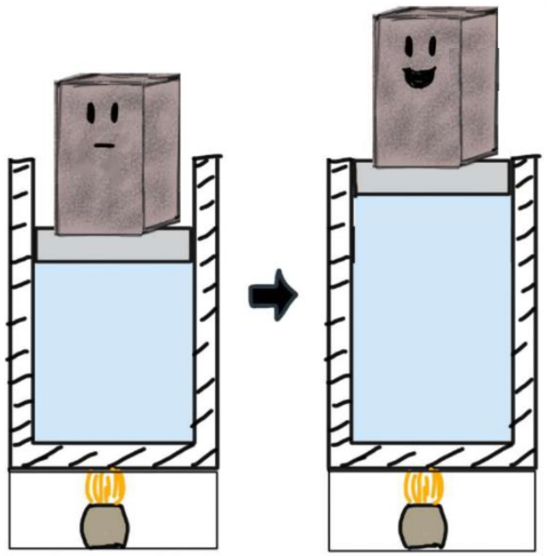
Lecture 17.

Isobaric, Isothermal and Adiabatic processes.



Last Time

- Ideal Gas Law

$$\boxed{PV = nRT}$$

- Internal energy:

$$\boxed{\Delta U = Q - W} \quad \boxed{\Delta U = nC_v\,\Delta T} \quad \boxed{U = \frac{\text{dof}}{2} nRT} \quad \boxed{C_v = \frac{3}{2}R}$$

ΔT $P, \Delta V$

- Work done by ideal gas:

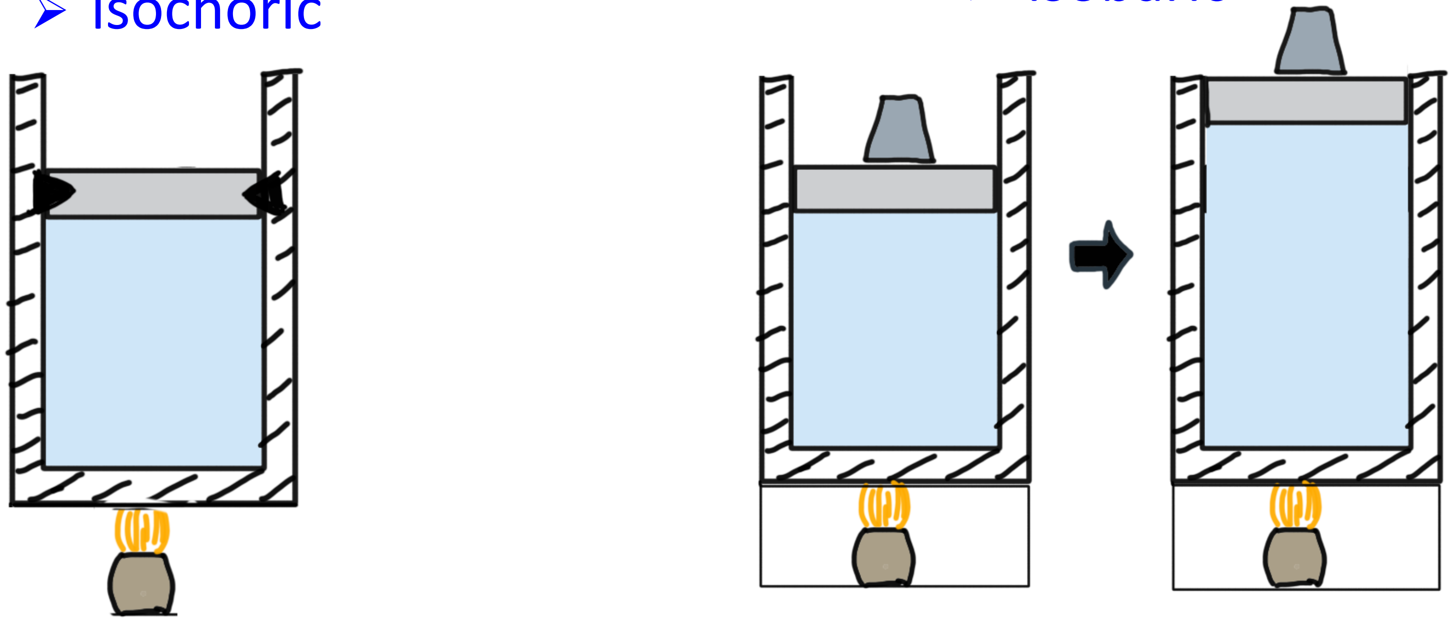
$$\boxed{W = \int P(V)\, dV}$$

$$\boxed{W = P \cdot \Delta V \text{ if } P = \text{const}}$$

- Terms:
  - isochoric: V constant
  - isobaric: P constant
  - isothermal: T constant
  - adiabatic: $Q = 0$ ✓

Q: In the two situations below, a gas is heated from 300K to 400K. Compare heat added in these two cases.

- isochoric
- isobaric



first case

second case

less heat

More heat

$Q = ?$

$\Delta U = Q - W$

$Q = \underbrace{\Delta U}_{\Delta T} + \underbrace{W_{gas}}_{P,\,\Delta V}$

Same

1) $W = 0$

2) $W > 0$

Last Time

Heat for Constant Pressure and Constant volume:

$\Delta U = Q - W$

isochoric

- Prove that for $V = const$: $Q = nC_v \Delta T$

$Q = \Delta U + \cancel{W}^{0}$; $W = P\Delta V = 0$ $\rightarrow$ $Q = \Delta U = nC_v \Delta T$

isobaric

- Prove that for $P = const$: $Q = nC_p \Delta T$ where $C_p = C_v + R$

$Q = \Delta U + W = nC_v \Delta T + P\Delta V = nC_v \Delta T + nR\Delta T =$

$P\Delta V = nR\Delta T$

$= n\underbrace{(C_v + R)}_{C_p}\Delta T$

- isochoric: V constant
- isobaric: P constant

Heat for Constant Pressure and Constant volume:

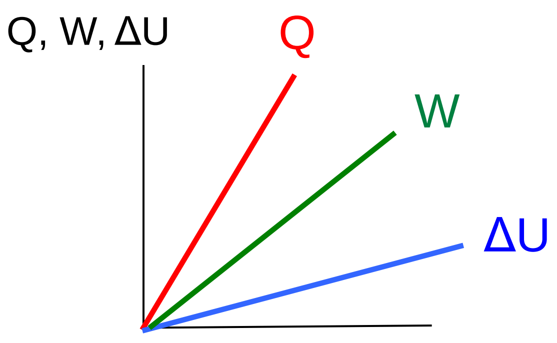
- Prove that for $V = const$: $Q = nC_v\Delta T$
  - $\Delta U = nC_v\Delta T = Q - W$
  - $W = P\Delta V = 0$
- Prove that for $P = const$: $Q = nC_p\Delta T$ where $C_p = C_v + R$

- $Q = \Delta U + W$
- $\Delta U = nC_v\Delta T$
- $W = P\Delta V = nR\Delta T$ (ideal gas law)
- So $Q = n(C_v + R)\Delta T$

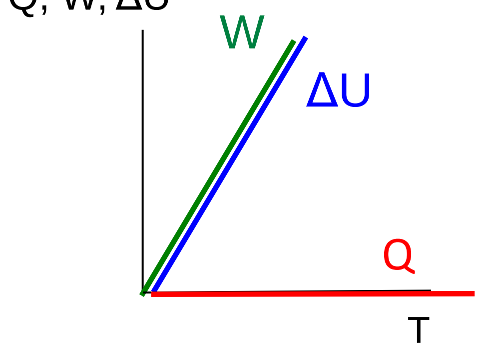
- Define: $\boxed{C_p = C_v + R}$
- We get: $\boxed{Q = nC_p\Delta T}$

- isochoric: V constant
- isobaric: P constant

Q: Given an isobaric process for an ideal gas, which graph best represents the heat transfer Q, work W, and internal energy ΔU, as the gas is heated?



A



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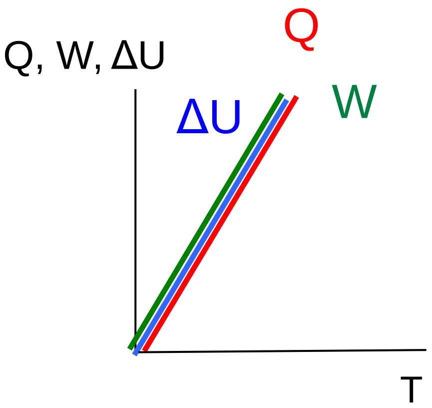
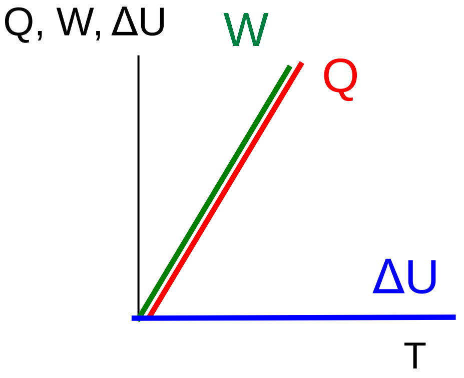
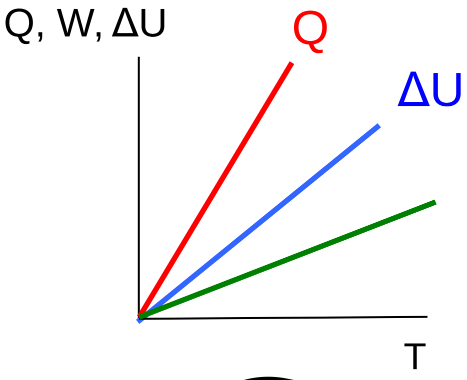


C



D



E

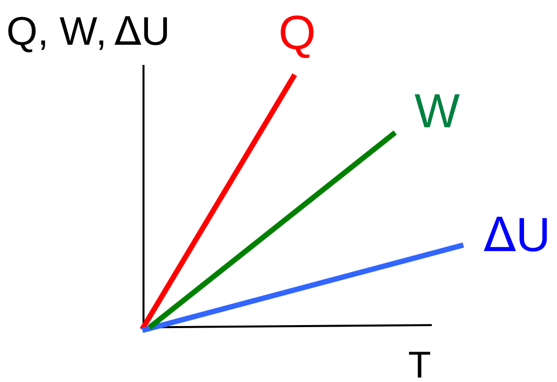
$$Q = \Delta U + W$$

$$\Delta U \overset{?}{=} n C_v \Delta T = n \frac{3R}{2} \Delta T$$

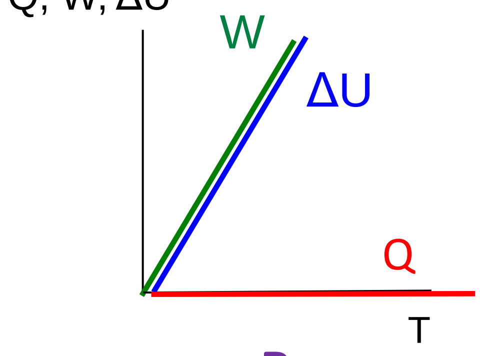
$$W \overset{?}{=} P \Delta V = n R \Delta T$$

- isobaric: P constant

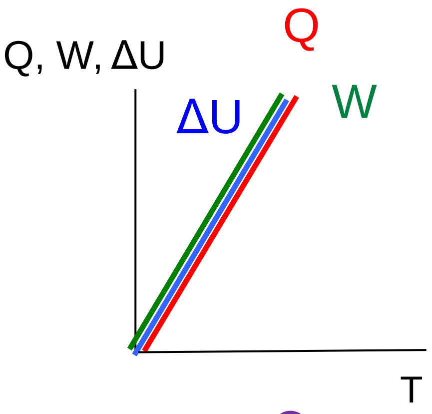
Q: Given an isobaric process for an ideal gas, which graph best represents the heat transfer Q, work W, and internal energy ΔU, as the gas is heated?



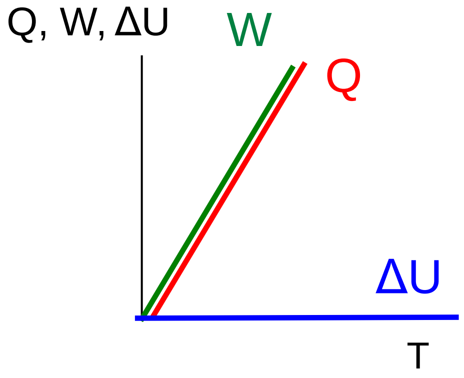
A



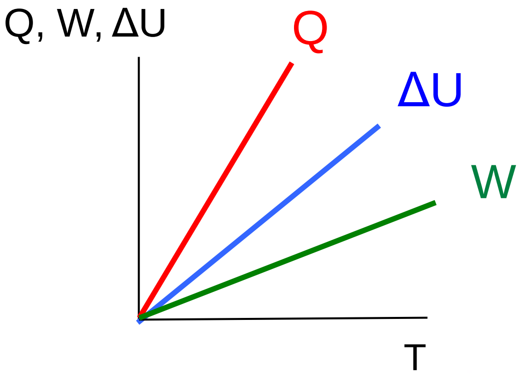
B



C



D



E ✓

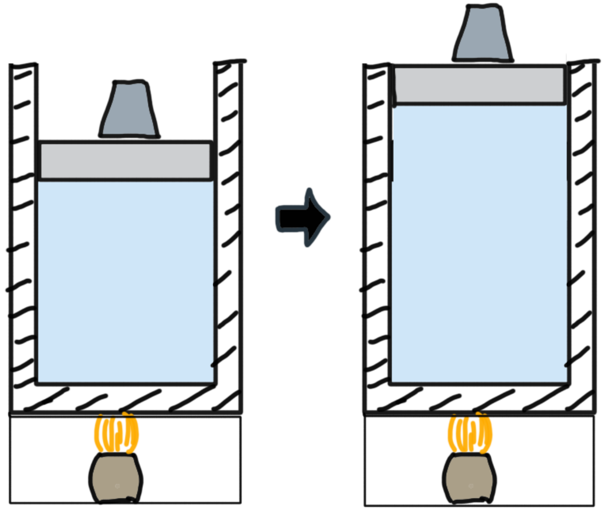
$$\Delta U = nC_v \Delta T$$

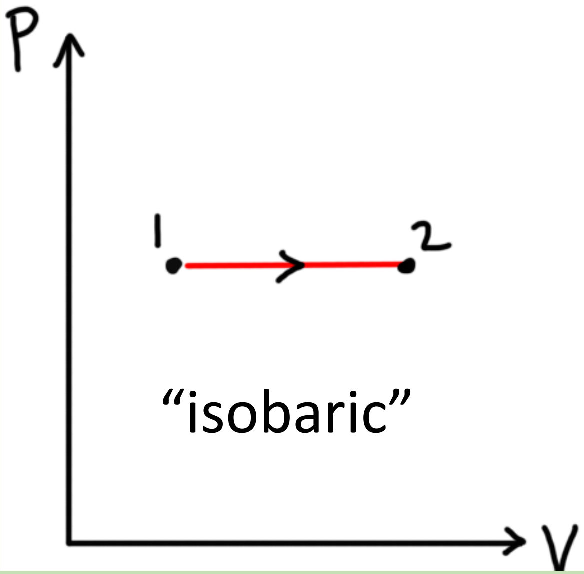
$$W = P\Delta V = nR\Delta T$$

$$Q = \Delta U + W$$

➢ isobaric: P constant

Constant Pressure: Summary

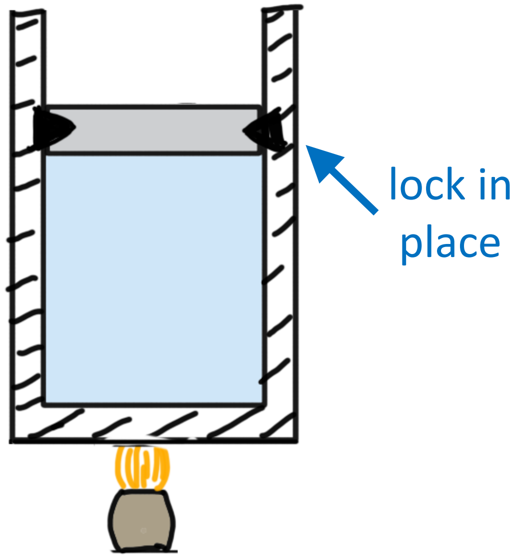




- Ideal Gas Law $\Longrightarrow \boxed{PV = nRT}$
  - $P, n = \text{const} \Longrightarrow \frac{V}{T} = \text{const}$ $\left(\frac{V_1}{T_1} = \frac{V_2}{T_2}\right)$ $(V \propto T)$
- $\Delta U = Q - W$
  - $\Delta U = nC_v \Delta T$
  - $W = P\Delta V$
  - $Q = nC_p \Delta T$
  - $C_p = C_v + R$

➢ isobaric: P constant

Constant Volume: Summary (Last Time)

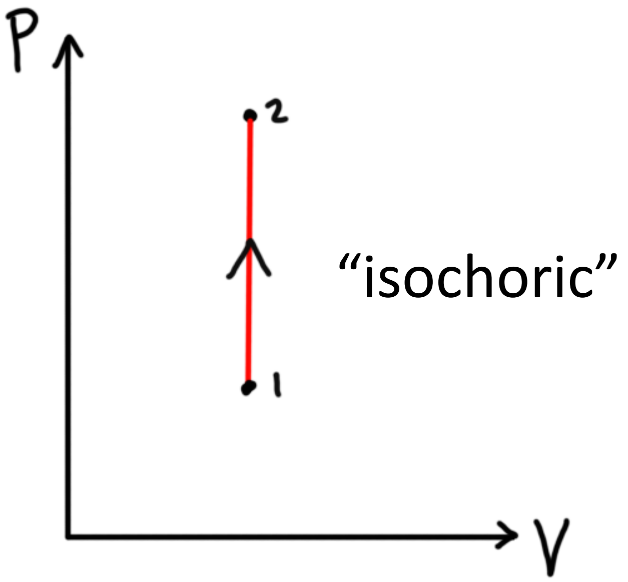


- Ideal Gas Law $\Longrightarrow PV = nRT$
  - ➢ n, V are constant $\Longrightarrow \frac{P}{T} = \text{const}$

$$\left(\frac{P_1}{T_1} = \frac{P_2}{T_2}\right)$$

$$(P \propto T)$$

- $\Delta U = Q - W$
  - ➢ $W = P\Delta V = 0$
  - ➢ So: $Q = \Delta U = nC_v\Delta T$

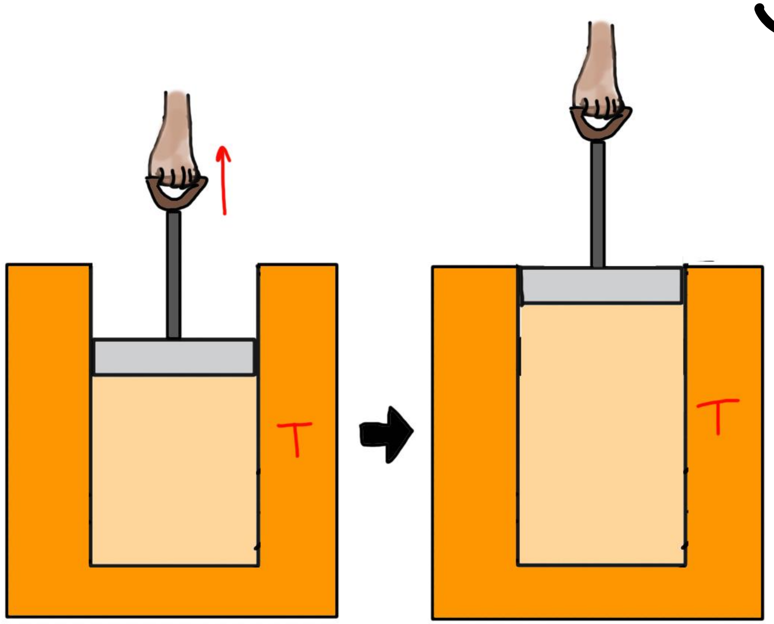


➢ isochoric: V constant

Q: Gas in a cylinder is slowly expanded while in contact with a heat reservoir so that its temperature remains constant. During this process, we can say that

→ isothermal

$Q = W > 0$



$\underbrace{\Delta U}_{\Delta T = 0 \;=\; 0} = Q - W$

A. Both Q and ΔU are 0

B. Q is 0 and ΔU is positive

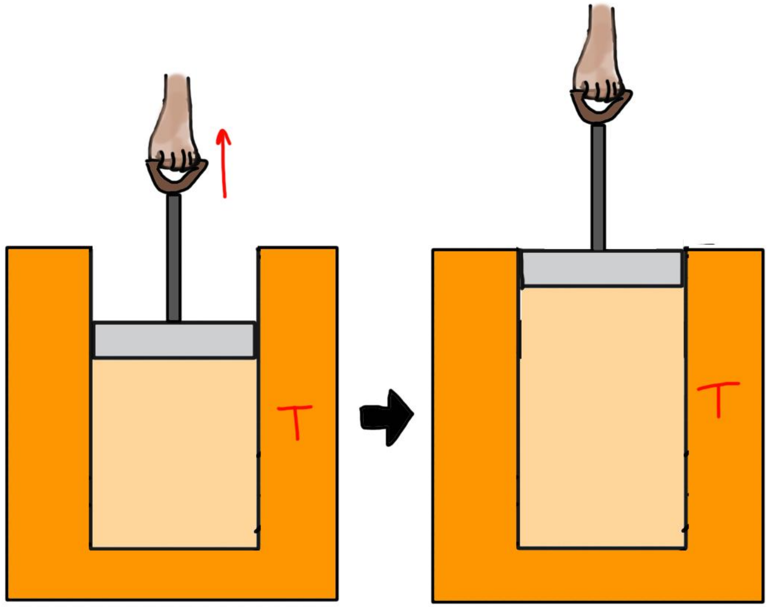
C. Q is 0 and ΔU is negative

D. ΔU is 0 and Q is positive

E. ΔU is 0 and Q is negative

➢ isothermal: T constant

Q: Gas in a cylinder is slowly expanded while in contact with a heat reservoir so that its temperature remains constant. During this process, we can say that



Constant T $\Longrightarrow \Delta U = 0$

W is positive (expansion)

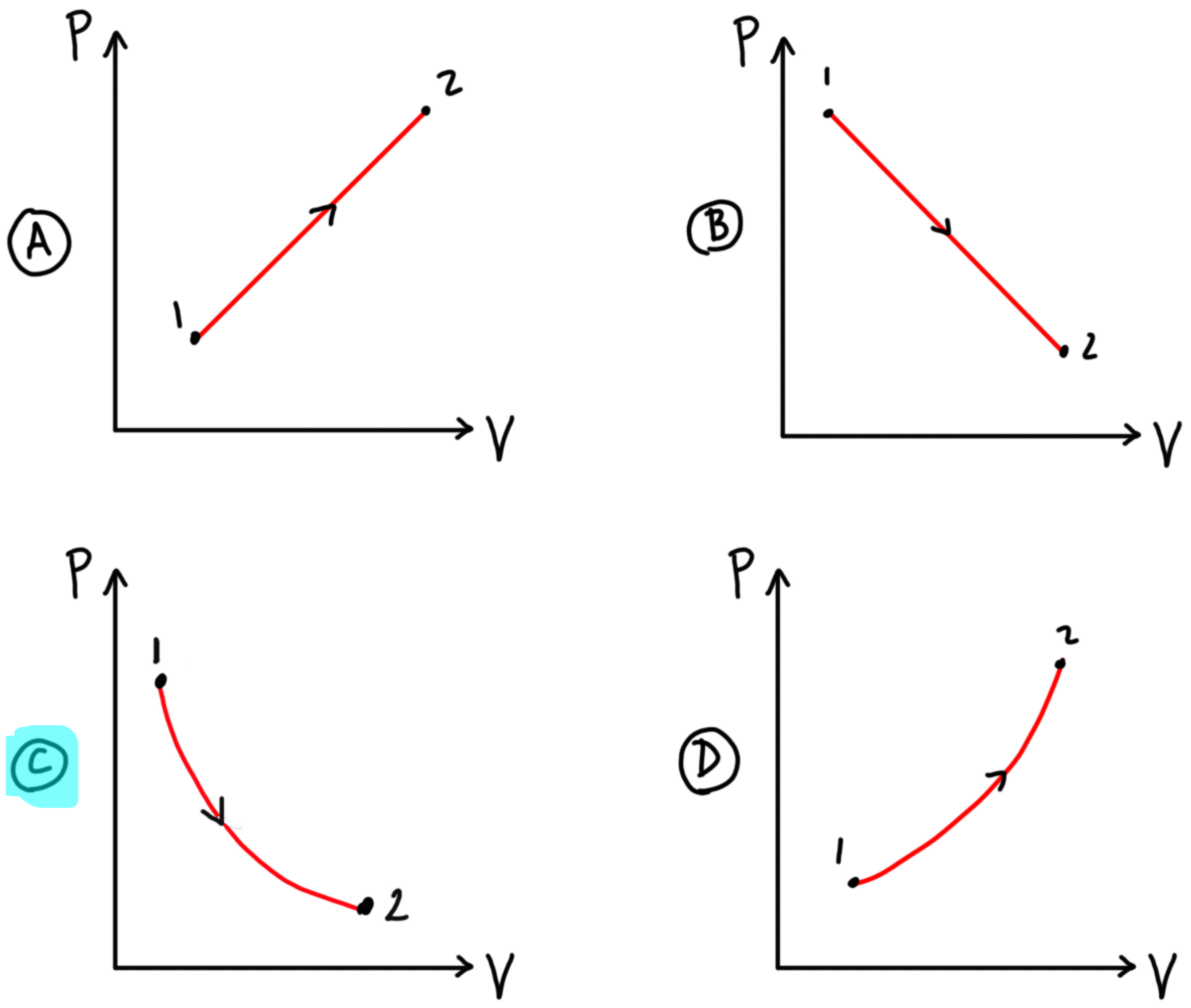
First Law: $\Delta U = Q - W$

So $Q = W > 0$

A. Both Q and ΔU are 0
B. Q is 0 and ΔU is positive
C. Q is 0 and ΔU is negative
D. ΔU is 0 and Q is positive ✓
E. ΔU is 0 and Q is negative

➢ isothermal: T constant

Q: Which graph could represent the expansion of an ideal gas at constant temperature?

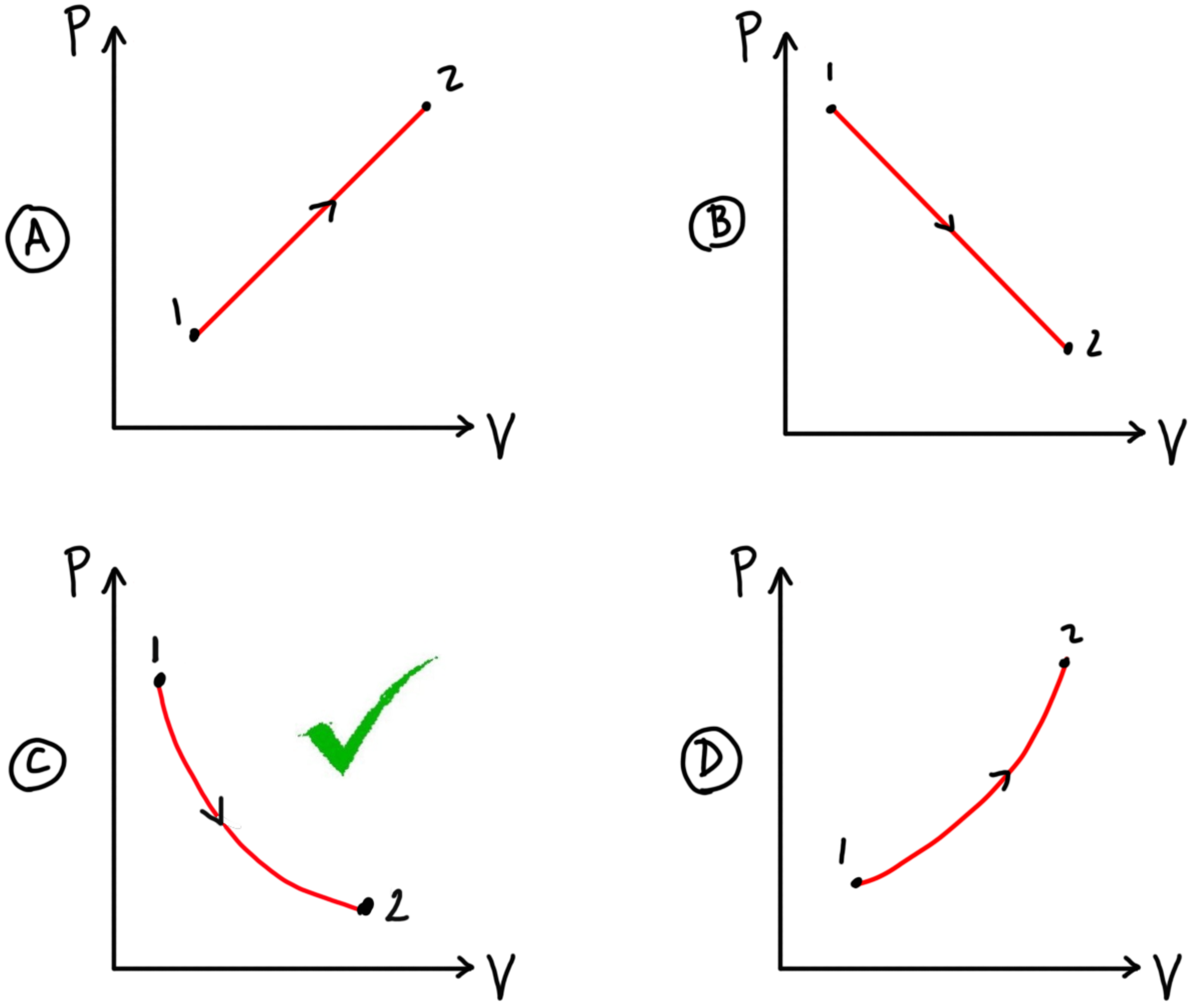


$$pV = nRT$$

$$p = \frac{\text{const}}{V}$$

➢ isothermal: T constant

Q: Which graph could represent the expansion of an ideal gas at constant temperature?



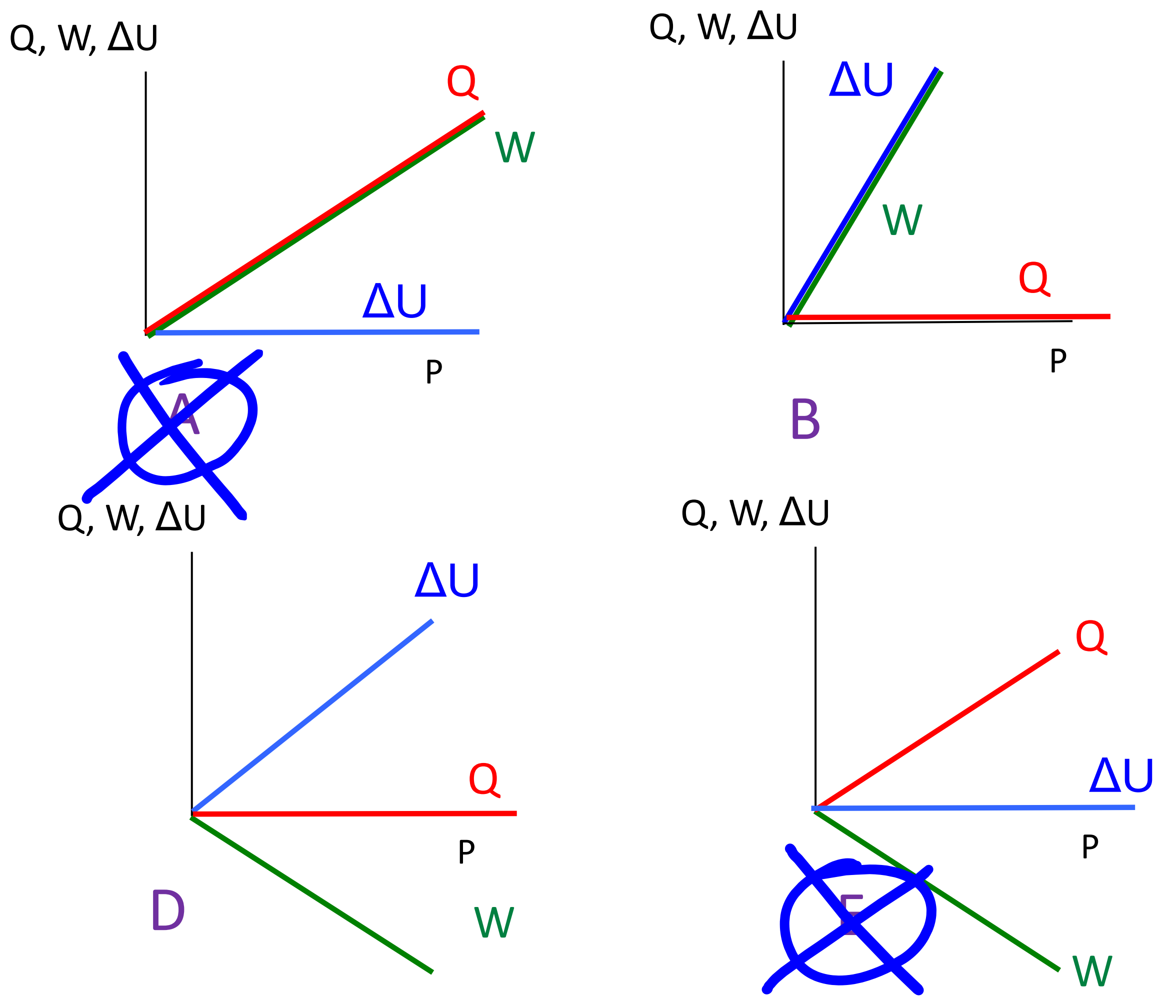
Have $PV = nRT$

So $\mathrm{P} = \dfrac{\mathrm{const}}{V}$

This looks like the $\frac{1}{x}$ function

➢ isothermal: T constant

Q: Given an isothermal process for an ideal gas, which graph best represents the heat transfer Q, work W, and internal energy ΔU, as the pressure increases?



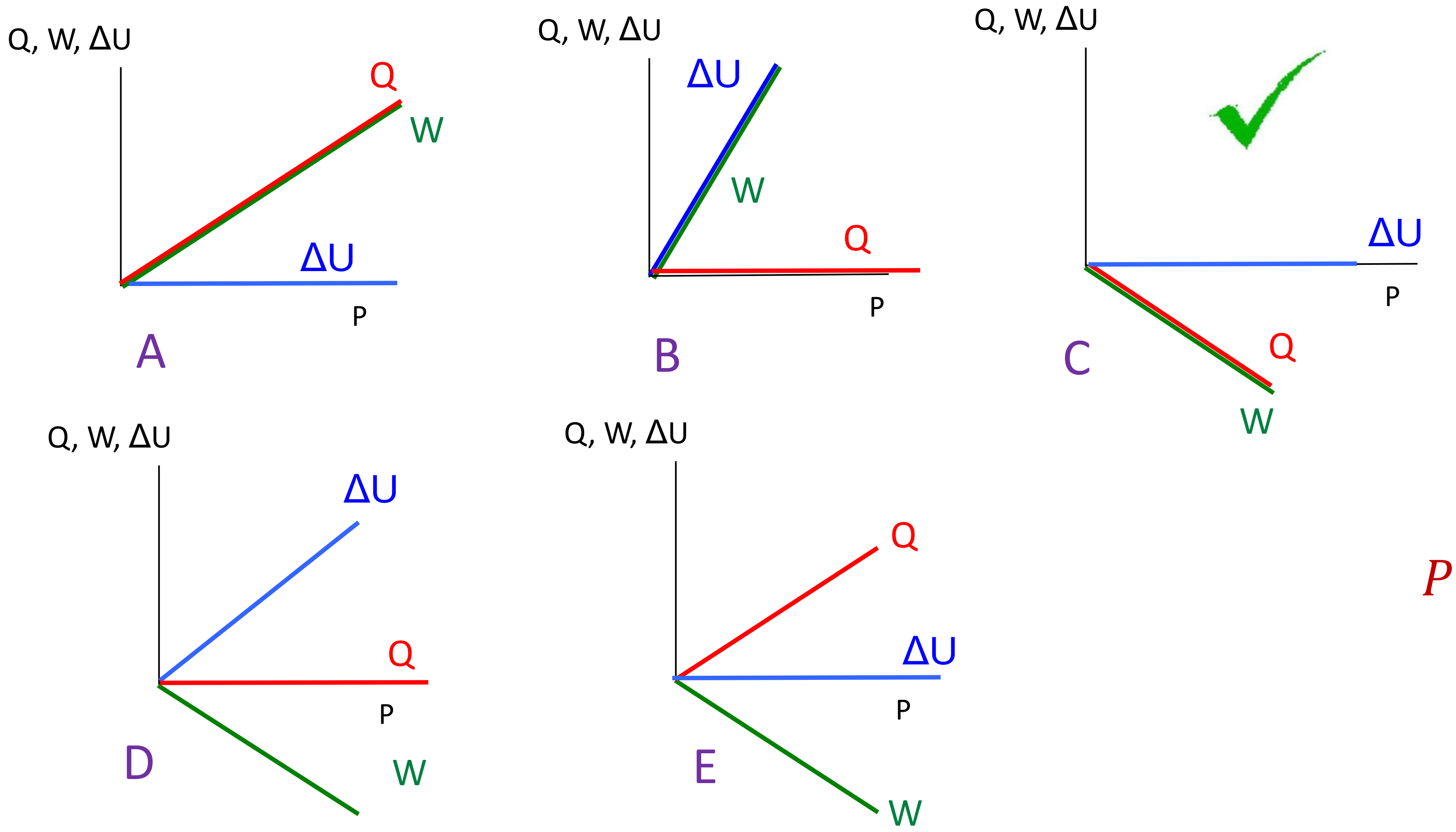
$P\uparrow \Rightarrow V\downarrow \Rightarrow ?$

$Q = W$

$0 = \underbrace{\Delta U}_{\Delta T} = Q - W$

➢ isothermal: T constant

Q: Given an isothermal process for an ideal gas, which graph best represents the heat transfer Q, work W, and internal energy ΔU, as the pressure increases?



$$\Delta U = nC_v\Delta T = 0$$

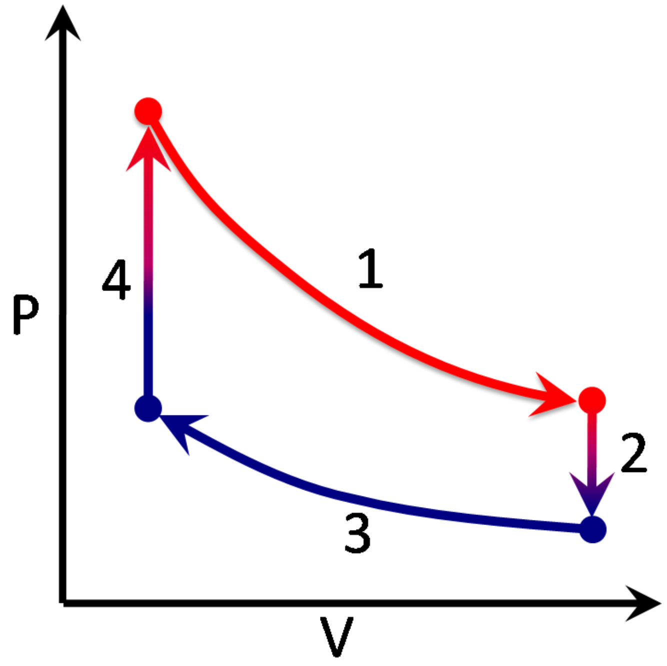
$$Q = W$$

$P \propto {}^{1}/_{V}$ so P increasing means V decreasing and $W < 0$

➢ isothermal: T constant

Q: In the picture, process 1 and 3 are isothermal. During how many of the four processes does (positive) heat flow into the gas?

$Q = \Delta U + W$ $\qquad PV = nRT$



4: $\Delta V = 0 \rightarrow W = 0 \qquad Q = \Delta U$

$P\uparrow \rightarrow T\uparrow \rightarrow \Delta U\uparrow \rightarrow Q\uparrow$

1: $\Delta U = 0 \rightarrow Q = W \Rightarrow Q > 0$

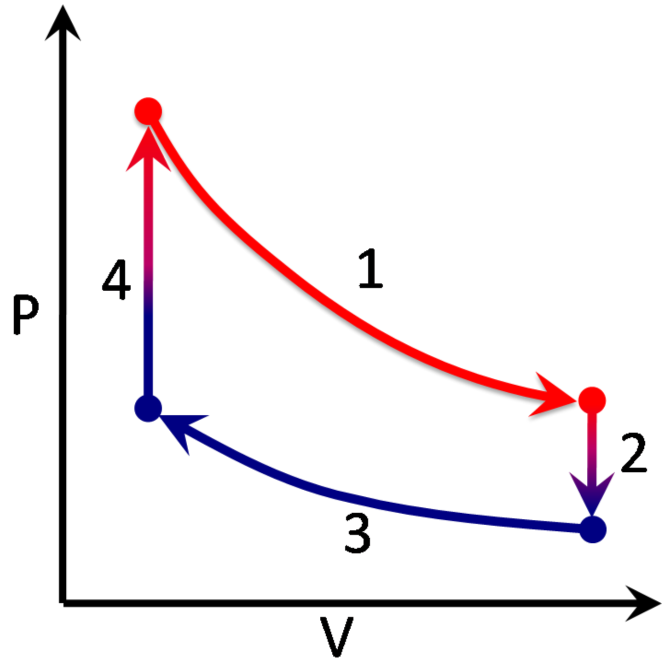
expanding $\rightarrow W > 0$

$\cancel{P\Delta V}$

A. 0
B. 1
C. 2
D. 3
E. 4

➢ isothermal: T constant

Q: In the picture, process 1 and 3 are isothermal. During how many of the four processes does (positive) heat flow into the gas?



A. 0
B. 1
C. 2
D. 3
E. 4

4: isochoric

$W = 0$ so $Q = \Delta U = nC_v\Delta T$

Positive Q since $P\uparrow$ implies $T\uparrow$ at constant V

1: isothermal

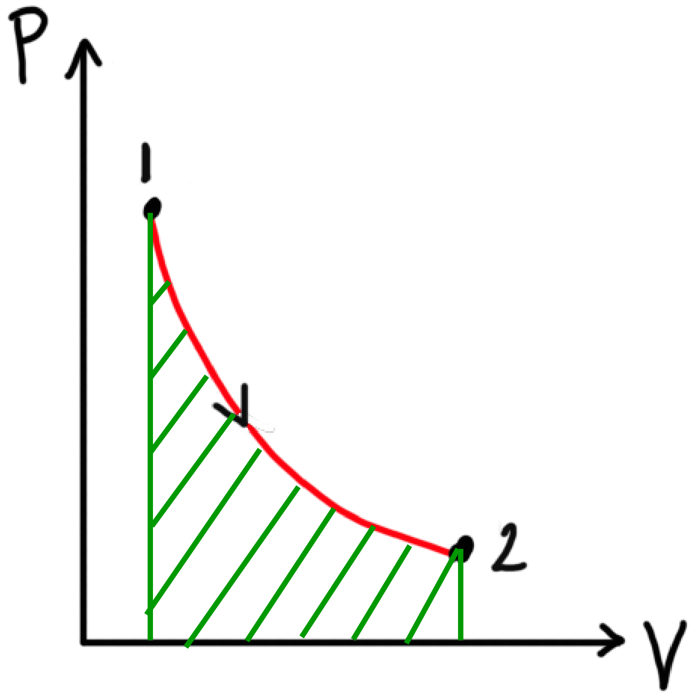
$\Delta U = 0$ so $Q = W > 0$ since expanding

2 & 3:

These are the reverse of 4 & 1, so $Q < 0$

- isothermal: T constant

Work for constant temperature



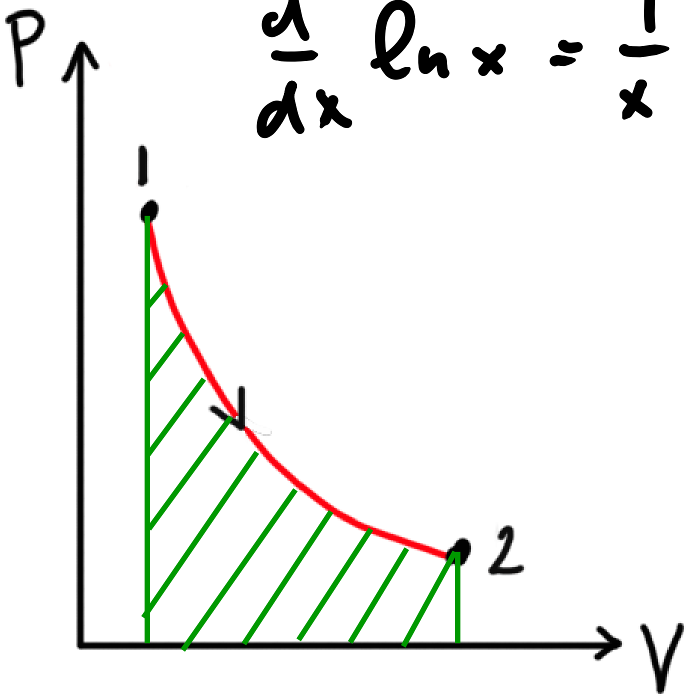
1) Find $P(V)$

2) Find $F(V)$ with $\frac{dF(V)}{dV} = P(V)$

3) Calculate $W = F\left(V_f\right) - F(V_i)$

$$W = \int_{V_i}^{V_f} P(V)\, dV$$

Work for constant temperature



$$\frac{d}{dx} \ln x = \frac{1}{x}$$

1) Find $P(V)$

Ideal Gas Law gives: $P(V) = \frac{nRT}{V}$ ← const

2) Find $F(V)$ with $\frac{dF(V)}{dV} = P(V)$

Can choose: $F(V) = nRT \ln(V)$

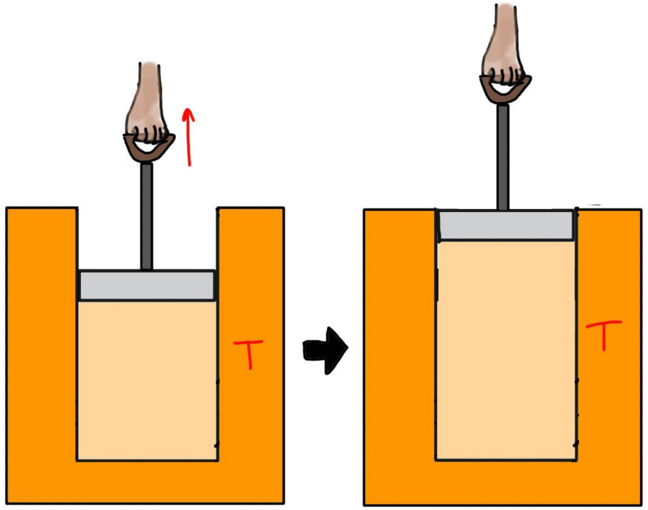
3) Calculate $W = F(V_f) - F(V_i)$

Get: $W = nRT \ln(V_f) - nRT \ln(V_i)$

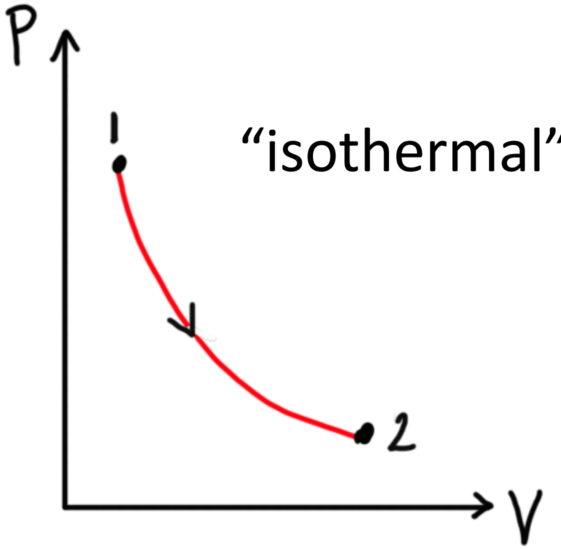
$$W = \int_{V_i}^{V_f} P(V)\, dV$$

$$\boxed{W = nRT \ln\left(\frac{V_f}{V_i}\right)}$$

Constant Temperature: Summary



- Ideal Gas Law $\Longrightarrow PV = nRT$
  - ➢ $PV = \text{const}$ so $P_1V_1 = P_2V_2$ $\quad (P \propto 1/V)$
- $\Delta U = Q - W$
  - ➢ $\Delta U = 0$
  - ➢ $Q = W =$ area under curve $= nRT \ln\left(\frac{V_f}{V_i}\right)$



"isothermal"

➢ isothermal: T constant