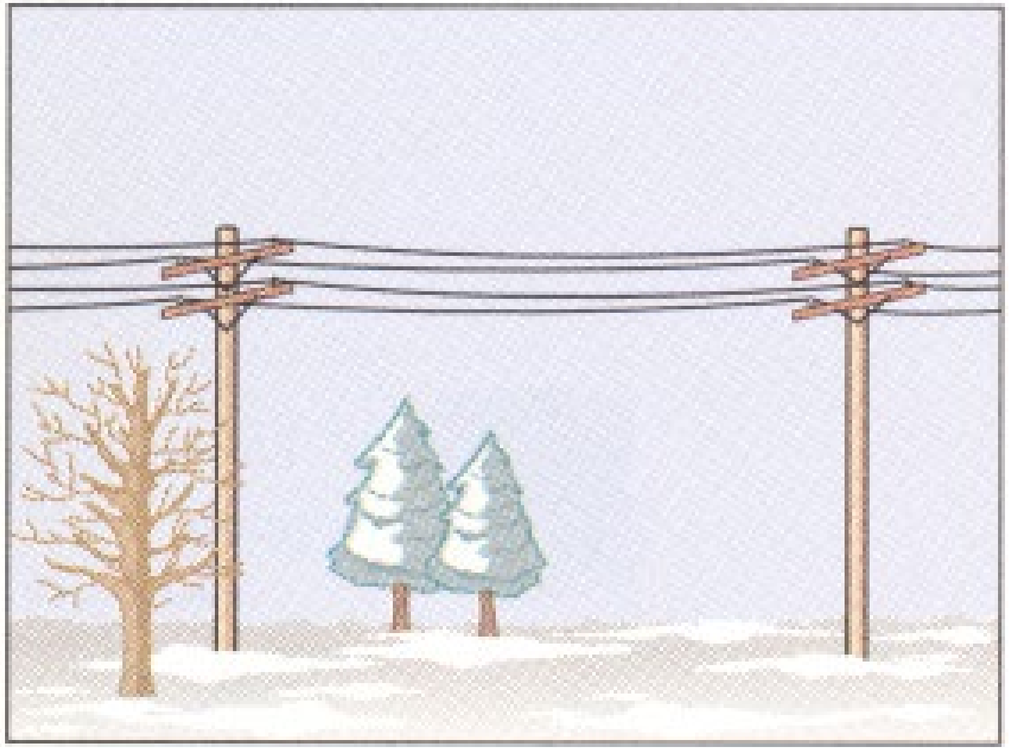
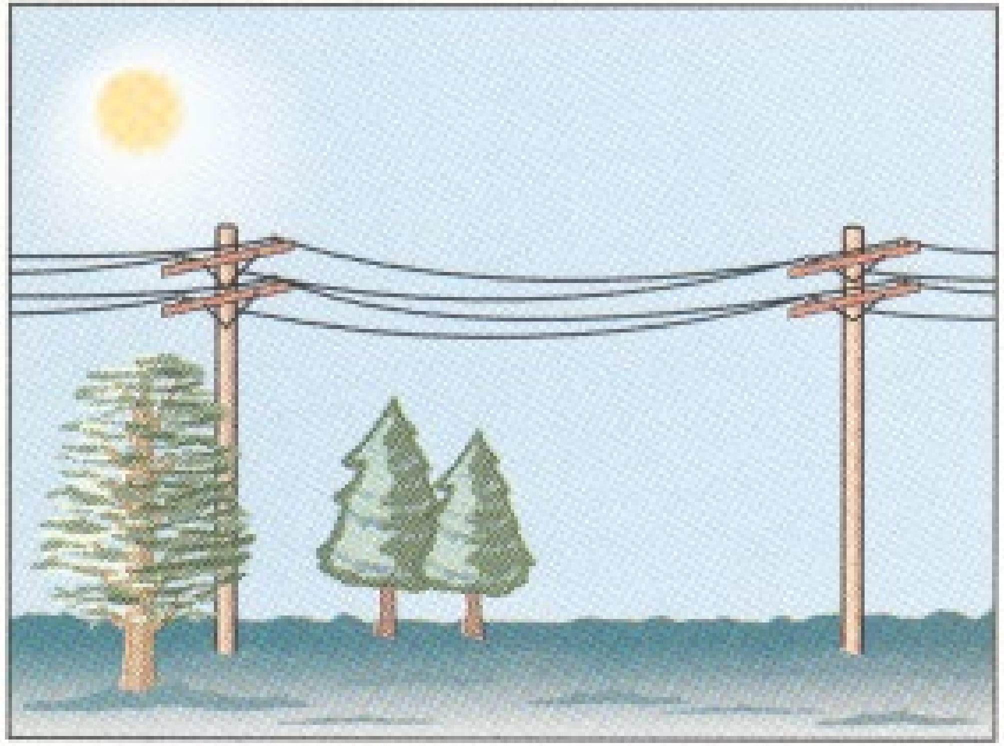
Announcements

- Homework Help Sessions with TAs: Monday, Tuesday 5-8pm
- This is an opportunity for you to discuss the homework questions with your classmates and ask questions to the TAs.
- Click on Zoom on the PHYS 157 Canvas site and then look under "Upcoming Meetings". Clicking on "Join" will open the zoom meeting.
- NOTE: These are completely optional and you are not required to participate. You can join whenever you like and stay as long as you want.
- Section 102 Office Hour: Tuesday 10:30-11:30 am, zoom (link on Canvas homepage)
- If you would like more practice beyond the Mastering Physics and Written Homework problems, you can have a look at the problems at the back of the chapters in the text. Answers are given for odd numbered problems.

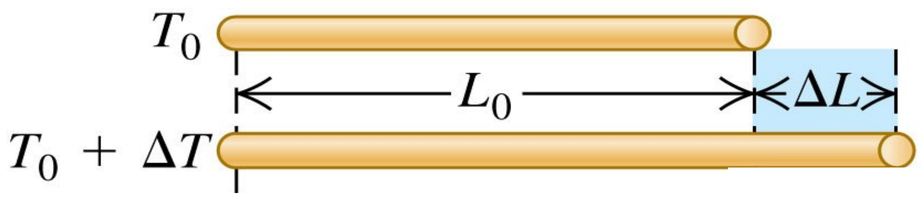
Lecture 5.
Thermal expansion





Last time:

- We talk about thermal expansion of solid objects
- So far we are interested in their linear expansion (1D)
- Thermal expansion depends on:
  - ➢ Change of temperature, ΔT
  - ➢ Initial length of the object, L_0
  - ➢ Material properties (captured by the coefficient of linear thermal expansion, α)



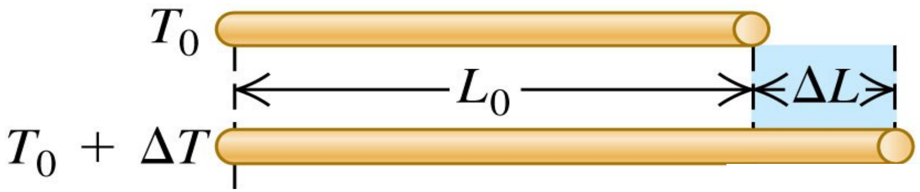
$$\Delta L = \alpha \, L_0 \, \Delta T$$

Coefficient of linear expansion is a material constant

TABLE 17.1 Coefficients of Linear Expansion

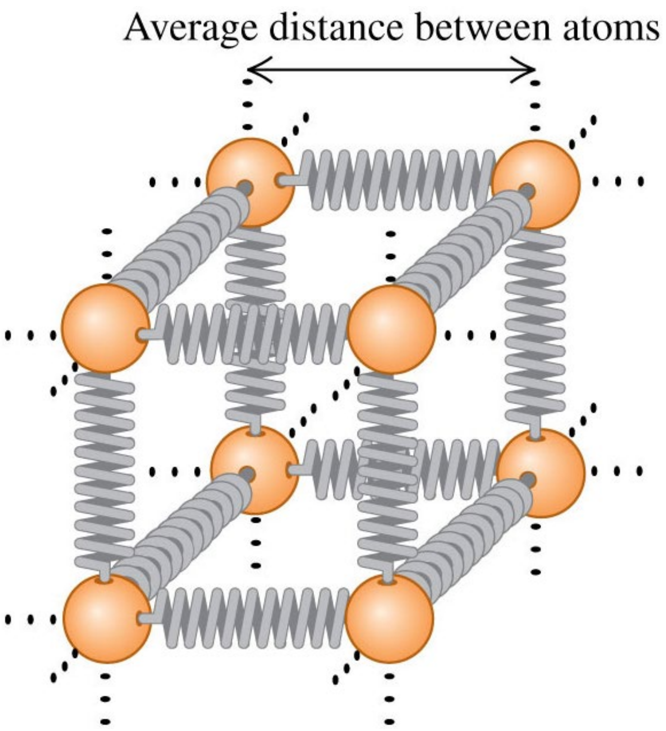
Material	α [K^{-1} or $(C°)^{-1}$]
Aluminum	2.4×10^{-5}
Brass	2.0×10^{-5}
Copper	1.7×10^{-5}
Glass	$0.4–0.9 \times 10^{-5}$
Invar (nickel–iron alloy)	0.09×10^{-5}
Quartz (fused)	0.04×10^{-5}
Steel	1.2×10^{-5}

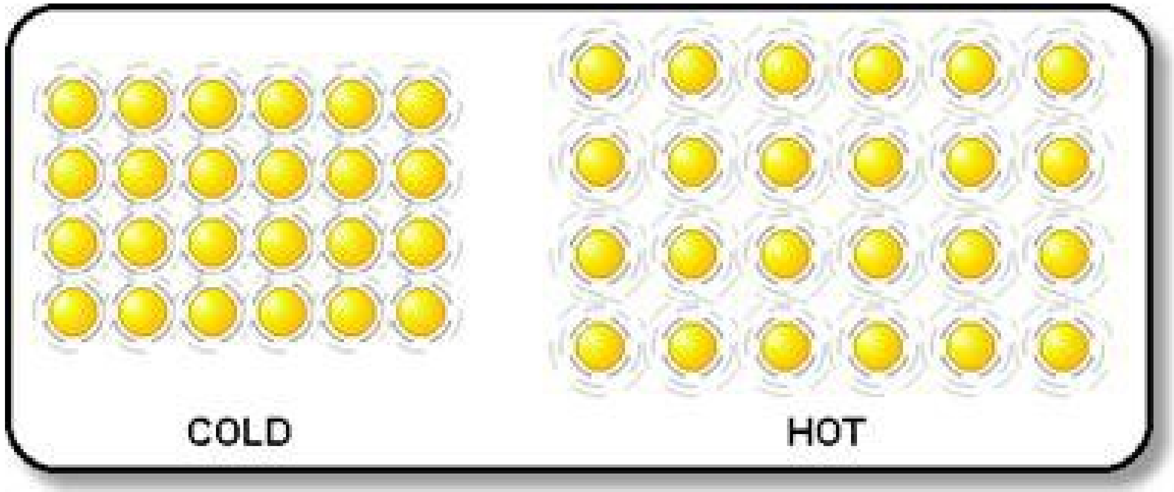
Textbook



$$\Delta L = \alpha L_0 \Delta T$$

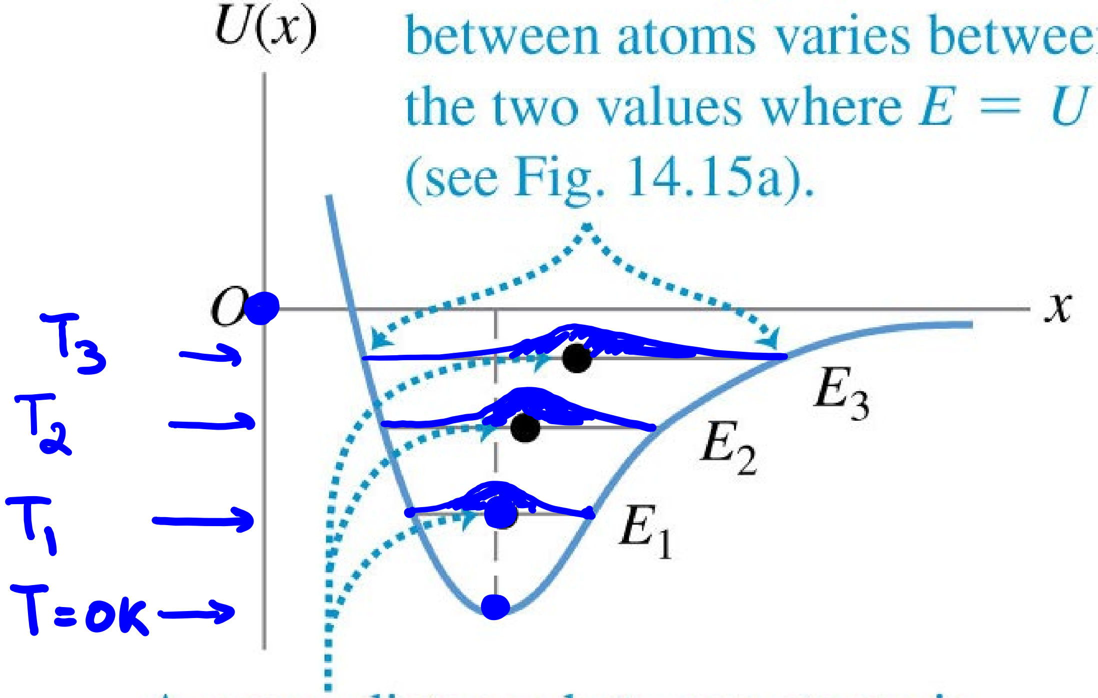
Why do materials usually expand when heated?





x = distance between atoms
● = average distance between atoms

For each energy E, distance between atoms varies between the two values where $E = U$ (see Fig. 14.15a).

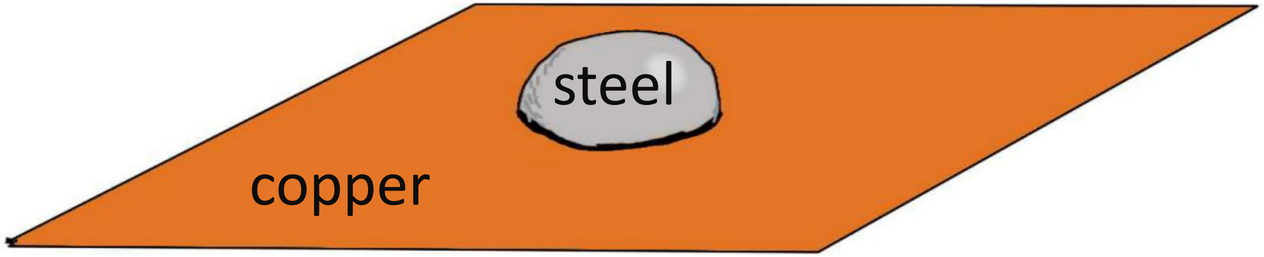


Average distance between atoms is midway between two limits. As energy increases from E_1 to E_2 to E_3, average distance increases.

Q: A steel ball does not quite fit through a hole in a copper plate.
If $\boxed{\alpha_{\text{steel}} < \alpha_{\text{copper}},}$ we could help the ball fit through the hole by:

Thermal expansion with a hole

A. Heating the system
B. Cooling the system
C. Either A or B will work
D. Neither A nor B will work

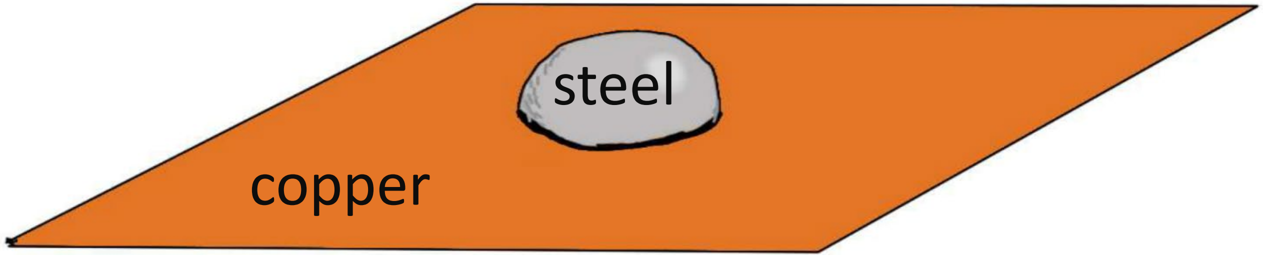


Ball and Ring Demo

Q: A steel ball does not quite fit through a hole in a copper plate. If $\alpha_{\text{steel}} < \alpha_{\text{copper}}$, we could help the ball fit through the hole by:

Thermal expansion with a hole

A. Heating the system ✓
B. Cooling the system
C. Either A or B will work
D. Neither A nor B will work

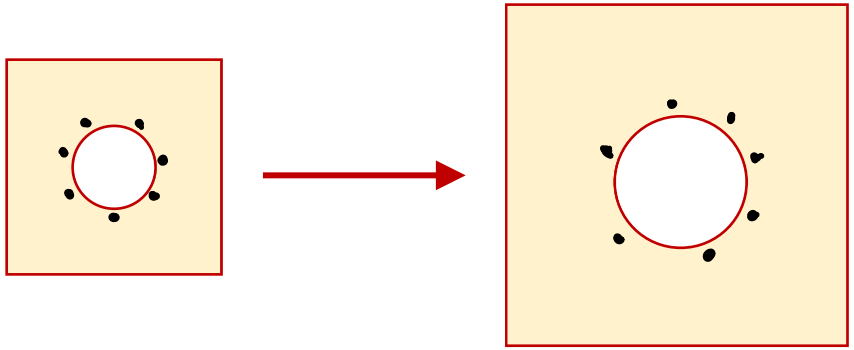


$$\Delta L = \alpha L_o \Delta T$$

Ball & hole both expand, but hole expands more since $\alpha_{\text{copper}} > \alpha_{\text{steel}}$

Q: Does the hole get larger or smaller when we heat the system? Why?

Hole grows in proportion to plate (same as if hole were filled)



If the radius of the steel ball at $T = 20^oC$ is $1.001\ cm$ and the radius of the hole in the copper plate is $1.000\ cm$, to what temperature must we heat the system before the ball falls through?

We have $\alpha_s = 1.2 \times 10^{-5} K^{-1}$ and $\alpha_c = 8 \times 10^{-5} K^{-1}$

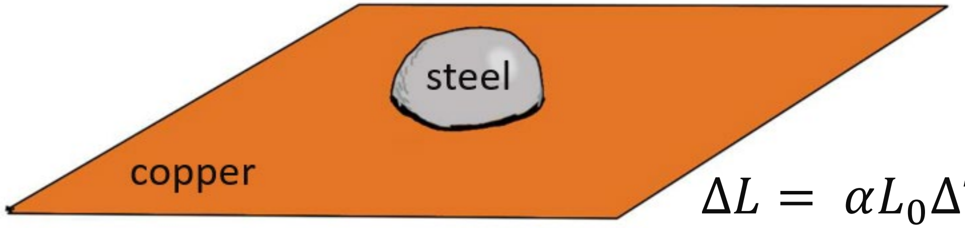
- Discuss a strategy for solving this.
- What should be true about ΔL_{ball} relative to ΔL_{hole}?

A. 5 °C

B. 15 °C

C. 25 °C

D. 50 °C

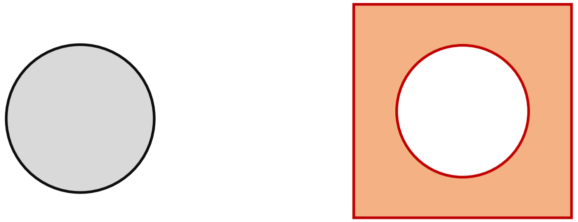


$\Delta L = \alpha L_0 \Delta T$

If the radius of the steel ball at $T = 20^oC$ is $1.001\ cm$ and the radius of the hole in the copper plate is $1.000\ cm$, to what temperature must we heat the system before the ball falls through?

We have $\alpha_s = 1.2 \times 10^{-5} K^{-1}$ and $\alpha_c = 8 \times 10^{-5} K^{-1}$

$R_b + \Delta R_b = R_h + \Delta R_h$ – goes through



$R_b + \alpha_s R_b \cdot \Delta T = R_h + \alpha_c R_h \Delta T$

$@T = 20\ ^oC \quad R_b = 1.001\ cm \quad R_h = 1\ cm$

$@T = ??\ ^oC \quad \underbrace{R_b' = R_b + \Delta R_b \quad R_h' = R_h + \Delta R_h}_{\text{same}}$

$R_b - R_h = (\alpha_c R_h - \alpha_s R_b)\,\Delta T$

$\Delta T = T_f - T_i =$

$\uparrow$ 20°C

$\Delta T = ?$

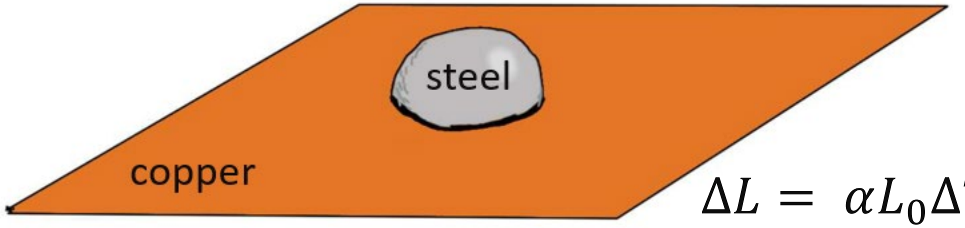
$$\Delta T = \frac{R_b - R_h}{\alpha_c R_h - \alpha_s R_b} = 14.7\ ^\circ C$$

A. 5 °C

B. 15 °C

C. 25 °C

D. 50 °C

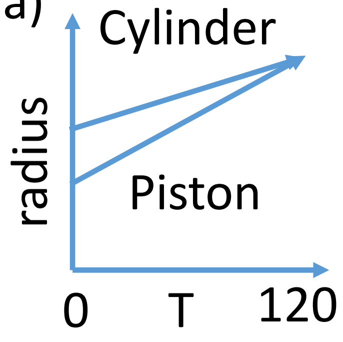


$\Delta L = \alpha L_0 \Delta T$

Q: In some car engines, the piston is aluminum ($\alpha = 2.4 \times 10^{-5}\ K^{-1}$), while the cylinder is cast iron ($\alpha = 1.2 \times 10^{-5}\ K^{-1}$). If the engine needs to operate between 0 °C and 120 °C, which of these is NOT a good design:

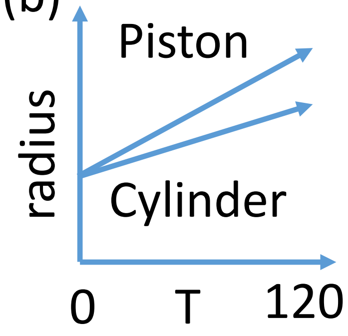
A. The piston barely fits in the cylinder at 120 °C

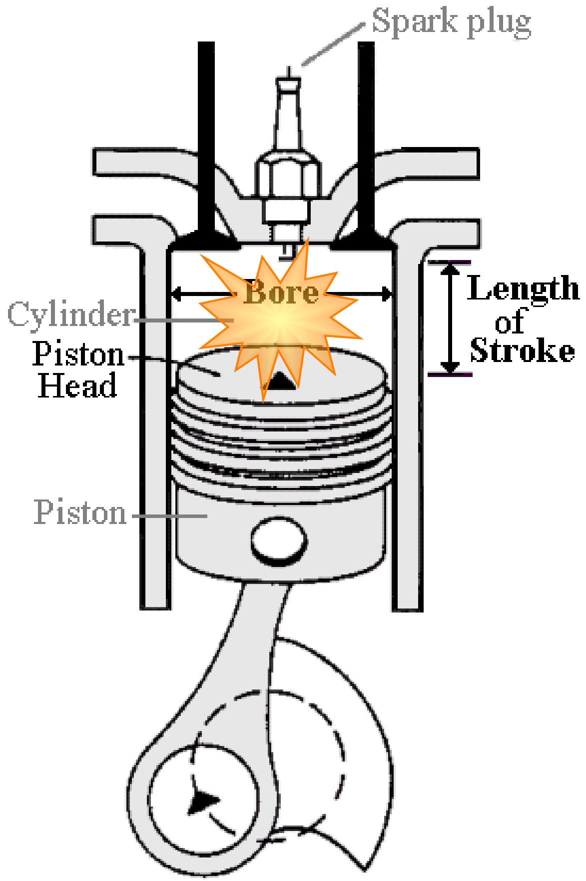
(a)



B. The piston barely fits in the cylinder at 0 °C

(b)





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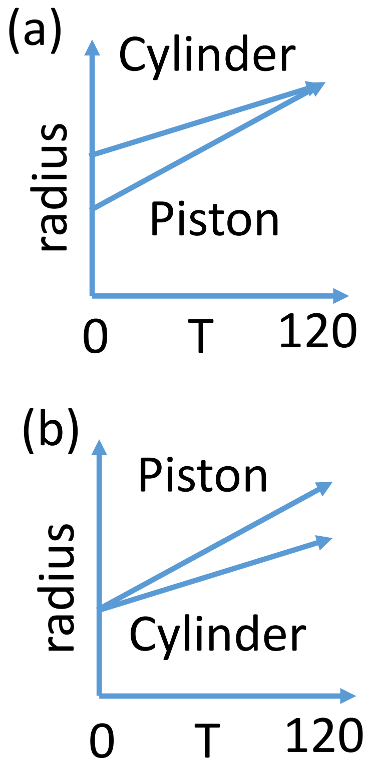
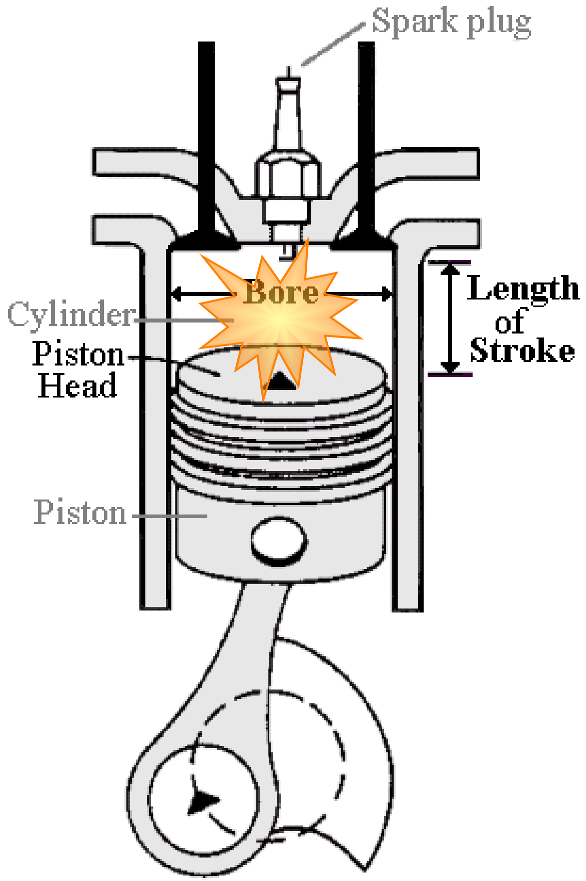
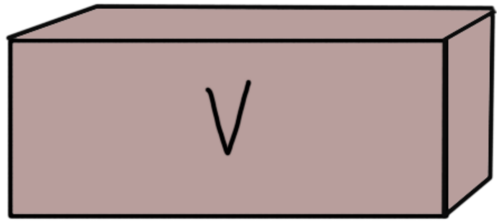


Piston expands more than cylinder as engine heats up, so wouldn't be able to move at higher temps

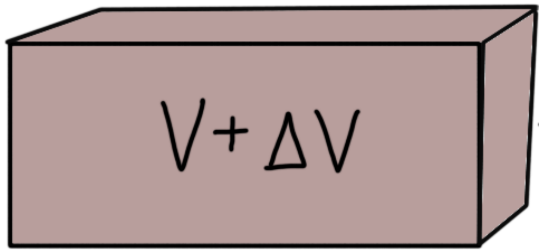


Extra: What do we need to worry about if the engine gets too hot? Too cold?

Volume expansion (3D)



Temperature
T



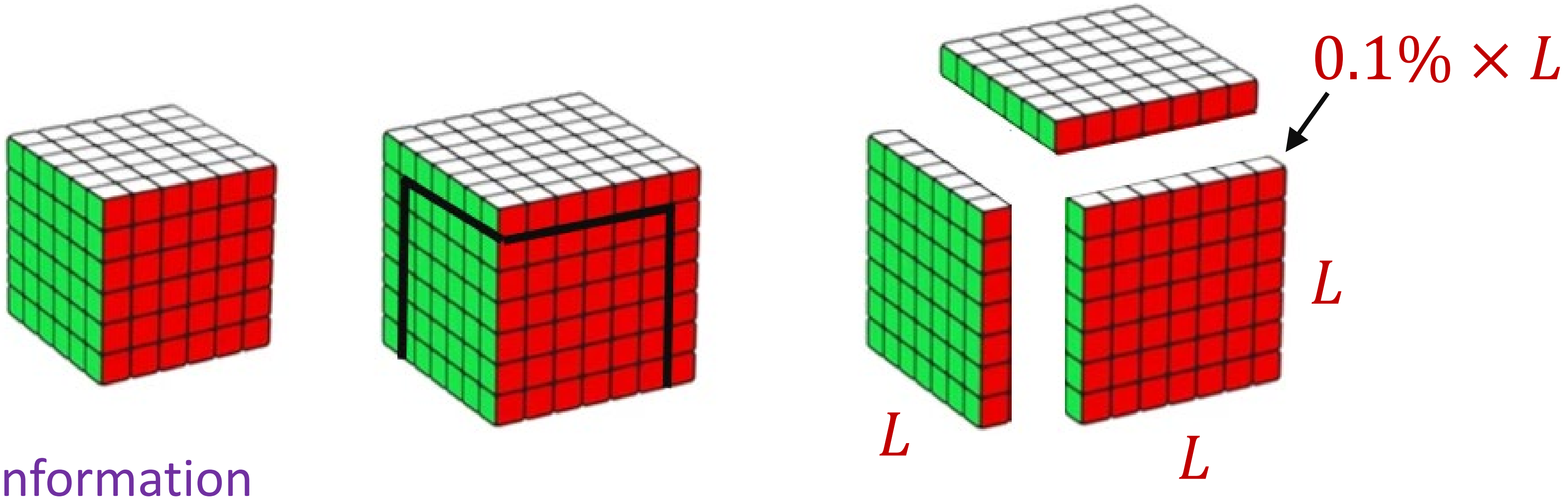
Temperature
$T + \Delta T$

$$\boxed{\Delta V = \beta\, V_0 \Delta T}$$

- β is the coefficient of volume expansion
- Units of β are K^{-1}
- assumes $\frac{\Delta V}{V_o}$ is small
- β can depend on temperature but the linear approximation is good over small temperature intervals
- Also applies to liquids

Q: When heated, each side of a cube of material expands by 0.1%. As a percentage of the original volume of the cube, the extra volume (shown in the third picture) after the expansion is:

A. 0.0000001%
B. 0.001%
C. 0.1%
D. 0.3%
E. There is not enough information



Look at the picture and use geometry to solve this!

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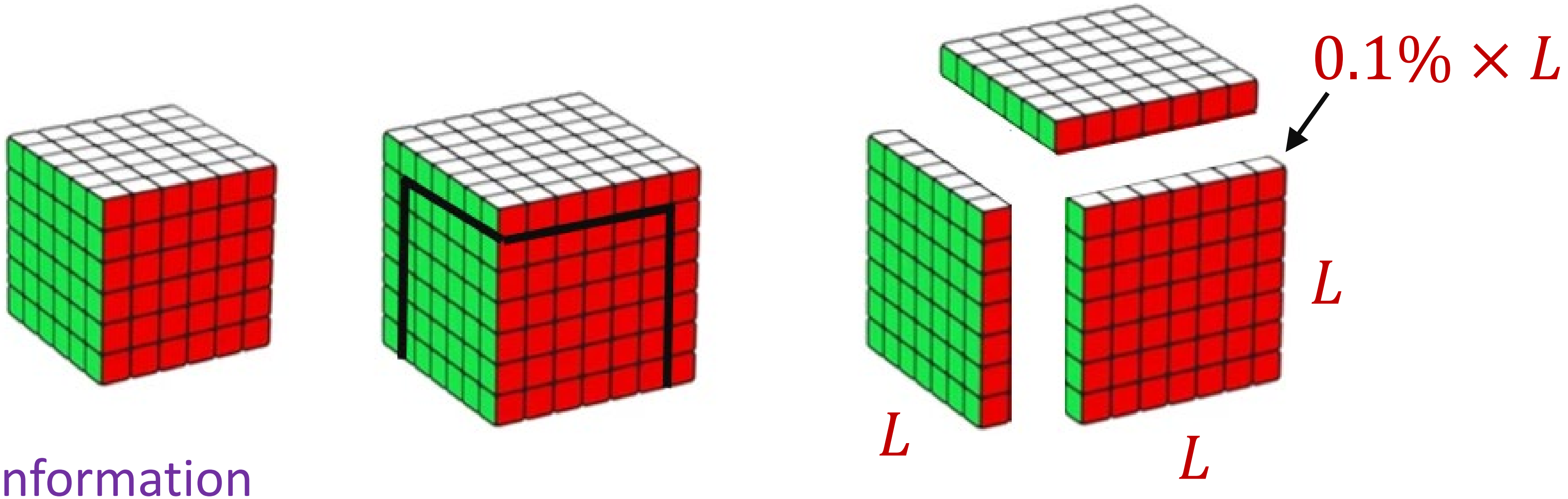


Since sides are L, L and $0.1\% \times L$, the volume of each part is $0.1\% \times L^3$.
So total increase is three times that, $\approx 0.3\% \times L^3$.

Look at the picture and use geometry to solve this!

Mathematical Derivation

$$\boxed{\Delta L = \alpha L_0 \Delta T}$$

$$\boxed{\Delta V = \beta V_0 \Delta T}$$

$i: \quad L \longrightarrow V_0 = L^3$

$f: \quad (L + \Delta L) \longrightarrow V' = (L + \Delta L)^3 =$

$$\boxed{\beta = 3\alpha}$$

$$= \underbrace{L^3}_{V_0} + \underbrace{3L^2\Delta L + 3L\Delta L^2 + \Delta L^3}_{\Delta V}$$

$$\frac{\Delta V}{V_0} \approx 3\,\frac{\Delta L}{L}$$

$$\beta \cancel{\Delta T} = 3\alpha \cancel{\Delta T}$$

✓

$$\frac{\Delta V}{V_0} = \frac{3L^2\Delta L + 3L\Delta L^2 + \Delta L^3}{L^3} = \underbrace{3\,\frac{\Delta L}{L}}_{10^{-3}} + \underbrace{\cancel{3\left(\frac{\Delta L}{L}\right)^2}}_{10^{-6}} + \underbrace{\cancel{\left(\frac{\Delta L}{L}\right)^3}}_{10^{-9}}$$

$0 \qquad 0$

Mathematical Derivation

$$\boxed{\Delta L = \alpha L_0 \Delta T}$$

$$\boxed{\Delta V = \beta V_0 \Delta T}$$

- Original volume: L^3
- New Volume: $(1.001 \times L)^3 \approx 1.003 \times L^3$
  - so 0.3% bigger
- More generally: $(L + \Delta L)^3 = \underbrace{L^3}_{V_0} + \underbrace{3L^2\Delta L + 3L(\Delta L)^2 + (\Delta L)^3}_{\Delta V}$

$$\frac{\Delta V}{V_0} = 3\frac{\Delta L}{L} + 3\left(\frac{\Delta L}{L}\right)^2 + \left(\frac{\Delta L}{L}\right)^3$$

This means $\beta = 3\alpha$

These are negligible compared to the first term if $\frac{\Delta L}{L}$ is small

Can we apply the same logic to a different shape (e.g. a sphere?)

$$i: \quad R \rightarrow V_0 = \frac{4\pi}{3} R^3$$

$$f: (R+\Delta R) \rightarrow V' = \frac{4\pi}{3}(R+\Delta R)^3 = \frac{4\pi}{3}\left(R^3 + 3R^2\Delta R + 3R\Delta R^2 + \Delta R^3\right)$$

$$= V_0 + \underbrace{\frac{4\pi}{3}\left(3R^2\Delta R + \cdots\right)}_{\Delta V}$$

$$\frac{\Delta V}{V} = \frac{\cancel{\frac{4\pi}{3}}\left(3 \cdot R^2\Delta R + 3R\Delta R^2 + \Delta R^3\right)}{\cancel{\frac{4\pi}{3}} R^3}$$

$$= 3\frac{\Delta R}{R} + \cancel{3\left(\frac{\Delta R}{R}\right)^2} + \cancel{\left(\frac{\Delta R}{R}\right)^3}$$

$$\boxed{\beta = 3\alpha}$$

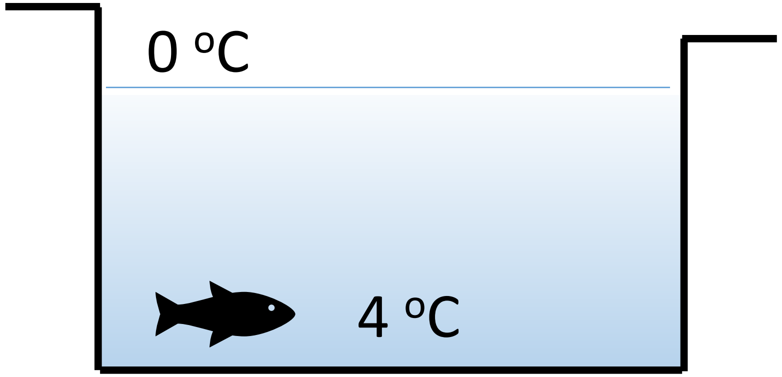
Volume expansion equations also apply to liquids, but…

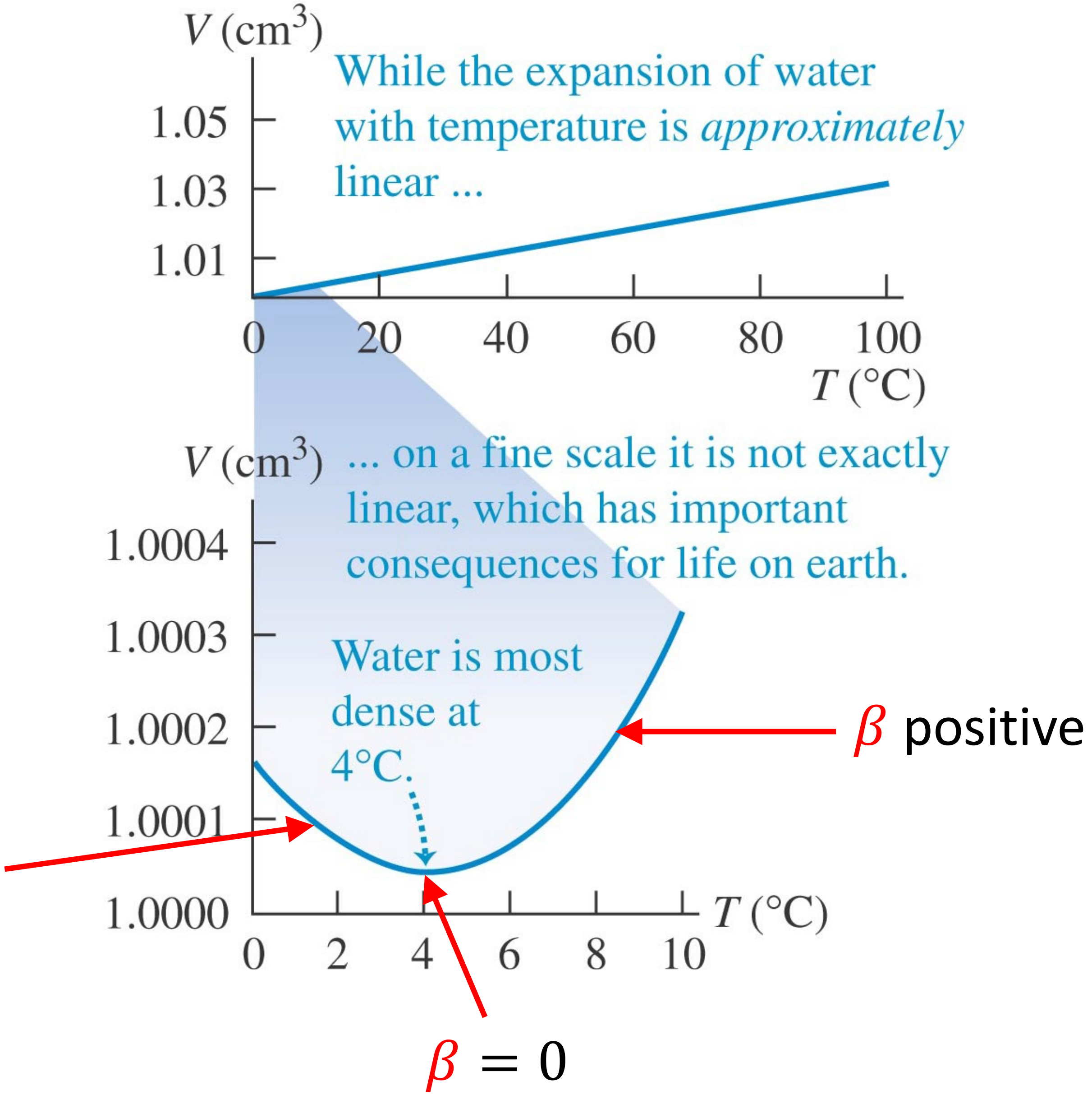
$$\boxed{\Delta V = \beta V_0 \Delta T} \quad \boxed{\beta = 3\alpha}$$

- Water: a special example

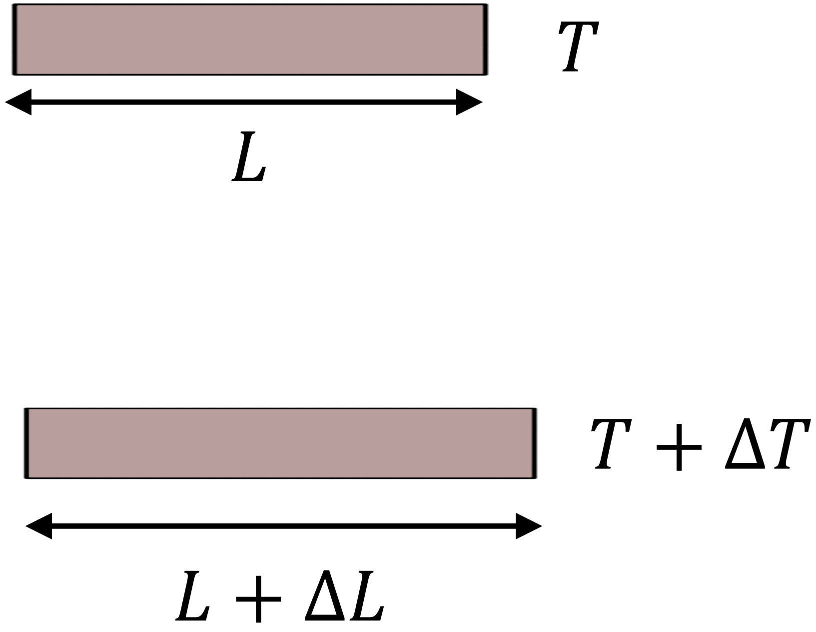
- Freezing water and glass bottles in a freezer

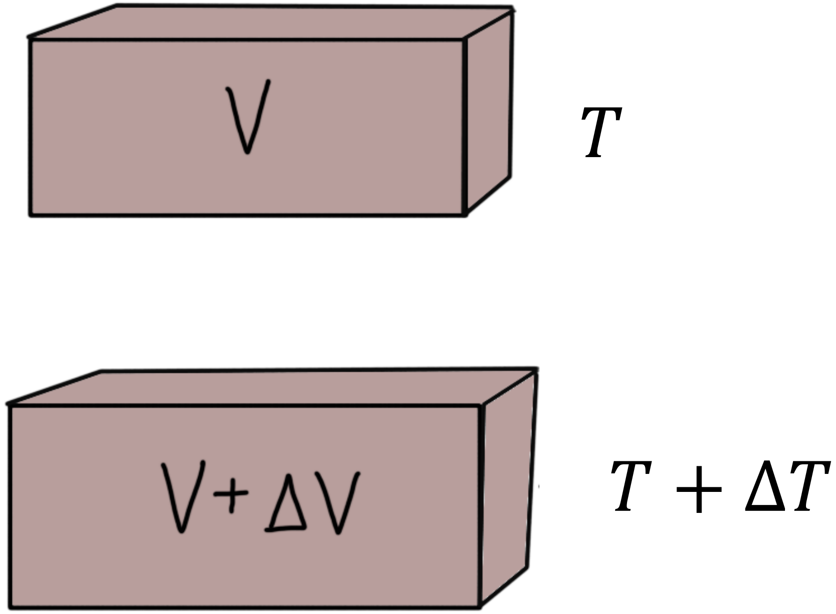
- Freezing water and life in lakes





Summary





$$\boxed{\Delta L = \alpha L_0 \Delta T}$$

$$\boxed{\Delta V = \beta V_0 \Delta T}$$

$$\beta = 3\alpha$$