Physics 157 Tutorial – Solutions

Problem 1: From an old midterm:

3. A cylinder contains Nitrogen gas ($C_v = 20.8$J/(mol K), $\gamma = 1.4$) at a pressure of 2.0 atm and a temperature of 300K. Its initial volume is 2.0 liters. The Nitrogen gas is next carried through the following processes: 1) It is heated at constant volume to a temperature of 450K. 2) It is expanded at constant temperature to a volume of 3.0 liters. 3) It is compressed at constant pressure until its volume is 2.0 liters, returning it to its initial state.

a) Draw the PV curve of this cycle. Be sure to label the temperature, pressure, and volume at each endpoint.

b) What is the efficiency of this heat engine? Use 1atm ≈ 100kPa

Note: efficiency of the engine is defined to be the **net** work done during the cycle divided by the heat **added**, ignoring any heat that flows out (i.e. parts where Q<0)

See the last page for hints and a step-by-step approach, but see how much you can do without using this! * Use initial state to find $n = \frac{PV}{RT} = \frac{200 \times 2}{8.31 \cdot 300} = 0.160$

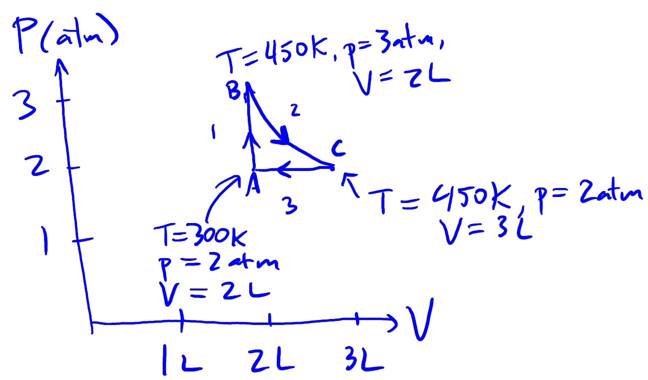
a) Initial state: $P = 2.0$ atm, $V = 2L$, $T = 300K$

process 1) const. volume, so $\frac{P}{T}$ constant. $\frac{P_f}{P_i} = \frac{T_f}{T_i} = 1.5$ so $P_f = 3.0$atm

process 2) const. T, so $P \cdot V = $ const $P_f \cdot V_f = P_i \cdot V_i = 3\text{atm} \cdot 2L = 6\text{atm} \cdot L$

$V_f = 3L$ so $P_f = 2$atm.

This is consistent with process 3 being a const. pressure process taking us back to the initial state



b) $W_1 = 0$ (const vol)

$W_2 = nRT_B \ln \frac{V_C}{V_B} = P_B \cdot V_B \ln \frac{V_C}{V_B}$

$= 600J \cdot \ln(3/2) = 243J$

$W_3 = P\Delta V = 200kPa \cdot (-1L) = -200J$

$W_{net} = 43J$

$\Delta U_1 = n C_v \Delta T = 0.160 \times 20.8 \times 150\ J = 500\ J$

so $Q_1 = \Delta U_1 + W_1 = 500\ J$

$\Delta U_2 = 0$ so $Q_2 = W_2 = 243J$

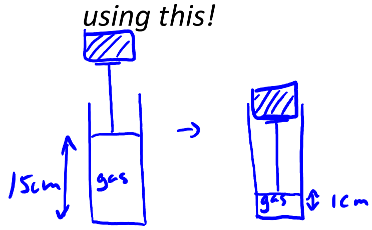
$\Delta U_3 = nC_v\Delta T = -500\ J$

so $Q_3 = \Delta U_3 + W_3 = -700\ J$

Efficiency is $\frac{W_{net}}{Q_{added}} = \frac{43J}{500J + 243J} = 0.058$

Problem 2) An ideal gas with $C_V = 3R$ is in a cylinder with area 0.25cm^2 with a piston that is originally 15cm above the bottom of the cylinder. The gas is initially at room temperature (20°C) and atmospheric pressure (100kPa). A 1.2 kg weight is placed on the top of the piston causing it to compress very quickly. At maximum compression, the piston is only 1cm above the bottom of the cylinder. What is the final temperature of the gas? You can ignore the mass of the piston and the gas, and ignore work done by the outside air.

See the last page for hints and a step-by-step approach, but see how much you can do without using this!



Since the compression is quick, the process is adiabatic, and $Q = 0$. This means $\Delta U = -W$, so we can calculate the temperature change as:

$$n C_v \Delta T = -W$$

Here, W is the work done by the gas, so $-W$ is the work done on the gas by the piston/weight. That amount of energy comes from the loss of potential energy:

$$-W = |\Delta E_{pot'l}| = |mg\Delta h| = 1.2\,kg \cdot 9.8\,m/s^2 \cdot 0.14\,m = 1.65\,J$$

To find n, we have: $n = \frac{V_i P_i}{R T_i}$ so $n \cdot R = \frac{(0.25\times10^{-4}\,m^2)\times(0.15\,m)\cdot 100\,kPa}{293\,K} = 0.00128\,J/K$

Finally: $$\Delta T = \frac{-W}{n C_v} = \frac{1.65\,J}{3nR} = \frac{1.65\,J}{3 \cdot 0.00128\,J/K} \approx 429.7\,K$$

$$\text{So } T_f = 293K + \Delta T \approx 723\,K$$

☆ We can also use $TV^{\gamma-1} = \text{constant}$ to get the same result ☆

Problem 3) For the cycle in the question 1), suppose that we are supplying heat by burning gasoline (this releases 34MJ/L), and we want to produce 1000W of mechanical power. What will be the required rate of fuel consumption in liters per hour? The efficiency is 0.058, so the heat added for 1000J of work is $\frac{1000J}{0.058} = 17.2\,kJ$. So we need 17.2 kJ/sec or 62 MJ/hour of heat. The fuel consumption is thus

$$\frac{62}{34} = 1.82\,L/hour.$$