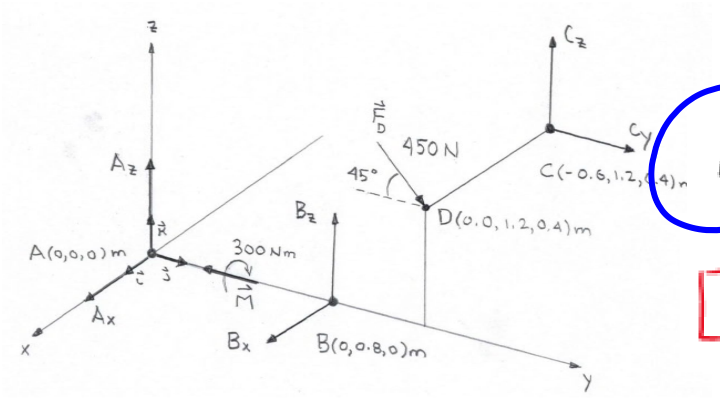
3D Equilibrium

W5-2. Determine the Cartesian components of these force reactions.



- Finalize:

$A_x + B_x = 0$ (1)

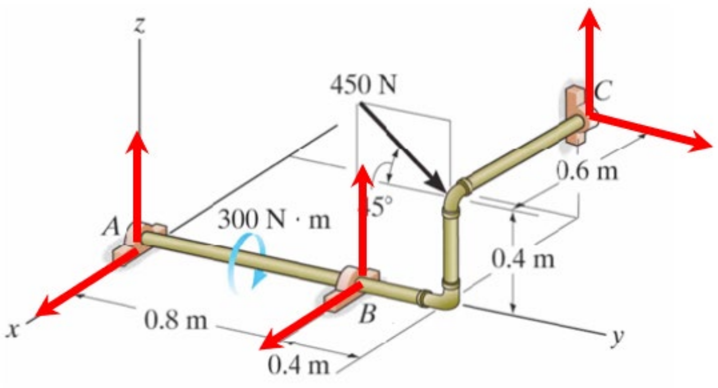
$\boxed{C_y = -318.2}$ (2)

$A_z + B_z + C_z = 450 \sin 45°$ (3)

$0.8 B_z + 1.2 C_z - 0.4 C_y = 509.12$ (4)

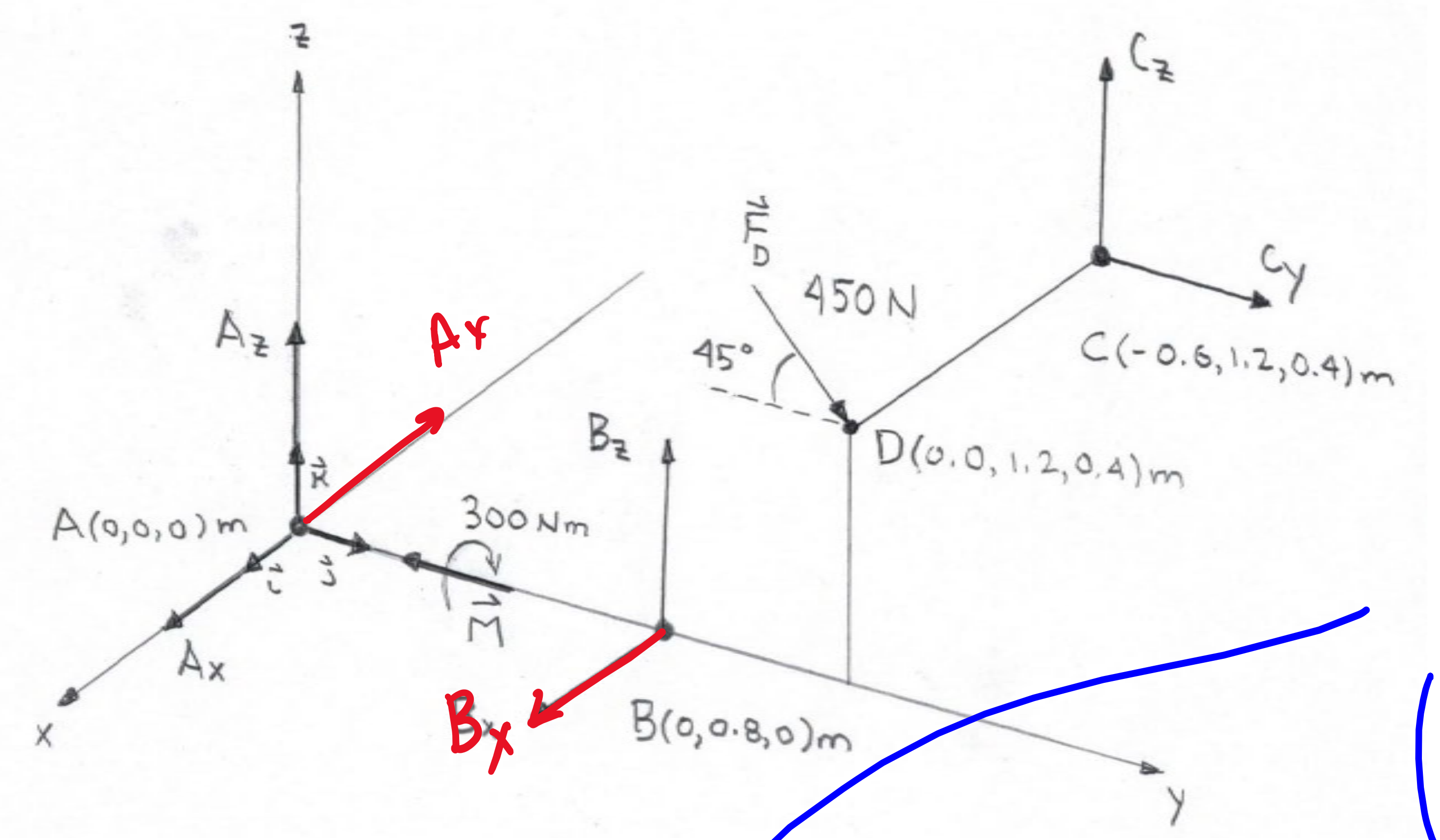
$\boxed{C_z = 500}$ (5)

$-0.8 B_x - 0.6 C_y = 0$ (6)



3D Equilibrium

W5-2. Determine the Cartesian components of these force reactions.



- Make sense:

act in the direction opposite to shown in the figure

ANSWER: $A_x = -239$ N $A_z = 90.9$ N $B_x = 239$ N $B_z = -273$ N $C_y = -318$ N $C_z = 500$ N

Q: What did we find, when we computed $\left(\vec{M}_B\right)_A$ and $\left(\vec{M}_C\right)_A$?

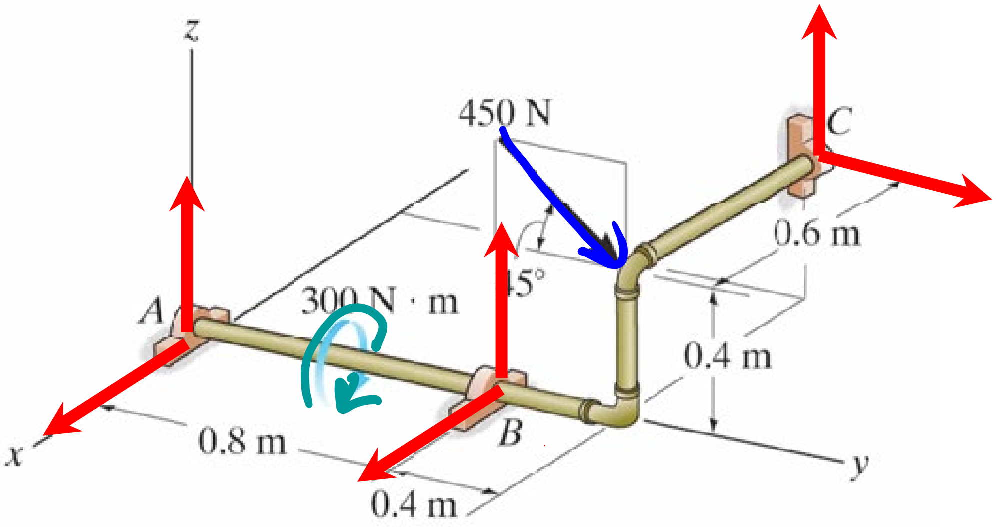
~~A.~~ Reaction moments.

✓ B. Moments of reaction forces

~~C.~~ Moments of active forces

~~D.~~ Reaction moments of reaction forces - ???

~~E.~~ Couple moments

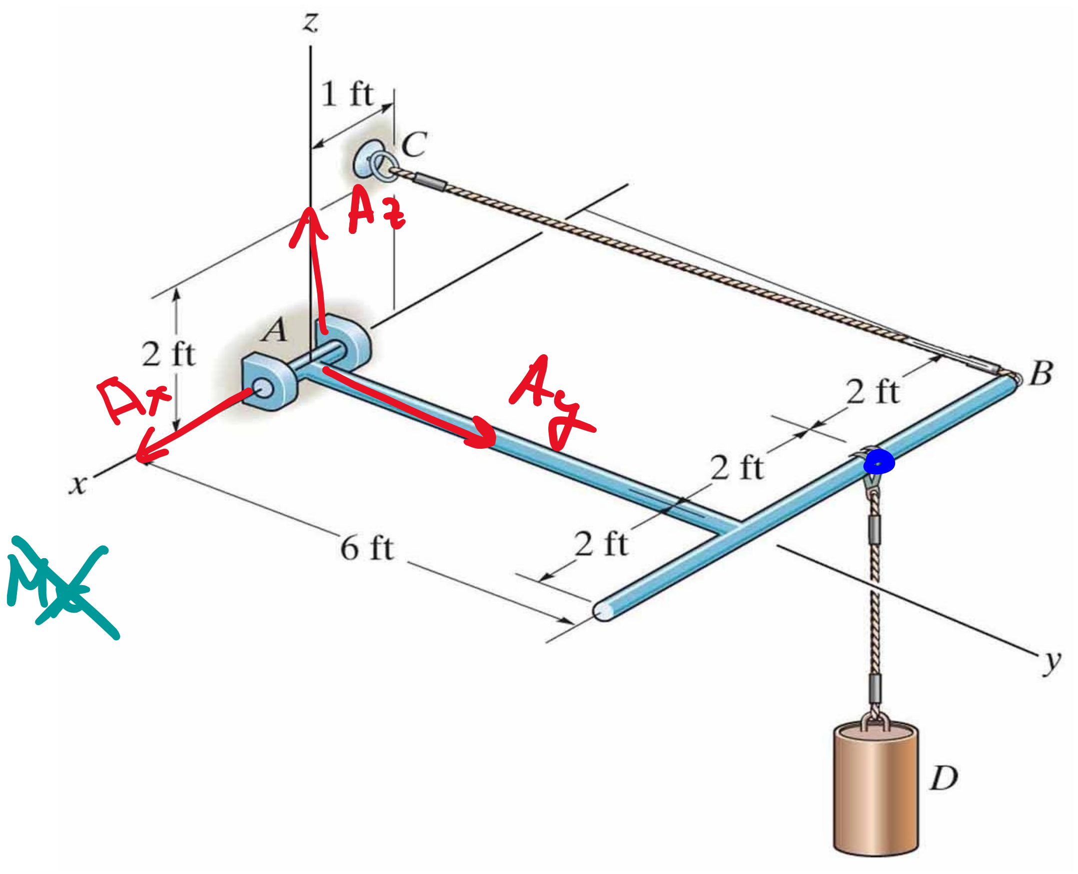


$$\left(\vec{M}_B\right)_A = \vec{r}_{AB} \times \vec{B}$$

$$\left(\vec{M}_C\right)_A = \vec{r}_{AC} \times \vec{C}$$

3D Equilibrium

W5-3. The member is supported by a pin at A and a cable BC. The load at D is 300 lb. Determine the x, y, z components of reaction at A and the tension in the cable BC.



A. x

B. y

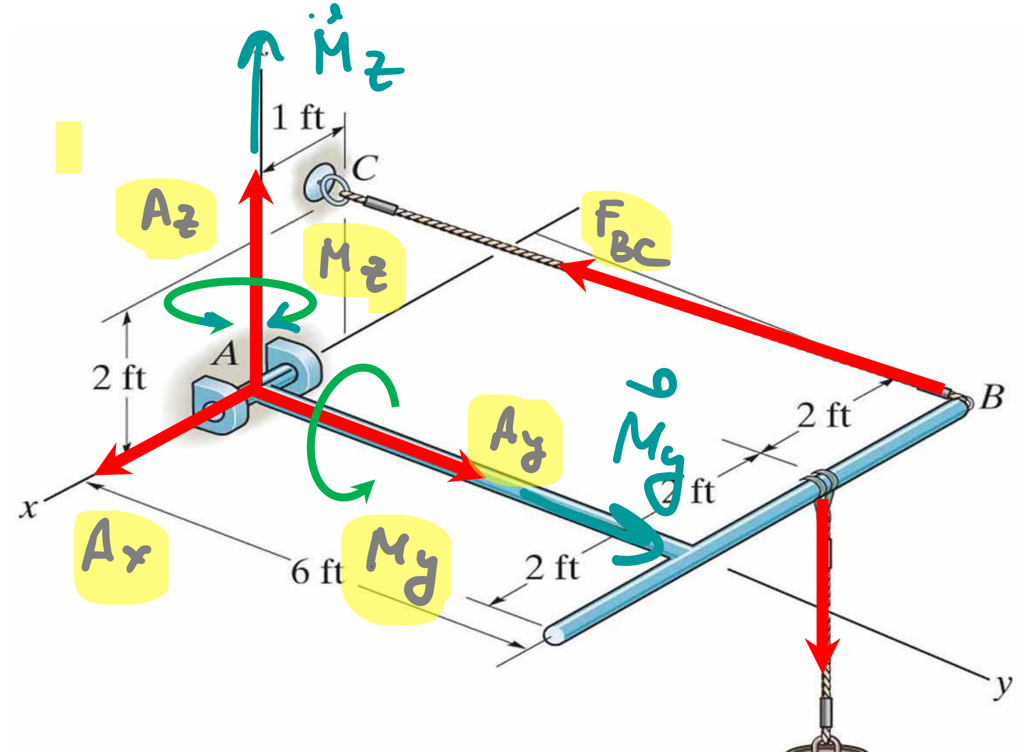
C. z

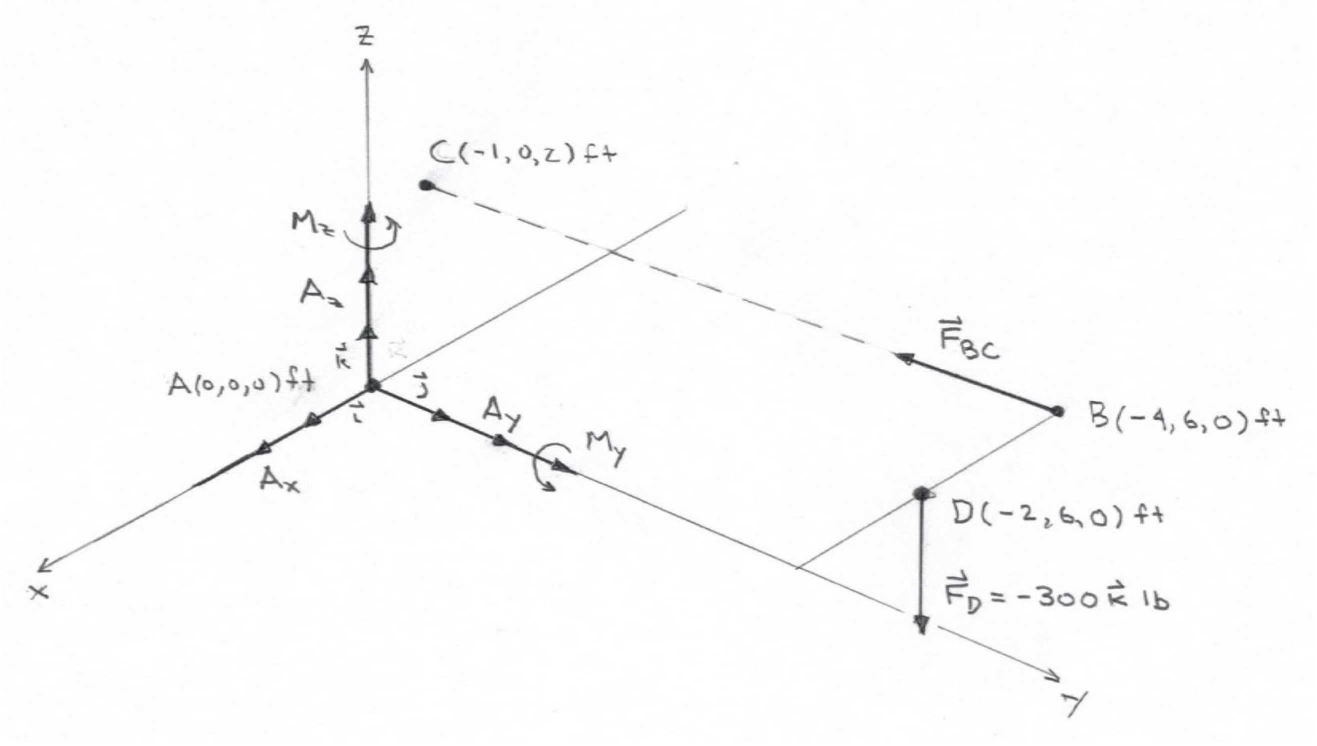
D xy

E None of the above

3D Equilibrium

W5-3. The member is supported by a pin at A and a cable BC. The load at D is 300 lb. Determine the x, y, z components of reaction at A and the tension in the cable BC.





6 Unkns

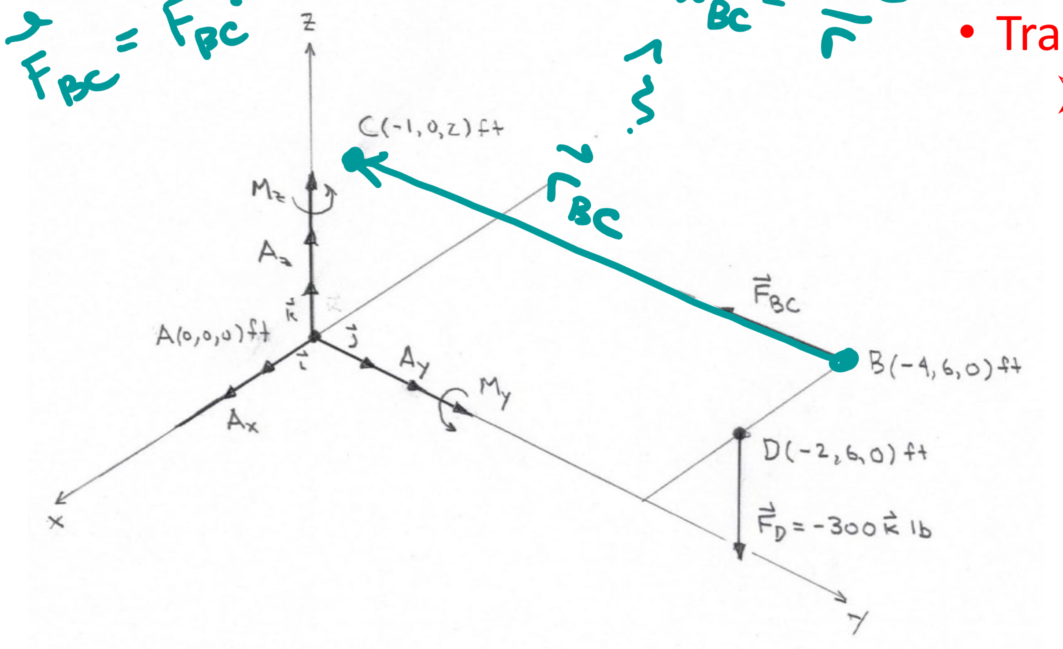
6 Equilibrium eqs

W

3D Equilibrium

W5-3. The member is supported by a pin at A and a cable BC. The load at D is 300 lb. Determine the x, y, z components of reaction at A and the tension in the cable BC.

$\vec{F}_{BC} = F_{BC} \cdot \vec{u}_{BC}$ $\qquad \vec{u}_{BC} = \dfrac{\vec{r}_{BC}}{r}$



- Translational equilibrium:
  - ➢ Forces?

$$\vec{A} = \vec{i}A_x + \vec{j}A_y + \vec{k}A_z$$

$$\vec{F}_D = -\vec{k}(300)$$

$$\vec{F}_R = 0 = \vec{A} + \vec{F}_D + \vec{F}_{BC}$$

$$\vec{F}_{BC} = \underbrace{\left(\frac{F_{BC}}{r_{BC}}\right)}_{B}\left[(3)\vec{i} + (-6)\vec{j} + (2)\vec{k}\right]$$

$$\vec{r}_{BC} = (3)\vec{i} + (-6)\vec{j} + (2)\vec{k} \; ; \quad r_{BC} = 7$$

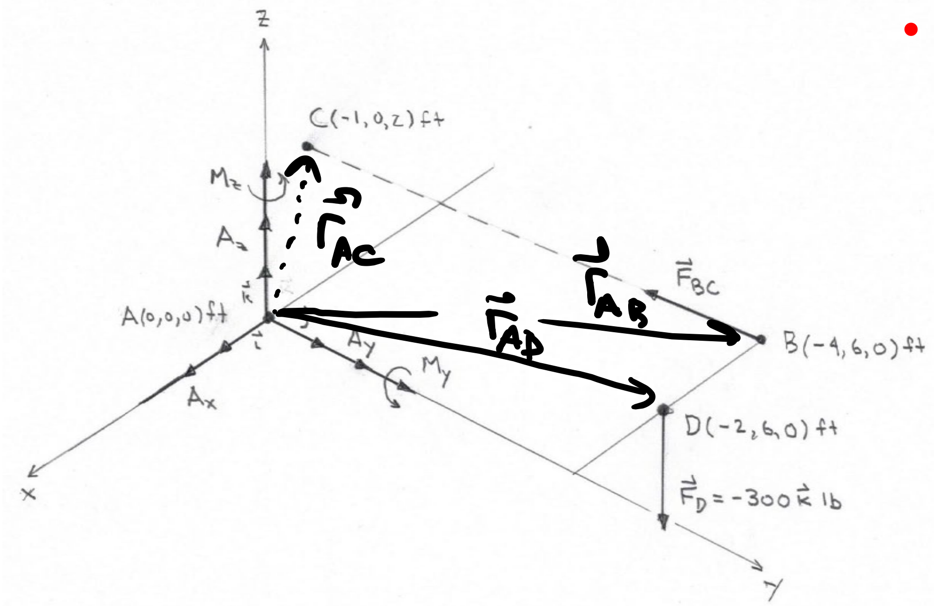
$x: \; \Sigma F_x = 0 : \quad A_x + 3B = 0 \qquad (1)$

$y: \; \Sigma F_y = 0 : \quad A_y - 6B = 0 \qquad (2)$

$z: \; \Sigma F_z : \quad A_z - 300 + 2B = 0 \qquad (3)$

3D Equilibrium

W5-3. The member is supported by a pin at A and a cable BC. The load at D is 300 lb. Determine the x, y, z components of reaction at A and the tension in the cable BC.



- Rotational equilibrium:
  - ➢ About which point?

$\vec{F}_D = -\vec{k} \cdot 300$ lb;

$\vec{F}_{BC} = B(3\vec{i} - 6\vec{j} + 2\vec{k})$

$$(\vec{M}_R)_A = 0 = (\vec{r}_{AD} \times \vec{F}_D) + (\vec{r}_{AB} \times \vec{F}_{BC}) + \vec{j} M_y + \vec{k} M_z$$

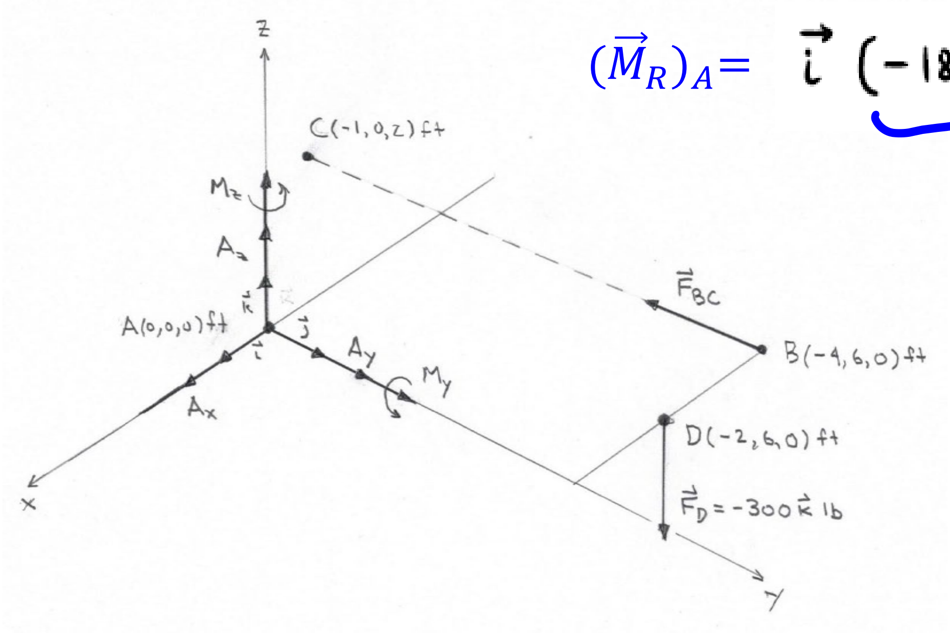
$$0 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 6 & 0 \\ 0 & 0 & -300 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 6 & 0 \\ 3B & -6B & 2B \end{vmatrix} + \vec{j} M_y + \vec{k} M_z$$

$$= \left(\vec{i}\,(-1800) - \vec{j}\,(600) + \vec{k} \cdot 0\right) + \left(\vec{i}\,(12B) - \vec{j}\,(-8B) + \vec{k}\,(24B - 18B)\right) + \vec{j} M_y + \vec{k} M_x$$

3D Equilibrium

W5-3. The member is supported by a pin at A and a cable BC. The load at D is 300 lb. Determine the x, y, z components of reaction at A and the tension in the cable BC.

$M_z = -900$ lb·ft



$$(\vec{M}_R)_A = \vec{i}\,(\underbrace{-1800 + 12B}_{=0\ (4)}) + \vec{j}\,(\underbrace{M_y - 600 + 8B}_{=0\ (5)}) + \vec{k}\,(\underbrace{M_z + 6B}_{=0\ (6)}) = 0$$

(4) ↪ $B = 150$

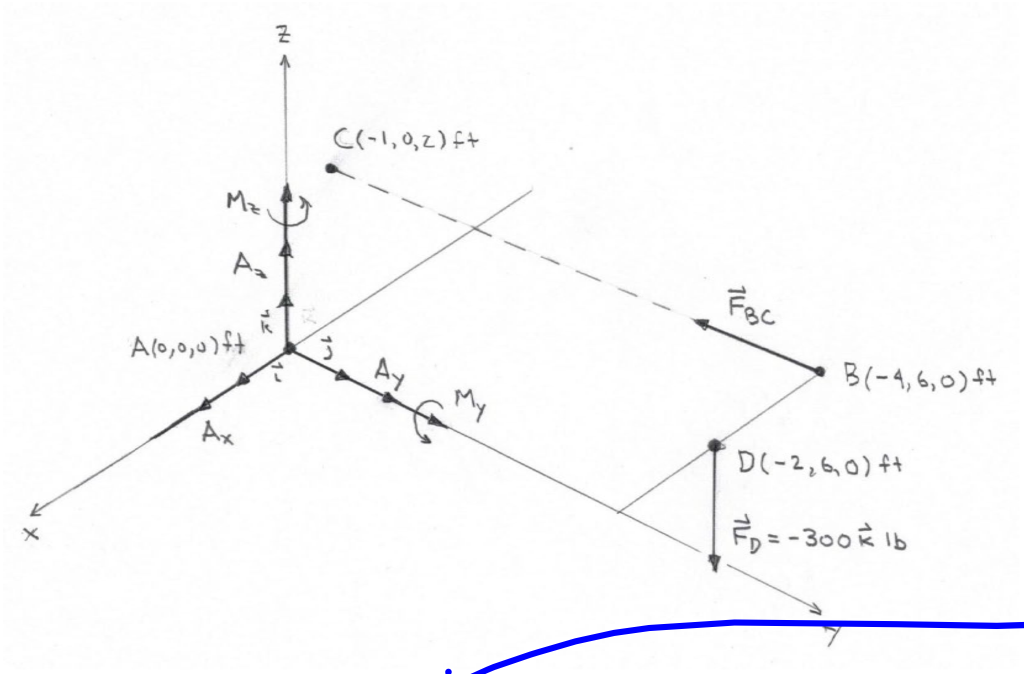
(5) ↪ $M_y = -600$ lb·ft

- $\vec{F}_{BC} = B\left[(3)\vec{i} + (-6)\vec{j} + (2)\vec{k}\right]$, $B = F_{BC}/7$

$F_{BC} = 7B = 150 \cdot 7 = 1050 = 1.05$ kip $= \boxed{1.05\ \text{kip}}$

3D Equilibrium

W5-3. The member is supported by a pin at A and a cable BC. The load at D is 300 lb. Determine the x, y, z components of reaction at A and the tension in the cable BC.



(1): $A_x = -3B$

(2): $A_y = 6B$

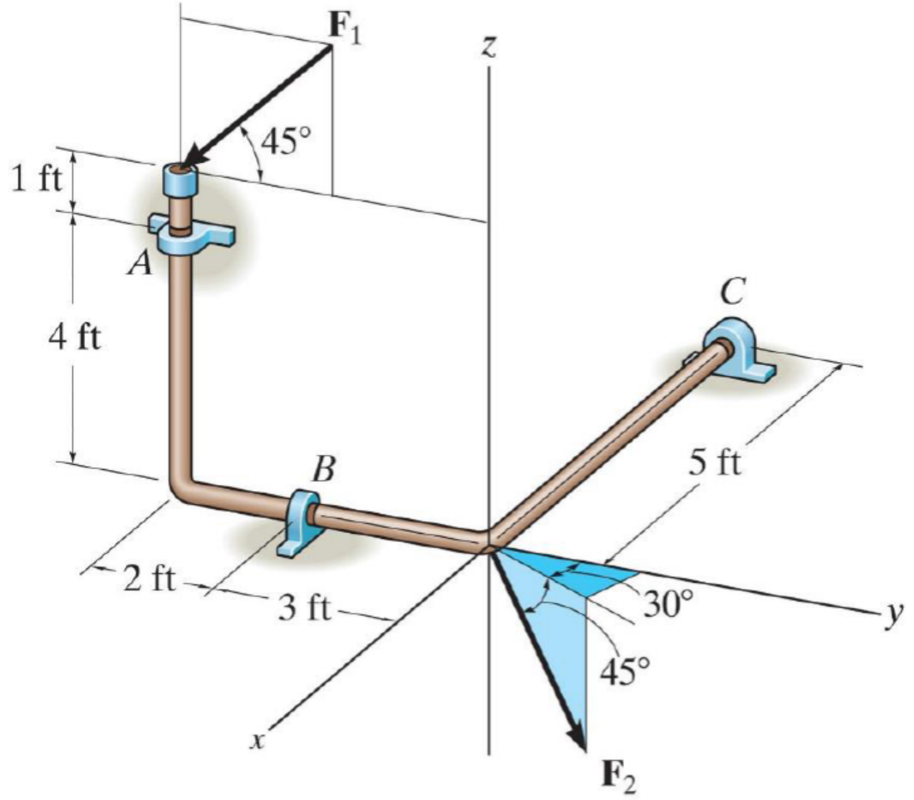
(3): $A_z = 300 - 2B$

$\boxed{B = 150}$ (4)

Answer: $A_x = -450$ lb, $A_y = 900$ lb, $A_z = 0$, $F_{BC} = 1.05$ kip, $M_y = -600$ lb ft, $M_z = -900$ lb ft

3D Equilibrium: Extra practice

E5-3. The bent rod is supported at A, B, C and by smooth journal bearings. The bearings are in proper alignment and only exert force reactions on the rod. The rod is subjected to forces as shown where $F_1 = 300$ lb and $F_2 = 250$ lb. The weight of the rod may be neglected. Determine the x, y, z components of reaction at the bearings.



6 equations that do not decouple

=> 6 x 6 matrix

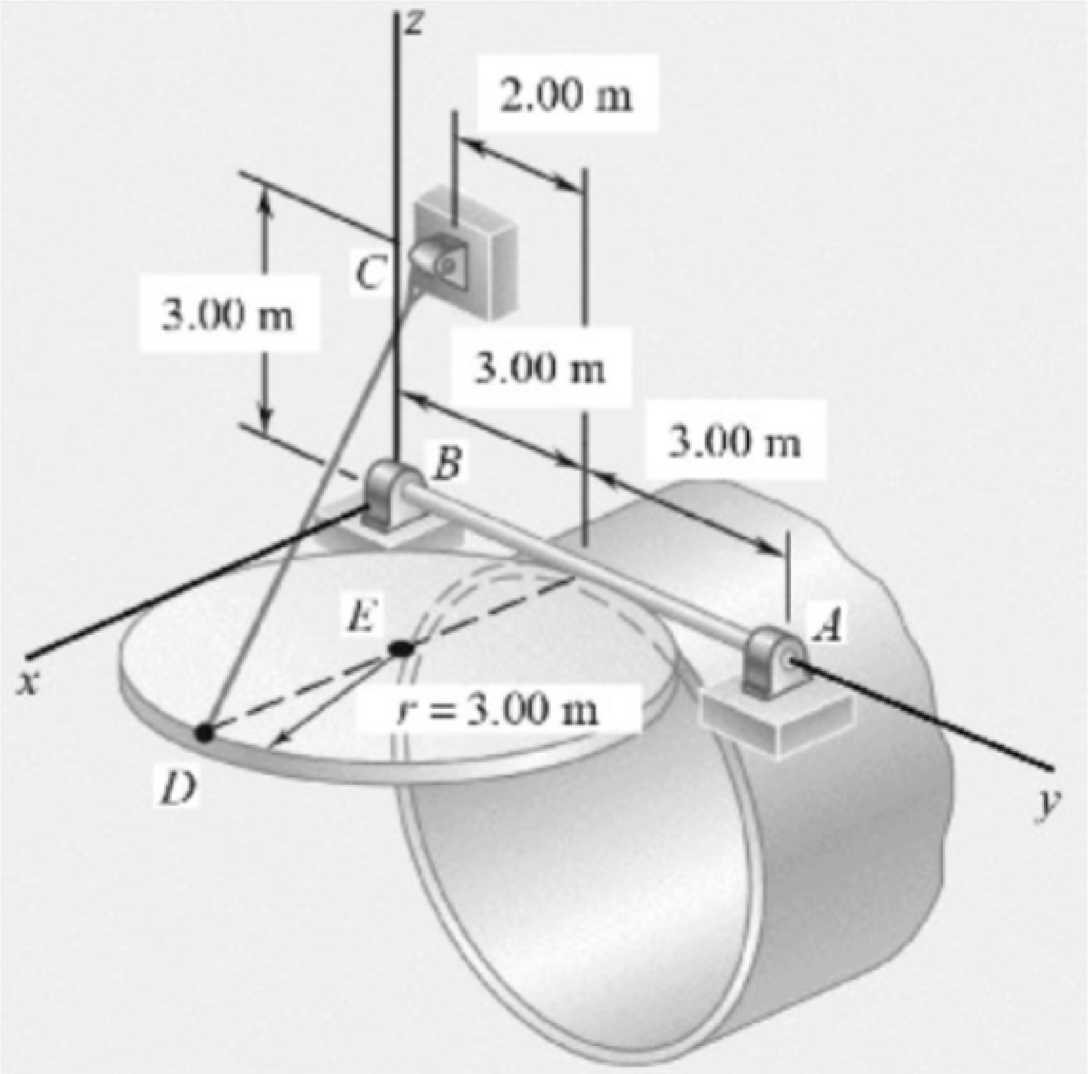
3D Equilibrium: Extra practice

midterm 2021

QUESTION 1 (20 marks)

The diagram below shows a rigid body *ABED* that is part of a hyperloop system. *ABED* consists of a disk-shaped plate with center at *E*, and a rod *AB*. These two components of *ABED* are rigidly attached to one another. *ABED* is supported in equilibrium by cable *CD*, a smooth journal bearing at *B* and a smooth thrust bearing at *A*. Note that the bearings at *A* and *B* are different types. The bearings are in proper alignment and only exert force reactions on the rod. The mass of the plate is 300 kg, and the corresponding weight acts at *E*. The mass of the rod can be neglected. Note that *ABED* is not in contact with the cylinder.

a) Draw a large, clear free-body diagram for *ABED*. (5 marks)
b) Determine Cartesian component force equations of equilibrium for *ABED*. (5 marks)
c) Determine a vector moment equation of equilibrium in determinant form for *ABED*. Take moments about *B*. (4 marks)
d) Determine Cartesian component moment equations of equilibrium for *ABED*. (3 marks)
e) Determine numerical values for the Cartesian components of reaction at *A* and *B* and the tension in the cable. (3 marks)



3D Equilibrium: Extra practice

QUESTION 1 (20 marks)

The diagram below shows a rigid body *ABED* that is part of a hyperloop system. *ABED* consists of a disk-shaped plate with center at *E*, and a rod *AB*. These two components of *ABED* are rigidly attached to one another. *ABED* is supported in equilibrium by cable *CD*, a smooth journal bearing at *B* and a smooth thrust bearing at *A*. Note that the bearings at *A* and *B* are different types. The bearings are in proper alignment and only exert force reactions on the rod. The mass of the plate is 300 kg, and the corresponding weight acts at *E*. The mass of the rod can be neglected. Note that *ABED* is not in contact with the cylinder.

a) Draw a large, clear free-body diagram for *ABED*. (5 marks)
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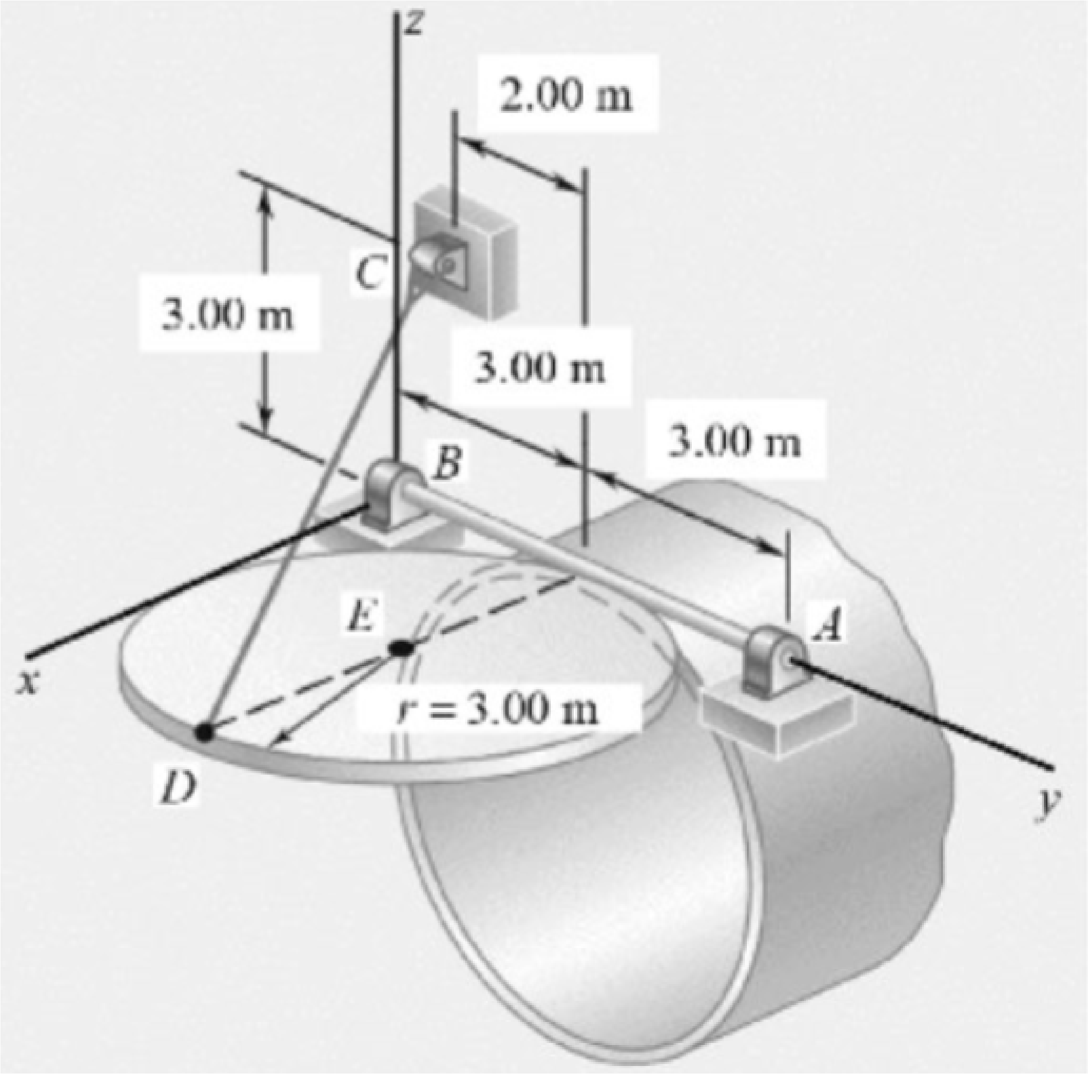
e) Determine numerical values for the Cartesian components of reaction at *A* and *B* and the tension in the cable. (3 marks)



Answer: $A_x = 490\ N;\ A_y = 981\ N;$
$A_z = 736\ N;\ B_x = 2.45\ kN;\ B_z = 736\ N$

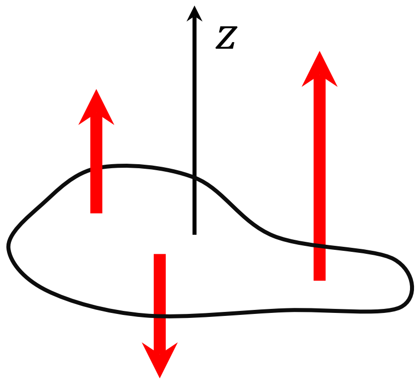
Be sure to check out the additional problems for this Chapter (and other Chapters) in:

- Extra practice worksheets
- Canvas -> ADDITIONAL INFORMATION -> Additional Problems

Doing as many Chapter 5 problems as possible will pay off for the midterm and final exams!

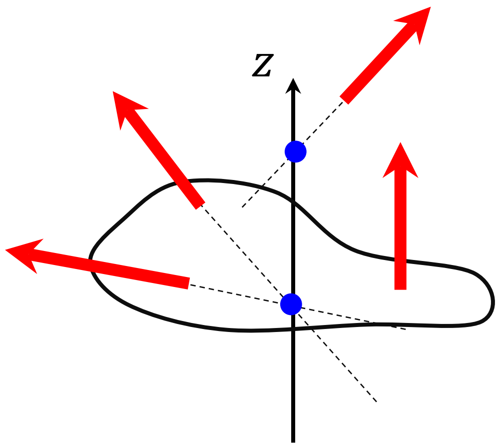
Some special cases: *Assume that we only have forces, no couple moments*

- Recap (*Week 4*):
  - ➢ The moment of a force about any point on its line of action is zero!
  - ➢ The moment of a force about any axis parallel to that force is zero!



- All forces are parallel to some axis:
  - All the moments about this axis are zero
  - Equation $\sum M_z = 0$ is automatically satisfied
  - You can have 5 unknowns maximum

- All forces have lines of action that are parallel or intersect some axis:



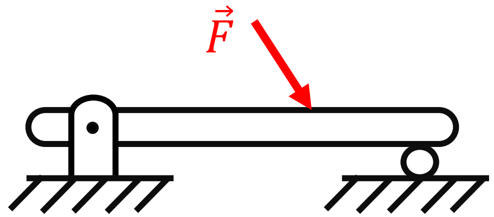
  - The scalar moment equation governing moments about this axis will be automatically satisfied ($d = 0$ for the forces whose lines of action intersect z)
  - You can have 5 unknowns maximum

- Section 5-4, Two- and Three-force members: Not covered. Optional, nice physics, enhances understanding

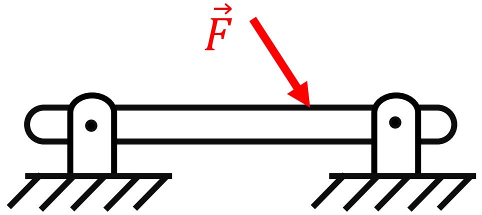
Section 5.7, self study

Number of Unknowns vs the Number of Equations

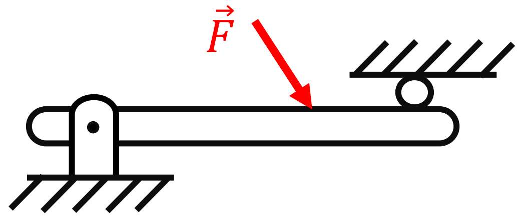
- You can have less than 6 unknowns if your system has special geometry
  - ➢ Our 2D problem, W5-1: 3 equations in 3 unknowns
  - ➢ Examples from the previous slide.
  - ➢ One more in HW 5
  - ➢ ……….

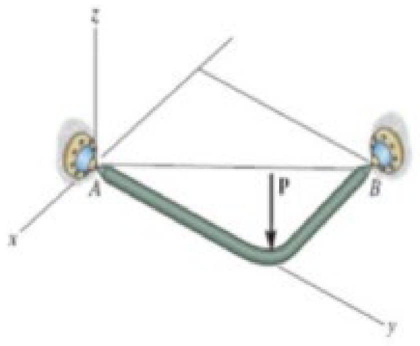


- You can have more than 6 unknowns if your system has redundant supports
  - ➢ Redundant supports: more than necessary to hold it in equilibrium
  - ➢ The system becomes "statically indeterminate"
  - ➢ Additional equations: from deformation condition at supports (beyond our scope)



- Improper constraints
  - ➢ 6 equations & 6 unknows, but no equilibrium
  - ➢ Instabilities (small departures from equilibrium "run away" rather than being "damped out")





❖ You can generally expect to be given problems that will have a solution (i.e. with the same number of unknowns as equations that are not automatically satisfied)

❖ Should note that real life will not always be as kind!!