Lecture 22.

Finding V from known $\vec{E}$ (continued).

Last Week:

$$\begin{tikzcd} \vec{F} \arrow[rr, leftrightarrow, "\Delta U = -\int_i^f \vec{F}\cdot d\vec{s}"] \arrow[d, leftrightarrow, "\vec{F} = q_{\underline{\pm}}\,\vec{E}"'] & & \Delta U \arrow[d, leftrightarrow, "\Delta U = q_{\underline{\pm}}\Delta V"] \\ \vec{E} \arrow[rr, leftrightarrow, "\Delta V = -\int_i^f \vec{E}\cdot d\vec{s}"'] & & \Delta V \end{tikzcd}$$

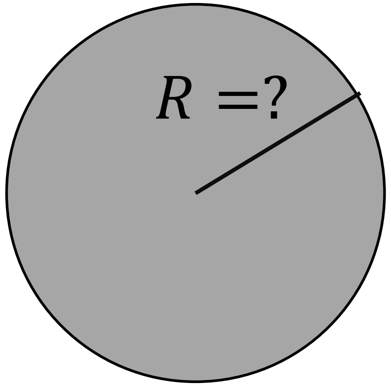
Q: A metal sphere carries a charge of 5×10^{-9} C. Its surface is at a potential of 400 V, relative to the potential far away. What is the radius of the sphere ?

- Potential – field connection:

$$\Delta V = V_f - V_i = -\int_{r=r_i}^{r=r_f} \vec{E} \cdot d\vec{s}$$

Last Time:

$V_s = 400\ V$



- From Gauss's law we know that the electric field outside the sphere is the same as for a point charge =>

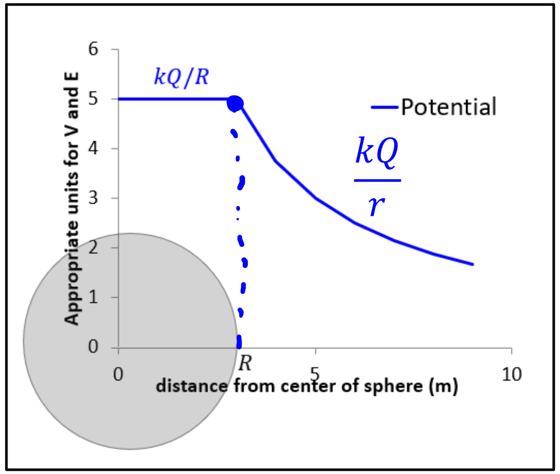
$$V_{\text{sphere}}(r > R) = \frac{kQ}{r}$$

A. 1.5 m
B. 0.9 m
C. 0.23 m
D. 0.11 m
E. 0.05 m

$$V_{\text{sphere}}(r < R) = 400\ V \equiv \frac{kQ}{R}$$

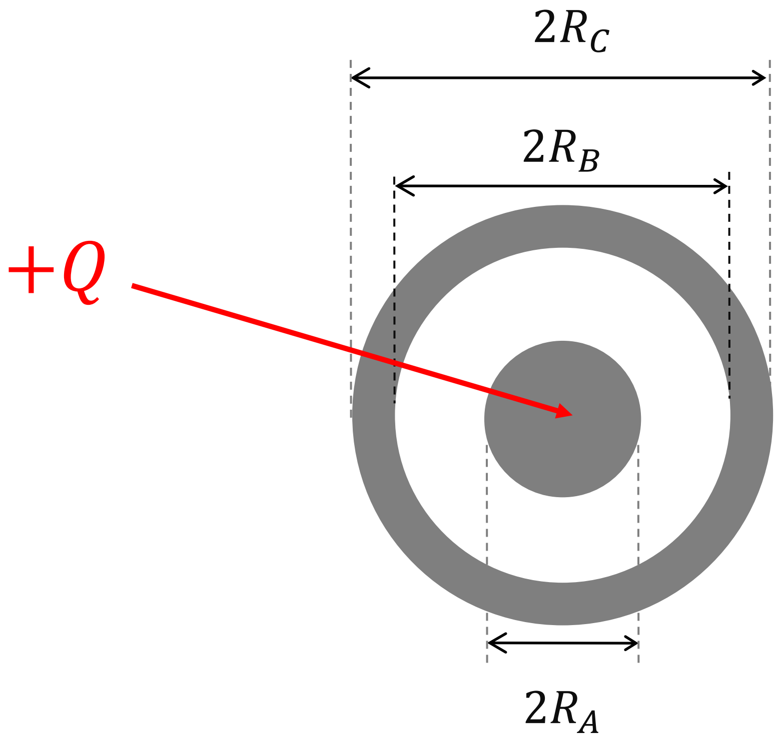
...and we know that potential is continuous!

$$V_{\text{sphere}}(r = R) = \frac{kQ}{R} = 400\ V \quad \Rightarrow \quad R = 0.11\ m$$



Q: A conducting sphere of radius R_A carrying charge $+Q$ is placed at the centre of an uncharged conducting spherical shell of inner radius R_B and outer radius R_C.

a) Find the electric field everywhere in space.
b) Find the electric potential everywhere in space. Calculate $V(R_A), V(R_B), V(R_C)$. Assume V is zero at infinity.



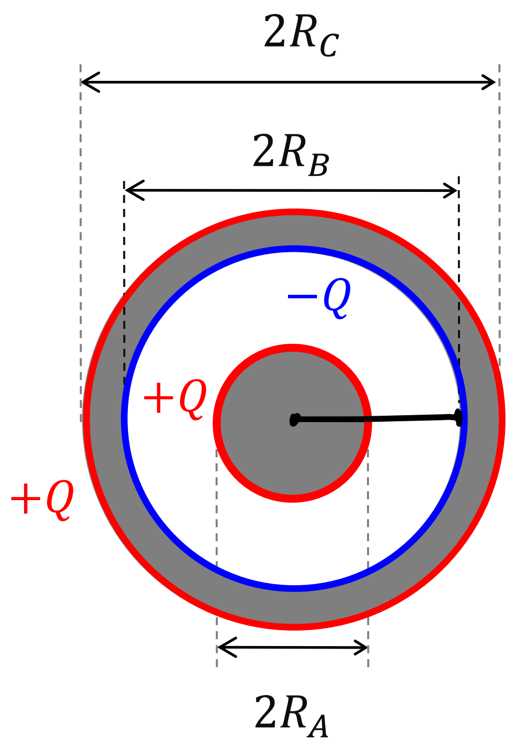
Q: A conducting sphere of radius R_A carrying charge $+Q$ is placed at the centre of an uncharged conducting spherical shell of inner radius R_B and outer radius R_C.

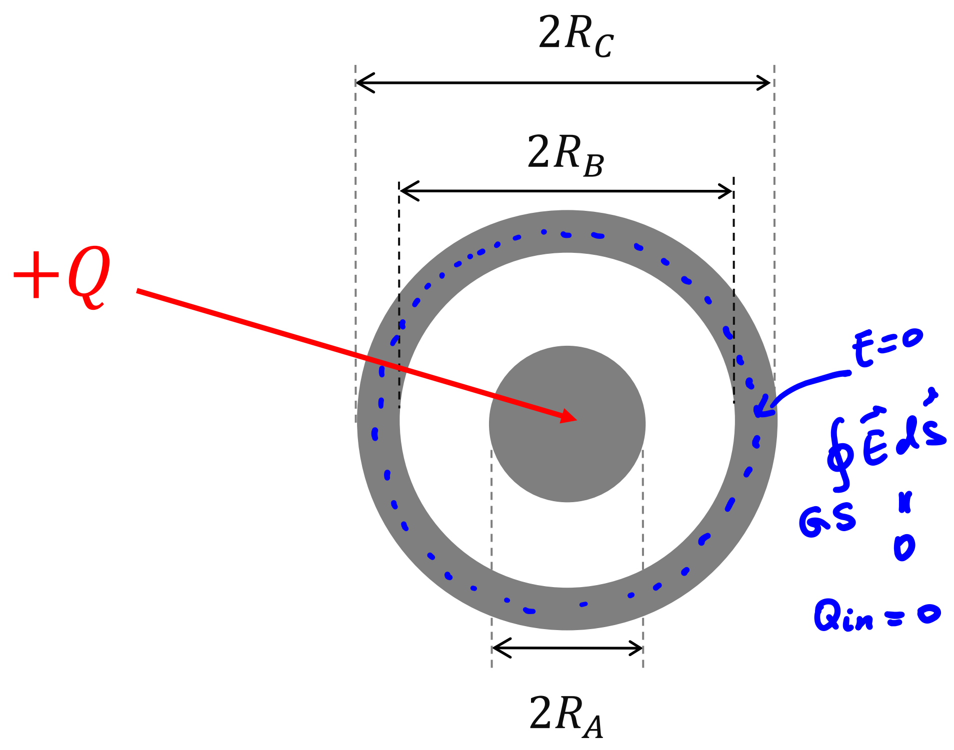
a) Find the electric field everywhere in space.

b) Find the electric potential everywhere in space. Calculate $V(R_A), V(R_B), V(R_C)$. Assume V is zero at infinity.

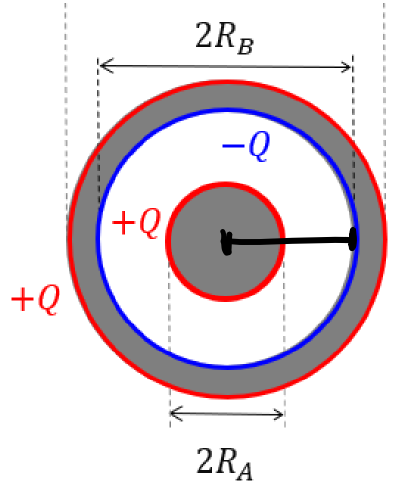
Warming-up:

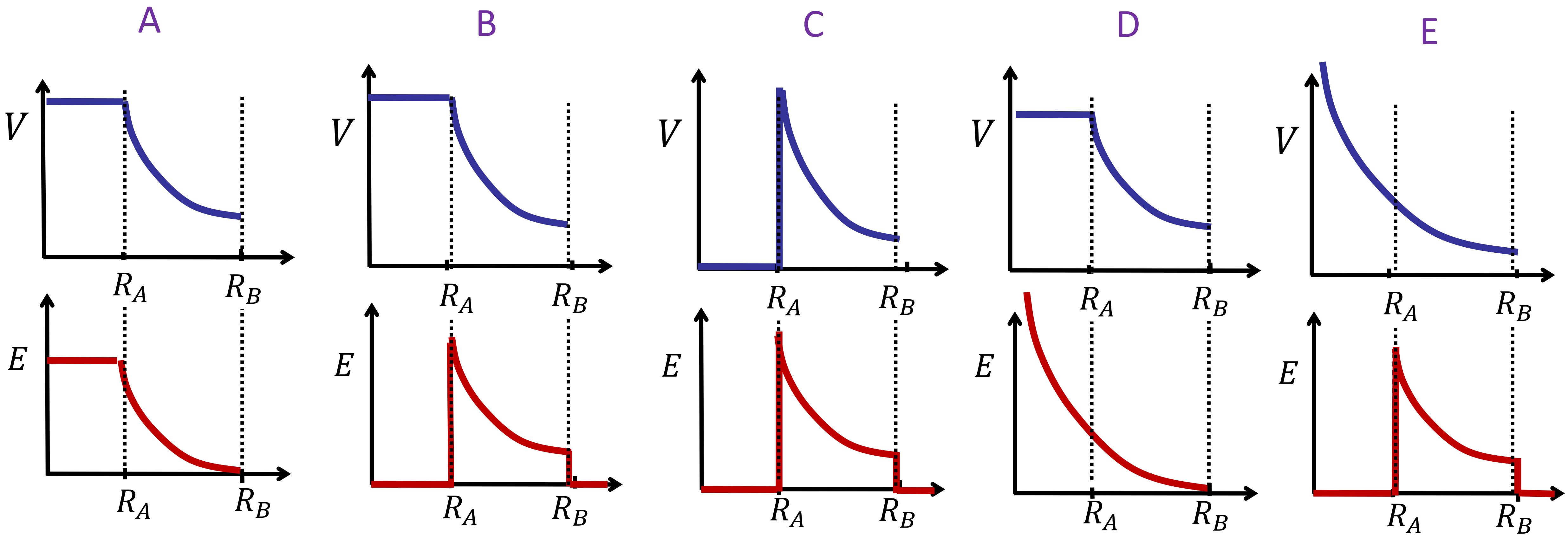
- What is the charge distribution?



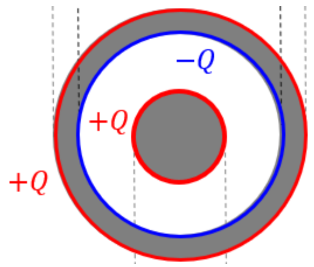


Q: Which of these graphs is closest to V and E in the range $0 < r < R_B$?





Q: Which of these graphs is closest to V and E in the range $0 < r < RB$?

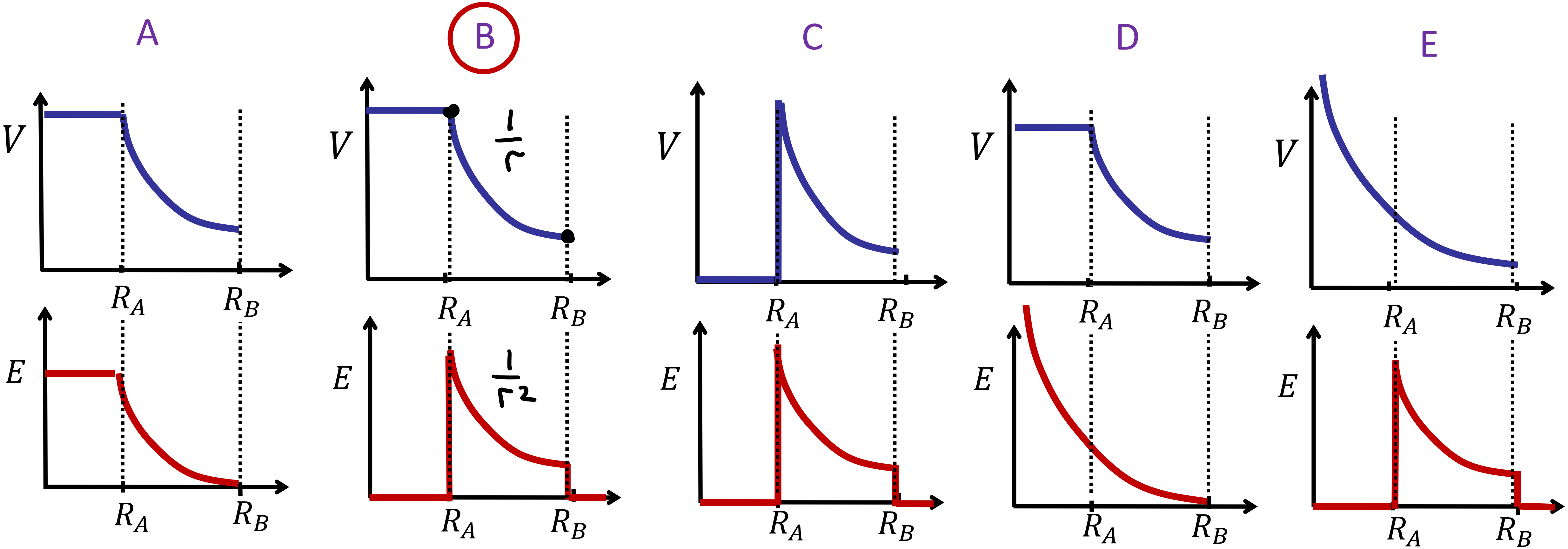


$E = 0$ inside a conductor!!

V is always continuous!!

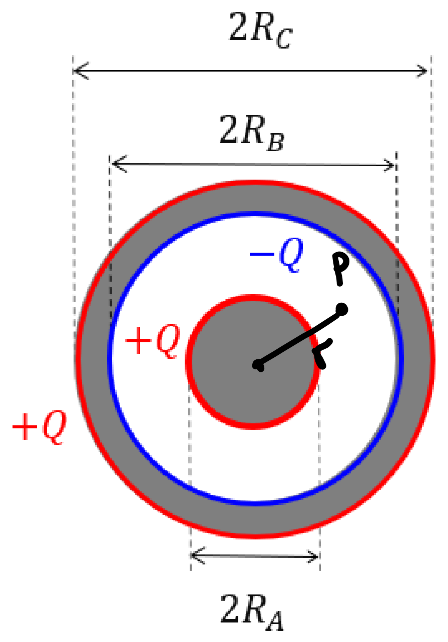
$E = 0$ inside a conductor!!

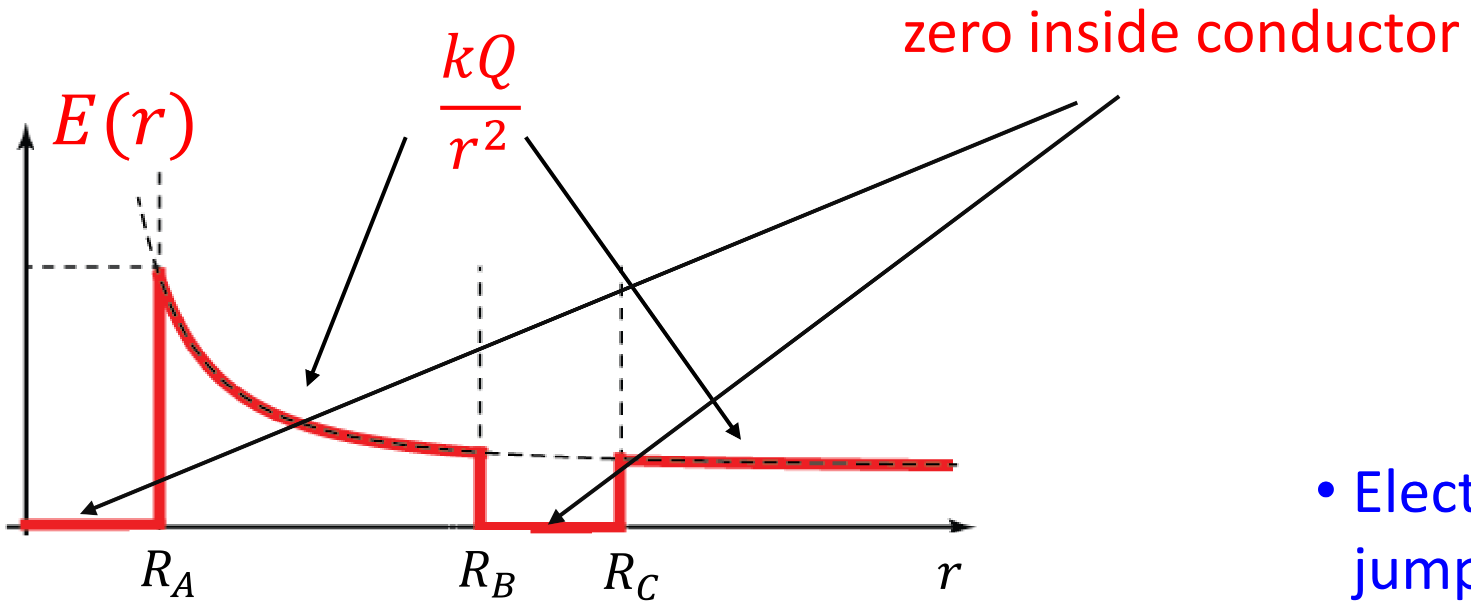
V is the same inside a conductor!!



We can find electric field in all intervals using the Gauss's law:

- $E(r) \cdot A_{\text{sphere}} = E(r) \cdot 4\pi r^2 = \dfrac{Q}{\varepsilon_0} \quad \Rightarrow \quad E(r) = \dfrac{kQ_{in}}{r^2}$
- $E = 0$ inside a conductor





- Electric field can have jumps at the boundaries.

Now we can find electric potential: $\Delta V(r) = V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$

Remember: integrate "outwards" so ds is positive

$d\vec{s} = d\vec{r}$

$\vec{E} \cdot d\vec{s} = E\,dr$

- Our choice: $V(\infty) = 0$

1. With $i = R_c,\ f = \infty$: $V(R_C) = \dfrac{kQ}{R_C}$

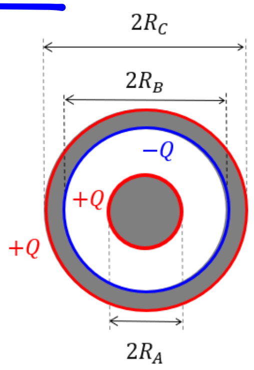
$$\underbrace{V_\infty}_{=0} - V_c = -\int_{R_c}^{\infty} \frac{kQ}{r^2} \cdot dr = \frac{k \cdot Q}{r}\Big|_{R_c}^{\infty} = -\frac{kQ}{R_c}$$

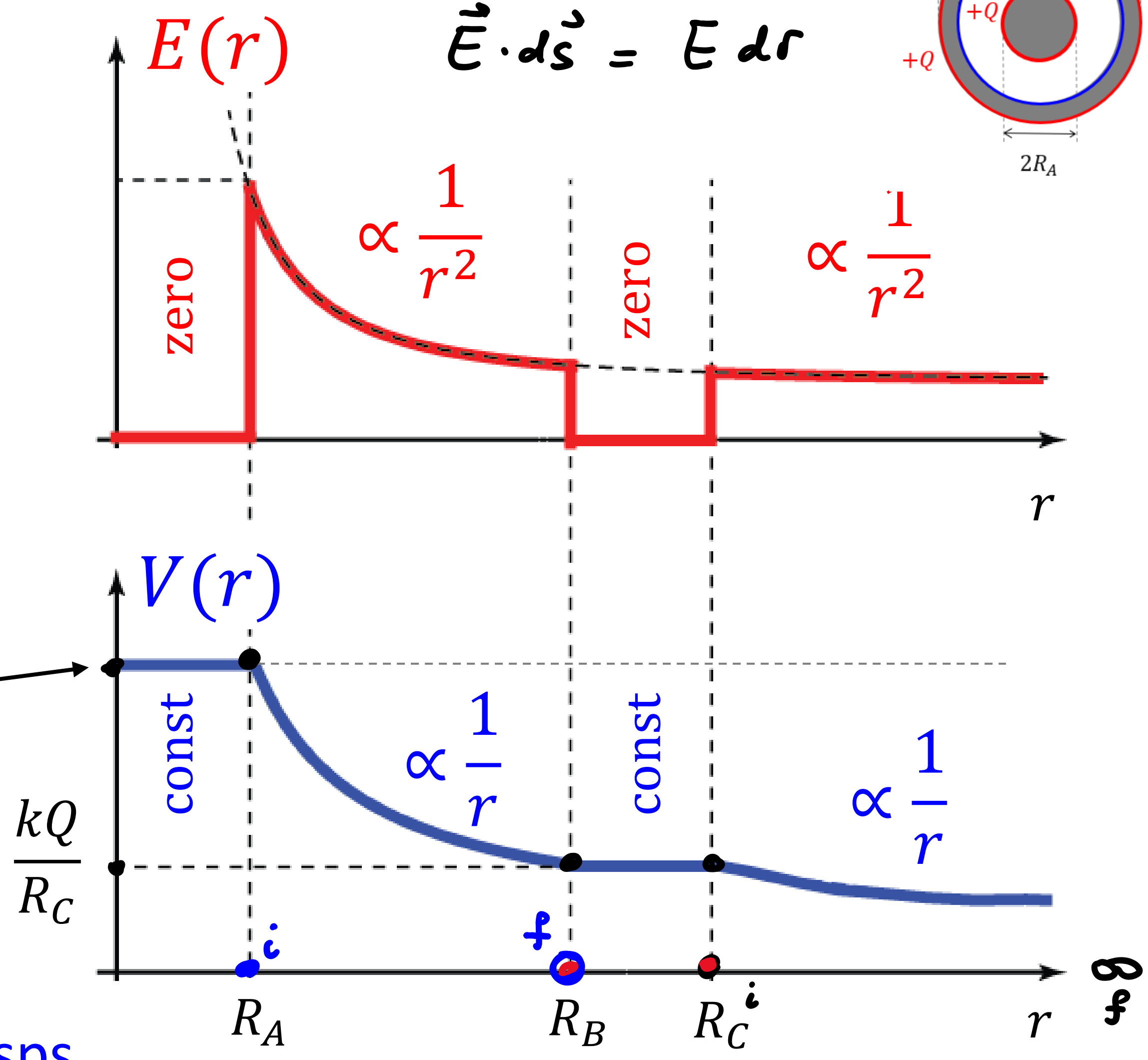
2. With $i = R_B,\ f = R_C$: $V(R_B) = V(R_C)$

$$V_c - V_B = -\int \underbrace{E}_{=0} dr \equiv 0$$

3. With $i = R_A,\ f = R_B$: $V(R_A) = \dfrac{kQ}{R_C} - \dfrac{kQ}{R_B} + \dfrac{kQ}{R_A}$

$$\underbrace{V_B}_{=\frac{kQ}{R_c}} - V_A = -\int_{R_A}^{R_B} \frac{kQ}{r^2} dr = \frac{kQ}{r}\Big|_{R_A}^{R_B} = \frac{kQ}{R_B} - \frac{kQ}{R_A}$$

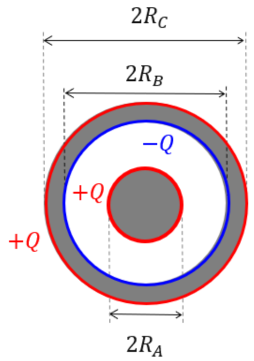




- Potential is always continuous, but it can have cusps.

Now we can find electric potential: $\Delta V(r) = V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$

Remember: integrate "outwards" so ds is positive



- Our choice: $V(\infty) = 0$

1. With $i = R_c,\ f = \infty$: $V(R_C) = \dfrac{kQ}{R_C}$

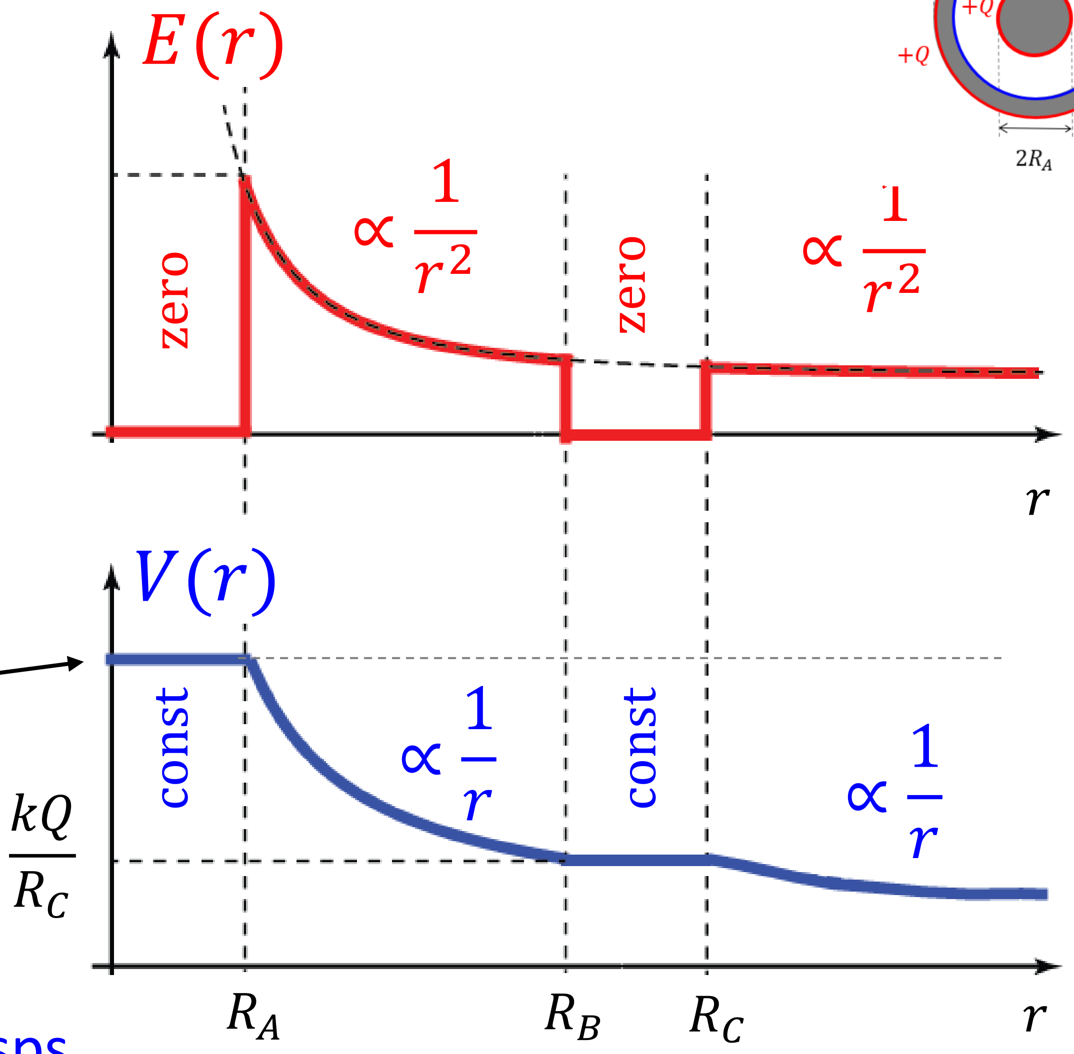
$$0 - V(R_C) = -\int_{R_C}^{\infty} \frac{kQ}{r^2}\, dr = \frac{kQ}{r}\bigg|_{R_C}^{\infty}$$

2. With $i = R_B,\ f = R_C$: $V(R_B) = V(R_C)$

$$V(R_C) - V(R_B) = 0$$

3. With $i = R_A,\ f = R_B$: $V(R_A) = \dfrac{kQ}{R_C} - \dfrac{kQ}{R_B} + \dfrac{kQ}{R_A}$

$$V(R_B) - V(R_A) = -\int_{R_A}^{R_B} \frac{kQ}{r^2}\, dr = \frac{kQ}{r}\bigg|_{R_A}^{R_B}$$

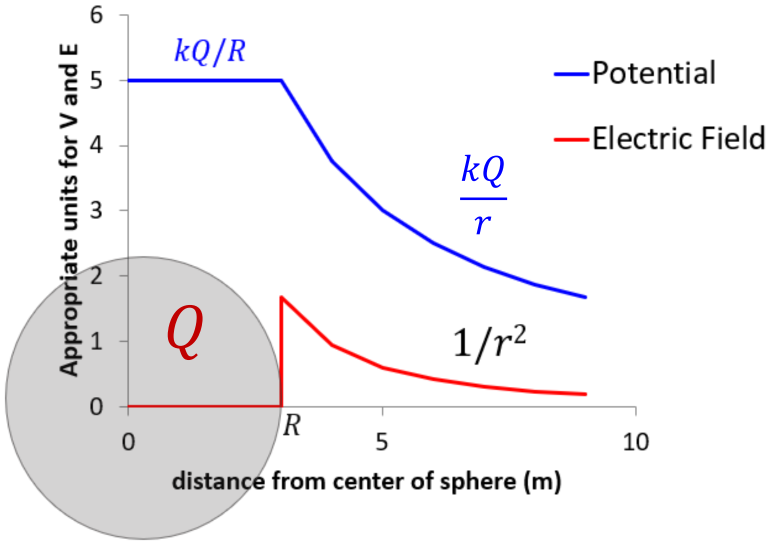


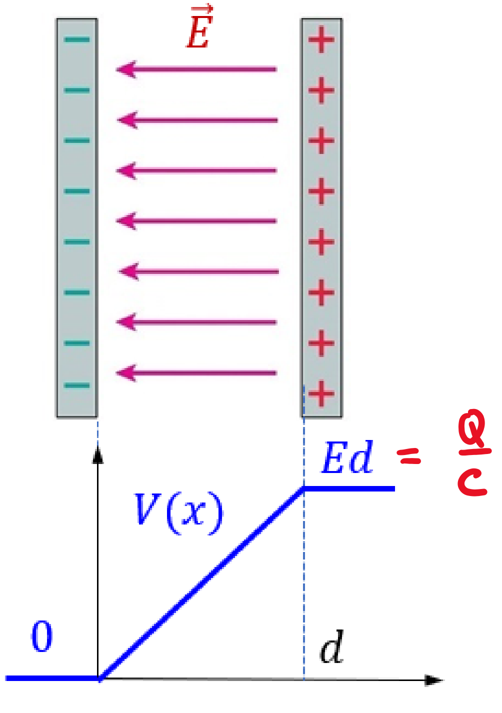
- Potential is always continuous, but it can have cusps.

Potential of various charged objects: Summary

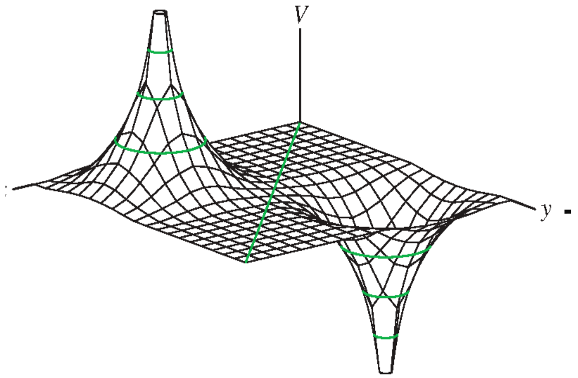
Using $\Delta V(r) = V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$ and expressions for electric field that we have derived before, we can show that:

- $V_{\text{spere}}(r) = \begin{cases} \dfrac{kQ}{R} & \text{for } r < R \\ \dfrac{kQ}{r} & \text{for } r > R \end{cases}$





- $V_{\text{capacitor}}(r)$: grows linearly across the plates



- $V_{\text{point charge}}(r) = \dfrac{kQ_{\pm}}{r}$

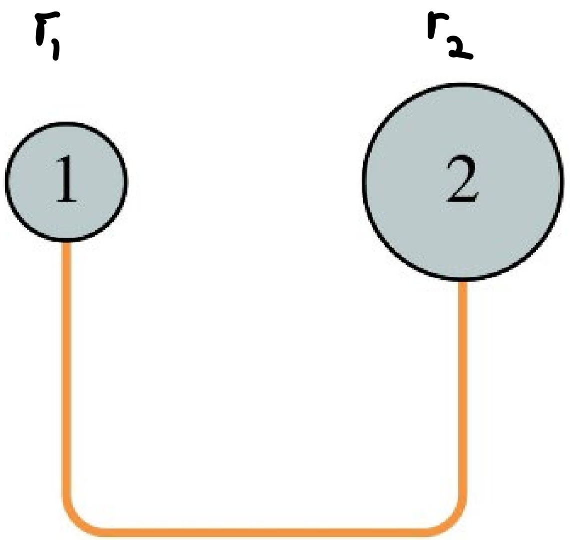
A charged conductor in equilibrium is all under the same potential ($\vec{E} = 0$ inside)

Q: Metal spheres 1 and 2 are connected by a metal wire.

a) Which is true about their charges?

$V_R = \frac{kQ}{R}$ – potential of a sphere with charge Q and radius R

$V_{R_1} = V_{R_2}$ (conductor is under the same potential!)



A. $Q_1 = Q_2$

B. $\frac{Q_1}{Q_2} = \frac{r_1}{r_2}$

C. $\frac{Q_1}{Q_2} = \frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$

D. $\frac{Q_1}{Q_2} = \frac{V_1}{V_2} = \frac{r_1^3}{r_2^3}$

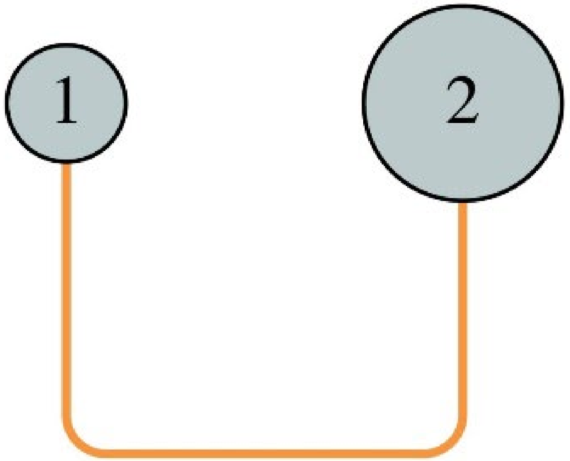
$\frac{kQ_1}{r_1} = \frac{kQ_2}{r_2}$

1) These two spheres + wire are "the same conductor" → the whole thing is under the same potential.

2) Due to the wire, the spheres are far enough from each other → we can approximate the potential of each by $V_R = kQ/R$.

Q: Metal spheres 1 and 2 are connected by a metal wire.

b) Which is true about the electric fields at their surfaces?



Q_1/Q_2

$\downarrow$

$$\frac{E_1}{E_2} = \frac{\cancel{k}Q_1}{r_1^2} \cdot \frac{r_2^2}{\cancel{k}Q_2} = \frac{\cancel{r_1}}{\cancel{r_2}} \cdot \frac{r_2^{\cancel{2}}}{r_1^{\cancel{2}}} = \frac{r_2}{r_1}$$

A. $E_1 = E_2$

B. $\frac{E_1}{E_2} = \frac{r_1}{r_2}$

C. $\frac{E_1}{E_2} = \frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$

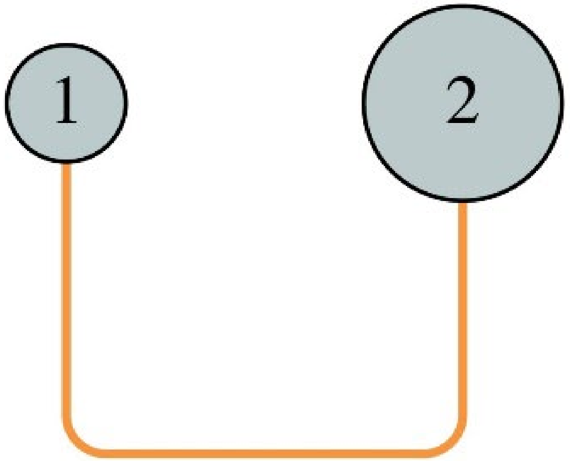
D. $\frac{E_1}{E_2} = \frac{r_2}{r_1}$

E. $\frac{E_1}{E_2} = \frac{A_2}{A_1} = \frac{r_2^2}{r_1^2}$

Q: Metal spheres 1 and 2 are connected by a metal wire.

c) Which is true about the charge density at their surfaces?

$\sigma = \frac{Q}{A}$



$$\frac{\sigma_1}{\sigma_2} = \frac{Q_1}{\cancel{4\pi} r_1^2} \cdot \frac{\cancel{4\pi} r_2^2}{Q_2} =$$

$$= \frac{Q_1}{Q_2} \frac{r_2^2}{r_1^2} = \frac{r_1}{r_2} \frac{r_2^2}{r_1^2}$$

A. $\sigma_1 = \sigma_2$

B. $\frac{\sigma_1}{\sigma_2} = \frac{r_1}{r_2}$

C. $\frac{\sigma_1}{\sigma_2} = \frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$

D. $\frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}$

E. $\frac{\sigma_1}{\sigma_2} = \frac{A_2}{A_1} = \frac{r_2^2}{r_1^2}$

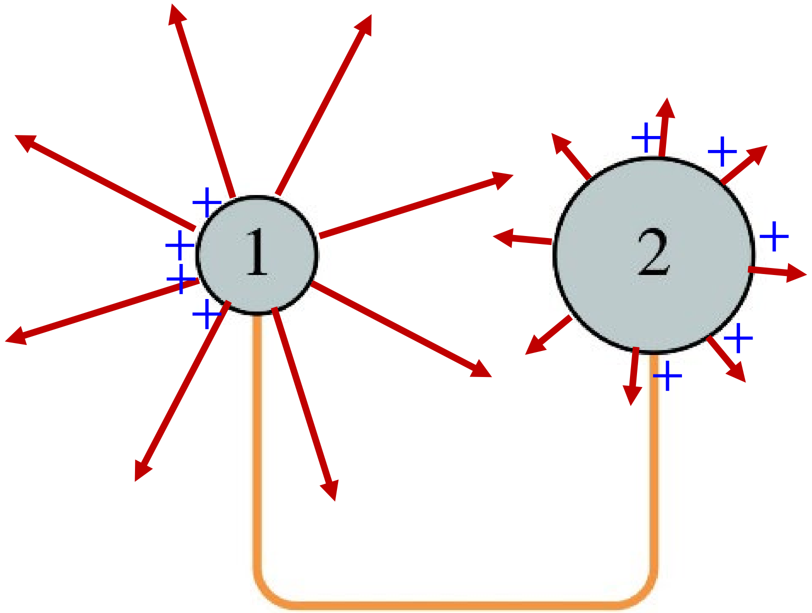
Q: Metal spheres 1 and 2 are connected by a metal wire.

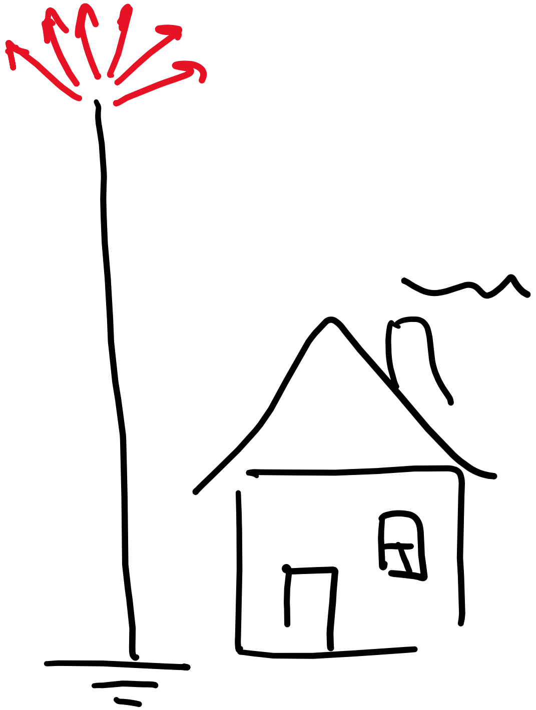
c) Which is true about the charge density at their surfaces?

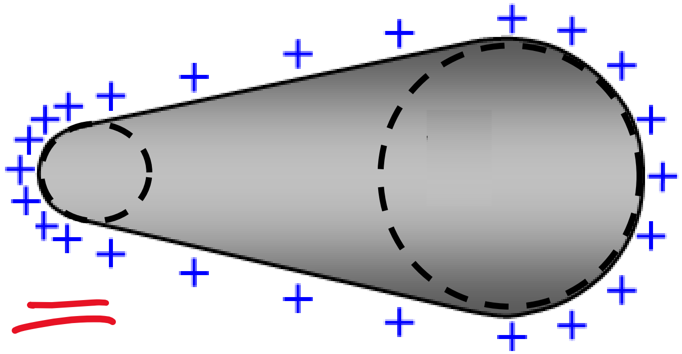
$$\frac{Q_1}{Q_2} = \frac{r_1}{r_2}$$

$$\frac{E_1}{E_2} = \frac{r_2}{r_1}$$

$$\frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1}$$







Note: Using these two spheres as a model of a conductor, we can get a qualitative idea why the surface charge density in conductors is larger at sharp corners.