Announcements:

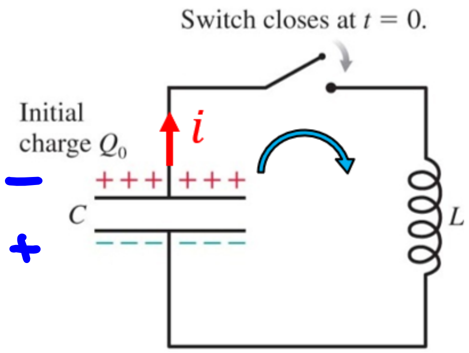
- Next Monday: 8:00 am class will have a midterm. Please wait in the lobby till ~8:55 (please wait until the TAs will open the doors and let you in)

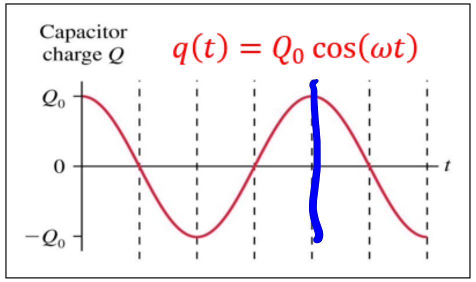
- Recordings: S201, S202 will be recorded since next Monday
  - Posted: on Fridays
  - Please use them wisely! Recordings bring more harm than benefit if:
    - Recordings replace live lectures
    - When you start thinking: “I did not understand it today, but it’s okay, I’ll watch the recording some time later”
    - Please check this information. Your decisions should be informed.

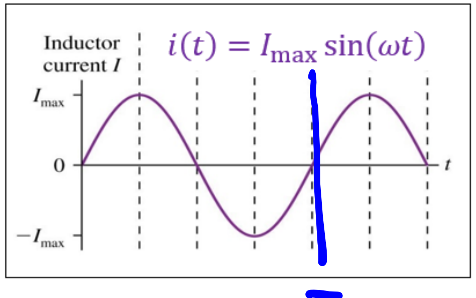
Lecture 9.

LC circuits and LCR circuits.
Energy in LC and LCR circuits.

Last Time:







T

$$U_C(t) = \frac{q(t)^2}{2C}$$

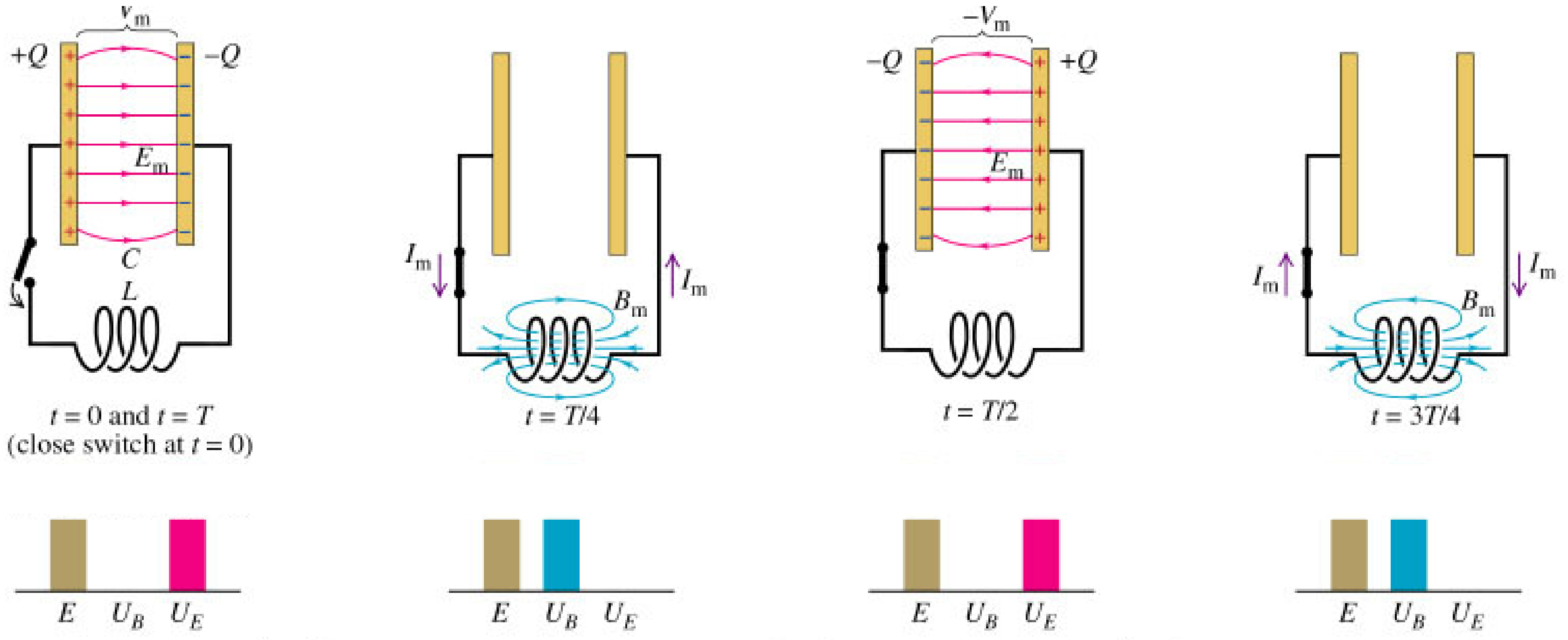
$$U_L(t) = \frac{L\, i(t)^2}{2}$$

$$\boxed{+\frac{q(t)}{C} + L\frac{d^2 q(t)}{dt^2} = 0}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\frac{Q_0}{\sqrt{LC}} = I_0$$

- Solution: $\boxed{q(t) = Q_0 \cos(\omega t)}$ $\boxed{i(t) = Q_0 \omega \sin(\omega t)}$



You’re asked to build an LC circuit that oscillates at ~10 kHz. In addition, the maximum current must be 0.1 A and the maximum energy stored in the capacitor must be 10^{-5} J. What values of inductance and capacitance must you use? (Pick the closest answer)

A. 0.2 mH and 0.1 μF
B. 2 mH and 0.1 μF
C. 2 mH and 1 μF
D. 20 mH and 1 μF
E. 20 mH and 10 μF

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- Relevant equations:

$$\omega = \frac{1}{\sqrt{LC}} = 2\pi f \qquad U_{L-C} = \frac{LI_{\max}^2}{2} = \frac{Q_0^2}{2C} \qquad I_{\max} = \frac{Q_0}{\sqrt{LC}}$$

- $L = 2\ mH\ \&\ C = 0.1\ \mu F$ give:

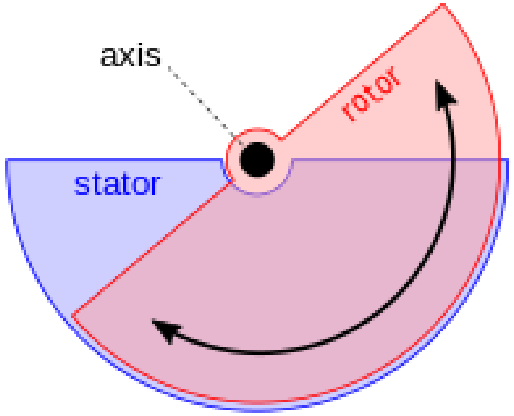
$$\omega = \frac{1}{\sqrt{LC}} = 71{,}000\,\frac{rad}{s} \quad \& \quad f = \frac{\omega}{2\pi} = 11.25\text{ kHz}$$ – the closest to 10 kHz.

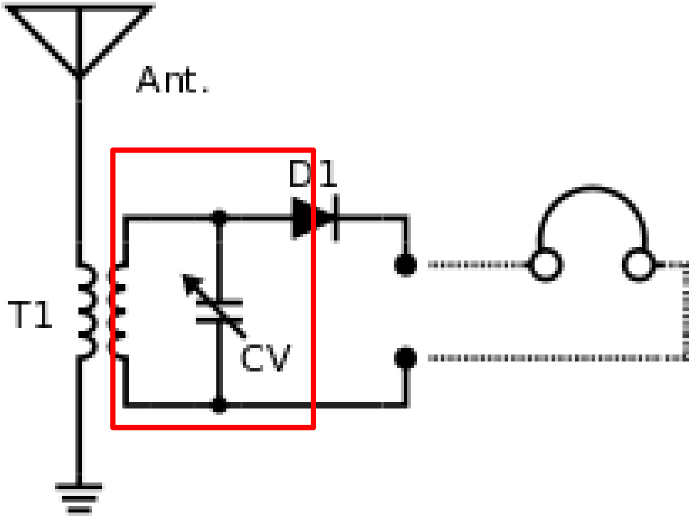
- If we use $L = 2\ mH$ and $I_{\max} = 0.1$ A:

the energy is equal to 10^{-5} J => it works!

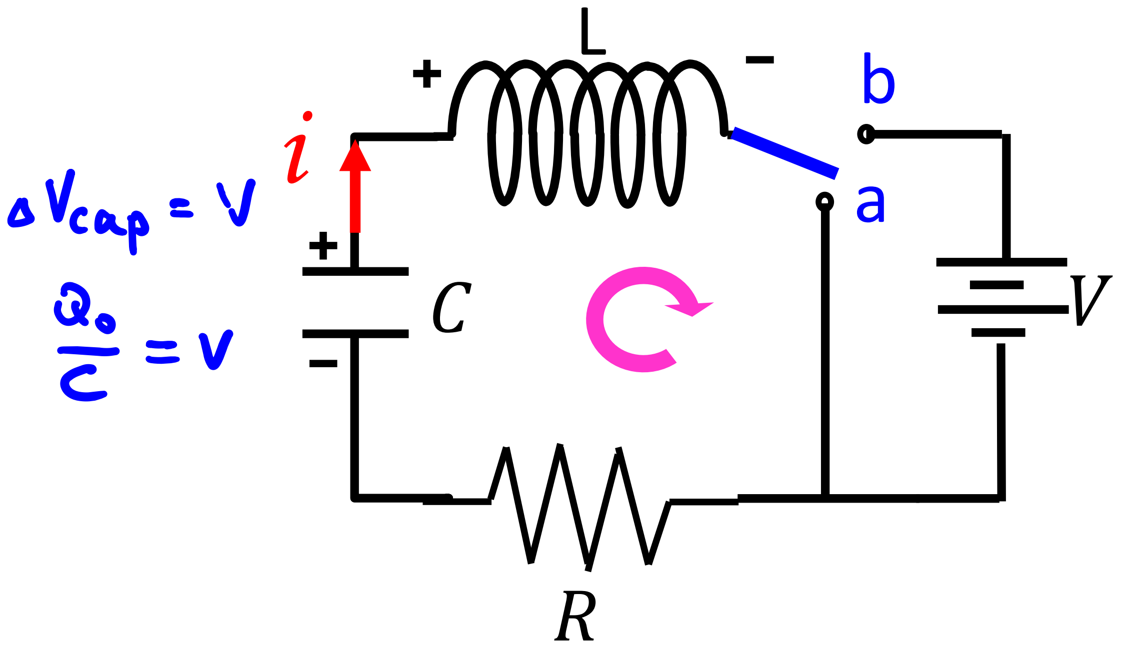
- Note that $I_{\max}$ (and hence U_{L-C}) are determined by the initial charge on the capacitor => you might need to think about how to restrict it, to not exceed maximum values

Application: Radio Receiver/Transmitter





RLC circuits



- Suppose $q(0) = Q_0$ and $i_0 = 0$ (switch connected to b for a long time)

- Then we move the switch from position "b" to "a" and discharge the capacitor (CW).

- Using Kirchhoff's loop voltage rule:

$$+\frac{q}{C} - L\frac{di}{dt} - Ri = 0$$

- Next, $i = -\frac{dq}{dt}$ (since $q \downarrow$ and $dq < 0$)

- We get:

$$\boxed{L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0}$$

equation for charge in this circuit

RLC circuits – generate damped current oscillations

$$\boxed{L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0} \quad \Rightarrow \quad \boxed{q(t) = A\,e^{-t/\tau_d}\cos(\omega' t + \phi_0)}$$

with $\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$ and $\tau_d = \frac{2L}{R}$

- It reminds us of:

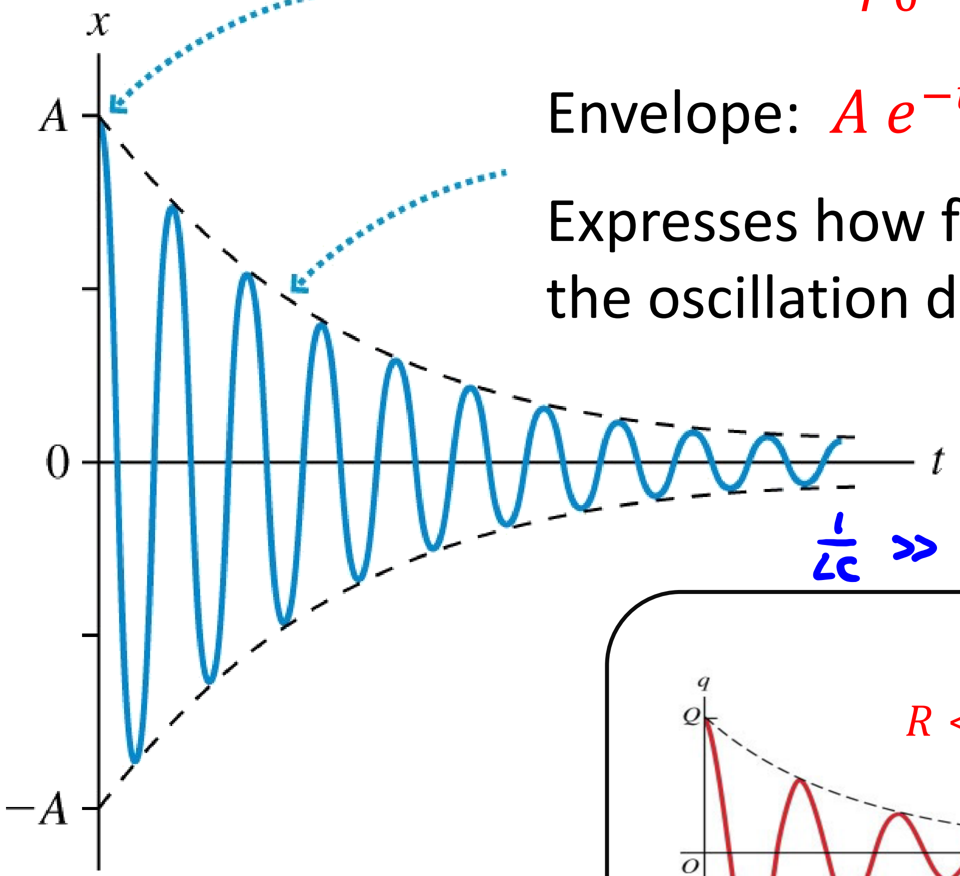
Phys 157

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \quad \Rightarrow \quad x(t) = A_0\,e^{-t/t_0}\cos(\omega' t + \phi_0)$$

with $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$ and $t_0 = \frac{2m}{b}$

- It's a damped oscillation!

Damped oscillations



Initial value: $A \cos \phi_0$

Envelope: $A\, e^{-t/\tau_d}$

Expresses how fast the oscillation decays

$$\boxed{q(t) = A\, e^{-t/\tau_d} \cos(\omega' t + \phi_0)}$$

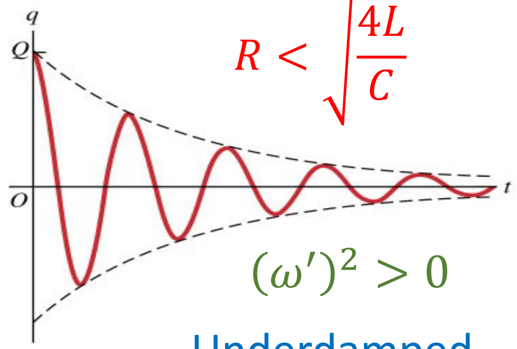
with $\quad \omega' = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}$

ideal

$\omega = \dfrac{1}{\sqrt{LC}}$

and $\quad \tau_d = \dfrac{2L}{R}$

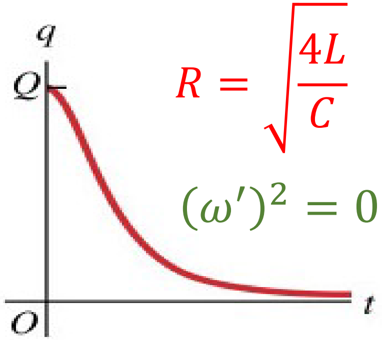
$\dfrac{1}{LC} \gg \dfrac{R^2}{4L^2}$ $\qquad \dfrac{1}{LC} = \dfrac{R^2}{4L^2}$ $\qquad \dfrac{1}{LC} \ll \dfrac{R^2}{4L^2}$



$R < \sqrt{\dfrac{4L}{C}}$

$(\omega')^2 > 0$

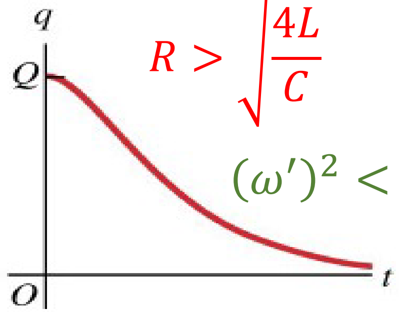
Underdamped circuit (small R)



$R = \sqrt{\dfrac{4L}{C}}$

$(\omega')^2 = 0$

Critically damped circuit (larger R)



$R > \sqrt{\dfrac{4L}{C}}$

$(\omega')^2 < 0$

Overdamped circuit (very large R)

Q: You charge the capacitor for a very long time, then move S to position 2 at $t = 0$.

a) Calculate I_L^{max} through the inductor.

b) What is the first time at which the current is maximum?

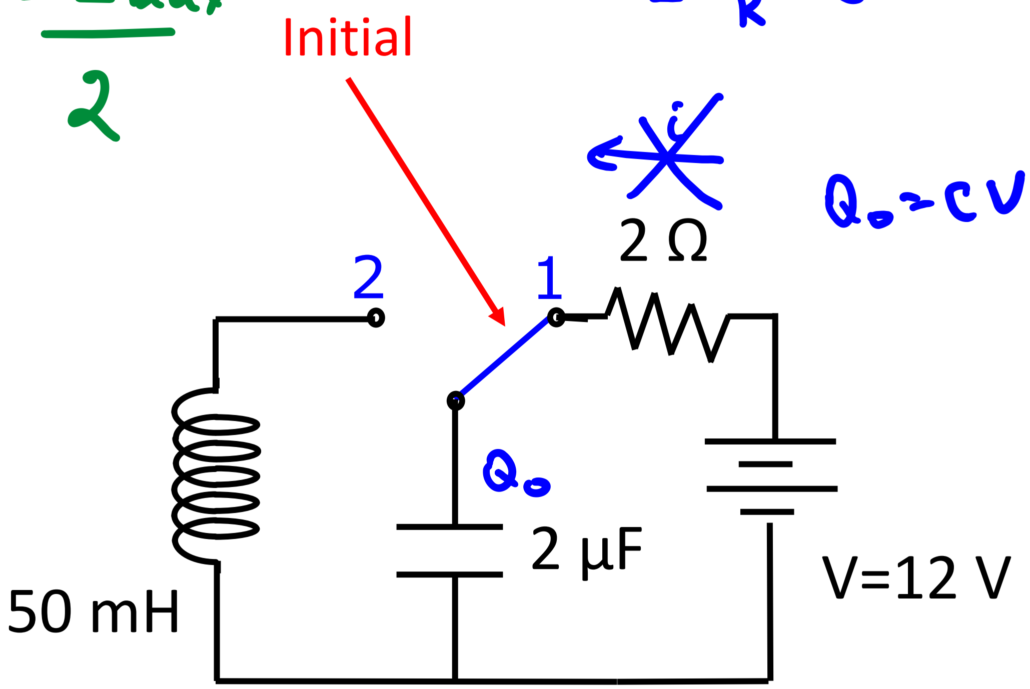
$$\frac{Q_0}{C} = \Delta V_C = V_{bat}$$

$$\Delta V_R = 0$$

$$Q_0 = CV$$

$$E_{tot} = \frac{q^2}{2C} + \frac{Li^2}{2} = \frac{Q_0^2}{2C} = \frac{L I_{max}^2}{2}$$

$$I_0 = \frac{Q_0}{\sqrt{LC}}$$



Q: You charge the capacitor for a very long time, then move S to position 2 at $t = 0$.

a) Calculate I_L^{max} through the inductor.

b) What is the first time at which the current is maximum?

- At $t = 0+$: $\Delta V_C = 12\ V$ (no current through the resistor => $\Delta V_R = 0$)

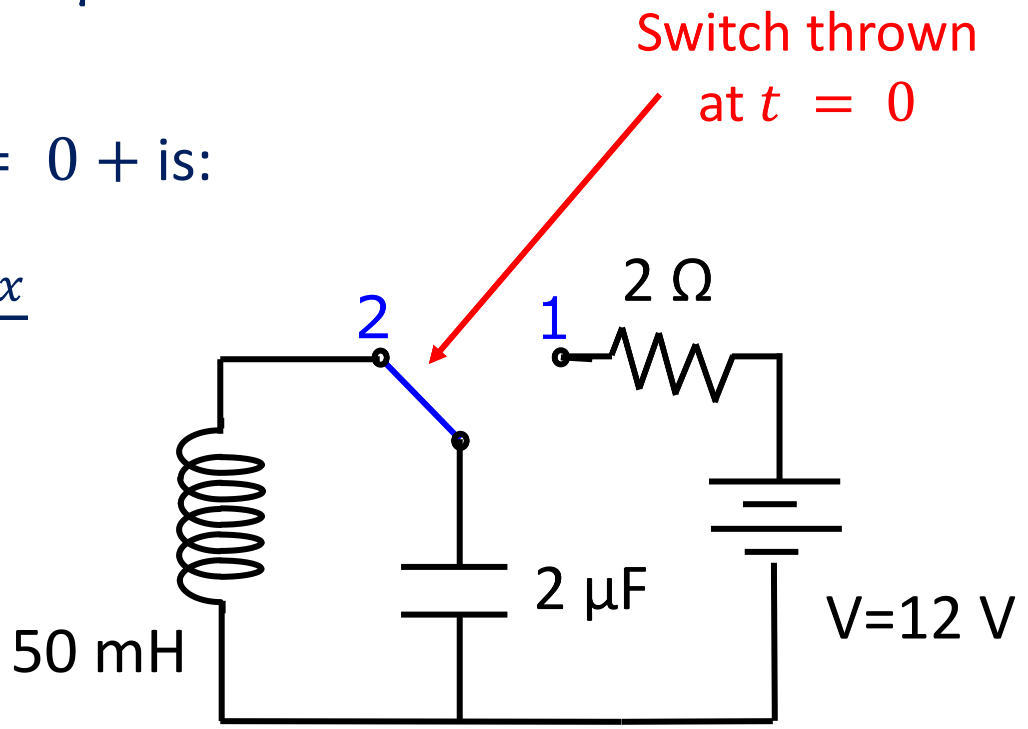
$$Q_0 = C\,\Delta V_{C,\text{initial}} = (2\mu F)(12\ V) = 24\ \mu C$$

- So, the (electric) energy stored in the capacitor at $t = 0+$ is:

$$U_C\,(t = 0+) = \frac{{Q_0}^2}{2C} = 1.44 \times 10^{-4}\,J = U_L = \frac{L I_{max}^2}{2}$$

- So the maximum current through the inductor is:

$$I_{max} = 75.9\ mA$$



Q: You charge the capacitor for a very long time, then move S to position 2 at $t = 0$.

a) Calculate I_L^{max} through the inductor.

b) What is the first time at which the current is maximum?

- The current in the inductor is maximum for the first time at:

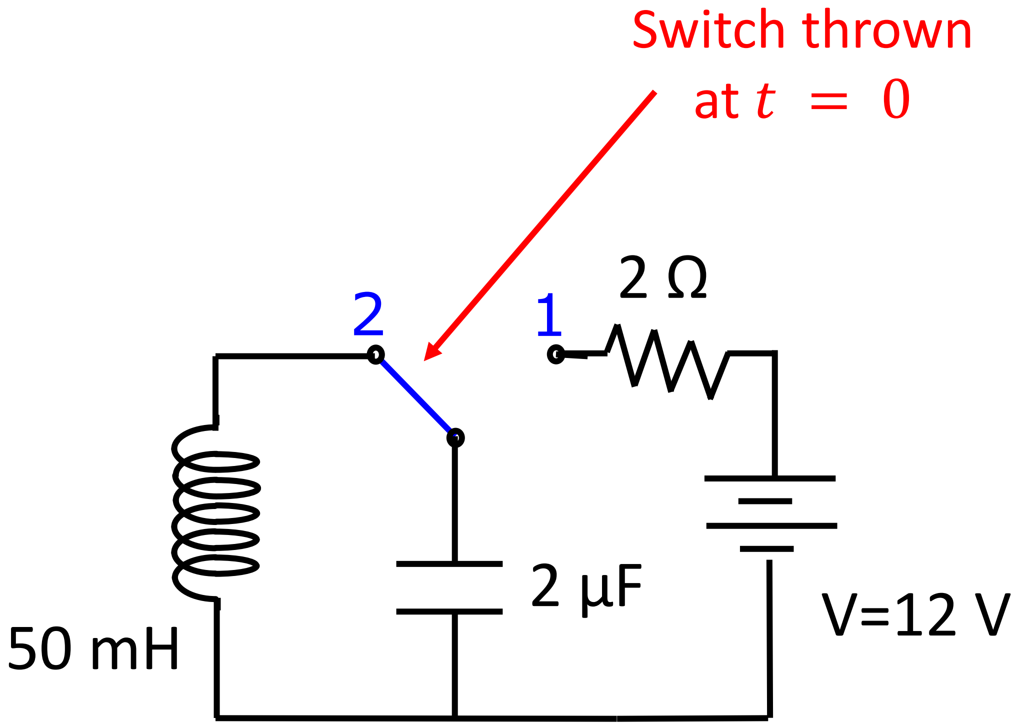
A. $t = \frac{T}{8}$

B. $t = \frac{T}{4}$

C. $t = \frac{T}{2}$

D. $t = \frac{3T}{4}$

E. $t = T$



Q: You charge the capacitor for a very long time, then move S to position 2 at $t = 0$.

a) Calculate I_L^{max} through the inductor.

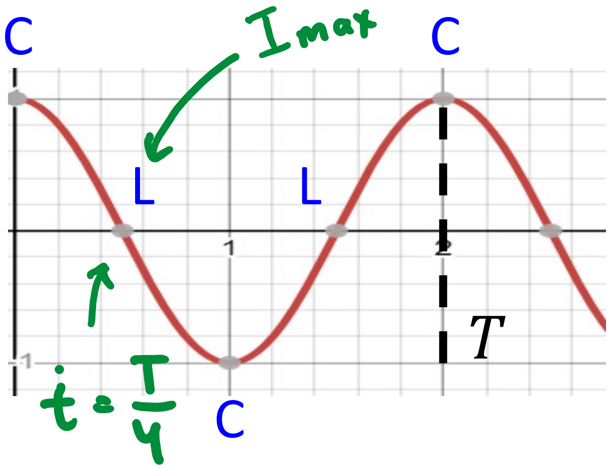
b) What is the first time at which the current is maximum?

$\omega = \frac{1}{\sqrt{LC}}$

$T = \frac{2\pi}{\omega}$

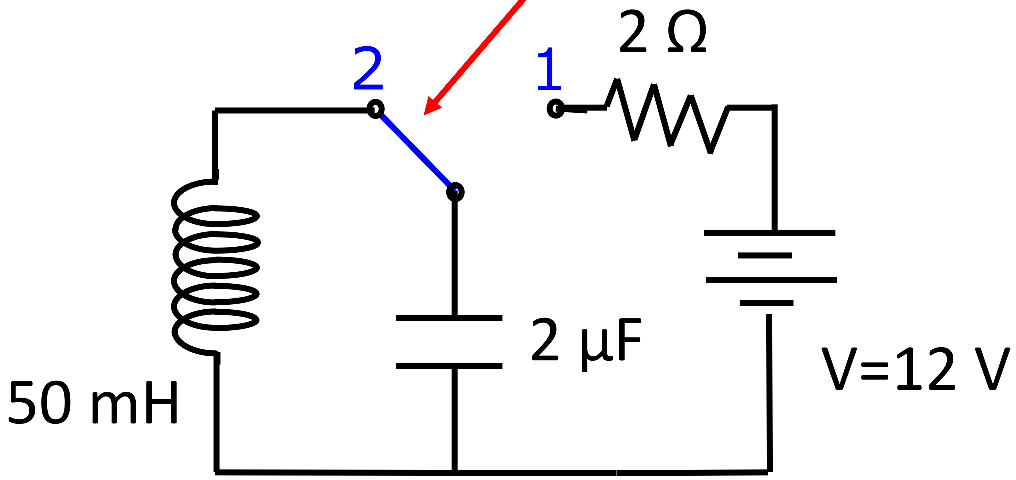
- After a time

$$t = \frac{T}{4} = \frac{1}{4} \cdot \frac{2\pi}{\omega} = \frac{\pi}{2}\sqrt{LC}$$



the energy initially stored in the capacitor is transferred to the inductor.

Switch thrown at $t = 0$



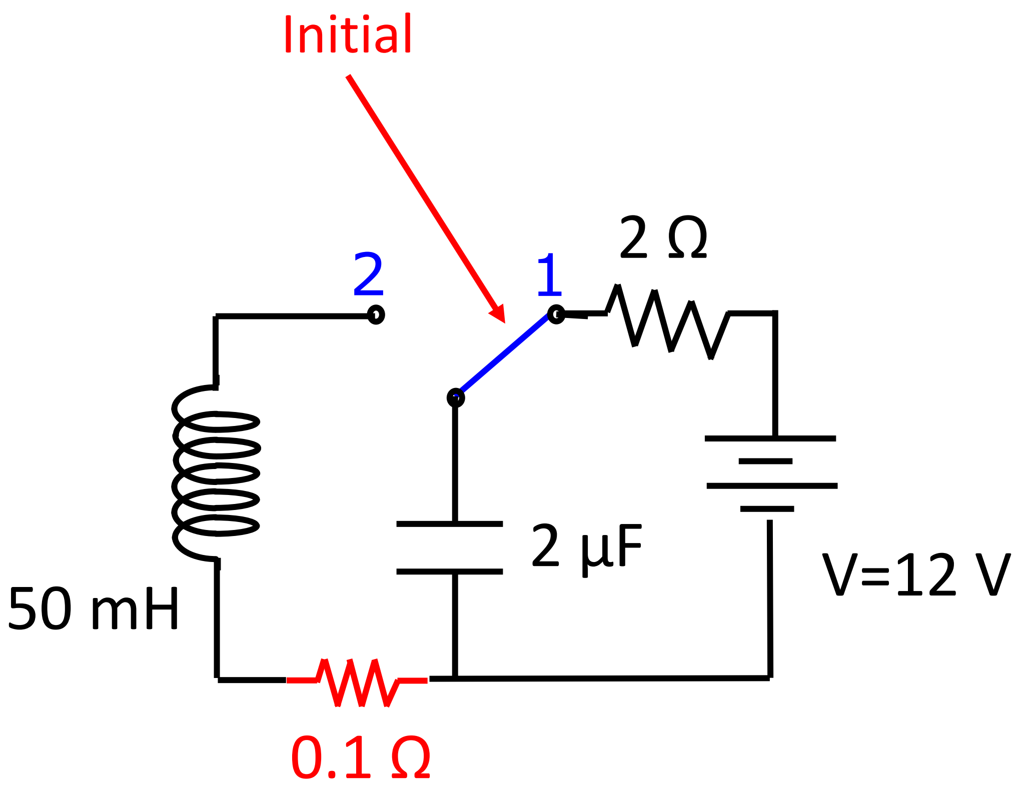
- Recall: $\omega = \frac{1}{\sqrt{LC}} = \frac{2\pi}{T}$

- Hence: $t = \frac{T}{4} = \frac{\pi}{2}\sqrt{LC} = 0.496\ ms$

Q: You charge the capacitor for a very long time, then move S to position 2 at $t = 0$.

In reality, room temperature wires always have a non-zero resistance. In this circuit, the resistance of the wires connecting L and C is R = 0.1 Ω.

Explain, in detail, how the 0.1 Ω resistance of the wire affects the behaviour of the circuit.



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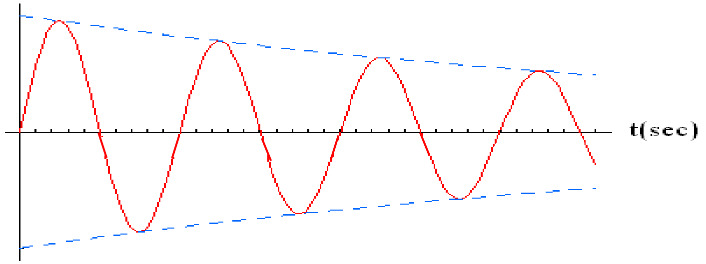
In reality, room temperature wires always have a non-zero resistance. In this circuit, the resistance of the wires connecting L and C is R = 0.1 Ω.

Explain, in detail, how the 0.1 Ω resistance of the wire affects the behaviour of the circuit.

- The small 0.1 Ω resistance in the RLC circuit will damp out the oscillations in the current.



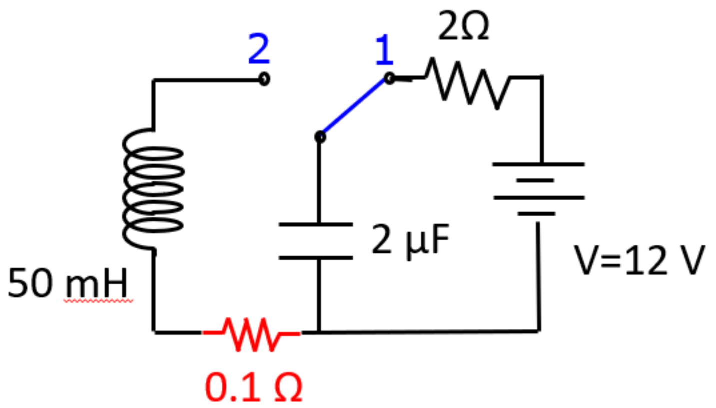
$$q(t) = Q_0 e^{-t/\tau_d} \cos(\omega' t) \qquad i(t) = \left|\frac{dq}{dt}\right| = B e^{-t/\tau_d} \sin(\omega' t)$$

- The small 0.1 Ω resistance will reduce the charge $q(t)$ on the capacitor to $1/e$ times its initial charge in a time $t = \tau_d = 2L/R = 1$ sec.

- The oscillation frequency will be lowered slightly,

  from $\omega = \sqrt{\frac{1}{LC}}$ to $\omega' = \sqrt{\frac{1}{LC} - \frac{(1\ \Omega)^2}{4L^2}}$.

- The oscillation will be under-damped: $\left(\frac{1}{\sqrt{LC}} \approx 3000\right) \gg \left(\frac{1\ \Omega}{2L} = 100\right)$



Q: At $t = 0$, the switch in the circuit below is quickly flipped from 1 to 2.

What resistance R is required to give an oscillatory frequency that is one-half the un-damped frequency?

$$\boxed{\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}$$

A. 5 Ω

B. 15 Ω

C. 25 Ω

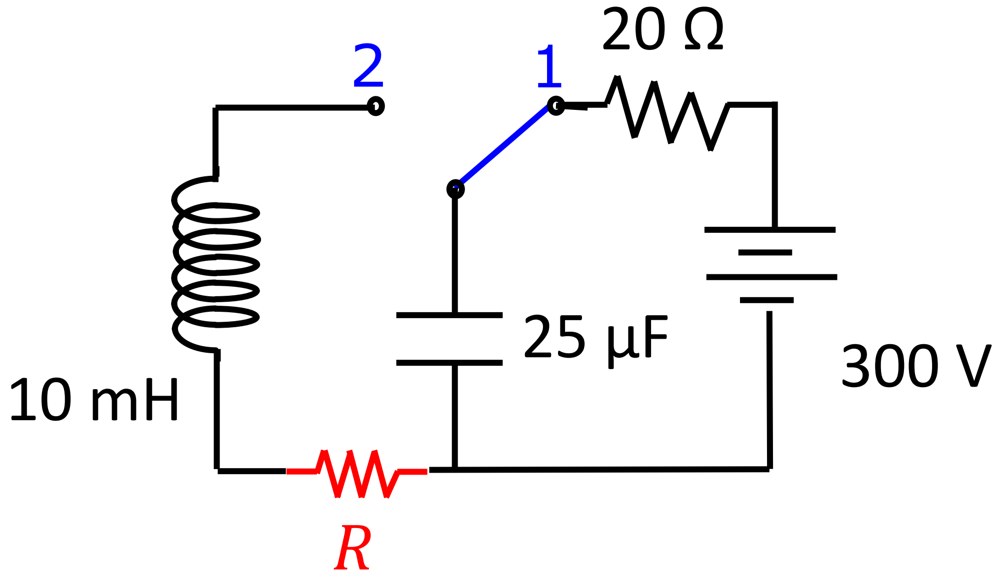
D. 35 Ω

E. 45 Ω

$$\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$\omega = 2\omega'$$



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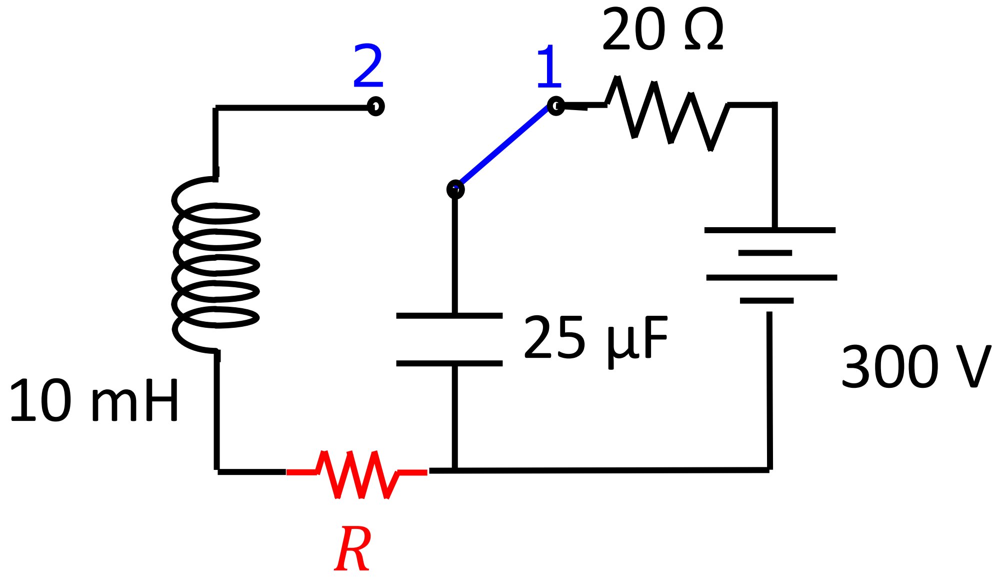
A. 5 Ω
B. 15 Ω
C. 25 Ω
D. 35 Ω
E. 45 Ω

$$\frac{\omega'}{\omega} = \frac{1}{2} \quad \Rightarrow \quad 2\omega' = \omega \quad \Rightarrow \quad 2\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = \sqrt{\frac{1}{LC}} \quad \Rightarrow$$

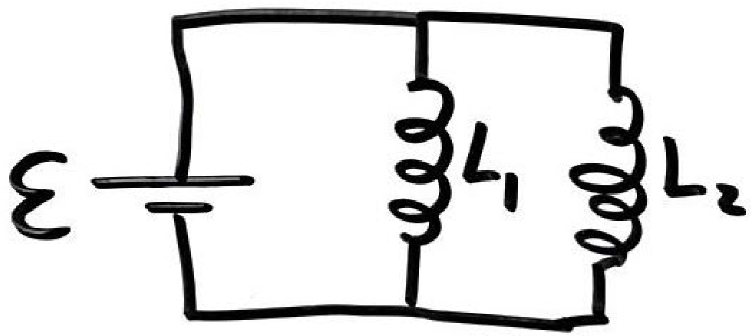
$$4\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right) = \frac{1}{LC} \quad \Rightarrow$$

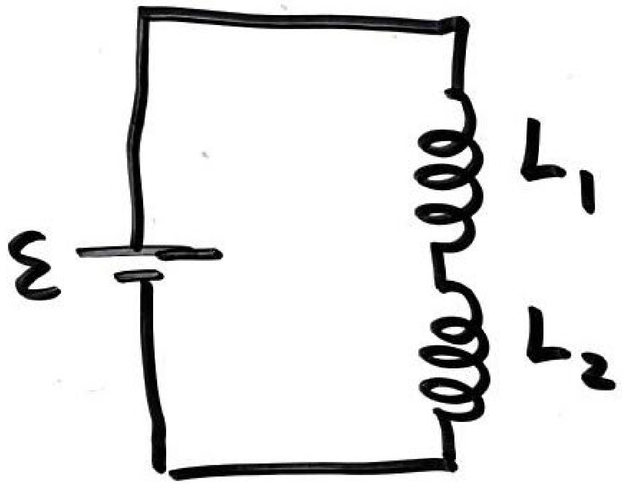
$$\frac{3}{LC} = \frac{4R^2}{4L^2} = \frac{R^2}{L^2} \quad \Rightarrow$$

$$R = \sqrt{\frac{3L}{C}} = 35\ \Omega$$



Combining Inductors





$$\boxed{\frac{1}{L_{eq,p}} = \frac{1}{L_1} + \frac{1}{L_2}}$$

$$\boxed{L_{eq,s} = L_1 + L_2}$$

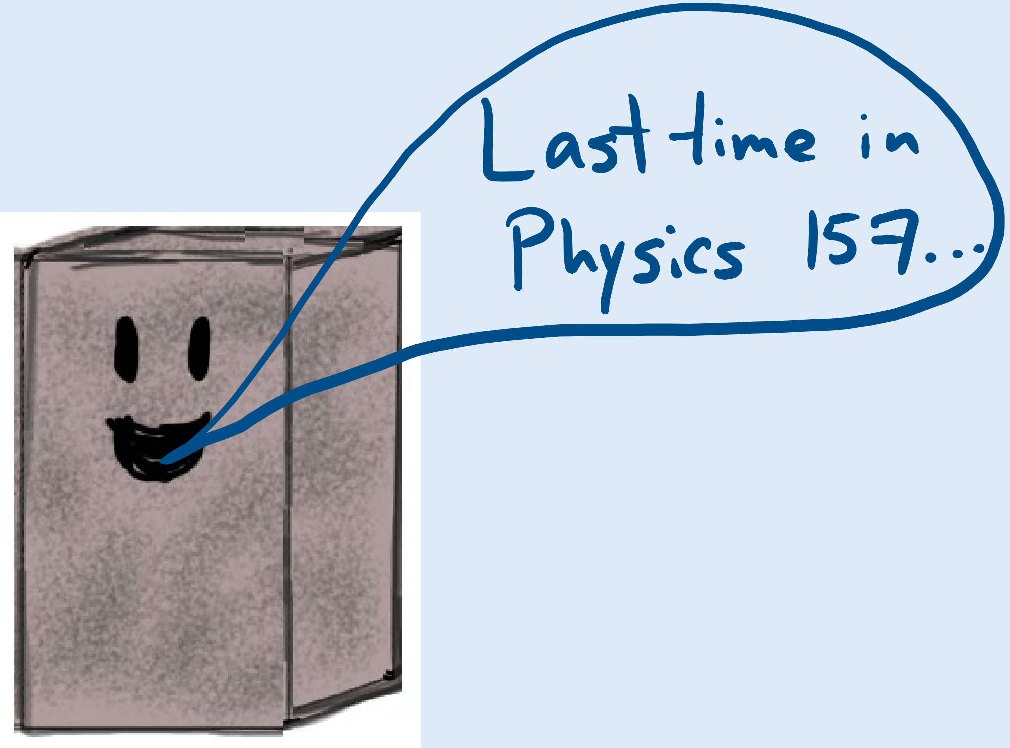
Inductors in parallel and series add just like resistors

Demo: Big Bang Capacitor

Let's wrap it up!

How many things can you see in one capacitor?

- Name 3 (or more) concepts relevant to this demo



How many things can you see in one capacitor?

- It stores electric charge! (How much?)
- It stores electric energy! (How much?)
- Where does the energy go when we shorten the circuit?
- Why do we damage it when we shorten it? (Its C dropped by 1/6 after 3 experiments!!)
- What will we get if we account for a small but finite resistance of the copper rod? How can we calculate this resistance?
- ………………………………………………………………………………

- End DC Circuits