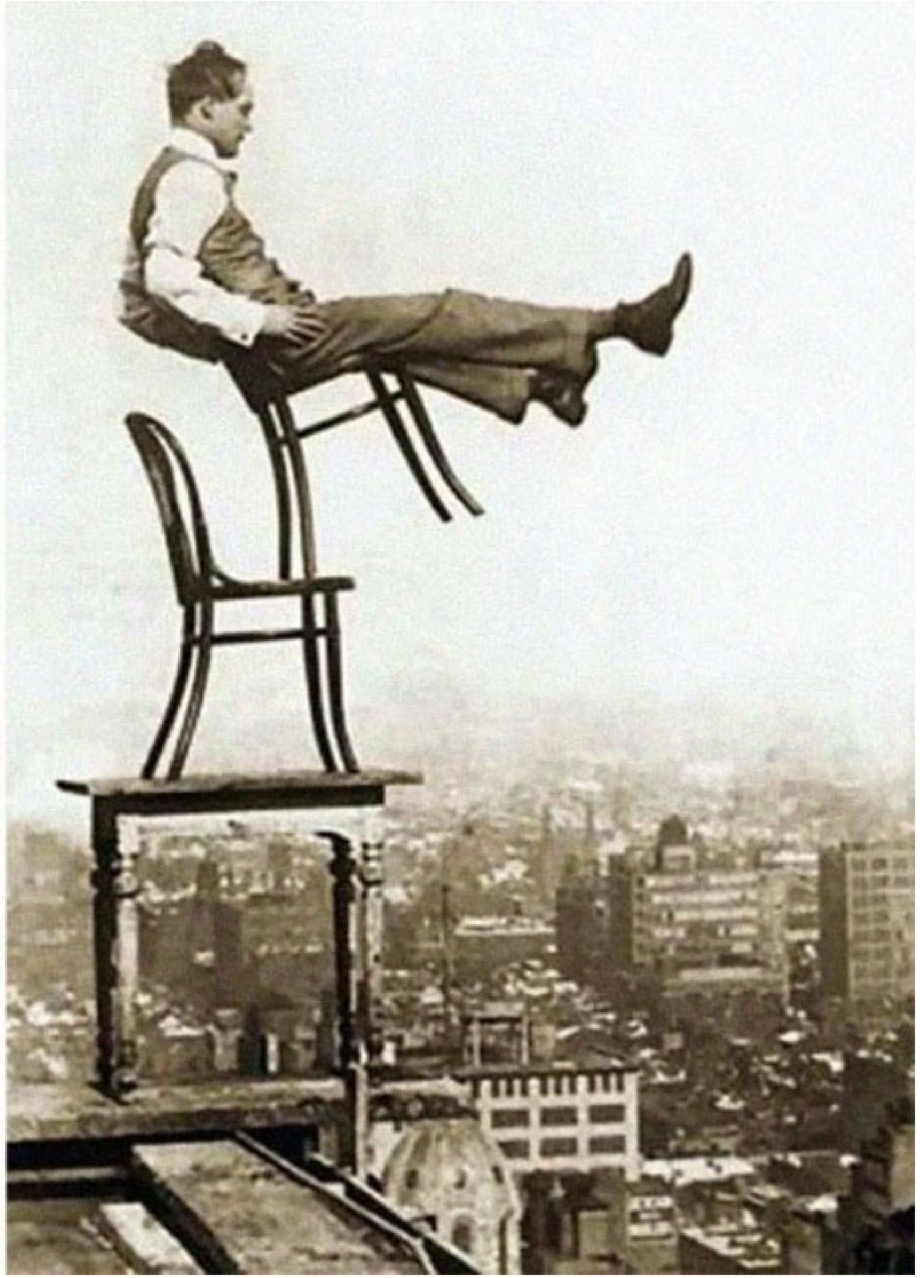
PHYS 170

Week 3: Equilibrium of a Particle. Moment of a Force.

Section 201 (Mon Wed Fri 12:00 – 13:00)



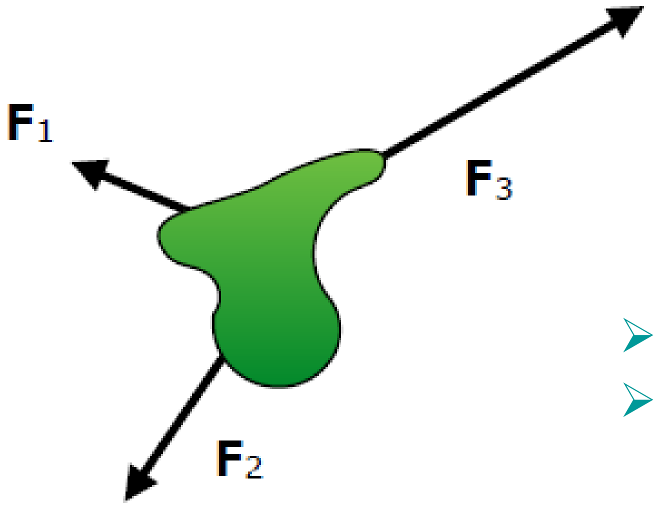
Equilibrium

Text: 3.1-3.2

Content:

- Conditions for equilibrium: $\sum F_x = \sum F_y = \sum F_z = 0$
- Free-Body Diagrams
- Types of forces

STATIC EQUILIBRIUM



- Equilibrium = absence of any kind of motion
- "Kinds of motion":
  - Translation (now)
  - Rotation (soon)

resultant force

- According to Newton's 2nd law, $\vec{F}_{\text{net}} = m\vec{a}$.
- $\vec{F}_{\text{net}} \neq 0$, then object has an acceleration => it moves!

- Hence, here is the condition of translational equilibrium: $\boxed{\vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n = 0}$

- Writing the same statement in components:

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0$$

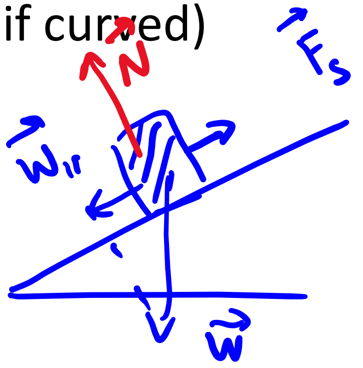
no motion along x | no motion along y | no motion along z

- NOTE: Three equations => can solve for at most **three** unknowns!
  (angles, magnitudes, components of forces in FBD)

SPECIAL FORCES

- **Weight $\vec{W}$**
  - Direction is down
  - Magnitude is mg, $g = 9.81$ N/kg (or m/s^2) in metric
  - Magnitude is pounds (lb) in US (mass is slugs, not pounds!)

- **Normal Force $\vec{N}$** (objects touch but not really connected)
  - Direction is perpendicular (="normal") to the contact surface (or its tangent if curved)
  - Magnitude is unknown, must be calculated



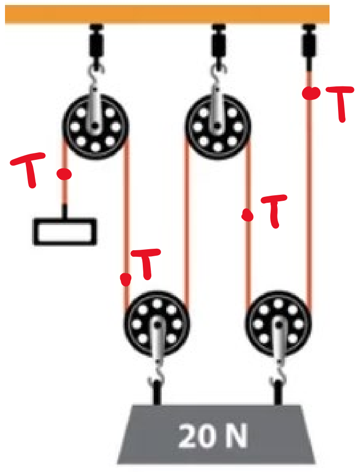
- **Static Friction Force $\vec{F}_s$** (objects touch as above, but don't slide)
  - Magnitude is unknown, except for $|\vec{F}_s| < \mu_s N$; must be calculated
  - Direction is parallel to surface but can be either direction ("Prevents expected motion")

- **Kinetic (sliding) friction $\vec{F}_k$**
  - has definite direction (opposite to the velocity)
  - and has definite magnitude: $|\vec{F}_k| = \mu_k N$

Typically, $\mu_k < \mu_s$.

SPECIAL FORCES (continued)

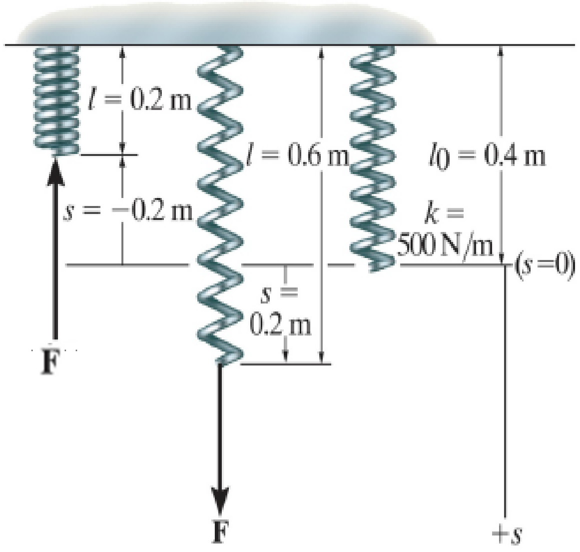
- **Rope Tension $\vec{T}$**
  - Direction of force = direction of rope
  - Tension value is the same throughout the rope, including through pulleys (approx.: massless, non-stretchable rope)



- **Spring Force $\vec{F}_{el}$**
  - Spring has a spring constant k and an unstretched length l_0.
  - Directed along the spring (approx.: non-flexible spring), in the direction opposite to the displacement (“restorative force”)
  - Its magnitude is equal to the spring constant k times the produced displacement from equilibrium, s:

$$\left|\vec{F}_{el}\right| = k\, s$$

Hooke's law



FREE-BODY DIAGRAMS (and The Art of Drawing Them)

Universe

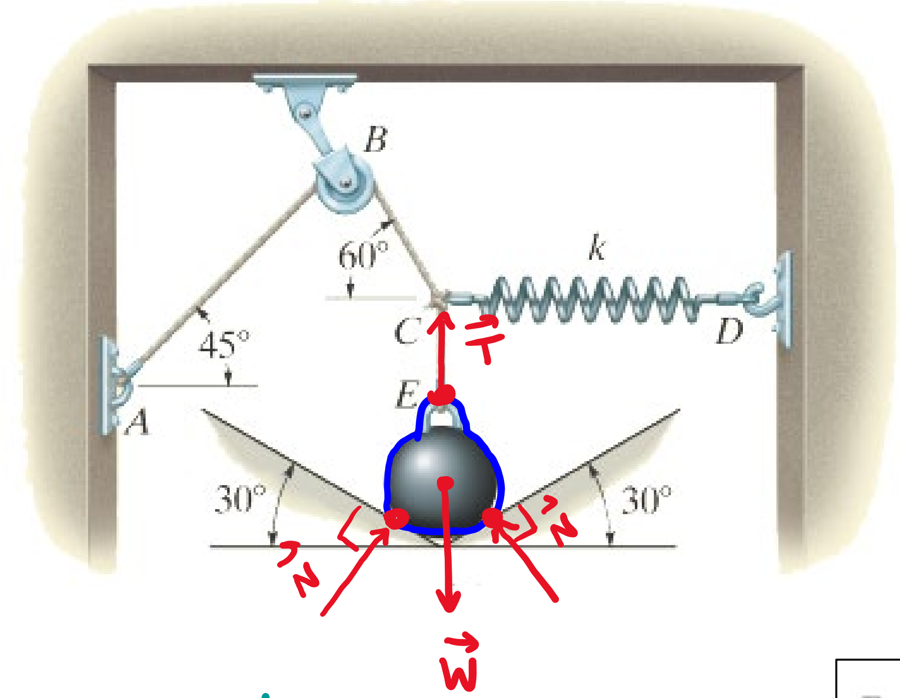
- **Particle vs Environment**:
  - Abstract the object as isolated from its environment. Draw it without any of the supports, braces, cables, springs etc. that might be attached to it.
  - If no rotation: object = "particle" / "rigid body" ← rotations

- **Show all forces acting on the object (Drawing Free Body Diagram, aka FBD)**:
  - Show on your diagram all external forces acting on the object. Internal forces cancel each other (Newton's 3rd law).
  - Include active forces, which tend to set the particle in motion, as well as reactive forces that are the result of constraints/supports that tend to prevent motion.
  - It may help to trace around the object's boundary to account for contact forces. Add gravity force if the object's mass is not zero.
  - Do NOT include forces that do not act directly on the object ! ←
  - Do NOT include forces that the object exerts on the environment ! ←

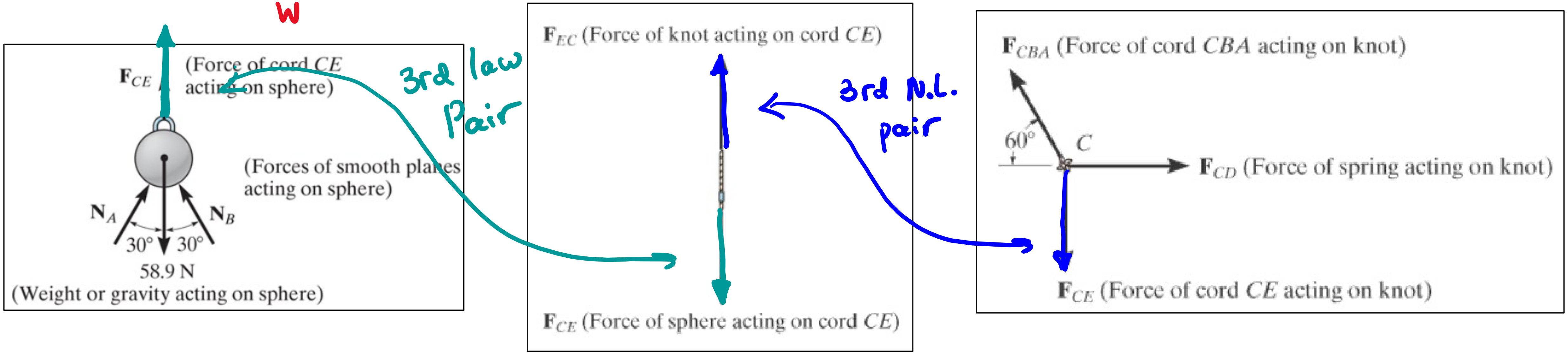
- **Label each force**.
  - It is often much more convenient to work with letters, not numbers. Introduce an appropriate notation for each force and label all the forces, show direction

- In equilibrium all forces must balance (component-wise).

FREE-BODY DIAGRAMS: Example from textbook



The sphere in the figure has a mass of 6 kg and is supported as shown. Draw a free-body diagram of <u>the sphere</u>, <u>the cord CE</u>, and <u>the knot at C</u>. Assume the spring and the rope to be massless.

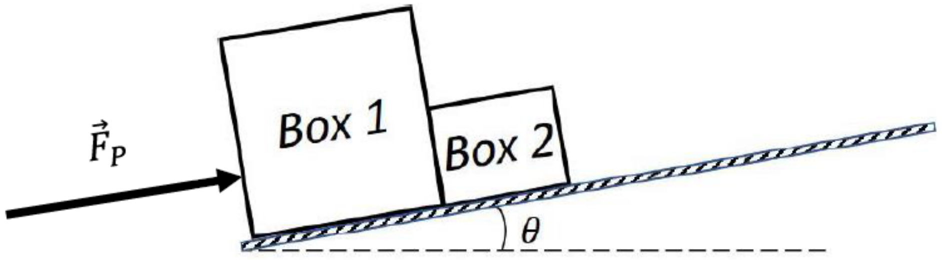


FREE-BODY DIAGRAMS: Example

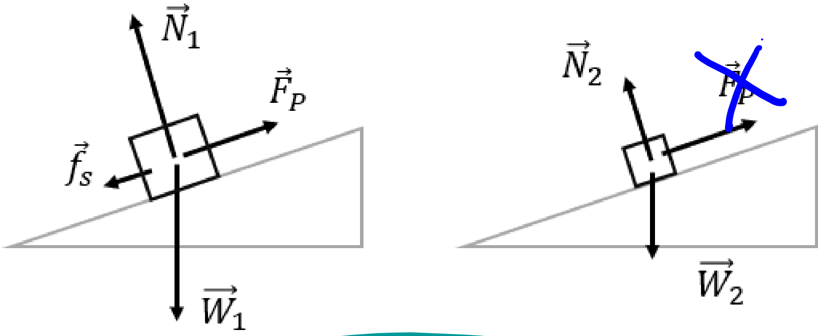
Two boxes are in equilibrium on a ramp. A force $\vec{F}_P$ pushes against box 1 up, parallel to the ramp.

There is no friction between box 2 and the ramp, but there is friction between box 1 and the ramp.

Which picture correctly represents the FBDs of the two boxes?

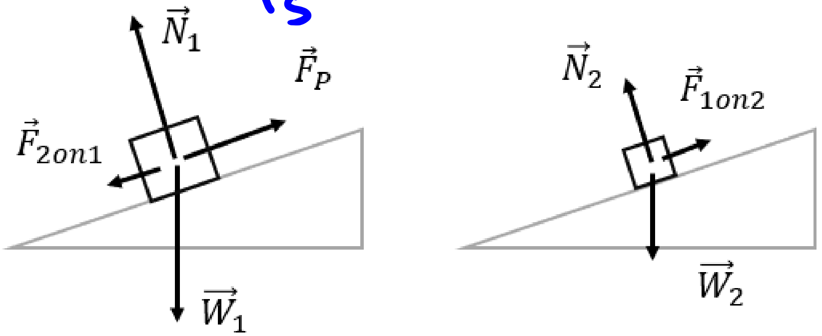


A.



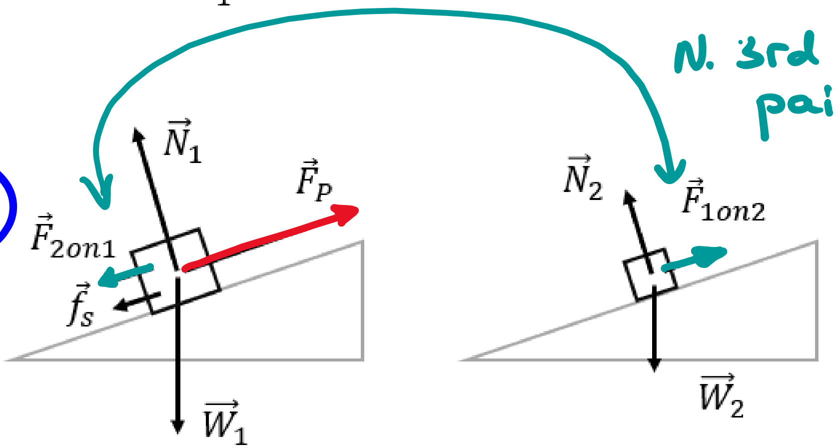
B.

f_s?

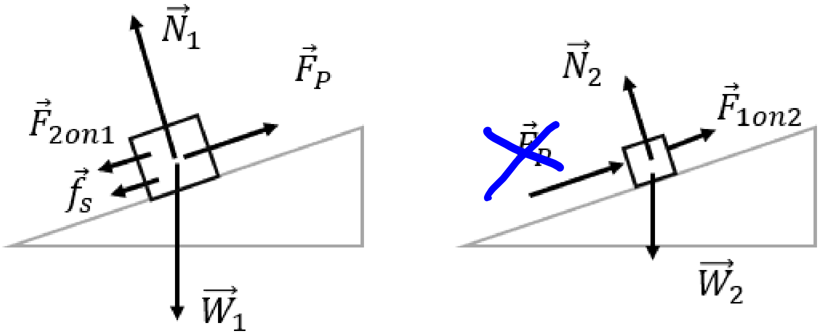


N. 3rd law pair

C.



D.



E. ☹

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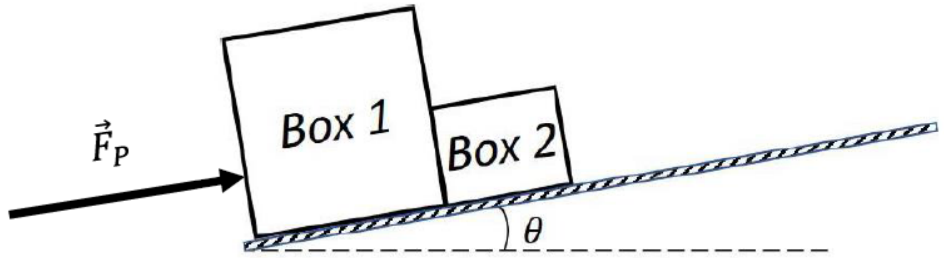
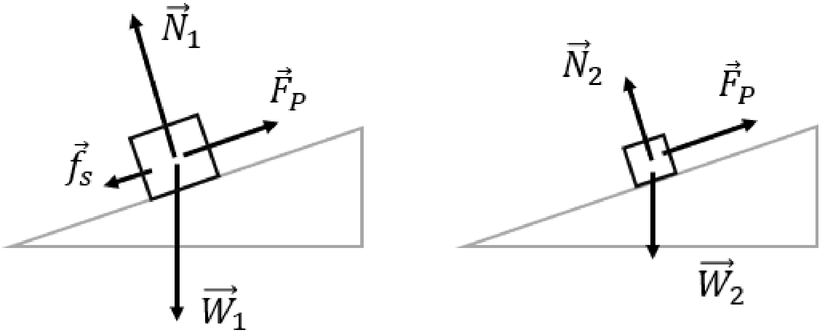


Which picture correctly represents the FBDs of the two boxes?

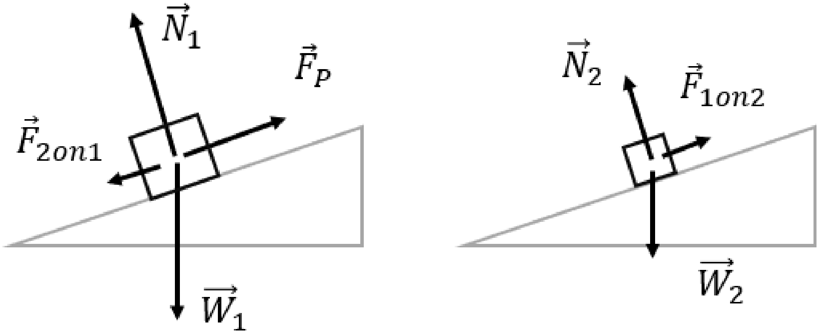
Force $\vec{F}_P$ does not belong to box 2 diagram, since it acts on box 1!
Forces that the two boxes exert onto each other are missing!

Friction force on box 2 is forgotten.

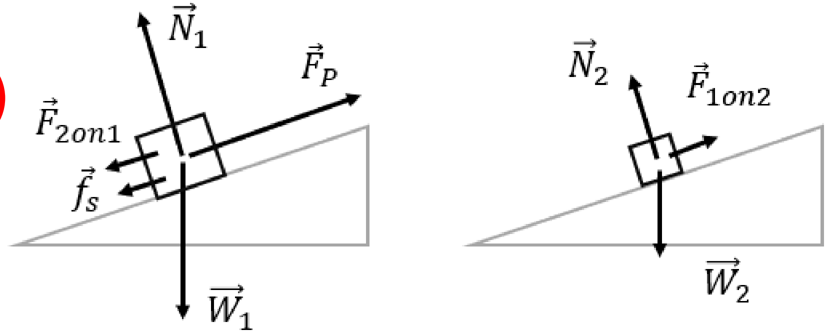
A.



B.

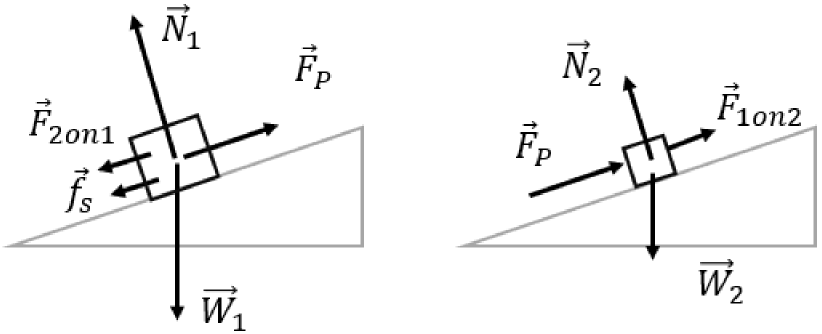


C.



Note $\vec{F}_{1on2}$ and $\vec{F}_{2on1}$, a Newton's 3rd law pair.

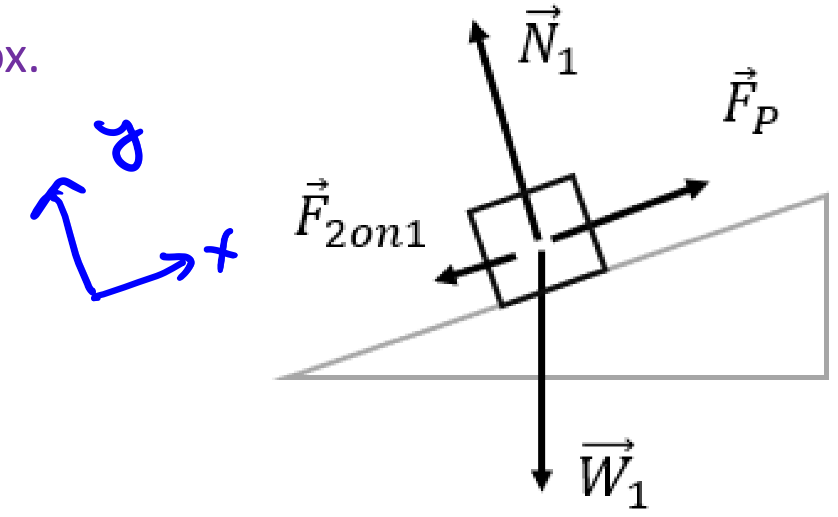
D.



Force $\vec{F}_P$ does not belong to box 2 diagram, since it acts on box 1!

E. ☹

Q: Write down translational equilibrium equation(s) for this box.



$$\vec{F}_R = 0$$

$$\vec{F}_R = \vec{F}_{2on1} + \vec{N}_1 + \vec{F}_p + \vec{W} = 0$$

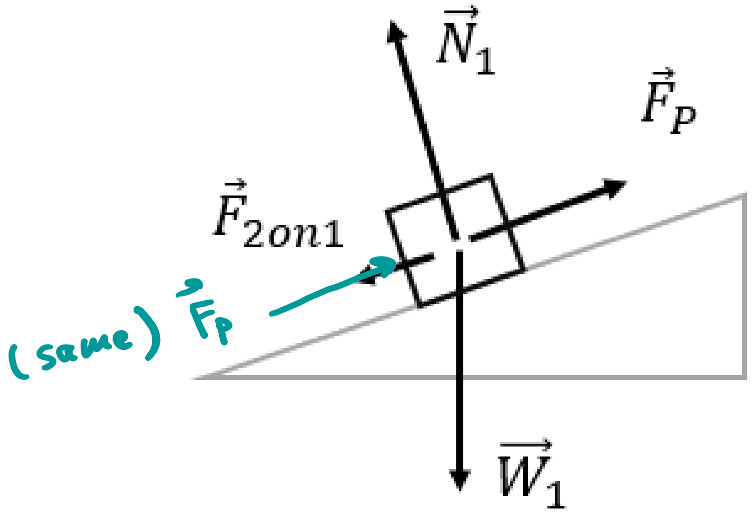
x: $F_{2on1,x} + N_{1,x} + F_{p,x} + W_x = 0$

y: $\cancel{F_{2on1,y}} + N_{1,y} + \cancel{F_{p,y}} + W_y = 0$

y 0

z: . . . . . .

Q: Write down translational equilibrium equation(s) for this box.



- We start with the vector form of the equilibrium equation:

$$\vec{F}_{2on1} + \vec{N}_1 + \vec{F}_P + \vec{W}_1 = 0$$

…and proceed to three scalar equilibrium equations in Cartesian components:

$$x: \quad F_{2on1,x} + N_{1,x} + F_{P,x} + W_{1,x} = 0 \qquad (1)$$

$$y: \quad F_{2on1,y} + N_{1,y} + F_{P,y} + W_{1,y} = 0 \qquad (2)$$

$$z: \quad F_{2on1,z} + N_{1,z} + F_{P,z} + W_{1,z} = 0 \qquad (3)$$ (Not applicable for this 2D problem)

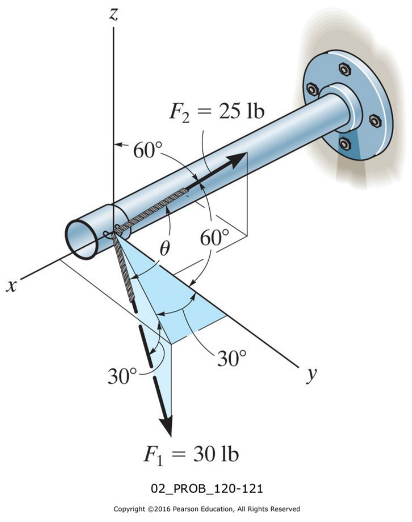
- After that, you need to find all the components explicitly and solve this system of equations for your unknowns.

DRAWING UNKNOWN FORCES

- If a force is unknown, we still need to draw an arrow for it in the Free Body Diagram. So we have to <u>**make a guess**</u> about which way it points.
  - ➢ If our solution yields a negative sign for some component, it just means that the actual component is opposite to the component that was guessed.
  - ➢ **This is perfectly OK**. There's **no need** to do neither of: (1) changing the direction of the arrow in the drawing, (2) setting up the equations again with the other sign, (3) solving them again!
  - ➢ Moreover, you may guess right for some components, and wrong for others. **That's perfectly OK**.

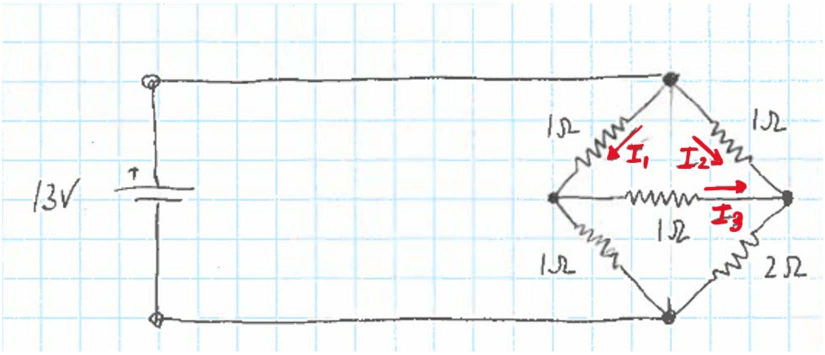
PHYS 170, Week 2:

W2-6. a) Projection of F_1 on F_2



$$F_{1 \text{ on } 2} = -5.44 \text{ lb}$$

PHYS 158, Week 1: Find all currents



$$I_1 = 6\,A, \quad I_2 = 5\,A, \quad I_3 = -1\,A.$$

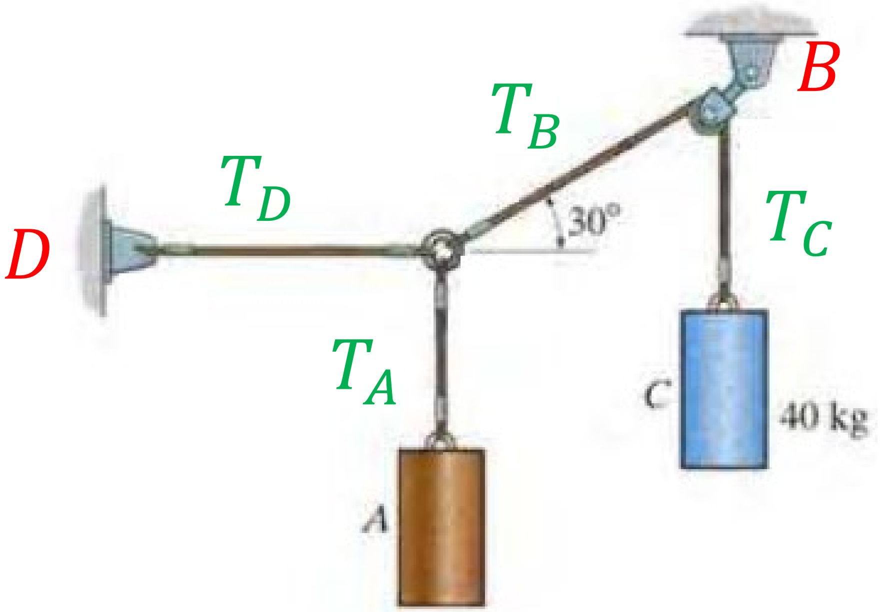
Equilibrium in 2D

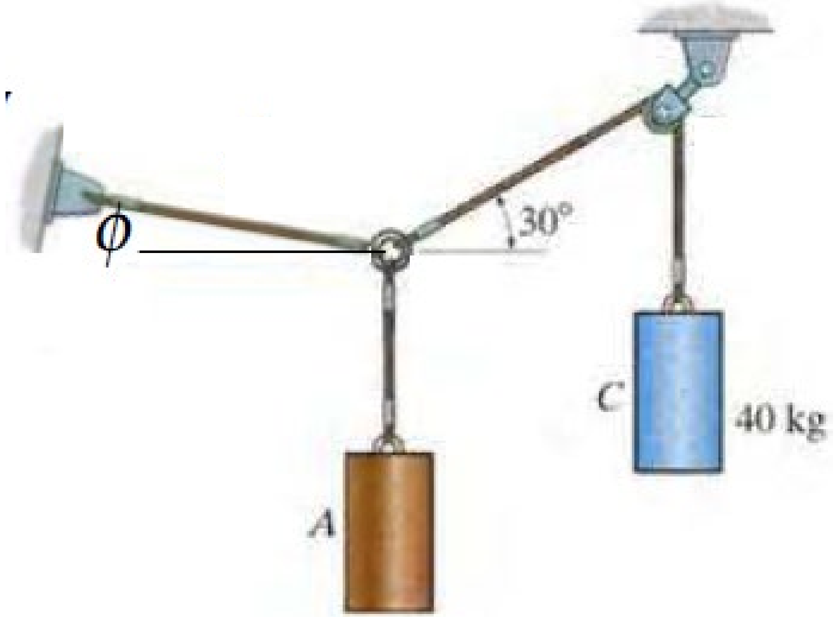
Text: 3.3

Content:

- Equilibrium for co-planar forces

E3-0. Find mass m_A, which provides equilibrium.





Good practice, but not 100% necessary for what follows.

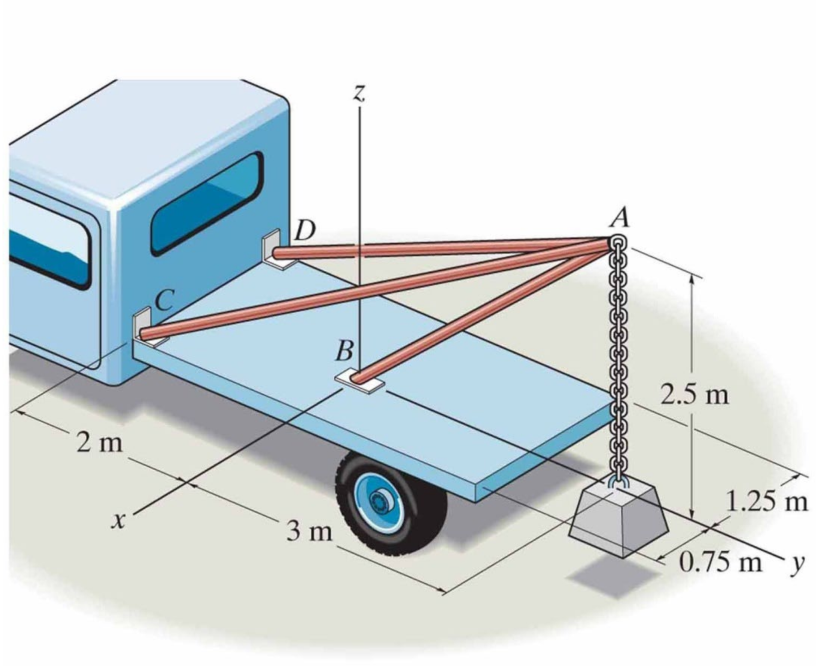
Equilibrium in 3D

Text: 3.4

Content:

- Practice on 3D equilibrium

W3-1. Determine the force acting along the axis of each of the three struts needed to hold the 500 kg block in equilibrium.



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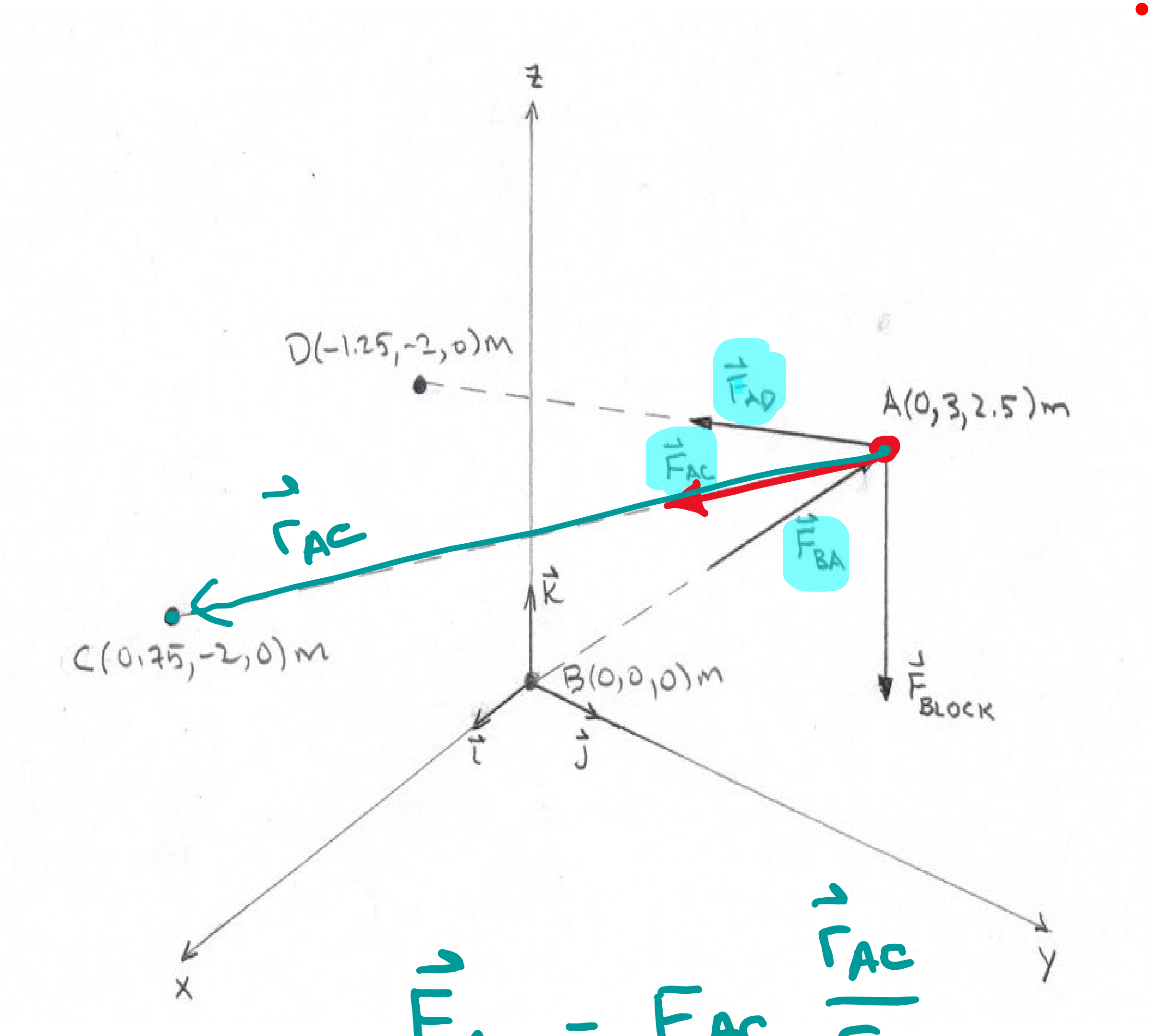
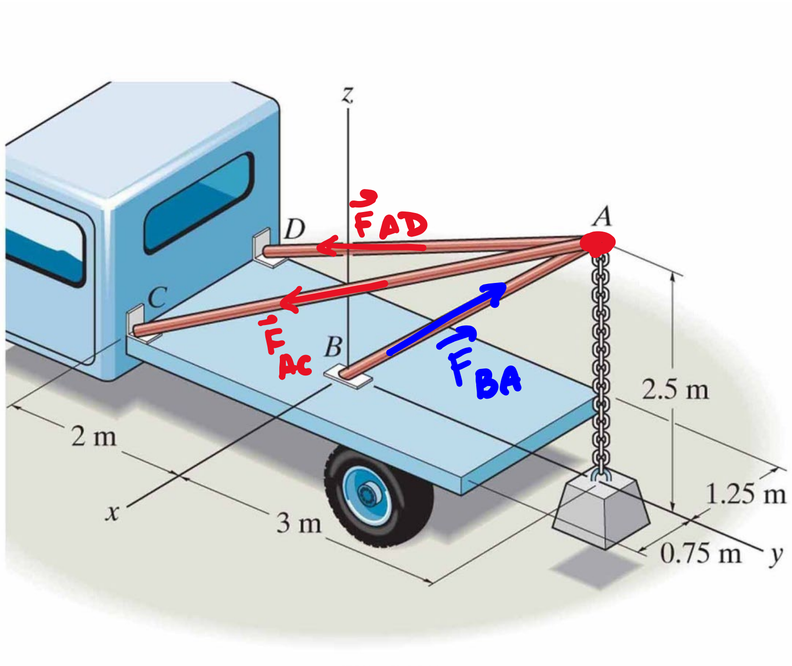
W3-1. Determine the force acting along the axis of each of the three struts needed to hold the 500 kg block in equilibrium.



$$\vec{F}_{AC} = F_{AC}\frac{\vec{r}_{AC}}{r_{AC}}$$

- FBD (up to 25% of your mark & impacts all other parts)
  - Big and clear ! (Half of the page)
  - Using straightedge !
  - All points / forces / axes / unit vectors labelled !
  - Coordinates of the points given next to them!
  - Directions of the forces chosen !



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