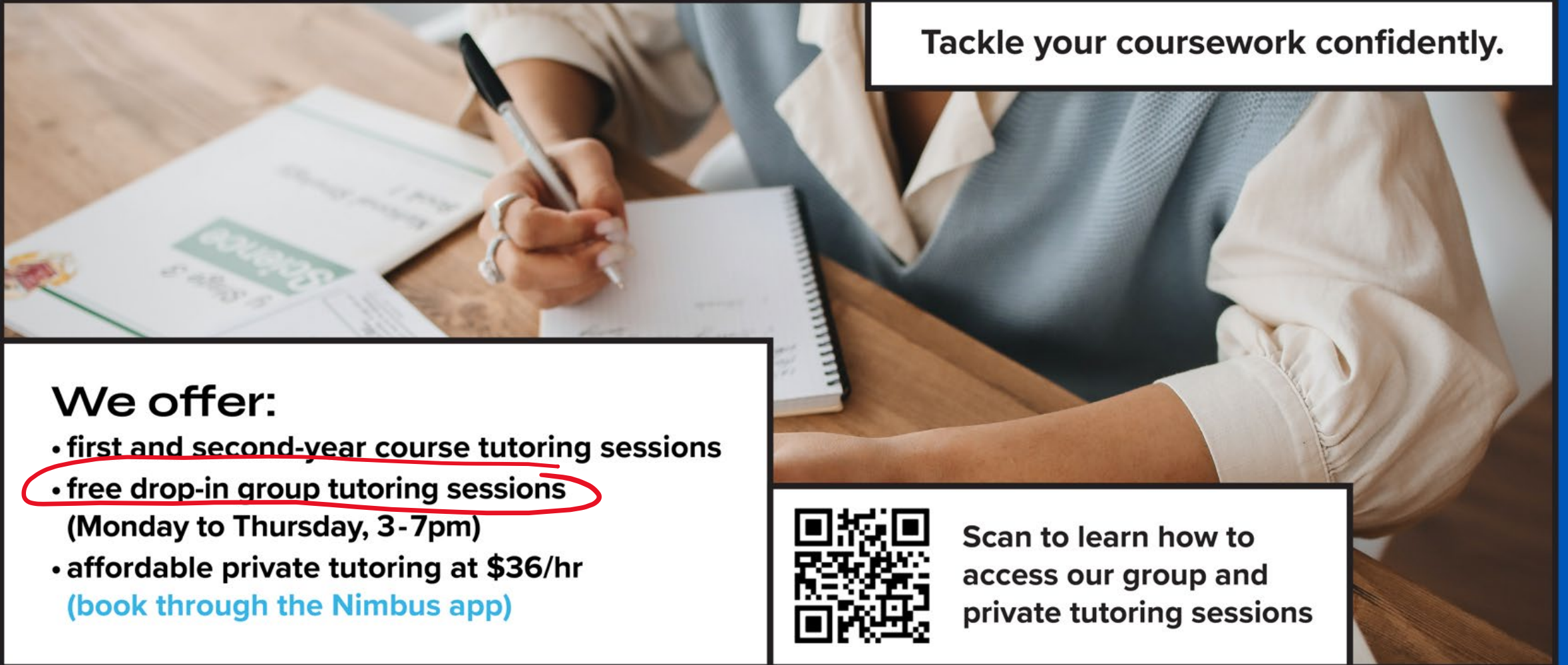


AMS TUTORING

ams.ubc.ca/tutoring



Tackle your coursework confidently.



We offer:

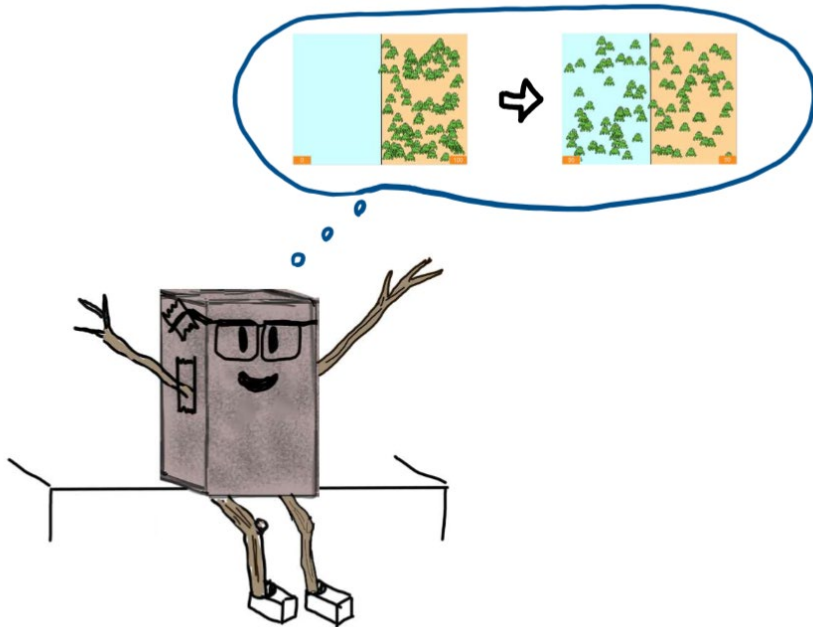
- first and second-year course tutoring sessions
- free drop-in group tutoring sessions
(Monday to Thursday, 3-7pm)
- affordable private tutoring at \$36/hr
(book through the [Nimbus app](#))



Scan to learn how to
access our group and
private tutoring sessions

Lecture 23.

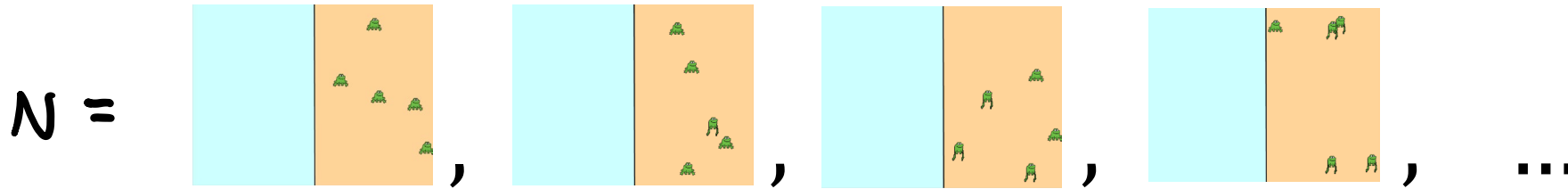
Entropy.



Last Time

- FROGS:

- **Macroscopic state:** e.g. (0,5) distribution of frogs
- **Microscopic state:** manifold of ways this macroscopic state can be arranged



- **ENTROPY** of a macroscopic configuration is:

$$S = \text{const} \times \ln[N]$$

N is the number of **microscopic** configurations of this **macroscopic** configuration

- GASES:

- **Macroscopic state:** pressure P , volume V , temperature T , internal energy \bar{U}
- **Microscopic state:** manifold of ways you can distribute U over gas molecules ([video](#))

Entropy: microscopic definition

- Combinatorics: $\# \text{config} = \underbrace{\left(\frac{N!}{n!(N-n)!} \right)} \times (10^5)^N$

$$\frac{100!}{0! 100!} = 1 \Rightarrow$$

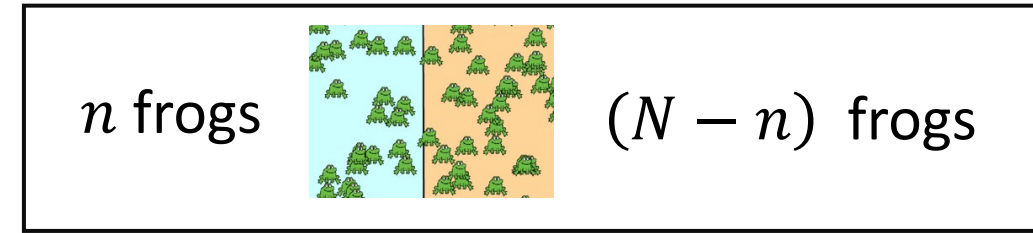
$\sim 10^{500}$ possible configurations of (0,100)

entropy is $\log(N) \sim 500$

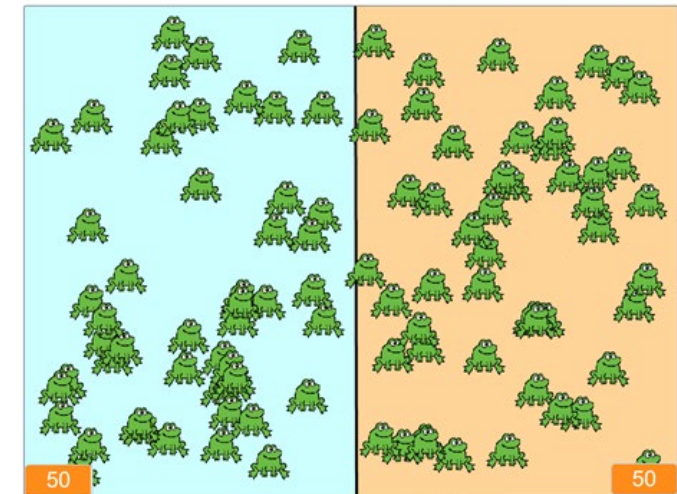
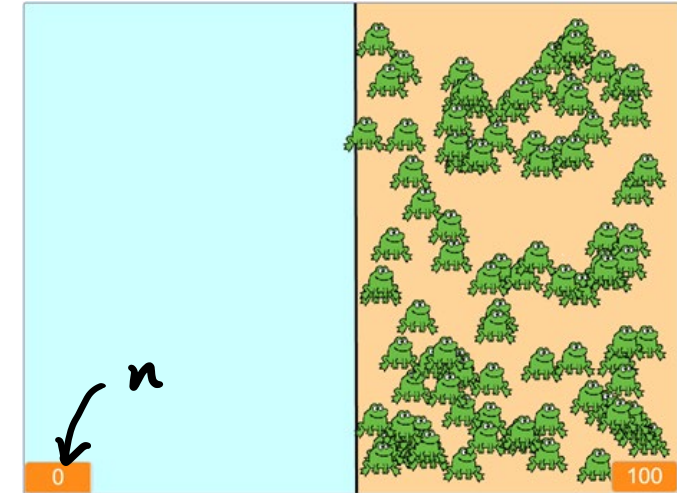
$$\frac{100!}{50! 50!} = 10^{29} \Rightarrow$$

$\sim 10^{529}$ possible configurations of (50,50)

entropy is $\log(N) \sim 529$ -- larger!



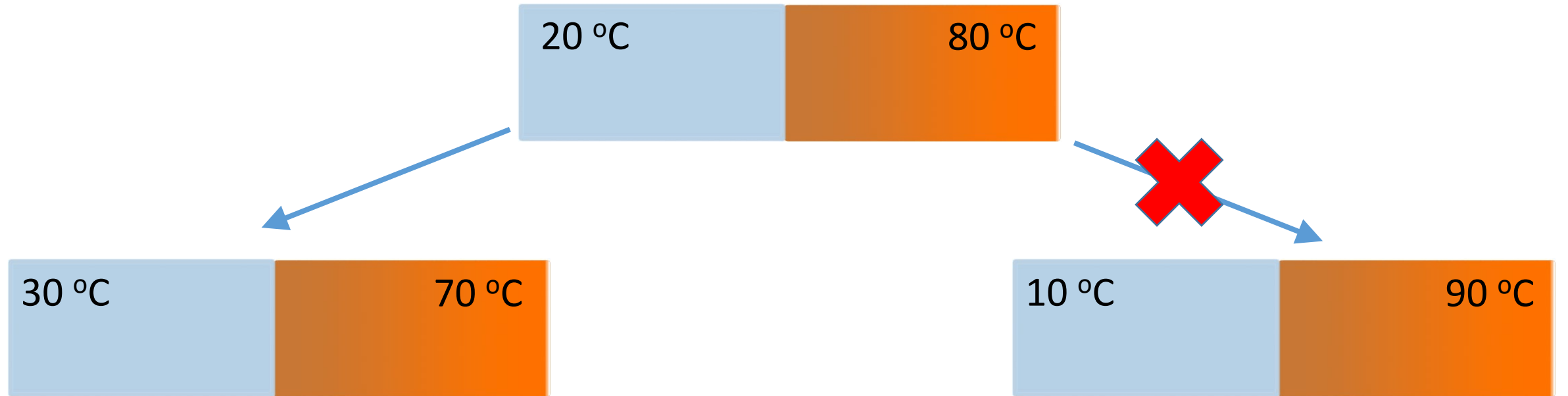
$N = 100$



2nd Law of Thermodynamics

Total entropy never decreases

(probability of decrease is vanishingly small)



Higher entropy

⇒ far more states with these Ts

$10^{1,000,000,000,000,000,000,000}$ times more likely

Lower entropy

⇒ far fewer states with these Ts

2nd Law of Thermodynamics

Total entropy never decreases

(probability of decrease is vanishingly small)



**The entropy of the system increases as
the time goes (or does not change)**



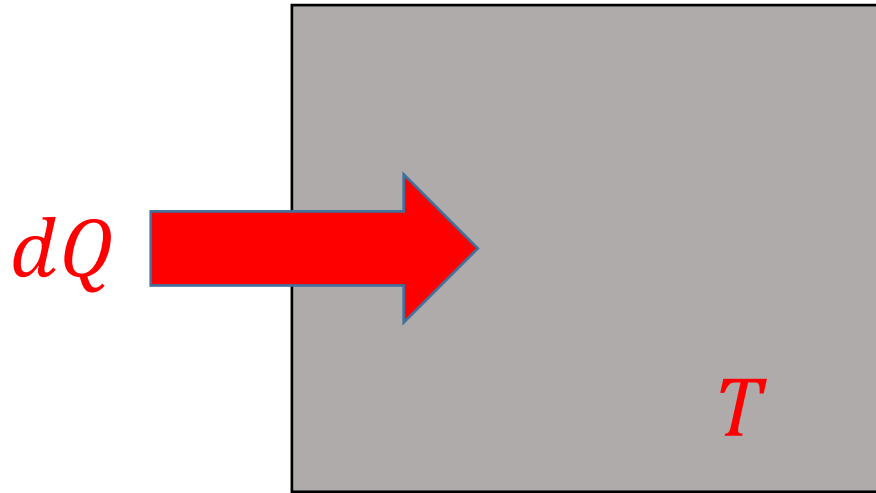
Measuring entropy is a way to
distinguish the past from the future



Time arrow!



Entropy: macroscopic definition



$$dS = \frac{dQ}{T} \leftarrow \text{const}$$

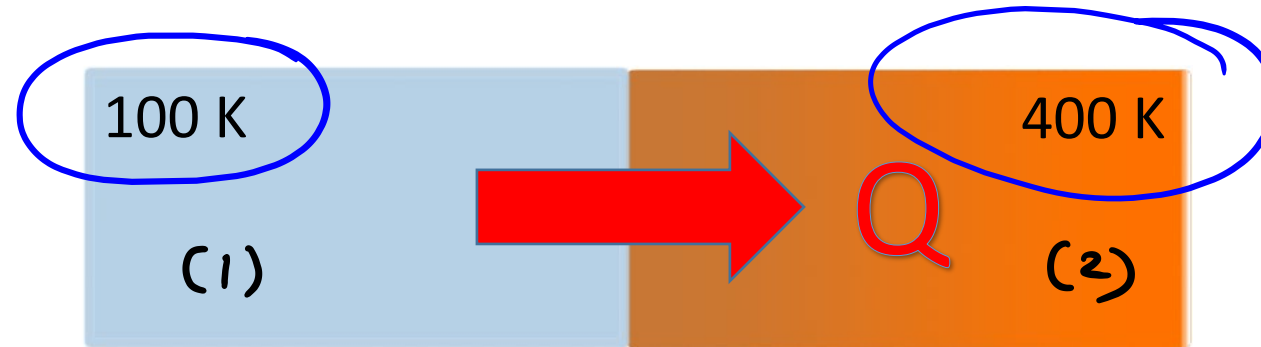
- dS is the change in entropy when dQ of heat is added to a system at temperature T
- Units of S are Joules/Kelvin

- Amazing result:

- We can prove this from the microscopic definition of S
- See examples 20.8 and 20.11 in the text, and/or watch this video:
<https://www.youtube.com/watch?v=t7gyi8NhgYg>



Q: Suppose that we had $\textcircled{1 \text{ J}}$ of energy flow from the cold object to the hotter object. What would be the change in entropy of the whole system?



$$dQ = -1 \text{ J}$$
$$T = 100 \text{ K}$$

$$dQ = +1 \text{ J}$$
$$T = 400 \text{ K}$$

$$dS = dS_1 + dS_2$$

$$dS = \frac{dQ}{T}$$

A. -0.0125 J/K

☒ B. -0.0075 J/K

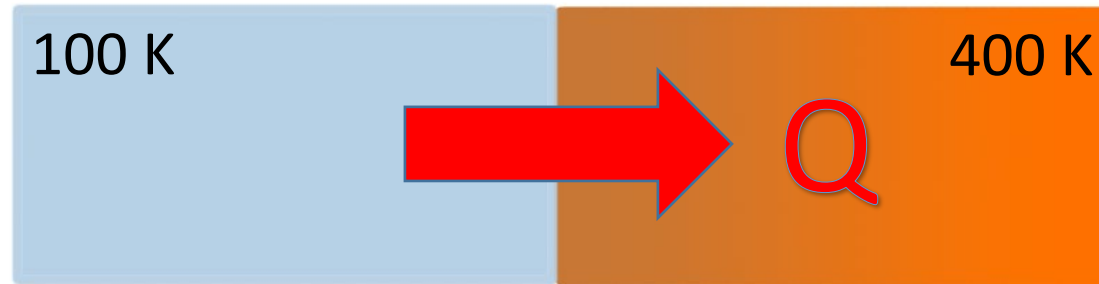
C. 0

D. 0.0075 J/K

E. 0.0125 J/K



Q: Suppose that we had 1 J of energy flow from the cold object to the hotter object.
What would be the change in entropy of the whole system?



$$\begin{aligned}\text{Have } dS &= dS_1 + dS_2 = \frac{dQ_1}{T_1} + \frac{dQ_2}{T_2} \\ &= \frac{-1 \text{ J}}{100 \text{ K}} + \frac{1 \text{ J}}{400 \text{ K}} = -0.0075 \text{ J/K}\end{aligned}$$

A. -0.0125 J/K

B. -0.0075 J/K

C. 0

D. 0.0075 J/K

E. 0.0125 J/K



↑
BAD!

Violates 2nd Law, so it won't happen!



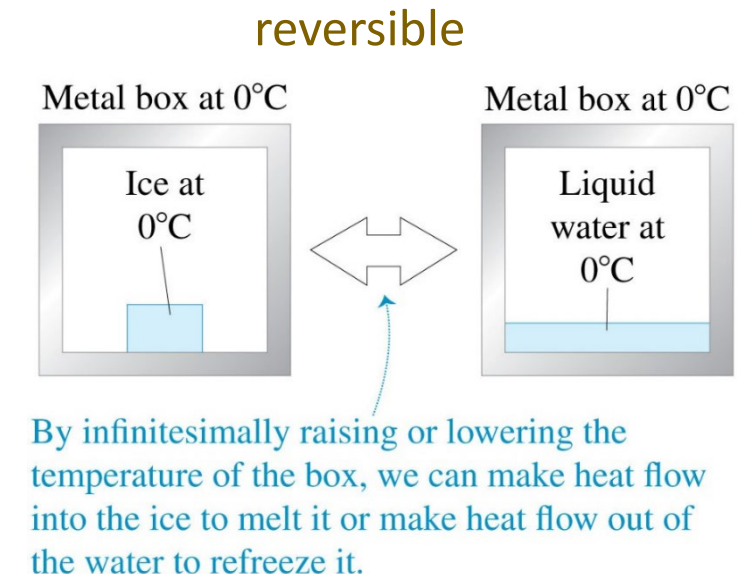
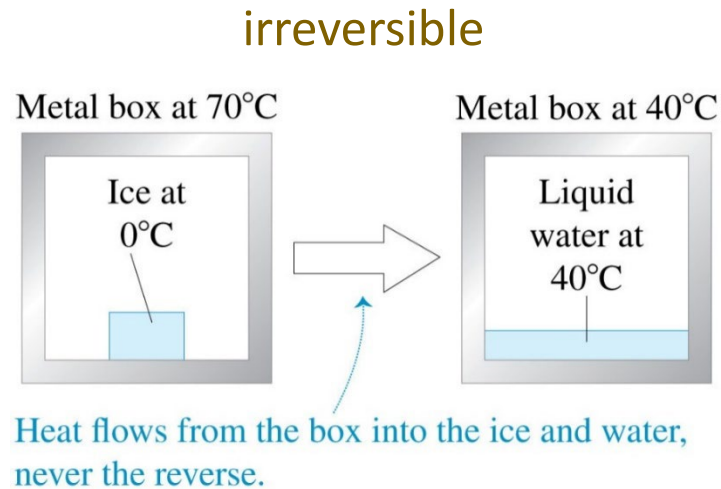
- Thermodynamic processes are mostly irreversible

- Entropy changes are either zero or positive – total entropy never decreases (this is because the larger the entropy, the higher the probability of the state)
- Therefore, in almost all processes, heat flows from high T to low T , increasing entropy
 - Reversing the process would require entropy to decrease, violating the 2nd Law!

- There are two kinds of evolution of a system:

- $dS = 0$: reversible
- $dS > 0$: irreversible

- Irreversible processes are associated with loss of “useful” energy (energy that we can control)



- Thermodynamic processes are mostly irreversible

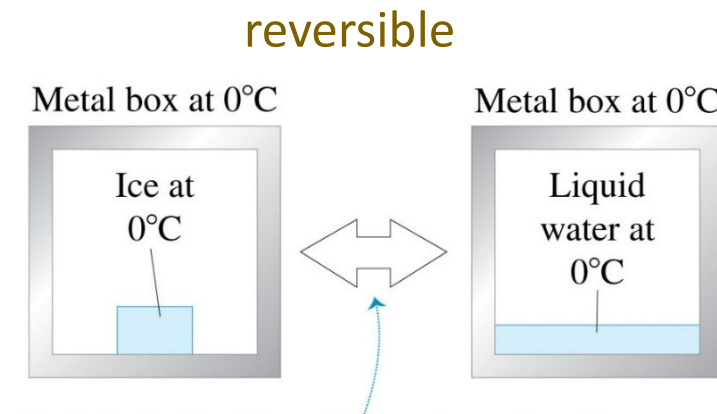
- Hence, reversible processes are the most useful for high efficiencies and COP

- There are only two kinds of reversible processes:

- Adiabatic processes: no heat transfer, so no change in entropy $\left(dS = \frac{dQ}{T}\right) \rightarrow dS = 0!$
- Isothermal processes: heat transfer between two objects at “nearly” the same T
- In both these cases, process can be reversed by infinitesimal change in conditions

- Note that if the entropy of the system does not change, then the entropy of environment usually increases!

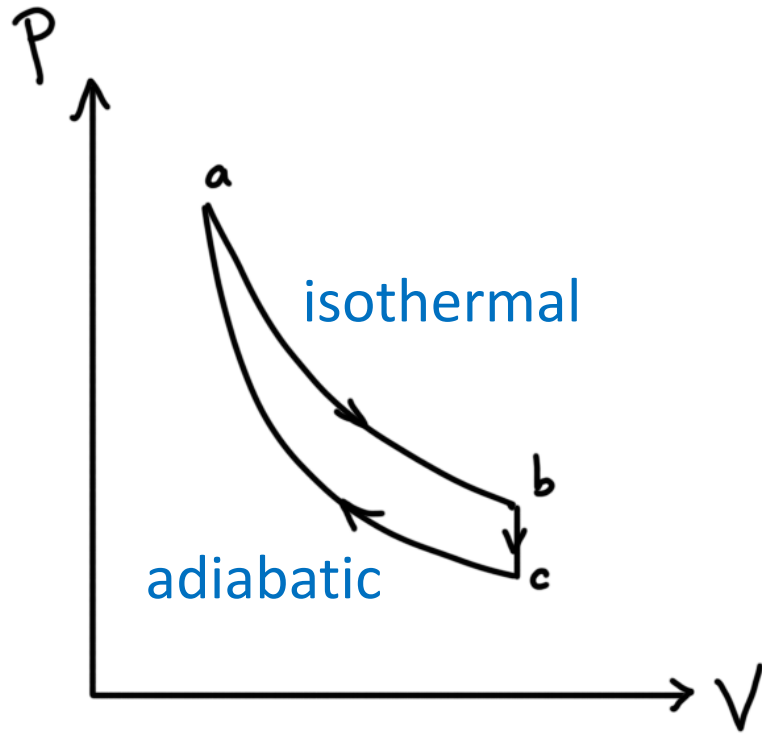
(we will see an example at the end of the week)



By infinitesimally raising or lowering the temperature of the box, we can make heat flow into the ice to melt it or make heat flow out of the water to refreeze it.



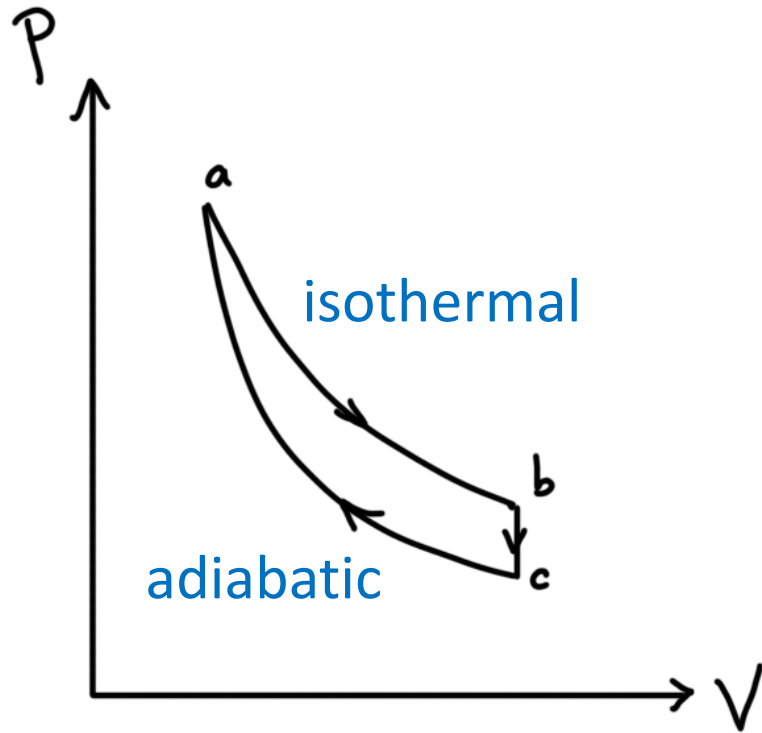
Q: In the cycle shown, what happens with entropy from $c \rightarrow a$?



- A. The entropy increases
- B. The entropy is constant
- C. The entropy decreases



Q: In the cycle shown, what happens with entropy from $c \rightarrow a$?



$$dS = \frac{dQ}{T}$$

$c \rightarrow a$ is adiabatic

$\Rightarrow dQ = 0$ and $dS = 0$

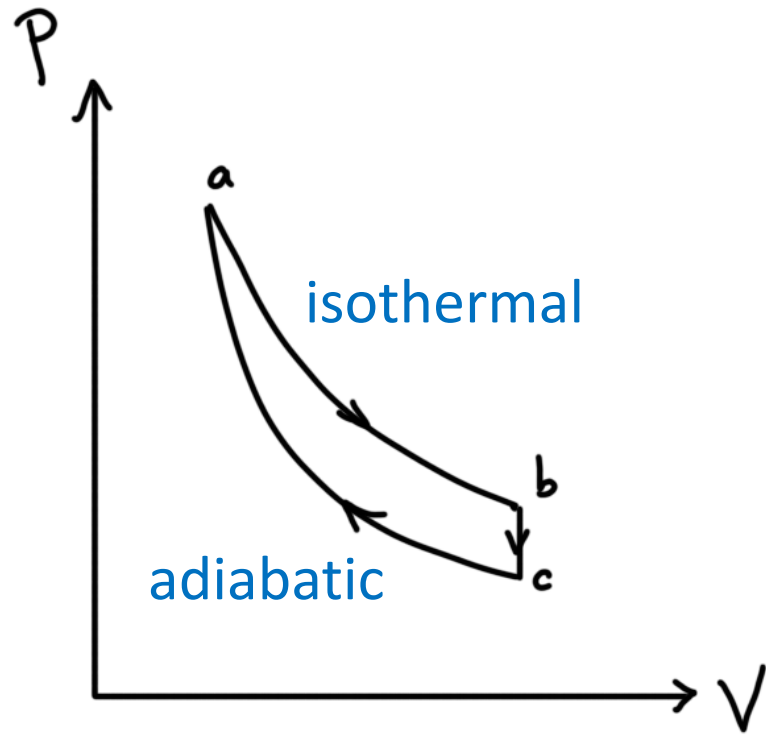
- A. The entropy increases
- B. The entropy is constant
- C. The entropy decreases



Reversible process



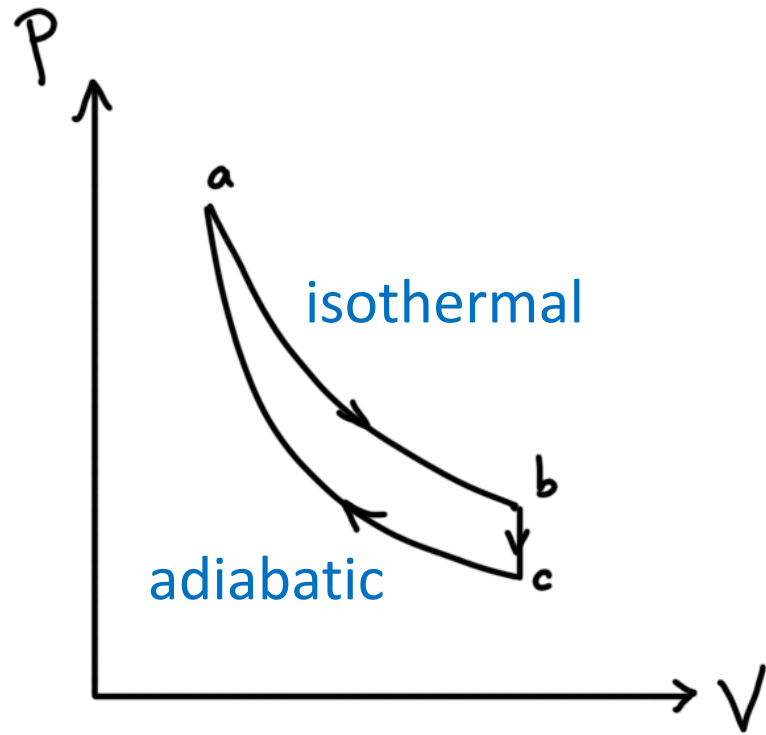
Q: In the cycle shown, heat Q enters the gas in the isothermal step $a \rightarrow b$ at temperature T . The entropy change during this step



- A. is equal to Q/T
- B. is equal to 0
- C. is equal to $-Q/T$
- D. cannot be determined from the information provided



Q: In the cycle shown, heat Q enters the gas in the isothermal step $a \rightarrow b$ at temperature T . The entropy change during this step



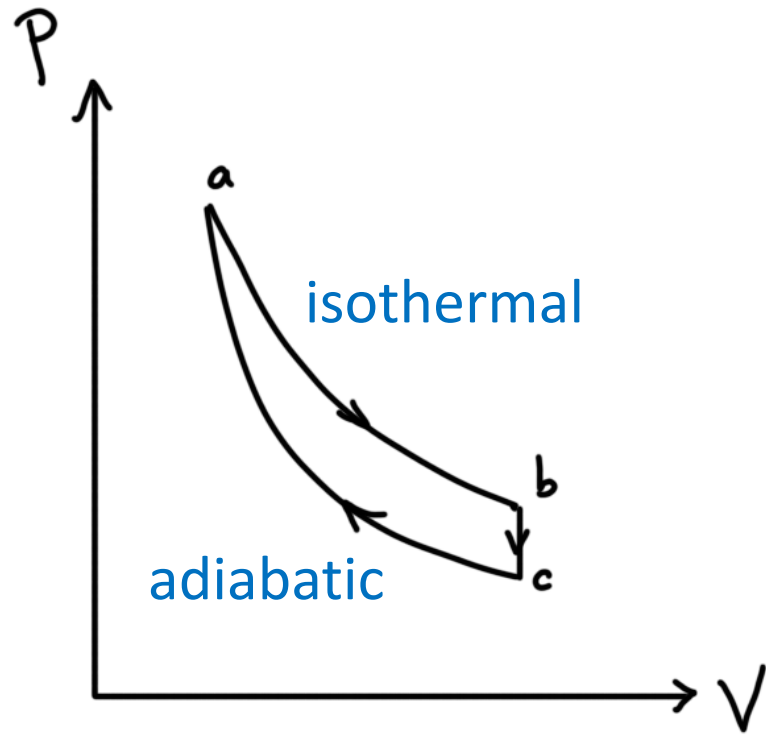
$$dS = \frac{dQ}{T}$$

T constant, so $\Delta S = \frac{Q}{T}$

- A. is equal to Q/T ✓
- B. is equal to 0
- C. is equal to $-Q/T$
- D. cannot be determined from the information provided



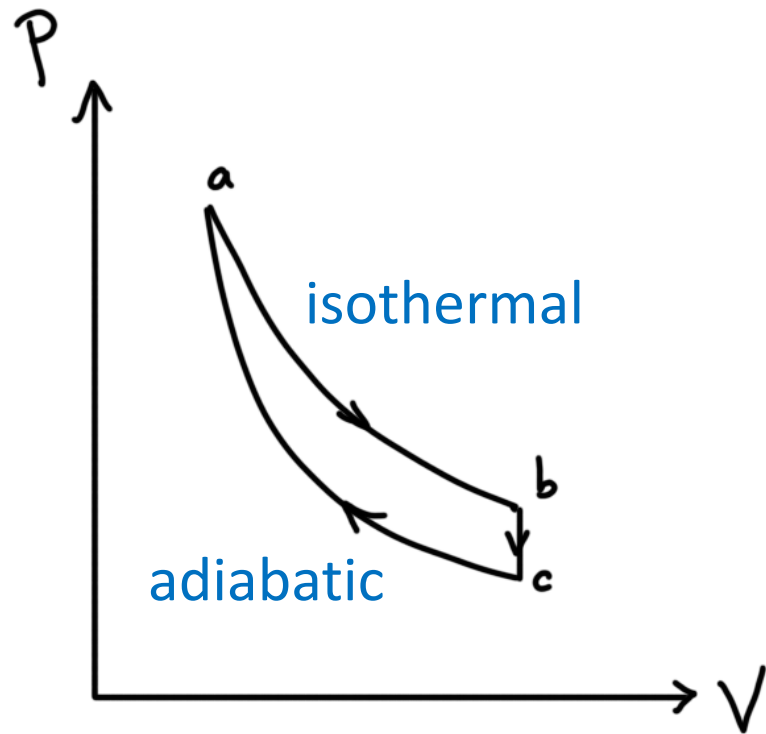
Q: In the cycle shown, heat Q enters the gas in the isothermal step $a \rightarrow b$ at temperature T . The entropy change from $b \rightarrow c$ is



- A. Q/T
- B. between 0 and Q/T
- C. 0
- D. between $-Q/T$ and 0
- E. $-Q/T$



Q: In the cycle shown, heat Q enters the gas in the isothermal step $a \rightarrow b$ at temperature T . The entropy change from $b \rightarrow c$ is



$$dS = \frac{dQ}{T}$$

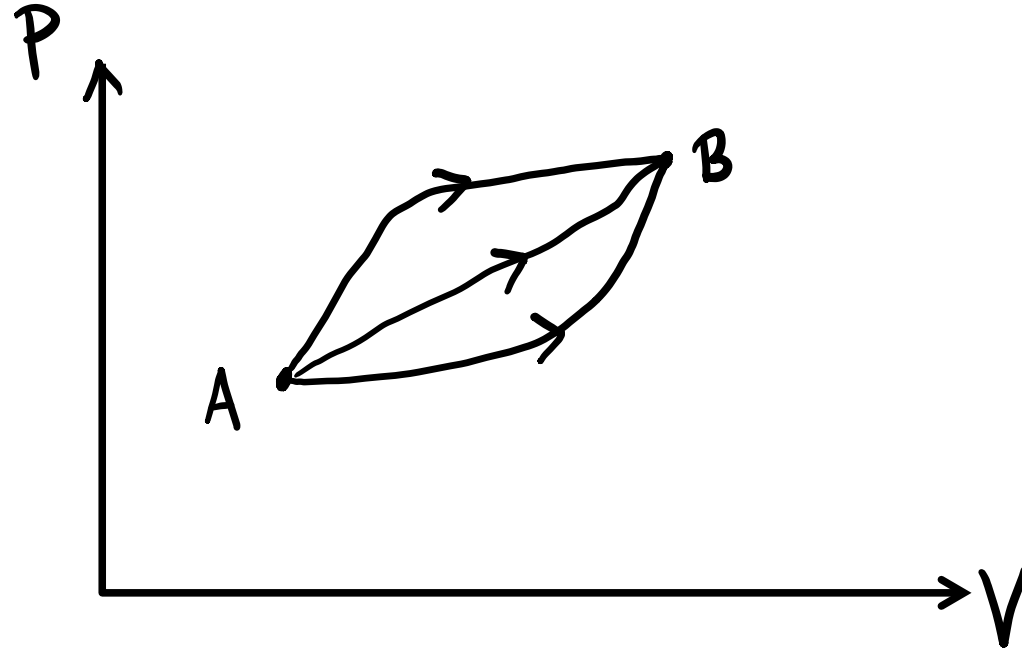
$S_c = S_a$ since $c \rightarrow a$ is adiabatic

So $\Delta S_{a \rightarrow b} + \Delta S_{b \rightarrow c} = 0$

$$\Delta S_{b \rightarrow c} = -\Delta S_{a \rightarrow b} = -\frac{Q}{T}$$

- A. Q/T
- B. between 0 and Q/T
- C. 0
- D. between $-Q/T$ and 0
- E. $-Q/T$ ✓

- Entropy is a **state variable** – like P, V, T, U



- ΔS is the same for all paths, and zero for a complete cycle
- But: S for environment usually increases!

- Entropy is **additive** (= “extensive property”)

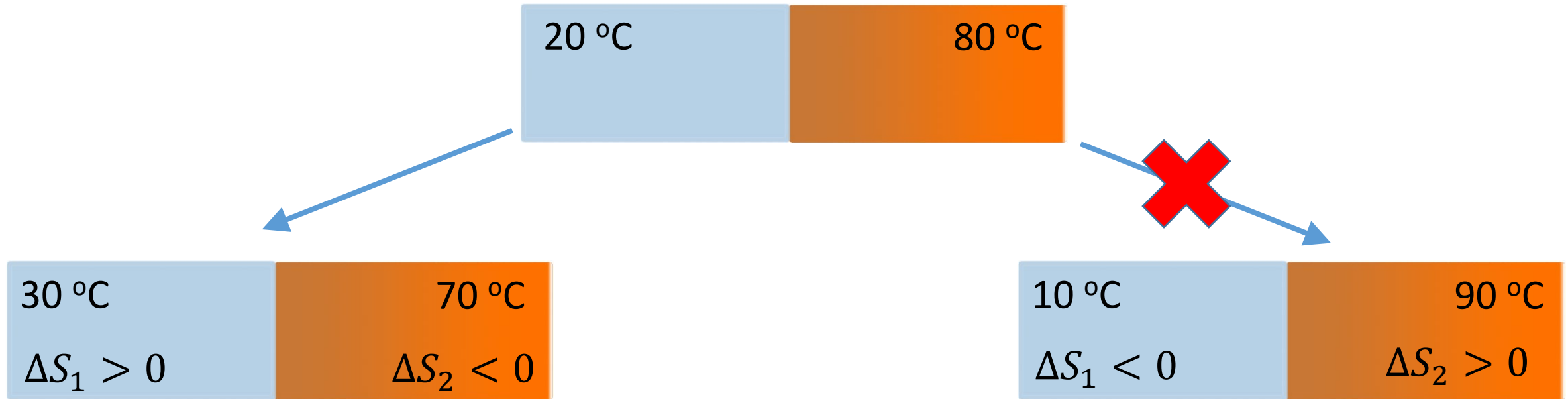


$$\triangleright S_{Total} = S_1 + S_2$$

2nd Law of Thermodynamics

Total entropy never decreases

(of whole system – a probability of a part can decrease!)



- In both cases, $|\Delta S_1| > |\Delta S_2|$ since $|Q_1| = |Q_2|$ but $T_1 < T_2$

$$\Delta S_{\text{Total}} > 0$$

$$\Delta S_{\text{Total}} < 0 \text{ violates 2}^{\text{nd}} \text{ law!}$$