- What is your personal preference in case of a longer Translink strike?

A. Strongly prefer in-person lectures to continue 30%

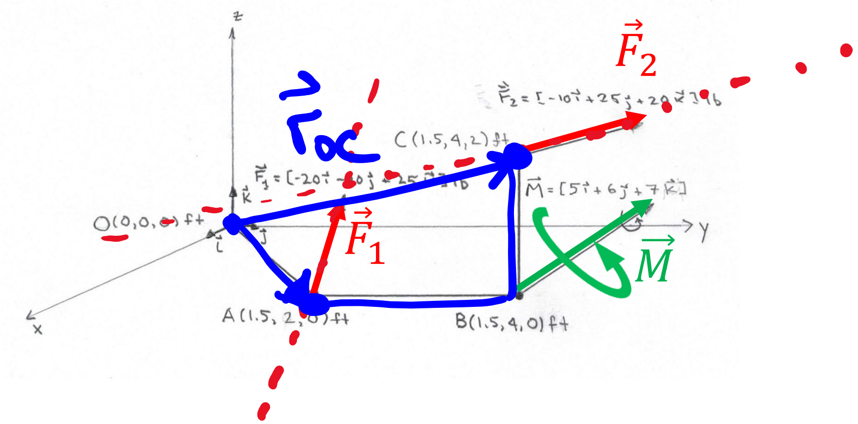
B. Simply can't get to the campus & need zoom lectures 49%

C. No strong preference, both ways work for me. 31%

❖ We will try to come up with a solution that addresses everybody's needs, and it's good to know how your preferences split.

W4-2. The pipe assembly is acted on by forces $\vec{F}_1$ and $\vec{F}_2$ and by a couple moment $\vec{M} = (5\vec{i} + 6\vec{j} + 7\vec{k})$ lb·ft.

3) Replace the force-couple system by a resultant force and couple moment at O. Express the results in Cartesian vector form.



$$\vec{F}_1 = (-20)\vec{i} + (-10)\vec{j} + (25)\vec{k} \qquad \vec{F}_2 = (-10)\vec{i} + (25)\vec{j} + (20)\vec{k}$$

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 = (-30.0)\vec{i} + (15.0)\vec{j} + (45.0)\vec{k}$$

$$\vec{M}_1 = \underline{\vec{r}_{OA}} \times \underline{\underline{\vec{F}_1}} \qquad \vec{M}_2 = \underline{\underline{\vec{r}_{OC}}} \times \underline{\underline{\vec{F}_2}}$$

$$\vec{r}_{OA} = (1.5)\vec{i} + (2)\vec{j} + (0)\vec{k} \qquad \vec{r}_{OC} = (1.5)\vec{i} + 4(\vec{j}) + (2)\vec{k}$$

$$\underline{\vec{M}_1} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.5 & 2 & 0 \\ -20 & -10 & 25 \end{vmatrix} = 50\vec{i} - 37.5\vec{j} + 25\vec{k}$$

$$\underline{\vec{M}_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1.5 & 4 & 2 \\ -10 & 25 & 20 \end{vmatrix} = 30\vec{i} - 50\vec{j} + 77.5\vec{k}$$

$$\vec{M}_R = \vec{M}_1 + \vec{M}_2 + \vec{M} = 85.0\vec{i} - 81.5\vec{j} + 110\vec{k}$$

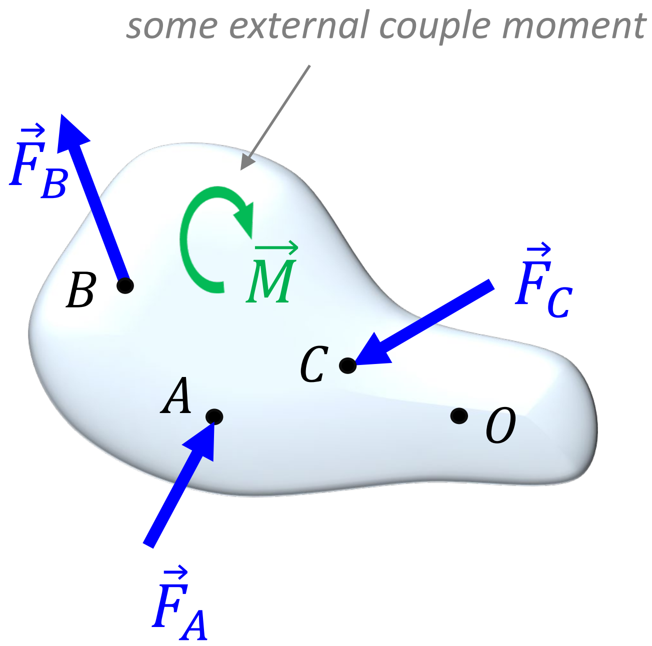
Further Simplification & Wrench

Text: 4.8

Content:

- Finding $\vec{F}_R$ and $\vec{M}_R$
- Equivalent system for coplanar forces
- Equivalent system for parallel forces
- General case: reduction to wrench
- Two vectors are parallel if…

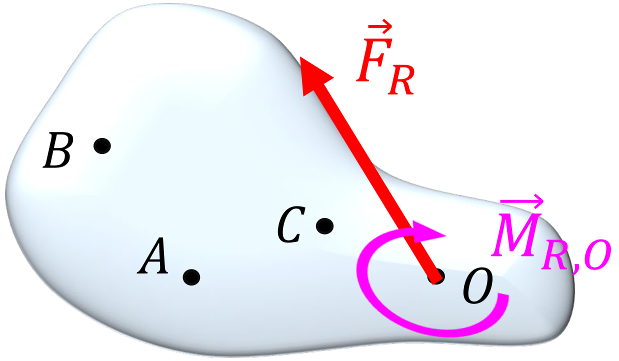
EQUIVALENT SYSTEMS: Take 2



- We will be interested in *simplifying* system of forces and couple moments acting on a body to a <u>single resultant force</u> and a <u>single couple moment</u> acting at some specified point O

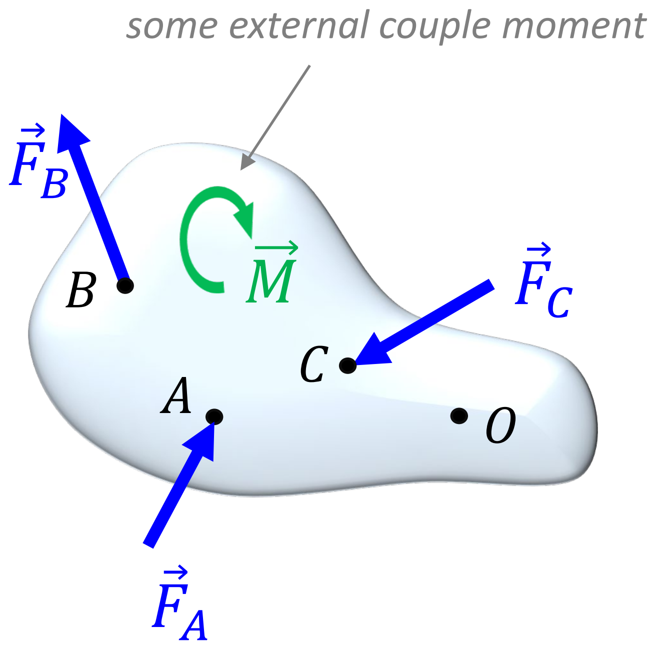
- We will mentally relocate all the forces to point O, maintaining equivalency (adding required couple moments).

- What will we get for $\vec{F}_R$ and $\vec{M}_{R,O}$?

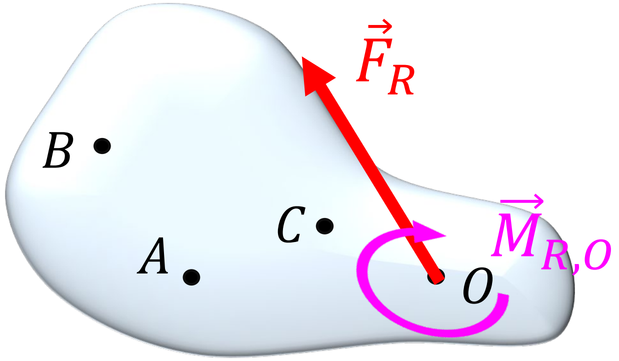


EQUIVALENT SYSTEMS: Take 2

- Resultant force (describes translation of the object):
  - ➢ $\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$. Just add the three vectors up.
  - ➢ $\vec{F}_R$ describes translation of the object. It does not depend on the location of the point O.



- Resultant moment about point O (describes rotation of the object about O):
  - ➢ Shift all the forces to point O and add compensating couple moments $\vec{M}_{A,O} = \vec{r}_{OA} \times \vec{F}_A$, $\vec{M}_{B,O} = \vec{r}_{OB} \times \vec{F}_B$, $\vec{M}_{C,O} = \vec{r}_{OC} \times \vec{F}_C$.
  - ➢ $\vec{M}_{A,O}$, $\vec{M}_{B,O}$, $\vec{M}_{C,O}$ are free vectors. Their magnitudes, however, depend on the location of point O, so it makes sense to apply them at point O.
  - ➢ $\vec{M}$ is a free vector => we can shift it to point O.
  - ➢ Now $\vec{M}_{R,O} = \vec{M}_{A,O} + \vec{M}_{B,O} + \vec{M}_{C,O} + \vec{M}$



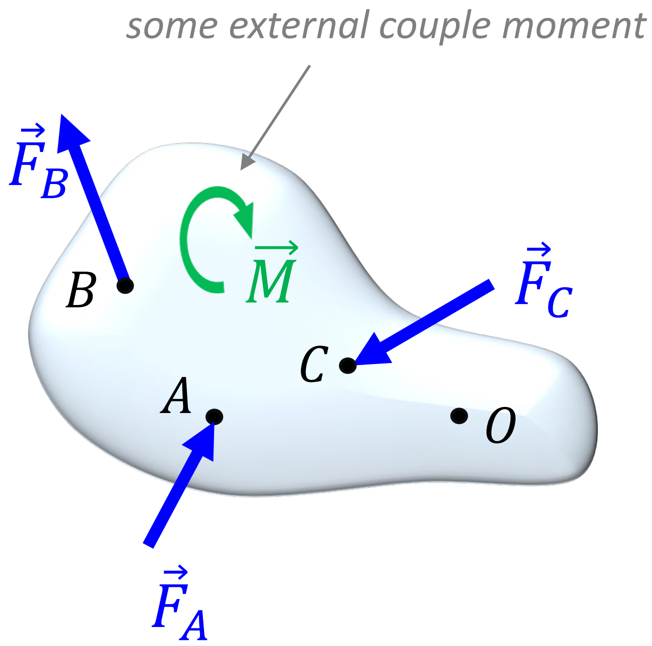
EQUIVALENT SYSTEMS: Take 2

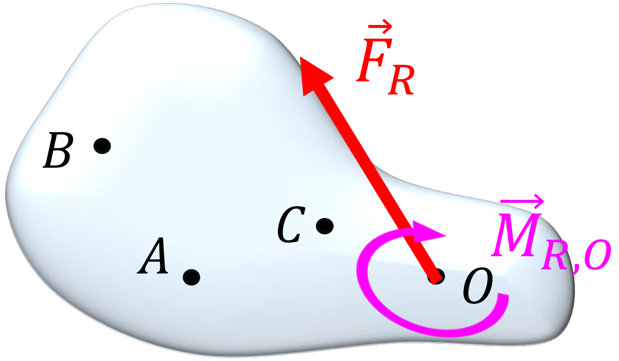
- We will be interested in *simplifying* system of forces and couple moments acting on a body to a single resultant force and a single couple moment acting at some specified point O

- We will mentally relocate all the forces to point O, maintaining equivalency (adding required couple moments).

- What will we get for $\vec{F}_R$ and $\vec{M}_{R,O}$?

  - $\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$. Just add the three vectors up.
  - $\vec{M}_{R,O} = \vec{M}_{A,O} + \vec{M}_{B,O} + \vec{M}_{C,O} + \vec{M}$: Sum of "compensating couple moments" ($\vec{M}_{A,O} = \vec{r}_{OA} \times \vec{F}_A$, $\vec{M}_{B,O} = \vec{r}_{OB} \times \vec{F}_B$, $\vec{M}_{C,O} = \vec{r}_{OC} \times \vec{F}_C$, depend on O) and the couple moments that





EQUIVALENT SYSTEMS: Take 2

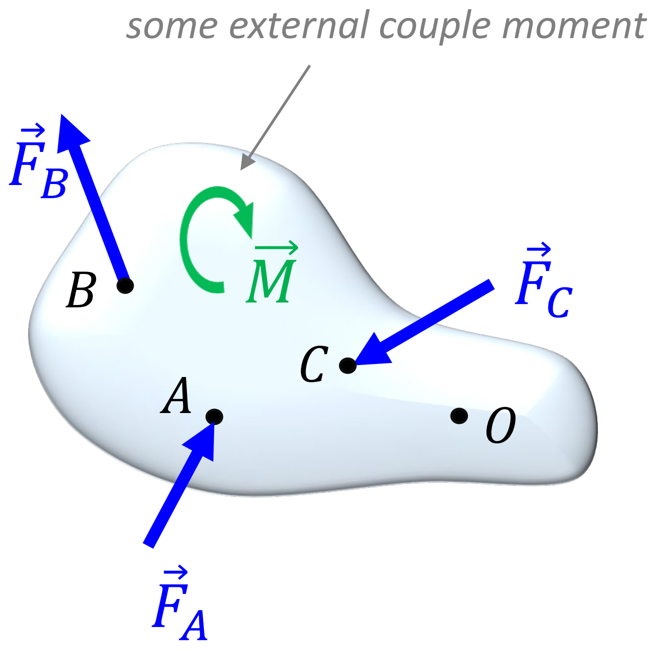
- Now, what determines the choice of the point O?

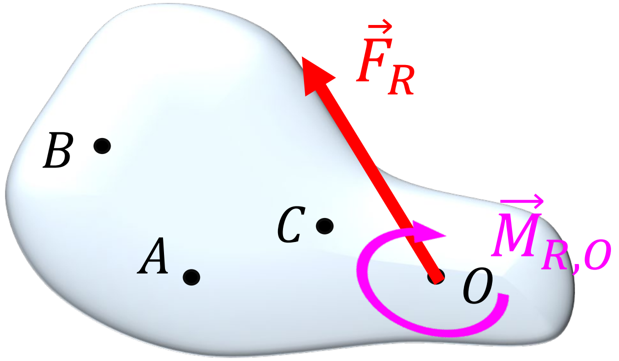
  - ➢ It depends. It may be determined by our convenience. Or by what we consider to have aesthetic appeal.

  - ➢ In the following examples, we are going to make equivalency transformations in order to find a point O such that it is characterized by a minimum number of vectors, namely:

    - ❖ We will want $\vec{M}_{R,O}$ be equal to zero, if possible. Then the only vector applied to the system is $\vec{F}_R$.

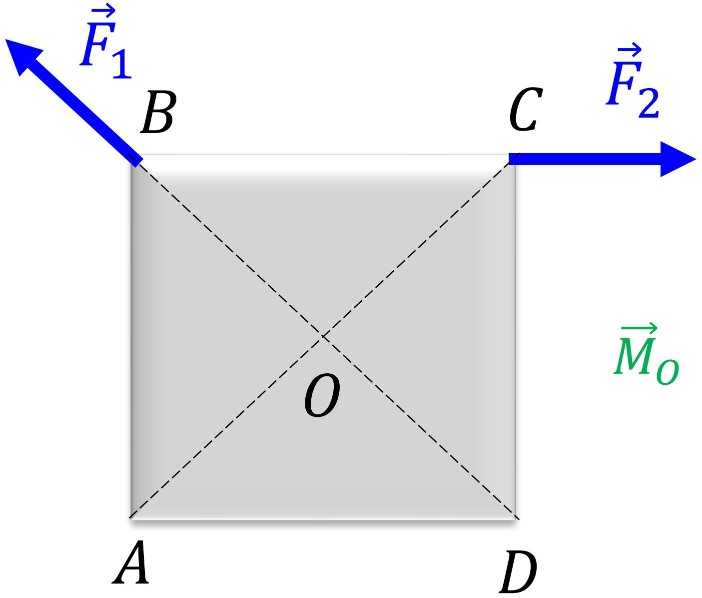
    - ❖ If it is not possible, we will want to find a point in which $\vec{M}_{R,O}$ is parallel to $\vec{F}_R$ (where *"the system is reduced to a wrench"*)





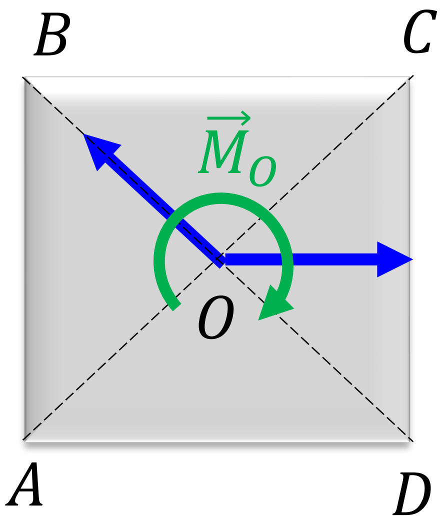
Example: Coplanar Forces

Task: Find a point P at which you can replace this system of coplanar forces by a single resultant force and zero resultant couple moment.



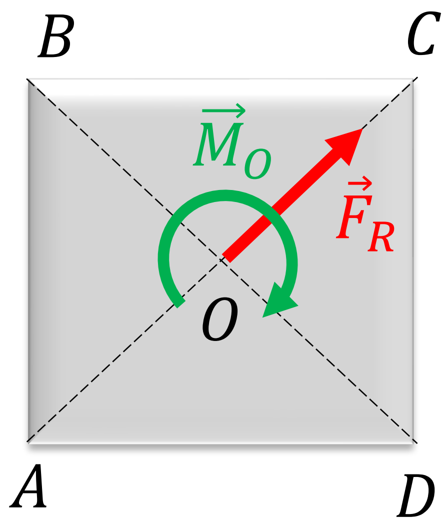
Initial system

$$\vec{M}_O = \vec{r}_{OC} \times \vec{F}_2$$

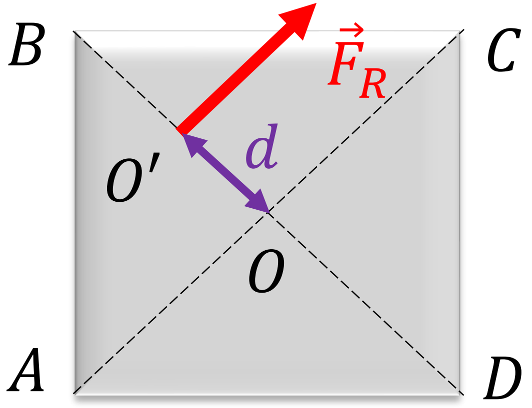


Shifting the forces to an arbitrary point O, adding compensating couple moment(s)

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2$$



One (resultant) force, and a couple moment



Shifting the resultant force by a perpendicular distance

$$d = M_O/F_R$$

(this would produce the moment M about point O)

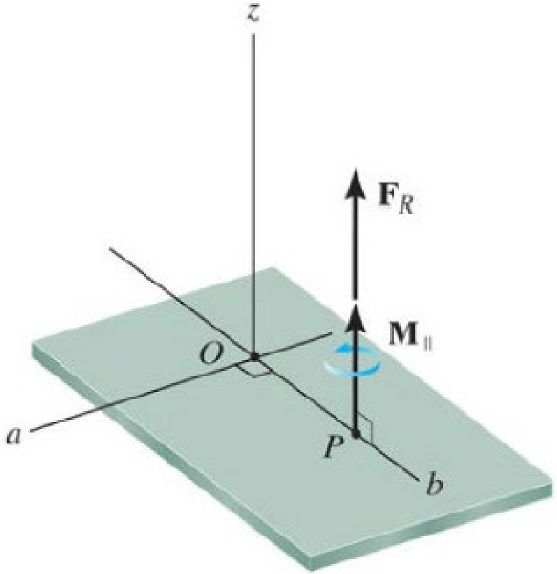
Bingo! We have replaced the initial system of two forces by an equivalent system described by only one force vector. ☺

WRENCH: Reduction to a Wrench

A screwdriver/drill

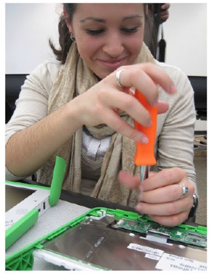
- If all the forces are <u>coplanar</u>, or when they all are <u>parallel to one and the same axis</u>, it is always possible to reduce a system of forces and couple moment to just one force vector! (Section 4.8)
  - ➢ In other words, you can always find a point where the resultant force will produce the desirable rotation effect

- In the general case (not one of the above two happy cases), it is still possible to find a point where the equivalent system will be described by only two vectors:
  - ➢ The resultant force $\vec{F}_R$
  - ➢ Net couple moment $\overrightarrow{M}_R$ parallel to $\vec{F}_R$



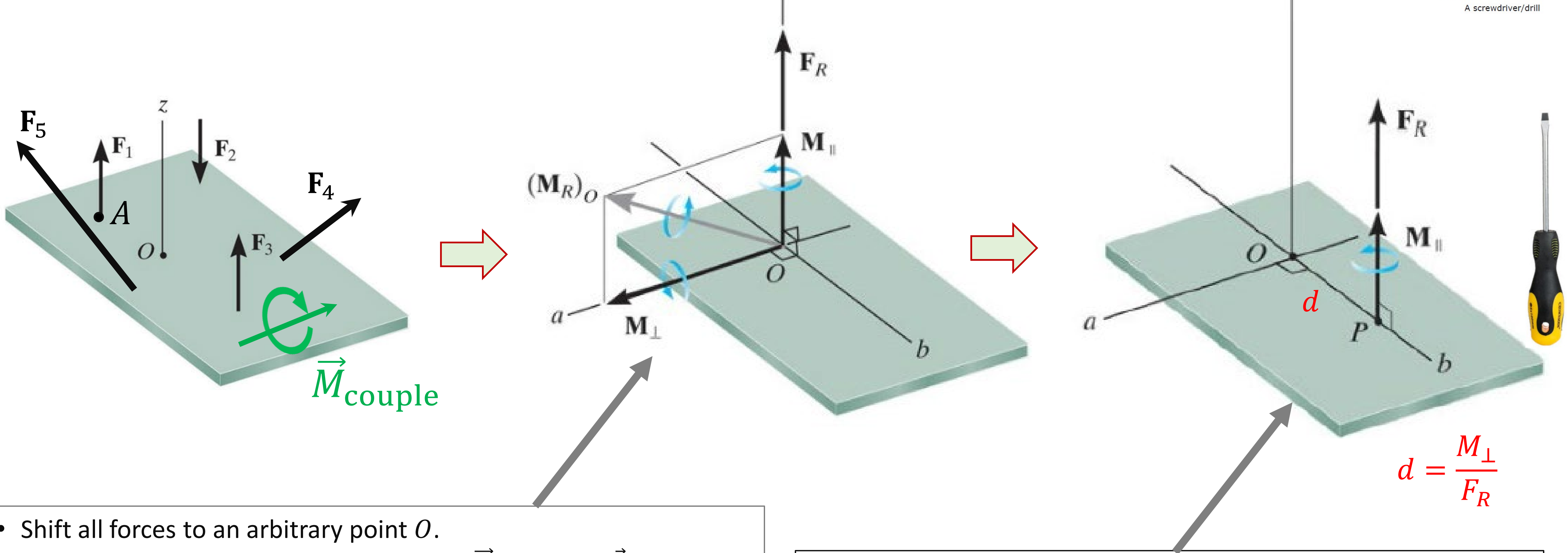
- "Wrench": $\vec{F}_R \,||\, \overrightarrow{M}_R$ (applied force is parallel to the direction of the rotation vector)

REDUCTION TO A WRENCH: (but it is NOT how we are going to do it in practice)



A screwdriver/drill

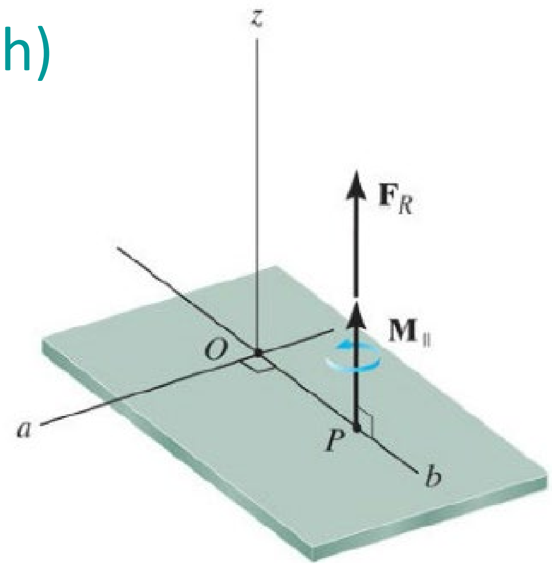
➢ Proof of Principle



- Shift all forces to an arbitrary point O.
- Add the compensating couple moments $\vec{M}_1 = \vec{r}_{OA} \times \vec{F}_1$, etc.
- Find the resultant force $\vec{F}_R = \sum \vec{F}_i$.
- Find the resultant couple moment at O: $\vec{M}_{R,O} = \sum \vec{M}_i + \vec{M}_{\text{couple}}$.
- Split $\vec{M}_{R,O}$ into two components: $\vec{M}_\perp \perp \vec{F}_R$ and $\vec{M}_\parallel \parallel \vec{F}_R$

- "Absorb" $\vec{M}_\perp$ into a shift of $\vec{F}_R$ to a point P, as we did before
- You are left with $\vec{F}_R$ and $\vec{M}_\parallel$: **a wrench**.

PROPERTIES OF PARALLEL VECTORS (will need for reducing to a wrench)



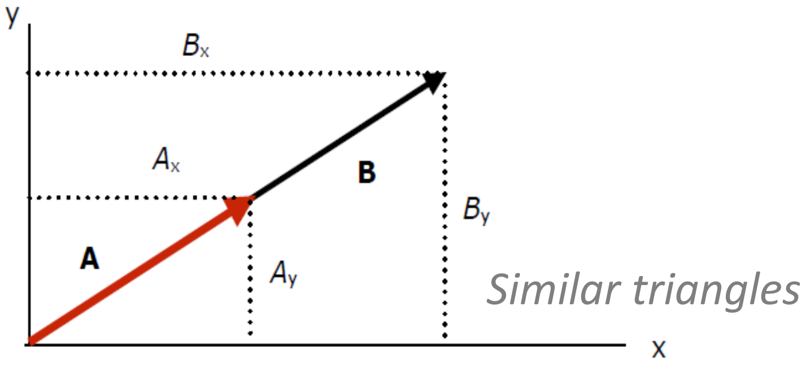
- Typical problem: determine $\vec{F}_R$, $\vec{M}_R = \vec{M}_\parallel$, and the coordinates of the point P.

- To do that, you will need to know **properties of parallel vectors** ($\vec{F}_R$ and $\vec{M}_\parallel$)

- Two vectors $\vec{A}$ and $\vec{B}$ are parallel if $\vec{B} = c\,\vec{A}$, where c is a scalar
  - If $c > 0$, they are parallel;
  - If $c < 0$, they are anti-parallel

- The constant c relates to their Cartesian components as follows:

$$\boxed{c = \frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}}$$

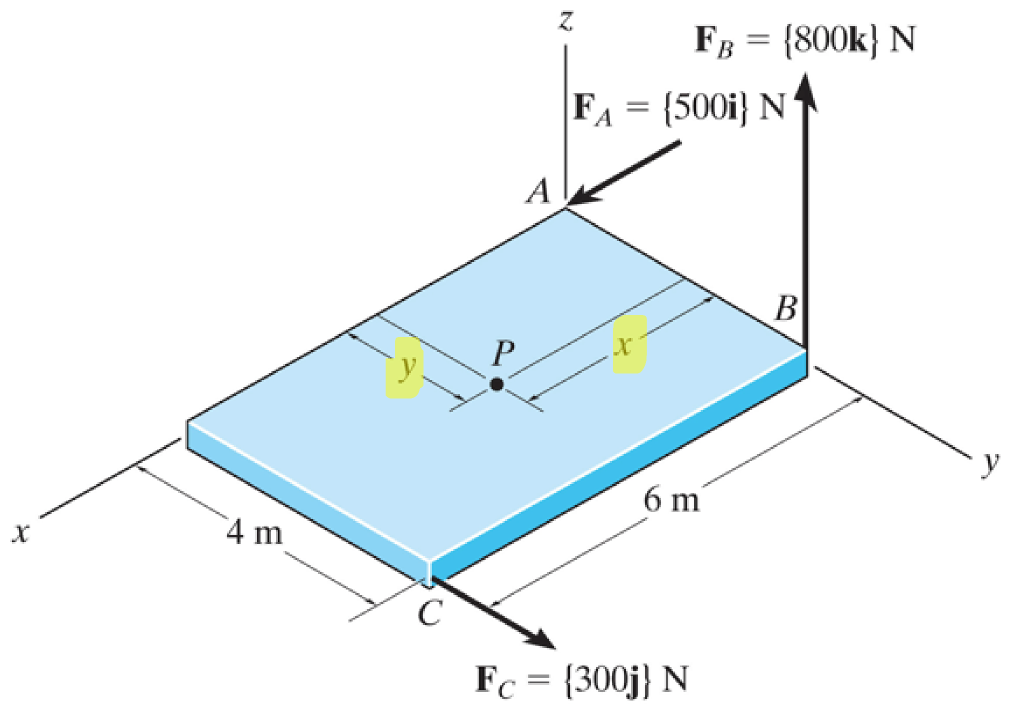


- Hence:

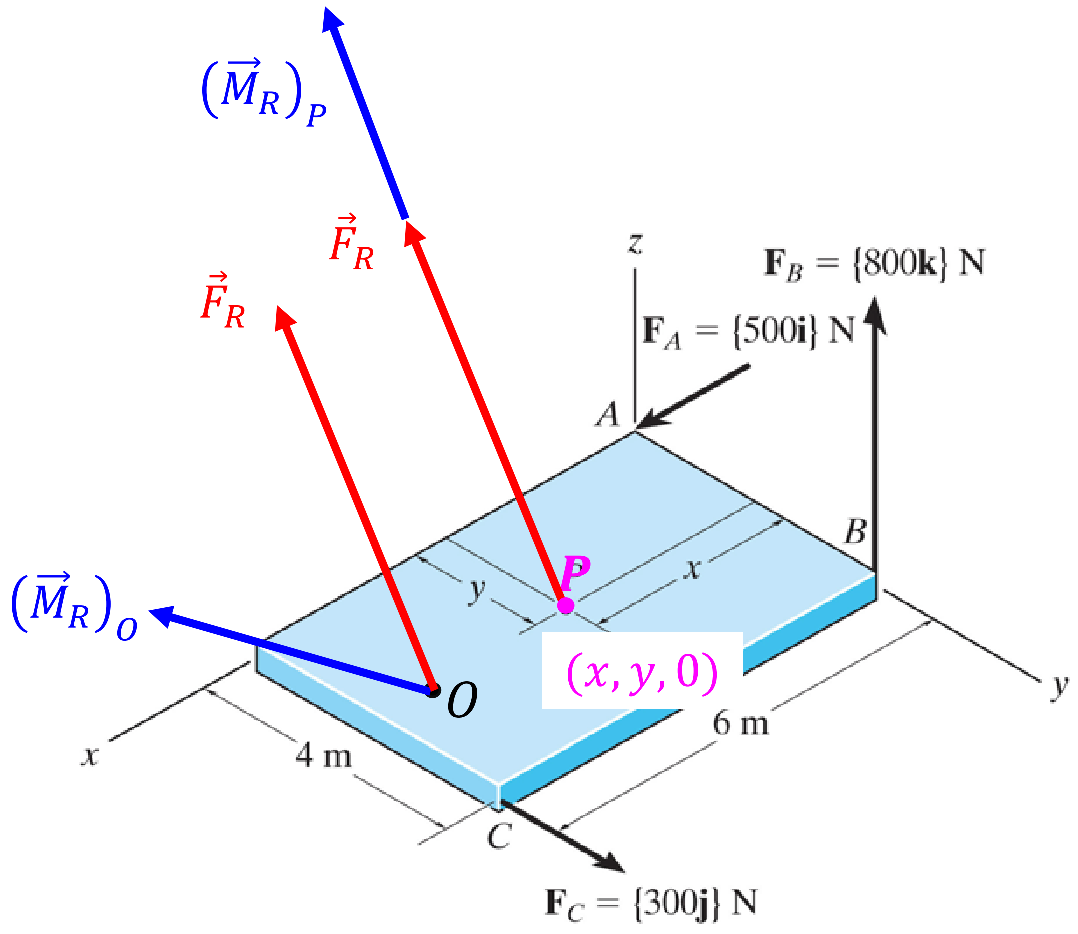
$$\frac{A_x}{B_x} = \frac{A_y}{B_y} \quad (1) \qquad \frac{A_y}{B_y} = \frac{A_z}{B_z} \quad (2)$$

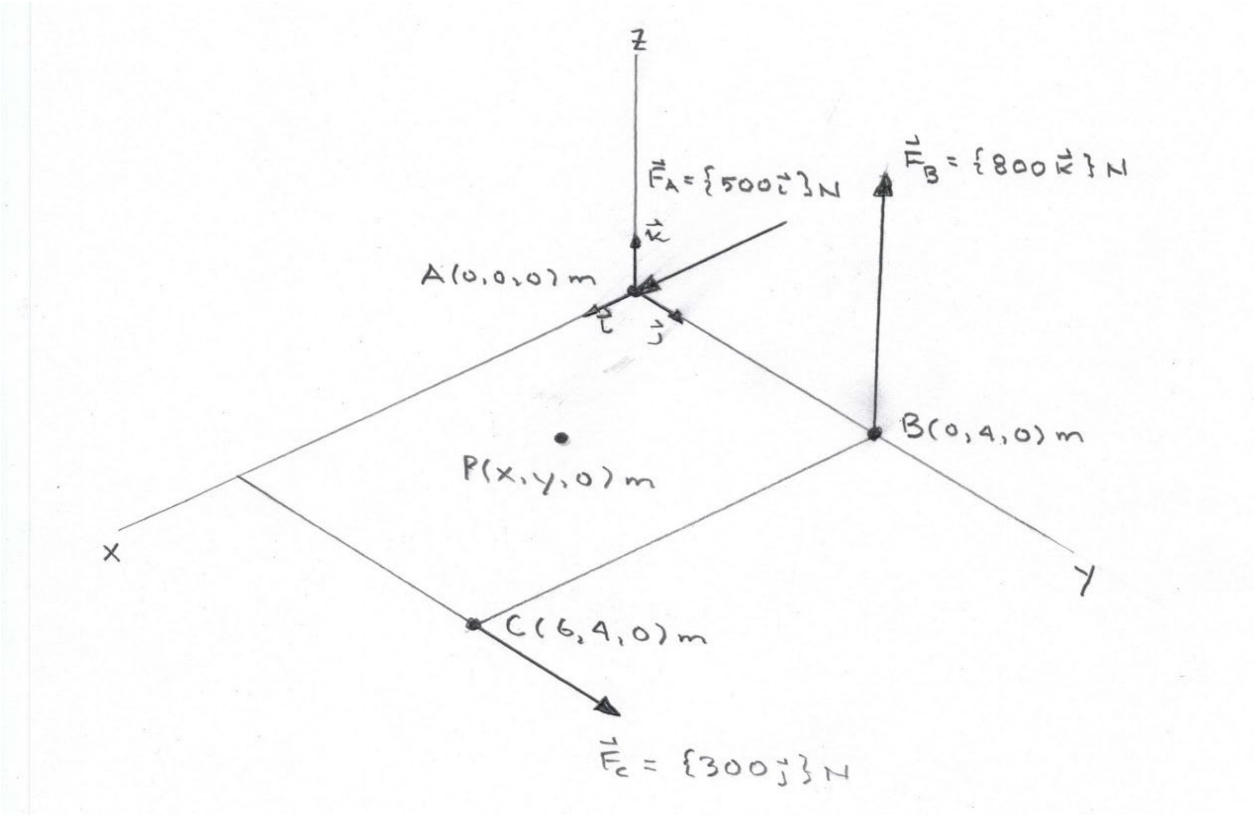
- If $c > 0$: $\vec{A} \uparrow\uparrow \vec{B}$
- If $c < 0$: $\vec{A} \uparrow\downarrow \vec{B}$

W4-3. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, y)$ where its line of action intersects the plate.

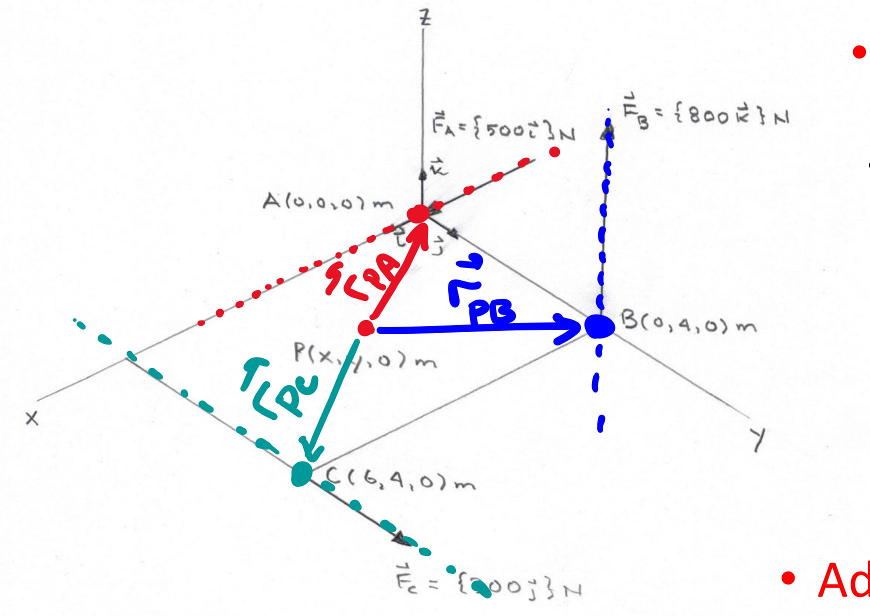


W4-3. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, y)$ where its line of action intersects the plate.





W4-3. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, y)$ where its line of action intersects the plate.



- Resultant force?

$$\vec{F}_A = (500)\vec{i} + (0)\vec{j} + (0)\vec{k}$$

$$\vec{F}_B = (0)\vec{i} + (0)\vec{j} + (800)\vec{k}$$

$$\vec{F}_C = (0)\vec{i} + (300)\vec{j} + (0)\vec{k}$$

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$= (500)\vec{i} + (300)\vec{j} + (800)\vec{k}$$

- Add couple moments:

$$\vec{M}_A = \vec{r}_{PA} \times \vec{F}_A \qquad \vec{r}_{PA} = (-x)\vec{i} + (-y)\vec{j} + (0)\vec{k}$$

$$\vec{M}_B = \vec{r}_{PB} \times \vec{F}_B \qquad \vec{r}_{PB} = (-x)\vec{i} + (4-y)\vec{j} + (0)\vec{k}$$

$$\vec{M}_C = \vec{r}_{PC} \times \vec{F}_C \qquad \vec{r}_{PC} = (6-x)\vec{i} + (4-y)\vec{j} + (0)\vec{k}$$

W4-3. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, y)$ where its line of action intersects the plate.

$$\vec{M}_A = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -x & -y & 0 \\ 500 & 0 & 0 \end{vmatrix} = \vec{i}(0) - \vec{j}(0) + \vec{k}(500y)$$

$$\vec{M}_B = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -x & 4-y & 0 \\ 0 & 0 & 800 \end{vmatrix} = \vec{i}(800(4-y)) - \vec{j}(-800x) + \vec{k}(0)$$

$$= \vec{i}(800(4-y)) + \vec{j}(800x) + \vec{k}(0)$$

$$\vec{M}_C = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6-x & 4-y & 0 \\ 0 & 300 & 0 \end{vmatrix} = \vec{i}(0) - \vec{j}(0) + \vec{k}(300(6-x))$$

$$\vec{M}_R = \vec{M}_A + \vec{M}_B + \vec{M}_C = \vec{i}(800(4-y)) + \vec{j}(800x) + \vec{k}(500y + 300(6-x))$$

W4-3. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, y)$ where its line of action intersects the plate.

$$\vec{F}_R = (500)\vec{i} + (300)\vec{j} + (800)\vec{k}$$

$$\vec{M}_R = [800(4 - y)]\vec{i} + (800x)\vec{j} + [500y + 300(6 - x)]\vec{k}$$

$\vec{F}_R \uparrow\uparrow \vec{M}_R$

$\boxed{F_R = 990 N}$

(1) $\frac{F_{Rx}}{M_{Rx}} = \frac{F_{Ry}}{M_{Ry}} \rightarrow \frac{5\cancel{00}}{\cancel{800}(4-y)} = \frac{3\cancel{00}}{\cancel{800}x}$

(2) $\frac{F_{Ry}}{M_{Ry}} = \frac{F_{Rz}}{M_{Rz}} \rightarrow \frac{3\cancel{00}}{8\cancel{00}x} = \frac{8\cancel{00}}{5\cancel{00}y + 3\cancel{00}(6-x)}$

$$\boxed{\begin{aligned} x &= 1.163 \\ y &= 2.061 \end{aligned}}$$

$(\vec{M}_R)_P \uparrow\uparrow \vec{F}_R$

$\vec{M}_R = 1551\vec{i} + 930.4\vec{j} + 2482\vec{k} \longrightarrow M_R(P) = 3.07 \text{ kN}$

$C = \frac{F_{Rx}}{M_{Rx}} = \frac{500}{1551} > 0$