Physics 157 Written Homework 7

In this homework set, you'll get some more practice with problems involving gas cycles as well as with entropy. We have tried to stress the benefit of solving problems symbolically and only putting in values at the very end. This problem set will also give you more practice doing that.

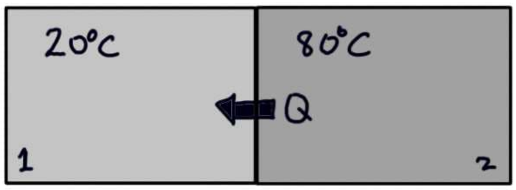
Things to keep in mind about entropy:

- for an infinitesimal process $dS = dQ/T$
- heat is the area under the curve of a TS diagram, with positive heat if S is increasing
- total entropy for a system never decreases, but entropy for certain parts can decrease
- entropy is a state variable, so $\Delta S = 0$ for a complete cycle, and ΔS is the same for any path between the same two endpoints
- The efficiency of an engine cycle plotted on a TS diagram is the area inside the cycle divided by the total area under the parts moving to the right

NOTE: This week's Written Homework 7 set includes a set of Multiple-Choice questions as well as one written problem.

- You will submit the answers to the Multiple-Choice questions on Mastering Physics as your Mastering Physics assignment 7. Your answers entered into Mastering Physics will count as your grade for Mastering Physics 7.
- Your solution for the written problem should be submitted as a separate pdf to Canvas (you don't have to include the questions entered into Mastering Physics in your pdf submission). The grade for your Written Homework 7 assignment will be based on the written problem solution you submit as a pdf.

Entropy Multiple-Choice questions 1-7: Answers to be entered in Mastering Physics 7:



Question 1: A small amount of heat flows from object 2 (initially at 80ºC) to object 1 (initially at 20ºC).

During this process, we can say that

A) The entropy of object 1 increases and the entropy of object 2 increases.
B) The entropy of object 1 increases, the entropy of object 2 decreases. The total entropy increases.
C) The entropy of object 1 increases, the entropy of object 2 decreases. The total entropy decreases.
D) The entropy of object 1 decreases, the entropy of object 2 increases. The total entropy increases.
E) The entropy of object 1 decreases, the entropy of object 2 increases. The total entropy decreases.

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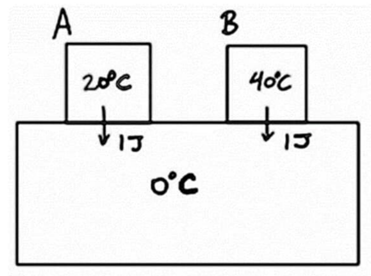
Question 2: Objects A and B with initial temperatures 20ºC and 40ºC are each placed on a block of metal with temperature 0ºC. In some small amount of time, 1 J of heat flows from each of the objects into the metal.

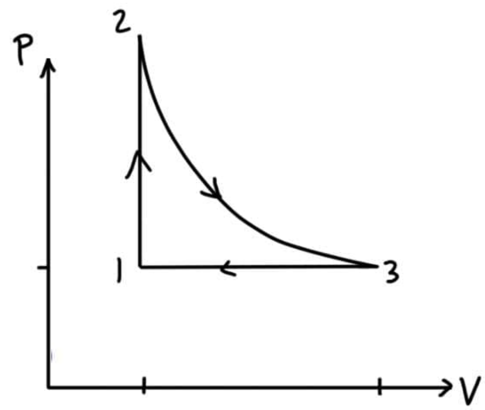


During this time, we can say that

A) The entropies of A and B each decrease, but the entropy of A has a larger change.
B) The entropies of A and B each decrease, but the entropy of B has a larger change.
C) The entropies of A and B each increase, but the entropy of A has a larger change.
D) The entropies of A and B each increase, but the entropy of B has a larger change.
E) The change in entropy of A is opposite to the change in entropy of B.

dS = dQ/T and here dQ is same for both but larger entropy change for smaller temperature.

Question 3: The pressure and volume for nitrogen gas in a cylinder undergoing a cyclic process are plotted in the diagram.

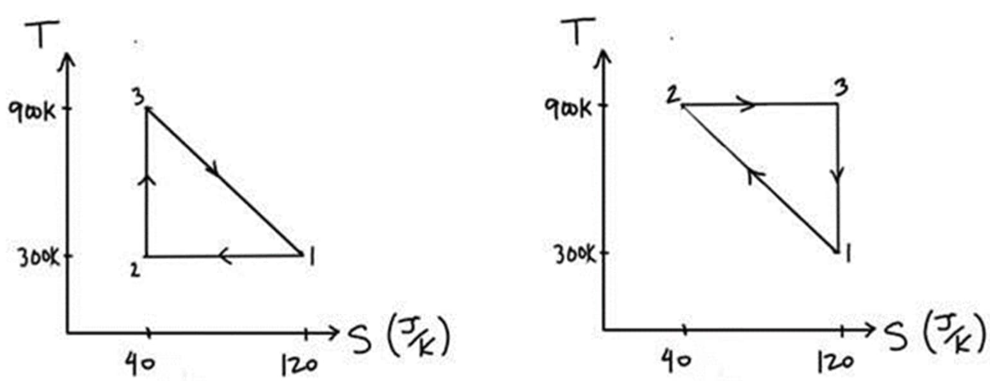


During the entire cycle $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$, we can say that

A) The entropy of the gas has a net increase.
B) The entropy of the gas has a net decrease.
C) The final entropy of the gas is the same as the initial entropy.

Entropy is a state variable!

Question 4: Two possible heat engine cycles are shown in the figure.



We can say that

A) The first cycle is more efficient
B) The second cycle is more efficient
C) The cycles have the same efficiency
D) There is not enough information to determine which cycle is more efficient.

Efficiency = enclosed area divided by total area below process with S increasing.

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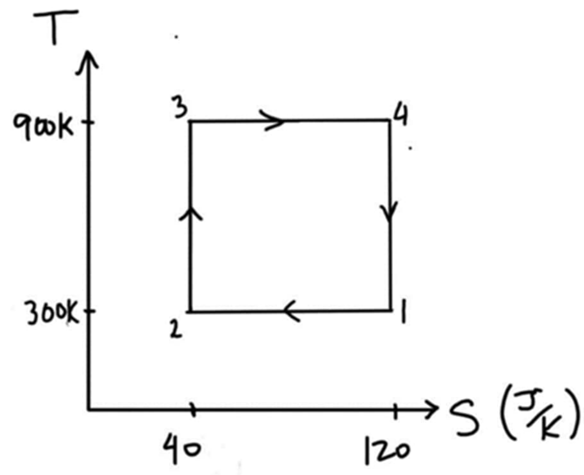
Question 5: A thermodynamic cycle for an ideal gas in a cylinder is shown on the TS diagram.



During the process 1 → 2, we can say that:

A) Heat flows into the gas.
B) Heat flows out of the gas.
C) Heat flows into the gas, then out of the gas, but there is no net flow of heat in or out during the process.
D) No heat flows in or out of the gas at any time during the process.

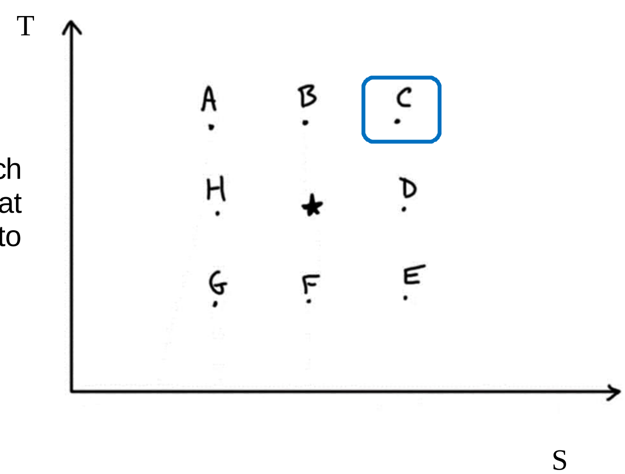
Process 1 → 2 goes toward smaller S, so heat goes out.

Question 6: For the cycle in the previous question, the efficiency is closest to

A) 25%
B) 33%
C) 50%
D) 66%
E) 75%

Efficiency = W / Qin = (area square) / (area under 3 → 4) = 2/3

Question 7: On the T-S diagram, the state marked with a star is the starting state for a constant pressure process in which the volume expands.



Which of the other states could be the ending state for such a process? (Note: you do not need to know precisely what a constant pressure process looks like on a TS diagram to answer this.)

P const, V increasing, so T increasing since V/T constant.
Q = $nC_p\Delta T > 0$ so S increasing

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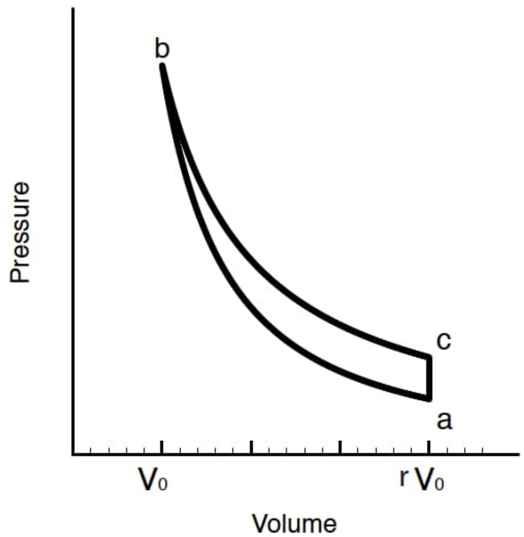
NOTE: You should submit your solution for the following written problem to Canvas as a separate pdf for your written homework 7 assignment.

Question 8: A new heat engine cycle can be described by an adiabatic compression (a → b), an isothermal expansion (b → c), and an isochoric cooling process (c → a), as shown in the graph. The compression ratio is r. For a diatomic ideal gas: $C_V = 5R/2$, $C_P = 7R/2$, γ = 1.4 **Assume the processes are reversible.**

a) Write down **the expressions** (not the values) for the heat transferred during each process.



Process A → B is adiabatic, so

$$Q_{A\to B} = 0 \tag{1}$$

Process B → C is isothermal, so

$$Q_{B\to C} = W_{B\to C} = nRT_B \ln\left(\frac{rV_o}{V_o}\right) = nRT_B \ln(r) \tag{2}$$

Process, C → A is isochoric, so

$$Q_{C\to A} = \Delta U_{C\to A} = nC_v\,\Delta T = nC_v\,(T_A - T_C) = nC_v\,(T_A - T_B) \tag{3}$$

There are really only two temperatures, T_A and T_B,since $T_C = T_B$. We also know that T_B will be higher than T_A after the adiabatic compression a → b. So, we can write all of our expressions in terms $T_A(\equiv T_{cold})$ and $T_B(\equiv T_{hot})$.

b) Find an **expression** for the efficiency of this cycle in terms of only the compression ratio.

Heat is absorbed during the isothermal process, b → c, since $Q_{B\to C}$ is > 0, and heat is expelled during the isochoric process, c → a, since $Q_{C\to A} < 0$. Thus

$$Q_{in} = Q_{B\to C} = nRT_B \ln(r) \tag{4}$$

The work done by the gas during the three processes is:

$$W_{A\to B} = -\Delta U_{A\to B} = -nC_V(T_B - T_A) \tag{5}$$

$$W_{B\to C} = nRT_B \ln\left(\frac{rV_o}{V_o}\right) = nRT_B \ln(r) \tag{6}$$

$$W_{C\to A} = 0 \tag{7}$$

The net work done by the gas is:

$$W_{net} = W_{A\to B} + W_{B\to C} + W_{C\to A} = -nC_V(T_B - T_A) + nRT_B \ln(r) \tag{8}$$

The efficiency is thus:

$$e = \frac{|W_{net}|}{|Q_{in}|} \tag{9}$$

$$e = \frac{(-nC_V(T_B-T_A)+nRT_B \ln(r))}{nRT_B \ln(r)} \tag{10}$$

$$e = \frac{\left(-\frac{5}{2}(T_B-T_A)+T_B \ln(r)\right)}{T_B \ln(r)} \tag{11}$$

$$e = 1 - \frac{5}{2}\frac{(T_B-T_A)}{T_B \ln(r)} \tag{12}$$

We can simplify this by writing T_A in terms of T_B by using:

$$T_A = T_B\,(V_B/V_A)^{\gamma-1} = T_B\,(1/r)^{\gamma-1} = T_B\, r^{1-\gamma} \tag{13}$$

Putting this into the expression for the efficiency gives:

$$e = 1 - \frac{5}{2}\frac{(1-r^{1-\gamma})}{\ln(r)} \tag{14}$$

c) If state a is at room temperature (300 K) and atmospheric pressure (100 kPa), and the compression ratio is 4, what is the efficiency for this heat engine?

We are given that $T_A = 300K$ and we can use (13) to find T_B:

$$T_B = T_A\left(\frac{rV_o}{V_o}\right)^{\gamma-1} = T_A\,(r)^{\gamma-1} = 300 \cdot (4)^{0.4} = 522\ K \tag{15}$$

Evaluating (14) for $r = 4$ gives

$$e = 1 - \frac{5}{2}\frac{(1-4^{-0.4})}{\ln(r)} = 0.23 \tag{16}$$

d) How does your answer in (c) compare to the efficiency that a Carnot engine running between the same two temperatures would have?

The Carnot efficiency is given by

$$e_{carnot} = 1 - \frac{T_{cold}}{T_{hot}} \tag{17}$$

We are given that $T_A = T_{cold} = 300K$ and we found in (15) $T_B = T_{hot} = 522K$. The Carnot efficiency is thus

$$e_{carnot} = 1 - \frac{300}{522} = 0.43 \tag{18}$$

This heat engine is less efficient than a Carnot engine operating between the same two temperatures.

(e) What is the entropy change for each process if V_o is 1L, and, as above, state a is at room temperature (300 K) and atmospheric pressure (100 kPa)?

Process A → B is adiabatic, so $Q_{A\to B} = 0$ and

$$\Delta S_{A\to B} = 0 \tag{19}$$

Process B → C is isothermal process, so we have $\Delta U_{B\to C} = 0$, and $Q_{B\to C} = W_{B\to C}$ and

$$\Delta S_{B\to C} = \frac{Q_{B\to C}}{T_B} = nR\ln\left(\frac{rV_o}{V_o}\right) = nR\ln(r) \tag{20}$$

The number of moles of gas can be found from the given parameters and the ideal gas law as

$$n = P_A V_A/(R\,T_A) = 100\cdot 10^3 \cdot 4 \cdot 1L \cdot (1m^3/1000\,L)/(8.31\cdot 300) = 0.16\,moles \tag{21}$$

With this, we have

$$\Delta S_{B\to C} = 0.16\cdot 8.31\cdot \ln(4) = +1.84\,J/K \tag{22}$$

Process C → A is isochoric process, so we have $W_{C\to A} = 0$, and $Q_{C\to A} = \Delta U_{C\to A}$ and

$$Q_{C\to A} = nC_v\,\Delta T \tag{23}$$

$$dQ_{C\to A} = nC_v\,dT \tag{24}$$

$$\Delta S_{C\to A} = \int_{T_B}^{T_A}\frac{nC_v\,dT}{T} = nC_v\ln\left(\frac{T_A}{T_B}\right) = 0.16\cdot\left(\frac{5}{2}\right)\cdot 8.31\cdot\ln\left(\frac{300}{522}\right) = -1.84\,J/K \tag{25}$$

Note that the total entropy change for the full cycle is $\Delta S = 0$, as expected, since S is a state variable.

f) The isochoric cooling process, c → a, occurs because the gas is brought into contact with an isothermal cold reservoir at temperature T_{cold} = 300K. What is the entropy change of this reservoir during the process c → a?

From (3) above, the heat flowing into the gas during, C → A is

$$Q_{C\to A} = \Delta U_{C\to A} = nC_v\,\Delta T = nC_v\,(T_A - T_C) = nC_v\,(T_A - T_B) \tag{26}$$

The heat flowing into the cold reservoir during C → A is

$$Q_{cold} = -Q_{C\to A} = -nC_v\,(T_A - T_B) \tag{26}$$

The change in entropy of the cold reservoir resulting from this is

$$\Delta S_{cold} = \frac{Q_{cold}}{T_A} = -\frac{Q_{C\to A}}{T_A} = -\frac{nC_v\,(T_A - T_B)}{T_A} = -\frac{0.16\cdot\left(\frac{5}{2}\right)\cdot 8.31\cdot(300-522)}{300} = +2.46\,J/K \tag{28}$$

f) What is the total entropy change of the gas plus cold reservoir during the process c → a? What does this tell you about what kind of process this is?

$$\Delta S_{total} = \Delta S_{C\to A} + \Delta S_{cold} = -1.84 + 2.46 = +0.62\,J/K \tag{29}$$

Since the total entropy change for this process is > 0, this process is irreversible.