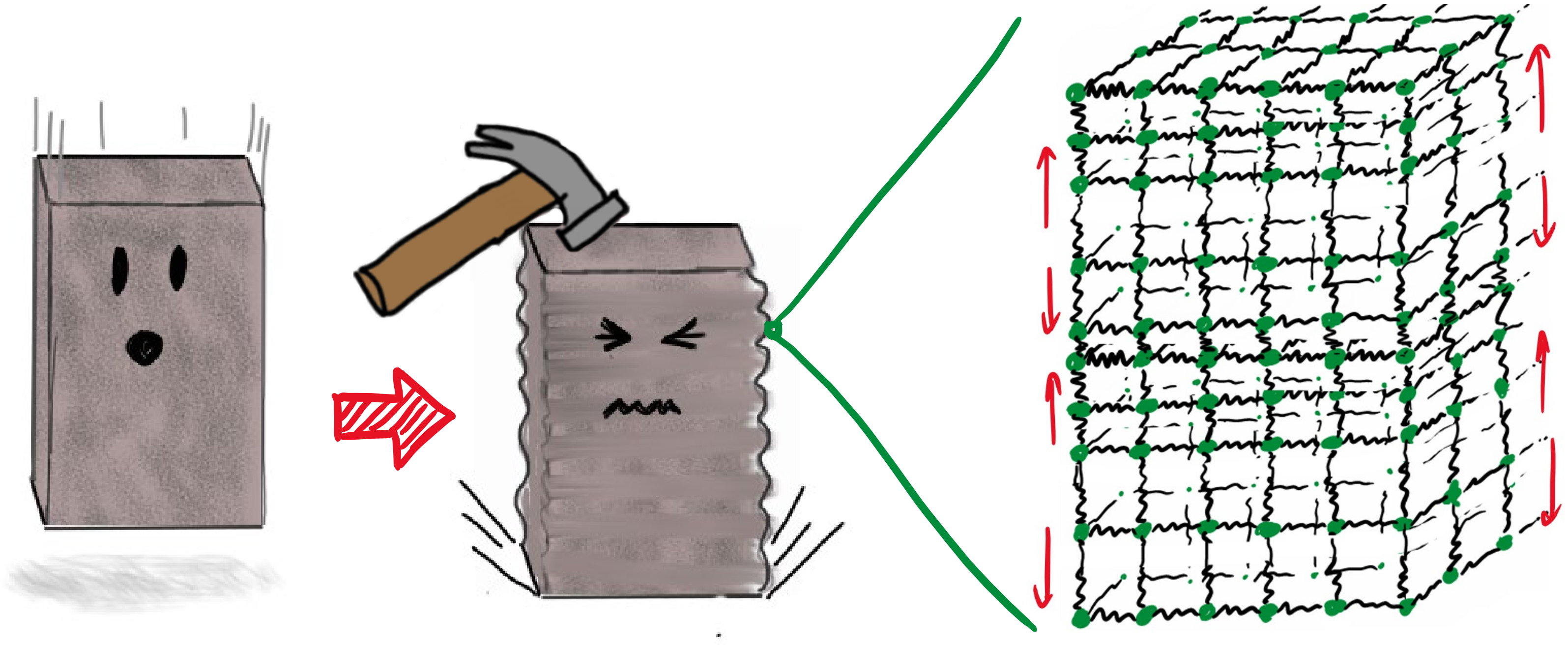
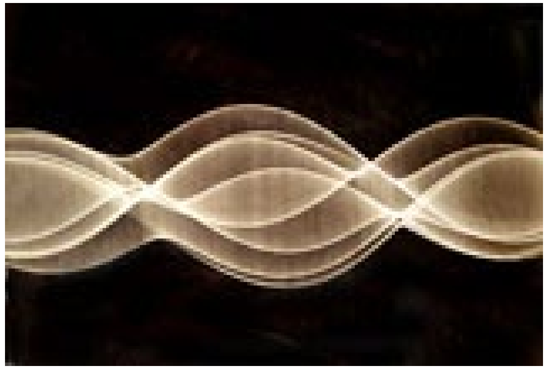
# Lecture 31.

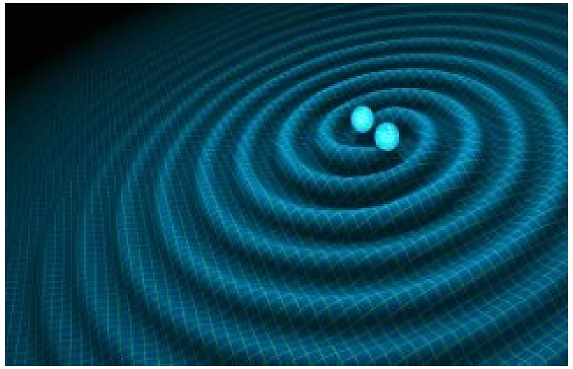
# A wave.

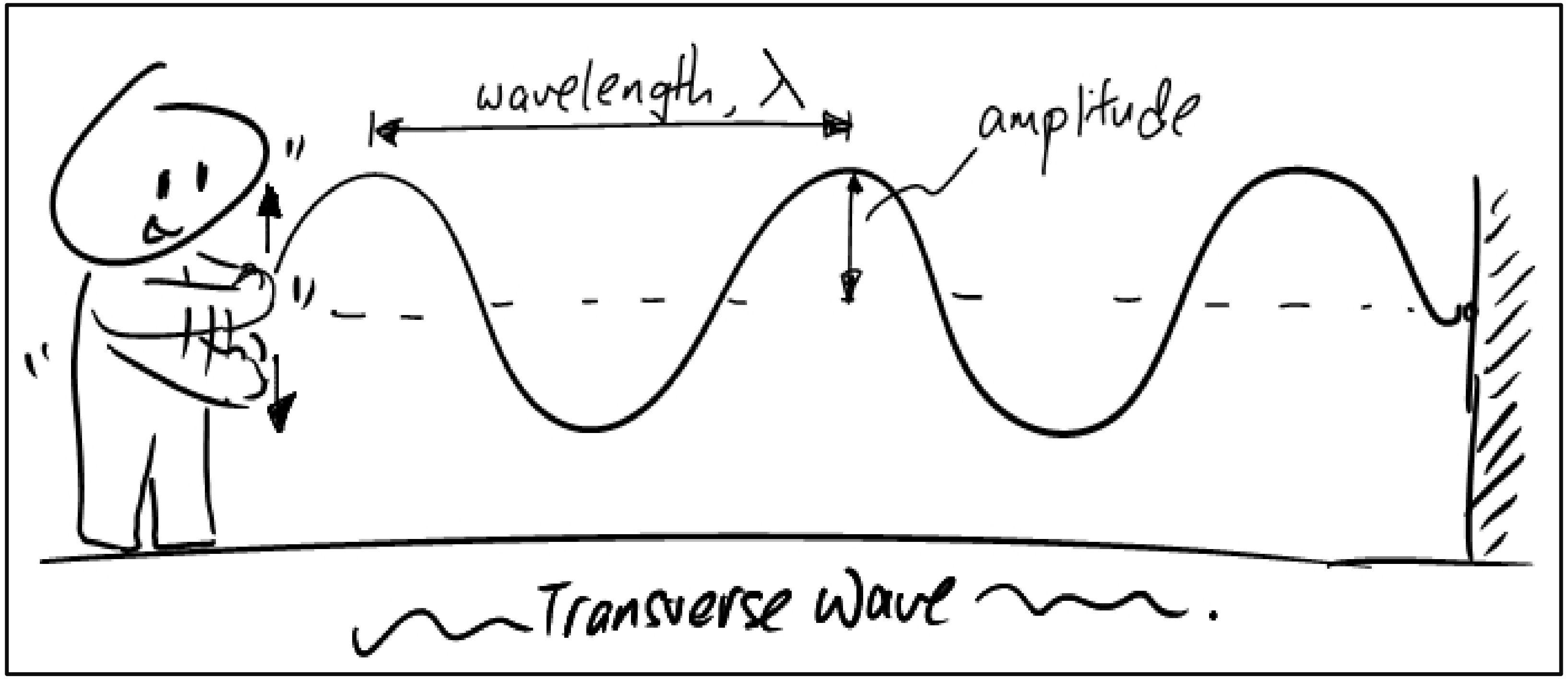
# Transverse and Longitudinal waves.



# A Wave

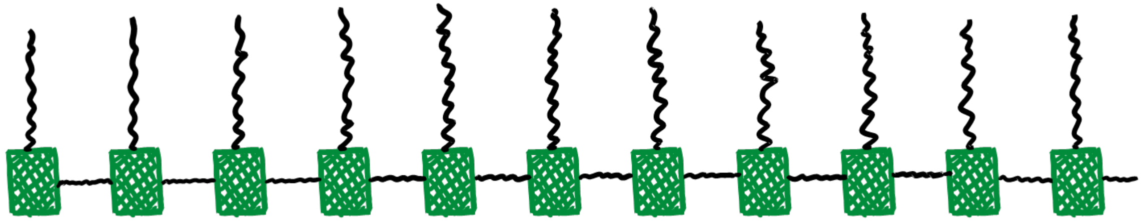






# Demo: A Mechanical Wave

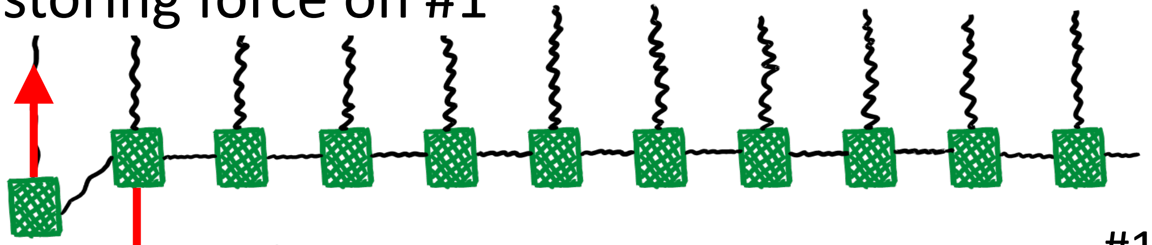
## Coupled oscillators



## Let's pull the leftmost down

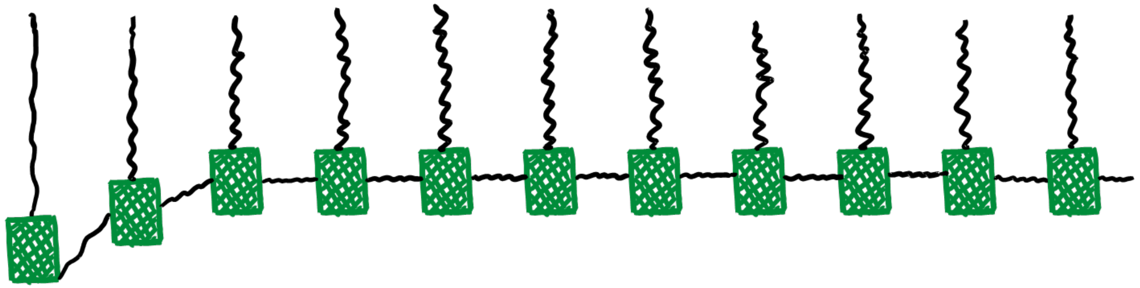
restoring force on #1

$F_1$

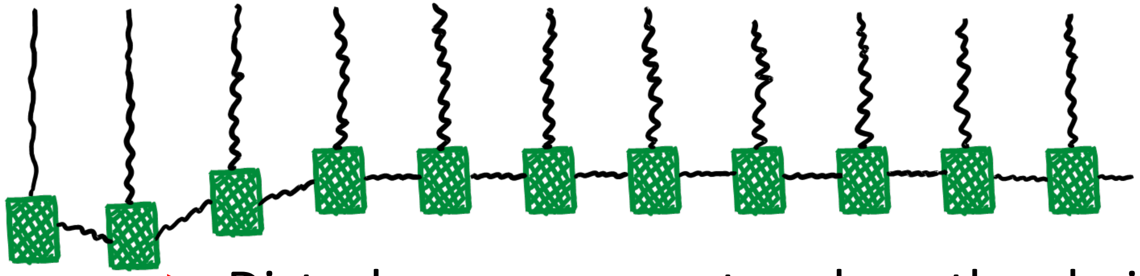


$F_2$: force displacing #2

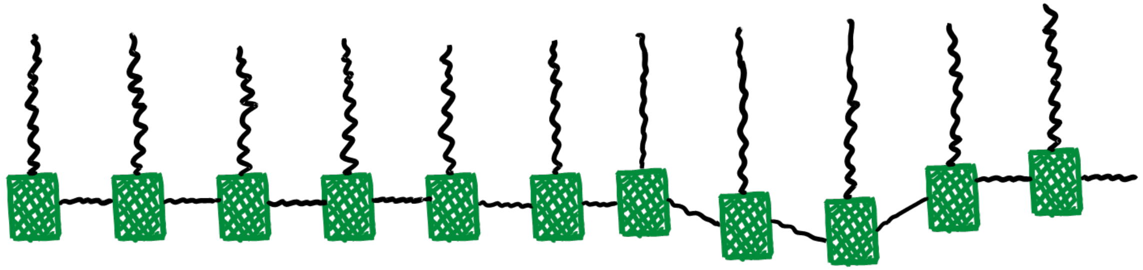
## #1 goes up, #2 goes down…



## …and pulls down #3, etc.

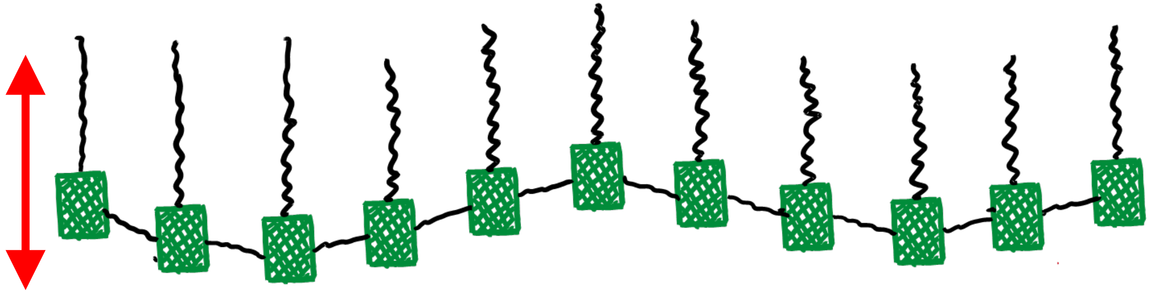


Disturbance propagates along the chain

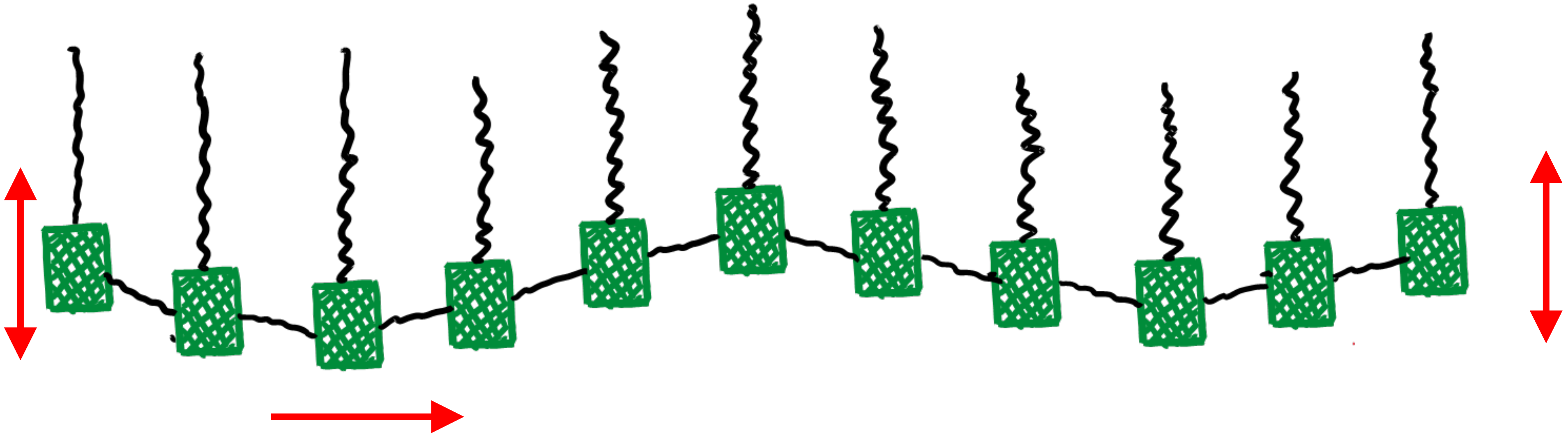


#1, 2, 3… settle down to equilibrium since energy is transferred to the other oscillators

## What if we keep driving #1?



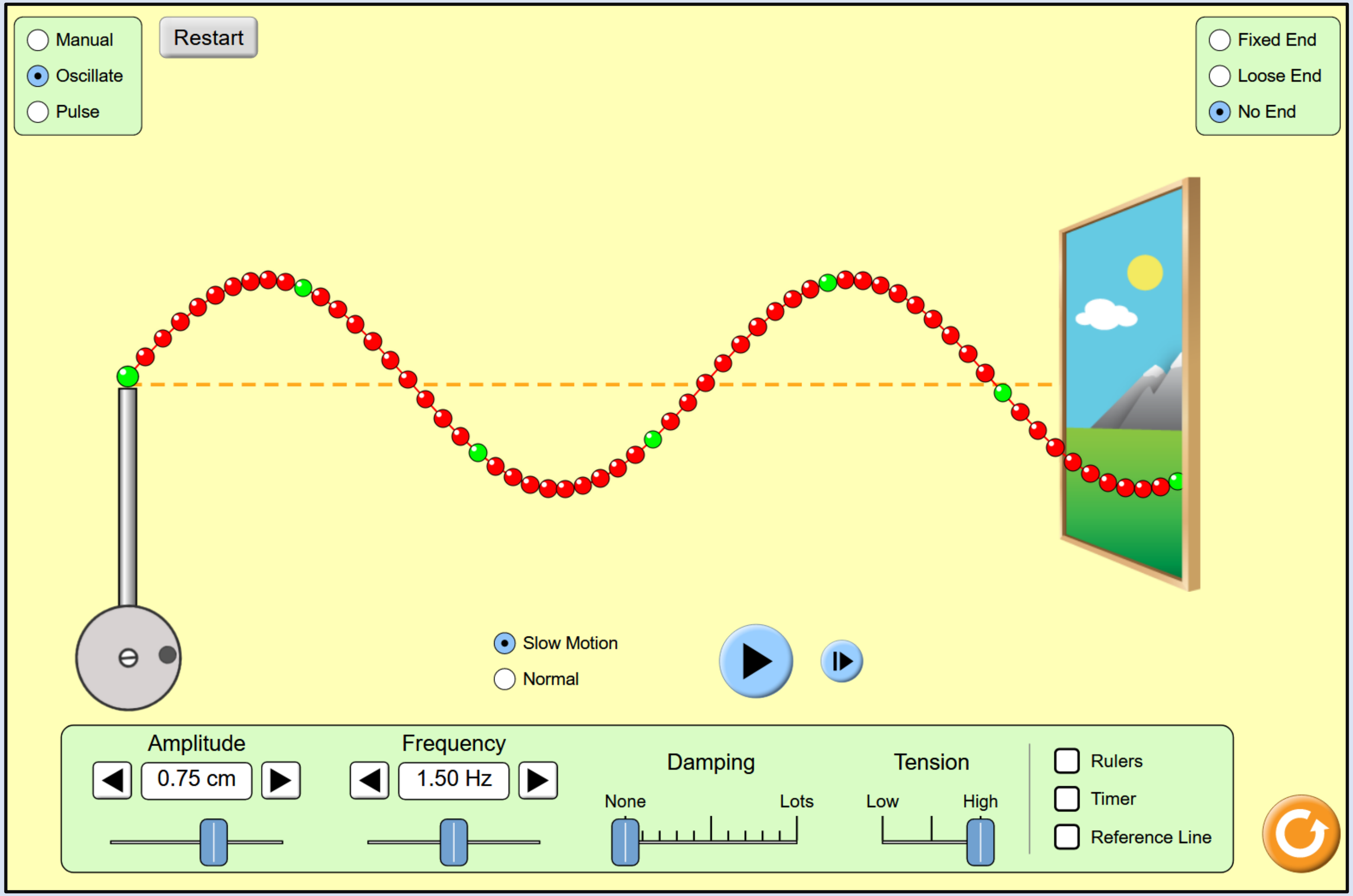
If we continuously drive the leftmost oscillator…



…we get a traveling wave!
(continuously adding energy to the system)

Transverse wave: oscillations perpendicular to direction wave travels

# Simulation of waves on a string




https://phet.colorado.edu/sims/html/wave-on-a-string/latest/wave-on-a-string_en.html

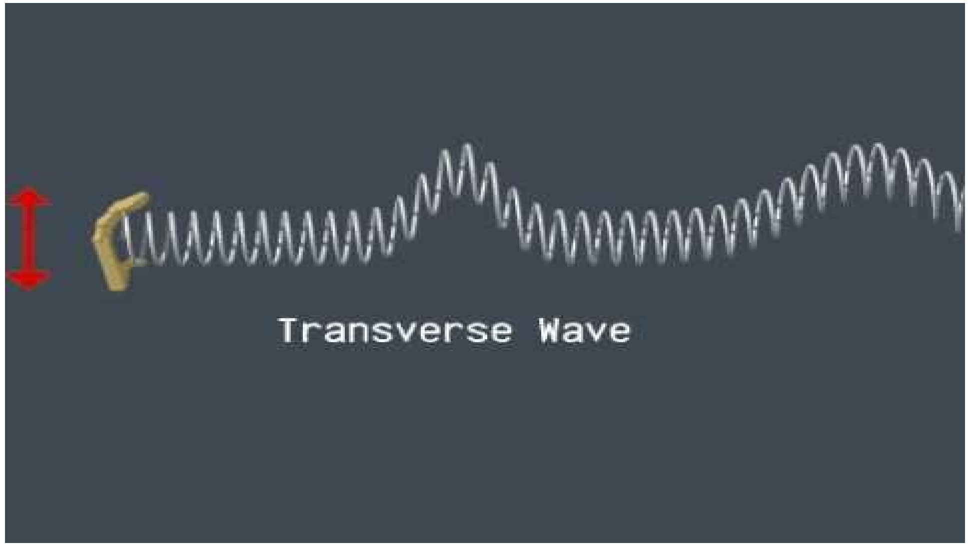
# A mechanical wave: Summary

- Many particles with a restoring force bringing each particle back to equilibrium if deflected (requires medium that will support this wave)
- The particles are coupled, so that the disturbance is transferred from one particle to another
- Disturbance needs a source

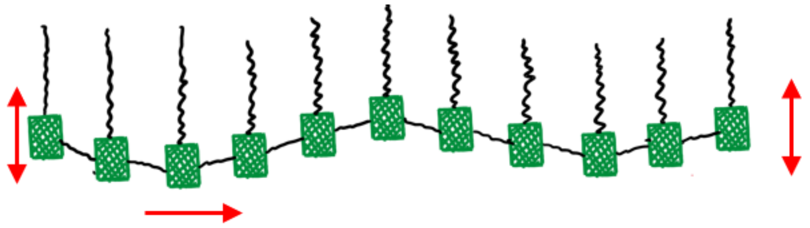

> Note that the particles of the medium do not actually propagate (they oscillate about their equilibrium positions). What travels through the medium is a "disturbance" – some sort of signal

# Mechanical waves can be transverse and longitudinal

- Transverse wave:

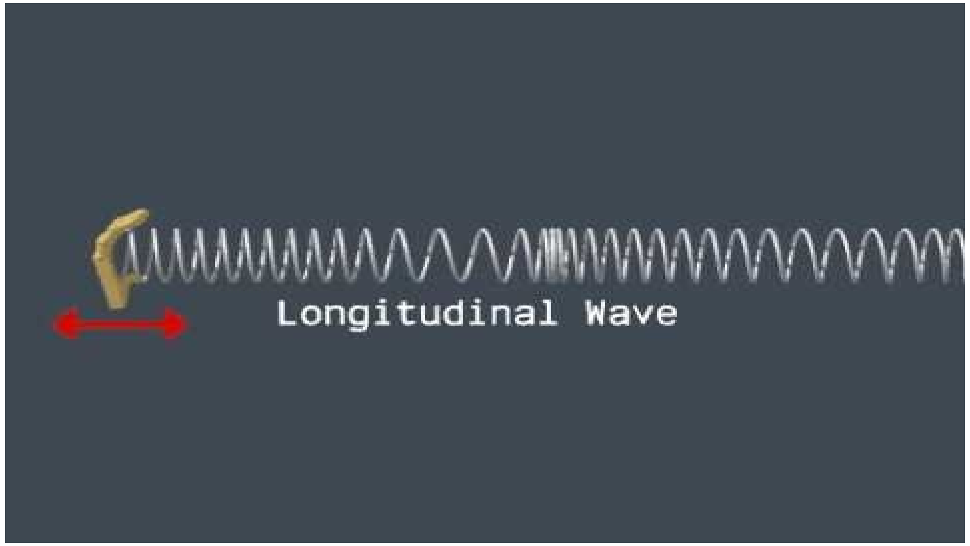



http://www.youtube.com/watch?v=UHcse1jJAto

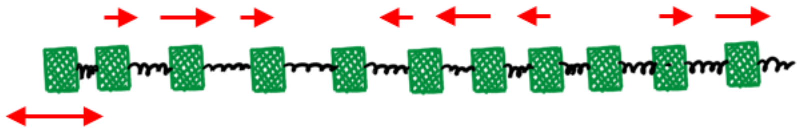


- oscillations are perpendicular to the direction wave travels

- Longitudinal wave:



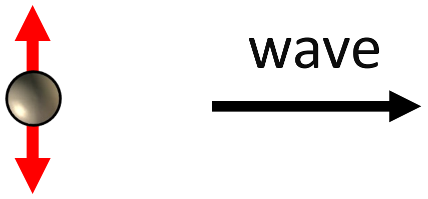

http://www.youtube.com/watch?v=aguCWnbRETU

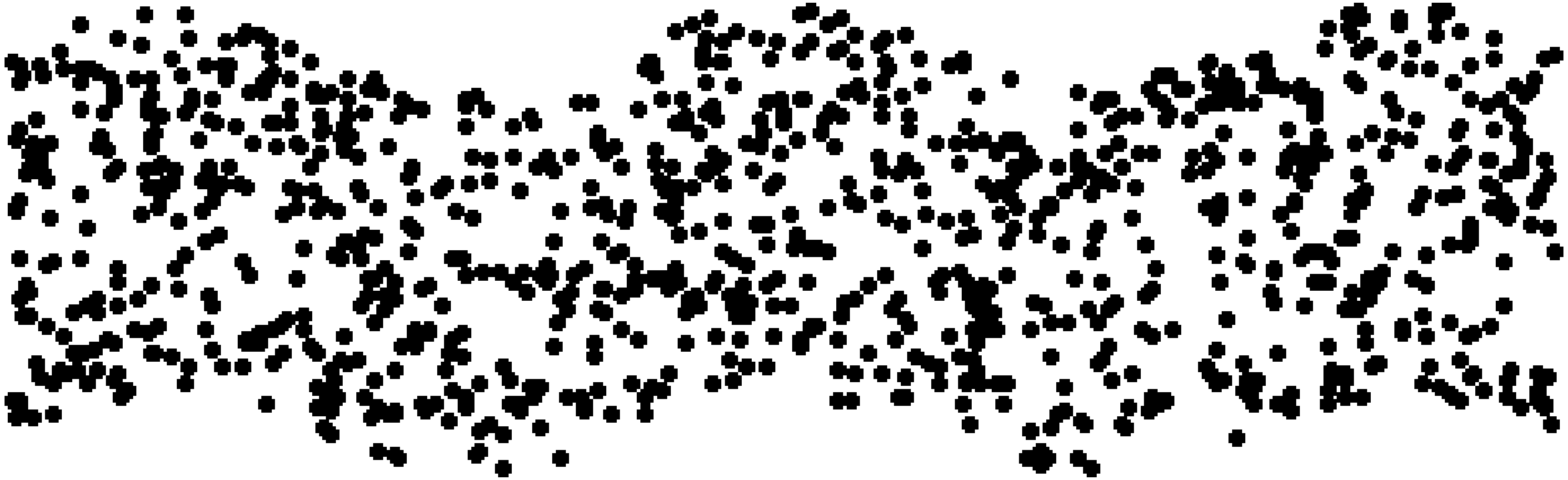


- oscillations are parallel to the direction wave travels

# Visualizing Transverse and Longitudinal Waves

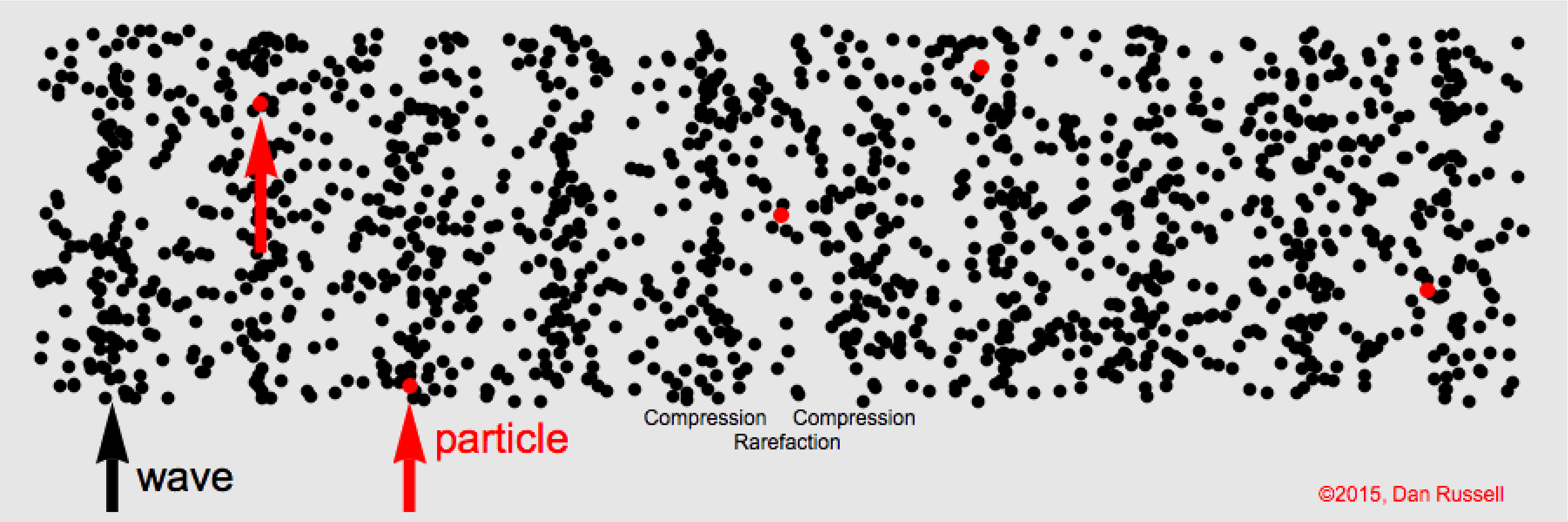
- Transverse Wave



- Longitudinal Wave

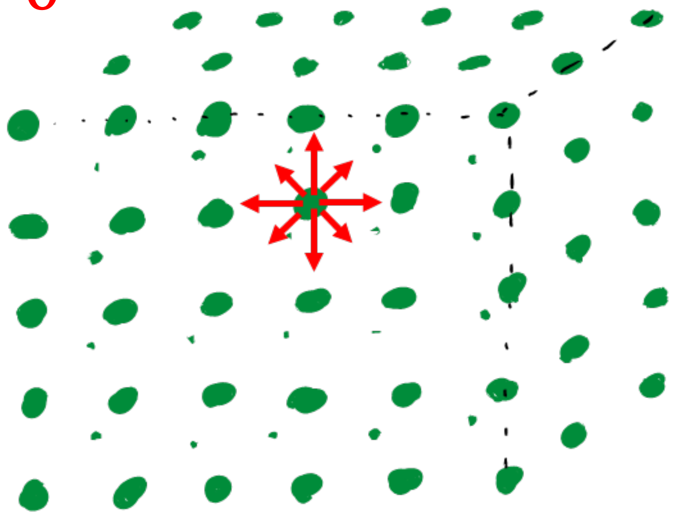

Density wave (alternating compressions and rarefactions)

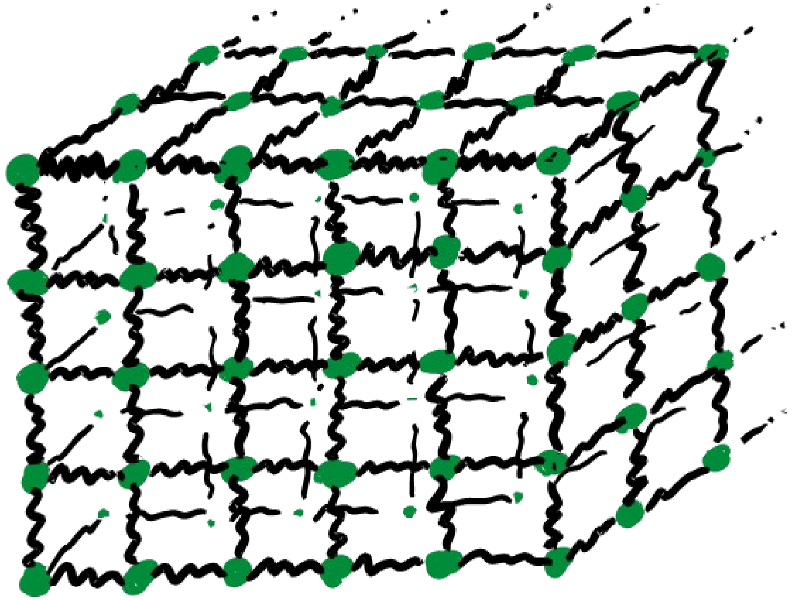



https://www.acs.psu.edu/drussell/Demos/waves/wavemotion.html

# Many physical systems act as coupled oscillators!

$F_{net} = 0$

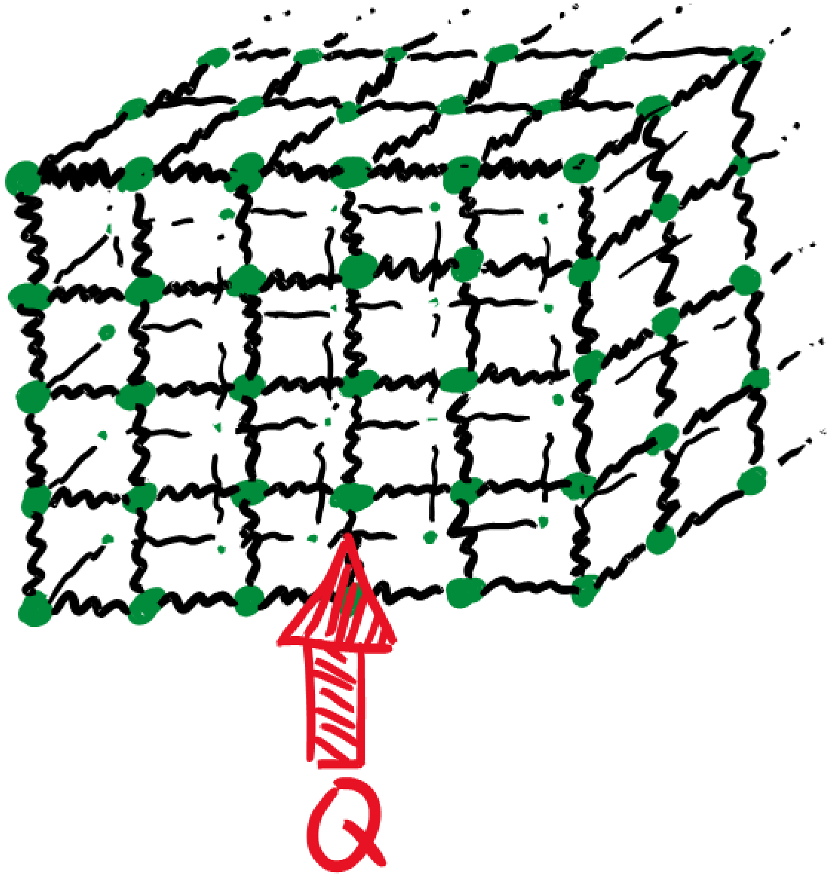


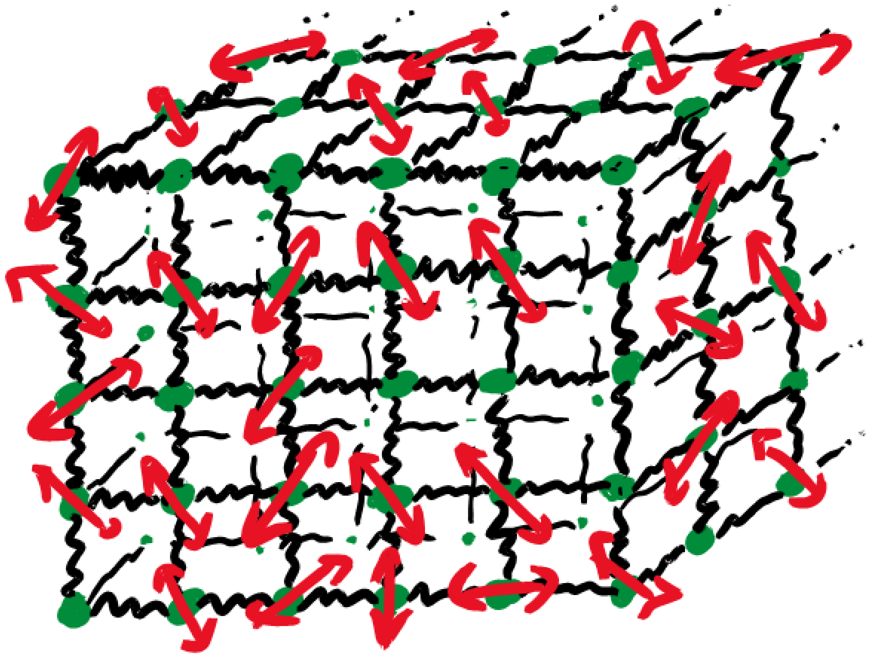


In a solid, each atom is in an equilibrium position…

…and is coupled to its neighbors by restoring electric forces => they are coupled oscillators!

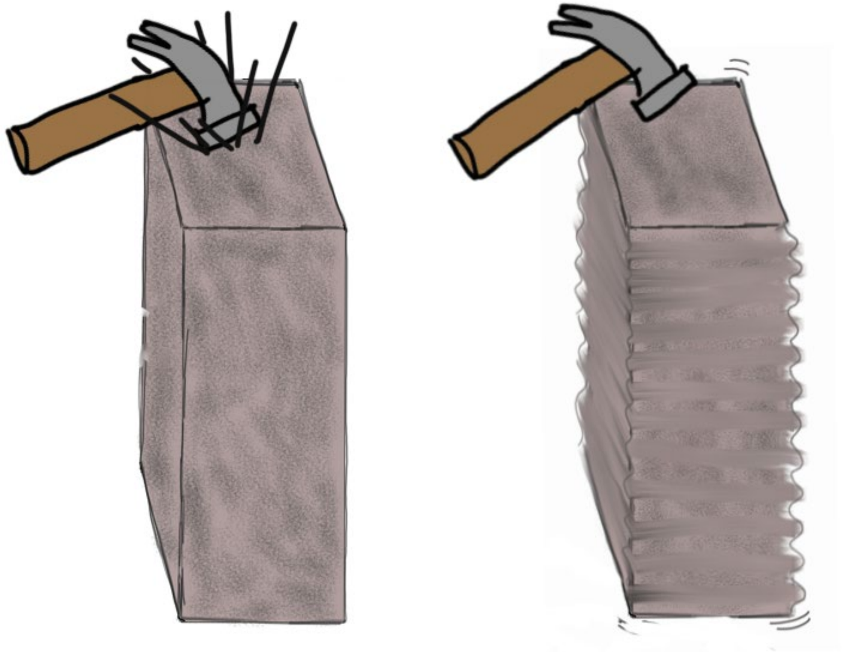
# Warm solid



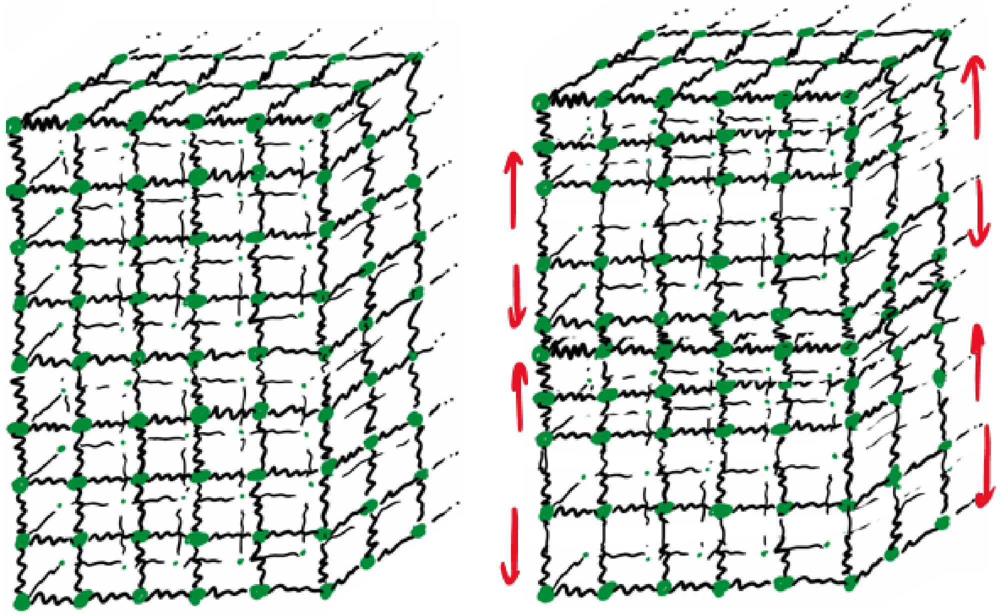
Each atom (oscillator) has small random oscillations about its equilibrium position

But: we can also have coordinated (driven) oscillations due to macroscopic external forces



equilibrium:

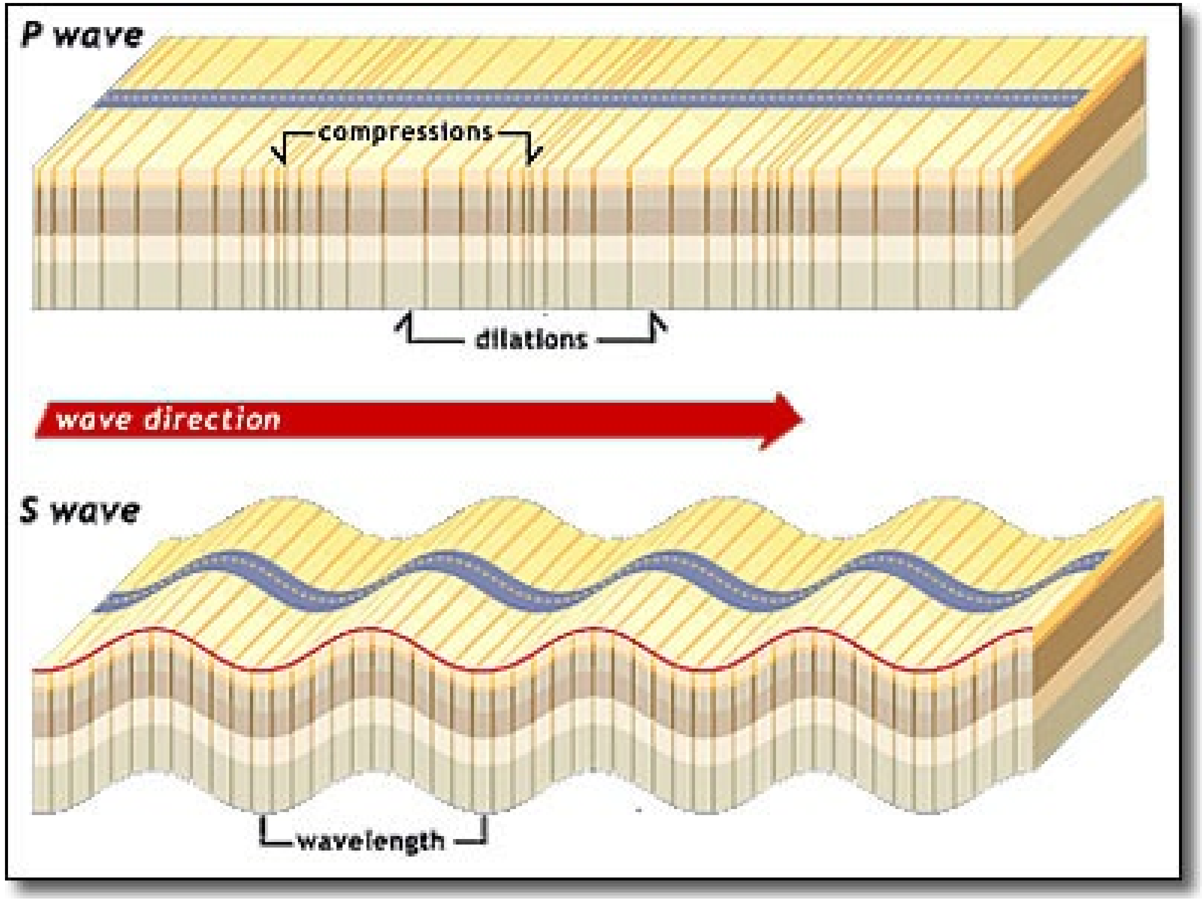
longitudinal vertical wave:



= sound wave in solid

# Solids support both longitudinal (compression) waves and transverse (shear) waves

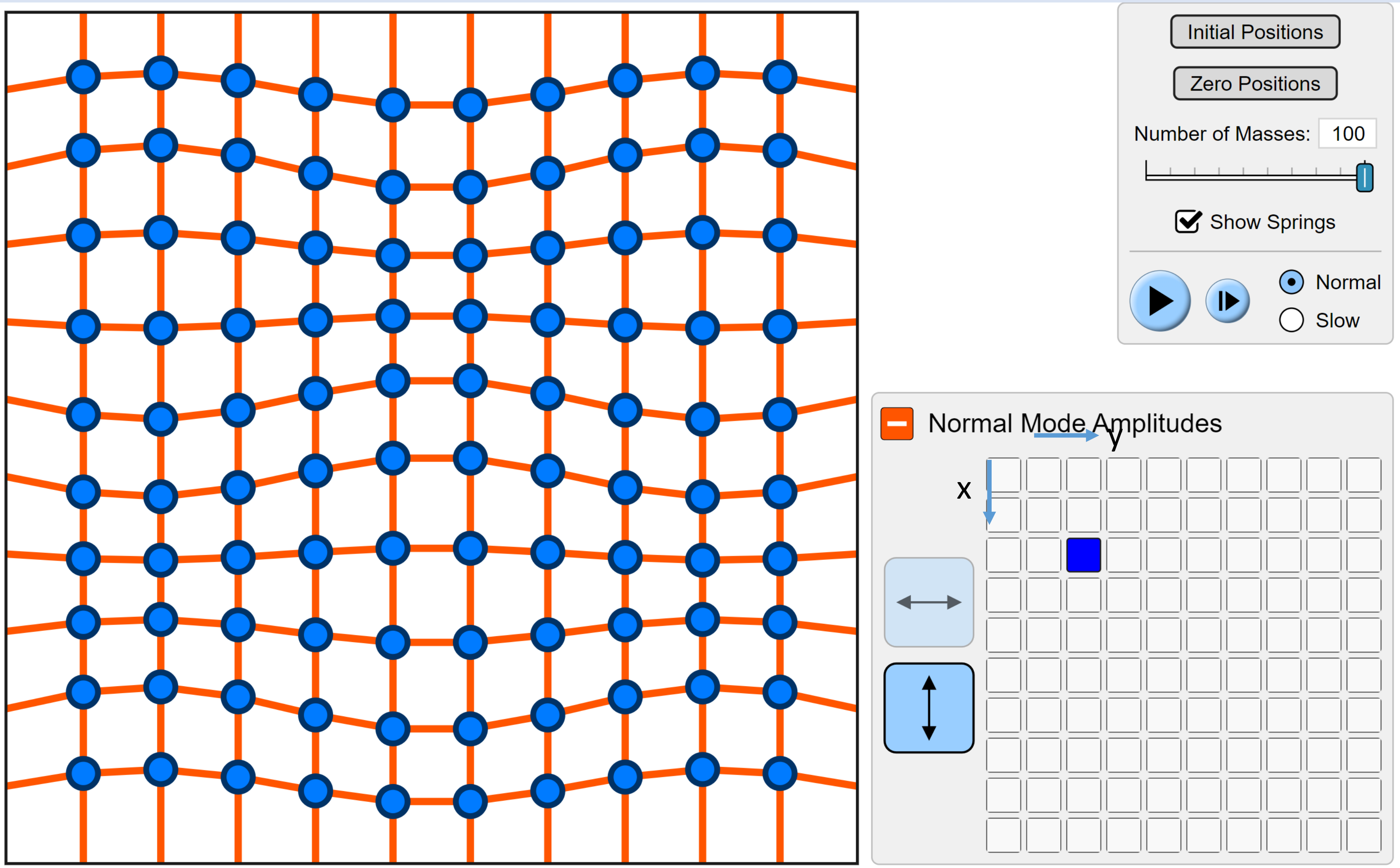
Example: *S* waves in earthquakes




Longitudinal
faster "primary"

Transverse
slower "secondary"

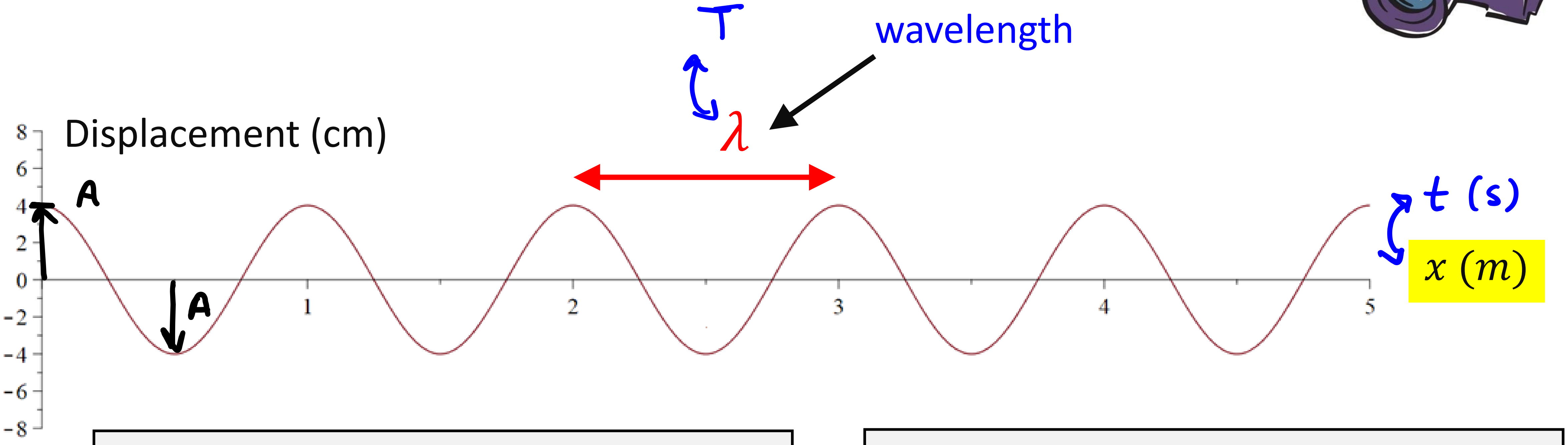
# Simulation of 1D and 2D coupled oscillators




https://phet.colorado.edu/sims/html/normal-modes/latest/normal-modes_all.html

# “Snapshot” graph

- Picture of the wave at an instant in time




?

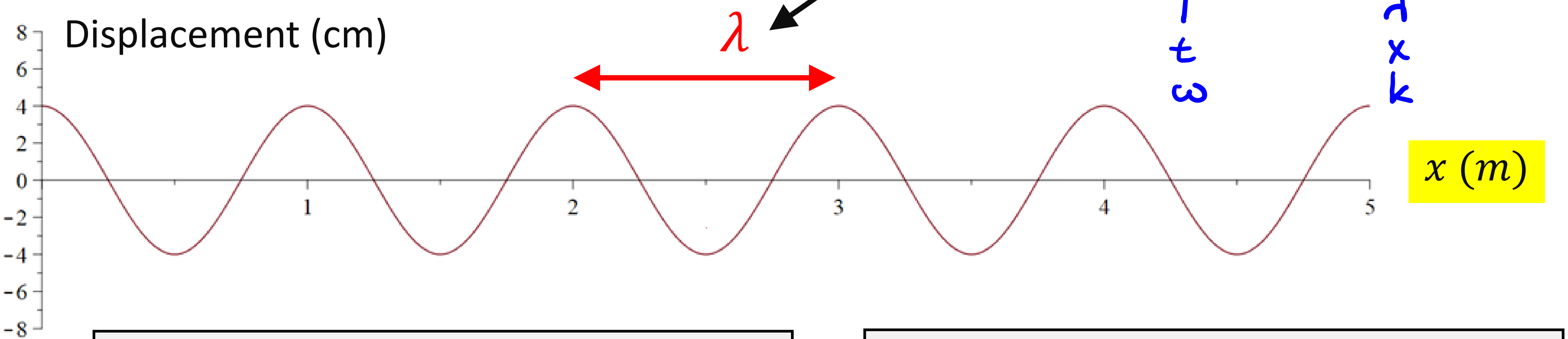
Simple Harmonic Oscillator:

$$\boxed{x(t) = A \cdot \cos(\omega t + \phi)}$$

angular frequency: $\omega = \dfrac{2\pi}{T}$

# “Snapshot” graph

- Picture of the wave at an instant in time




| SHO : | wave |
|---|---|
| $T$ | $\lambda$ |
| $t$ | $x$ |
| $\omega$ | $k$ |

**Snapshot of a wave:**

$$D(x) = A \cdot \cos(kx + \phi)$$

wave number: $k = \dfrac{2\pi}{\lambda}$

**Simple Harmonic Oscillator:**

$$x(t) = A \cdot \cos(\omega t + \phi)$$

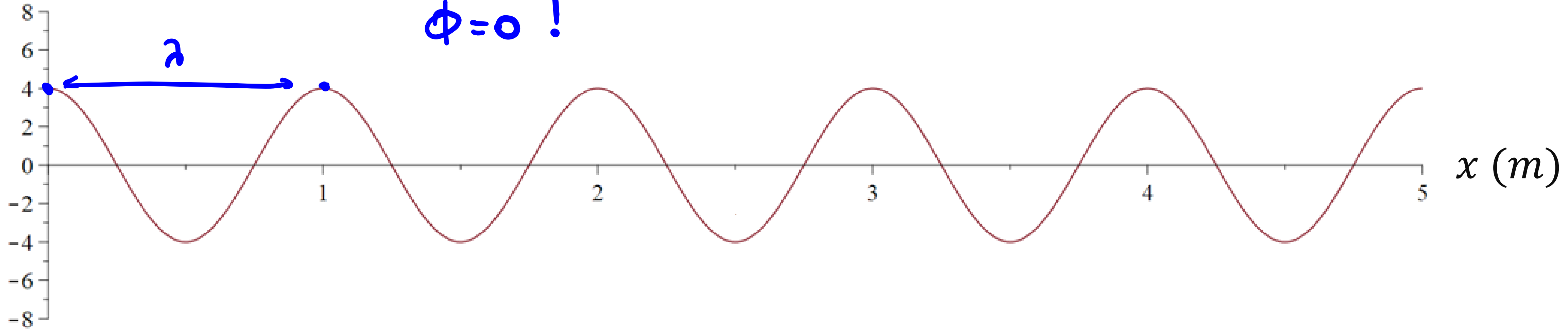
angular frequency: $\omega = \dfrac{2\pi}{T}$

Q: The picture shows a wave on a string at some time $t = 0$. Which of the following represents the displacement of the string as a function of position at $t = 0$?

$D(x) = A\cos(kx + \phi)$ $\quad k = \frac{2\pi}{\lambda}$

Displacement (mm)

$\phi = 0$ !




A. $4\text{ mm}\cos\left(\frac{x}{1\,m}\right)$

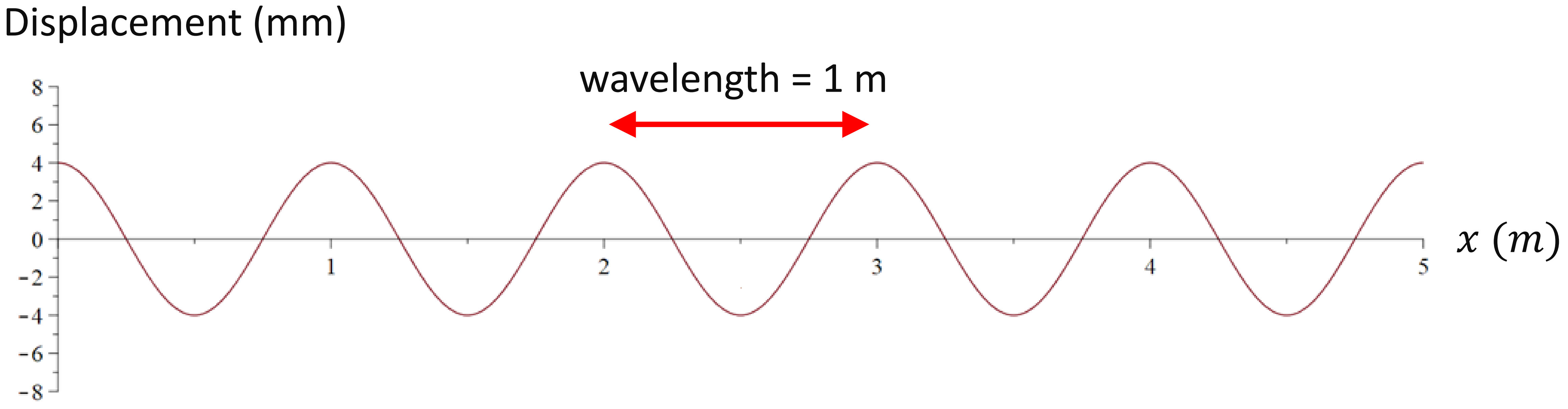
B. $4\text{ mm}\cos(1\,m \cdot x)$

C. $4\text{ mm}\cos\left(\frac{2\pi}{1\,m} \cdot x\right)$

D. $4\text{ mm}\cos\left(\frac{1\,m}{2\pi} \cdot x\right)$

E. $4\text{ mm}\cos(x - 1\,m)$

Q: The picture shows a wave on a string at some time $t = 0$. Which of the following represents the displacement of the string as a function of position at $t = 0$?




A. $4\text{ mm}\cos\left(\frac{x}{1\ m}\right)$

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C. $4\text{ mm}\cos\left(\frac{2\pi}{1\ m}\cdot x\right)$ ✓

D. $4\text{ mm}\cos\left(\frac{1\ m}{2\pi}\cdot x\right)$

E. $4\text{ mm}\cos(x - 1\ m)$

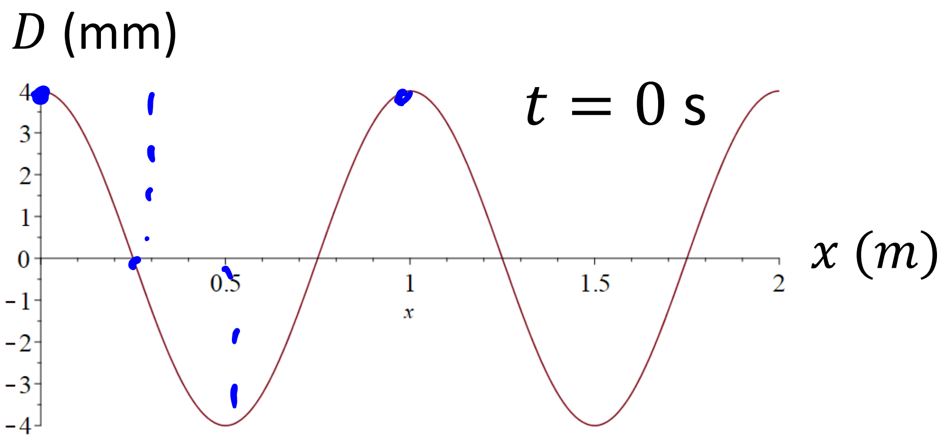
Just like for D vs t in an oscillator, but here $t$ is replaced by $x$ and $T$ is replaced by $\lambda$

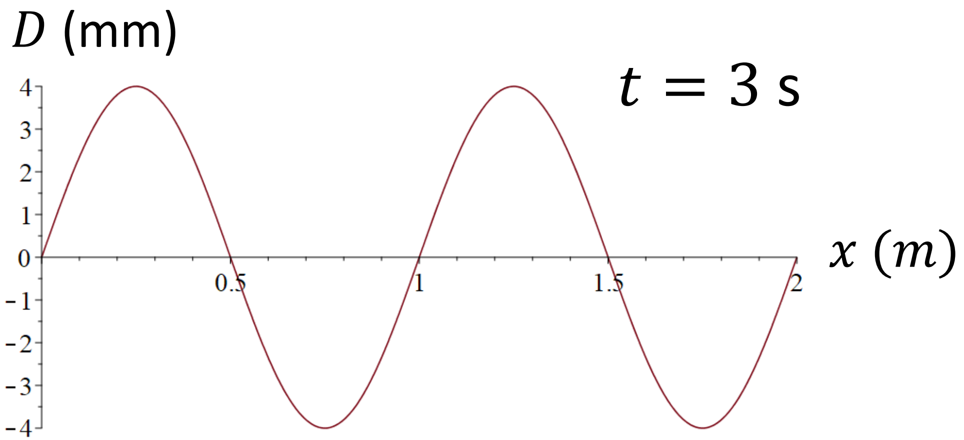
So $D(x) = A \cdot \cos\left(\frac{2\pi}{\lambda} \cdot x\right)$

Q: At $t = 0$, the displacement as a function of position for the wave shown is $D(x) = 4\text{ mm}\cos\left(\frac{2\pi}{1\,m}\cdot x\right)$. At $t = 3$ s, the wave has moved to the right, as shown in the second graph. The displacement as a function of position is now:

$D(x) = A\cos(kx + \phi)$

A. $D(x) = 4\text{ mm}\cdot\cos\left(\frac{2\pi}{1\,m}\cdot x - 3\,s\right)$
B. $D(x) = 4\text{ mm}\cdot\cos\left(\frac{2\pi}{1\,m}\cdot x + 3\,s\right)$
C. $D(x) = 4\text{ mm}\cdot\cos\left(\frac{2\pi}{1\,m}\cdot x - \frac{\pi}{2}\right)$ ✓
D. $D(x) = 4\text{ mm}\cdot\cos\left(\frac{2\pi}{1\,m}\cdot x + \frac{\pi}{2}\right)$
E. $D(x) = 4\text{ mm}\cdot\cos\left(\frac{2\pi}{1\,m}\cdot x + \frac{2\pi}{3\,s}\right)$




Q: How will it change as the time goes?

Q: At $t = 0$, the displacement as a function of position for the wave shown is $D(x) = 4\text{ mm} \cos\left(\frac{2\pi}{1\,m} \cdot x\right)$. At $t = 3$ s, the wave has moved to the right, as shown in the second graph. The displacement as a function of position is now:

- Shifted $\frac{1}{4}$ period to the right, so phase is $-\frac{2\pi}{4} = -\frac{\pi}{2}$

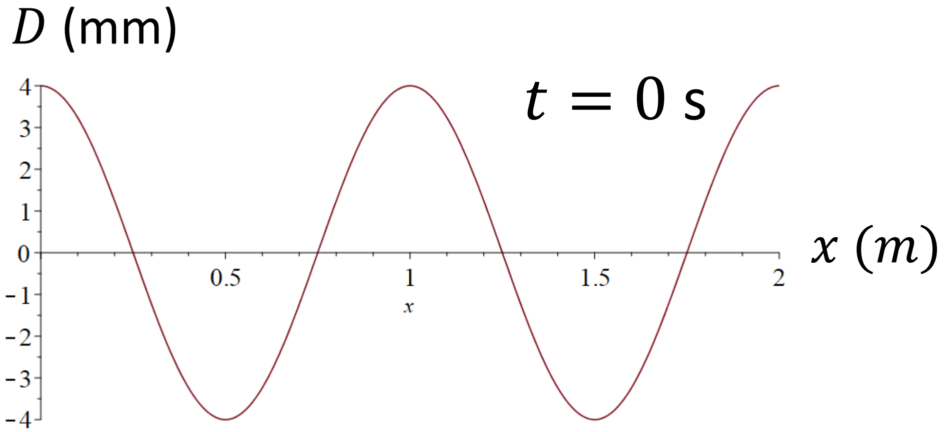
A. $D(x) = 4\text{ mm} \cdot \cos\left(\frac{2\pi}{1\,m} \cdot x - 3\,s\right)$

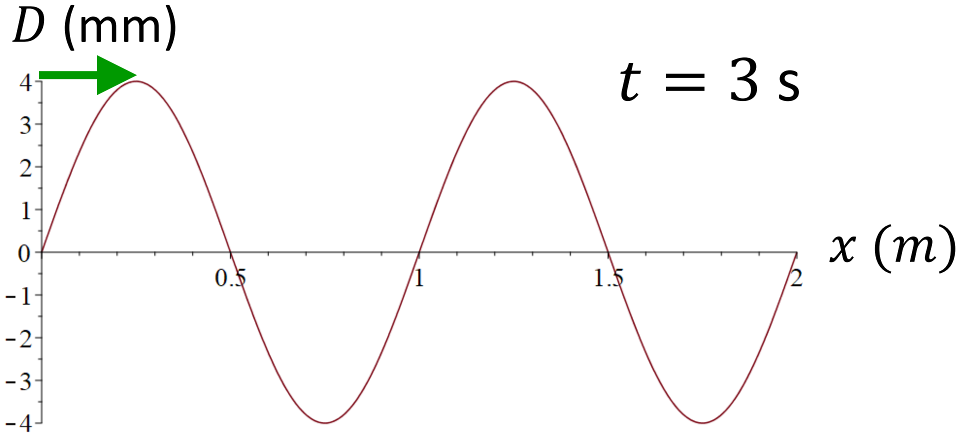
B. $D(x) = 4\text{ mm} \cdot \cos\left(\frac{2\pi}{1\,m} \cdot x + 3\,s\right)$

C. $D(x) = 4\text{ mm} \cdot \cos\left(\frac{2\pi}{1\,m} \cdot x - \frac{\pi}{2}\right)$ ✔

D. $D(x) = 4\text{ mm} \cdot \cos\left(\frac{2\pi}{1\,m} \cdot x + \frac{\pi}{2}\right)$

E. $D(x) = 4\text{ mm} \cdot \cos\left(\frac{2\pi}{1\,m} \cdot x + \frac{2\pi}{3\,s}\right)$




Q: How will it change as the time goes?

- As the time goes, the phase will increase