GENERAL STRATEGY (for any impending motion problem with many objects):

- Draw all the FBDs. Pay attention to:
  - The direction of the friction forces (they oppose potential motion);
  - The locations of the normal forces (they are shifted from the center if there is a tendency for rotation)
  - The 3rd Newton's law $(\vec{N}, \vec{F})$-pairs at the interfaces between the objects in your system
- Perform "equations vs unknowns" analysis:
  - Set up equilibrium equations. Count the unknowns and the equilibrium equations.
  - Find out how many impending motion equations you need to close your system of equations.
  - Write down all the restrictions ("$F \leq \mu N$, x inside the body" for all the objects and $(\vec{N}, \vec{F})$-pairs)
- Identify possible scenarios of breaking the equilibrium. The number of points at which the equilibrium breaks should be equal to the number of impending motion equations you need to close your system of equations.
- Pick one scenario, add the required number of impending motion equations in accordance with this scenario, and find the unknowns.
- Check if the remaining restrictions are satisfied.
  - If yes: You win ☺ => The End. If not: ☹ Pick a different scenario and repeat the last steps until you win.

Restrictions & Impending motion equation

Q: How can we mathematically express the following:

- The block tips over the lower right corner.

A. $F \leq \mu N, \quad x \leq \frac{a}{2}$

B. $F = \mu N, \quad x \leq \frac{a}{2}$

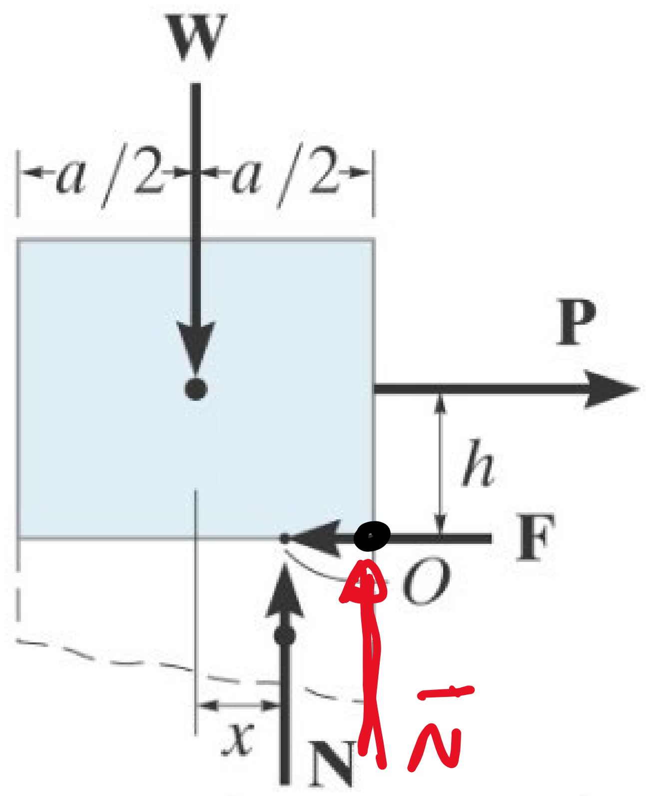
C. $F \leq \mu N, \quad x = \frac{a}{2}$

D. $F = \mu N, \quad x < \frac{a}{2}$

E. $F < \mu N, \quad x = \frac{a}{2}$

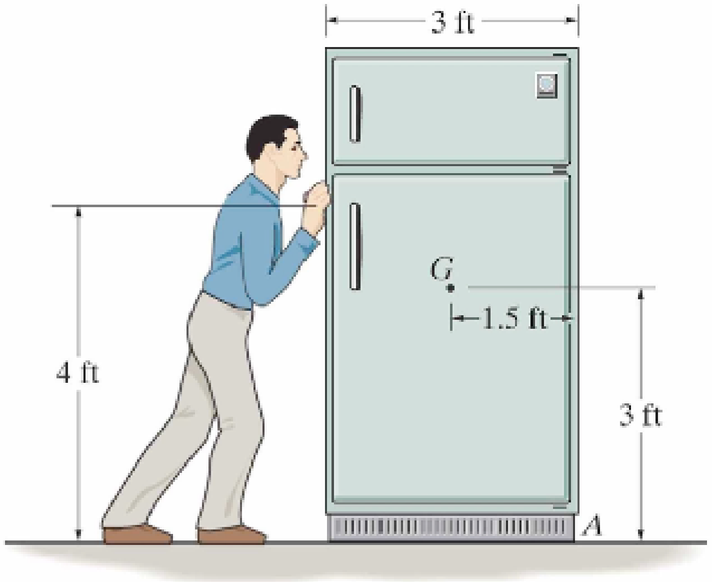
restriction

impending motion eq



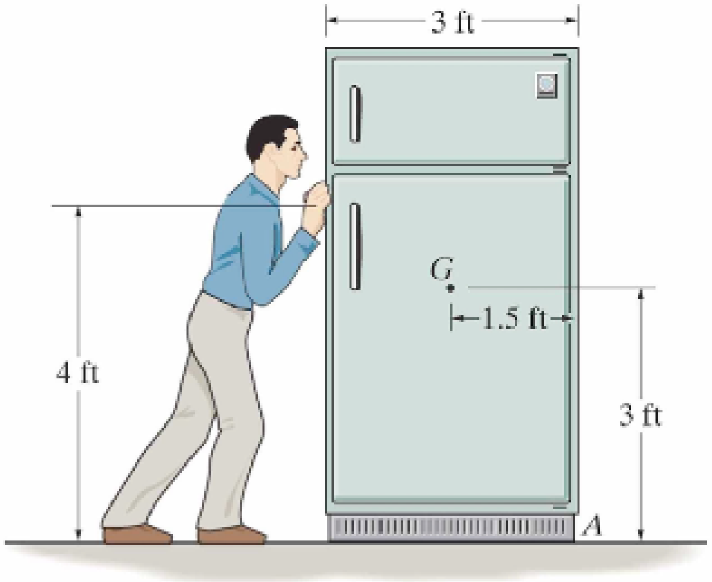
WARMING UP: Let's move a refrigerator

W6-1. The refrigerator weighs 180 lb and rests on a tile floor. The coefficient of static friction μ_R between the refrigerator and the floor is 0.25. The man weighs 150 lb. The coefficient of static friction μ_M between his shoes and the floor is 0.6. The man pushes horizontally on the refrigerator. Can the man move the refrigerator?



- Drawing FBD: Which directions of the friction forces and normal forces make sense?

WARMING UP: Let's move a refrigerator



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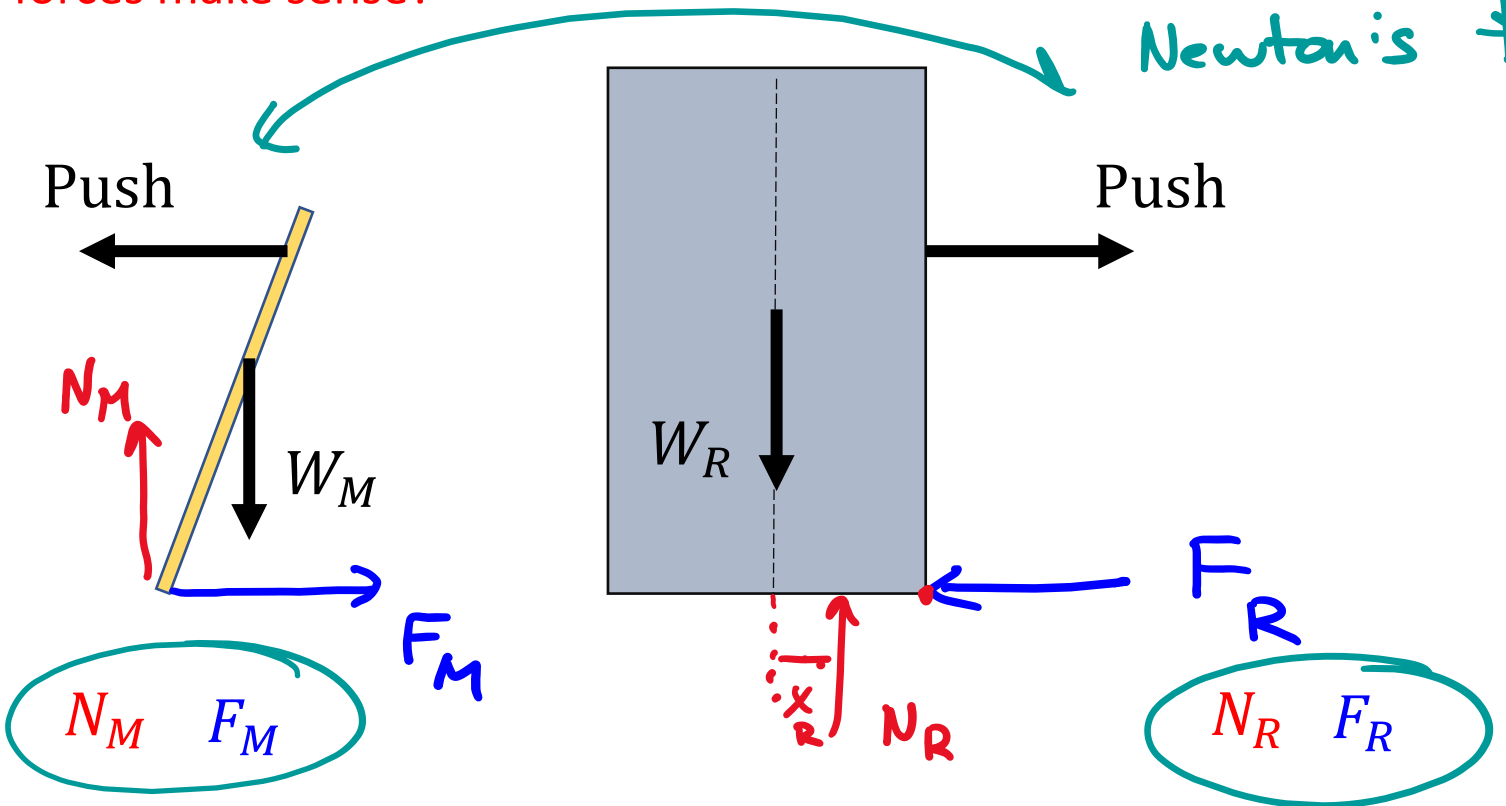

Newton's third law pair!

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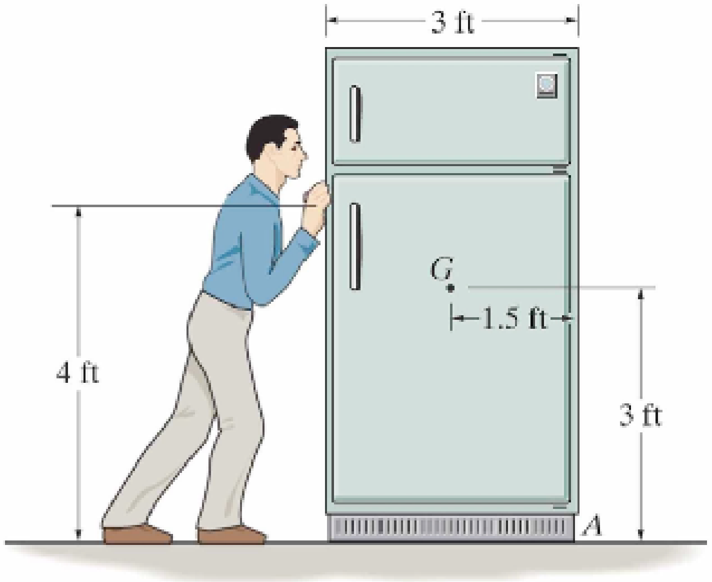

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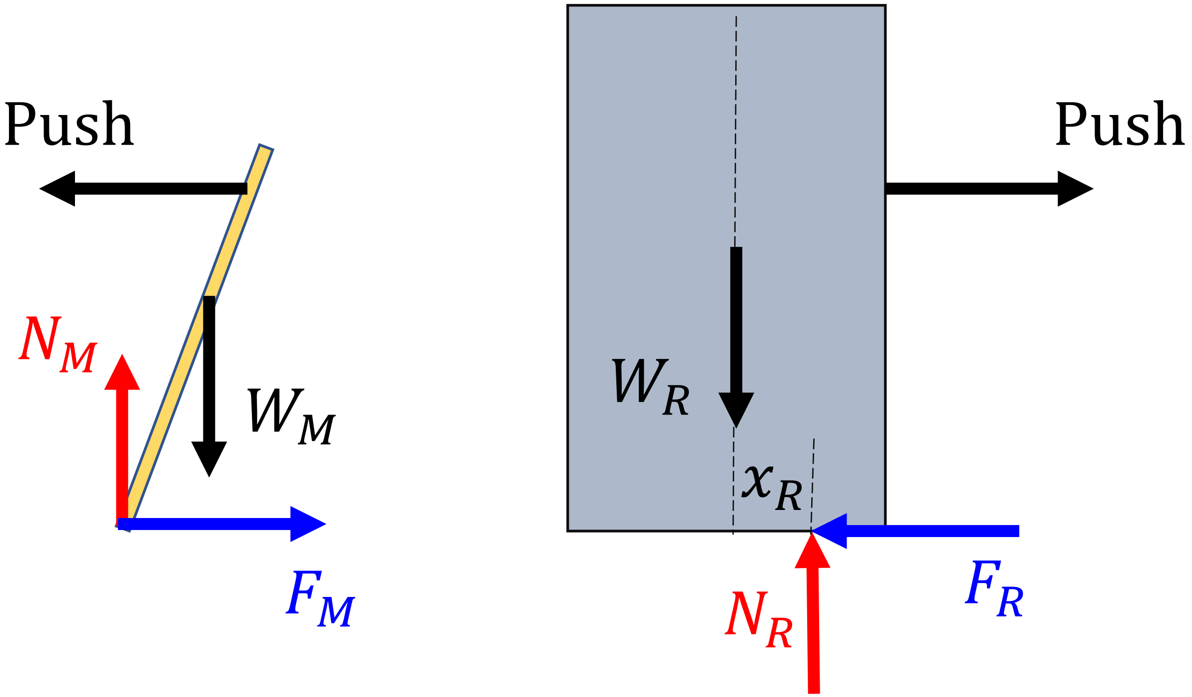

$\vec{F}_R, \vec{F}_M$: oppose potential motion

$\vec{N}_R, \vec{N}_M$: normal to the surface of contact

$\vec{N}_R$: shifted from the center to maintain rotational equilibrium.

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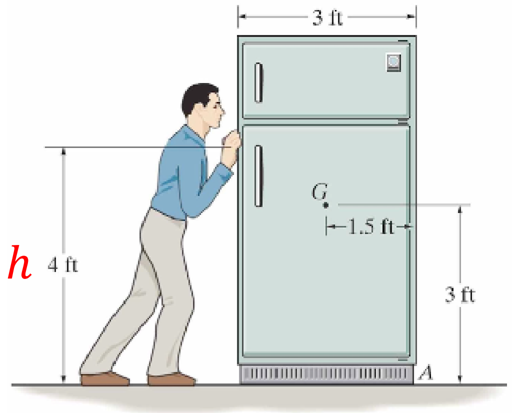
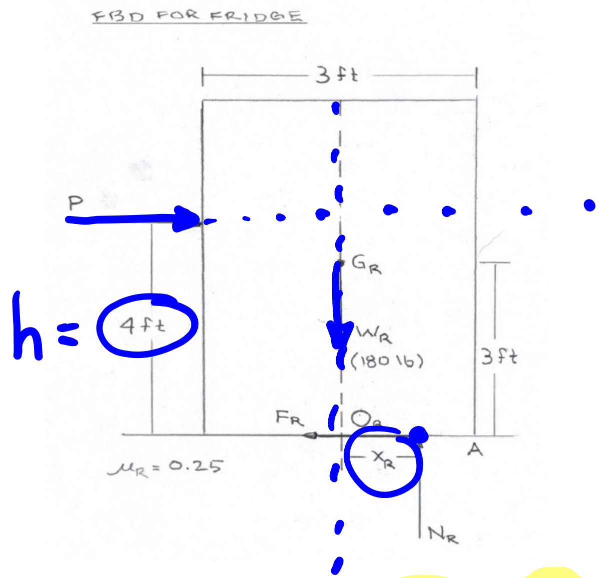
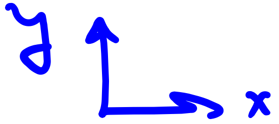
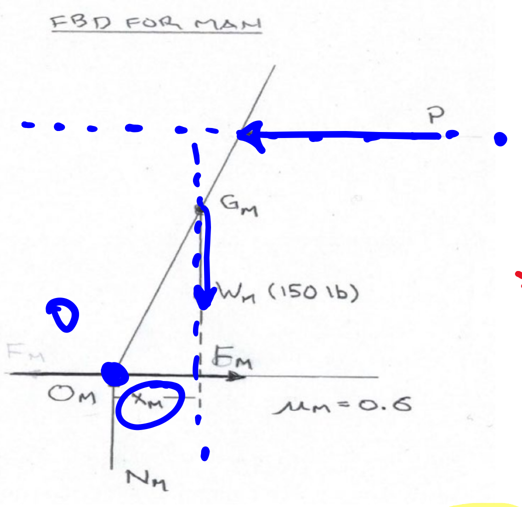






- Equilibrium equations:



7 unknowns

6 eqs

(1) $\Sigma F_x = 0 \qquad P - F_R = 0$

(2) $\Sigma F_y = 0 \qquad N_R - W_R = 0$

(3) $\Sigma M_z = 0 \qquad -Ph + W_R x_R = 0$

(4) $\Sigma F_x = 0 \qquad F_M - P = 0$

(5) $\Sigma F_y = 0 \qquad N_M - W_M = 0$

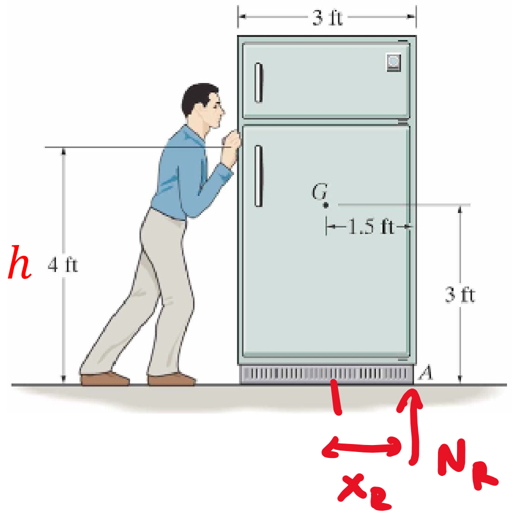
(6) $\Sigma M_z = 0 \qquad -W_M x_M + Ph = 0$

- Restrictions:

$x_R \leq 1.5$ ft $\qquad F_R \leq \mu_R N_R \qquad F_M \leq \mu_M N_M$

- Unknowns vs Equations?

W6-1. The refrigerator weighs 180 lb and rests on a tile floor. The coefficient of static friction μ_R between the refrigerator and the floor is 0.25. The man weighs 150 lb. The coefficient of static friction μ_M between his shoes and the floor is 0.6. The man pushes horizontally on the refrigerator.



So far we have: $F_R \le \mu_R N_R$, $x_R \le 1.5$ ft; $F_M \le \mu_M N_M$,

- Possible scenarios, Impending motions equations, and restrictions:

? A. Ref slides (Ref ~~tips~~ Man ~~slide~~)

(7) $\boxed{F_R = \mu_R N_R}$ (*) $x_R \le 1.5$ (**) $F_M \le \mu_M N_M$

B. Man slides (Ref ~~slide~~ Ref ~~tips~~)

$F_M = \mu_M N_M$ $F_R \le \mu_R N_R$ $x_R \le 1.5$

C. Ref tips (Ref ~~slide~~ Man ~~slides~~)

$x_R = 1.5$ ft $F_R \le \mu_R N_R$ $F_M \le \mu_M N_M$

- In your opinion, which of them is most viable? Why?

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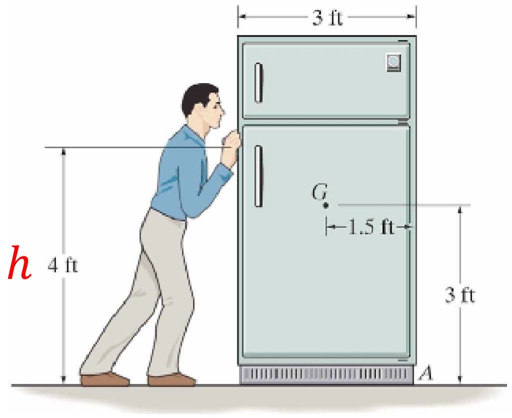


- Solve & CHECK:

Fridge:

$\Sigma F_x = 0: \quad P = F_R \quad (1)$

$\Sigma F_y = 0: \quad N_R = W_R \quad (2)$ (W_R = 180)

$\Sigma M_{O_R} = 0: \quad W_R x_R - P \cdot 4 = 0 \quad (3)$

Man:

$\Sigma F_x = 0: \quad P = F_M \quad (4)$

$\Sigma F_y = 0: \quad N_M = W_M \quad (5)$ (W_M = 150)

$\Sigma M_{O_M} = 0: \quad P \cdot 4 - W_M x_M = 0 \quad (6)$

$F_R = \mu_R \cdot N_R \quad (7)$

$F_M \leq \mu_M N_M \quad (*)$

$x \leq 1.5 \text{ ft} \quad (**)$

$$P = \overset{(1)}{F_R} \overset{(7)}{=} \mu_R N_R \overset{(2)}{=} \mu_R W_R = 45 \text{ lb}$$

$(*) \quad F_M \leq \mu_M N_M$

$45 \text{ lb} \leq 0.6 \cdot 150 = 90$ Yes!!

$(**) \quad x_R \leq 1.5$

$x_R = \dfrac{4 \cdot 45}{180} = 1 \leq 1.5$

Yes!!

- What if we would have started with a different scenario?

(A) ASSUME: Fridge tips: $x_R = 1.5$ ft (7') – instead of (7) ← impending tipping condition

Then: (3): $P = \frac{x_R W_R}{4} = 67.5$ lb ⇒

(1) and (4): $F_R = F_M = 67.5$ lb

CHECK: $(F_M = 67.5 \text{ lb}) < (\mu_M N_M = 0.6 \cdot 150 = 90 \text{ lb})$ – OK (man is not sliding)

$(F_R = 67.5 \text{ lb}) > (\mu_R N_R = 0.25 \cdot 180 = 45 \text{ lb})$ – DOES NOT WORK ⇒ the fridge will slide before it tips.

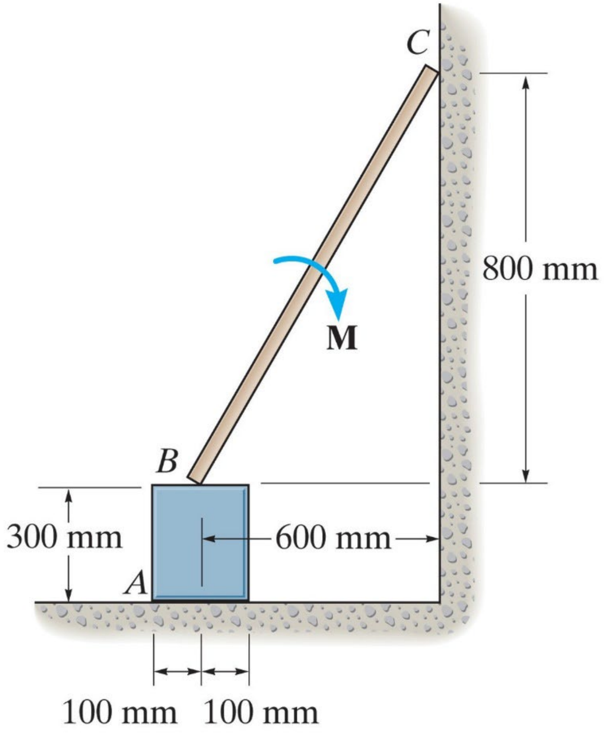
(B) ASSUME: Man slides: $F_M = \mu_M \cdot N_M = 90$ lb (7'') ← impending sliding for the man.

Then: $P = F_R = F_M = 90$ lb ⇒ $(F_R = 90 \text{ lb}) > (\mu_R N_R = 45 \text{ lb})$ – WRONG (the fridge is also sliding)

$\left(x_R = \frac{4P}{W_R} = 2\text{ft}\right) > (O_R A = 1.5 \text{ ft})$ – WRONG

- From Piazza,
  PHYS 170 2020 W2

W6-2. The uniform 6 kg slender rod rests on the top center of the 3 kg block. The coefficients of static friction at A, B, and C are $\mu_A = 0.4$, $\mu_B = 0.6$ and $\mu_C = 0.3$. Determine the largest couple moment which can be applied to the rod without causing motion of the rod.



- Drawing FBD: Which directions of the friction forces and normal forces make sense?

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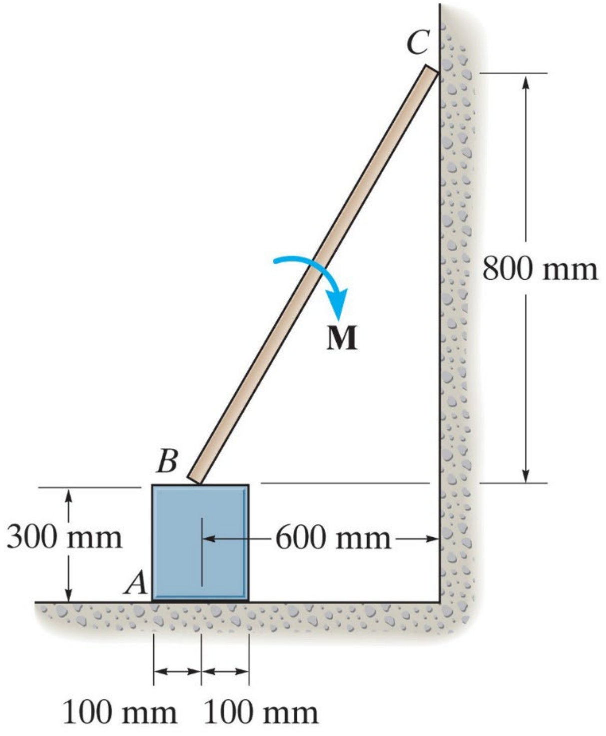

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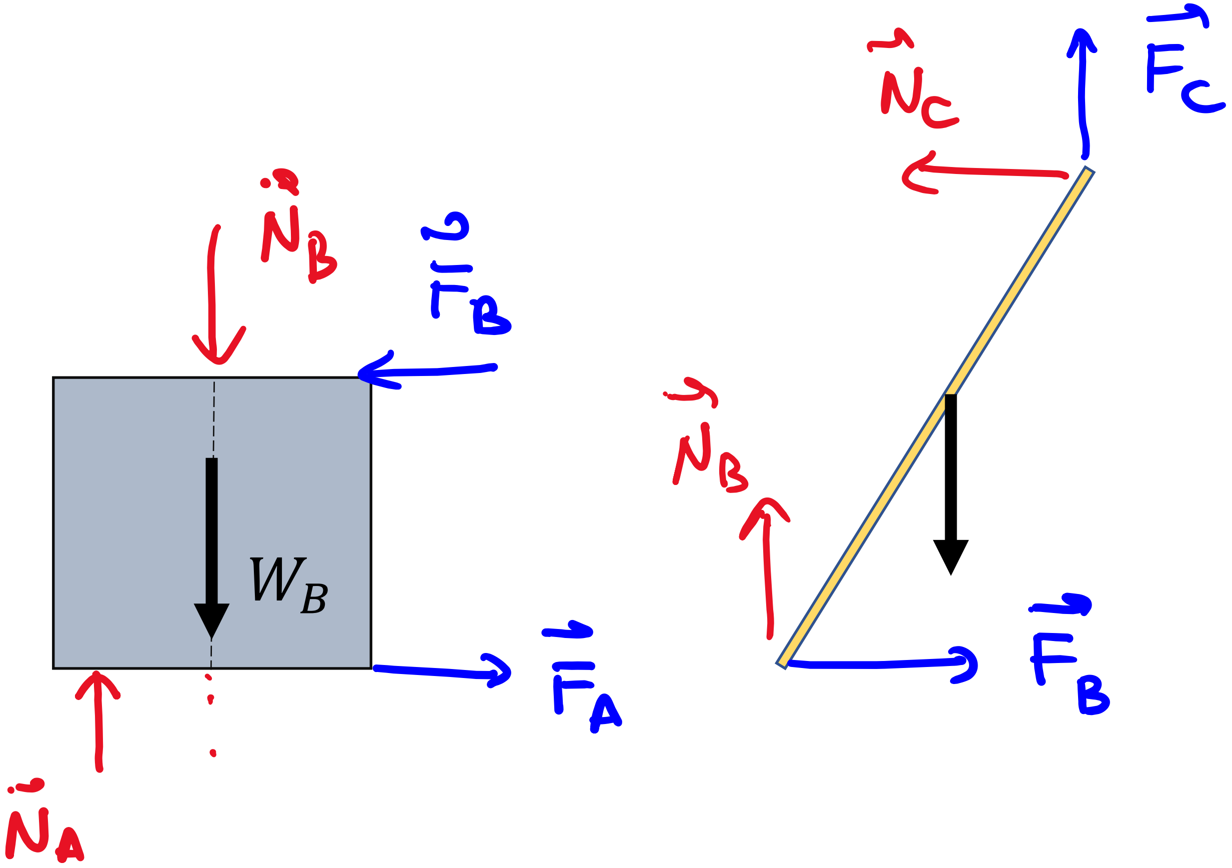

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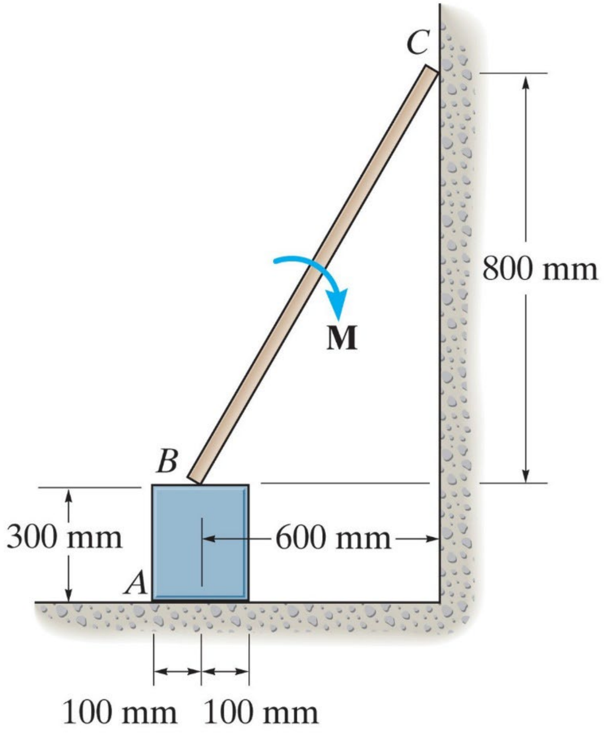

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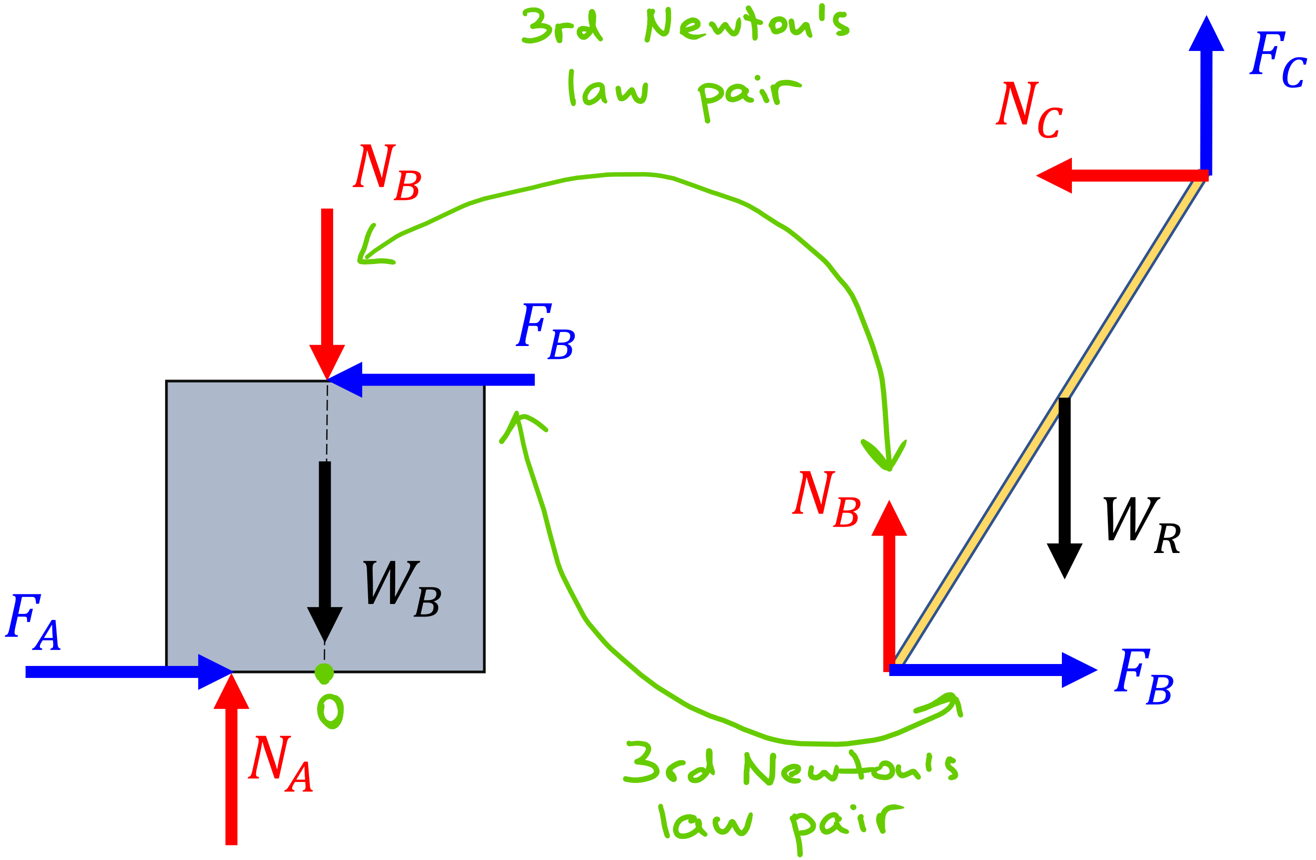

Location of N_A is determined by rotational equilibrium (consider the moments of all the forces acting on the block about the point O).

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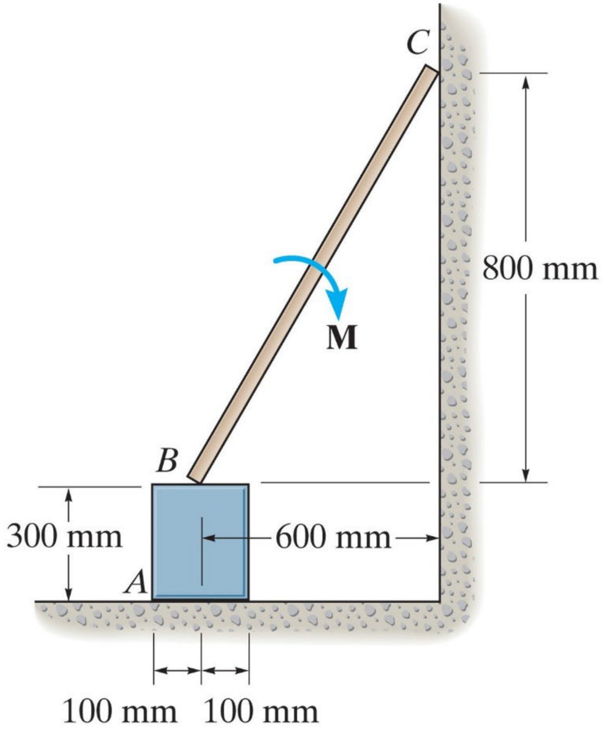
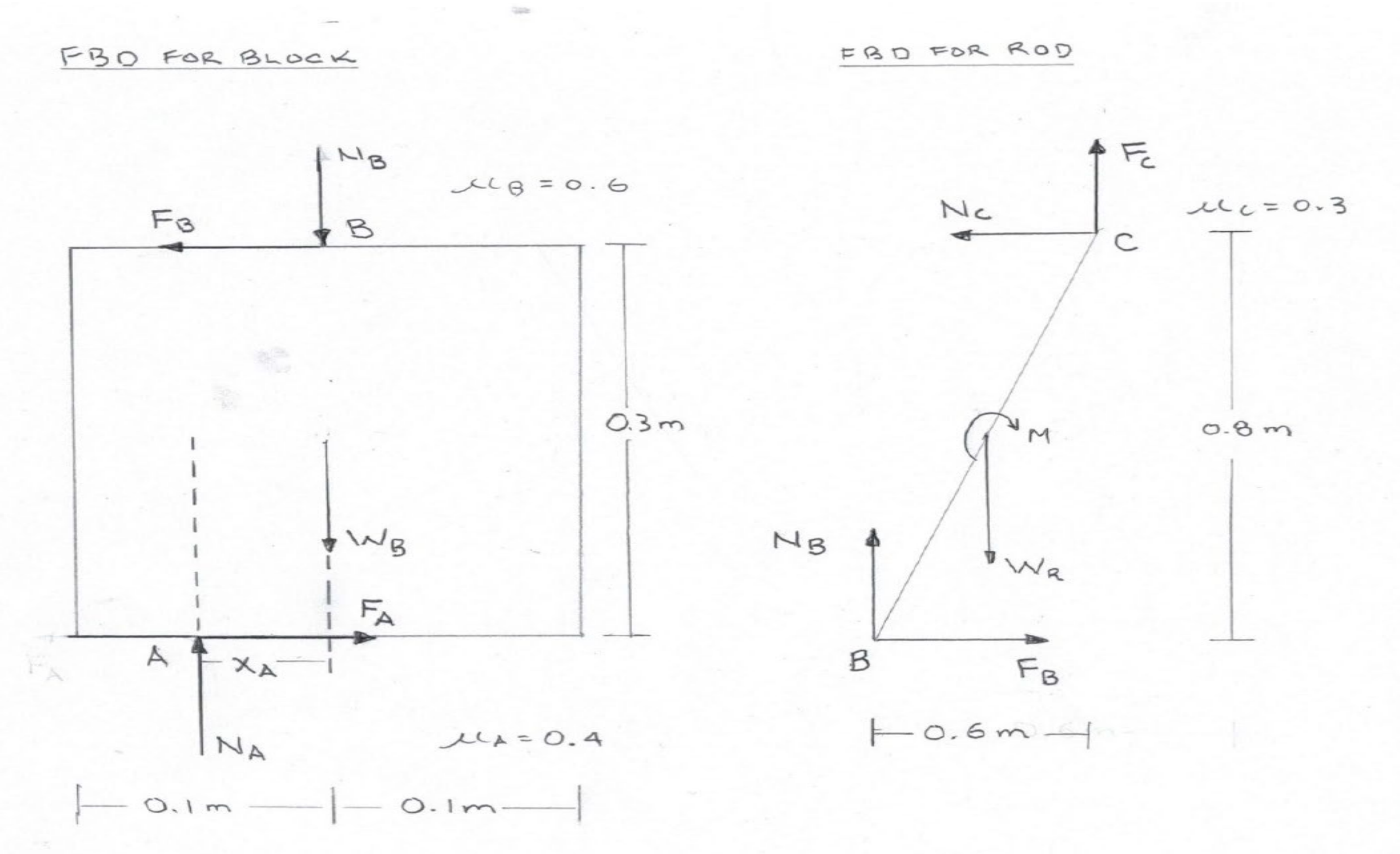




W6-2. 6 kg slender rod, 3 kg block, $\mu_A = 0.4$, $\mu_B = 0.6$ and $\mu_C = 0.3$. M=?

- Equilibrium equations:

Rod:

(1) $\Sigma F_x = 0 \qquad F_B - N_C = 0$

(2) $\Sigma F_y = 0 \qquad N_B + F_C - W_R = 0$

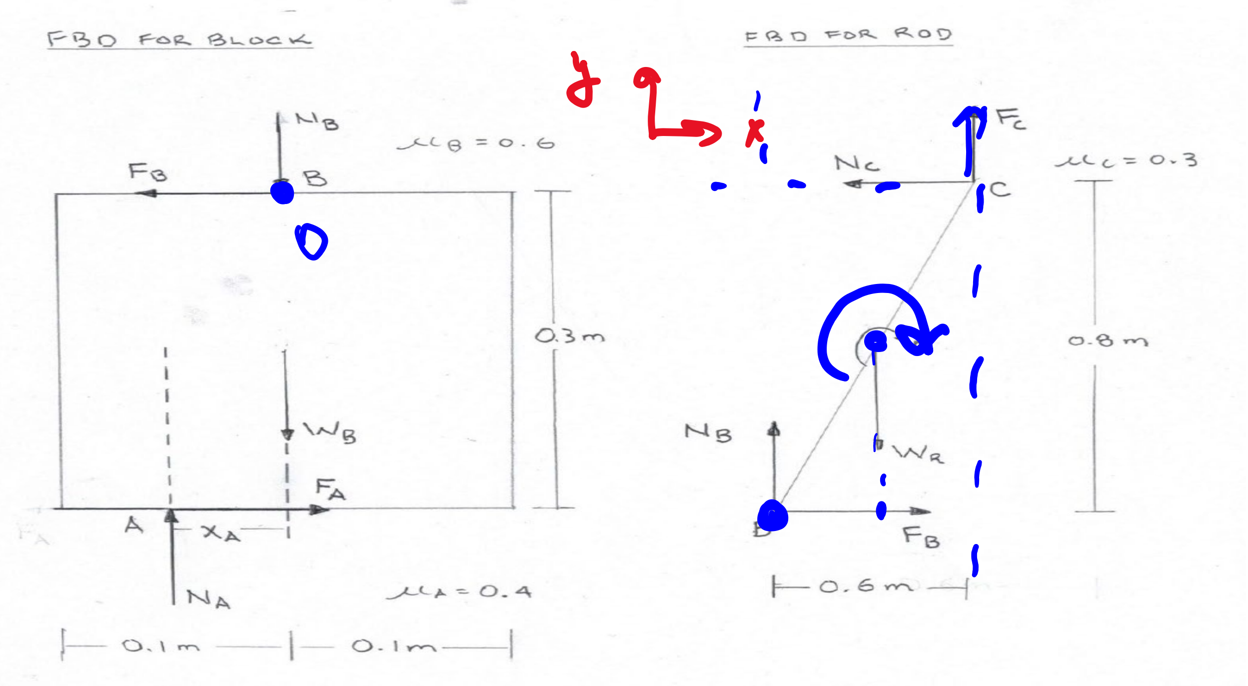
(3) $\Sigma M_z = 0 \qquad - W_R\, 0.3 + N_C\, 0.8 + F_C \cdot 0.6 - M = 0$

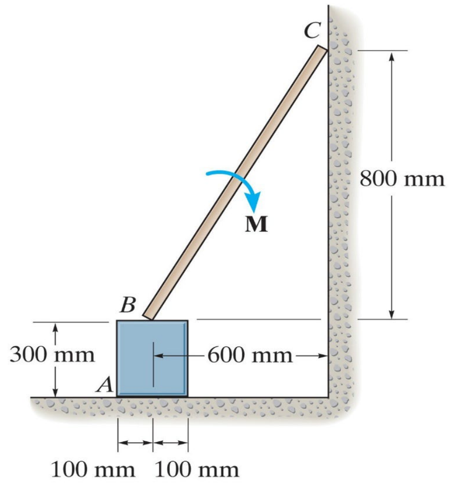
Block:

(4) $\Sigma F_x = 0 \qquad F_A - F_B = 0$

(5) $\Sigma F_y = 0 \qquad N_A - N_B - W_B = 0$

(6) $\Sigma M_z = 0 \qquad F_A\, 0.3 - N_A\, x_A = 0$





- Restrictions:

- Unknowns: