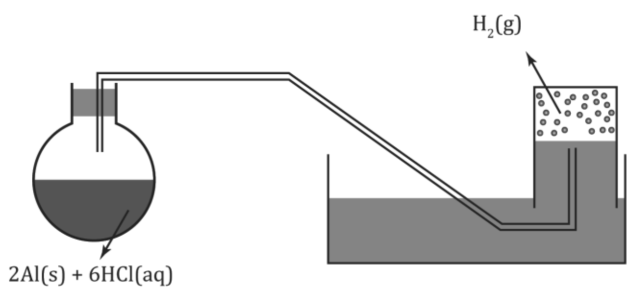
1. Use the ideal gas law to calculate the pressure, in atm, exerted by 1.00 mol of $Cl_2(g)$ confined to a volume of 2.00 L at 273 K.

$$P = \frac{nRT}{V} = \frac{(1.00\ \text{mol})(0.08206\ \text{L atm mol}^{-1}\text{K}^{-1})(273\ \text{K})}{2.00\ \text{L}} = 11.2\ \text{atm}$$

2. The reaction of aluminum with HCl produces hydrogen gas. 35.5 mL of H_2 is collected in a sealed container over water at 26 °C, and the pressure is measured to be 755 mmHg. How many moles of H_2 were produced? (The vapour pressure of water at 26 °C is 25.2 mmHg.)

$$2\ Al(s) + 6\ HCl(aq) \rightarrow 2\ AlCl_3(aq) + 3\ H_2(g)$$



$$P_{H_2} = P_{meas} - P_{H_2O} = 755\ \text{mmHg} - 25.2\ \text{mmHg} = 729.8\ \text{mmHg}$$

$$P_{H_2} = 729.8\ \text{mmHg} \times \frac{1\text{atm}}{760\ \text{mmHg}} = 0.960\text{atm}$$

$$n_{H_2} = \frac{PV}{RT} = \frac{(0.960\ \text{atm})(0.0355\ \text{L})}{(0.08206\ \text{L atm mol}^{-1}\ \text{K}^{-1})(299\ \text{K})} = 0.00139\ \text{mol}$$

Thus, 0.00139 mol of hydrogen gas is produced.

3. Good Question. Aerospace engineers sometimes write the gas law in terms of the mass of the gas rather than the number of moles.

$$PV = mR_{specific}T$$

In such a formulation, the molar mass of the gas must be incorporated into the value of the gas constant (the gas constant will change for different gases, represented by $R_{specific}$ in the above equation).

a) Briefly describe an experiment that can be performed to determine $R_{specific}$. Assume the gas behaves ideally.

A gas with known mass can be placed in a container of known and constant volume. The pressure of the container can be monitored as the temperature is increased. If the gas behaves ideally, a plot of pressure vs. temperature would be linear with a slope equal to m x $R_{specific}$. Since the mass is known $R_{specific}$ can be calculated.

b) Suggest a reason why this approach may be attractive for aerospace engineers.

The most common gas encountered in aerospace engineering is air. Because air is a mixture of gases, finding the partial pressures is tedious and non-trivial. In this way an engineer only needs to measure the mass of air to determine its pressure or volume.

c) Assume the mole fractions of O_2 and N_2 in air are 0.21 and 0.79, respectively. Calculate the average molar mass of air (the mass of one mole of air) and use this number to calculate $R_{specific}$ for air in $m^2\,s^{-2}\,K^{-1}$.

Comparing the usual ideal gas law with the one written above gives

$$PV = nRT = mR_{specific}T$$

so that

$$R_{specific,air} = \frac{n_{air}R}{m_{air}} = \frac{R}{M_{air}}$$

with M_{air} the molar mass of air, that is the mass per mole of air. From the given composition of air,

$$1\ \text{mol air} = 0.79\ \text{mol}\ N_2 + 0.21\ \text{mol}\ O_2$$

$$\text{mass of 1 mol air} = 0.79\ \text{mol}\ N_2 \times 28.01\frac{g}{\text{mol}} + 0.21\ \text{mol}\ O_2 \times 32\frac{g}{\text{mol}} = 28.86\ g$$

This gives $M_{air} = 28.86\ g/mol$ which when substituted in the equation above for $R_{specific,air}$ gives (remembering that $1\ J = 1\ kg\ m^2/s^2$)

$$R_{specific,air} = \frac{R}{M_{air}} = \frac{8.314\frac{kg\cdot m^2}{s^2\cdot K\cdot mol}}{0.02886\frac{kg}{mol}} = 288.2\frac{m^2}{s^2 K}$$

4. A piston in a car engine is maintained at constant pressure during a combustion reaction. The combustion reaction is given by:

$$C_8H_{18}(g) + 12.5\ O_2(g) \rightarrow 8\ CO_2(g) + 9\ H_2O(l)$$

Using the ideal gas law, identify possible reason(s) that would cause the piston volume to change.

This reaction is exothermic (produces heat), which causes the gas in the piston to expand. Moreover, the temperatures inside a piston can exceed 100 °C causing the water product to become a gas. This, along with the CO_2 formed, means that the number of gas molecules increases during combustion.

5. Use the van der Waals equation to calculate the pressure, in atm, exerted by 1.00 mol of $Cl_2(g)$ confined to a volume of 2.00 L at 273 K given the van der Waals constants for chlorine are: a = 6.49 L^2 atm mol^{-2}; b = 0.0562 L mol^{-1}. Compare this with the pressure in Question 1 calculated assuming ideal conditions. What is the likely cause of the deviation from ideal behaviour?

Rearrange the van der Waals equation to isolate the pressure giving

$$\boldsymbol{P = \frac{nRT}{V - nb} - \frac{n^2 a}{V^2}}$$

$$\boldsymbol{n^2 a = (1.00\ \text{mol})^2\ \times\ 6.49\ \frac{\text{L}^2\text{atm}}{\text{mol}^2} = 6.49\ \text{L}^2\ \text{atm}}$$

$$\boldsymbol{nb = 1.00\ \text{mol} \times 0.0562\ \text{L mol}^{-1} = 0.0562\ \text{L}}$$

$$\boldsymbol{P = \frac{1.00\ \text{mol} \times 0.08206\ \text{L atm mol}^{-1}\text{K}^{-1}\ \times\ 273\ \text{K}}{(2.00\ - 0.0562)\ \text{L}} - \frac{6.49\ \text{L}^2\ \text{atm}}{(2.00\ \text{L})^2}}$$

$$\boldsymbol{P = 11.5\ \text{atm} - 1.62\ \text{atm} = 9.9\ \text{atm}}$$

The pressure calculated using only the *b* term would give a non-ideal pressure of 11.5 atm, close to the ideal condition of 11.2 atm. Incorporating the *a* term reduces this value by 1.62 atm. Thus the interactions related to *a*, attractive intermolecular forces, likely cause deviation from ideal behaviour.

6. Good Question. Quicklime (CaO) is produced by the thermal decomposition of calcium carbonate ($CaCO_3$) in a 700.00 L reactor designed to withstand up to 3 atm of pressure. What is the maximum mass of $CaCO_3$ that can be safely decomposed in this reactor at 25°C? Assume CO_2 behaves non-ideally (molar mass of $CaCO_3$ = 100.09 g/mol, a_{CO2}= 3.592 atm L^2/mol^2, b_{CO2} = 0.04267 L/mol).

$$CaCO_3(s) \rightarrow CaO(s) + CO_2(g)$$

Since CO_2 is the only species responsible for producing the maximum pressure in the reactor, one has to determine, using the van der Waals equation, how many moles of CO_2 are necessary to produce a pressure of 3 atm, that is

$$\left(P + a\left(\frac{n^2}{V^2}\right)\right)(V - nb) = nRT$$

$$\left(3\text{ atm} + 3.592\,\frac{\text{atm}\cdot\text{L}^2}{\text{mol}^2}\left(\frac{n^2}{700^2\text{L}^2}\right)\right)\left(700\text{ L} - n\cdot 0.04267\,\frac{\text{L}}{\text{mol}}\right)$$

$$= n\cdot 0.08206\,\frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}}(298.15\text{ K})\ \text{ so that}$$

$$2100\text{ atm}\cdot\text{L} - 0.1280\,n\,\frac{\text{atm}\cdot\text{L}}{\text{mol}} + 0.005131\,n^2\,\frac{\text{L atm}}{\text{mol}^2} - 3.128\times 10^{-7}n^3\,\frac{\text{atm}\cdot\text{L}}{\text{mol}^3} = 24.47\,n\,\frac{\text{L}\cdot\text{atm}}{\text{mol}}$$

Rearranging this equation, and dividing out the L atm units gives

$$-3.128\times 10^{-7}\frac{n^3}{\text{mol}^3} + 0.005131\frac{n^2}{\text{mol}^2} - 24.59\frac{n}{\text{mol}} + 2100 = 0$$

Solving this cubic equation for *n* gives $n = 86.97$ mol

Note that you will NOT be expected to solve cubic equations in CHEM 154 exams. Knowing the amount of carbon dioxide then allows one to calculate the number of moles of calcium carbonate to produce it, and hence the mass as

$$86.97\text{ mol }CO_2\times\frac{1\text{ mol }CaCO_3}{1\text{ mol }CO_2}\times\frac{100.09\text{ g }CaCO_3}{1\text{mol }CaCO_3} = 8705\text{ g }CaCO_3$$
$$= 8.70\text{ kg of }CaCO_3$$