

## Physics 157 Tutorial 3

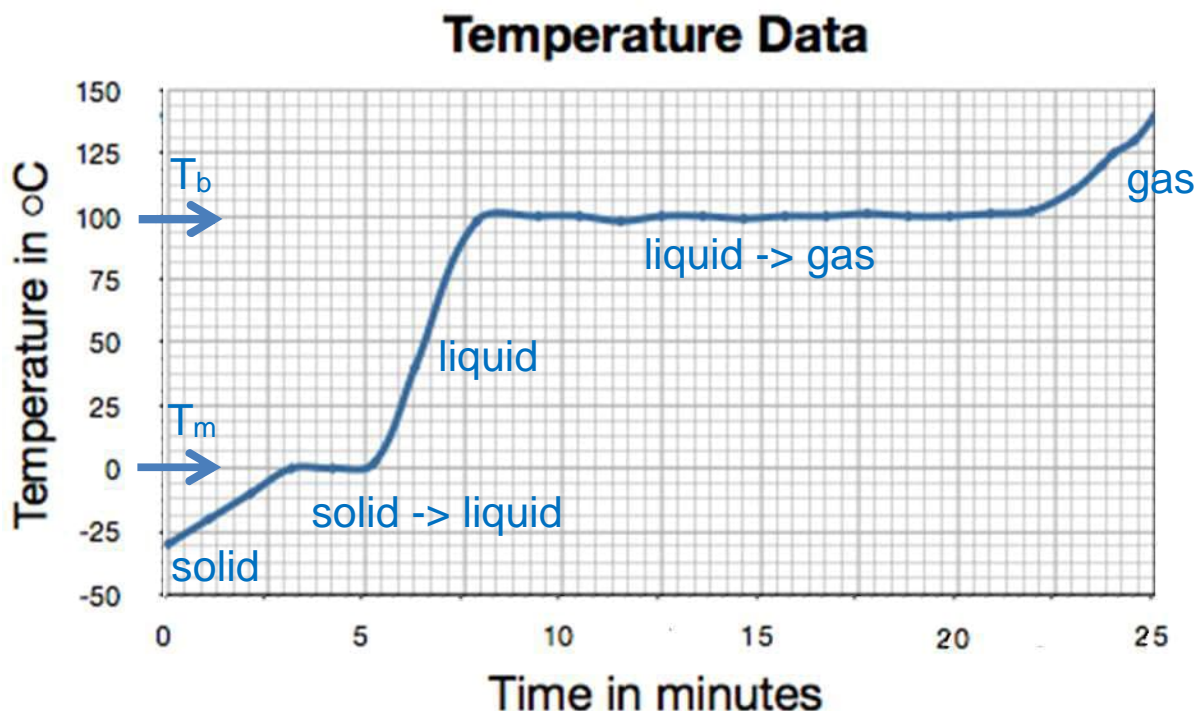
Name:

Student number:

Group member names:

In this tutorial, you will get some practice with problems involving heat transfer and how this leads to temperature changes and phase changes, as well as problems involving heat conduction. Hints and useful formulae are at the back.

**Question 1:** Exhausted after a challenging week of classes, you decide to relax by heading out to a restaurant with your friends. Soon after ordering, a server comes by and places a glass filled with a mysterious transparent liquid in front of you on the table. “What’s going on?” you think to yourself. “I didn’t order that. Are they going to charge me for that? Is someone trying to poison me?” You don’t want to cause a major scene, so you decide to remain quiet about this very disturbing development. However, when nobody is looking, you furtively transfer the liquid to a thermos for later analysis. You place it in the freezer when you get home to preserve the sample. Later, you measure the mass of the sample to be 200 g, and with the material in a sealed container, you add heat at a constant rate of 500 W ( $1\text{W} = 1\text{ J/s}$ ), measuring the temperature as a function of time. Your data is below:



a) Label each part of the graph above, indicating the phase of the material and whether a phase change is occurring. What are the melting and boiling temperatures?

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b) How much heat flows into the material between 10 and 14 minutes?  
*See the hint on the last page if you are stuck.*

Power is  $500 \text{ W} = 500 \text{ J/s}$ . Heat is  $500 \text{ J/s} \times 240 \text{ s} = 120,000 \text{ J}$

c) Approximately what is the specific heat in the liquid phase, in  $\text{J/kg}\cdot\text{K}$ ?

Have  $Q = m c \Delta T$ , so  $C = Q/(m \Delta T)$ .

For  $\Delta T = 100 \text{ K}$ , we have  $Q = 150 \text{ s} \times 500 \text{ W} = 75,000 \text{ J}$ .

Also,  $m = 0.2 \text{ kg}$ , so  $c = 75,000 \text{ J} / (0.2 \text{ kg} \times 100 \text{ K}) \sim 3.75 \times 10^3 \text{ J/kg}\cdot\text{K}$

d) Compared to this, is the specific heat for the solid phase smaller or larger?

Smaller slope, so larger specific heat (more  $Q$  for same  $\Delta T$ )

e) What is the latent heat of vaporization for this material?

Have  $L = Q_{\text{vap}}/m$  where  $Q = 14 \text{ min} \times 60 \text{ s/min} \times 500 \text{ W} = 420,000 \text{ J}$

So,  $L = 2.1 \times 10^6 \text{ J/kg}$

f) Compared to this, is the latent heat of fusion smaller or larger?

Smaller (less  $Q$  for same mass)

g) You begin to suspect the mysterious liquid may be water. Are the values you calculated consistent with this?

For water,  $T_m = 0 \text{ }^\circ\text{C}$ ,  $T_b = 100 \text{ }^\circ\text{C}$ ,  $c_{\text{liq}} = 4190 \text{ J/kg}\cdot\text{K}$  and  $L_v = 2.256 \times 10^6 \text{ J/kg}$ . These are quite close to the values estimated from the graph, so the mysterious liquid is likely to be water.

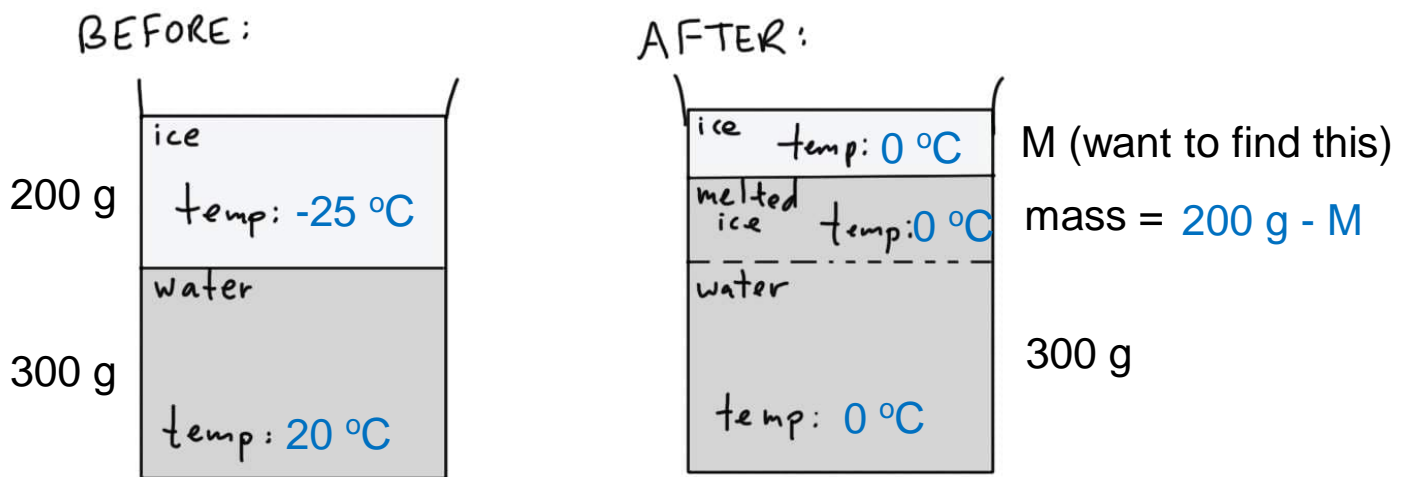
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**Question 2:** In this question, we'll work through a problem that's very similar to a number of past midterm and exam questions:

Suppose you start with 200 g of ice at temperature  $-25\text{ }^{\circ}\text{C}$ , and place this into 300 g of tap water, initially at  $20\text{ }^{\circ}\text{C}$ . Eventually the drink reaches equilibrium. What mass (if any) of ice is left?

Note: You can neglect the heat capacity of the cup and heat exchange with the environment. Useful constants:  $c_{\text{ice}} = 2100\text{ J/kg}\cdot\text{K}$ ,  $c_{\text{water}} = 4190\text{ J/kg}\cdot\text{K}$ ,  $L_{\text{fusion-water}} = 334,000\text{ J/kg}$ .

A good first step is to draw before and after pictures to help you visualize the situation, and label them with known and unknown quantities. For now, we will assume that there is ice left at the end:



a) Assuming that some ice is left at the end, fill in the temperatures on the diagram above, as well as the mass of the ice that has melted. (*hint at the back*)

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To analyze this, a good start is to treat each part separately and understand the net heat added to each part during the process. Q will be negative for parts that cool. (*hint at the back*)

b) For the water initially present, what is Q for the process (answer in Joules):

$$Q_{\text{water}} = m_{\text{water}} c_{\text{water}} \Delta T_{\text{water}} = 0.3 \text{ kg} \times 4190 \times (-20) = -25,140 \text{ J}$$

How much heat is added to the ice up to the time it starts melting?

$$Q_{\text{ice, warming}} = m_{\text{ice}} c_{\text{ice}} \Delta T_{\text{ice}} = 0.2 \text{ kg} \times 2100 \times 25 = 10,500 \text{ J}$$

How much heat is added to the ice that melts? *Hint: what is the appropriate mass to use here?*

$$Q_{\text{ice, melting}} = m_{\text{melted}} L_f = (0.2 \text{ kg} - M) \times 334,000 \text{ J/kg}$$

c) What must be the sum of the quantities in part b? Why?

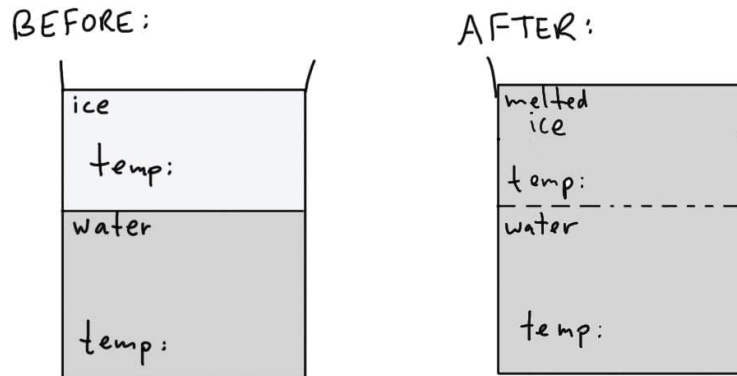
Zero because of energy conservation.

d) Using the idea from c, and your results from b, write down an equation and solve it for the mass M

$$\begin{aligned} -25,140 \text{ J} + 10,500 \text{ J} + 66,800 \text{ J} - M \times 334,000 \text{ J/kg} &= 0 \\ \rightarrow M &= 0.156 \text{ kg} \end{aligned}$$

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If it happened that your result was negative, that would tell us that there is actually no ice left over, and the final temperature will be greater than zero. In this case, the problem is actually simpler, since we can start with the following picture:



e) As an example of this (if you have time), work out the final temperature of water if we start with 50 g of ice at  $-25\text{ }^{\circ}\text{C}$  and 450 g of water at  $20\text{ }^{\circ}\text{C}$ . Use similar steps to what you did above, and show your work.

*Hints: In this case, you have an unknown variable  $T$  (the final temperature) and you should have three contributions to the heat for the melted ice component.*

Water:  $Q_{\text{water}} = m_{\text{water}} \times c_{\text{water}} \times (T - 20)$

Ice:  $Q_{\text{ice}} = m_{\text{ice}} c_{\text{ice}} \times 25 + m_{\text{ice}} L_f + m_{\text{ice}} c_{\text{water}} T$

$Q_w + Q_{\text{ice}} = 0$

$(m_{\text{water}} + m_{\text{ice}}) c_{\text{water}} T = 20 m_{\text{water}} c_{\text{water}} - 25 m_{\text{ice}} c_{\text{ice}} - m_{\text{ice}} L_f$

$2095 \text{ J/K} \times T = 18385 \text{ J}$

$T = 8.8\text{ }^{\circ}\text{C}$

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**Question 3:** You're feeling hungry once again. Still unsettled by a previous restaurant experience, and trying to save some money, you decide to make some pasta at home. You put 2 L of water into an aluminum pot with a base of area  $100 \text{ cm}^2$  and thickness  $0.4 \text{ cm}$ , put it on the stove, and turn on the heat. You start browsing Netflix, and just as you begin to watch an episode of your favorite vampire romance series, the water starts boiling. Unfortunately, the steam from your pot is nothing compared to the steaminess of the opening scene, and you become completely absorbed in the episode, forgetting about the water, the pasta, and even your hunger. It's not until the closing credits roll, 45 minutes later, that you remember the pot on your stove, and just at that moment, your smoke alarm sounds since the water has just boiled away and the (somewhat dirty) pot has started to smoke.

What was the temperature of the burner under your pot?

(constants:  $k_{Al} = 205 \text{ W/m}\cdot\text{K}$ ,  $L_v = 2256 \times 10^3 \text{ J/kg}$ )

*If you are stuck, look at the hints on the last page and answer those questions one at a time.*

The heat required to boil off all the water is:

$$Q = mL = 2 \text{ L} \times 1 \text{ kg/L} \times 2256 \times 10^3 \text{ J/kg} = 4.51 \times 10^6 \text{ J}$$

This takes 45 minutes, so the heat current into the pot is:

$$H = Q / \Delta t = 4.51 \times 10^6 \text{ J} / (45 \text{ min} \times 60 \text{ s/min}) = 1.67 \times 10^3 \text{ J/s}$$

The heat current is related to the temperature difference by:

$$H = k A (T_H - T_C) / L$$

Here  $T_C = 100 \text{ }^\circ\text{C}$ ,  $L$  is  $4 \times 10^{-3} \text{ m}$ ,  $A = 0.01 \text{ m}^2$ ,  $k = 205 \text{ W/m}\cdot\text{K}$ , and  $T_H$  is what we want to find. Rearranging, we get:

$$T_H = H L / (k \cdot A) + T_C = 103.3 \text{ }^\circ\text{C}$$

NOTE: This is lower than the actual temperature since we have assumed that all the heat from the burner goes into the water.

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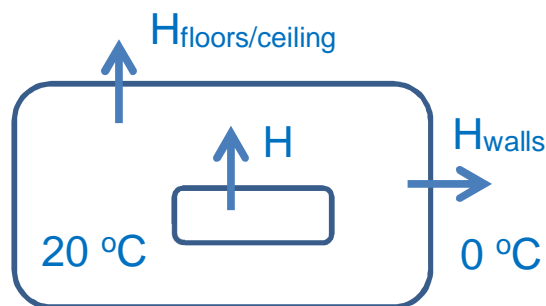
**Question 4:** A box-shaped single-story home has a height of 4 m and horizontal dimensions 10 m by 10 m. The insulation layer around the house has an R-value of 15 for the walls and 20 for the floor and ceiling. Estimate the power of a furnace that would be required to keep this house at 20 °C if the outside temperature is 0 °C.

(Recall that the R-value is related to the thickness  $L$  and conductivity  $k$  of an insulation layer by  $R = L/k$ . The R-value is this quantity in units  $\text{ft}^2 \cdot \text{F}^\circ \cdot \text{hours}/\text{BTU}$ . To find  $L/k$  in SI units from the R value, we multiply by 0.1761. For example, an R value of 10 means that  $L/k$  is  $1.761 \text{ m}^2 \cdot \text{K}/\text{W}$ . The ratio  $k/L$  that appears in the equation for heat current is just the inverse of this).

*If you are stuck, look at the hints on the last page and answer those questions one at a time.*

Since the temperature inside the house is remaining constant, the heat flowing out of the house must equal the heat from the furnace.

The heat flowing out is the sum of the heat through the walls and the heat through the floors/ceiling. So



$$H_{\text{furnace}} = 4 \times H_{\text{wall}} + 2 \times H_{\text{ceiling}}$$

The area of a wall is  $4 \times 10 = 40 \text{ m}^2$ . The area of the ceiling is  $100 \text{ m}^2$ . We have:

$$H_{\text{wall}} = k A_{\text{wall}} (T_H - T_C) / L = A_{\text{wall}} (T_H - T_C) / R_{\text{wall}} = 40 \times 20 / (15 \times 0.1761) \text{ J/s} = 303 \text{ J/s}$$

$$\text{Also, } H_{\text{ceiling}} = A_{\text{ceiling}} (T_H - T_C) / R_{\text{ceiling}} = 568 \text{ J/s}$$

So we need

$$H_{\text{furnace}} = (4 \times 303 + 2 \times 568) \text{ W} = 2348 \text{ W} \text{ (assuming there are no leaks, open windows, etc!)}$$

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### Useful formulae:

Relation between heat  $Q$  flowing in to a material and temperature change ( $c$  = specific heat):

$$Q = m c \Delta T$$

Relation between heat flowing in to a material and amount of mass changing phase ( $L$  = latent heat for that phase change):

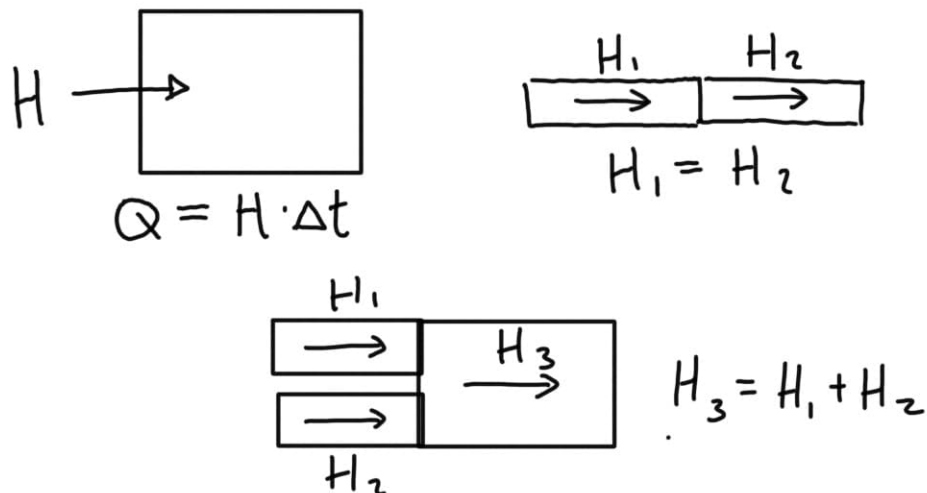
$$Q = m L$$

### Conduction problem tips:

As usual, it's helpful to draw a picture and label it, and break the system into parts. For the part or parts of the system through which heat is flowing, you will need the basic relation:

$$H = k A (T_H - T_C) / L$$

Finally, you will need to relate the heat currents and heats for the various parts. For example, if a heat current  $H$  is going into a system, the amount of heat that enters in time  $\Delta t$  is  $Q = H \Delta t$ . If heat is flowing through two adjacent objects in series, the heat current is the same for both. If they are side-by-side, the heat currents add up.





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**Hints:** wait until you are stuck before using the hints, then try them one at a time. For each hint, write the missing letters in the blank spaces, in order. For example: putting p,y,c,t,r,l in \_\_\_h\_\_\_si\_\_\_s\_\_\_uto\_\_\_ia\_\_\_ in gives “physics tutorial”

**Hint for question 1b):** missing letters: z, o, n, l, k, g, h, f, g, o

Hint: It is not \_\_\_er\_\_\_, and you do \_\_\_ot need to \_\_\_oo\_\_\_ at the \_\_\_rap\_\_\_ to \_\_\_i\_\_\_ure it \_\_\_ut.

**Hint for question 2a):** missing letters: v, y, h, g, e, l, b, m

Hint: E\_\_\_er\_\_\_t\_\_\_in\_\_\_ is in \_\_\_qui\_\_\_i\_\_\_riu\_\_\_ in the final picture.

**Hint for question 2b):** missing letters: u, w, q, t, n, p, g

Hint: In each part, \_\_\_se one of the t\_\_\_o e\_\_\_ua\_\_\_io\_\_\_s at the to\_\_\_ of this pa\_\_\_e.

### Hints for question 3:

- How much heat is required to boil all the water?
- What is the heat current through the bottom of the pot?
- What is the temperature of the water while it is boiling away?
- If you know the heat current, can you figure out the difference between the burner temperature and the water temperature?

### Hints for question 4:

- How does the power of the furnace relate to the total heat flowing out of the house?
- How can we calculate the total heat flowing out of the house if we have different R values for different surfaces?
- What do we use for A in the heat formula?