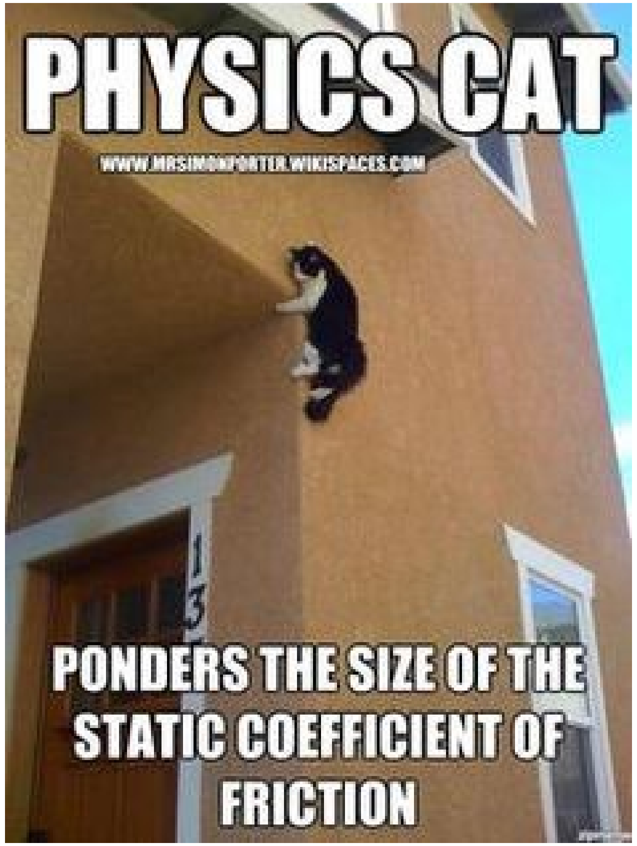
PHYS 170

Week 6: Friction

Section 201 (Mon Wed Fri 12:00 – 13:00)

Static Friction



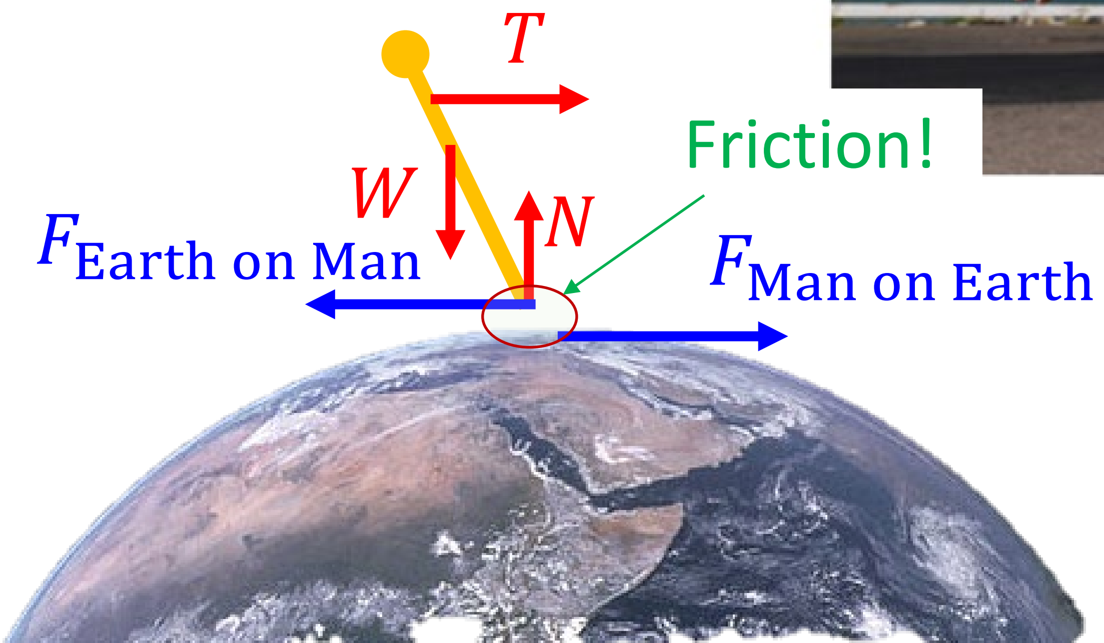
Text: 8.1-8.2

Content:

- Friction: microscopic mechanism
- Static friction and kinetic friction
- Various scenarios of breaking equilibrium
- Impending motion: conditions and restrictions
- Impending motion problems: General strategy

Why can he pull the track?

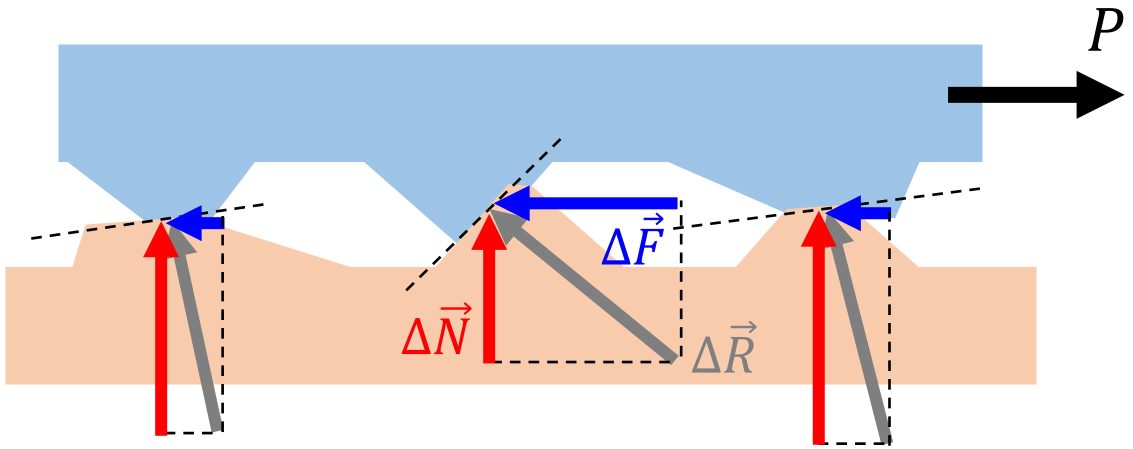
FBD for this person:

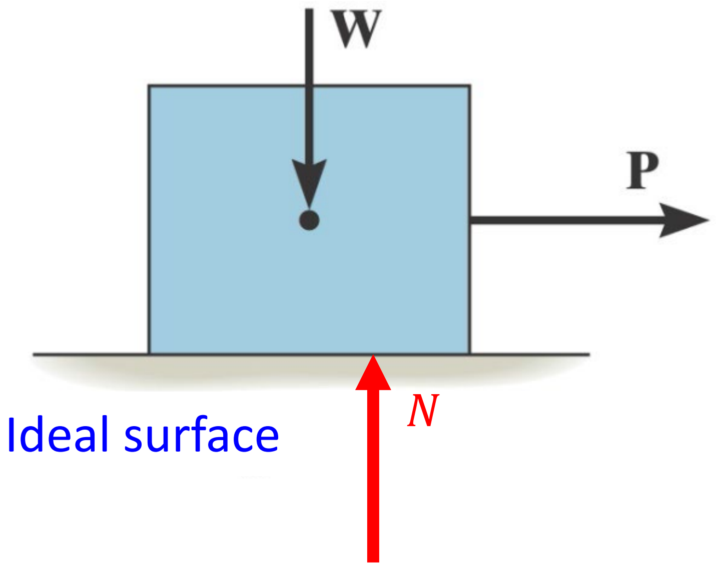


- Propulsion force forward on the man from the Earth:
  - thanks Newton's 3rd law!
- His feet do not slip due to static friction

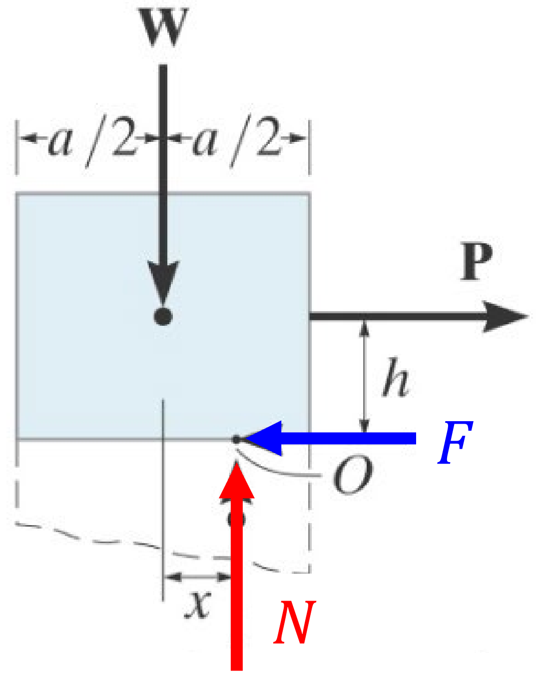
MICROSCOPIC VIEW OF DRY FRICTION

Rough surface: zoom on the contact area





- On microscopic scale, friction develops as a result of "interlocking" contacts between irregularities in the surfaces that are in contact
- Each contact point produces a tiny reactive force $\Delta\vec{R}_n$
- Each $\Delta\vec{R}_n$ has a (horizontal) frictional component $\Delta\vec{F}_n$ and a (vertical) normal component $\Delta\vec{N}_n$ (here n labels the microscopic contact)
- Combining these microscopic $\Delta\vec{N}$'s and $\Delta\vec{F}$'s we will get macroscopic resultant normal force and resultant friction force: $\vec{F} = \sum \Delta\vec{F}_n, \quad \vec{N} = \sum \Delta\vec{N}_n$



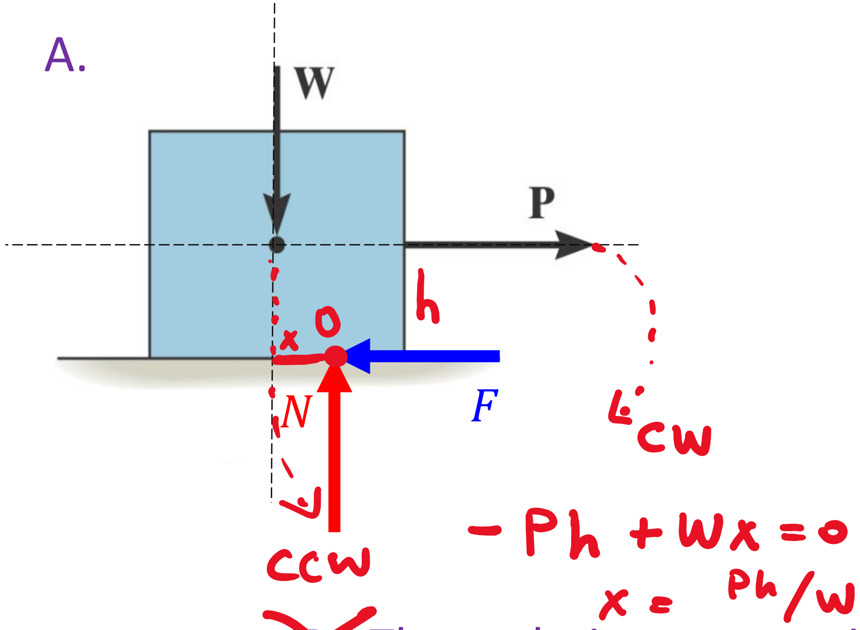
- Note that $\vec{N}$ and $\vec{F}$ are "two sides of the same coin" – therefore, they always appear together: $(\vec{N}, \vec{F})$

LOCATION OF $\vec{F}$ and $\vec{N}$

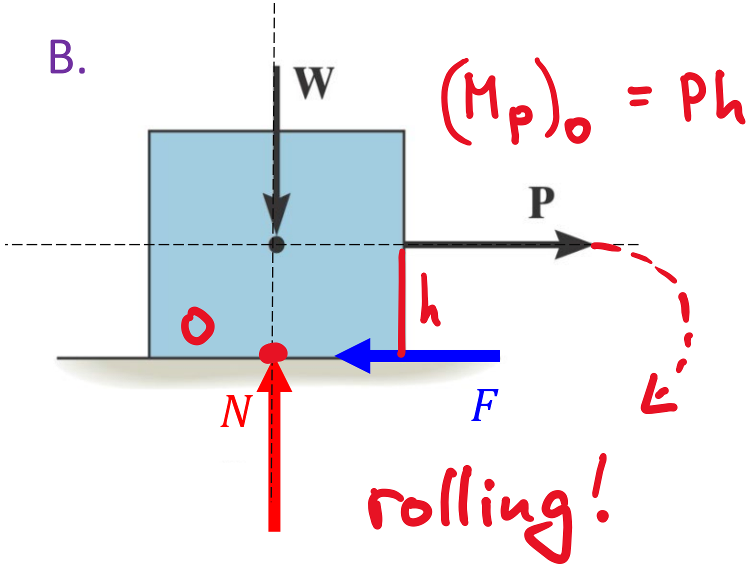
- By construction, $\vec{F}$ is tangent to the interface, and $\vec{N}$ is normal (= "perpendicular") to the interface
- Since $\vec{F}$ opposes motion, it acts in the direction opposite to that in which the motion would occur in the absence of friction

Q: Which of these choices is correct? Why?

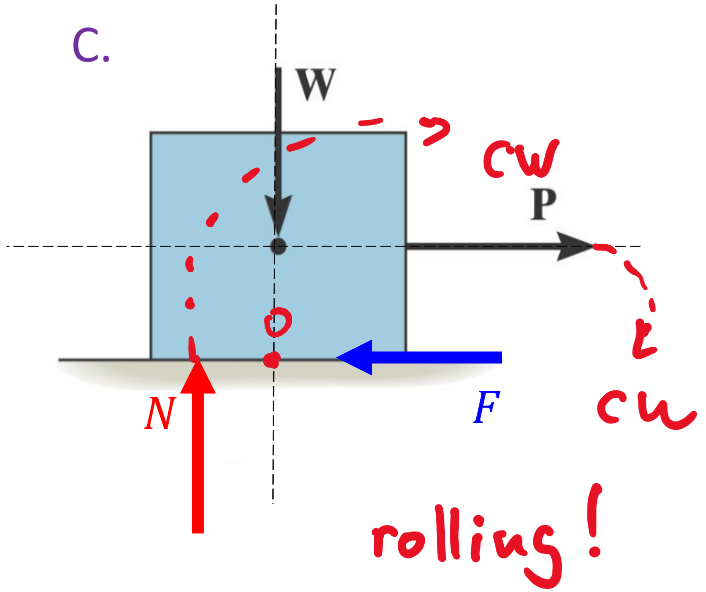
A.



B.



C.



~~D.~~ These choices are equivalent.

E. Depends on the circumstances.

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Only in case A the moments created by all the forces can balance out to zero. Below it is shown by calculating the moments explicitly.

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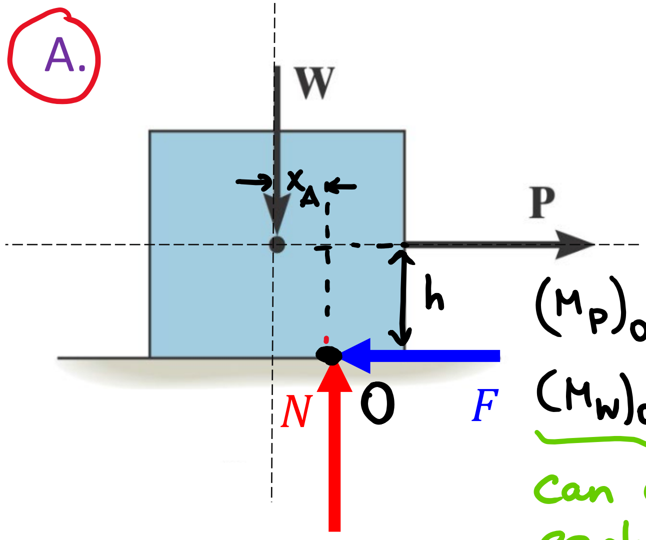
A.

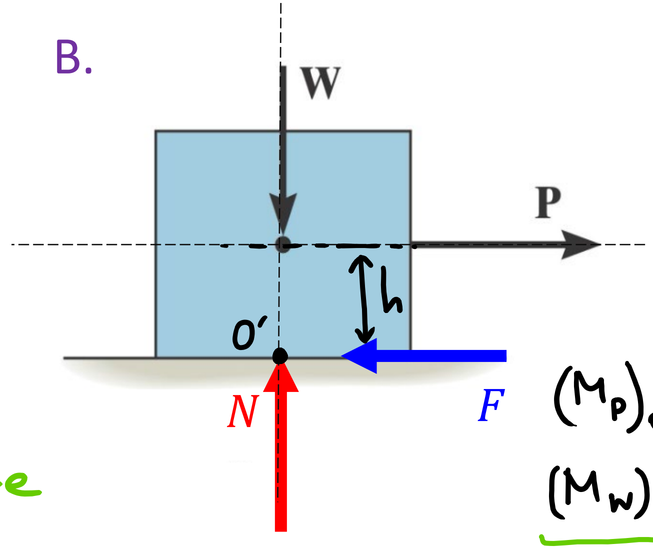


$(M_P)_O = -Ph$

$\underbrace{(M_W)_O = +Wx_A}$

can compensate each other

B.



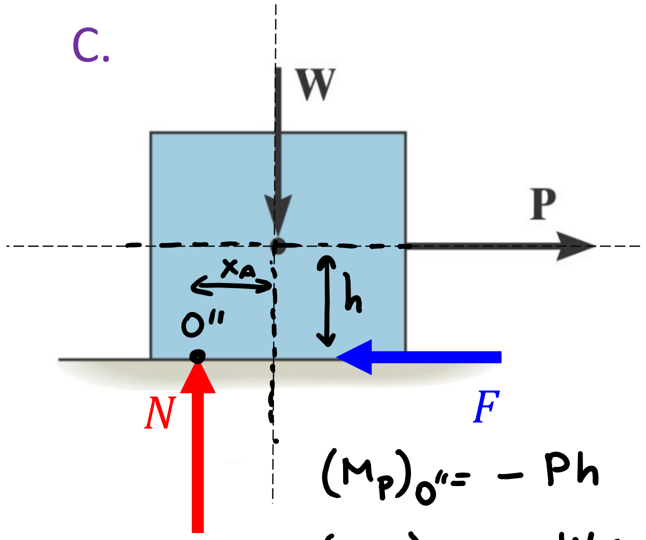
$(M_P)_{O'} = -Ph$

$(M_W)_{O'} = 0$

$\underbrace{\phantom{(M_W)_{O'} = 0}}$

$(M_R)_{O'}$ negative

C.



$(M_P)_{O''} = -Ph$

$(M_W)_{O''} = -Wx_A$

$(M_R)_{O''}$ negative

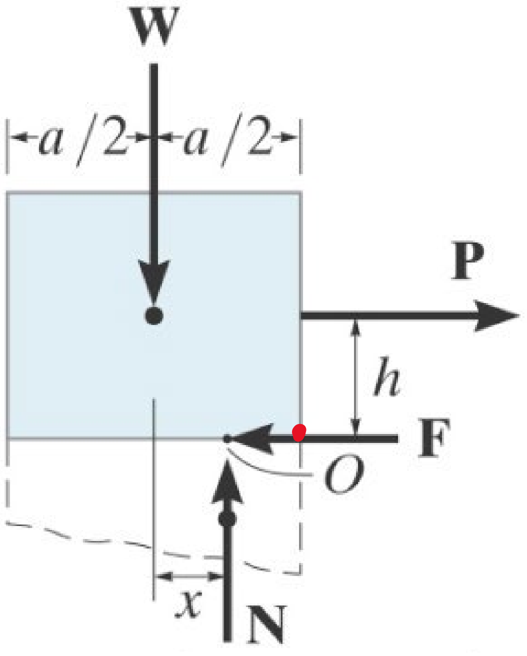
D. These choices are equivalent.

They are not! (why?)

E. Depends on the circumstances.

EQUILIBRIUM: Summary

Assume that the body is at equilibrium



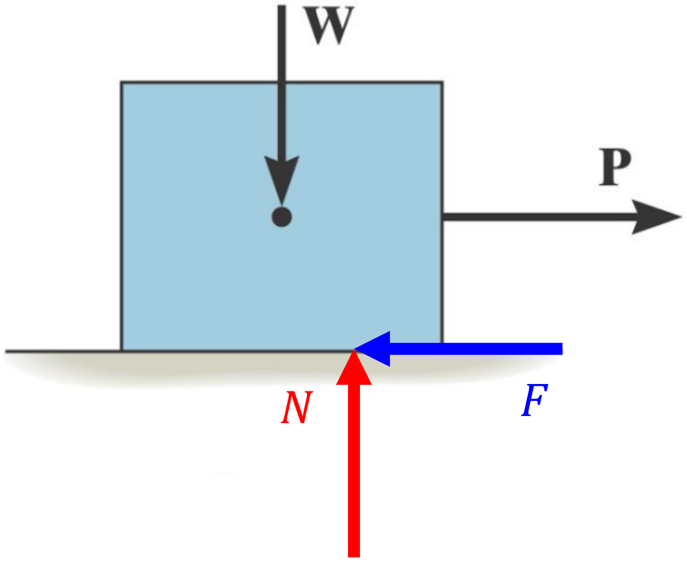
- $\vec{F}$ is always tangent to the interface.
  - Which of two possible directions?
  - Equilibrium ⇒ $\vec{F}$ prevents motion, which would occur in the absence of friction ⇒ Opposite to the direction in which the object would move or even moves, if the equilibrium is broken.

- $\vec{N}$ is always normal (= "perpendicular") to the interface
  - Where on the interface?
  - To maintain rotational balance!
  - Hint: typically, you need to shift the normal force towards the potential tipping point
  - Example: Consider moment equilibrium about point O: $Wx = Ph \Rightarrow x = Ph/W$
  - This equality "$x = Ph/W$" is **NOT** universal, since you may have more forces and more moments, you always need to look carefully at your system at hand!

FRICTION: Properties

Q: The weight of the block is 100 N.

a) You pull with a force $P = 20$ N. The object does not move due to friction. What is the friction force?



Static friction coefficient: $\mu_S = 0.5$

a) A. 20 N

b) B. 40 N

C. 50 N

D. 60 N

c) E. Something else

$F_{s,max} = \mu_s N$

$N = W$

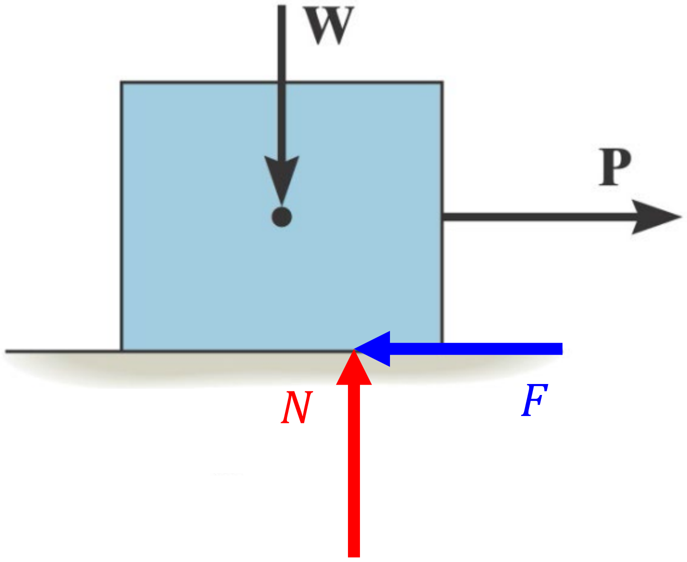
$F_{s,max} = 0.5 \cdot 100 = 50$ N

$F_k = \mu_k N$

b) What happens if you start pulling with a force $P = 40$ N?

c) $P = 60$ N?

FRICTION: Properties



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a) You pull with a force $P = 20$ N. The object does not move due to friction. What is the friction force?

A. 20 N ← a)

B. 40 N ← b)

C. 50 N

D. 60 N

E. Something else

← c)

b) What happens if you start pulling with a force $P = 40$ N?

c) $P = 60$ N?

From translational equilibrium:

$$F = P$$

This holds until friction force reaches its maximum value:

$$F_s = \mu_s N = \mu_s W = 0.5 \cdot 100 = 50\,N$$

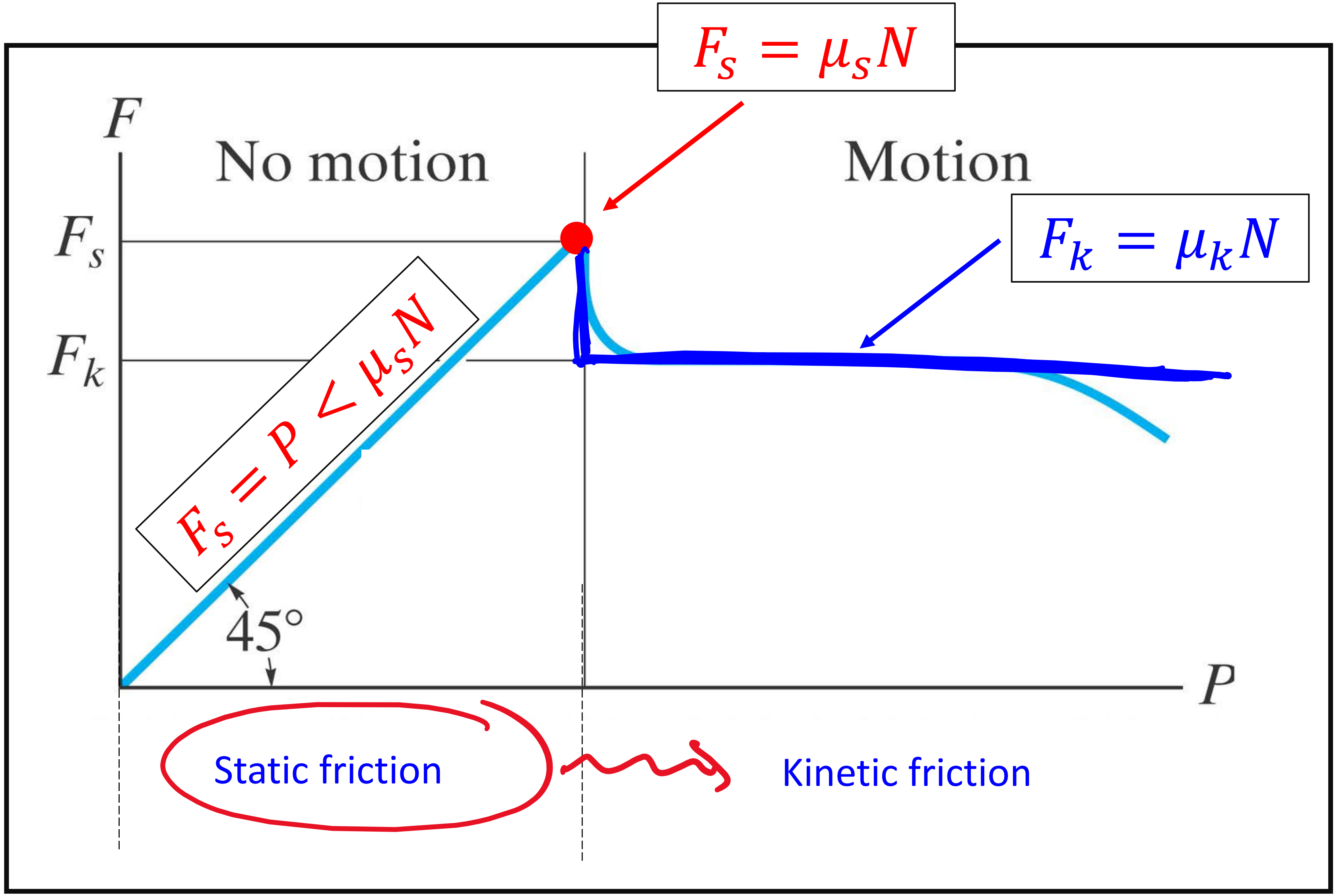
from FBD, $N = W$

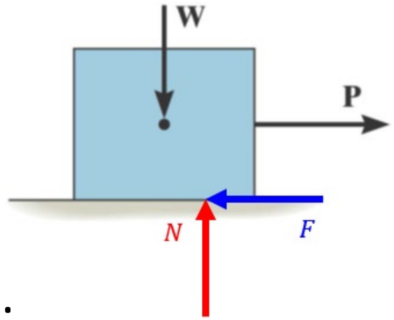
If $P > F_s$, the object starts into motion, with the friction force determined by kinetic friction coefficient:

$$F = \mu_k N$$

$$\mu_k \neq \mu_s$$

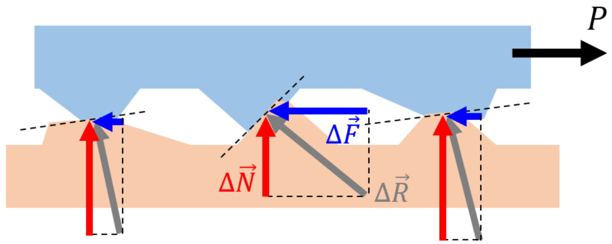
FRICTION: Properties





Kinetic friction:

- When: $P > \mu_s N$
- Why: points of contact start "riding over" one another; sticking-unsticking at high points



$$\mu_k < \mu_s < 1$$

IMPENDING MOTION: When the equilibrium is about to break

- Impending motion: your object is still static, but if you increase the load, even infinitesimally, it will lose equilibrium.

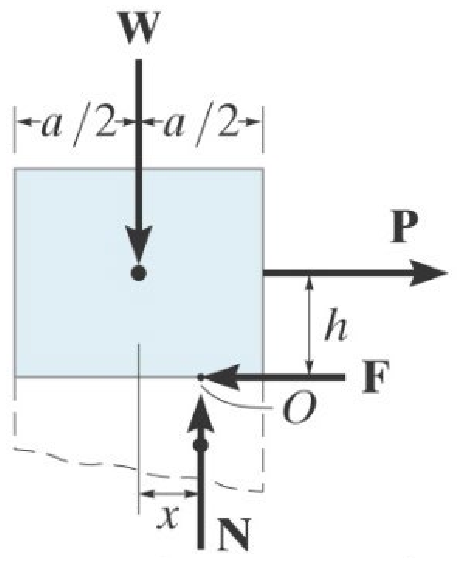
- How exactly will it lose equilibrium?
  - It can slip (breaking translational equilibrium)…
  - … or it can tip (breaking rotation equilibrium)

- So what will the object prefer to do?

- It depends! ☺ We are going to explore various scenarios, to find out which of them is the "weak link in the chain"

Simple example: Pulling a box

- Data: W, μ_s, a, h,. What is the value of P above which the equilibrium will be broken?



- Equilibrium:
  - Translational equilibrium ('no slipping') if: $P = F$, and $F \leq F_{max} = \mu_s N$
  - Rotational equilibrium ('no tipping') if: $Wx = Ph$, and $x \leq x_{max} = \frac{a}{2}$ ("the normal force is applied inside the object")

- Slipping
  - What if $P > F_{max} = \mu_s N$?
  - The box will slide.

?

- Tipping
  - What if, instead, $P > \frac{W x_{max}}{h} = \frac{W(a/2)}{h}$?
  - The box will tip.

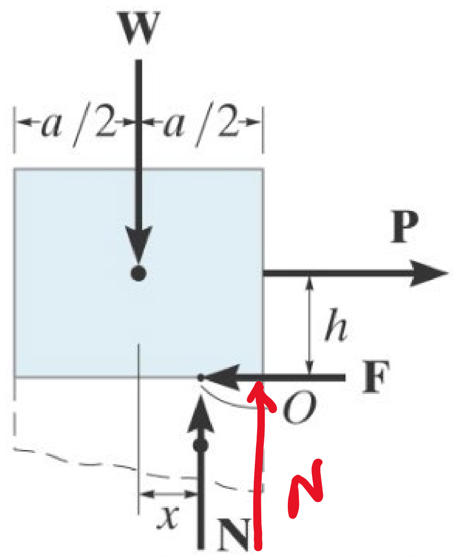
- So, will it slip or tip?

> - There are two scenarios of breaking the equilibrium: slipping and tipping
> - Which one wins depends on the parameters: which of the conditions $P > \mu_s N$ (slipping) or $P > \frac{W(a/2)}{h}$ (tipping) will be satisfied first (i.e. with minimum P).

Simple example: Pulling a box

- Approach:

- Data: $W, \mu_s, a, h,$. What is the value of P above which the equilibrium will be broken?



➢ Assume some magnitude of P, and assume that the equilibrium holds. We can write down 3 (in 2D) equilibrium equations in terms of the forces P, F, N, W and the arms x, h. Clearly, in equilibrium we must have:

$$F \leq \mu_s N, \qquad x \leq a/2$$

➢ Now let us (mentally) increase the force P and see <u>which equilibrium will break first</u>:

❖ Translational: this will happen if the friction force F saturates first, reaching its maximum possible value, $F = \mu_s N$: If this happens, friction cannot keep the object against slipping after P is increased, even infinitesimally. So the slipping scenario is characterized by equations:

$$F = \mu_s N, \qquad x \leq a/2$$

❖ Rotational: this will happen if the arm x saturates first, reaching its maximum possible value, $x = a/2$: If this happens, the moment created by normal force cannot keep the object against tipping after P is increased, even infinitesimally. So the tipping scenario is characterized by equations:

$$F \leq \mu_s N, \qquad x = a/2$$

How to formalize this?

- Equilibrium:
  - ➢ $F \leq \mu_s N$ (static, not dynamic, friction regime)
  - ➢ $x \leq \frac{a}{2}$ (normal force is applied inside the body)

We will call $F \leq \mu_S N$ and $x \leq \frac{a}{2}$ "restrictions". They must be satisfied, unless the object is moving already.

Assume that P starts growing ⇒ F and x will change. At some point:

- Equilibrium is about to break (aka "Impending motion" regime)

Note that "<" corresponds to stable equilibrium, and "=" to the impending motion regime.

- That's where the fun starts... We are at a fork:

➢ $F = \mu_s N$ (friction force saturates)

➢ $x \leq \frac{a}{2}$ (N is inside the body, prevents rotation)

Slipping!

➢ $F \leq \mu_s N$ (friction force still keeps the box)

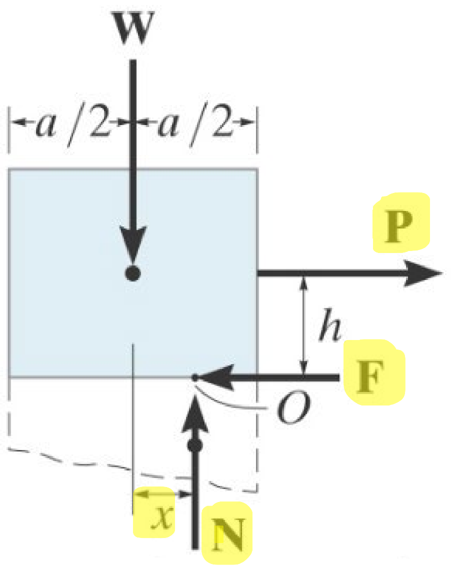
➢ $x = \frac{a}{2}$ (N can't oppose rotation anymore)

Tipping!

We will call the equalities $F = \mu_S N$ and $x = \frac{a}{2}$ "impending motion equations". They will help us to understand how exactly the equilibrium is broken (which scenario is realized).

GENERAL STRATEGY (for our simple example):

- Data: W, μ_s, a, h,. What is the value of P above which the equilibrium will be broken?



Impending motion problem for this box:

- Draw FBD(s) [place N correctly!!!]
- Equations vs unknowns analysis:
  - Unknowns (4): P, N, F, x.
  - Equilibrium equations (3): $\sum F_x = 0, \quad \sum F_y = 0, \quad \sum (M_z)_O = 0$
  - Impending motion equations: (4)-(3) = 1 ⇒ we need one more!!!
  - Write down the restrictions: $F \leq \mu_s N$ and $x \leq \frac{a}{2}$.
- Identify possible scenario(s): slipping along horizontal / tipping at the right corner
  - ASSUME SLIPPING: (eq. #4) $F = \mu_s N$ => find P, N, F, x => CHECK remaining restriction ($x \leq a/2$)
    - Restriction satisfied? => the box will slip. You win ☺ => The End.
    - Restriction is not satisfied? ☹ =>
  - ASSUME TIPPING: (eq. #4) $x = a/2$ => find P, N, F, x => CHECK remaining restriction ($F \leq \mu_s N$)
    - Restriction satisfied? => the box will tip. You win ☺ => The End.
    - Restriction is not satisfied? => Check your work ☹ ☹ ☹ ☹ …….

GENERAL STRATEGY (for any impending motion problem with many objects):

- Draw all the FBDs. Pay attention to:
  - The direction of the friction forces (they oppose potential motion);
  - The locations of the normal forces (they are shifted from the center if there is a tendency for rotation)
  - The 3rd Newton's law $(\vec{N}, \vec{F})$-pairs at the interfaces between the objects in your system
- Perform "equations vs unknowns" analysis:
  - Set up equilibrium equations. Count the unknowns and the equilibrium equations.
  - Find out how many impending motion equations you need to close your system of equations.
  - Write down all the restrictions ("$F \leq \mu N$, x inside the body" for all the objects and $(\vec{N}, \vec{F})$-pairs)
- Identify possible scenarios of breaking the equilibrium. The number of points at which the equilibrium breaks should be equal to the number of impending motion equations you need to close your system of equations.
- Pick one scenario, add the required number of impending motion equations in accordance with this scenario, and find the unknowns.
- Check if the remaining restrictions are satisfied.
  - If yes: You win ☺ => The End. If not: ☹ Pick a different scenario and repeat the last steps until you win.