PHYSICS 158 HW-10 – due Wednesday, Apr 3rd at 11:59pm

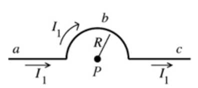
You should upload your work to Gradescope using the link on the CANVAS website. Please indicate the page # on which you start each problem. Use a printer/scanner or a scanning app on your phone to reduce the file size. Make sure that it is readable so that it can be graded. Label your file with your **ActID# -- Last Name**. Also please write your **ActID# -- Last Name** along with your **Tutorial #, Date** at the top of each page. Use the physical or numerical constants from your book unless the value is given in the problem. **Please circle all your answers.**

The written HW is designed to help you develop your problem solving and presentation skills. You must include a diagram or sketch (for every problem). You do not need to write out all the words for the problem -- you should use your diagram to define your variables. You must include a brief statement indicating what the problem has asked you to calculate. You can copy and paste the statement of the problem into your work, if you like. Your TA will grade two problems in detail (5 points each). The other problems will be given either 0 or 1 point (for a good effort). You could lose 1 point overall if your submission is **NOT** neat.

1. (Difficulty: *) (a) Calculate the magnitude & direction of the magnetic field at the point P in terms of $R, I_1, and\ I_2$.
(b) Evaluate for the case when $I_1 = I_2$.

IDENTIFY: Calculate the magnetic field vector produced by each wire and add these fields to get the total field.

SET UP: First consider the field at P produced by the current I_1 in the upper semicircle of wire. See Figure 28.33a.

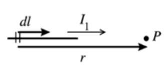


Consider the three parts of this wire:
a: long straight section
b: semicircle
c: long, straight section

Figure 28.33a

Apply the Biot-Savart law $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{l} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I d\vec{l} \times \vec{r}}{r^3}$ to each piece.

EXECUTE: Part a: See Figure 28.33b.

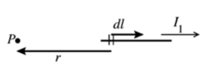


$d\vec{l} \times \vec{r} = 0$,
so $dB = 0$.

Figure 28.33b

The same is true for all the infinitesimal segments that make up this piece of the wire, so $B = 0$ for this piece.

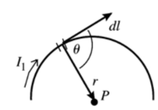
Part c: See Figure 28.33c.



$d\vec{l} \times \vec{r} = 0$,
so $dB = 0$ and $B = 0$ for this piece.

Figure 28.33c

Part b: See Figure 28.33d.



$d\vec{l} \times \vec{r}$ is directed into the paper for all infinitesimal segments that make up this semicircular piece, so $\vec{B}$ is directed into the paper and $B = \int dB$ (the vector sum of the $d\vec{B}$ is obtained by adding their magnitudes since they are in the same direction).

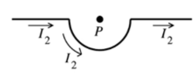
Figure 28.33d

$|d\vec{l} \times \vec{r}| = r dl \sin\theta$. The angle θ between $d\vec{l}$ and $\vec{r}$ is 90° and $r = R$, the radius of the semicircle. Thus $|d\vec{l} \times \vec{r}| = R\,dl$.

$$dB = \frac{\mu_0}{4\pi}\frac{I|d\vec{l} \times \vec{r}|}{r^3} = \frac{\mu_0 I_1}{4\pi}\frac{R}{R^3}dl = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)dl.$$

$$B = \int dB = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)\int dl = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)(\pi R) = \frac{\mu_0 I_1}{4R}.$$

(We used that $\int dl$ is equal to πR, the length of wire in the semicircle.) We have shown that the two straight sections make zero contribution to $\vec{B}$, so $B_1 = \mu_0 I_1/4R$ and is directed into the page.



For current in the direction shown in Figure 28.33e, a similar analysis gives $B_2 = \mu_0 I_2/4R$, out of the paper.

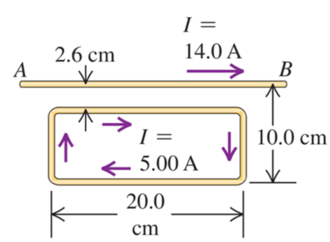
Figure 28.33e

$\vec{B}_1$ and $\vec{B}_2$ are in opposite directions, so the magnitude of the net field at P is $B = |B_1 - B_2| = \frac{\mu_0|I_1 - I_2|}{4R}$.

EVALUATE: When $I_1 = I_2$, $B = 0$.

2. (Difficulty: *) The long straight wire AB carries a current $I = 14.0\ Amps$. The rectangular loop carries a current $I = 5.0\ Amps$.

a) Calculate the magnitude and direction of the NET force acting on the loop.
b) Calculate the net Torque acting on the loop.



IDENTIFY: Consider the forces on each side of the loop.

SET UP: The forces on the left and right sides cancel. The forces on the top and bottom segments of the loop are in opposite directions, so the magnitudes subtract.

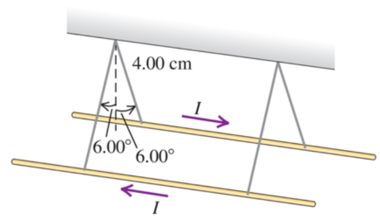
EXECUTE: $F = F_t - F_b = \left(\frac{\mu_0 I_{\text{wire}}}{2\pi}\right)\left(\frac{Il}{r_t} - \frac{Il}{r_b}\right) = \frac{\mu_0 I l I_{\text{wire}}}{2\pi}\left(\frac{1}{r_t} - \frac{1}{r_b}\right)$.

$F = \frac{\mu_0(5.00\text{ A})(0.200\text{ m})(14.0\text{ A})}{2\pi}\left(-\frac{1}{0.100\text{ m}} + \frac{1}{0.026\text{ m}}\right) = 7.97\times10^{-5}$ N. The force on the top segment is toward the wire, so the net force is toward the wire.

EVALUATE: The net force on a current loop in a uniform magnetic field is zero, but the magnetic field of the wire is not uniform; it is stronger closer to the wire.

Since both forces are in the plane of the loop the net Torque acting on the loop will be 0.

3. (Difficulty: **) Two long, parallel wires (with mass per unit length = 0.0125 kg/m) carrying equal but opposite currents are suspended from a common axis by 4.0 cm strings as shown. If each string makes an angle of 6° with the vertical, calculate the magnitude of the current.



IDENTIFY: Apply $\Sigma \vec{F} = 0$ to one of the wires. The force one wire exerts on the other depends on I so $\Sigma \vec{F} = 0$ gives two equations for the two unknowns T and I.

SET UP: The force diagram for one of the wires is given in Figure 28.61.

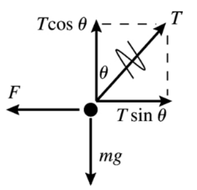


Figure 28.61

The force one wire exerts on the other is $F = \left(\dfrac{\mu_0 I^2}{2\pi r}\right)L$, where $r = 2(0.040\text{ m})\sin\theta = 8.362\times10^{-3}\text{ m}$ is the distance between the two wires.

EXECUTE: $\Sigma F_y = 0$ gives $T\cos\theta = mg$ and $T = mg/\cos\theta$.

$\Sigma F_x = 0$ gives $F = T\sin\theta = (mg/\cos\theta)\sin\theta = mg\tan\theta$.

And $m = \lambda L$, so $F = \lambda Lg\tan\theta$.

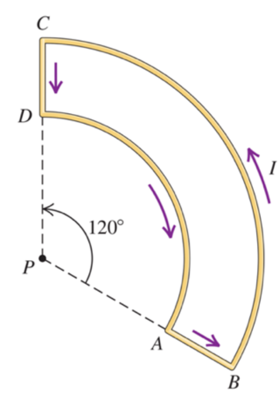
$$\left(\frac{\mu_0 I^2}{2\pi r}\right)L = \lambda Lg\tan\theta.$$

$$I = \sqrt{\frac{\lambda g r\tan\theta}{(\mu_0/2\pi)}}.$$

$$I = \sqrt{\frac{(0.0125\text{ kg/m})(9.80\text{ m/s}^2)(\tan 6.00°)(8.362\times10^{-3}\text{ m})}{2\times10^{-7}\text{T}\cdot\text{m/A}}} = 23.2\text{ A}.$$

EVALUATE: Since the currents are in opposite directions the wires repel. When I is increased, the angle θ from the vertical increases; a large current is required even for the small displacement specified in this problem.

4. Calculate the magnitude and direction of the magnetic field at point P due a current I = 12.0 A in the wire. Assume that the radii of the segments AD, BC = 20.0 cm, 30.0 cm and the length of segments AB, CD = 10.0cm.



IDENTIFY: Both arcs produce magnetic fields at point P perpendicular to the plane of the page. The field due to arc DA points into the page, and the field due to arc BC points out of the page. The field due to DA has a greater magnitude than the field due to arc BC. The net field is the sum of these two fields.

SET UP: The magnitude field at the center of a circular loop of radius a is $B = \dfrac{\mu_0 I}{2\pi a}$. Each arc is 120°/360° = 1/3 of a complete loop, so the field due to each of them is $B = \dfrac{1}{3}\dfrac{\mu_0 I}{2\pi a} = \dfrac{\mu_0 I}{6\pi a}$.

EXECUTE: The net field is

$$B_{net} = B_{20} - B_{30} = \frac{\mu_0(12.0\text{ A})}{6\pi}\left(\frac{1}{0.200\text{ m}} - \frac{1}{0.300\text{ m}}\right) = 4.19\times10^{-6}\text{ T} = 4.19\ \mu\text{T}.$$

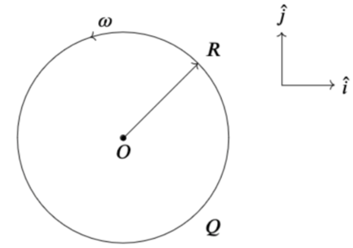
Since $B_{20} > B_{30}$, the net field points into the page at P.

EVALUATE: The current in segments CD and AB produces no magnetic field at P because its direction is directly toward (or away from) point P.

5. (Difficulty: **) A toy company is planning on rolling out a new product called the magnetic hula-hoop. Assume the hoop is made of a non-conducting material and has a radius R and a uniform charge density λ.

a) If the hoop is spun around the point O with a constant angular velocity ω, what is the magnetic field at the center of the hoop ? Remember $v = \omega R$.

b) If the speed of the ring is no longer constant, but instead is expressed by the function $\omega(t) = \omega_0 e^{-at}$ what is the direction of the induced emf in the ring ? (Please write your answer as either CW or CCW).



Solution:

a) We can recognize that current is simply the rate of change (or flow) of charge. In the case of the spinning ring we are creating a flow of charge so we can write the current as

$$I = \frac{dQ}{dt}$$

We can then use chain rule to rewrite this in terms of the variables that we have.

$$I = \frac{dQ}{dt} = \frac{dQ}{ds}\frac{ds}{dt} = \lambda \cdot v$$
$$v = R\omega \Rightarrow I = \lambda R\omega$$

We can then use the Biot-Savart law to find the magnetic field at the center of the ring.

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}\times\hat{r}}{r^2}$$
$$d\vec{l}\times\hat{r} = dl\hat{k}$$
$$dl = Rd\theta$$
$$r = R$$
$$\vec{B} = \frac{\mu_0 \lambda R\omega}{4\pi}\int_0^{2\pi}\frac{Rd\theta}{R^2}\hat{k} = \boxed{\frac{\mu_0\lambda\omega}{2}\hat{k}}$$

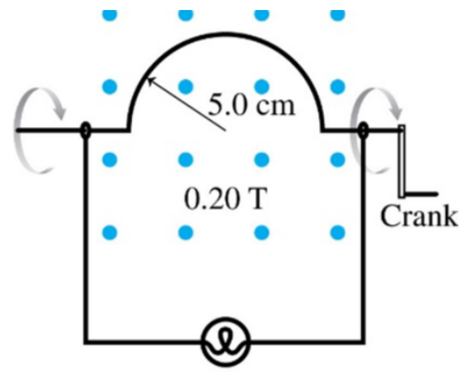
b) We found in part (a) that the magnetic field is pointing out of the page. We also found that the magnetic field is proportional to the angular velocity of the ring so as ω decreases over time then the magnetic field strength will also decrease.

$$\frac{d\vec{B}}{dt} < 0\hat{k}$$
$$\Phi_B = \vec{B}\cdot\vec{A} = BA$$
$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{dB}{dt}A > 0(\text{out of page})$$

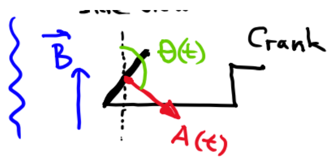
We can see that the induced emf is positive and is pointing out of the page. This means that the induced current will be flowing in the counterclockwise direction.

6. (Difficulty: **) The picture shows the main idea behind a generator. Mechanical force or torque is used to rotate a wire loop in a magnetic field. Note that here only a half loop is rotated.
a) Find an expression for the induced current as a function of time if you turn the crank with frequency f. Assume that the semicircle is at its highest point (current position) at $t = 0$ s.

b) With what frequency will you have to turn the crank to generate 4.0 W of electrical peak power ? To calculate the peak power, use the value corresponding to the amplitude of the periodic current found in part a.



a) $I(t)=\frac{\mathcal{E}(t)}{R}$; $\quad \mathcal{E}(t)=\left|\frac{d\Phi_m(t)}{dt}\right|$;



$\Phi_m(t) = BA\cos\theta(t)$

$A=\frac{\pi r^2}{2}$; $\quad \theta(t)=\frac{2\pi}{T}t = 2\pi f t$

since θ varies from 0 to 2π within time $T = 1/f$

$\Phi_m(t) = \frac{\pi r^2 B}{2}\cos(2\pi f t)$.

$$I(t) = \frac{B}{R}\frac{\pi r^2}{2}\left|\frac{d}{dt}\cos(2\pi f t)\right| = \frac{B}{R}\frac{\pi r^2}{2}\cdot 2\pi f\cdot \sin(2\pi f t) =$$

$$= \underbrace{\boxed{\frac{B\pi^2 r^2 f}{R}}}_{\equiv I_0}\sin(2\pi f t)$$

$\equiv I_0$, the amplitude of the current.

b) $P = I_0^2 R = \frac{B^2\pi^4 r^4 f^2}{R^{\cancel{2}}}\cdot\cancel{R} \Rightarrow$

$$\boxed{f=\frac{\sqrt{PR}}{B\pi^2 r^2}} \qquad f=\frac{\sqrt{4W\cdot 1\Omega}}{0.2T\cdot(3.14)^2\cdot 25\cdot 10^{-4}\,m^2} =$$

$$= 405\ Hz$$