Lecture 26.

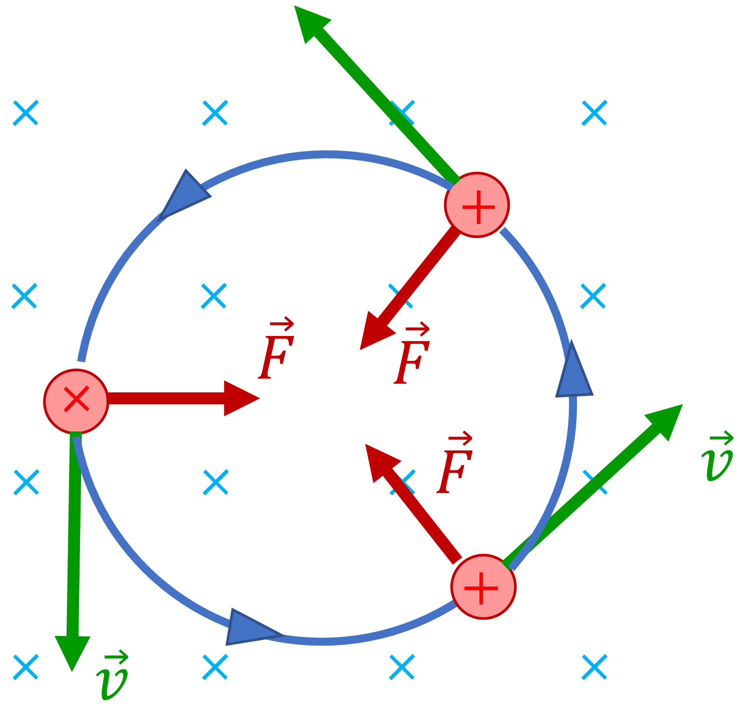
Cyclotron motion in 2D and 3D.
Hall effect.
Force on a current-carrying wire.

Last Time:

Magnetic force: $\vec{F}_B = q_{\pm}\, \vec{v} \times \vec{B}$

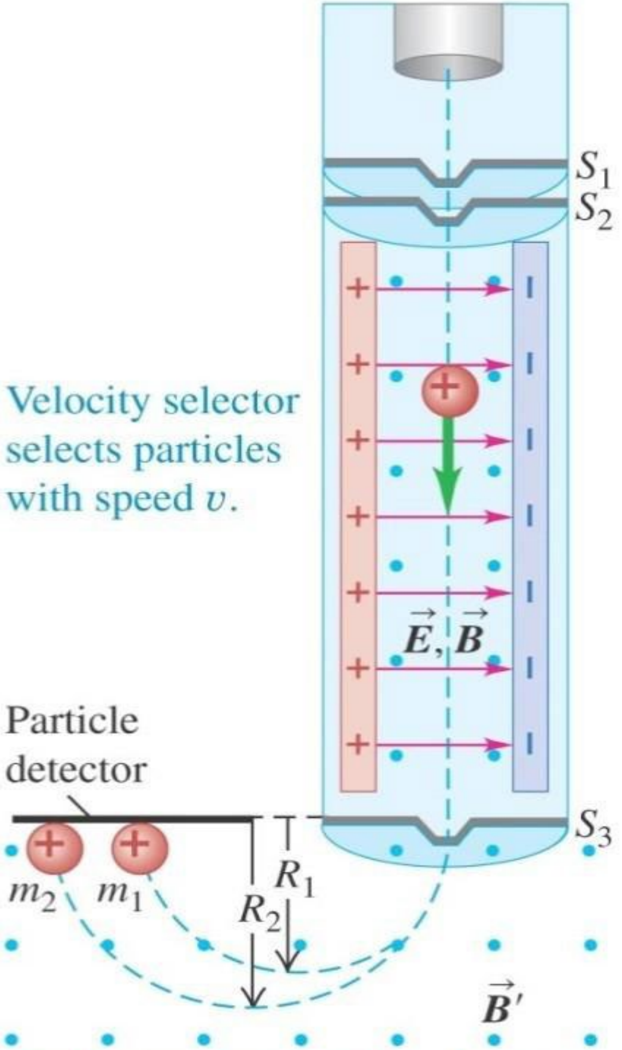
Cyclotron motion:

- Radius: $r_{cyc} = \dfrac{mv}{qB}$
- Frequency: $f_{cyc} = \dfrac{qB}{2\pi m}$



Crossed E- and B-fields: $\vec{F} = q_{\pm}\vec{E} + q_{\pm}\vec{v} \times \vec{B}$

Q: Assume that the electric field strength E of the velocity selector of a mass spectrometer is set to 5.65×10^3 N/C, while the magnetic field strength B is set to 0.224 T. After exiting the velocity selector, the charged particle is exposed only to the magnetic field and moves in a circular orbit with radius 2.9 cm. Assuming that the particle is singly charged (i.e. carries charge e), find the mass of the particle.



Magnetic field separates particles by mass; the greater a particle's mass, the larger is the radius of its path.

- $E = 5.65 \times 10^3 \frac{N}{C}$
- $B = 0.224\,T$
- $r = 2.9\,cm$
- $e = 1.6 \cdot 10^{-19}\,C$
- $m = ?$

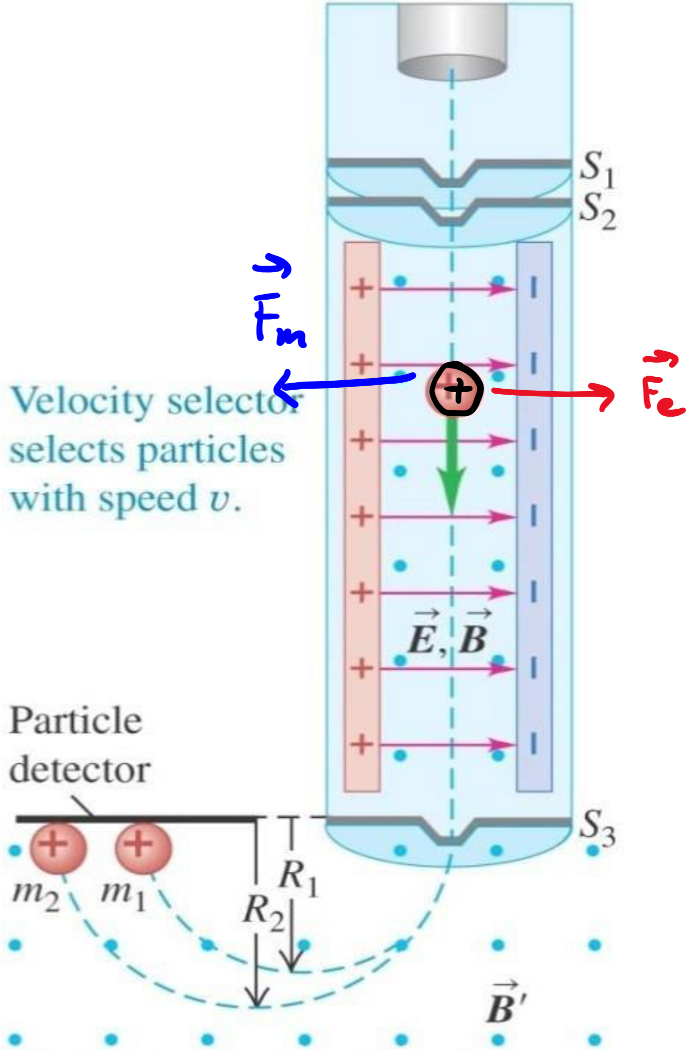
$$r_{cyc} = \frac{mv}{qB} \qquad f_{cyc} = \frac{qB}{2\pi m}$$

$$\vec{F}_m = e\,\vec{v} \times \vec{B} = evB\,\overbrace{\sin\theta}^{1}$$

$$m = \frac{qBr}{v} = \frac{qBrB}{E} = 4.12 \cdot 10^{-26}\,kg$$

$$F_e = F_m$$

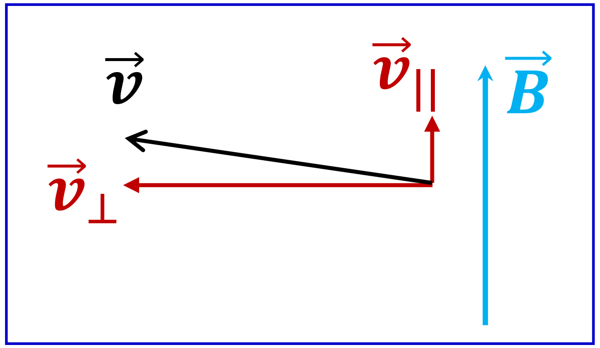
$$\cancel{q}E = \cancel{q}vB \longrightarrow v = \frac{E}{B}$$



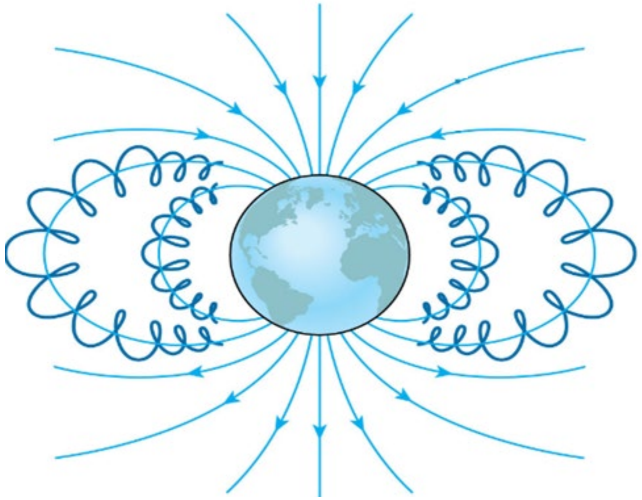
Magnetic field separates particles by mass; the greater a particle's mass, the larger is the radius of its path.

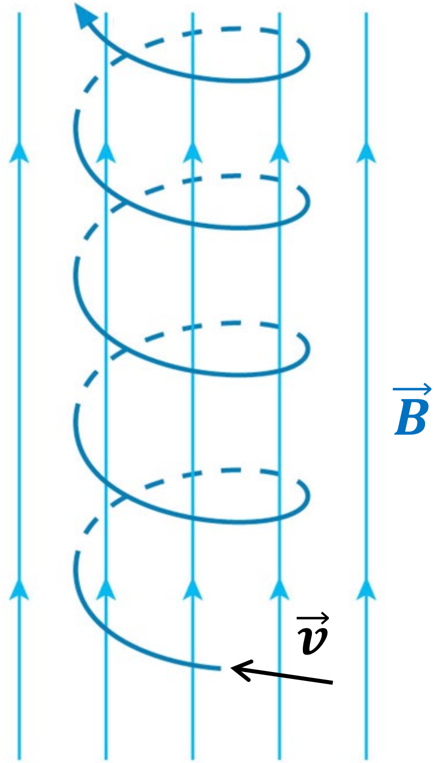
Motion of Charged Particles in Magnetic Fields: 3D

- What if $\vec{v}$ not exactly perpendicular to $\vec{B}$?
- Split the velocity into two components, $v_\perp$ and $v_{||}$!
  - ➢ circular motion due to $v_\perp$,
  - ➢ plus, linear motion due to $v_{||}$.



- The trajectory is a helix!





Example: Charged particles in the field of the Earth

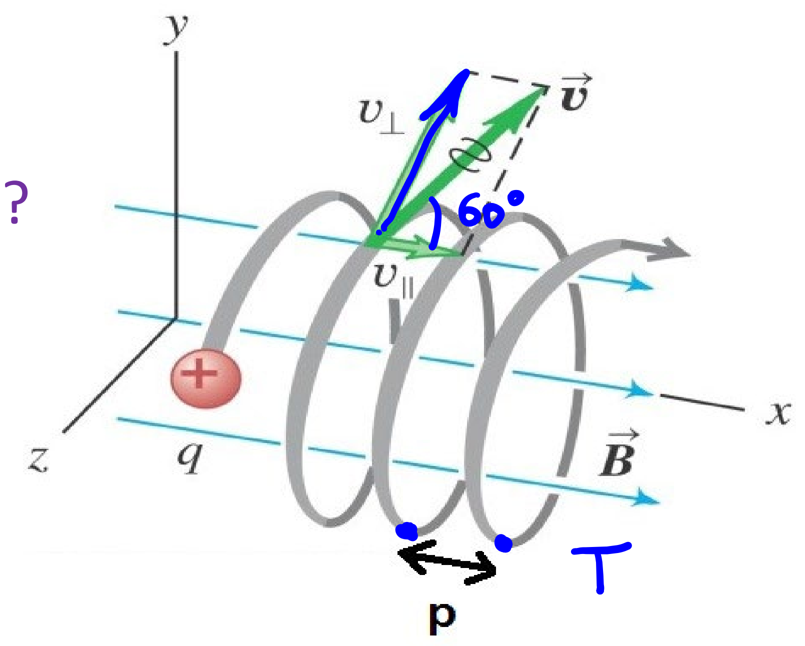
Q: A proton enters a region of magnetic field at an angle of 60^0 with respect to the magnetic field lines. The proton's speed is v = 5.0 x 10^6 m/s and the magnetic field strength is 20 mT. (m_p = 1.67 x 10^{-27} kg, e = 1.6 x 10^{-19} C.)

B

r_{cyc}

a) What is the radius of the proton's spiral trajectory?

b) What is the spacing p ('pitch') between two adjacent turns?



a) $r_{cyc} = \dfrac{m v_\perp}{qB} = 2.26\text{ m}$

$\Delta s = v t$

b) $p = v_\parallel T_{cyc} = v_\parallel \dfrac{1}{f_{cyc}} =$

$= v \cos 60^\circ \dfrac{2\pi m}{eB} = 8.2\text{ m}$

$v_\perp = v \sin 60^\circ$ (cycl)

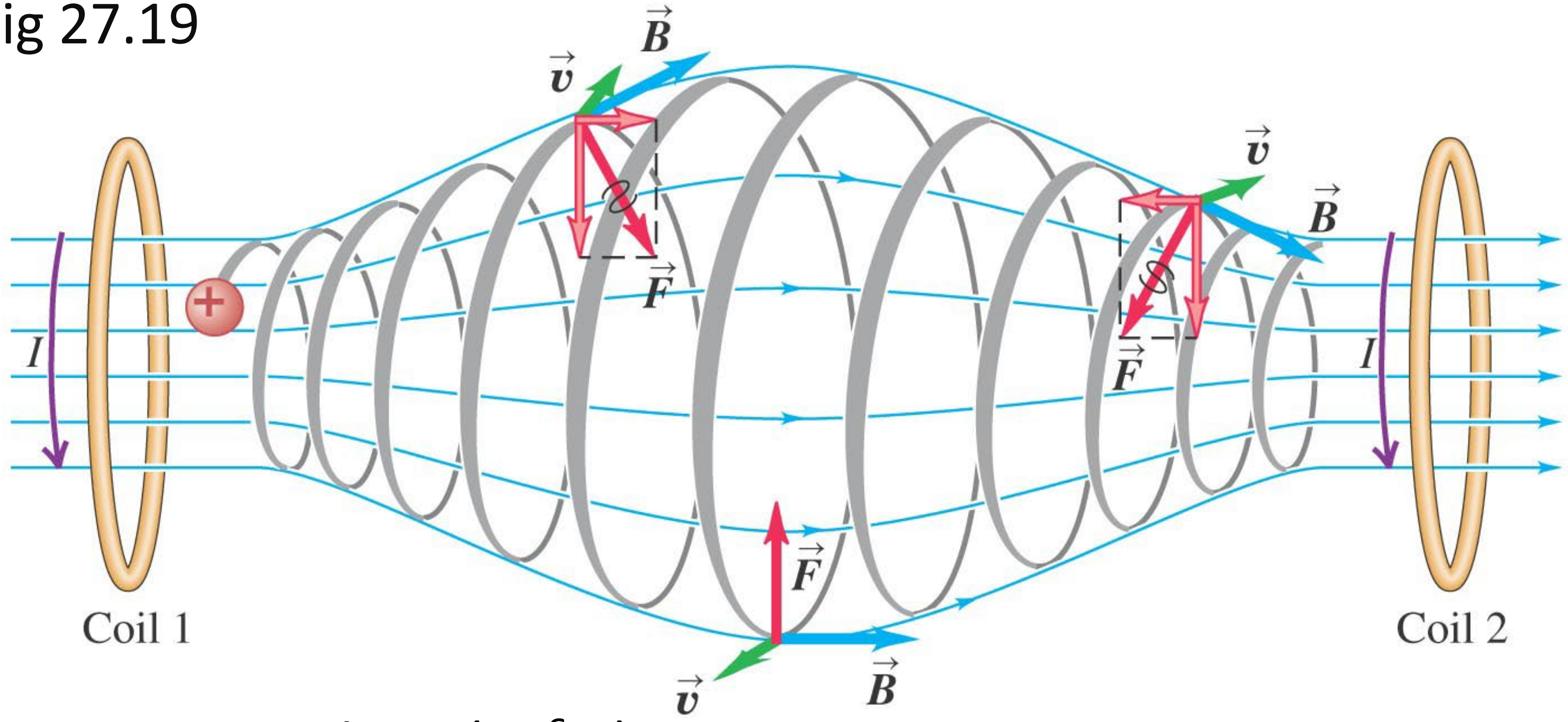
$v_\parallel = v \cos 60^\circ$ (linear motion along $\vec{B}$)

$$r_{cyc} = \frac{mv}{qB} \qquad f_{cyc} = \frac{qB}{2\pi m}$$

Magnetic Bottle (Ion Trap)

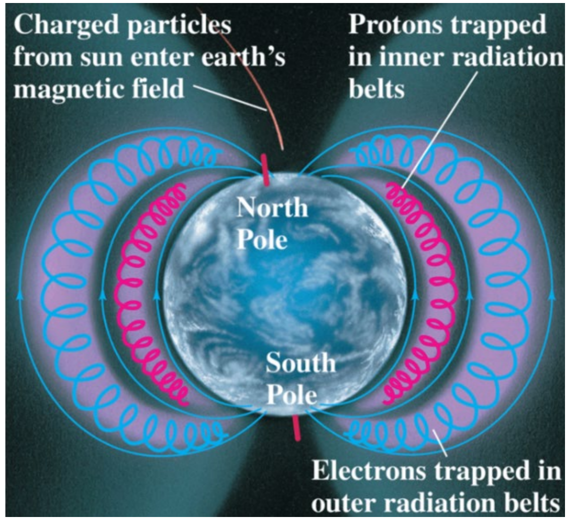
$$r_{cyc} = \frac{mv}{qB} \qquad f_{cyc} = \frac{qB}{2\pi m}$$

Fig 27.19



Trapping ionized gas (10^6 K)

Earth's Van Allen belt
(aurora borealis/australis)



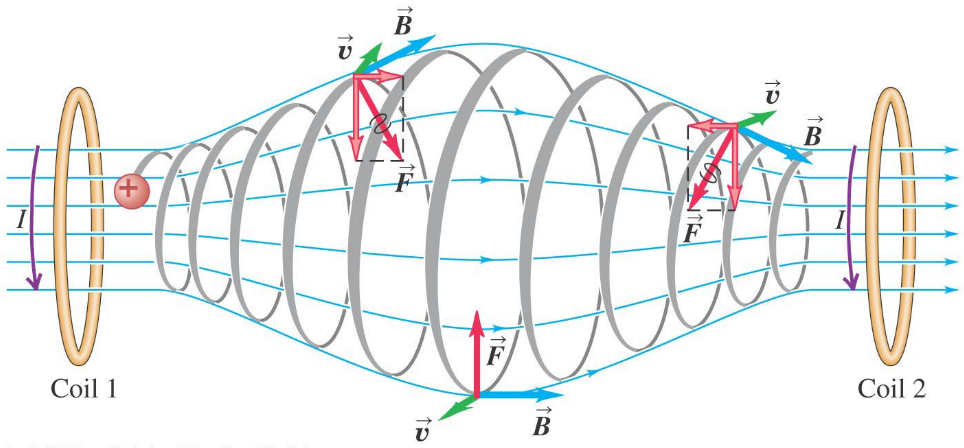
A charged particle enters a region with a non-uniform B-field such as a Magnetic Ion Trap. What can you say about the motion of the charged particle in such a magnetic trap?

A. Its speed will decrease
B. It will experience a force directed towards the region where the magnetic field is the weakest
C. Its speed will remain constant
D. Need to know details of the B-field to answer this question
E. Both B and C will occur

A charged particle enters a region with a non-uniform B-field such as a Magnetic Ion Trap. What can you say about the motion of the charged particle in such a magnetic trap?

- Note: $v_{\parallel}$ changes, but v_{total} remains constant

$v_{\perp}$ changes



A. Its speed will decrease

✓ B. It will experience a force directed towards the region where the magnetic field is the weakest

? C. Its speed will remain constant

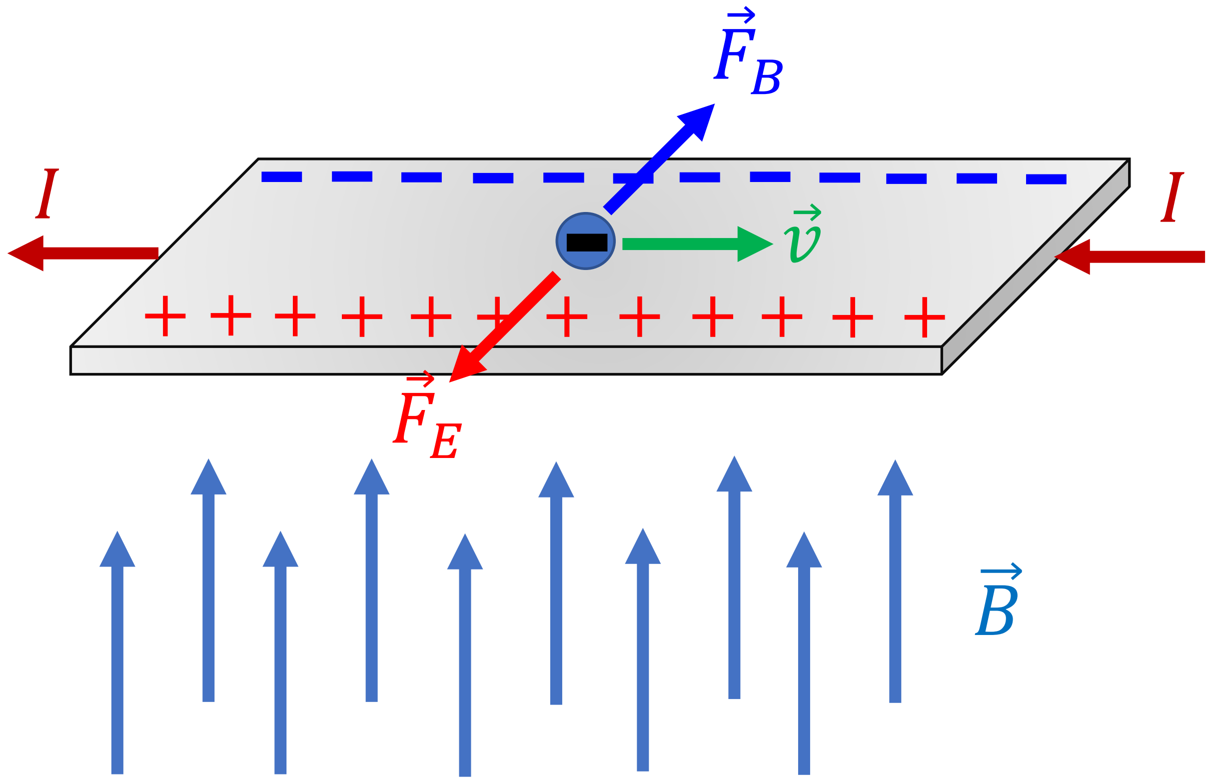
D. Need to know details of the B-field to answer this question

E. Both B and C will occur

The Hall effect

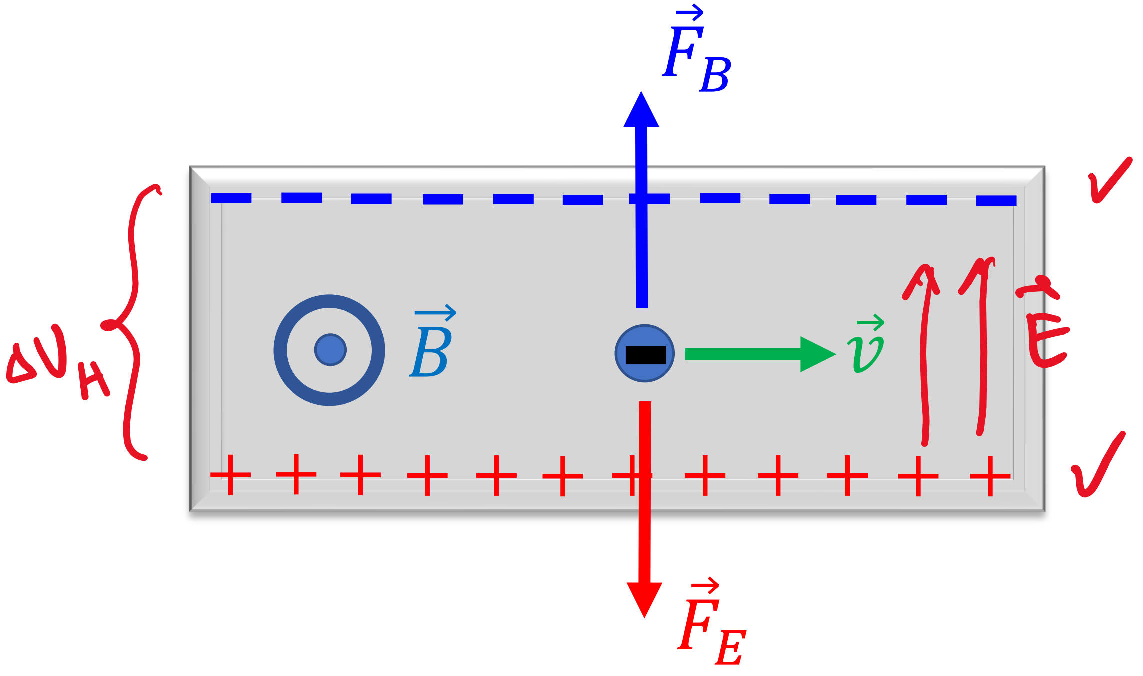
- A potential difference ("*the Hall voltage*") develops across a plane conductor in a perpendicular magnetic field when current is passing through the conductor.

Example:



- B-field up, (conventional) current to the left;
- Actual charge carriers (electrons) are moving to the right;
- The magnetic force on them is into the page
- They get deflected towards the back side of the conductor
- "Lack of negative charge" accumulates on the front side
- The E-field builds up across the conductor!
- Electric force develops that stabilizes the flow of electrons.

- Magnetic force deflects moving conduction electrons to one side.
- Positive charge (missing electrons) accumulate on the other side.
- Electric field, and hence a voltage, build up across the conductor



- Steady state is reached when $F_B = F_E$.
- After the steady state is reached, the charges can flow without being deflected, since for them $F_B = F_E$.

- Potential difference ΔV_H ('the Hall voltage') is proportional to magnetic field strength.

Q: Electric current is flowing through a plane conductor of width w to the right as shown in the figure (the conventional current flows to the right, so that the electrons move to the left). The magnetic field of a magnitude B points out of the page. Assume that positive and negative charges accumulate at opposite surfaces of the bar, i.e. all positive and negative charges are separated by the same distance w.

a) Find the expression for the Hall voltage.

b) What is the Hall voltage ΔV_H if w = 5.0 mm, B = 0.3 T and the drift speed of the electrons is v = 0.1 mm/s?

$\vec{F}_m = e_- \vec{v} \times \vec{B} \rightarrow evB$ – magn.

$F_e = F_m$

$\downarrow \qquad \downarrow$

$\cancel{e}E = \cancel{e}vB$

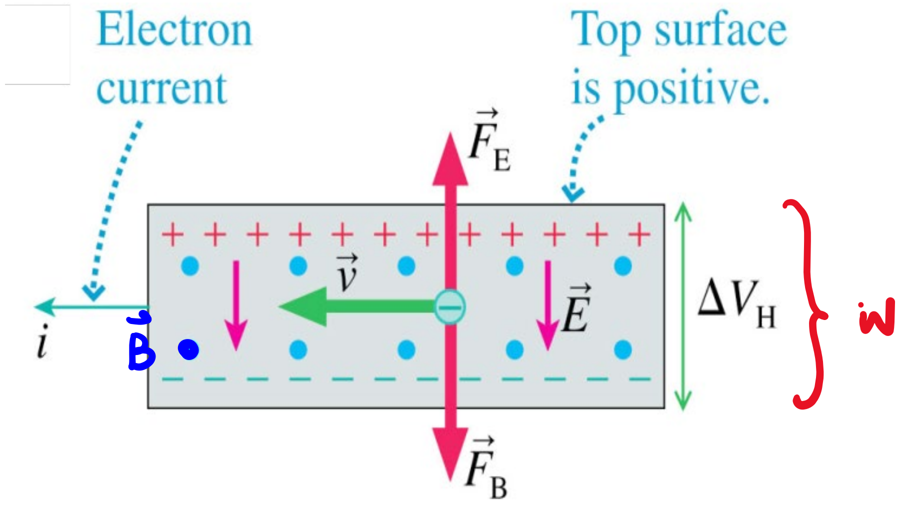
$\Delta V = -\int \vec{E} \cdot d\hat{r}$

$|\Delta V| = \int E\,dy = E \cdot w$

$E = \frac{|\Delta V|}{w}$

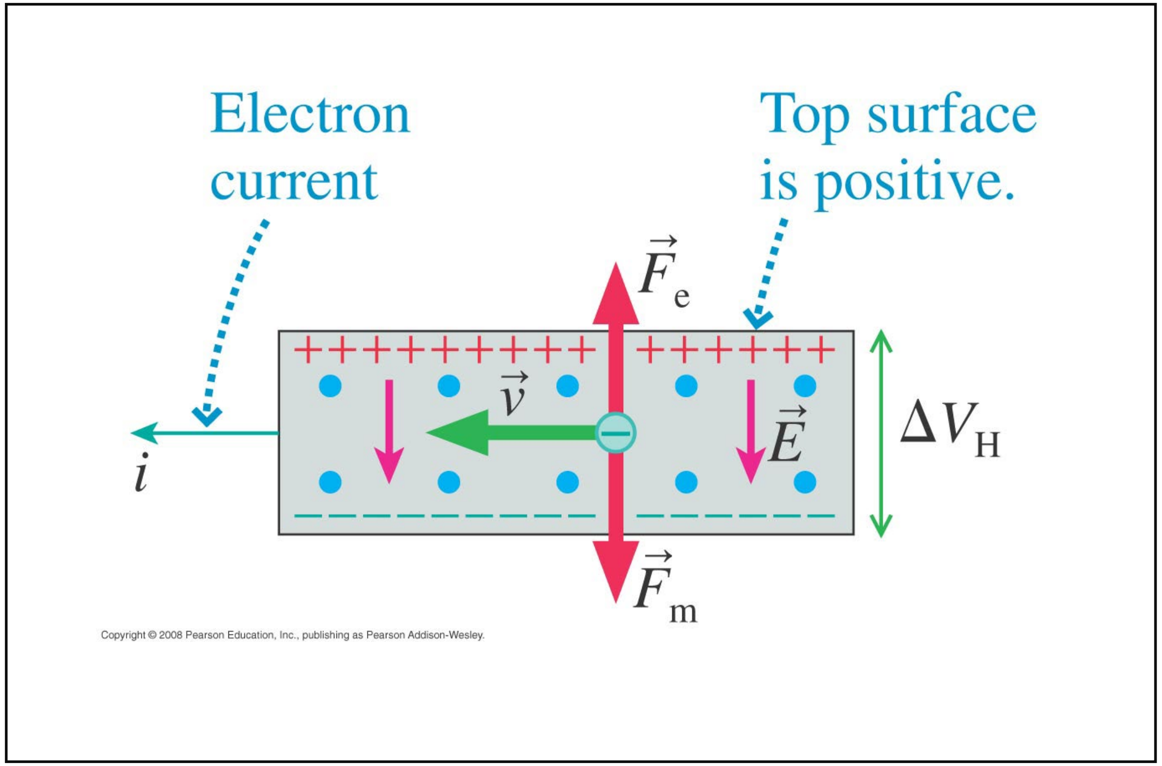
$\frac{\Delta V_H}{w} = vB \rightarrow \boxed{\Delta V_H = wvB}$

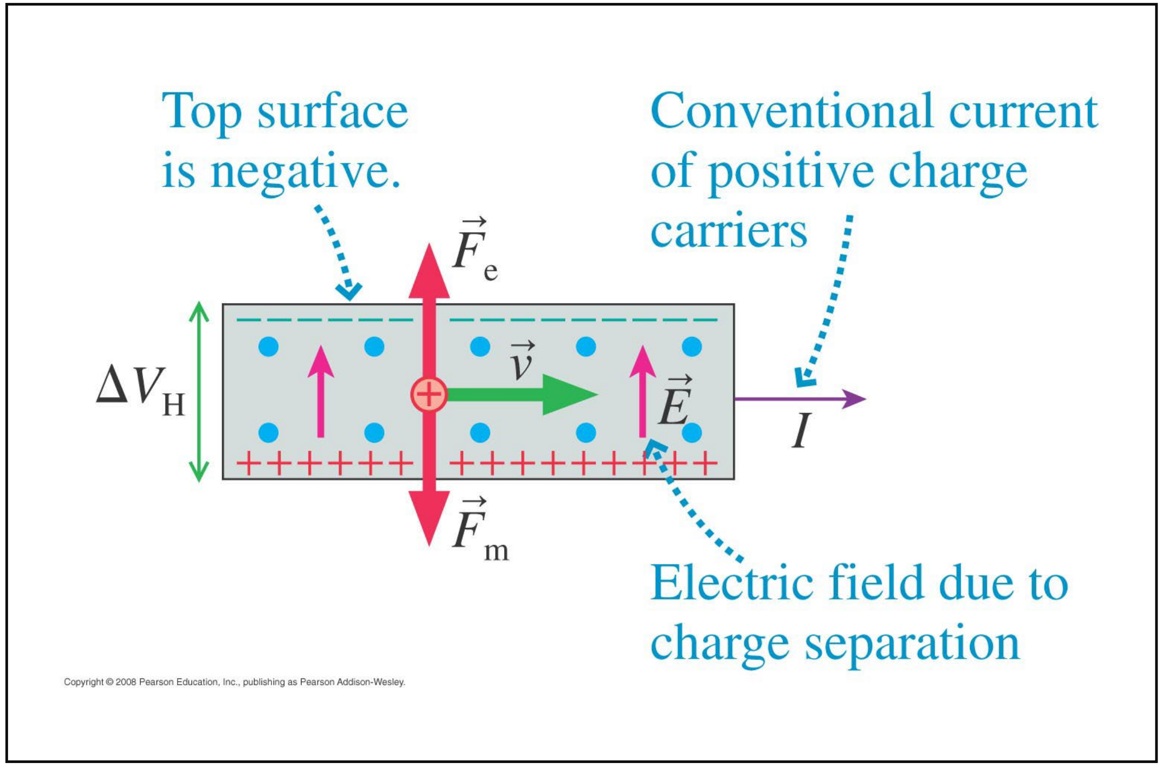
$\rightarrow 1.5 \cdot 10^{-7}$ V



Applications

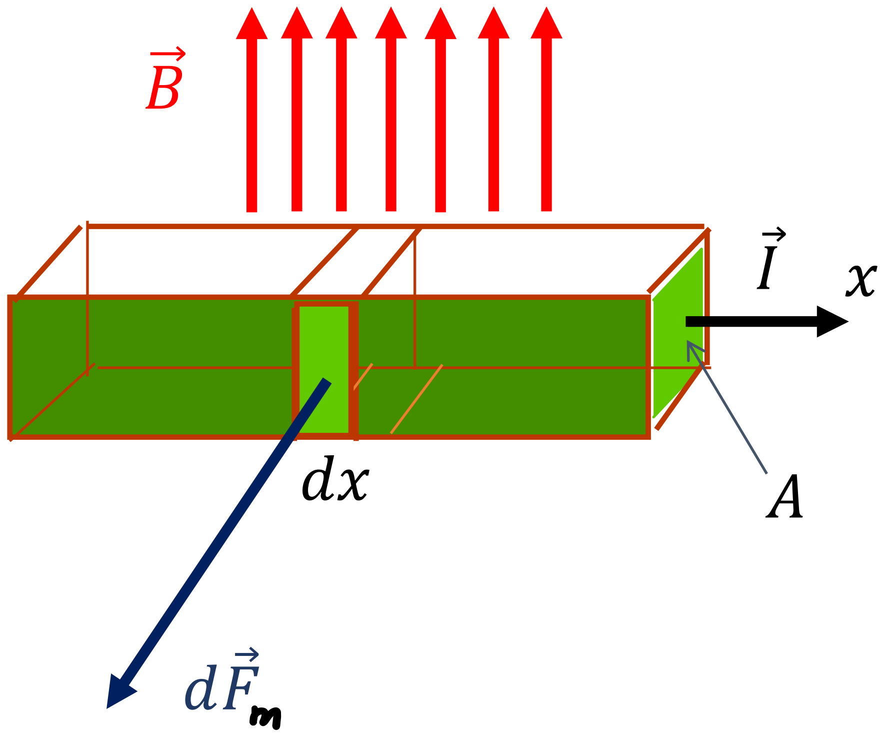
- The Hall effect can be used to:
  - measure magnetic fields using the equation $\Delta V_H = wvB$;
  - verify experimentally the sign of the charge carriers:





Note that in both these cases the conventional current is to the right!

Force on a "current segment"



- Consider a force acting on a wire segment dx carrying current I in a magnetic field $\vec{B}$.

$$d\vec{F} = dq\,\vec{v} \times \vec{B} = dq \frac{d\vec{x}}{dt} \times \vec{B} = \frac{dq}{dt}\, d\vec{x} \times \vec{B} = I\, d\vec{x} \times \vec{B}$$

$$\boxed{\vec{F} = \int I\, d\vec{x} \times \vec{B}}$$

- $d\vec{x}$ is a segment of the wire in the direction of the current.

- If $\vec{B} = \text{const}$: $\boxed{F = ILB \sin\theta_{\vec{I},\vec{B}}}$ with L = length of the wire segment immersed in the field & θ = angle between the wire and the B-field

dir: "$\vec{I}$" $\times \vec{B}$

- This force on a current-carrying conductor will be the basis for discussing electric motors, generators, alternators, etc !!