PHYS 170

Week 5: Equilibrium of a Rigid Body

Section 201 (Mon Wed Fri 11:00 – 12:00)

Equilibrium and Reaction Forces & Moments

Text: 5.1-5.7

Content:

- Reaction forces and reaction moments
- Equilibrium: six equations
- Drawing three-body diagrams
- Types of support and their reactions
- Equilibrium in 2D and 3D

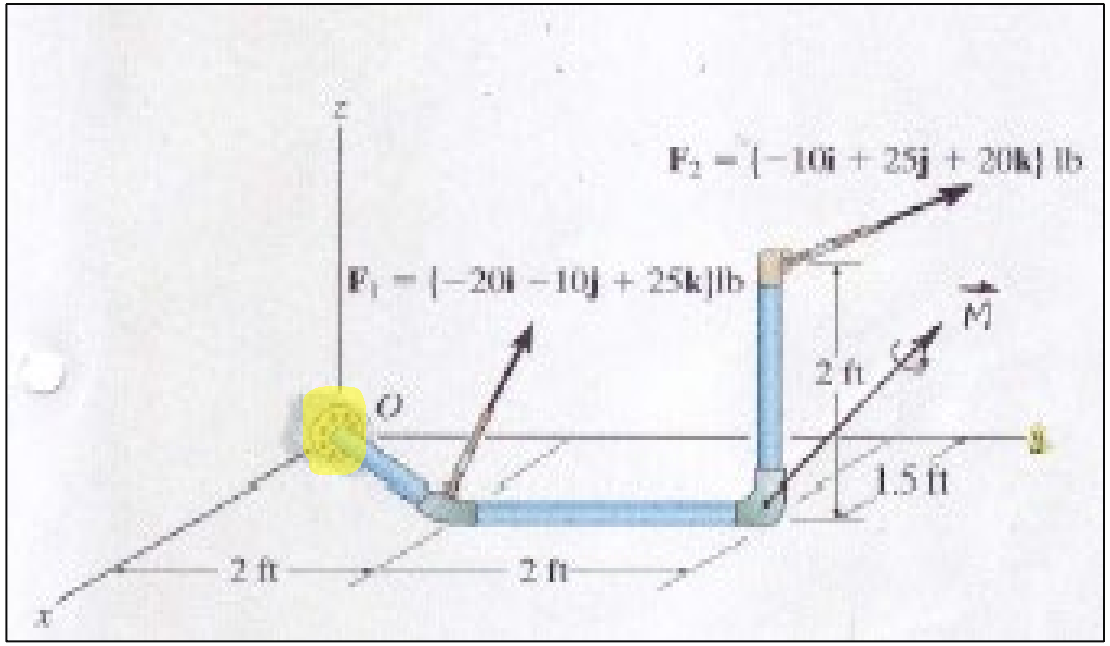
Look at this pipe from W4-2.

- It has two forces and one force couple applied to it.
- We have found that $\vec{F}_R = (-30)\vec{i} + (15)\vec{j} + (45)\vec{k}$ and $\left(\vec{M}_R\right)_O = (85.0)\vec{i} - (81.5)\vec{j} + (110)\,\vec{k}$

Wait… If $\vec{F}_R \neq 0$, and $\left(\vec{M}_R\right)_O \neq 0$, does it mean that the object undergoes translational motion and also rotates about point O?

A. Only translational motion.
B. Only rotational motion.
C. Both.
D. Neither.

Explain your answer.



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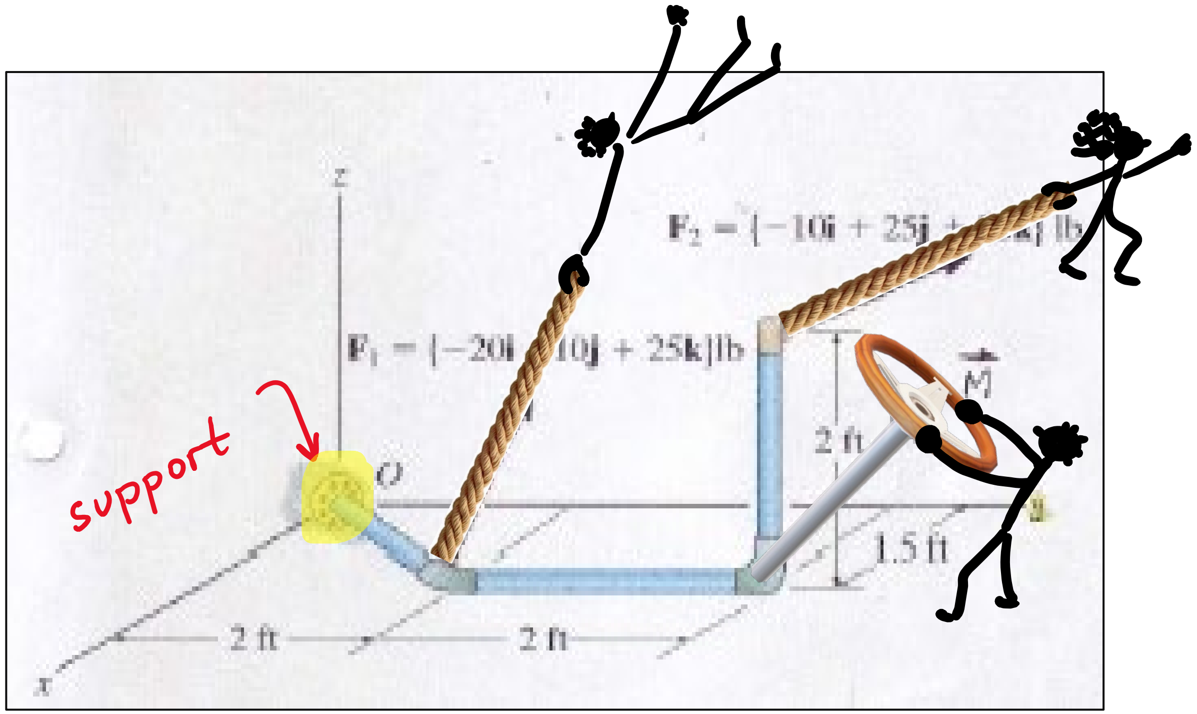


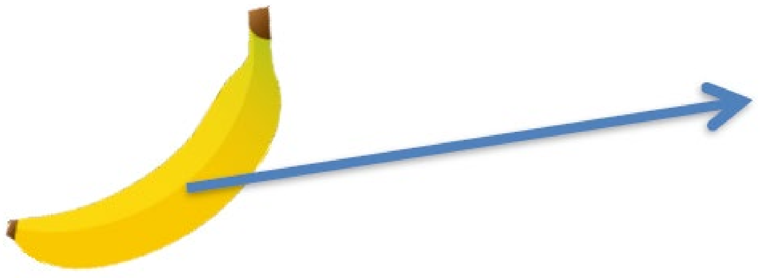
A. Only translational motion.
B. Only rotational motion.
C. Both.
D. Neither.

What is shown here are NOT the net force and the net moment, but only external forces and external moment (applied by some external "agents"). In addition to them, there must be a force and a couple moment produced by <u>the support</u> at the origin ("reactions"). The object stays in equilibrium since reactions balance out the external forces and the external moment.

REACTIONS

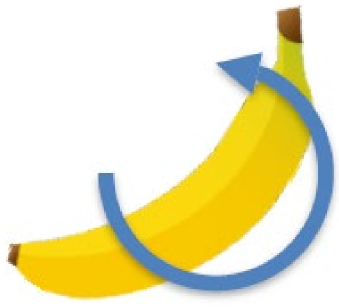
Forces can make an object shift and rotate, and moments can make an object rotate. But forces and moments can also prevent an object from motion and rotation!

- A reaction force resists the translational motion of an object. (it stops the object from moving all at once in any direction)



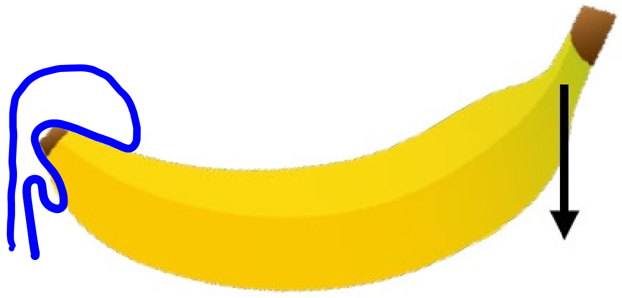
Reaction forces prevent this.

- A reaction moment resists the rotational motion of an object. (it stops the objects from spinning)

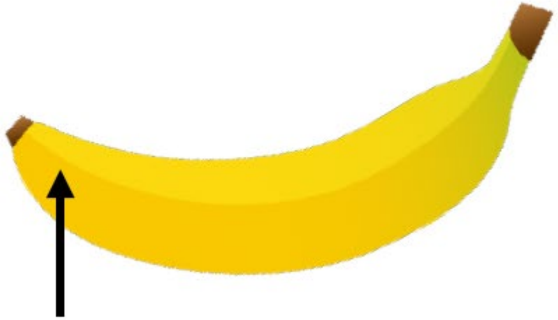


Reaction moments prevent this.

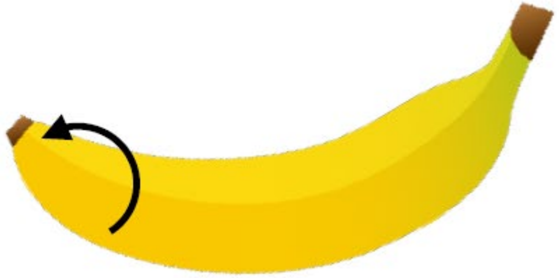
Q: You want to peel a banana. To do so you act a force on the right end as shown, and grip the left end with your hand to maintain equilibrium. Which load could represent your left hand?



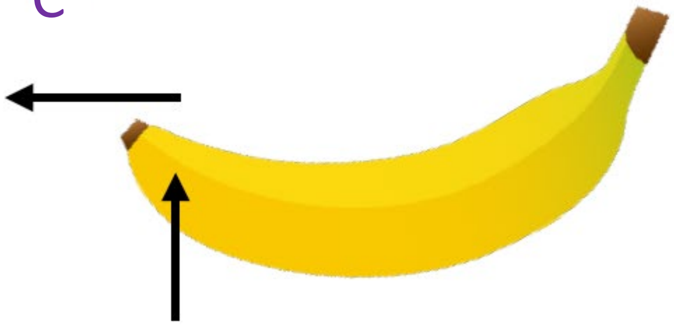
A



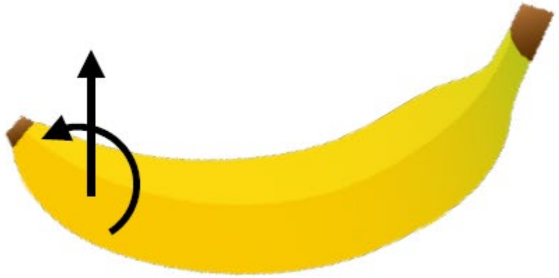
B



C



D



E. None of them

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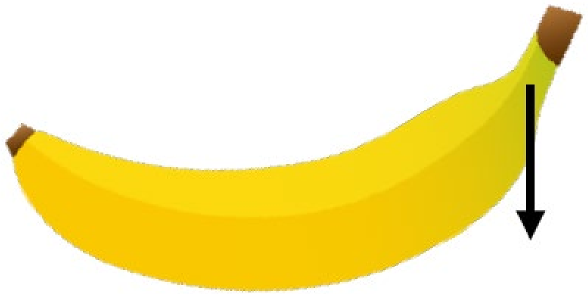
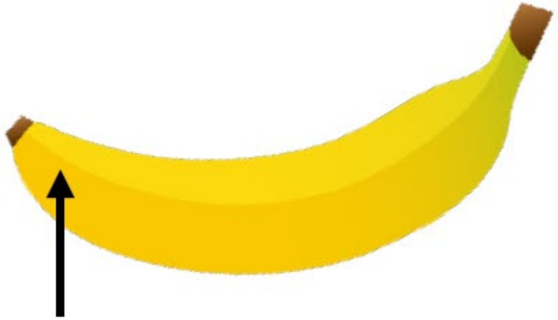


A



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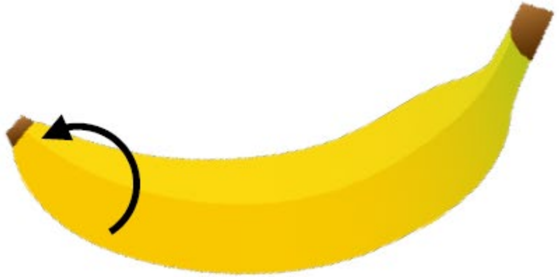
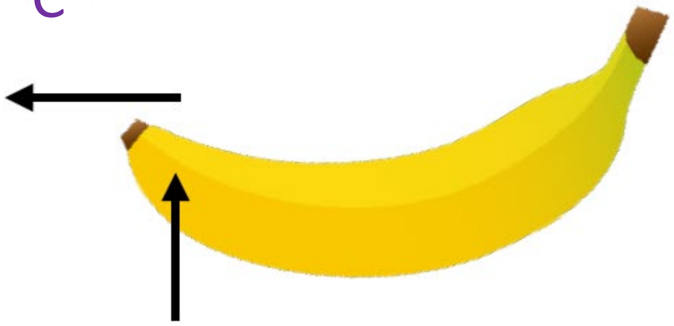


E. None of them

C



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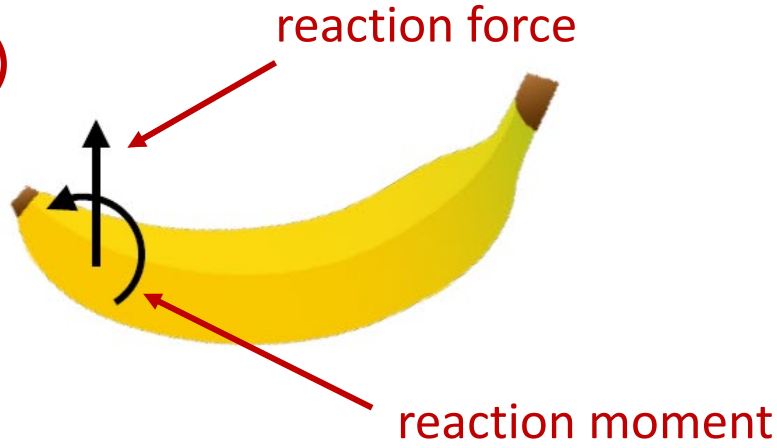
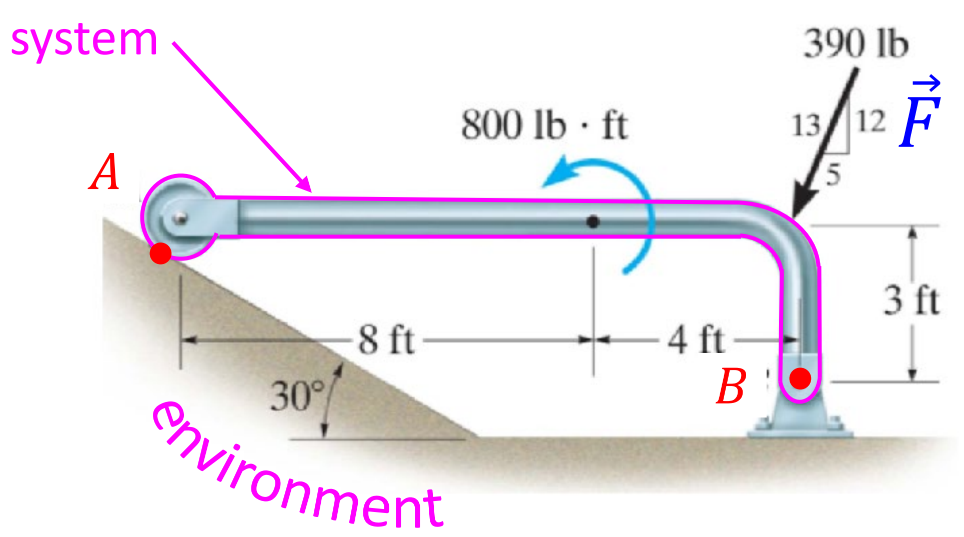


FORCES: Types (same is true for moments!)



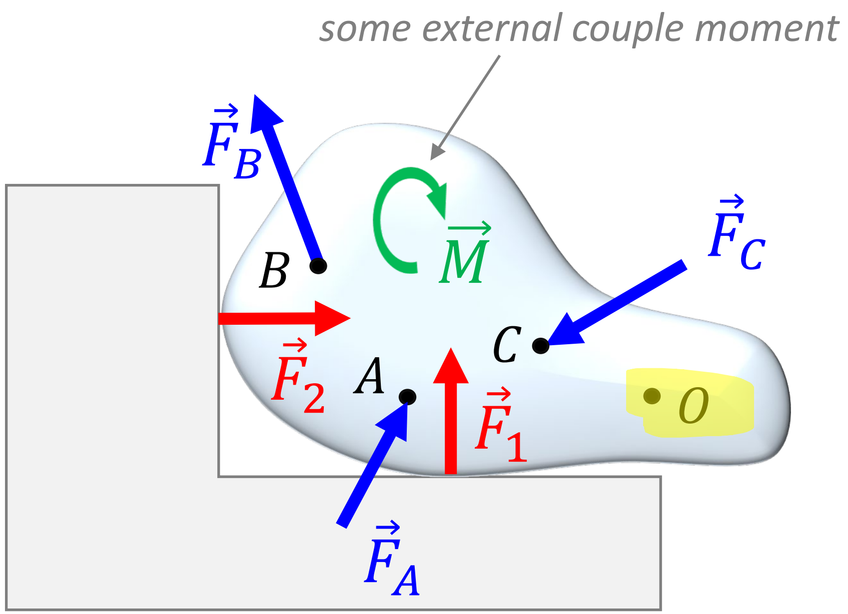
- **External forces**: Active forces (in the figure: $\vec{F}$)
  - ➢ They are applied to your object by some external agent
  - ➢ For massive objects, includes gravity force
  - ➢ We take them into account

- **External forces**: Reactive forces (supports at A and B)
  - ➢ Your system acts on the environment, and the environment acts back on your system
  - ➢ Appear at the "supports" = points of contact between your system and environment (A and B)
  - ➢ We take them into account

- **Internal forces**: act between different parts of your system
  - ➢ The wheel acts on the rod, and the rod acts on the wheel
  - ➢ They are antiparallel and equal in magnitude (Newton's 3rd law) and hence mutually cancel
  - ➢ That is why we disregard them

- An object is in translational equilibrium when all active forces are compensated by reaction forces
- Similarly, rotational equilibrium means that all active moments are compensated by reaction moments

EQUILIBRIUM: Conditions



- Translational equilibrium:

$$\vec{F}_R = \sum \vec{F} = 0$$

$$\begin{cases} \sum F_x = 0 & \text{(no motion along x)} \\ \sum F_y = 0 & \text{(no motion along y)} \\ \sum F_z = 0 & \text{(no motion along z)} \end{cases}$$

- Rotational equilibrium:
  - ➢ Pick a point (let us call it O)

$$\vec{M}_{R,O} = \sum \vec{M}_{\text{couple}} + \sum \vec{M}_O = 0$$

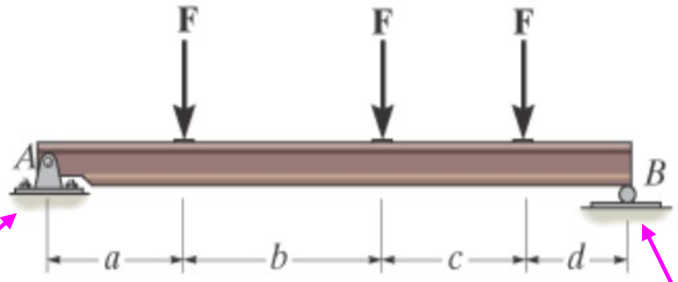
(produced by active and reactive forces)

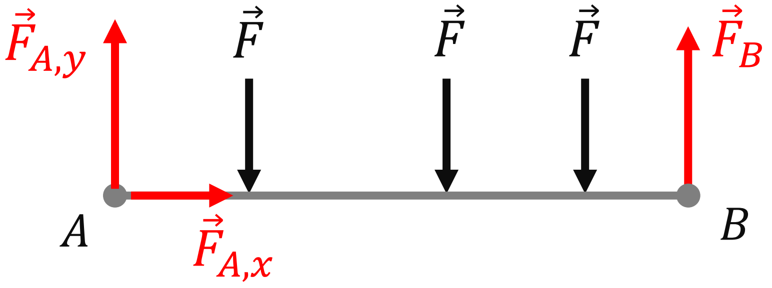
$$\begin{cases} \sum M_{\text{couple},x} + \sum M_{O,x} = 0 & \text{(no rotations about x-axis)} \\ \sum M_{\text{couple},y} + \sum M_{O,y} = 0 & \text{(no rotations about y-axis)} \\ \sum M_{\text{couple},z} + \sum M_{O,z} = 0 & \text{(no rotations about z-axis)} \end{cases}$$

- An object is in equilibrium if both these conditions are satisfied

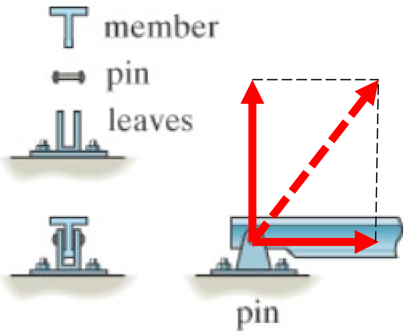
TRANSLATING REAL LIFE TO A DIAGRAM: Reaction Forces and Moments

- A reaction force is developed by a support that restricts the translation of its attached member
- A reaction couple moment is developed when rotation of the attached member is prevented





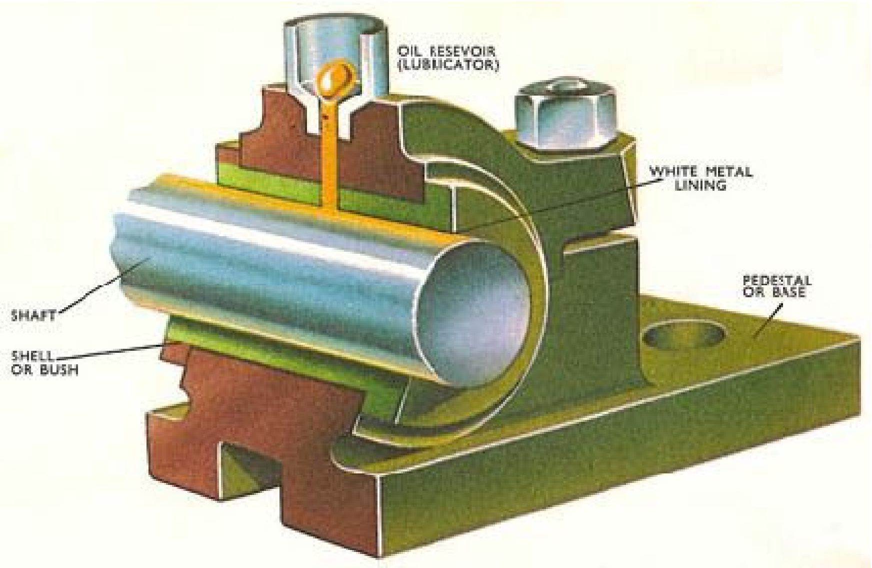
- ➢ Prevents motion downwards and to the left
- ➢ "Pin" (induces two forces)



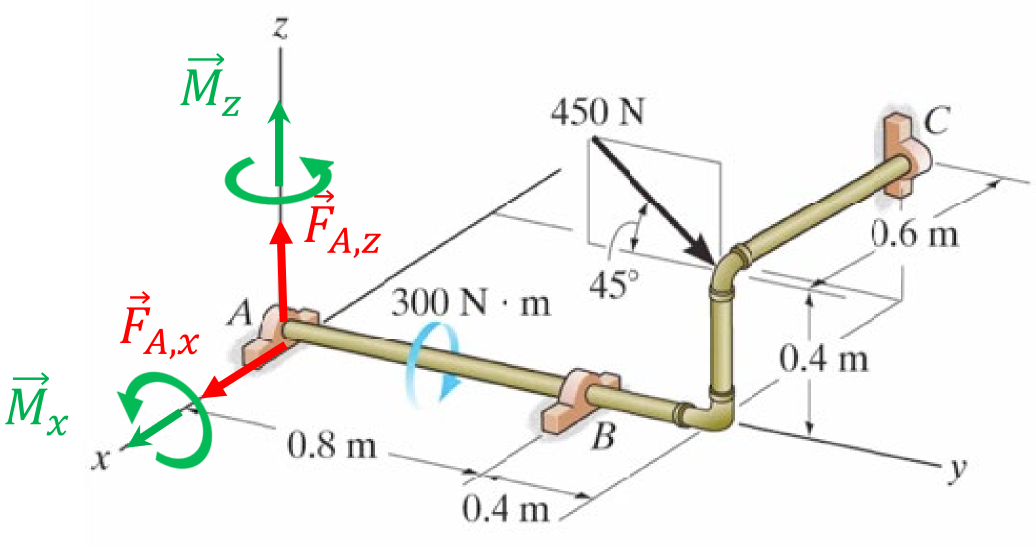
- ➢ Only prevents motion downwards (we neglect friction)
- ➢ "Roller" (induces normal force)

TRANSLATING REAL LIFE TO A DIAGRAM: Reaction Forces and Moments

- A reaction force is developed by a support that restricts the translation of its attached member
- A reaction couple moment is developed when rotation of the attached member is prevented



journal bearing



- ➢ Can move along y, cannot move along x or z;
- ➢ Can rotate about y, cannot rotate about x or z.

TRANSLATING REAL LIFE TO A DIAGRAM: Reaction Forces and Moments

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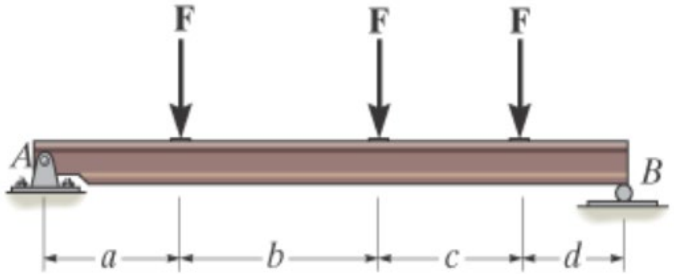
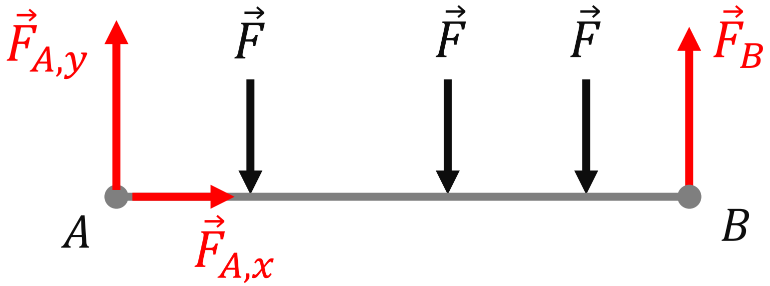
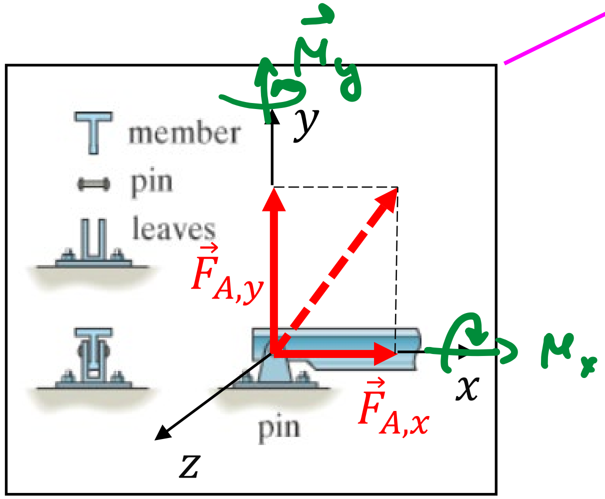






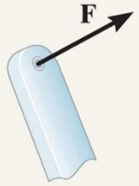
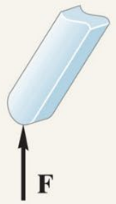
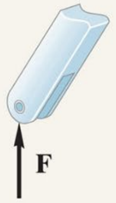
- Note that the pin also prevents rotations about the horizontal and vertical axes (it only allows for rotations about the axis perpendicular to the page). Why did we not add $\vec{M}_{A,x}$ and $\vec{M}_{A,y}$?

- Well, because here there is no external rotation tendency about x- and y- axes! Note that all the forces create rotation tendency about z-axis only. Therefore:

  ➢ No external moment / force => there is nothing to react to!

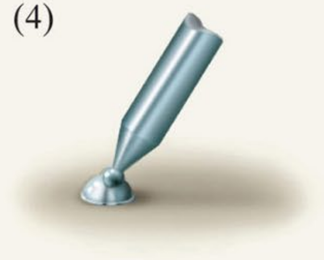
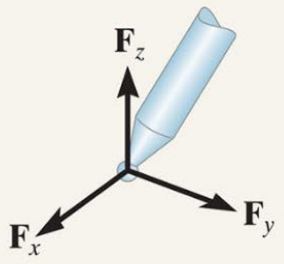
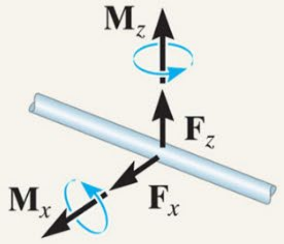
Textbook, Table 5-2: types of support and reaction forces / reaction moments created

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(1)  cable		One unknown. The reaction is a force which acts away from the member in the known direction of the cable.
(2)  smooth surface support		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.
(3)  roller		One unknown. The reaction is a force which acts perpendicular to the surface at the point of contact.

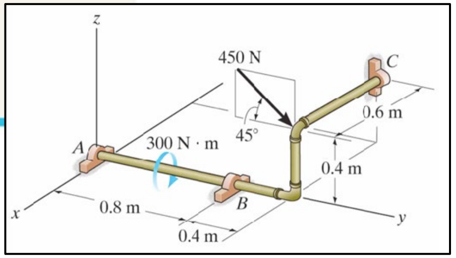
Textbook, Table 5-2: types of support and reaction forces / reaction moments created

TABLE 5–2 Supports for Rigid Bodies Subjected to Three-Dimensional Force Systems

Types of Connection	Reaction	Number of Unknowns
(4)  ball and socket		Three unknowns. The reactions are three rectangular force components.
(5)  single journal bearing		Four unknowns. The reactions are two force and two couple-moment components which act perpendicular to the shaft. Note: The couple moments are *generally not applied* if the body is supported elsewhere. See the examples.

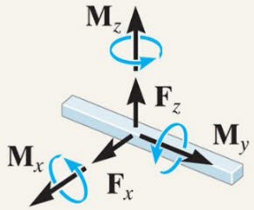
Here there is just one force, but you don't know its direction (it is determined by the combination of forces acting on the body) => Assume 3 forces (components) and find them

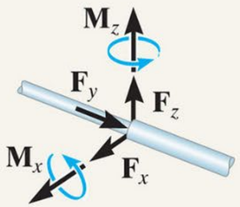
See e.g. in W5-2: "The bearings are in ***proper alignment*** *and only exert force reactions on the rod "*

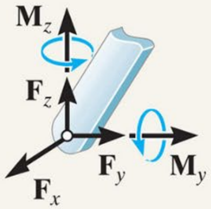


Textbook, Table 5-2: types of support and reaction forces / reaction moments created

<table>
<tr><th colspan="3">TABLE 5–2 Continued</th></tr>
<tr><th>Types of Connection</th><th>Reaction</th><th>Number of Unknowns</th></tr>
<tr><td>(6)

single journal bearing with square shaft</td><td></td><td>Five unknowns. The reactions are two force and three couple-moment components. Note: The couple moments are generally not applied if the body is supported elsewhere. See the examples.</td></tr>
<tr><td>(7)

single thrust bearing</td><td></td><td>Five unknowns. The reactions are three force and two couple-moment components. Note: The couple moments are generally not applied if the body is supported elsewhere. See the examples.</td></tr>
<tr><td>(8)

single smooth pin</td><td></td><td>Five unknowns. The reactions are three force and two couple-moment components. Note: The couple moments are generally not applied if the body is supported elsewhere. See the examples.</td></tr>
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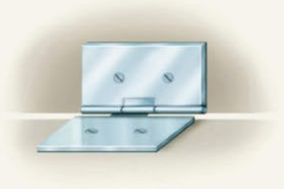
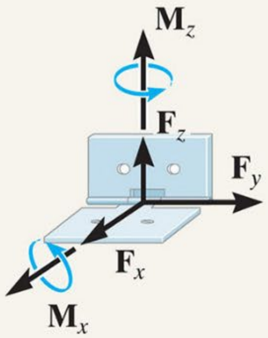
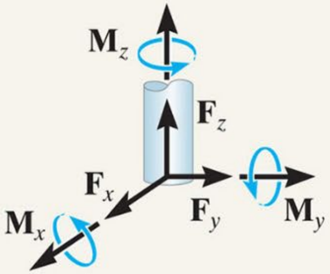
Textbook, Table 5-2: types of support and reaction forces / reaction moments created

TABLE 5–2 Continued

Types of Connection	Reaction	Number of Unknowns
(9)  single hinge		Five unknowns. The reactions are three force and two couple-moment components. *Note*: The couple moments *are generally not applied* if the body is supported elsewhere. See the examples.
(10)  fixed support		Six unknowns. The reactions are three force and three couple-moment components.

SOLVING EQUILIBRIUM PROBLEMS

- Approximation:
  - Rigid body approximation (no bending, no deformations)
  - No friction for now (we will add it next week!)
  - Some elements are assumed to be massless => then no gravity force for them $m \rightarrow \vec{W}$
- Procedure:
  - Draw a Free Body Diagram (please read the next slide carefully). Add active forces and couple moments.
  - At each contact point, in accordance with the type of the support, introduce reaction forces / reaction couple moments ("reactions") that prevent possible translations / rotations.
  - You can check Table 5-2 in your textbook (also reproduced above) to figure out which reactions to add, or (better!) figure them out on your own (Q: How??)
  - So, our FBD should contain all external (active) forces and couple moments, and all reaction forces and reaction moments.
  - When we draw the reaction forces and moments, we assume some (usually arbitrary) directions.
  - We set up the system of equilibrium equations (rotational + translational) and solve for the unknowns.
  - If some components of the reaction force(s) or moment(s) appear to be negative, we adjust our initial assumption(s) about their direction. You don't need to redo the problem.

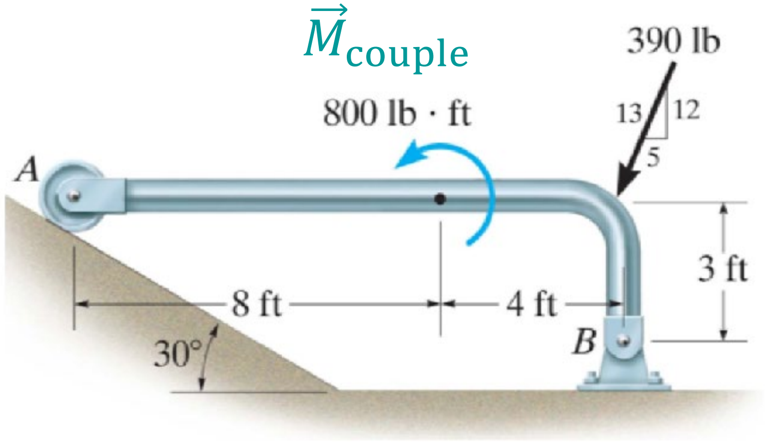
Procedure for Drawing Free Body Diagram (FBD)

- Draw properly labelled right-handed coordinate system
  - Include unit vectors, with tails at origin
- Draw outlined shape
  - Abstract the particle as isolated or cut “free” from its environment, by drawing its outlined shape (i.e. without any of the supports, braces, cables, springs etc. etc. that might be attached to it), “stick”/line diagram generally suffices—a point (“ball”) for problems in this chapter
- Show all forces
  - Show on your diagram all forces acting on the particle. Includes active forces, which tend to set the particle in motion, as well as reactive forces that are the result of constraints/supports that tend to prevent motion
  - **CRUCIAL POINT:** Must account for all forces, may help to trace around the particle’s boundary. Can often make use of the fact that particle is in equilibrium, i.e. that “forces must balance”.
- Identify/label each force
  - Known forces should be labeled with magnitudes & directions. Letters are used to represent magnitudes and directions of unknown forces

Our TAs will refer to this slide when marking your tutorials and exams

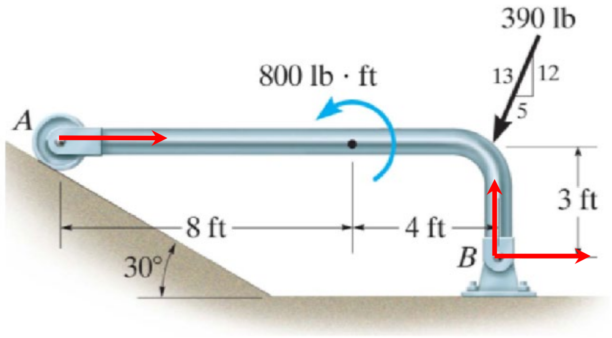
WARMING UP: 2D Equilibrium

W5-1. Find the reaction forces and moments at points A and B at equilibrium.

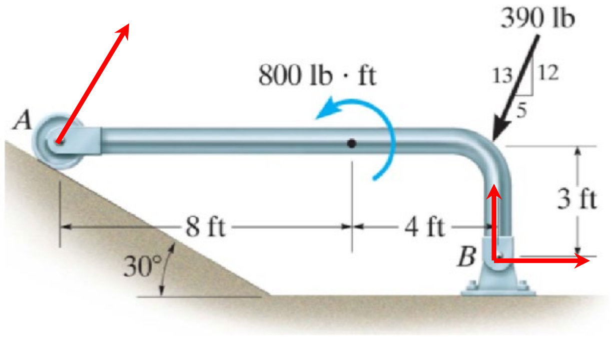


Q: Which picture is correct?

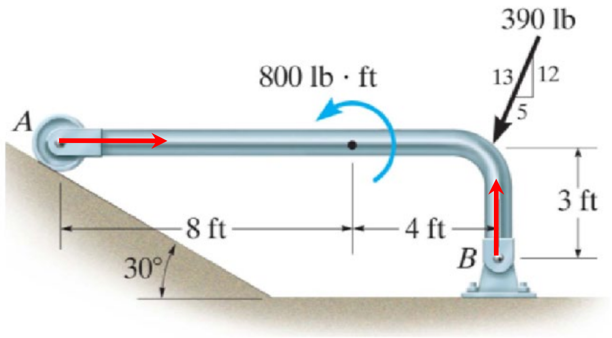
A



C



B



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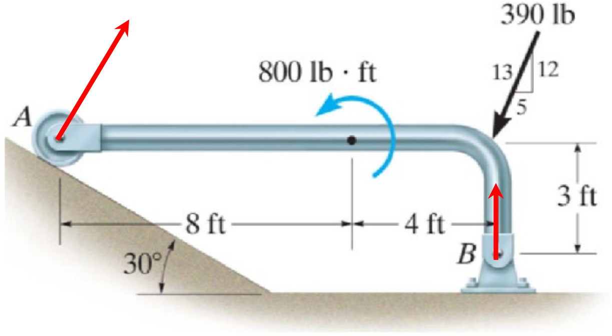
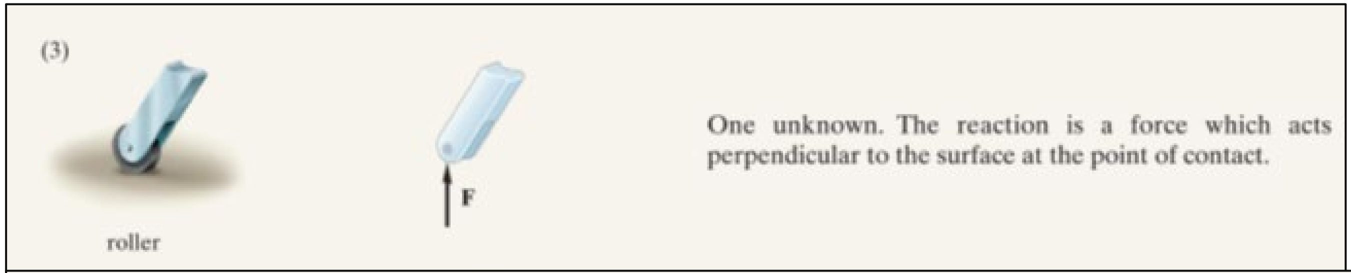
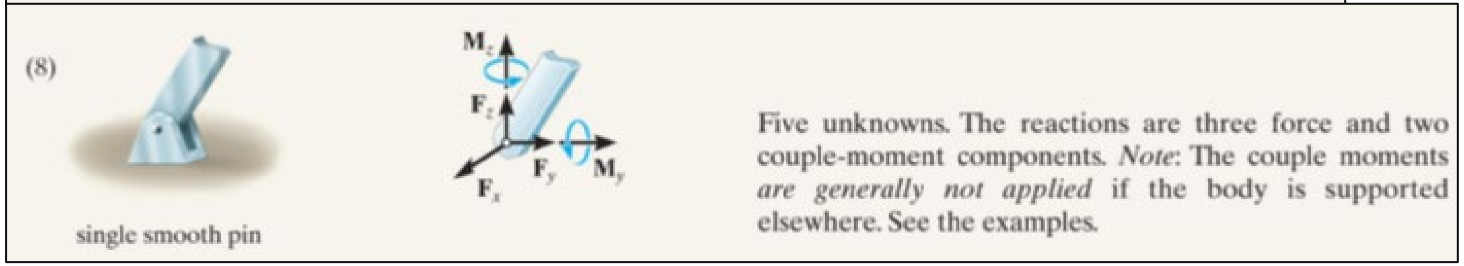
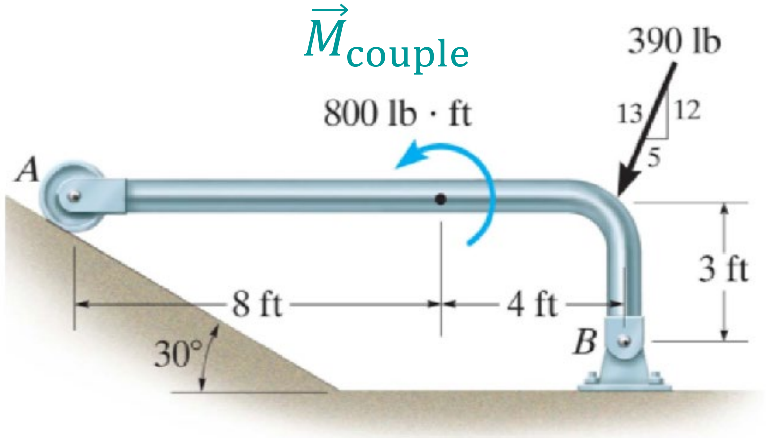


E. There should be reaction couple moment(s), not only forces!

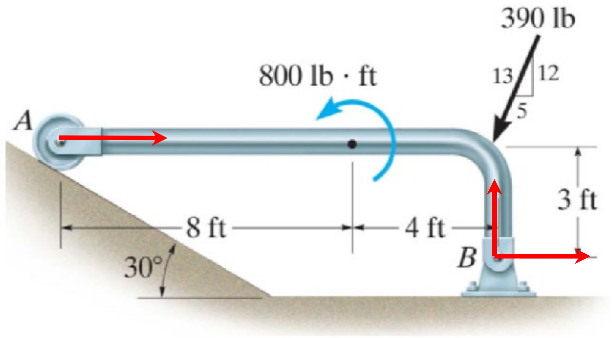
WARMING UP: 2D Equilibrium



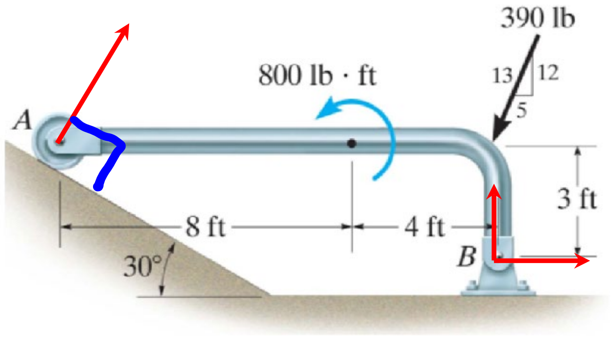




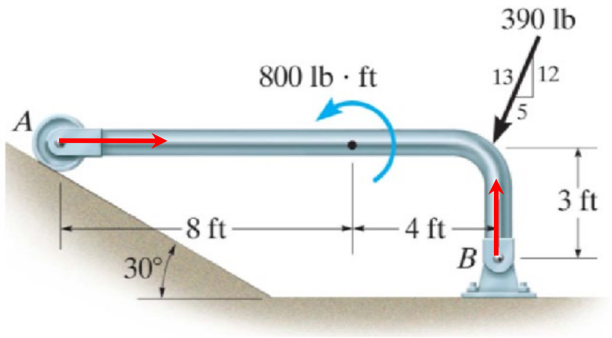
A



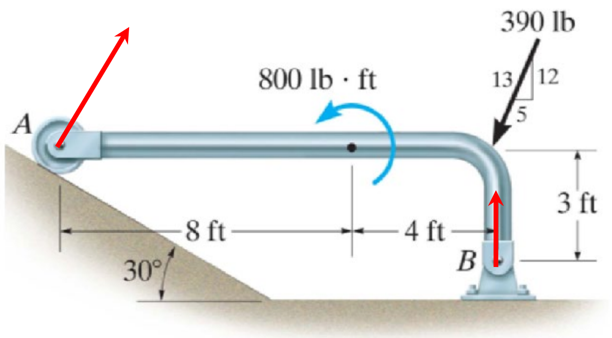
C



B



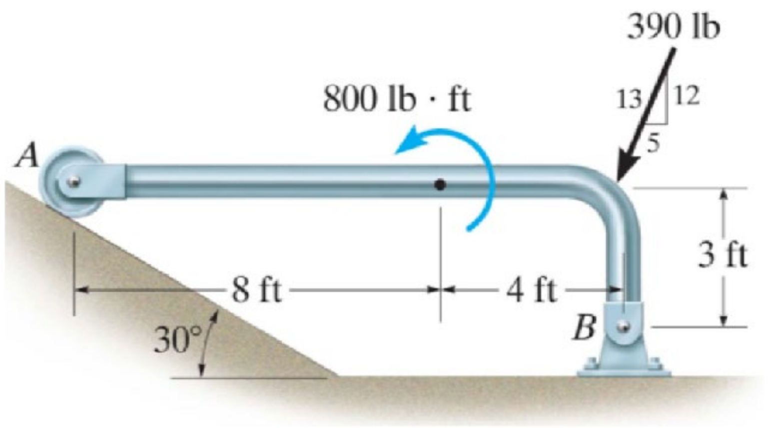
D



E. There should be reaction couple moment(s), not only forces!

WARMING UP: 2D Equilibrium

W5-1. Find the reaction forces and moments at points A and B at equilibrium.



Q: We need all the moments, i.e. (i) exerted by the reaction forces from the previous slide, and (ii) the external couple moment $\vec{M}_{\text{couple}}$= 800 lb ft to cancel. Which point we must choose as O to calculate our moments about?

A. A
B. B
C. Where the 800 ft lb moment is acting.
D. Where the 390 N force is acting.
E. Any of the above is fine

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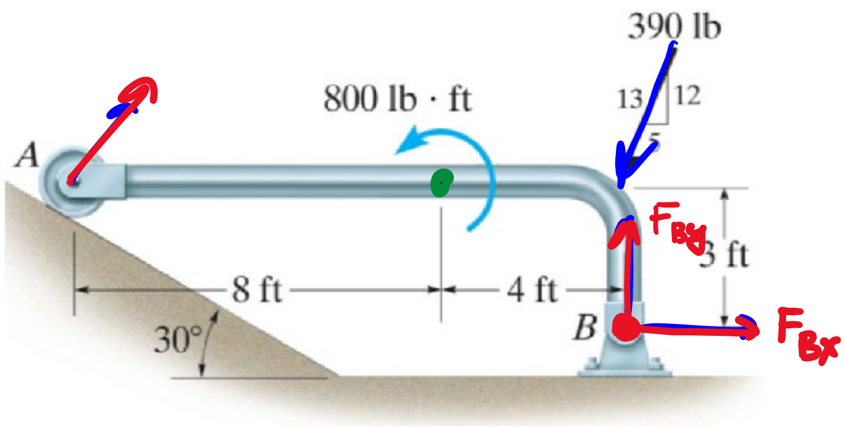

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There is no "must". A couple moment is a free vector and can be associated with any point. That said, it is possible to argue that the simplest choice is B, since it eliminates two unknown force components *(the moment of a force about a point on its line of action is zero)*.