Announcement

- No class on Monday (holiday)
- A make-up class on Thursday (here at Hebb 100, at 13:00 pm)
- Survey: your feedback (best over the long weekend)

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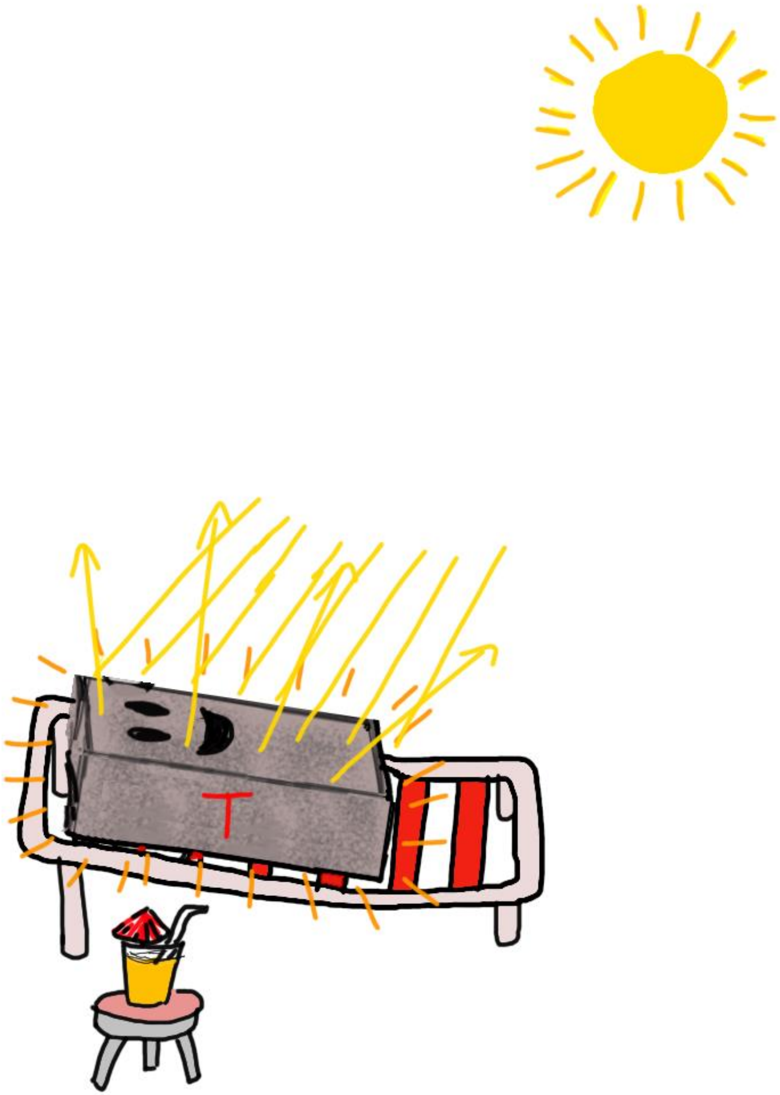


Lecture 13.

Energy balance and earth climate

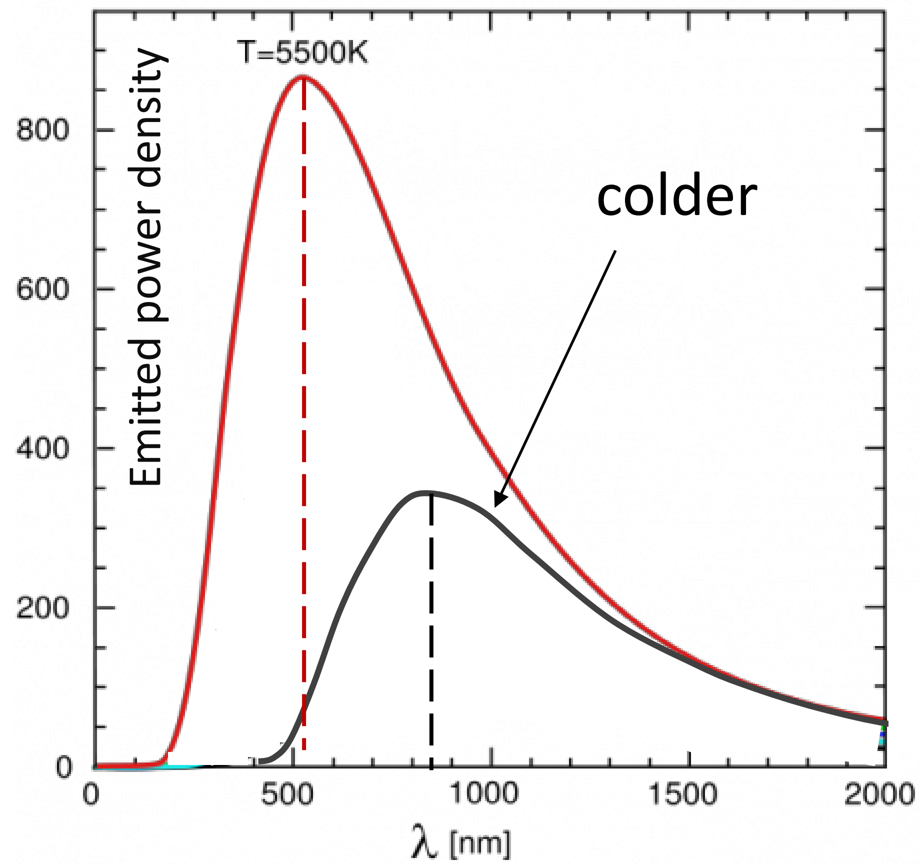
Last Time

- Wien's law

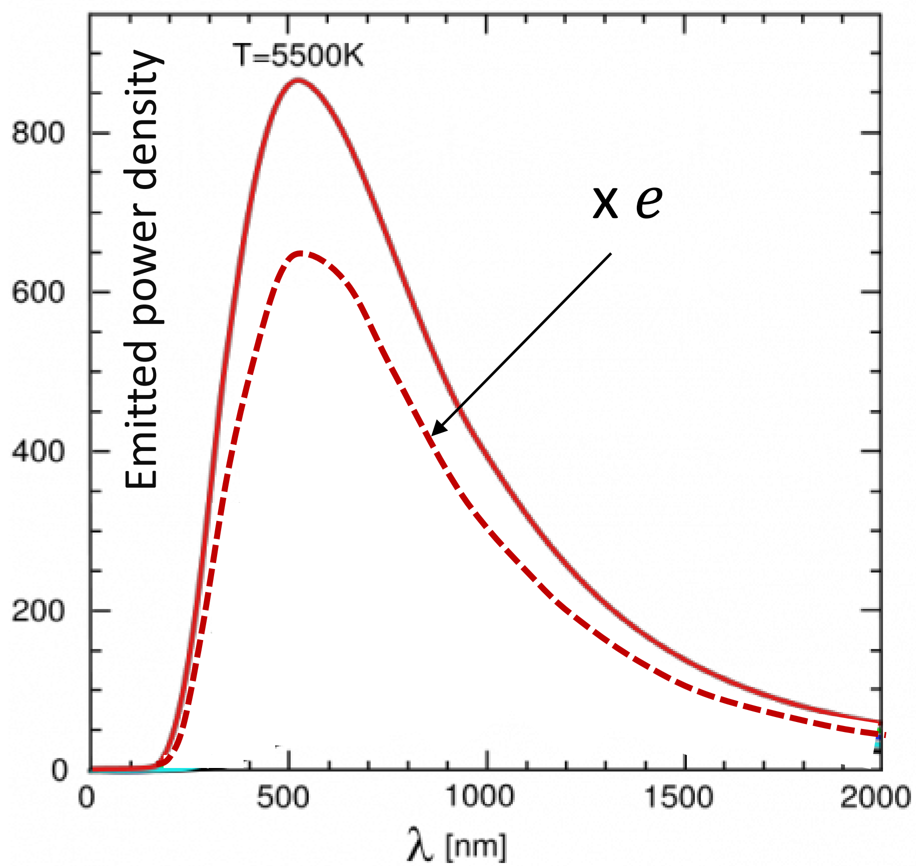
$$\boxed{\lambda_{max} = \frac{b}{T}}$$

- Stefan-Boltzmann law

$$\boxed{H = \frac{Q}{\Delta t} = Ae\sigma T^4}$$



- Emissivity



Q: Yoltar heats their little planet (far from any stars) with a 1GW heater. If they wish to double the equilibrium surface temperature of their planet, they should increase the power of their heater to:

$$H_{in} = H_{out}$$

$$H = Ae\sigma T^4$$

A. 1.21 GW

B. 2 GW

C. 4 GW

D. 8 GW

E. 16 GW

Hint: where does the energy from the heater go?

$H = Ae\sigma T^4$

Q: Yoltar heats their little planet (far from any stars) with a 1GW heater. If they wish to double the equilibrium surface temperature of their planet, they should increase the power of their heater to:

Steady state:

power from heater = power radiated

To double T, need 16 x P

A. 1.21 GW
B. 2 GW
C. 4 GW
D. 8 GW
E. 16 GW

Hint: where does the energy from the heater go?

A harder problem, but *really* interesting!

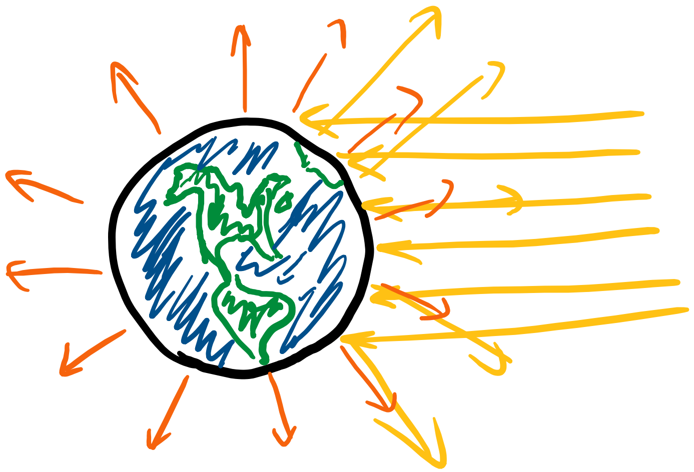
A planet with radius r = 6400 km lies at a distance R = 150,000,000 km from a yellow star with temperature T = 5700K and radius R_S = 695,000 km.

Which planet are we talking about?

The earth!

Let's use our knowledge of thermodynamics to estimate the surface temperature of our planet!

Energy balance for the earth



- Model:

$$H_{in} \rightarrow \boxed{\text{constant T}} \rightarrow H_{out}$$

- Key relation for steady state heat flow:

$$\boxed{H_{in} = H_{out}}$$

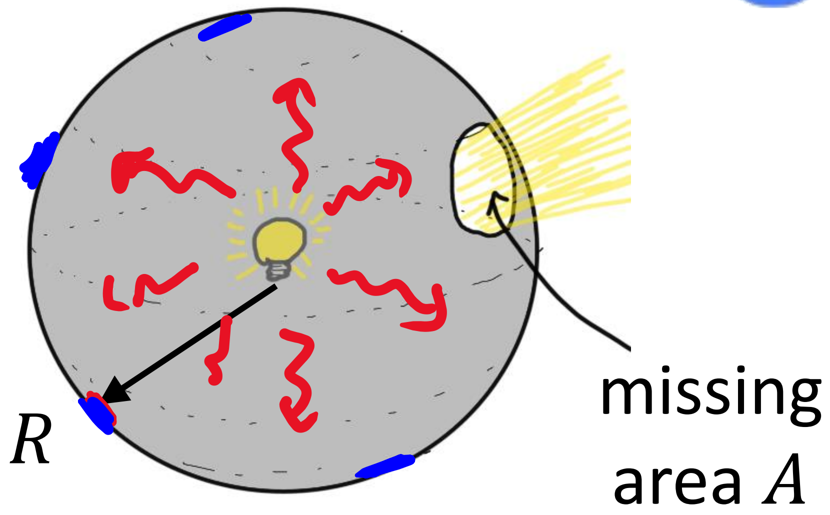
- H_{in} = absorbed sunlight = ??
- H_{out} = IR radiation = $Ae\sigma T^4$

➢ The emissivity e of earth is close to 1.

What is H_{in}?

Q: A light bulb producing 100 W of radiation is placed at the center of a sphere of perfectly absorbing material, with radius R. A hole is cut into the sphere, removing an area A of material. Assume the light from the bulb spreads out uniformly in all directions.

What is the rate of energy flow through the hole?



$$P_{bulb} \frac{A_{catch}}{A_{sphere}}$$

distributed

A. 100W

B. 100W × A

C. 100W × A/R^2

D. 100W × $A/(\pi R^2)$

E. 100W × $A/(4\pi R^2)$

Remember: Area of circle: πR^2
Area of sphere: $4\pi R^2$

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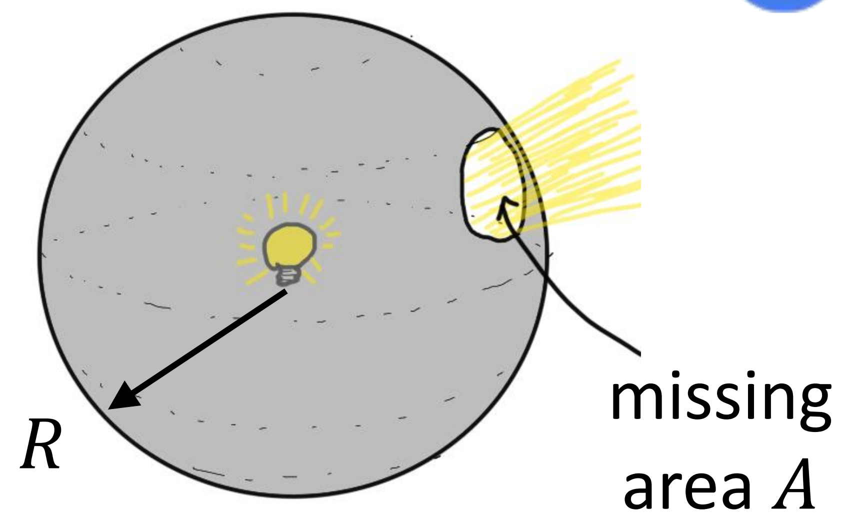
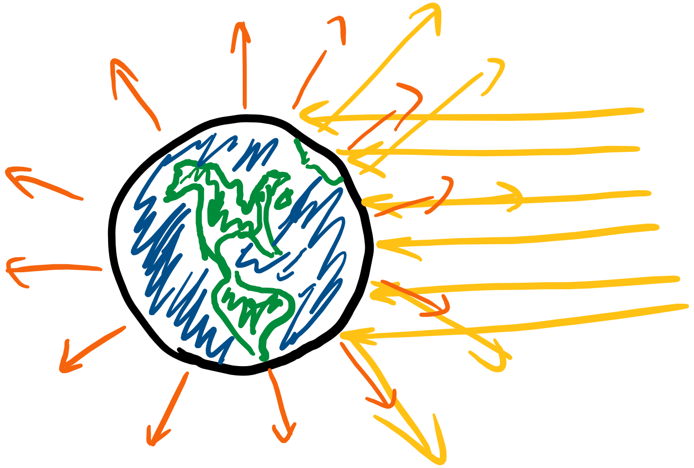


Light spreads out uniformly
Power leaving bulb = power reaching sphere
Hole covers fraction $A/(4\pi R^2)$ of sphere
So power of light coming out is 100W x $A/(4\pi R^2)$

A. 100W
B. 100W × A
C. 100W × A/R^2
D. 100W × $A/(\pi R^2)$
E. 100W × $A/(4\pi R^2)$ ✓

Remember: Area of circle: πR^2
Area of sphere: $4\pi R^2$

Albedo



- Albedo is the overall average reflection coefficient for solar radiation incident on an object
- Albedo of the Earth is about 0.3 which means that the Earth as a whole reflects 30% of solar radiation

Picture of earth taken by Apollo 17, December 7, 1972

Q: Let us put these two pieces together:

R_{sun}

- The power from the sun is: $H_{sun} \approx A_{sun}\sigma T_{sun}^4$
- Albedo (reflection coefficient) is α_e

What is the power H_{in} of solar radiation absorbed by the Earth? Answer in terms of H_{sun}, earth's albedo, a_e, (fraction of sunlight reflected) and the parameters r_e and R_{s-e} as shown.

spread

Hint: think about the earlier clicker question.

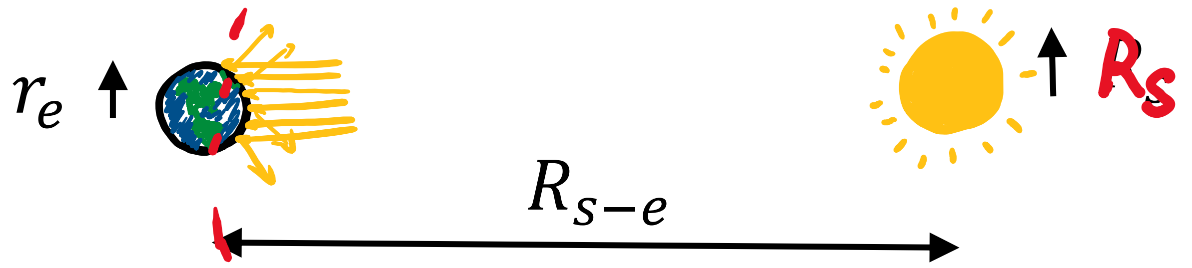
A. $H_{sun}\dfrac{\pi r_e^2}{4\pi R_{s-e}^2}a_e$

B. $H_{sun}\dfrac{\pi r_e^2}{4\pi R_{s-e}^2}(1-a_e)$

C. $H_{sun}\dfrac{2\pi r_e^2}{4\pi R_{s-e}^2}a_e$

D. $H_{sun}\dfrac{2\pi r_e^2}{4\pi R_{s-e}^2}(1-a_e)$

E. $H_{sun}\dfrac{4\pi r_e^2}{4\pi R_{s-e}^2}a_e$



πR_e^2

$(1-\alpha)$

$$H_{abs} = H_{sun} \cdot \frac{A_{catch}}{A_{spread}} \cdot (\text{account of reflection})$$

$4\pi R_{s-e}^2$

$e_{sun} \approx 1$
$A_{sun} = 4\pi R_s^2$

Catching light

- Regular earth:

- we catch sunlight with a cross-section
- We emit with surface area

- Flat earth:

- ➢ Each blocks the same amount of sunlight
- ➢ What matters for catching radiation is the cross-sectional area: πr_e^2
- ➢ What matters for emitting radiation is the surface area: $4\pi r_e^2$

Q: Let us put these two pieces together:

- The power from the sun is: $H_{sun} \approx A_{sun} \sigma T_{sun}^4$
- Albedo (reflection coefficient) is α_e

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Hint: think about the earlier clicker question.

A. $H_{sun} \dfrac{\pi r_e^2}{4\pi R_{s-e}^2} a_e$

B. $H_{sun} \dfrac{\pi r_e^2}{4\pi R_{s-e}^2} (1 - a_e)$ ✓

C. $H_{sun} \dfrac{2\pi r_e^2}{4\pi R_{s-e}^2} a_e$

D. $H_{sun} \dfrac{2\pi r_e^2}{4\pi R_{s-e}^2} (1 - a_e)$

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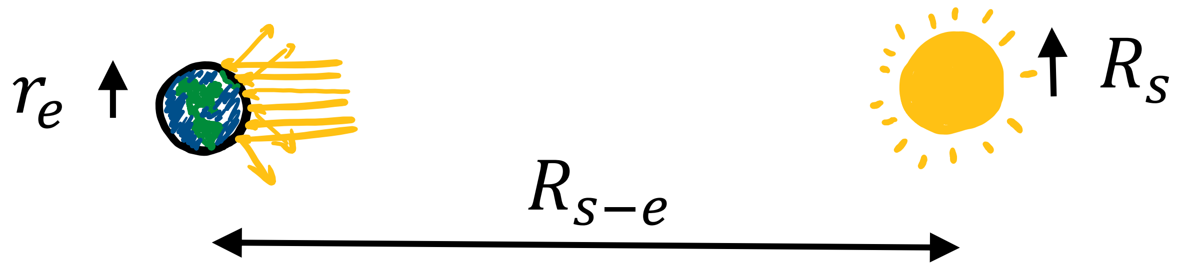


Cross-sectional area of earth is πr_e^2

Power hitting earth is: $H_{sun} \dfrac{\pi r_e^2}{4\pi R_{s-e}^2}$

Fraction (1-a_e) is absorbed, so:

$$H_{in} = H_{sun} \frac{\pi r_e^2}{4\pi R_{s-e}^2} (1 - a_e)$$

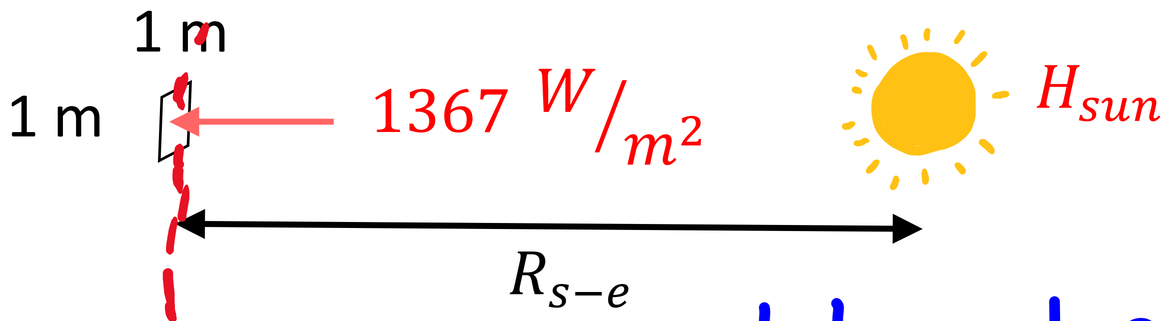
$e_{sun} \approx 1$
$A_{sun} = 4\pi R_s^2$

The solar constant

- At earth's orbit, the power per unit area, or **intensity**, of sunlight is:

$$I_{sc} = \frac{H_s}{4\pi R_{s-e}^2} = 1367\ {}^{W}/_{m^2}$$

- I_{sc} is called the "solar constant"
  - ➢ Doubling the distance would mean 1/4 the intensity



- In terms of I_{sc}, the heat current into the earth due to sunlight is:

$$H_{in} \approx \overbrace{\pi r_e^2}^{\text{catchment area}}\ \overbrace{(1 - a_e)}^{\text{reflection}}\ I_{sc} \longleftarrow \text{source intensity}$$

Q: The diameter of Saturn is ~ 10x that of Earth and it is ~ 10x as far away from the Sun. The power of solar radiation hitting Saturn compared to that hitting Earth is:

A. 100x smaller
B. 10x smaller
C. the same
D. 10x bigger
E. 100x bigger

$$H_{in, Sat} = H_{sun} \frac{A_{catch}}{A_{spread}} =$$

$$= H_{sun} \frac{\pi R_{sat}^2}{4\pi R_{sun-sat}^2} = H_{sun} \frac{\pi (\cancel{10} R_e)^2}{4\pi (\cancel{10} R_{sun-e})^2}$$

100

100

Q: The diameter of Saturn is ~ 10x that of Earth and it is ~ 10x as far away from the Sun. The power of solar radiation hitting Saturn compared to that hitting Earth is:

A. 100x smaller
B. 10x smaller
C. the same
D. 10x bigger
E. 100x bigger

Intensity of sunlight (power per unit area): $I = \frac{H_{sun}}{4\pi R^2}$

10x R $\Rightarrow I_j = \frac{1}{100} I_e$

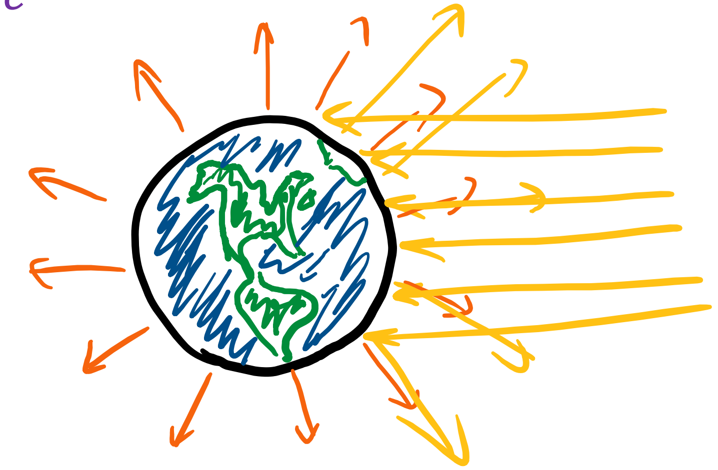
Total power (area x Intensity) and $A = \pi r^2$

10x r $\Rightarrow A_j = 100 A_e$

So, $H_j = H_e$

Here R = distance from the source to the planet (determines "spread area"), and r is the radius of the planet (determines "catching area").

Q: Calculate the equilibrium surface temperature T_e in terms a_e, I_{sc}, r_e, R_{s-e}, σ, and the emissivity e_e



Our model:

- Steady state: $H_{in} = H_{out}$

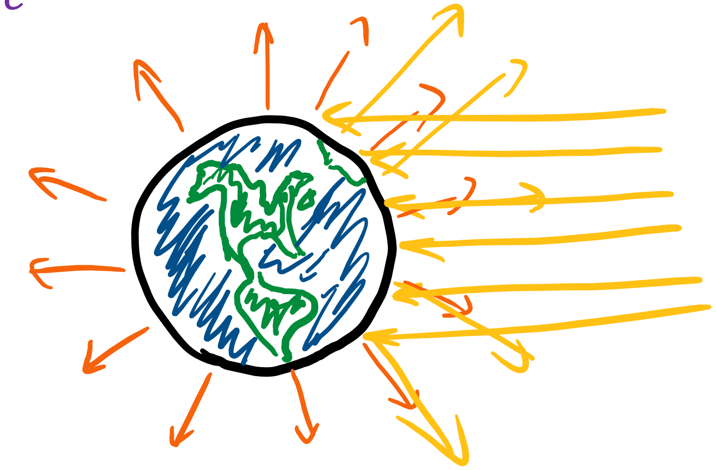
A. $T_e = \left[\frac{a_e I_{sc}}{4 e_e \sigma}\right]^{1/4}$

B. $T_e = \left[\frac{(1-a_e) I_{sc}}{4 e_e \sigma}\right]^{1/2}$

C. $T_e = \left[\frac{(1-a_e) I_{sc}}{4 e_e \sigma}\right]^{1/4}$

D. $T_e = \left[\frac{\pi r_e^2}{4\pi R_{s-e}^2} \frac{(1-a_e) I_{sc}}{e_e \sigma}\right]^{1/4}$

Q: Calculate the equilibrium surface temperature T_e in terms $a_e, I_{sc}, r_e, R_{s-e}, \sigma,$ and the emissivity e_e



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Our model:

- Steady state: $H_{in} = H_{out}$
- H_{in} = Absorbed sunlight ≈ $\pi r_e^2 (1 - a_e) I_{sc}$
- H_{out} = IR radiation = $A e \sigma T^4$

surf, earth

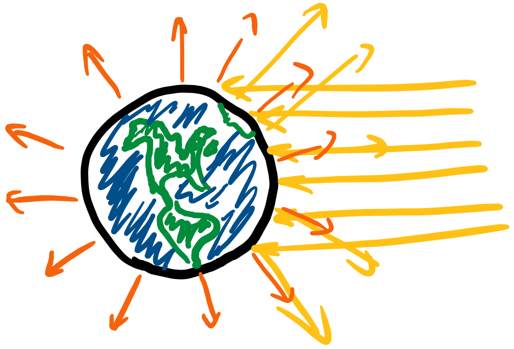
$$\pi r_e^2 (1 - a_e) I_{sc} = 4\pi r_e^2 e_e \sigma T_e^4$$

$$T_e = \left[\frac{(1 - a_e) I_{sc}}{4 e_e \sigma}\right]^{1/4}$$

Equilibrium surface temperature of earth: Summary

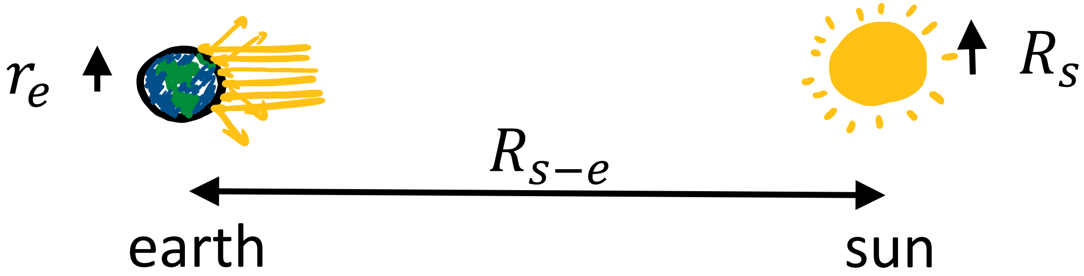
$H_{in} \rightarrow$ constant T $\rightarrow H_{out}$

- Steady state: $H_{in} = H_{out}$



- H_{in} = Absorbed sunlight $\approx \pi r_e^2(1-a_e)I_{sc}$
- H_{out} = IR radiation = $Ae\sigma T^4$

$A_{\substack{sun\\surf}}\, e\sigma T_{sun}^4$



- $I_{sc} = \frac{H_s}{4\pi R_{s-e}^2} = 1367\ {}^{W}/_{m^2}$

- Plug in and get: $\pi r_e^2(1-a_e)I_{sc} = 4\pi r_e^2 e_e\sigma T_e^4 \Rightarrow$

$$\boxed{T_e = \left[\frac{(1-a_e)I_{sc}}{4e_e\sigma}\right]^{1/4}}$$

Does this make sense?

$$\boxed{T_e = \left[\frac{(1-a_e)I_{sc}}{4e_e\sigma}\right]^{1/4}}$$

- Surface of earth has $e_e \approx 1$ for IR radiation and

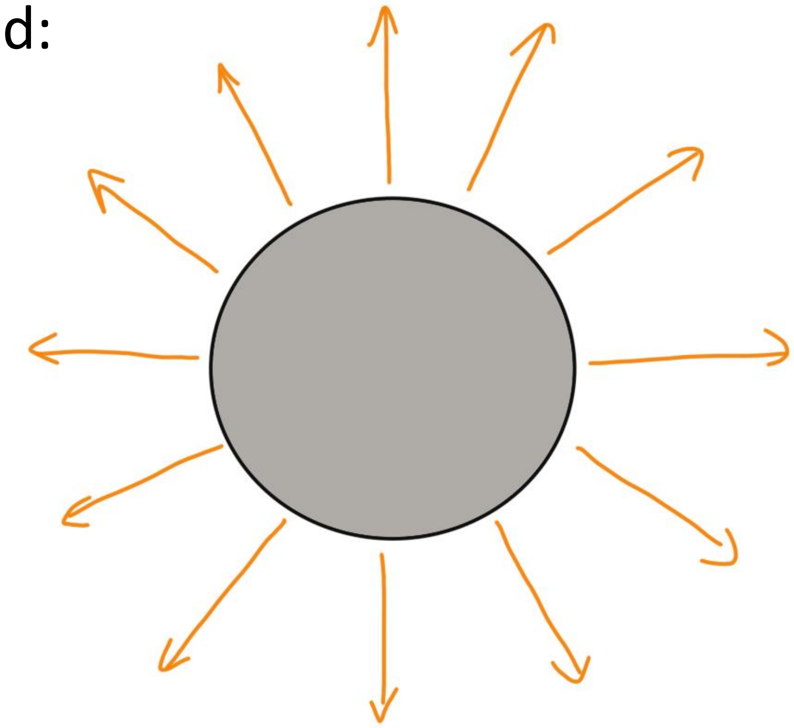
$$I_{sc} = 1367\ {}^{W}\!/_{m^2} \qquad a_e = 0.3 \qquad \sigma = 5.67x10^{-8}\,\frac{W}{m^2K^4}$$

- These give: $T_e \cong -18^oC$
- Something is off…..
- What have we neglected?
  - ➢ Heating from earth's core (like Yoltar)?
  - ➢ Nope; that amounts to 47 TW whereas H_{in} ~ 173,000 TW !!
- There is something else going on…

Actual surface temperature is larger due to the greenhouse effect

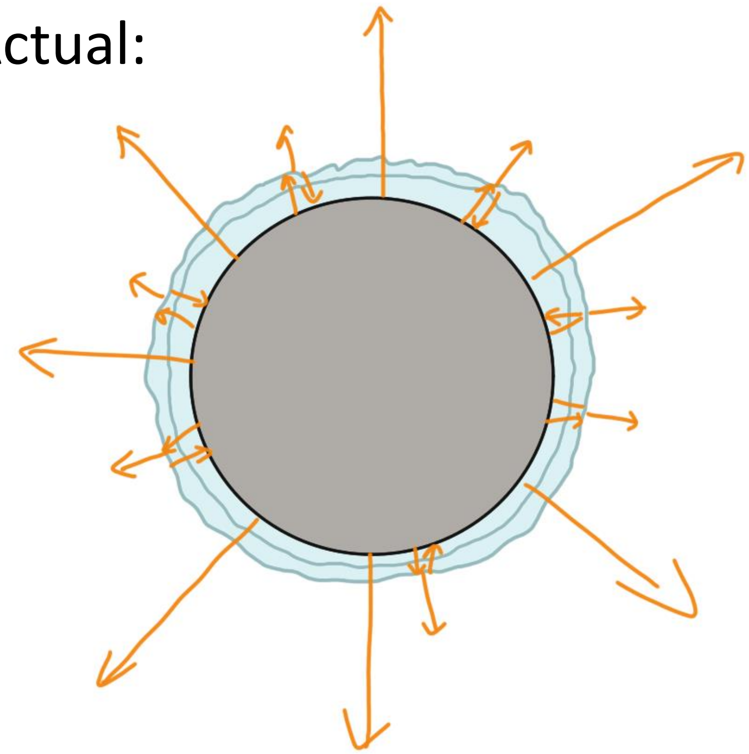
- **Greenhouse effect**: Some IR radiation is absorbed by "greenhouse gases" and re-emitted back to earth

- We assumed:



$e \approx 1$

- Actual:



$e \approx 0.6$

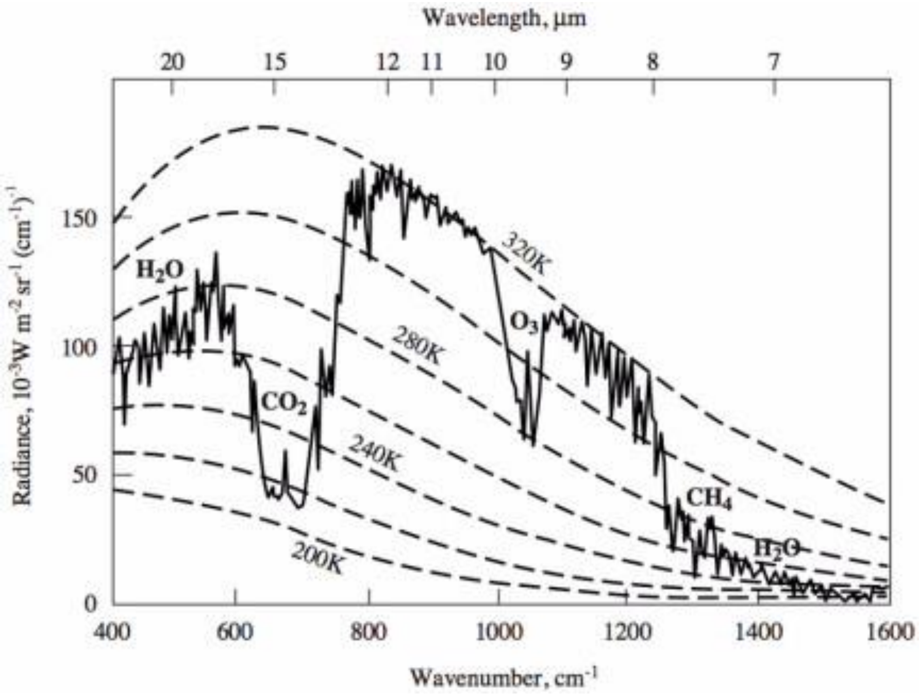
Greenhouse Effect

$$T_e = \left[\frac{(1 - a_e)I_{sc}}{4e_e\sigma}\right]^{1/4}$$

- Lower e $\Rightarrow$ higher T
- Greenhouse effect: $e \approx 0.6 \Rightarrow T \cong 14.5^o C$
- Actual average surface temperature of earth $T \cong 13.9^o C$
  - ➢ Our simple calculation is amazingly close to the actual value!
- But e can decrease due to, e.g., increasing CO_2 concentration in atmosphere
  - ➢ Resulting in Global Warming…

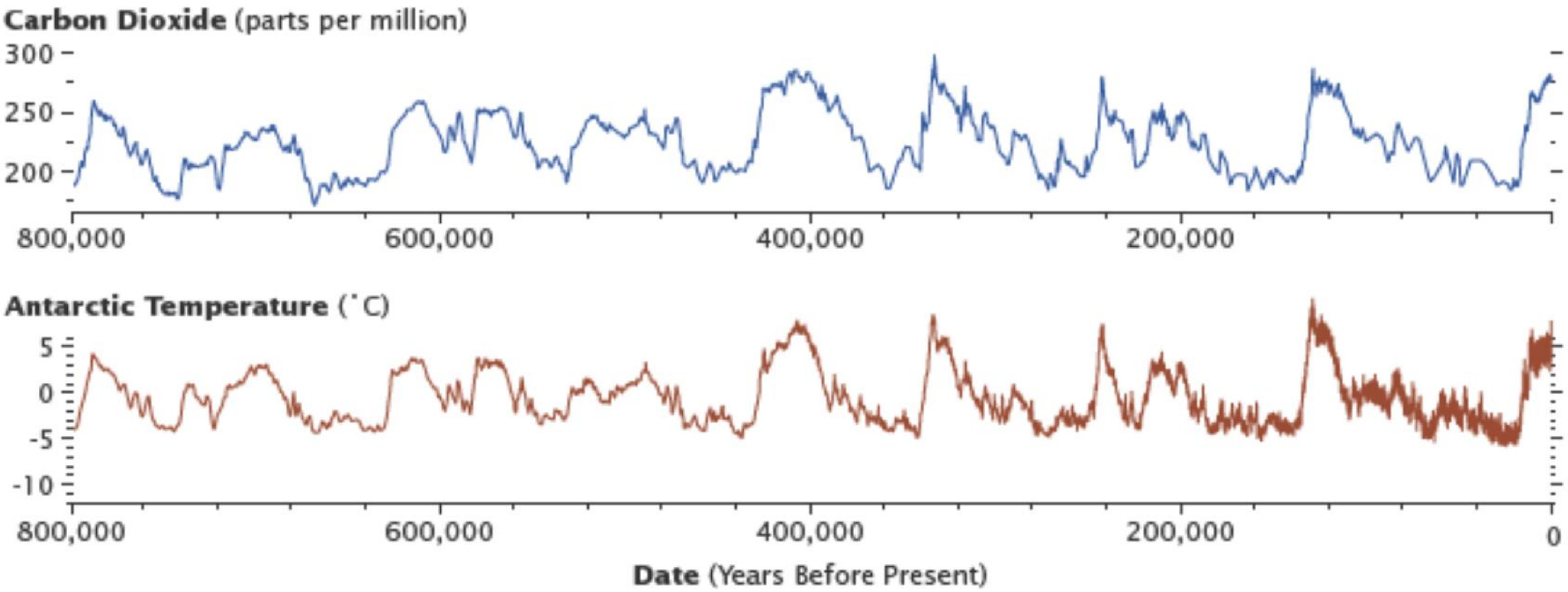
Global Warming

- Greenhouse gases, especially CO_2, absorb IR radiation emitted by the earth and re-emit it back to earth, effectively reducing the earth's emissivity



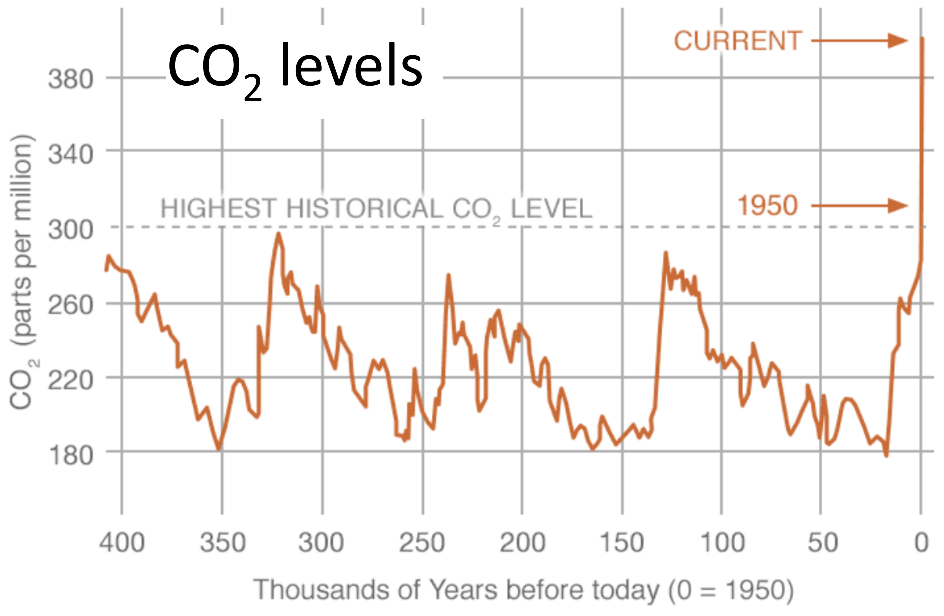
- In addition, as earth warms, white, sunlight reflecting ice in the Arctic disappears, lowering our albedo and further increasing the temperature

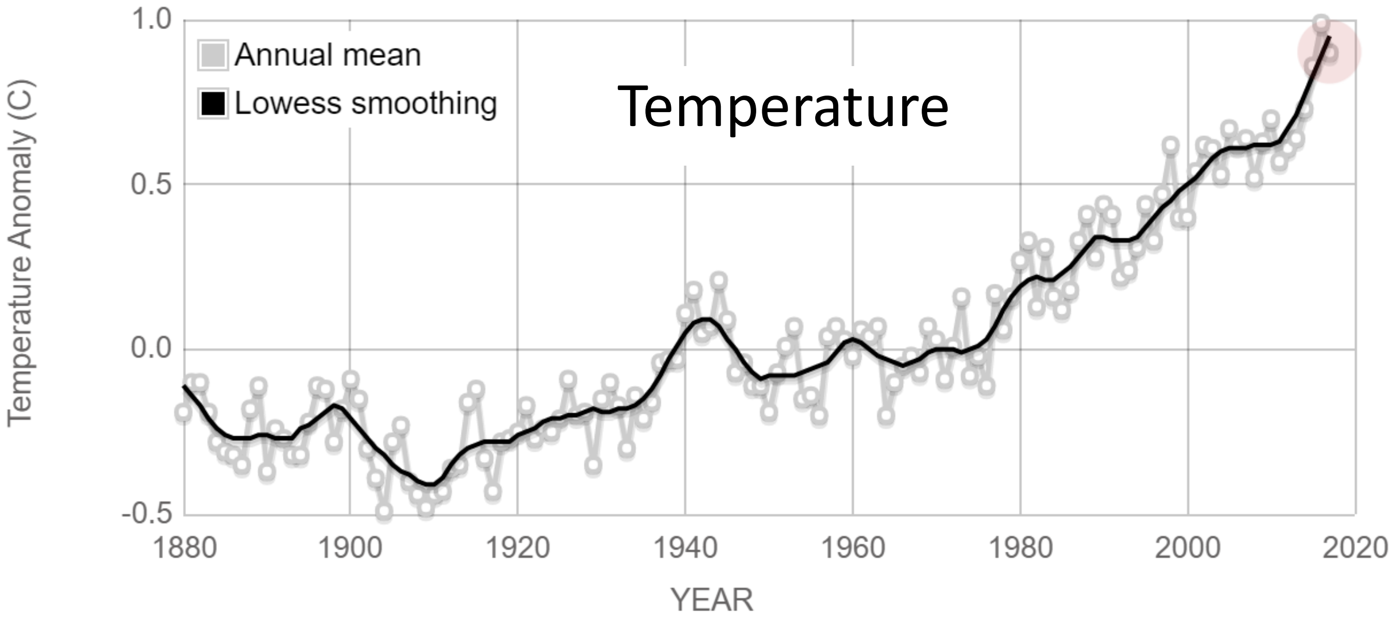
CO_2 correlates closely with temperature



- There is nothing guaranteed about our current temperature of ~ 14°C
- With plausible changes to our atmosphere (emissivity) and albedo, we can dial up pretty much any temperature we like, from freezing to boiling

CO_2 correlates closely with temperature





- Almost all climate scientists believe this rise due to human activity

Temperature of Venus…

- Venus has an albedo of 0.750 and lets assume it's emissivity is ~1. The radius of Venus is 6052 km. The Solar constant at Earth is 1367 W/m^2 and the distance from Venus to the Sun is 0.72 times the Earth to Sun distance.
- What is the temperature of Venus?

A. Click A if you are done

B. Click B if you are stuck

Temperature of Venus…

- Venus has an albedo of 0.750 and lets assume it's emissivity is ~1. The radius of Venus is 6052 km. The Solar constant at Earth is 1367 W/m^2 and the distance from Venus to the Sun is 0.72 times the Earth to Sun distance.
- What is the temperature of Venus?

We still have: $H_{in} = H_{out}$

$$\pi r_v^2 (1 - a_v) I_v = 4\pi r_v^2 e_v \sigma T_v^4$$

$$T_v = \left[\frac{(1 - a_v) I_v}{4 e_v \sigma}\right]^{1/4}$$

I_v is different than I_{SC} :

Isc

$$I_v = \frac{H_s}{4\pi R_{s-v}^2} = \left(\frac{R_{s-e}^2}{R_{s-v}^2}\right)\left(\frac{H_s}{4\pi \left(R_{s-e}^2\right)}\right) = \frac{R_{s-e}^2}{R_{s-v}^2} I_{SC} = \left(\frac{1}{0.72}\right)^2 1367 = 2637 \frac{W}{m^2}$$

$$T_v = -41^o\ C$$

- Actual temperature is $T_v = 464^o\ C$
- What could we be missing?
- Atmosphere of Venus is dense and predominately CO_2
- $\Longrightarrow$ effective emissivity ~ 0.01 !!
- Huge greenhouse effect!!