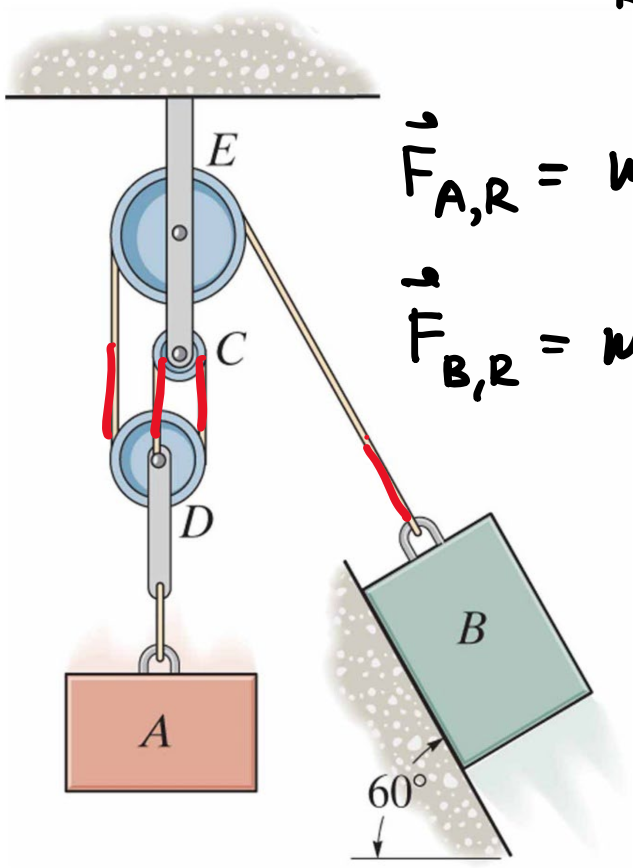
W9-1. The mass of block A is 100 kg. The mass of block B is 60 kg. The coefficient of kinetic friction between block B and the inclined plane is 0.4. A and B are released from rest. Determine the acceleration of block A and the tension in the cord. Neglect the mass of the pulleys and the cord.

Last Time

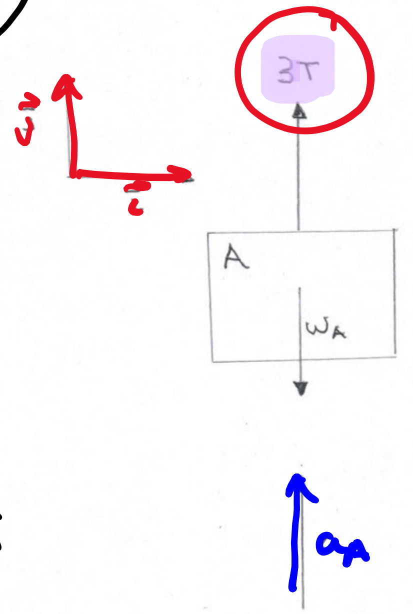
$\vec{F}_R = m\vec{a}$ → 2 eq

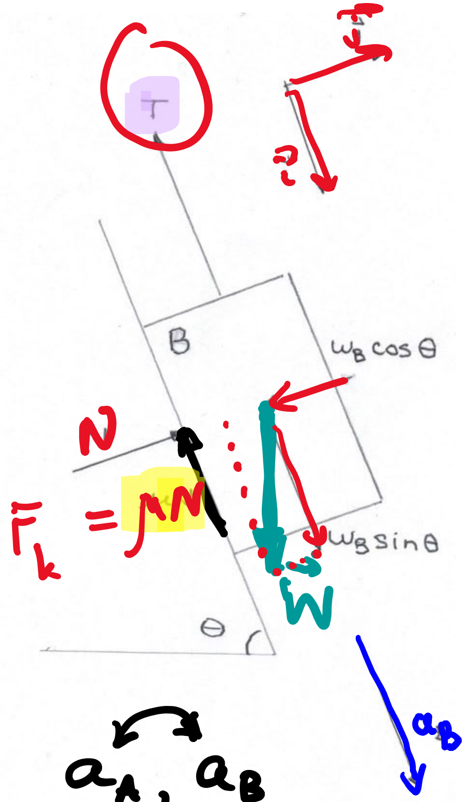


$\vec{F}_{A,R} = m_A \vec{a}_A$

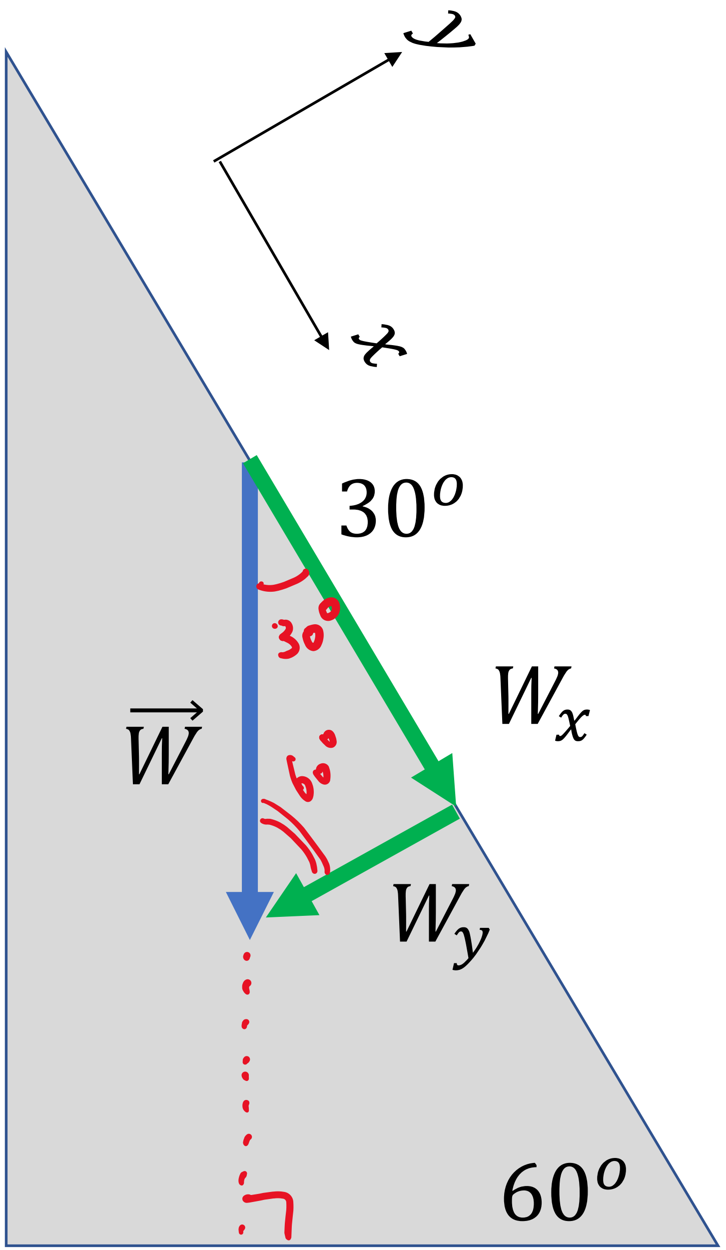
$\vec{F}_{B,R} = m_B \vec{a}_B$

2 eq





dependent motion : a_A, a_B



$$W_x = W\cos 30^o = W\sin 60^o$$

$$W_y = -W\sin 30^o = -W\cos 60^o$$

W9-1. The mass of block A is 100 kg. The mass of block B is 60 kg. The coefficient of kinetic friction between block B and the inclined plane is 0.4. A and B are released from rest. Determine the acceleration of block A and the tension in the cord. Neglect the mass of the pulleys and the cord.

$\vec{F}_{R,A} = m_A \vec{a}_A$

x: N/A

(1) y: $3T - W_A = m_A a_A$

$\vec{F}_{R,B} = m_B \vec{a}_B \rightarrow \vec{a}_B = (a_B, 0)$

x: $W_B \sin\theta - \mu N - T = m_B a_B$ (2)

y: $N - W_B \cos\theta = 0$ (3)

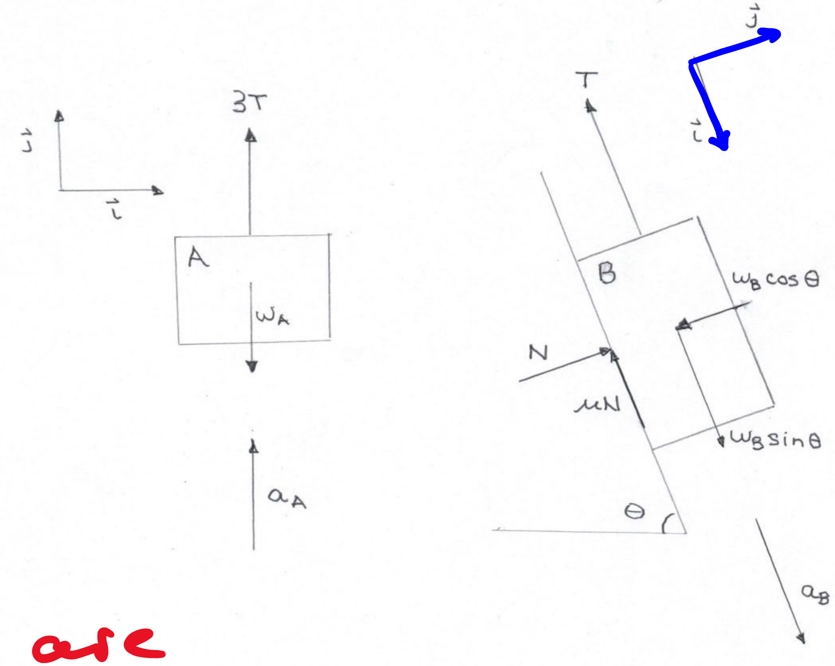
3 eqs, 4 unknowns!

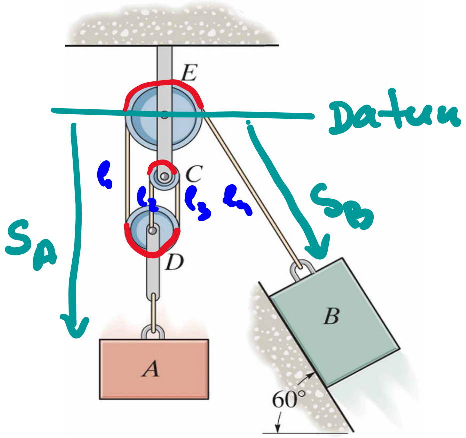
$\frac{d^2}{dt^2}\left(s_B + 3 s_A = 0\right) \rightarrow$ on your own!

$\ddot{s}_B + 3\ddot{s}_A = 0$

$a_B = \begin{cases} 3a_A & \text{A.} \\ -3a_A & \text{B.} \end{cases}$

a_A, a_B are positive numbers!





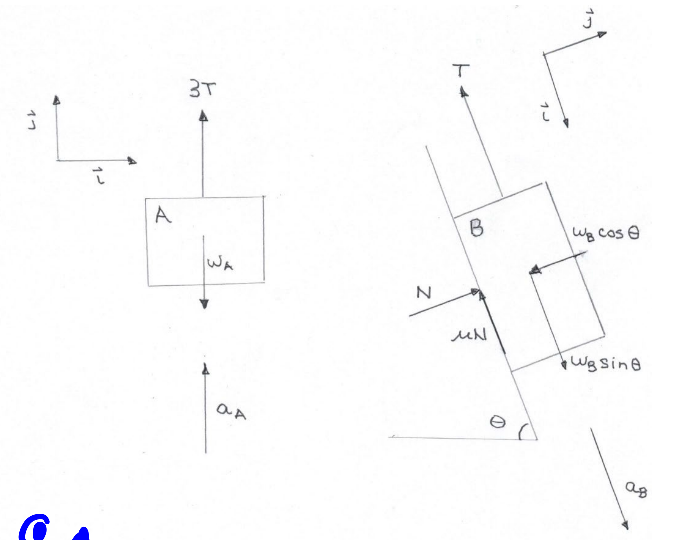
W9-1. The mass of block A is 100 kg. The mass of block B is 60 kg. The coefficient of kinetic friction between block B and the inclined plane is 0.4. A and B are released from rest. Determine the acceleration of block A and the tension in the cord. Neglect the mass of the pulleys and the cord.

$$3T - W_A = m_A a_A$$

$$N = W_B \cos\theta$$

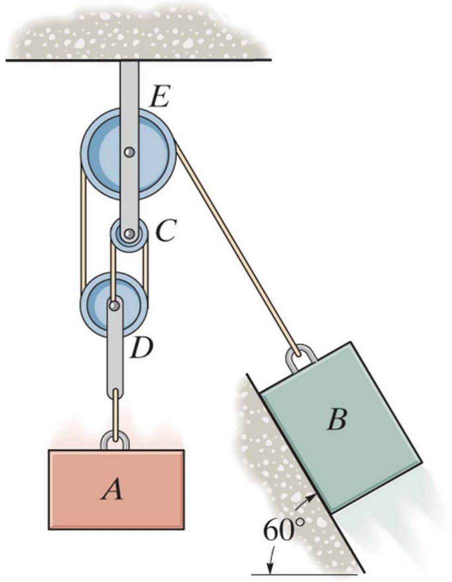
$$W_B \sin\theta - \mu N - T = m_B a_B$$

$$a_B = 3a_A$$

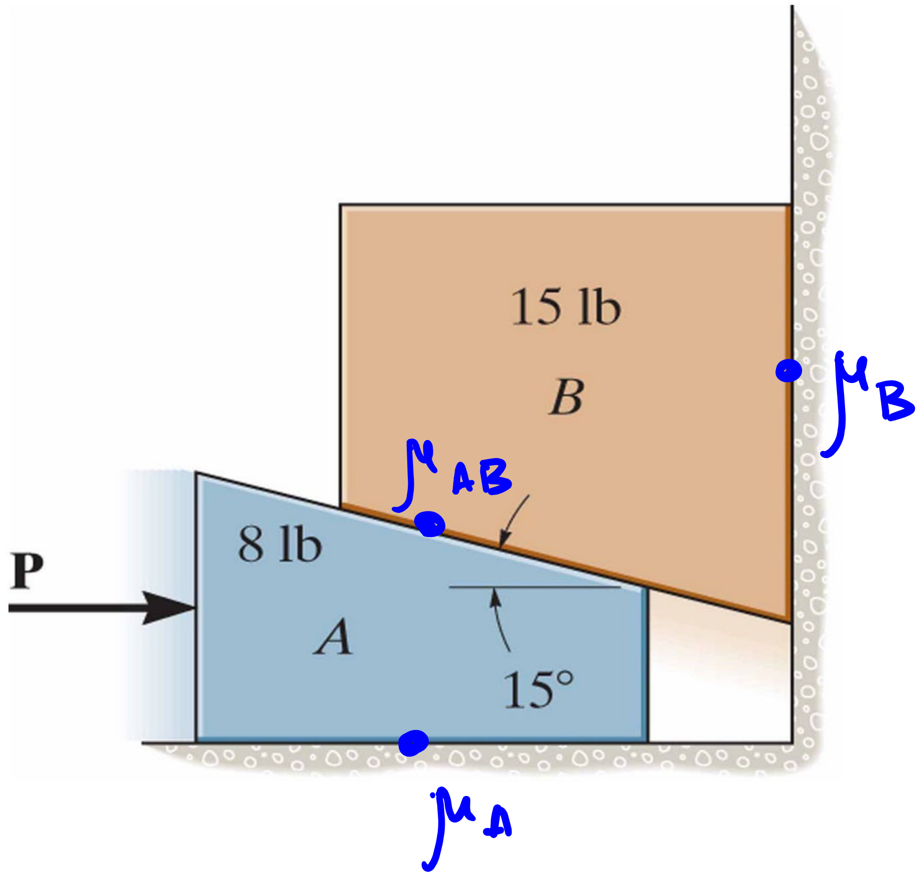


$$\left[3m_B(\sin 60^\circ - \mu\cos 60^\circ) - m_A\right]g = \underline{(m_A + 9m_B)}\,a_A$$

$$\boxed{a_A = 305\ \frac{mm}{s^2}} > 0 \rightsquigarrow \boxed{T = 337\ N}$$



W9-2. A horizontal force $P = 20$ lb is applied to block A. The coefficients of kinetic friction between block A and the horizontal surface, between the two blocks, and between block B and the vertical surface are 0.1, 0.2 and 0.3, respectively. Determine the acceleration of each block and all normal forces.



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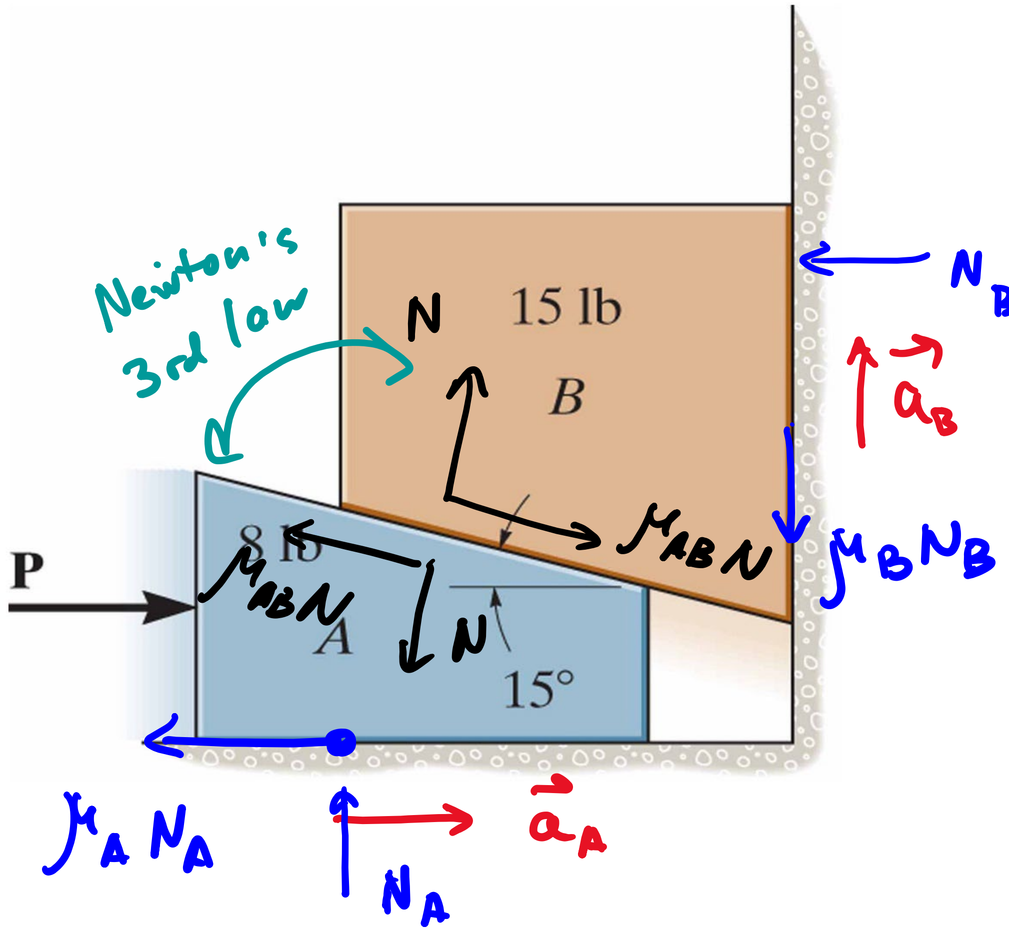


Direction of accelerations?

A. a_A left, a_B up

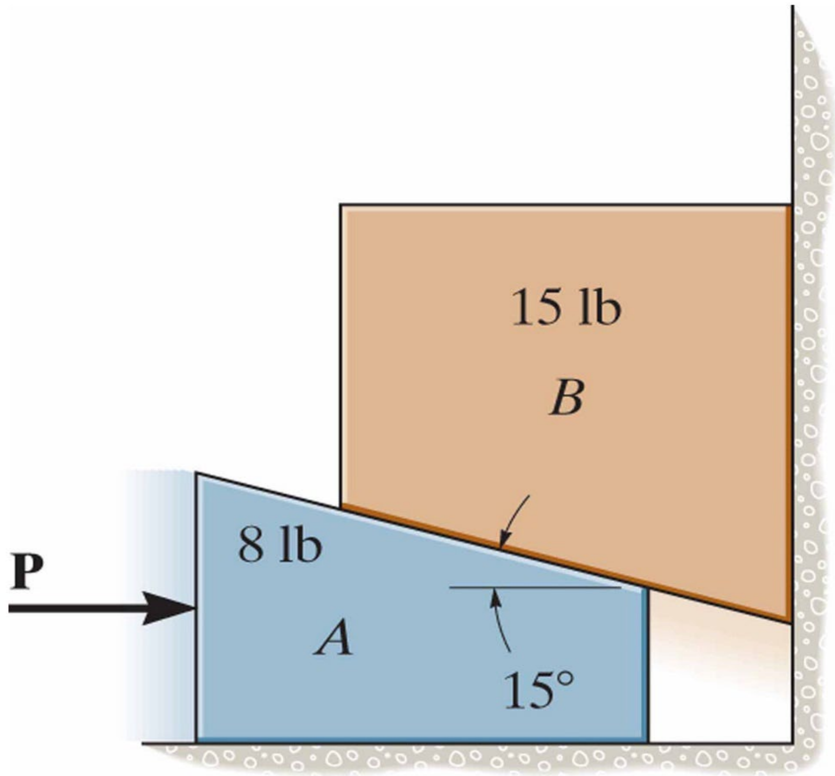
B. a_A right, a_B down

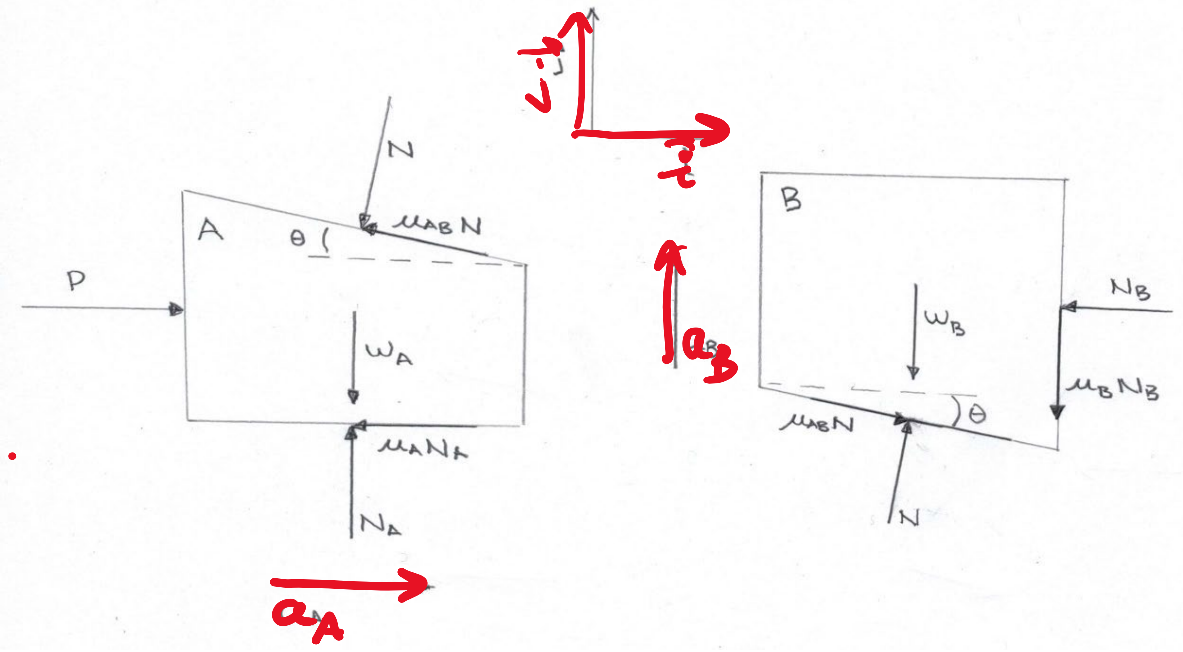
C. a_A left, a_B down

D. a_A right, a_B up

E. None of the above (one of them is at an angle)

W9-2. A horizontal force $P = 20$ lb is applied to block A. The coefficients of kinetic friction between block A and the horizontal surface, between the two blocks, and between block B and the vertical surface are 0.1, 0.2 and 0.3, respectively. Determine the acceleration of each block and all normal forces.





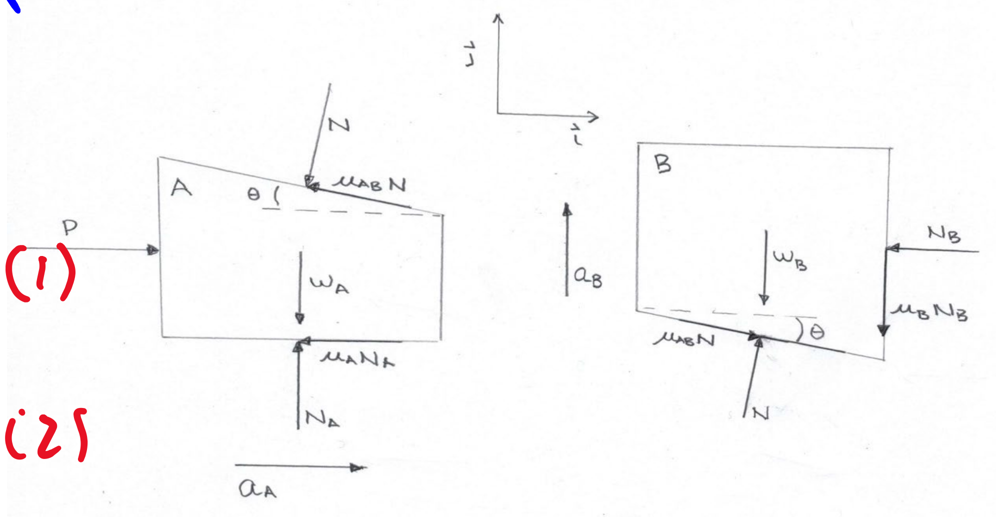
W9-2. $P = 20$ lb, $W_A = 8$ lb, $W_B = 15$ lb, θ=15°, $\mu_A = 0.1$, $\mu_{AB} = 0.2$, $\mu_B = 0.3$.

$a_A, a_B, N_A, N_B, N = ?$

A: $\vec{F}_{RA} = m_A \vec{a}_A$ $\qquad \boxed{a_{A,y} = 0}$

x: $P - N\sin\theta - \mu_{AB} N \cos\theta - \mu_A N_A = m_A a_A \qquad (1)$

y: $N_A - W_A + \mu_{AB} N \sin\theta - N\cos\theta = 0 \qquad (2)$



B: $\vec{F}_{RB} = m_B \vec{a}_B$ $\qquad \boxed{a_{B,x} = 0}$

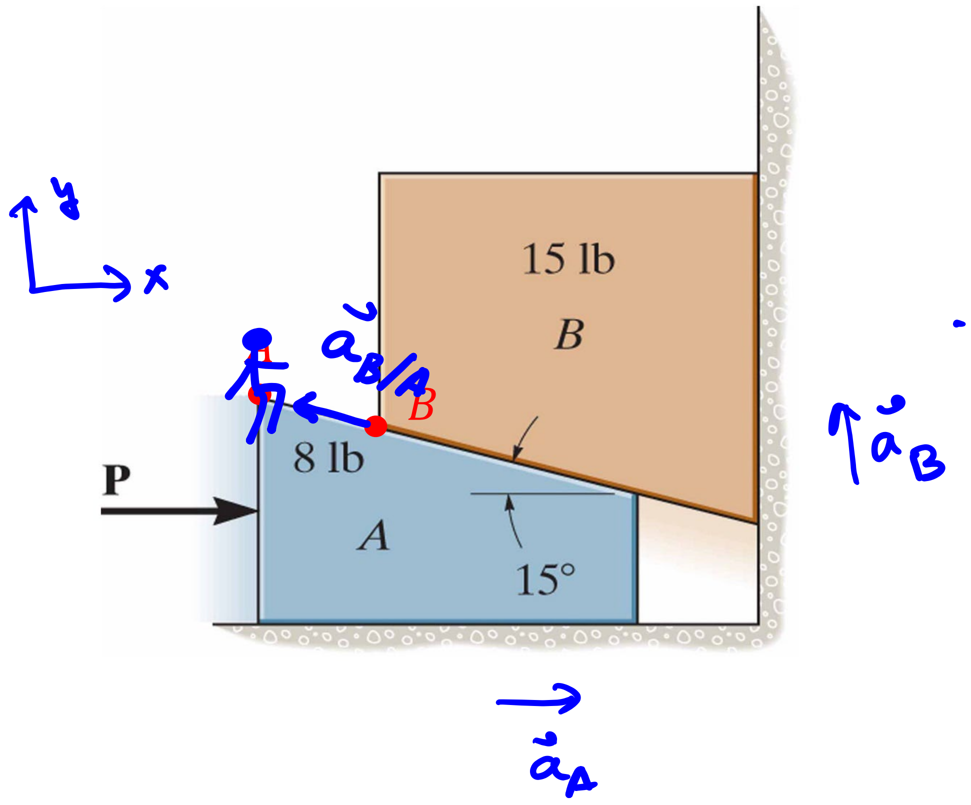
x: $N\sin\theta + \mu_{AB} N\cos\theta - N_B = 0 \qquad (3)$

y: $N\cos\theta - \mu_{AB} N\sin\theta - \mu_B N_B - W_B = m_B a_B \qquad (4)$

4 eqs, 5 unknowns!

W9-2. $P = 20$ lb, $W_A = 8$ lb, $W_B = 15$ lb, θ=15°, $\mu_A = 0.1$, $\mu_{AB} = 0.2$, $\mu_B = 0.3$. $\vec{a}_{B/A} = ?$

$a_A, a_B, N_A, N_B, N = ?$



- Assume you are sitting at point A of block A and look at the point B of block B that is moving towards you. What is $\vec{a}_{B/A}$?

$$\vec{a}_{B/A} = -\vec{i}\, \underline{a_{B/A}} \cos\theta + \vec{j}\, \underline{a_{B/A}} \sin\theta$$

$$= \vec{i}\,(-a_{B/A}\cos\theta) + \vec{j}\,(a_{B/A}\sin\theta)$$

- How $a_{B/A}$ is connected to a_A and a_B?

A. $\vec{a}_{B/A} = \vec{a}_A + \vec{a}_B$

B. $\vec{a}_{B/A} = \vec{a}_A - \vec{a}_B$

C. $\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$ $= (\vec{j}\, a_B) - (\vec{i}\, a_A) =$

D. Help!

$$= \vec{i}\,(-a_A) + \vec{j}\,(a_B)$$

$a_{B/A} \cos\theta = a_A$ → $a_{B/A} = \dfrac{a_A}{\cos\theta}$

$a_{B/A} \sin\theta = a_B$ → $a_B = \dfrac{a_A}{\cos\theta} \sin\theta = \boxed{a_A \tan\theta = a_B}$

W9-2. $P = 20$ lb, $W_A = 8$ lb, $W_B = 15$ lb, θ=15°, $\mu_A = 0.1$, $\mu_{AB} = 0.2$, $\mu_B = 0.3$.

$a_A, a_B, N_A, N_B, N = ?$

$$P - \mu_{AB} N \cos\theta - N \sin\theta - \mu_A N_A = m_A a_A;$$

$$N_A + \mu_{AB} N \sin\theta - N \cos\theta - m_A g = 0;$$

$$\mu_{AB} N \cos\theta + N \sin\theta - N_B = 0;$$

$$N \cos\theta - \mu_B N_B - m_B g - \mu_{AB} N \sin\theta = m_B a_B;$$

$$a_B = a_A \tan\theta$$

Solver: $a_A = 26.0\,\frac{ft}{s^2}$; $a_B = 6.97\,\frac{ft}{s^2}$; $N_A = 26.0\ lb$; $N_B = 10.6\ lb$; $N = 23.4\ lb$

Let’s summarize:

- Starting point: Newton’s 2nd law

$$\begin{cases} \vec{F}_{R,A} = m_A \vec{a}_A \\ \vec{F}_{R,B} = m_B \vec{a}_B \end{cases} \Rightarrow$$

Project, get 4 equations in Cartesian components:

$A,x: \quad P - \mu_{AB} N \cos\theta - N \sin\theta - \mu_A N_A = m_A a_A$

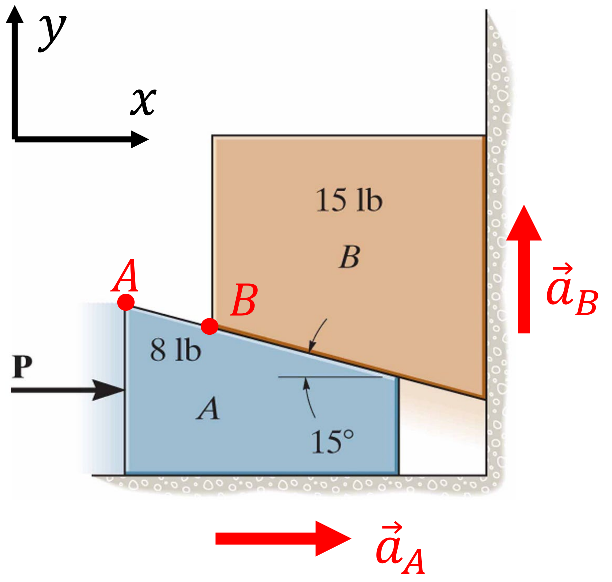
$A,y: \quad N_A + \mu_{AB} N \sin\theta - N\cos\theta - m_A g = 0 = m_A \underline{a_{A,y}}$

$B,x: \quad \mu_{AB} N \cos\theta + N \sin\theta - N_B = 0 = \underline{m_B a_{B,x}}$

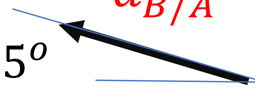
$B,y: \quad N\cos\theta - \mu_B N_B - m_B g - \mu_{AB} N \sin\theta = m_B \underline{\underline{a_B}} = m_B a_{B,y}$

4 eqs, 5 unknowns

- Two accelerations are not independent!
- Relative motion to the rescue:

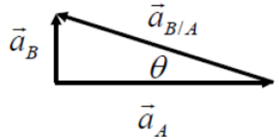


- Assume you are sitting in point A of block A and look at the point B of block B that is mowing towards you. What is $\vec{a}_{B/A}$?



$\Rightarrow \quad a_{B/A\_x} = -a_{B/A}\cos\theta \qquad a_{B/A\_y} = a_{B/A}\sin\theta$

- From Monday’s lecture: $\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$



$\Rightarrow \quad a_{B/A\_x} = a_{B,x} - a_{A,x} = -a_A \qquad a_{B/A\_y} = a_{B,y} - a_{A,y} = a_B$

- We finally get: $a_A = a_{B/A}\cos\theta, \quad a_B = a_{B/A}\sin\theta$