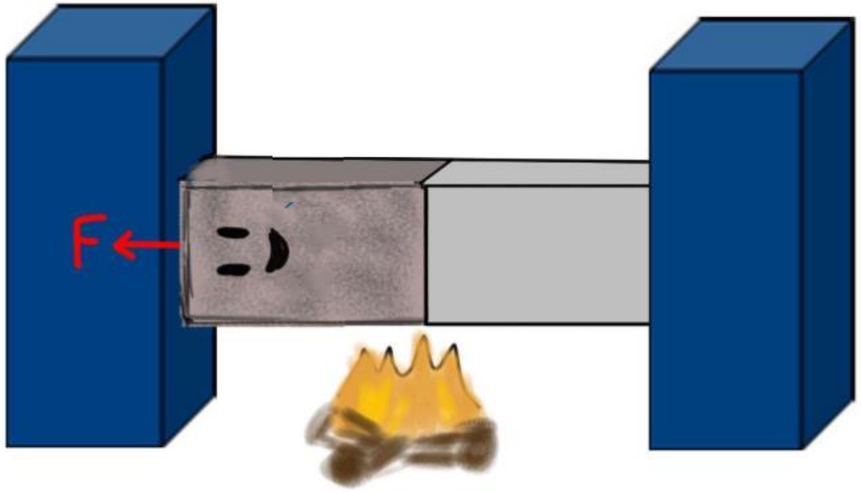
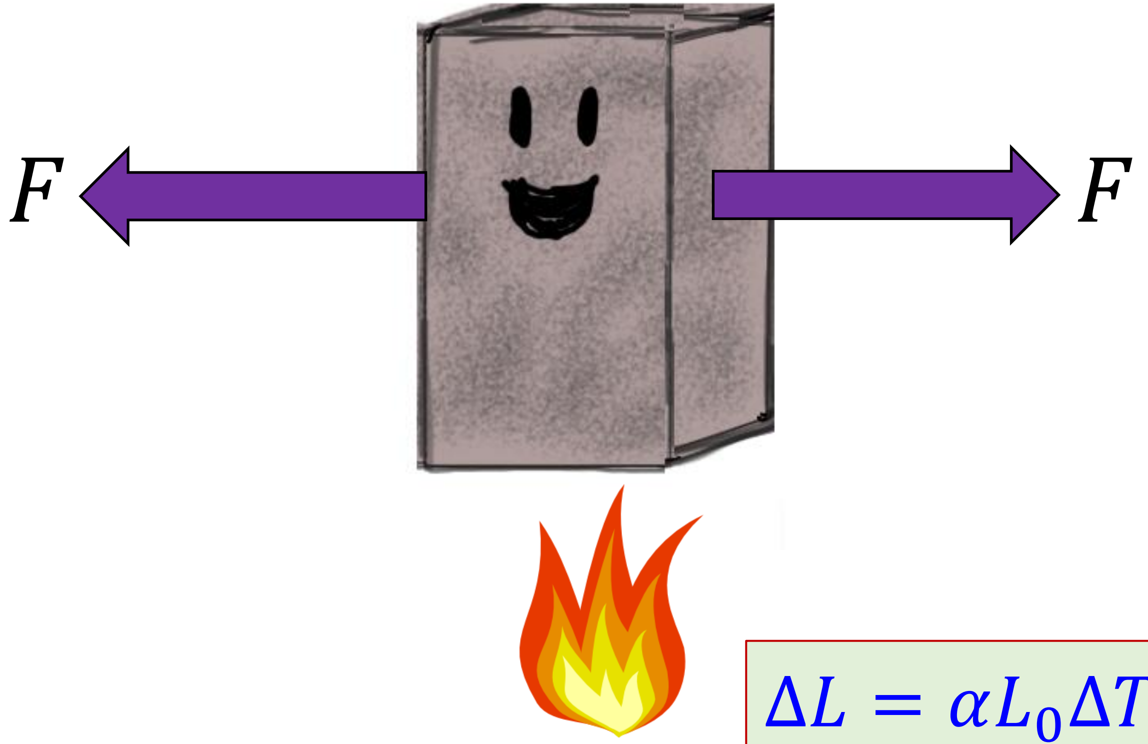


Lecture 7.
Thermal expansion &
Mechanical compression

Last time:



$$\frac{F}{A} = Y\,\frac{\Delta L}{L_0}$$

$$\Delta L = \alpha L_0 \Delta T$$

Q: A steel rod of length L_0 is heated by temperature ΔT. How would you determine how much stress (force per unit area) is required to compress the rod back to its original length?

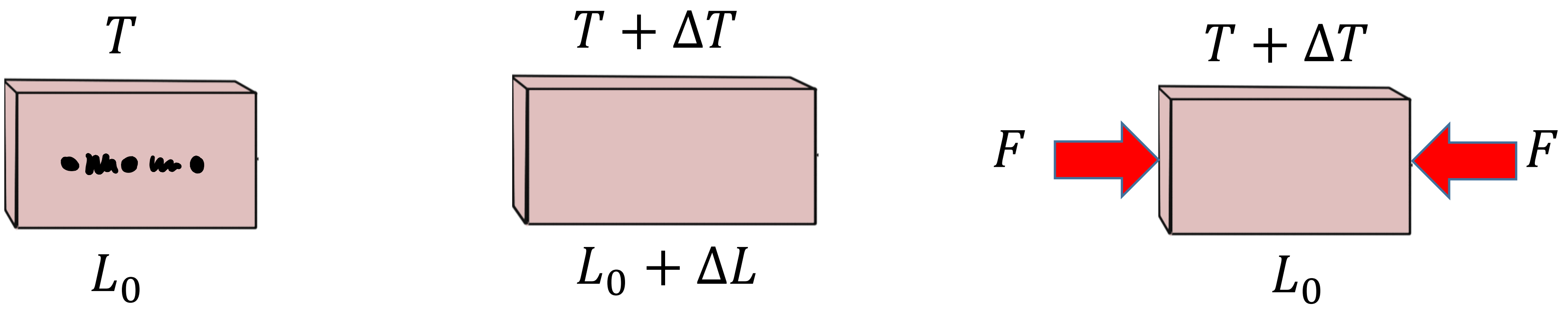
A. $Y \alpha L_0 \Delta T$

B. $Y \alpha \Delta T$

C. $Y L_0 \Delta T$

D. $\alpha L_0 \Delta T$

E. $Y \alpha L_0$



$$\boxed{\Delta L_T = \alpha L_0 \Delta T}$$

Thermal expansion

$$\boxed{\frac{F}{A} = Y \frac{\Delta L_F}{L_0}}$$

Stress vs strain

$$|\Delta L_F| = |\Delta L_T|$$

$$\frac{F}{A} = Y \frac{\Delta L_T}{L_0}$$

Q: A steel rod of length L_0 is heated by temperature ΔT. How would you determine how much stress (force per unit area) is required to compress the rod back to its original length?

A. $Y\alpha L_0\,\Delta T$
B. $Y\,\alpha\,\Delta T$ ✓
C. $Y\,L_0\,\Delta T$
D. $\alpha\,L_0\,\Delta T$
E. $Y\,\alpha\,L_0$

Total change in length is the sum of the change due to temperature and the change due to applied stress:

$$\Delta L = \Delta L_{\text{thermal}} + \Delta L_{\text{stress}}$$

Change in length under heating is:

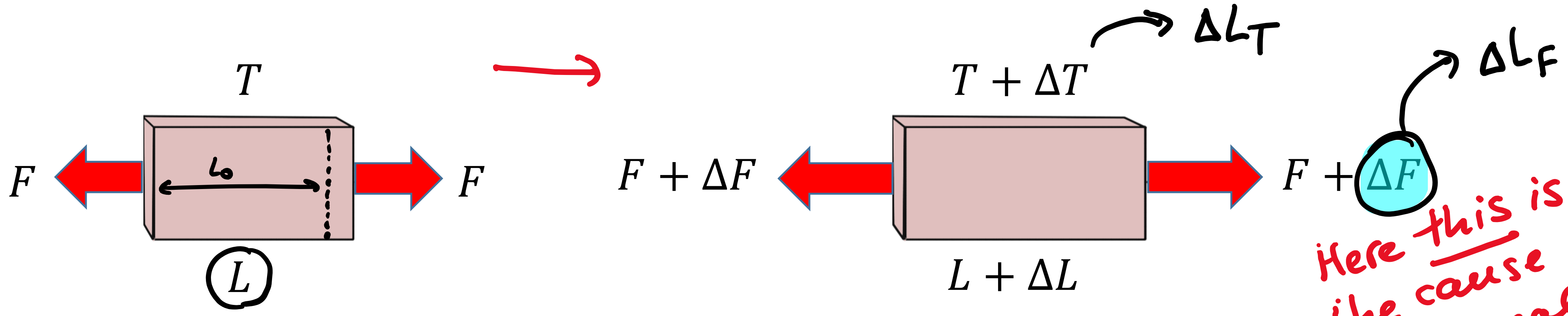
$$\Delta L_{\text{thermal}} = \alpha L_0 \Delta T$$

To reverse this change, need an <u>equal but opposite amount</u> of change in length due to the applied stress:

$$\Delta L_{\text{stress}} = \frac{F}{A}\frac{L_0}{Y} = -\Delta L_{\text{thermal}} = -\alpha L_0 \Delta T$$

- Magnitude: $|F/A| = Y\alpha\Delta T$
- Direction: compression (inwards)

Q: A copper wire under a tension force of F and at temperature T initially has a length L. If we heat up the wire by ΔT and also change the tension force by ΔF, how would you determine how much the length of the wire changes (total expansion)?



Here this is the cause of additional mechanical expansion

?

$$\Delta L_{tot} = \Delta L_T + \Delta L_F$$

A. $\Delta L = \alpha L \Delta T + \frac{L}{Y}\frac{F}{A}$

B. $\Delta L = \alpha\, L \Delta T + \frac{L}{Y}\frac{\Delta F}{A}$

C. $\Delta L = \alpha L \Delta T + \frac{L}{Y}\frac{(F+\Delta F)}{A}$

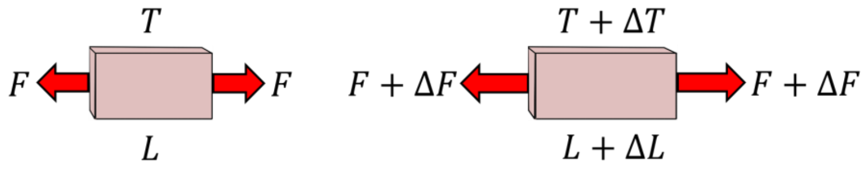
L

D. Something else

effect

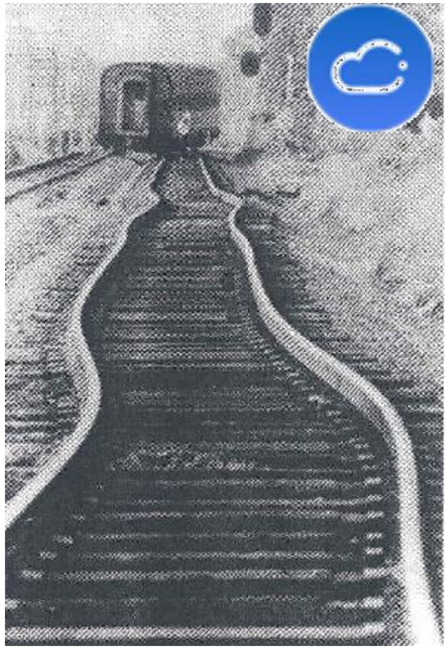
$$\underbrace{\left(\frac{F}{A}\right)}_{\text{cause}} = Y \left(\frac{\Delta L}{L_0}\right) \; ; \quad L_0 \to L_0 + \Delta L$$

Q: A copper wire under a tension force of F and at temperature T initially has a length L. If we heat up the wire by ΔT and also change the tension force by ΔF, how would you determine how much the length of the wire changes (total expansion)?



- Treat the change in length from thermal expansion and the change in length from the force increase separately.
- Total change in length is the sum of the change due to temperature and the change due to applied stress: $\Delta L = \Delta L_T + \Delta L_F$
- Change in length under heating is: $\Delta L_T = \alpha L \Delta T$
- Change in length due to applied stress: $\frac{\Delta F}{A} = Y \frac{\Delta L_F}{L} \Rightarrow \Delta L_F = \frac{\Delta F}{A} \frac{L}{Y}$
- Total expansion: $\Delta L = \Delta L_T + \Delta L_F = \alpha L \Delta T + \frac{L}{Y} \frac{\Delta F}{A}$
    - ➢ Note that ΔL_F is due to ΔF, not $F + \Delta F$ (since it is ΔF that causes this change)!

10 m long steel train rails are laid end to end on a winter day (0 ^oC). If the engineer forgot to leave gaps for thermal expansion, roughly how much force is generated at the ends of each rail due to thermal stress when the temperature reaches 30 ^oC?



- Cross-sectional area of the rail: $0.01\ m^2$
- $Y_{\text{steel}} = 20 \times 10^{10}\text{Pa}$
- $\alpha_{\text{steel}} = 1.2 \times 10^{-5} K^{-1}$

A. 700 N

B. 7,000 N

C. 70,000 N

D. 700,000 N

E. 7,000,000 N

Extra: How much gap should have been left?

$$\boxed{\Delta L_F = \frac{\Delta F}{A}\frac{L}{Y}}$$

$$\boxed{\Delta L_T = \alpha L \Delta T}$$

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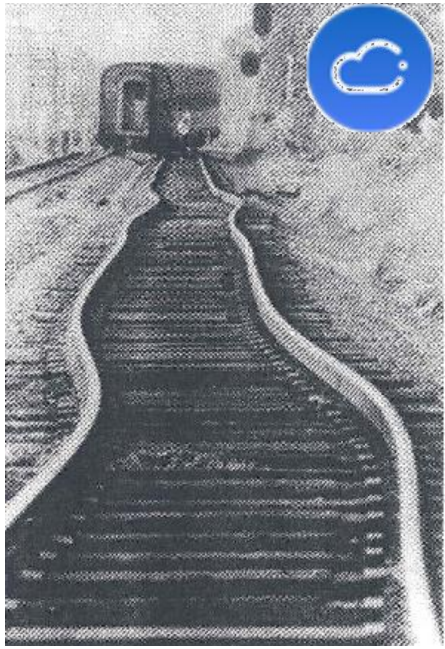

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A. 700 N

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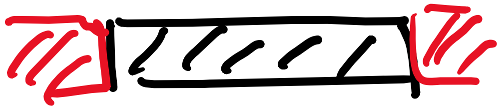
$$\Delta L_T = \alpha L_0 \Delta T \approx 3.6\, mm = \Delta L$$



$$\Delta F = A\, Y \frac{\Delta L}{L} = (10^{-2}) \times 20 \cdot 10^{10} \times \frac{3.6 \cdot 10^{-3}}{10} = 7.2 \times 10^5\ \text{N}$$

~ 70 tons of weight!!

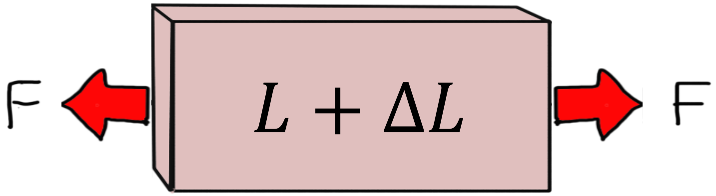
Extra: How much gap should have been left?

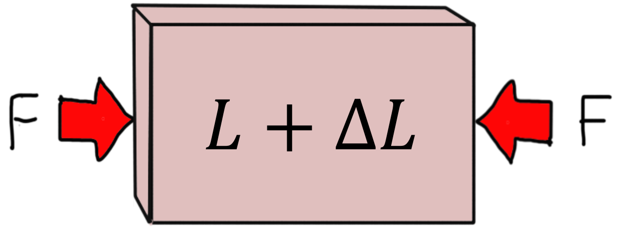
$$\boxed{\Delta L_F = \frac{\Delta F}{A}\frac{L}{Y}} \quad \boxed{\Delta L_T = \alpha L \Delta T}$$

Handling signs of F and ΔL

- Strain:

$$\boxed{\Delta L_F = \frac{F}{A}\frac{L}{Y}}$$



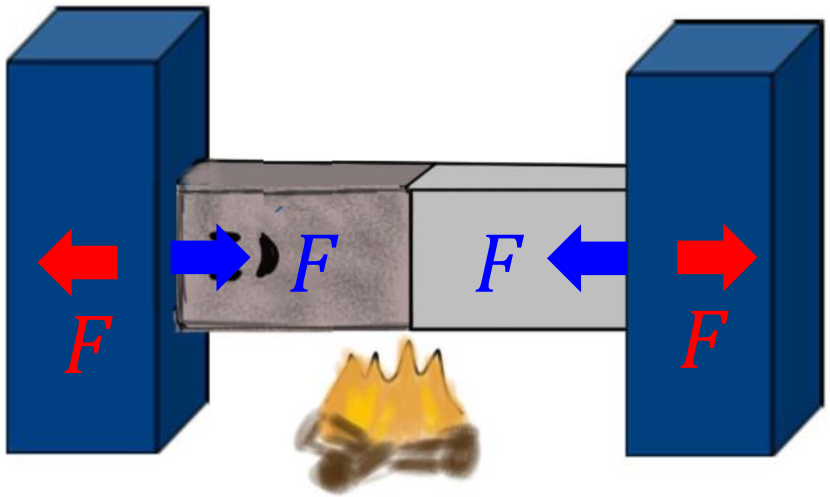


Two alternative approaches for handling signs of F and ΔL:

- Take tensile forces as positive, compressive forces as negative, and use $\Delta L = \frac{F}{A}\frac{L}{Y}$, OR
- Take all forces as positive (regardless of direction) and use $\Delta L = -\frac{F}{A}\frac{L}{Y}$ for compressive forces and $\Delta L = \frac{F}{A}\frac{L}{Y}$ for tensile forces

- Thermal expansion: $\boxed{\Delta L_T = \alpha L \Delta T}$
  - Increasing temperature gives positive ΔL, decreasing temperature gives negative ΔL

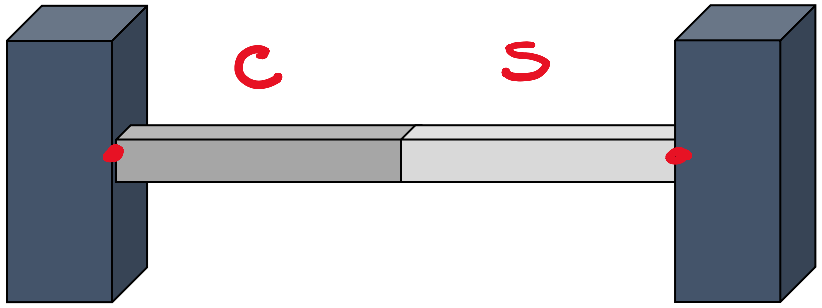
Heating a restricted object



- An object is sitting between two rigid walls
- You start heating the object; it expands (thermal expansion)
- It starts pressing on the walls (exerts forces $\vec{F}$)
- The walls act back on the object and exert on it forces, $\vec{F}$, equal in magnitude and opposite in direction (Newton's 3rd law)

A compound bar consisting of a copper rod with a length of 1 m and cross-section area of 2.00 cm^2 placed end to end with a steel rod with length 1 m and cross-sectional area 2.00 cm^2. The compound rod is placed between two rigid walls. Initially there is no stress in the bars at room temperature 20 ^{o}C.

Find the force on each wall at 40 ^{o}C.



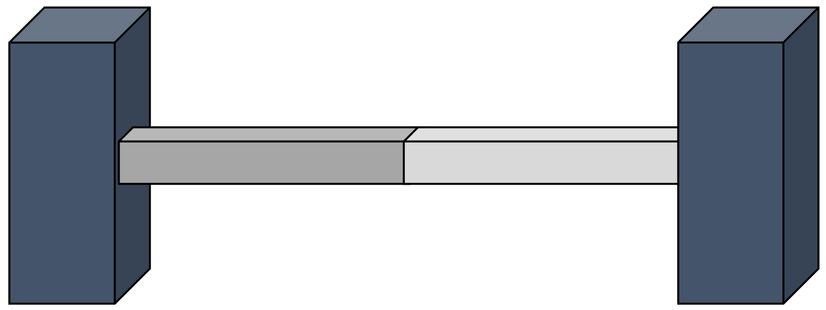
A harder problem

Solving strategy (might vary, so be creative!):

- Identify your system & sub-systems
- Visualize: draw a picture for your system and for each part of your system (each sub-system). If you want to describe changes, "before" and "after" pictures might be especially useful
- Introduce notations, identify relevant equations
- Write down relationships between variables within each system / sub-system, and the relationships connecting different parts of your systems

A compound bar consisting of a copper rod with a length of 1 m and cross-section area of 2.00 cm^2 placed end to end with a steel rod with length 1 m and cross-sectional area 2.00 cm^2. The compound rod is placed between two rigid walls. Initially there is no stress in the bars at room temperature 20^o C.

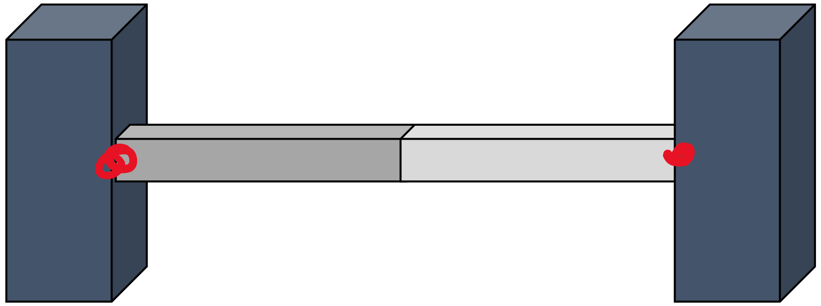
As the system is heated, we expect that:



A. Both rods will increase in length
B. Both rods will decrease in length
C. No change in rod’s lengths
D. One rod will get longer and the other rod will get shorter

A compound bar consisting of a copper rod with a length of 1 m and cross-section area of 2.00 cm^2 placed end to end with a steel rod with length 1 m and cross-sectional area 2.00 cm^2. The compound rod is placed between two rigid walls. Initially there is no stress in the bars at room temperature 20° C.

As the system is heated, we expect that:



Rigid walls => fixed total distance => A and B are wrong.

C: possible, but to react identically they need to have the same α and Y.

But they are different materials, so it is likely that α and Y are different, and they will likely change relative length => D.

A. Both rods will increase in length
B. Both rods will decrease in length
C. No change in rod's lengths
D. One rod will get longer and the other rod will get shorter ✓

Step 1: System

- Visualize what will happen
- Draw a before/after picture
- Gives names to known & unknown quantities & label diagram
- Understand which quantities are changing and which are fixed

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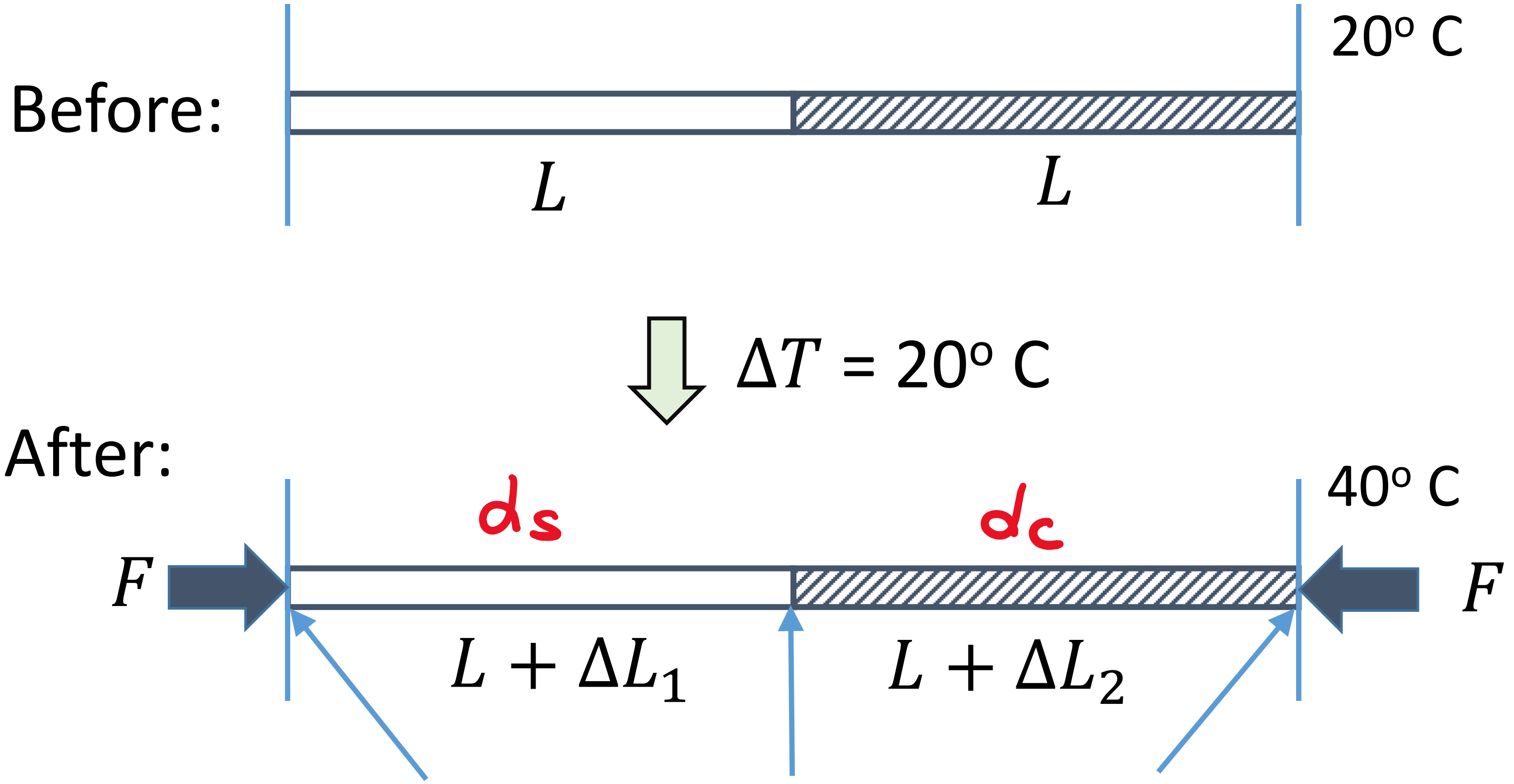
$\Delta L_T = \alpha L_0 \Delta T$



Total length is fixed: $\Delta L_1 + \Delta L_2 = 0$

We want to find F

Compression forces acting on ends of rods

Step 2: Parts of the system

- Isolate the parts of the system
- For each part, draw a before/after free body diagrams, making use of Newton’s Laws to relate forces

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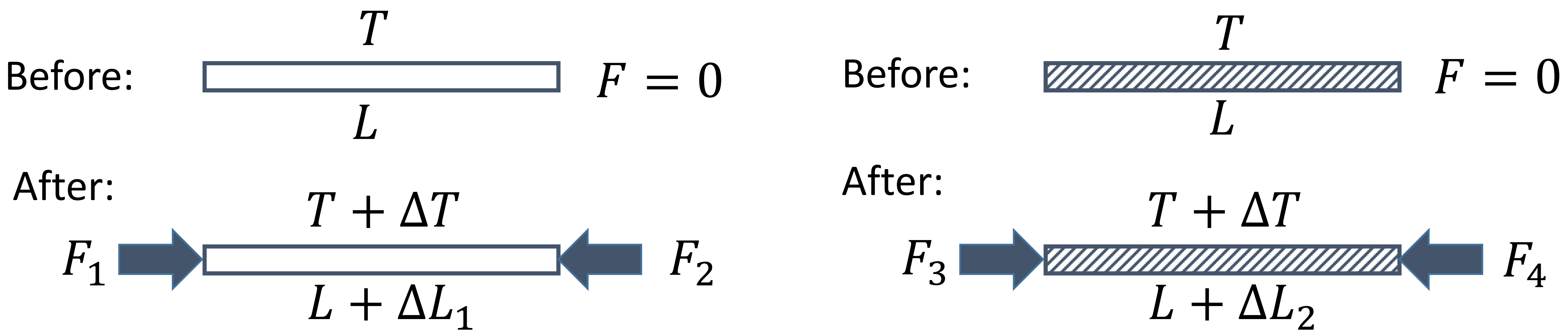
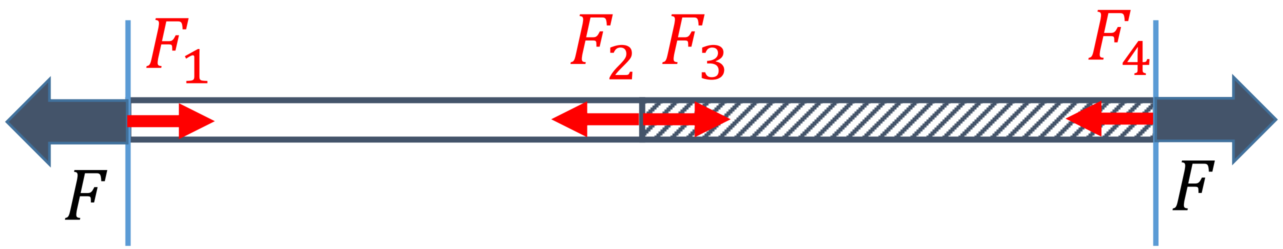


- Question: What are F_1, F_2, F_3, and F_4 in terms of F, the magnitude of the forces on the two walls, and why?



$F_1 = F$ Newton's 3rd law

$F_2 = F_1$ left bar not accelerating

$F_3 = F_2$ Newton's 3rd law

$F_4 = F_3$ right bar not accelerating

$F_4 = F$ Newton's 3rd law

Step 3: Equations

- For each part, write an equation relating the change in length to the changes in temperature and forces

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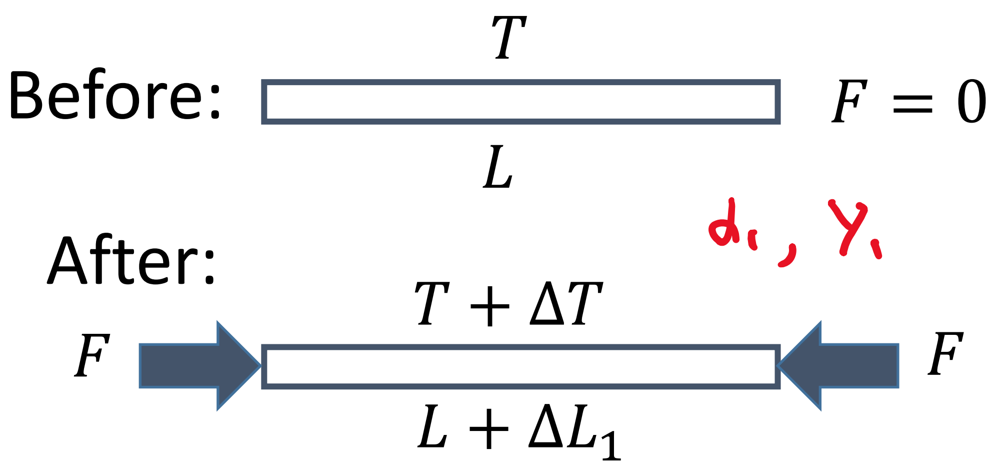

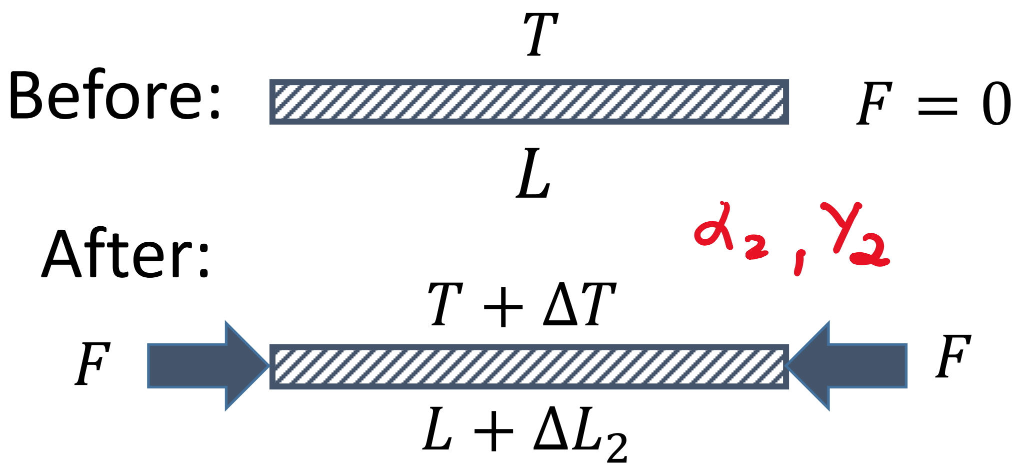

One of them is positive, one negative

$$\Delta L_1 = \alpha_1 L \Delta T - \frac{F}{A}\frac{L}{Y_1} \qquad \Delta L_2 = \alpha_2 L \Delta T - \frac{F}{A}\frac{L}{Y_2}$$

F is positive, the signs express compression