Physics 157 Written Homework 4

Learning Goals for this assignment

In this homework set, you'll get some practice with problems involving radiation.

Specific skills you will practice include:

- To be able to interpret spectrum graphs, calculating the relative power for different ranges of wavelengths and deduce the temperature of an object via Wien's law in the case of a thermal spectrum.
- To predict the power in electromagnetic radiation emitted by an object given its temperature, area, and emissivity.
- To calculate the equilibrium temperature of a radiating object when the ingoing energy is supplied directly via a power source or absorbed from some external source of radiation. To correctly take into account emissivity and albedo in these calculations.
- To be able to calculate the intensity (power per unit area) of radiation from a spherically symmetric source given the power of the source and the distance to the source.

Tips:

- The thermal radiation from an object with temperature T has a peak wavelength of $\lambda = b/T$, where b = 2.9 mm-K.
- For an object whose temperature is not changing, we have $H_{net} = 0$, or $H_{in} = H_{out}$. This is just energy conservation.
- Objects with temperature T radiate energy from their surface at a rate $H = A\ e\ \sigma\ T^4$. The area A is the total surface area of the part of the object at temperature T. The emissivity e is a property of the surface material.
- If the object is in an environment with temperature T_{env}, it will absorb radiation at a rate $H = A\ e\ \sigma\ (T_{env})^4$.
- For an object radiating with power H uniformly in all directions, the intensity or power per area of the radiation at a distance R from the object is $I = H/(4\ \pi\ R^2)$. The relative intensity at two different distances is then determined by $I_2/I_1 = (R_1/R_2)^2$.
- The solar constant I_{SC} is defined to be the intensity of sunlight in the vicinity of the Earth, and has a value of 1367 W/m^2.
- For an object in radiation with intensity I coming from a distant source, the amount it will absorb is $H = A\ I\ (1 - a)$, where a is the albedo or fraction of light reflected, and A here is the area of the incoming beam that is blocked off by the object (e.g. $A = \pi\ R^2$ for both a sphere of radius R and a disk of radius R perpendicular to the light rays).

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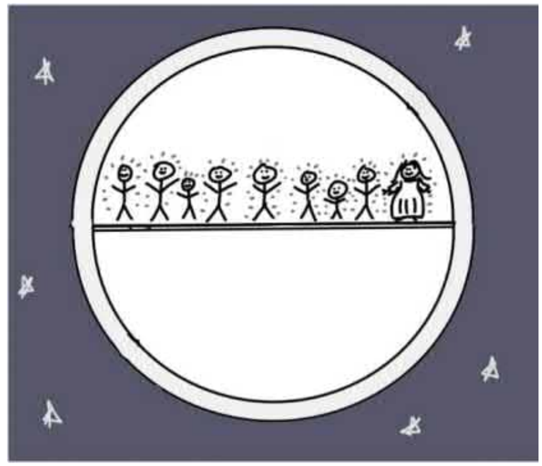
Question 1: You are getting married and decide to hold your wedding in space! Your spacecraft is a spherical shell (R = 5 m) of material whose outside surface has emissivity 1 (for infrared wavelengths) and albedo 0.4.



The 100 people at your wedding generate a total of 100 kW of heat, all of which reaches the surface of your spacecraft. Additionally, the craft absorbs heat from a nearby star (distance 10^{10} m, emissivity = 1), whose light has an intensity of 750 W/m^2 at the position of your spacecraft.

a) Assuming that the outer surface of your spacecraft has a uniform equilibrium temperature T, determine this temperature.

Hint: you don't need to worry about inward radiation from the shell, since all of this is absorbed by the shell again.

At equilibrium, the heat current in to the surface of your spacecraft must equal the heat current out due to radiation:

$H_{in} = H_{out}$

We have:

$H_{in} = H_{people} + H_{starlight}$

We are given that H_{people} = 100 kW.
We have (since (1-a) is the fraction of light absorbed):

$H_{starlight} = I_{star}$ x (area blocked off by ship) x (1-a)

= 750 kW/m^2 x π $(5\ m)^2$ x 0.6 = 35.3 kW

Finally,

$H_{out} = A\ e\ \sigma\ T^4 = 4\ \pi\ (5\ m)^2 \times 1 \times 5.67 \times 10^{-8}\ (W/m^2\cdot K^4) \times T^4$

Combining equations and solving for T gives:

$T = [135.3\ kW / (4\ \pi\ (5\ m)^2 \times 5.67 \times 10^{-8})]^{1/4} = 295\ K$

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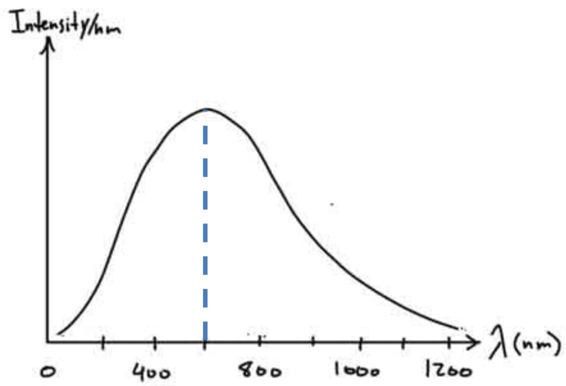
b) One of your wedding guests is an astronomer. They decide to measure the spectrum of light from the nearby star and find the result shown below. Using this information, plus the information from part (a), determine the radius of the star.



The wavelength of the peak in the spectrum is λ_{max} ~ 600 nm. From Wien's Law:

$T = b / \lambda_{max} = 2.9\text{x}10^{-3}$ m·K / $600\text{x}10^{-9}$ m = 4833 K

The heat current from the star is

$H_{star} = A_{star} \text{ x } \sigma \text{ x } T^4$ (have e = 1)

$= 4 \pi R_{star}^2 \sigma T^4$

This is related to the intensity at your spacecraft by:

$I_{star} = H_{star} / (4 \pi D^2)$

where D is the distance to the star.

So:

$I_{star} = (R_{star}^2 / D^2) \sigma T^4$

and:

$R_{star} = [(D^2 I_{star}) / (\sigma T^4)]^{1/4}$

$= [((10^{10} \text{ m})^2 \text{ x } 750 \text{ W/m}^2) / (5.67\text{x}10^{-8} \text{ W/m}^2\cdot\text{K}^4 \text{ x } 4833^4)]^{1/2}$

$= 4.92 \text{ x } 10^7$ m

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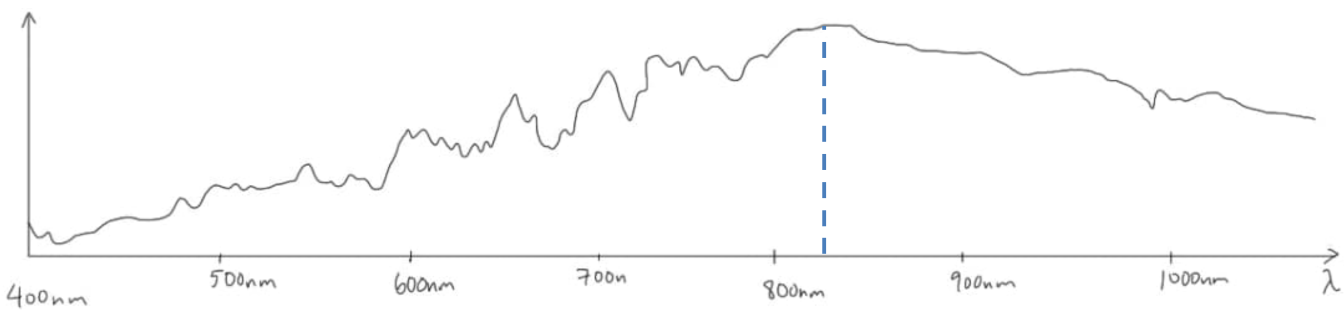
Question 2: The graph below shows an approximation to the spectrum of electromagnetic radiation from the star Betelgeuse (brightest star in the constellation Orion). This star is known to be a distance of 643 light years from the Earth (1 light year is the distance light travels in a year). The intensity of electromagnetic radiation (power per area) from Betelgeuse as measured on Earth is 8 x 10^{-11} times the intensity of electromagnetic radiation from the sun (the solar constant). You can assume the emissivity of Betelgeuse is e = 1.



a) Using this information, estimate the radius of Betelgeuse.

We can find the radius of Betelgeuse, r_b, if we can estimate its total power radiated and its temperature.

We have:

$$H_b = A_b \times \sigma \times T_b^4 \qquad (e_b = 1)$$

$$= 4 \pi r_b^2 \sigma T_b^4 \qquad (1)$$

So:

$$r_b = [H_b / (4 \pi \sigma T_b^4)]^{1/2}$$

To find T_b, we can use the spectrum. We see that the wavelength for the peak in the spectrum is $\lambda_b \sim 825$ nm. Using Wien's Law, this suggests that the temperature of Betelgeuse is:

$$T_b = b / \lambda_b = 0.2898 \text{ cm·K} / 825 \text{ nm} = 3.5 \times 10^3 \text{ K} \qquad (2)$$

To find the total power, we can use that the intensity of radiation at Earth is:

$$I_b = 8 \times 10^{-11} I_{sc} = 8 \times 10^{-11} \times 1367 \text{ W/m}^2 = 1.1 \times 10^{-7} \text{ W/m}^2$$

This is related to the total power H by:

$$I_b = H_b / (4 \pi d_b^2)$$

where d_b = 643 light years = 6.1 x 10^{18} m.

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So,

$H_b = 1.1 \times 10^{-7}$ W/m$^2 \times 4\,\pi \times (6.1 \times 10^{18}$ m$)^2$ W $= 5.1 \times 10^{31}$ W (3)

Combining (1), (2), and (3) gives:

$r_b = 6.9 \times 10^{11}$ m ~ 995 r_{sun} ~ 4.5 $d_{earth\text{-}sun}$

b) How many times larger is this than the Sun's radius?

The sun's radius is ~ 696,340 km, so r_b ~ 995 r_{sun}

c) How does this compare to the distance between the Sun and the Earth?

The sun's radius is ~ 1.5×10^8 km, so r_b ~ 4.6 $d_{earth\text{-}sun}$!!