Lecture 19.

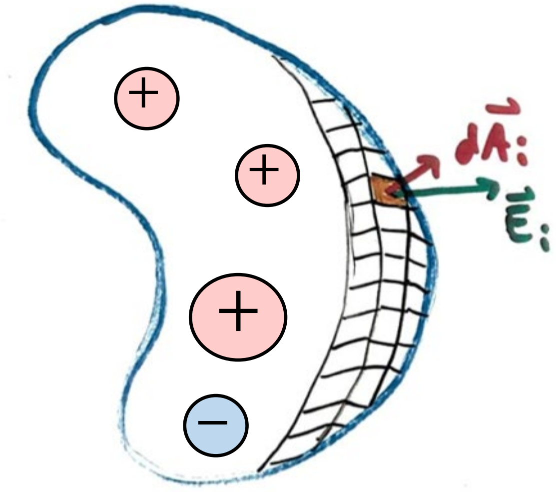
Gauss’s law in conductors.

Gauss's law

➢ Net electric flux through a <u>closed surface</u> = charge inside that surface/ε_0

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0}$$

Gauss's law relates <u>electric field</u> with <u>charges</u> that create this field

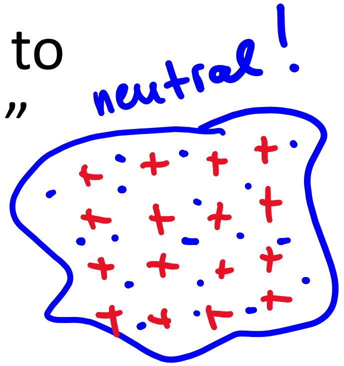


➢ Applications:

- Electric field created by highly symmetric charge distributions (Week 6)
- Electric field and electric charges in conductors (today)

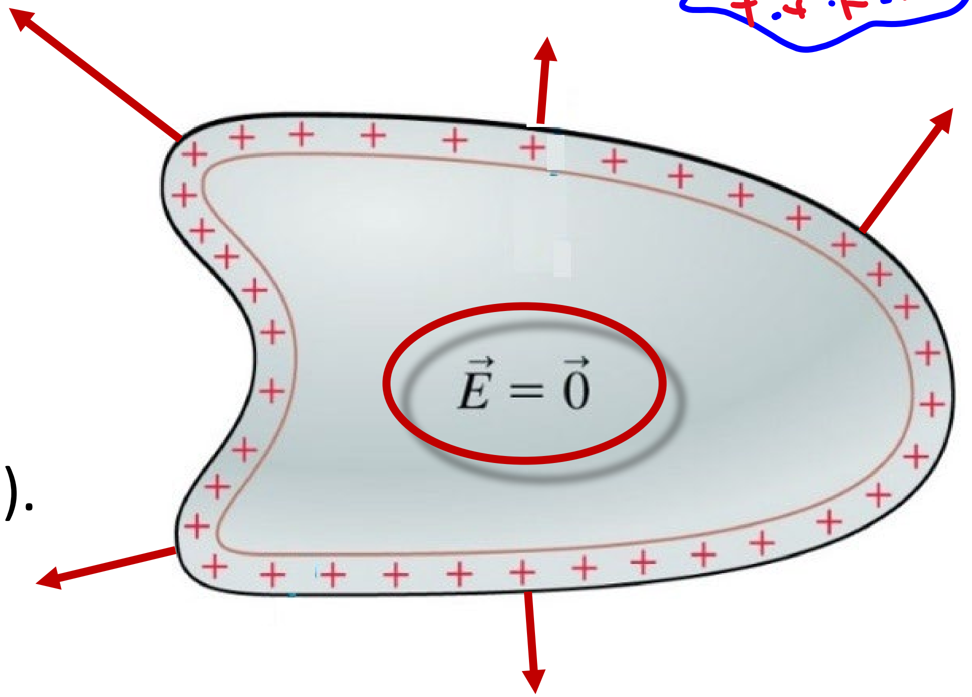
Conductors in Electrostatic Equilibrium

- A conductor is a material in which the negative charges (electrons) are free to move around, while positively charged ions form its rigid immobile "skeleton"



- Electrostatic equilibrium = charges do not move (= no currents)

- Electric properties of conductors in equilibrium:



1. There is no net charge inside a conductor
   (excess charge can only sit on its surface).

2. There is no electric field inside a conductor ($E_{in} = 0$).

3. E-field on the surface of a conductor is perpendicular to it.

Conductors in Electrostatic Equilibrium

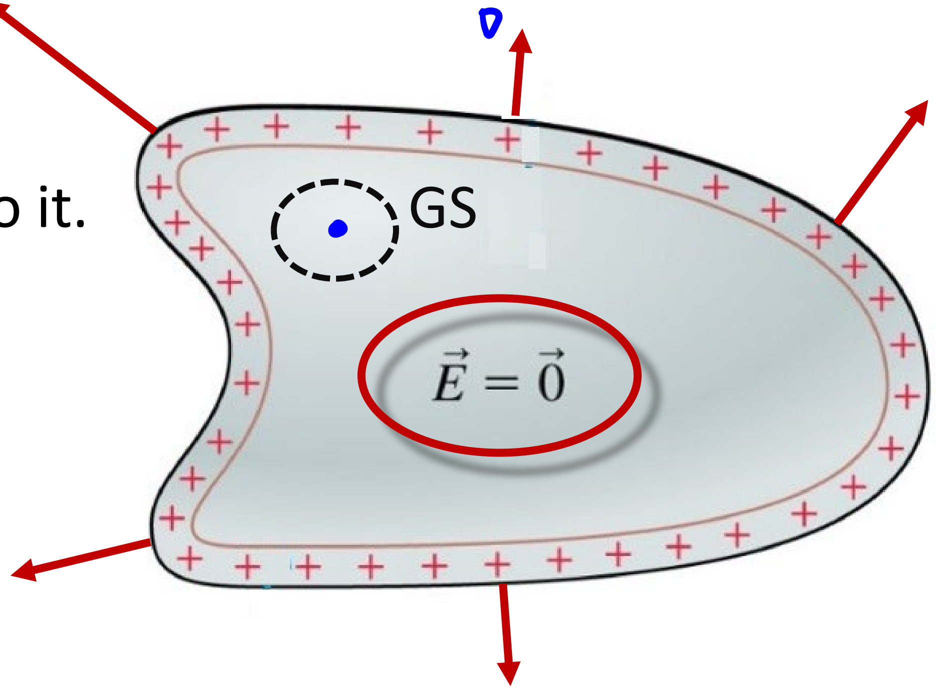
2. There is no electric field inside a conductor ($E_{in} = 0$):

- Equilibrium = Electrons do not move = there is no force on them
- There is no force on them & $\vec{F} = q\vec{E}$ $\Rightarrow$ $\vec{E} \equiv 0$

$\oint \vec{E} \cdot d\vec{A} = 0$

3. E-field on the surface of a conductor is perpendicular to it.

- Tangential component of E-filed would mean surface current => only normal component is possible



1. There is no <u>net</u> charge inside a conductor
(excess charge can only sit on its surface).

- $\vec{E} \equiv 0$ => $\Phi_e = 0$ for any GS => $Q_{in} = 0$ for any GS inside the conductor

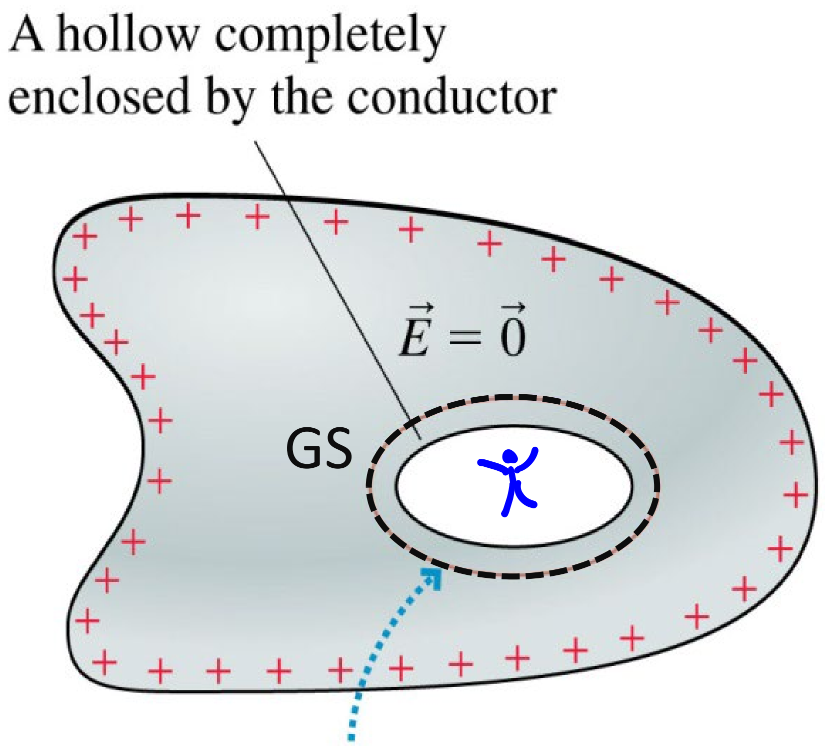
Screening

- Can excess charges reside on the interior surface of a charged conductor?

(! here we assume that there is no charge placed inside the hole !)

No.

- Field is zero everywhere inside the conductor
- Total flux through the Gaussian surface, which surrounds the cavity, is zero
- No charge can sit on the interior surface

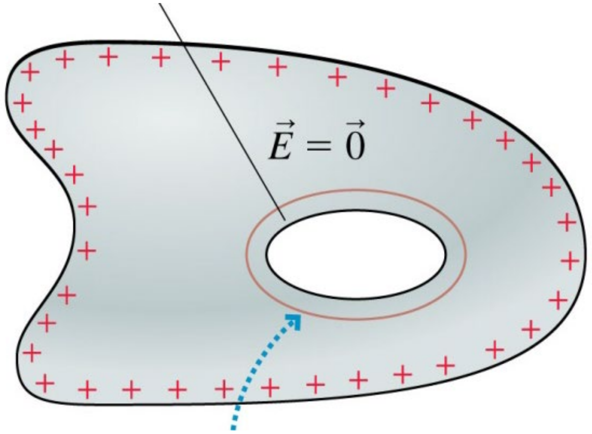


The flux through the Gaussian surface is zero. There's no net charge inside, hence no charge on this interior surface.

This property has important applications

Screening

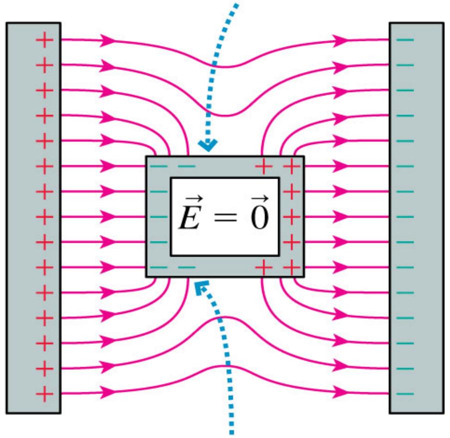
- Due to "$E = 0$ inside a conductor" law, any conducting shell "insulates" an empty cavity inside it from any outside electricity
  - ➢ This phenomenon is called "screening"



- Faraday cage

Demo!

The conducting box has been polarized and has induced surface charges.

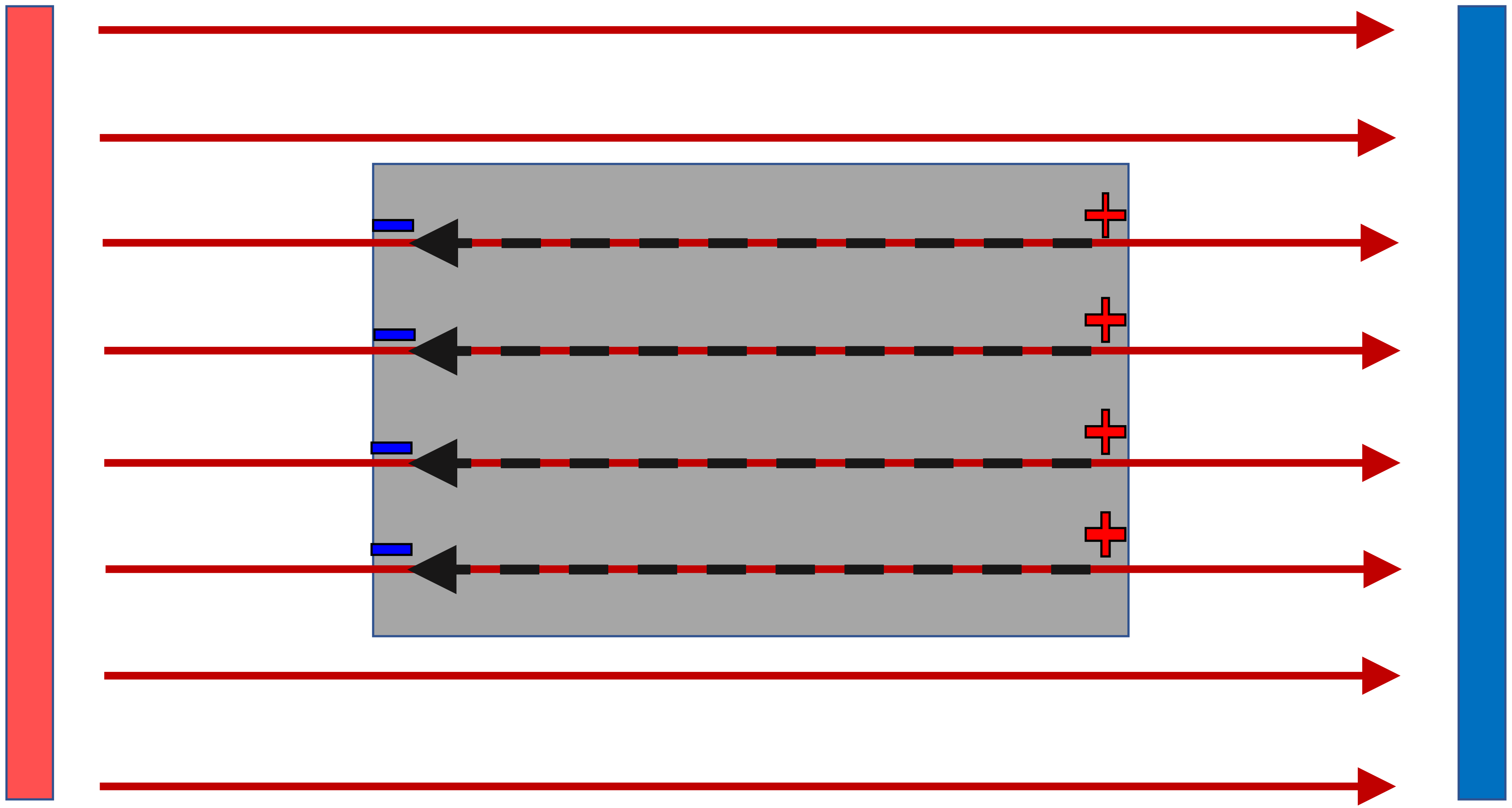


The electric field is perpendicular to all conducting surfaces.

- Another example of a Faraday's cage: a car.

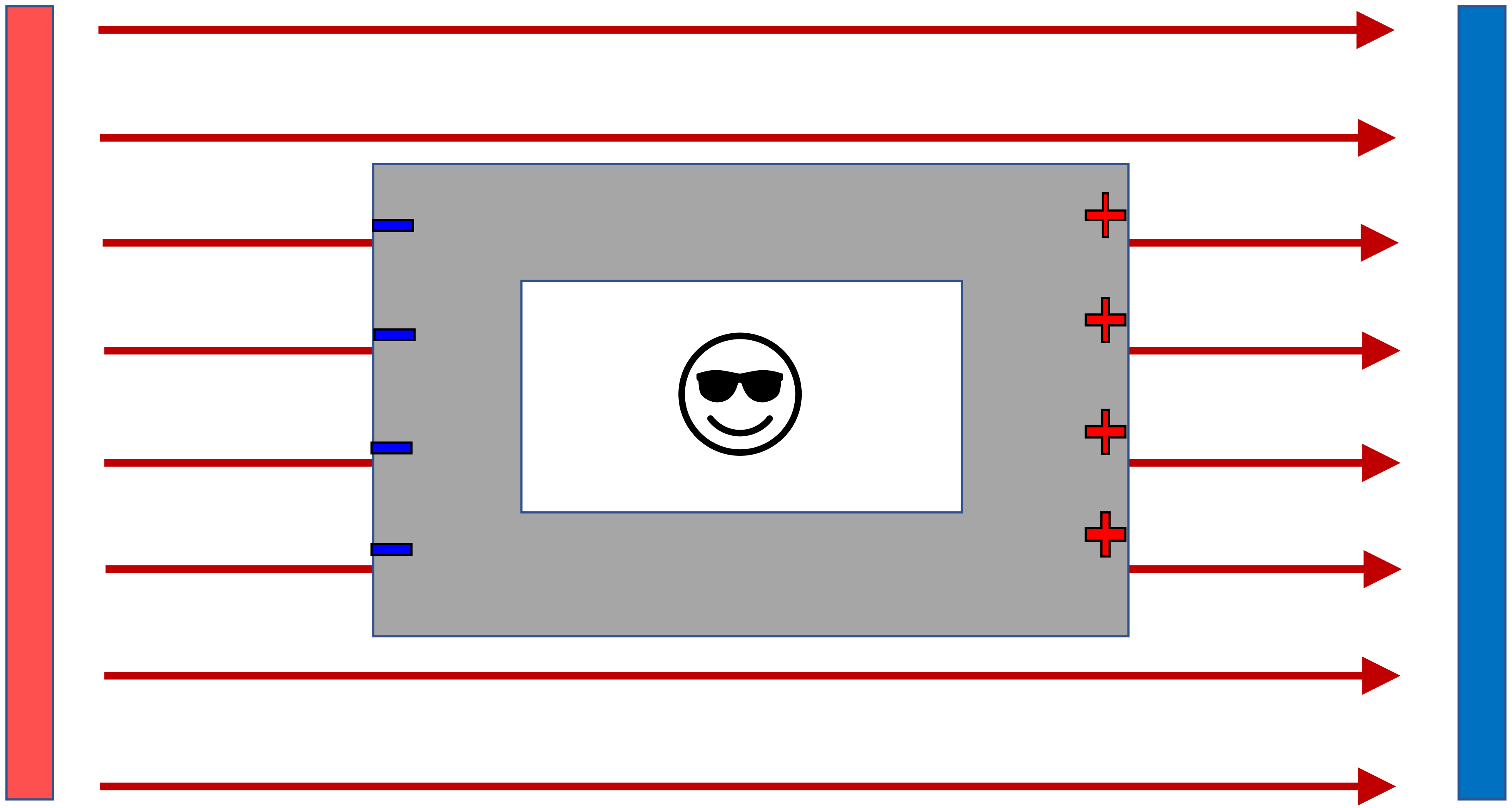
Video on screening:
https://www.youtube.com/watch?v=MnZD5Fi0VJM

Faraday’s cage



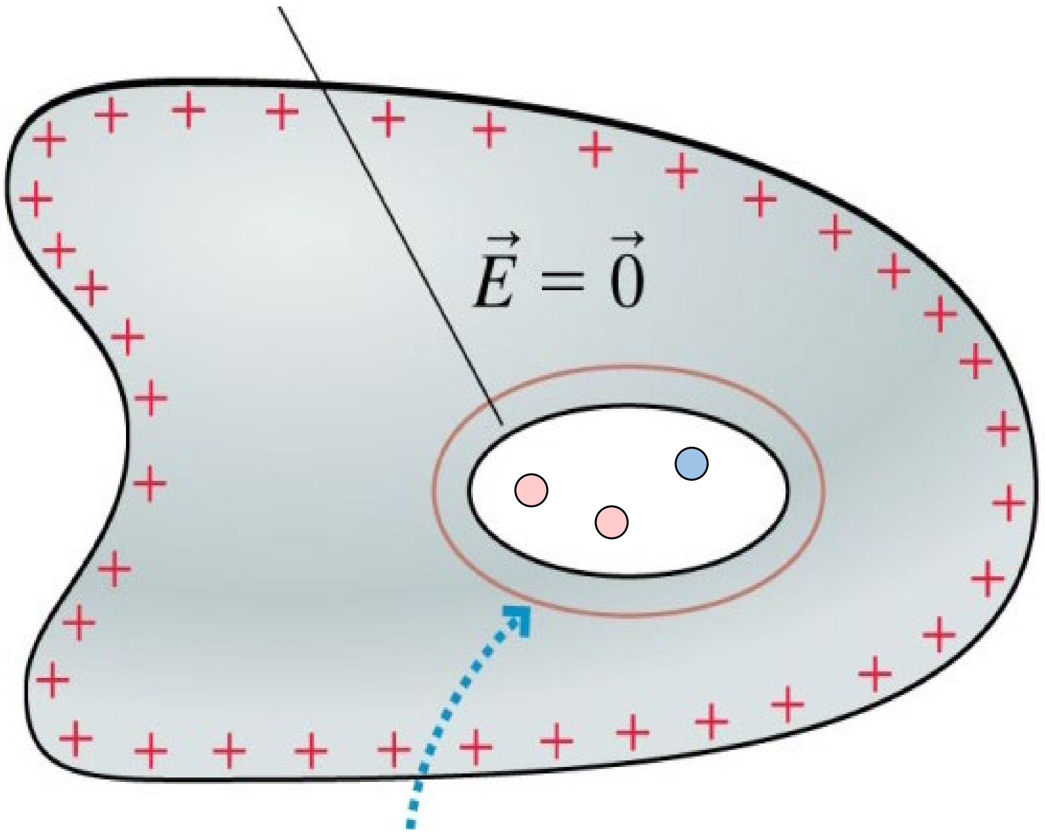
Faraday's cage

Polarized!



Yet another scenario: what if a charge is placed in the cavity surrounded by a conducting shell?

- Due to "$E = 0$ inside a conductor" law, any conducting shell "insulates" an empty cavity inside it from any outside electricity
  - ➢ This phenomenon is called "screening"



- But what if the cavity contains charges?

Q: Consider a neutral spherical metal shell of inner radius R_1. A point charge $Q = -5.0\ mC$ is located at its center.

Do we expect to have charges on the inner and outer surfaces of this shell?

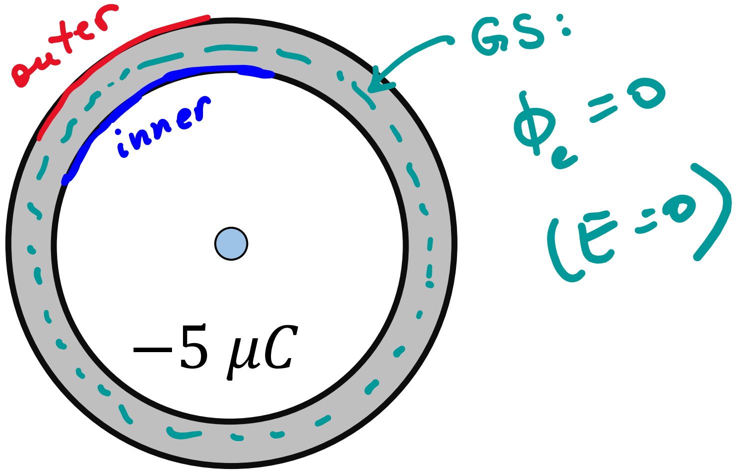
Inner surface:

A. Zero

B. +5.0 μC → $\phi_{net} = 0$!

C. −5.0 μC

D. Something else



Outer surface:

A. Zero

B. +5.0 μC

C. −5.0 μC → neutral !

D. Something else

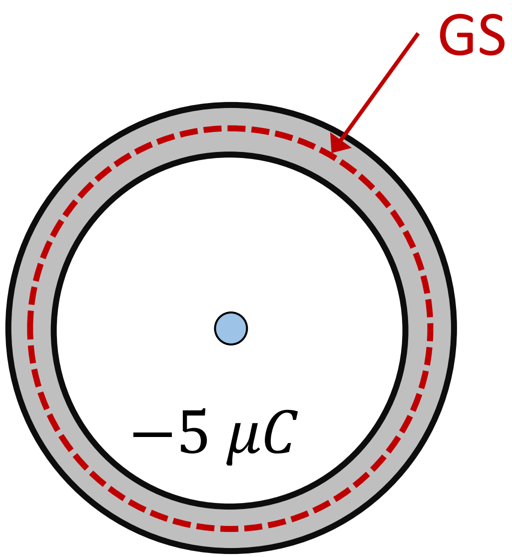
Q: Consider a neutral spherical metal shell of inner radius R_1. A point charge $Q = -5.0\,mC$ is located at its center.

Do we expect to have charges on the inner and outer surfaces of this shell?

Inner surface:

A. Zero
B. +5.0 μC
C. −5.0 μC
D. Something else

- Consider the GS shown in the figure. We know that $E = 0$ everywhere inside the conductor =>
- $E = 0$ everywhere on this GS =>
- $\Phi_e = \oint_{GS} \vec{E} \cdot d\vec{A} = 0$ =>
- This GS must enclose zero net charge.



Outer surface:

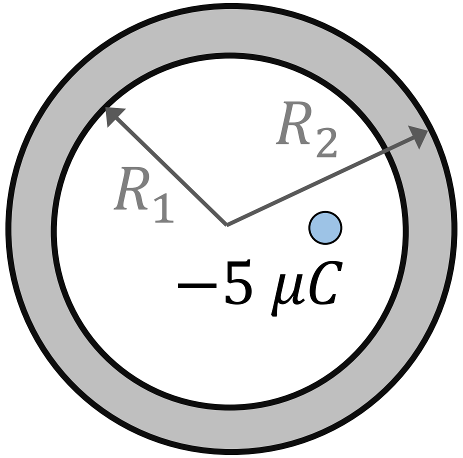
A. Zero
B. +5.0 μC
C. −5.0 μC
D. Something else

- The shell was initially neutral => if +5 uC goes to its inner surface, then -5 uC should go to its outer surface (remember that no charge can reside inside the conductor, it only can sit on its surfaces)

Q: Consider a neutral spherical metal shell of inner radius R_1. A point charge $Q = -5.0\ mC$ is located at $r = R_1/2$.

a) Calculate the induced charges on the inner and outer surfaces of the shell.

b) Sketch the E-field lines inside and outside the metal shell.

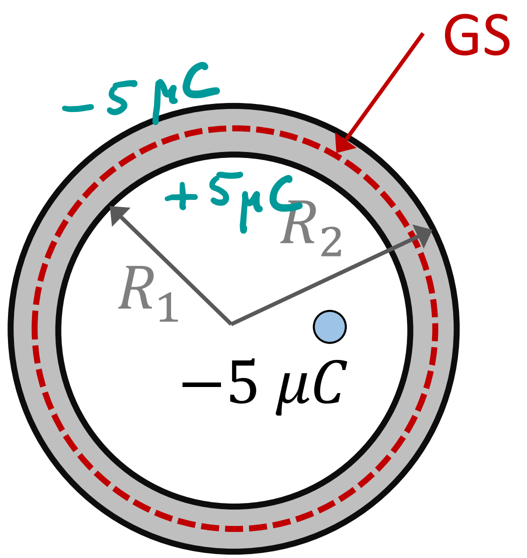


Q: Consider a neutral spherical metal shell of inner radius R_1. A point charge $Q = -5.0\ mC$ is located at $r = R_1/2$.

a) Calculate the induced charges on the inner and outer surfaces of the shell.

b) Sketch the E-field lines inside and outside the metal shell.

- Here the logic is the same as with the charge sitting in the center:
  - $E = 0$ inside the conductor under static equilibrium =>
  - Flux through this GS is zero =>
  - Enclosed charge is zero =>
  - $Q_{inner} = +5\ \mu C;$
  - The shell is neutral =>
  - $Q_{outer} = -5\ \mu C$ (charge conservation)

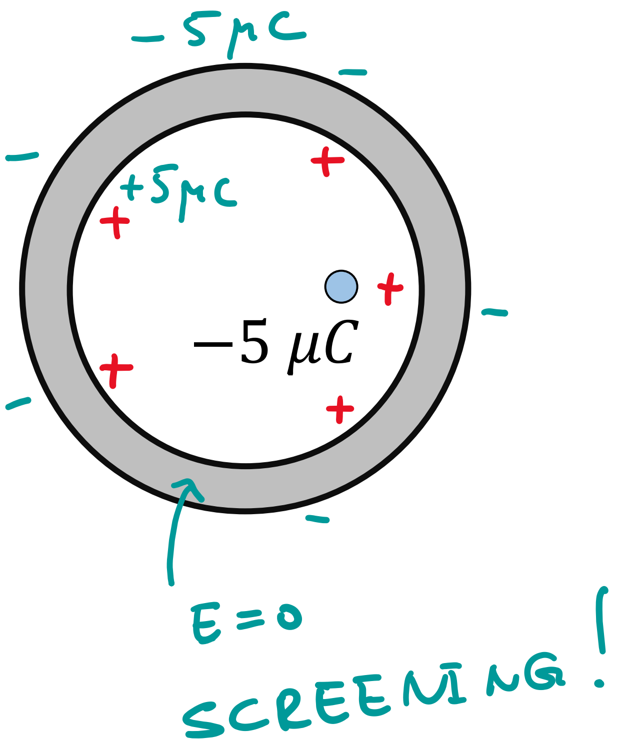


- How is the charge distributed over the surfaces, uniformly or non-uniformly? (This is the key to figuring out E-filed)

Q: Consider a neutral spherical metal shell of inner radius R_1. A point charge $Q = -5.0\ mC$ is located at $r = R_1/2$.

a) Calculate the induced charges on the inner and outer surfaces of the shell.

b) Sketch the E-field lines inside and outside the metal shell.



Charge distribution is:

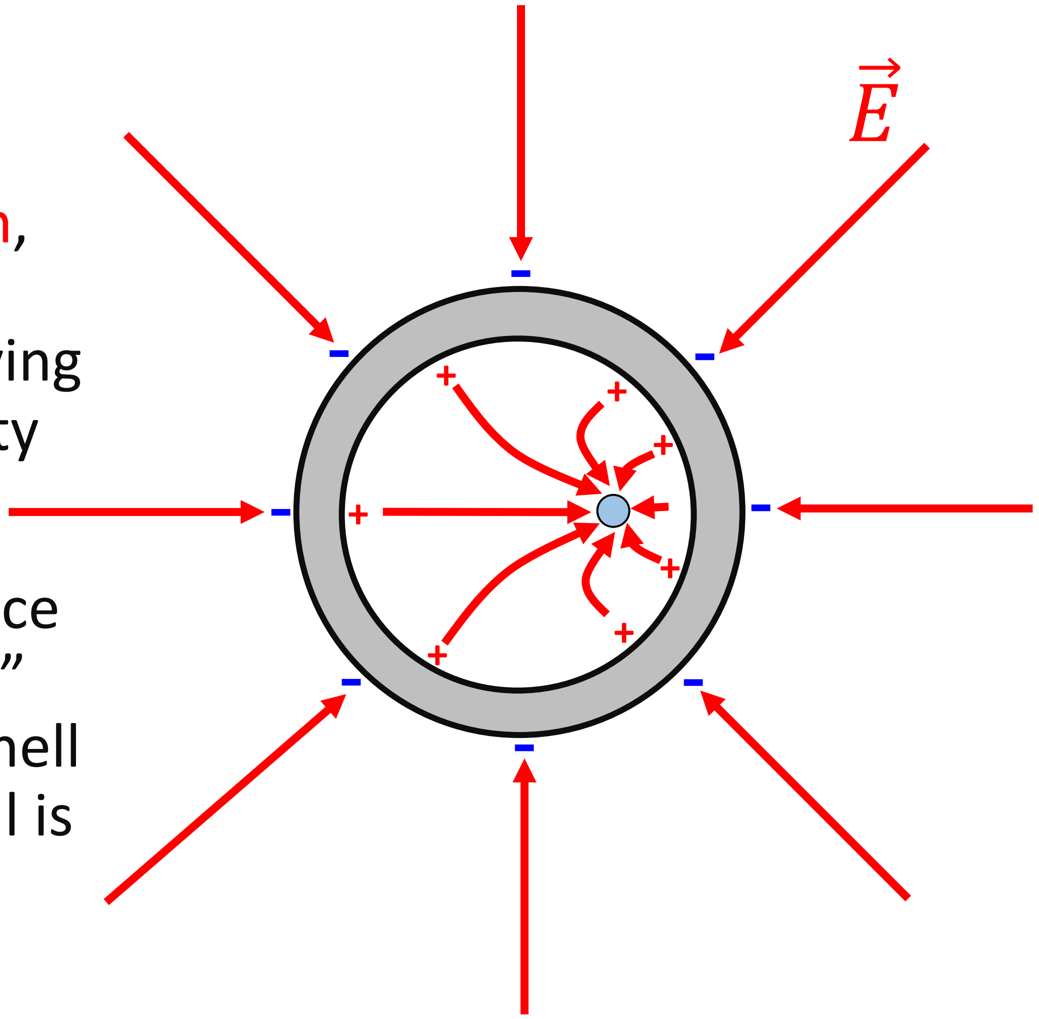
A. Uniform on the inner shell, uniform on the outer shell
B. Uniform on the inner shell, non-uniform on the outer shell
C. Non-uniform on the inner shell, uniform on the outer shell
D. Non-uniform on the inner shell, non-uniform on the outer shell
E. I really don't know!!!

Q: Consider a neutral spherical metal shell of inner radius R_1. A point charge $Q = -5.0\, mC$ is located at $r = R_1/2$.

a) Calculate the induced charges on the inner and outer surfaces of the shell.

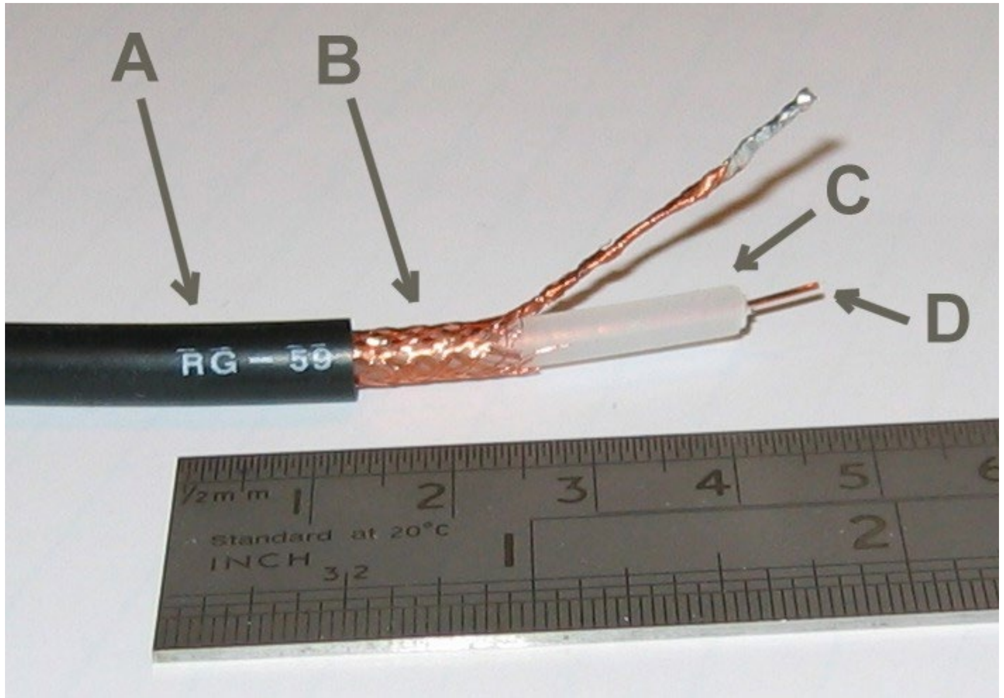
b) Sketch the E-field lines inside and outside the metal shell.

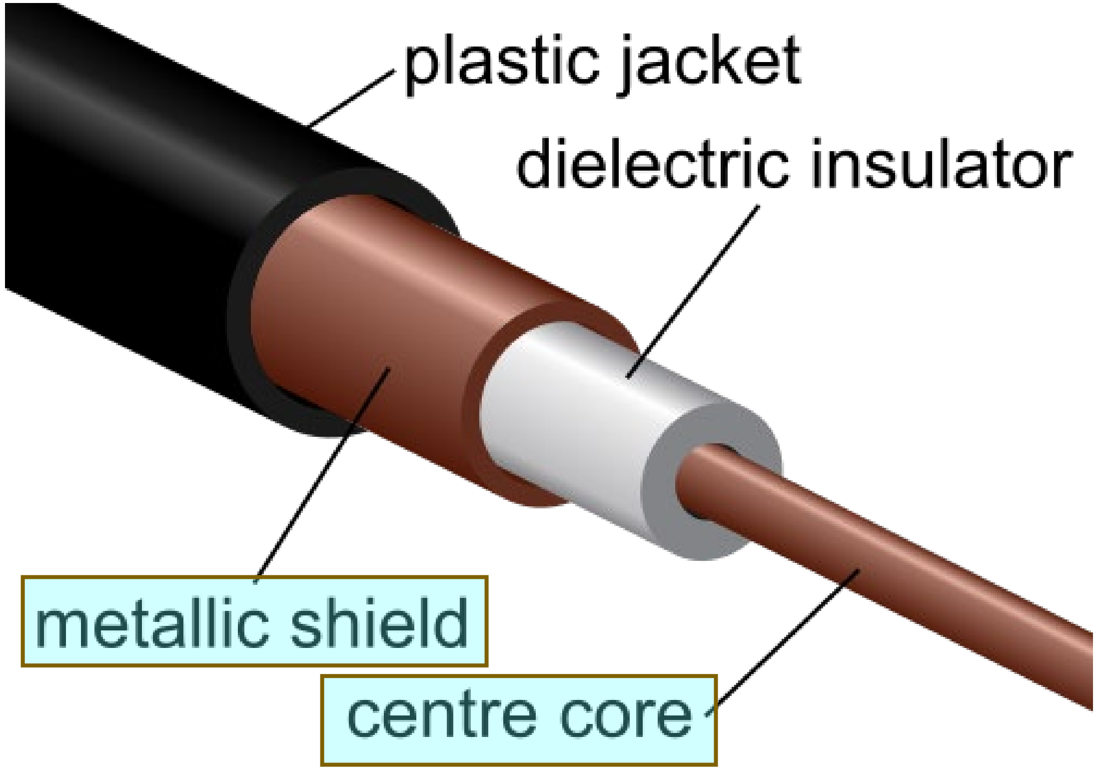
- $Q_{inner} = +5\ \mu C,\ \ Q_{outer} = -5\ \mu C$



- Charge distribution on the inner surface is non-uniform, since closer to Q electrons feel more repulsion and are pushed further away along the inner shell surface, leaving more + closer to Q. Hence, the electric field in the cavity is asymmetric (note field lines ⊥ to the surface)
- Charge distribution on the outer surface is uniform, since the charges sitting on the outer surface "know nothing" about what's going on in the cavity: $E = 0$ inside the shell => screening! Hence, the electric field outside the shell is symmetric, and identical to the field that a charge Q sitting in the center of the cavity would produce.

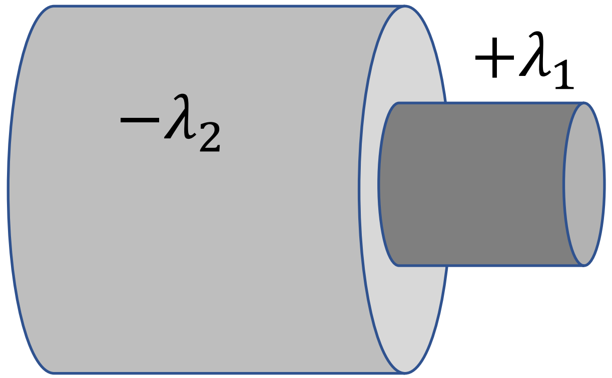
Coaxial Cable

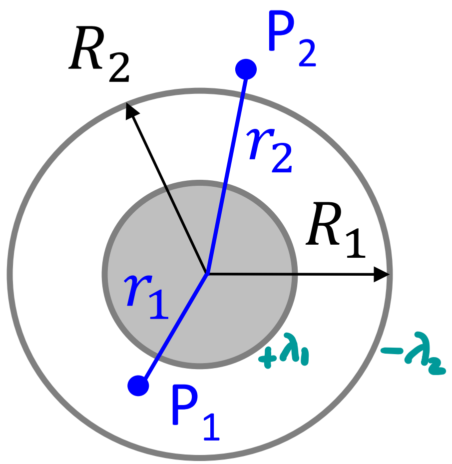




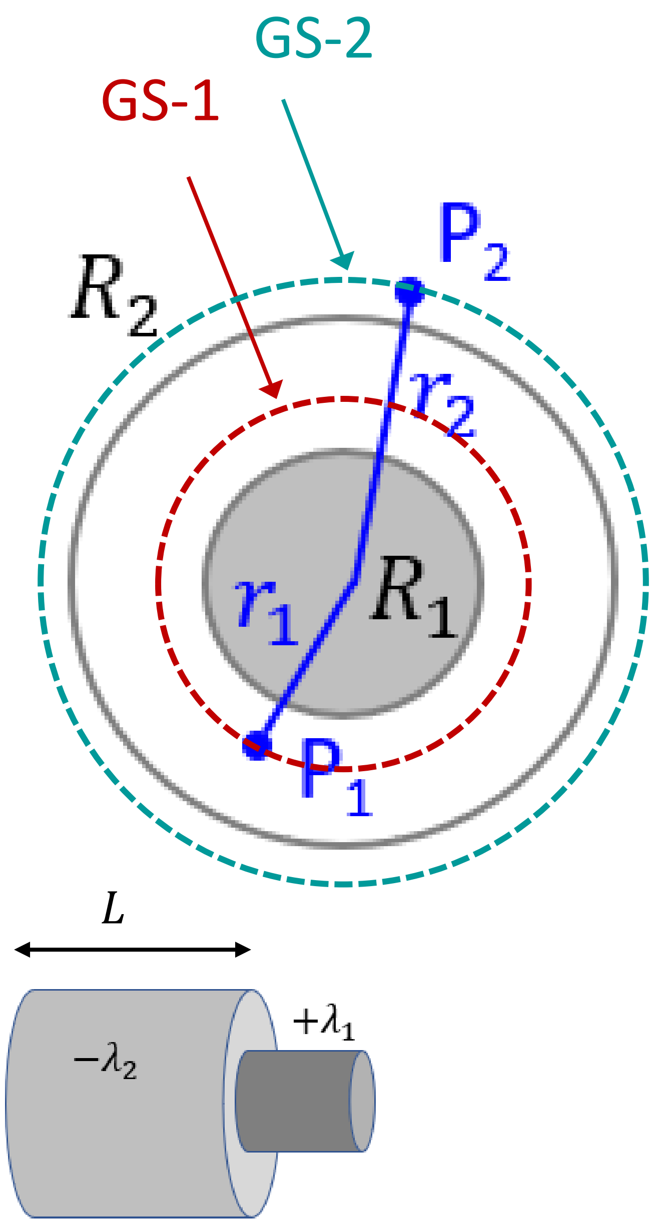
Coaxial Cable

The linear charge density on the centre core of a coaxial cable is $+\lambda_1$, and it is $-\lambda_2$ on its metallic shield. Ignore the dielectric insulator and the plastic jacket. Calculate electric field outside the thin metallic shield (radius R_2) and just outside the central core of radius R_1.





Coaxial Cable



1) Electric field between two conductors:

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = E(r_1)A_{\text{side}} = E(r_1) \cdot L \cdot 2\pi r_1$$

$$Q_{in} = \lambda_1 L$$

$$E(r_1) \cdot \cancel{L} \cdot 2\pi r_1 = \frac{\lambda_1 \cancel{L}}{\varepsilon_0}$$

$$\boxed{E(R_1 < r < R_2) = \frac{\lambda_1}{2\pi\varepsilon_0 \cdot r}}$$

2) Electric field outside the thin metal shied:

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = E(r_2)A_{\text{side}} = E(r_2) \cdot L \cdot 2\pi r_2$$

$$Q_{in} = (\lambda_1 - \lambda_2)L$$

$$E(r_2) \cdot \cancel{L} \cdot 2\pi r_2 = \frac{(\lambda_1 - \lambda_2)\cancel{L}}{\varepsilon_0}$$

$$\boxed{E(r > R_2) = \frac{(\lambda_1 - \lambda_2)}{2\pi\varepsilon_0 \cdot r} = 0 \text{ if } \lambda_1 = \lambda_2}$$

Summary: Properties of Conductors in Electrostatic Equilibrium (= no currents)

- The electric field is always zero inside a conductor.
- As a result of that (Gauss's law) all excess charge is distributed over the conductor's surface(s). Inside, a conductor is neutral (i.e. no NET charge).
- Inner surfaces of a conductor carry no charge (screening), unless there is a charge "sitting" inside the cavity (pinned somewhere NOT on the conductor's surface). Such an external charge will polarize the conductor and attract equal but opposite charge to its inner surface.
- The electric field outside a conductor is always perpendicular to the surface at each point.
- The surface charge density, and hence the density of E-field lines, is greatest where the radius of curvature of the surface is smallest (we did not discuss it here, since the proof from considering electric potential – a new concept that follows this week).