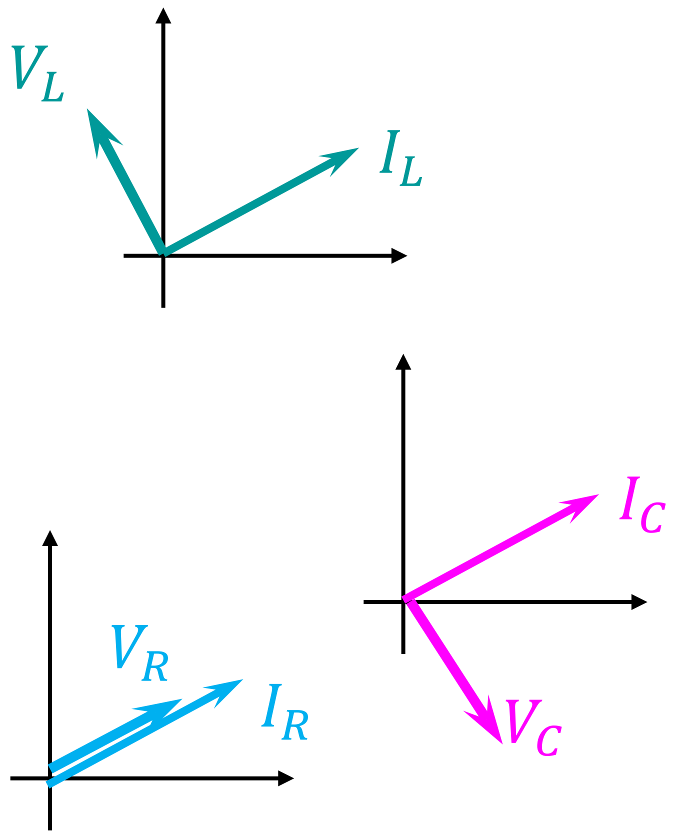
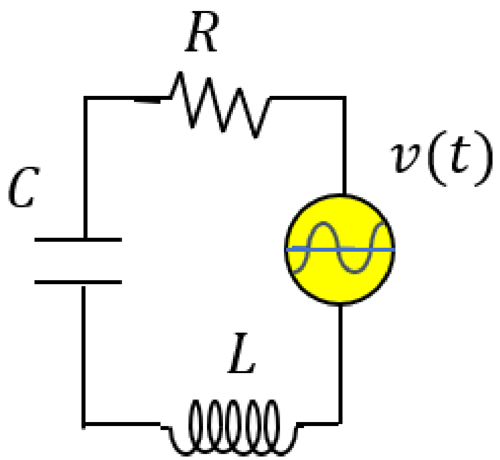
Lecture 13.

Parallel AC circuits.
Coulomb force.

Last Time

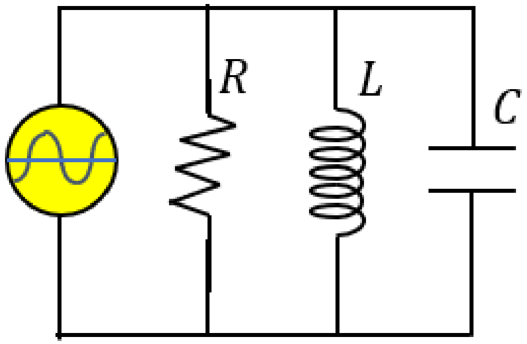
Phasors for AC circuit elements





Series:

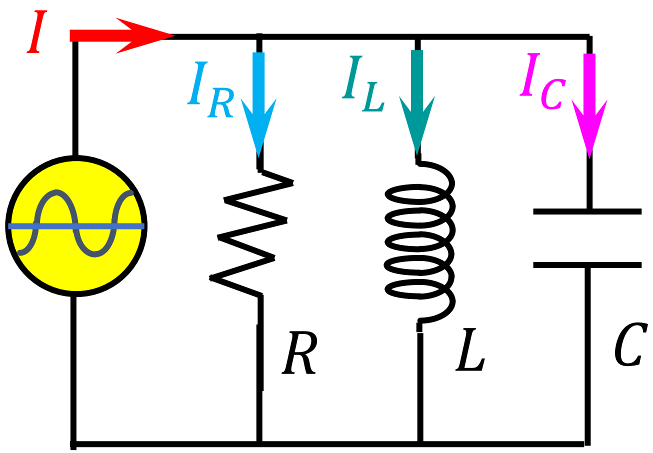
- Common current
- $v(t) = v_R(t) + v_L(t) + v_C(t)$



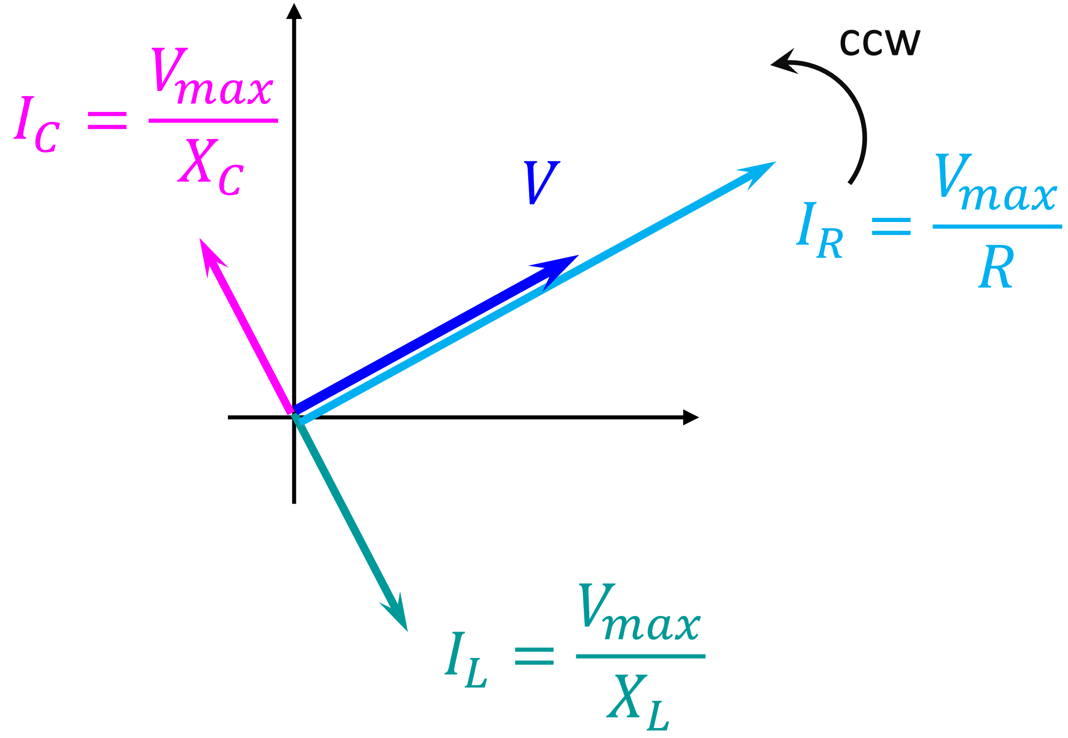
Parallel:

- Common voltage
- $i(t) = i_R(t) + i_L(t) + i_C(t)$

AC RLC parallel circuit

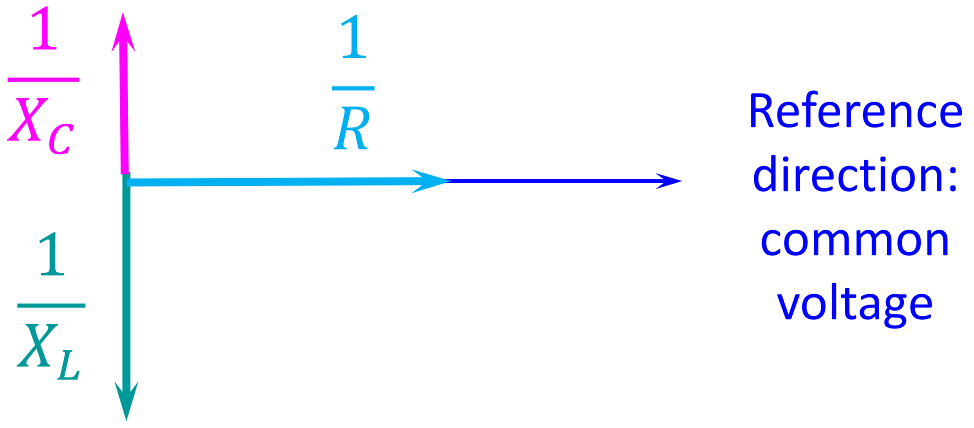


- Let's combine the elementary phasor pairs, taking into account that they have common voltage (same magnitude and direction of $\vec{V}$):



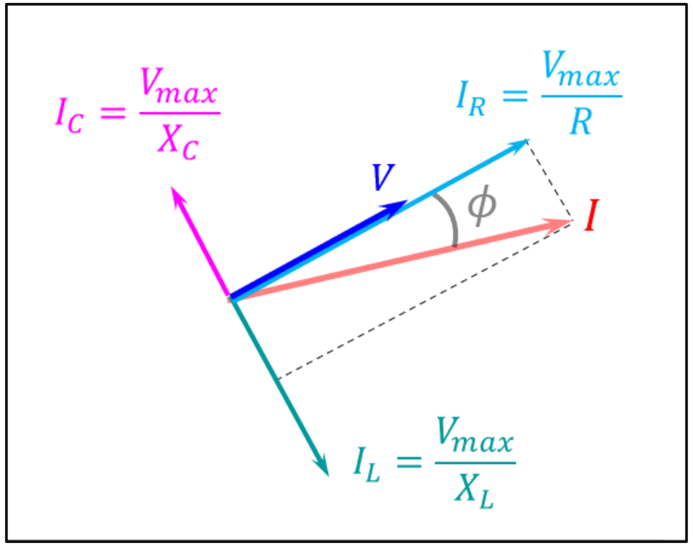
$$\vec{I} = \vec{I}_R + \vec{I}_L + \vec{I}_C$$

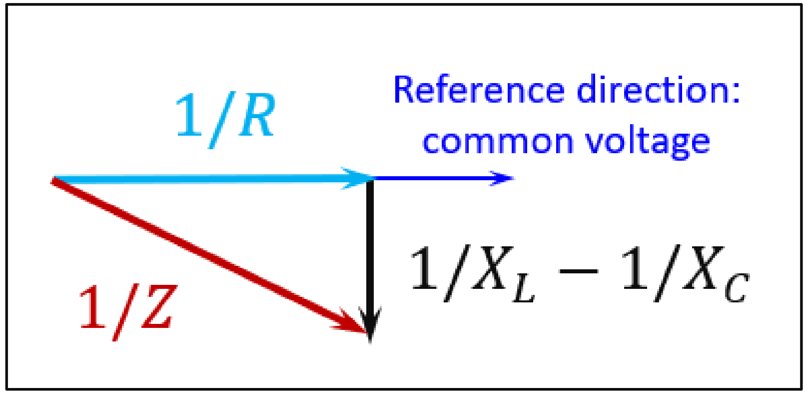
- Rescale (divide by common V_{max}), rotate:

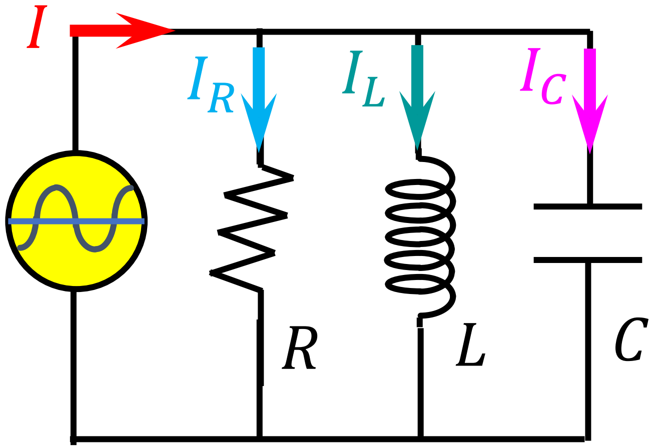


- Build the impedance triangle from reciprocal reactances!

AC RLC parallel circuit (impedance derived)







$$\vec{I} = \vec{I}_R + \vec{I}_L + \vec{I}_C$$

$$I^2 = I_R^2 + (I_L - I_C)^2$$

$$I = \sqrt{\left(\frac{\cancel{V}}{R}\right)^2 + \left(\frac{\cancel{V}}{X_L} - \frac{\cancel{V}}{X_C}\right)^2} = \frac{\cancel{V}}{Z}$$

- Hence:

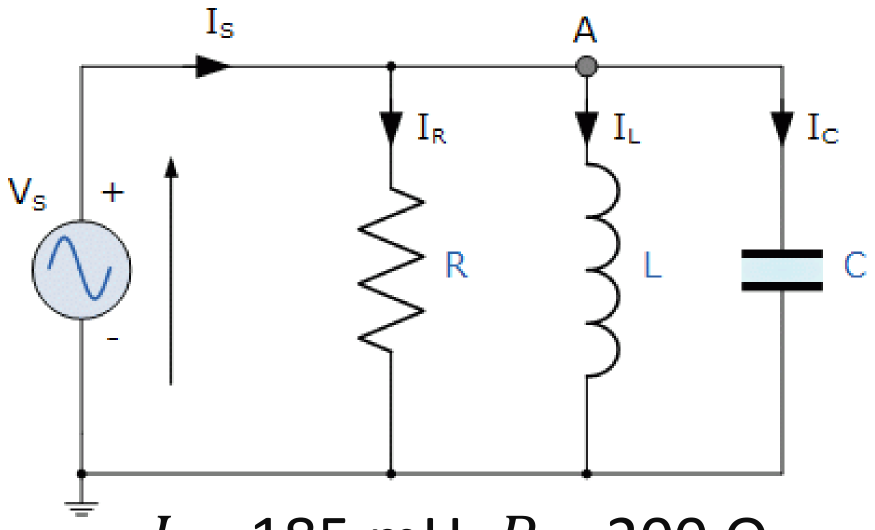
$$\frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

- $Z = Z(\omega)$:

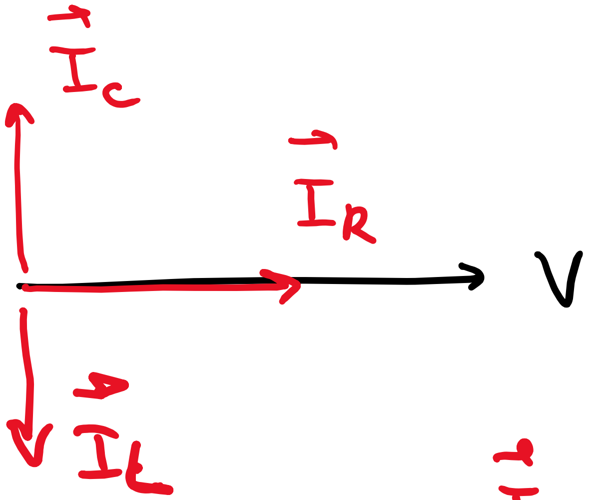
$$\frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

Q: An AC source with $V_{peak} = 150\ V$ and $f = 50\ Hz$ drives the following parallel RLC circuit.

What must be the capacitance C if the current is in phase with the source voltage? Pick the closest answer.



L = 185 mH, R = 200 Ω



A. 1 μF
B. 5 μF
C. 100 μF
D. 50 μF
E. 2 F

$$\vec{I} = \vec{I}_R + \vec{I}_c + \vec{I}_L$$

$$I_c = I_L$$

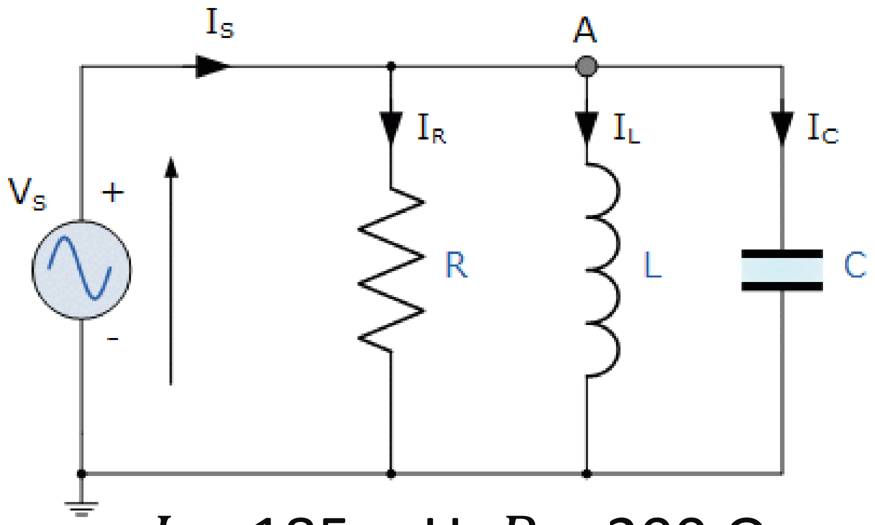
$$V_{max}/X_c = \frac{V_{max}}{X_c}$$

$$X_L = X_c$$

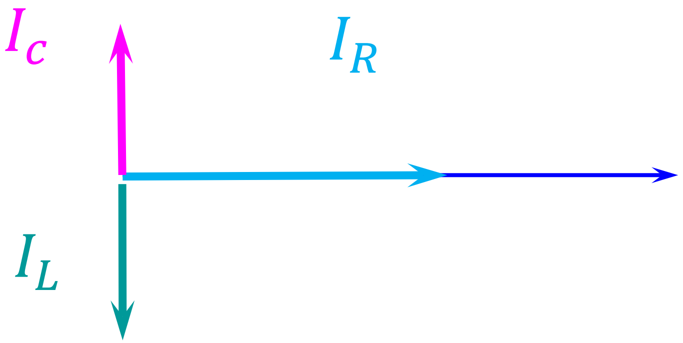
$$\omega L = \frac{1}{\omega C}$$

Q: An AC source with $V_{peak} = 150\ V$ and $f = 50\ Hz$ drives the following parallel RLC circuit.

What must be the capacitance C if the current is in phase with the source voltage? Pick the closest answer.



L = 185 mH, R = 200 Ω



$\phi = \widehat{(\vec{I}, \vec{V})}$

A. 1 µF

B. 5 µF

C. 100 µF

D. 50 µF

E. 2 F

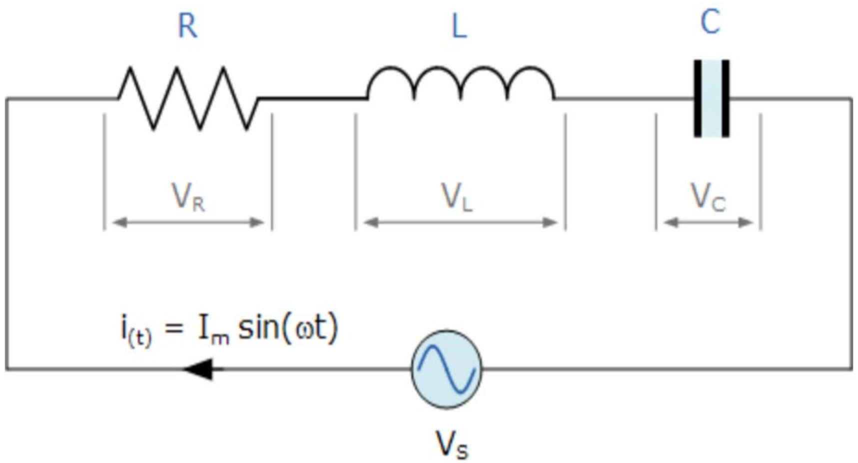
- For total current I to be parallel to V, we need $I_C = I_L$

- $\tan\phi = \frac{1/X_C - 1/X_L}{1/R} = \underline{0}$

- $X_C = X_L \quad \Rightarrow \quad C = \frac{1}{\omega^2 L}$

AC circuits summary

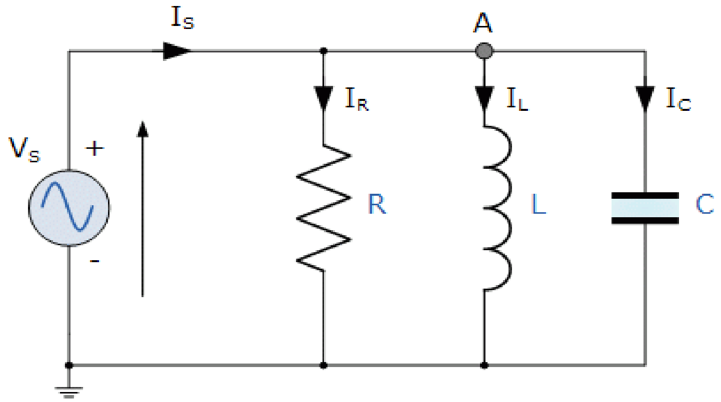
Series RLC



$$Z = \sqrt{R^2 + (X_L - X_C)}$$

$$\tan(\phi) = \frac{X_L - X_C}{R}$$

Parallel RLC

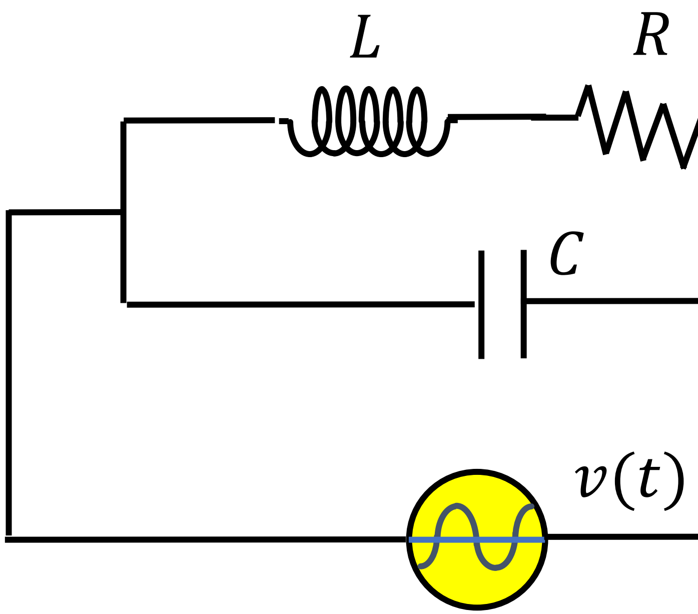


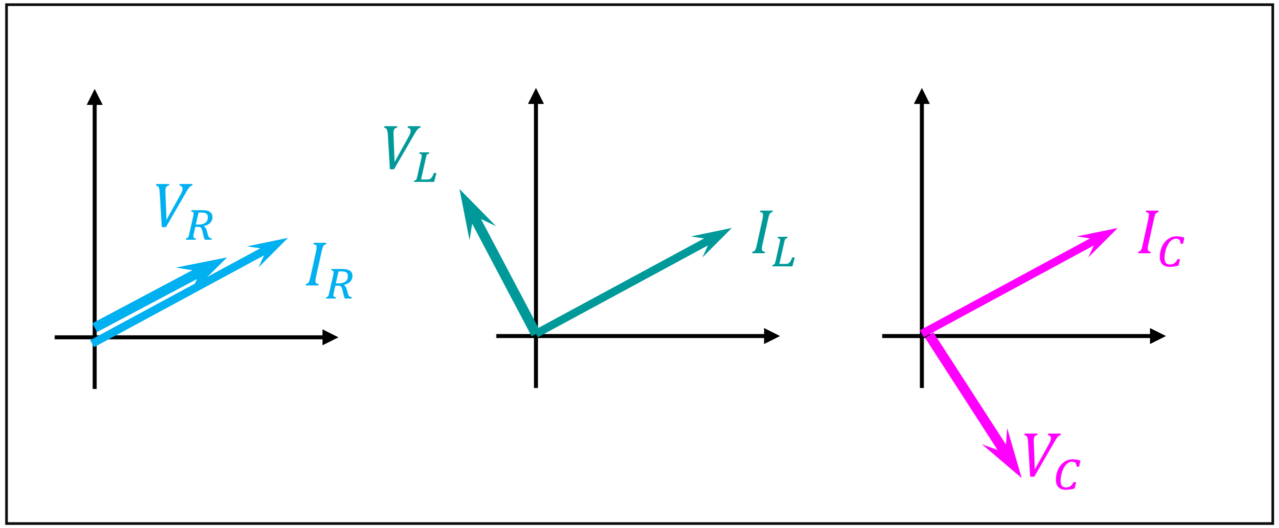
$$\frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

$$\tan(\phi) = \frac{1/X_C - 1/X_L}{1/R}$$

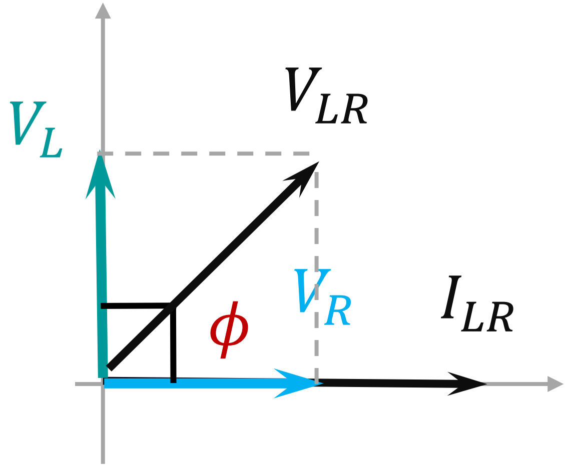
$$V = IZ, \quad V_R = I_R R, \quad I_L = I_L X_L, \quad I_C = I_C X_C, \quad X_L = \omega L, \quad X_C = 1/\omega C$$

More complex circuits:

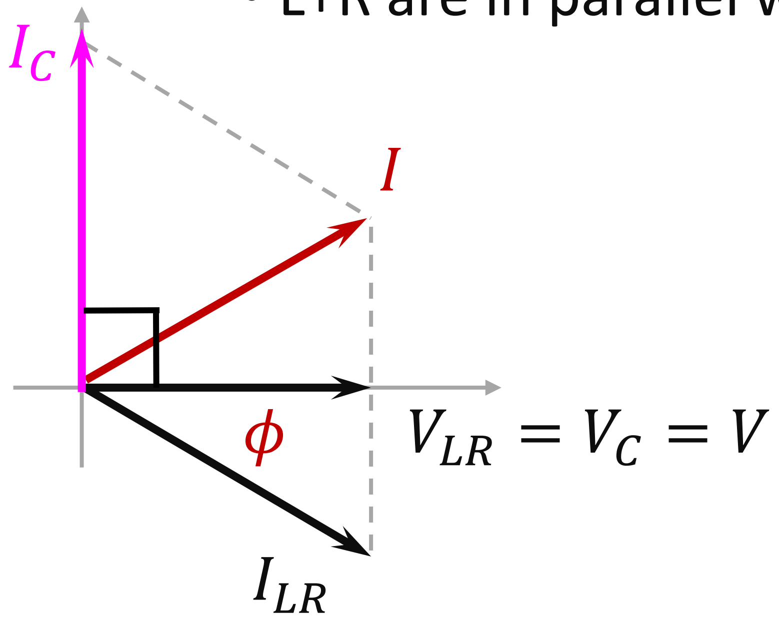




- L and R are in series:



- L+R are in parallel with C:



- Adding vectors / Using complex numbers

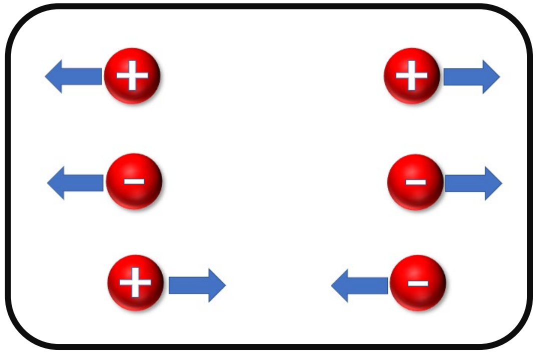
End of circuits

Electric force and Electric field

Text: Ch 21

- Ch 21.1 Electric charge, Conservation of charge
- Ch 21.2 Conductors and insulators, Charging by induction
- Ch 21.3 Coulomb Law
- Ch 21.4 Electric field
- Ch 21.5 Superposition of E-field, Field due to continuous charge distribution (ring of charge, charged line segment)
- Ch 21.6 Electric field lines
- Ch 21. 7 Electric dipole: force and torque

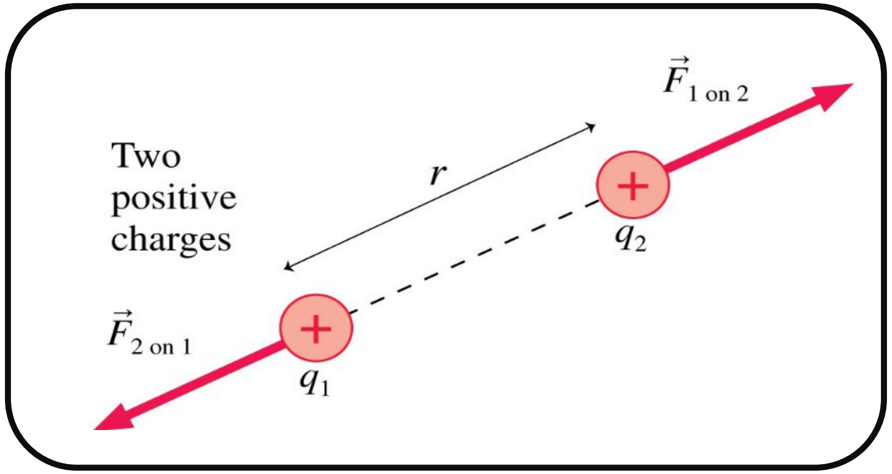
- Experiment:
  - ➢ Like charges repel
  - ➢ Unlike (opposite) charges attract



- Hence, there is a force between charged objects: long-range, distance-dependent

Coulomb Law (for two point charges):

$$F_{1\text{ on }2} = F_{2\text{ on }1} = k\frac{|q_1||q_2|}{r^2}$$



- Larger charges => larger force
- Smaller distance => larger force

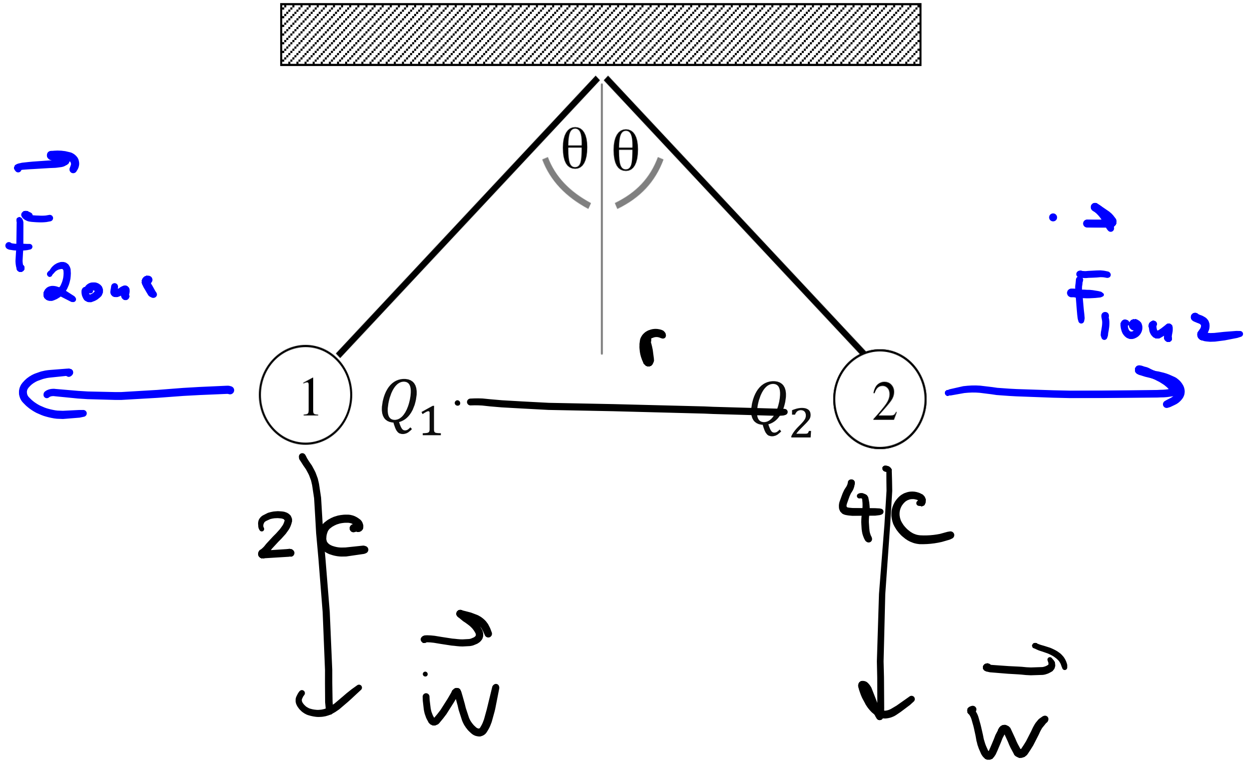
- Electrostatic constant: $k = 9 \times 10^9$ N m^2/C^2
- Vacuum permittivity: $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 / N m^2

$$k = \frac{1}{4\pi\varepsilon_0}$$

Q: Two equal mass small pith balls are charged, and hang on strings as shown:

$$F_{1\,\text{on}\,2} = k\,\frac{(2C)(4C)}{r^2}$$

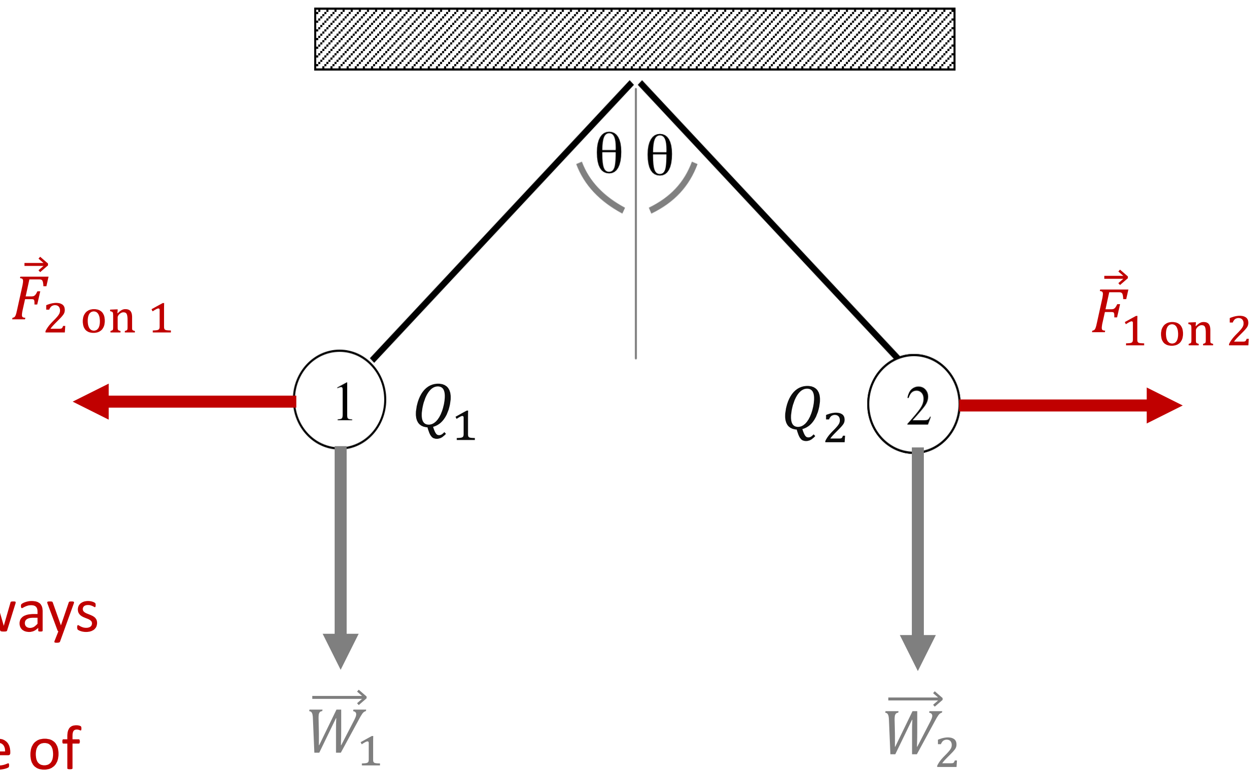
$$F_{2\,\text{on}\,1} = k\,\frac{(4C)(2C)}{r^2}$$



What can you say about the magnitudes of the charges Q_1 and Q_2 on the two balls?

A. $|Q_1|$ must equal $|Q_2|$
B. $|Q_1|$ cannot equal $|Q_2|$
C. Can't tell -- not enough information.

Q: Two equal mass small pith balls are charged, and hang on strings as shown:



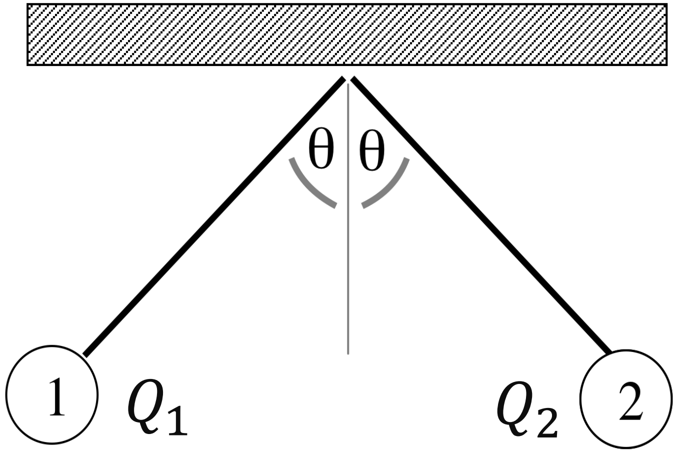
$$F_{1\text{ on }2} = F_{2\text{ on }1} = k\frac{|q_1||q_2|}{r^2}$$

- The two forces, $\vec{F}_{2\text{ on }1}$ and $\vec{F}_{1\text{ on }2}$, are always equal in magnitude (Newton's 3rd law).
- Both charges contribute to the magnitude of this mutual force.

What can you say about the magnitudes of the charges Q_1 and Q_2 on the two balls?

A. $|Q_1|$ must equal $|Q_2|$
B. $|Q_1|$ cannot equal $|Q_2|$
C. Can't tell -- not enough information.

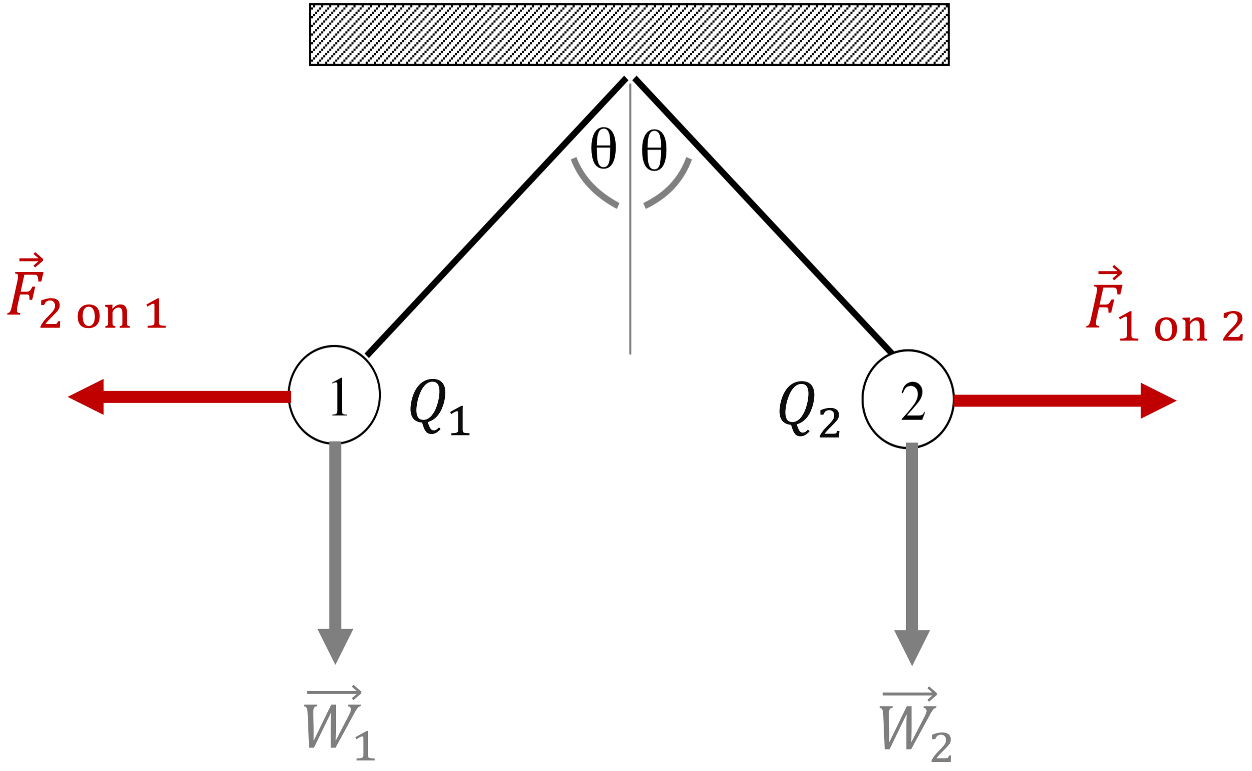
Q: Two equal mass small pith balls are charged, and hang on strings as shown:



What can you say about the signs of the charges Q_1 and Q_2 on the two balls?

A. Both must be "+"
B. Both must be "–"
C. Sign $Q_1 \neq$ Sign Q_2
D. Both charges must have the same sign (but we can't tell if they're both "+", or both "–")

Q: Two equal mass small pith balls are charged, and hang on strings as shown:



- Like charges repel (both + and −), unlike charges attract

What can you say about the signs of the charges Q_1 and Q_2 on the two balls?

A. Both must be "+"

B. Both must be "−"

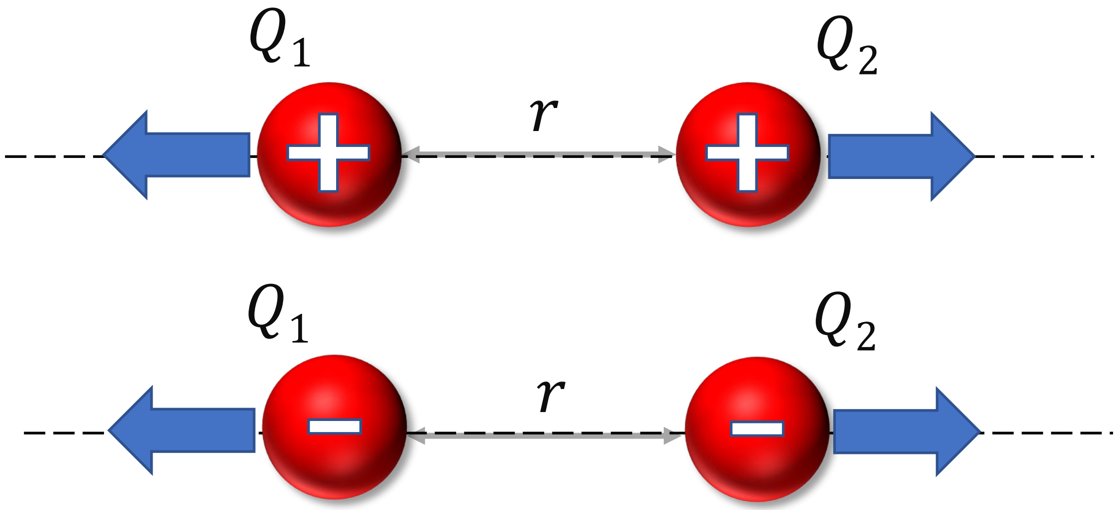
C. Sign $Q_1 \neq$ Sign Q_2

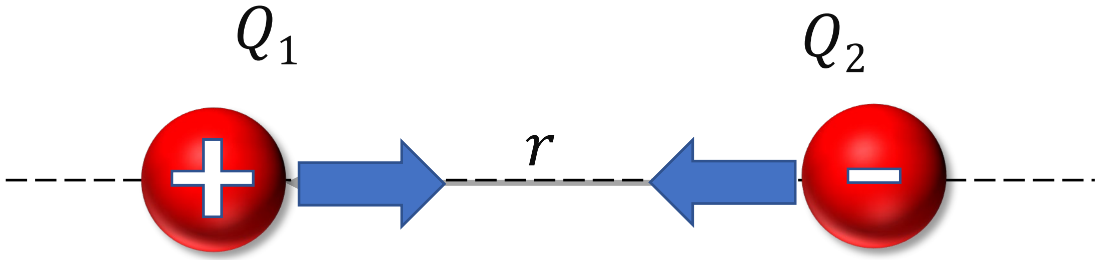
D. Both charges must have the same sign (but we can't tell if they're both "+", or both "−")

Electrostatic Attraction & Repulsion

$\cancel{\vec{F} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}}$

- How to compute the magnitude and the direction of the Coulomb force properly?





$$|\vec{F}_{12}| = |\vec{F}_{21}| = k\frac{|q_1||q_2|}{r^2}$$

1. Find the distance between the charges.

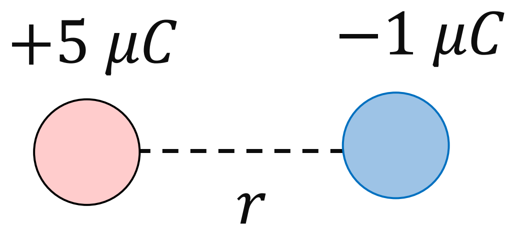
2. Draw a line passing through the two charges.

3. Draw the force $\vec{F}_{21}$ on Q_1 due to Q_2 with its tail at location 1, pointing either towards Q_2 or away from Q_2. Pick the direction using the rule “Like charges repel, unlike charges attract”

4. Repeat the procedure for $\vec{F}_{12}$.

Q: Two identical metal balls (with equal mass & radius) have charges of +5 μC and -1 μC. They are fixed in space, and each feels a force of magnitude F.

Now you bring them together so they touch, then you move them back to their original positions.



What is the magnitude of the new force they feel?

A. F (stays the same)

B. $\frac{4}{5}F$

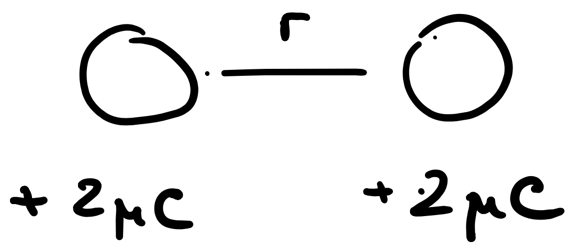
C. $\frac{1}{5}F$

D. $5F$

E. $4F$

$$F = k\frac{(5\mu C)(1\mu C)}{r^2}$$

$$F' = k\frac{(2\mu C)(2\mu C)}{r^2}$$

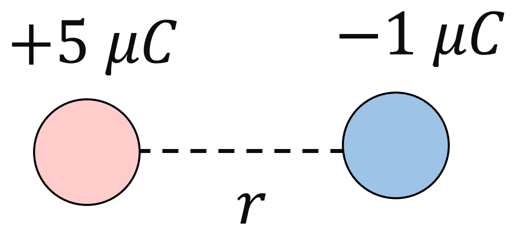


identical!!!

By symmetry

Q: Two identical metal balls (with equal mass & radius) have charges of +5 μC and -1 μC. They are fixed in space, and each feels a force of magnitude F.

Now you bring them together so they touch, then you move them back to their original positions.



What is the magnitude of the new force they feel?

$$\left|\vec{F}_{\text{before}}\right| = k\frac{|+5||-1|}{r^2}$$

A. F (stays the same)

B. $\frac{4}{5}F$

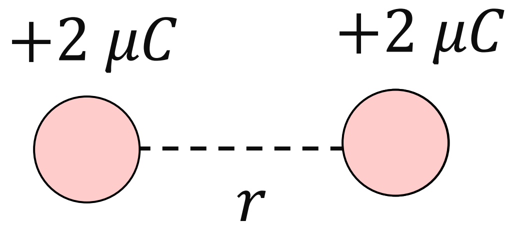
C. $\frac{1}{5}F$

D. $5F$

E. $4F$

- Spheres are identical (!!!)
  => charge will split evenly

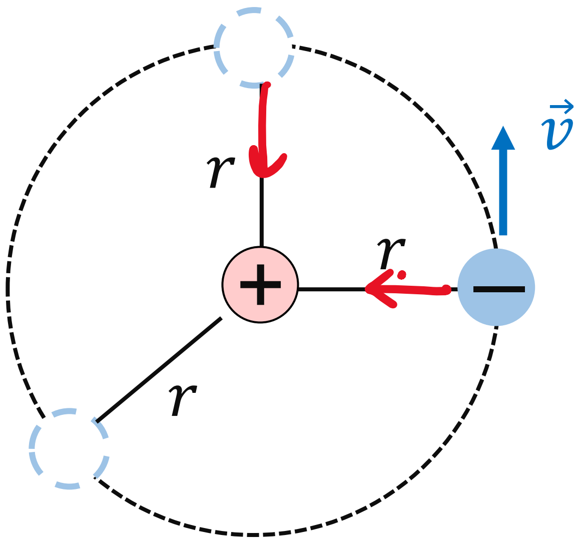
- Net charge conserves =>
  $Q_1' = Q_2' = \frac{5-1}{2} = 2\ \mu C$



$$\left|\vec{F}_{\text{after}}\right| = k\frac{|+2||+2|}{r^2}$$

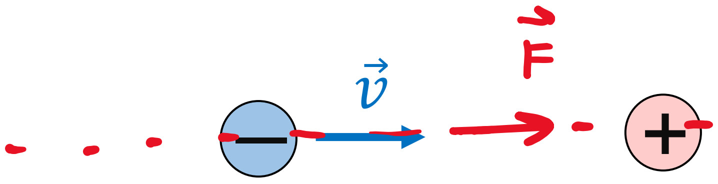
Which aspect(s) of the electric force on the negative point charge will remain constant as it moves:

a) a circular orbit around a positive point charge



$$F = k\frac{q_1 q_2}{r^2}$$

b) a straight line directly toward a stationary positive point charge



r - changes

A. Magnitude

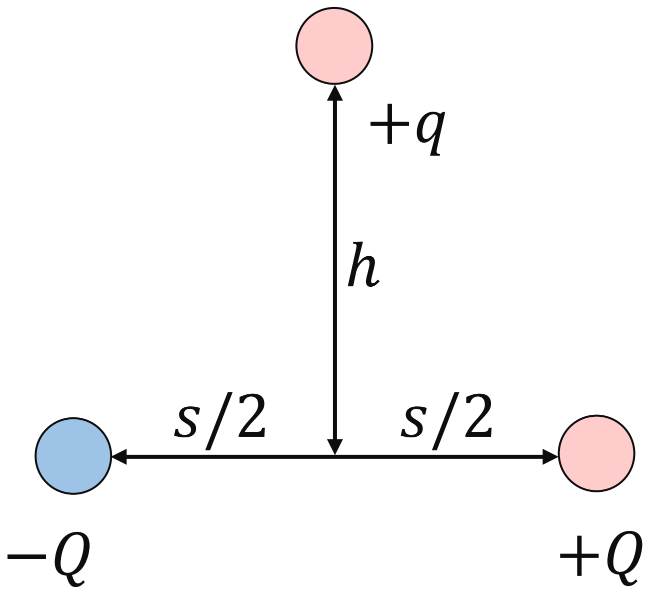
B. Direction

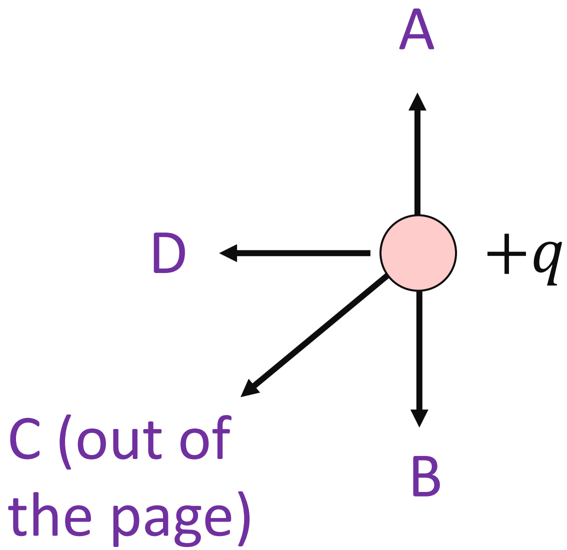
C. Magnitude and direction

D. Neither magnitude nor direction

Q: Three charges, $+q$, $-Q$ and $+Q$, are fixed in the x,y plane as shown below. What happens to the $+q$ charge if it is free to move? Consider only the instant when it is released.

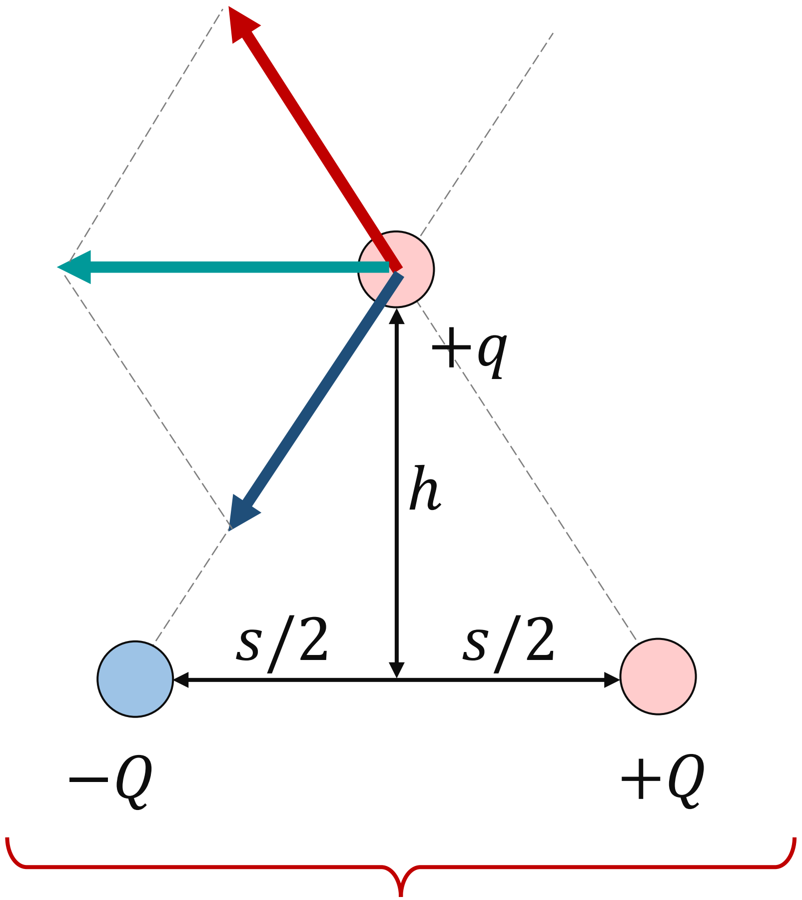
$\underline{\underline{\vec{F}}} = m\underline{\underline{\vec{a}}}$

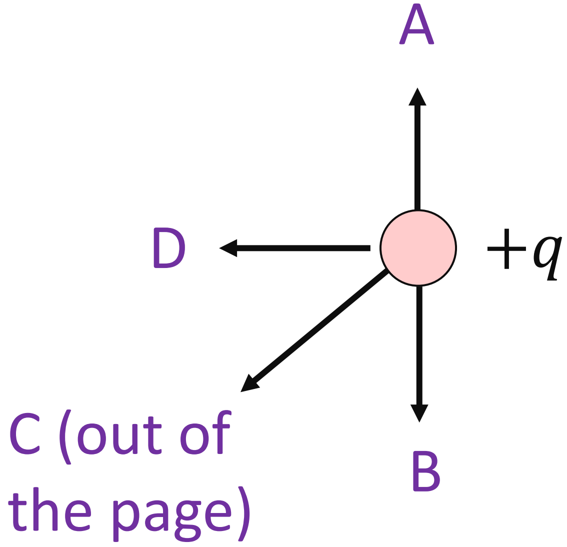




A. $+q$ will experience no acceleration
B. $+q$ will accelerate along direction A
C. $+q$ will accelerate along direction B
D. $+q$ will accelerate along direction C
E. $+q$ will accelerate along direction D

Q: Three charges, $+q$, $-Q$ and $+Q$, are fixed in the x,y plane as shown below. What happens to the $+q$ charge if it is free to move? Consider only the instant when it is released.





Dipole: Two charges of the same magnitude but opposite charge at a small distance, s

A. $+q$ will experience no acceleration
B. $+q$ will accelerate along direction A
C. $+q$ will accelerate along direction B
D. $+q$ will accelerate along direction C
E. $+q$ will accelerate along direction D