Name and student number:

Group members:

Physics 157 Tutorial 2 – Young's modulus

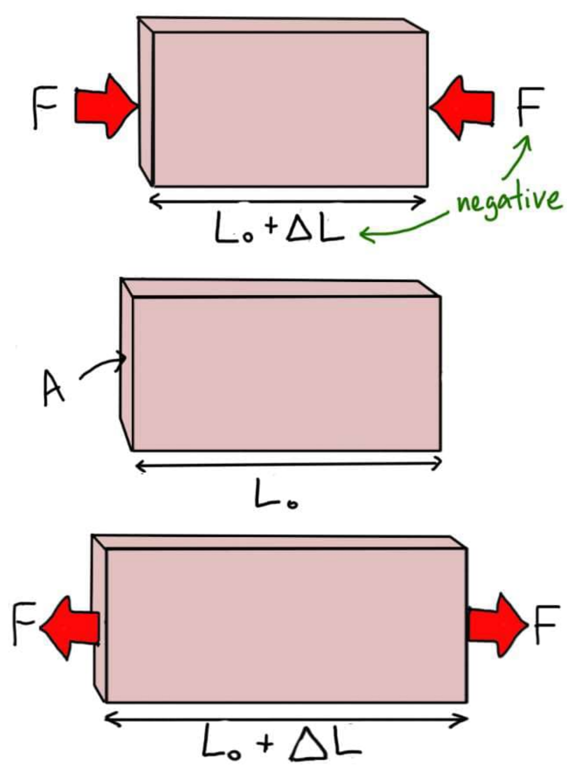
Today, we'll think about how the length of an object changes when stretching or compressing forces act on it. There is also a problem (#9) in which the length is determined by changes in both stress and temperature. You should be sure to try that problem, and ask the TA if you are stuck.

You will work in groups of three or four, but each person should complete their own worksheet to hand in.

Ask the TA to come to your group if you are stuck.

You've probably learned about Hooke's Law for a spring. This says that if we exert a pushing or pulling force F on the spring, the change in length ΔL is related to the force by

(1) $$F = k\,\Delta L$$



Here, F is positive for pulling and negative for pushing. The number k is a constant (the "spring constant") that is larger for stiffer springs.

Actually, *almost all materials obey a law like this* (as long as the forces aren't too large); they are just usually not as stretchy as a spring, so it's harder to measure ΔL. For a block of some solid material, like a brick, the constant k in the equation above depends on the material***, but also on the dimensions of an object***.

Let's understand how:

Question 1: First, let's think about how k depends on the cross-sectional area A, shown in the middle figure above.

Suppose we have an object with twice the cross-sectional area as the one in the pictures above, but the same length. If we also apply twice as much compressive force as in the first picture above (shown at the right), we expect that the amount of compression ΔL will be

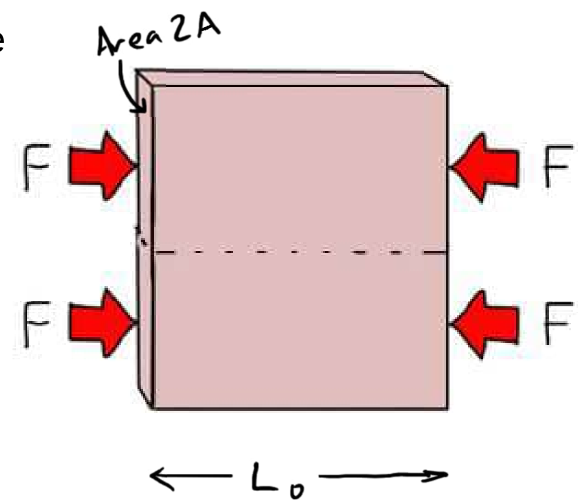
a) Twice as much as in the first picture above

b) Half as much as in the first picture above

c) The same as in the first picture above

Why?

The new brick is just like 2 of the old ones glued on top of one another. Since we are also using twice the force, we should get the same compression as before.



Hint: we can't use equation (1) to answer this because the constant k may not be the same for the two cases. Instead, compare the two pictures and think of a direct argument for your answer.

Question 2: For each quantity below, circle one of the options to say how that quantity in the example of question 1 compares to the original case:

F:	same as before	double	half
ΔL:	same as before	double	half
k = F/ ΔL:	same as before	double	half
A:	same as before	double	half

Question 3: Based on your answer to question 2, would you say that k is

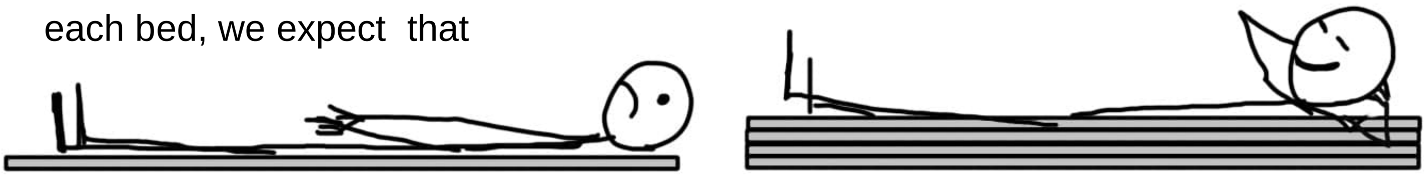
a) proportional to the area A (i.e. doubling A doubles k)

b) inversely proportional to the area A (i.e. doubling A halves k)

c) independent of the area A (i.e. doubling A doesn't change k)

Check with a TA to make sure you have the right answer here!

Question 4: Now let's understand how k depends on the length/thickness of our object if we keep the cross-sectional area fixed. First, imagine a bed made from a thin foam mattress. Now imagine a second bed made from several of these stacked on top of one another. If two people of the same mass lay on each bed, we expect that



a) The second bed will compress significantly less than the first

b) The second bed will compress the same amount as the first

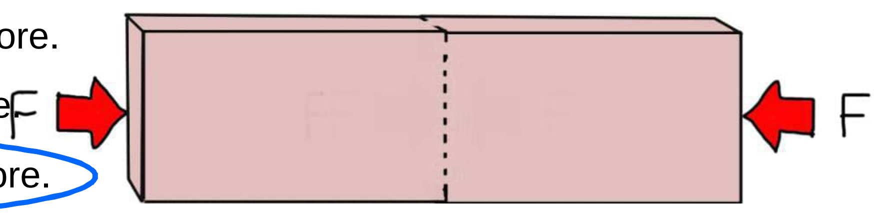
c) The second bed will compress significantly more than the first

Question 5: The figure below shows an object with twice the length and the same cross-sectional area as the object in the first picture on the previous page. If we apply the same force as in that case, there will be:

a) The same compression as before.

b) Half the compression as before.

c) Twice the compression as before.



Hint: Use your intuition from question 4! Question 5 is like the situation with two mattresses, except things are rotated so the forces are horizontal.

Can you give a convincing argument for your answer?

Each half of the new brick is the same size and feels the same forces as the original brick, so will compress the same amount as the original one. The total ΔL is then twice as much as before.

Question 6: For each quantity below, circle one of the options to say how that quantity in the example of question 5 compares to the original case:

F:	same as before	double	half
ΔL:	same as before	double	half
$k = F/\Delta L$:	same as before	double	half
L_o:	same as before	double	half

Question 7: Based on your answer to question 6, would you say that k is

a) Proportional to the length $\mathbf{L_o}$ (i.e. doubling $\mathbf{L_o}$ doubles k)

b) Inversely proportional to the length $\mathbf{L_o}$ (i.e. doubling $\mathbf{L_o}$ halves k)

c) Independent of the length $\mathbf{L_o}$ (i.e. doubling $\mathbf{L_o}$ doesn't change k)

Check with a TA to make sure you have the right answer here!

Hopefully, you have convinced yourself that k should be proportional to area (if length is kept fixed) and inversely proportional to length (if area is kept fixed). This allows us to write an equation

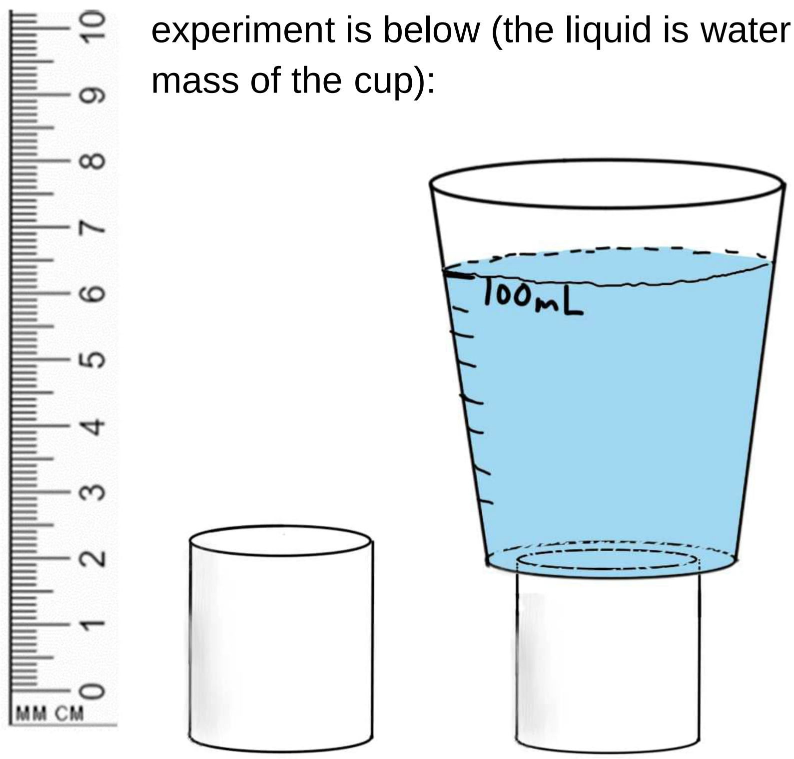
(2) $$k = Y\frac{A}{L_o}$$

where **Y** is a constant that does not depend on the length or the area. This constant is called the **Young's modulus**, and is a basic property of the material that our object is made of. It tells us how stiff that material is. Substituting the expression for k in (2) into equation (1), we get:

(3) $$\frac{F}{A} = Y\frac{\Delta L}{L_0}$$

The left side is the force per unit area that we apply. This is called the **STRESS**. The ratio $\Delta L/L_0$ is fractional change in length of the object. It is called the **STRAIN**. So equation (3) says that stress is proportional to strain, and the Young's modulus Y is the proportionality constant.

Question 8: A group of first year engineering students decide to measure the Young's modulus of a marshmallow. A picture of their experiment is below (the liquid is water, and you can ignore the mass of the cup):



Calculate Y. In your calculations, use SI units of meters, kg, N so that your final result for Y will be in N/m^2. Your result doesn't need to be really accurate.

The mass of water is ~ 0.1L x 1 kg/L = 0.1 kg. The height of the left marshmallow is ~ 3.0 cm and the height of the right marshmallow is ~ 2.7 cm. The diameter of each marshmallow is ~ 2.7 cm.

So we have $\Delta L = 0.3$ cm, and $L_o = 3.0$ cm.

The area is $\pi R^2 \sim 3.14 \times (0.0135\text{ m})^2 = 5.7 \times 10^{-4}\text{ m}^2$.

The force is $F = M_{water}\, g = (0.1\text{ kg} \times 9.8\text{ m/s}^2) \sim 1\text{ N}$

From $F/A = Y\, \Delta L/L$, we get $Y = (1\text{ N}/ 5.7 \times 10^{-4}\text{ m}^2) \times (1/ 0.1) \sim 1.7 \times 10^4\text{ N/m}^2$.

(Anything from $1.5 – 3 \times 10^4\text{ N/m}^2$ is reasonable)

Question 9: At a particle accelerator facility, a 16,000 kg detector is to be installed on top of four aluminum legs, each with cross sectional area $0.001 m^2$. The legs are fabricated at 20^oC to a height of 0.10000 m. After the detector is in place, the scientists adjust the temperature of the legs in order to compensate for the compression due to the weight of the detector. If they would like the final height of the legs to be 0.10000 m, what should be the temperature of the legs?

For aluminum, $Y = 7\times10^{10}$, $\alpha = 2.4\times10^{-5}$

The net change in height is

$$\Delta L = \Delta L_{th} + \Delta L_{st} \tag{1}$$

$$= \alpha L_o \Delta T + \frac{1}{Y} L_o \frac{\Delta F}{A} \tag{2}$$

Where $\Delta F = -mg$. We want $\Delta L = 0$, so:

$$\Delta T = \frac{-\Delta F}{\alpha T A} = \frac{16{,}000\ kg \cdot 9.8\ m/s^2}{2.4x10^{-5}K^{-1} \cdot 0.001m^2 x 7x10^{10} N/m^2} = 23^o C \tag{3}$$

Thus, the temperature should be $20^oC + \Delta T = 43^oC$.

Part b) If the legs need to be kept within the range 0.10000 m ± 0.00005 m, what range of temperatures are allowable for the aluminum legs?

Here, we want $-0.00005\ m \le \Delta L \le 0.00005\ m$. Thus, we have:

$$\alpha L_o \Delta T_{max} + \frac{1}{Y} L_o \frac{\Delta F}{A} = 0.00005\ m \tag{4}$$

$$\alpha L_o \Delta T_{min} + \frac{1}{Y} L_o \frac{\Delta F}{A} = -0.00005\ m \tag{5}$$

Solving, we get $\Delta T_{max} = 44^o C$ and $\Delta T_{min} = 2.5^o C$.

So, we need $22.5^oC \le T \le 64^oC$.