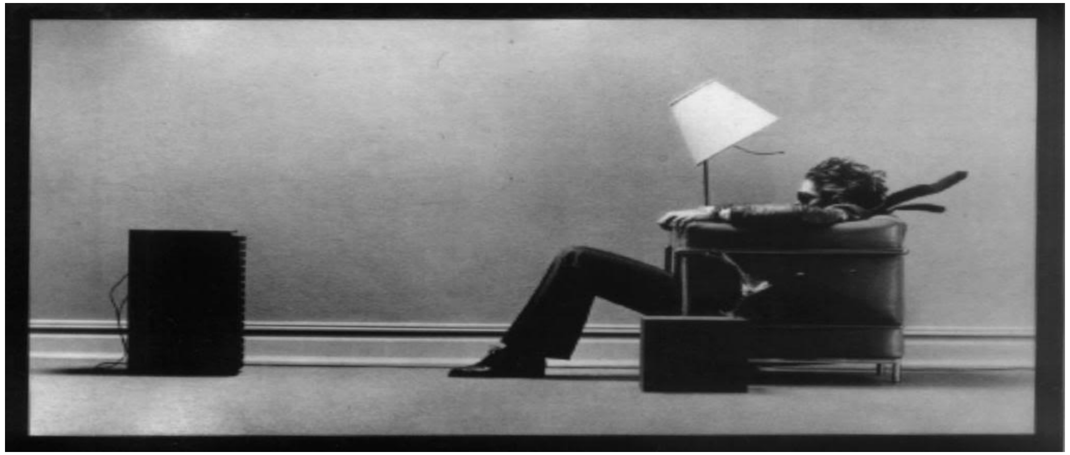
Lecture 34.

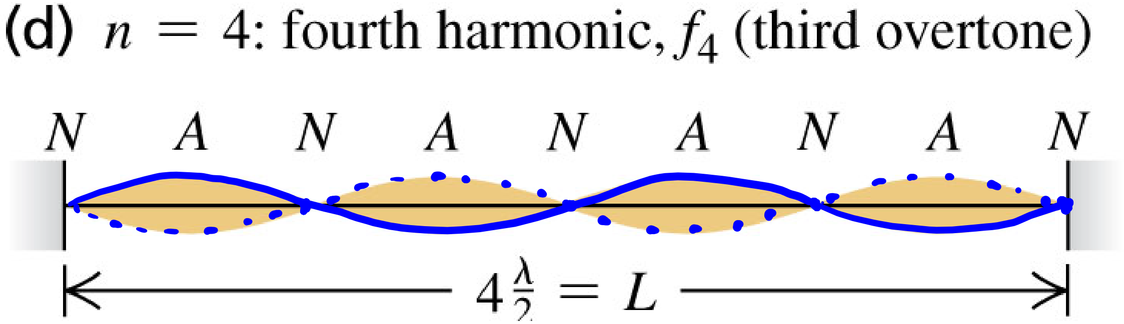
Standing waves on a string (finish).

Sound waves.



Last Time

- Standing wave. Example: 4th harmonics



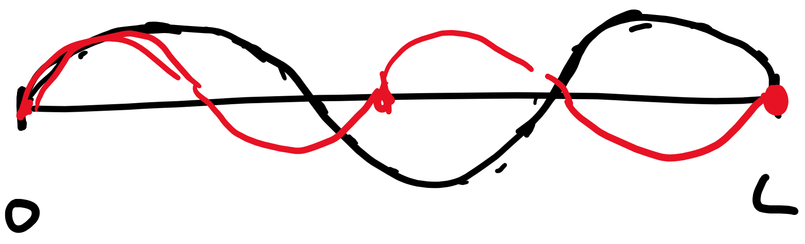
- Wavelength – from geometry: $\frac{\lambda}{2} n = L$ ($n = 4$ => 4th harmonic)

- Frequency – from wavelength, λ, and speed of the wave, v:

$$v = \sqrt{\frac{T}{\mu}} = f\lambda$$

Recap

Q: A string driven at some frequency ω supports a standing wave (third harmonic). What will happen if you loosen the string by an appropriate amount, while keeping driving it at the same frequency?



3rd harmonic

$$3 \cdot \frac{\lambda}{2} = L$$

$T \downarrow \rightarrow ?$

$$\downarrow \lambda \cdot f = \downarrow v = \sqrt{\frac{T}{\mu}} \downarrow$$

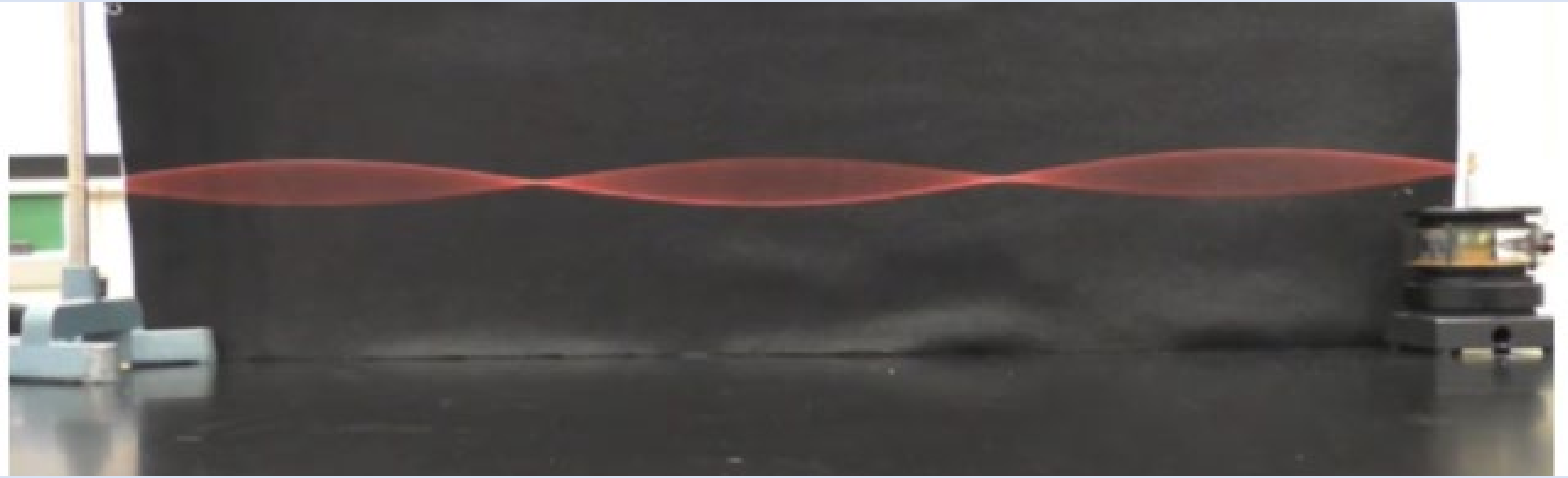
same

A. You can hit the second harmonic

B. You can hit the forth harmonic

C. There won't be any other harmonics at this frequency.

Demo: Standing wave on a string



Recap

Q: A string driven at some frequency ω supports a standing wave (third harmonic). What will happen if you loosen the string by an appropriate amount, while keeping driving it at the same frequency?

- Third harmonic (initial): $3 \cdot \frac{\lambda}{2} = L$
- $v = \sqrt{\frac{T}{\mu}} = f\lambda$
- When T goes down, v goes down, too. Since the frequency f remains the same, the wavelength should decrease. If the new wavelength is such that $4 \cdot \frac{\lambda'}{2} = L$, we will observe 4th harmonic.

A. You can hit the second harmonic
B. You can hit the forth harmonic
C. There won't be any other harmonics at this frequency.

Bonus: find the ratio of $T_{\mathrm{old}}/T_{\mathrm{new}}$ at which this happens.

Questions left:

Q1: How does a wave “know” to fit a string?

Q2: Okay, we know that on a string $v = \sqrt{T/\mu}$. But we are talking about a <u>standing wave</u>, where nothing moves in the horizontal direction.

What for the speed is that???

We will need: Reflection of travelling waves

- Whenever a travelling wave encounters a boundary, it is reflected, sending a travelling wave in the opposite direction. Net signal is the sum of incident and reflected waves.
- The nature of the reflected wave depends on the nature of the boundary – we call this the "boundary condition"

Fixed end

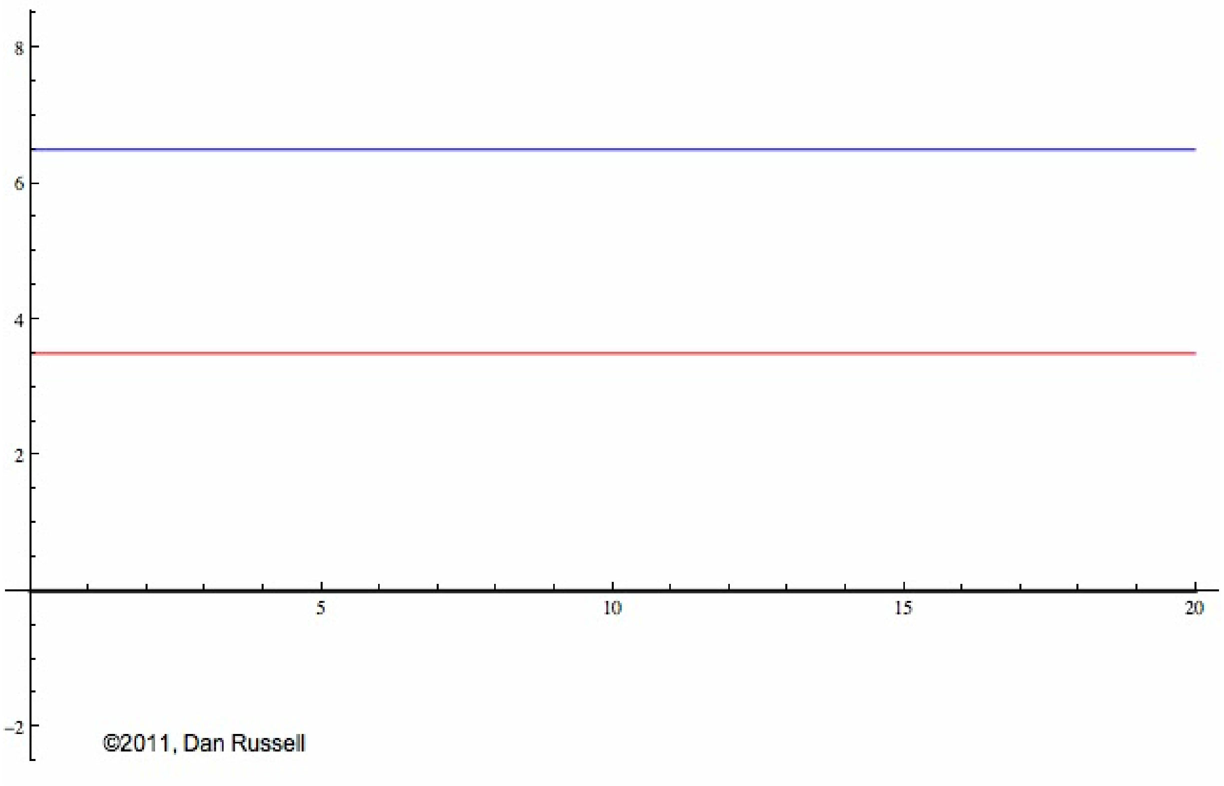
Open end

  - A wave reflected from a fixed boundary condition is inverted (180^{o} phase shift) so that the right and left travelling waves sum to zero at the fixed end
  - A wave reflected from an open boundary condition has the same phase (0^{o} phase shift) so that the right and left waves sum to twice the amplitude at the open end

- See https://www.acs.psu.edu/drussell/Demos/reflect/reflect.html

Standing Wave is superposition of two sine waves travelling past each other in opposite directions

- Mechanism: Each wave adds up with its own reflected pattern!



Right-moving wave, incident:

$A\cos(kx - \omega t) =$

$A\,\cancel{\cos(kx)}\cos(\omega t) + A\sin(kx)\sin(\omega t)$

Left-moving wave, reflected:

$A\cos(kx + \omega t) =$

$-(A\,\cancel{\cos(kx)}\cos(\omega t) - A\sin(kx)\sin(\omega t))$

$$\boxed{\text{SUM} = 2A\sin(kx)\sin(\omega t)}$$

At a resonant frequency & wavelength, <u>their superposition on a string is stable</u>. At other frequencies and wavelengths, their superposition averages out to zero.

https://www.acs.psu.edu/drussell/Demos/superposition/superposition.html

Animation courtesy of Dr. Dan Russell, Grad. Prog. Acoustics, Penn State

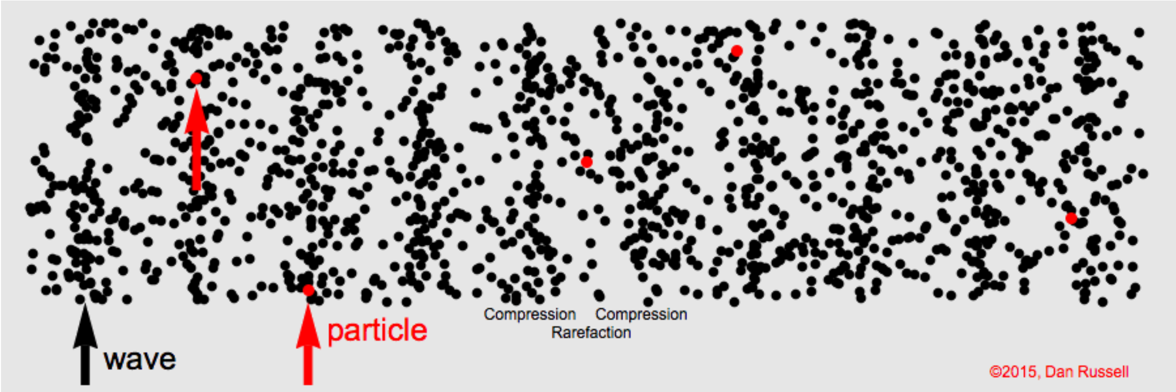
Summary:

- Superposition of waves: $D(x,t) = D_1(x,t) + D_2(x,t)$
- Standing waves: when a wave fits the geometry of the system
- Speed of wave is determined by physical properties of a system: $v = \sqrt{T/\mu}$. Hence, $\lambda f = \sqrt{T/\mu}$.
- Reflection of waves at fixed end / open end boundary conditions & Standing wave as a superposition of two harmonic waves propagating in opposite directions.

Longitudinal (sound) waves

- Sound is simply any longitudinal wave in a medium
- The audible range of frequencies for humans is ~20 Hz to 20,000 Hz
- For a sinusoidal sound wave traveling in the x-direction, the wave function $D(x, t)$ gives the instantaneous displacement of a particle in the medium at position x and time t
- In a longitudinal wave the displacements are parallel to the direction of travel of the wave, not perpendicular as in a transverse wave

- Sound wave is best visualized as a density (=pressure) wave



Displacement and pressure in sound waves

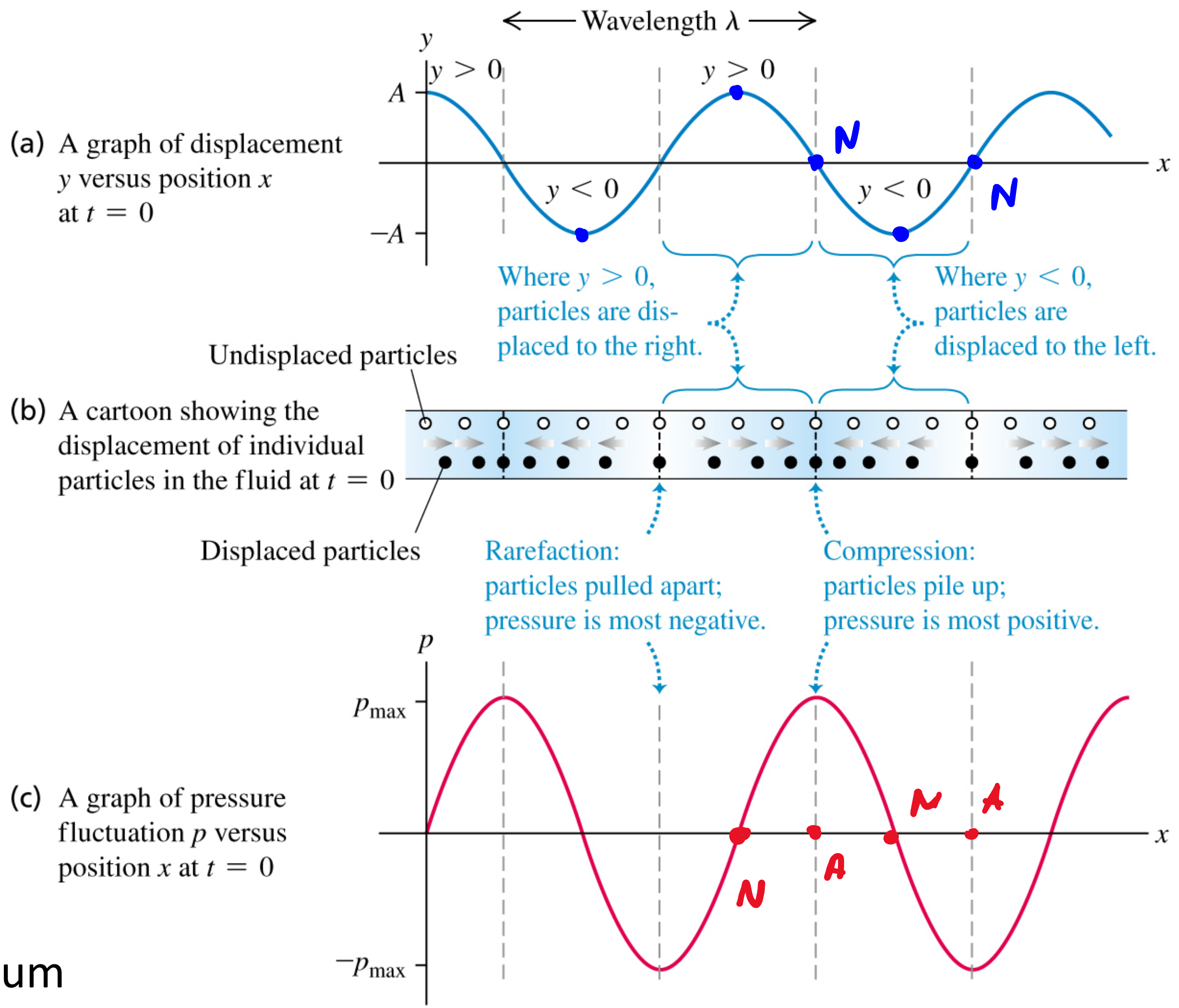
- Displacement:

$$D(x,t) = A\cos(kx - \omega t)$$

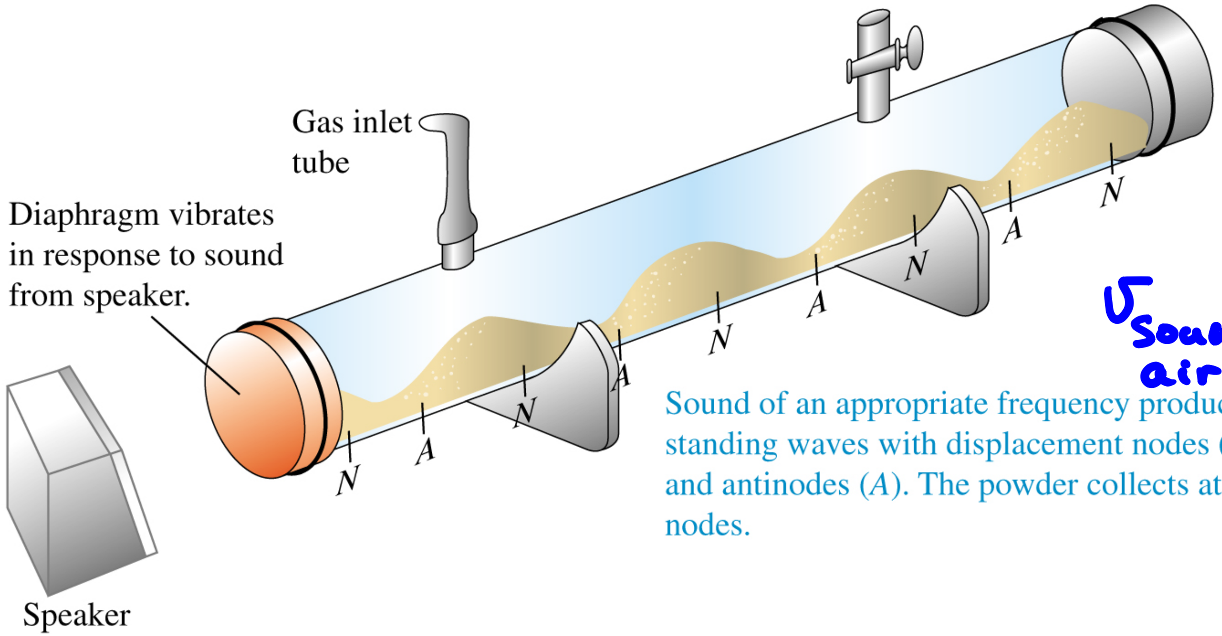
- Pressure:

$$p(x,t) = -B\frac{\partial D(x,t)}{\partial x}$$
$$= BkA \cdot \sin(kx - \omega t)$$

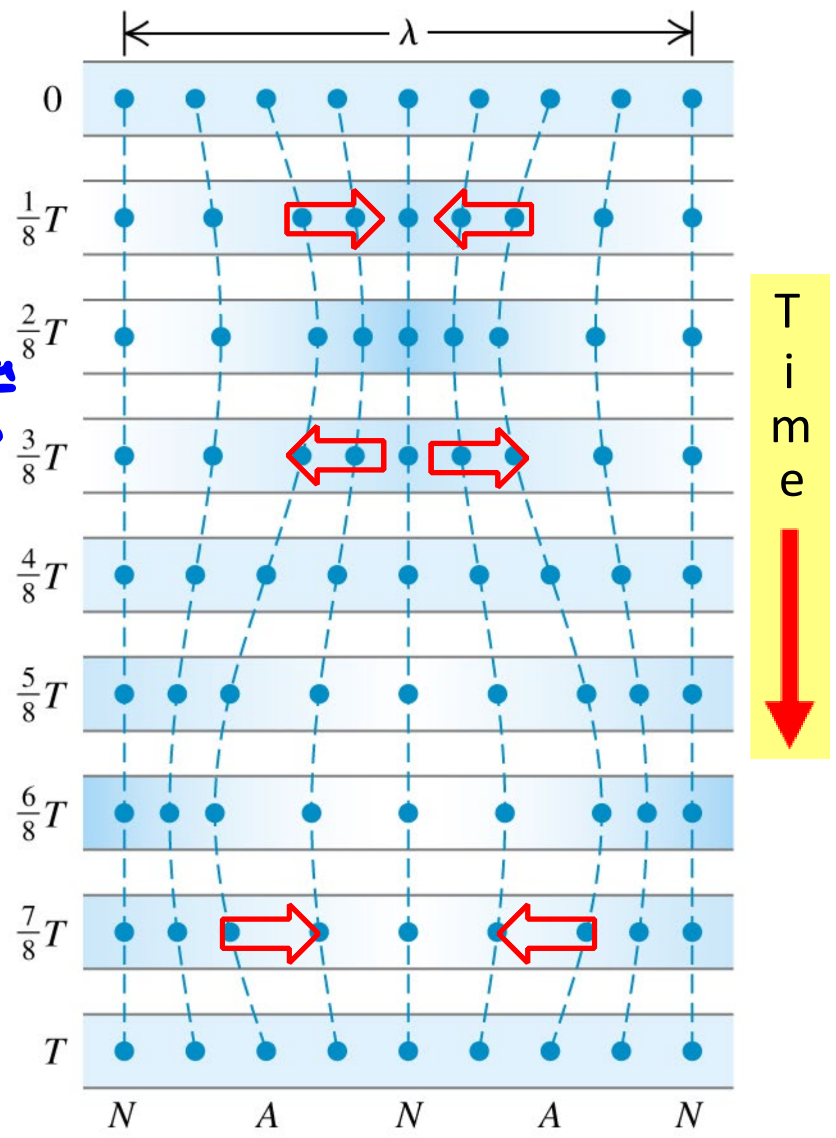
B is the bulk modulus of the medium



Longitudinal standing sound waves



$v_{\text{sound air}} \approx 343 \frac{m}{s}$



N = a displacement node = a pressure antinode
A = a displacement antinode = a pressure node

- Displacement Node (particles don't move) at same place as Pressure Anti-Node (where pressure fluctuates most)
- Used to measure v_{sound} in gases (one of the experiments done in PHYS 159)

https://www.acs.psu.edu/drussell/Demos/StandingWaves/StandingWaves.html

Q: At a maximum compression in a sound wave, which of the following are true?

A. particles are displaced by the maximum distance in the same direction as the wave is moving
B. particles are displaced by the maximum distance in the direction opposite to the direction the wave is moving
C. particles are displaced by the maximum distance in the direction perpendicular to the direction the wave is moving
D. the particle displacement is zero
E. more than one of the above can be true, depending on circumstances

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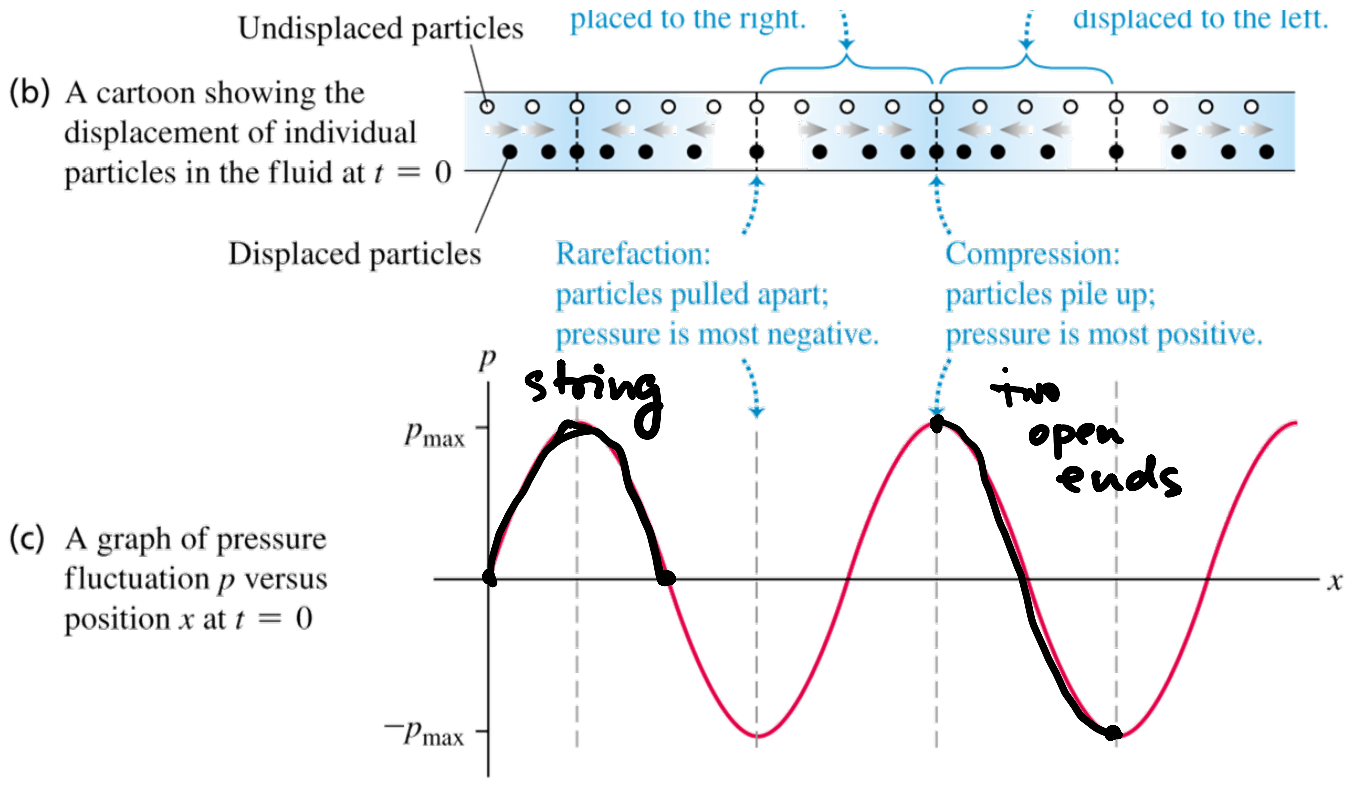
E. more than one of the above can be true, depending on circumstances

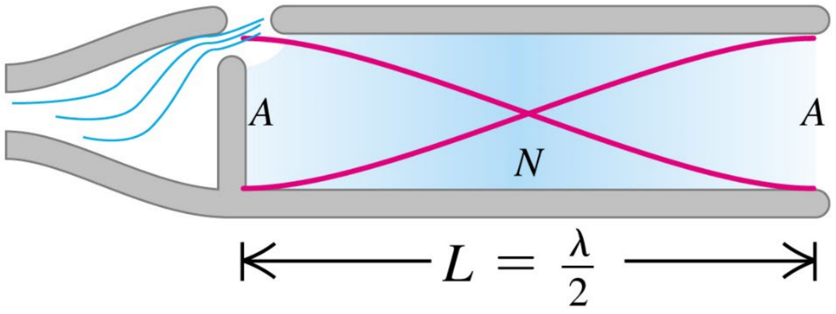


Boundary Conditions: Two Open Ends

Open End → Pressure Node → Displacement Anti-Node

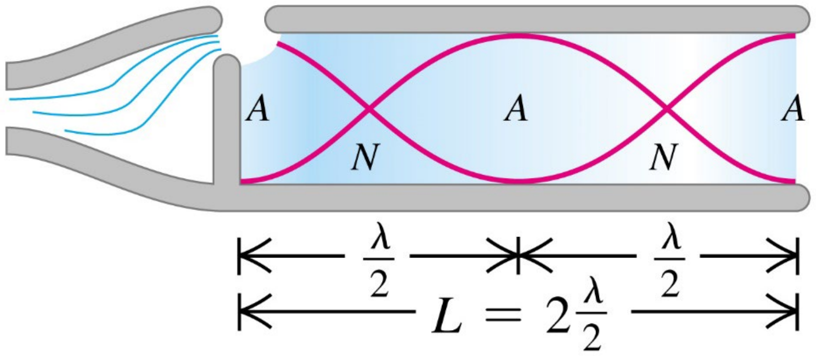
- Fundamental:

$$\lambda_1 = 2L \qquad f_1 = \frac{v}{2L}$$



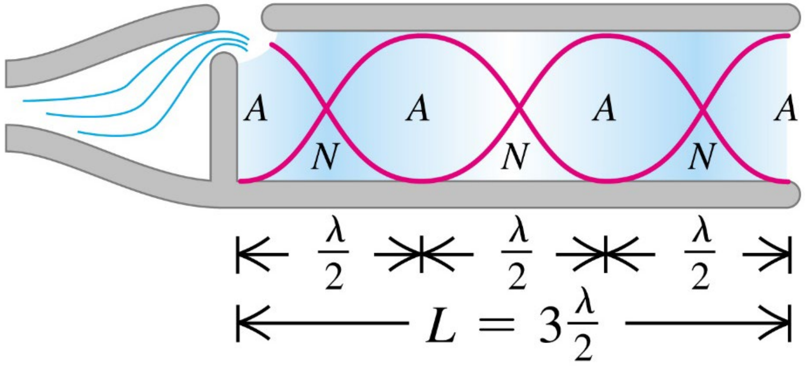
- Second Harmonic:

$$\lambda_2 = L \qquad f_2 = \frac{v}{L}$$



- Third Harmonic

$$\lambda_3 = \frac{2L}{3} \qquad f_3 = \frac{3v}{2L}$$



All harmonics are possible

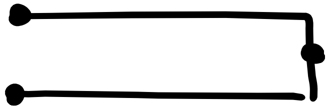
$$n = 1, 2, 3, 4, \ldots$$

$$\lambda_n = 2L/n$$

$$f_n = n\frac{v}{2L} = nf_1$$

$f_n \lambda_n = v = 343 \frac{m}{s}$

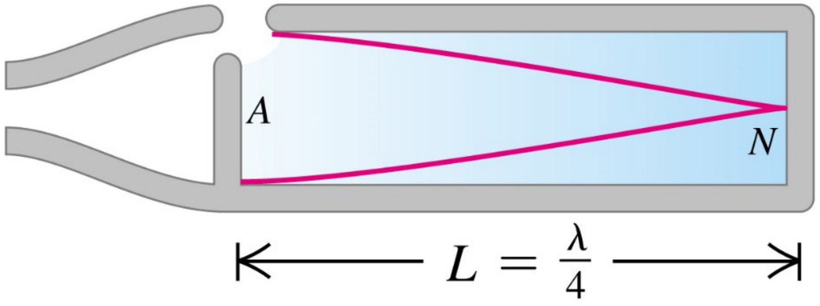
Boundary Conditions: One Open End – One Closed End



Closed End → Pressure Anti-Node → Displacement Node

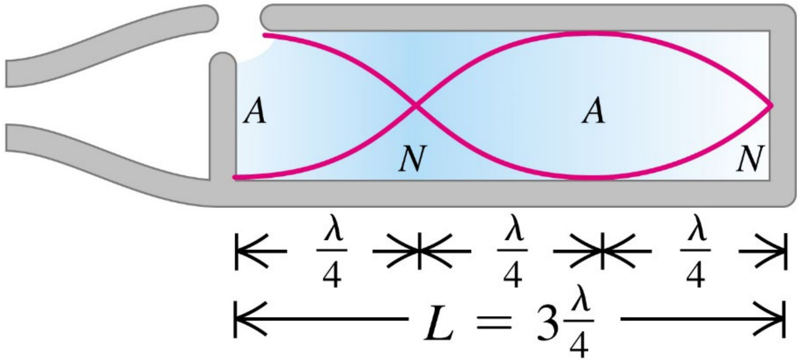
- Fundamental:

$$\lambda_1 = 4L \qquad f_1 = \frac{v}{4L}$$



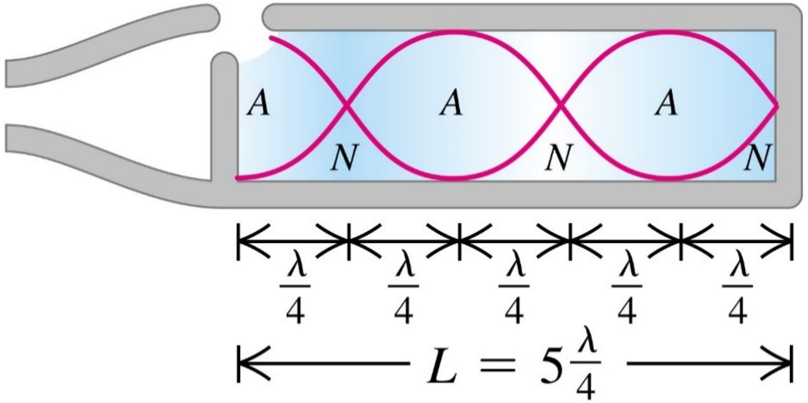
- Third Harmonic:

$$\lambda_2 = \frac{4L}{3} \qquad f_2 = \frac{3v}{4L}$$



- Fifth Harmonic:

$$\lambda_3 = \frac{4L}{5} \qquad f_3 = \frac{5v}{4L}$$



Only odd harmonics are possible

$$n = n_{odd} = 1, 3, 5, \ldots$$

$$\lambda_n = 4L/n$$

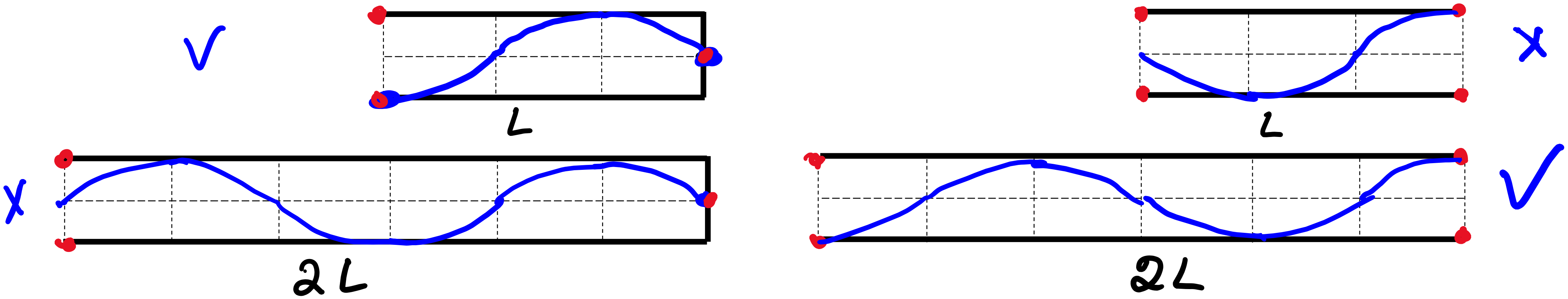
$$f_n = n\frac{v}{4L} = nf_1$$

Q: An organ pipe of length L has one closed end and a fundamental frequency of 220 Hz. For which of the following organ pipes will there be a resonance if a tuning fork of frequency 660 Hz is sounded next to the pipe?

A. An organ pipe of length L with one closed end
B. An organ pipe of length $2L$ with one closed end
C. An organ pipe of length L with two open ends
D. An organ pipe of length $2L$ with two open ends
E. More than one of the above

Q: An organ pipe of length L has one closed end and a fundamental frequency of 220 Hz. For which of the following organ pipes will there be a resonance if a tuning fork of frequency 660 Hz is sounded next to the pipe? $f\lambda = 343$ m/s

Way 1:



A. An organ pipe of length L with one closed end

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C. An organ pipe of length L with two open ends

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E. More than one of the above

Q: An organ pipe of length L has one closed end and a fundamental frequency of 220 Hz. For which of the following organ pipes will there be a resonance if a tuning fork of frequency 660 Hz is sounded next to the pipe?

Way 2: We know that $f_1(L) \equiv v/4L = 220\ \mathrm{Hz}$

One Closed End (n odd): $f_n = n\, v/4L_{\mathrm{tube}}$

$f_1(L) = 220\ \mathrm{Hz}$ $\quad f_1(2L) = 110\ \mathrm{Hz}$

$f_3(L) = 660\ \mathrm{Hz}$ $\quad f_3(2L) = 330\ \mathrm{Hz}$

$f_5(2L) = 550\ \mathrm{Hz}$

$f_7(2L) = 770\ \mathrm{Hz}$

Two Open Ends (all n): $f_n = n\, v/2L_{\mathrm{tube}}$

$f_1(L) = 440\ \mathrm{Hz}$ $\quad f_1(2L) = 220\ \mathrm{Hz}$

$f_2(L) = 880\ \mathrm{Hz}$ $\quad f_2(2L) = 440\ \mathrm{Hz}$

$f_3(2L) = 660\ \mathrm{Hz}$

A. An organ pipe of length L with one closed end
B. An organ pipe of length $2L$ with one closed end
C. An organ pipe of length L with two open ends
D. An organ pipe of length $2L$ with two open ends
E. More than one of the above ✓