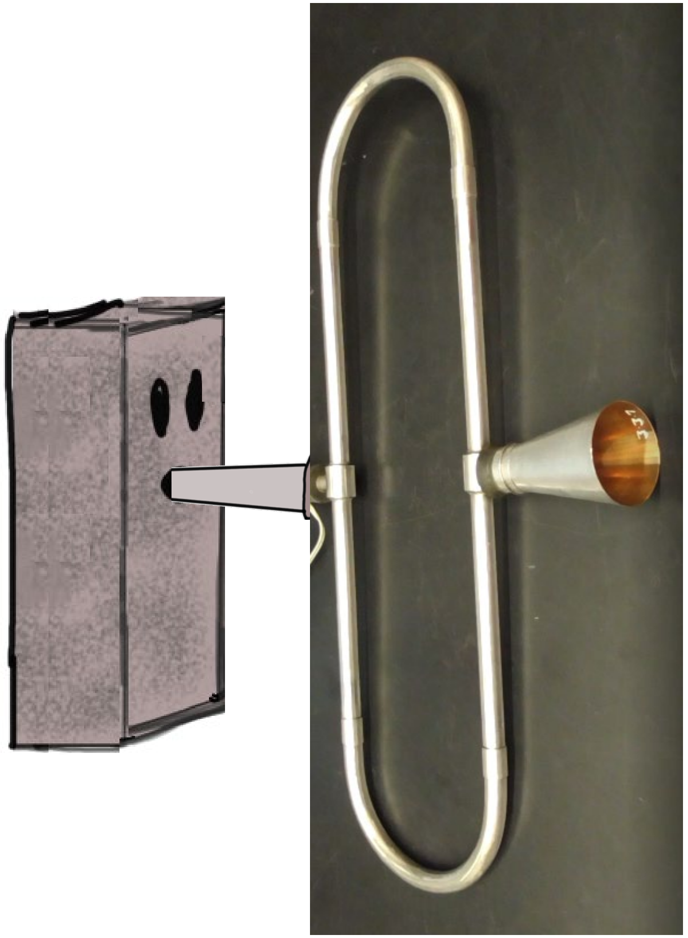
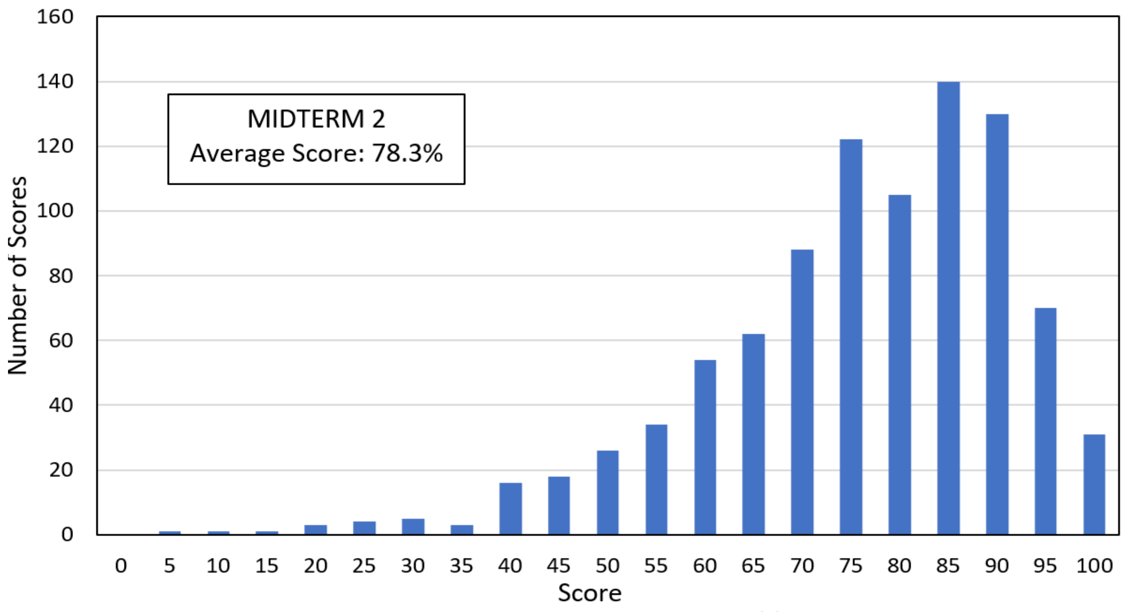
Lecture 33.

Standing sound waves (finish).

Interference.



Midterm 2 (2023)



If you didn’t do well on the written part:

- On the final, demonstrate what you DO know for the written problems, so we can give you credit for it, even if you aren't sure, or are stuck, on some parts.

If you want to adjust your learning strategy:

- The best way to learn is deliberate practice
- Talk with other people about PHYS 157 stuff
- Try “teaching” mini-lectures/tutorials on PHYS 157 stuff

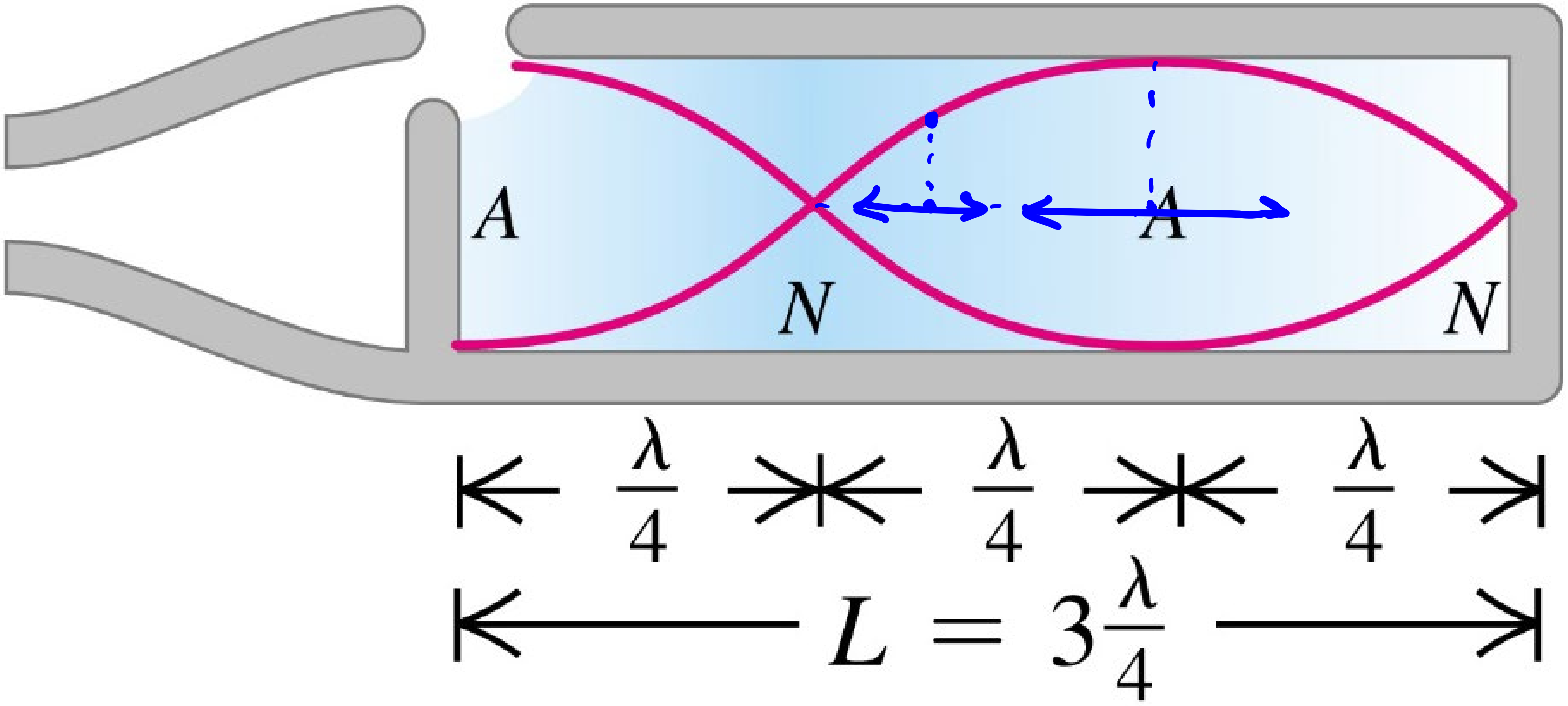
Interact with the teaching team and your peers:

- My time: Tuesdays 10:30 onwards
- Cannot attend? Check other instructors
- HW help session (Mon-Tue 5:00 pm)
- Hebb 112 is an undergrad drop-in center: Come to do your homework with your classmates, chat about difficult concepts, ask your questions, help your peers

Overall course score

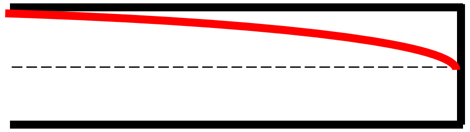
Reading Quizzes	3%
Tutorials	6%
MP Homework	6%
Written Homework	9%
Quizzes	18%
First midterm	13%
Second midterm	13%
Final exam	32%

Last Time:



A
N
A
N
$\frac{\lambda}{4}$
$\frac{\lambda}{4}$
$\frac{\lambda}{4}$
$L = 3\frac{\lambda}{4}$

Last Time:



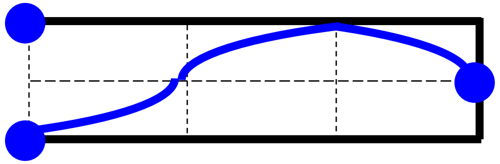
$f = 220\text{ Hz}$ $\quad f\lambda = 343\,\frac{m}{s}$ $\quad f = 660\text{ Hz}$

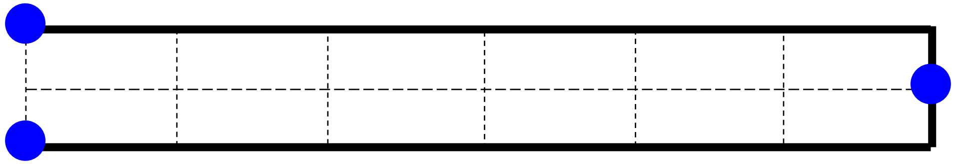
One Closed End

$$f_n = n_{odd}\underbrace{\left(\frac{v}{4L}\right)}_{=\,220\text{ Hz}} = n_{odd} f_1$$

1, 3, 5, 7...

$f_1(L) = 220\text{ Hz}$
$f_3(L) = 660\text{ Hz}$





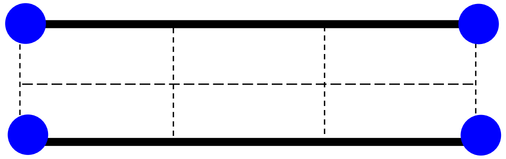
$f_1(2L) = 220/2 = 110\text{ Hz}$
$f_3(2L) = 330\text{ Hz}$
$f_5(2L) = 550\text{ Hz}$
$f_7(2L) = 770\text{ Hz}$

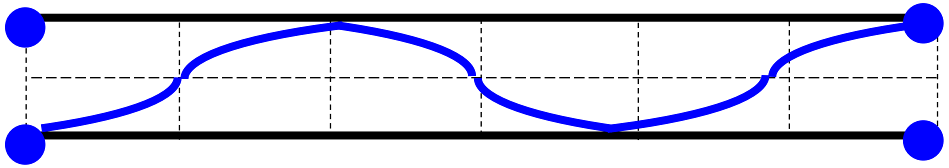
Two Open Ends

n = 1, 2, 3, 4...

$$f_n = n\frac{v}{2L} = nf_1$$

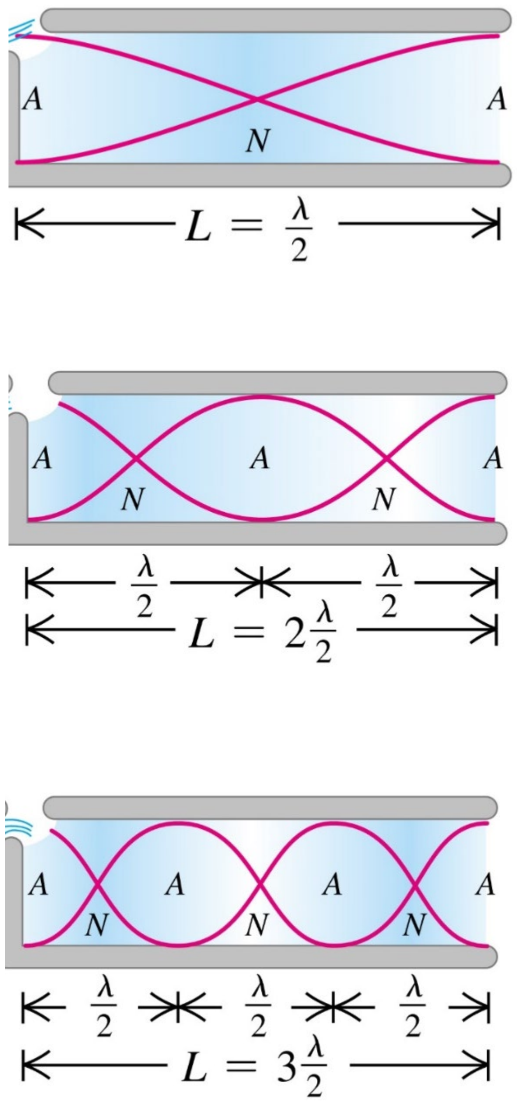
$f_1(L) = 220 \cdot 2 = 440\text{ Hz}$
$f_2(L) = 880\text{ Hz}$





$f_1(2L) = 220 \cdot 2/2 = 220\text{ Hz}$
$f_2(2L) = 440\text{ Hz}$
$f_3(2L) = 660\text{ Hz}$

Boundary Conditions: Sounds Waves in a Pipe (Summary)



Two Open Ends

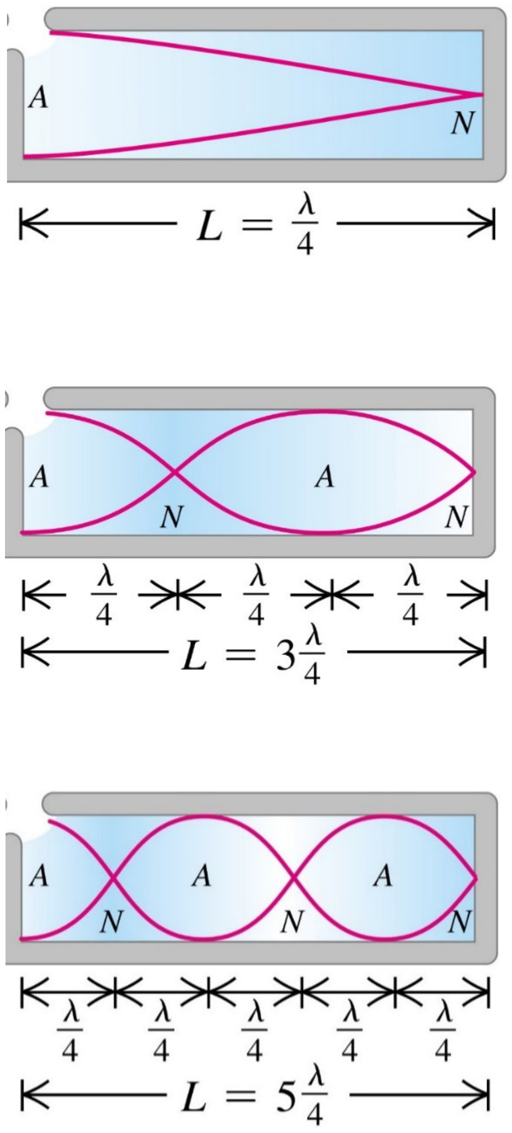
All harmonics are possible

$$n = 1, 2, 3, 4, \ldots$$

$$\lambda_n = 2L/n$$

$$f_n = n\frac{v}{2L} = nf_1$$

Same formulas (but different pictures) for Two Closed Ends



One Closed End

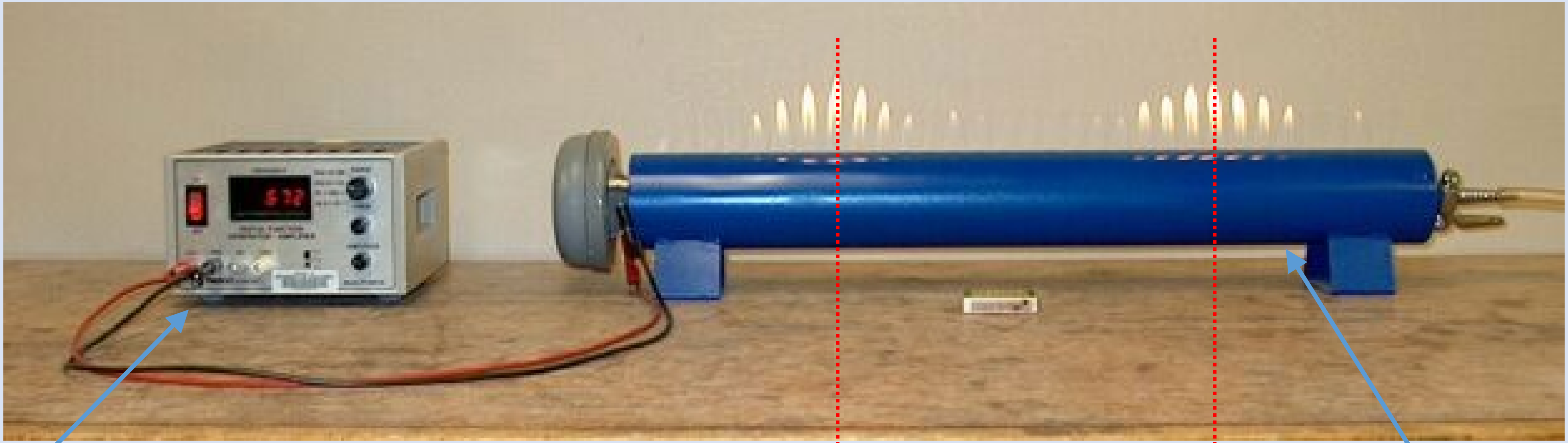
Only odd harmonics are possible

$$n_{odd} = 1, 3, 5, \ldots$$

$$\lambda_n = 4L/n$$

$$f_n = n\frac{v}{4L} = nf_1$$

Ruben's Tube: Visualizing Longitudinal Standing Waves



Function Generator (varies f)

$f = {}^{v}/_{\lambda}$

${}^{\lambda}/_{2}$

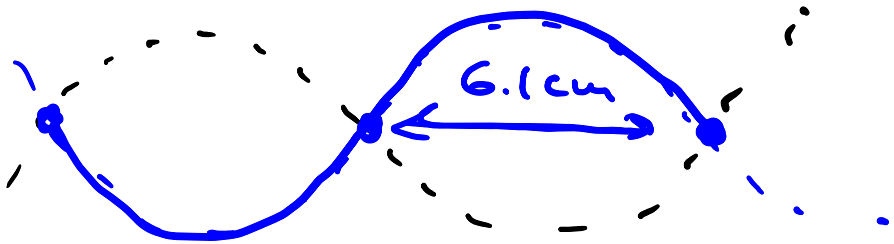
Tube filled with propane

- Speed of sound in propane is $v \approx 250\ m/s$
- Should see $\lambda \approx 0.5\ m$ for $f \approx 500\ Hz$
- A full understanding requires very complicated fluid dynamics (well beyond the level of this course)

Theory: https://www.youtube.com/watch?v=BbPgy4sHYTw

Q: The rotating plate in a microwave is used to avoid 'cold spots' in the food. Without the rotating plate you will notice cold spots 6.1 cm apart. These cold spots are at the nodes of the standing wave inside the oven.

What is the frequency of the microwaves?



A. $2.5 \times 10^{7}\ Hz$

B. $5.0 \times 10^{7}\ Hz$

C. $2.5 \times 10^{9}\ Hz$

D. $5.0 \times 10^{9}\ Hz$

E. $10.0 \times 10^{9}\ Hz$

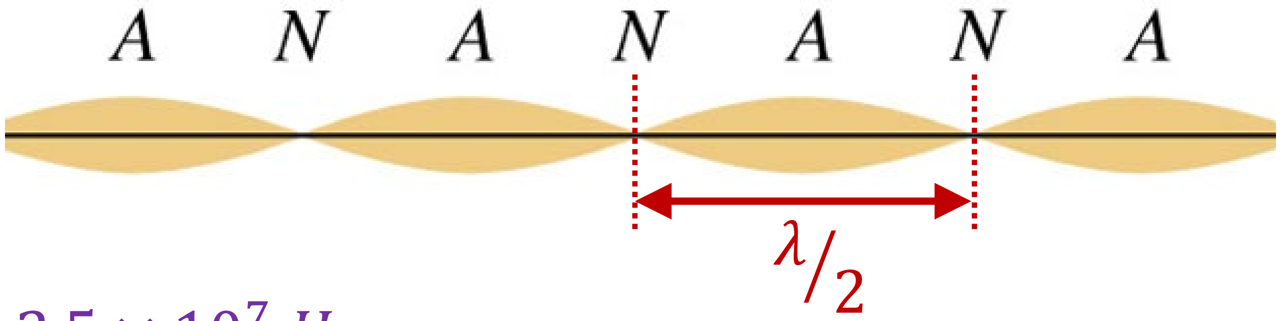
$6.1\ cm = \lambda/2$

$c = \lambda \cdot f$

NOTE: Microwaves travel at the speed of light, which is $3 \times 10^{8}\ m/s$

Q: The rotating plate in a microwave is used to avoid ‘cold spots’ in the food. Without the rotating plate you will notice cold spots 6.1 cm apart. These cold spots are at the nodes of the standing wave inside the oven.

What is the frequency of the microwaves?



A. $2.5 \times 10^7\ Hz$

B. $5.0 \times 10^7\ Hz$

C. $2.5 \times 10^9\ Hz$ ✓

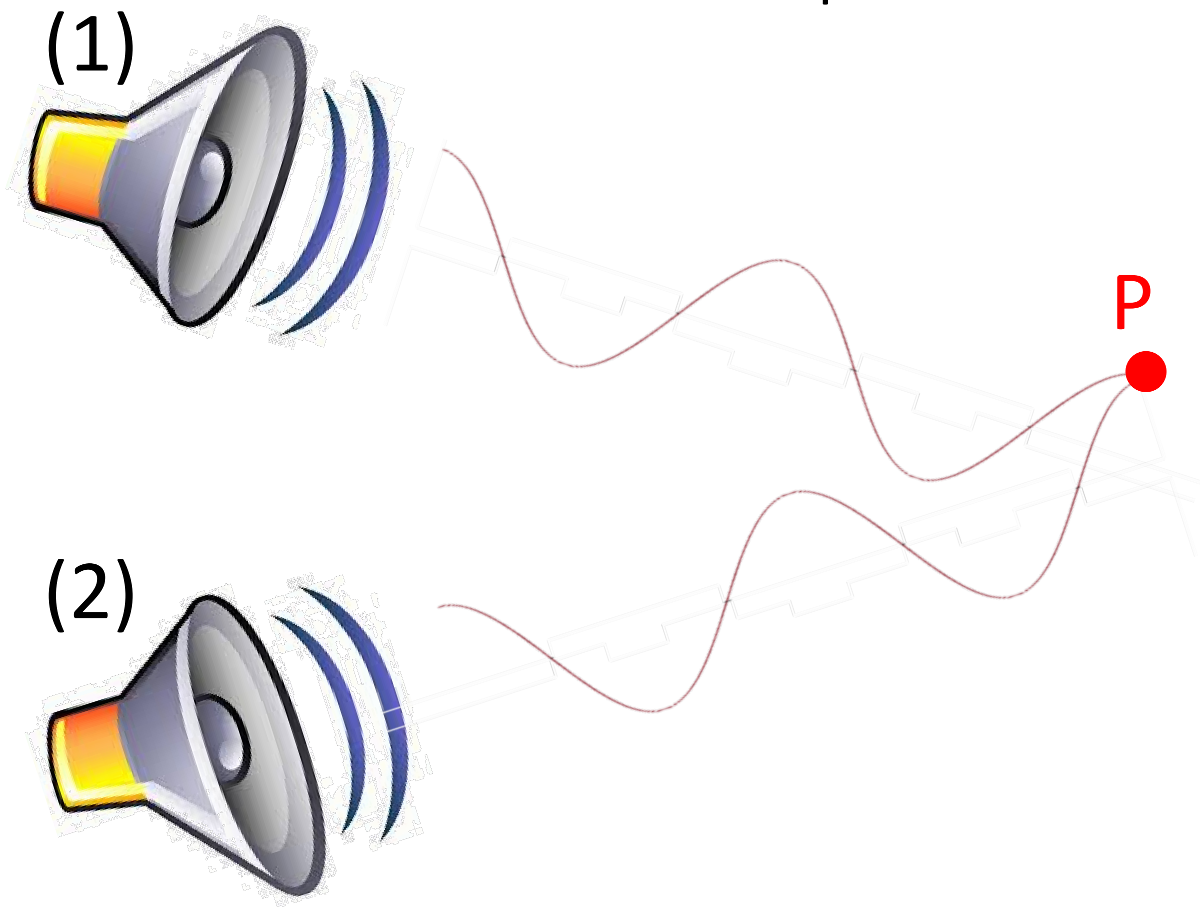
D. $5.0 \times 10^9\ Hz$

E. $10.0 \times 10^9\ Hz$

$$f = \frac{v}{\lambda} = \frac{3 \times 10^8\ m/s}{12.2 \times 10^{-2}\ m} \approx 2.5 \times 10^9\ Hz$$

NOTE: Microwaves travel at the speed of light, which is $3 \times 10^8\ m/s$

Interference of waves



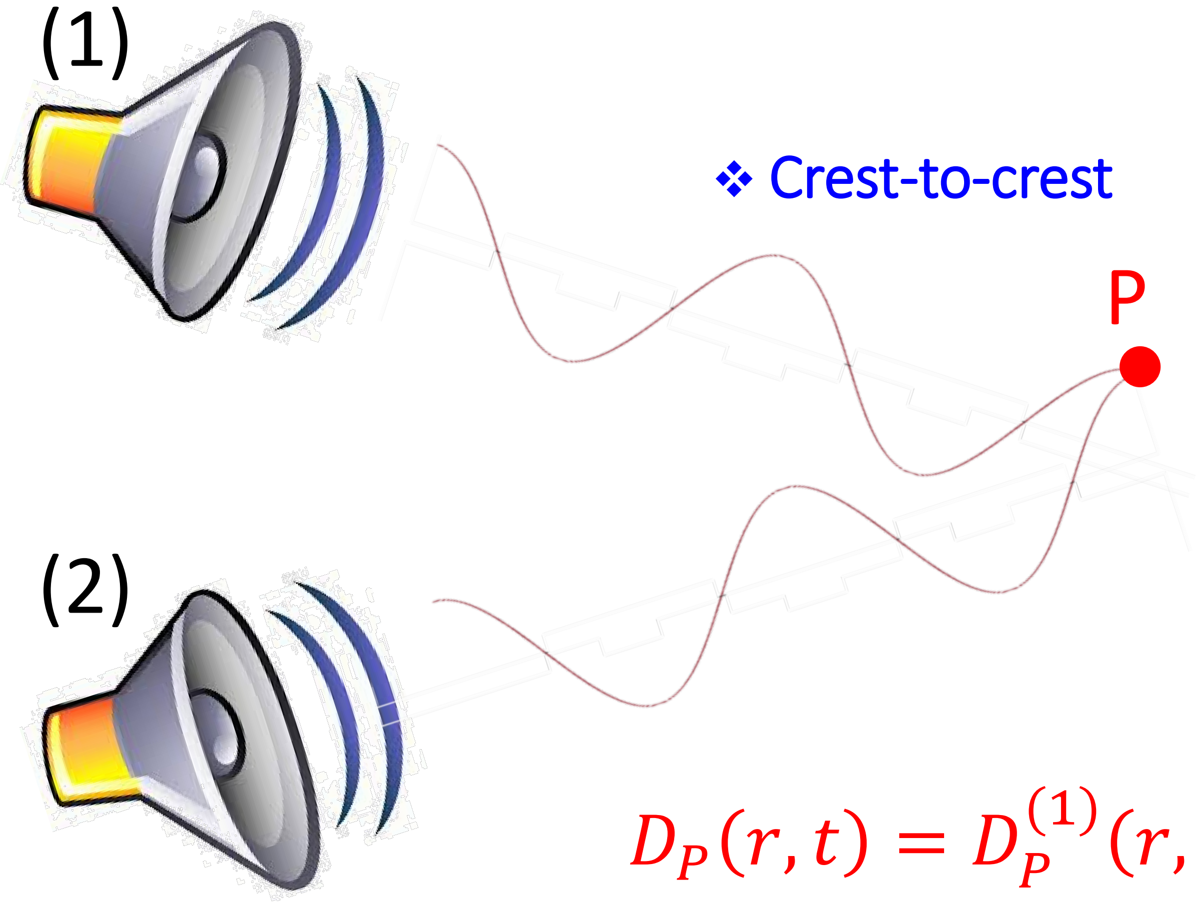
- Consequence of the Principle of Superposition

- Assume we have waves from two sources

- Displacement at point P is the sum of the displacements from the two individual waves

Interference of waves

"in phase" at point P

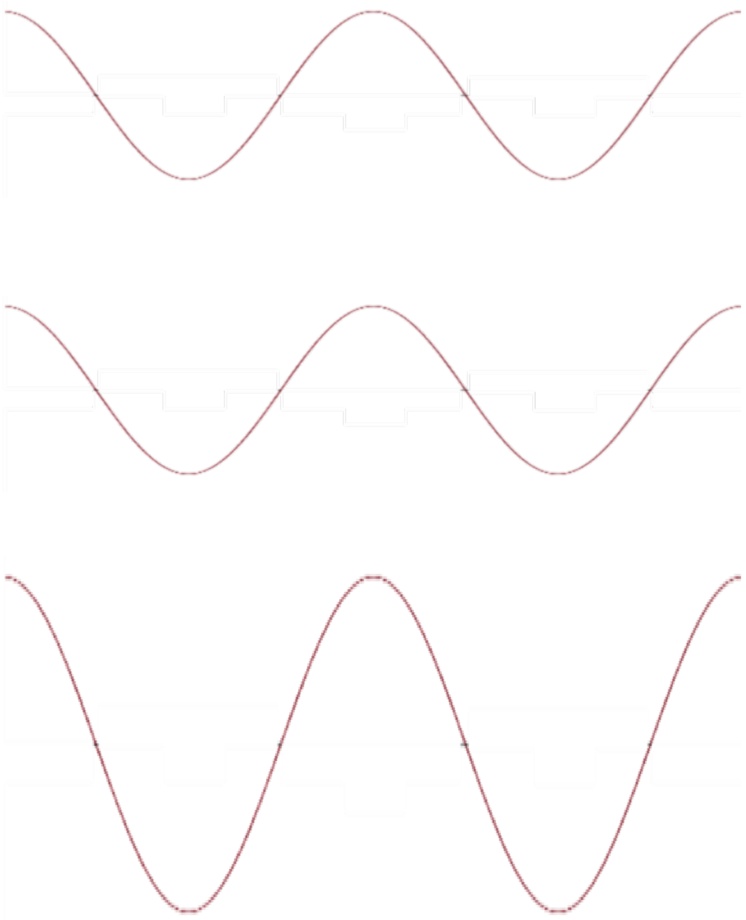


❖ Crest-to-crest

$D_P^{(1)}(r,t)$:

$D_P^{(2)}(r,t)$:

$D_P(r,t) = D_P^{(1)}(r,t) + D_P^{(2)}(r,t)$:

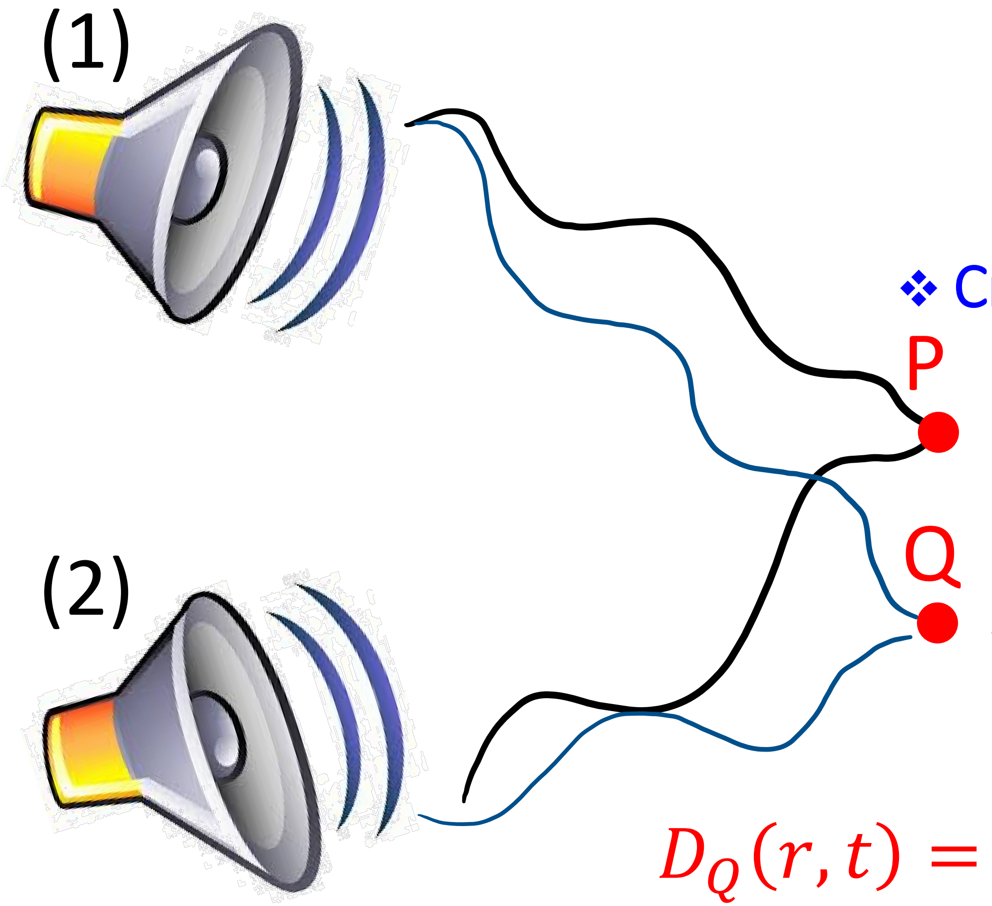


➢ Constructive Interference

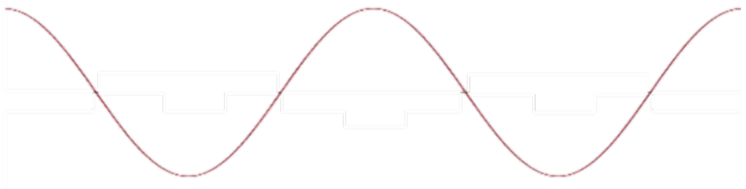
The two waves enhance each other

Interference of waves

"out of phase" at point Q

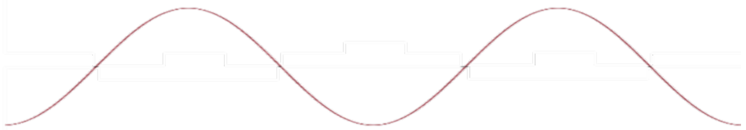


$D_Q^{(1)}(r,t)$:



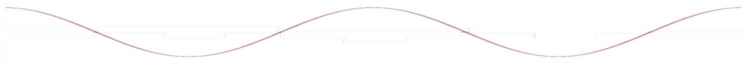
- ❖ Crest-to-crest

$D_Q^{(2)}(r,t)$:



- ❖ Crest-to-trough

$$D_Q(r,t) = D_Q^{(1)}(r,t) + D_Q^{(2)}(r,t):$$



- ➢ Destructive Interference

The two waves suppress each other

Phase of a wave

- Initial phase of each wave

(related to the initial condition, i.e. $D(r = 0, t = 0)$)

$$D_1(r_1, t) = A\cos(\underbrace{kx - \omega t + \phi_1}_{\Phi_1(x,t)})$$

$$D_2(r_2, t) = A\cos(\underbrace{kx - \omega t + \phi_2}_{\Phi_2(x,t)})$$

- Phase of each wave: $\Phi(x, t) = kx - \omega t + \phi$

- The result of interference (crest-to-crest, or crest-to-trough, or something in between) is determined by the phase difference, $\Delta\Phi = \Phi_2 - \Phi_1$ between two waves.

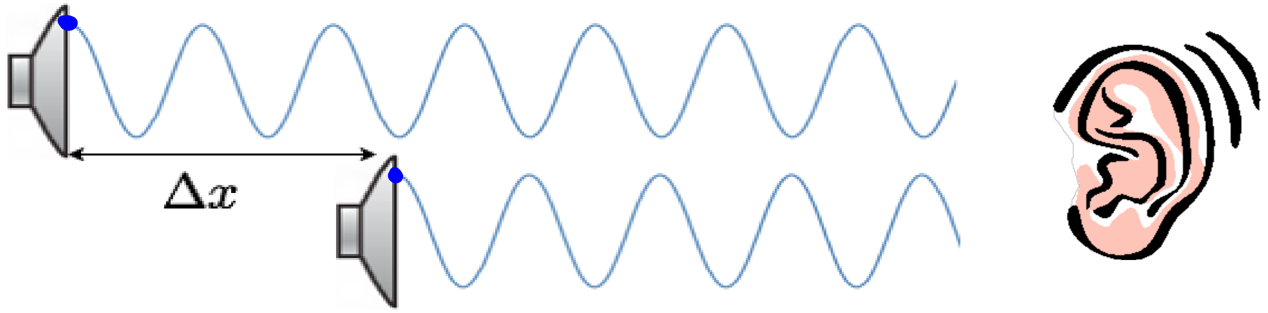
Path difference & phase difference for two waves

$$\boxed{\Phi(x,t) = kx - \omega t + \phi}$$

- The path difference: the distance one wave travels compared to another

different travelled distance

same initial phase



same ω, same k

$$\Phi_1(x,t) = kx_1 - \omega t + \phi$$

$$\Phi_2(x,t) = kx_2 - \omega t + \phi$$

- Here the wave from the top speaker must travel further than the wave from the bottom speaker before they reach the observer.

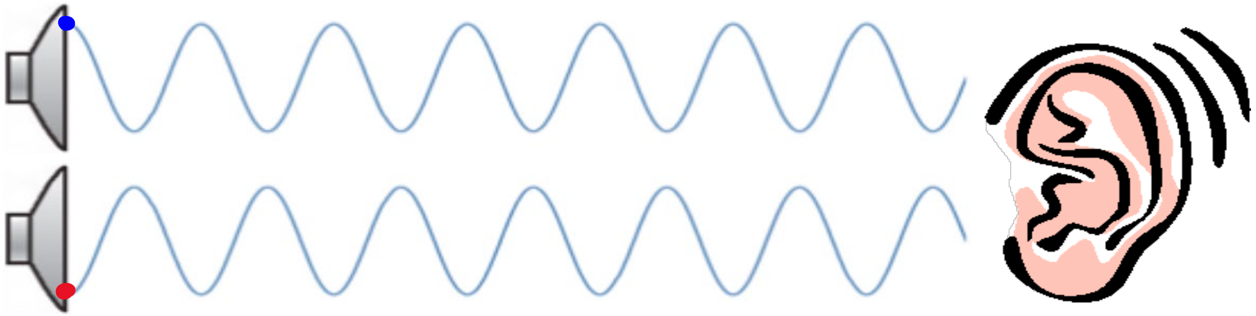
- This creates a phase difference: $$\boxed{\Delta\Phi_{\text{path}} = k\,\Delta x}$$

Initial phase difference & phase difference for two waves

- The inherent (initial) phase difference is phase difference between two sources

same ω, same k, same travelled distance

different initial phases



$$\Phi_1(x,t) = kx - \omega t + \phi_1$$

$$\Phi_2(x,t) = kx - \omega t + \phi_2$$

- These speakers are producing waves that are out of phase by $\Delta\phi_0 = \pi$ radians

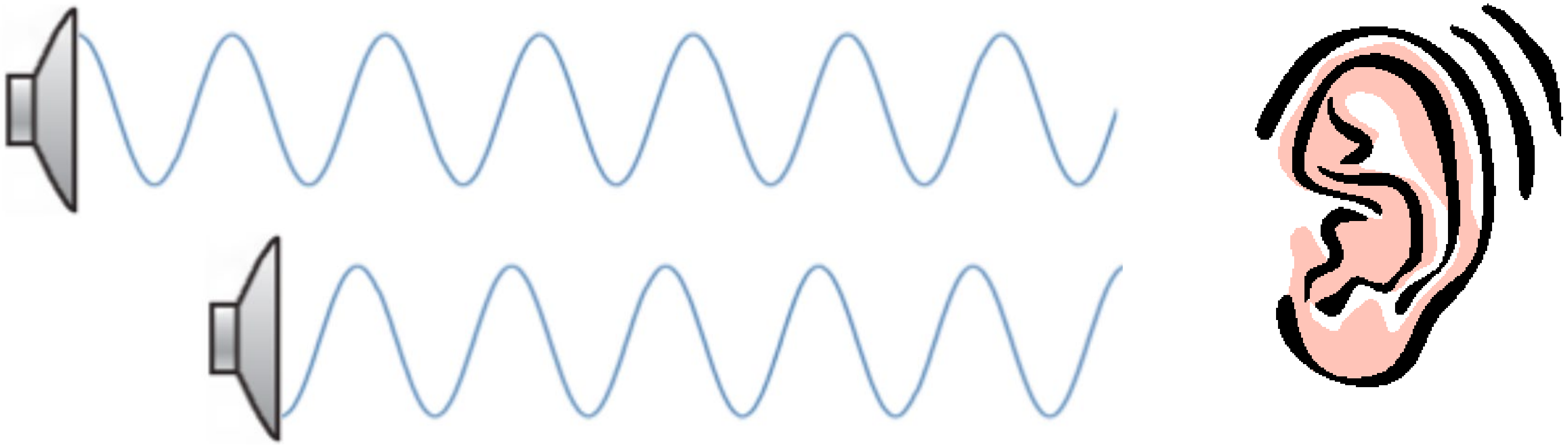
- This creates a phase difference: $\boxed{\Delta\Phi_{\text{initial phase}} = \Delta\phi_0}$ $= \phi_2 - \phi_1$

Total phase difference between two waves

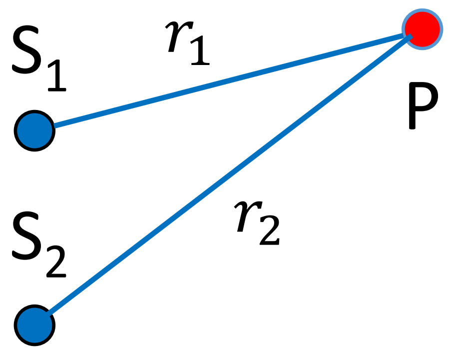
- The overall phase difference between two waves is going to be given by the sum of the initial phase difference and the phase difference due to different path lengths:

$$\boxed{\Delta\Phi \;=\; k\Delta x + \Delta\phi_0 \;=\; 2\pi\frac{\Delta x}{\lambda} + \Delta\phi_0}$$

$k = \frac{2\pi}{\lambda}$



Adding two waves mathematically



$$D_1(r_1, t) = A\cos(kr_1 - \omega t + \phi_1) = A\cos\Phi_1(r_1, t)$$

$$D_2(r_2, t) = A\cos(kr_2 - \omega t + \phi_2) = A\cos\Phi_2(r_2, t)$$

- Then $D_{\text{total}} = D_2 + D_1$ is:

❖Trig: $\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{b-a}{2}\right)$

$$D_{\text{total}} = 2A\cos\left(\frac{\Phi_1+\Phi_2}{2}\right)\cos\left(\frac{\Phi_1-\Phi_2}{2}\right)$$

cos = ±1 : constructive

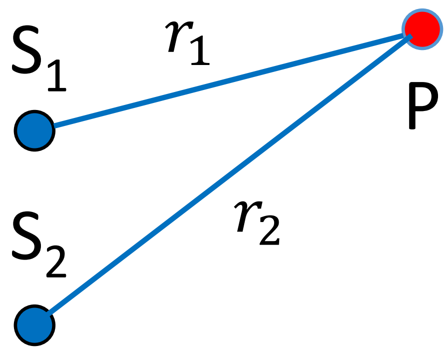
cos = 0 : destructive

$$D_{\text{total}} = 2A\underbrace{\cos\left(\frac{k(r_2+r_1)}{2} - \omega t + \frac{(\phi_2+\phi_1)}{2}\right)}_{\text{Travelling wave!}}\underbrace{\overbrace{\cos\left(\frac{1}{2}(k\Delta r_{12} + \Delta\phi_{12})\right)}}_{\text{Amplitude modulation – depends on } \Delta r \text{ and } \Delta\phi\text{, not time}}$$

$$k\Delta r_{12} = \frac{2\pi}{\lambda}(r_2 - r_1)$$

$$\Delta\phi_{12} = (\phi_2 - \phi_1)$$

Constructive and destructive interference



$$D_{\text{total}} = 2A\cos\left(\frac{\Phi_1+\Phi_2}{2}\right)\cos\left(\frac{\Phi_1-\Phi_2}{2}\right)$$

$$k\Delta r_{12} = \frac{2\pi}{\lambda}(r_2 - r_1)$$

$$\Delta\phi_{12} = (\phi_2 - \phi_1)$$

$$D_{\text{total}} = 2A\underbrace{\cos\left(\frac{k(r_2+r_1)}{2} - \omega t + \frac{(\phi_2+\phi_1)}{2}\right)}_{\text{Travelling wave!}}\underbrace{\cos\left(\frac{1}{2}(k\Delta r_{12} + \Delta\phi_{12})\right)}_{\text{Amplitude modulation – depends on } \Delta r \text{ and } \Delta\phi \text{, not time}}$$

- For maximum amplitude: $\cos(\dots) = 1$ & for minimum amplitude: $\cos(\dots) = 0$. Hence,

Constructive interference:	$k\Delta r_{12} + \Delta\phi_{12} = n \cdot 2\pi$	(crest-to-crest)
Destructive interference:	$k\Delta r_{12} + \Delta\phi_{12} = n_{odd} \cdot \pi$	(crest-to-trough)

Interference: Summary

- There can be both path difference (Δr_{12}) and a phase difference ($\Delta\phi_{12}$) between two sources: (here we used $k = 2\pi/\lambda$)

$$\boxed{\begin{array}{ll} \text{Constructive Interference:} & 2\pi(\Delta r_{12}/\lambda) + \Delta\phi_{12} = n \cdot 2\pi \\ \text{Destructive Interference:} & 2\pi(\Delta r_{12}/\lambda) + \Delta\phi_{12} = n_{odd} \cdot \pi \end{array}}$$

$n = 0, 1, 2, \ldots$
$n_{odd} = 1, 3, 5, \ldots$

- We can express this in terms of effective distance: (multiply by $\lambda/2\pi$)

$$\boxed{\begin{array}{ll} \text{Constructive Interference:} & \Delta r_{12} + \lambda(\Delta\phi_{12}/2\pi) = n \cdot \lambda \\ \text{Destructive Interference:} & \Delta r_{12} + \lambda(\Delta\phi_{12}/2\pi) = n_{odd} \cdot \lambda/2 \end{array}}$$

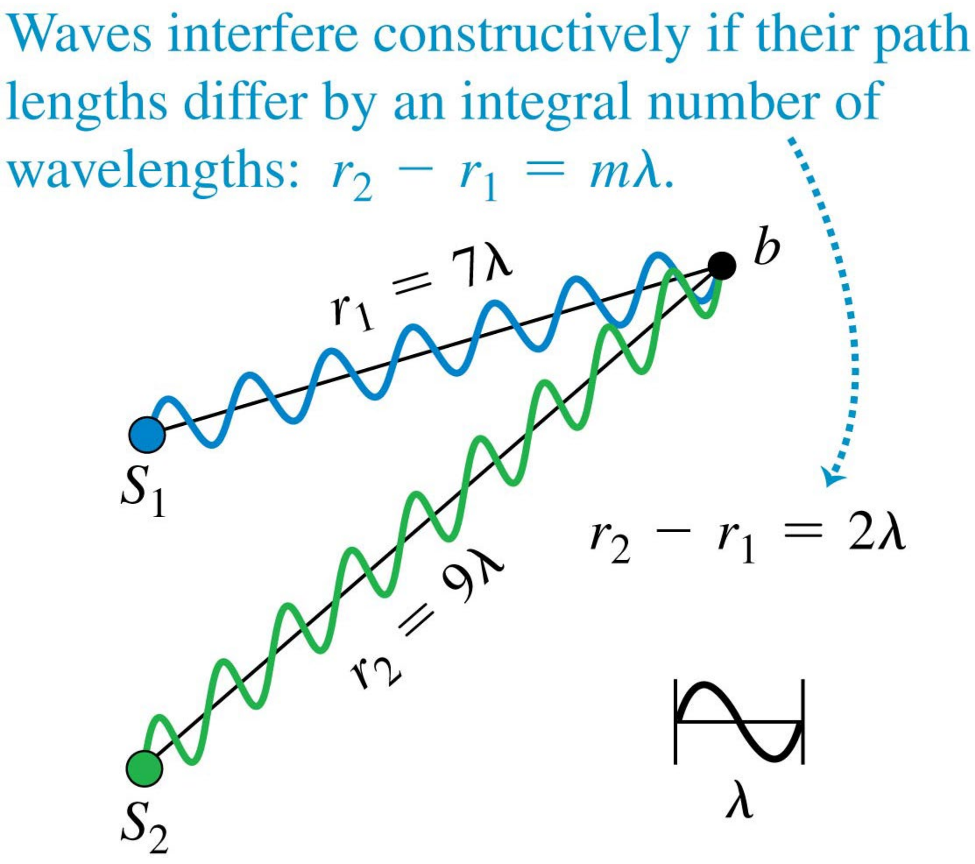
$n = 0, 1, 2, \ldots$
$n_{odd} = 1, 3, 5, \ldots$

- In the special case that the two sources are in phase, $\Delta\phi_{12} = 0$, so

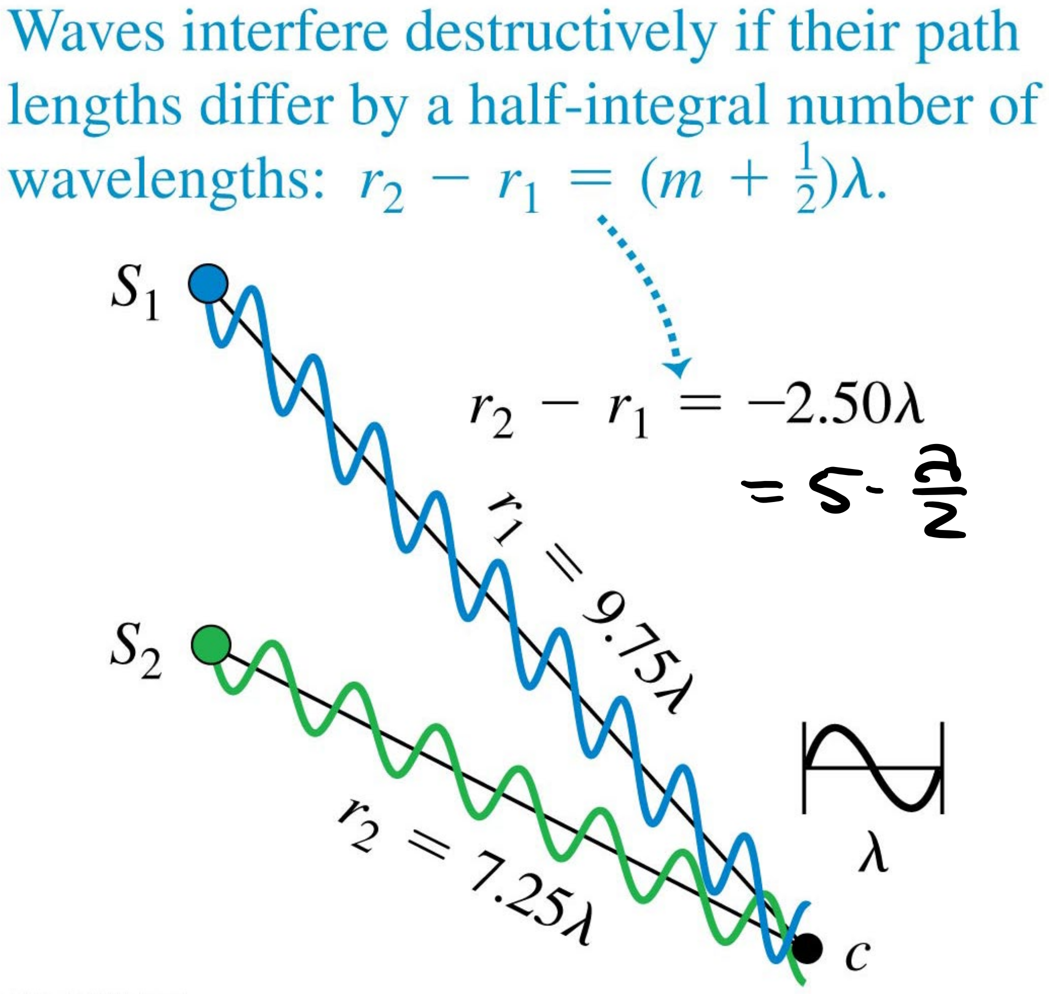
Constructive Interference: $\Delta r_{12} = n \cdot \lambda$
Destructive Interference: $\Delta r_{12} = n_{odd} \cdot \lambda/2$

$n = 0, 1, 2, \ldots$
$n_{odd} = 1, 3, 5, \ldots$

Example: Interference due to path differences for in-phase sources



$$\Delta r_{12} = r_2 - r_1 = \lambda, 2\lambda, \ldots$$



NOTE: If S_1 and S_2 are exactly in phase with each other, then $\Delta\phi_{12} = 0$, and the phase difference at b & c is only due to the path length difference, Δr_{12}

$$\Delta r_{12} = r_2 - r_1 = \frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots$$

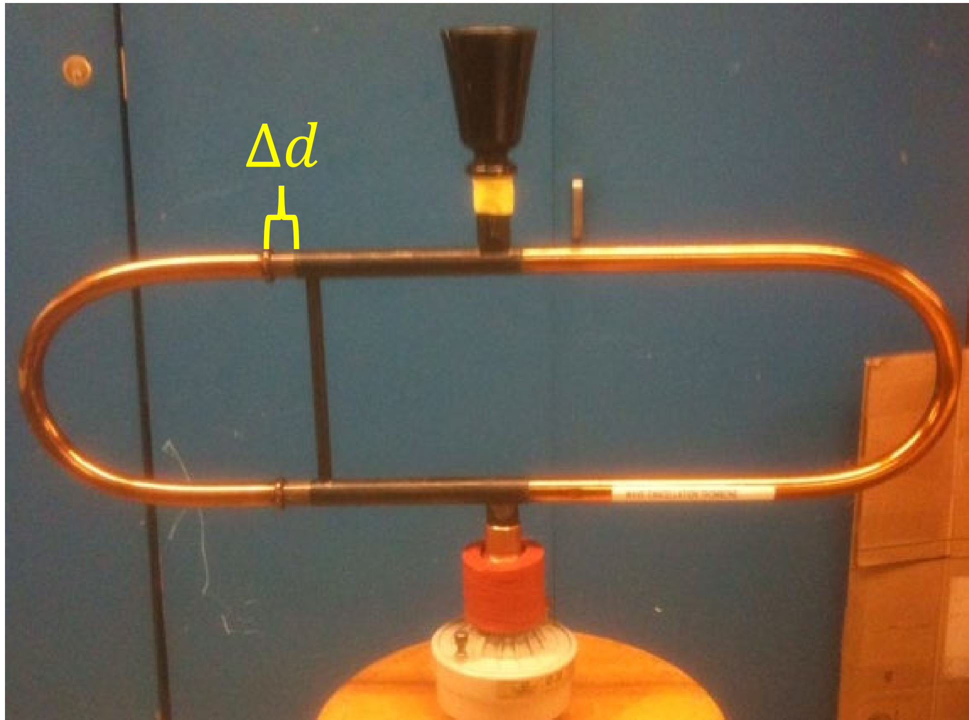
Q: If the sound frequency is 3 kHz, what distance, Δd, do we have to pull out the slide of the trombone to go from one minimum to the next minimum? You can take the speed of sound to be 346 m/s.

$$\Delta r_{12} = 1\frac{\lambda}{2} \quad - \text{1st min}$$

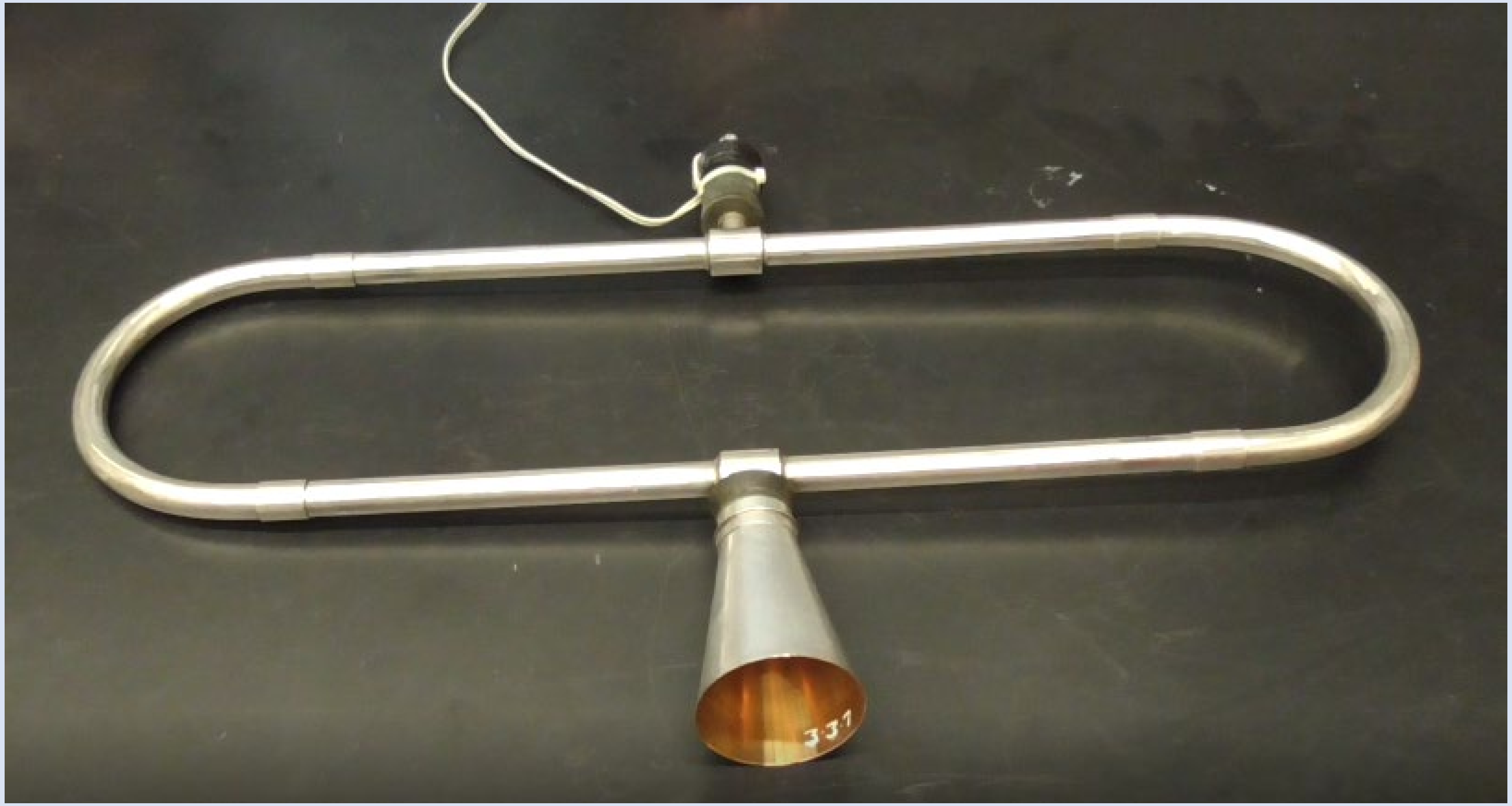
$$\Delta r_{12} = 3\frac{\lambda}{2} \quad - \text{2nd min}$$

$$(3-1)\frac{\lambda}{2} = \lambda = 2\Delta d$$

A. $0\ cm$
B. $2.9\ cm$
C. $5.8\ cm$
D. $8.7\ cm$
E. $11.5\ cm$



Demo: Interference trombone



Q: If the sound frequency is 3 kHz, what distance, Δd, do we have to pull out the slide of the trombone to go from one minimum to the next minimum? You can take the speed of sound to be 346 m/s.

- The sound travelling both sides starts off with the same phase
- The path length difference between the two sides, Δr, increases by $2\Delta d$
- Condition for destructive Interference:

  $$\Delta r = n_{odd}\,\lambda/2 \qquad n = 1, 3, 5, \ldots$$

- From one minimum to the next minimum, $\Delta n = 2$

  $$\Delta r = 2\Delta d = \Delta n\,\lambda/2 = \lambda = v/f$$

- So

  $$\Delta d = v/(2f) = (346\,m/s)/(2 \cdot 3 \cdot 10^3/s) = 5.77\,cm$$

A. $0\,cm$

B. $2.9\,cm$

C. $5.8\,cm$ ✓

D. $8.7\,cm$

E. $11.5\,cm$