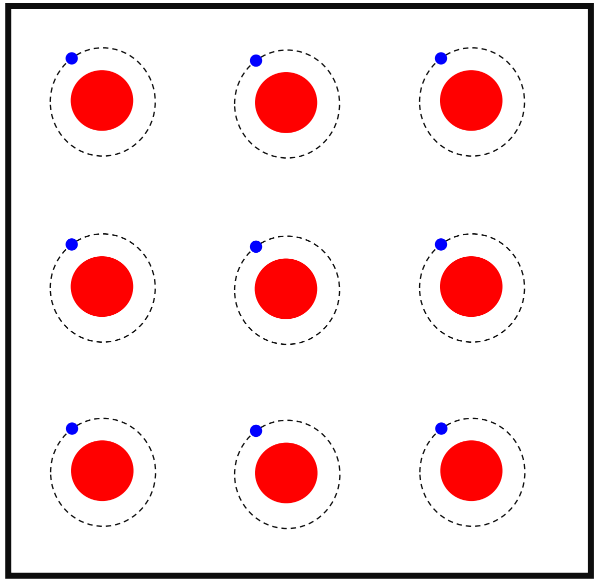
Lecture 23.

Electric properties of dielectrics. Polarization.

- Text: Ch 24.4 – 6

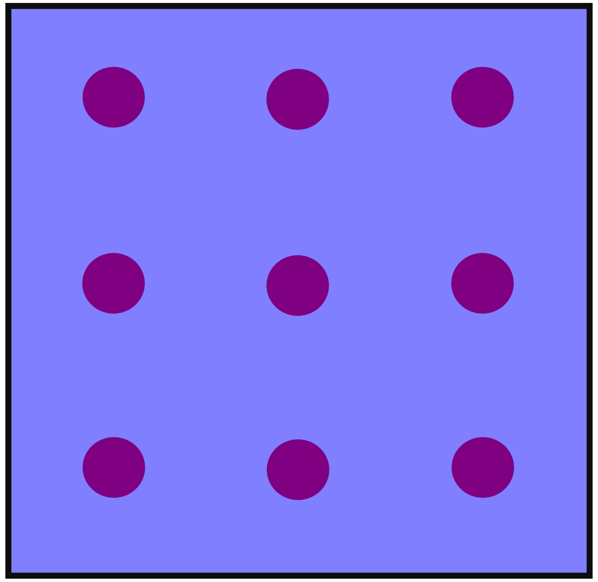
Insulator / Dielectric



- All electrons are tightly bound to their parent atoms. They cannot travel away from them.
- No charge transfer!

Conductor

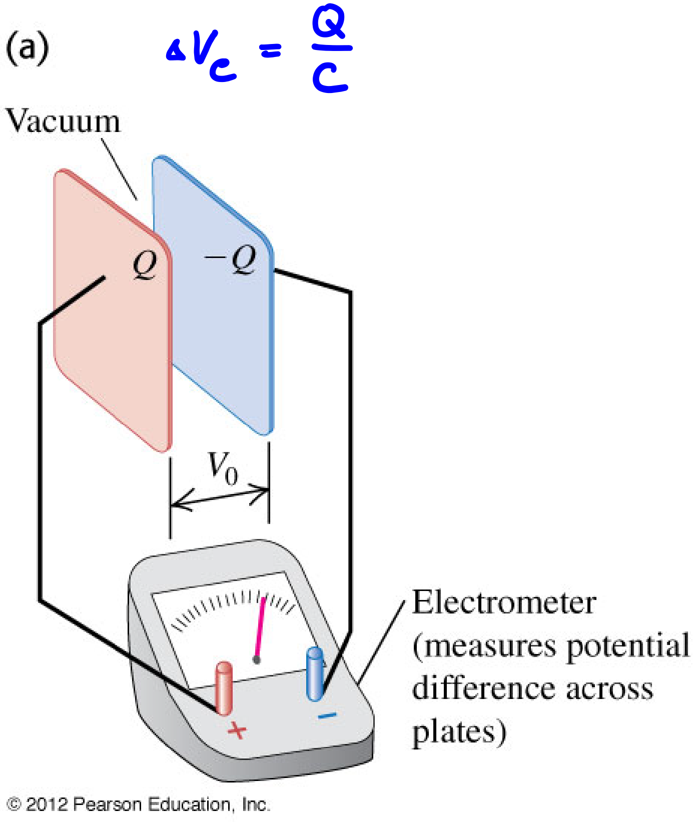
$m_{ion} \gg m_e$

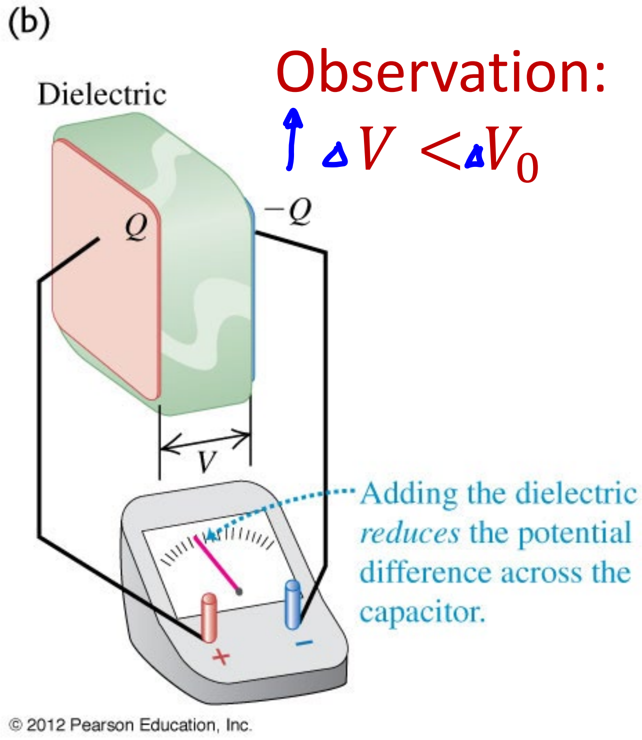


- Electrons $(-)$ are mobile (they are light!)
  - ➢ "Sea of electrons"
- Ions $(+)$ are fixed (they are heavy!)
  - ➢ "Ionic lattice"

Dielectrics do interesting things

$\Delta V_c = \frac{Q}{C}$





Observation:

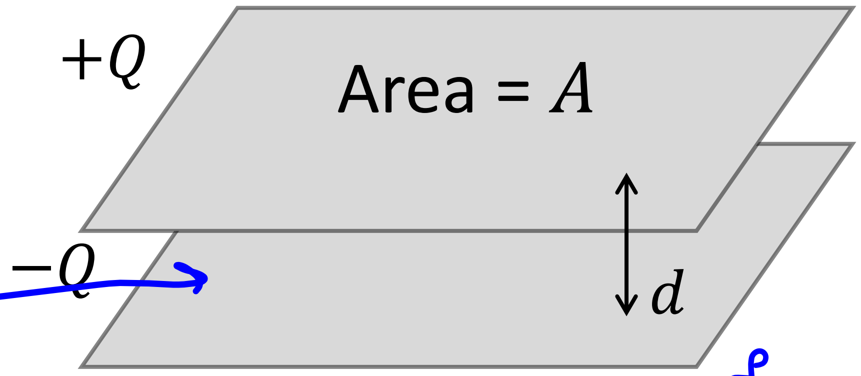
$\uparrow \Delta V < \Delta V_0$

Note that here $Q = \text{const}$ (charges do not have any place to go from the plates)

Parallel Plate Capacitor: Capacitance (Recap)

- Inside the gap region:

$$E_+ = \frac{\sigma}{2\varepsilon_0} \downarrow \quad \& \quad E_- = \frac{\sigma}{2\varepsilon_0} \downarrow$$



- Superposition principle: $\vec{E} = \vec{E}_+ + \vec{E}_-$

$$E = \frac{\sigma}{\varepsilon_0} \downarrow$$

$$\Delta V_c = -\oint \vec{E} \cdot d\vec{s}$$

$$\boxed{Q = C \Delta V_c}$$

$$\sigma = Q/A$$

- Hence, $\Delta V = Ed = \frac{\sigma d}{\varepsilon_0} = \frac{Q}{A}\frac{d}{\varepsilon_0}$

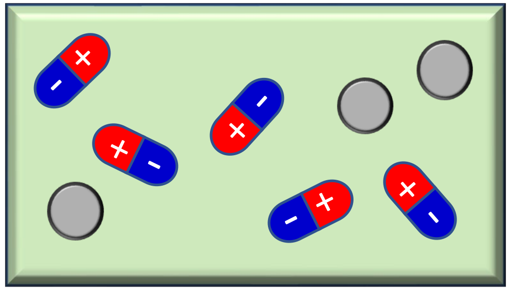
- Therefore: $\boxed{C_{\parallel} = \frac{Q}{\Delta V} = \frac{A\varepsilon_0}{d}}$

- C depends only on the geometry!

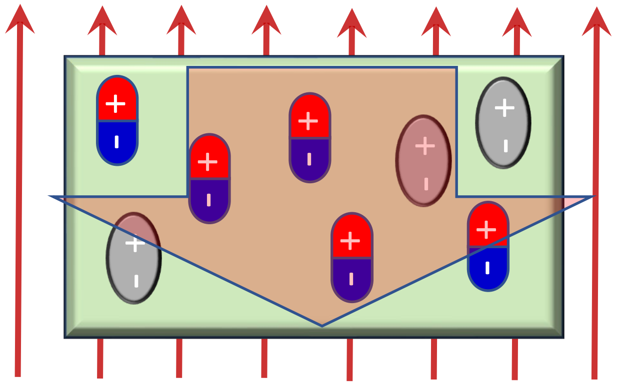
- How will this picture change if we fill the capacitor with a dielectric?

Dielectric materials in external electric field: Polarization

$E_{\text{ext}} = 0$

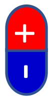


$\vec{E}_{\text{ext}}$



- This dielectric is made of:
  - ➢ Polar molecules (tiny dipoles) 
  - ➢ Atoms / non-polar molecules 

- They are randomly oriented. The average electric field from the dipoles is zero, and the atoms are neutral

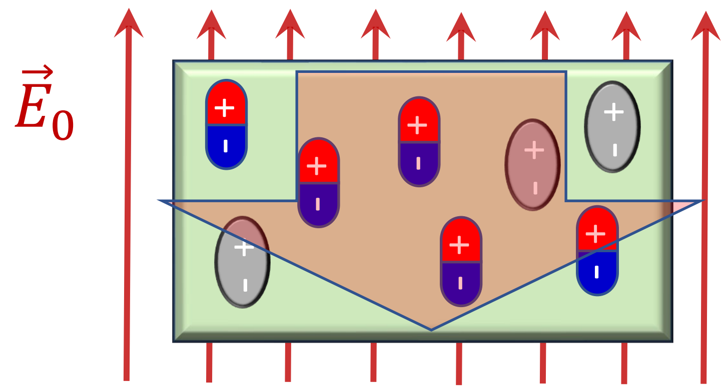
- Effect of the external field:
  - ➢ Polar molecules align with the field 
  - ➢ Atoms get polarized, and also align with the field 

- This alignment creates in internal electric field opposite to the external field (from + to -)

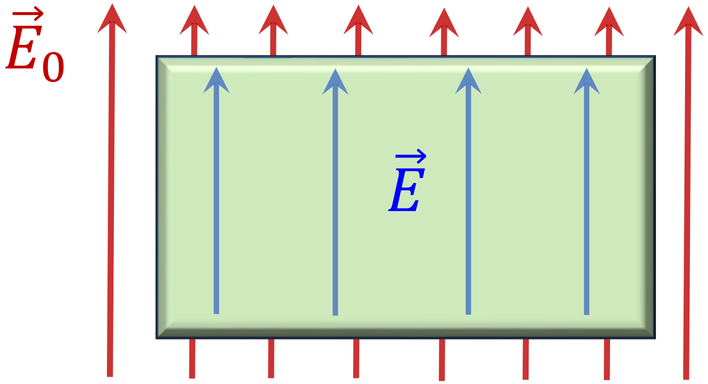
- $\vec{E}_{\text{net,diel}} = \vec{E}_{\text{ext}} + \vec{E}_{\text{int}} \quad < \quad \vec{E}_{\text{ext}} \qquad \Rightarrow$

- Polarization weakens external E-field inside a dielectric!

Dielectric materials in external electric field: Polarization



- Due to polarization of dielectric, the net electric field inside it ($\vec{E}$) is always less than what it would have been without the dielectric ($\vec{E}_0$), i.e. in empty space

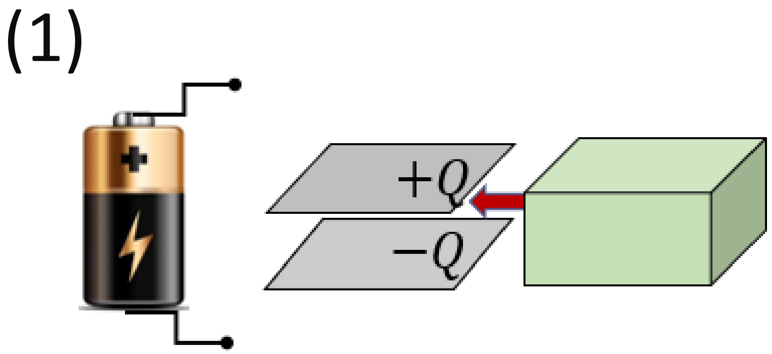


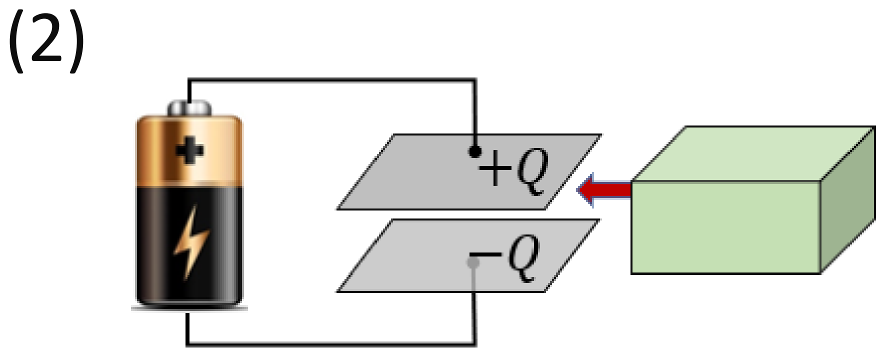
- This can be expressed as

$$E = \frac{E_0}{K}$$

with $K > 1$ being a material-dependent coefficient (dielectric constant)

Q: You have two identical capacitors. Cap 1 has been fully charged and then disconnected from the battery (=> it carries a fixed charge), while Cap 2 is always connected to a battery (=> it has a fixed voltage across its plates). You stick a dielectric into each. What will happen with the electric energy stored in each of the two capacitors?





A. U_1 goes up, U_2 goes up.

B. U_1 goes up, U_2 goes down.

C. U_1 goes down, U_2 goes up.

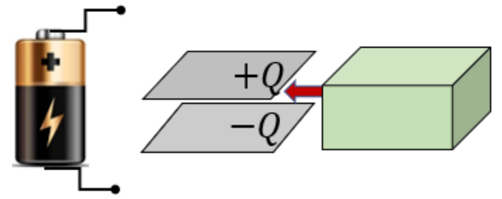
D. U_1 goes down, U_2 goes down.

E. They don't change.

$$U_c = \frac{Q^2}{2C} = \frac{C\,\Delta V_c^2}{2}$$

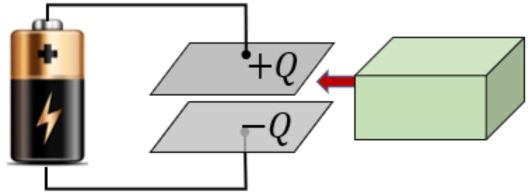
PHYS 158 Quest: Push a dielectric into a capacitor!

a) A parallel-plate capacitor is fully charged by a battery and then disconnected from it. A dielectric is then inserted into the capacitor. How will all these quantities change?



(1)

b) Same for capacitor which gets filled with a dielectric while it is still connected to a battery.



(2)

A. Increase
B. Decrease
C. Stays the same
D. Not yet there
E. Don't know where to start

$$U = \frac{Q^2}{2C} = \frac{C(\Delta V)^2}{2}$$

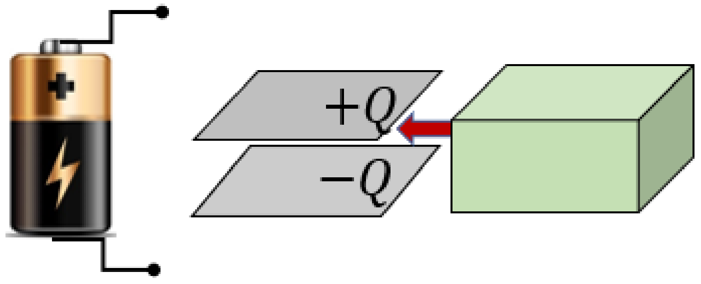
$$C = \frac{\varepsilon_0 A}{d}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

$$Q = C\Delta V$$

$$\Delta V = Ed$$

Dynamical simulation of what happens inside a capacitor:
https://phet.colorado.edu/en/simulation/capacitor-lab



$\Delta V = E \cdot d$

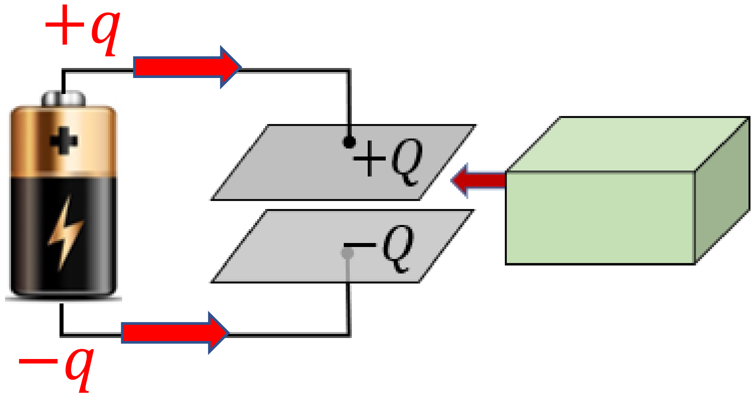
$\rightarrow Q = C\,\Delta V$

$U = \dfrac{Q^2}{2C} = \dfrac{C\,\Delta V^2}{2}$

A B

A. #1

B. #2



$$Q = \text{const}$$

$$E = \frac{E_0}{K} \downarrow \quad \text{(polarization)}$$

$$\Delta V = Ed = \frac{E_0}{K} d = \frac{\Delta V_0}{K} \downarrow$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{(\Delta V_0/K)} = KC_0 \uparrow$$

$$U_1 = \frac{Q^2}{2C} = \frac{Q^2}{2KC_0} = \frac{U_0}{K} \downarrow$$

$$Q = Q_0 K \uparrow$$

$$E = \frac{\Delta V}{d} = \text{const} \qquad \text{(polarization ???)}$$

$$\Delta V = \text{const}$$

$$C = \frac{Q}{\Delta V} = \frac{Q_0 K}{\Delta V_0} = KC_0 \uparrow$$

$$U_2 = \frac{C\Delta V^2}{2} = \frac{KC_0\,\Delta V^2}{2} = KU_0 \uparrow$$

Q: A parallel plate capacitor has capacitance $C_0 = 2\,pF$ when there is vacuum between the plates. If Silicon dioxide, a dielectric with $K = 3.9$, is inserted between the plates the capacitance becomes…

A. C = 0.5 pF

B. C = 1.95 pF

C. C = 7.8 pF

D. C = 12.5 pF

E. Depends on the experiment

Q: A parallel plate capacitor has capacitance $C_0 = 2\,pF$ when there is vacuum between the plates. If Silicon dioxide, a dielectric with $K = 3.9$, is inserted between the plates the capacitance becomes...

- We have seen that in both cases ($Q = const$ and $V = const$), the capacitance of a capacitor increased after the dielectric has been inserted: $C_0 \rightarrow C = KC_0$
- In general, capacitance is determined by the geometry and the material of the capacitor, and does not depend on the conditions of the experiment (what we keep constant).

A. C = 0.5 pF

B. C = 1.95 pF

C. C = 7.8 pF

D. C = 12.5 pF

E. Depends on the experiment

$$Q = C\,\Delta V$$

$$\sigma A = C\,Ed = C\frac{E_0}{K}d = C\frac{d}{K}\frac{\sigma}{\varepsilon_0}$$

Polarization

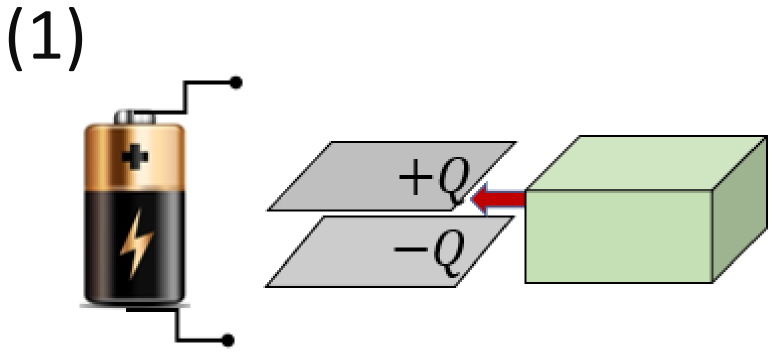
$$E_0 = \sigma/\varepsilon_0$$

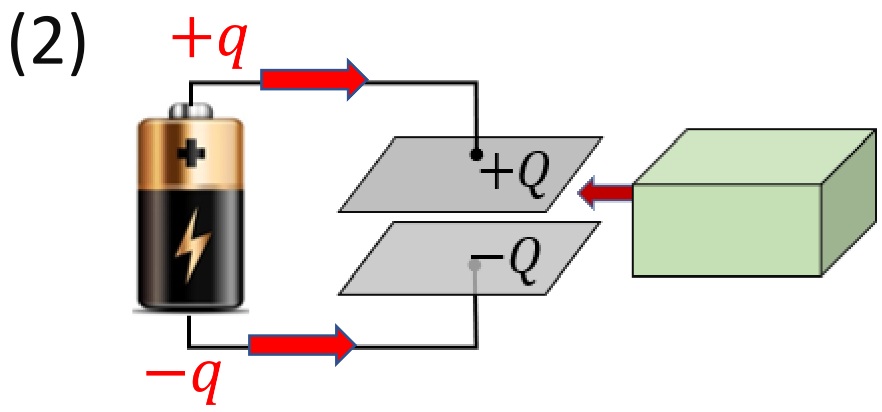
Result:

$$\boxed{C = \frac{(K\varepsilon_0)A}{d} = \frac{\varepsilon A}{d}}$$

with $\varepsilon = K\varepsilon_0$ being dielectric permittivity

Making sense





- With dielectric, the capacitance of a capacitor increases: $C_0 \;\rightarrow\; C = KC_0$
  - ➢ Practical applications!

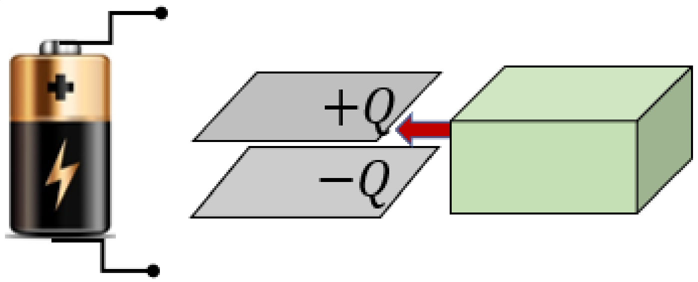
- We can write: $C = \dfrac{K\varepsilon_0 A}{d} = \dfrac{\varepsilon A}{d}$ with $\varepsilon = K\varepsilon_0$ being dielectric permeability

➢ In case (1), $U \downarrow$. Where did the energy go?

➢ In case (2), $U \uparrow$. Where did the energy come from?

Making sense

(1)



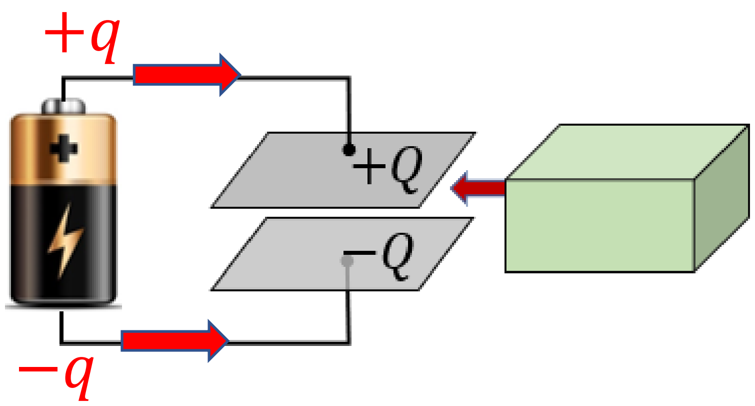
- $U \downarrow$. Where did the energy go?

A: It is spent in different ways.

- Aligning tiny dipoles with E-field
- Pulling the dielectric into the capacitor due to fringe effects!

$$F_x = -\frac{dU}{dx}$$

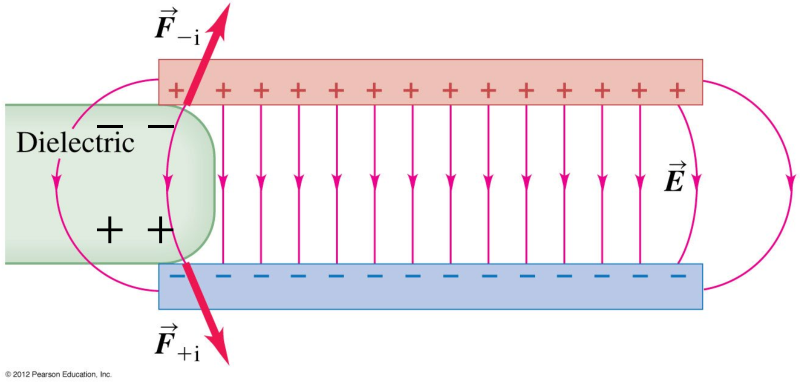
(2)

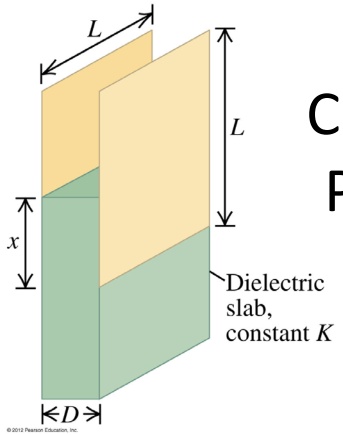


- $U \uparrow$. Where did the energy come from?

A: From extra charges pulled out of the battery.

- Electric field inside the capacitor polarizes it, and then interacts with the induced charge distribution





Challenge Problem 24.72

Electric energy and its connection to forces

- Consider a parallel plate capacitor filled with air (battery disconnected, $Q = \text{const}$):

$$U_0 = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

- Total Stored Energy in the E-field between the capacitor plates

$$F_x = -\frac{dU_0}{dx}$$

$$\text{Energy Field Density} = \frac{\text{Energy}}{\text{Volume}}$$

$$\text{Volume between plates} = Ad$$

$$C = \frac{A\varepsilon_0}{d}$$

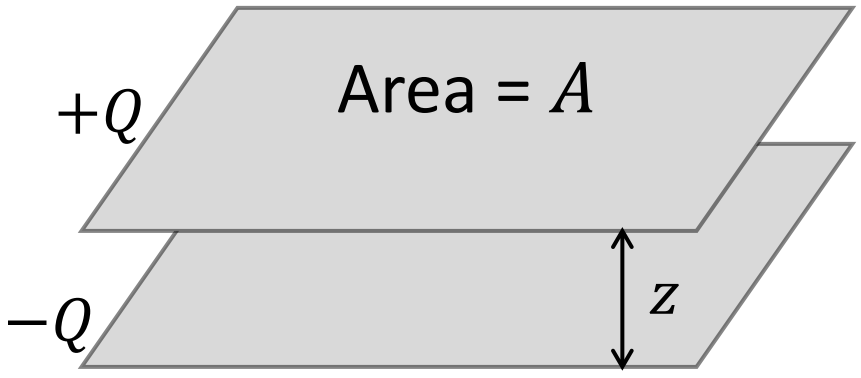
$$u_0 = \frac{U_0}{Ad} = \frac{1}{2}\left(\frac{A\varepsilon_0}{d}\right)V^2\frac{1}{Ad} = \frac{\varepsilon_0}{2}\left(\frac{V}{d}\right)^2 = \boxed{\frac{1}{2}\varepsilon_0 E_0^2 = u_0}$$

- With a dielectric inside: $\varepsilon_0 \to \varepsilon = K\varepsilon_0, \quad E_0 \to E = \frac{E_0}{K}$

$$u_0 \to \boxed{u = \frac{1}{2}\varepsilon E^2} = \frac{u_0}{K}$$

Electrostatic attraction: Simple example

Q: A parallel-plate capacitor has a plate area A and a plate separation of z. The charge on each plate has a magnitude Q. There is no battery connected to the plates.

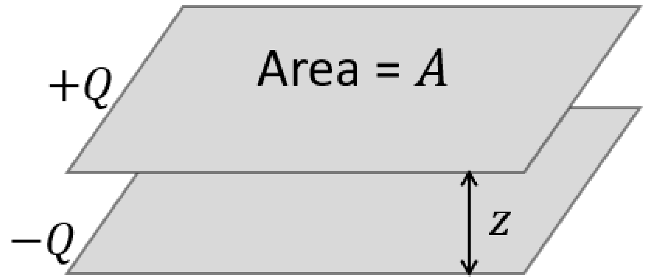


Find the total force acting on the top plate.

$\vec{F} = q_{\pm} \vec{E}$ difficult!

- Hint: Coulomb law, $\vec{F} = q\vec{E}$, is not very useful here (it will require a lot of integration).
- Use $F_z = -\dfrac{dU}{dz}$ instead!

Electrostatic attraction: Simple example



- Energy stored in a capacitor: $U(z) = \dfrac{Q^2}{2C}$

- For a parallel plate capacitor
  (z is the distance between the plates) $C(z) = \dfrac{A\varepsilon_0}{z} = \dfrac{A}{4\pi k\, z}$

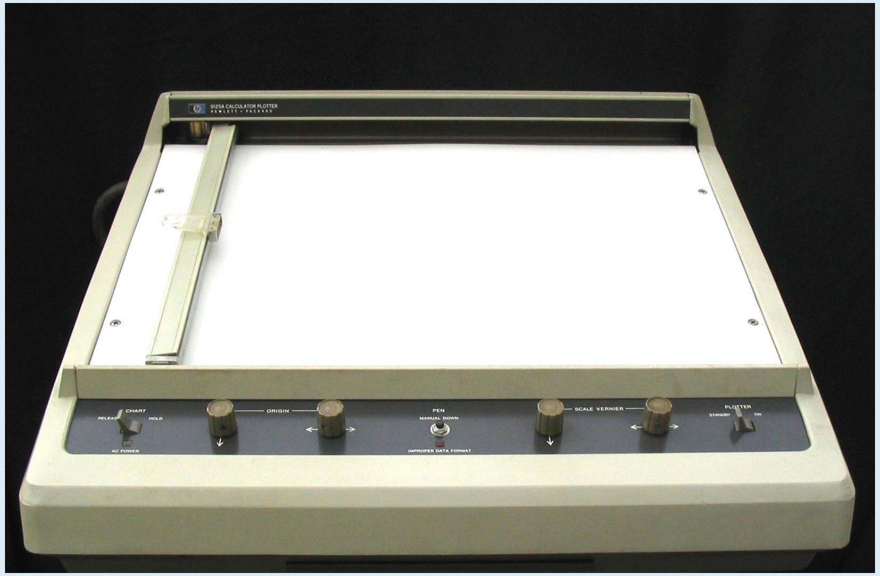
- Hence for a fixed charge the energy stored in a parallel plate capacitor is:

$$U = \frac{Q^2}{2C} = \frac{Q^2}{2}\frac{4\pi k z}{A} = \frac{Q^2\, z}{2A\varepsilon_0}$$

- The force acting between the plates is: $F_z = -\dfrac{dU}{dz} = -\dfrac{1}{2}\, Q^2\, \dfrac{4\pi k}{A} = -\dfrac{Q^2}{2A\varepsilon_0}$

- Answer: Magnitude: $F = \dfrac{Q^2}{2A\varepsilon_0}$ Direction: attractive (positive & negative plates)

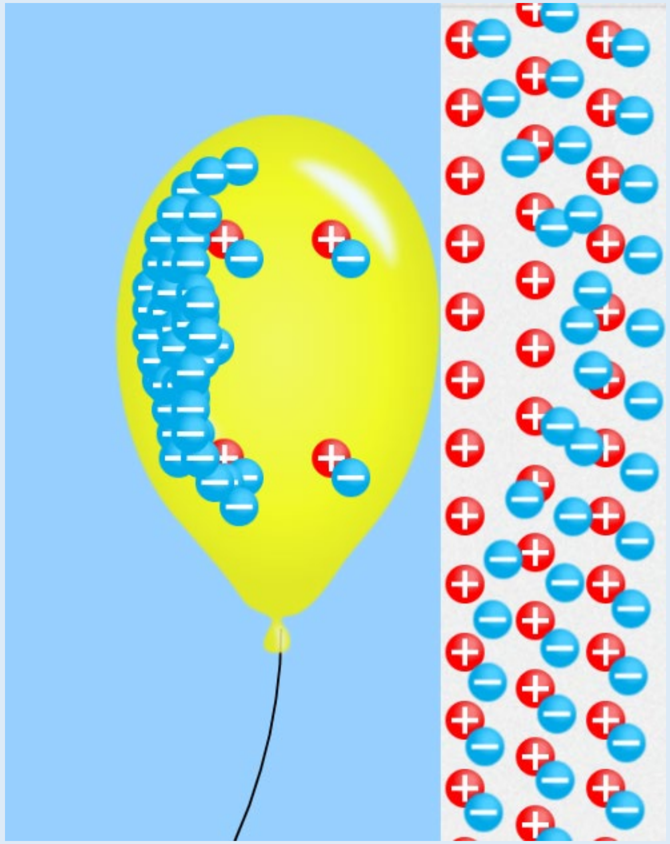
Electrostatic attraction: Examples

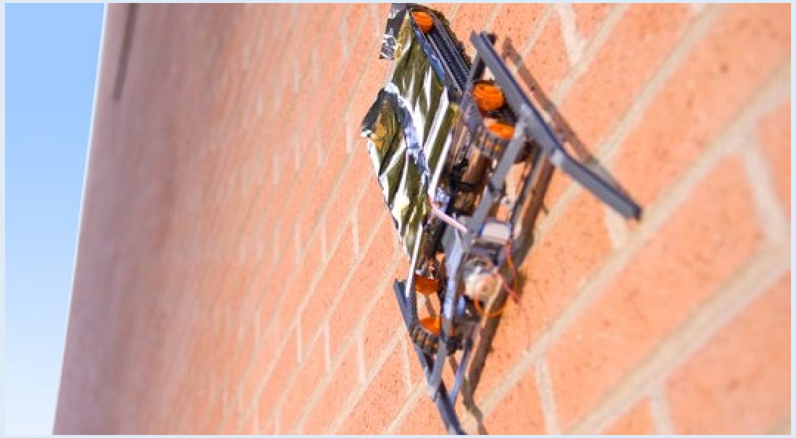


https://phet.colorado.edu/sims/html/balloons-and-static-electricity/latest/balloons-and-static-electricity_en.html

Paper plotter

DEMO !





Robot that can climb walls using electrostatic attraction

0.5 to 1.5 N / cm^2

https://www.youtube.com/watch?v=I4DHfNtZGts&amp=&lr=1

End of Electricity

Start Magnetism next week