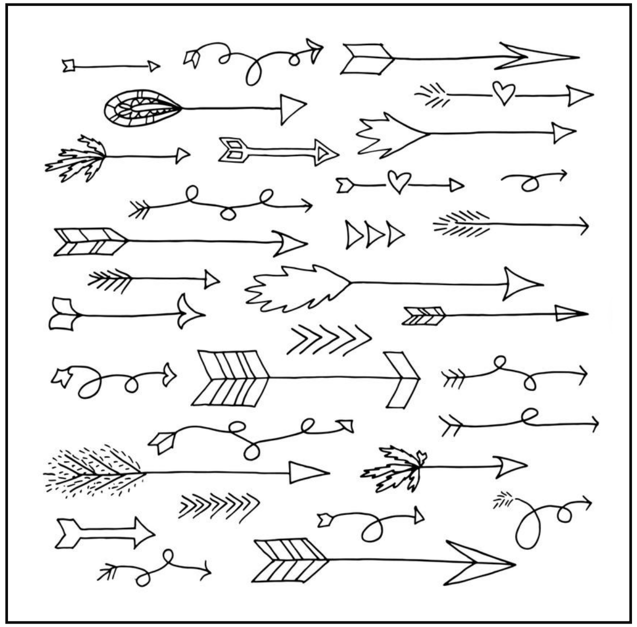
Force Vectors in 2D



Text: 2.1-2.4

Content:

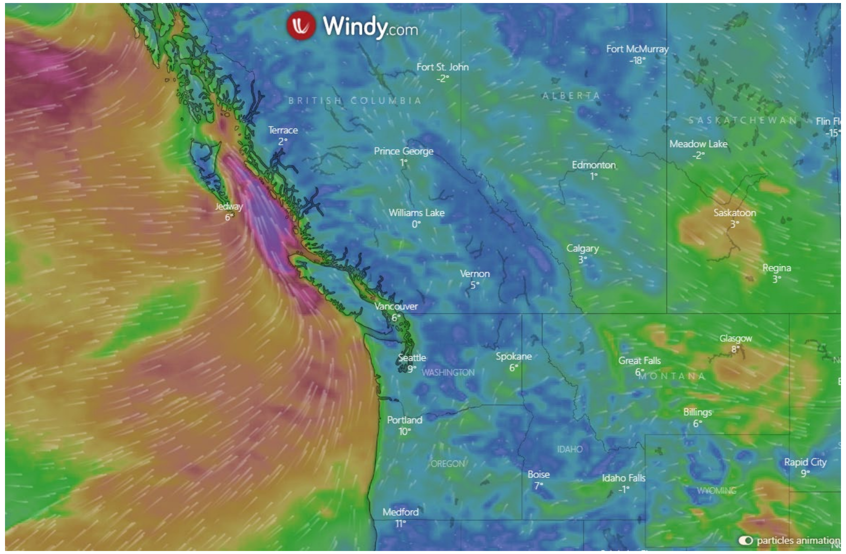
- Vectors (magnitude & direction) and scalars (magnitude)
- Description of Vectors in 2D:
  - Magnitude and Angle
  - Cartesian coordinates: Unit vectors, Projections of a vector
  - Conversion between these two descriptions
- Multiplying vector by a scalar
- Adding up vectors: Resultant vector
  - Graphically
  - Using Sine and Cosine theorems
  - (!) In components
- Practice (W1-1, W1-2 , W1-3)

SCALARS AND VECTORS

- **Scalar:**
  - Quantities characterized by one number (positive, negative or zero)
  - Examples: Mass, volume, length, temperature, density…..

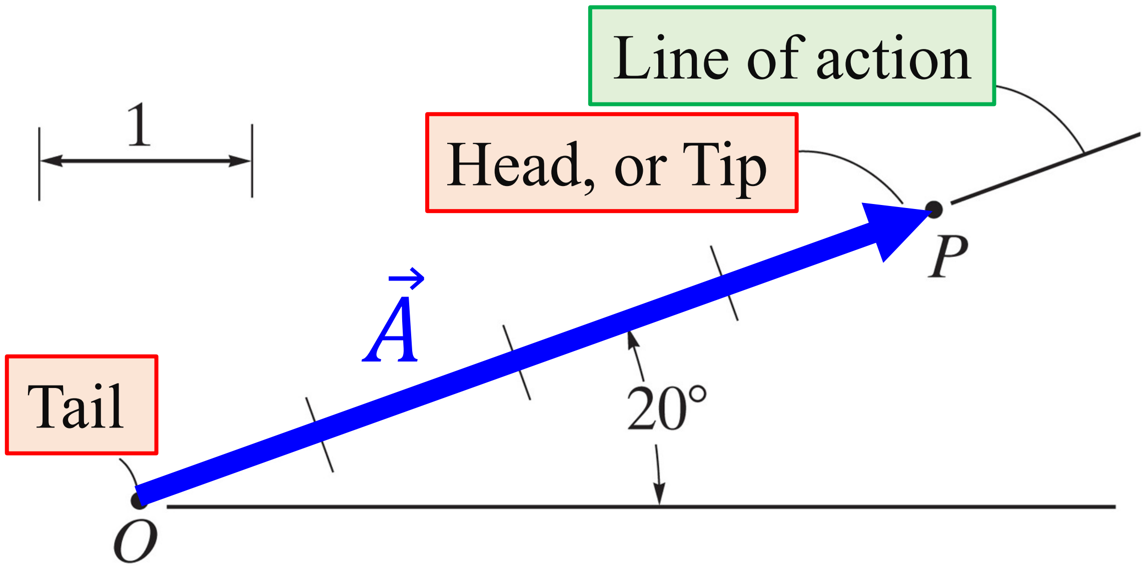
- **Vector: more complex object**
  - Has **magnitude** and **direction**
  - Examples: position, velocity, force, moment, …..
  - Notations:
    - For a vector: $\mathbf{A}$ (convenient when typing) or $\vec{A}$ (convenient when writing)
    - For vector’s **magnitude**: A or $|\vec{A}|$
    - Note: “magnitude” = “absolute value” = “strength” (of a force): always a positive number!

- **Matrices, tensors: even more complex mathematical objects**



- *Temperature is a scalar*
- *Wind velocity is a vector: it has a magnitude (20 km/h) and direction (North-East)*

GRAPHICAL REPRESENTATION



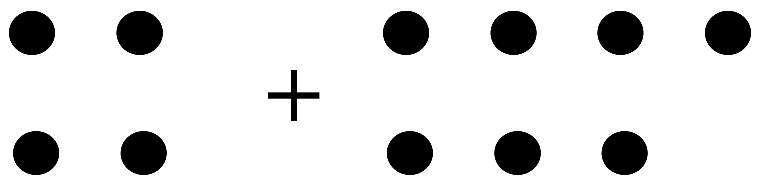
Vector $\vec{A}$:

- Magnitude: 4 units
- Direction: 20º above horizontal

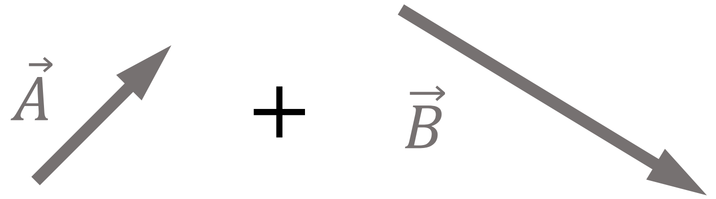
(reference direction)

VECTOR ADDITION

- Adding scalars:
  - $4 + 7 \ = \ 11$
  - $8 - 2 = 6$
  - We are adding / subtracting **magnitudes** – simple!



- Adding vectors:
  - $???$
  - We need to combine entities that have **magnitudes** and **direction** – more difficult!

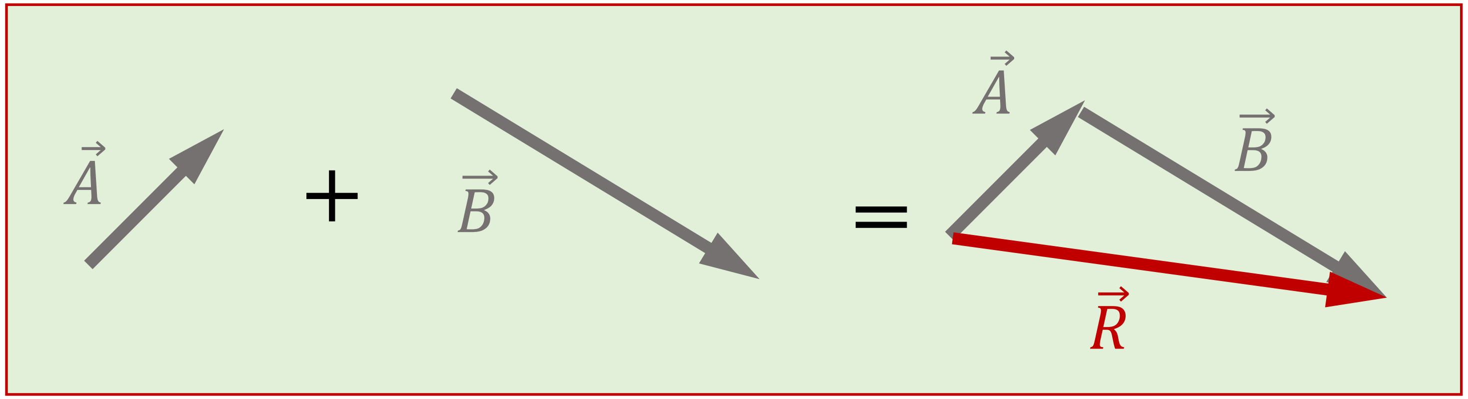


- Two methods (approaches):
  - Graphical
  - Using vector’s components

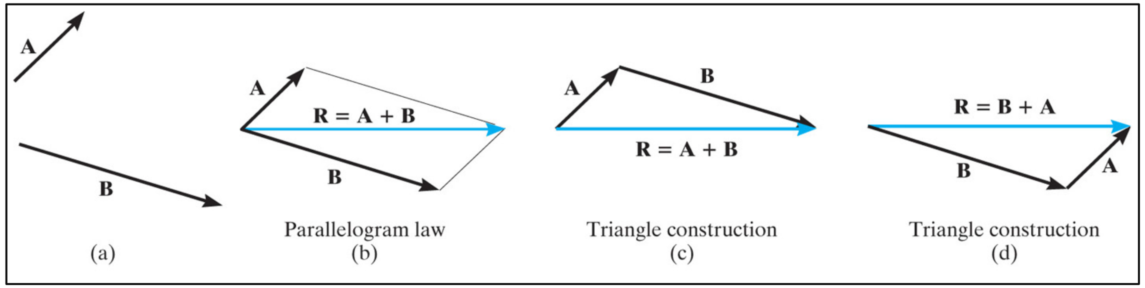
VECTOR ADDITION: Graphical approach

- If you have vectors $\vec{A}$ and $\vec{B}$, what is the resultant vector, $\vec{R} = \vec{A} + \vec{B}$?

- You can move vectors around via “parallel translations” (that preserve both vector’s magnitude and direction)
- Vectors add “tip to tail”



- Convince yourself that:
  - $\vec{A} + \vec{B} = \vec{B} + \vec{A}$



Equivalent approaches:

- “Parallelogram”
- “Triangle”

VECTOR ADDITION: Sense making

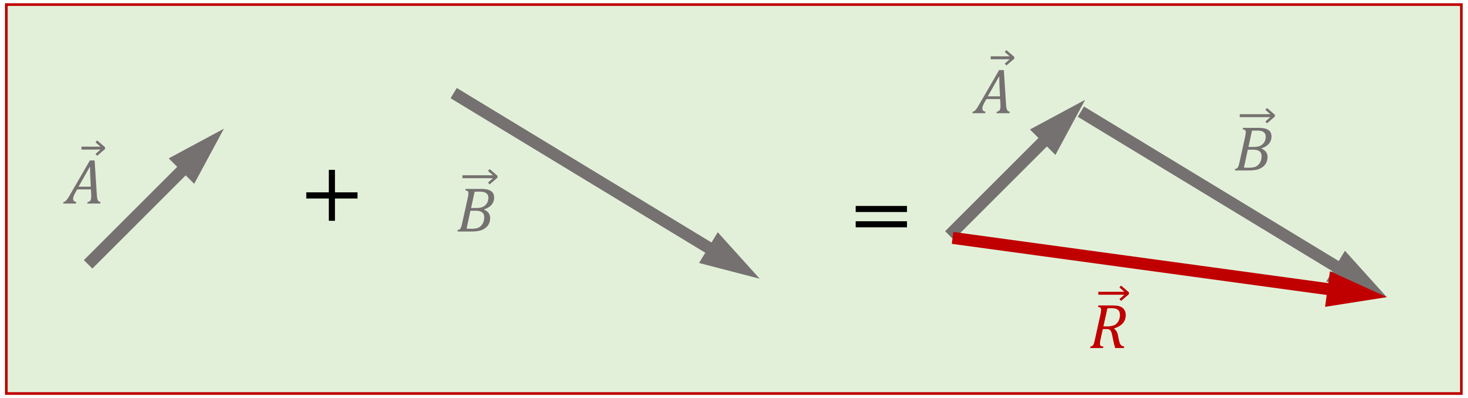
Q: Let $\vec{R} = \vec{A} + \vec{B}$. Which is correct:

A. $R = A + B$

B. $R > A + B$

C. $R < A + B$

D. Depends on the circumstances



VECTOR ADDITION: Sense making

Q: Let $\vec{R} = \vec{A} + \vec{B}$. Which is correct:

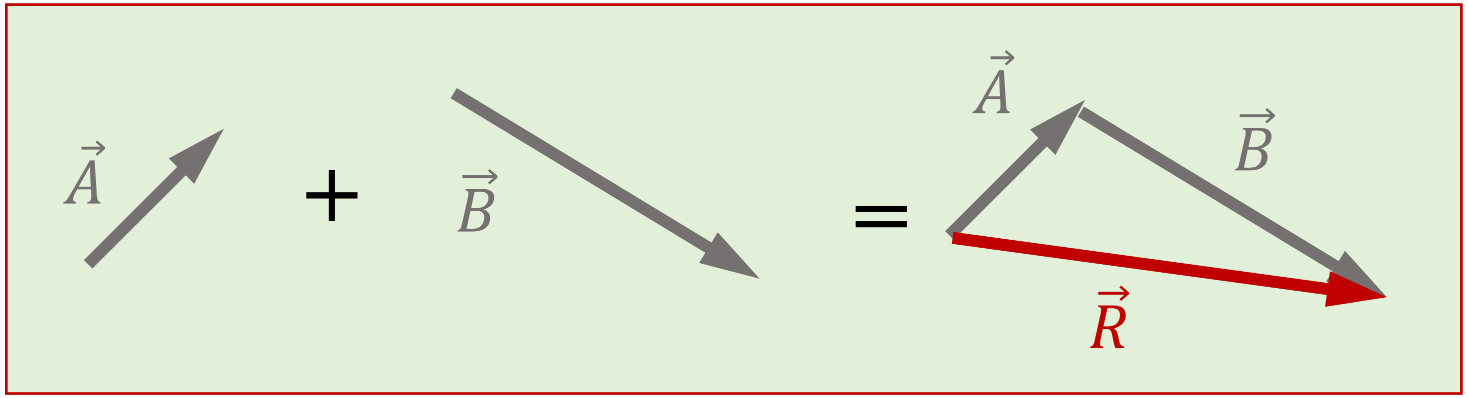
A. $R = A + B$

B. $R > A + B$

C. $R < A + B$

D. Depends on the circumstances

$R = |\vec{R}|$ is the magnitude (the length) of the vector $\vec{R}$, and the came holds for the symbols A and B. From the vector triangle we see that its one side is always shorter than the some of other two sides (since it is the shortest distance between the two points).



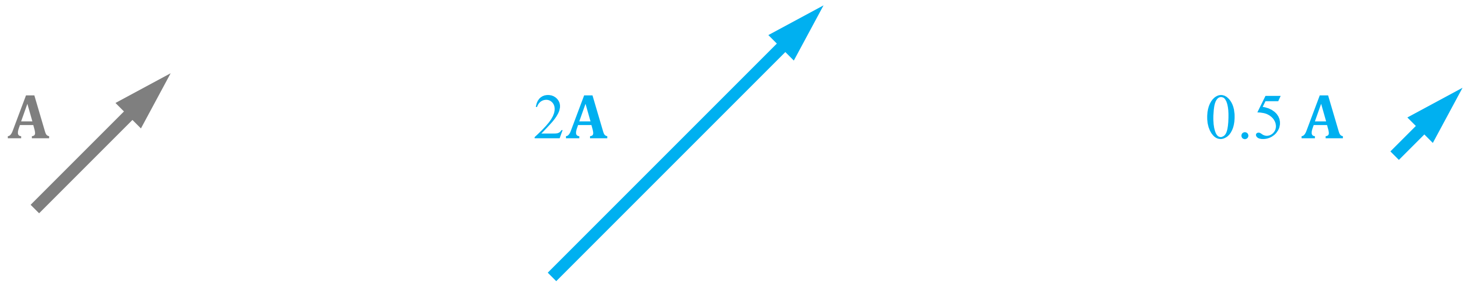
Q. Do you know how to express R if you know what the vectors $\vec{A}$ and $\vec{B}$ are?

MULTIPLYING A VECTOR BY A SCALAR: Graphical approach

- Q: If you have a vector $\vec{A}$ and a scalar b, what is $b\vec{A}$?
- A: Vector times scalar is a vector!

- Multiplying vector by a scalar **changes its magnitude** and **preserves its line of action**

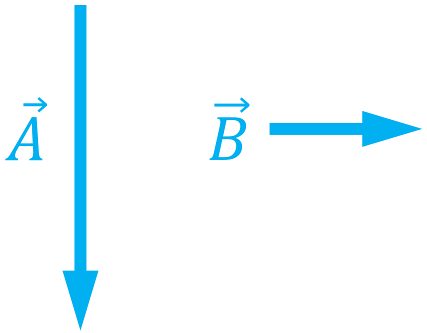
- **Positive scalars**: preserves vector's direction, scales the magnitude

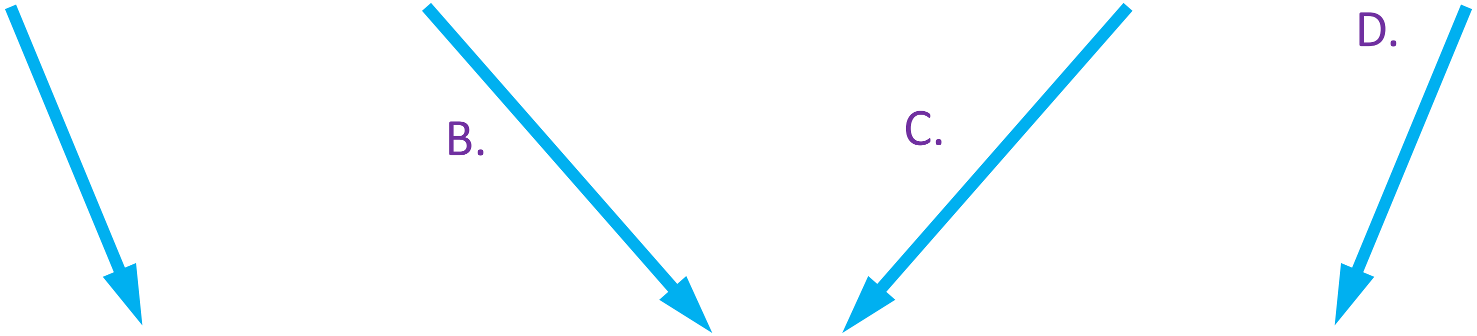


- **Negative scalars**: flips vector's direction, scales the magnitude

- Later we will learn how to multiply a vector by a vector…

Q: Given these two vectors, which of the following is the vector $\vec{R} = \vec{A} - 2\vec{B}$?





A.

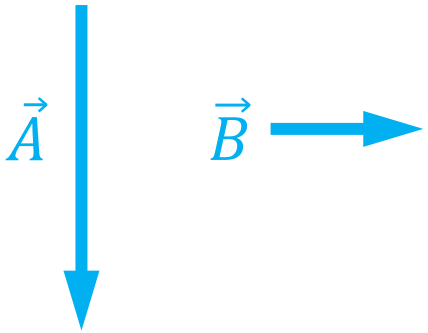
B.

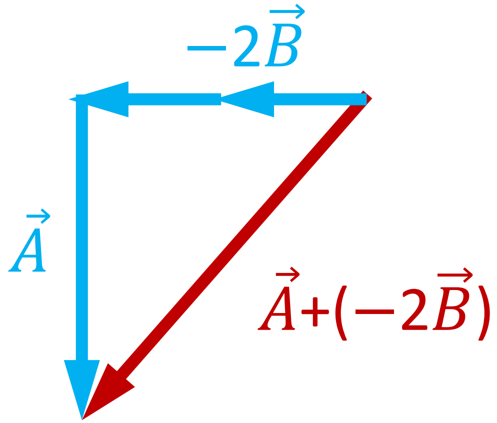
C.

D.

E. Some other vector

Q: Given these two vectors, which of the following is the vector $\vec{R} = \vec{A} - 2\vec{B}$?

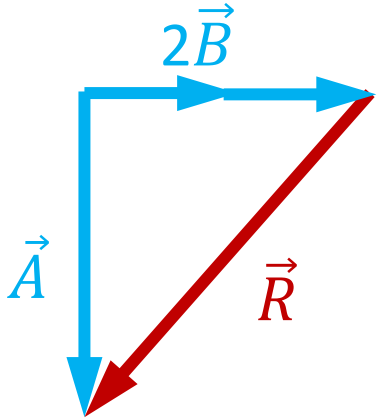




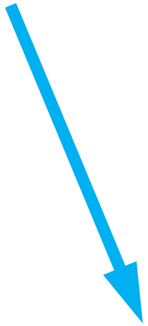
Solution 1.

Solution 2.

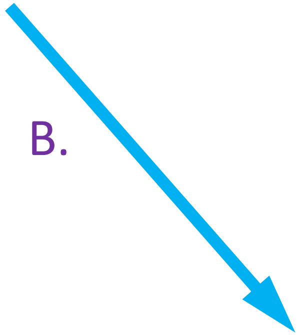
- Rewrite: $\vec{A} = \vec{R} + 2\vec{B}$
- Find $\vec{R}$ such that it is true



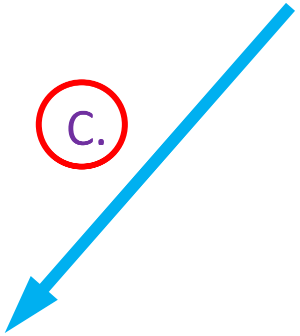
A.



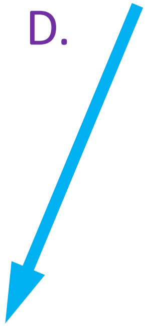
B.



C.

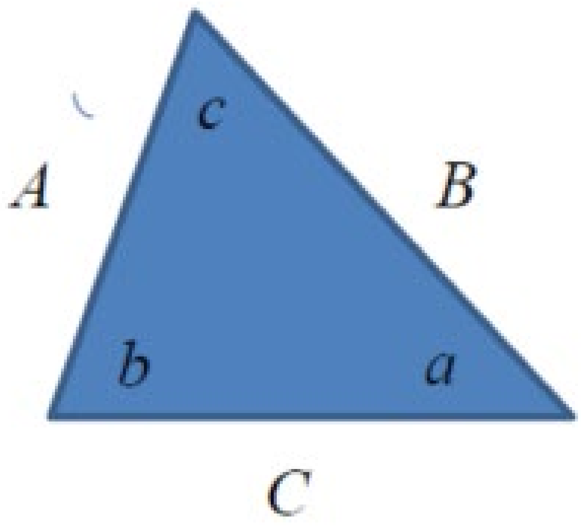


D.



E. Some other vector

VECTOR ADDITION-2: Sine law & Cosine law



- **Sine law**: $$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

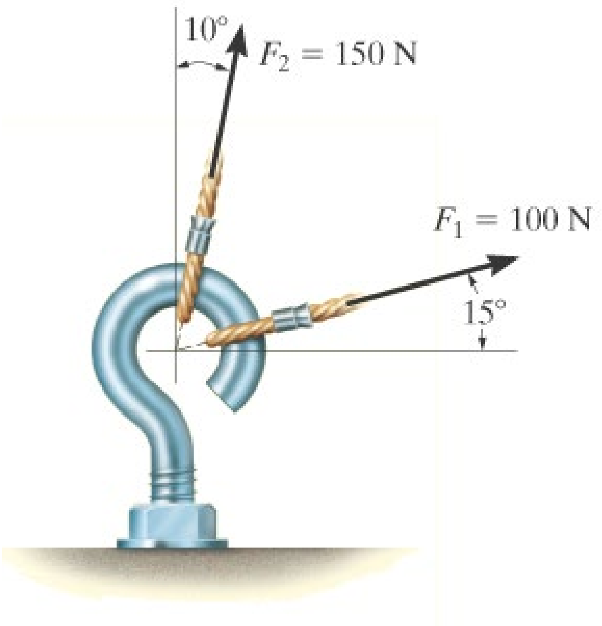
- **Cosine law**:
$$A^2 = B^2 + C^2 - 2BC\cos a$$
$$B^2 = A^2 + C^2 - 2AC\cos b$$
$$C^2 = A^2 + B^2 - 2AB\cos c$$

- These theorems can be used to compute missing elements of a triangle
- Vectors add up using the triangle rule
  - ➢ We can use these theorems to add vectors up!

W1-1a: The screw eye is subject to two forces, $\vec{F}_1$ and $\vec{F}_2$. Determine the direction and the magnitude of the resultant force. Use sine and cosine theorems.

Textbook:

Example 2.1

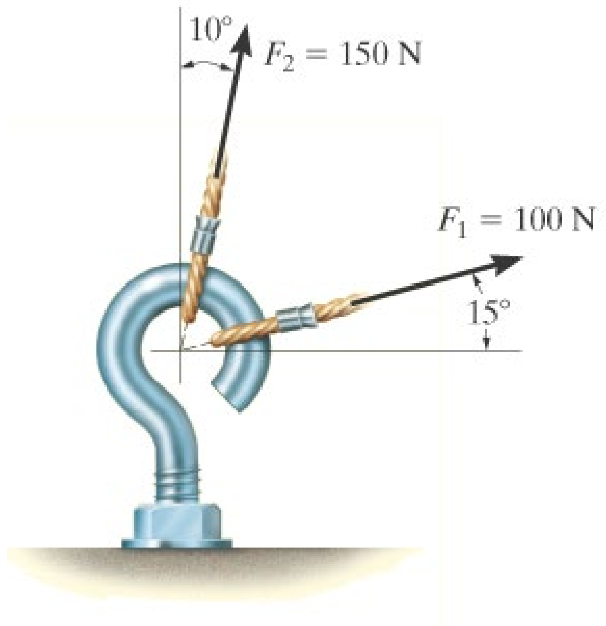


$$F_1 = 100\ N$$
$$F_2 = 150\ N$$

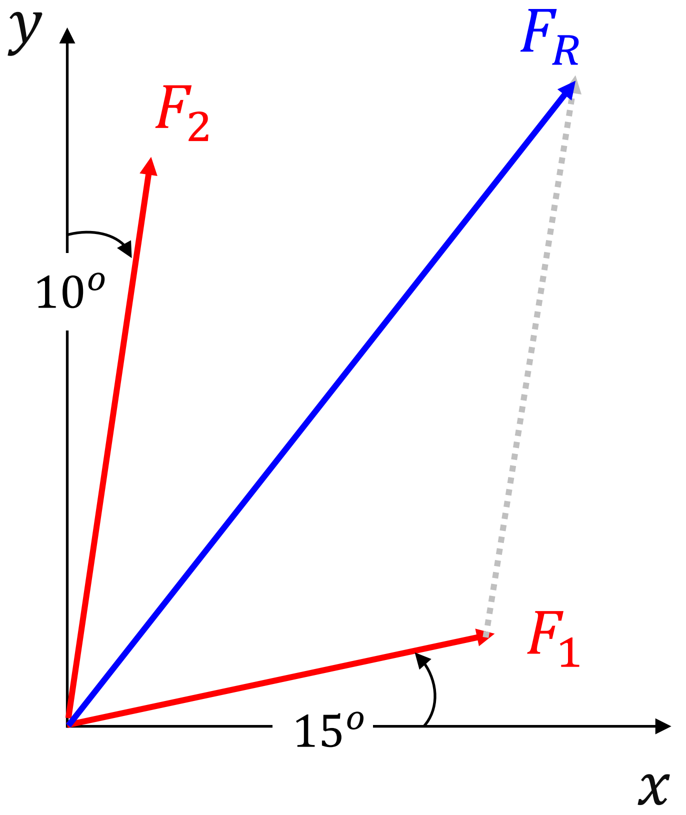
W1-1a: The screw eye is subject to two forces, $\vec{F}_1$ and $\vec{F}_2$. Determine the direction and the magnitude of the resultant force. Use sine and cosine theorems.

Textbook:

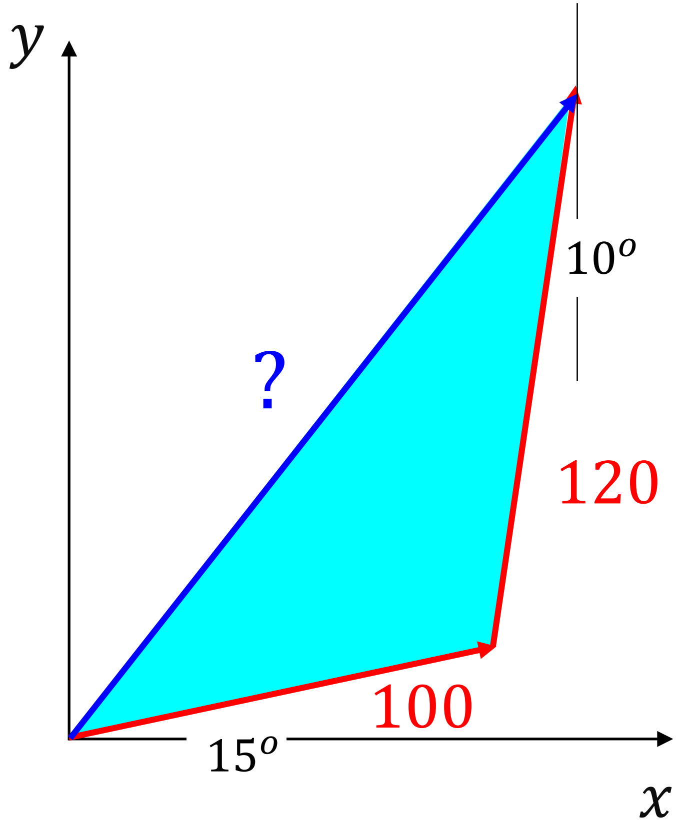
Example 2.1



$$F_1 = 100\ N$$
$$F_2 = 150\ N$$

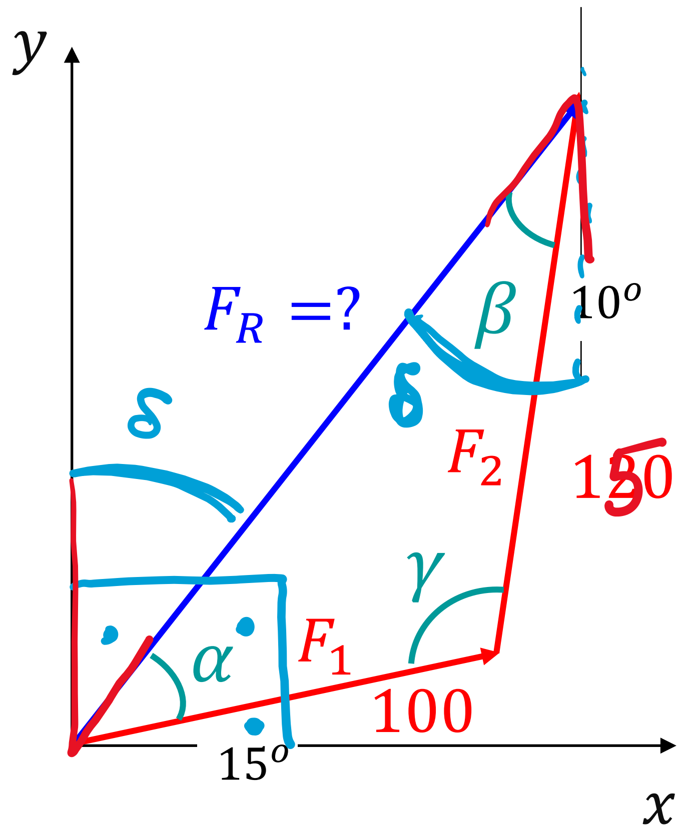


- Sketch F_R



- Apply sine and cosine theorems to find its longest side

W1-1a: The screw eye is subject to two forces, $\vec{F}_1$ and $\vec{F}_2$. Determine the direction and the magnitude of the resultant force. Use sine and cosine theorems.



$$\rightarrow \quad F_R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos\gamma$$

$$\gamma = ?$$

$$(\alpha + \beta) + \gamma = 180^\circ \quad \rightarrow \quad \gamma = 180^\circ - 65^\circ = 115^\circ \, !$$

$$15^\circ + \alpha + \delta = 90^\circ$$

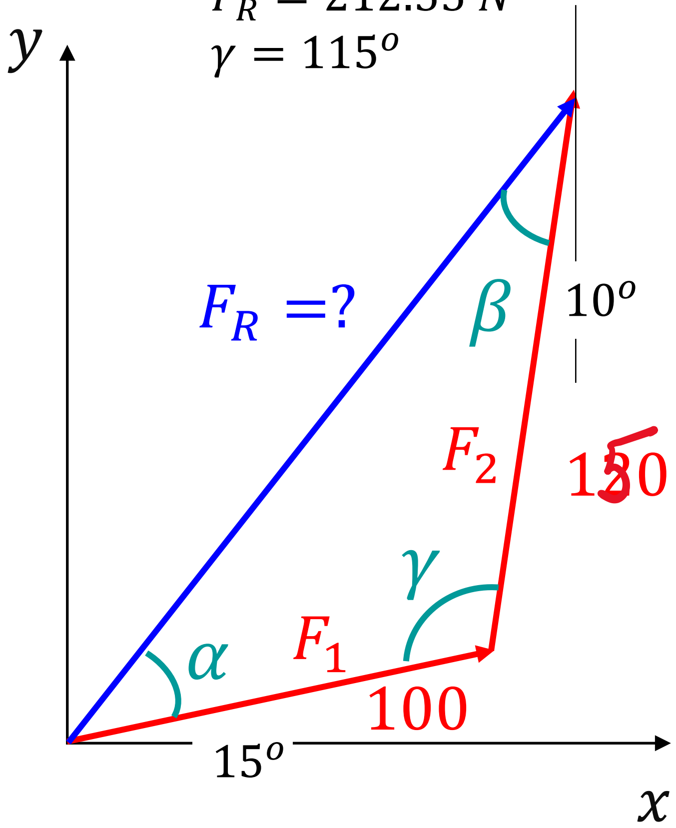
$$\delta \hookrightarrow \beta + 10^\circ$$

$$15^\circ + \underbrace{\alpha + \beta} + 10^\circ = 90^\circ \quad \rightarrow \quad \alpha + \beta = 65^\circ$$

$$F_R = 212.55$$

W1-1a: The screw eye is subject to two forces, $\vec{F}_1$ and $\vec{F}_2$. Determine the direction and the magnitude of the resultant force. Use sine and cosine theorems.

$F_1 = 100\ N$
$F_2 = 150\ N$
$F_R = 212.55\ N$
$\gamma = 115^o$



$$\frac{F_2}{\sin\alpha} = \frac{F_R}{\sin\gamma} \rightarrow \sin\alpha = \sin\gamma\, \frac{F_2}{F_R} =$$

$$= (\sin 115^\circ)\, \frac{150}{212.55} \rightarrow \alpha = 39.76$$

$$\text{angle} = \alpha + 15^\circ = 54.76$$

$$\boxed{\begin{array}{l} F_R = 213\ N \\ \text{angle} = 54.8^\circ \end{array}}$$

Angle from positive-x direction:

A. 39.8°
B. 49.8°
C. 54.8°
D. 61.2°
E. Something else

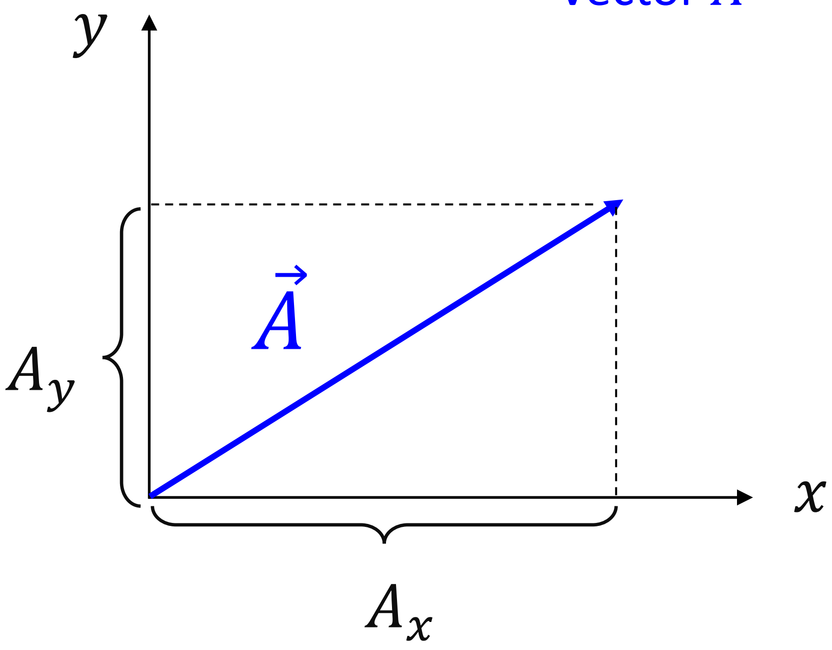
• Sine and cosine theorem can help us to calculate a sum of **two** vectors using the properties of a triangle made from these two vectors and the resultant vector.

• What if you need to add up 10 vectors?

• You can apply this approach to **pairs** of vectors: first add vectors 1 and 2, then combine the result with vector 3, then combine the result with vector 4… Too long!

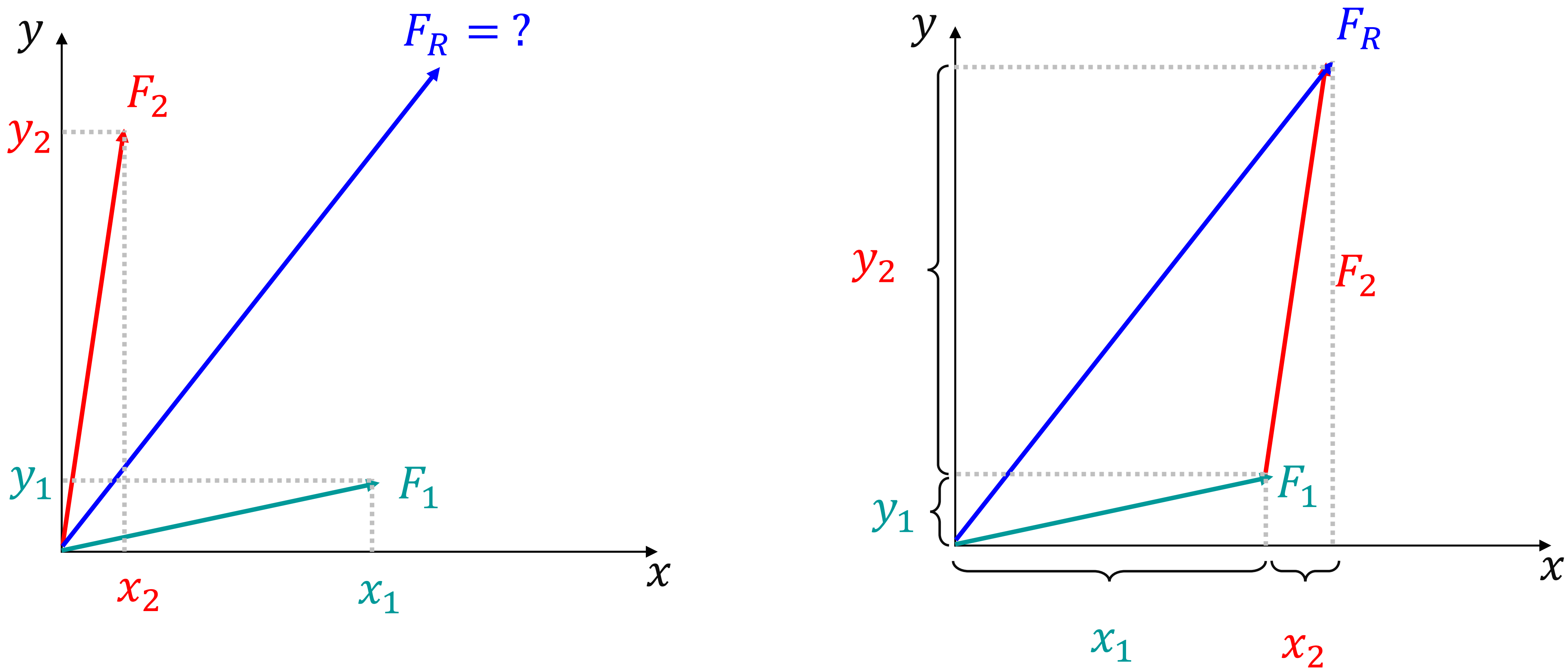
- We are now going to discuss how to represent a vector in Cartesian components.
- Knowing vector components, you know everything about the vector.
- One of the applications of components is that knowing a components of vectors it will be **very easy to add up any number of vectors** following a standard procedure.

Vector $\vec{A}$



A_x, A_y – its x and y components

➢ That's how you can add vectors using components:



Q: What are the components of the resultant vector, $\vec{F}_R = \vec{F}_1 + \vec{F}_2$?

$F_{R,x} =$ $x_1 + x_2$

$F_{R,y} =$ $y_1 + y_2$

- If you know Cartesian components of two vectors, you can find Cartesian components of their sum => you know everything about their sum.

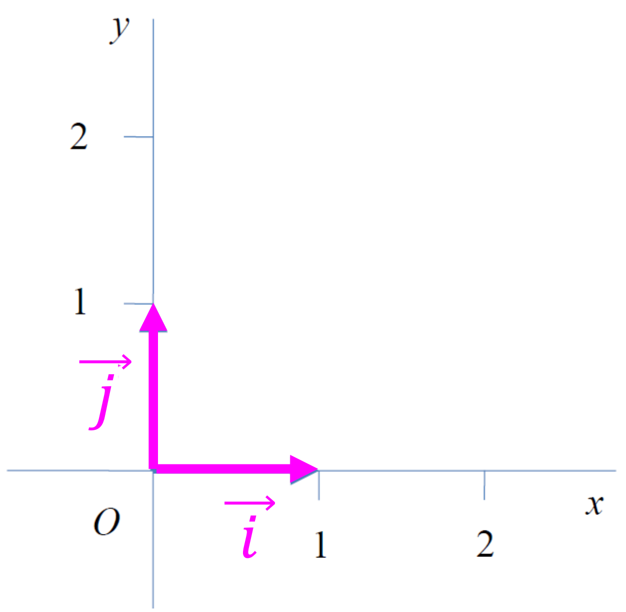
- Now we will talk about the components:
  - Expressing vectors using $\vec{i}\,,\vec{j}$ -vectors
  - Positive / negative components
  - How to find the magnitude of a vector from its components
  - How to find the direction of a vector from its components

- …and will come back to vector addition after that.

CARTESIAN COORDINATE SYSTEM: Two Dimensions (2D)

- **Cartesian coordinate system in 2D**:

- "Cartesian" = having something to do with René Descartes (1596-1650), who introduced the coordinate system with **perpendicular axes**



- Perpendicular axes, x and y
- **Unit vectors** (= "orts", = "basis vectors"):
  - ➢ $\vec{i}$ -- vector of length 1 in the positive direction of x-axis
  - ➢ $\vec{j}$ -- vector of length 1 in the positive direction of y-axis

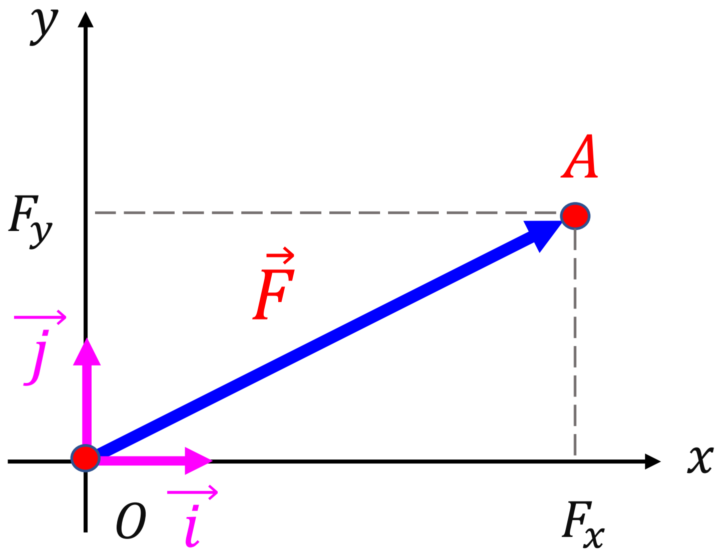
VECTORS IN A CARTESIAN COORDINATE SYSTEM: Two Dimensions

- Position vector: A vector with the tail at the origin, $(0,0)$, and with the tip having some coordinates, $A = (F_x, F_y)$.

- (F_x, F_y) are the Cartesian components of the vector $\vec{F}$

- Any vector $\vec{F}$ in 2D can be uniquely **resolved into two components** along x and y:

$$\boxed{\vec{F} = \vec{i}\ F_x + \vec{j}\ F_y}$$



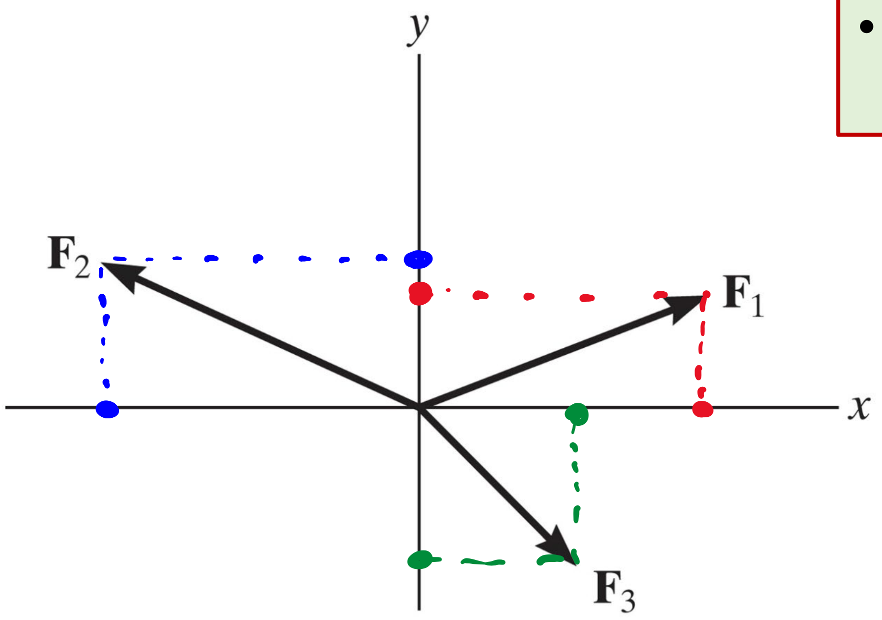
- Simply said: to get from O to A,
  - You make F_x steps in $\vec{i}$-direction (along x-axis), and
  - You make F_y steps in $\vec{j}$-direction (along y-axis),

- Note that $\vec{F} = \vec{F}_x + \vec{F}_y$,

with $\vec{F}_x = \vec{i}\ F_x$ and $\vec{F}_y = \vec{j}\ F_y$.

CARTESIAN COMPONENTS OF A VECTOR: Two Dimensions

- Cartesian components F_x and F_y can be positive or negative:



- If a vector points in a positive / negative direction of an axis, its projection on this axis will be positive / negative

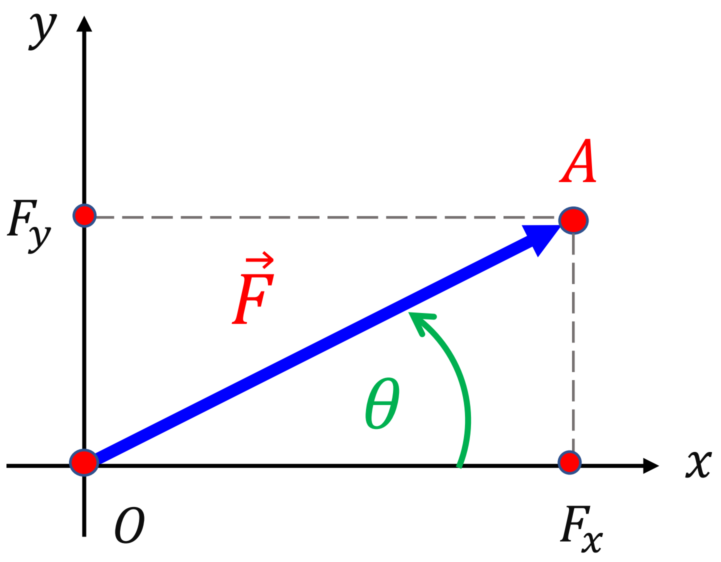
Q: Positive or negative?

- F_{1x} > 0, F_{1y} > 0
- F_{2x} < 0, F_{2y} > 0
- F_{3x} > 0, F_{3y} < 0

- These are **coplanar vectors** (vectors that lay in the same plane) => it is a 2D problem

- Each position vector in 2D is fully defined by just two numbers:
  - Either by its two components: (F_x, F_y)
  - Or by its magnitude and direction angle: (F, θ)

- Conversion between (F, θ)-representation and (F_x, F_y)-representation of a vector is:



- Trigonometry:

$$F_x = F\cos\theta \quad \text{and} \quad F_y = F\sin\theta$$

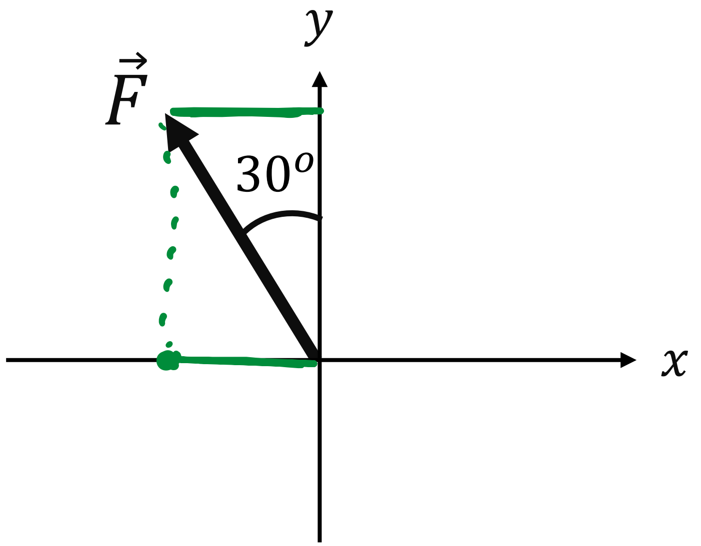
$$\tan\theta = \frac{F_y}{F_x}$$ (to be used with care, see below)

- Pythagoras theorem:

$$F = |\vec{F}| = \sqrt{F_x^2 + F_y^2}$$

Q: Find the component of $\vec{F}$ in the $\vec{i}$-direction.

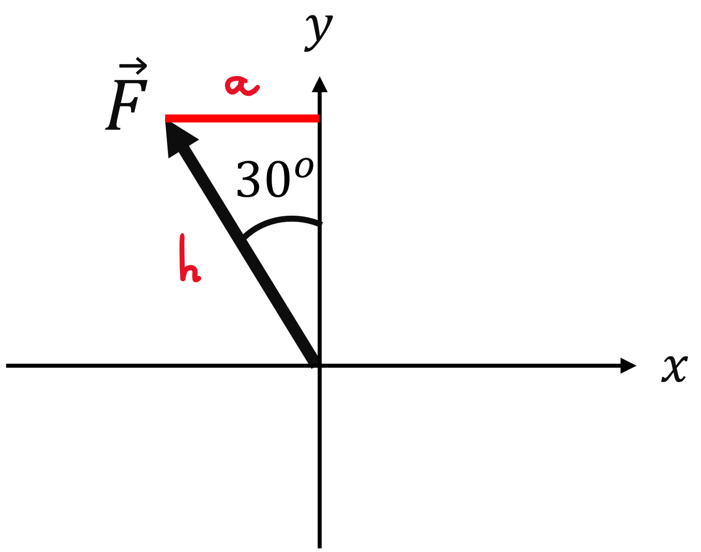
A. $F\cos(30^o)$
B. $F\sin(30^o)$
C. $-F\cos(30^o)$
D. $-F\sin(30^o)$
E. Not sure



$F_x = - \sin 30^\circ$

Q: Find the component of $\vec{F}$ in the $\vec{i}$-direction.

A. $F\cos(30^o)$
B. $F\sin(30^o)$
C. $-F\cos(30^o)$
D. $-F\sin(30^o)$
E. Not sure



$$\sin 30^\circ = \frac{a}{h} \rightarrow$$

$$a = h \cdot \sin 30^\circ$$

$$F_x = -a = -h \cdot \sin 30^\circ$$

sin, not cos. Always check which angle is given.

minus, since the x-component points in negative-x direction

Q: Given the vector $\vec{F} = -5\vec{i} + 3\vec{j}$, find:

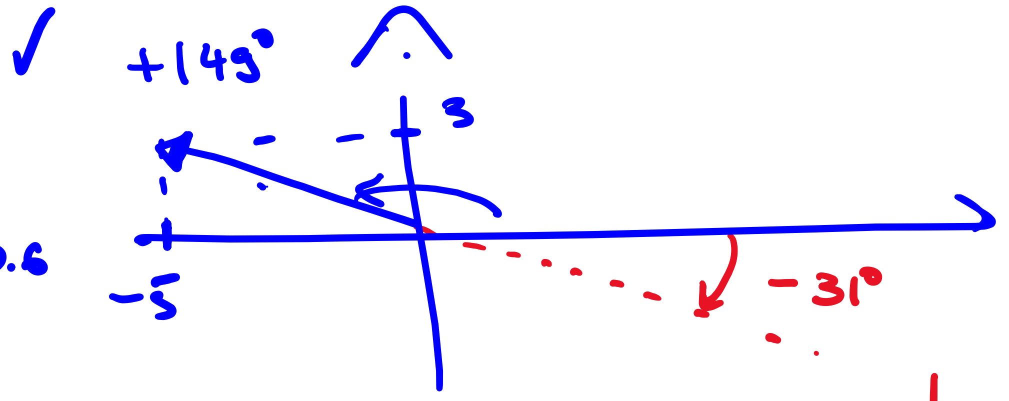
$$\tan\theta = \frac{F_y}{F_x}$$

a) Its magnitude:

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{5^2 + 3^2} = 5.83$$

b) The angle θ from the positive x-axis for the direction in which $\vec{F}$ points.

A. -59°
B. -31°
C. 31°
D. 59°
E. 149°



$$\tan = \frac{3}{-5} = -0.6$$

$$\tan = \frac{-3}{5} = -0.6$$

Calc: $-90° < \varphi < 90°$

Q: Given the vector $\vec{F} = -5\vec{i} + 3\vec{j}$, find:

$$\tan\theta = \frac{F_y}{F_x}$$

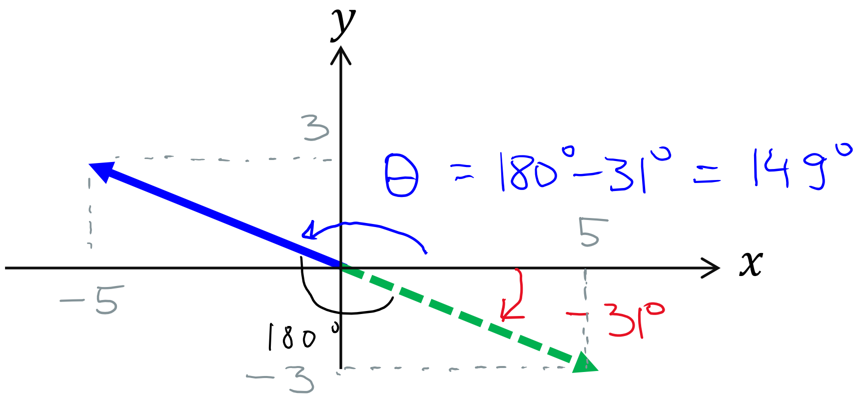
a) Its magnitude:

$$F = \sqrt{5^2 + 3^2} = 5.83$$

b) The angle θ from the positive *x*-axis in which $\vec{\boldsymbol{F}}$ points.

A. -59º
B. -31º
C. 31º
D. 59º
E. 149º

- We know that: $\tan(\theta) = \frac{F_y}{F_x} = \frac{(+3)}{(-5)} \quad \Rightarrow \quad \theta = \arctan\left(\frac{+3}{-5}\right) = \arctan(-0.6)$
- Plugging this into the calculator gives: $\theta = \arctan\left(\frac{+3}{-5}\right) = \arctan(-0.6) = -31^o \Rightarrow$ B
- Does this answer make sense or not? According to the figure, $\theta = -31^o$ corresponds to a vector with $F_x = +5$ and $F_y = -3$, whereas we actually have $F_x = -5$ and $F_y = +3$!
- The problem here is that the calculator cannot tell $\arctan\left(\frac{-3}{5}\right)$ from $\arctan\left(\frac{3}{-5}\right)$.



- It always returns the angle that lays between -90^o and $+90^o$, thus (wrongly) stating that $\theta = -31^o$.
- The correct angle is found as: $\theta = -31^o + 180^o = 140^o$.
- Message: don't trust calculators blindly! Always draw a picture and check that your answer makes sense.