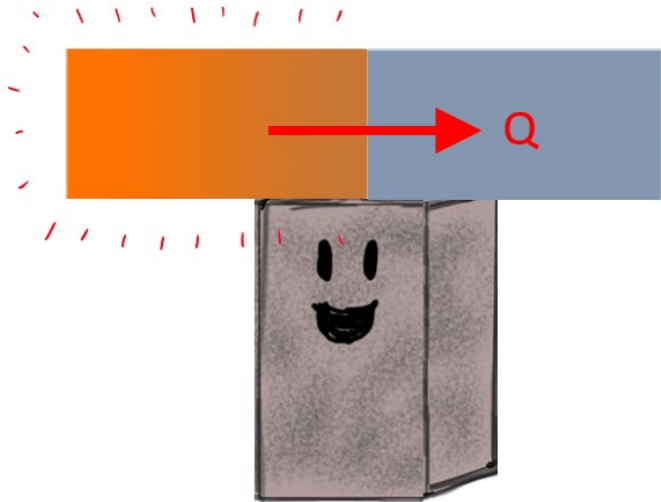
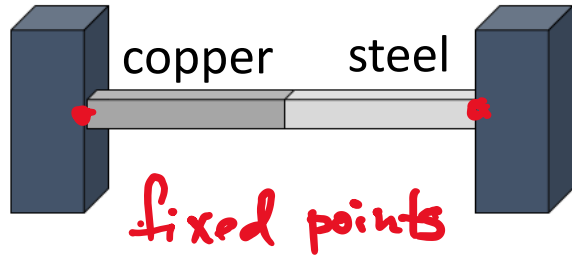


Lecture 8. Heat



Last Time:



$$T: 20^{\circ}\text{C} \Rightarrow 40^{\circ}\text{C}$$

$$F_{\text{from wall}} = ?$$

$$\Delta L_1 + \Delta L_2 = 0$$

Step 3: Equations

- For each part, write an equation relating the change in length to the changes in temperature and forces

Before: $\frac{T}{L}$ $F = 0$

After: $\frac{T + \Delta T}{L + \Delta L_1}$

F \rightarrow \leftarrow F

Before: $\frac{T}{L}$ $F = 0$

After: $\frac{T + \Delta T}{L + \Delta L_2}$

F \rightarrow \leftarrow F

One of them is positive, one negative

$$\Delta L_1 = \alpha_1 L \Delta T - \frac{F L}{A Y_1}$$

$$\Delta L_2 = \alpha_2 L \Delta T - \frac{F L}{A Y_2}$$

F is positive, the signs express compression

Step 4

- Collect equations and solve for unknowns

A. $F = 3.4 \times 10^1 N$

B. $F = 6.5 \times 10^2 N$

C. $F = 8.2 \times 10^3 N$

D. $F = 9.1 \times 10^4 N$

E. Something else

Step 4 • Collect equations and solve for unknowns

$$\Delta L_1 + \Delta L_2 = 0 \quad (1)$$

$$\Delta L_1 = \alpha_1 L \Delta T - \frac{F L}{A Y_1} \quad (2)$$

$$\Delta L_2 = \alpha_2 L \Delta T - \frac{F L}{A Y_2} \quad (3)$$

A. $F = 3.4 \times 10^1 N$

B. $F = 6.5 \times 10^2 N$

C. $F = 8.2 \times 10^3 N$

D. $F = 9.1 \times 10^4 N$

E. Something else

• Plug (2) and (3) into (1):

$$\alpha_1 L \Delta T + \alpha_2 L \Delta T - \frac{F L}{A Y_1} - \frac{F L}{A Y_2} = 0$$

• Solve:

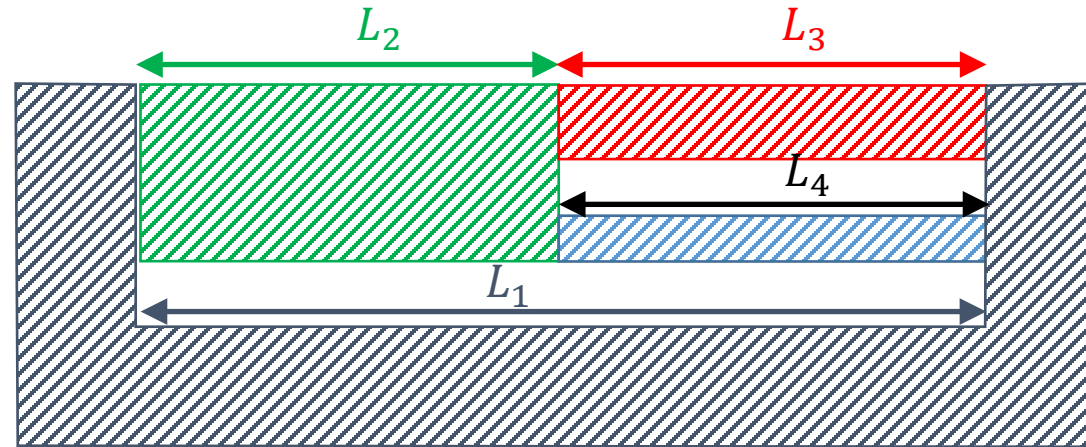
• Isolate terms with F :

$$\frac{FL}{A} \left(\frac{1}{Y_1} + \frac{1}{Y_2} \right) = (\alpha_1 + \alpha_2) L \Delta T$$

$$F = \frac{(\alpha_1 + \alpha_2) A \Delta T}{\left(\frac{1}{Y_1} + \frac{1}{Y_2} \right)} = 8.2 \times 10^3 N$$



If this system is heated, what constraints must be satisfied by the four quantities ΔL_1 , ΔL_2 , ΔL_3 , and ΔL_4 ? (all objects are attached to each other and cannot separate)



A. $\Delta L_1 + \Delta L_2 + \Delta L_3 + \Delta L_4 = 0$

B. ΔL_1 = $\Delta L_2 + \Delta L_3 = \Delta L_2 + \Delta L_4$

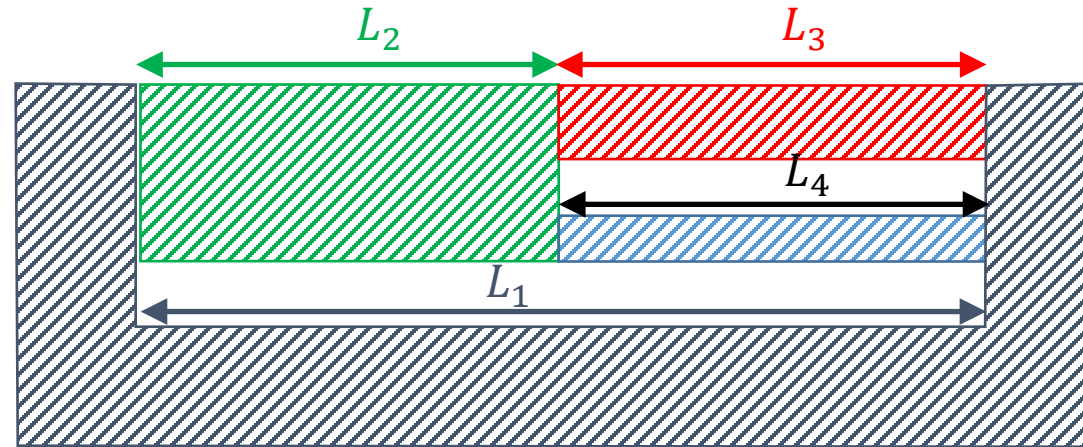
C. $\Delta L_2 + \Delta L_3 = 0 = \Delta L_2 + \Delta L_4$

D. (a) and (b)

E. (b) and (c)



If this system is heated, what constraints must be satisfied by the four quantities ΔL_1 , ΔL_2 , ΔL_3 , and ΔL_4 ? (all objects are attached to each other and cannot separate)



A. $\Delta L_1 + \Delta L_2 + \Delta L_3 + \Delta L_4 = 0$

B. $\Delta L_1 = \Delta L_2 + \Delta L_3 = \Delta L_2 + \Delta L_4$ ✓

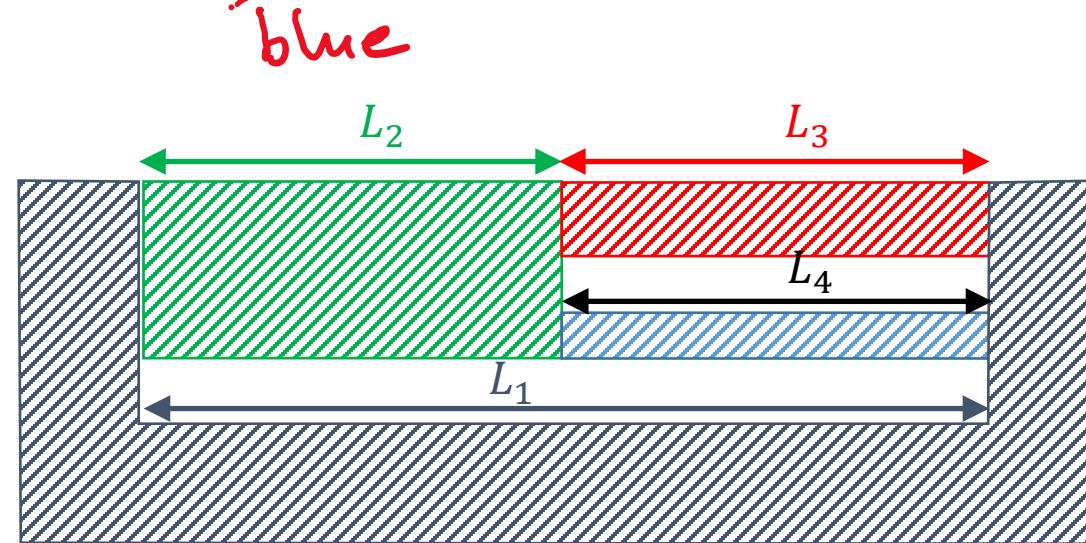
C. $\Delta L_2 + \Delta L_3 = 0 = \Delta L_2 + \Delta L_4$

D. (a) and (b)

E. (b) and (c)



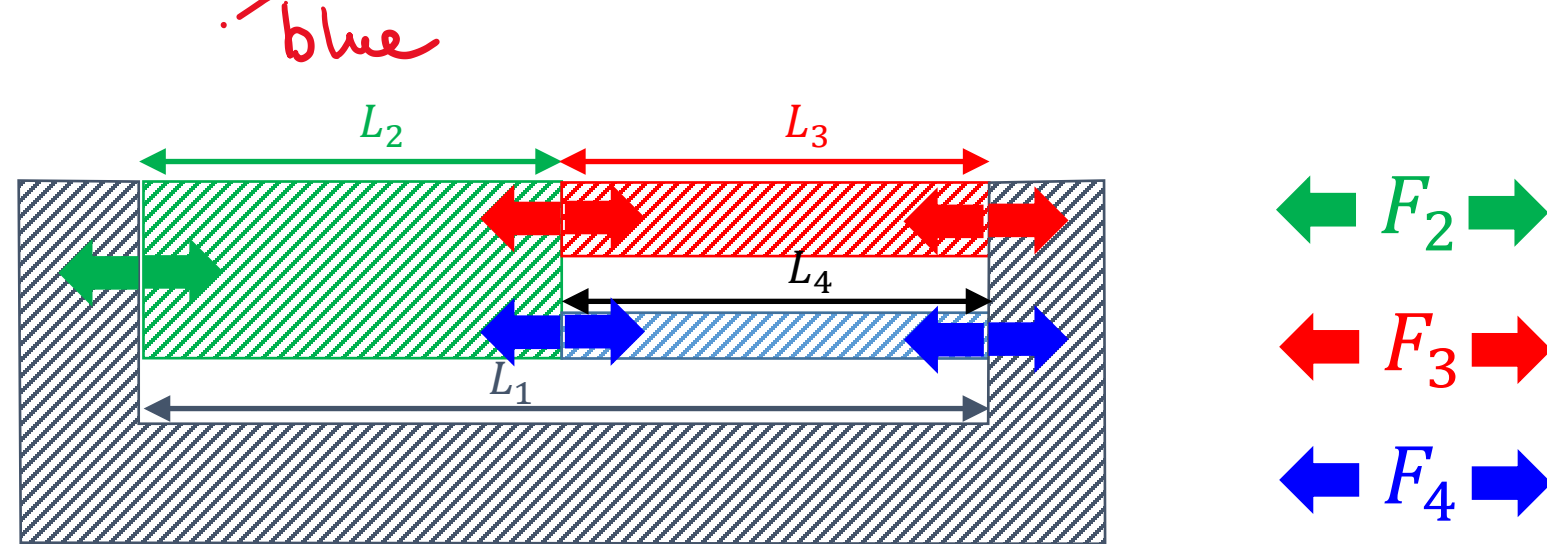
If this system is heated, what relations do we have between the compressive forces F_2 , F_3 and F_4 on the green, red, and ~~black~~ objects?



- A. $F_2 + F_3 + F_4 = 0$
- B. $F_2 = F_3 = F_4$
- C. $F_2 = F_3 + F_4$
- D. $F_3 = F_4$
- E. Help!



If this system is heated, what relations do we have between the compressive forces F_2 , F_3 and F_4 on the green, red, and black objects?



A. $F_2 + F_3 + F_4 = 0$

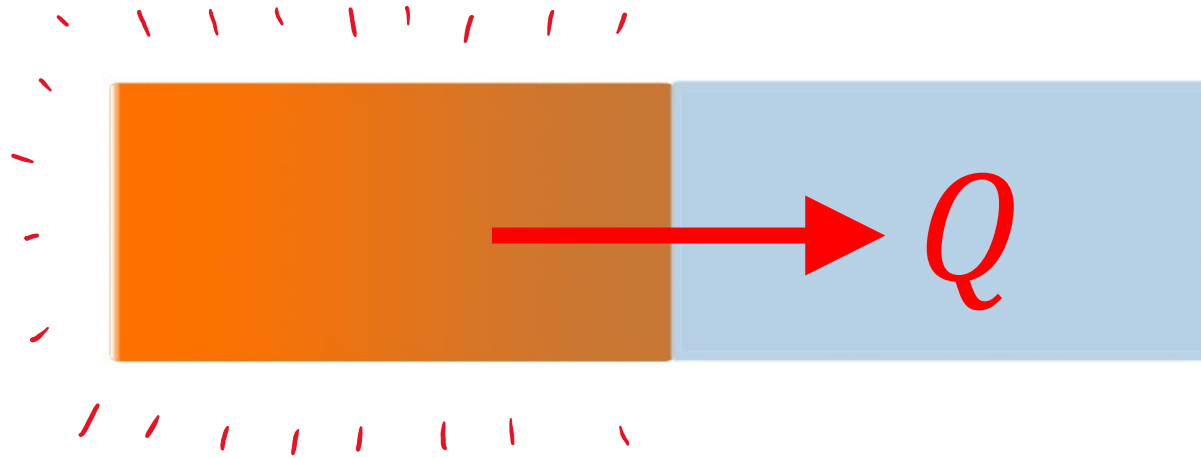
B. $F_2 = F_3 = F_4$

C. $F_2 = F_3 + F_4$ ✓

D. $F_3 = F_4$

E. Help!

Heat



Q = heat: amount of energy transferred due to temperature differences

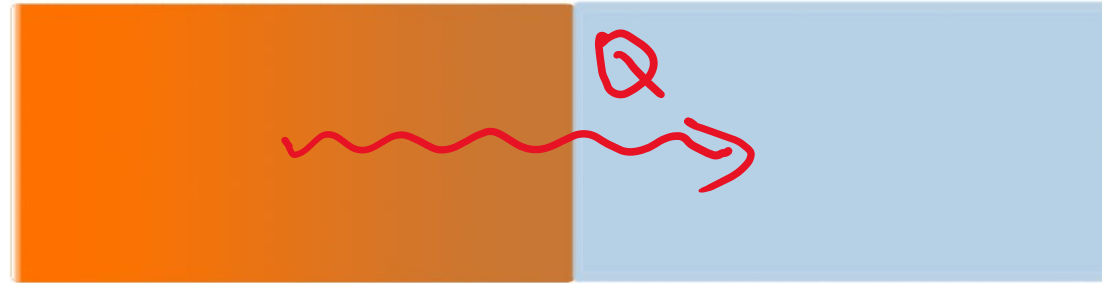
- Q is energy!!! Measured in Joules (J)
- T is temperature (NOT energy); measured in degrees Celsius ($^{\circ}\text{C}$) or kelvin (K)



Two objects with the same mass are put in thermal contact and insulated from their environment. If the initial temperatures are $100\text{ }^{\circ}\text{C}$ and $0\text{ }^{\circ}\text{C}$, the final equilibrium temperature will be

$$T = 100^{\circ}\text{C}$$

$$T = 0^{\circ}\text{C}$$



- A. $50\text{ }^{\circ}\text{C}$
- B. Somewhere between $0\text{ }^{\circ}\text{C}$ and $100\text{ }^{\circ}\text{C}$, but not necessarily $50\text{ }^{\circ}\text{C}$
- C. Not necessarily between $0\text{ }^{\circ}\text{C}$ and $100\text{ }^{\circ}\text{C}$



Two objects with the same mass are put in thermal contact and insulated from their environment. If the initial temperatures are $100\text{ }^{\circ}\text{C}$ and $0\text{ }^{\circ}\text{C}$, the final equilibrium temperature will be

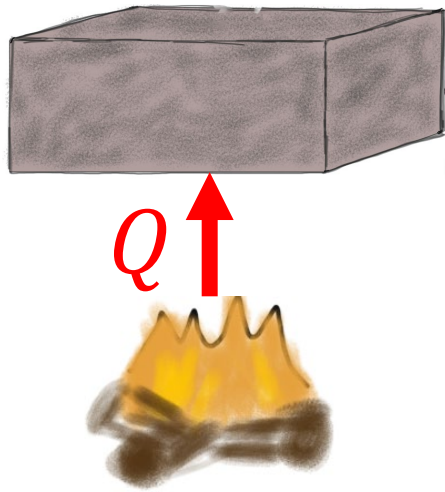


- Heat flows from the hotter object to the cooler object
- Temperature of the hot object decreases and the cold object increases, until they are the same temperature, somewhere between 0°C and 100°C
- Not necessarily 50°C since some materials require more energy for a given temperature change

- A. $50\text{ }^{\circ}\text{C}$
- B. Somewhere between $0\text{ }^{\circ}\text{C}$ and $100\text{ }^{\circ}\text{C}$, but not necessarily $50\text{ }^{\circ}\text{C}$ ✓
- C. Not necessarily between $0\text{ }^{\circ}\text{C}$ and $100\text{ }^{\circ}\text{C}$

Specific heat

- Heat required to raise the temperature of a material of mass m is determined by its **specific heat**, c :



$$\overset{1\text{ kg}}{Q} = m \overset{1\text{ K}}{\underline{\underline{c}}} \Delta T$$

- Q = heat added
- m = mass
- c = **specific heat**
- ΔT = change in temperature

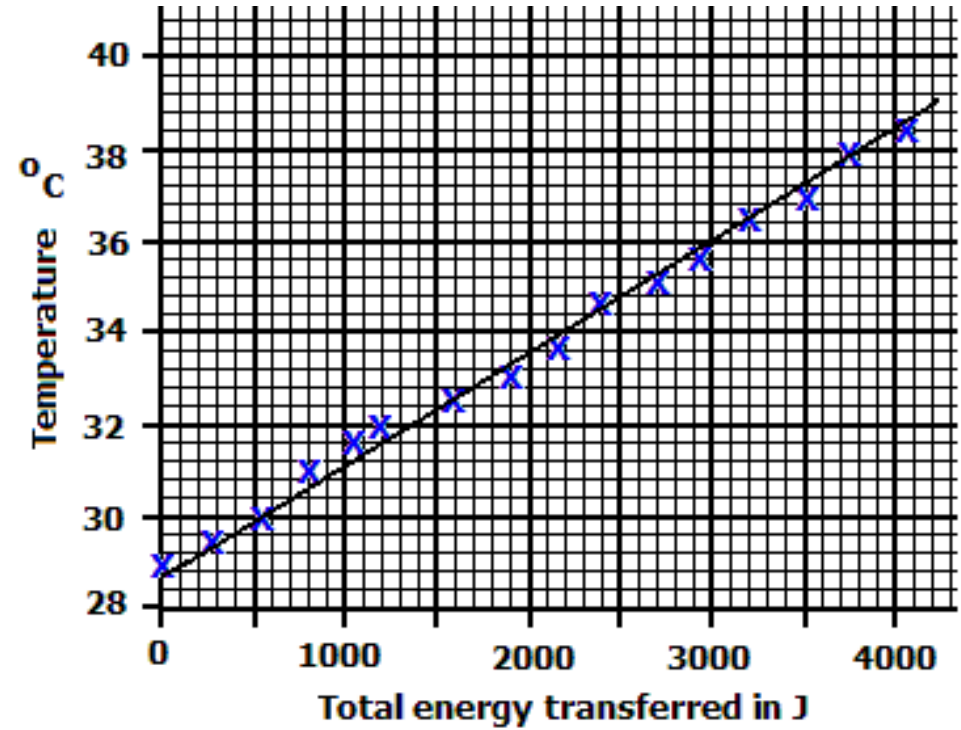
- c is a measure of the energy required to heat 1 kg of material by 1 K (material constant!)
- Units of c are $\frac{\text{J}}{\text{kg}\cdot\text{K}}$



Q: Heat is added to two kilograms of a liquid, and data for the temperature vs energy transfer is shown. If we took data for another liquid with the same mass but a larger specific heat, the slope of this graph would be

- A. Larger
- B. Smaller
- C. The same
- D. Any of the above are possible.

$$Q = m c \Delta T$$



Extra: What is the specific heat of the original liquid?



Q: Heat is added to two kilograms of a liquid, and data for the temperature vs energy transfer is shown. If we took data for another liquid with the same mass but a larger specific heat, the slope of this graph would be

- A. Larger
- B. Smaller ✓
- C. The same
- D. Any of the above are possible.

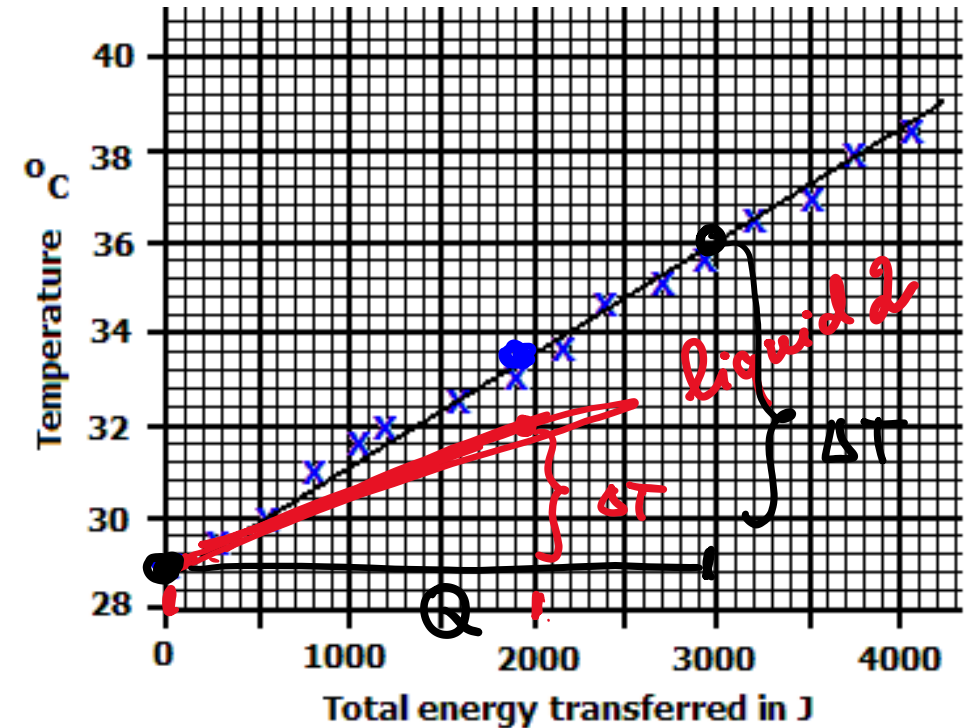
div ↑ ↓

$$Q = m c \Delta T$$

$$c = \frac{Q}{m \Delta T}$$

↘ 2 kg

Smaller ΔT for same Q , so smaller slope (rise/run)



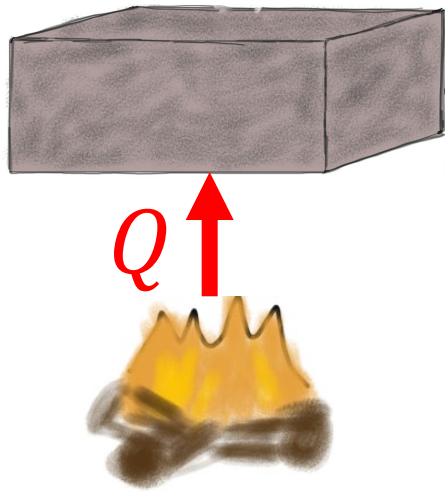
Extra: What is the specific heat of the original liquid?

$$c \approx \frac{3000}{2 \cdot 7} \approx 200 \frac{\text{J}}{\text{kg} \cdot \text{K}}$$

Molar heat capacity

$$1 \text{ mole} = 6 \cdot 10^{23} \text{ particles}$$

- Heat required to raise the temperature of n moles of a substance is determined by its **molar heat capacity**, C :



$Q = m c \Delta T$
or:
 $Q = n C \Delta T$

specific heat

molar heat capacity

- n = # of moles
- C = molar heat capacity (also called 'molar specific heat')

- c in $\frac{\text{J}}{\text{kg} \cdot \text{K}}$: energy required to heat 1 kg of material by 1 K
- C in $\frac{\text{J}}{\text{mol} \cdot \text{K}}$: energy required to heat 1 mole of material by 1 K

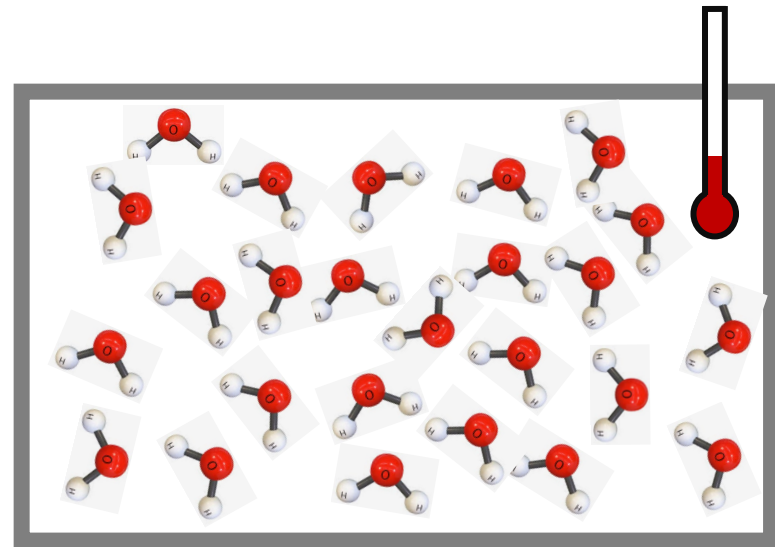
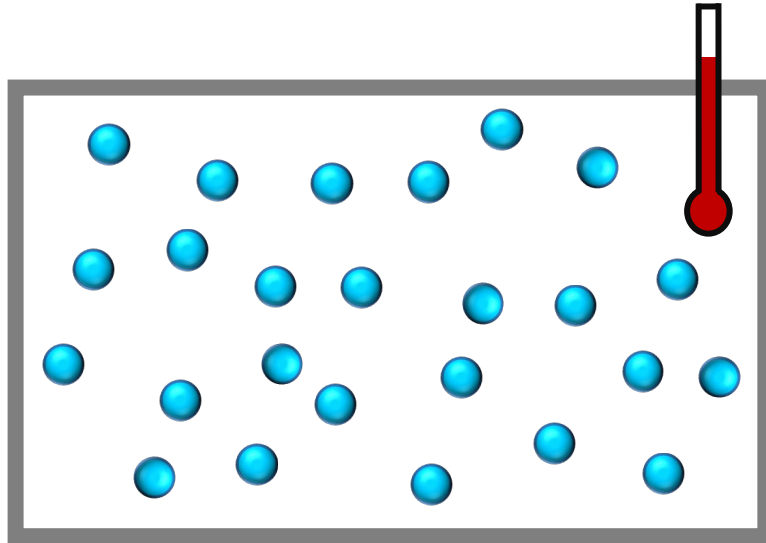
Specific Heat values

Table 17.3 Approximate Specific Heats and Molar Heat Capacities
(Constant Pressure)

Substance	Specific Heat, c (J/kg · K)	Molar Mass, M (kg/mol)	Molar Heat Capacity, C (J/mol · K)
Aluminum	910	0.0270	24.6
Beryllium	1970	0.00901	17.7
Copper	390	0.0635	24.8
Ethanol	2428	0.0461	111.9
Ethylene glycol	2386	0.0620	148.0
Ice (near 0°C)	2100	0.0180	37.8
Iron	470	0.0559	26.3
Lead	130	0.207	26.9
Marble (CaCO ₃)	879	0.100	87.9
Mercury	138	0.201	27.7
Salt (NaCl)	879	0.0585	51.4
Silver	234	0.108	25.3
Water (liquid)	4190	0.0180	75.4

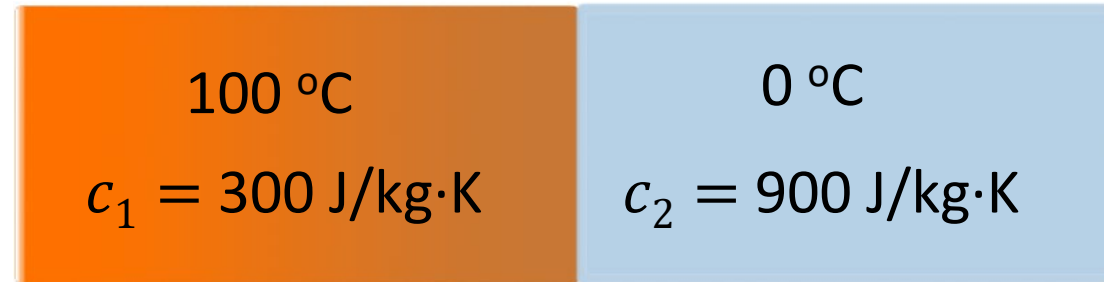
Why is heat capacity larger for some materials?

- Temperature is proportional to average kinetic energy of molecules



- For more complicated materials, part of energy added goes to rotations/vibrations, etc..., so it takes more Q to increase their kinetic energy (= rise temperature)

Q: Two objects with the same mass are put in thermal contact but insulated from their environment. If the initial temperatures are $100\text{ }^{\circ}\text{C}$ and $0\text{ }^{\circ}\text{C}$, and the specific heats are $c_1 = 300\text{ J/kg}\cdot\text{K}$ and $c_2 = 900\text{ J/kg}\cdot\text{K}$, calculate the final equilibrium temperature.



$$0^{\circ}\text{C} < T < 100^{\circ}\text{C}$$

$$T = ?$$

Hint: isolate the parts and think of what is happening to each part separately. Draw a picture and label it.

$$Q = m c \Delta T$$

Step 1: System

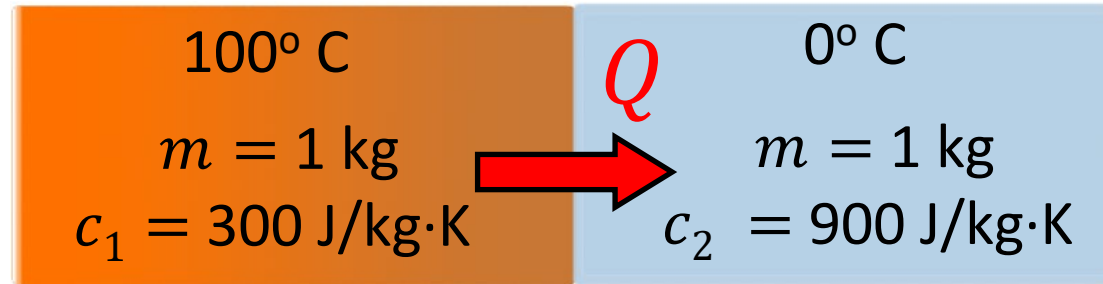
- Visualize what will happen
- Draw a before/after picture
- Gives names to known & unknown quantities & label diagram
- Understand which quantities are changing and which are fixed

- Next: for each part, determine how much heat was added

Step 1: System

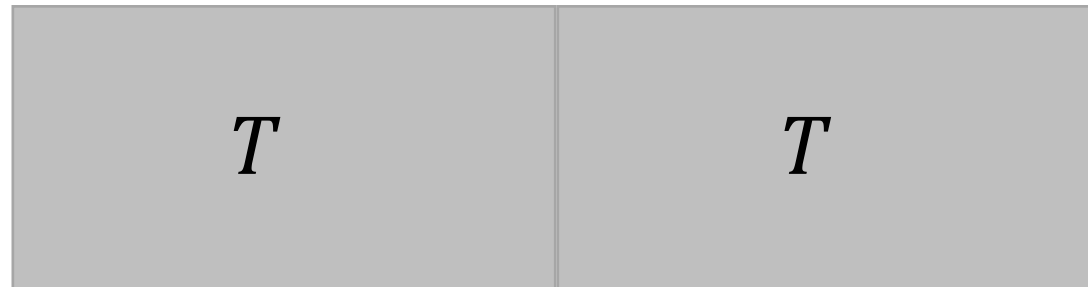
- Visualize what will happen
- Draw a before/after picture
- Gives names to known & unknown quantities & label diagram
- Understand which quantities are changing and which are fixed

Before:



Heat given off by 1st
is equal to heat
received by 2nd

After:



We want
to find T

- Next: for each part, determine how much heat was added

Step 2: Parts of the system

- Isolate the parts of the system
- For each part, draw a before/after picture

Step 2: Parts of the system

- Isolate the parts of the system
- For each part, draw a before/after picture

Before:

$$\begin{aligned}T_1 &= 100\text{ }^{\circ}\text{C} \\ m &= 1\text{ kg} \\ c_1 &= 300\text{ J/kg}\cdot\text{K}\end{aligned}$$

Transferred heat: $Q_1 < 0$

After:

T

Before:

$$\begin{aligned}T_1 &= 0\text{ }^{\circ}\text{C} \\ m &= 1\text{ kg} \\ c_2 &= 900\text{ J/kg}\cdot\text{K}\end{aligned}$$

Tranferred heat: Q_2

After:

T

$$Q = m c \Delta T$$

$$Q_2 > 0$$

- How are Q_1 and Q_2 related?

$$Q_1 + Q_2 = 0$$

- $Q > 0$ if T increases (heat received)
- $Q < 0$ if T decreases (heat spent)

Step 3: Equations

- For each part, write an equation relating the change in length to the changes in temperature and forces



$$Q = m c \Delta T$$

Step 3: Equations

- For each part, write an equation relating the change in length to the changes in temperature and forces

no forces here! 😊



Before:

$$\begin{aligned} T_i &= 100^\circ\text{C} \\ m &= 1 \text{ kg} \\ c_1 &= 300 \text{ J/kg}\cdot\text{K} \end{aligned}$$

T_i

Q: For the object initially at 100°C , transferred heat is:

A. $Q_1 = 300 \text{ J/K} \cdot T$

B. $Q_1 = 300 \text{ J/K} \cdot 100^\circ\text{C}$

C. $Q_1 = 300 \text{ J/K} \cdot (T - 100^\circ\text{C}) \cdot 1 \text{ kg}$

D. $Q_1 = 300 \text{ J/K} \cdot (100^\circ\text{C} - T)$

E. $Q_1 = 300 \text{ J/K} \cdot (T + 100^\circ\text{C})$

$$\Delta T = T_f - T_i$$

After:

$$T$$

T_f

Transferred heat: Q_1

$$Q_1 + Q_2 = 0$$

Q: What is Q_2 ?

$$\Delta T = T_{f,2} - T_{i,2} =$$

$$Q_2 = m c_2 (T - 0^\circ\text{C}) = 900 \cdot 1 \cdot T = 900 T \quad [\text{J}]$$

$$Q = m c \Delta T$$

Step 3: Equations

- For each part, write an equation relating the change in length to the changes in temperature and forces



Q: For the object initially at 100° C, transferred heat is:

Before:

$$\begin{aligned}T_1 &= 100^\circ\text{C} \\m &= 1 \text{ kg} \\c_1 &= 300 \text{ J/kg}\cdot\text{K}\end{aligned}$$

Transferred heat: Q_1

After:

T

$$\Delta T = T_f - T_i$$

- A. $Q_1 = 300 \text{ J/K} \cdot T$
- B. $Q_1 = 300 \text{ J/K} \cdot 100^\circ\text{C}$
- C. $Q_1 = 300 \text{ J/K} \cdot (T - 100^\circ\text{C})$ ✓
- D. $Q_1 = 300 \text{ J/K} \cdot (100^\circ\text{C} - T)$
- E. $Q_1 = 300 \text{ J/K} \cdot (T + 100^\circ\text{C})$

$$Q_i = m c \Delta T \quad (\text{NOTE: } \Delta T = T_F - T_I)$$

$$m = 1 \text{ kg}$$

$$c_1 = 300 \text{ J/kg}\cdot\text{K}$$

$$\Delta T = (T - 100^\circ\text{C})$$

Q_i is negative (hotter object loses heat)

Q: What is Q_2 ?

$$Q_2 = m C_2 (T - 0^\circ\text{C}) > 0$$

$$Q = m c \Delta T$$

Step 4 • Collect equations and solve for unknowns

$$Q = m c \Delta T$$

Step 4 • Collect equations and solve for unknowns

- We have:

$$Q_1 = 300 \text{ J/K} \cdot (T - 100^\circ \text{ C}) \text{ - negative}$$

$$Q_2 = 900 \text{ J/K} \cdot (T - 0^\circ \text{ C}) \text{ - positive}$$

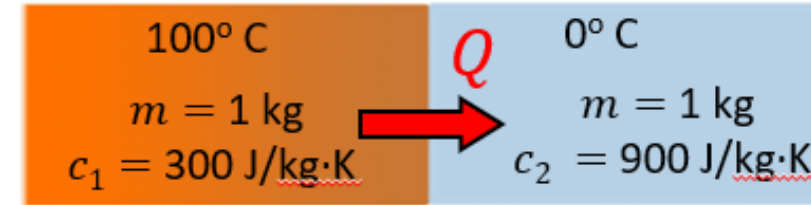
- How are Q_1 and Q_2 related? Why?

Energy conservation! Heat given off by 1st is equal to heat received by 2nd

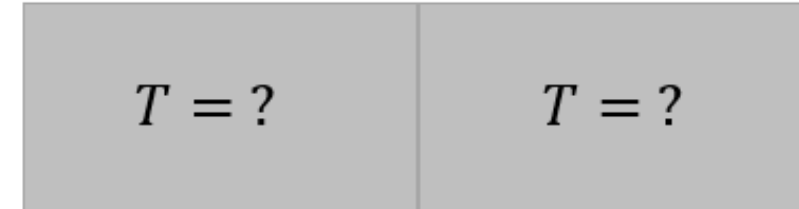
$$Q_1 + Q_2 = 0$$

$$1200 \frac{\text{J}}{\text{K}} \cdot T - 30,000 \text{ J} = 0 \Rightarrow T = 25^\circ \text{ C}$$

Before:

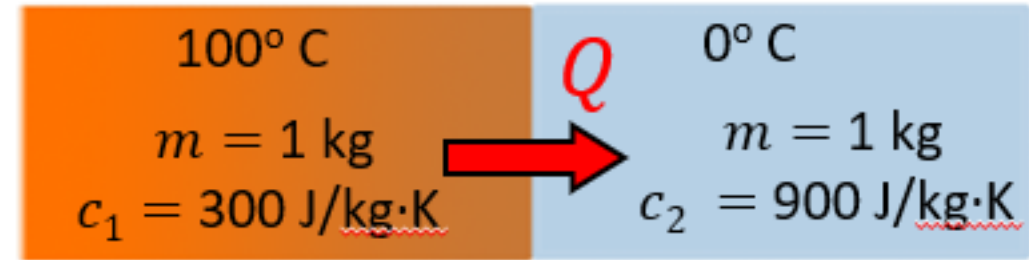


After:

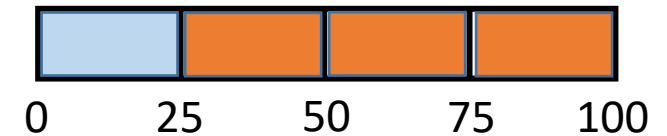


$$Q = m c \Delta T$$

Same problem done very quickly



- Intuitively: c_2 is $3 \times c_1$, so same magnitude of heat will cause 1/3 of the temperature change
- 25° is 1/3 of 75° and these add to 100° C



- General solution for the final temperature:

$$T = \left(\frac{m_1 c_1}{m_1 c_1 + m_2 c_2} \right) T_1 + \left(\frac{m_2 c_2}{m_1 c_1 + m_2 c_2} \right) T_2$$

$$Q = m c \Delta T$$

const $\times 3 \uparrow$ $\times 3 \downarrow$

Weighted average
of T_1 and T_2