plate

Q: Which of the voltage drops are equal to  $\Delta V_C = -\frac{Q}{C}$  ?



- A. 3 and 4
- B. 1 and 2
- C. 1 and 4
- D. 2 and 3
- E. Help!

Q: Which of the voltage drops are equal to  $\Delta V_C = -\frac{Q}{C}$  ?



- Choice of initial (i) and final (f) points is dictated by accepted travel direction
- Positively charged plate is at a higher voltage than negative plate
- The voltage drop is negative if  $V_f < V_i$

- A. 3 and 4
- B. 1 and 2
- C. 1 and 4
- D. 2 and 3**
- E. Help!



## Capacitors in circuits: Connected in Parallel

- Combination of capacitors can yield a capacitor that can store a **higher total charge**.
- In the circuit diagram you can replace the combination with an **equivalent capacitor** that has the “**equivalent capacitance**”.

- Example: **Three capacitors in parallel**.

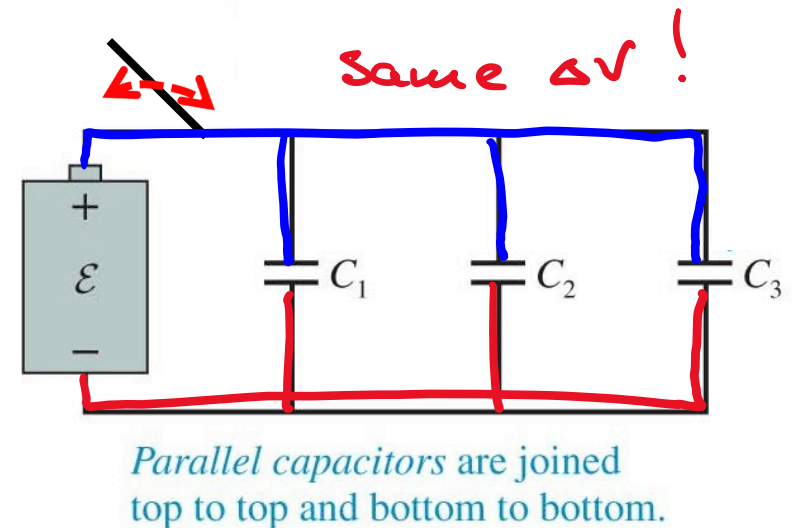
- Total charge stored in the combination:  $Q_P = Q_1 + Q_2 + Q_3$
- The potential difference due to the battery is the same across each capacitor:

$$\Delta V_1 = \Delta V_2 = \Delta V_3 = \varepsilon$$

- Using  $Q = C \Delta V$  for each capacitor, we get:

$$Q_P = (C_1 + C_2 + C_3)\varepsilon \equiv C_{eq}\varepsilon$$

- Hence:  $C_{eq} = C_P = C_1 + C_2 + C_3$



## Capacitors in circuits: Connected in Series

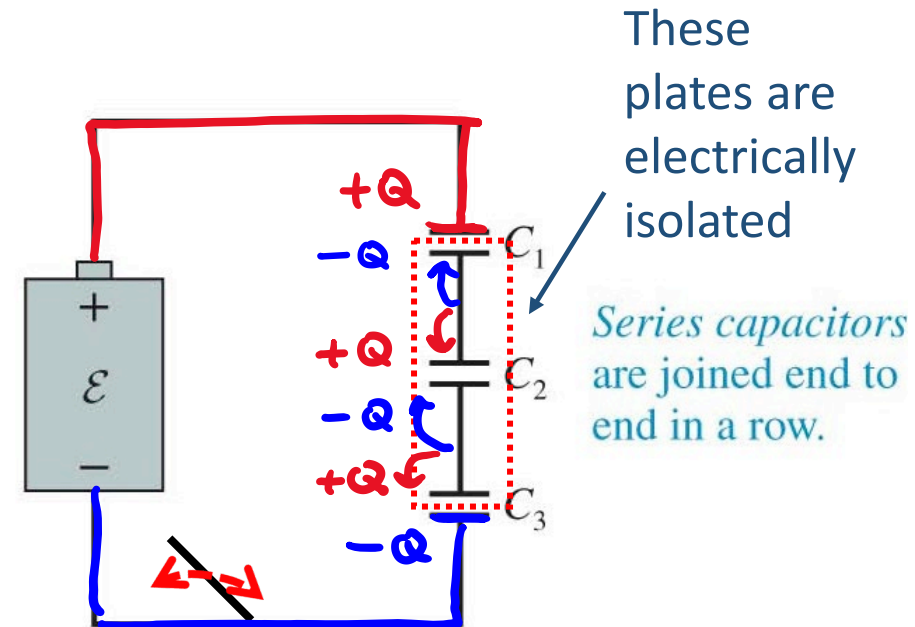
- Interestingly, these three capacitors must all **store the same charge**.
- Only two outer plates are connected to the battery: battery deposits charge  $\pm Q$  there.
- Other plates are **electrically isolated** from the battery: the battery cannot deposit any charge there! There is only **induced charge** on these isolated plates ( $\mp Q$  pulled out of the neutral wire connecting the capacitors)
- Hence, each capacitor stores the **same charge  $Q$**  – even if their capacitances are different!

- Same charge:  $Q_1 = Q_2 = Q_3 = Q$

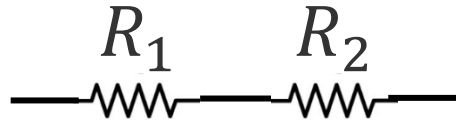
- Voltage drops add:  $\Delta V_1 + \Delta V_2 + \Delta V_3 = \varepsilon$   
$$\frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = \frac{Q}{C_{eq}}$$

- Hence:

$$\frac{1}{C_{eq}} = \frac{1}{C_P} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



Resistors in series:

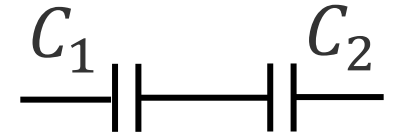


$$R_{eq,s} = R_1 + R_2$$

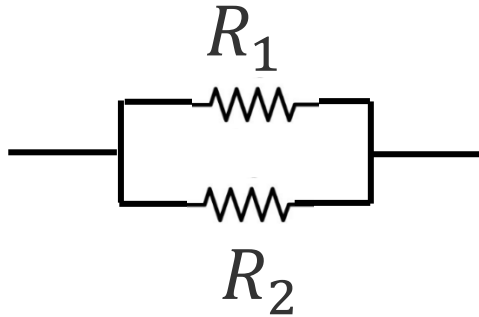
Reminder

$$\frac{1}{C_{eq,s}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Capacitors in series:

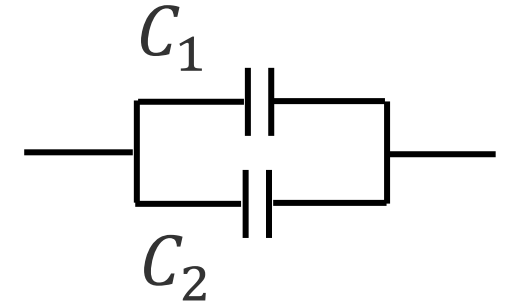


Resistors in parallel:



$$\frac{1}{R_{eq,p}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Capacitors in parallel:

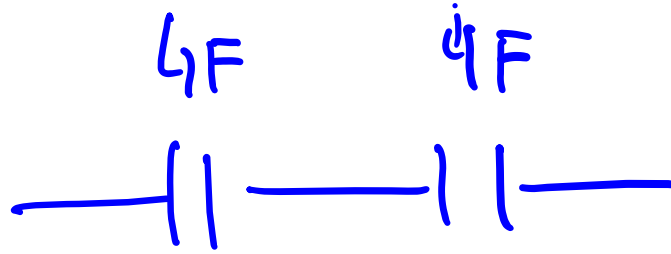


- Note that for two (but not more!) components it is convenient to use:

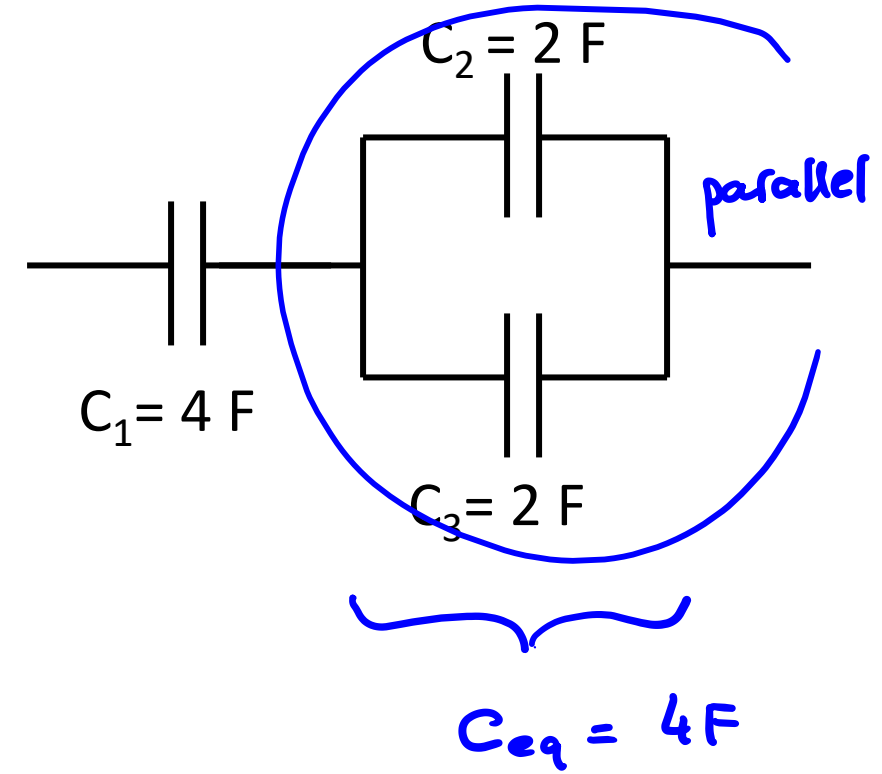
$$R_{eq,p} = \frac{R_1 R_2}{R_1 + R_2}$$

$$C_{eq,s} = \frac{C_1 C_2}{C_1 + C_2}$$

Q: What is the effective total capacitance of this set of capacitors?



$$C_{eq} = \frac{4 \cdot 4}{4 + 4} = 2F$$



A: 0 F

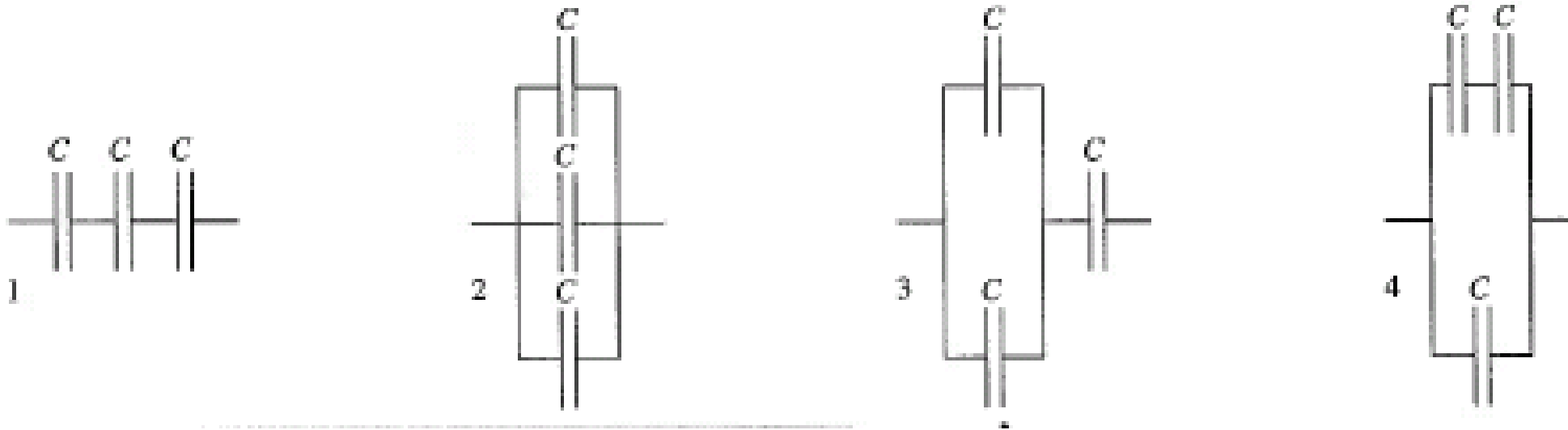
B: 5 F

C: 8 F

D: 4 F

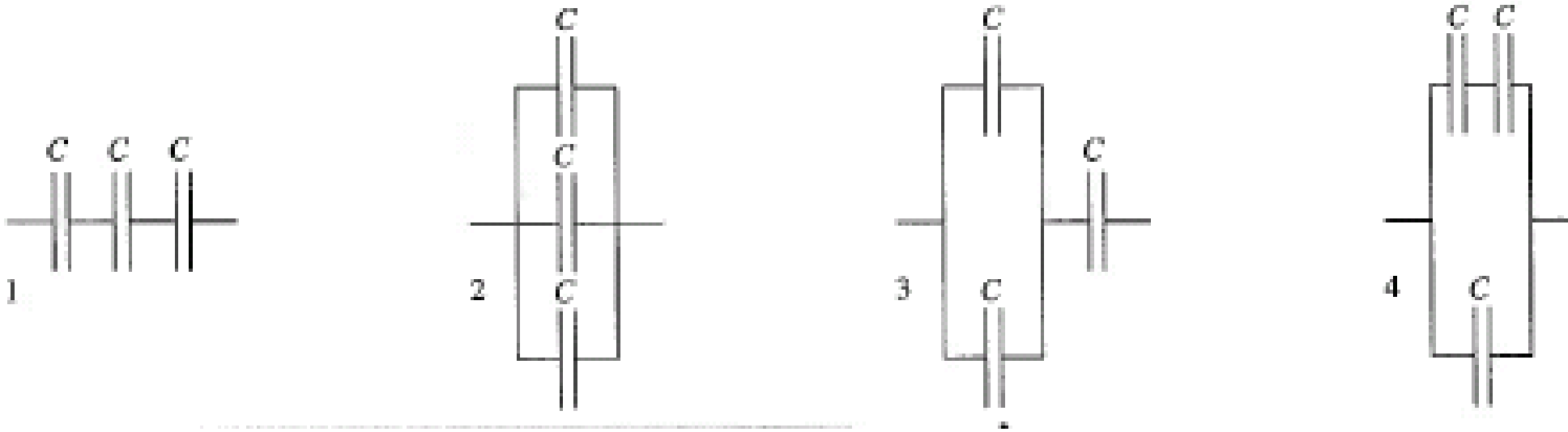
E: Something else

Q: Rank the equivalent capacitance of these four groups of capacitors



- A.  $2 > 4 > 3 > 1$
- B.  $1 > 4 > 3 > 2$
- C.  $2 > 3 > 4 > 1$
- D.  $3 = 4 > 2 > 1$
- E.  $3 > 4 > 1 > 2$

Q: Rank the equivalent capacitance of these four groups of capacitors



$$\bullet \frac{1}{C_1} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} \Rightarrow C_1 = \frac{C}{3}$$

$$\bullet C_2 = C + C + C = 3C$$

$$\bullet \frac{1}{C_3} = \frac{1}{2C} + \frac{1}{C} = \frac{3}{2C} \Rightarrow C_3 = \frac{2C}{3} = 0.67C$$

$$\bullet C_4 = C + \frac{C \cdot C}{C + C} = \frac{3C}{2} = 1.5C$$

A.  $2 > 4 > 3 > 1$

B.  $1 > 4 > 3 > 2$

C.  $2 > 3 > 4 > 1$

D.  $3 = 4 > 2 > 1$

E.  $3 > 4 > 1 > 2$

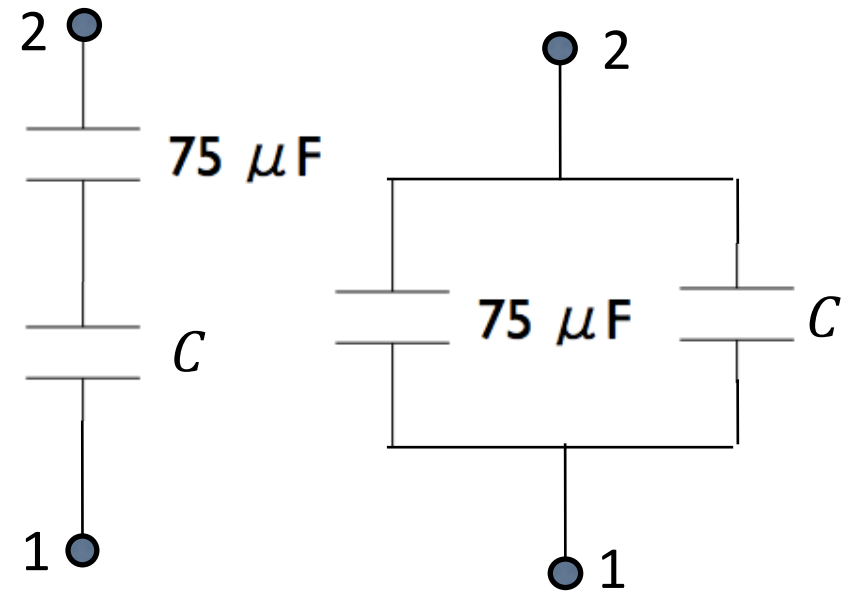


Q: You are asked to come up with a capacitance of  $50\ \mu\text{F}$  between 1 & 2 but you only have a box of  $75\ \mu\text{F}$  capacitors available.

You start with one  $75\ \mu\text{F}$  capacitor. What combination  $75\ \mu\text{F}$  capacitor(s) can you add to it, so that the effective capacitance of the circuit is  $50\ \mu\text{F}$ ? How will you attach this extra combination,  $C$ , in parallel or in series? What its value should be?

How do you build it using only  $75\ \mu\text{F}$  caps?

- A. Add  $C = 25\ \mu\text{F}$  in series
- B. Add  $C = 25\ \mu\text{F}$  in parallel
- C. Add  $C = 75\ \mu\text{F}$  in series
- D. Add  $C = 150\ \mu\text{F}$  in series
- E. Add  $C = 150\ \mu\text{F}$  in parallel



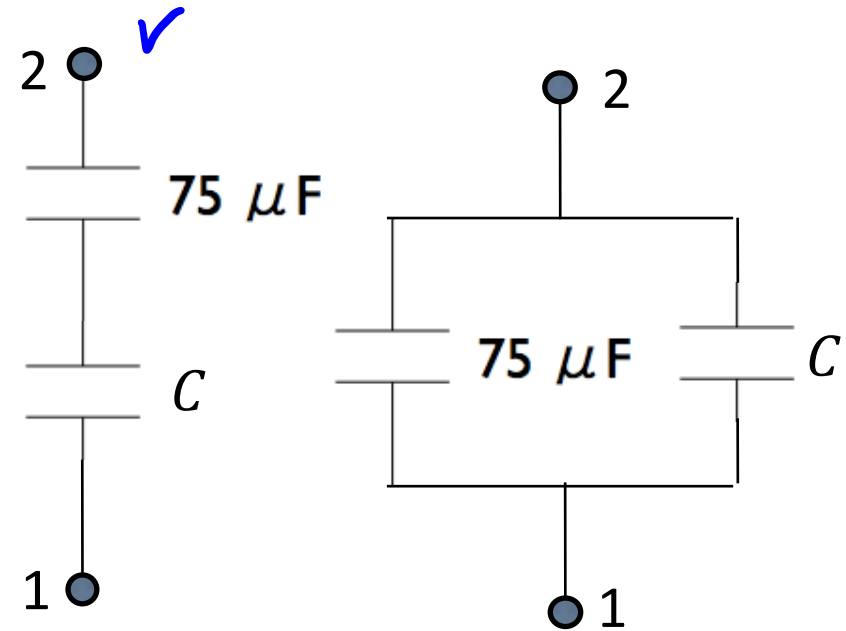
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- A. Add  $C = 25\ \mu\text{F}$  in series
- B. Add  $C = 25\ \mu\text{F}$  in parallel
- C. Add  $C = 75\ \mu\text{F}$  in series
- D. Add  $C = 150\ \mu\text{F}$  in series**
- E. Add  $C = 150\ \mu\text{F}$  in parallel

“In parallel” can only increase the equivalent capacitance, while “in series” makes it smaller



$$\frac{1}{50} = \frac{1}{75} + \frac{1}{C} \Rightarrow C = 150\ \mu\text{F}.$$

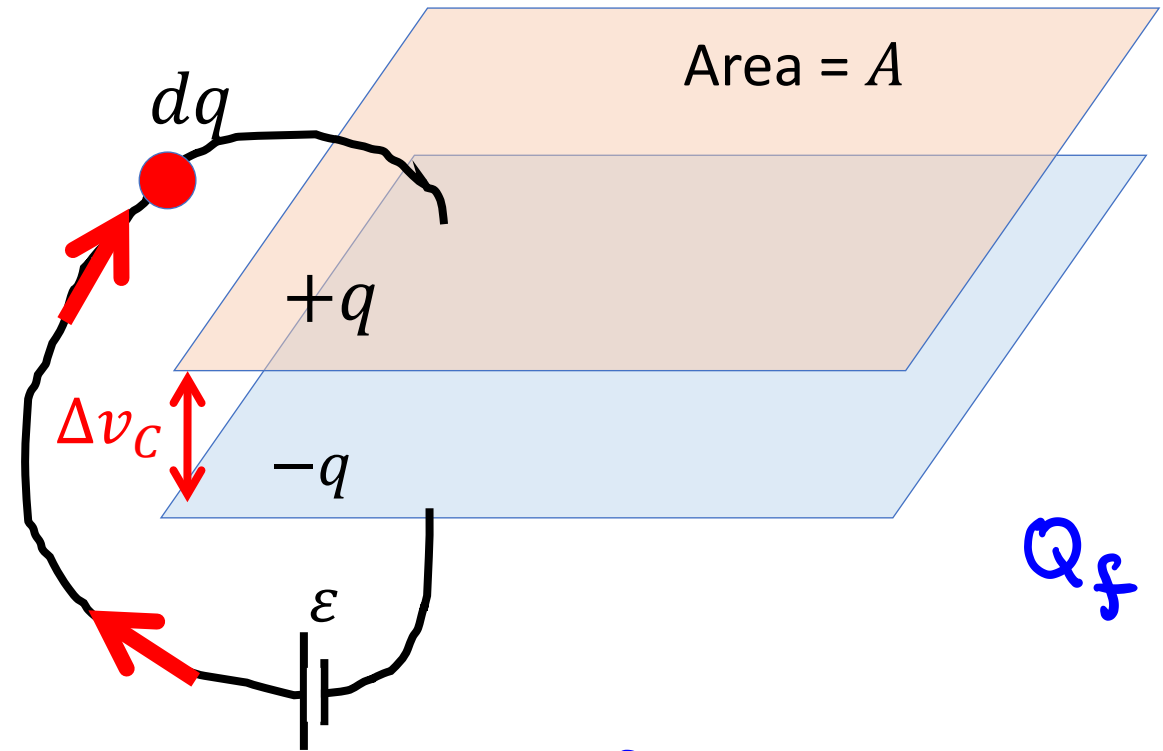
Using two  $75\ \mu\text{F}$  capacitors in parallel will give you a  $150\ \mu\text{F}$  capacitor.

## Energy stored in a Capacitor

- Battery charges the capacitor “step by step”, by transferring small amounts of charge,  $dq$
- Here the charge already accumulated is  $q$ . It creates voltage  $\Delta v_C = q/C$  across the plates.
- The work done by the EMF to move the “next” portion of charge  $dq$  is:

$$dW = \Delta v_C dq = dU_C \quad (U_C \text{ is electric energy stored in the capacitor})$$

- Total work is the integral of  $dW$ , and it is equal to electric energy stored in a capacitor charged to a charge  $Q_f$  (here  $\Delta V_f = Q_f/C$  is the final voltage across its plates):



$$Q_f = C \Delta V_f = C \mathcal{E}$$

$$U_C = \int_0^{Q_f} \frac{q}{C} dq = \frac{Q_f^2}{2C} = \frac{C \Delta V_f^2}{2}$$