PHYS 170
Week 4: Moment about an Axis. Moment of a Couple. Equivalent Systems

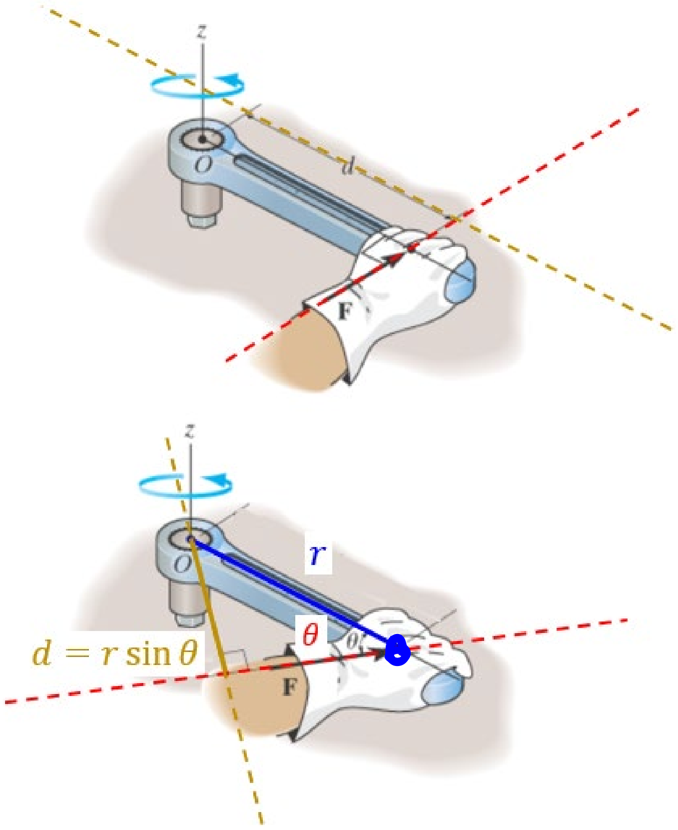
Section 201 (Mon Wed Fri 12:00 – 13:00)

Last Time:

Moment of a force

- Scalar definition

  (useful in 2D)

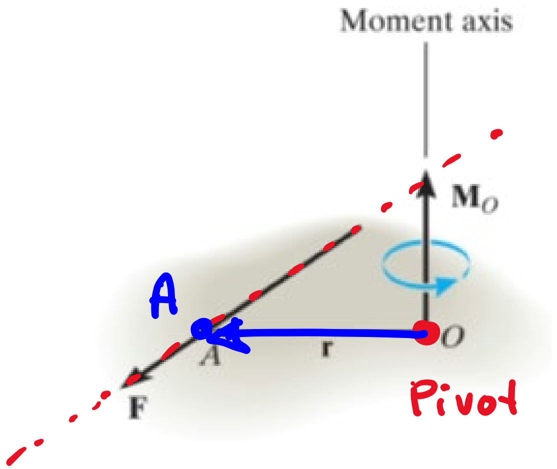


$$M_O = \pm Fd$$
$$= \pm Fr \sin\theta$$

- d = arm (perpend. distance from O to the line of action of $\vec{F}$)
- r = connects O with arbitrary point on the line of action of $\vec{F}$
- + for CCW, − for CW

- Vector definition

  (useful in 3D)

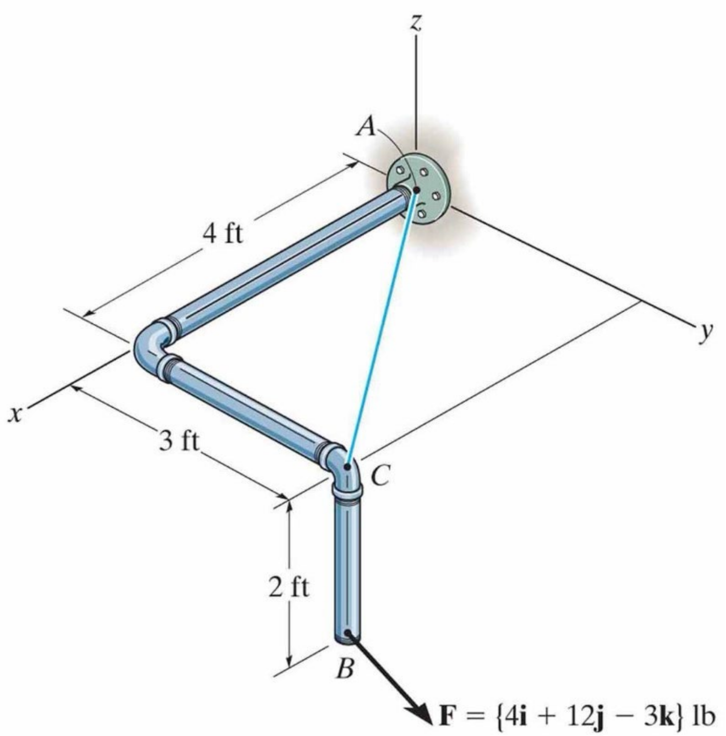


$$\vec{M}_O = \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

- r = connects O with arbitrary point on the line of action of $\vec{F}$

W4-1. Force $\vec{F}$ is acting on the pipe as shown.

1) Determine the moment of $\vec{F}$ about A **(W3-4a)**
2) Determine the moment arm (lever arm) for $\vec{F}$ **(W3-4b)**
3) Determine the magnitude of the moment of $\vec{F}$ about an axis extending from A to C



PROB04_054-055.jpg

W4-1. Force $\vec{F}$ is acting on the pipe as shown.

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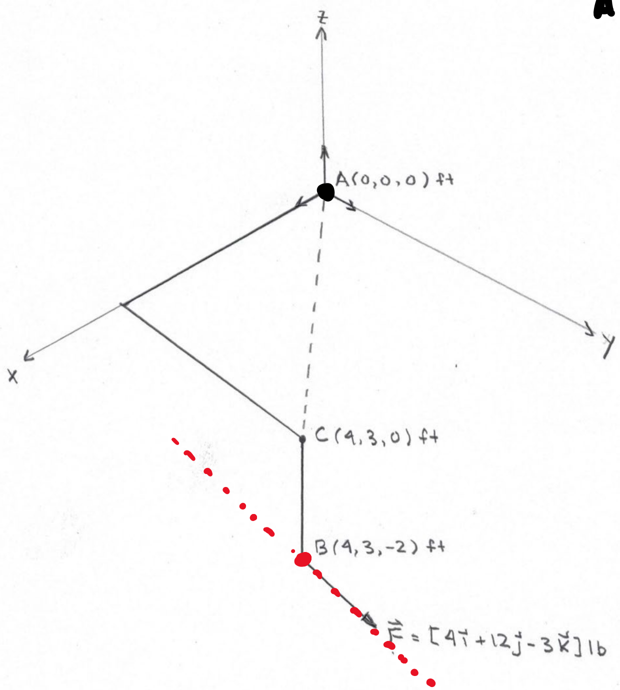

$$\vec{M}_A = \vec{r}_{AB} \times \vec{F}$$

$$\vec{r}_{AB} = (4)\vec{i} + (3)\vec{j} + (-2)\vec{k}$$

$$\vec{M}_A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix} = \begin{aligned} &+\vec{i}\,[(3)(-3) - (-2)(12)] \\ &-\vec{j}\,[(4)(-3) - (-2)(4)] \\ &+\vec{k}\,[(4)(12) - (3)(4)] = \end{aligned}$$

$$= [(15)\vec{i} + (4)\vec{j} + (36)\vec{k}] \ ft \cdot lb$$

W4-1. Force $\vec{F}$ is acting on the pipe as shown.

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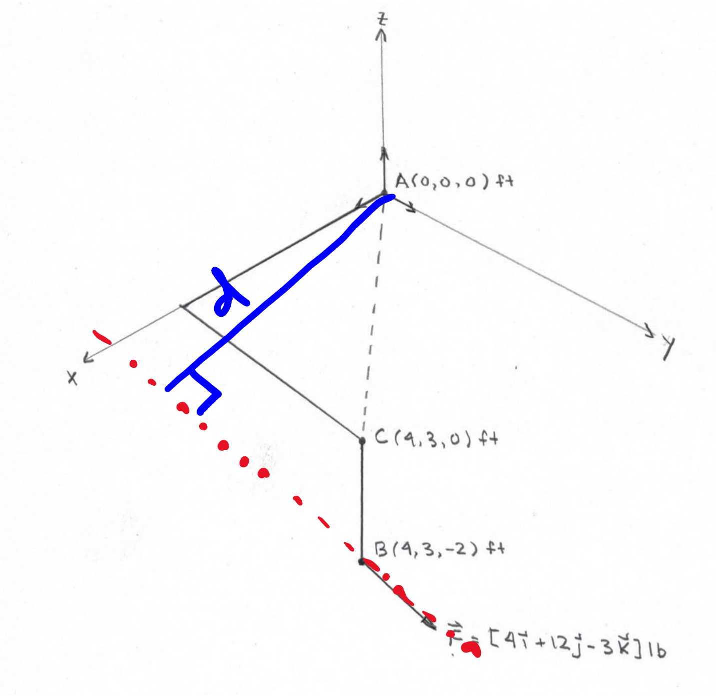

$$\vec{M}_A = [(15)\vec{i} + (4)\vec{j} + (36)\vec{k}] \text{ lb ft}$$

$$M_A = \sqrt{(15)^2 + 4^2 + 36^2} = 39.20$$

$$\vec{F} = [(4)\vec{i} + (12)\vec{j} + (-3)\vec{k}] \text{ lb}$$

$$F = \sqrt{(4)^2 + (12)^2 + (3)^2} = 13$$

$$M_A = F \cdot d$$

$$d = \frac{M_A}{F} = \frac{39.20}{13} = 3.02 \text{ ft}$$

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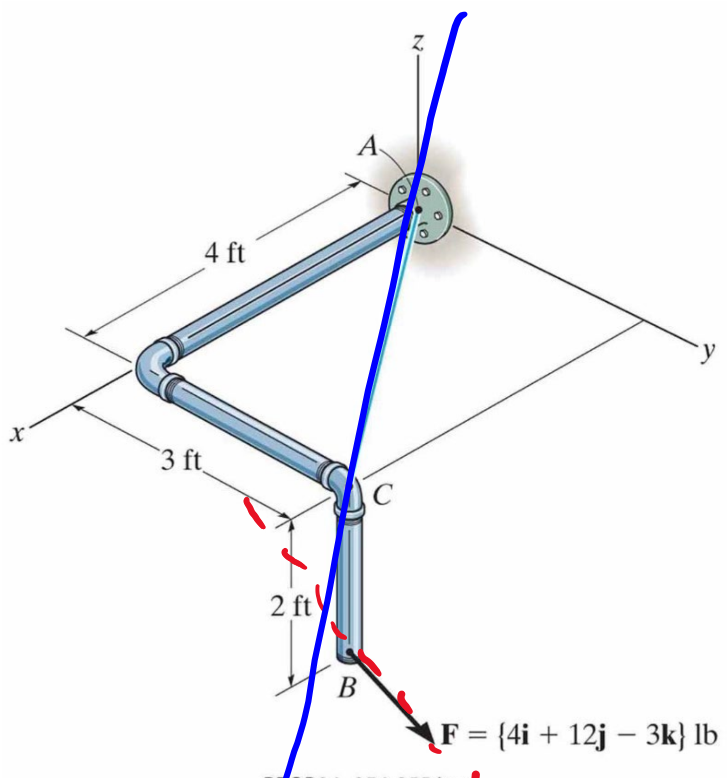

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- Our definition of the moment of a force, $\vec{M} = \vec{r} \times \vec{F}$, was developed for the moment of the force $\vec{F}$ about a point ("pivot point"), since $\vec{r}$ is a vector connecting <u>the pivot point</u> with an arbitrary point on the line of action of the force $\vec{F}$
- Let us now define the moment of a force about an axis!

Moment of a Force about a Specified Axis

Text: 4.5

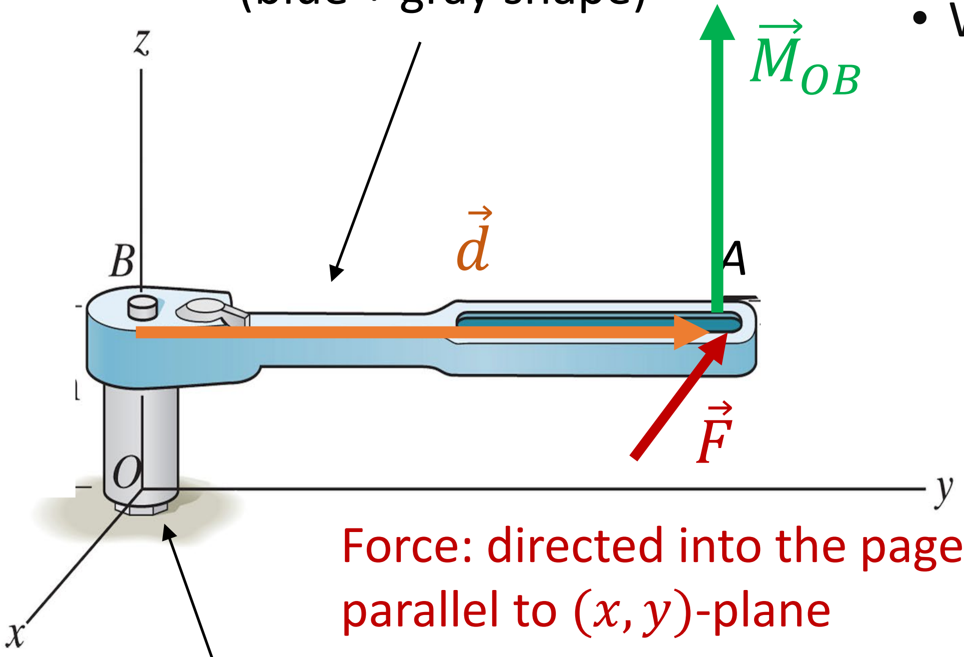
Content:

- Moment about a specified axis
- Connection with a moment about a point

Moment about an axis

- Sometimes the constraints of a system allow motion only about certain axes. Example: a wrench used for changing tires. Here the rotation is only possible about the axis OB

Lug wrench
(blue + gray shape)



Force: directed into the page, parallel to (x, y)-plane

Nut: in the origin, rotates in the (x, y)-plane

- We can define the arm of this rotation by finding a vector $\vec{d}$ which:
  - ➢ connects the axis OB with the line of action of the force, and
  - ➢ is <u>perpendicular</u> to OB

- Then: $\boxed{\vec{M}_{OB} = \vec{d} \times \vec{F}}$

- $\vec{M}_{OB}$ is parallel to OB – makes sense!

- Q: In general in 3D, it might be very difficult to figure out what $\vec{d}$ is!!! Any others way to find $\vec{M}_{OB}$??

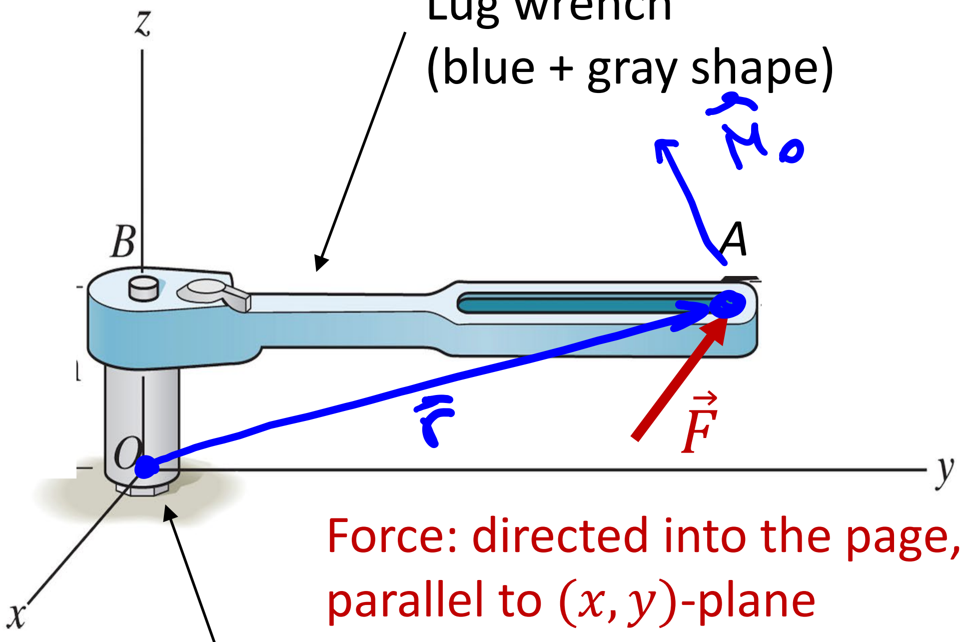
- A: Yes! Use projections.

- Before we proceed:

Q: Let’s pick a point at the rotation axis, OB. It can be <u>any</u> point, for which we know its coordinates. For the sake of example, let it be point O.

What can you say about the moment of the force applied at point *A* <u>about point *O*</u>?

Lug wrench
(blue + gray shape)



Force: directed into the page, parallel to (x, y)-plane

Nut: in the origin, rotates in the (x, y)-plane

$\vec{M}_O$ points in:

A. The $+z$ direction

B. The $-z$ direction

C. Some other direction.

- Before we proceed:

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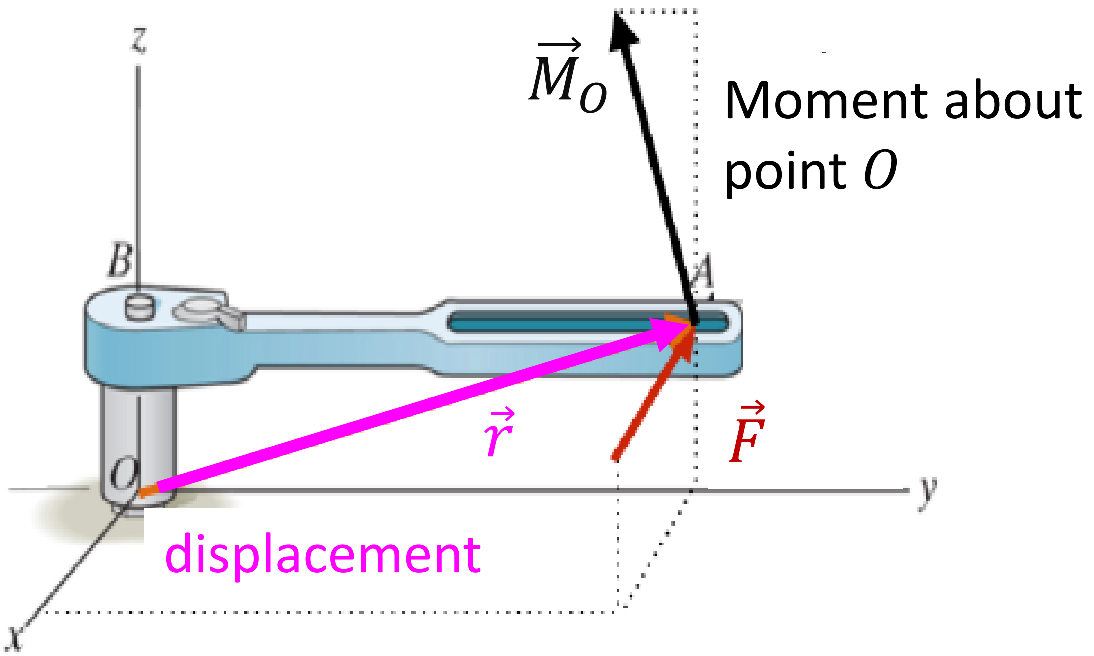


- The arm for $\vec{M}_O$ is $\vec{r}_{OA}$, not $\vec{r}_{BA}$!

$\vec{M}_O$ points in:

A. The $+z$ direction

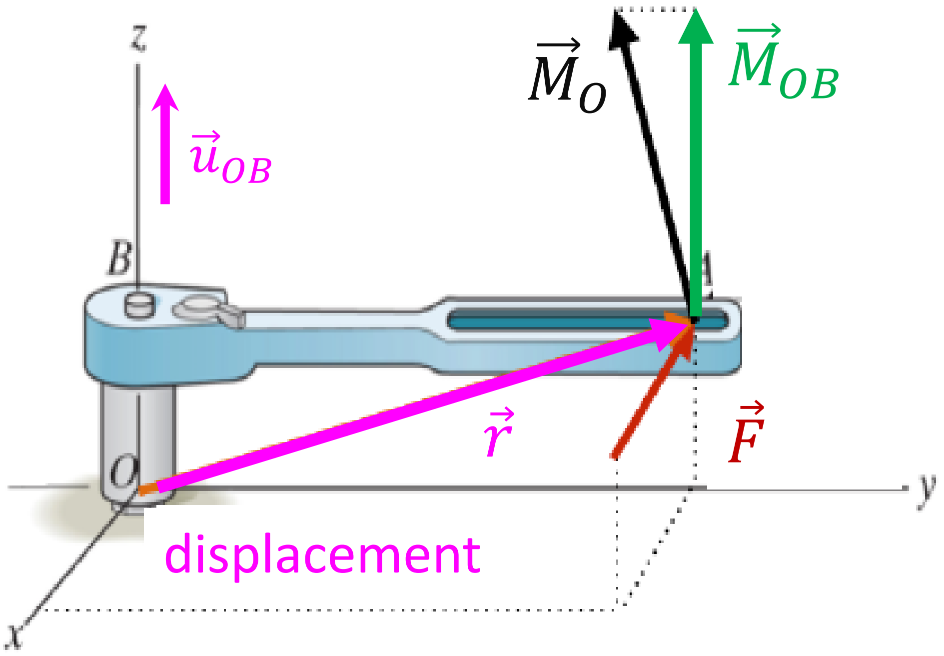
B. The $-z$ direction

C. Some other direction.

- Recall the definition of the moment of a force about a point:

  - Connect the pivot point, O, with an <u>arbitrary point</u> on the line of action of the force $\vec{F}$ => get the displacement vector $\vec{r}$
  - Compute $\vec{M}_O = \vec{r} \times \vec{F}$

MOMENT ABOUT AN AXIS: General procedure

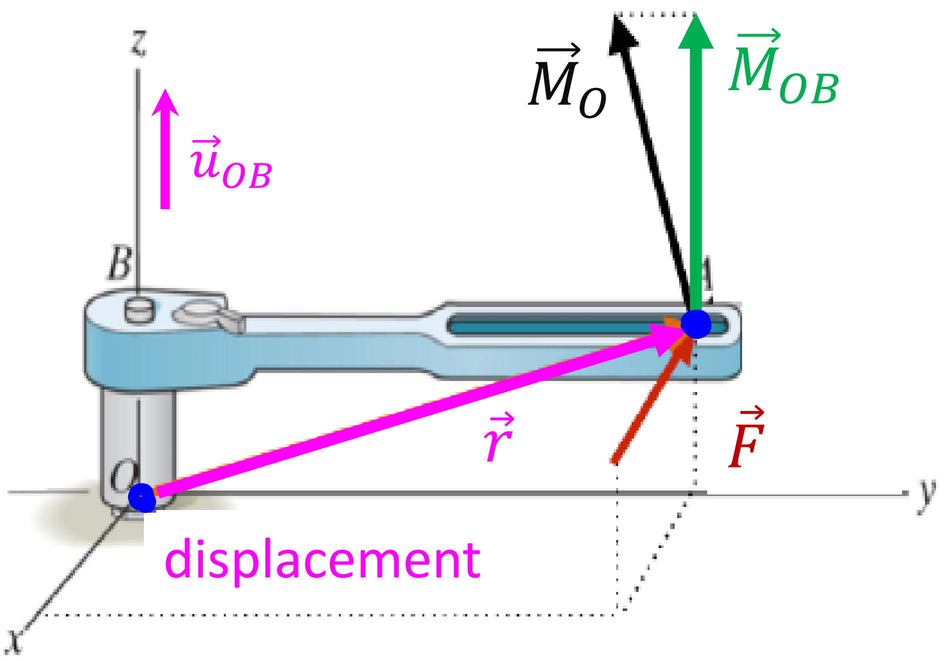


- Q: Is there any connection between the moment of the force $\vec{F}$ about point O, $\vec{M}_O$, and the moment of te force $\vec{F}$ about the axis OB, $\vec{M}_{OB}$?

- A: Yes! The moment $\vec{M}_{OB}$ it the projection of the moment $\vec{M}_O$ on the axis OB.

- Physical meaning: Here the actual rotation is about z-axis because of the system constraints =>
- Only the z-component of the moment $\vec{M}_O$ contributes to the actual rotation.

MOMENT ABOUT AN AXIS: General procedure



- Pick an arbitrary point at the rotation axis OB (say, O)
- Compute the displacement vector $\vec{r}$ connecting this point with the line of action of the force $\vec{F}$
- Compute the moment about point O:

$$\vec{M}_O = \vec{r} \times \vec{F}$$

- Project $\vec{M}_O$ onto the axis OB to find $\vec{M}_{OB}$
- To project, you can use the dot product between $\vec{M}_O$ and the unit vector in the direction of the axis OB, $\vec{u}_{OB}$:

$$\vec{M}_{OB} = (\vec{M}_O \cdot \vec{u}_{OB})\vec{u}_{OB}$$

Magnitude: $\boxed{M_{OB} = \vec{u}_{OB} \cdot \vec{M}_O}$

Vector form: $\boxed{\vec{M}_{OB} = (\vec{u}_{OB} \cdot \vec{M}_O)\, \vec{u}_{OB}}$

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1) Determine the moment of $\vec{F}$ about A **(W3-4a)**
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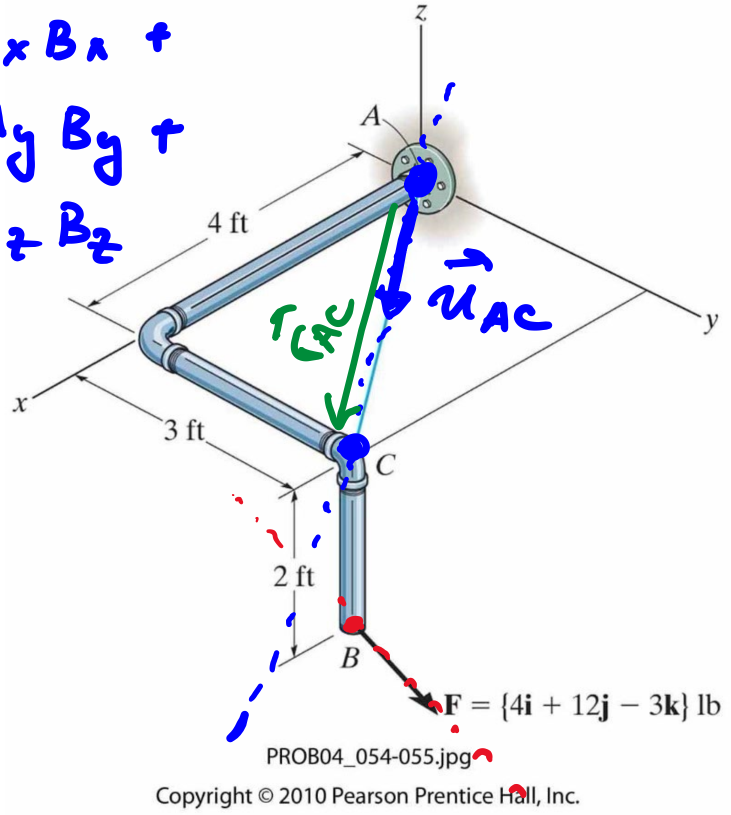
3) Determine the magnitude of the moment of $\vec{F}$ about an axis extending from A to C

$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$

$$\vec{M}_A = [(15)\vec{i} + (4)\vec{j} + (36)\vec{k}] \text{ lb ft}$$



$$M_{AC} = (\vec{u}_{AC} \cdot \vec{M}_A) \qquad \vec{u}_{AC} = \frac{\vec{r}_{AC}}{r_{AC}}$$

$$\vec{r}_{AC} = (4)\vec{i} + (3)\vec{j} + (0)\vec{k} \qquad r_{AC} = \sqrt{\dots} = 5$$

$$\vec{u}_{AC} = \left(\frac{4}{5}\right)\vec{i} + \left(\frac{3}{5}\right)\vec{j} + (0)\vec{k}$$

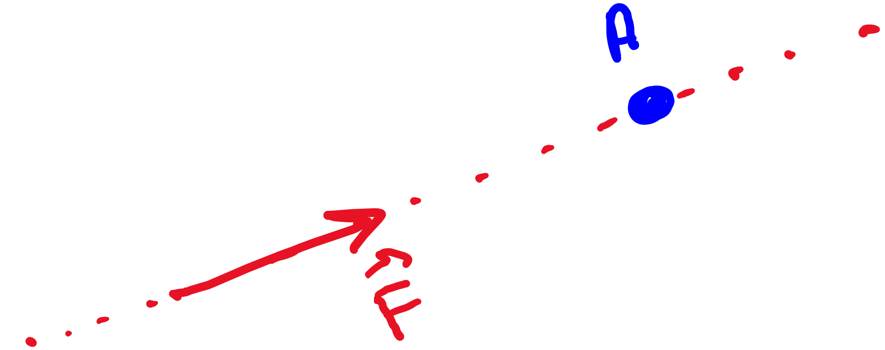
$$M_{AC} = \left(\frac{4}{5}\cdot 15 + \frac{3}{5}4 + 0\cdot 36\right) = 14.4$$

$$\text{lb}\cdot\text{ft}$$

MOMENT OF A FORCE: Special cases

Remember that we have: $M_a = \vec{u}_a \cdot \vec{M}_O = \vec{u}_a \cdot (\vec{r}_{OA} \times \vec{F})$

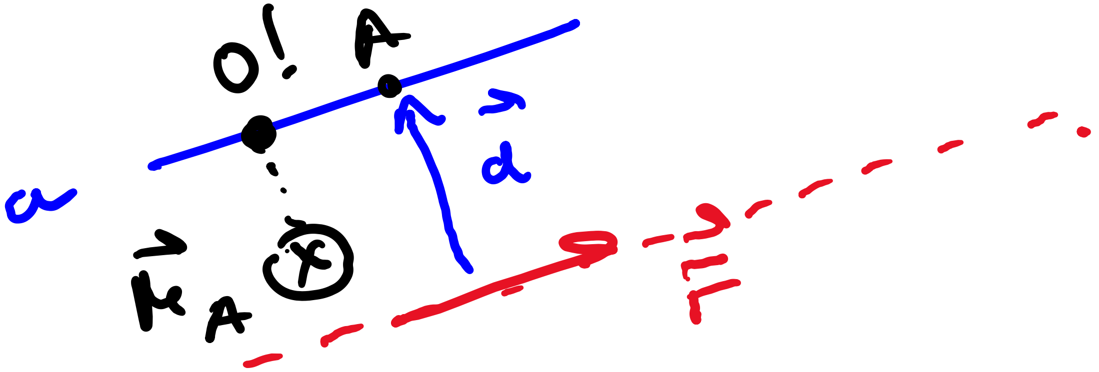
- Q: What is the moment of a force about a point on its line of action?



$\vec{M}_A = 0$

$d = 0$!

- Q: A force is parallel to the specified axis. What is its moment about that axis?



$M_a = \vec{M}_A \cdot \vec{u}_a$

MOMENT OF A FORCE: Special cases

Remember that we have: $M_a = \vec{u}_a \cdot \vec{M}_O = \vec{u}_a \cdot (\vec{r}_{OA} \times \vec{F})$

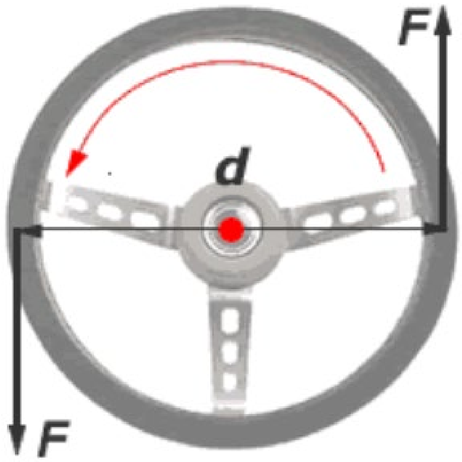
- Q: What is the moment of a force about a point on its line of action?

  - $M_O = Fd = 0$, since $d = 0$
  - The moment of a force about any point on its line of action is zero!
  - The force only pushes/pulls on that point, but it cannot produce any rotation about it.

- Q: A force is parallel to the specified axis. What is its moment about that axis?

  - For any point O in this axis: $\vec{M}_O = (\vec{r}_{OA} \times \vec{F})$ is perpendicular to $\vec{F}$ ⇨ $\vec{M}_O \perp \vec{u}_a$
  - The dot product of two perpendicular vectors is zero ⇨ $M_a = \vec{u}_a \cdot \vec{M}_O = 0$
  - The moment of a force about any axis parallel to that force is zero!

Moment of a Couple



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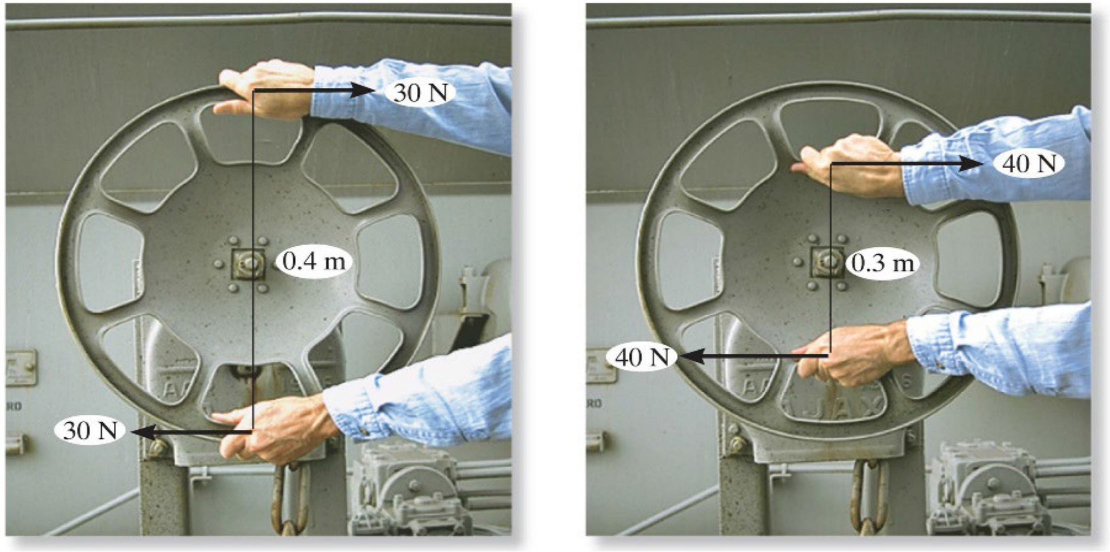
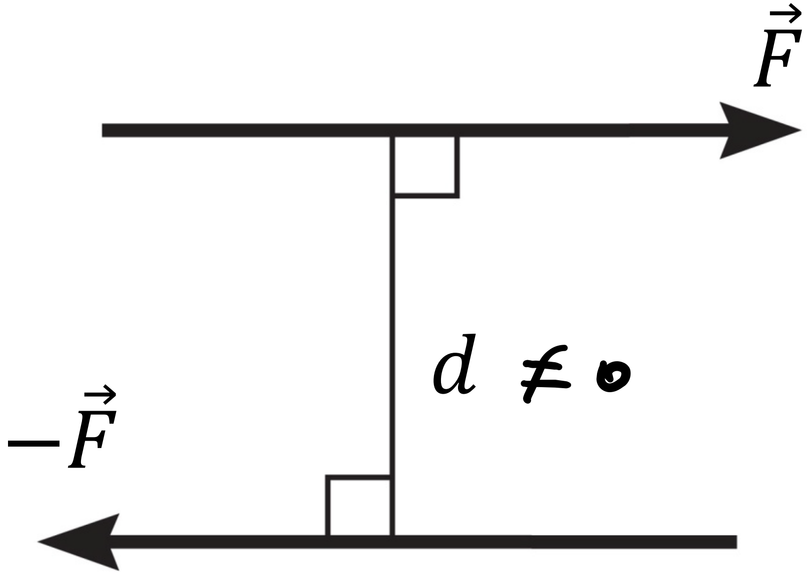
Content:

- A couple and couple moment
- Moment of a couple is a free vector

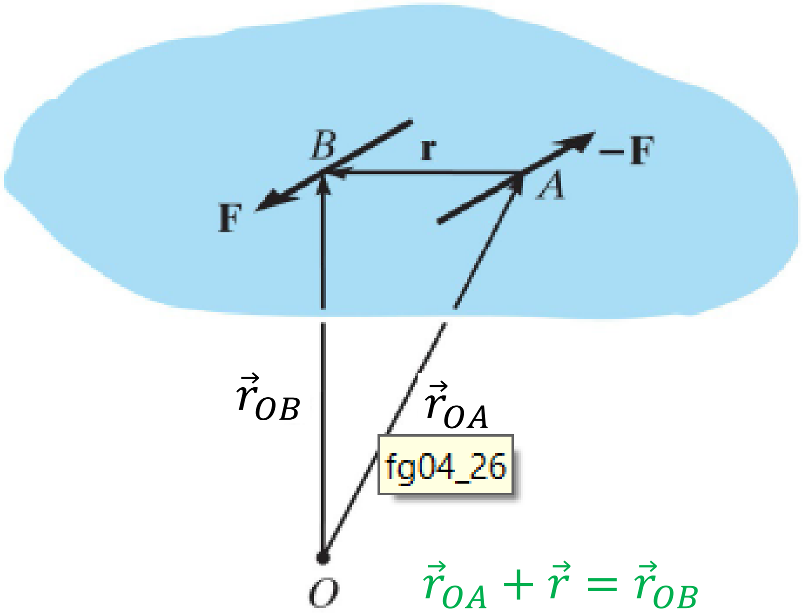
MOMENT OF A COUPLE: Intro





- **Couple (definition)**: Two parallel forces that have the **same magnitude**, **opposite directions**, and which are **separated by a perpendicular distance d**
  - Note that **net force** of this couple is **zero**…
  - …but the **moment** of this couple is **not zero**
    - Hence, a couple produces rotation, but cannot cause displacement

MOMENT OF A COUPLE: about which point?



$\vec{r}_{OA} + \vec{r} = \vec{r}_{OB}$

- Let us take an arbitrary point O and calculate the moment of this couple about it:

$$\vec{M}_O = \vec{r}_{OB} \times \vec{F} + \vec{r}_{OA} \times \left(-\vec{F}\right) = (\vec{r}_{OB} - \vec{r}_{OA}) \times \vec{F} = \vec{r} \times \vec{F} \equiv \vec{M}$$

- A moment of a couple does NOT depend on the choice of the point about which it is calculated

- Note: this is in a striking contrast with the moment of <u>one force</u> about a point or about an axis, which **changes** if you change the point about which you calculate this moment!