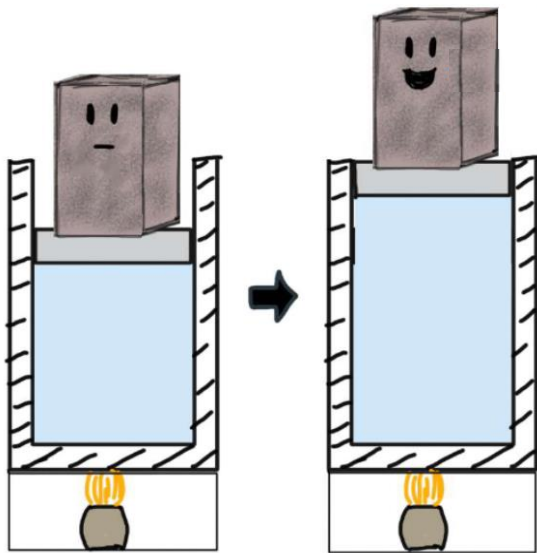Lecture 16.

Internal energy.

Isochoric and Isobaric processes.

From before

- Ideal Gas Law

$$PV = nRT$$

- 1st Law of Thermodynamics

$$\Delta U = Q - W$$

net change in the internal energy of the object

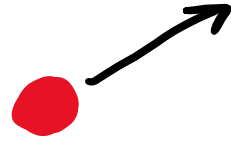heat supplied to the object (thermodynamics)

work done by the object (mechanics)

$$W = P \cdot \Delta V$$

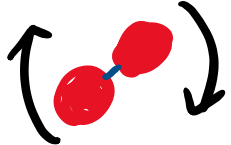Internal energy of a gas, $U(T, n)$ ← ideal gas

$$\Delta U = Q - W$$

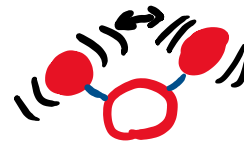- Internal energy of an object (including gas) is a sum of:
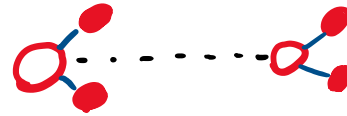
  ➢ Translational motion of molecules

  ➢ Their rotational energy

  ➢ Their vibrational energy

  E_{kin}

  ⇐   Internal energy
  of <u>ideal gas</u> is determined
  only by its temperature T

  $$E_{kin} = E_{kin}^{trans} + E_{kin}^{vib} + E_{kin}^{rot}$$

  ➢ Electrostatic potential energy
  of their interaction

  ⇐   Neglect in an <u>ideal gas</u>
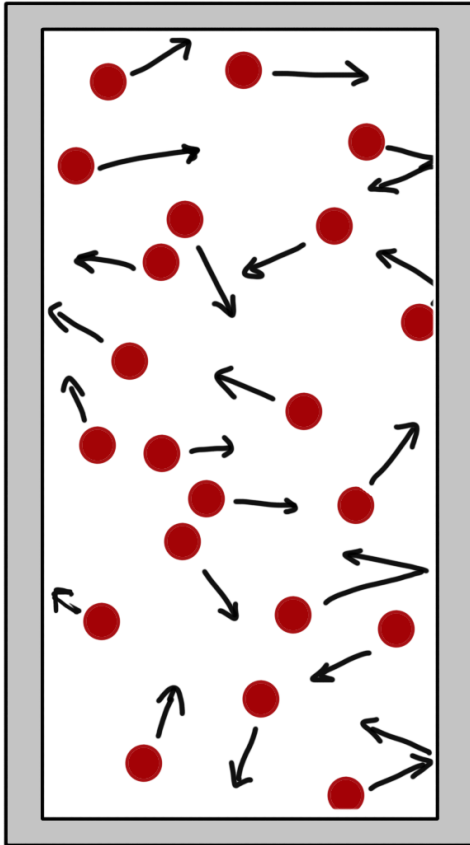 (it's a part of ideal gas model)

- Main equation:

  $$\Delta U = nC_v \Delta T$$

  ➢ C_v is molar specific heat
  measured at $V = $ const

  ➢ Larger for more complex molecules

Example: Internal energy of a monatomic ideal gas

$$PV = nRT$$

- $U = N \cdot E_{kin}^{avg}$ is total kinetic energy of molecules (ideal gas!)

- $E_{kin}^{avg} = \text{const} \cdot T$

- $\text{const} = \dfrac{3}{2}\dfrac{R}{N_A}$, where

  Translations: 3 dof

  ➤ R is the ideal gas constant
  ➤ N_A is Avogadro's number
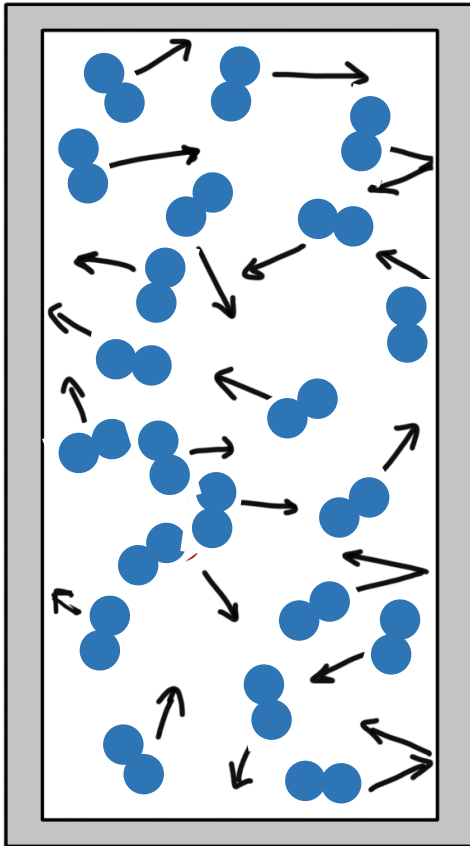 ➤ 3 comes from the number of degrees of freedom (dof)

Result:

$$U = \frac{3}{2}nRT$$

From previous slide:

$$\Delta U = nC_v\Delta T \ \Rightarrow$$

$$C_v = \frac{3}{2}R$$

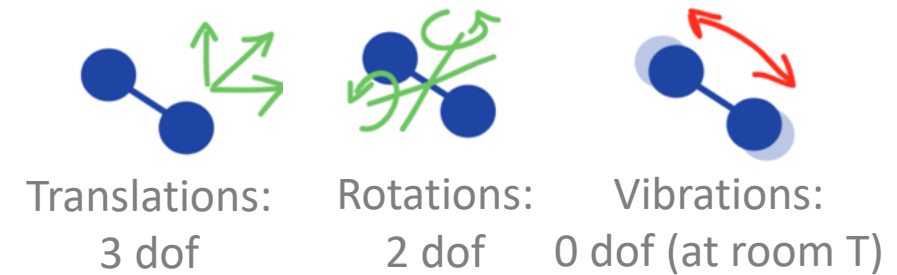# Example: Internal energy of a diatomic ideal gas

- $U = N \cdot E_{kin}^{avg}$ is total kinetic energy of molecules (ideal gas!)

- $E_{kin}^{avg} = \text{const} \cdot T$

- $\text{const} = \dfrac{5}{2}\dfrac{R}{N_A}$, where

  - R is the ideal gas constant
  - N_A is Avogadro's number
  - 5 comes from the number of degrees of freedom (dof)

Translations: 3 dof

Rotations: 2 dof

Vibrations: 0 dof (at room T)

Result: $U = \dfrac{5}{2}nRT$

From previous slide:

$\Delta U = nC_v\Delta T \Rightarrow$

$C_v = \dfrac{5}{2}R$

Internal Energy: Summary

- Important equations:

$$\Delta U = Q - W$$

$$\Delta U = nC_v \, \Delta T$$

$$U = \frac{\text{dof}}{2} nRT$$

$$W = \int P \, dV$$

- Add to that

$$W = P \cdot \Delta V$$

$$PV = nRT$$

to find various characteristics of ideal gas under various circumstances

- Ideal Gas: U determined from T and n only

- U is a "state variable":
 - ➤ Determined from P, V, T, n (for non-ideal gas)
  - ➤ ΔU only depends on initial & final state (not on the path)
    - ❖ Q and W do depend on the path, but not their difference, ΔU!

Q: A container with a partition in the middle is filled halfway with an ideal monatomic gas. If the partition is removed instantaneously so that the gas is allowed to fill the box, the final temperature of the gas will be:

$$\Delta U = Q - W$$

"0"   "0"   "0"

$$\downarrow$$

$$\Delta T = 0$$

$$\Delta U = n C_v \Delta T$$

"0"   "0"
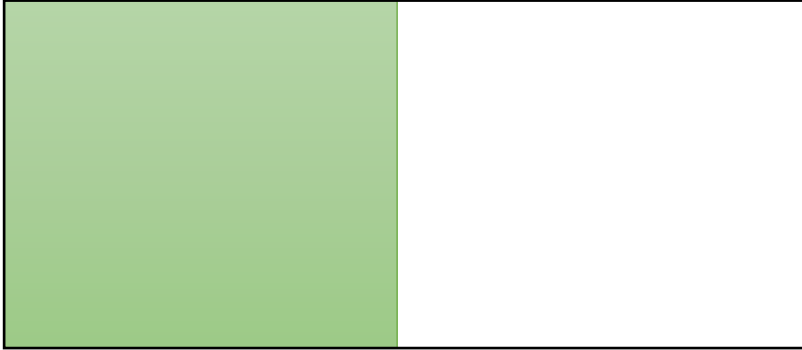
"Free expansion"

A. lower than the original temperature

B. the same as the original temperature

C. higher than the original temperature

D. I have no idea how to think about this

Q: A container with a partition in the middle is filled halfway with an ideal monatomic gas. If the partition is removed instantaneously so that the gas is allowed to fill the box, the final temperature of the gas will be:
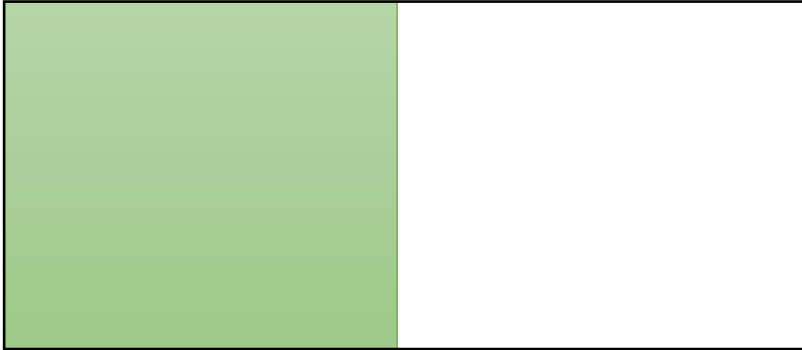
Energy conserved:

$$W = 0 \quad Q = 0 \quad \Delta U = Q - W = 0$$

$\Rightarrow$ Kinetic energy per molecule doesn't change, (or you can use $\Delta U = n C_v \, \Delta T$)

$\Rightarrow$ Temperature doesn't change

A. lower than the original temperature
B. the same as the original temperature ✔
C. higher than the original temperature
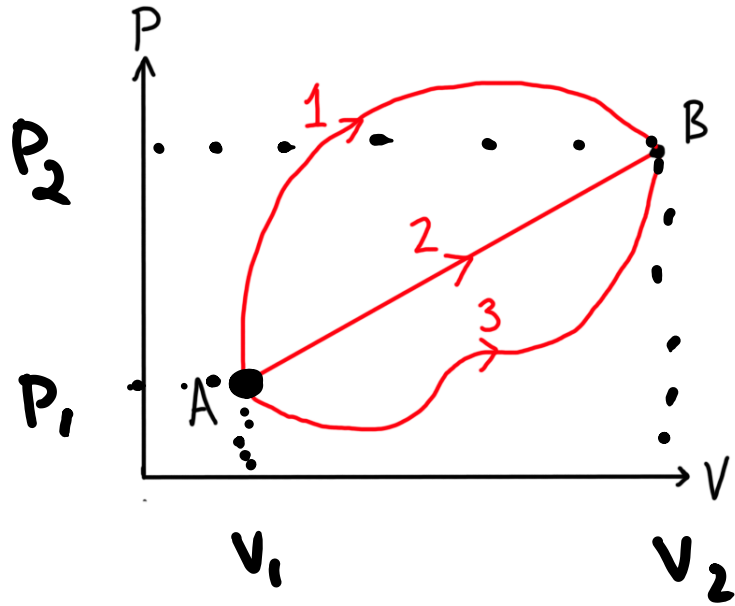D. I have no idea how to think about this

"Free expansion"

Q: The graph shows three possible processes for an ideal gas going from A to B.
For which path is the change ΔU largest?

A. Path 1

B. Path 2

C. Path 3

D. They are all the same

E. We don't have enough information to answer

$\Delta U = ?$

$\Delta T = ?$

$T_1 = \dfrac{P_1 V_1}{nR}$

$T_2 = \dfrac{P_2 V_2}{nR}$

$PV = nRT$

Same $\Delta T \longrightarrow$ same ΔU

Q: The graph shows three possible processes for an ideal gas going from A to B.
For which path is the change ΔU largest?
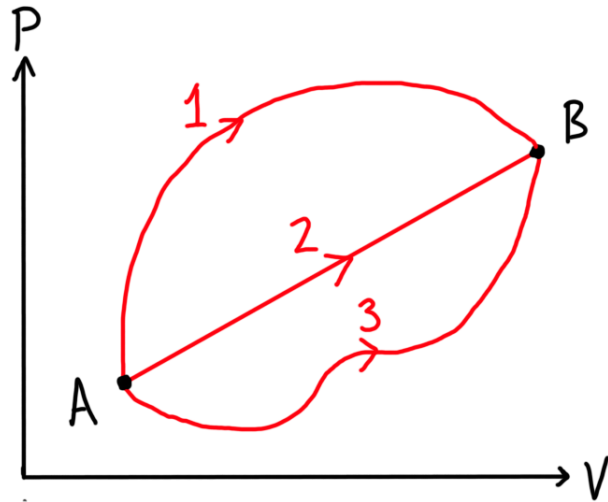
$$\Delta U = nC_v\Delta T$$

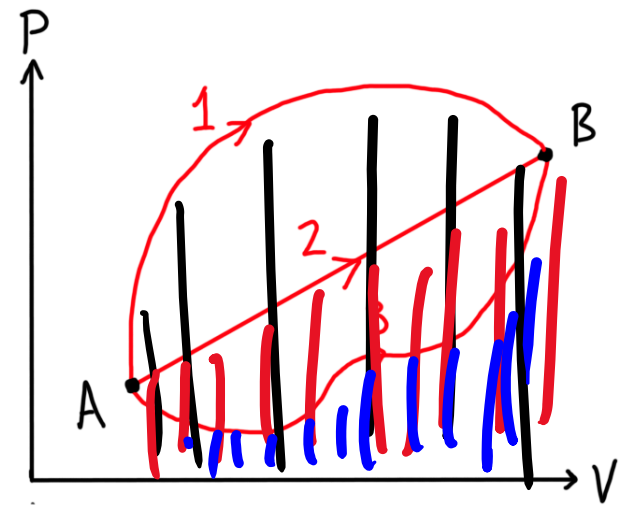T is determined by P, V via $PV = nRT$

Same change in $P, V \Longrightarrow$ same change in T

So, ΔU is the same

A. Path 1

B. Path 2

C. Path 3

D. They are all the same ✓

E. We don't have enough information to answer

Q: The graph shows three possible processes for an ideal gas going from A to B. For which path is Q (the heat added) the largest?

$$\Delta U = Q - W \longrightarrow$$

$$Q = \Delta U + W$$

$$\underbrace{\quad}_{\text{same}} \quad \underbrace{\quad}_{\text{area}}$$

under graph

A. Path 1

B. Path 2

C. Path 3

D. They are all the same

E. We don't have enough information to answer

Q: The graph shows three possible processes for an ideal gas going from A to B.
For which path is Q (the heat added) the largest?

P

1

B

2

3

A

V

$$Q = \Delta U + W$$

ΔU is the same for all paths

$\Delta U = nC_v\Delta T$

T is determined by P, V via $PV = nRT$

Same change in $P, V \Longrightarrow$ same change in T

So, ΔU is the same

W largest for path 1 (largest area under graph)

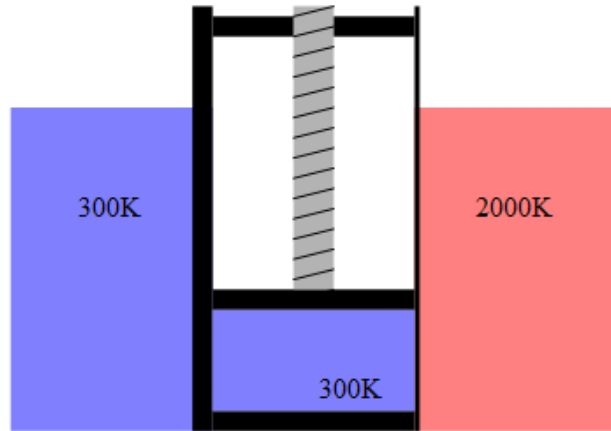$\Longrightarrow Q$ largest for path 1

A. Path 1 ✔️

B. Path 2

C. Path 3

D. They are all the same

E. We don't have enough information to answer

Analyzing thermodynamic processes

By Gonfer - Own work, CC BY-SA 2.5,
https://commons.wikimedia.org/w/index.php?curid=10901965

- Want to:
  - ➢ Calculate temperature, pressure, volume: Ideal Gas Law: $PV = nRT$
  - ➢ Calculate work: $W = P\Delta V$ (or area under P-V curve)
  - ➢ Calculating change in U: $\Delta U = nC_v\Delta T$
  - ➢ Calculate heat added: First Law: $Q = \Delta U + W$

- We'll understand these for:
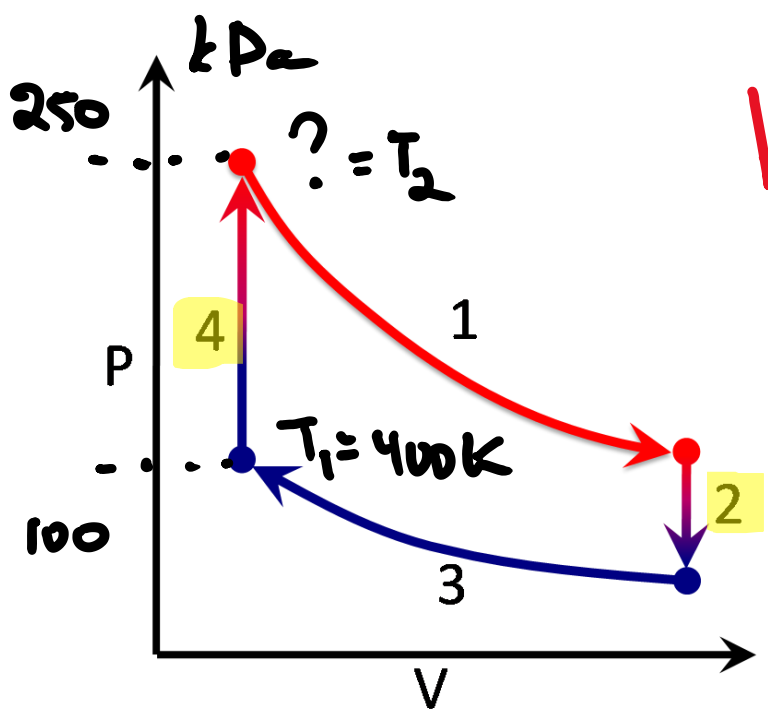 - ➢ isochoric: V constant
  - ➢ isobaric: P constant
  - ➢ isothermal: T constant
  - ➢ adiabatic: $Q = 0$
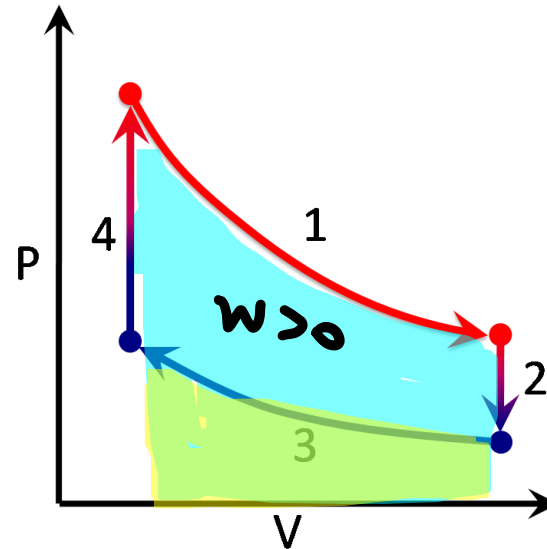
Note: You don't need to memorize the formulas that we will derive for each process, rather understand how it all works!

Q: In the process 4, the pressure of an ideal gas increases from 100 kPa to 250 kPa. If the initial temperature is 400 K, the final temperature is

Kelvin !!!

$$PV = nRT$$

$$\frac{T}{P} = const = \frac{V}{nR}$$

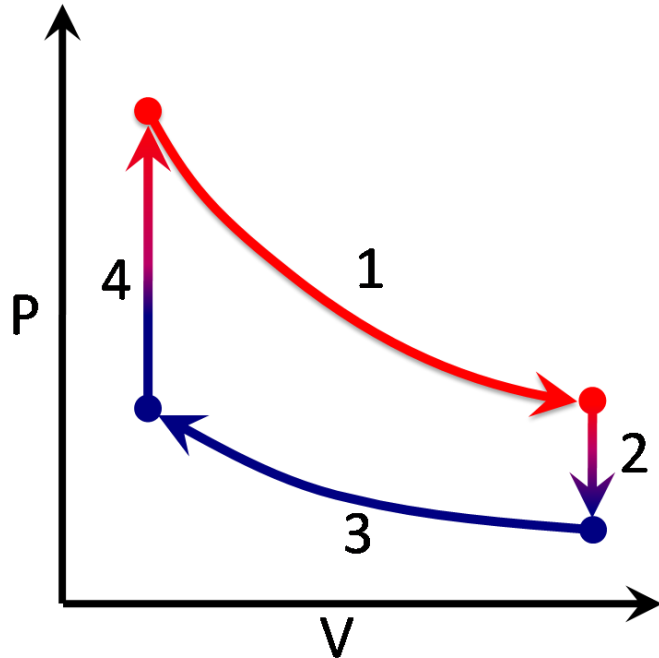A. 160 K

B. 400 K

$$\frac{T_2}{P_2} = \frac{T_1}{P_1} \rightarrow T_2 = T_1 \frac{P_2}{P_1} = \cdots$$

C. 600 K

D. 800 K

E. 1000 K

➢ isochoric: V constant

Q: In the process 4, the pressure of an ideal gas increases from 100 kPa to 250 kPa.
If the initial temperature is 400 K, the final temperature is

T in Kelvin !!!

Ideal Gas law: $PV = nRT$ $\implies$ $\dfrac{P}{T} = \dfrac{nR}{V}$

n, V, R are constant $\implies P/T$ is constant

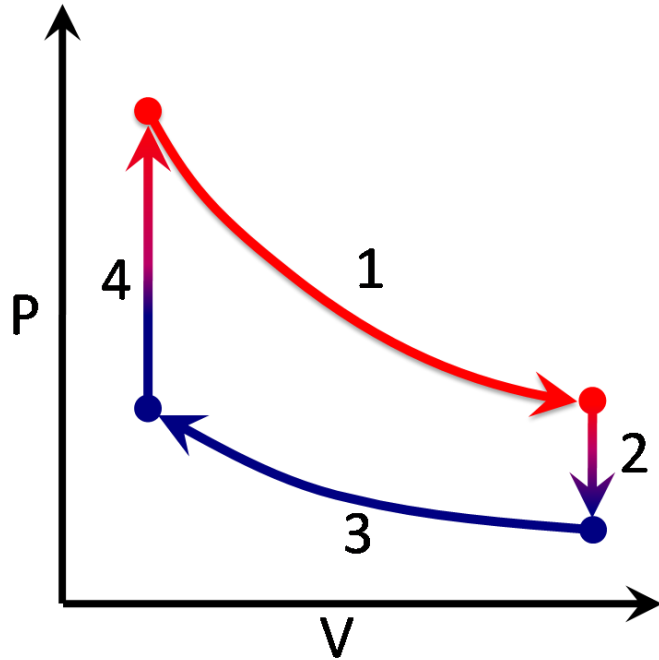$\dfrac{P_2}{T_2} = \dfrac{P_1}{T_1} \implies T_2 = T_1 \dfrac{P_2}{P_1} = 1000$ K

A. 160 K

B. 400 K

C. 600 K

D. 800 K

E. 1000 K ✓

➢ isochoric: V constant

Q: During process 4, we can say that

$$\Delta U = Q - \cancel{W}$$

$$\leqslant$$

$$0$$

area under
4 is zero
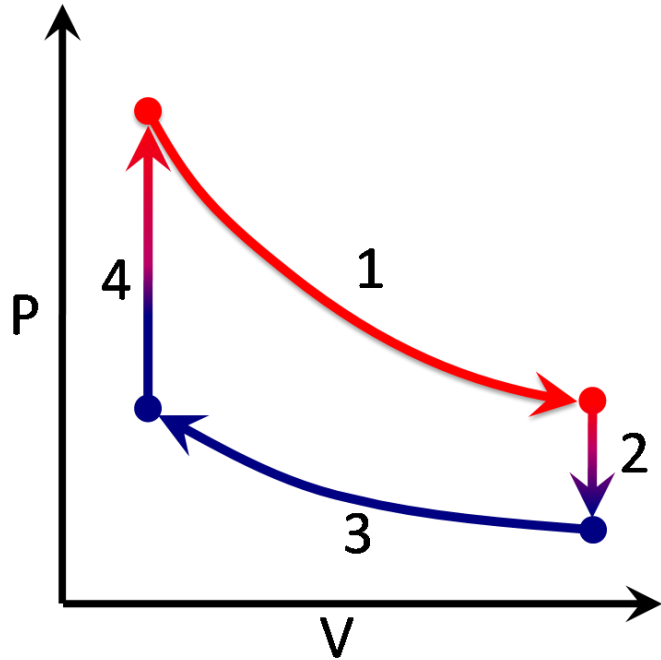
A. $Q = W$

B. $Q = \Delta U$

C. $\Delta U = -W$

D. None of the above

➢ isochoric: V constant

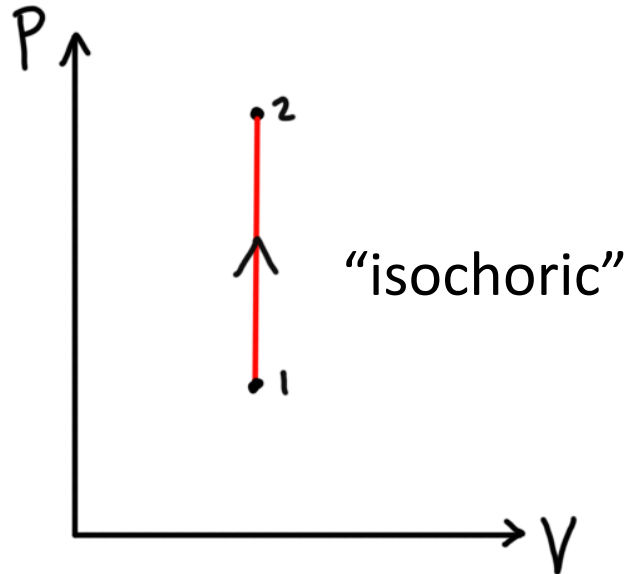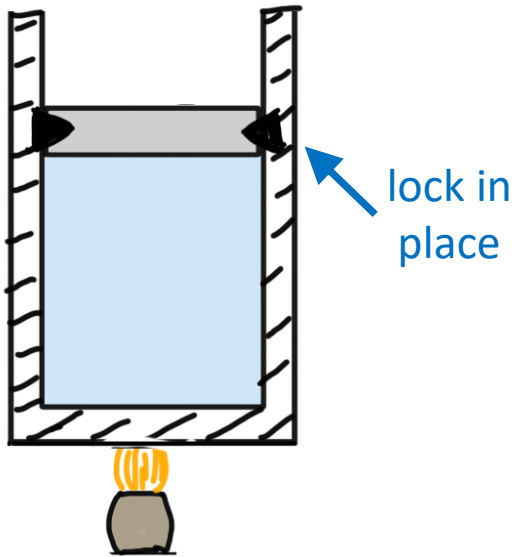Q: During process 4, we can say that

First Law: $\Delta U = Q - W$

For constant V, $W = 0$

So $\Delta U = Q$

A. $Q = W$

B. $Q = \Delta U$ ✔

C. $\Delta U = -W$

D. None of the above

➢ isochoric: V constant

Constant Volume: Summary

lock in place

"isochoric"

- Ideal Gas Law $\implies PV = nRT$

  ➢ n, V are constant $\implies \dfrac{T_2}{T_1} = \dfrac{P_2}{P_1}$ $(P \propto T)$
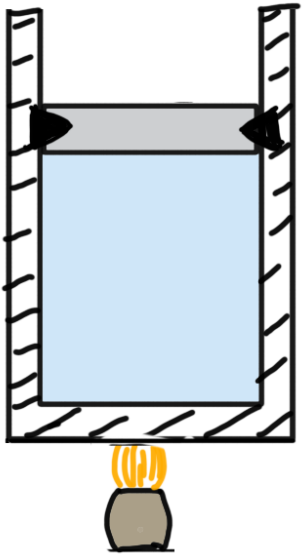
- $\Delta U = Q - W$

  ➢ $W = P\Delta V = 0$
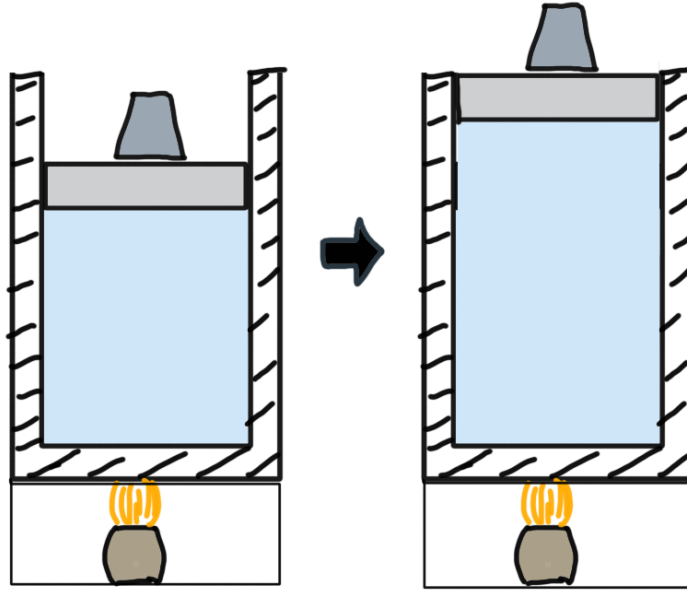
  ➢ So: $Q = \Delta U = nC_v\Delta T$

  ➢ isochoric: V constant

Q: In the two situations below, a gas is heated from 300K to 400K. We can say that the heat added

first case

second case

$$\Delta U = Q - W$$

$$Q = \Delta U + W$$

$W = ?$ $\Delta U = ?$

$W = ?$ $\Delta U = ?$

A. is the same in both cases
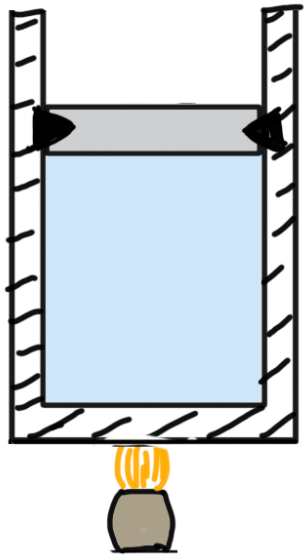
same → same ΔT

B. is greater in the first case where the volume is held fixed

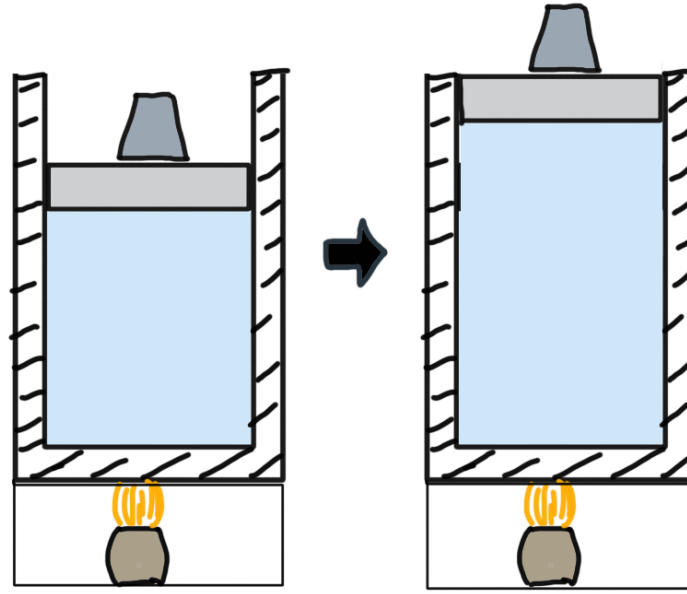C. is greater in the second case where pressure is fixed

➢ isobaric: P constant

Q: In the two situations below, a gas is heated from 300K to 400K.
We can say that the heat added

first case   second case

First Law: $Q = \Delta U + W$

ΔU same for both (same ΔT)

W positive for second case

So Q larger for second case

A. is the same in both cases

B. is greater in the first case where the volume is held fixed

C. is greater in the second case where pressure is fixed ✔

➢ isobaric: P constant