MINDFULNESS MATTERS

Dr. Ishan Shivanand

Reframe your mindset & build a positive outlook

Manage high-pressure situations & anxiety

Learn how to become mentally resilient!

MARCH 27th, 2024

5:00-7:00 PM

Chemical and Biological Engineering Building (CHBE) - 101

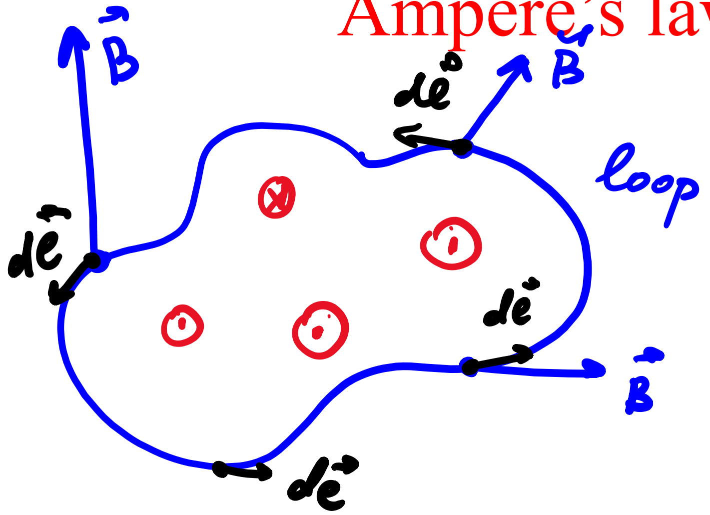
Food & refreshments will be provided.

DE-STRESS WITH US!

RSVP!

Lecture 30.

Ampere's law and its applications.



$$\oint_{\text{loop}} \vec{B}\cdot d\vec{\ell} = \mu_0 I_{\text{encl}}$$

- Ch 28: Ampère's Law (Sect 28.6)

Ampere's law

- Let us start with the field produced by a straight wire:

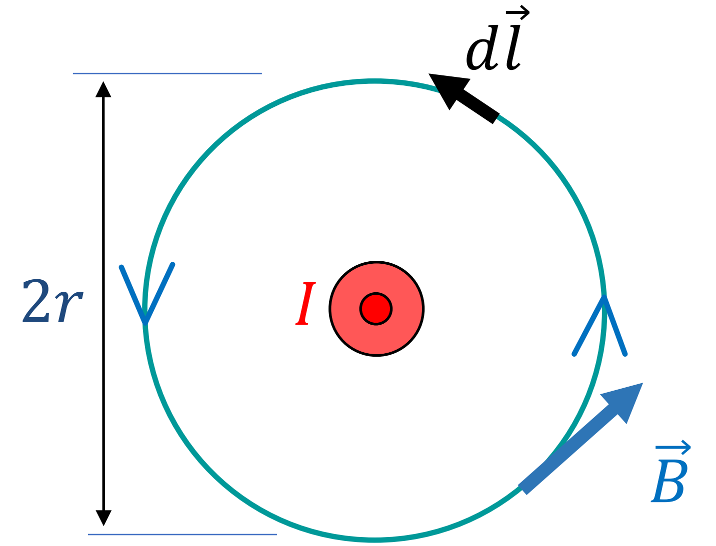
Consider the integral of the magnetic field along a circular line surrounding a current-carrying wire:

$$\oint \vec{B}\cdot d\vec{l} = \oint B\, dl = B\oint dl = B\cdot 2\pi r = \frac{\mu_0 I}{\cancel{2\pi r}}\cancel{2\pi r}$$

$\vec{B} \uparrow\uparrow d\vec{l}$

$|\vec{B}| = const$ along the circle

$B = \frac{\mu_0 I}{2\pi r}$ for the field of a wire



closed path with an enclosed area

Therefore:

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\mathrm{encl}}}$$

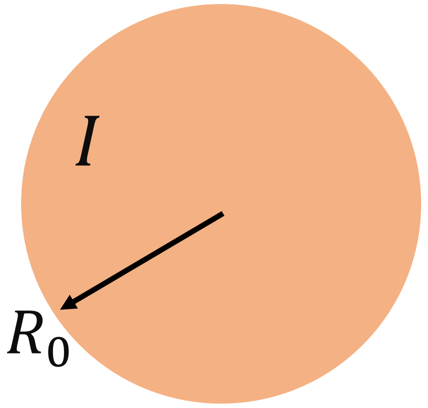
- This relation proved to be true for:
  - Any shape of the integration path
  - Any distribution of currents passing through the loop
- Useful: when we have enough symmetry!

Ampère's law: application

a) Calculate $B_{in}(r < R_0)$ inside a long solid wire carrying a uniformly distributed current I.

b) Calculate $B_{out}(r > R_0)$ outside the long solid wire.

(wire is perpendicular to the page)



End view

Ampère's law: application

a) Calculate $B_{in}(r < R_0)$ inside a long solid wire carrying a uniformly distributed current I.

b) Calculate $B_{out}(r > R_0)$ outside the long solid wire.

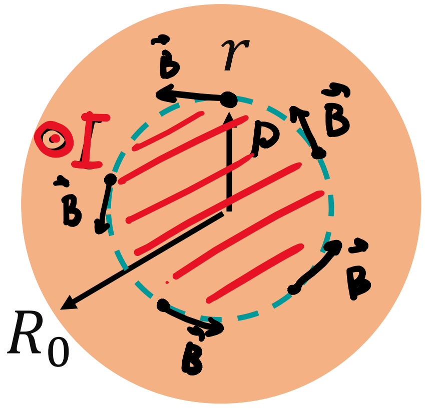
- Choose a circular Ampèrian circular loop with radius $r < R_0$.

- Evaluating $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ around this closed loop gives:

$$B_{in}(r) \cdot 2\pi r = \mu_0 \, I_{\text{encl}}$$

- Finding I_{encl}: $\dfrac{I_{\text{encl}}}{I} = \dfrac{\pi r^2}{\pi R_0^2} \quad \Rightarrow \quad I_{encl} = I \, \dfrac{r^2}{R_0^2}$

- Therefore: $B_{in}(r) = \dfrac{\mu_0 I}{2\pi R_0^2} r$



End view

Ampère's law: application

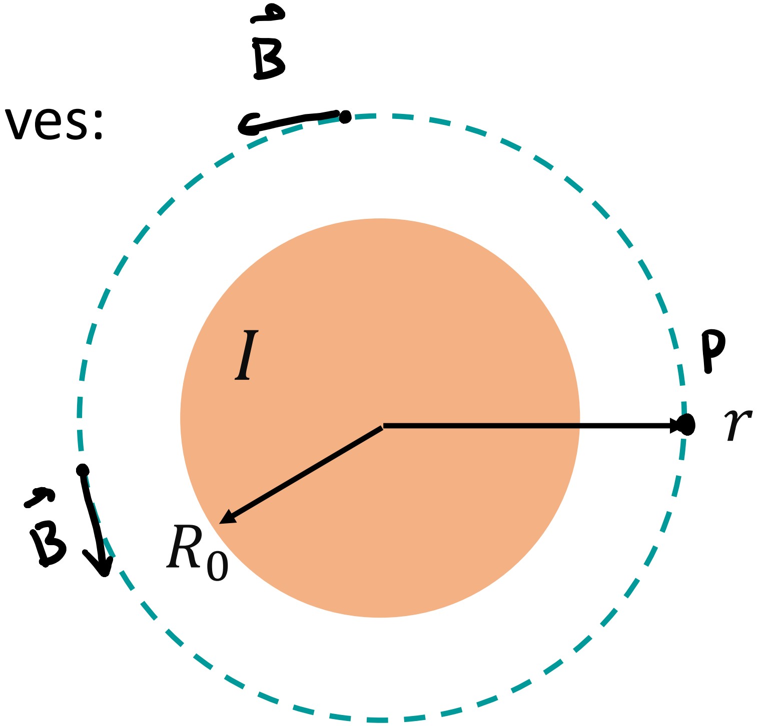
a) Calculate $B_{in}(r < R_0)$ inside a long solid wire carrying a uniformly distributed current I.

b) Calculate $B_{out}(r > R_0)$ outside the long solid wire.

- Choose a circular Ampèrian circular loop with radius $r > R_0$.
- Evaluating $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ around this closed loop gives:

$$B_{out}(r) \cdot 2\pi r = \mu_0 \, I_{\text{encl}}$$

- Finding I_{encl}: $\quad I_{encl} = I$
- Therefore: $\quad B_{out}(r) = \dfrac{\mu_0 I}{2\pi r}$



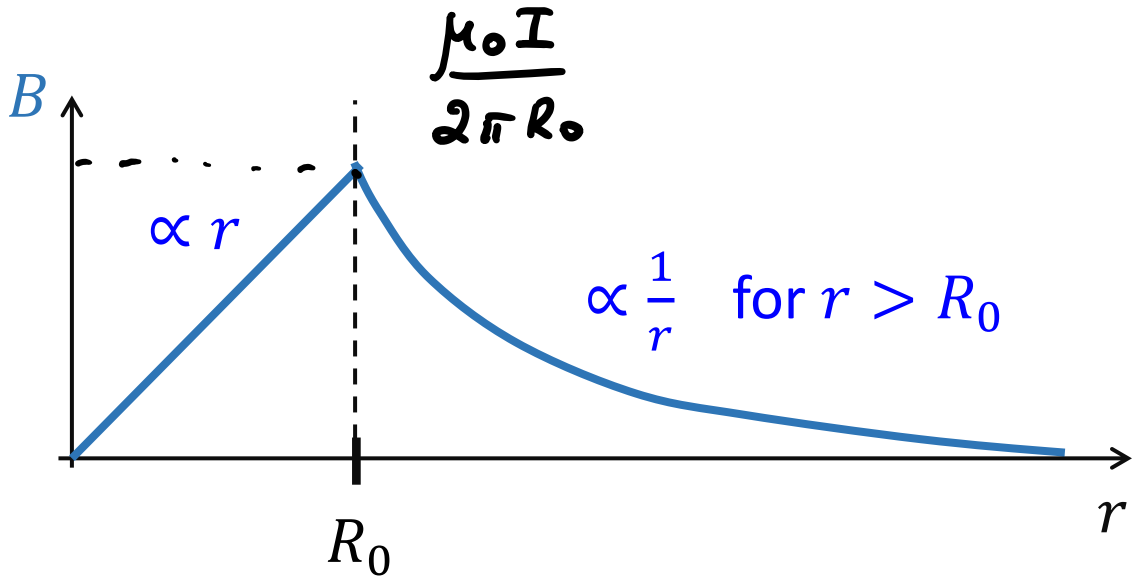
End view

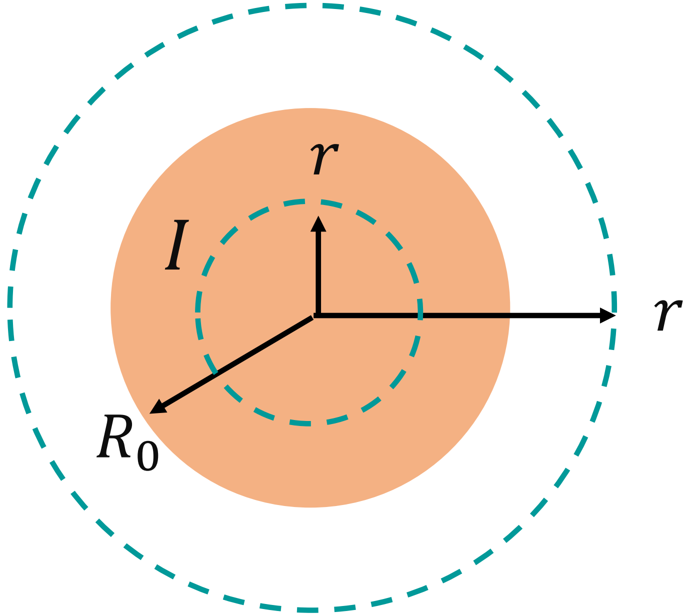
Ampère's law: application

a) Calculate $B_{in}(r < R_0)$ inside a long solid wire carrying a uniformly distributed current I.

b) Calculate $B_{out}(r > R_0)$ outside the long solid wire.

- Magnetic field inside a current-carrying wire:



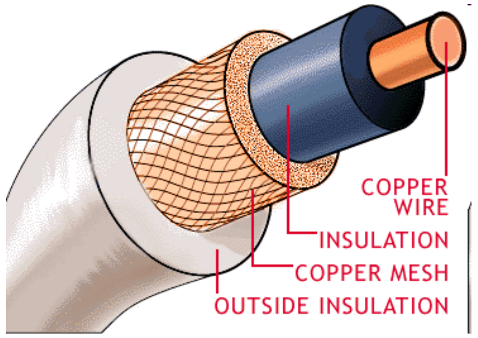


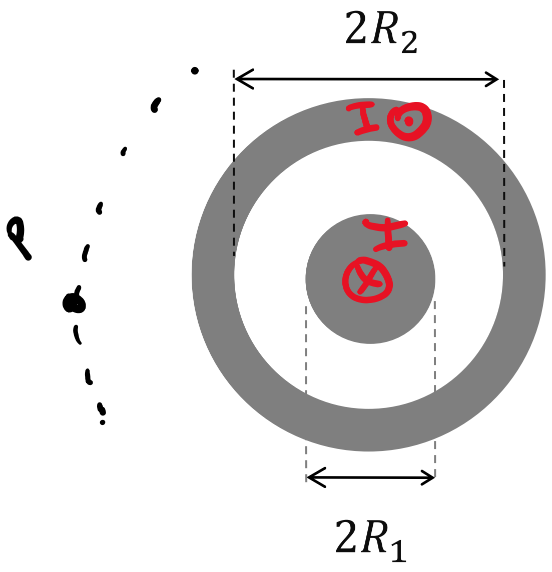
End view

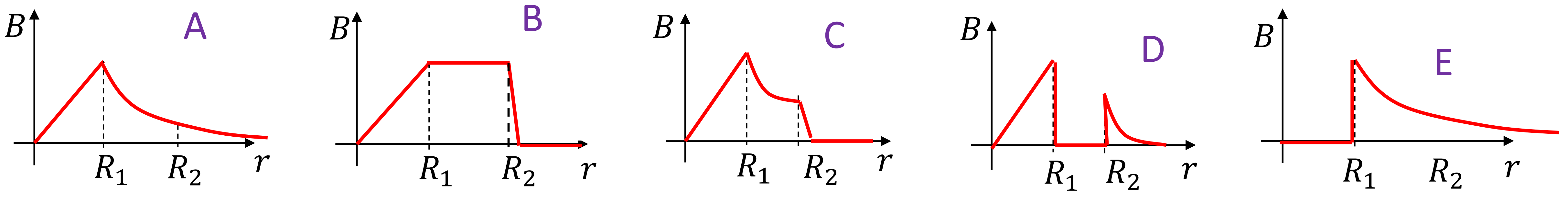
Coaxial Cable

Q: A coaxial cable consists of a wire (radius R_1) surrounded by an insulating sleeve and another cylindrical conducting shell (inner radius R_2) and finally another insulating sleeve. The wire and the shell carry the same current I but in opposite directions.

Which diagram represents correctly the magnetic field inside and outside the coaxial cable?







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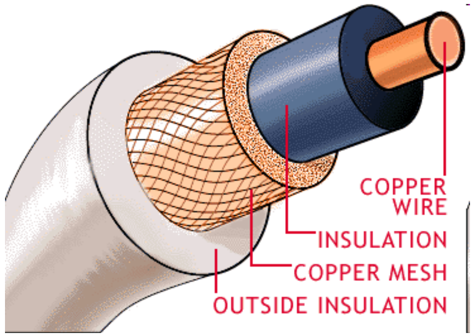
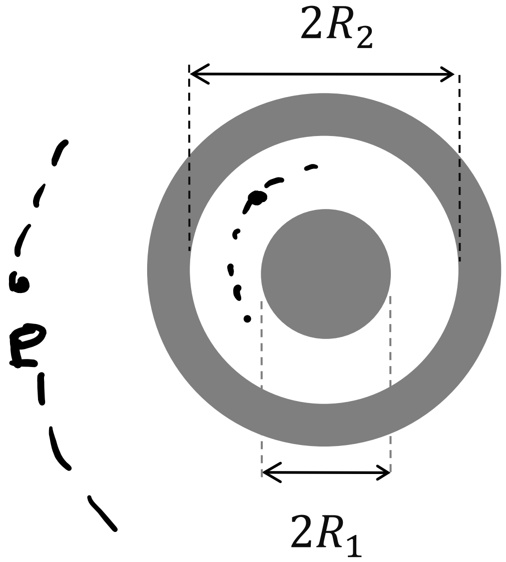
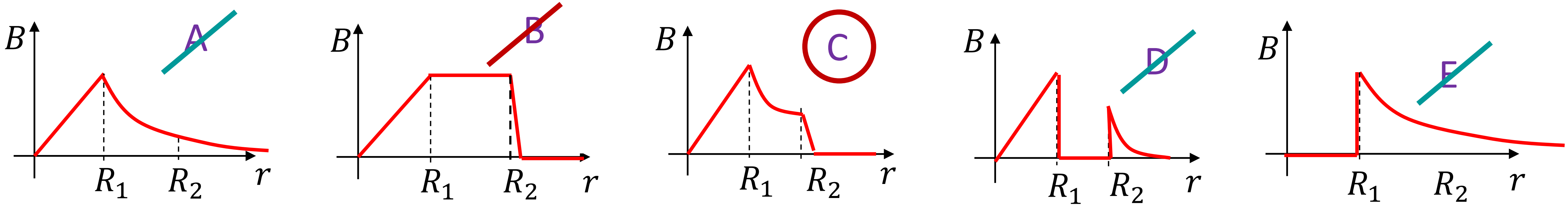


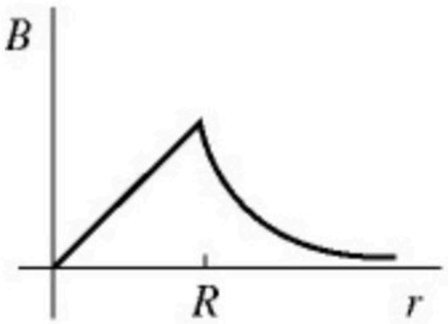
Which diagram represents correctly the magnetic field inside and outside the coaxial cable?



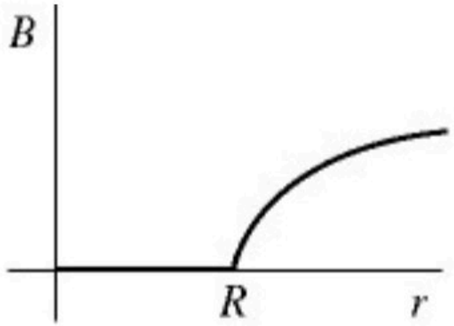
- Outside the shield: $I_{\text{encl}} = 0 \quad \Rightarrow \quad B = 0$
- Linear increase inside the wire (previous questions)
- $1/r$ decay outside the wire (previous questions)



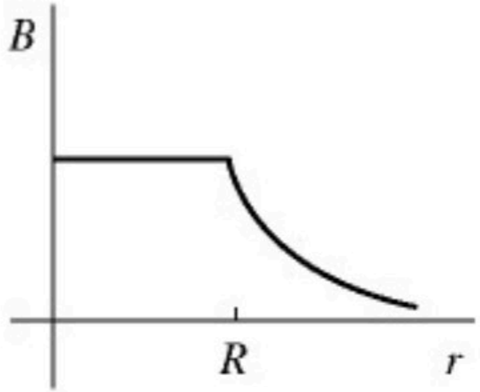
Q: A very long, hollow, <u>thin-walled</u> conducting cylindrical shell (like a pipe) of radius R carries a current along its length uniformly distributed throughout the thin shell. Which one of the graphs most accurately describes the magnitude B of the magnetic field produced by this current as a function of the distance r from the central axis?



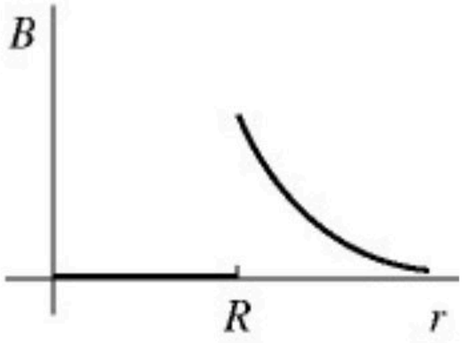
A



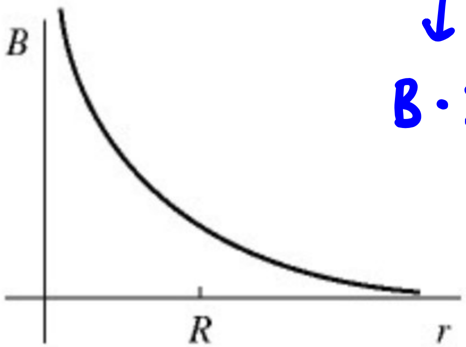
B



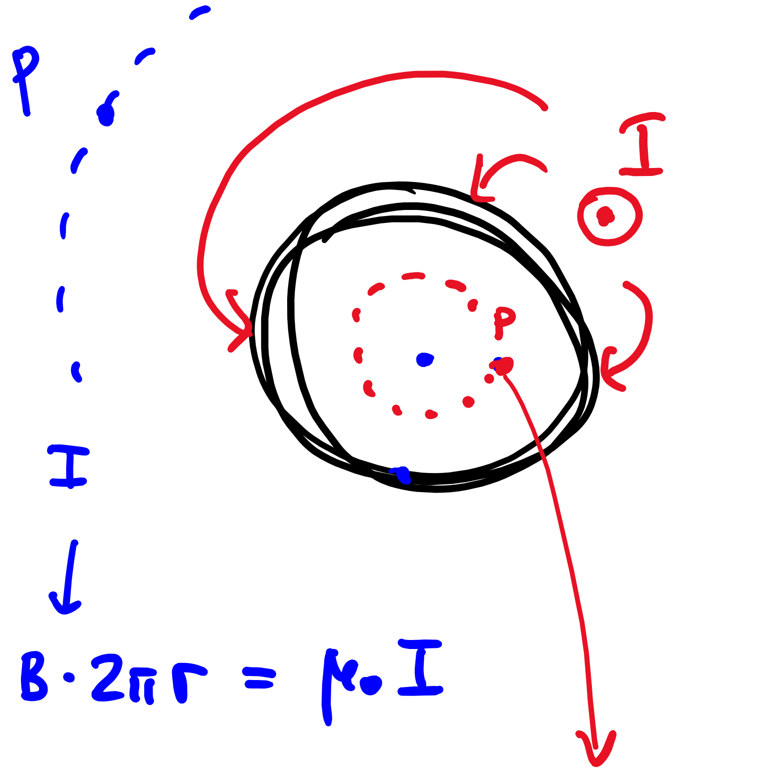
C



D



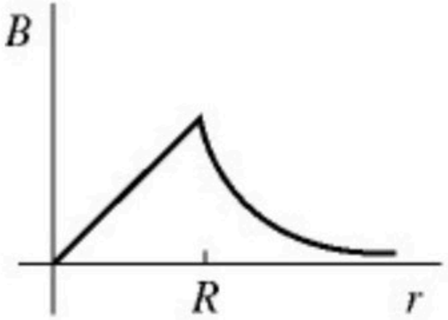
E



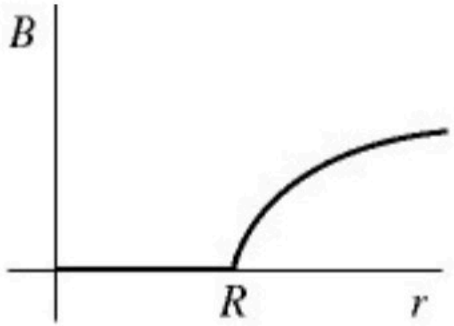
$B \cdot 2\pi r = \mu_0 I$

$B \cdot 2\pi r = 0$

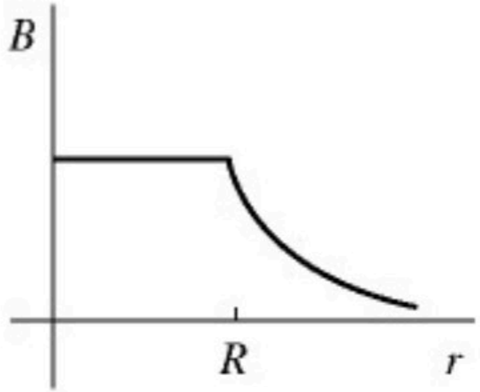
Q: A very long, hollow, thin-walled conducting cylindrical shell (like a pipe) of radius R carries a current along its length uniformly distributed throughout the thin shell. Which one of the graphs most accurately describes the magnitude B of the magnetic field produced by this current as a function of the distance r from the central axis?



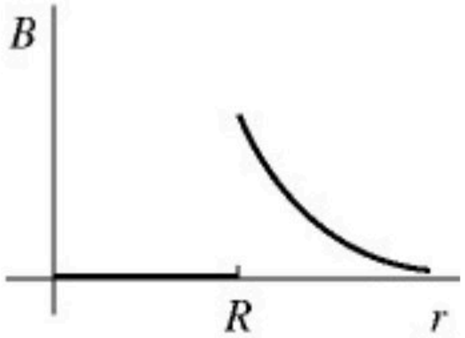
A



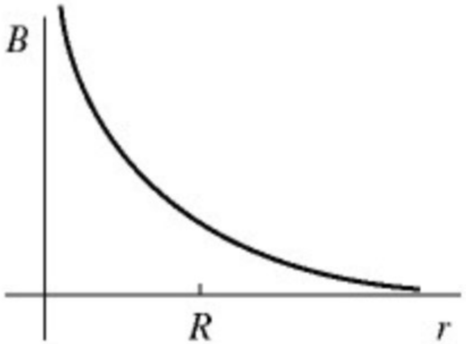
B



C



D

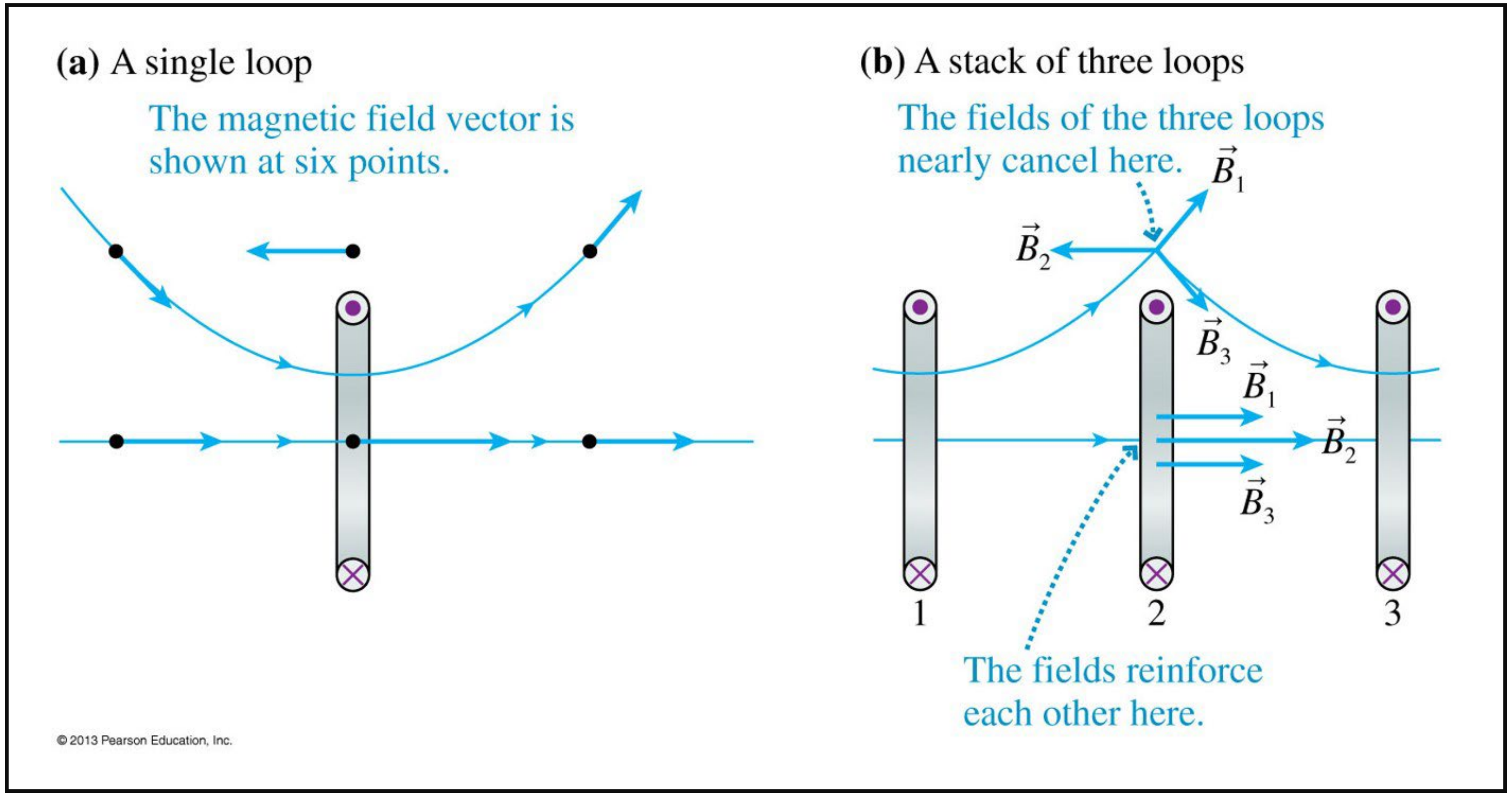


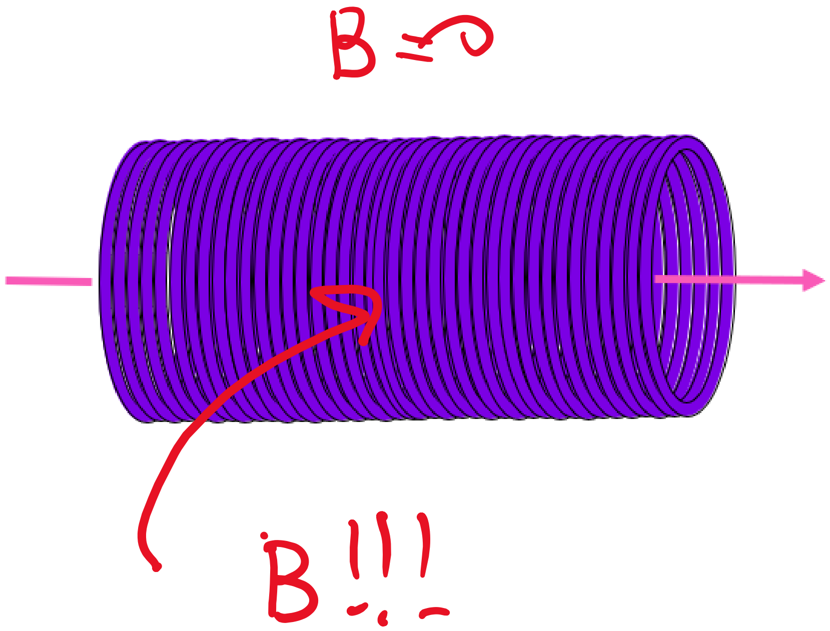
E

- $1/r$ decay outside the shell (same B-field as a wire)
- $B = 0$ inside (no current enclosed)

Solenoid

- The helical "winding" of a solenoid is like a sequence of circular loops, very slightly – negligibly – tilted.

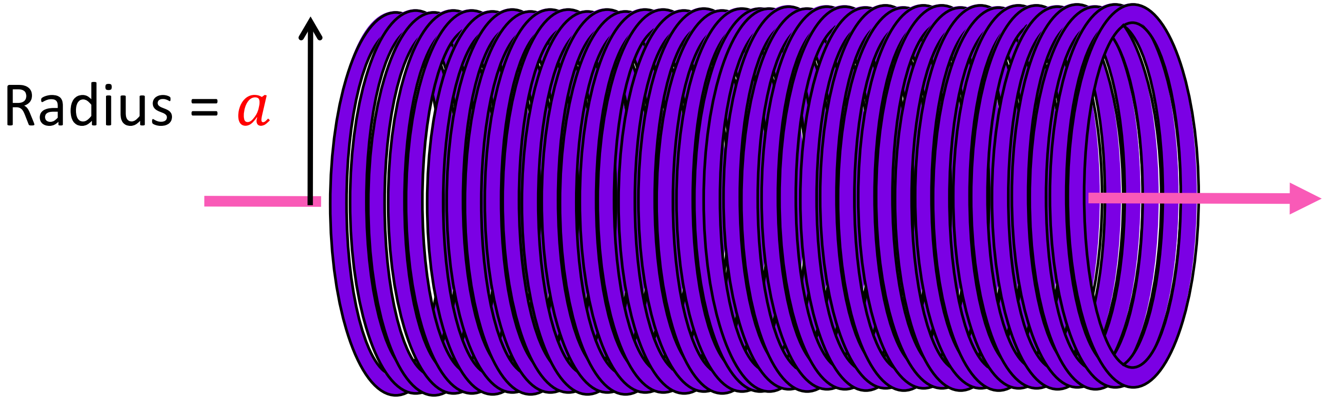


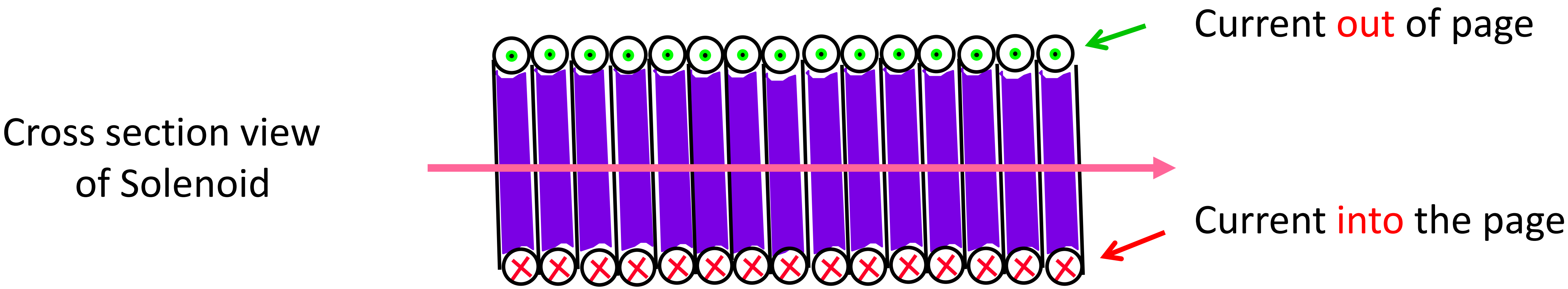


- Magnetic fields due to individual loops cancel outside the solenoid, and enhance each other inside it.

Solenoid

Q: Using Ampère's law, calculate the magnetic field (inside) a long solenoid (i.e. far from its ends). Assume the solenoid has n loops per meter.





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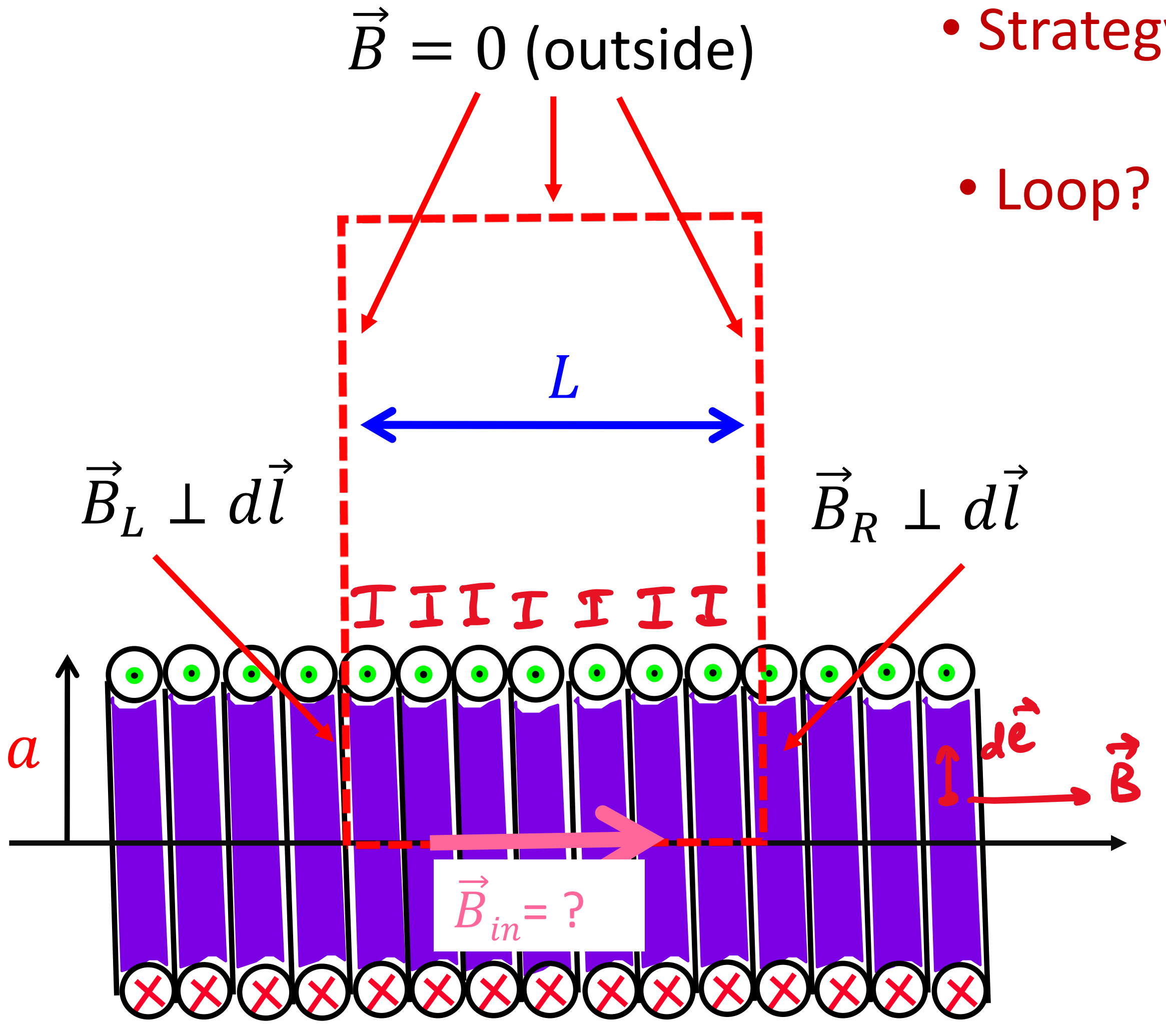


- Strategy: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\mathrm{encl}}$
- Loop?
  - From symmetry, the B-field inside the solenoid is parallel to its axis.
  - Using RHR, B_{inside} points to the right.
  - We also expect that the B-field is very small outside the solenoid (neglect ends)
  - …and $\vec{B} \perp d\vec{l}$ at the vertical sides inside

$\oint \vec{B} \cdot d\vec{l} = B_{in} L$ $\qquad$ $I_{\mathrm{encl}} = InL$

$$\boxed{B_{in} = \mu_0 I n}$$

Finite Length Solenoid

Unfortunately, Engineers have to deal with finite length solenoids!

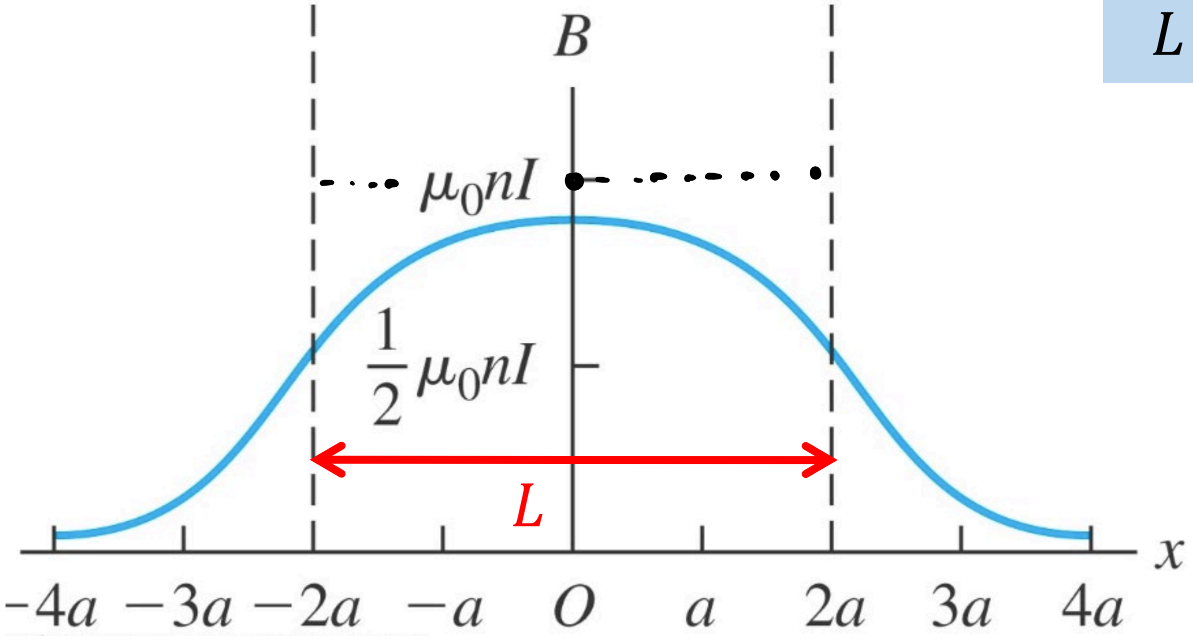
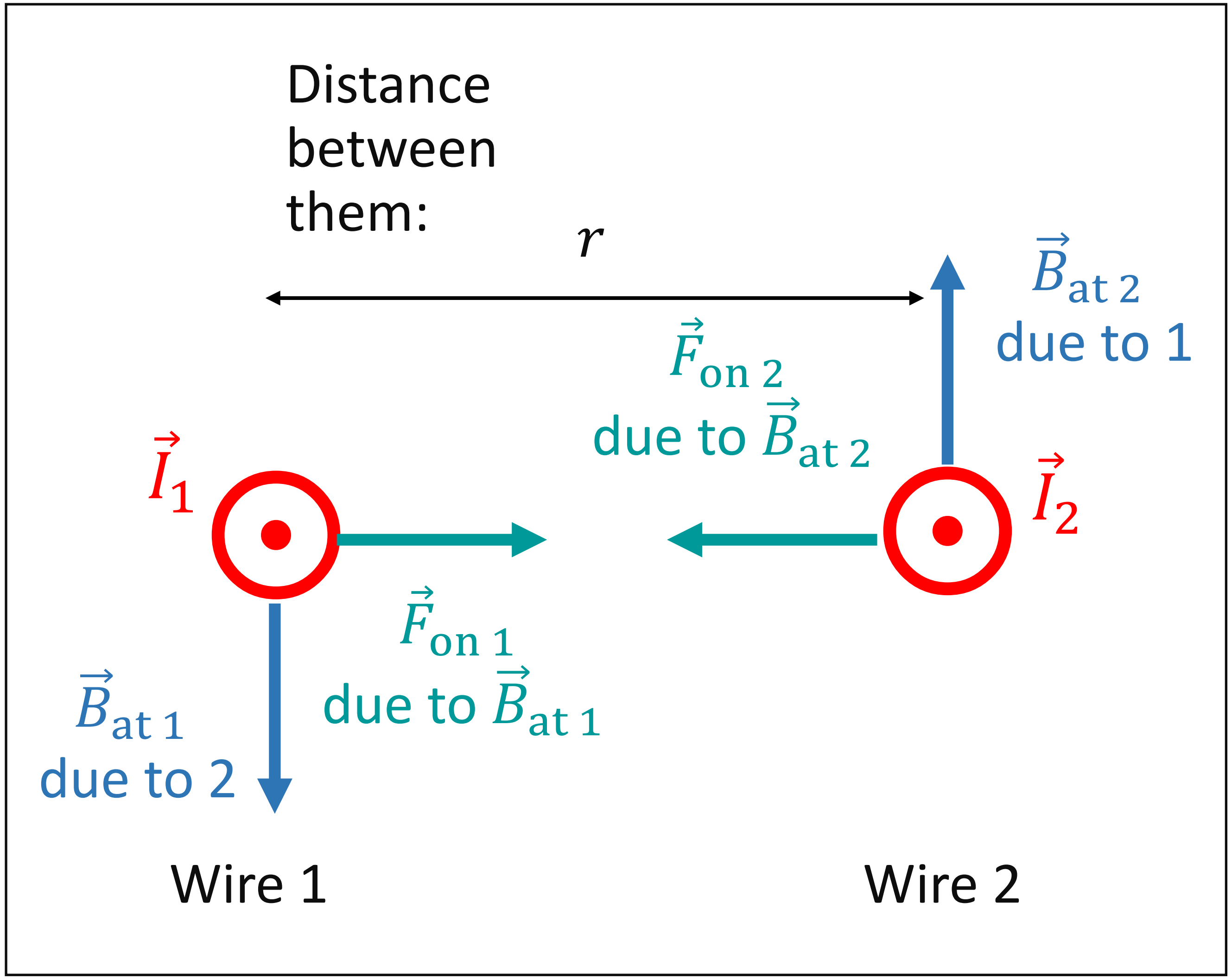


Fig 28.24
$L = 4a$

Look at website -- http://www.falstad.com/vector3dm

Force between two long straight wires of length L



- Find the force of their interaction per unit length

$$\frac{F_{\text{on 2}}}{L} = I_2 \underbrace{B_{\text{at 2}}}_{\text{due to 1}} = I_2\left(\frac{\mu_0}{2\pi}\frac{I_1}{r}\right)$$

$$\frac{F_{\text{on 1}}}{L} = I_1 \underbrace{B_{\text{at 1}}}_{\text{due to 2}} = I_1\left(\frac{\mu_0}{2\pi}\frac{I_2}{r}\right)$$

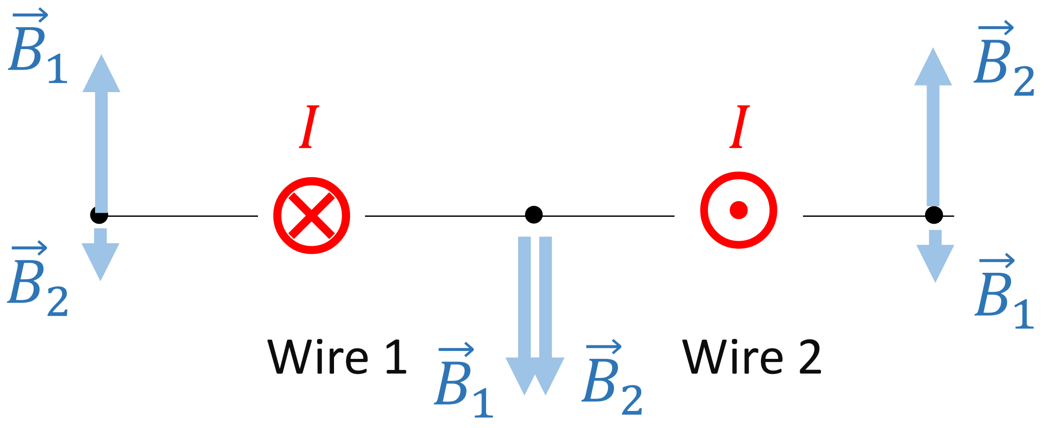
$$\frac{F_{\text{on 1}}}{L} = \frac{F_{\text{on 2}}}{L} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}$$

$\boxed{\vec{B}_{\text{wire}}\text{: RHR}}$

$$\boxed{\underbrace{B_{\infty\,\text{wire}}}_{\text{due 2}} = \frac{\mu_0}{2\pi}\frac{I_2}{r}}$$

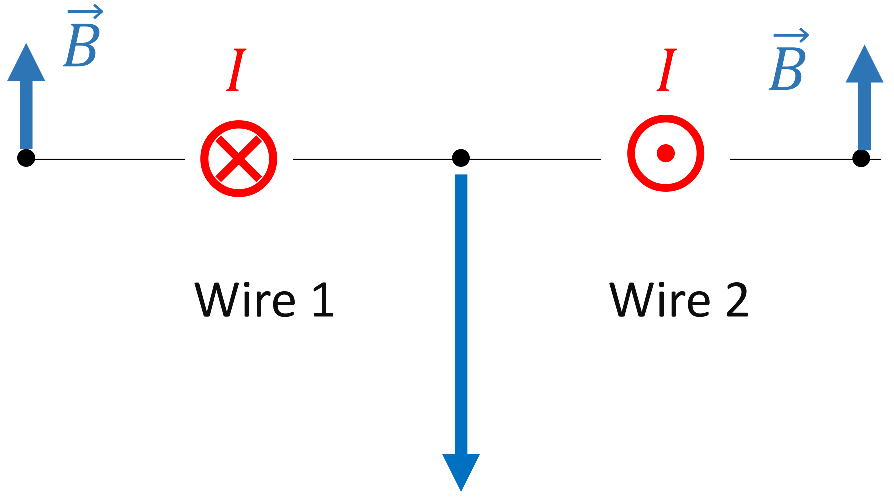
$$\boxed{\vec{F}_{\text{on wire}} = L\,"\vec{I}_1" \times \vec{B}_{\text{due 2}}}$$

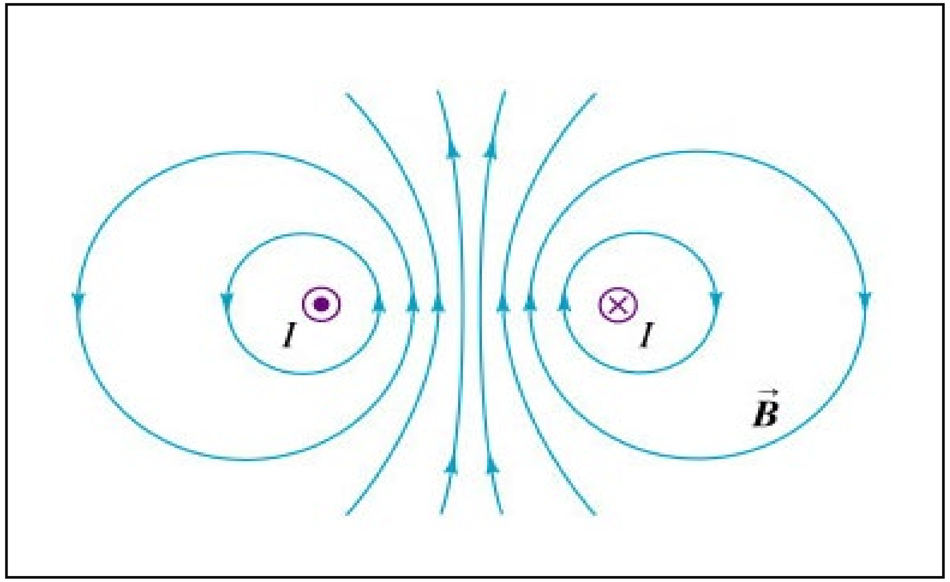
Magnetic Field of Two Straight Wires



- $\vec{B}_1$ and $\vec{B}_2$ add in central region
- $\vec{B}_1$ and $\vec{B}_2$ partially cancel out in the outer regions

- Use Superposition!

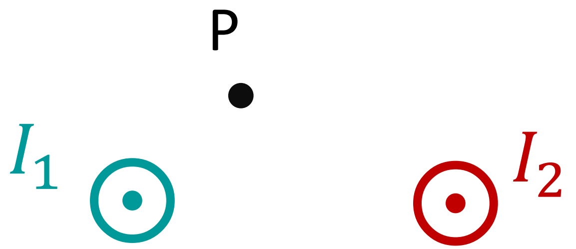




Magnetic Dipole Field

$$B_{\infty\text{ wire}} = \frac{\mu_0}{2\pi}\frac{I}{r}$$

Q: Consider two parallel wires carrying currents I_1 and I_2 as shown below. Which method(s) can be used to compute the total B-field at the point P?



A. Only Biot Savart Law
B. Only Ampere's Law
C. Both (A) and (B)
D. None of these methods can be used

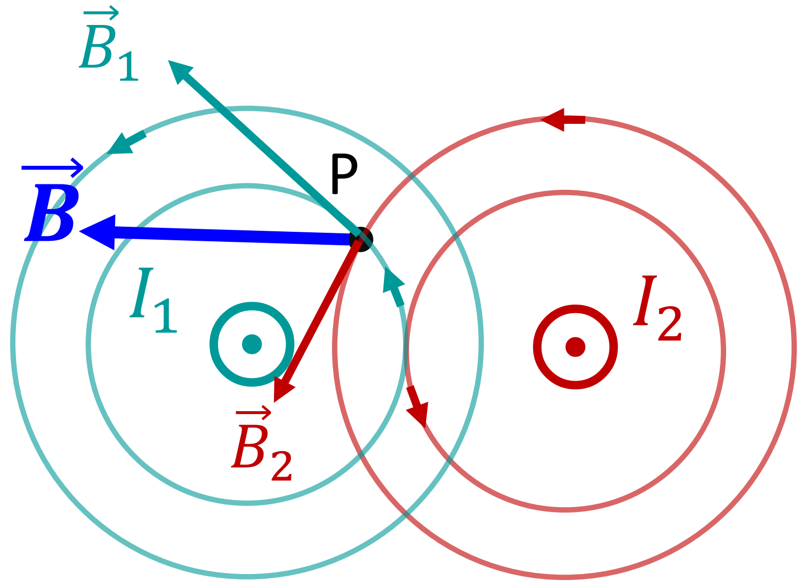
Biot-Savart

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}$$

Ampere

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$$

Q: Consider two parallel wires carrying currents I_1 and I_2 as shown below. Which method(s) can be used to compute the total B-field at the point P?



- We can still use Ampère's Law in this situation BUT we must use it for each wire separately so that B_1 and B_2 are constant along each of the circular paths !!

- Then we can add up $\vec{B}_1$ and $\vec{B}_2$ at every point in space -- using SUPERPOSITION and SYMMETRY !!

A. Only Biot Savart Law

B. Only Ampere's Law

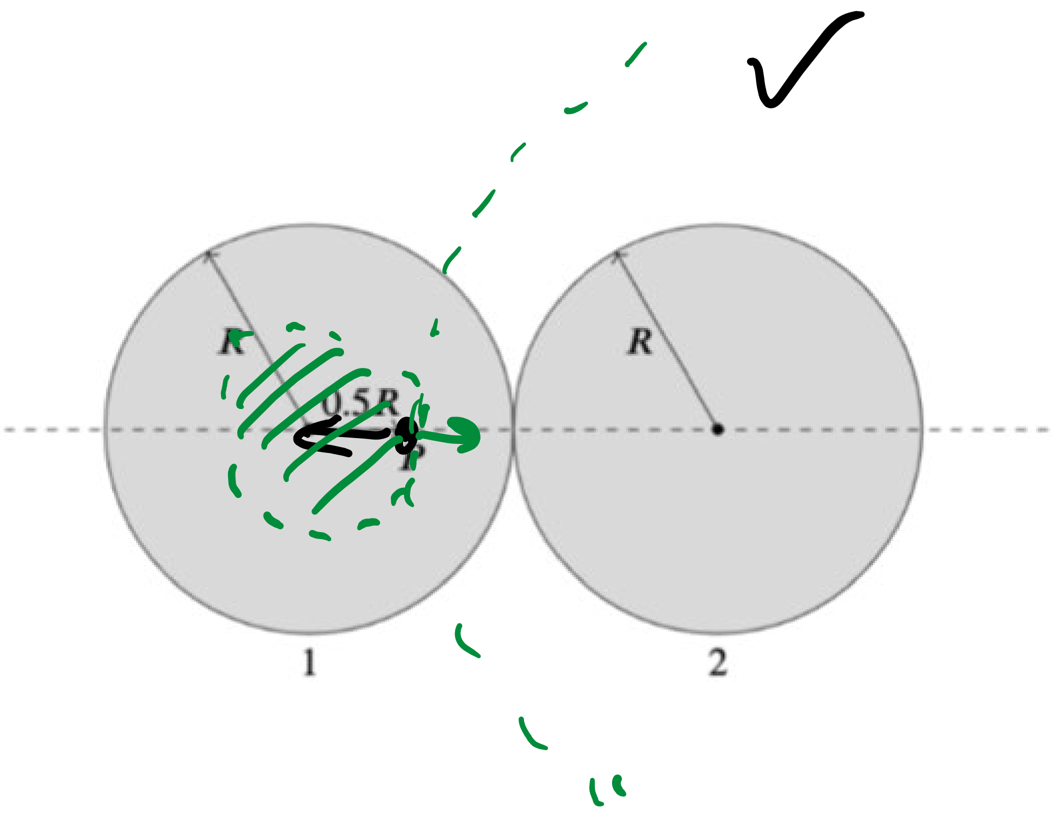
C. Both (A) and (B)

D. None of these methods can be used

Biot-Savart	Ampere
$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{l} \times \hat{r}}{r^2}$	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$

• Quote: Then we can add up $\vec{B}_1$ and $\vec{B}_2$ at every point in space

-- using SUPERPOSITION and SYMMETRY !!



• Another example: in HW-7, you found $\vec{E}$ at P by applying Gauss's law to each of these two spheres separately, and using superposition principle!