Problem C4.2 (☆): An L-R-C series circuit, the components have the following values: $L = 2.0\ mH$, $C = 18\ \mu F$, and $R = 15\ \Omega$. The AC power source has an RMS voltage of $120\ V$ with a frequency of $3.2\ kHz$.

(a) Draw the impedance triangle for this circuit.

(b) Calculate the impedance Z and the phase angle ϕ.

Your friend gives you a new power source with the same RMS voltage, but an unknown frequency.

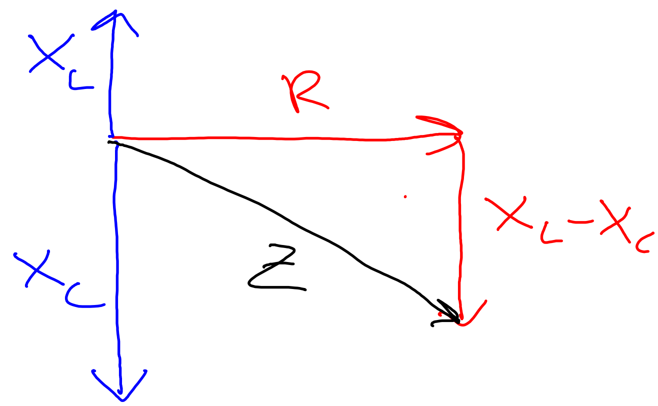
(c) You observe the power factor of $\cos\phi = 0.819$. What is the frequency of this source?

(a)

$$X_C = \frac{1}{\omega C} = 17.4\,\Omega$$

$$X_L = \omega L = 6.4\,\Omega$$

$$R = 15\,\Omega$$



(b)

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \underline{18.6}\ \Omega$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = -0.631 \text{ rad}$$

(c)

$$\cos\phi = 0.819 \Rightarrow \phi = 0.611 \text{ rad}$$

$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$\therefore\ \omega^2 LC - RC\tan\phi\,\omega - 1 = 0$$

$$\omega = \underline{8077} \text{ or } \cancel{-3700} \text{ rad/s}$$

negative soln. unphysical

Problem C4.3 (☆☆): You are given an L-R-C series circuit with unknown values for L, R, and C. You do have, however, an AC voltage source with $V_{RMS} = 8\ V$ and a tunable frequency ω. You also have an Ammeter that measures the RMS current I_{RMS} and the power factor $\cos\phi$.

(a) Devise a procedure to determine L, R, and C by measuring the circuit with the tools at hand.

(a) The RMS current and phase angle are given by

$$I_{RMS}(\omega) = \frac{V_{RMS}}{\sqrt{Z^2 + (\omega L - \frac{1}{\omega C})^2}}$$

$$\tan\phi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

Measuring I_{RMS} and ϕ at several frequencies should be sufficient to determine R, L, & C.

A key simplification occurs at the resonance frequency $\omega_0 = \sqrt{\frac{1}{LC}}$, where $I_{RMS} = \frac{V_{RMS}}{R}$.

This is but one of many ways.

Problem C4.3 continued (☆☆): Using the same setup, suppose you measured I_{RMS} as a function of frequency and found that the maximum RMS current is $40\ mA$ at $\omega_0 = 12.5\ kHz$.

- **(b)** What is the resistance R? What does this tell you about L and C?
- **(c)** What is the power factor at $\omega = \omega_0$?
- **(d)** In addition you find that at $\omega_1 = 17\ kHz$ the power factor is 0.5. Based on this information, what are the values of L and C?

(b) At resonance, $I_{RMS}(\omega_0) = \dfrac{V_{RMS}}{R}$

$\therefore R = 200\ \Omega$

At resonance, $\omega_0 = \sqrt{\frac{1}{LC}}$. Hence we have one equation to determine L & C.

(c) At resonance, $\tan\phi = \dfrac{\cancel{\omega L - \frac{1}{\omega C}}^{\,0}}{R} = 0$

$\therefore \cos\phi = 1$

(d) At 17 kHz, $\cos\phi = 0.5 \Rightarrow \phi = 60°$

$\therefore \tan(60°) = \sqrt{3} = \dfrac{\omega_1 L - \frac{1}{\omega_1 C}}{R}$

From (a): $\omega_0^2 = \dfrac{1}{LC}$

$L\left(\omega_1 - \dfrac{\omega_0^2}{\omega_1}\right) = \sqrt{3}\,R \Rightarrow L = 44.3\ mH$

$C = \dfrac{1}{L\omega_0^2} = 144\ nF$