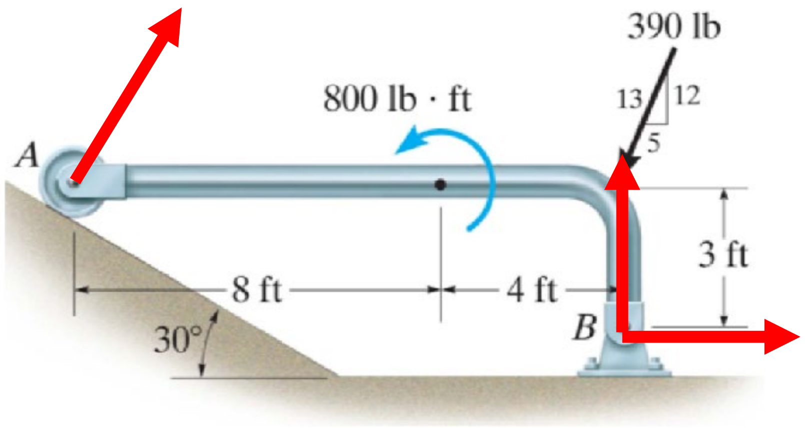
WARMING UP: 2D Equilibrium

W5-1. Find the reaction forces and moments at points A and B at equilibrium.



Last Time:

Q: We need all the moments, i.e. (i) exerted by the reaction forces from the previous slide, and (ii) the external couple moment $\vec{M}_{\text{couple}}$= 800 lb ft to cancel. Which point we must choose as O to calculate our moments about?

A. A
B. B
C. Where the 800 ft lb moment is acting.
D. Where the 390 N force is acting.
E. Any of the above is fine

There is no "must". A couple moment is a free vector and can be associated with any point. That said, it is possible to argue that the simplest choice is B, since it eliminates two unknown force components *(the moment of a force about a point on its line of action is zero)*.

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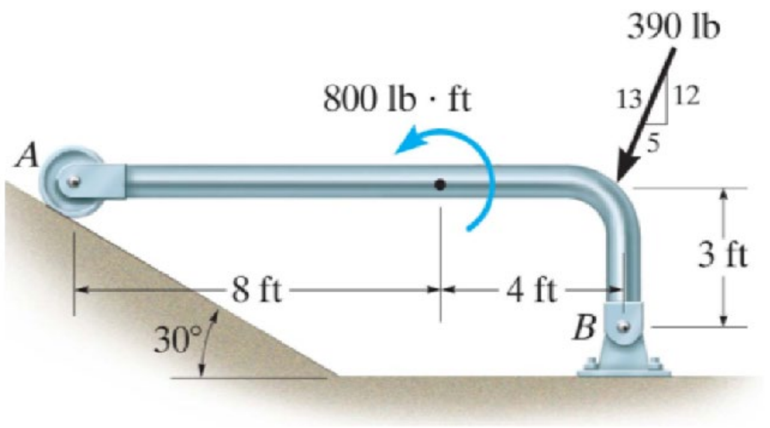
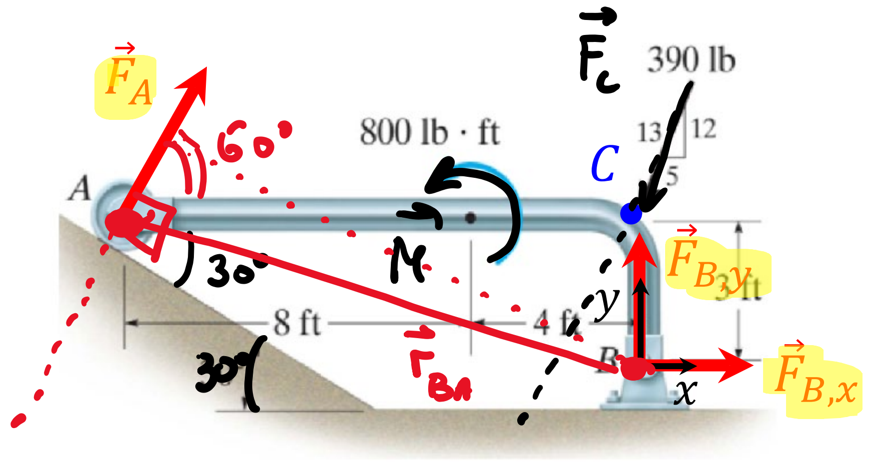


WARMING UP: 2D Equilibrium

z $\odot$



W5-1. Find the reaction forces and moments at points A and B at equilibrium.

- Translational equilibrium: $\vec{F}_R = \vec{F}_A + \vec{F}_B + \vec{F}_c = 0 \leftarrow$

$x: \quad \Sigma F_x = 0 \qquad F_A \cos 60° + F_{Bx} - 390 \cdot 5/13 = 0 \quad (1)$

$y: \quad \Sigma F_y = 0 \qquad F_A \sin 60° + F_{By} - 390 \cdot 12/13 = 0 \quad (2)$

$z: \quad N/A$

- Rotational equilibrium: $(\vec{M}_R)_B = (\vec{M}_A)_B + \cancel{(\vec{M}_B)_B}^{0} + (\vec{M}_c)_B + \vec{M} = 0$

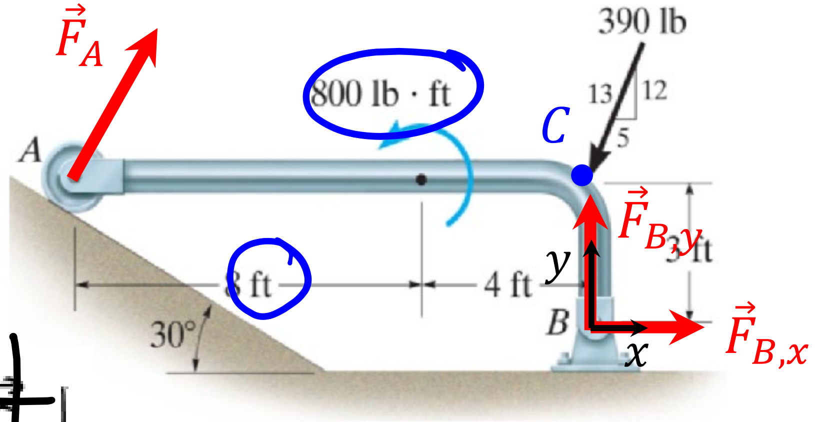
$$(\vec{M}_A)_B = \vec{r}_{BA} \times \vec{F}_A = \qquad\qquad (\vec{M}_c)_B = \vec{r}_{Bc} \times \vec{F}_c =$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -12 & 3 & 0 \\ F_A \cos 60° & F_A \sin 60° & 0 \end{vmatrix} \qquad = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 3 & 0 \\ -390\frac{5}{13} & -390\frac{12}{13} & 0 \end{vmatrix}$$

$\vec{M} = 800\,\vec{k}$

WARMING UP: 2D Equilibrium

- Rotational equilibrium (continued):



$$(\vec{M}_R)_B =$$

$$= \vec{k}\cdot 800 + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -12 & 3 & 0 \\ F_A\cos 60^\circ & F_A \sin 60^\circ & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 3 & 0 \\ -390\cdot\frac{5}{13} & -390\,\frac{12}{13} & 0 \end{vmatrix}$$

RHR ⇒ out of the page ⇒ along positive z

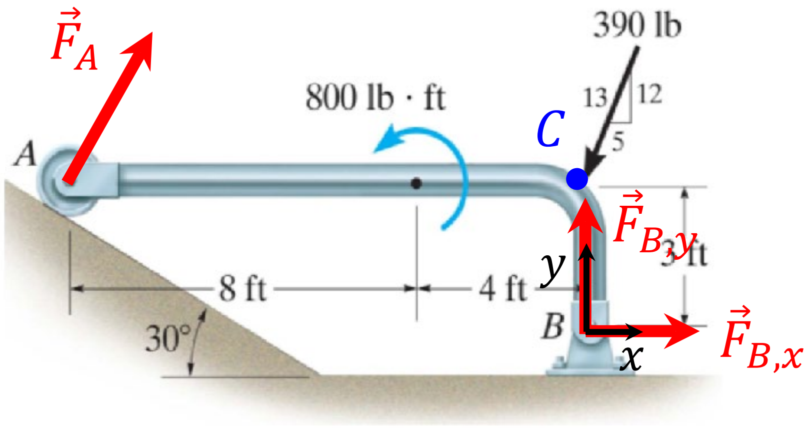
$$= \vec{k}\cdot 800 + \vec{k}\left(-12\cdot F_A \sin 60^\circ - 3F_A\cos 60^\circ\right) + \vec{k}\left(0 - 3\cdot\left(-390\,\frac{5}{13}\right)\right) =$$

$$\underbrace{\vec{k}\left[800 + 390\,\frac{3\cdot 5}{13} - F_A\left(3\cos 60^\circ + 12\sin 60^\circ\right)\right]} = \vec{M}_R$$

(3) $M_z = 0$

$$F_A = \frac{800 + 390\,\frac{15}{13}}{3\cos 60^\circ + 12\sin 60^\circ} = 105.11 \text{ lb}$$

WARMING UP: 2D Equilibrium



- Finalize:

$$F_A \cos 60° + F_{Bx} - 390 \cdot \frac{5}{13} = 0 \qquad (1)$$

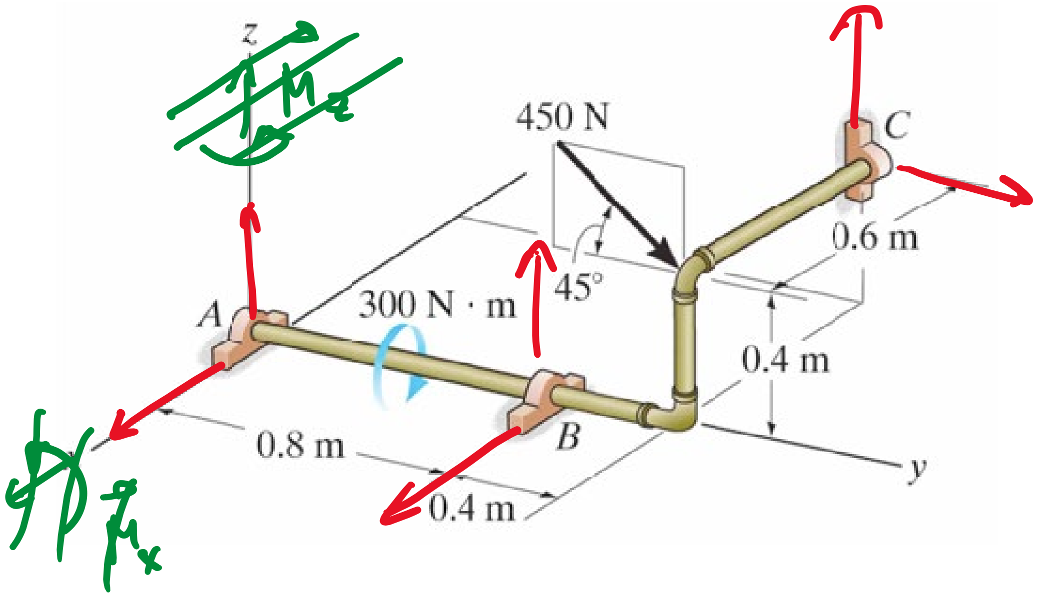
$$F_A \sin 60° + F_{By} - 390 \cdot \frac{12}{13} = 0 \qquad (2)$$

- $F_A = 105.11\ lb$

$F_{Bx} = 97.4\ lb \qquad F_{By} = -269\ lb \qquad F_A = 105\ lb$

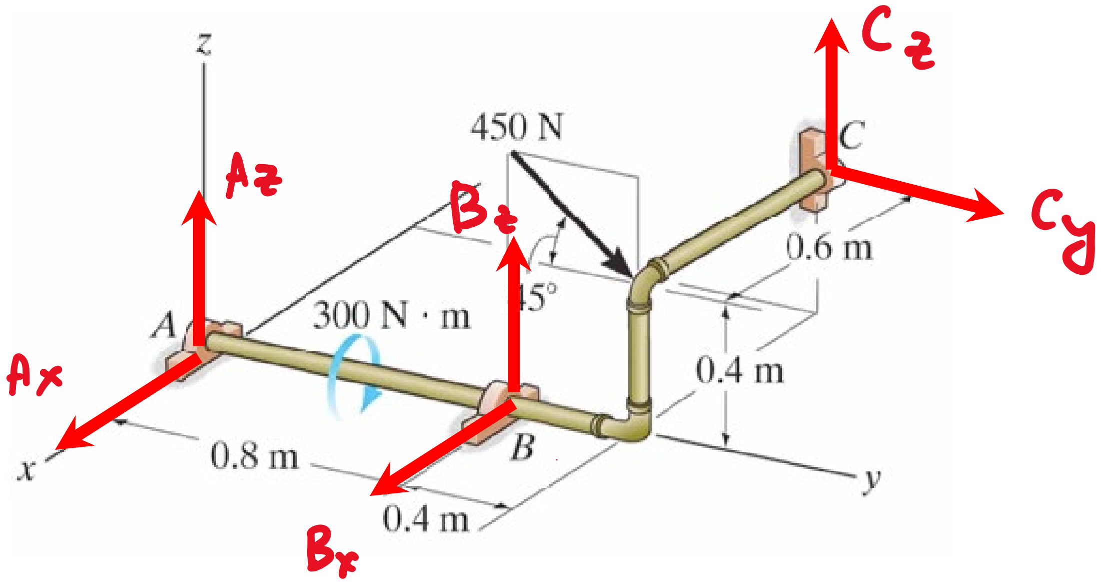
3D Equilibrium

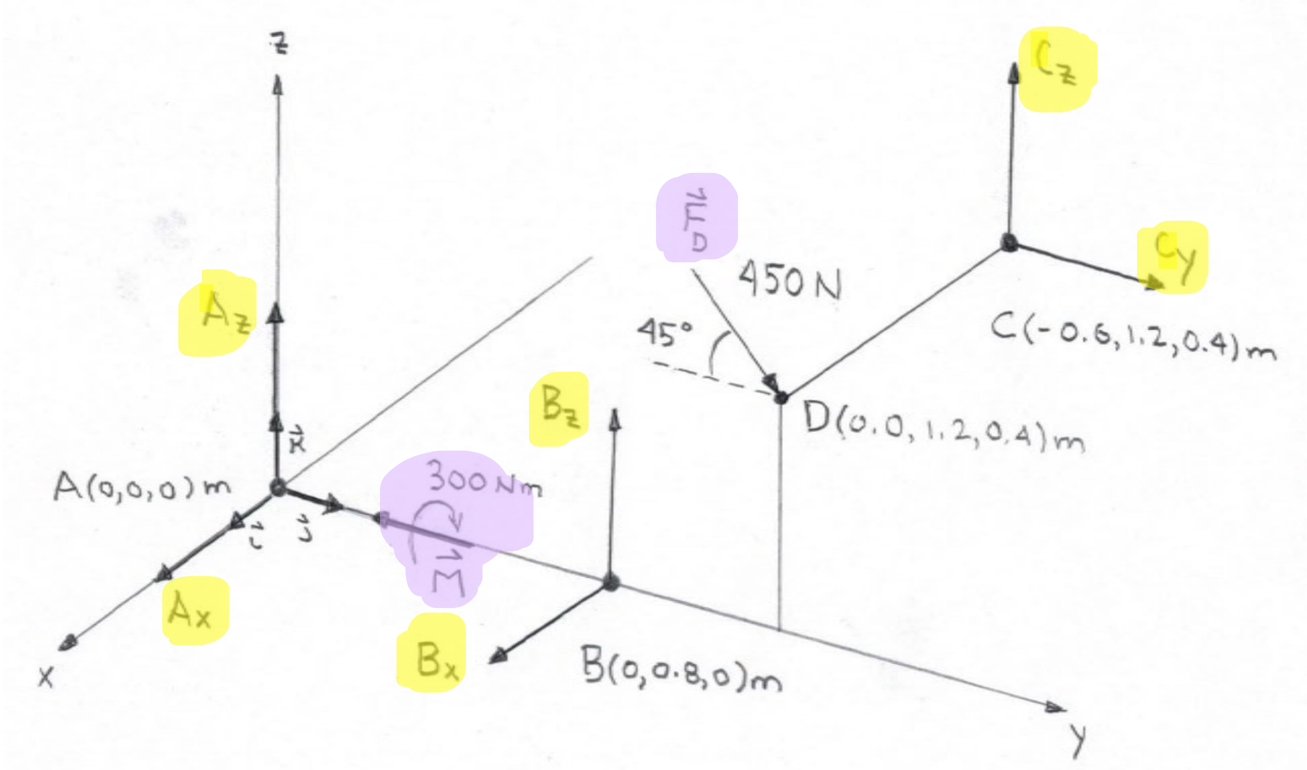
W5-2. The pipe assembly is subjected to a force and couple moment as shown and is held in equilibrium by smooth journal bearings at A, B, and C. The bearings are in proper alignment and only exert force reactions on the pipe assembly. The weight of the pipe assembly may be neglected. Determine the Cartesian components of these force reactions.



3D Equilibrium

W5-2. The pipe assembly is subjected to a force and couple moment as shown and is held in equilibrium by smooth journal bearings at A, B, and C. The bearings are in proper alignment and only exert force reactions on the pipe assembly. The weight of the pipe assembly may be neglected. Determine the Cartesian components of these force reactions.

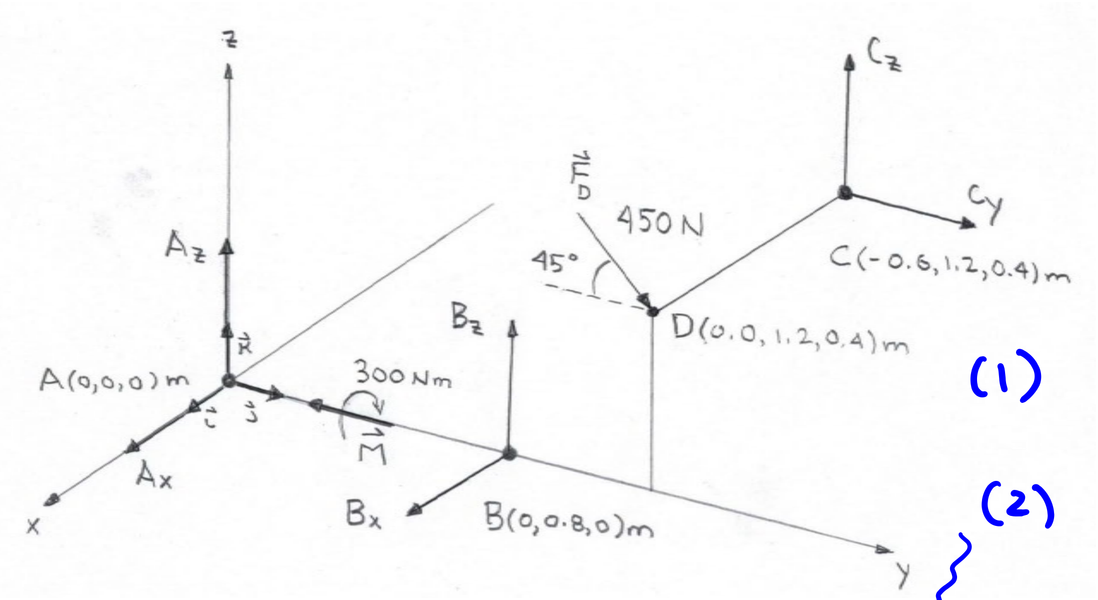




- proper alignment
- weight neglected

3D Equilibrium

W5-2. Determine the Cartesian components of these force reactions.



- Translational equilibrium:

$$\vec{F}_R = \vec{A} + \vec{B} + \vec{C} + \vec{F}_D = 0$$

(1) $x: \ \Sigma F_x = 0 \qquad A_x + B_x = 0$

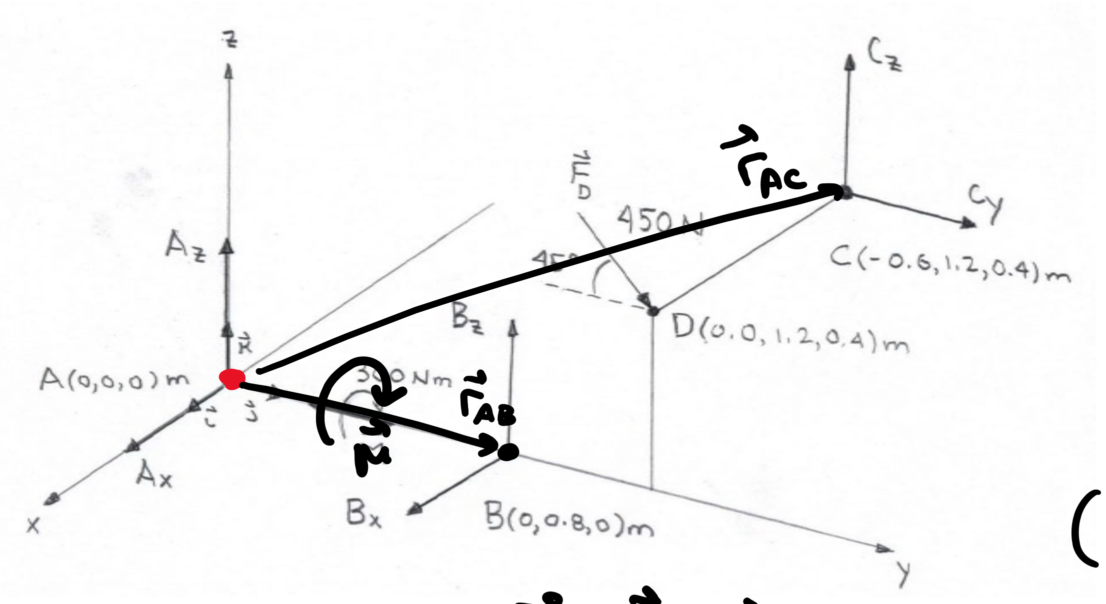
(2) $y: \ \Sigma F_y = 0 \qquad C_y + 450 \cos 45^\circ = 0$

(3) $z: \ \Sigma F_z = 0 \qquad A_z + B_z + C_z - 450 \cos 45^\circ = 0$

$$C_y = -\frac{450}{\sqrt{2}} = -318.2 \text{ N}$$

3D Equilibrium

W5-2. Determine the Cartesian components of these force reactions.



- Rotational equilibrium:
  - ➢ About which point? A

$$(\vec{M}_R)_A = \cancel{(\vec{M}_A)_A}^{\,0} + (\vec{M}_B)_A + (\vec{M}_C)_A + (\vec{M}_D)_A + \vec{M} = 0$$

$$(\vec{M}_B)_A = \vec{r}_{AB} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0.8 & 0 \\ B_x & 0 & B_z \end{vmatrix}$$

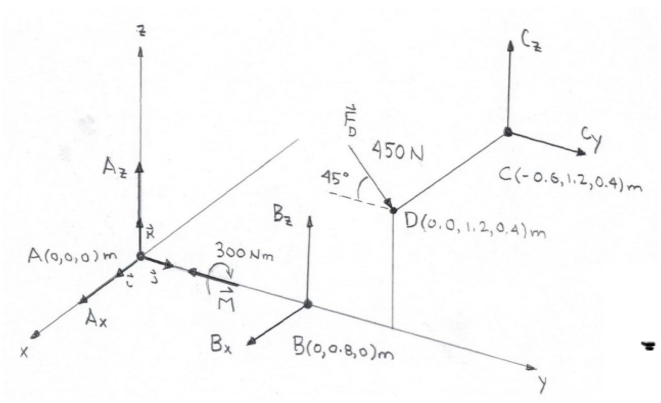
$$(\vec{M}_C)_A = \vec{r}_{AC} \times \vec{C} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -0.6 & 1.2 & 0.4 \\ 0 & C_y & C_z \end{vmatrix}$$

$$(\vec{M}_D)_A = \vec{r}_{AD} \times \vec{F}_D = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1.2 & 0.4 \\ 0 & \frac{450}{\sqrt{2}} & -\frac{450}{\sqrt{2}} \end{vmatrix}$$

$$\vec{M} = -300\vec{j}$$

3D Equilibrium

W5-2. Determine the Cartesian components of these force reactions.



$$(\vec{M}_R)_A = -\vec{j}\cdot 300 + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0.8 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -0.6 & 1.2 & 0.4 \\ 0 & C_y & C_z \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1.2 & 0.4 \\ 0 & 450\cos 45^\circ & -450\sin 45^\circ \end{vmatrix}$$

$$= -\vec{j}\cdot 300 + \left(\vec{i}\cdot(0.8B_z) - \vec{j}\cdot(0) + \vec{k}(-0.8B_x)\right) + \left(\vec{i}(1.2C_z - 0.4C_y) - \vec{j}(-0.6C_z) + \vec{k}(-0.6C_y)\right) +$$

$$+ \left(\vec{i}(-1.2\cdot 450\sin 45^\circ - 0.4\cdot 450\cos 45^\circ) - \vec{j}(0) + \vec{k}(0)\right) =$$

$$= \vec{i}\left[0.8B_z + 1.2C_z - 0.4C_y - 1.2\frac{450}{\sqrt{2}} - 0.4\frac{450}{\sqrt{2}}\right] = 0 \qquad (4)$$

$$\vec{j}\left[-300 + 0.6C_z\right] = 0 \qquad (5)$$

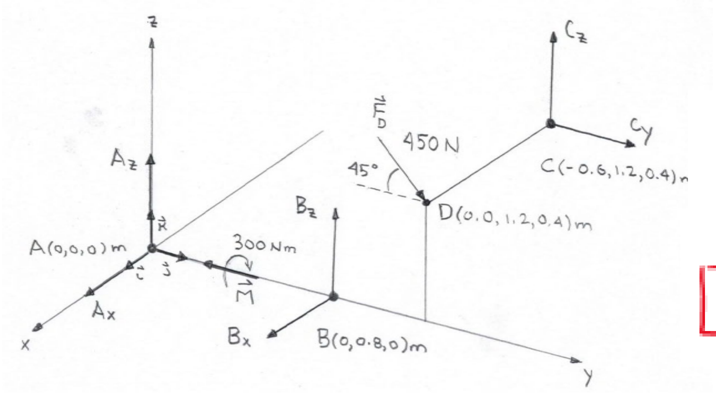
$$\vec{k}\left[-0.8B_x - 0.6C_y\right] = 0 \qquad (6)$$

$$= \vec{M}_R = 0$$

$$C_z = 500.0$$

3D Equilibrium

W5-2. Determine the Cartesian components of these force reactions.



- Finalize:

$A_x + B_x = 0$ (1)

$0.8\,B_z + 1.2\,C_z - 0.4\,C_y = 509.12$ (4)

$\boxed{C_y = -318.2}$ (2)

$\boxed{C_z = 500}$ (5)

$A_z + B_z + C_z = 450 \sin 45°$ (3)

$-0.8\,B_x - 0.6\,C_y = 0$ (6)

(6) $\rightarrow$ $B_x = \cdots$

(4) $\rightarrow$ $B_z = \cdots$

(1) $\rightarrow$ $A_x = \cdots$

(3) $\rightarrow$ $A_z = \cdots.$