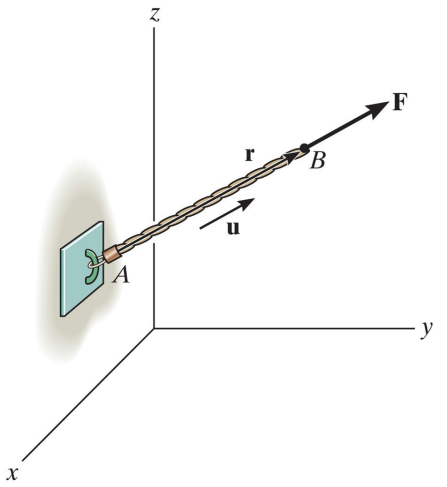
Position vectors
and
Force directed along a line



Text: 2.7-2.8

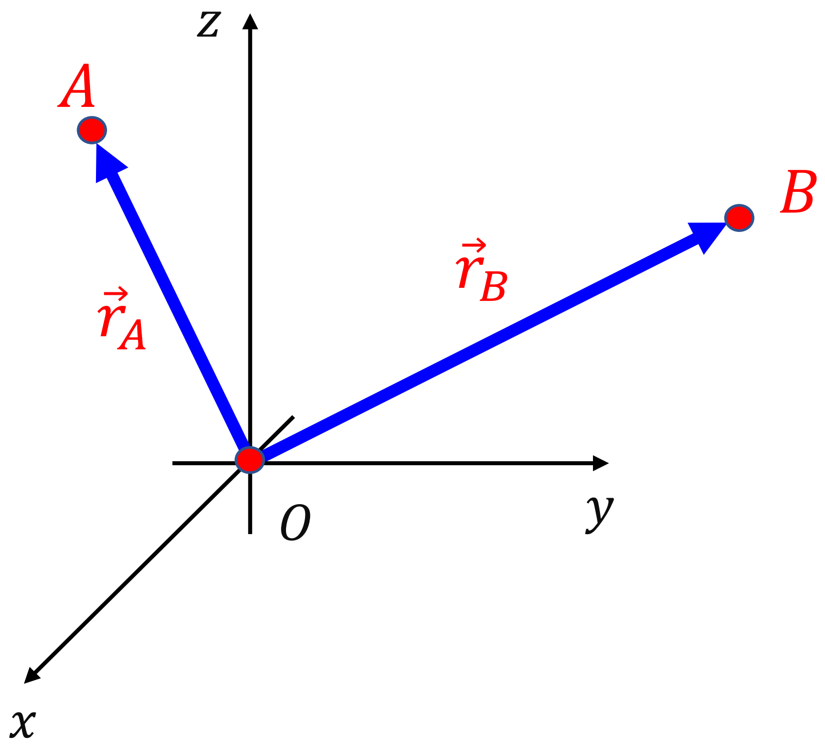
Content:

- Position vectors and displacement vectors
- Vector directed along a line
- Solving systems of linear equations
- Practice: W2-4

POSITION VECTOR

Notations for the coordinates of a point: $P(x, y, z)$

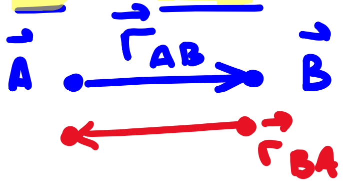
For example: $A(1, -1, 4)$, $B(0, 5, 3.5)$



- A **position vector** is a vector that **goes from the coordinate origin O to a point**.

DISPLACEMENT VECTOR

- We will denote as $\vec{r}_{AB}$ a vector that goes from point A to point B (= the "**displacement vector**")
- Once again: $\vec{r}_{AB} \equiv \vec{r}_{A\text{ to }B}$ (a vector from B to A has the opposite sign: $\vec{r}_{BA} = -\vec{r}_{AB}$!!)



Q: Given two points A and B, the vector $\vec{r}_{AB}$ from A to B is given by the expression:

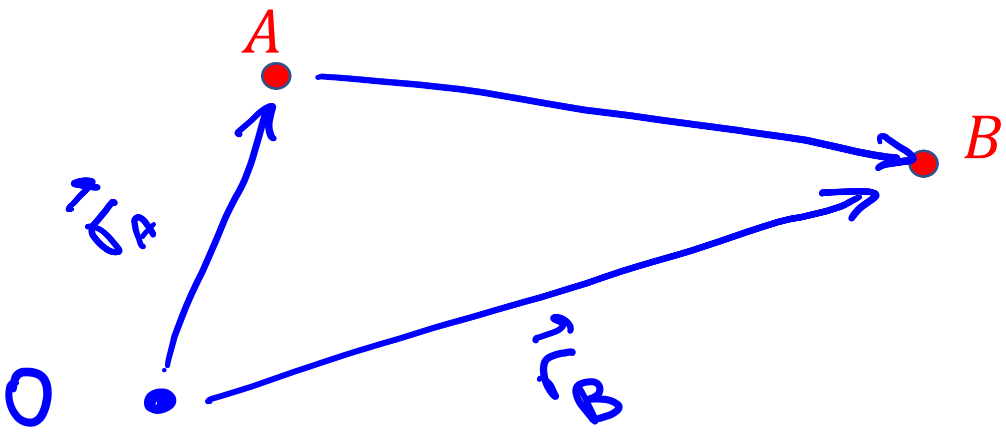
A. $\vec{r}_{AB} = \vec{r}_A + \vec{r}_B$

B. $\vec{r}_{AB} = \vec{r}_A - \vec{r}_B$

C. $\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$

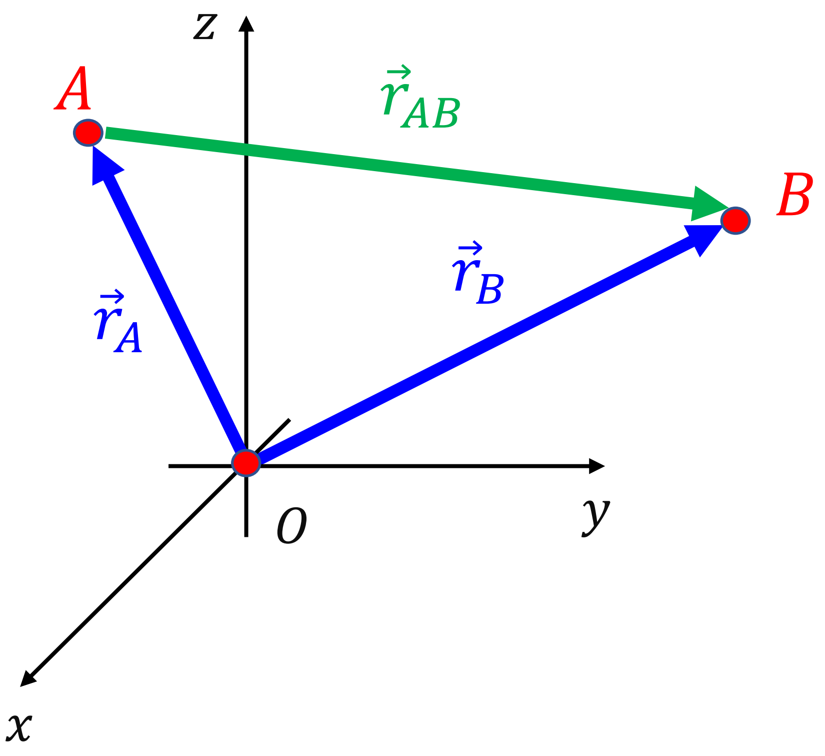
D. The origin isn't defined, so we can't tell.

tip - tail



$\vec{r}_A + \vec{r}_{AB} = \vec{r}_B$

DISPLACEMENT VECTOR



- Choose arbitrary coordinate origin O.
- Draw a vector from A to B – **displacement vector $\vec{r}_{AB}$**

- We see that $\vec{r}_A + \vec{r}_{AB} = \vec{r}_B$.
- Hence, $\boxed{\vec{r}_{AB} = \vec{r}_B - \vec{r}_A} =$

$$(\vec{i}x_B + \vec{j}y_B + \vec{k}z_B) - (\vec{i}x_A + \vec{j}y_A + \vec{k}z_A)$$

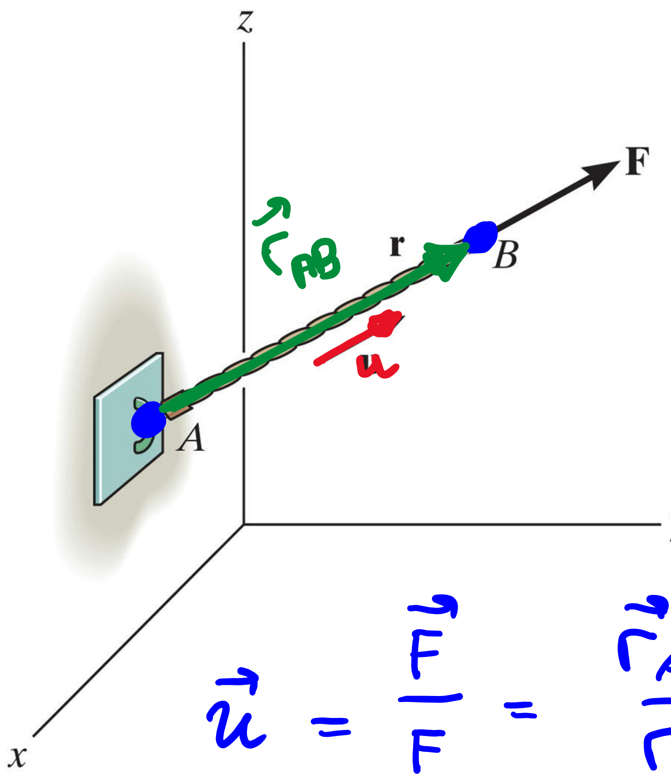
- *This result does not depend on the choice of the point O*
- *Don't accidentally reverse A and B when working quickly !*

- If $A = A(x_A, y_A, z_A)$ and $B = B(x_B, y_B, z_B)$, then:

$$\boxed{\vec{r}_{AB} = \vec{i}\,(x_B - x_A) + \vec{j}\,(y_B - y_A) + \vec{k}(z_B - z_A)}$$

tip (B) – tail (A)

FORCE VECTOR DIRECTED ALONG A LINE



- Will often (cables, struts…) encounter situation where force vector direction is defined by two points, $A(x_A, y_A, z_A)$ and $B(x_B, y_B, z_B)$, lying on the line of action.

- The force vector $\vec{F}$ has the same direction as the displacement vector $\vec{r}_{AB}$ passing through these two points.

- If $\vec{u}$ is a unit vector that defines this direction, we have:

$$\boxed{\vec{F} = F\,\vec{u}} \quad \text{and} \quad \boxed{\vec{r}_{AB} = r_{AB}\,\vec{u}}$$

$$\vec{u} = \frac{\vec{F}}{F} = \frac{\vec{r}_{AB}}{r_{AB}}$$

- Hence, $$\boxed{\vec{F} = F\,\frac{\vec{r}_{AB}}{r_{AB}}}$$

➢ $\vec{r}_{AB} = \vec{r}_B - \vec{r}_A = \vec{i}\,(x_B - x_A) + \vec{j}\,(y_B - y_A) + \vec{k}(z_B - z_A),$

➢ $r_{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$

Practice: A force with magnitude F acts along a line defined by points $A(x_A, y_A, z_A)$ and $B(x_B, y_B, z_B)$.

Write the expression for $\vec{F}$ in Cartesian components.

$$\vec{F} = F\,\frac{\vec{r}_{AB}}{r_{AB}}$$

$$\vec{r}_{AB} = (x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k}$$

$$\therefore\ r_{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

$$\vec{F} = \frac{F}{r_{AB}}\left\{(x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k}\right\}$$

$$\vec{F} = F\,\frac{\vec{r}_{AB}}{r_{AB}}$$

Practice: A force with magnitude F acts along a line defined by points $A(x_A, y_A, z_A)$ and $B(x_B, y_B, z_B)$.

Write the expression for $\vec{F}$ in Cartesian components.

Solution: We are going to make use of $\vec{F} = F\,\dfrac{\vec{r}_{AB}}{r_{AB}}$ where $\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$

- Step 1: Write down the coordinates of A and B: $A = (x_A, y_A, z_A)$ $B = (x_B, y_B, z_B)$

- Step 2: Compute $\vec{r}_{AB}$ and $r_{AB} = |\vec{r}_{AB}|$:

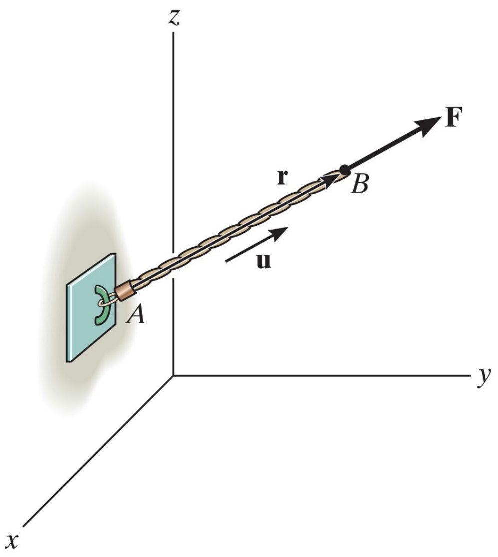
$$\vec{r}_{AB} = \vec{i}\,(x_B - x_A) + \vec{j}\,(y_B - y_A) + \vec{k}(z_B - z_A)$$

$$r_{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

- Step 3: You are done!

$$\vec{F} = F\,\vec{u}_{AB} = F\,\frac{\vec{r}_{AB}}{r_{AB}} = F\,\frac{\vec{i}\,(x_B - x_A) + \vec{j}\,(y_B - y_A) + \vec{k}(z_B - z_A)}{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}$$

Calculational trick to use with vectors of unknown magnitude but known direction - 1



- Assume that you want to find the magnitude F of a force $\vec{F}$ that passes through two points, (x_A, y_A, z_A) and (x_B, y_B, z_B), on its line of action:

$$\vec{F} = F\,\frac{\vec{r}_{AB}}{r_{AB}}$$

$$\vec{r}_{AB} = \vec{i}\,(x_B - x_A) + \vec{j}\,(y_B - y_A) + \vec{k}(z_B - z_A)$$

$$r_{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

- The components of the vector $\vec{r}_{AB}$ are usually some "nice" whole numbers, but the components of the unit vector, $\vec{r}_{AB}/r_{AB}$, are usually irrational numbers

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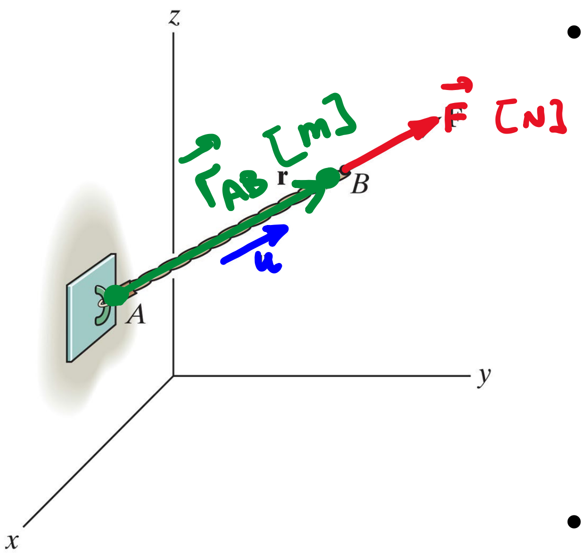
- Note that if you write the vector $\vec{F}$ as $\vec{F} = F\,\frac{\vec{r}_{AB}}{r_{AB}}$, you will have an unknown F multiplied by a set of irrational numbers – inconvenient, and higher chances to make a mistake! ☹

- But: There is **a trick** which can make your life easier.

Calculational trick to use with vectors of unknown magnitude but known direction - 1



- Assume that you want to find the magnitude F of a force $\vec{F}$ that passes through two points, (x_A, y_A, z_A) and (x_B, y_B, z_B), on its line of action:

$$\vec{F} = F\frac{\vec{r}_{AB}}{r_{AB}}$$

$$\vec{r}_{AB} = \vec{i}\,(x_B - x_A) + \vec{j}\,(y_B - y_A) + \vec{k}(z_B - z_A)$$

$$r_{AB} = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

- The components of the vector $\vec{r}_{AB}$ are usually some “nice” whole numbers, but the components of the unit vector, $\vec{r}_{AB}/r_{AB}$, are usually irrational numbers

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- **The idea** is to introduce a new scalar unknown, $X = F/r_{AB}$, and express the vector $\vec{F}$ as:

$$\vec{F} = \left(\frac{F}{r_{AB}}\right)\vec{r}_{AB} = (X)\,\vec{r}_{AB}$$

- Now $\vec{F}$ is expressed as an unknown, X, multiplied by a set of whole numbers – much better! ☺

- The (small) price to pay is that, after you find X, you need also to find F as $F = X\,r_{AB}$.

W2-4. The magnitude of the resultant force is 1300 N and acts along the axis of the strut from A towards O. Determine the magnitude of each of the three forces acting on the strut. Set x=0 and z=5.5 m.

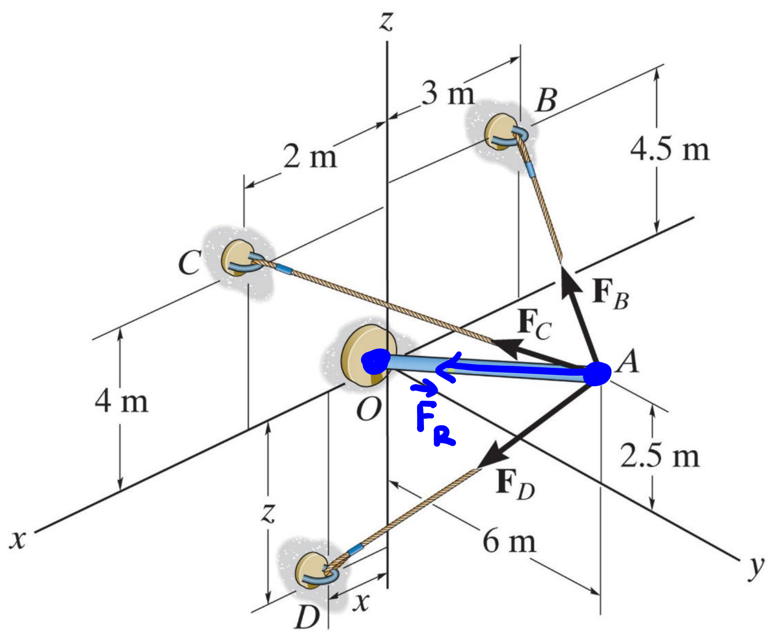

Figure: 02_P108-109

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W2-4. The magnitude of the resultant force is 1300 N and acts along the axis of the strut from A towards O. Determine the magnitude of each of the three forces acting on the strut. Set x=0 and z=5.5 m.

F_R, $\vec{u}_R$ $\rightarrow$ $\vec{F}_R$ = known

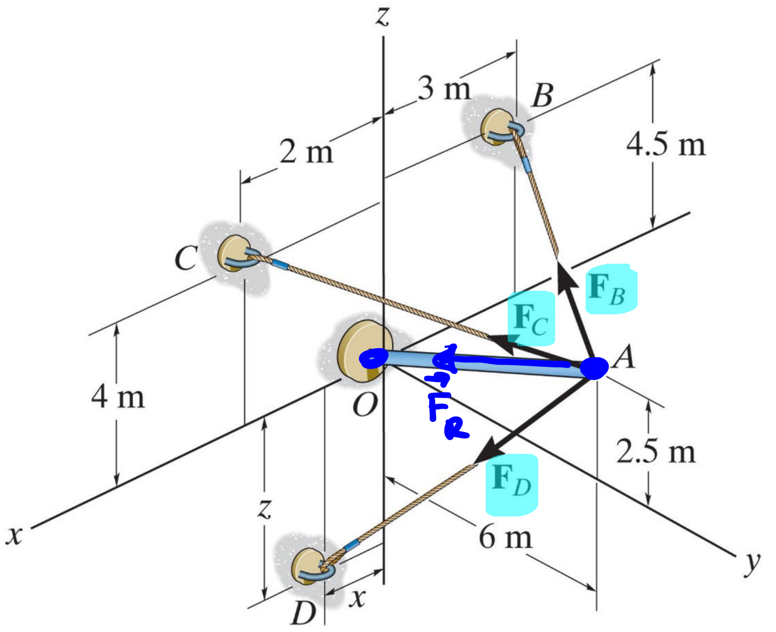

Figure: 02_P108-109

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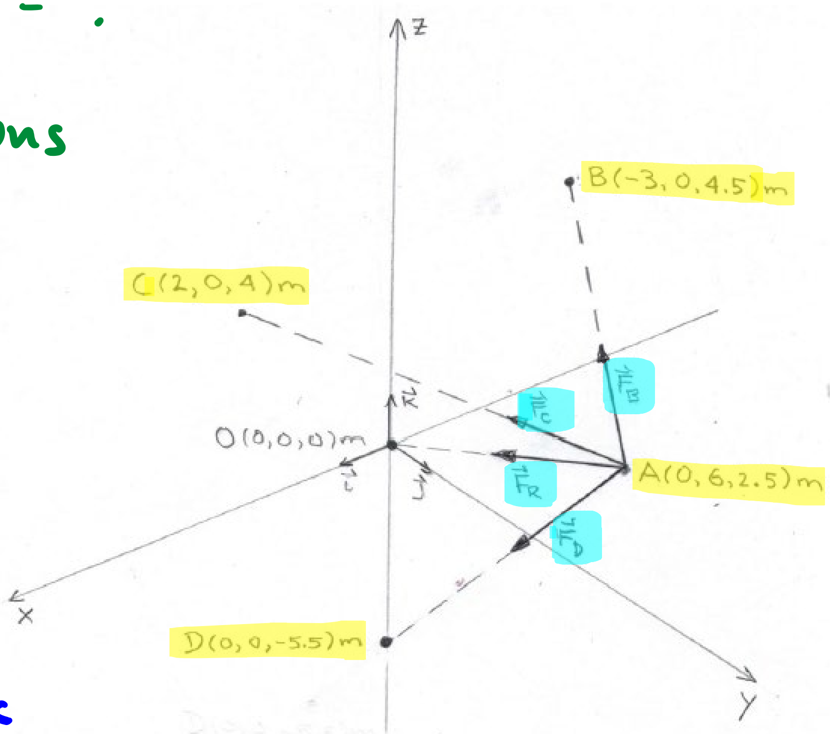
F_B, F_C, F_D = ?

3 unknowns

3 eqs ?



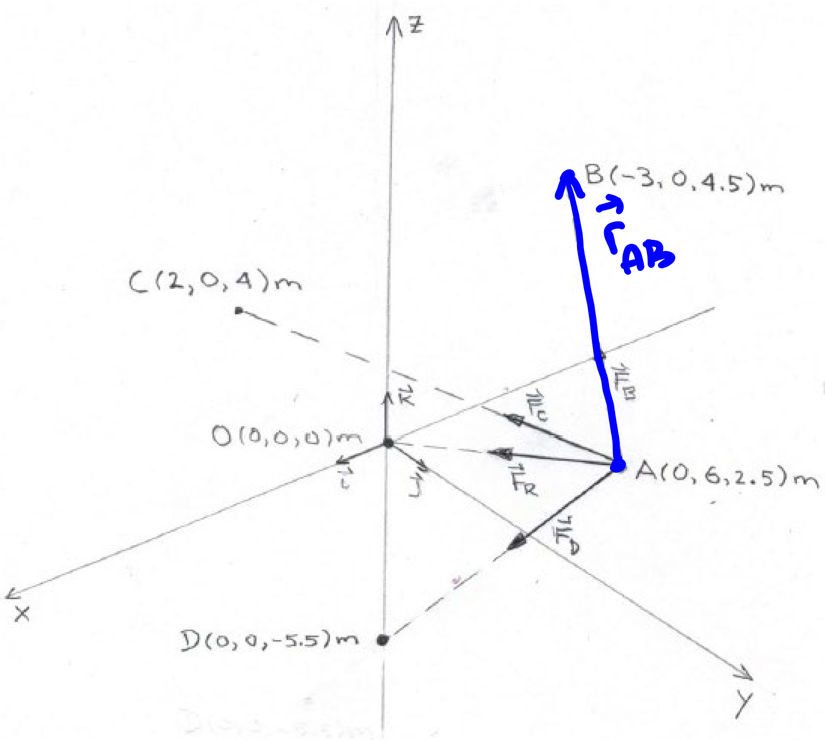
$$\vec{F}_R = \vec{F}_C + \vec{F}_B + \vec{F}_D$$

(1) x: $F_{Rx} = F_{Cx} + F_{Bx} + F_{Dx}$

(2) y: $F_{Ry} = F_{Cy} + F_{By} + F_{Dy}$

(3) z: F_{Rz} = . . .

W2-4. The magnitude of the resultant force is 1300 N and acts along the axis of the strut from A towards O. Determine the magnitude of each of the three forces acting on the strut. Set x=0 and z=5.5 m.



- The coordinates of the relevant points:

A	(0,	6,	2.5)
B	(-3,	0,	4.5)
C	(2,	0,	4)
D	(0,	0,	-5.5)

- Compute $\vec{r}_{AB}$, etc.

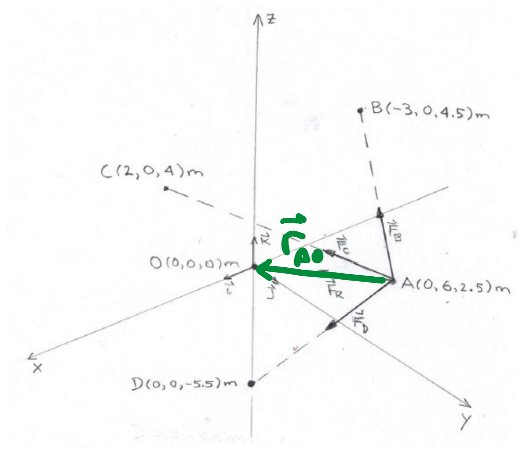
tip	tail
B	A
C	A
D	A

$\vec{F}_B \uparrow\uparrow$ $\vec{r}_{AB} = (\overset{x_B - x_A}{-3})\vec{i} + (\overset{y_B - y_A}{-6})\vec{j} + (\overset{z_B - z_A}{2})\vec{k}$

$\vec{F}_C \uparrow\uparrow$ $\vec{r}_{AC} = (2)\vec{i} + (-6)\vec{j} + (1.5)\vec{k}$

$\vec{F}_D \uparrow\uparrow$ $\vec{r}_{AD} = (0)\vec{i} + (-6)\vec{j} + (-8)\vec{k}$

W2-4. The magnitude of the resultant force is 1300 N and acts along the axis of the strut from A towards O. Determine the magnitude of each of the three forces acting on the strut. Set x=0 and z=5.5 m.



- Compute r_{AB}, etc.

$$\vec{r}_{AB} = (-3)\,\vec{i} + (-6)\,\vec{j} + (2)\,\vec{k} \qquad r_{AB} = \sqrt{(-3)^2 + (-6)^2 + (2)^2} = 7$$

$$\vec{r}_{AC} = (2)\,\vec{i} + (-6)\,\vec{j} + (1.5)\,\vec{k} \qquad r_{AC} = \sqrt{(2)^2 + (-6)^2 + (1.5)^2} = 6.5$$

$$\vec{r}_{AD} = (0)\,\vec{i} + (-6)\,\vec{j} + (-8)\,\vec{k} \qquad r_{AD} = \sqrt{0^2 + (-6)^2 + (-8)^2} = 10$$

- Compute forces & introduce B, C, D:

$$\vec{F}_B = \overbrace{\left(\frac{F_B}{r_{AB}}\right)}^{B}\vec{r}_{AB} = B\left((-3)\vec{i} + (-6)\vec{j} + (2)\vec{k}\right)$$

$$\vec{F}_C = \overbrace{\left(\frac{F_C}{r_{AC}}\right)}^{C}\vec{r}_{AC} = C\left((2)\vec{i} + (-6)\vec{j} + (1.5)\vec{k}\right)$$

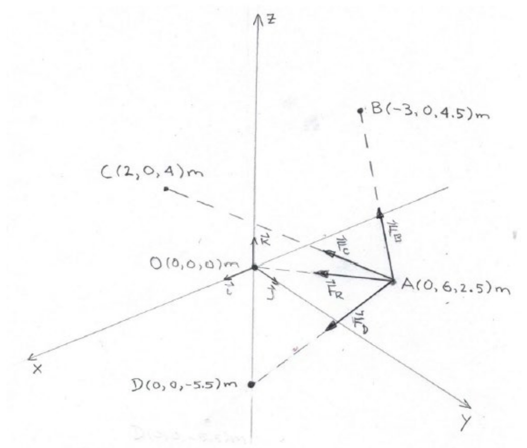
$$\vec{F}_D = \overbrace{\left(\frac{F_D}{r_{AD}}\right)}^{D}\vec{r}_{AD} = D\left((0)\vec{i} + (-6)\vec{j} + (-8)\vec{k}\right)$$

our forces in Cartesian components!

$$\underbrace{\vec{F}_B + \vec{F}_C + \vec{F}_D}_{} = \underbrace{\vec{F}_R}_{}$$

?

W2-4. The magnitude of the resultant force is 1300 N and acts along the axis of the strut from A towards O. Determine the magnitude of each of the three forces acting on the strut. Set x=0 and z=5.5 m.



- We know that the resultant of these three forces acts along AO and $F_R = 1300$ N. Let's write $\vec{F}_R$ in Cartesian components.
- A(0, 6, 2.5) & O(0,0,0)

 tail tip

$$\vec{r}_{AO} = \vec{i}\,(0-0) + \vec{j}\,(0-6) + \vec{k}\,(0-2.5)$$

- $\vec{r}_{AO} = \vec{i}\,(0) + \vec{j}\,(-6) + \vec{k}\,(-2.5)$
- $r_{AO} = \sqrt{(0)^2 + (-6)^2 + (-2.5)^2} = 6.5$

200

$$\vec{F}_R = \frac{F_R}{r_{AO}}\,\vec{r}_{AO} = \left(\frac{1300}{6.5}\right)\left[(0)\vec{i} + (-6)\vec{j} + (-2.5)\vec{k}\right]$$

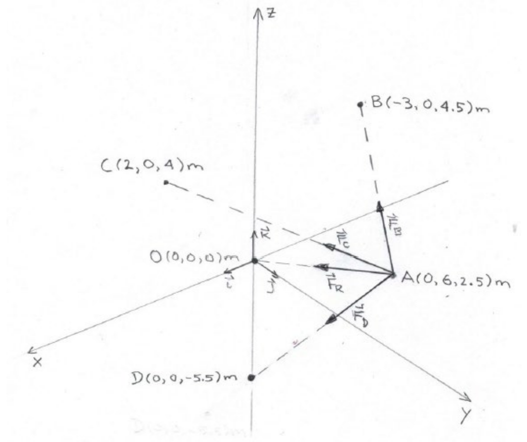
$$200\left[(0)\vec{i} + (-6)\vec{j} + (-2.5)\vec{k}\right]$$

$$= 0(\vec{i}) - 1200\vec{j} - 500\vec{k}$$

$$= (0)\,\vec{i} + (-1200)\,\vec{j} + (-500)\,\vec{k} \longrightarrow$$

$\vec{F}_R$ in Cartesian components!

W2-4. The magnitude of the resultant force is 1300 N and acts along the axis of the strut from A towards O. Determine the magnitude of each of the three forces acting on the strut. Set x=0 and z=5.5 m.



- Almost there! What remains: Find the force magnitudes F_B, F_C, F_D.

$$\vec{F}_B = B\,[(-3)\,\vec{i} + (-6)\,\vec{j} + (2)\,\vec{k}] \qquad \frac{F_B}{r_{AB}} = B = F_B/7 \longrightarrow F_B = 7B$$

$$\vec{F}_C = C\,[(2)\,\vec{i} + (-6)\,\vec{j} + (1.5)\,\vec{k}] \qquad \frac{F_C}{r_{AC}} = C = F_C/6.5 \qquad F_C = 6.5C$$

$$\vec{F}_D = D\,[(0)\,\vec{i} + (-6)\,\vec{j} + (-8)\,\vec{k}] \qquad \frac{F_D}{r_{AD}} = D = F_D/10 \qquad F_D = 10D$$

$$\vec{F}_R = 200\,[(0)\,\vec{i} + (-6)\,\vec{j} + (-2.5)\,\vec{k}]$$

$$\left.\begin{aligned} x&: -3B + 2C + 0D = 0 \\ y&: -6B - 6C - 6D = -1200 \\ z&: 2B + 1.5C - 8D = -500 \end{aligned}\right\}$$

$B = 45.36$

$C = 68.04$

$D = 86.60$

$$\boxed{\begin{aligned} F_B &= 318\text{ N} \\ F_C &= 442\text{N} \\ F_D &= 866\text{ N} \end{aligned}}$$

not forget!

Comment: Solving systems of linear equations

- General rule: the number of equations should be equal to the number of unknowns.
- Write the equations in the standard form, e.g.:

$$\begin{aligned} -3x + 2y &= 0 \\ -6x - 6y - 6z &= -1200 \\ 2x + 1.5y - 8z &= -500 \end{aligned}$$

- Here x, y, z are **variables**, everything else are coefficients. Collect them in a matrix, with the first column being x-coefficients, etc., and the last column being the right-hand side coefficients:

$$M = \begin{bmatrix} -3 & 2 & 0 & 0 \\ -6 & -6 & -6 & -1200 \\ 2 & 1.5 & -8 & -500 \end{bmatrix}$$

Each **row** of this matrix corresponds to one equation; row elements must be entered consistently (i.e. *x* coefficients in first column, *y* coefficients in second etc.); right-hand side of the equation is always the last element of the row.

Reduced Row Echelon Form method produces a series of linear operations to transform this matrix to another form:

$$M = \begin{bmatrix} 1 & 0 & 0 & d_1 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 1 & d_3 \end{bmatrix}$$

from which you will get: $x = d_1, \quad y = d_2, \quad z = d_3$. In our numerical example, $d_1 = x = 45.36, \quad d_2 = y = 68.04, \quad d_3 = 86.60$

- Worked example of solving a 3x3 system using **rref** on Canvas: “Additional information”→TI-3x3-solve-example.pdf”
- Alternative (<u>not for the exams!!!</u>): Use numerical solver of a system of linear equations