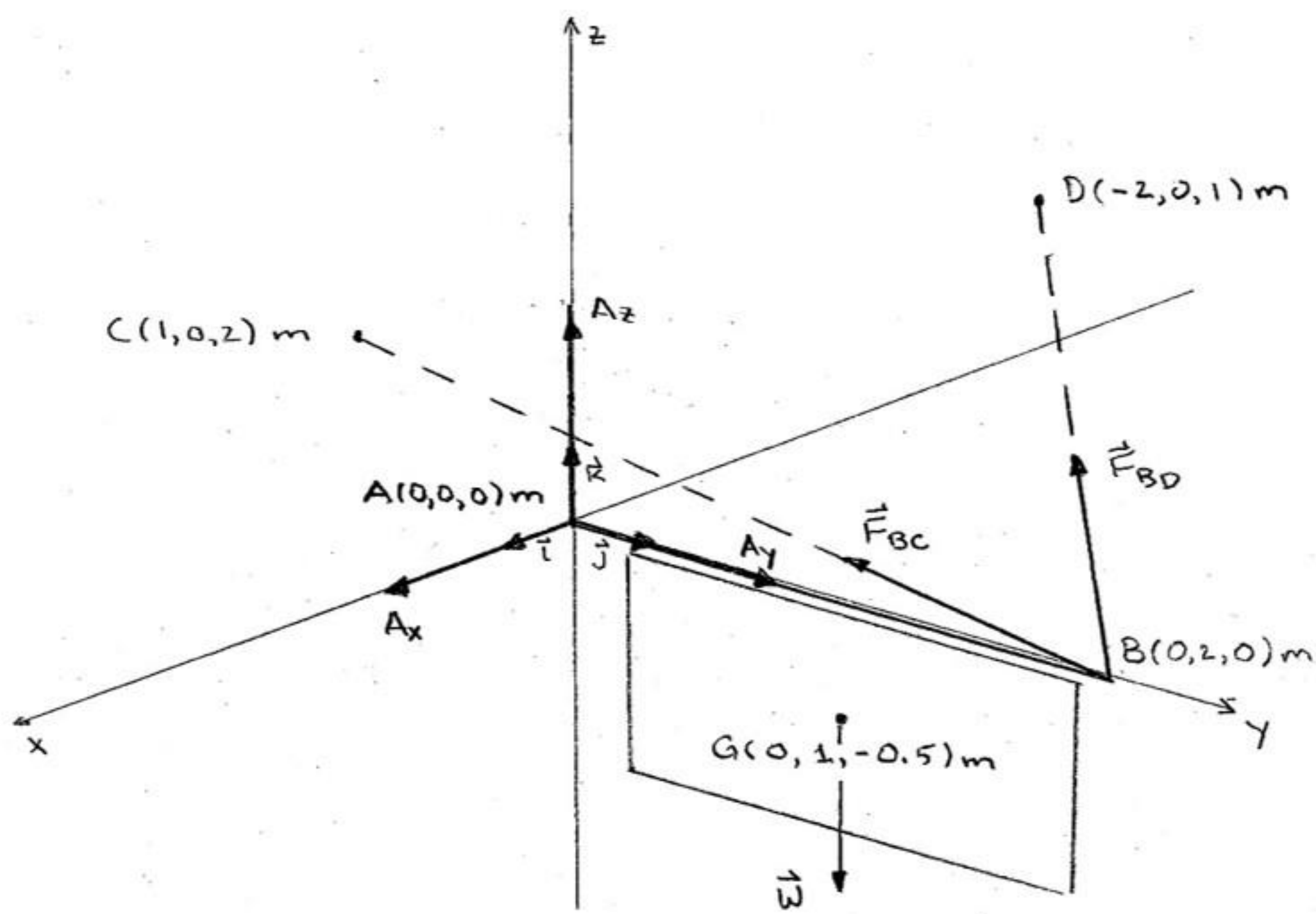
SOLUTION TO QUESTION 1 (15 MARKS) page 1

a) Free-Body Diagram (4 marks)



Coordinates:

$A(0,0,0)$ m
$B(0,2,0)$ m
$C(1,0,2)$ m
$D(-2,0,1)$ m
$G(0,1,-0.5)$ m

SOLUTION TO QUESTION 1 (15 MARKS) page 2

b) Cartesian component force equations of equilibrium (4 marks)

Forces and moments (suppressing units):

$$\vec{F}_A = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$$

$$\vec{F}_{BC} = \left(\vec{i} - 2\vec{j} + 2\vec{k}\right)X \qquad X = F_{BC} / \sqrt{1^2 + 2^2 + 2^2} = F_{BC} / 3$$

$$\vec{F}_{BD} = \left(-2\vec{i} - 2\vec{j} + \vec{k}\right)Y \qquad Y = F_{BD} / \sqrt{2^2 + 2^2 + 1^2} = F_{BD} / 3$$

$$\vec{W} = -(100)(9.81)\vec{k} = -981\vec{k}$$

$$\sum F_x = 0: \qquad A_x + X - 2Y = 0 \qquad (1)$$

$$\sum F_y = 0: \qquad A_y - 2X - 2Y = 0 \qquad (2)$$

$$\sum F_z = 0: \qquad A_z + 2X + Y - 981 = 0 \qquad (3)$$

c) Vector moment equation of equilibrium (2 marks)

$$\vec{r}_{AB} = 2\vec{j} \qquad r_{AG} = \vec{j} - 0.5\vec{k}$$

$$(\vec{M}_R)_A = \sum \vec{M} + \sum(\vec{r} \times \vec{F}) =$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & 0 \\ 1 & -2 & 2 \end{vmatrix} X + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & 0 \\ -2 & -2 & 1 \end{vmatrix} Y + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & -0.5 \\ 0 & 0 & -981 \end{vmatrix} = 0$$

SOLUTION TO QUESTION 1 (15 MARKS) page 3

d) Cartesian component moment equation of equilibrium: (3 marks)

$$4X + 2Y - 981 = 0 \quad (4)$$
$$0 = 0 \quad (5)$$
$$-2X + 4Y = 0 \quad (6)$$

e) Numerical values of the components of reaction at A and the tensions in the cables (2 marks)

$X = 196.2$ $\quad$ $Y = 98.1$

$F_{BC} = 589$ N $\quad$ $F_{BD} = 294$ N

$A_x = 0$ N $\quad$ $A_y = 589$ N

$A_z = 490$ N

SOLUTION TO QUESTION 1 (15 MARKS) page 4

Alternate approach

b) Cartesian component force equations of equilibrium (4 marks)

Forces and moments (suppressing units)

$$\vec{F}_A = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$$

$$\vec{F}_{BC} = \frac{\vec{i} - 2\vec{j} + 2\vec{k}}{\sqrt{1^2 + 2^2 + 2^2}} F_{BC} = \frac{1}{3}F_{BC}\vec{i} - \frac{2}{3}F_{BC}\vec{j} + \frac{2}{3}F_{BC}\vec{k}$$

$$\vec{F}_{BD} = \frac{-2\vec{i} - 2\vec{j} + \vec{k}}{\sqrt{2^2 + 2^2 + 1^2}} F_{BD} = -\frac{2}{3}F_{BD}\vec{i} - \frac{2}{3}F_{BD}\vec{j} + \frac{1}{3}F_{BD}\vec{k}$$

$$\vec{W} = -(100)(9.81) = -981\vec{k}$$

$$\sum F_x = 0: \qquad A_x + \frac{1}{3}F_{BC} - \frac{2}{3}F_{BD} = 0 \qquad (1)$$

$$\sum F_y = 0: \qquad A_y - \frac{2}{3}F_{BC} - \frac{2}{3}F_{BD} = 0 \qquad (2)$$

$$\sum F_z = 0: \qquad A_z + \frac{2}{3}F_{BC} + \frac{1}{3}F_{BD} - 981 = 0 \qquad (3)$$

c) Vector moment equation of equilibrium (2 marks)

$$\vec{r}_{AB} = 2\vec{j} \qquad r_{AG} = \vec{j} - 0.5\vec{k}$$

SOLUTION TO QUESTION 1 (15 MARKS) page 5

$$(\vec{M}_R)_A = \sum \vec{M} + \sum (\vec{r} \times \vec{F}) =$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & 0 \\ \frac{1}{3} & -\frac{2}{3} & \frac{2}{3} \end{vmatrix} F_{BC} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & 0 \\ -\frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{vmatrix} F_{BD} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & -0.5 \\ 0 & 0 & -981 \end{vmatrix} = 0$$

c) Cartesian component moment equations of equilibrium: (3 marks)

$$\frac{4}{3}F_{BC} + \frac{2}{3}F_{BD} - 981 = 0 \qquad (4)$$

$$0 = 0 \qquad (5)$$

$$-\frac{2}{3}F_{BC} + \frac{4}{3}F_{BD} = 0 \qquad (6)$$

d) Numerical values of the components of reaction at A and the tensions in the cables: (2 marks)

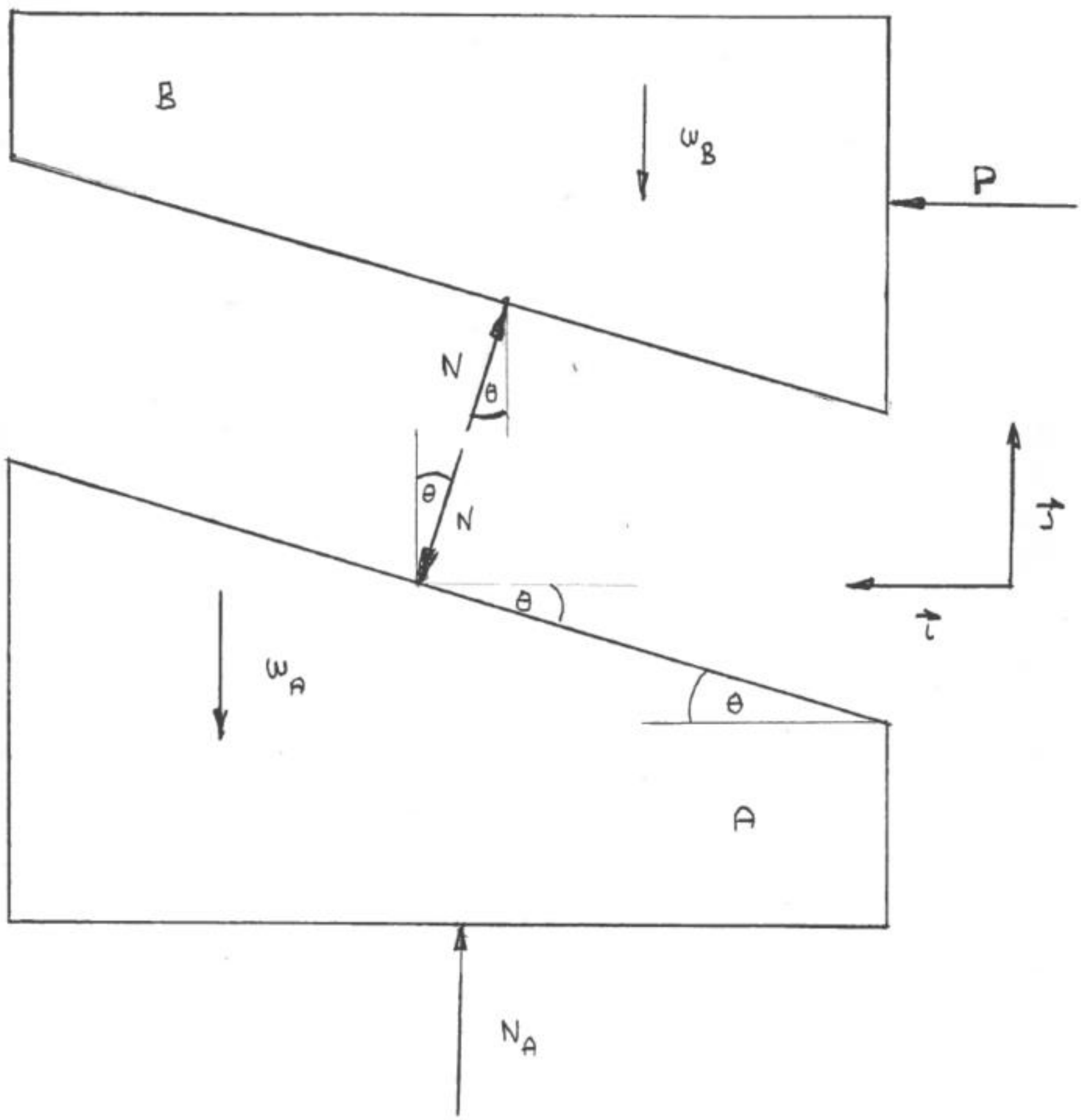
$F_{BC} = 589\text{ N}$ $F_{BD} = 294\text{ N}$

$A_x = 0\text{ N}$ $A_y = 589\text{ N}$

$A_z = 490\text{ N}$

SOLUTION TO QUESTION 2 (page 1)

a) Free-Body Diagrams



(6 marks)

<u>SOLUTION TO QUESTION 2</u> (page 2)

Data

$m_A = 5$ kg $\qquad m_B = 10$ kg $\qquad g = 9.81$ m/s^2

$\theta = 20^\circ$ $\qquad P = 150$ N $\qquad a_{B/A} = 3.2657$ m/s^2

The x-axis is positive to the left. The y-axis is positive up.

$\vec{i}$ points to the left. $\vec{j}$ points to the up.

b) Equations of motion

<u>Block B</u>

$\Sigma F_x = ma_x$: $\qquad P - N\sin\theta = m_B a_{Bx}$

$$P - N\sin\theta = m_B(a_A + a_{B/A}\cos\theta) \qquad (1)$$

$\Sigma F_y = ma_y$: $\qquad N\cos\theta - m_B g = m_B a_{By}$

$$N\cos\theta - m_B g = m_B a_{B/A}\sin\theta \qquad (2)$$

<u>Block A</u>

$\Sigma F_x = ma_x$: $\qquad N\sin\theta = m_A a_A \qquad (3)$

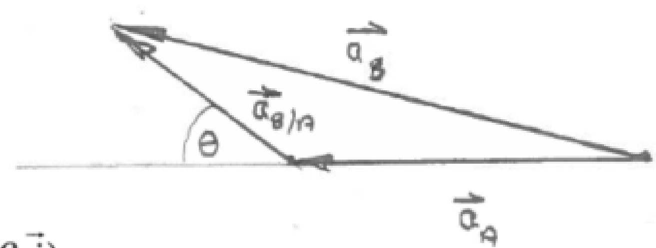
$\Sigma F_y = ma_y$: $\qquad N_A - N\cos\theta - m_A g = 0 \qquad (4)$

SOLUTION TO QUESTION 2 **(page 3)**

c) Accelerations

$$\vec{a}_A = a_A \vec{i}$$

$$\vec{a}_B = a_{Bx} \vec{i} + a_{By} \vec{j}$$



$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

$$\vec{a}_{B/A} = a_{B/A}(\cos\theta \vec{i} + \sin\theta \vec{j})$$

$$a_{Bx} = a_A + a_{B/A}\cos\theta$$

$$a_{By} = a_{B/A}\sin\theta$$

((b) and Accelerations: 6 marks)

Accelerations and normal forces

$$a_A = 7.95 \text{ m/s}^2$$

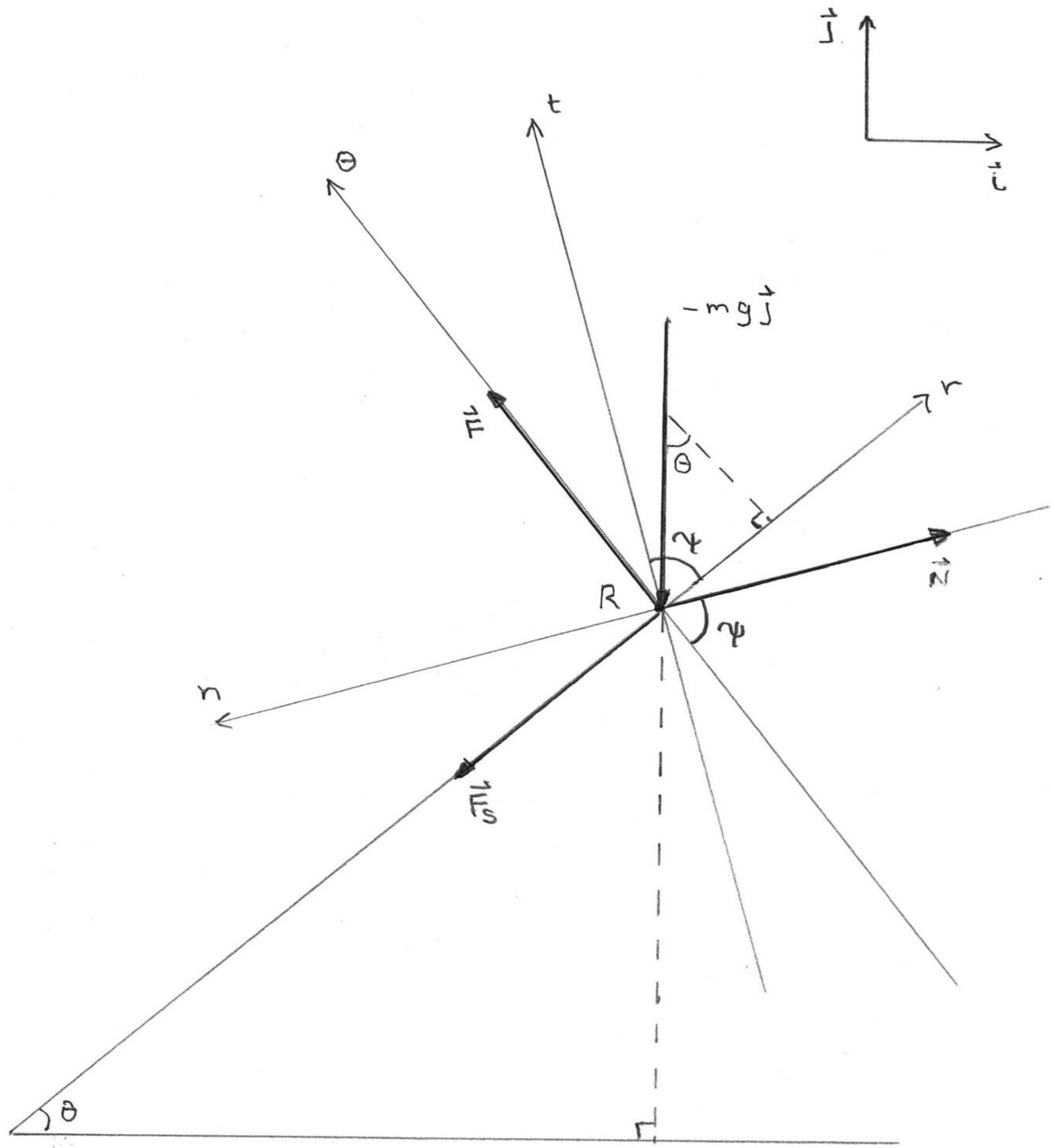
$$N = 116 \text{ N}$$

$$N_A = 158 \text{ N}$$

(3 marks)

SOLUTION TO QUESTION 3 (15 MARKS) page 1

a) Free body diagram (4 marks)



SOLUTION TO QUESTION 3 (15 MARKS) page 2

b) Angle ψ between $\vec{u}_r$ and $\vec{u}_t$ (2 marks)

$$\psi = \arctan\left(\frac{r}{dr/d\theta}\right) = 56.2^\circ$$

c) Numerical values for a_r and a_θ (2 marks)

$$a_r = \ddot{r} - r\dot{\theta}^2 = -5.71 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 4.90 \text{ m/s}^2$$

d) Equations of motion for R (4 marks)

$$-k(r - r_0) + N\sin\psi - mg\sin\theta = ma_r$$

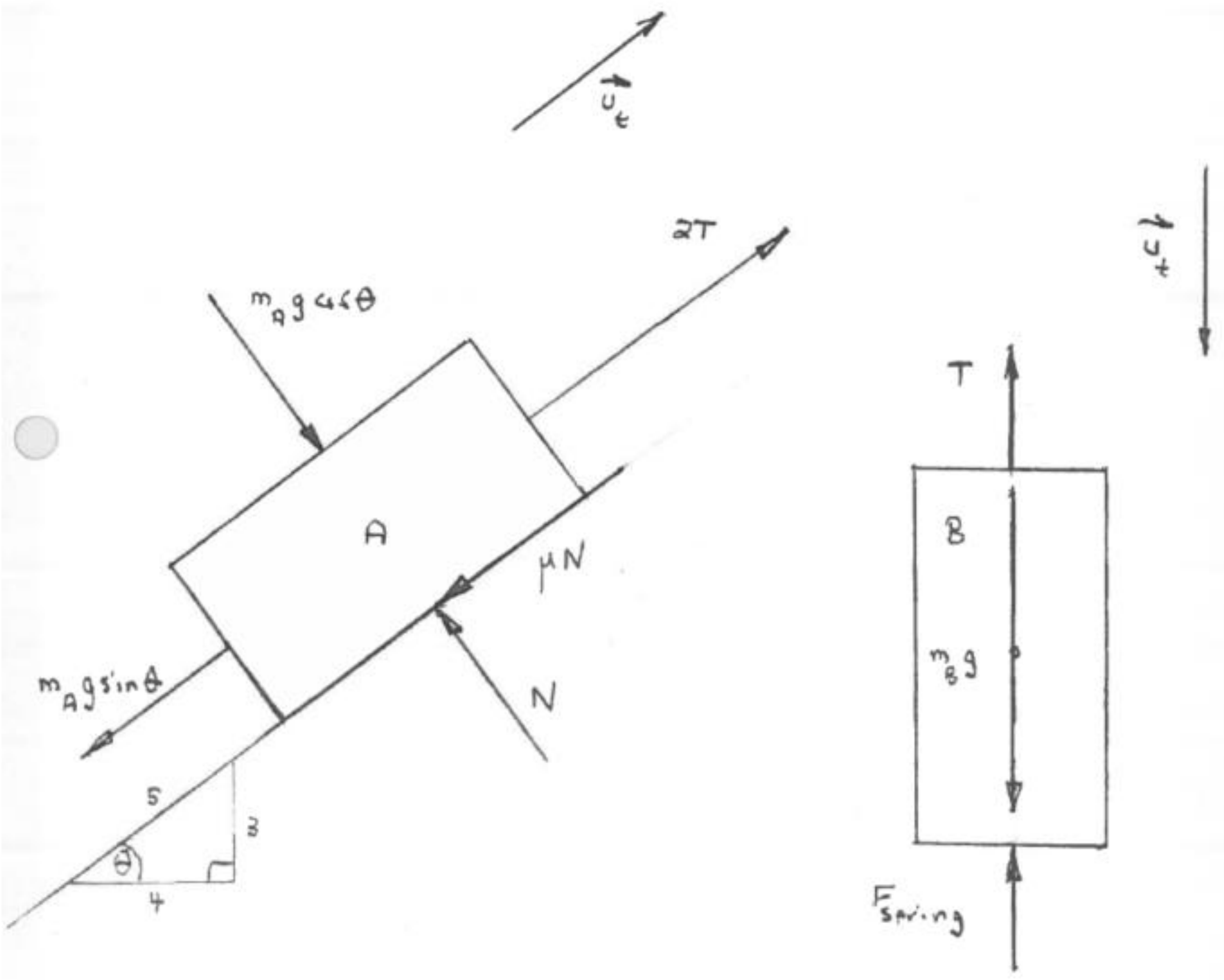
$$F - N\cos\psi - mg\cos\theta = ma_\theta$$

e) Numerical values for F and N (3 marks)

$$F = 28.2 \text{ N} \qquad N = 6.07 \text{ N}$$

<u>SOLUTION TO QUESTION 4</u> (page 1)

a) <u>Free-body diagrams</u>



(4 marks)

SOLUTION TO QUESTION 4 (page 2)

Data $m_A = 6$ kg $\quad m_B = 10$ kg $\quad g = 9.81$ m/s^2

$\theta = \tan^{-1}(3/4) \quad k = 20$ N/m $\quad \mu = 0.3$

$v_{A1} = v_{B1} = 0 \quad h_{A1} = h_{B2} = 0 \quad \Delta s_B = 2$ m

Dependent motion

$$2\Delta s_A = \Delta s_B \tag{1}$$

$$2v_A = v_B \tag{2}$$

Principle of Work and Energy

$$\frac{1}{2}mv_1^2 + V_1 + (U_{other})_{1\to 2} = \frac{1}{2}mv_2^2 + V_2 \tag{3}$$

V is the potential energy due to gravity and the spring force.

$(U_{other})_{1\to 2}$ is the work done by tension and the friction force.

b) Block A $\quad \vec{u}_t$ points up the plane

$$(2T - \mu m_A g\cos\theta)\Delta s_A = \frac{1}{2}m_A v_{A2}^2 + mgh_{A2} \tag{4}$$

$$\Delta s_A = 1\text{ m} \qquad h_{A2} = \Delta s_A \sin\theta \tag{5}$$

<u>SOLUTION TO QUESTION 4</u> (page 3)

c) <u>Block *B*</u> $\vec{u}_t$ points down

$$m_B g h_{B1} - T\,\Delta s_B = \frac{1}{2} m v_{B2}^2 + \frac{1}{2} k (\Delta s_B)^2 \tag{6}$$

$$h_{B1} = \Delta s_B = 2\text{ m} \tag{7}$$

d) <u>Speed and tension</u>

Add equations (4) and (6):

$$m_B g h_{B1} - \mu m_A g \cos\theta\,\Delta s_A = \frac{1}{2} m_A v_{A2}^2 + \frac{1}{2} m_B v_{B2}^2 + m_A g h_{A2} + \frac{1}{2} k (\Delta s_B)^2 \tag{8}$$

Solve equation (8) for v_{A2}:

$$v_{A2} = 2.15\text{ m/s}$$

Solve equation (4) or (6) for T:

$$T = 31.7\text{ N}$$

(b) 4 marks)
(c) 4 marks)
(d) 3 marks)