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Physics 157 Tutorial – Solutions

This tutorial gives you practice thinking about ideal gases as thermodynamics systems. This week, we'll focus on using the ideal gas law to calculate properties of a system after changes in pressure/volume/temperature with one of these quantities fixed, and on calculating the work done by a gas during some process.

Important formulae and tips:

The ideal gas law relates the pressure of a gas to its volume, temperature, and number of moles n via:

$$P V = n R T$$

When we have some process that changes some of these quantities but keeps the number of moles fixed, it is often useful to relate the initial quantities to the final quantities by

$$P_1 V_1 / T_1 = P_2 V_2 / T_2$$

This simplifies even more when pressure, volume, or temperature are fixed during the process.

During any process, we define **W** to be the work done **by** the gas. For a constant pressure process, this is equal to

$$W = P \Delta V$$

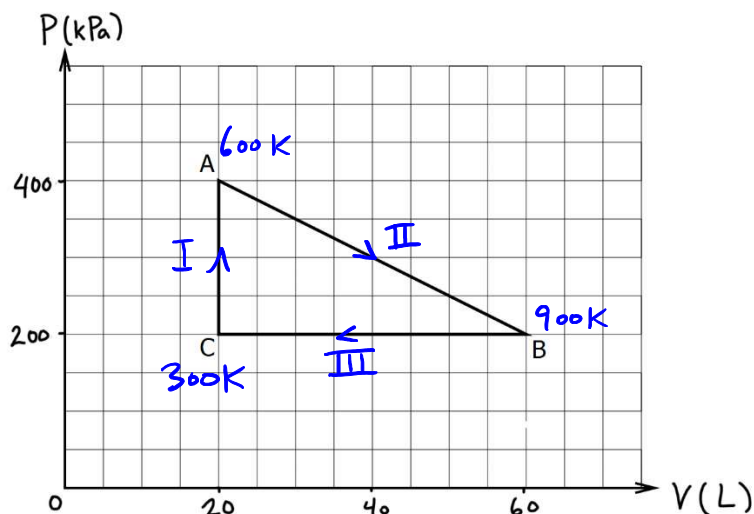
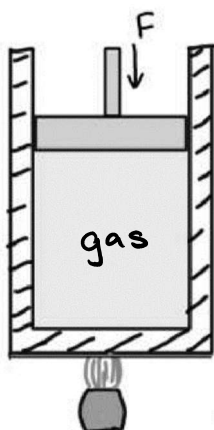
so the work is positive when the gas expands, and negative when the gas is compressed. When the pressure is changing, we need to calculate the work by breaking the process up into small parts with approximately constant pressure, and using that the work for each infinitesimal part is

$$dW = P dV$$

The total work can then be written as an integral between initial and final volumes:

$$W = \int_{V_i}^{V_f} P(V) dV$$

where $P(V)$ is the function that gives the pressure P at volume V . A simple way to understand this is that it **W is the area under the graph of P vs V**, taken between the initial and final points with a +/- sign if the volume increases/decreases.



Problem 1: A certain amount of gas, initially at 300K is contained in a cylinder with a movable piston on top. The graph above shows the pressure vs volume for a cyclical process in which:

I) The gas is heated with the piston fixed in place.

II) The gas is heated further while the forces that were holding the piston in place are removed in a controlled way.

III) The gas is cooled down, letting the piston move freely with the outside pressure at atmospheric pressure.

a) Label the three segments of the graph above with I, II, and III, showing which segment corresponds to which process. Draw an arrow on each segment to show which direction we are moving.

b) Calculate the temperature at the points A, B, and C.

In process I, V is const, so $T_A/T_C = P_A/P_C = 2 \Rightarrow T_A = 600K$

In process III, P is constant, so $T_B/T_C = V_B/V_C = 3 \Rightarrow T_B = 900K$

$$T_A = 600K$$

$$T_B = 900K$$

$$T_C = 300K \text{ (given)}$$

c) Calculate the work W done by the gas for each segment, indicating whether the work is positive or negative. Also calculate the net work for the cycle:

Use $W = \text{area under graph}$, with $+/-$ for expanding/contracting



note: $1kPa \times 1L = 1J$

$$W_I = 0$$

$$W_{II} = 12000J$$

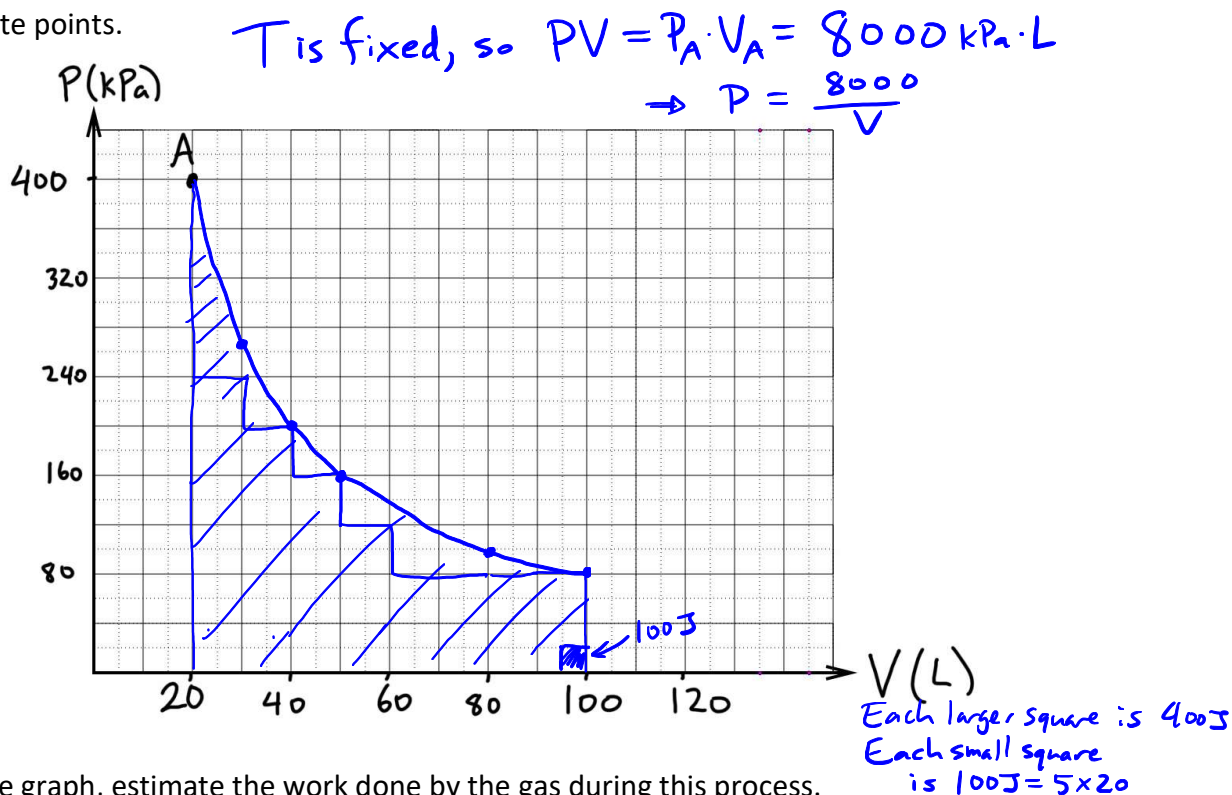
$$W_{III} = -8000J$$

$$W_{net} = 4000J$$

d) How many moles of gas are there? $n = \frac{PV}{RT} = \frac{200 \cdot 20}{8.31 \cdot 300} = 1.60 \text{ moles}$

Problem 2: The cylinder of gas from the previous cycle is taken when it is in the state B, and placed in a container of steam at the same temperature as the gas inside. The gas is now allowed to expand with its temperature remaining fixed, until the volume has increased by a factor of five.

a) Draw this process on the graph below, showing the final point and several accurately drawn intermediate points.



b) Using the graph, estimate the work done by the gas during this process.

$$W = (26 \text{ big squares}) + (24 \text{ small squares}) = 26 \times 400 + 24 \times 100 \text{ J} \approx 12,800 \text{ J}$$

c) Now we'll do a more accurate calculation of the work using the integral formula. First, we need to write an expression that gives the pressure in terms of the volume. Write this expression below. V should be the only variable appearing in your expression:

From above
 $P(V) = \frac{8000 \text{ J}}{V}$

d) Now calculate the work by the integral formula on page 1. The basic method is

i) Find a function $F(V)$ which has the property that its derivative $F'(V)$ equals $P(V)$

ii) Use the basic calculus result that $\int_{V_i}^{V_f} P(V) dV = F(V_f) - F(V_i)$

$$F(V) = 8000 \times \ln(V)$$

$$W = 8000 \times \ln(100) - 8000 \times \ln(20) = 12875 \text{ J}$$

Compare your answers to c) and d)!

Problem 3 (extra if you have time; involves some material from next week but included here since no tutorials next week): For these kinds of problems, the other thing that you will often be asked to do is to find the heat **Q** that enters the gas during each step. To do this, we need to use two extra pieces of physics:

i) When heat **Q** flows into a gas during a process, some of this energy is used up when the gas does work, and the rest increases the energy **U** of the gas itself (e.g. the kinetic energy of the molecules). This is summarized by the **First Law of Thermodynamics**:

$$Q = \Delta U + W$$

ii) The change in energy ΔU of an ideal gas can be calculated directly from the change in temperature by

$$\Delta U = n C_v \Delta T$$

where C_v is the molar heat capacity at constant volume¹ and $C_v = 3R/2$ for a monatomic ideal gas.

Your basic solution strategy to find **Q** is to first calculate **W** and ΔU and then use the First Law to find **Q**.

a) For the process in question 1, calculate ΔU and **Q** for each step? assuming $C_v = \frac{3}{2}R$

$$\Delta U_I = n C_v \Delta T = \frac{3}{2} n R \Delta T = \frac{3}{2} \cdot \left(\frac{40}{3}\right) \cdot 300 = 6000 \text{ J}$$

$$Q_I = \Delta U_I + W_I = 6000 \text{ J}$$

Note: from Q1 we have
 $nR = \frac{PV}{T} = \frac{40}{3} \text{ J/K}$

$$\Delta U_{II} = n C_v \Delta T = \frac{3}{2} n R \Delta T = \frac{3}{2} \left(\frac{40}{3}\right) \cdot 300 = 6000 \text{ J}$$

$$Q_{II} = \Delta U_{II} + W_{II} = 6000 \text{ J} + 12000 \text{ J} = 18000 \text{ J}$$

$$\Delta U_{III} = n C_v \Delta T = \frac{3}{2} n R \Delta T = \frac{3}{2} \left(\frac{40}{3}\right) (-600) = -12000 \text{ J}$$

$$Q_{III} = \Delta U_{III} + W_{III} = -12000 \text{ J} - 8000 \text{ J} = -20000 \text{ J}$$

b) What is the net heat $Q_I + Q_{II} + Q_{III}$ that enters the gas during the cycle? Is this greater than or less than the net work done?

$$Q_{\text{net}} = (6000 + 18000 - 20000) \text{ J} = 4000 \text{ J} = W_{\text{net}}$$

Expected since $\Delta U = 0$ for a full cycle

c) For the process in question 2, calculate ΔU and **Q**.

$$\Delta U = 0 \quad (\text{constant temperature})$$

$$Q = W = 12875 \text{ J}$$

¹ Remember that C_v was defined by $Q = n C_v \Delta T$ for a gas at constant volume. Here, there is no work, so $Q = \Delta U$.