Lecture 29.

B-field due to a wire (short segment and long wire).
“Two-step logic” in magnetic problems.
Ampere’s law.

Announcement

Midterm 2 – next Monday

What is on MT2?

- All electricity (Coulomb law and on)
- Cutoff: Up to Lorentz force (Week 9)

Resources?

- Lectures, textbook, HW, tutorial problems…
- Posted practice exams

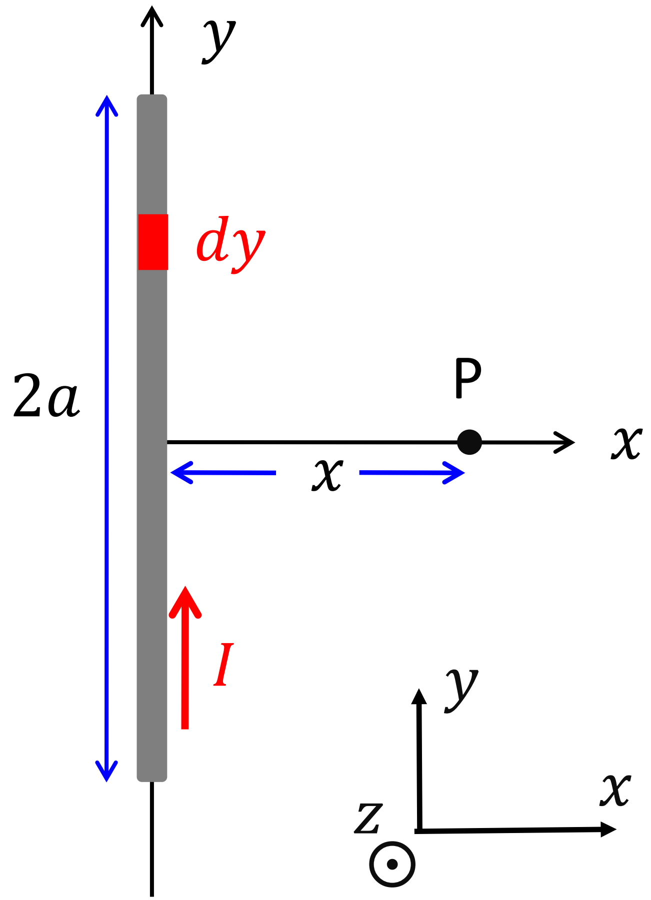
Help sessions?

- Monday 17:30-18:30 (Henn 318)
- Wednesday 17:00 - 18:00 (Henn 318)
- Friday 17:00 - 18:00 (Henn 318) & on Zoom
- Saturday 14:00 - 16:00 (Hebb 112)

Magnetic field of a short straight wire

Last Time

Q: Use the Biot-Savart Law to compute magnetic field B created by a short current segment of length $2a$ with current I at the symmetry axis of the wire. Your answer should be a vector.



Exercise: Before you do the math, think about how to solve the problem and write a few sentences outlining your strategy.

- First consider a small wire segment dy => find the field $d\vec{B}$ produced by it at P => considering symmetry, integrate its components to get the resultant field at P

You might need:

$$\int \frac{x\,dy}{(x^2+y^2)^{3/2}} = \frac{y}{x\sqrt{x^2+y^2}}$$

Biot-Savart law:

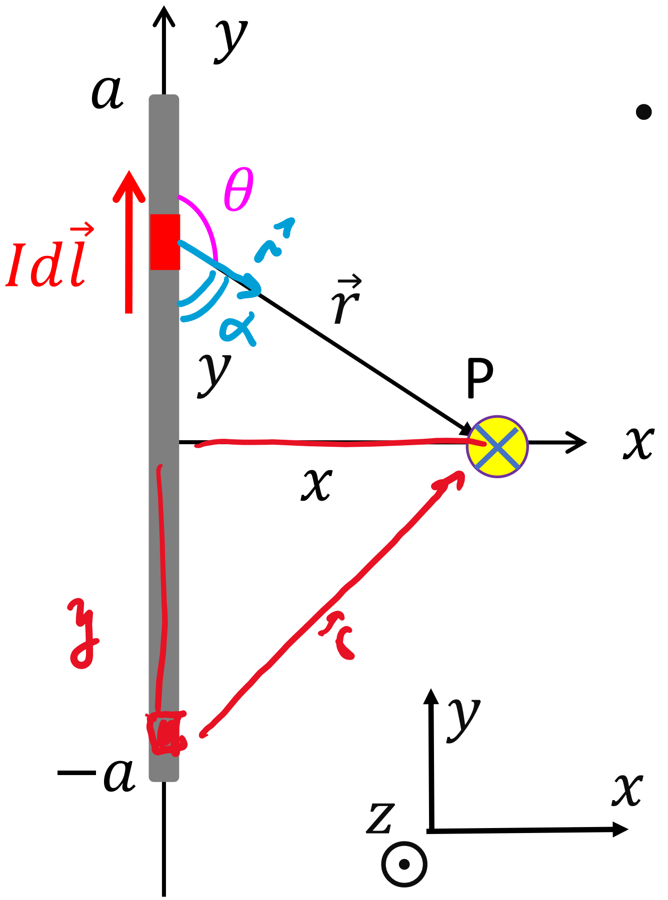
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$$

Magnetic field of a short straight wire

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I|d\vec{l} \times \hat{r}|}{r^2}$

$\sin\theta = \sin(\pi - \theta) = x/r$

Q: Use the Biot-Savart Law to compute magnetic field B created by a short current segment of length $2a$ with current I at the symmetry axis of the wire. Your answer should be a vector.



- Direction? $d\vec{l} \times \hat{r} \Rightarrow$ along $(-\vec{k})$ = into the page
- Magnitude? $\left|I\, d\vec{l} \times \hat{r}\right| = I\, dl\, \underbrace{|\hat{r}|}_{1} \sin\theta = I\, dy \sin\theta$

$$\vec{B}_{\text{at P}} = \int d\vec{B} = \int \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2} = (-\hat{k}) \frac{\mu_0 I}{4\pi} \int_{-a}^{a} \frac{\overbrace{\sin\theta}^{a\ \sin\alpha}\ dy}{r^2}$$

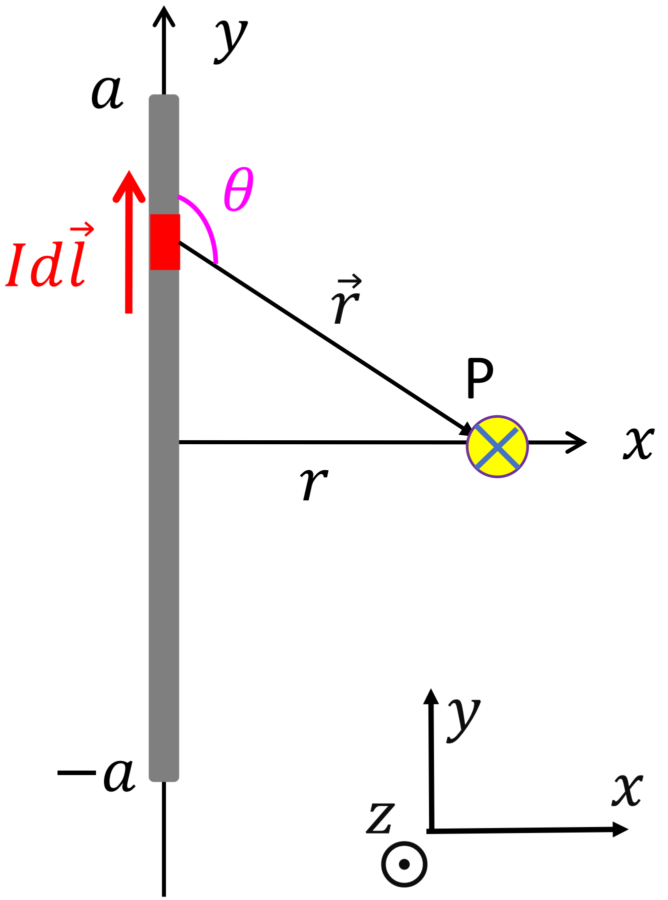
$$= (-\hat{k}) \frac{\mu_0 I}{4\pi} \int \frac{x\, dy}{r^3} = (-\hat{k}) \frac{\mu_0 I\, x}{4\pi} \int_{-a}^{a} \frac{dy}{(x^2 + y^2)^{3/2}}$$

$$\boxed{\vec{B}_{\text{at P}} = -\frac{\mu_0 I}{4\pi} \frac{2a}{x\sqrt{x^2 + a^2}} \hat{k}}$$

- y = integration variable
- x = parameter

Magnetic field of an infinitely long straight wire

Q: Use the Biot-Savart Law to compute magnetic field B created by an infinitely long current-carrying wire with current I at a perpendicular distance r from the wire.



- Finite wire: $\vec{B}_{\text{at P}} = -\frac{\mu_0 I}{4\pi}\frac{2a}{r\sqrt{\cancel{r^2}+a^2}}\hat{k}$

- Take limit: $a \gg r$ (neglect r^2 in comparison with a^2)

$$\frac{\mu_0 I}{4\pi}\frac{2a}{r\sqrt{r^2+a^2}} \rightarrow \frac{\mu_0 I}{4\pi}\frac{2\cancel{a}}{r\cancel{\sqrt{a^2}}} = \rightarrow \frac{\mu_0 I}{2\pi r}$$

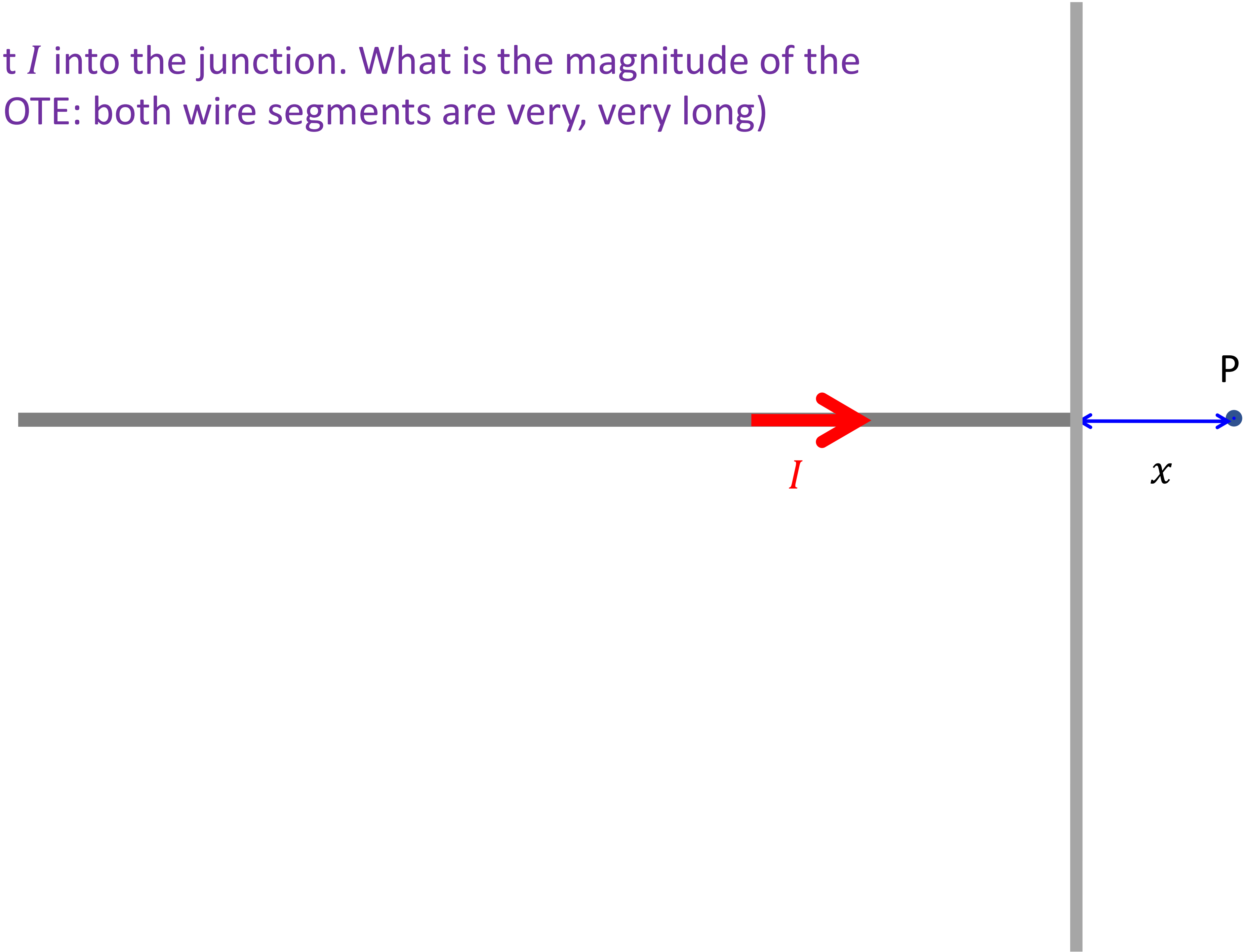
perp distance from the wire

$$\boxed{\vec{B}_{\text{at P}} = -\frac{\mu_0 I}{2\pi r}\hat{k}}$$

looong wire

We will see very shortly that there is a much easier way to compute the B-field for a long straight wire – Ampere's Law

A wire carries current I into the junction. What is the magnitude of the B-field at point P? (NOTE: both wire segments are very, very long)



A. $B = \frac{2\mu_0 I}{\pi x}$

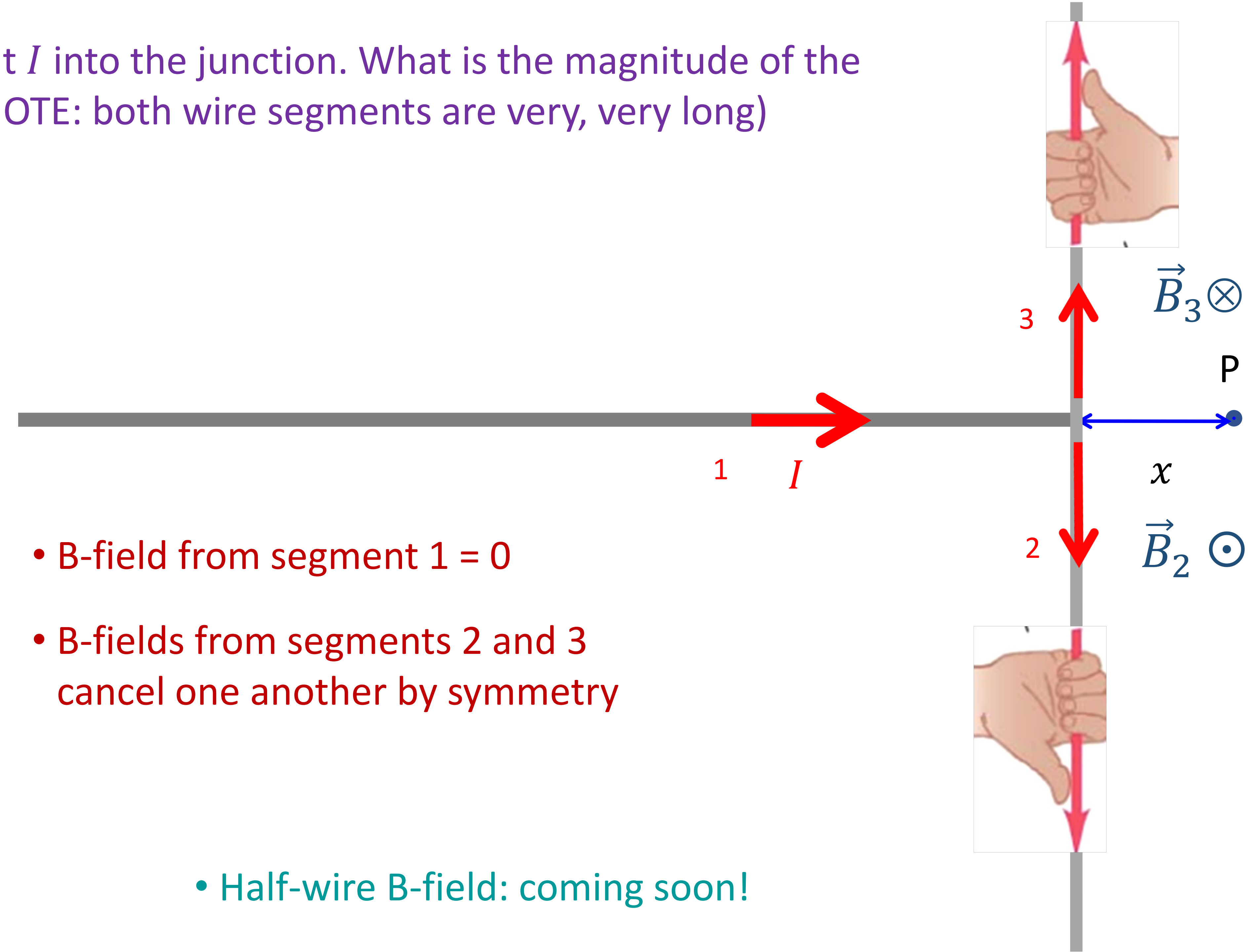
B. $B = \frac{\mu_0 I}{\pi x}$

C. $B = \frac{\mu_0 I}{2\pi x}$

D. $B = \frac{\mu_0 I}{4\pi x}$

E. $B = 0$

A wire carries current I into the junction. What is the magnitude of the B-field at point P? (NOTE: both wire segments are very, very long)



A. $B = \frac{2\mu_0 I}{\pi x}$

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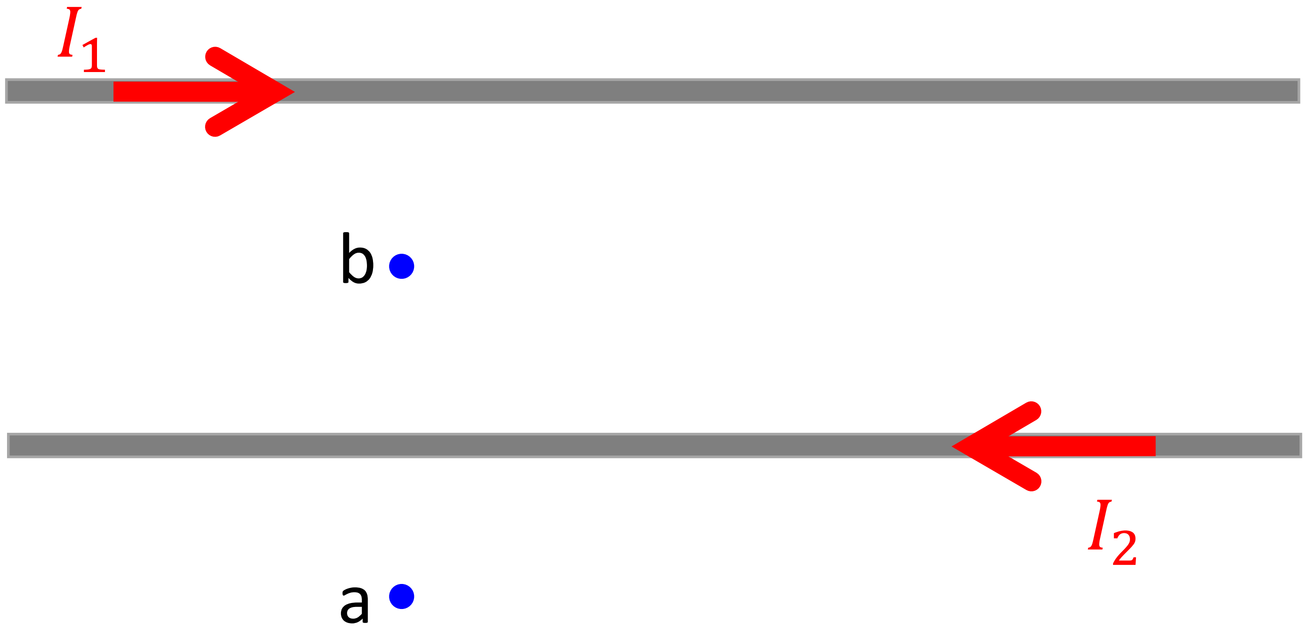
D. $B = \frac{\mu_0 I}{4\pi x}$

E. $B = 0$

- B-field from segment 1 = 0
- B-fields from segments 2 and 3 cancel one another by symmetry

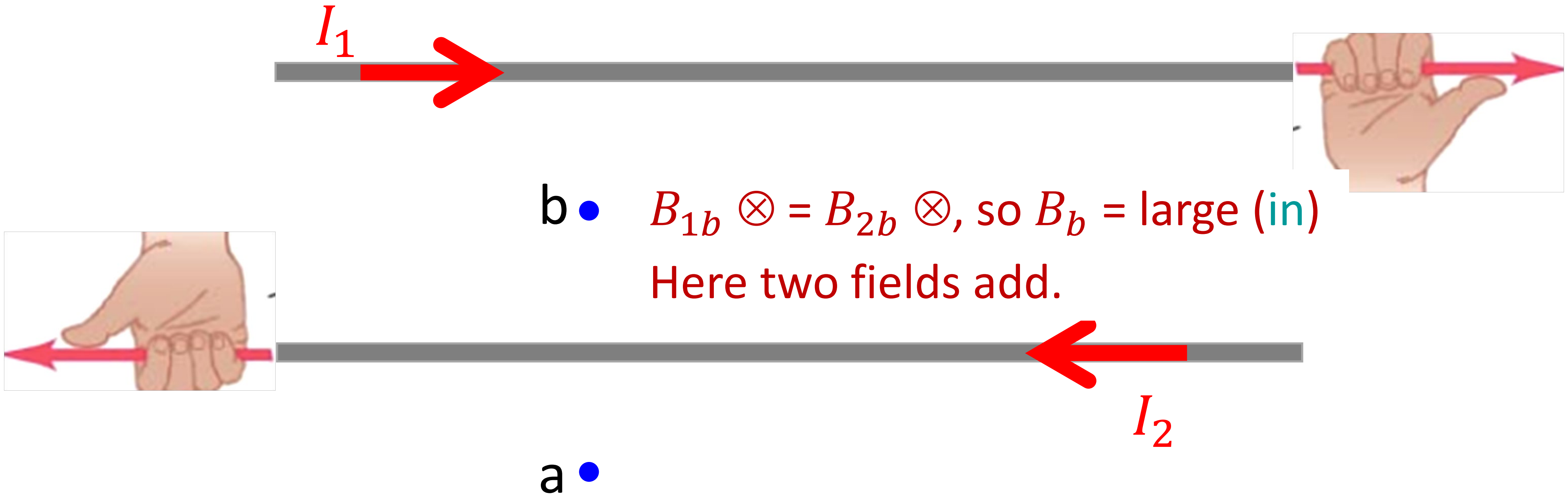
- Half-wire B-field: coming soon!

Q: Compare the magnitude and direction of the B-fields at points a and b. ("out"=out of the page, "in"=into the page). NOTE: both wire segments are infinitely long and $I_1 = I_2$.



A. B_a (in) < B_b (in)
B. B_a(out) < B_b (out)
C. B_a (out) > B_b (in)
D. B_a (out) < B_b (in)
E. B_a (in) > B_b (out)

Q: Compare the magnitude and direction of the B-fields at points a and b. ("out"=out of the page, "in"=into the page). NOTE: both wire segments are infinitely long and $I_1 = I_2$.



$B_{1b}\otimes = B_{2b}\otimes$, so B_b = large (in)

Here two fields add.

$B_{1a}\otimes < B_{2a}\odot$, so B_a = small (out)

Here two fields partially subtract.

A. B_a (in) < B_b (in)

B. B_a(out) < B_b (out)

C. B_a (out) > B_b (in)

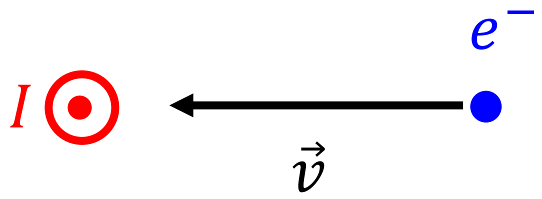
D. B_a (out) < B_b (in)

E. B_a (in) > B_b (out)

$$B_{\text{wire}}(r) = \frac{\mu_0 I}{2\pi r}$$

Q: A long straight wire carries current I out of the page.

An electron travels towards the wire from the right. What is the direction of the force on the electron?



A. $\otimes$

B. $\odot$

C. $\leftarrow$

D. $\rightarrow$

E. $\downarrow$

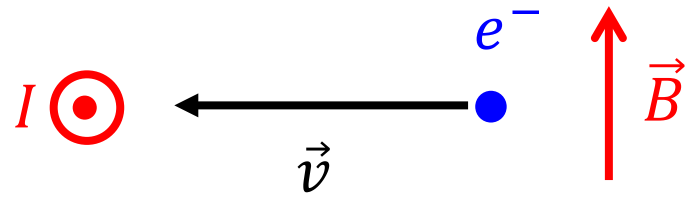
Q: A long straight wire carries current I out of the page.

An electron travels towards the wire from the right. What is the direction of the <u>force</u> on the electron?

$$B = \frac{\mu_0 I}{2\pi r}$$

Force on the electron:

$$\vec{F}_m = q_{\pm}\, \vec{v} \times \vec{B}$$



A. $\otimes$

B. $\odot$

C. $\leftarrow$

D. $\rightarrow$

E. $\downarrow$

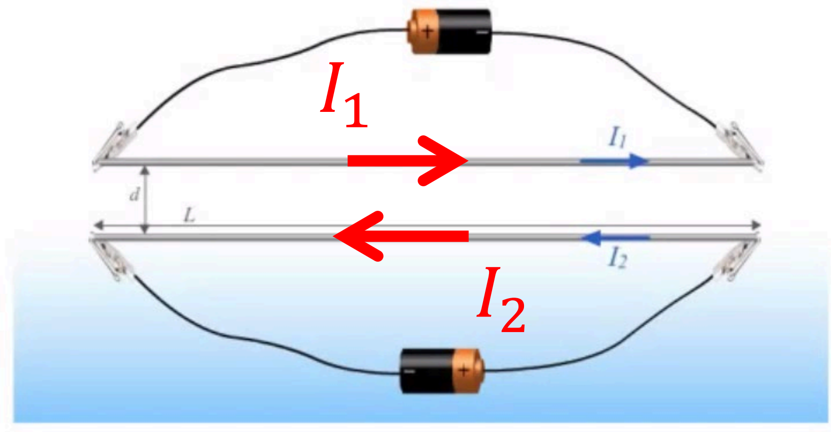
- Why there is a force on the electron?? – Because it is a charged particle moving in a magnetic field.

- Who creates this magnetic field?? – The wire!

- "Two-step logic": object #1 creates magnetic field at the location of object #2 => object #2 experiences magnetic force!

Q: Two wires carry current in opposite directions. What the wires will do?

A. Attract
B. Repel
C. No effect on each other



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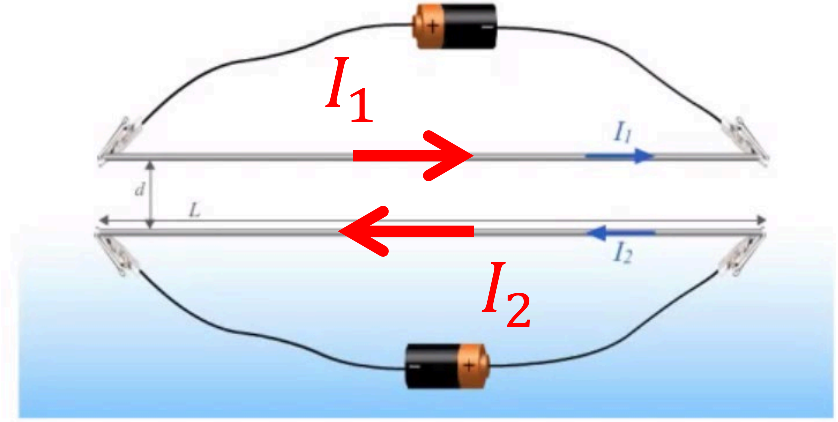
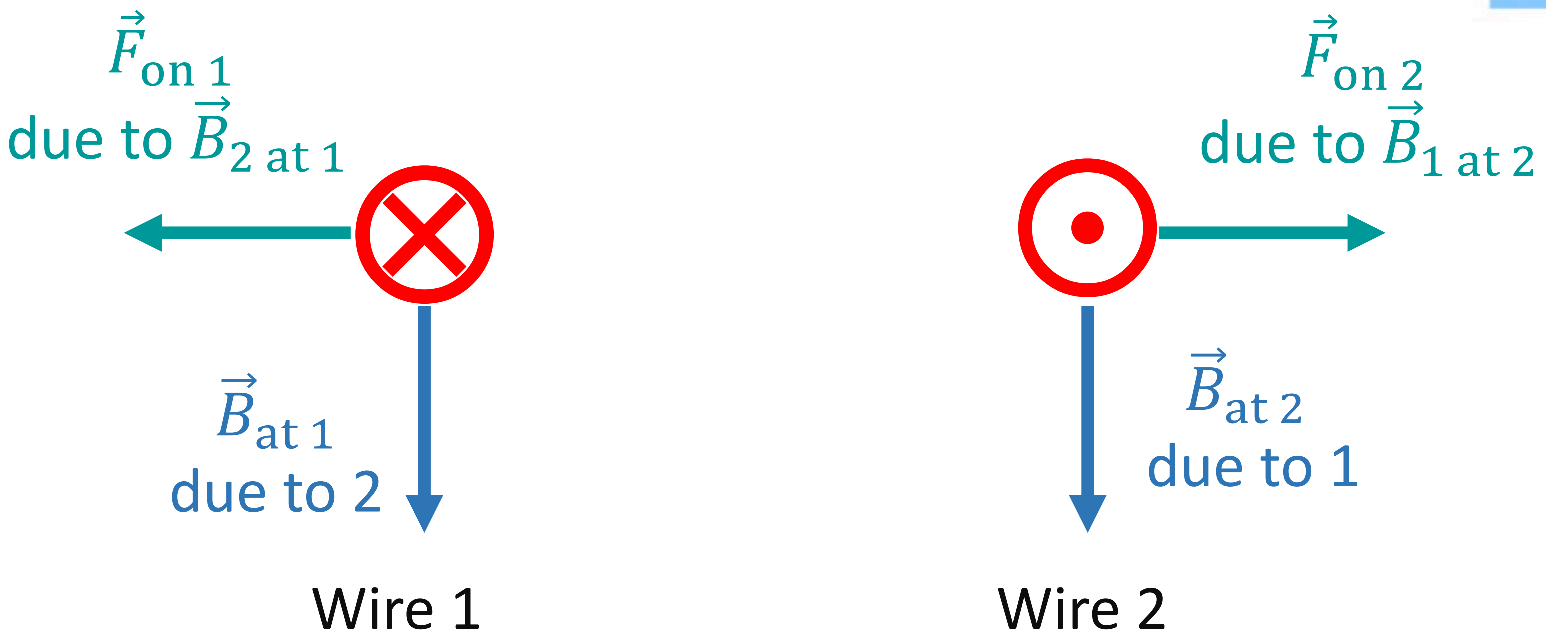


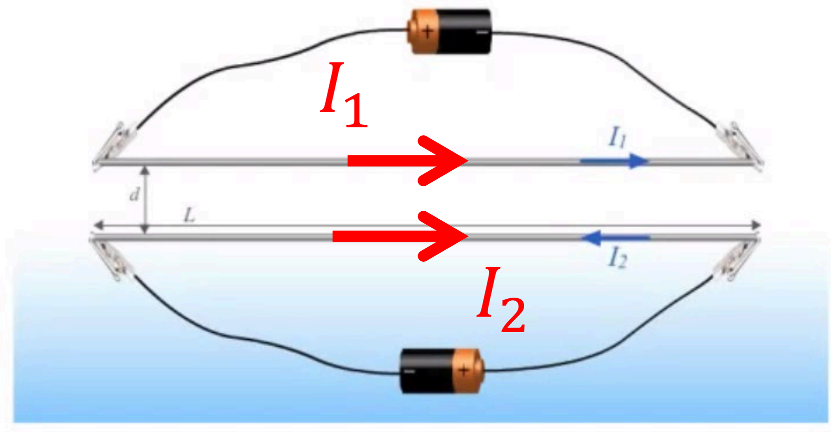


$\vec{B}_{\text{wire}}$: RHR

$\vec{F}_{\text{on wire}} = L\,"\vec{I}" \times \vec{B}$

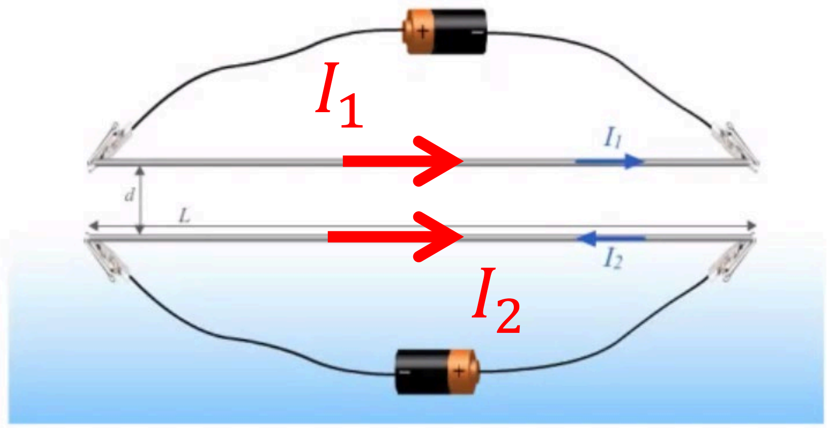
Q: Two wires carry current in the same directions. What the wires will do?

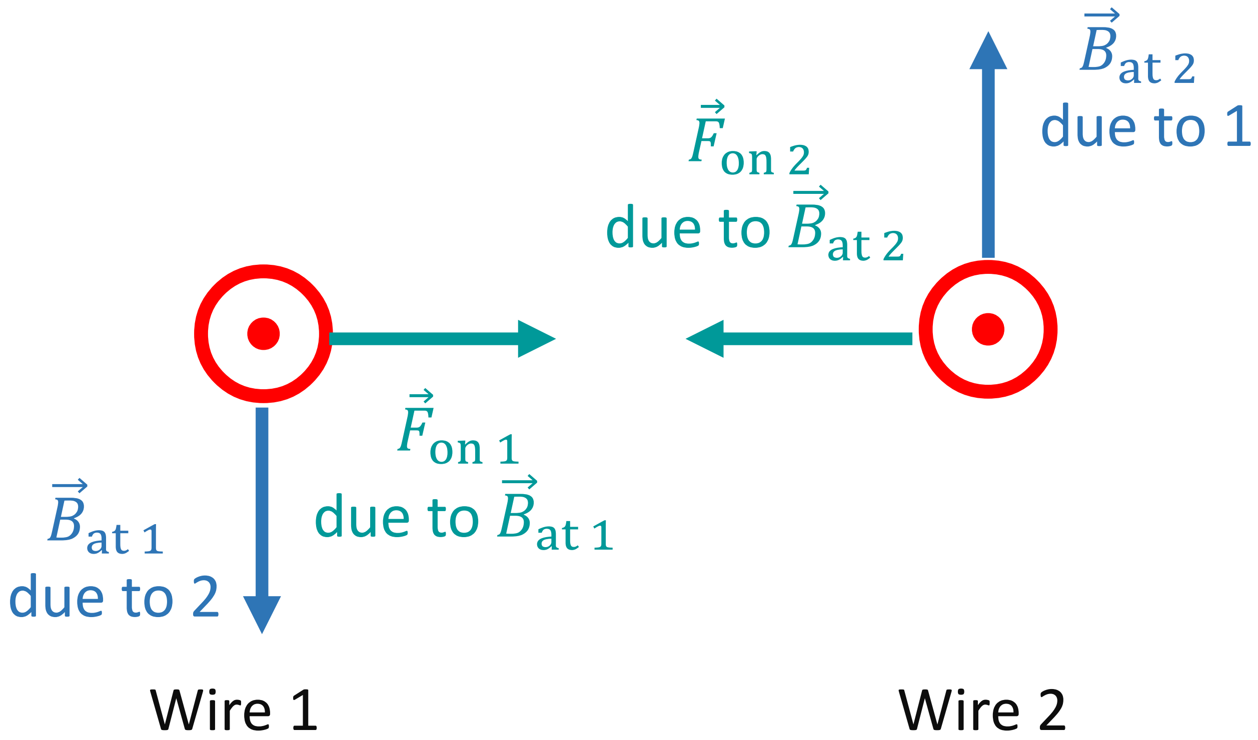
A. Attract
B. Repel
C. No effect on each other



Q: Two wires carry current in opposite directions. What the wires will do?

A. Attract
B. Repel
C. No effect on each other



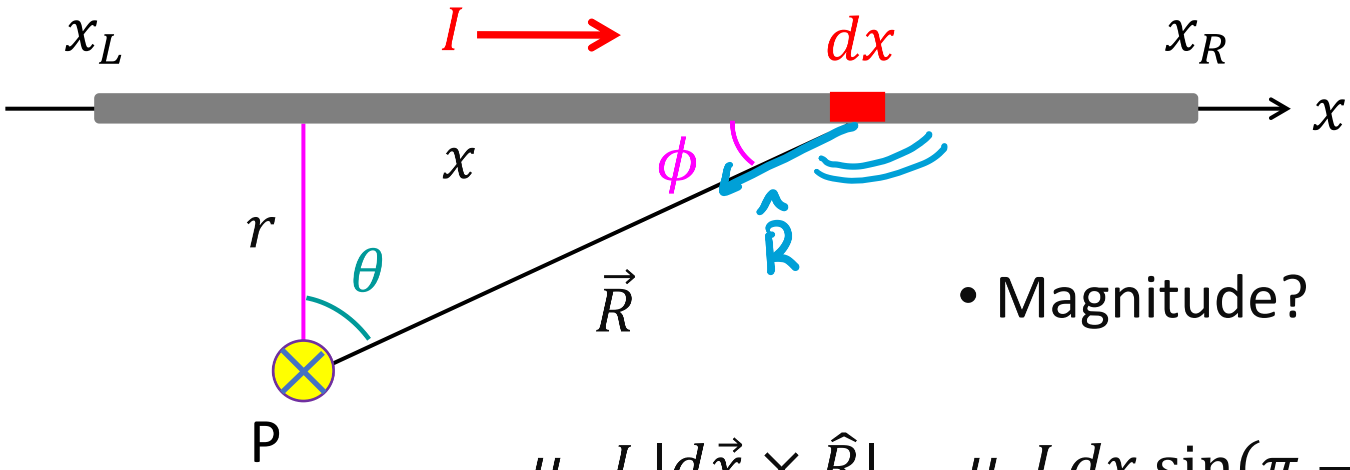


$\vec{B}_{\text{wire}}$: RHR

$\vec{F}_{\text{on wire}} = L\ "\vec{I}" \times \vec{B}$

B-Field of a short straight wire (off-center)

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{x} \times \hat{R}}{R^2}$$



- Direction? into the page (RHR)

- Magnitude?

$$dB = \frac{\mu_0}{4\pi}\frac{I\,|d\vec{x} \times \hat{R}|}{R^2} = \frac{\mu_0 I}{4\pi}\frac{dx\,\sin(\pi-\phi)}{R^2} = \frac{\mu_0 I}{4\pi}\frac{dx\,\sin(\phi)}{R^2} = \frac{\mu_0 I}{4\pi}\frac{dx\,\cos(\theta)}{R^2}$$

- Note: integration is much easier using θ:

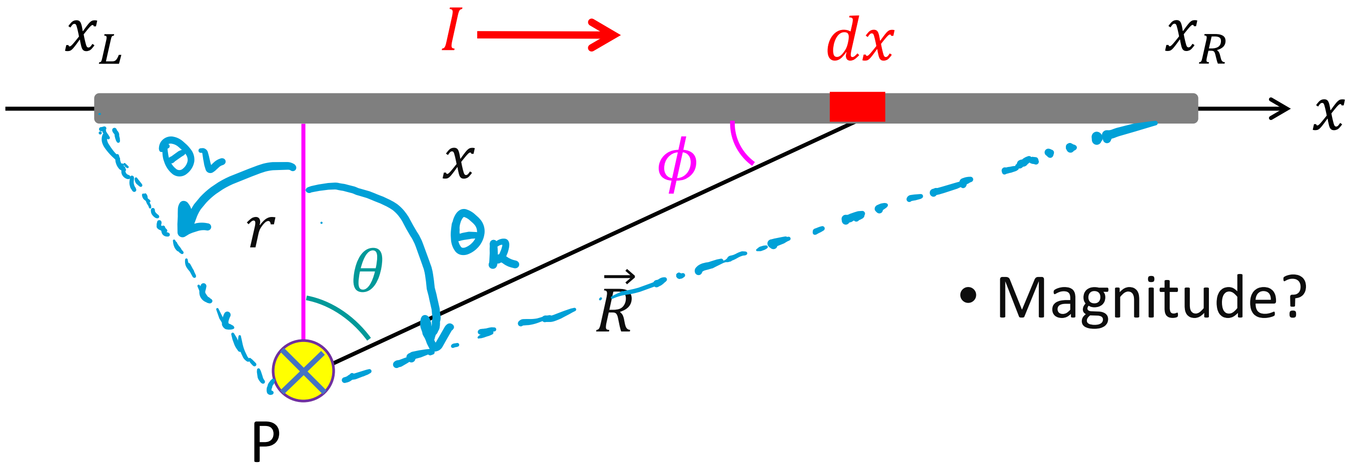
$$x = r\tan\theta \;\Rightarrow\; \frac{dx}{d\theta} = \frac{r}{\cos^2\theta} \;\Rightarrow\; dx = \frac{r\,d\theta}{\cos^2\theta} \;\Rightarrow\; dB = \frac{\mu_0 I}{4\pi}\frac{\cancel{r}\,d\theta\cos(\theta)}{\cos^2(\theta)}\frac{1}{R^2}$$

$r^{\cancel{2}}$

… and $R\cos(\theta) = r \;\Rightarrow\;$ $$dB_{\text{wire}} = \frac{\mu_0 I}{4\pi}\frac{d\theta\cos(\theta)}{r}$$

B-Field of a short straight wire (off-center)

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{x} \times \hat{R}}{R^2}$$



- Direction? into the page (RHR)

- Magnitude? $dB_{\text{wire}} = \dfrac{\mu_0 I}{4\pi} \dfrac{d\theta \cos(\theta)}{r}$

$$B_{\text{wire}} = \frac{\mu_0}{4\pi} \frac{I}{r} \int_{\theta_L}^{\theta_R} d\theta \cos(\theta)$$

$$\boxed{B_{\text{wire}} = \frac{\mu_0}{4\pi} \frac{I}{r} \left(\sin(\theta_R) - \sin(\theta_L)\right)}$$

$\sin(\theta_R) = \dfrac{x_R}{\sqrt{x_R^2 + r^2}}$,

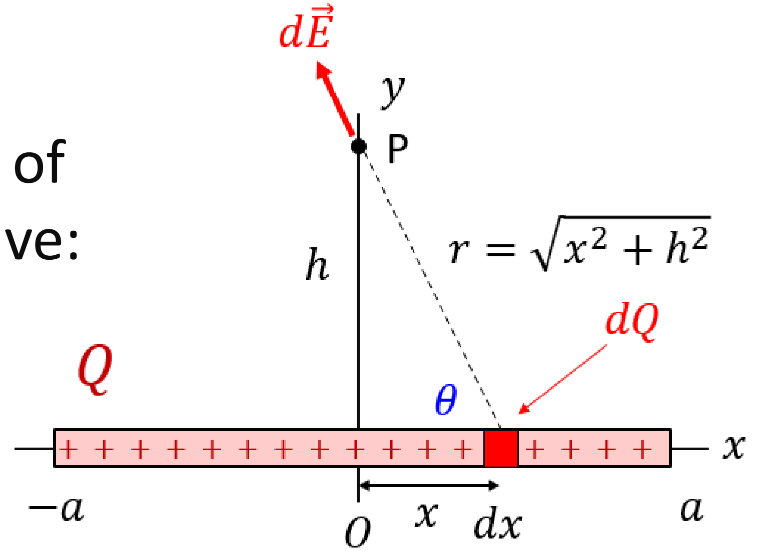
etc.

- Infinite wire: $\sin(\theta_R) = +1$, $\sin(\theta_L) = -1$, and we get: $\boxed{B_{\text{LOOOONG wire}} = \dfrac{\mu_0}{2\pi} \dfrac{I}{r}}$

- Semi-Infinite wire with $x_L = 0$: $\sin(\theta_R) = +1$, $\sin(\theta_L) = 0$: $\boxed{B_{\text{half-wire}} = \dfrac{\mu_0}{4\pi} \dfrac{I}{r}}$

Did you enjoy this derivation?

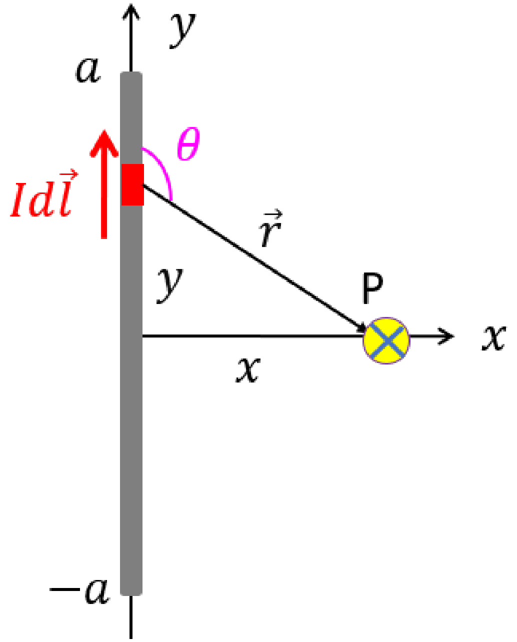
- Let's recall: we could derive the electric field of a very long line of charge by integrating Coulomb law – but there was an alternative:



Gauss's law $$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

- Requires symmetry to obtain E by integrating electric flux over a closed surface with an enclosed volume

- Solving for B_P for a very long current-carrying wire (i.e. $a \gg x$) using Biot-Savart's Law was also hard, but there is a MUCH easier alternative!



Ampere's law $$\Phi_m = \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{in}$$

- Requires symmetry to obtain B by integrating magnetic flux over a closed path with an enclosed area