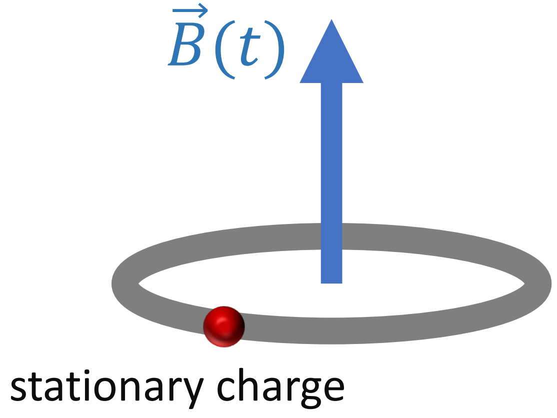
Lecture 34.

Are B-field and E-field connected?
Displacement current.
Maxwell’s correction to Ampere’s law.

Last Time:

Everything is going so well so far…



- But: why changing magnetic flux creates electric current??
- What exactly moves the charges around??

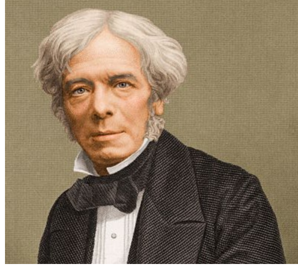
- Does the induced emf appear only in a loop?
  Can it appear in a piece of metal?

- Yes. Example: Eddy currents.

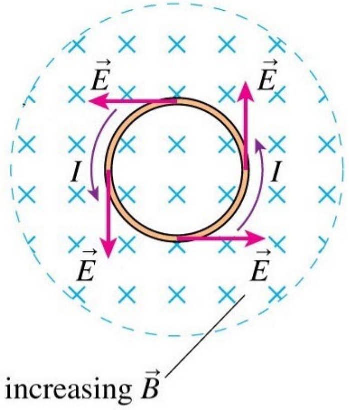
- Can induced emf appear in air? In vacuum?
- Yes, it can.

Q: what physical entity exerts a force on electric charges?

What causes creation of emf in changing magnetic field?



- Charge motion is driven by electric field.
- Varying magnetic field creates electric field! In a wire, in a metal, in vacuum – everywhere.
  - emf is defined as work done per unit charge: $d\varepsilon = dW/q$
  - Work of electric force: $dW = \vec{F} \cdot d\vec{l} = q\vec{E} \cdot d\vec{l}$
  - If a charge moves around a entire loop, as in this figure, the work is:



$$W_{\text{closed curve}} = q \oint \vec{E} \cdot d\vec{l}$$

$$\varepsilon = \oint \vec{E} \cdot d\vec{l}$$

- Note: non-electrostatic E-field is non-conservative

Therefore: Faraday's law in integral form

$$\boxed{\varepsilon = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}}$$

Faraday's law: (now a more sophisticated version describing the electric field induced by changing magnetic field)

- At the age of 14 he became an apprentice to a local bookbinder and bookseller
- In 1812, at the age of 20 and at the end of his apprenticeship, Faraday attended lectures by the eminent English chemist Humphry Davy
- Faraday subsequently sent Davy a 300-page book based on notes that he had taken during these lectures. Davy's reply was immediate, kind, and favourable.

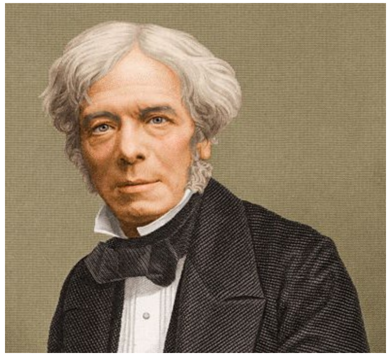
https://en.wikipedia.org/wiki/Michael_Faraday

$$\varepsilon = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

- In 1813, when Davy damaged his eyesight in an accident with nitrogen trichloride, he decided to employ Faraday as an assistant.

Faraday's law: (integral form)

$$\boxed{\varepsilon = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}}$$

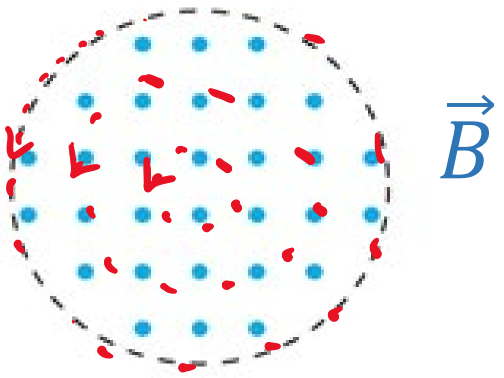


It is the first time when E-field and B-field appear in the same equation.

Conclusion:

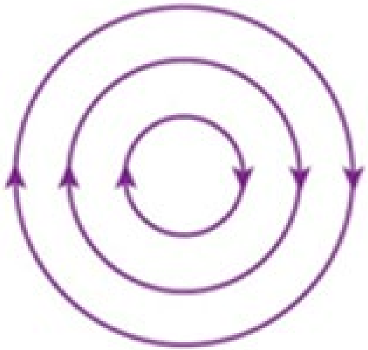
- We now know two sources of E-field:
  - electric charges
  - changing magnetic field

- Direction of induced E-field: Lenz's law + RHR!
- I.e. its direction is such that the induced current (in case it would have been created) would result in opposing the change of the magnetic flux.

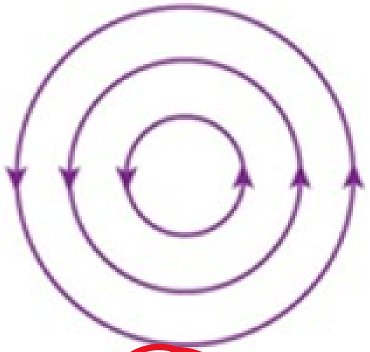


Q: The magnetic field is decreasing.
How does the induced electric field look like?

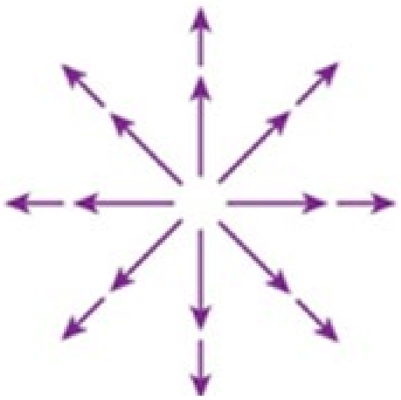
$\vec{B}_{ext} \odot$ decr $\rightarrow$ $\vec{B}_{ind} = \odot$



A.



B.



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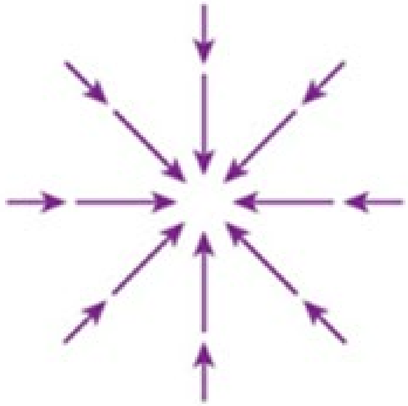


D.

E. Something else

Equations for E-field and B-field: Let's put them together

- Gauss's law for electricity

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

Electric charges are sources of E-field

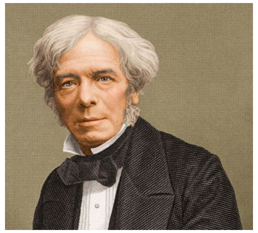
- Gauss's law for magnetism

$$\oint \vec{B} \cdot d\vec{A} = 0$$

There are no magnetic monopoles

- Faraday's law

$$\varepsilon = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$



Changing B-field is a source of E-field

- Ampere's law

?

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{net}$$

Electric current is a source of B-field

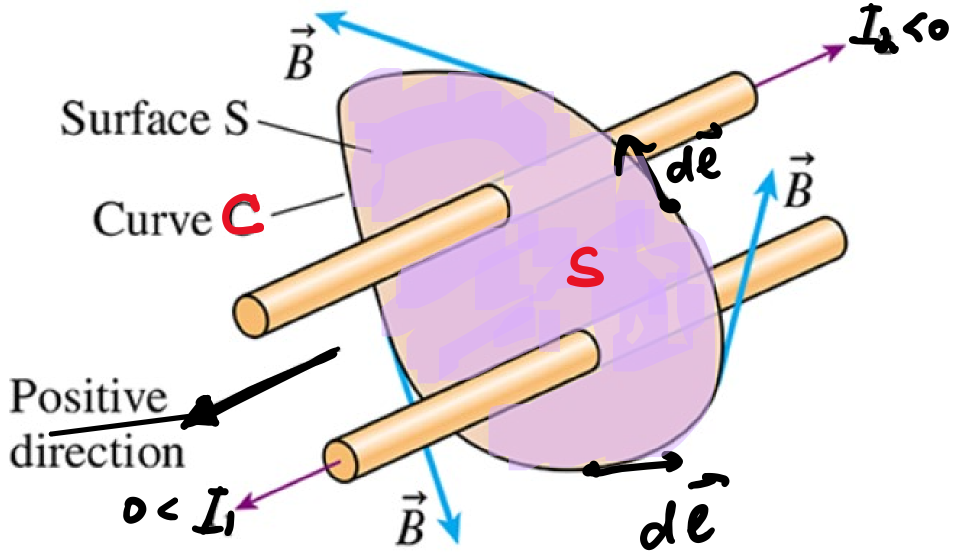
- One important piece is still missing !!!

Ampere's law: (Integral form)

- It tells us that electric currents produce magnetic field. Okay.

$$\oint_C \underbrace{\vec{B}\cdot d\vec{l}}_{10} = \mu_0 \underbrace{I_{net}}_{S}$$

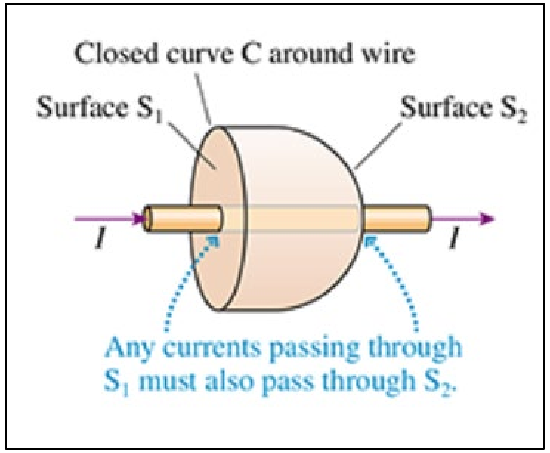
- Ampere's law assumes a choice of:
  - ➢ An integration path (curve C, along which we integrate $\vec{B}$)
  - ➢ …and the surface (surface S) that C captures. We will say that I_{net} is the net current passing through that surface.



- Positive direction for the current: RHR with how you go along the loop

Ampere's law: (Going 3D)

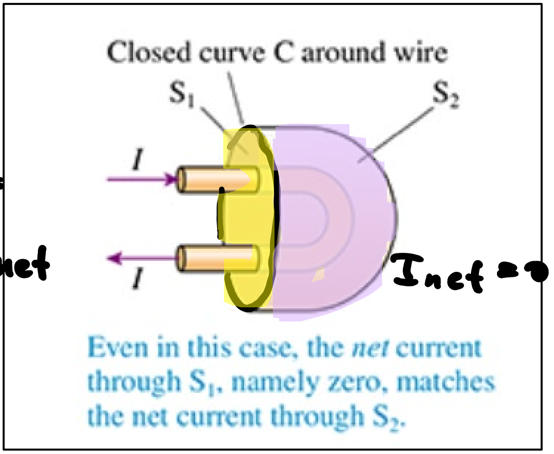
- In fact, surface S is not required to be flat. It can be curved. It just must be any surface restricted by the curve C.
  - ➢ Indeed, any current passing through S_1 (flat) also passes through S_2 (curved).



- This is even true if the wire bends:
  - ➢ Here the net current passing through S_1 (flat) is zero, as well as the current passing through S_2 (curved).

$\oint \vec{B}\cdot d\vec{l} = 0 =$

$= \mu_0 I_{net}$



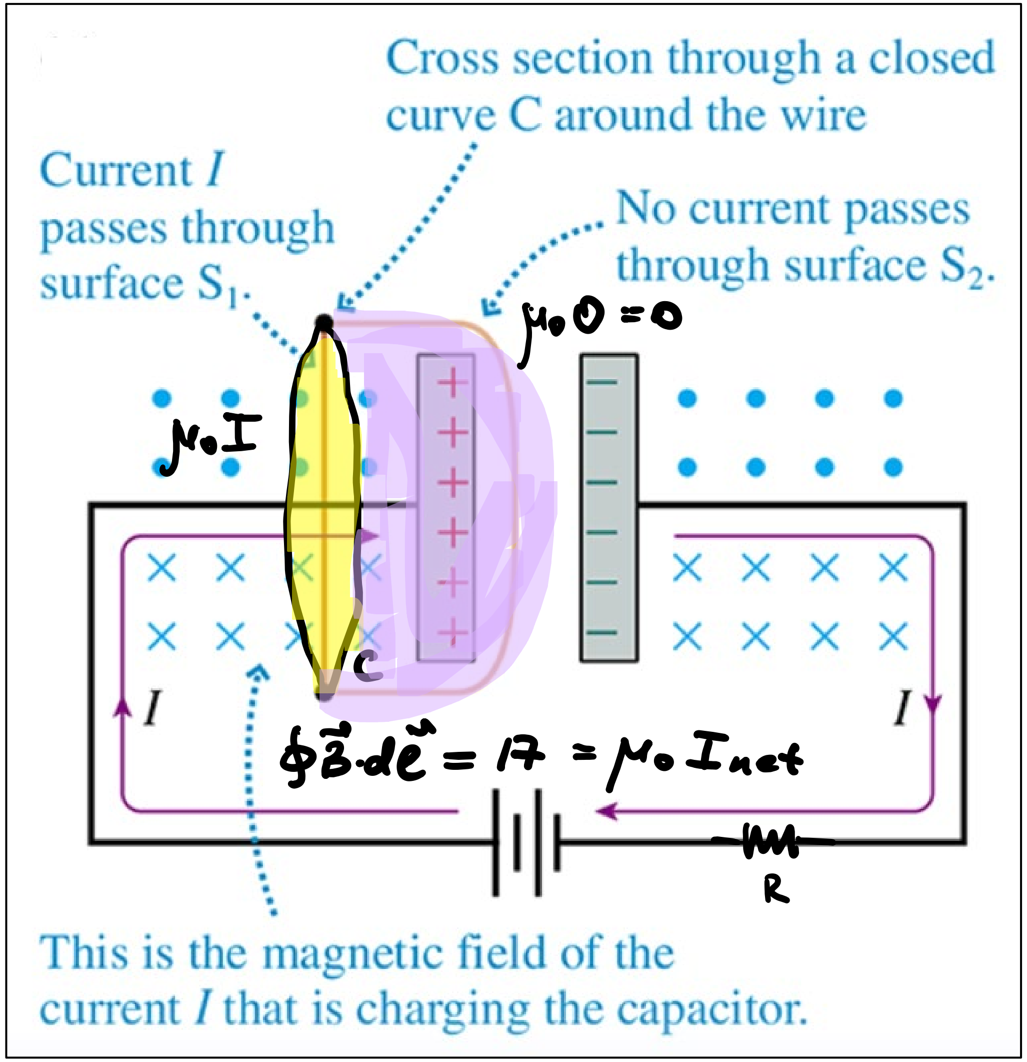
$I_{net} = 0$

- According to Ampere's law, if two surfaces are bound by the same closed curve, the current through them must be the same (since the integral of the B-field over the curve is for them the same).

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

Ampere's law: (Going 3D)

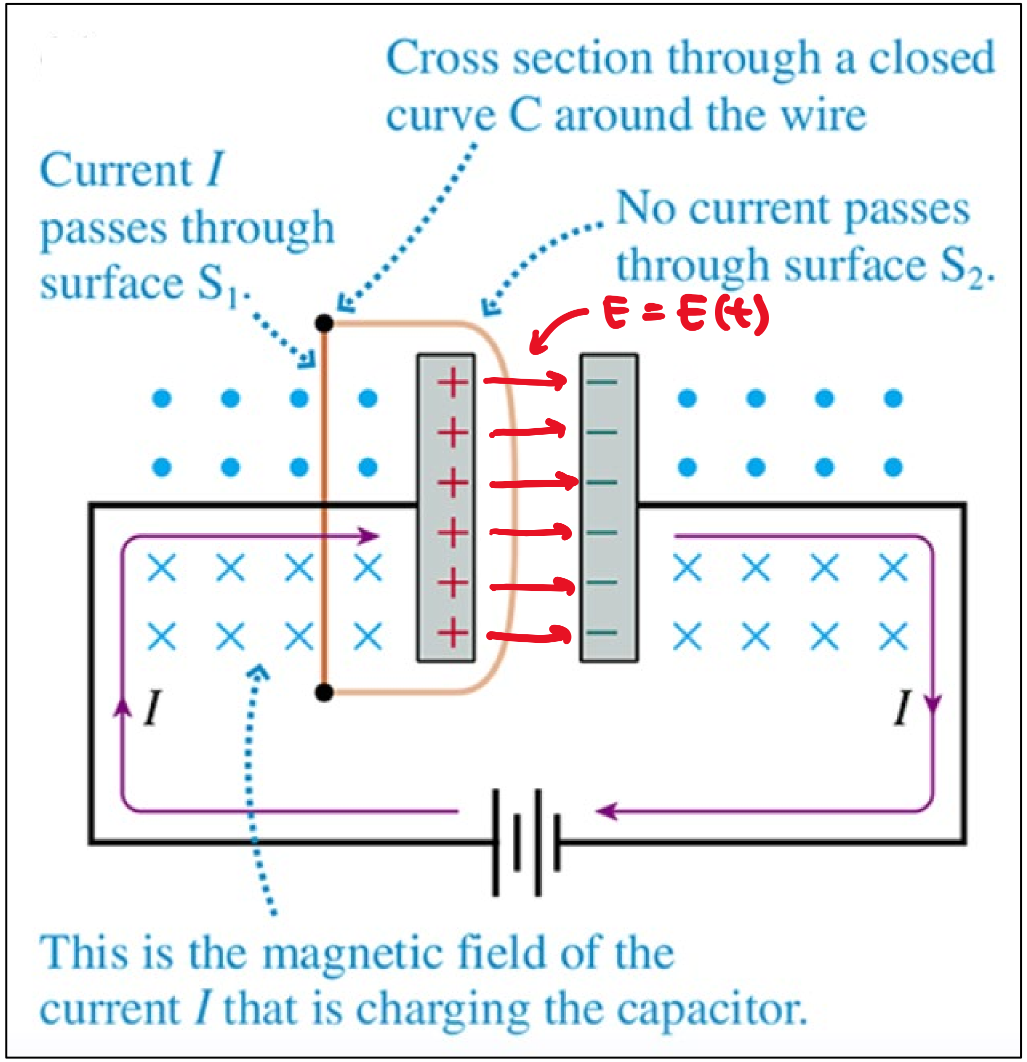
- But: we are in trouble!
- Consider a charging capacitor, and a semi-sphere S_2 embracing one of the plates (say, with positive charge):
  - ➢ the integral over the curve gives $\mu_0 I$ for its flat surface, but zero for the spherical surface...
  - ➢ Then what is $\oint \vec{B} \cdot d\vec{l}$ over the curve C??



- We need to revisit Ampere's law, that states that B-field relates to currents...

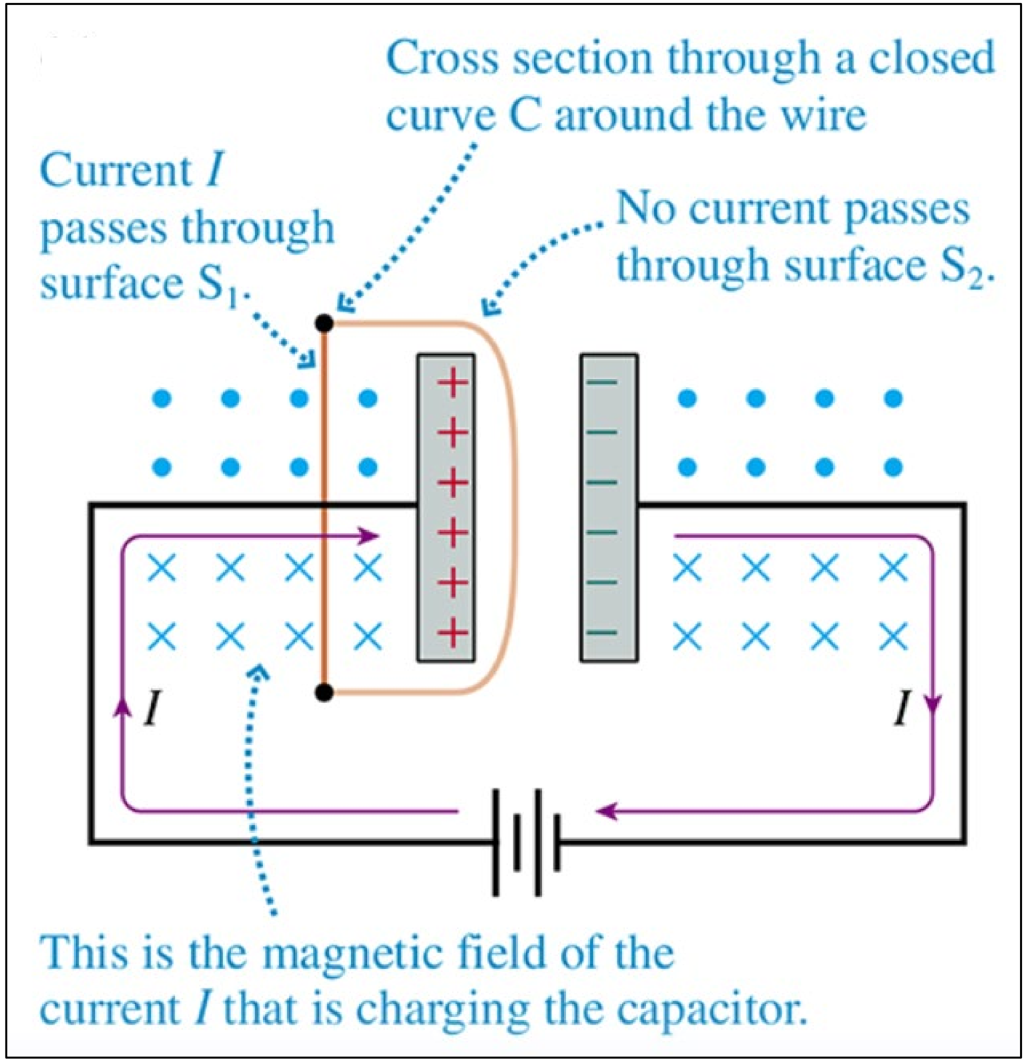
Ampere's law: (Going 3D)

- Ideas:
  - ➢ Assume that Ampere's law is valid in many situations, and only needs fine-tuning
  - ➢ Note that electric field through S_2 changes. Can this observation be useful?
  - ➢ Wait… There are two sources of E-field:
    - ❖ (i) electric charges
    - ❖ (ii) changing B-field
  - ➢ What if, likewise, there are two sources of B-field:
    - ❖ (i) electric currents, and
    - ❖ (ii) changing E-field?



Ampere's law: (Let's fix it)

- Assumption:
  - B-field can be created not only by current (=moving charge), but also by changing electric flux (that's what J.C. Maxwell has recognized in 1855).



  - Let us quantify this assumption.

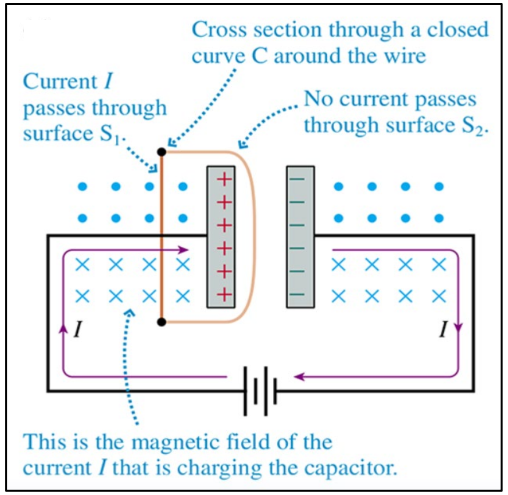
Q: Show that for a charging capacitor, the current I that charges it and the electric flux Φ_e inside it are connected by:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

$$\boxed{\frac{d\Phi_e}{dt} = \frac{I}{\varepsilon_0}}$$

$$\frac{d}{dt}\Phi_e = \frac{d}{dt}\left(A \cdot E(t)\right) = \cancel{A}\frac{d}{dt}\frac{Q(t)}{\varepsilon_0 \cancel{A}} = \frac{1}{\varepsilon_0}\frac{dQ(t)}{dt} = \frac{I}{\varepsilon_0}$$

$$I = \underbrace{\left[\varepsilon_0 \frac{d\Phi_e}{dt}\right]}$$

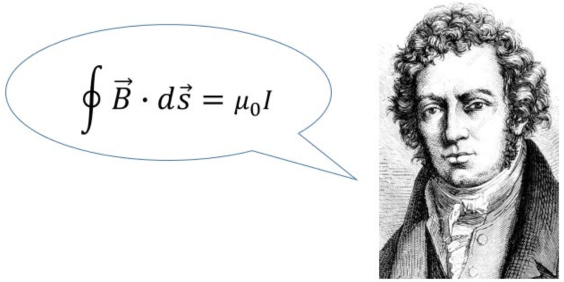


Ampere's law (revisited):

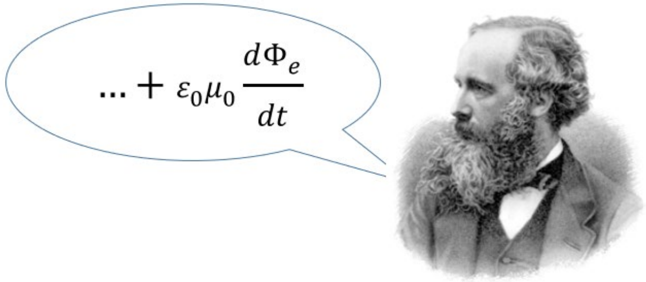
• We found, for a charging capacitor : $\frac{d\Phi_e}{dt} = \frac{I}{\varepsilon_0}$

• Therefore, the quantity $\varepsilon_0 \frac{d\Phi_e}{dt}$:

- Has units of current, [A]
- It passes through S_2, and
- Equals the current I that passes through S_1



• We can use $\varepsilon_0 \frac{d\Phi_e}{dt}$, which we will call "displacement current" I_{disp} *) the same way as I in the Ampere's law, and claim changing E-field to be another source of B-field!



Ampere-Maxwell's law

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I_{\text{ext}} + I_{\text{disp}} \right)}$$

*) Nothing is actually being displaced. The name appears for historical reasons from fluid model of electricity. Moreover, there is no current – there is only change of the electric flux!

Ampere-Maxwell's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(I_{\text{ext}} + I_{\text{disp}} \right)$$

$$I_{\text{disp}} = \varepsilon_0 \frac{d\Phi_e}{dt}$$

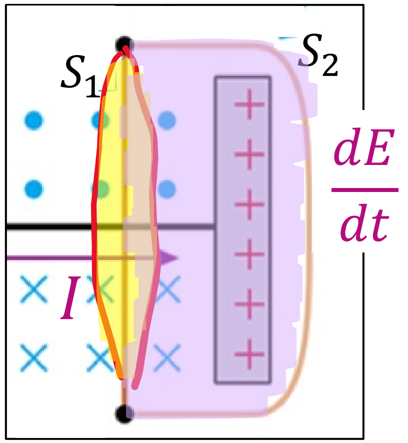
- It states that there are two sources of B-field, currents and changing E-field, that should be considered on the same footing, always.

- Let's see how it works:

  - For S_1: $\oint \vec{B} \cdot d\vec{l} = \mu_0 (I + 0)$
  - For S_2: $\oint \vec{B} \cdot d\vec{l} = \mu_0 \left(0 + I_{\text{disp}} \right)$

$\oint \vec{B} \cdot d\vec{\ell} = 15 = ?$

$I_{disp} = I$



- Note: displacement current is not a flow of charge. It looks like there is the "same" current "through" a capacitor, but it is just changing electric field, there is no actual charge transfer (since air inside a capacitor is an insulator) !!!

What is the direction of the B-field induced by changing electric field?

- As before, the induced B-field and I_{disp} must obey RHR.

- How do we define the direction of I_{disp} ??

$$I_{\text{disp}} = \varepsilon_0 \frac{d\Phi_e}{dt} = (\varepsilon_0 A)\frac{dE}{dt}$$

- increasing E: $\frac{dE}{dt} > 0 \Rightarrow$ corresponds to $I_{\text{disp}} \uparrow\uparrow E$
- decreasing E: $\frac{dE}{dt} < 0 \Rightarrow$ corresponds to $I_{\text{disp}} \uparrow\downarrow E$

The direction of I_{disp} depends on:

(1) the direction of $\vec{E}$, and

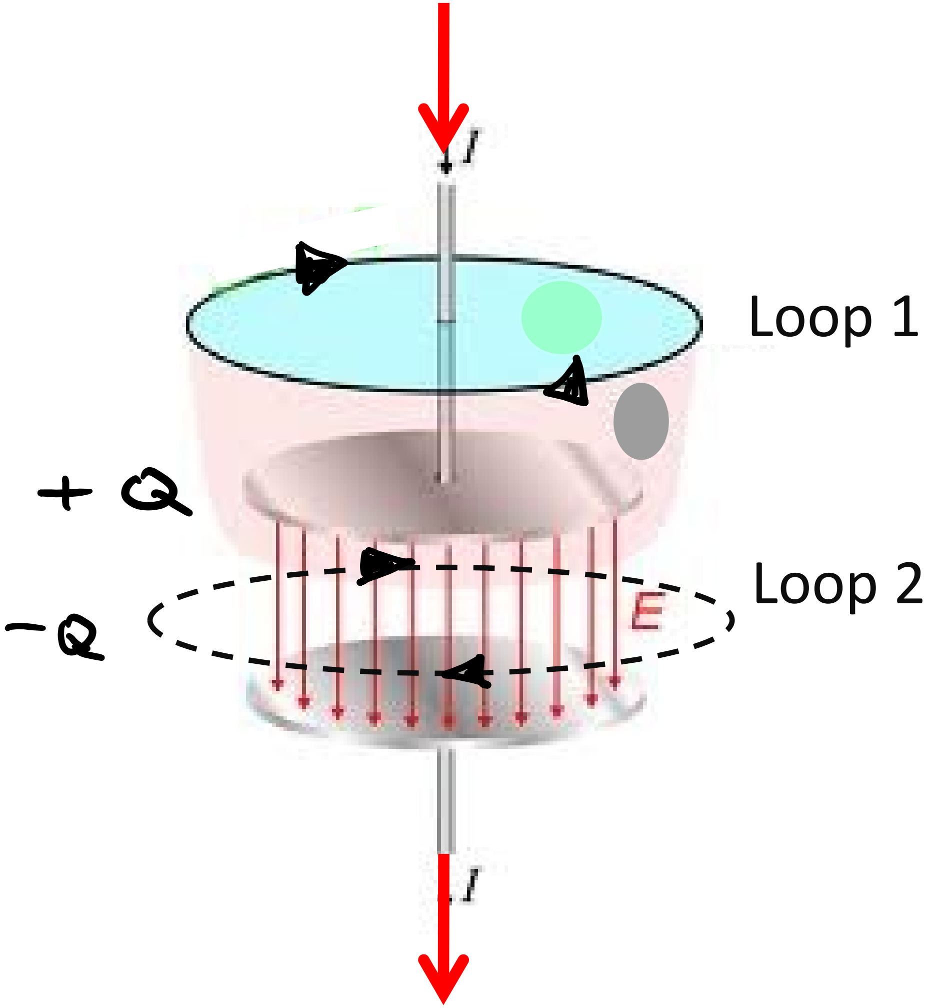
(2) on whether it is increasing or decreasing!

Q: An electrical current I charges a capacitor as shown. In this case, a displacement current I_{disp} flows between the capacitor plates because $q(t)$ is changing as a function of time.

Using Ampere's law, what can you say about the magnetic fields $\vec{B}_1$ & $\vec{B}_2$ arounds loop 1 & 2? Assume that you look at the capacitor from the top.

$\vec{E}\downarrow$ increas $\rightarrow$ $I_{disp}\downarrow$

$\dot{I}_{disp} \uparrow\uparrow \vec{E}$



A. $\vec{B}_1$ CW & $\vec{B}_2$ CW

B. $\vec{B}_1$ CW & $\vec{B}_2$ CCW

C. $\vec{B}_1$ CCW & $\vec{B}_2$ CW

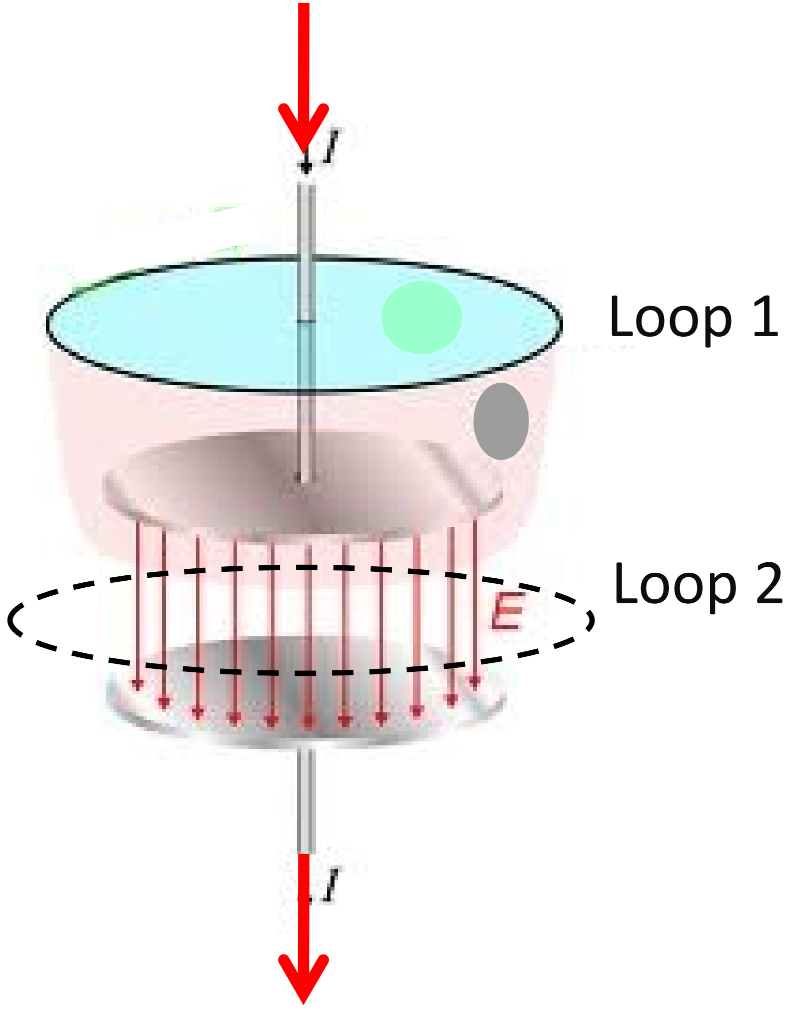
D. $\vec{B}_1$ CCW & $\vec{B}_2$ CCW

E. $\vec{B}_1$ CW & $\vec{B}_2 = 0$

Q: An electrical current I charges a capacitor as shown. In this case, a displacement current I_{disp} flows between the capacitor plates because $q(t)$ is changing as a function of time.

Using Ampere's law, what can you say about the magnetic fields $\vec{B}_1$ & $\vec{B}_2$ arounds loop 1 & 2? Assume that you look at the capacitor from the top.

- Loop 1: RHR for current I => $B_1 \neq 0$, CW
- Loop 2: E is increasing (charging capacitor) => $I_{\text{dis}} \uparrow\uparrow \vec{E}$ => $B_2 \neq 0$, CW (RHR)



A. $\vec{B}_1$ CW & $\vec{B}_2$ CW
B. $\vec{B}_1$ CW & $\vec{B}_2$ CCW
C. $\vec{B}_1$ CCW & $\vec{B}_2$ CW
D. $\vec{B}_1$ CCW & $\vec{B}_2$ CCW
E. $\vec{B}_1$ CW & $\vec{B}_2 = 0$

Q: These figures show a side and top view of a capacitor with charge Q and electric and magnetic fields $\vec{E}$ and $\vec{B}$ at time t. At this time, the charge is:

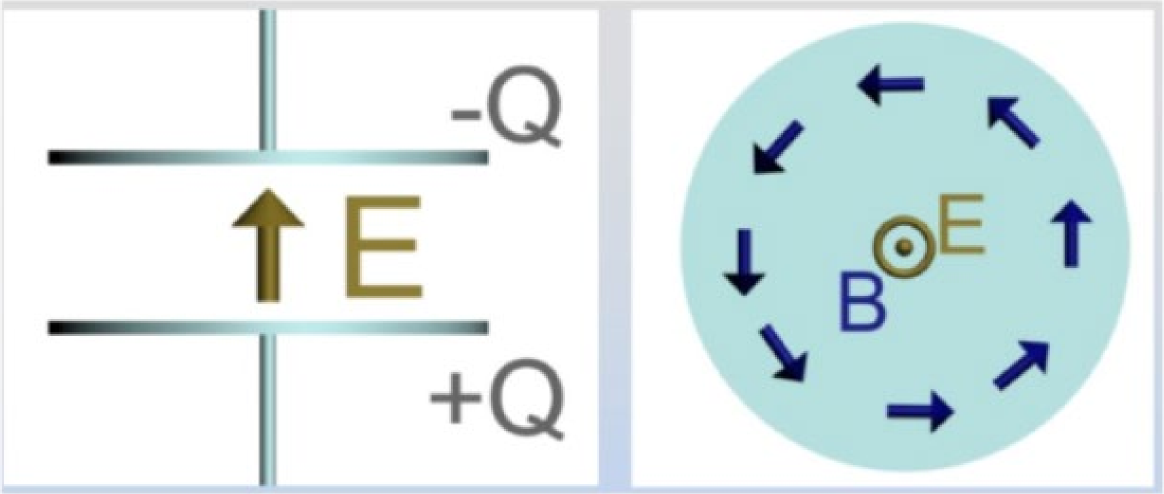
A. Increasing

B. Decreasing

C. Does not change

$\vec{B}_{ind}$ ccw $\longrightarrow$ I_{ind} $\odot$

$\vec{I}_{ind} \uparrow\uparrow \vec{E} \longrightarrow E \uparrow$



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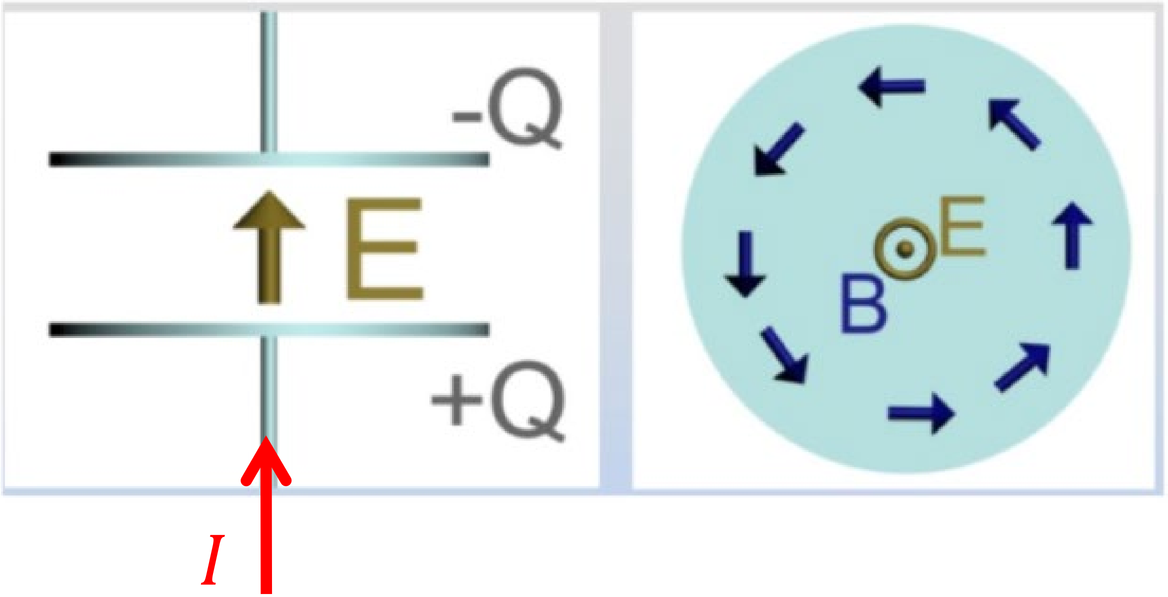
A. Increasing

B. Decreasing

C. Does not change

We are looking from the top

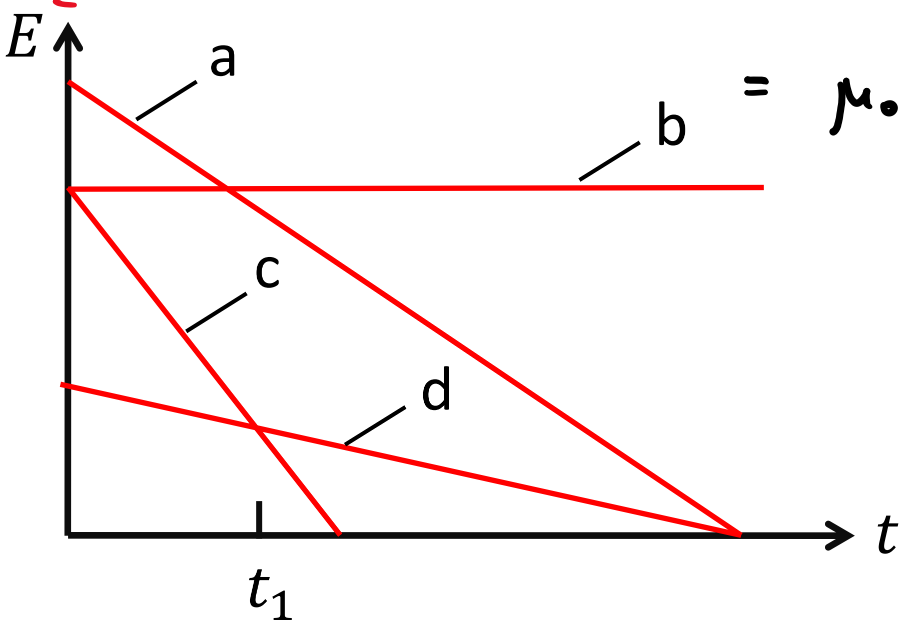


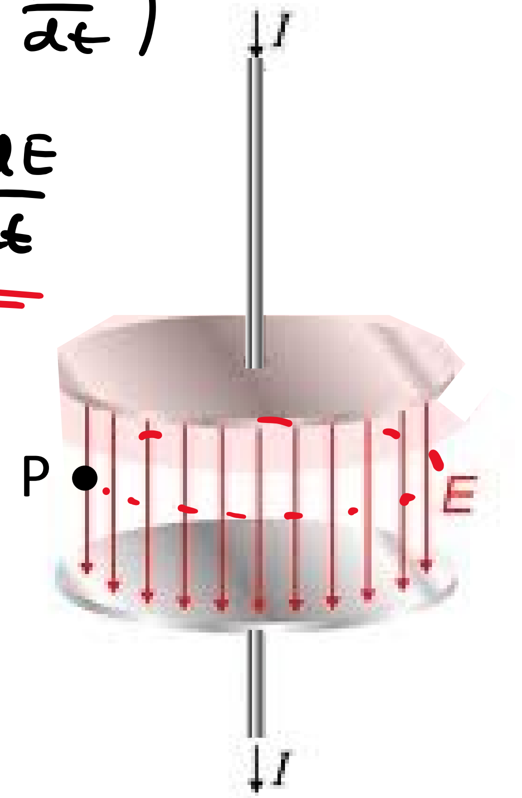
- $\vec{B}$ CCW =>
- I_{disp} out of page, i.e. parallel to $\vec{E}$ =>
- $\left(\frac{dE}{dt} > 0\right) \Rightarrow E$ is increasing =>
- Q is increasing.

Q: The electric field strength in four identical capacitors is shown as a function of time. Rank in order, from largest to smallest, the magnetic field strength at the outer edge of the capacitor (point P) at time t_1.

$$B \cdot 2\pi r = \oint \vec{B} \cdot d\vec{\ell} = \mu_0 \left(\cancel{I} + \varepsilon_0 \frac{d\Phi_e}{dt} \right)$$

$$= \mu_0 \varepsilon_0 A \cdot \frac{dE}{dt}$$





A. $B_a > B_b > B_c > B_d$

B. $B_a = B_b > B_c = B_d$

C. $B_a > B_b = B_c > B_d$

D. $B_b > B_a > B_d > B_c$

E. $B_c > B_a > B_d > B_b$

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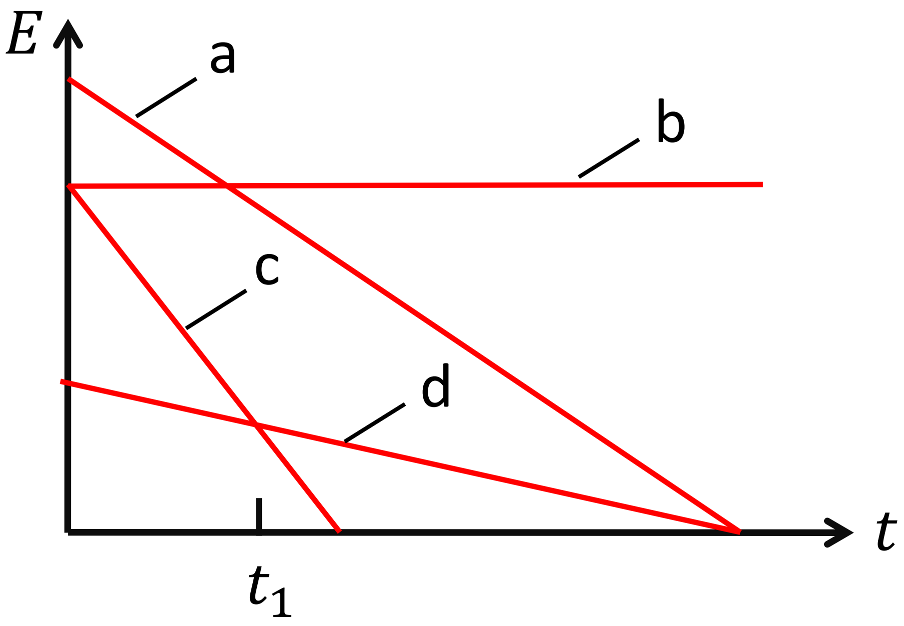
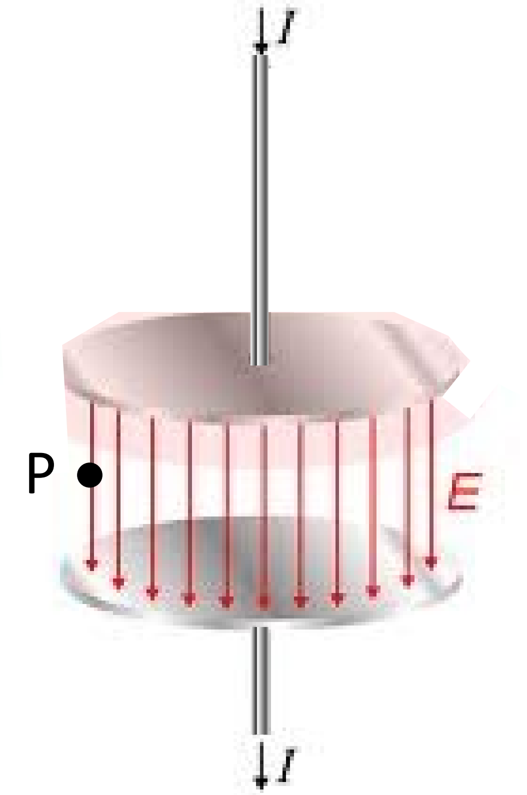




A. $B_a > B_b > B_c > B_d$

B. $B_a = B_b > B_c = B_d$

C. $B_a > B_b = B_c > B_d$

D. $B_b > B_a > B_d > B_c$

E. $B_c > B_a > B_d > B_b$

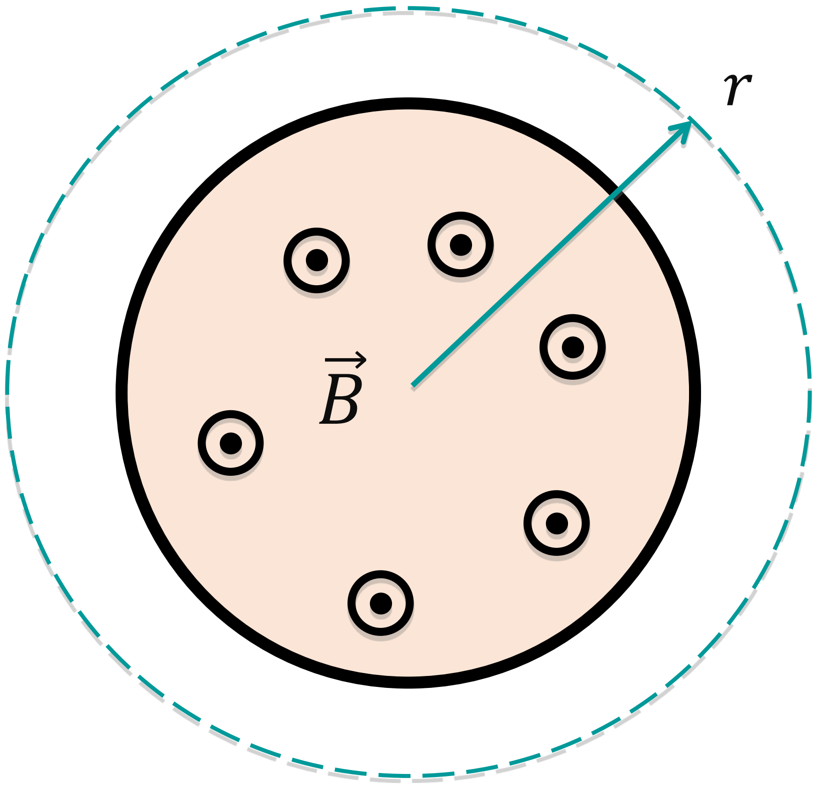
$$\oint \vec{B} \cdot d\vec{l} = 2\pi r\, B = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 A \frac{dE}{dt}$$

=> just rank the slopes

Q: The current in an infinitely long solenoid with uniform magnetic field $\vec{B}$ inside is increasing so that the magnitude B increases in time as $B = B_0 + kt$.

In what direction is the induced E-field on a circular loop of radius r outside the solenoid, as shown?

???



A. CW
B. CCW
C. The induced E is zero
D. Radial, inwards
E. Radial, outwards