Lecture 35.

Paramagnetism and Diamagnetism.
Superconductivity.
Maxwell's equations.

Last Time:

- Faraday's law: Changing B-field is a source of E-field

$$\varepsilon = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

- Direction of induced E-field:

  Lenz's law + RHR (in imaginary current loop)

- Ampere's law: Electric current & changing E-field are sources of B-field

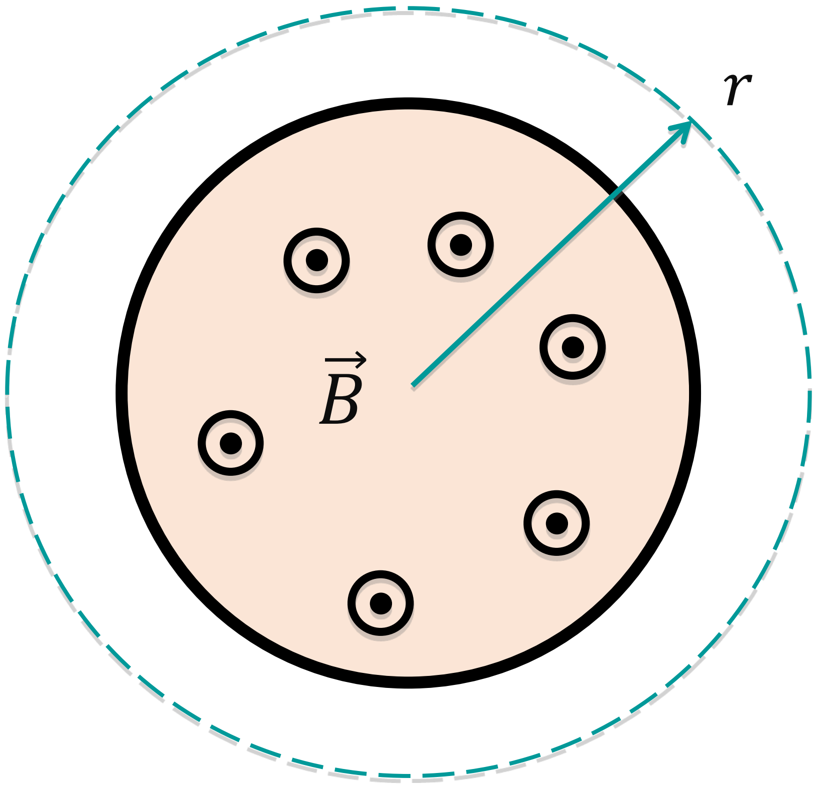
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{ext} + \mu_0 \left(\varepsilon_0 \frac{d\Phi_E}{dt} \right)$$

- Direction of induced B-field:

  RHR with external current or
  with "displacement current"

Q: The current in an infinitely long solenoid with uniform magnetic field $\vec{B}$ inside is increasing so that the magnitude B increases in time as $\boxed{B = B_0 + kt.}$

In what direction is the induced E-field on a circular loop of radius r outside the solenoid, as shown?

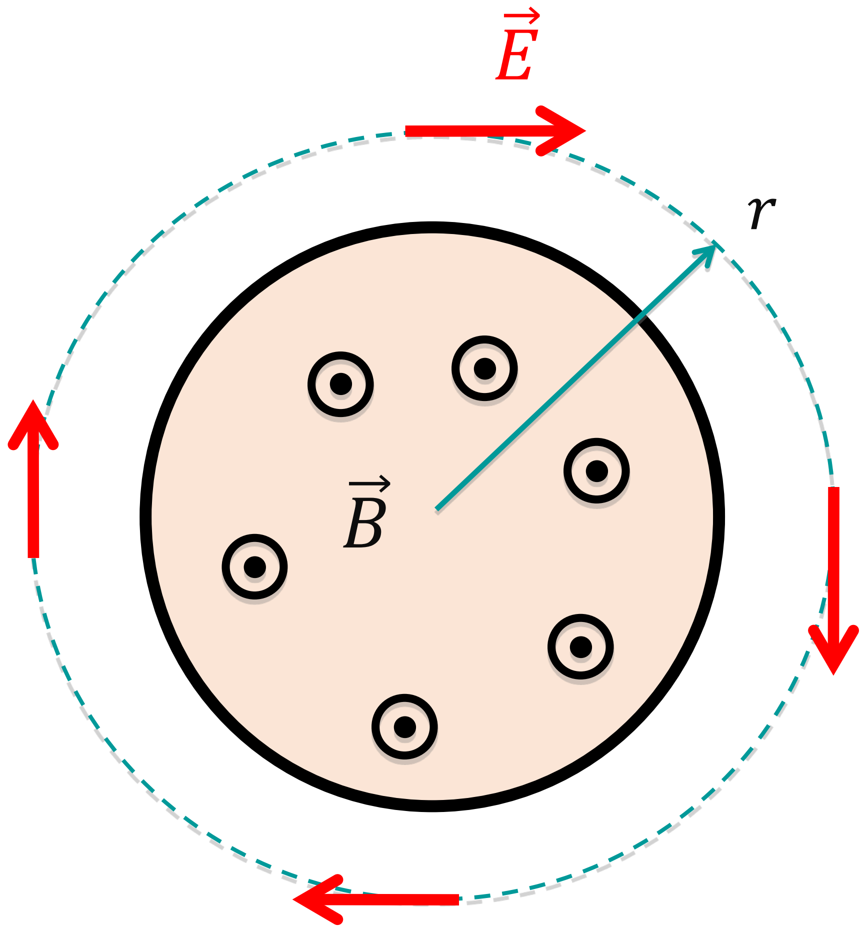


A. CW
B. CCW
C. The induced E is zero
D. Radial, inwards
E. Radial, outwards

Q: The current in an infinitely long solenoid with uniform magnetic field $\vec{B}$ inside is increasing so that the magnitude B increases in time as $B = B_0 + kt$.

In what direction is the induced E-field on a circular loop of radius r outside the solenoid, as shown?

- Magnetic flux out of the page increases =>
- Need magnetic flux into the page (Lenz's law) =>
- If there were a loop at r, the current would have been CW => E-field CW



A. CW

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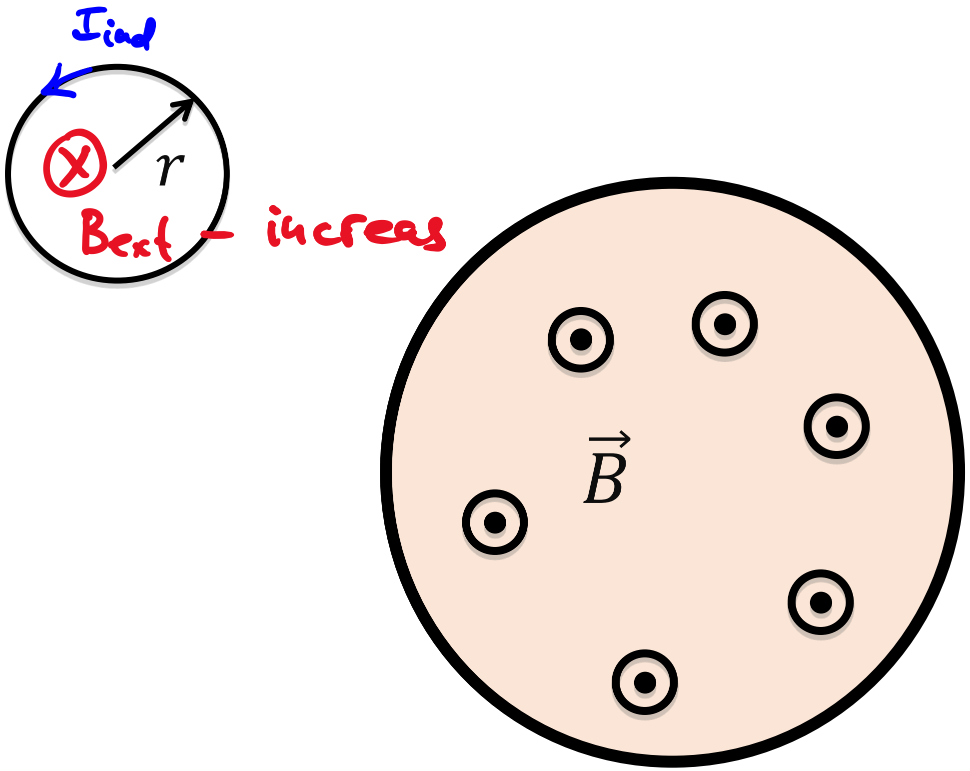
Q: The current in an infinitely long solenoid with uniform magnetic field $\vec{B}$ inside is increasing so that the magnitude B increases in time as $B = B_0 + nt$.

A small circular loop of radius r is placed outside the solenoid as shown. What is the direction and magnitude of emf around the small loop?



A. $\varepsilon = n \cdot \pi r^2$, CW

B. $\varepsilon = n \cdot \pi r^2$, CCW

C. $\varepsilon = n \cdot 2\pi r$, CW

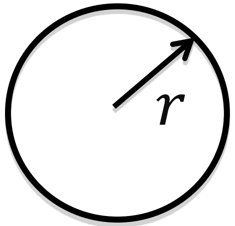
D. $\varepsilon = n \cdot 2\pi r$, CCW

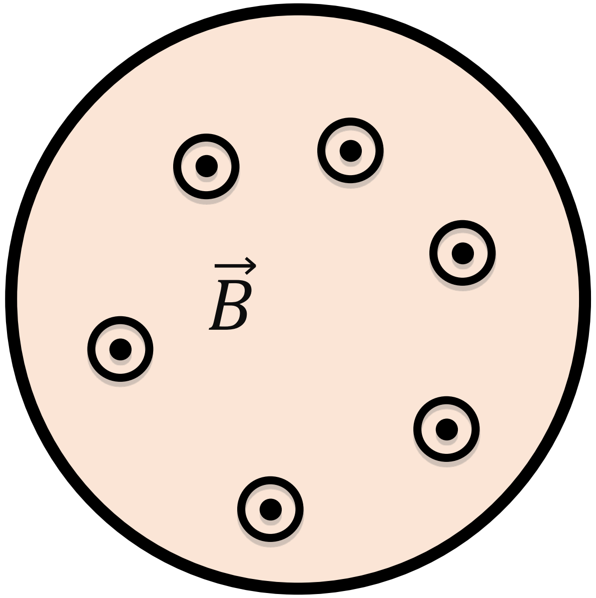
E. Zero

Q: The current in an infinitely long solenoid with uniform magnetic field $\vec{B}$ inside is increasing so that the magnitude B increases in time as $B = B_0 + nt$.

A small circular loop of radius r is placed outside the solenoid as shown. What is the direction and magnitude of emf around the small loop?

- For infinite solenoid, $B_{out} \equiv 0$.
- But, for a finite solenoid:
  - ➢ B-flux at loop: $\otimes$ and increasing
  - ➢ Need B-field at loop $\odot$ =>
  - ➢ I_{ind} CCW
  - ➢ $|\varepsilon| = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi r^2 \frac{dB_{out}}{dt}$





A. $\varepsilon = n \cdot \pi r^2$, CW

B. $\varepsilon = n \cdot \pi r^2$, CCW

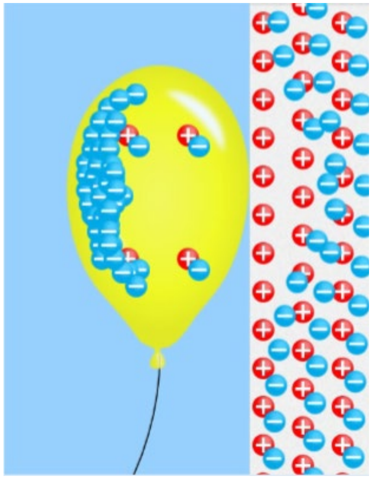
C. $\varepsilon = n \cdot 2\pi r$, CW

D. $\varepsilon = n \cdot 2\pi r$, CCW

E. Zero

Paramagnetic and Diamagnetic Materials

Q: Does this remind you of something?



Maybe electric polarization?

(interaction between a charged and a neutral object)

paramagnetic

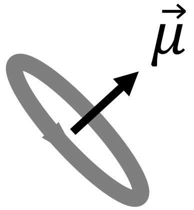
(O_2, Al, Ca, …)

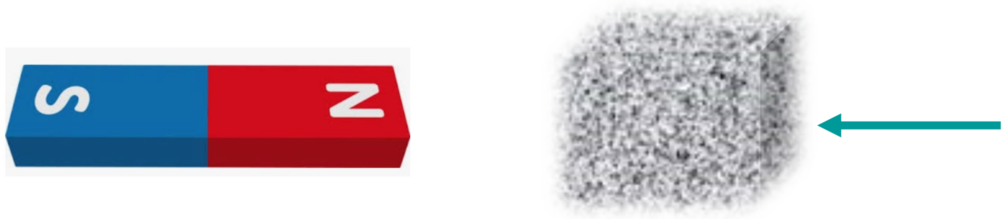
diamagnetic

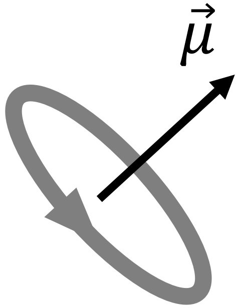
(water, wood, organic, copper)

Paramagnetic and Diamagnetic Materials: Electronic structure

Think about each valence electron as a tiny loop current
– or, more strictly, as a tiny magnetic moment

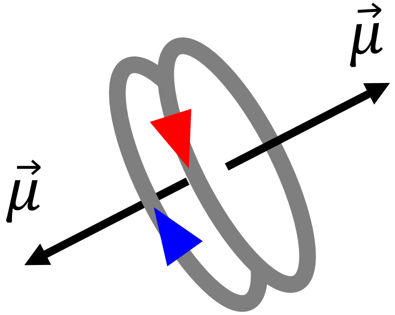






paramagnetic

there are unpaired electrons

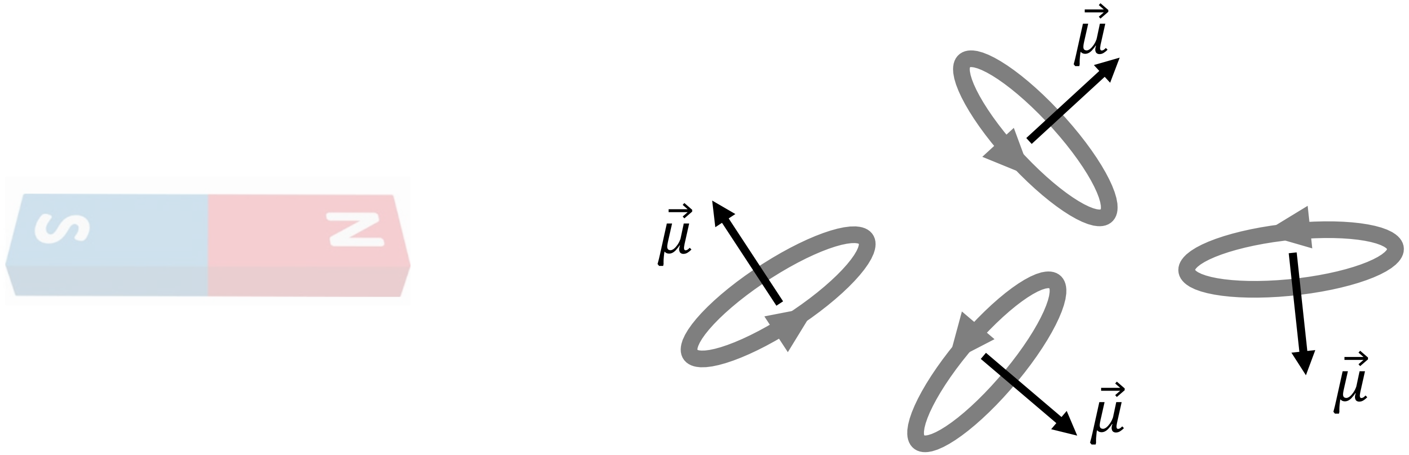


diamagnetic

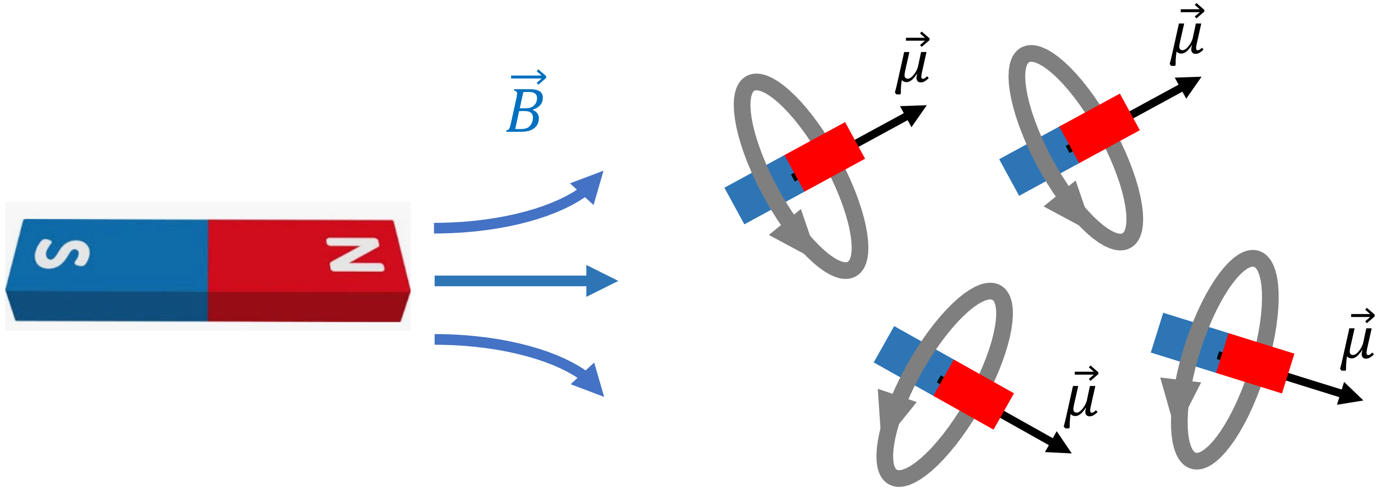
all electrons are paired

Paramagnetic Materials

there are unpaired electrons



- When there is no external B field, they are randomly oriented => net magnetic moment is zero

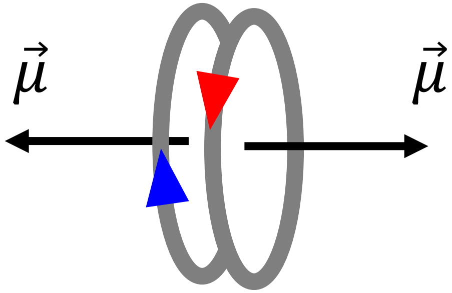


- External B field aligns them, and net (macroscopic) magnetic moment appears

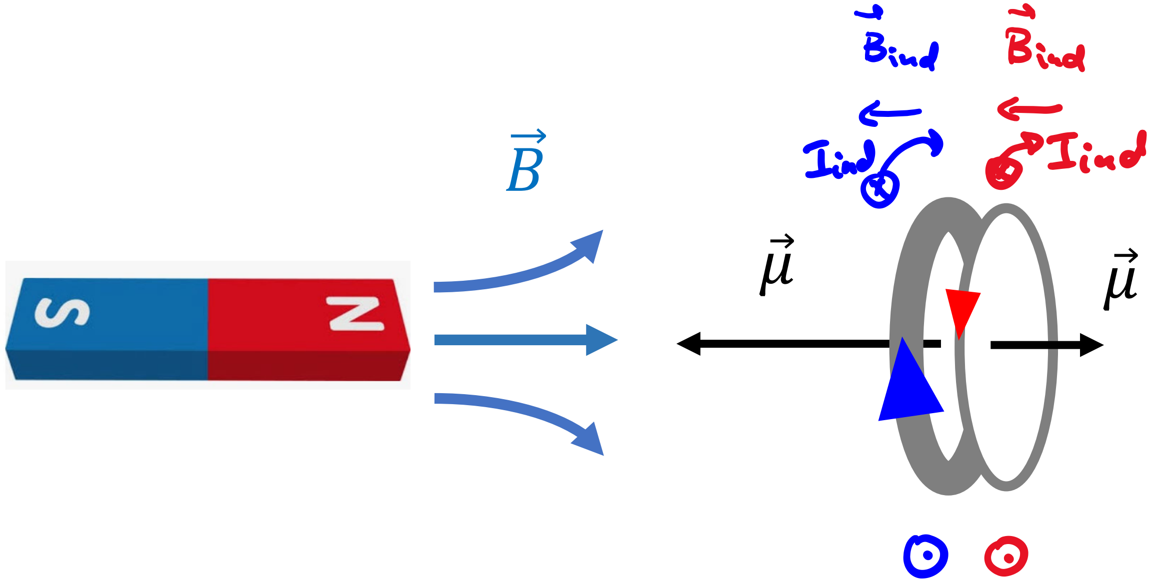
- Attraction!

Diamagnetic Materials

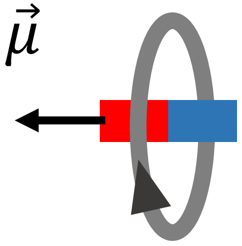
all electrons are paired



- No net micro-magnetic moment ☹
- But: there is magnetic induction!!!
- What will each tiny loop do to oppose the change?



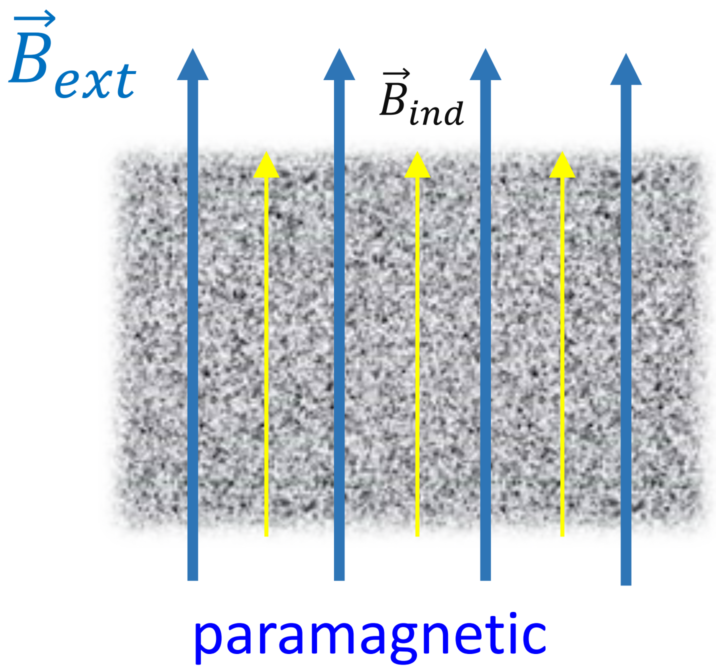
- Net magnetic moment to the left!

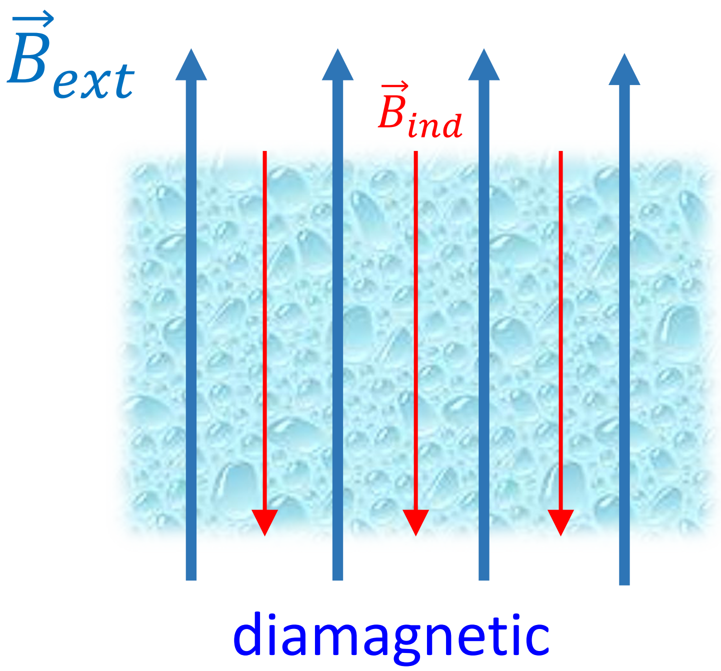


- Repulsion!

Paramagnetic and Diamagnetic Materials

- Hence, paramagnetic and diamagnetic materials respond differently to external magnetic field by producing different magnetisation:





- Induced magnetic fields are tiny and disappear when you remove $\vec{B}_{ext}$

Superconductivity

- Not on the exam, but cool

- There is, however, a situation when diamagnetism is so strong that it expels magnetic field out of the medium entirely: superconductivity.

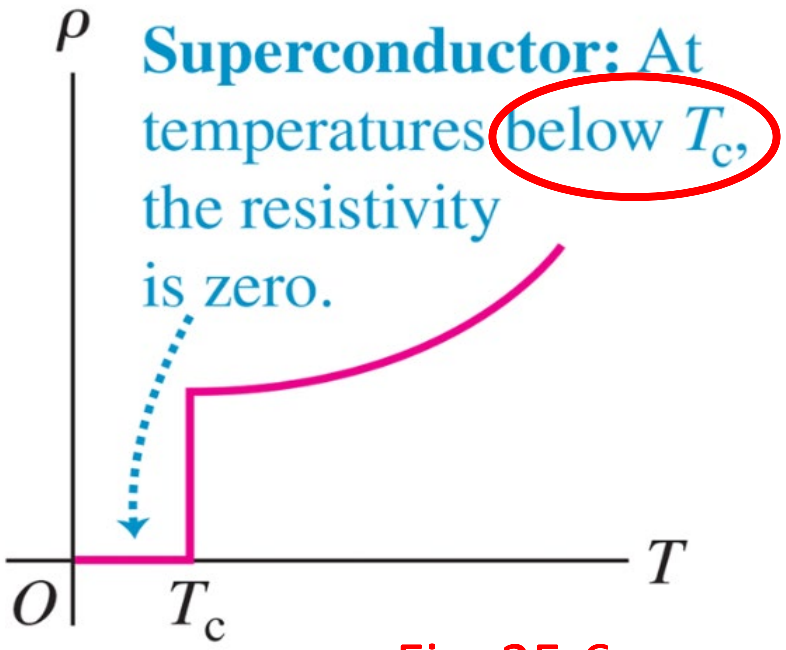
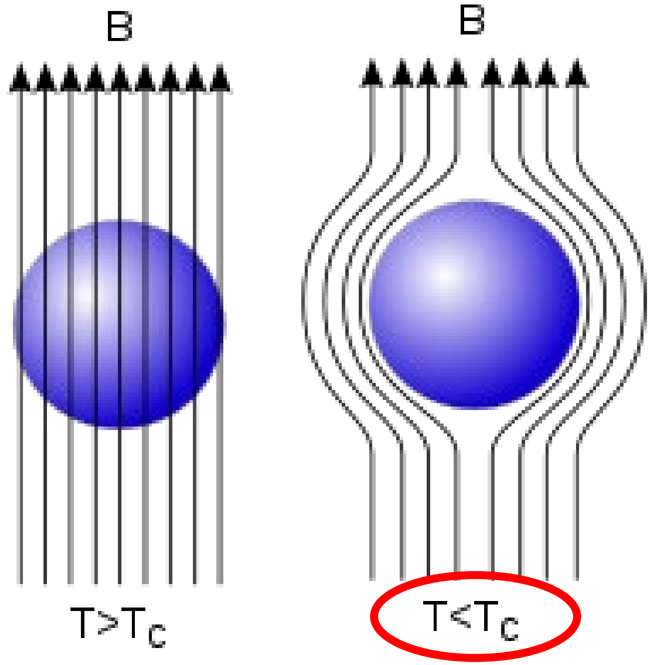


Fig. 25.6

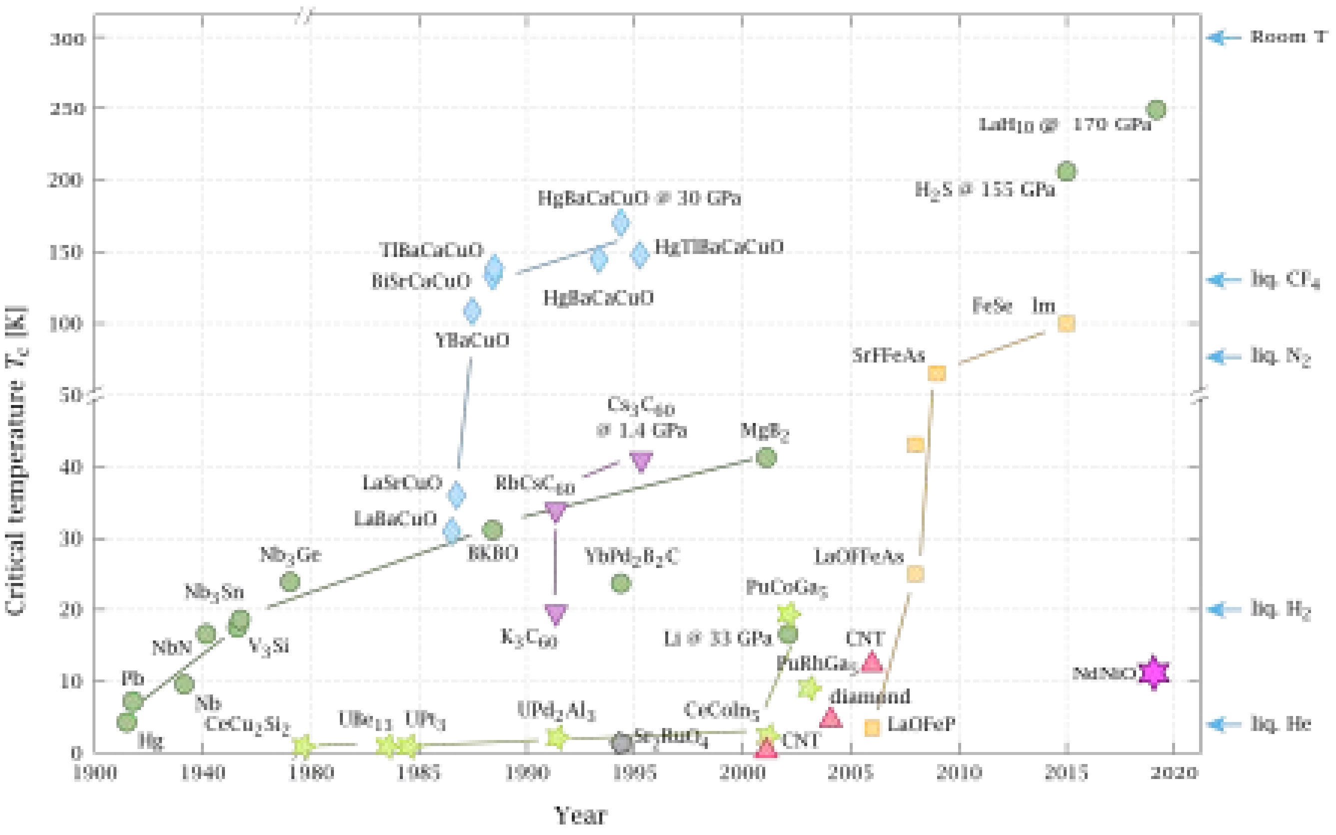
- Resistivity becomes zero. Literally.



- B field becomes expelled from SC (Meissner effect)

Superconductivity

- What is the critical temperature for superconductivity?



From Wikipedia

- First discovery:

  Heike Kamerlingh Onnes (1911)
  at T < 4.2 K

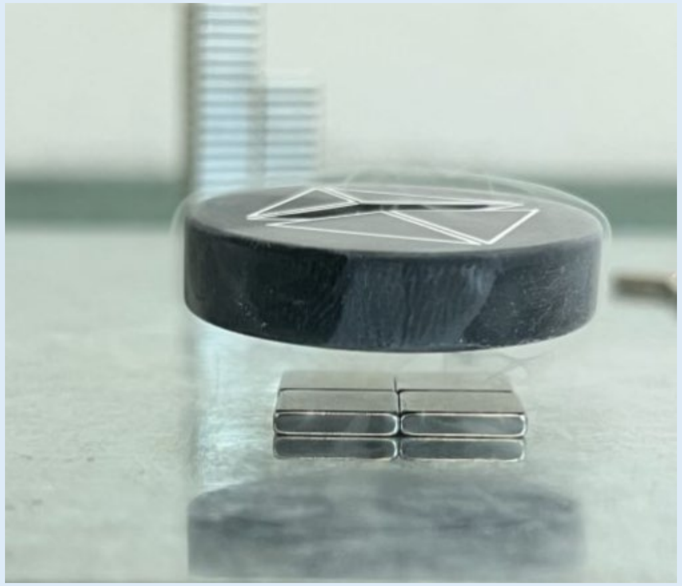
- High-temperature SC:

  at T > 77 K = -238 C (liquid nitrogen boiling point)

- Room-temperature SC:

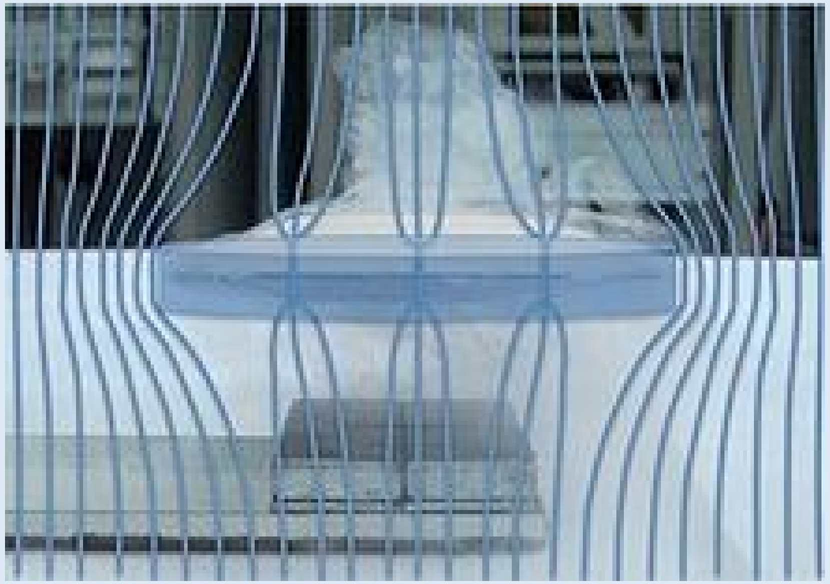
  Just think about what we could achieve with that…

Superconductivity: Magnetic levitation



DEMO!

- Explanation: magnetic levitation happens due to flux pinning aka quantum locking



- Strong B_{ext} overcomes the Meissner expulsion and penetrates the superconductor as tiny magnetic threads (fluxons). Magnetic field is constrained in these discrete threads.
- Superconductor "does not want" the fluxons to move around since it dissipates energy => superconductor is "locked" by these threads in mid air!

https://www.ted.com/talks/boaz_almog_the_levitating_superconductor

https://www.youtube.com/watch?v=kSoC1Sjj74U

Superconductivity: Magnetic levitation

https://en.wikipedia.org/wiki/Maglev

Different maglev systems achieve levitation in different ways, which broadly fall into two categories: electromagnetic suspension (EMS) and electrodynamic suspension (EDS). The power needed for levitation is typically not a large percentage of the overall energy consumption of a high-speed maglev system.[7] Instead, overcoming drag takes the most energy.

Despite over a century of research and development, there are only six operational maglev trains today — three in China, two in South Korea, and one in Japan.[8][9]

There are two competing efforts for high-speed maglev systems, i.e., 300–620 km/h.

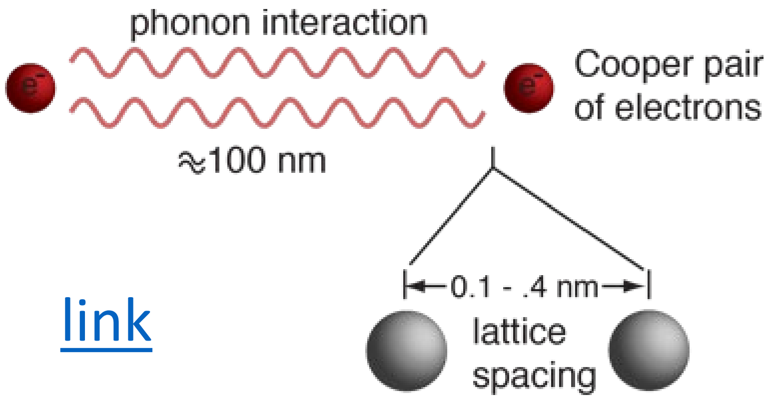
- The first is based on the Transrapid technology used in the Shanghai maglev train
  - 2006: 500 km/h, testing on a new 1.5-kilometre test track (Tongji University, northwest of Shanghai).
  - 2019: 600 km/h, testing.
  - 2021: 600 km/h, operating (Qingdao[65])

- A second, incompatible high-speed prototype was constructed by Max Bögl and Chengdu Xinzhu Road & Bridge Machinery Co. Ltd. and in 2021. Developed at Southwest Jiaotong University in Chengdu, the Super Bullet Maglev design uses high-temperature superconducting magnets, is designed for 620 km/h and was demonstrated on a 165-metre test track.[67]

Superconductivity: How??

- Macroscopic theory of SC: V.L. Ginzburg and L.D. Landau (1950)
- Full microscopic theory of SC: J. Bardeen, L. Cooper and J.R. Schrieffer (1957)

- Explanation: Superconducting current is a superfluid of Cooper pairs (electrons coupled with each other through lattice vibrations, aka phonons)



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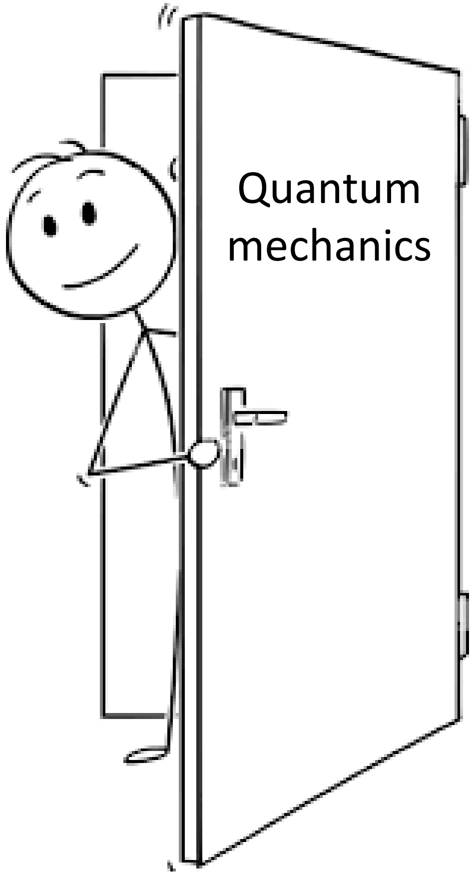
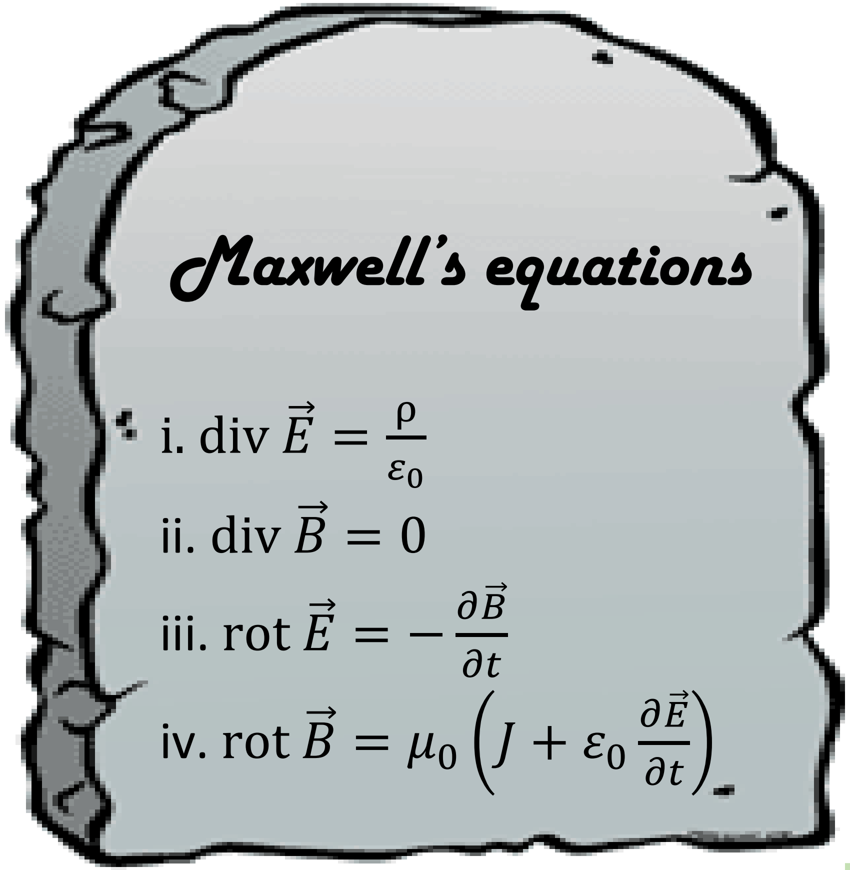


Maxwell’s Equations



$$\ldots + \varepsilon_0 \mu_0 \frac{d\Phi_e}{dt}$$

- System of Maxwell’s equations
- Electromagnetic waves

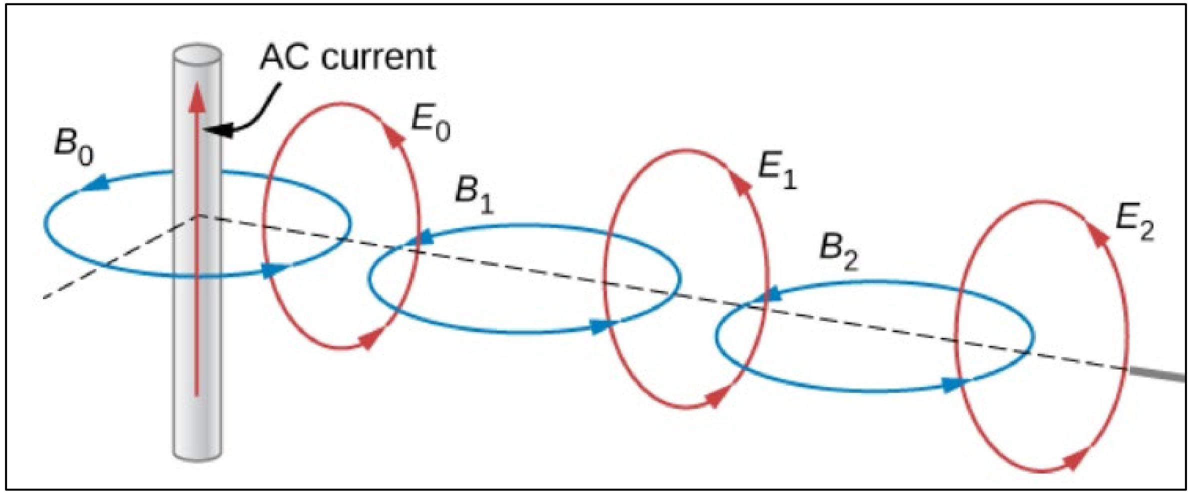
Law	Integral form	Differential form	Meaning
Gauss's law	$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0}$	$\text{div}\,\vec{E} = \frac{\rho}{\varepsilon_0}$	Electric charges produce electric field.
Gauss's law for magnetism	$\oint \vec{B} \cdot d\vec{A} = 0$	$\text{div}\,\vec{B} = 0$	There are no magnetic monopoles ("magnetic charges").
Faraday's law	$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$	$\text{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$	Electric field can be also produced by changing magnetic field.
Ampère-Maxwell law	$\oint \vec{B} \cdot d\vec{l} =$ $= \mu_0 I_{ext} + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$	$\text{rot}\,\vec{B} =$ $= \mu_0 \left(J + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$	Magnetic field is produced by electric currents and changing electric field.

Maxwell's Equations & Electromagnetic Field

- You don't need to worry about the "Differential form" column. Though in practice Maxwell's equations are used in their differential form, this is beyond the scope of this course. What we will need is the reduced table below.

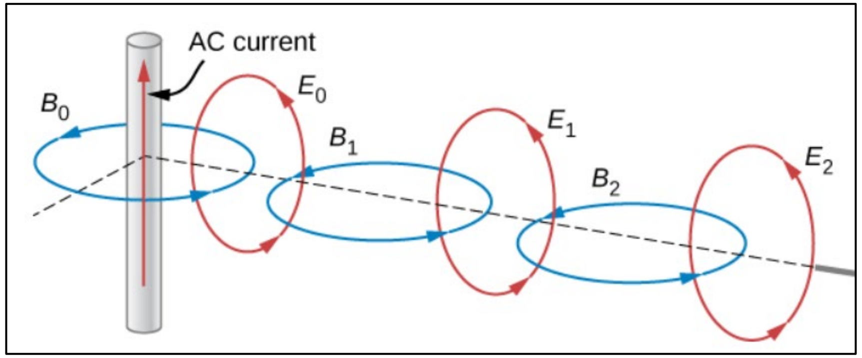
- We see that changing B-field is a source for E-filed, and simultaneously changing E-field is a source for B-field

- Hence, E- and B-fields are coupled (if one of them appears and changes, the other appears and changes, too!



Gauss's law	$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0} \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0}$
Gauss's law (magnetism)	$\oint \vec{B} \cdot d\vec{A} = 0$
Faraday's law	$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$
Ampère-Maxwell law	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{ext} + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt}$

Maxwell's Equations & Electromagnetic Field



- E- and B-fields are coupled
- More than that: it is not always possible to tell electric field from magnetic field…
- In the dynamic limit, there is no "electric" or "magnetic" fields, there is one entity called "electromagnetic field", with electric and magnetic fields being its inseparable sides (like two sides of the same coin)

Gauss's law	$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0} \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{in}}}{\varepsilon_0}$
Gauss's law (magnetism)	$\oint \vec{B} \cdot d\vec{A} = 0$
Faraday's law	$\oint \vec{E} \cdot d\vec{s} = -\cancel{\frac{d\Phi_m}{dt}}$
Ampère-Maxwell law	$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{ext} + \cancel{\varepsilon_0\mu_0 \frac{d\Phi_e}{dt}}$

Q: Then how comes that we have studied them separately for 11 weeks??

A: In the static limit, E field and B field decouple.