Physics 157 Written Homework 3

Learning Goals for this assignment

Classes this past week have focused on the flow of heat and its effect on changes in temperature and phase. In this week's homework you will get practice applying the quantitative relationships that govern these effects. Specific skills you will practice include:

- To calculate the amount of heat required to change the temperature of an object by a fixed amount, or to change the phase of a given amount of material.
- For systems with multiple parts initially at different temperatures and/or in different phases, to calculate the final equilibrium temperature and/or portion of the system in each phase.
- Given data for the temperature as a function of heat added, or the temperature as a function of time for a heat source with a specified power, to deduce the specific heat and/or latent heat of a material.
- To calculate the rate of heating/cooling/phase change in situations where heat transfer is taking place via conduction.

Heat/ phase change problems:

In these problems, it's again useful to consider each component of the system separately. Here, a good starting point is to write down an equation for the heat that flows into the system (this may be negative). This will always be of the form $Q = m\,c\,\Delta T$ or $Q = mL$, or some combination. For example, with ice at -20 degrees Celsius going to water at 30 degrees Celsius, we'll have the heat to bring the ice to the melting point, plus the heat to melt the ice, plus the heat to bring the resulting water up to 30 degrees.

If some ice melts and some doesn't you can treat these as separate components of the system.

Next, write down any equations that relate the parts. We always have that the sum of the heats added to each part equals the total heat added to the whole system (which is often zero).

We also have that all parts of the system must have the same final temperature. If the system contains both ice and water at the end, this temperature is 0 degrees.

Physics 157 Written Homework 3

Question 1: One of the Mastering Physics questions ask you to write up the solution and hand it in as part of your Written Homework assignment. For that MP problem, you will still put your answer in MP, but you will write out the solution and submit it as the answer for Q1 of your written homework assignment. It will be graded based on your explanation.

Most of the Mastering Physics questions are fairly straight forward. They are there to help make sure you understand the basic steps that will appear in more complicated problems. Writing out the solution for one of them will give you extra practice actually writing down the solution to a problem since you will be graded based on your written solutions for some of the problems on midterms and the exam.

Note: Different students will have different numbers for mass of steam, the initial mass of water, and the initial temperature of the water in this problem.

Problem 17.97

In a container of negligible mass, 4.50×10^{-2} kg of steam at 100 $^\circ\mathrm{C}$ and atmospheric pressure is added to 0.230 kg of water at 54.0 $^\circ\mathrm{C}$.

Part A

If no heat is lost to the surroundings, what is the final temperature of the system?

In a problem like this, we don't yet know whether steam is left at the end or not. So, let's work out the problem assuming that some amount of steam, $m_{steam\text{-}final}$, is left at the end and then check to see if our result is reasonable. With our assumption, steam is left at the end, so the final temperature must be 100 ^{o}C.

Physics 157 Written Homework 3

Part B

At the final temperature, how many kilograms are there of steam?

During this process, the amount of heat added to the initial mass of water is:

$$Q_{initial\text{-}water} = M_{water} \times c_{water} \times (100 - 54\ K)$$

The mass of steam that condenses into water is ($m_{steam\text{-}initial} - m_{steam\text{-}final}$), which requires latent heat of vaporization given by (this is negative because steam is condensing instead of water vaporizing):

$$Q_{steam\text{-}condensing} = (m_{steam\text{-}final} - m_{steam\text{-}initial}) \times Lv$$

We can also invoke energy conservation to obtain a relation between these two:

$$Q_{initial\text{-}water} + Q_{steam\text{-}condensing} = 0$$

Solving for $m_{steam\text{-}final}$ gives:

$$m_{steam\text{-}final} = m_{steam\text{-}initial} - M_{water} \times c_{water} \times (100 - 54\ K) / L_v = 2.54 \times 10^{-2}\ kg$$

Note that we obtained a positive value for the final mass of steam, so our initial assumption that some steam is left was correct. If we had obtained a negative value, then we would know that the initial mass of steam all condensed to water. In that case, we would only have one unknown (the final temperature of the water, $T_{water\text{-}final}$). How would you solve for $T_{water\text{-}final}$?

Part C

How many kilograms are there of liquid water?

At the end, the total mass of the water will be the initial mass of water plus the mass of steam that condensed into water:

$$M = M_{water\text{-}initial} + (M_{initial\text{-}steam} - M_{final\text{-}steam}) = 0.2496\ kg$$

Physics 157 Written Homework 3

Question 2: Freezing Lake

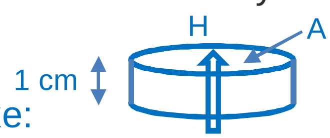
In many parts of Canada, people play hockey in the winter on frozen lakes. However, it is not considered safe by the Canadian Red Cross to play hockey on ice until the ice is 20 cm thick. On a winter day with a temperature −10°C, it is observed that the lake has a 1 cm thick layer of ice over the entire surface.

Constants which may be of use: The thermal conductivity of ice is 2.18 W/m·K. Density of ice is 916.7 kg/m^3 and of water is 1000 kg/m^3. Latent heat of fusion of ice is 334x10^3 J/kg.

Part A

If the air temperature remains at −10°C, find the rate in cm per hour at which ice is initially added to the layer. The thermal conductivity of ice is 2.18W/m·K. Answer in cm/hour.

Think about some area A of the lake:



Heat current is:

$H = k_{ice} A (T_H - T_C)/L = 2.18$ W/m·K x A x 10K/0.01m = 2.18x10^3 W/m^2 x A

In 1 hour, energy leaving ice is:

Q = H x 1 hour = 3600 s x 2.18x10^3 W/m^2 x A = A x 7.85 x 10^6 J/m^2

Mass of ice added in 1 hour is:

$M = Q / L_f$ = A x (7.85 x 10^6 J/m^2) / (334x10^3 J/kg) = A x 23.5 kg/m^2

Volume of ice added in 1 hour is:

V = M/density = A x 23.5 kg/m^2 / 916.7 kg/m^3 = A x 0.0256 m

Thickness of ice added in 1 hour is:

V/A = 0.0256 m

So, the rate ice is added is 2.56 cm/hour

Part B

Find the rate in cm per hour at which ice is added to the layer as a function of the thickness X, where X is the number of centimeters.

If you repeat the calculations using X cm instead of 1 cm for thickness, the answer will be (2.56 / X) cm/hour.

Physics 157 Written Homework 3

Part C

Estimate the time it takes before you can safely play hockey.

Hint: using your result from the previous question, think about how the rate varies as the ice gets thicker. The rate will be very different depending on the thickness. We are not looking for an exact answer for the total time it takes before you can play safely. Think about the time it takes for a "typical" thickness.

An exact solution would require an integration. However, a reasonably good estimate of the time can be obtained by estimating the time it takes for ice having a "typical" (constant) value of the thickness during the freezing process. Since the thickness grows from 1 cm to 20 cm over the course of the hour, you can get a reasonable estimate by choosing a "typical" thickness of, e.g., 10 cm and treating the thickness as constant at this value for the entire hour. This will over-estimate the time when the thickness is smaller, and under-estimate the time when the thickness is larger, and these two errors will tend to cancel out.

Using this approach and taking a "typical" thickness of 10 cm, the "typical" rate would be 0.256 cm/hour. At this rate, adding 19 cm of ice would take 19/0.256 = 74 hours, which is about 3 days.

Here is the exact solution for the total time T:

$$T = \int_1^{20} \frac{dt}{dr} dr = \int_1^{20} \frac{r}{2.56} dr = \frac{r^2}{2x2.56} \Big|_1^{20} = \frac{(400-1)}{(2x2.56)} = 78\ hours$$

We see that our simplified approach gave quite a good estimate compared to the exact approach. This is often the case, so keep this technique in mind for when you need to make an approximate estimate.