Announcements:

- Deadline for all HW assignments: Sunday => Wednesday

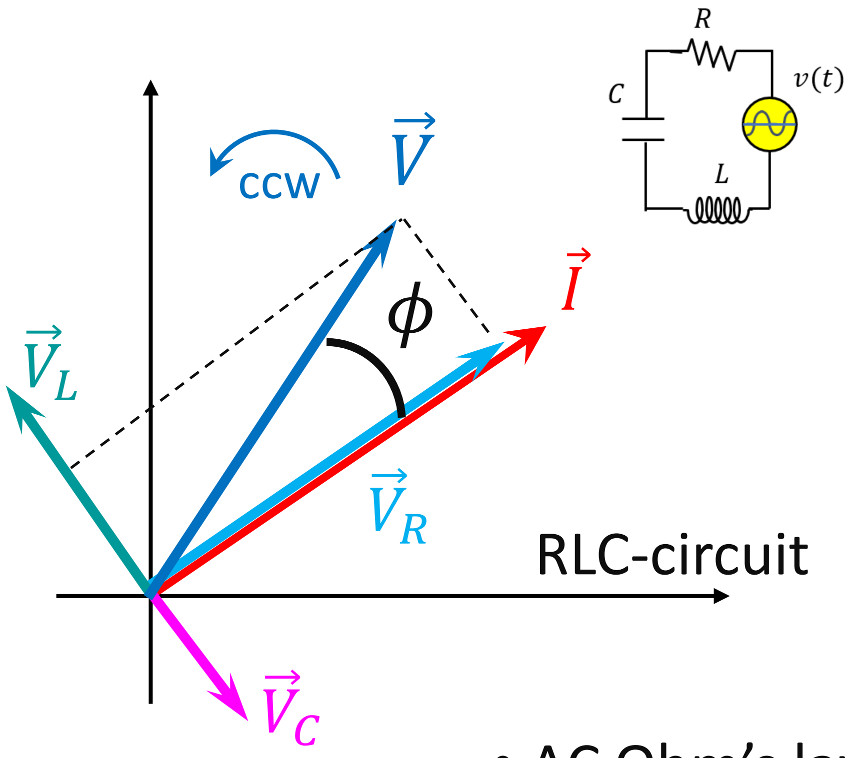
- In short: you can nominate students for an award

Lecture 11.

AC circuits and Impedance Triangle. Practice.

Last Time:

$$v_{\text{source}}(t) = V_{max}\cos(\omega t)$$

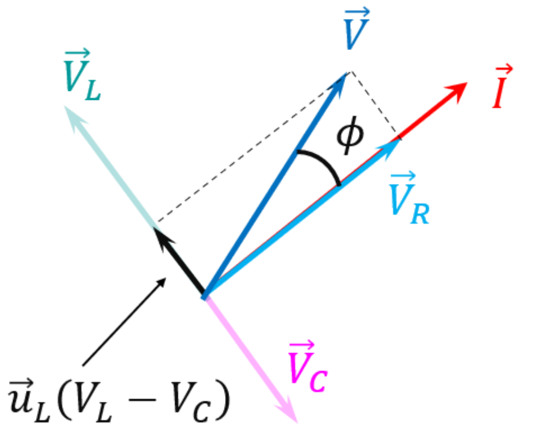


- Phasors – abstract vectors that help us trace phase delays between the current and the voltage drops
- Their projection on the x-axis is the instantaneous voltage / instantaneous current
- "Resistances":
  - $V_R = IX_R \quad (X_R = R)$
  - $V_L = IX_L \quad (X_L = \omega L)$
  - $V_C = IX_C \quad (X_C = 1/\omega C)$
- Impedance: $Z = \sqrt{X_R^2 + (X_L - X_C)^2}$
- AC Ohm's law: $V_{max} = IZ$
- Phase V to I: $\tan\phi = \frac{X_L - X_C}{R}$

AC RLC series circuit: Impedance Triangle

$$Z = \sqrt{X_R^2 + (X_L - X_C)^2}$$

$$\tan(\phi) = \frac{X_L - X_C}{X_R}$$



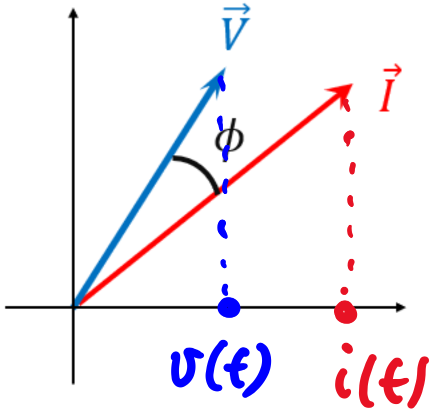
$X_R = R$

$X_L = \omega L$

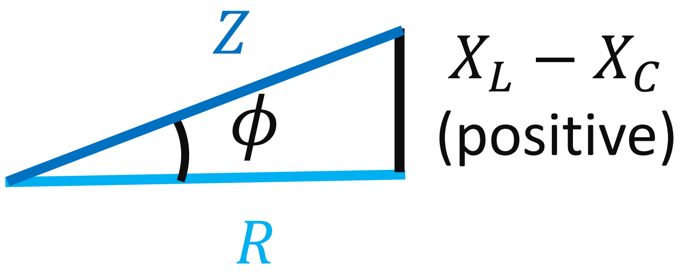
$X_C = 1/\omega C$

$V_{max} = I_{max}\, Z$

- If $X_L > X_C$, current lags voltage:

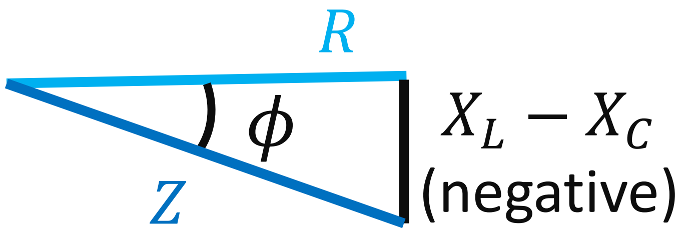


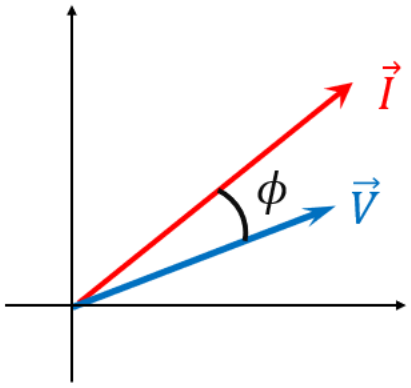
"Impedance triangles"



- $i(t) = I_{max} \cos \omega t$
- $v(t) = (I_{max} Z) \cos(\omega t + \phi)$

- If $X_L < X_C$, current leads voltage:





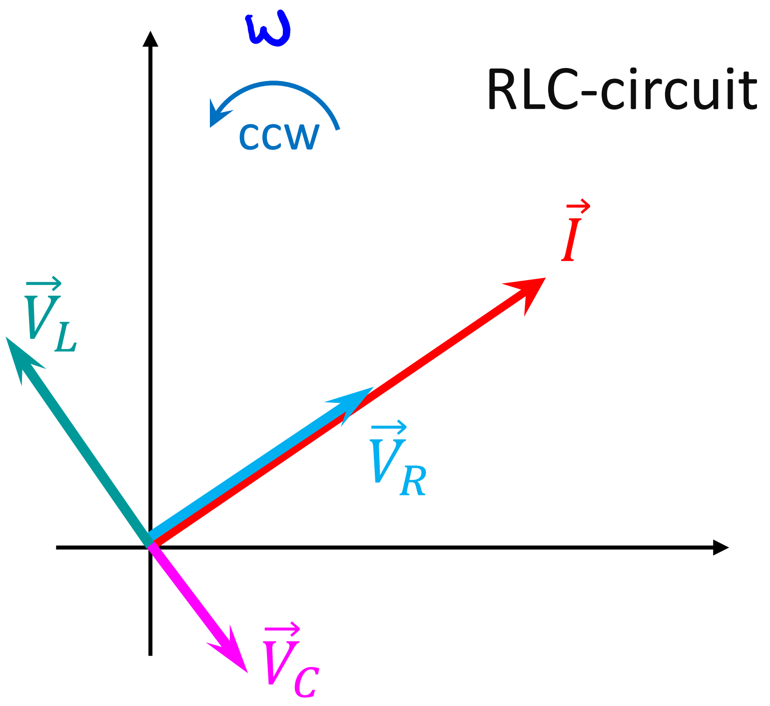
- $i(t) = I_{max} \cos \omega t$
- $v(t) = (I_{max} Z) \cos(\omega t - |\phi|)$

AC RLC series circuit: Peak and Instantaneous values

$X_R = R$

$X_L = \omega L$

$X_C = 1/\omega C$



- Peak values across elements:

$$V_R = RI_{max} \qquad V_L = X_L I_{max} \qquad V_C = X_C I_{max}$$

➢ Note that the do not occur at the same time!

- Peak value of the source voltage: $V_{max} = I_{max}\, Z$

- Instantaneous values:

$$v_{\text{source}}(t) = v_R(t) + v_L(t) + v_C(t)$$

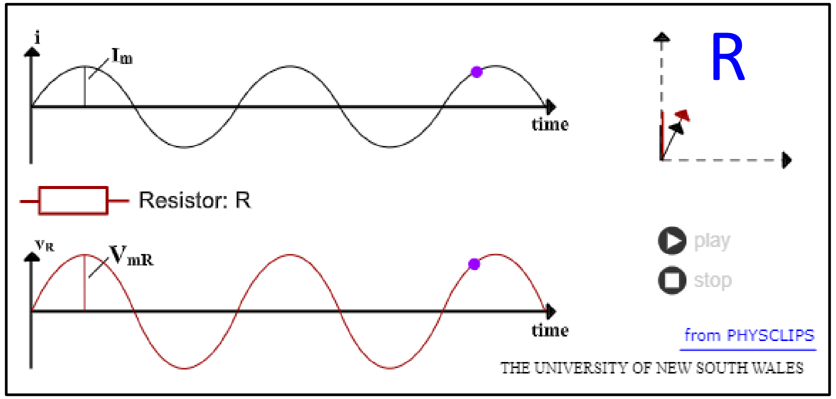
$$= V_R \cos(\omega t) + V_L \cos(\omega t + \pi/2) + V_C \cos(\omega t - \pi/2)$$

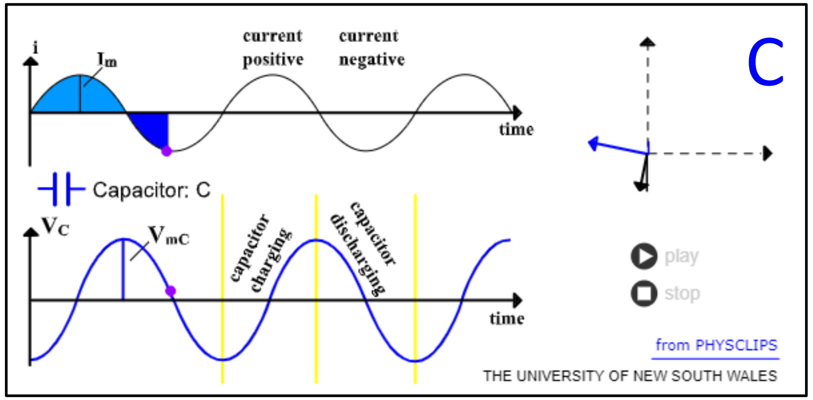
$$= V_{max} \cos(\omega t + \phi)$$

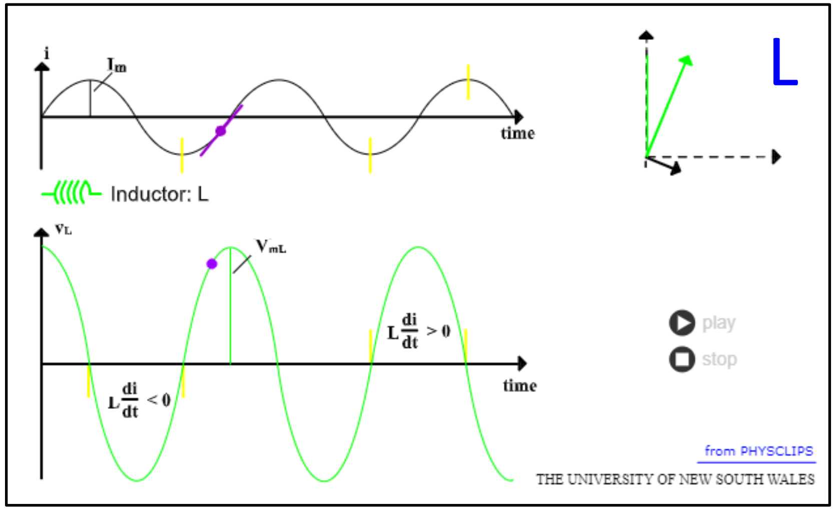
Phasor animations

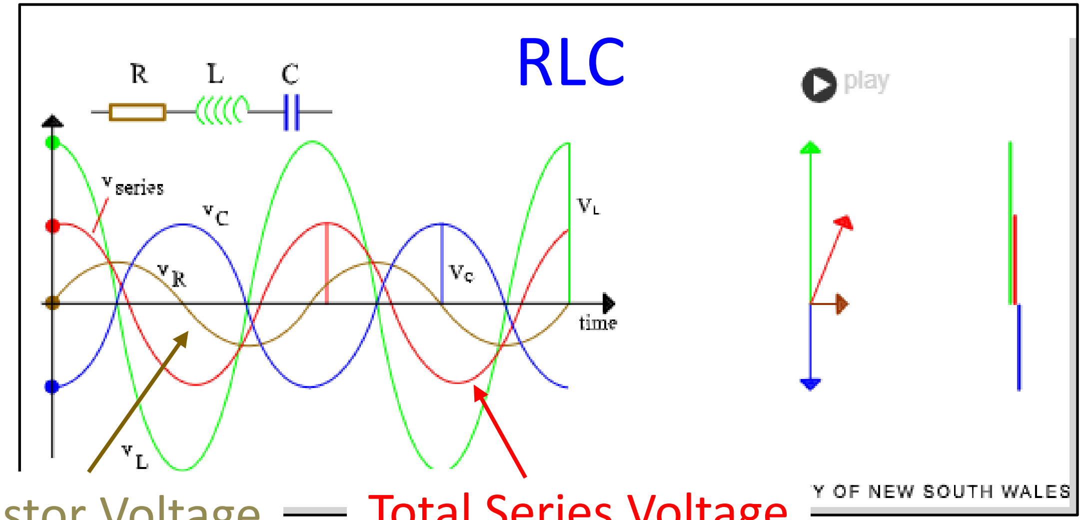
University of New South Wales, Australia

https://www.animations.physics.unsw.edu.au/jw/AC.html



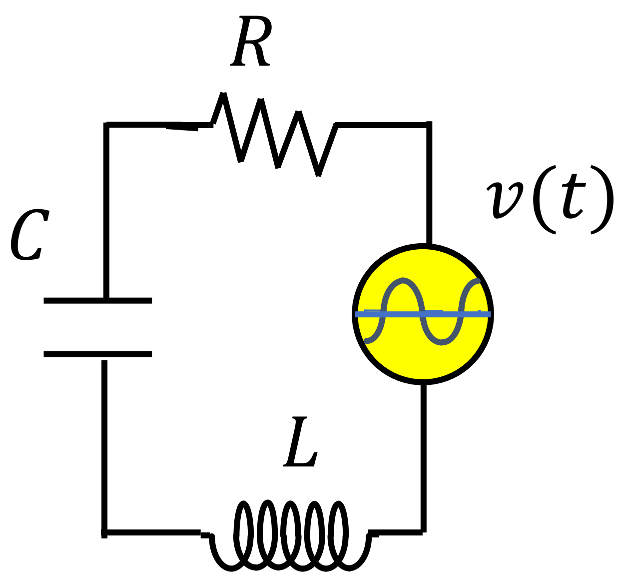






Resistor Voltage — Total Series Voltage

Q: What statement below about an RLC circuit is accurate?



A. The instantaneous voltage v is the same across all three circuit elements

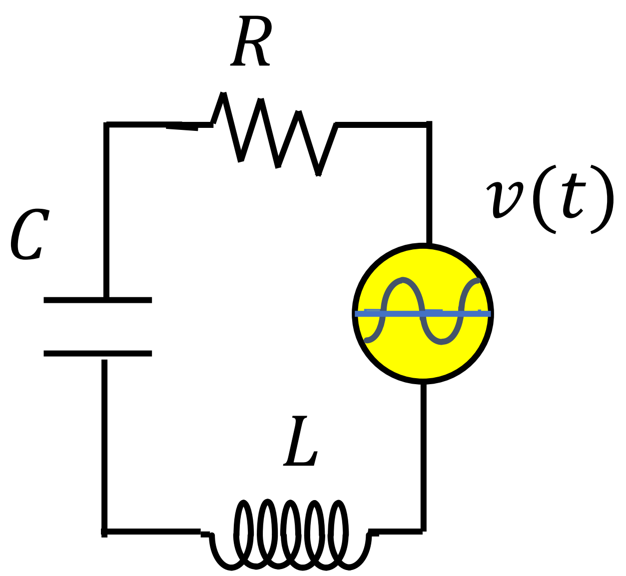
B. The instantaneous current i is the same across all three circuit elements

C. The instantaneous voltage v AND the current i are both the same across all three circuit elements

D. The instantaneous voltage v AND the current i are NOT always the same across all three circuit elements

E. Not sure

Q: What statement below about an RLC circuit is accurate?



- All three elements (R, L, and C) in the circuit are in series, so the instantaneous current through them must be the same.

A. The instantaneous voltage v is the same across all three circuit elements

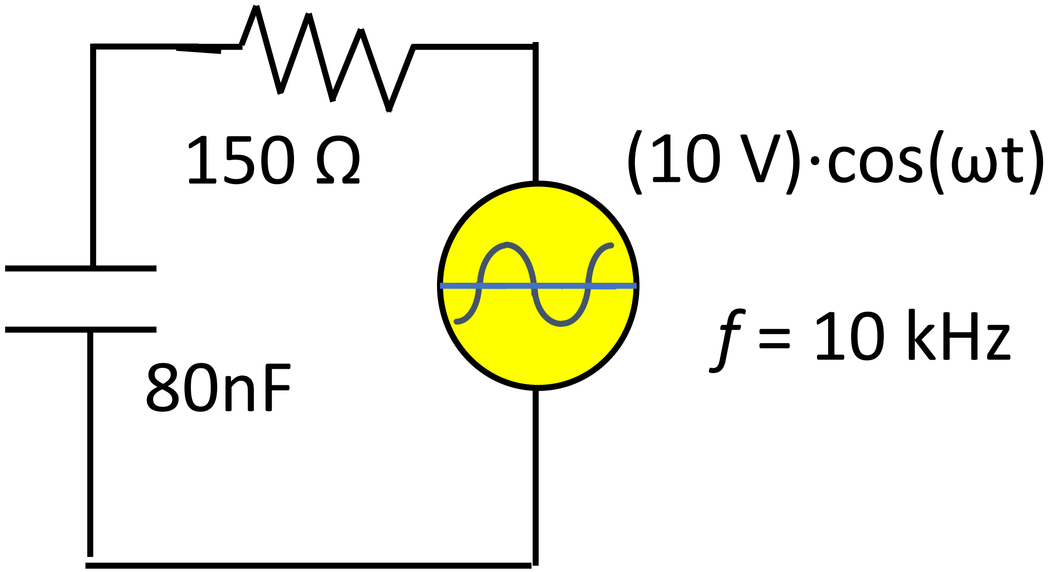
B. The instantaneous current i is the same across all three circuit elements

C. The instantaneous voltage v AND the current i are both the same across all three circuit elements

- The instantaneous voltages v_R, v_L and v_C don't have to be the same, but $v_{\text{source}}(t) = v_R(t) + v_L(t) + v_C(t)$ (algebraic sum at each moment of time).
- Peak voltages add up as vectors: $\vec{V}_{\text{source}} = \vec{V}_R + \vec{V}_L + \vec{V}_C$

Q: a) What is the peak voltage across the resistor, V_R^{peak}, in this circuit?

b) What is the peak voltage across the capacitor, V_C^{peak}?

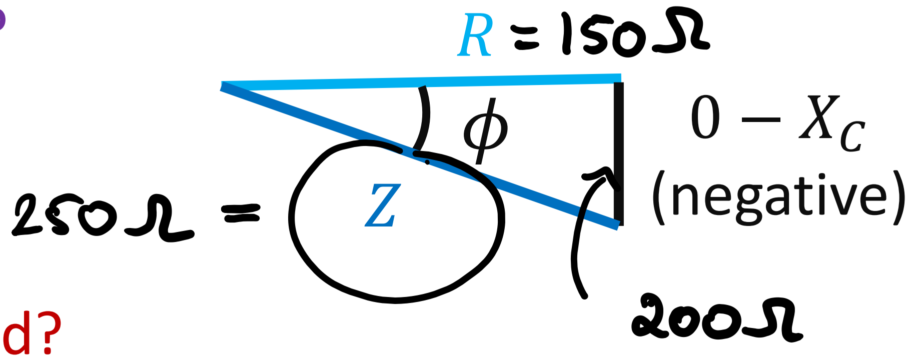


A. 0 V
B. 4 V
C. 6 V
D. 8 V
E. 10 V

Q: a) What is the peak voltage across the resistor, V_R^{peak}, in this circuit?

b) What is the peak voltage across the capacitor, V_C^{peak}?

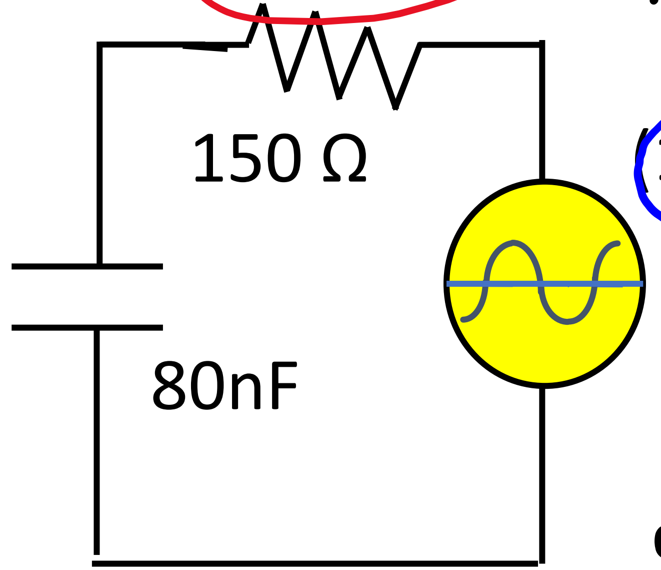
$X_c = \frac{1}{\omega C} = 200\,\Omega$



- Start with drawing the phasor diagram!
- Who is in your diagram? Who is ahead, who is behind?
- What do you know in this diagram, what you need to find?

$I_{max} = \frac{V_{max}}{Z}$

$V_R = I_{max} R = V_{max} \frac{R}{Z} = 6$

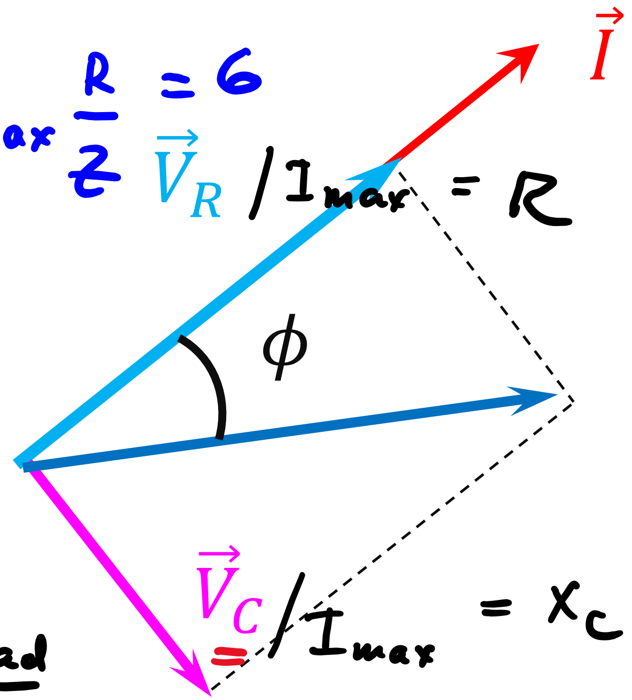


$\underbrace{(10\text{ V})}_{V_{max}} \cdot \cos(\omega t)$

$f = 10$ kHz

$\downarrow$

$\omega = 2\pi f = 2\pi \cdot 10^4 \frac{\text{rad}}{s}$



$V_{max} \frac{X_c}{Z} = V_c = I_{max} X_c = 8\text{ V}$

A. 0 V

B. 4 V

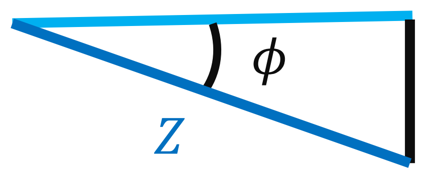
C. 6 V

D. 8 V

E. 10 V

Q: a) What is the peak voltage across the resistor, V_R^{peak}, in this circuit?

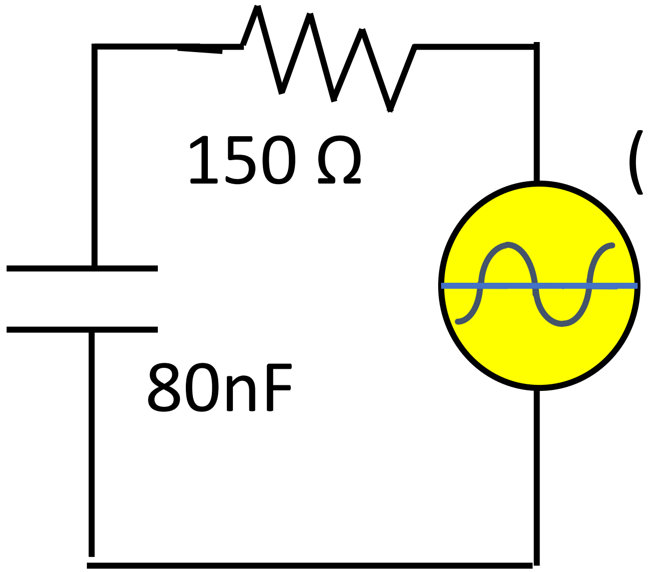
b) What is the peak voltage across the capacitor, V_C^{peak}?



Current in the circuit is: $I_{peak} = \dfrac{V_{peak}}{Z}$ Both R and C affect the current !!!

f = 10 kHz => $\omega = 2\pi f$

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = \sqrt{(150)^2 + \left(\frac{1}{2\pi \cdot 10^4 \cdot 8 \cdot 10^{-8}}\right)^2} = 250\ \Omega$$



$$V_R = I_{peak} R = V_{peak}\frac{R}{Z} = 10\,\frac{150}{250} = 6\ \text{V}$$

$$V_C = I_{peak} X_C = V_{peak}\frac{(1/\omega C)}{Z} = 10\,\frac{200}{250} = 8\ \text{V}$$

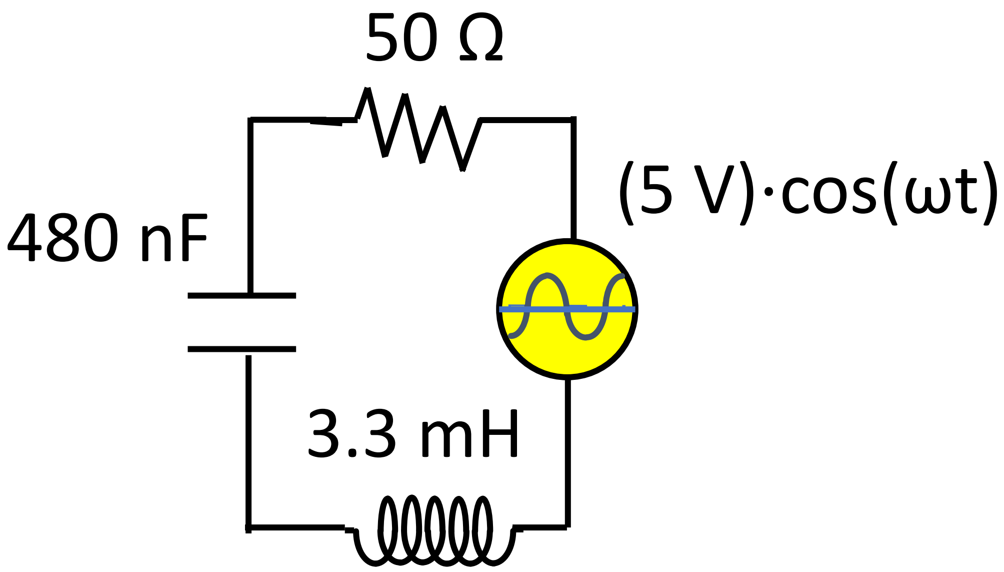
Q: For this RLC circuit:

a) Determine the impedance

b) Determine I_{peak}

c) Determine ϕ at f = 3kHz, 4kHz, & 5kHz.

Does the current lead or lag the voltage?

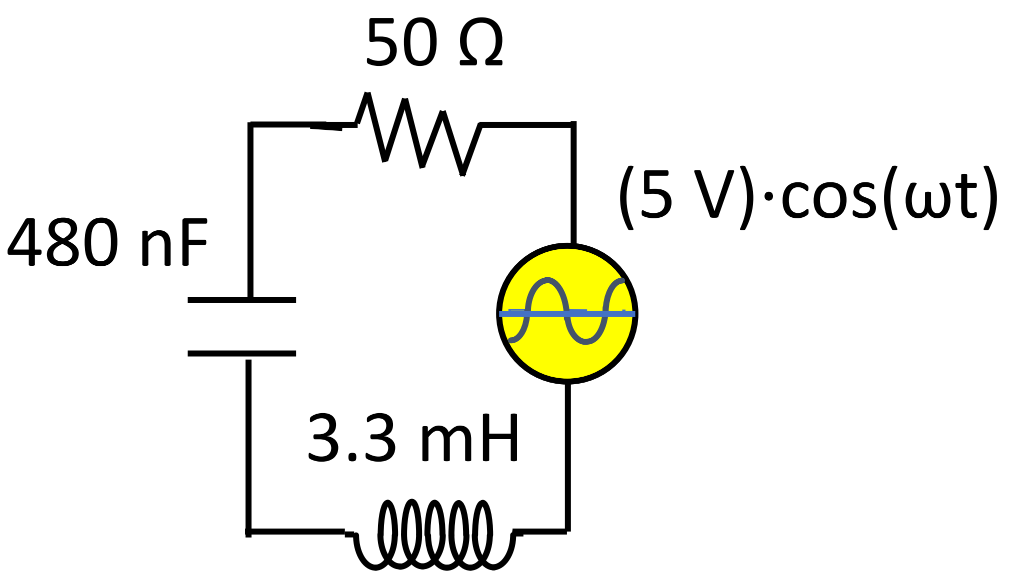


Q: For this RLC circuit:

a) Determine the impedance

b) Determine I_{peak}

c) Determine ϕ at f = 3kHz, 4kHz, & 5kHz



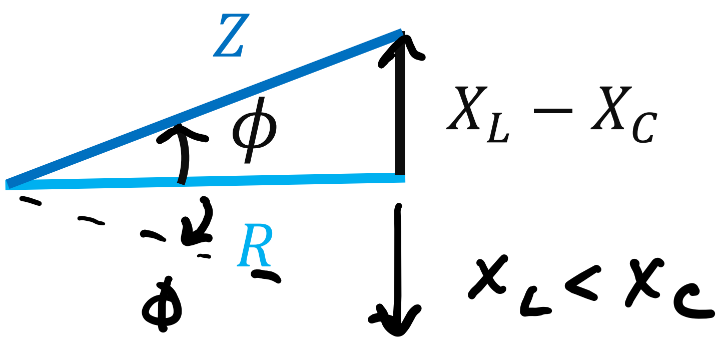
$\omega = 2\pi f$

- Set up impedance triangle:

$X_R = R$

$X_L = \omega L$

$X_C = 1/\omega C$



- Note that angle ϕ can be positive or negative, we have to compute X_L and X_C to know for sure

- Now:

$$Z^2 = R^2 + (X_L - X_C)^2$$

$$Z(\omega) = \sqrt{(20\ \Omega)^2 + \left(\omega(3.3\ mH) - \frac{1}{\omega(480\ nF)}\right)^2}$$

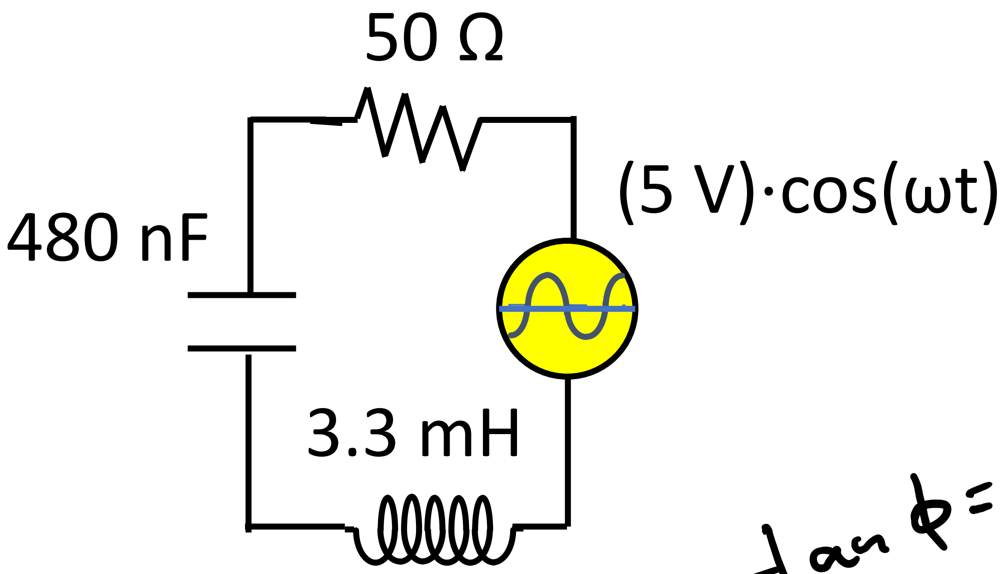
Q: For this RLC circuit:

a) Determine the impedance

b) Determine I_{peak}

c) Determine ϕ at $f = \underline{3\text{kHz}}$, $\underline{4\text{kHz}}$, & $\underline{5\text{kHz}}$.

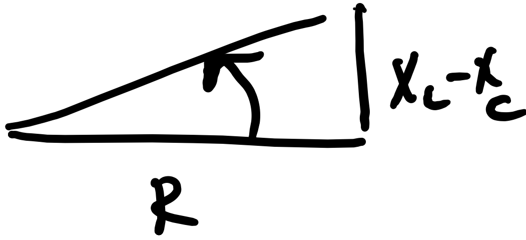
Does the current lead or lag the voltage?



$\tan\phi = \frac{X_L - X_C}{R}$

- I_{peak} is linked to V_{peak} of the source voltage by the impedance:

$f \rightarrow \omega$



$$I_{peak}(\omega) = \frac{V_{peak}}{Z} = \frac{5\ V}{\sqrt{(50\ \Omega)^2 + \left(\omega(3.3\ mH) - \frac{1}{\omega(480\ nF)}\right)^2}}$$

The peak current depends on the oscillation frequency $\omega = 2\pi f$

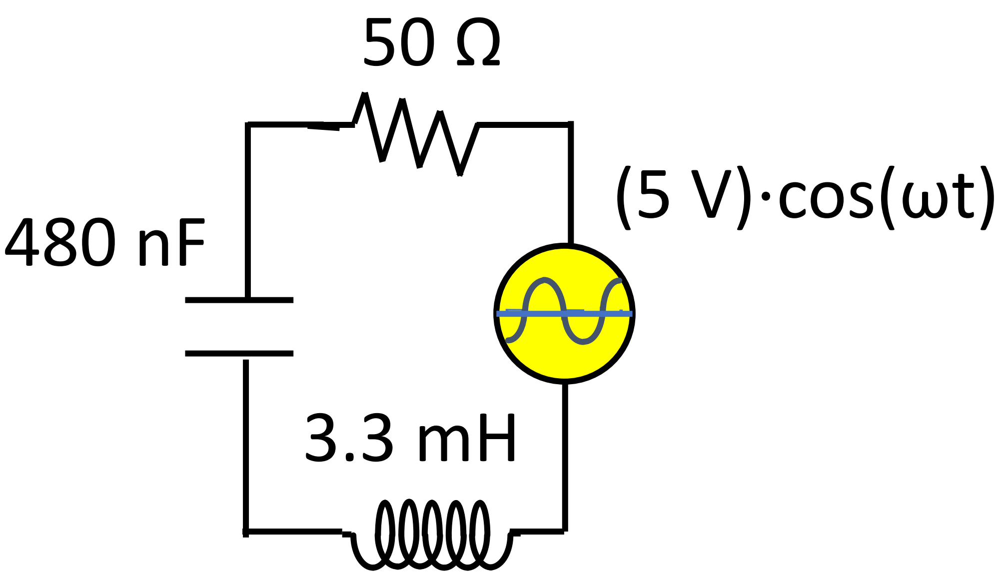
Q: For this RLC circuit:

a) Determine the impedance

b) Determine I_{peak}

c) Determine ϕ at f = 3kHz, 4kHz, & 5kHz.

Does the current lead or lag the voltage?

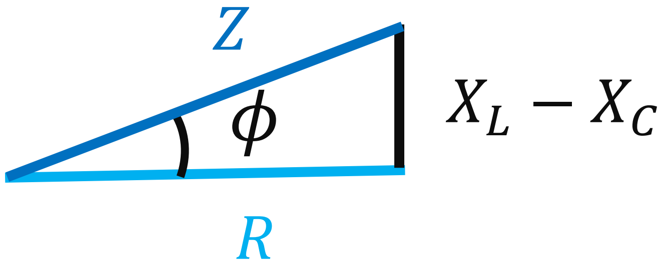


- Set up impedance triangle:

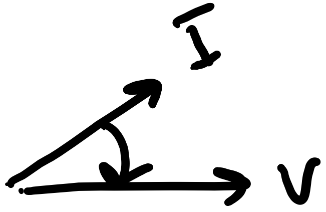
$$X_R = R$$

$$X_L = \omega L$$

$$X_C = 1/\omega C$$



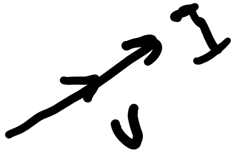
f => $\omega = 2\pi f$

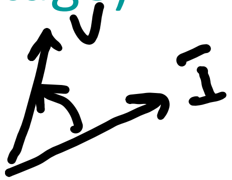


- Phase:

$$\phi(\omega) = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

- $\phi(3\ \mathrm{kHZ}) = -44^o$ (Current leads voltage)
- $\phi(4\ \mathrm{kHZ}) = 0^o$ (Current in phase with voltage)
- $\phi(5\ \mathrm{kHZ}) = +37^o$ (Current lags voltage)

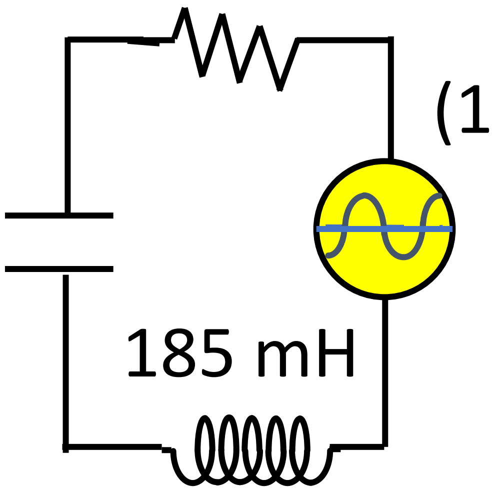




Q: An AC circuit with $V_{peak} = 150$ V and $f = 50$ Hz drives this RLC circuit.

a) What is the peak voltage across the resistor?

b) What is the peak voltage across the inductor?



$f = 50\ H \longrightarrow \omega = 2\pi f = 341.2\ \frac{rad}{s}$

$1/\omega C = X_c = \cdots\ 50\ \Omega$

$L\cdot\omega = X_L = \cdots\ 58.1\ \Omega$

$Z = \cdots\ 40.8\ \Omega \quad = \sqrt{R^2 + (X_L - X_c)^2}$

A. 0 V

B. 4 V

C. 120 V

D. 145 V

E. 215 V

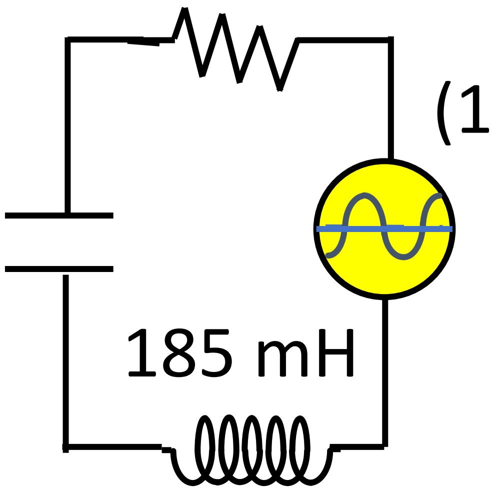
$V_R = \cdots\ V_{max}\frac{R}{Z} = 145\ \Omega$

$V_L = \cdots\ V_{max}\frac{X_L}{Z} = 215\ \Omega$

Q: An AC circuit with $V_{peak} = 150$ V and $f = 50$ Hz drives this RLC circuit.

a) What is the peak voltage across the resistor?

b) What is the peak voltage across the inductor?



- Compute I_{peak} with Ohm's law: $I_{peak} = \frac{V_{peak}}{Z}$

$$\left.\begin{aligned} X_L &= 2\pi(50)(185\times10^{-3}) = 58\ \Omega \\ X_C &= 49\ \Omega \end{aligned}\right\} \quad Z = \sqrt{40^2 + (58.1 - 49)^2} = 41\ \Omega$$

A. 0 V
B. 4 V
C. 120 V
D. 145 V
E. 215 V

- Applying Ohm's law to R and L:

$$V_R = I_{peak}R = V_{peak}\frac{R}{Z} = 150\cdot\frac{40}{41} = 146\ \cancel{\Omega}\ \text{V}$$

$$V_L = I_{peak}X_L = V_{peak}\frac{X_L}{Z} = 150\cdot\frac{58}{41} = 212\ \cancel{\Omega}\ \text{V}$$

larger than Vmax???
(next time)