Lecture 3.

Ohm's law (continued).
Grounding.
Electric power.
Terminal voltage vs EMF.

Last Time:

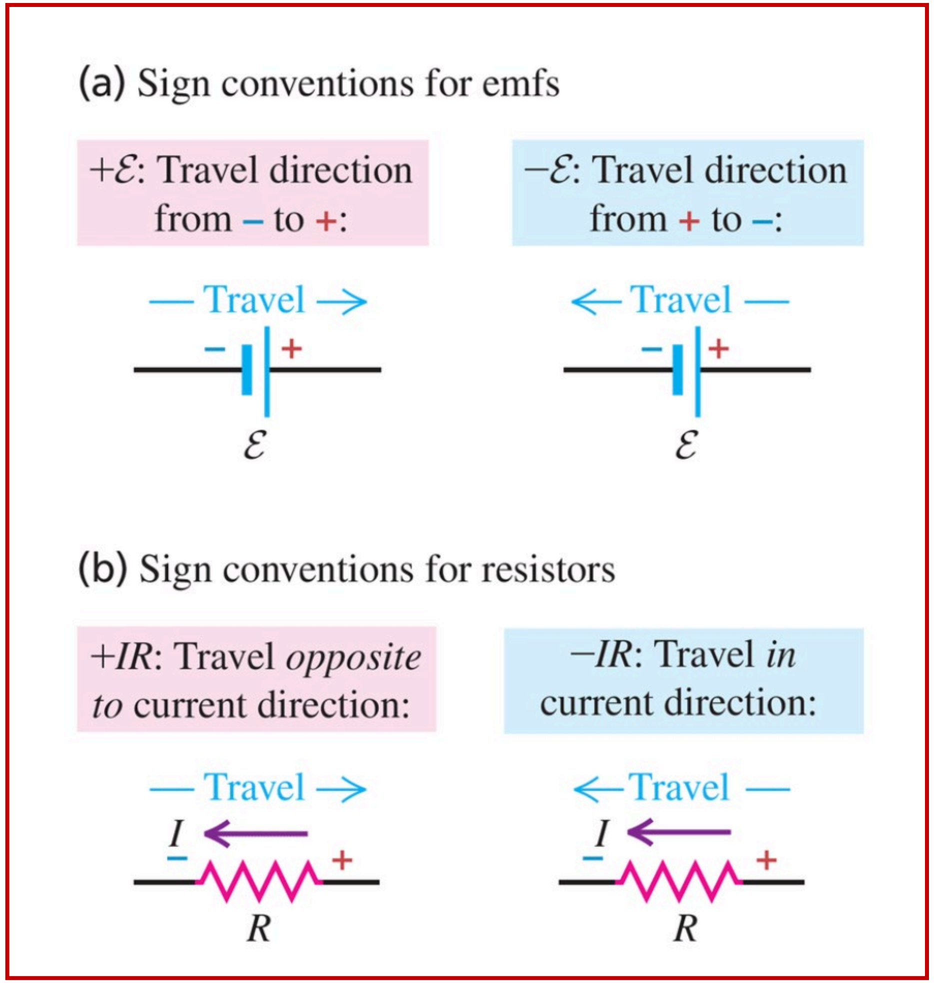
- Ohm's law

$$\boxed{\Delta V_R = IR}$$

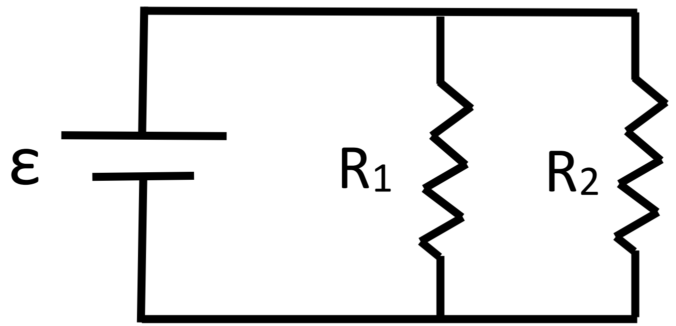
- Kirchhoff's loop law

$$\boxed{\sum\nolimits_{\text{loop}} \Delta V = 0}$$

- Sign conventions for voltage drops



Simple DC Circuit with resistors.



In this circuit, $\varepsilon = 12\,V, R_1 = 2\,\Omega$, and $R_2 = 3\Omega$.

a) Calculate the current through R_1.
b) What is the current supplied by the battery?

A. 2.4 A
B. 4.0 A
C. 6.0 A
D. 8.0 A
E. 10.0 A

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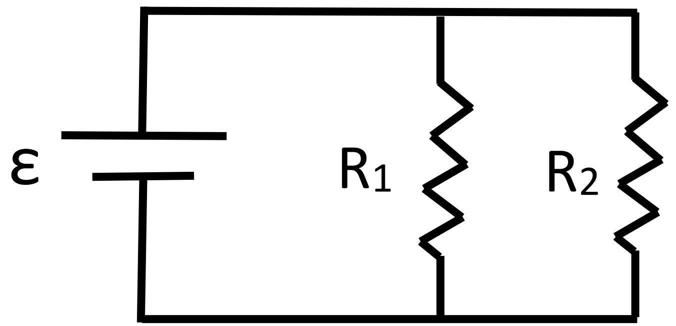

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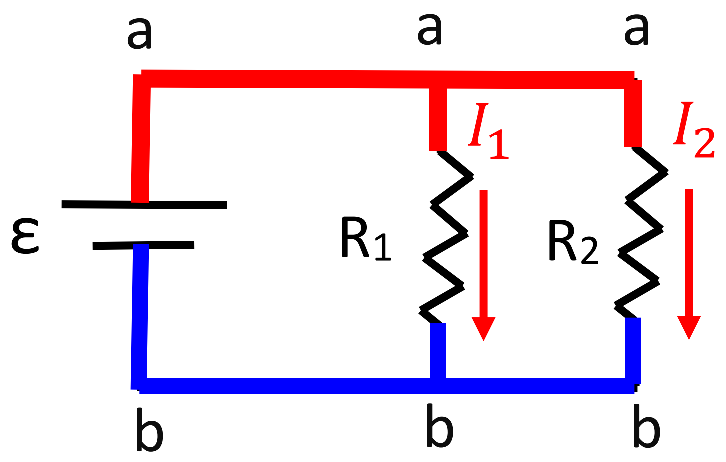
- You can solve part b) using equivalent resistance:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \Rightarrow \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = 1.2\ \Omega \quad \Rightarrow \quad I_{tot} = \frac{\varepsilon}{R_{eq}} = 10\ A$$

- To answer part a), you need to figure out how this current splits between the two resistors – possible, but difficult and long! ☹

Simple DC Circuit with resistors.



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A. 2.4 A
B. 4.0 A
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- All three components are under the same voltage
  (same voltage at all points of an ideal wire!)

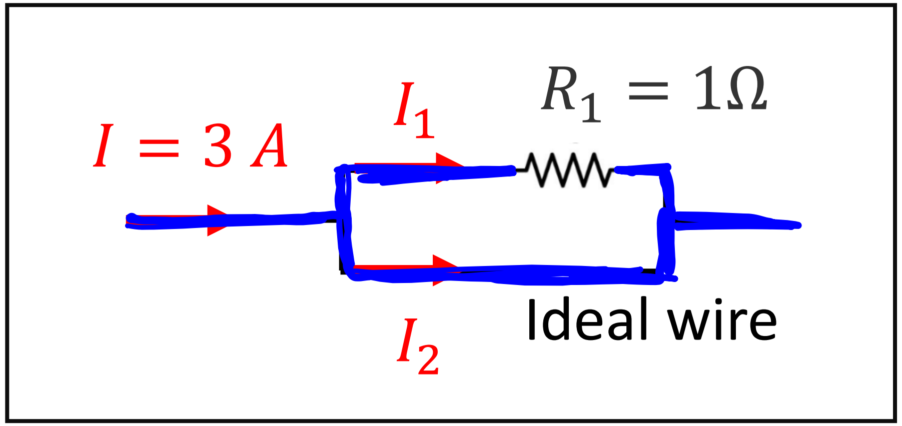
$$I_1 = \frac{\varepsilon}{R_1} = 6\,A \qquad I_2 = \frac{\varepsilon}{R_2} = 4\,A$$

$$I_{tot} = I_1 + I_2 = 10\,A$$

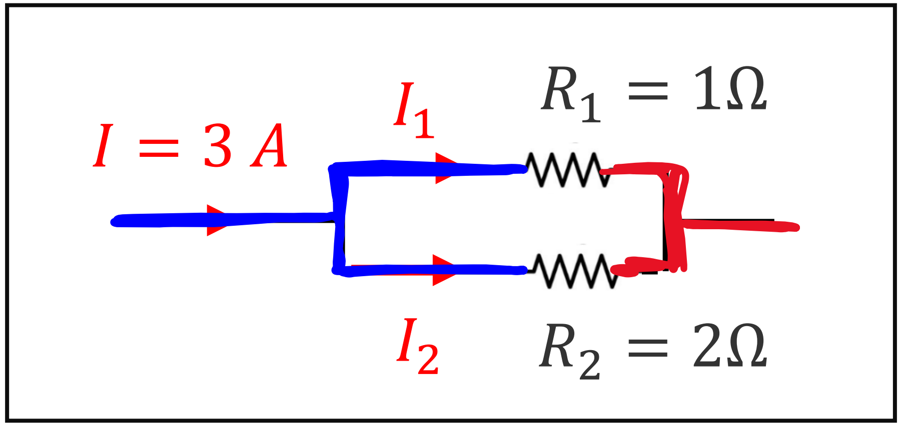
How current splits?

- Common misconceptions.

Compare:



$I_{R1} = \frac{\Delta V_{R1}}{R_1} = 0$



Find currents I_1 and I_2.

A. $I_1 = 0, \quad I_2 = 3\ A$

B. $I_1 = 1\ A, \ I_2 = 2\ A$

C. $I_1 = 2\ A, \ I_2 = 1\ A$

D. $I_1 = 3\ A, \ I_2 = 0$

E. Splits evenly ($1.5\ A$ each)

i) $I_1 + I_2 = 3A$

$\Delta V_{comb} = I_1 R_1 = I_2 R_2$

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B. $I_1 = 1\ A, \ I_2 = 2\ A$

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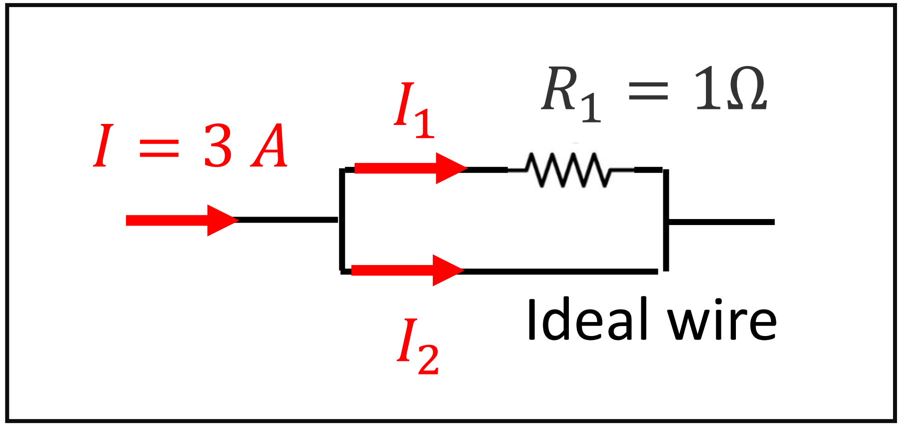
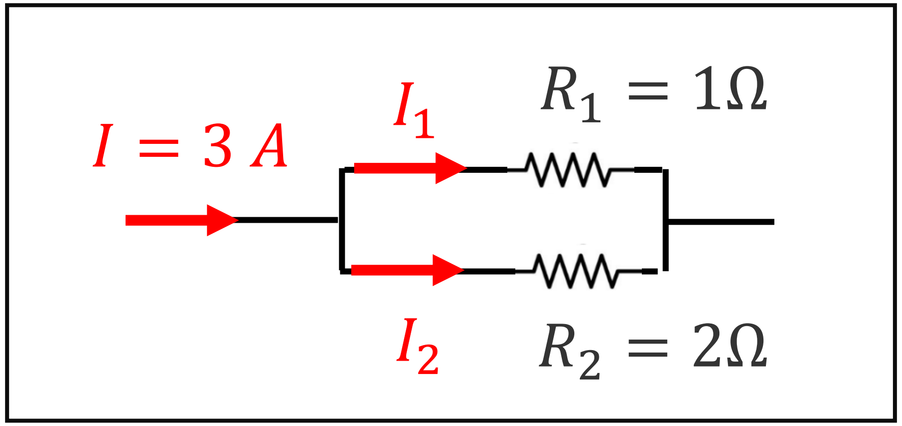




$\Delta V_{R1} = \Delta V_{\mathrm{wire}}$ (in parallel)

$\Delta V_{\mathrm{wire}} = 0$ (ideal wire!) => $\Delta V_{R1} = 0$

$$I_1 = \frac{\Delta V_{R1}}{R_1} = \frac{0\ V}{1\ \Omega} = 0$$

$I_2 = I = 3\ A$

All current goes into the ideal wire!

$\Delta V_{R1} = \Delta V_{R2}$ (in parallel)

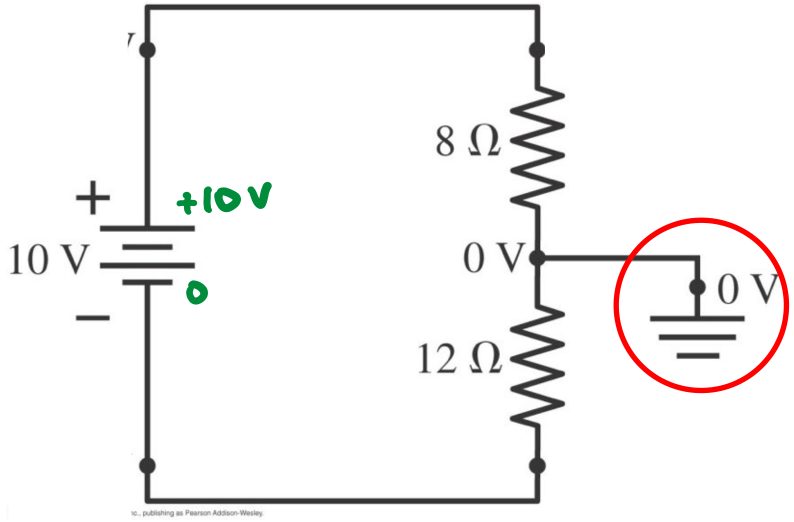
$$R_{eq} = \frac{1 \cdot 2}{1+2} = \frac{2}{3}\Omega \quad \Rightarrow \quad \Delta V_{1,2} = IR_{eq} = 2V$$

$$I_1 = \frac{\Delta V_{R1}}{R_1} = \frac{2\ V}{1\ \Omega} = 2\ A$$

$$I_2 = \frac{\Delta V_{R2}}{R_2} = \frac{2\ V}{2\ \Omega} = 1\ A$$

Current splits inversely proportional to R!

Grounding



- Grounding the circuit does not change any of the currents, but it does allow us to define specific values for the voltage at each point in the circuit.

- This allows us to join different circuits together, without any unexpected currents between them.

- Spoiler: grounding simply means connecting a point at your circuit to a big conductor (it can be Earth, but not necessarily). Later we will learn that the whole conductor always stays under the same "electric potential" => it has the same voltage everywhere, and this voltage is imposed onto the wires touching it.

Q: a) Find the current at each corner in the circuit.

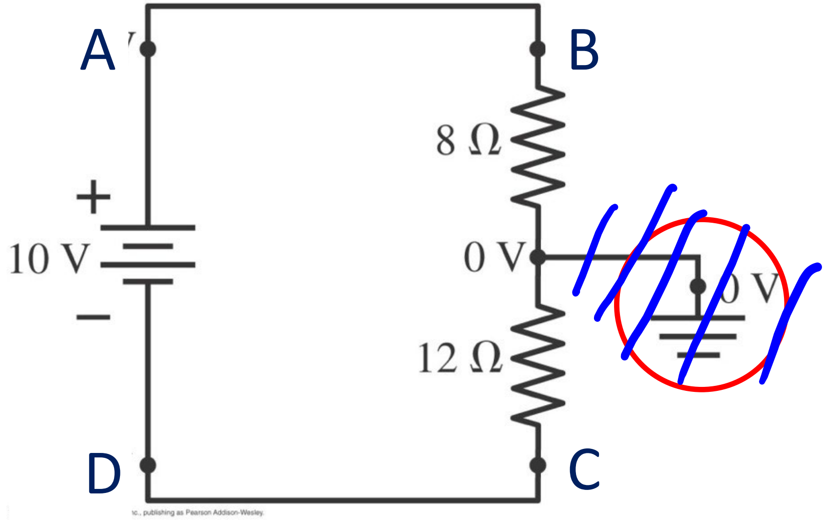
A. $I_A = 1.25\ A,\quad I_B = 1.25\ A,\quad I_C = 0.83\ A,\quad I_D = 0.83\ A$

B. $I_A = 1.25\ A,\quad I_B = 1.25\ A,\quad I_C = 1.25\ A,\quad I_D = 1.25\ A$

C. $I_A = 0.5\ A,\quad I_B = 0.5\ A,\quad I_C = 0.5\ A,\quad I_D = 0$

D. $I_A = 0.83\ A,\quad I_B = 0.83\ A,\quad I_C = 0.83\ A,\quad I_D = 0.83\ A$

E. $I_A = 0.5\ A,\quad I_B = 0.5\ A,\quad I_C = 0.5\ A,\quad I_D = 0.5\ A$



0.5 A

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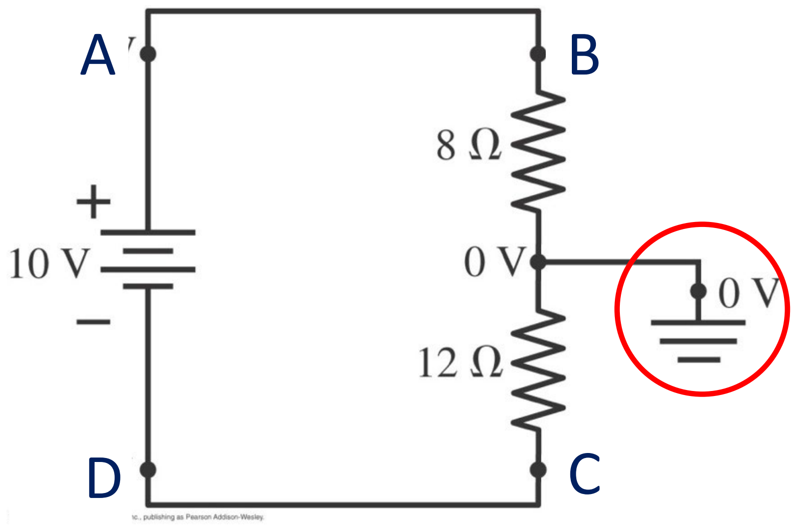


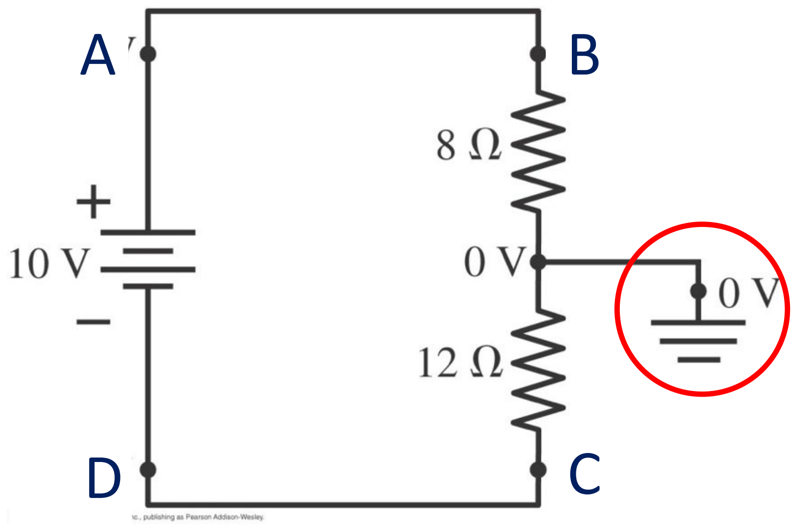
Circuit: two resistors in series

$$R_{eq} = R_{8\Omega} + R_{12\Omega} = 20\ \Omega$$

$$I_{tot} = \frac{V}{R_{eq}} = \frac{10\ V}{20\ \Omega} = 0.5\ A; \qquad I_{tot} = I_A = I_B = I_C = I_D$$

Q: b) Find the voltage at each corner in the circuit.

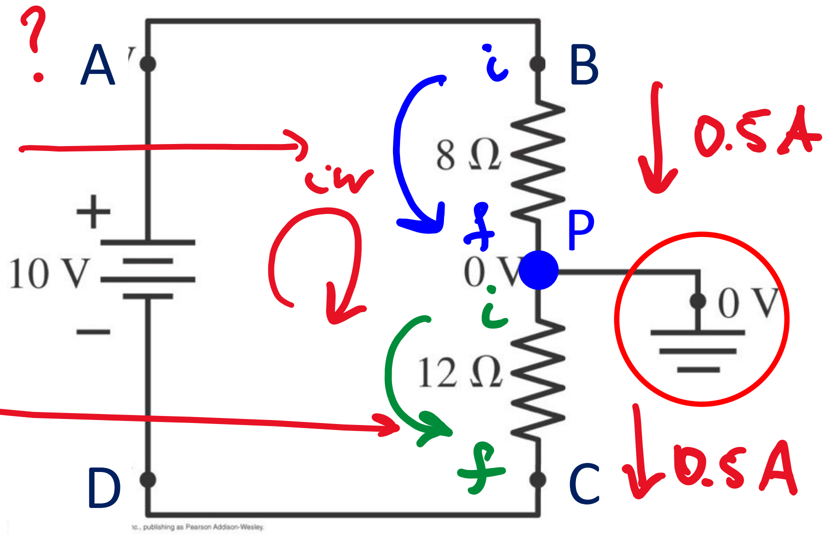
A. $V_A = 10\,V$, $V_B = 10\,V$, $V_C = 0$, $V_D = 0$
B. $V_A = 10\,V$, $V_B = 6\,V$, $V_C = 4\,V$, $V_D = 0A$
C. $V_A = 4\,V$, $V_B = 4\,V$, $V_C = 6\,V$, $V_D = 6\,V$
D. $V_A = 4\,V$, $V_B = 4\,V$, $V_C = -6\,V$, $V_D = -6\,V$
E. Something else



Q: b) Find the voltage at each corner in the circuit.

$V_P - V_B = -0.5\cdot 8 = -4$

$\Delta V_{8\Omega} = ?$



A. $V_A = 10\ V,\quad V_B = 10\ V,\quad V_C = 0,\quad V_D = 0$
B. $V_A = 10\ V,\quad V_B = 6\ V,\quad V_C = 4\ V,\quad V_D = 0A$
C. $V_A = 4\ V,\quad V_B = 4\ V,\quad V_C = 6\ V,\quad V_D = 6\ V$
D. $V_A = 4\ V,\quad V_B = 4\ V,\quad V_C = -6\ V,\quad V_D = -6\ V$
E. Something else

$V_C - V_P = \Delta V_{12\Omega} = \dots$

$-0.5\cdot 12 =$

$= -6 = V_C - V_P$

1) $V_A = V_B$ and $V_C = V_D$ (these pairs belong to same ideal wire)

2) $\Delta V_{8\Omega} = V_P - V_B = -IR_{8\Omega} = -0.5\cdot 8 = -4\ V \quad\Rightarrow\quad V_B = 4\ V$

$\Delta V_{12\Omega} = V_C - V_P = -IR_{12\Omega} = -0.5\cdot 12 = -6\ V \quad\Rightarrow\quad V_C = -6\ V$

$\Delta V = V_f - V_i$

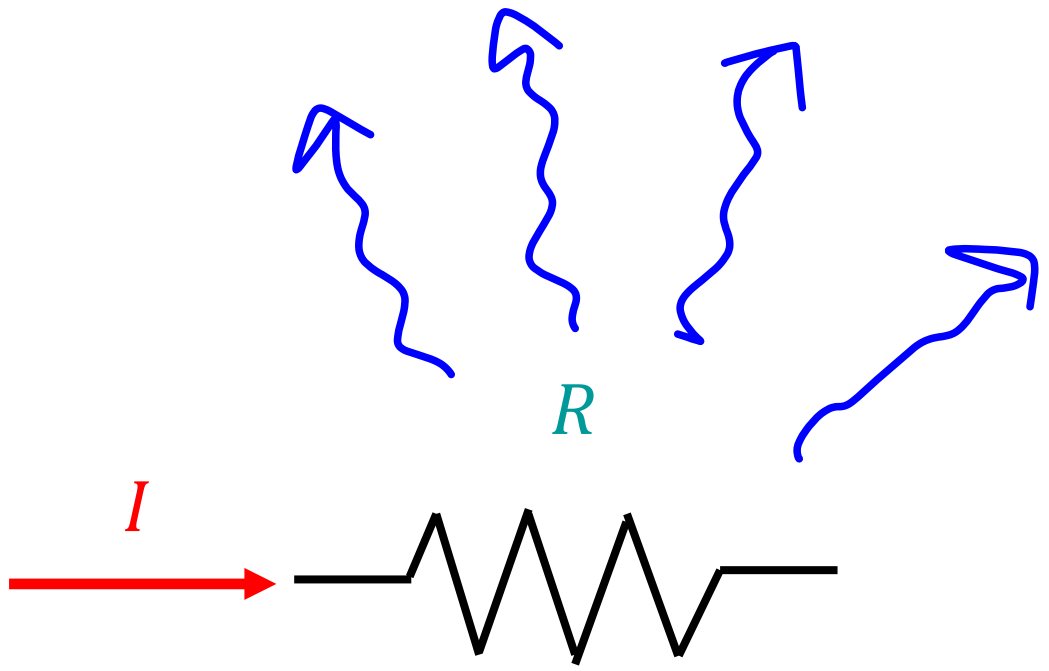
Electric power dissipated by a resistor

$$P = \frac{\Delta U}{\Delta t} = \frac{\text{Energy}}{\text{time}}$$

- Power: amount of energy dissipated by a resistor per unit time.

$$\boxed{P_R = I\Delta V_R = I^2 R = \frac{(\Delta V_R)^2}{R}}$$

- These three forms are equivalent, since $\Delta V_R = IR$



- Light bulb: brightness = dissipated power

Q: Bulbs a, b, and c are identical, and are all glowing.

Suppose an additional wire is now connected between point 1 and point 2. What happens to each bulb? Does it get brighter, dimmer, stay the same, or go out?

A. a = ↑, b = ↑, c = ↓ (brighter = ↑, dimmer = ↓)

B. a = ↓, b = ↓, c = out

C. a = ↑, b = out , c = out

D. a = ↓, b = out, c = ↓

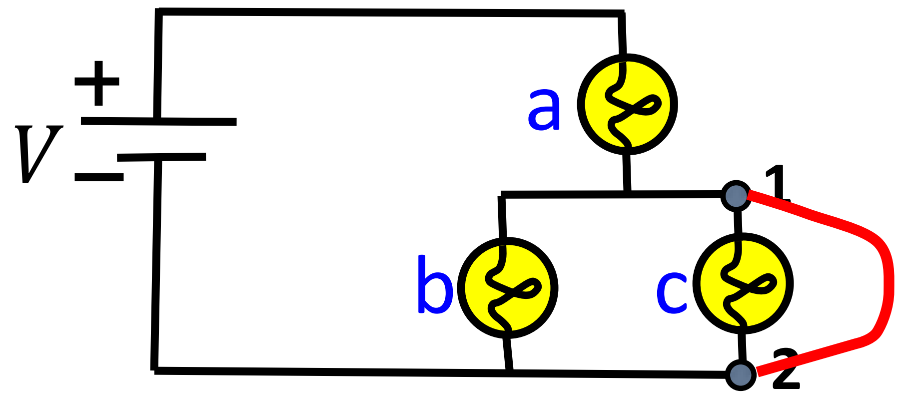
E. a = ↓, b = out, c = out

$$P_A = I_{tot}^2 \cdot R \uparrow$$

Before: $R_{eq} = R + \frac{R}{2} = \frac{3R}{2}$

After: $R_{eq} = R$

$R_{eq} \downarrow \rightarrow I_{tot} \uparrow$



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C. a = ↑, b = out , c = out

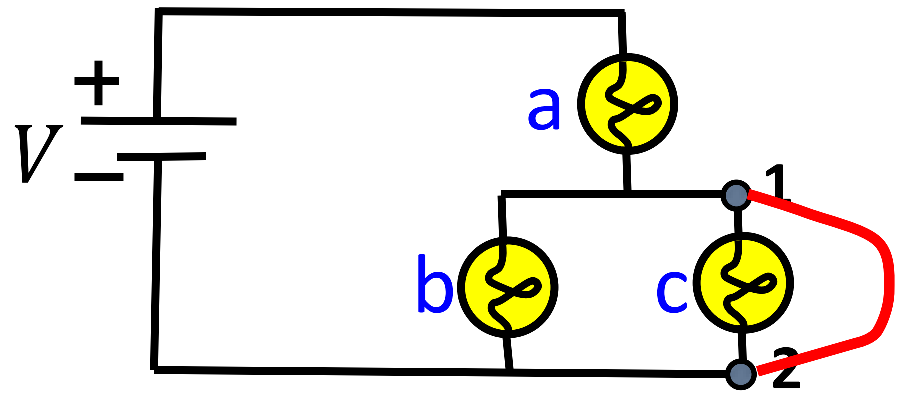
D. a = ↓, b = out, c = ↓

E. a = ↓, b = out, c = out

This wire is assumed to be ideal => same potential along the whole wire => $\Delta V_{12} = 0$ => $I_b = \frac{\Delta V_{12}}{R_B} = 0$, and $I_c = \frac{\Delta V_{12}}{R_C} = 0$, so both of these bulbs go out (all current now goes through the wire).

- Without wire: $R_{eq} = R_a + R_{bc}$.
- With the wire: $R_{eq} = R_a$ =>

$R_{eq} \downarrow$ => $I_{tot} \uparrow$. The current through bulb a increases (since it receives I_{tot}) => **bulb a gets brighter**.

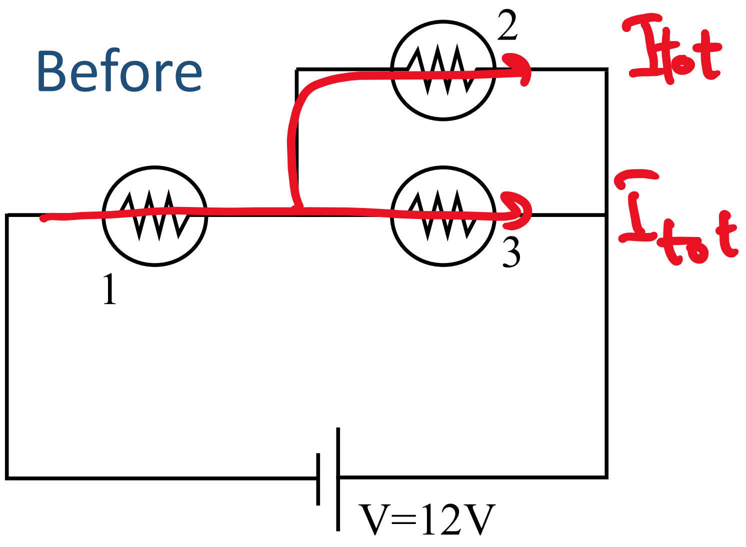


Q: a) What happens to the brightness of bulb 1, when bulb 2 burns out? Bulb 1 gets:

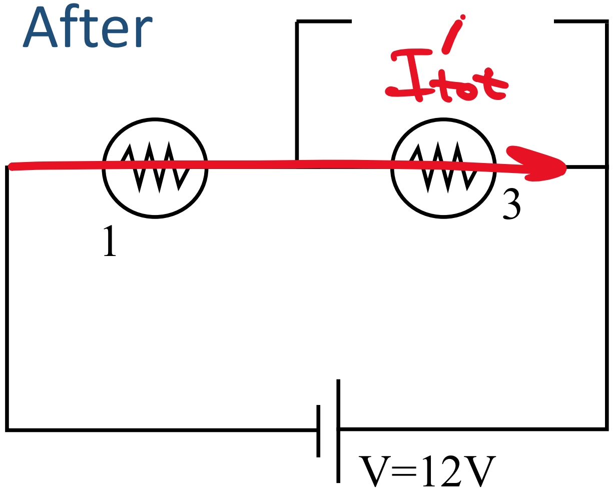
A. Dimmer
B. Brighter
C. Stays the same
D. Goes out
E. Need to know R

$R_{eq} \uparrow \rightarrow I_{tot} \downarrow \rightarrow$ Dimmer

OK



b) What happens to bulb 3?

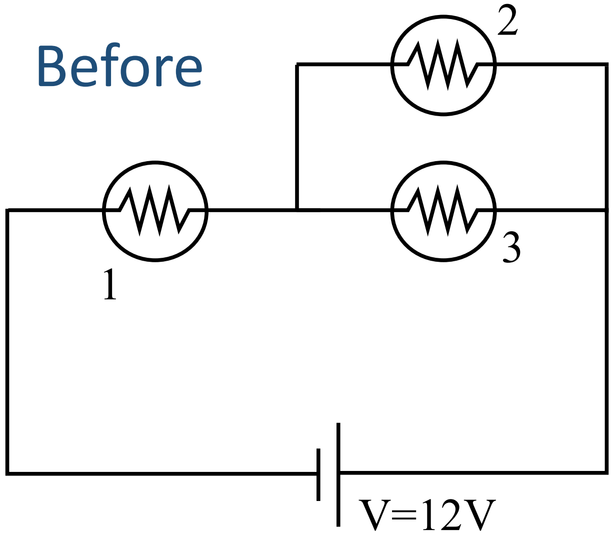


Q: a) What happens to the brightness of bulb 1, when bulb 2 burns out?

b) What happens to bulb 3?

$$\boxed{P_R = I^2 R}$$

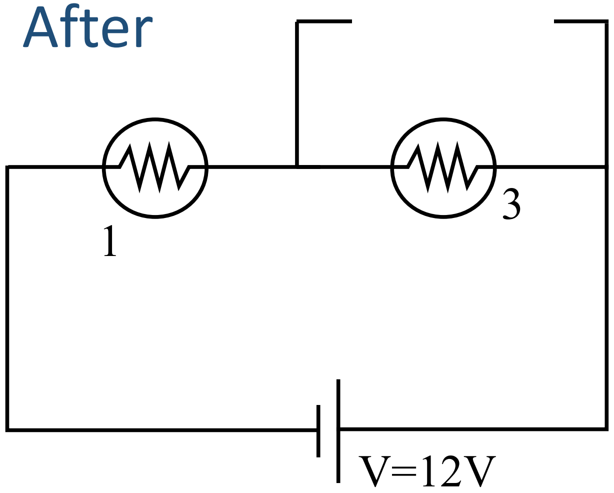
Before



$$R_{eq} = R + \frac{R \cdot R}{R + R} = \frac{3R}{2}$$

$$I_{tot} = \frac{V}{R_{eq}} = \frac{2V}{3R} = I_1; \qquad I_2 = I_3 = \frac{I_1}{2} = \frac{V}{3R}$$

After



$$R'_{eq} = R + R = 2R$$

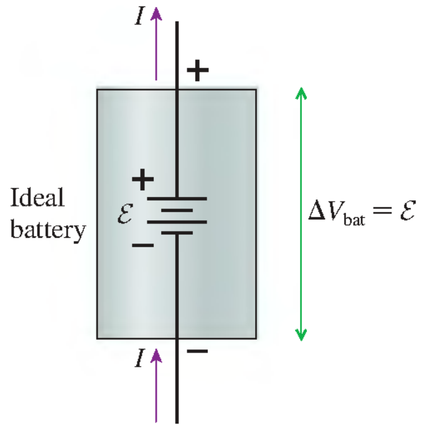
$$I'_{tot} = \frac{V}{R'_{eq}} = \frac{V}{2R} = I'_1 = I'_3$$

Compare ($P_R = I_R^2 R$):

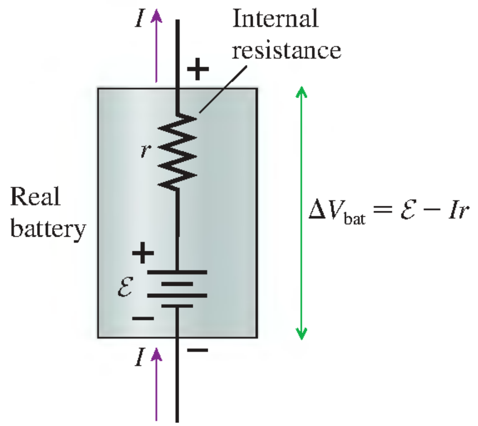
- $I_2 \downarrow \Rightarrow \quad P_2 \downarrow$: off
- $I_1 \downarrow \Rightarrow \quad P_1 \downarrow$: dimmer
- $I_3 \uparrow \Rightarrow \quad P_1 \uparrow$: brighter

Real batteries

- Battery produces electromotive force (emf): ε (from chemical reaction)
- But: user gets terminal voltage: ΔV_{bat}
  - Ideal battery: $\varepsilon = \Delta V_{bat}$
  - Real battery: $\varepsilon > \Delta V_{bat}$ (runs hot while you use it)
  - Losses can be modelled by an "internal resistance", r



- What is the terminal voltage for a circuit with emf ε, internal resistance r and an external load R?

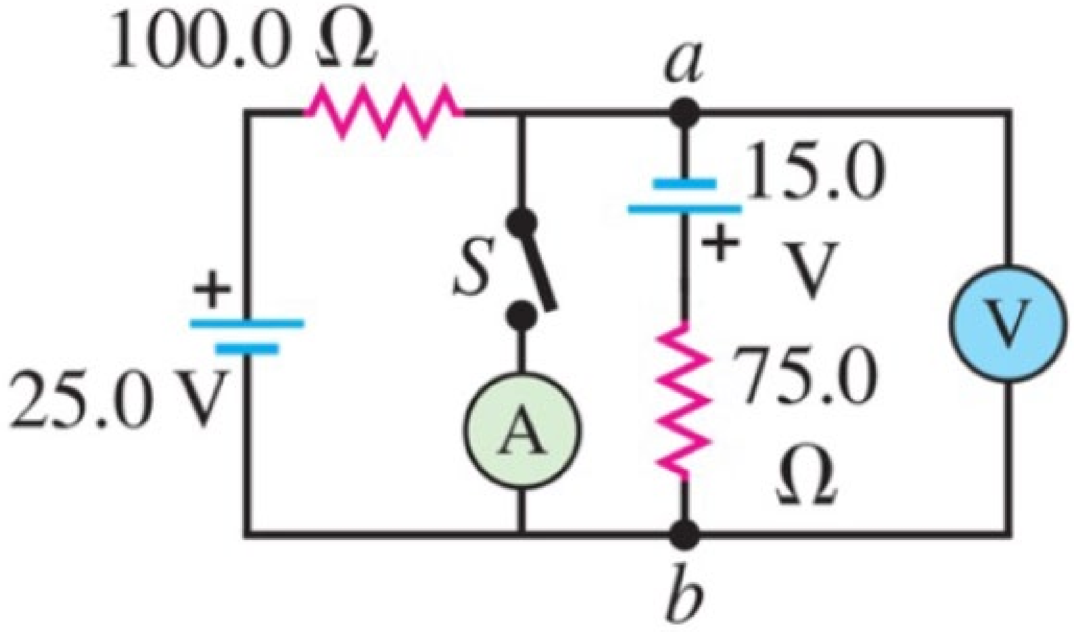


$$\Delta V_{bat} = \varepsilon - Ir \qquad I = \frac{\varepsilon}{R_{eq}} = \frac{\varepsilon}{R + r}$$

$$\boxed{\Delta V_{bat} = \varepsilon - \frac{\varepsilon}{R + r} r = \frac{R}{R + r} \varepsilon}$$

- Terminal voltage depends on the external load!

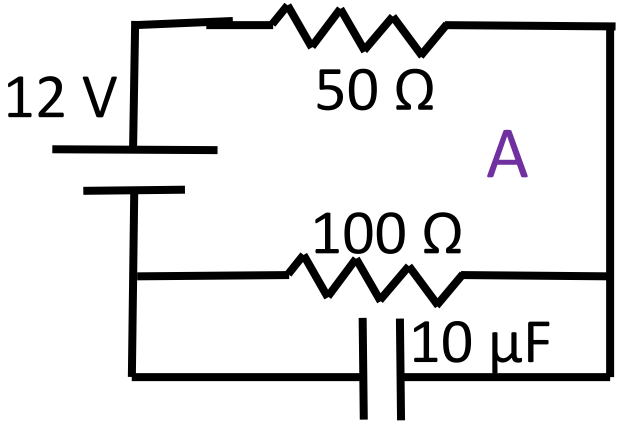
Electrical measuring instruments: Ch 26.3 (on your own)

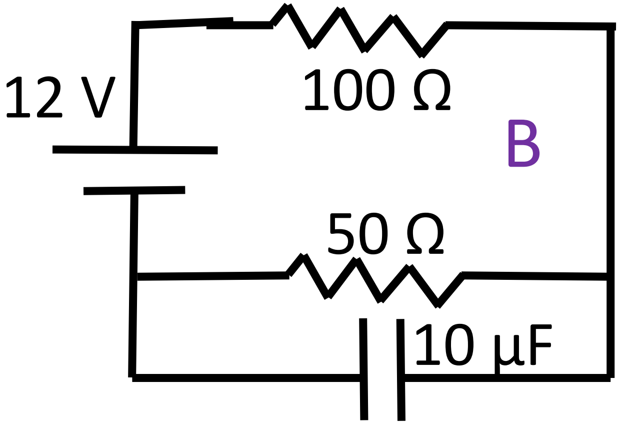


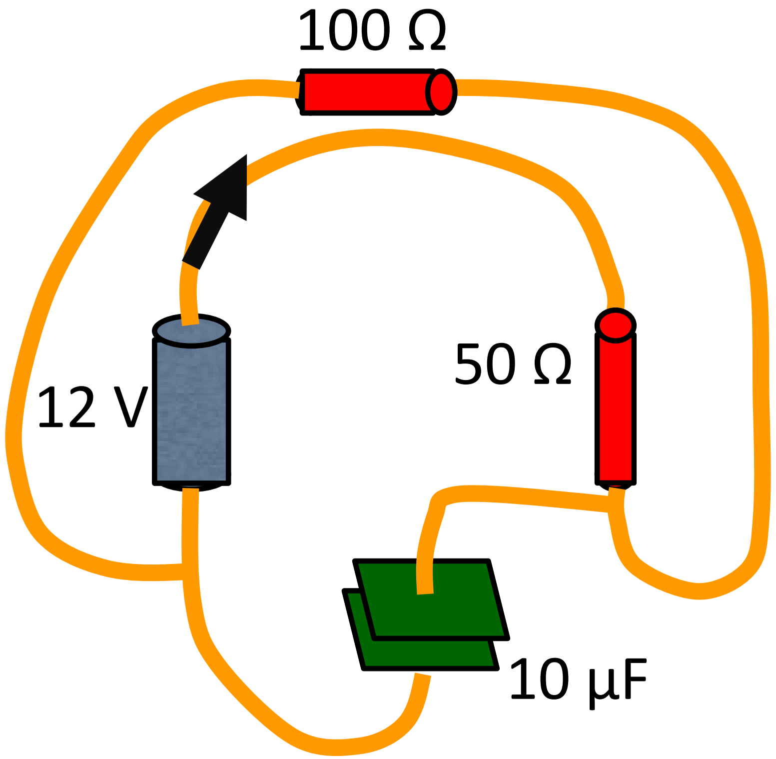
- Ammeter Ⓐ measures current (in series)
- Voltmeter Ⓥ measures voltage drop (in parallel)

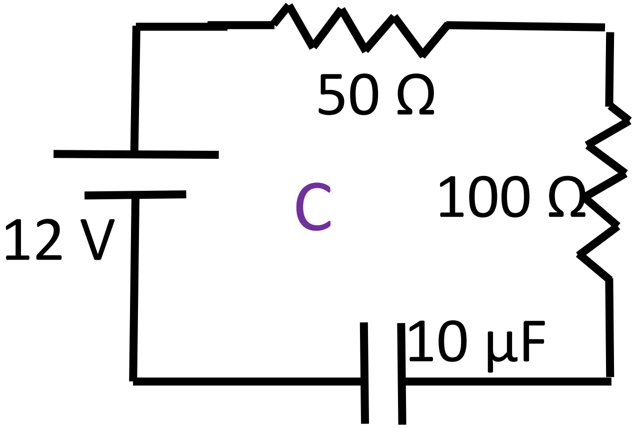
Q: You arrive in the lab and you find a 12 V battery, a 10 μF capacitor, and two resistors wired together as shown on the right. In order to correctly analyze the circuit response, you should redraw the circuit yourself (on paper).

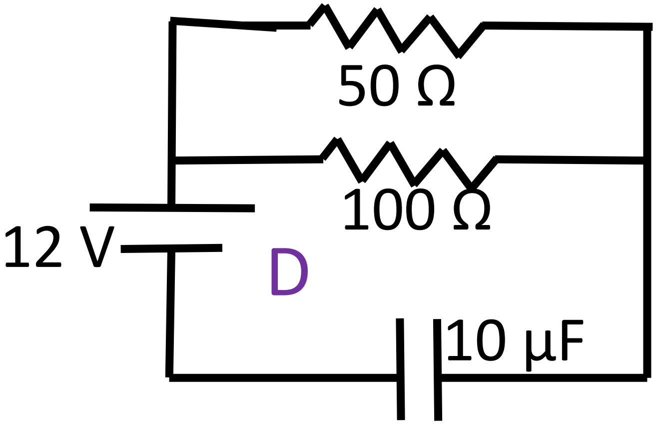
Which circuit diagram below is the correct one?











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