This event is one where students come in to provide feedback or "Beef" about their current classes and receive free pizza in return for their participation. "Feedback" can include a variety of things extending from lecture pacing, to test material, to office hours and tutorial help.

PHYS 158 Survey

https://ubc.ca1.qualtrics.com/jfe/form/SV_0P7ghGFNHR3A06O

Please leave us your feedback about how the course goes so far!

$\vec{E} \longrightarrow V\,?$

$$\Delta V = V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

Lecture 23.

Finding $\vec{E}$ from known V.

$V \longrightarrow \vec{E}\,?$

Now: assume that we know V and want to find $\vec{E}$

Challenge: V is a scalar, $\vec{E}$ is a vector.

- How can we reproduce a vector (3 components!) from one single number?
- We can start with finding the projection of $\vec{E}$ on some arbitrary direction (since a projection is a scalar, too)

$$\Delta V(r) = -\int_i^f \vec{E}\cdot d\vec{r} \quad \Rightarrow \quad E_r$$

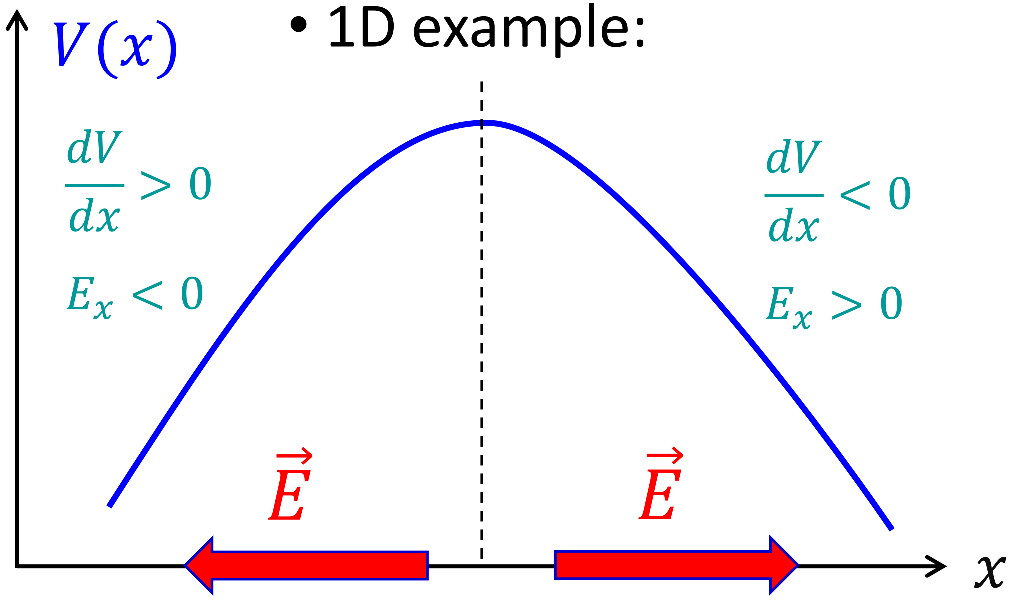
$$= -E\, dr \cos\theta$$

$$\boxed{dV = -\vec{E}\cdot d\vec{r}} = -E_r\, dr \quad \Rightarrow$$

E_r: projection of $\vec{E}$ onto the direction $d\vec{r}$

$$\boxed{E_r = -\frac{dV}{dr}}$$

- 1D example:



- E-field always points downhill!

Now: assume that we know V and want to find $\vec{E}$

Challenge: V is a scalar, $\vec{E}$ is a vector.

- How can we reproduce a vector (3 components!) from one single number?
- Now we can generalize it for 3D case by considering projections of $\vec{E}$ onto directions x, y, z:

$$E_x = -\frac{dV(x,y,z)}{dx} \qquad E_y = -\frac{dV(x,y,z)}{dy} \qquad E_z = -\frac{dV(x,y,z)}{dz}$$

Strictly speaking, these are partial derivatives: $E_x = -\frac{\partial V(x,y,z)}{\partial x}$, etc.

$\nabla_x = \frac{\partial}{\partial x}$

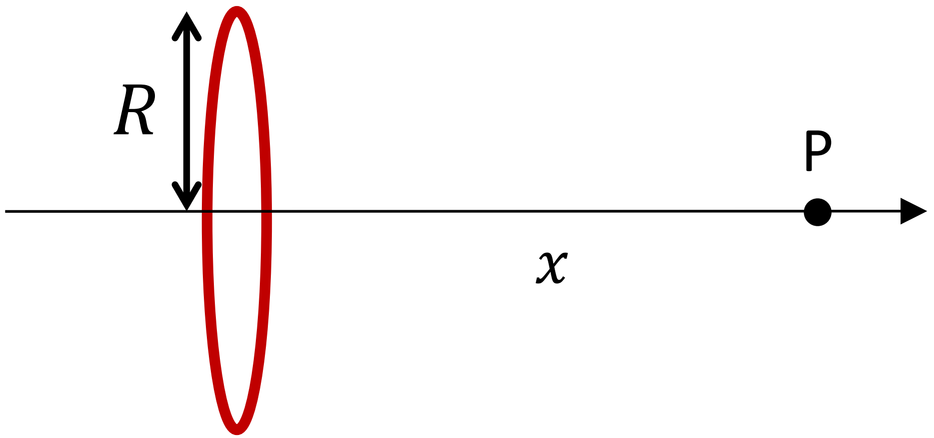
$\vdots$

nabla

In 3D: $\vec{E} = -\nabla V(x,y,x) = -\vec{i}\,\frac{\partial V}{\partial x} - \vec{j}\,\frac{\partial V}{\partial y} - \vec{k}\,\frac{\partial V}{\partial x}$ ('gradient')

Since V is usually quite easy to calculate (it's a scalar!), it might be easier to find the electric field, $\vec{E}$, from its derivatives than from Gauss's law of by integrating vectors!

Q: A conducting ring of radius R has a total charge Q. Find the electric potential, $V(x)$ at the point P on its axis. Assume V is zero at infinity.



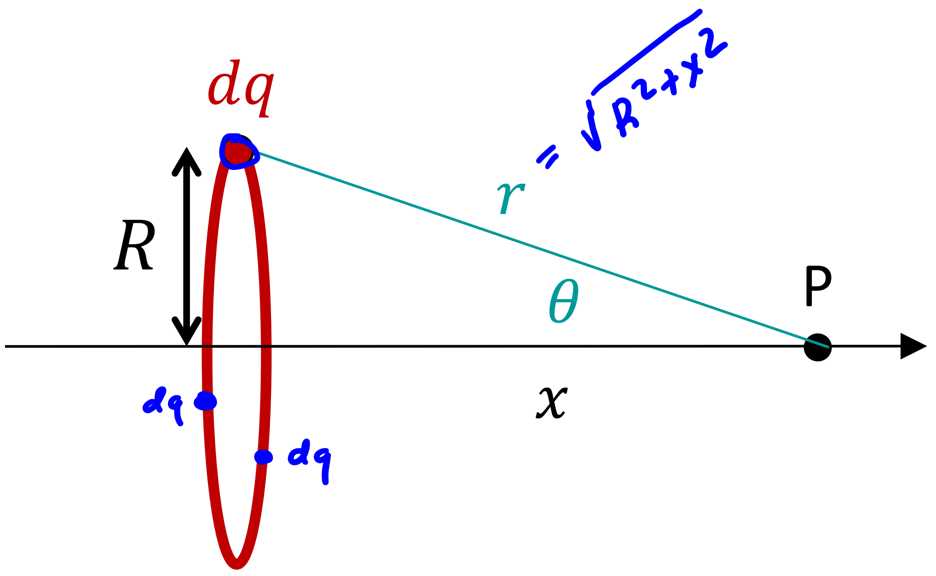
- Mentally cut the ring into small point charges, each of size dq.

Q: For the small charge dq (see picture), what is the potential dV at point P?

Assume V is zero at infinity.

$q: \quad V(r) = \frac{kq}{r}$

$dq: \quad V(P) = \; ?$



A. $dV = \frac{k\,dq}{(x^2+R^2)^{1/2}}$

B. $dV = \frac{k\,dq}{x^2+R^2}$

C. $dV = \frac{k\,dq\cos\theta}{(x^2+R^2)^{1/2}}$

D. $dV = \frac{k\,dq\cos\theta}{x^2+R^2}$

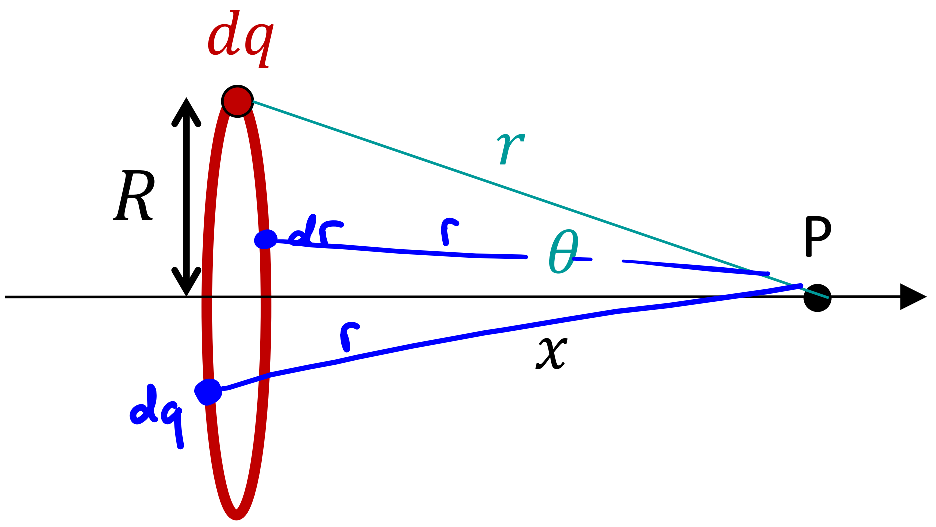
- Mentally cut the ring into small point charges, each of size dq.

Q: What is the potential V at point P? Assume V is zero at infinity.

$$dV = \frac{k\,dq}{r} \quad \text{with } r = \sqrt{x^2 + R^2}. \qquad \text{Hence,} \quad dV = \frac{k\,dq}{(x^2+R^2)^{1/2}}.$$

$$V = \int_{\text{ring}} dV = \int_{\text{ring}} \frac{k\,dq}{(x^2+R^2)^{1/2}} =$$

$$= \frac{k}{(x^2+R^2)^{1/2}} \int_{\text{ring}} dq$$



$$\boxed{V(x) = \frac{k\,Q}{(x^2+R^2)^{1/2}}}$$

+ const, which we set to be zero (since $V(\infty) = 0$) ✓

- Now let's use the potential of the ring on the x-axis, namely,

$$V(x) = \frac{k\,Q}{(x^2+R^2)^{1/2}}$$

$V(x)$: $E_y = -\frac{\partial V}{\partial y} = 0$

$E_z = -\frac{\partial V}{\partial z} = 0$

to compute the electric field of the ring on the x-axis.

$$\frac{d}{dx}\frac{f}{g} = \frac{f'g - g'f}{g^2}$$

A. $E_x = \frac{k\,Q}{(x^2+R^2)^{3/2}}$

B. $E_x = \frac{k\,Q\,x}{(x^2+R^2)^{3/2}}$

C. $E_x = \frac{k\,Q}{(x^2+R^2)^{1/2}}$

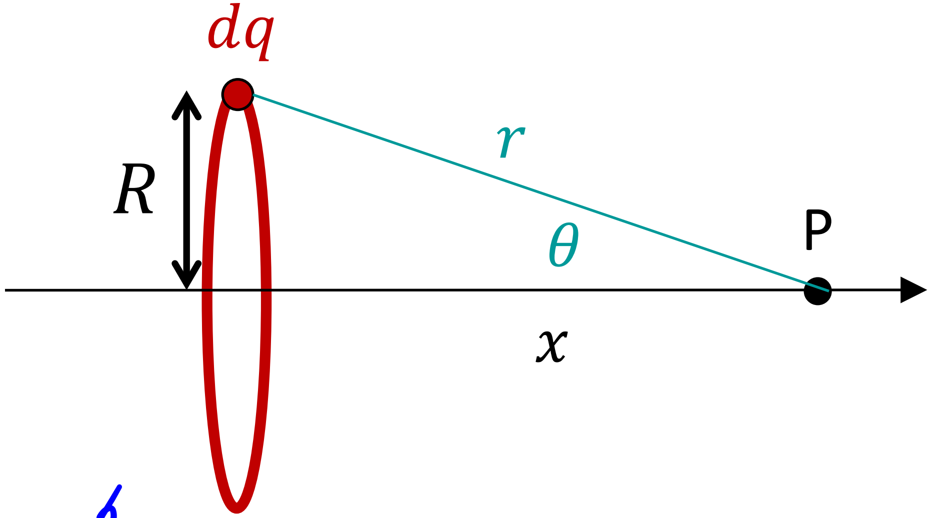
D. $E_x = \frac{k\,Q\,x}{(x^2+R^2)^{1/2}}$

$$E_x = -\frac{\partial V}{\partial x} =$$

$$= -\frac{\partial}{\partial x}\frac{kQ}{(x^2+R^2)^{1/2}} =$$

$$= -kQ\,\frac{-1\cdot\frac{\partial}{\partial x}\sqrt{x^2+R^2}}{(x^2+R^2)} =$$

$$= kQ\,\frac{\cancel{2}x}{(x^2+R^2)\cdot\cancel{2}\sqrt{x^2+R^2}}$$



- Now let's use the potential of the ring on the x-axis, namely,

$$V(x) = \frac{k\,Q}{(x^2 + R^2)^{1/2}}$$

to compute the electric field of the ring on the x-axis.

$$E_x = -\frac{dV(x)}{dx}$$

$$= -\frac{d}{dx}\left(\frac{k\,Q}{(x^2 + R^2)^{1/2}}\right)$$

$$= \frac{k\,Q\,x}{(x^2 + R^2)^{3/2}}$$

A. $E_x = \frac{k\,Q}{(x^2+R^2)^{3/2}}$

B. $E_x = \frac{k\,Q\,x}{(x^2+R^2)^{3/2}}$

C. $E_x = \frac{k\,Q}{(x^2+R^2)^{1/2}}$

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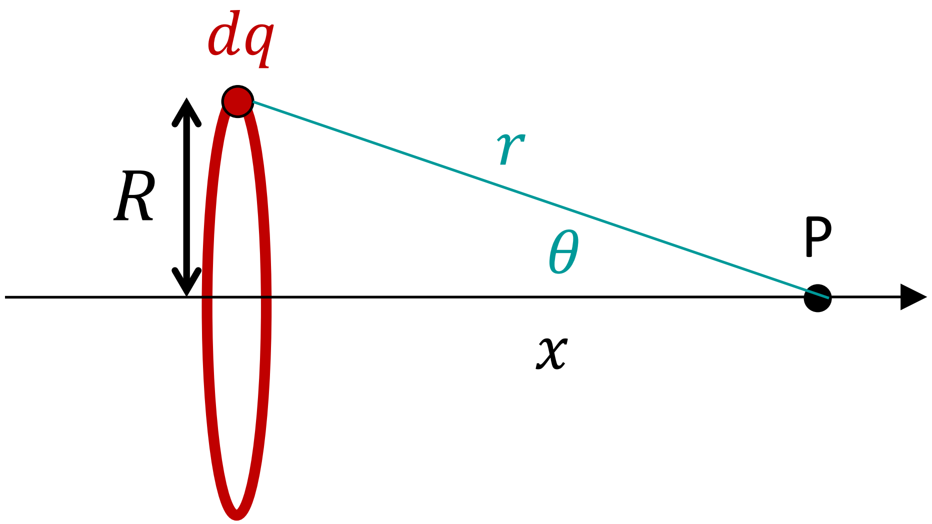
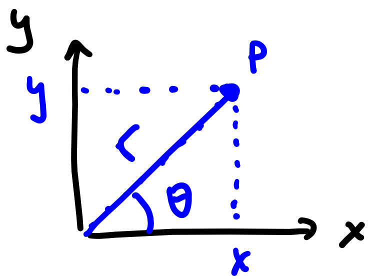


Q: Assume that a potential is given as $V(x,y) = \frac{kq}{\sqrt{x^2+y^2}} = \frac{kq}{r} = V_{p.ch.}$

a) What are the x- and y-components of the electric field?

b) What is the magnitude of the field (electric field strength) at that point?



$r = \sqrt{x^2+y^2}$

$$E_z = -\frac{\partial V(x,y)}{\partial z} = 0$$

$$E_x = -\frac{\partial}{\partial x}\frac{kq}{\sqrt{x^2+y^2}} = -kq\frac{-\cancel{2}x}{(x^2+y^2)\cancel{2}\sqrt{x^2+y^2}} = \frac{kqx}{(x^2+y^2)^{3/2}} = \frac{kq}{r^2}\cdot\frac{x}{r} = \frac{kq}{r^2}\cdot\cos\theta$$

$$E_y = -\frac{\partial}{\partial y}\frac{kq}{\sqrt{x^2+y^2}} = \cdots\cdots\cdots = \frac{kqy}{(x^2+y^2)^{3/2}} = \frac{kq}{r^2}\cdot\frac{y}{r} = \frac{kq}{r^2}\cdot\sin\theta$$

$$E = \sqrt{E_x^2+E_y^2} = \sqrt{\left(\frac{kq}{r^2}\right)^2\underbrace{[\cos^2\theta+\sin^2\theta]}_{1}} = \frac{kq}{r^2} \equiv E_{p.ch.}\ !$$

Capacitance

Recap

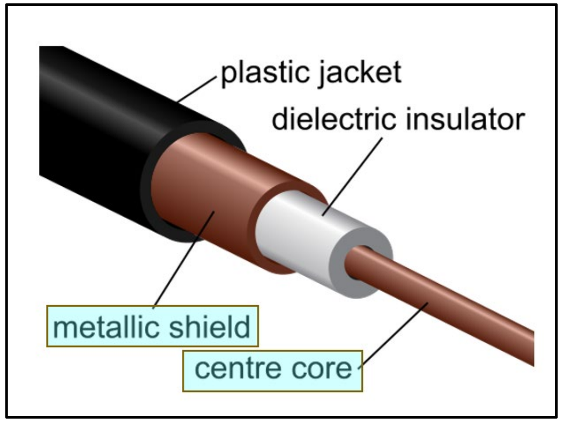
- In a charged capacitor the voltage created across its plates and the charge on its plates are proportional to each other:

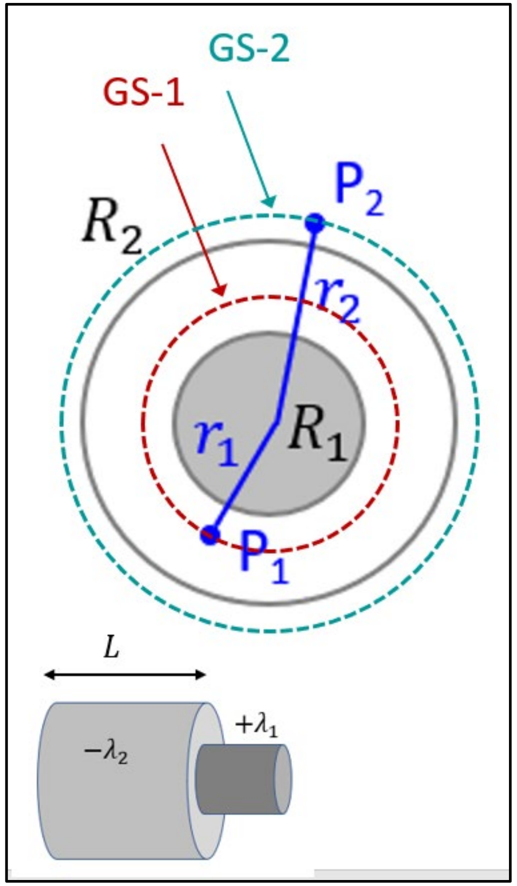
$$\boxed{Q = C\,\Delta V_C}$$

- Capacitance: $C = Q/\Delta V_C$

(meaning: how much charge we can store at a given voltage)

- C depends only on geometry





$$E(R_1 < r < R_2) = \frac{\lambda_1}{2\pi\varepsilon_0 \cdot r}$$

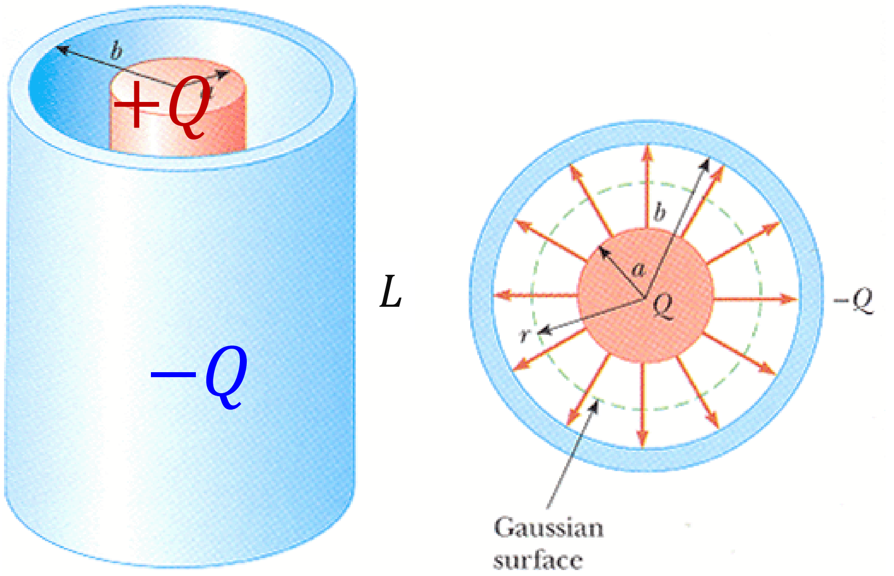
$$E(r > R_2) = \frac{(\lambda_1 - \lambda_2)}{2\pi\varepsilon_0 \cdot r}$$

$$= 0 \text{ if } \lambda_1 = \lambda_2$$

Q: Consider two co-centric cylinders (a core and a shield). The outer radius of the core is a, and the inner radius of the shield is b. The length of the cylinders is L.

Assume there is a charge $+Q$ on the central core and $-Q$ on the metallic shield. Assume air between them. Assume the shield is grounded.

What is the capacitance of this capacitor?



Q: Consider two co-centric cylinders (a core and a shield). The outer radius of the core is a, and the inner radius of the shield is b. The length of the cylinders is L.

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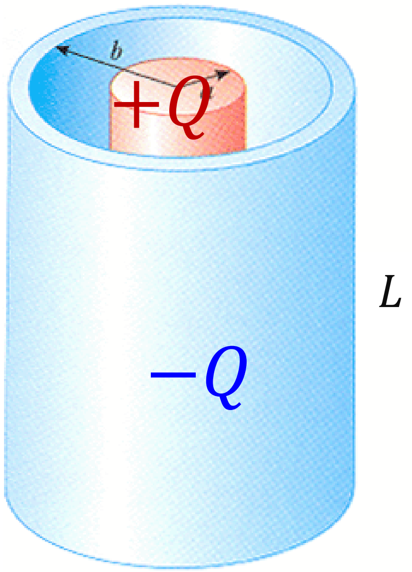
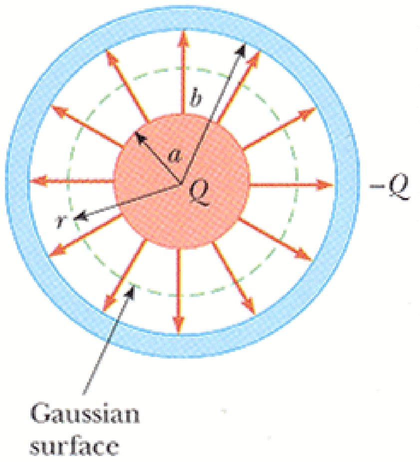


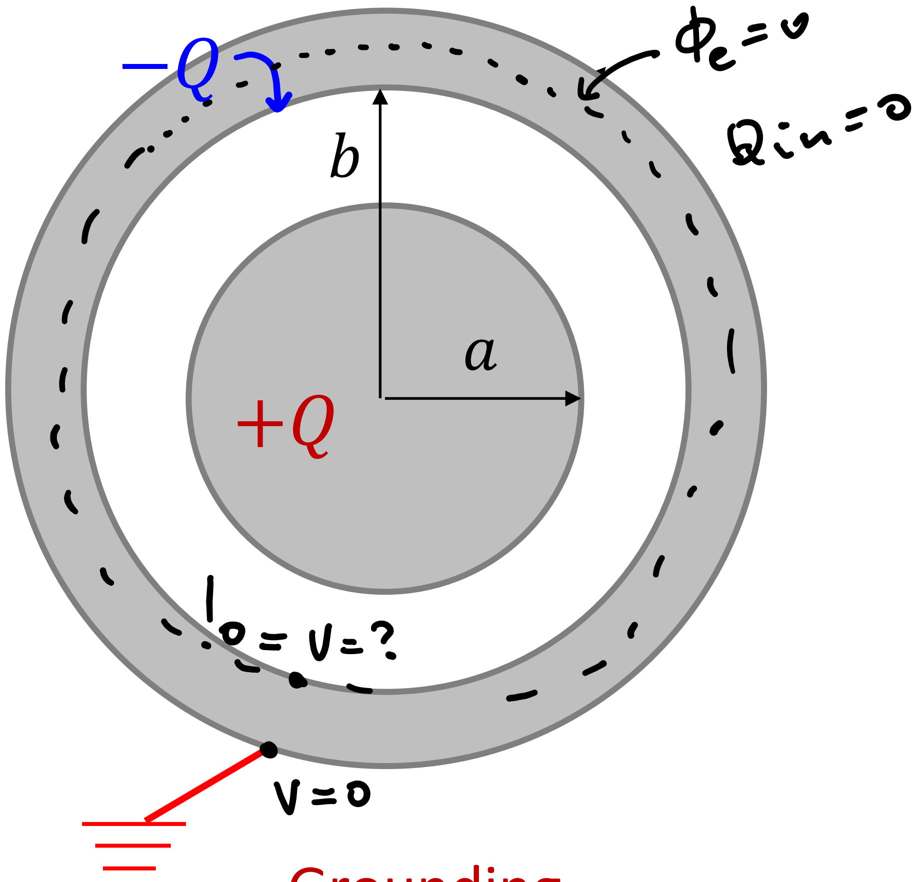


Strategy:

- Find the electric field (Gauss's law):
  1) between the core and the shield, E_1
  2) outside the metallic shield, E_2.

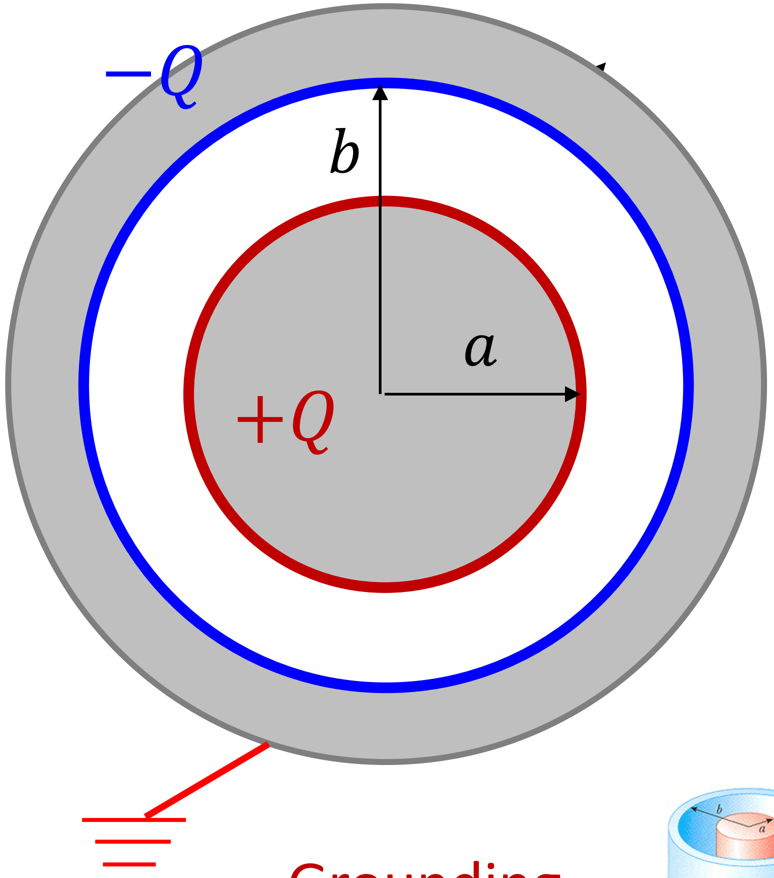
- $\vec{E}(r) \Rightarrow \; V(r)$

- $V(r)$ will be proportional to Q => we will find capacitance, $C = \frac{Q}{\Delta V}$

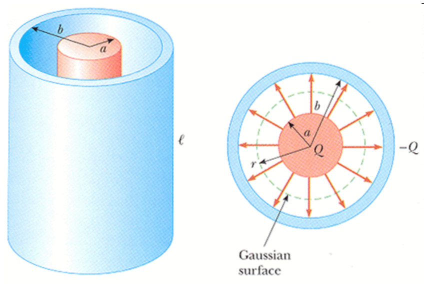


Grounding.
What’s its role?

Charge distribution



Grounding.
Here $V = 0$!



- From Gauss's law: $\boxed{E = \frac{2k\lambda}{r}}$ with $\lambda = \frac{Q_{in}}{L}$

- Electric potential ⇔ Electric field:

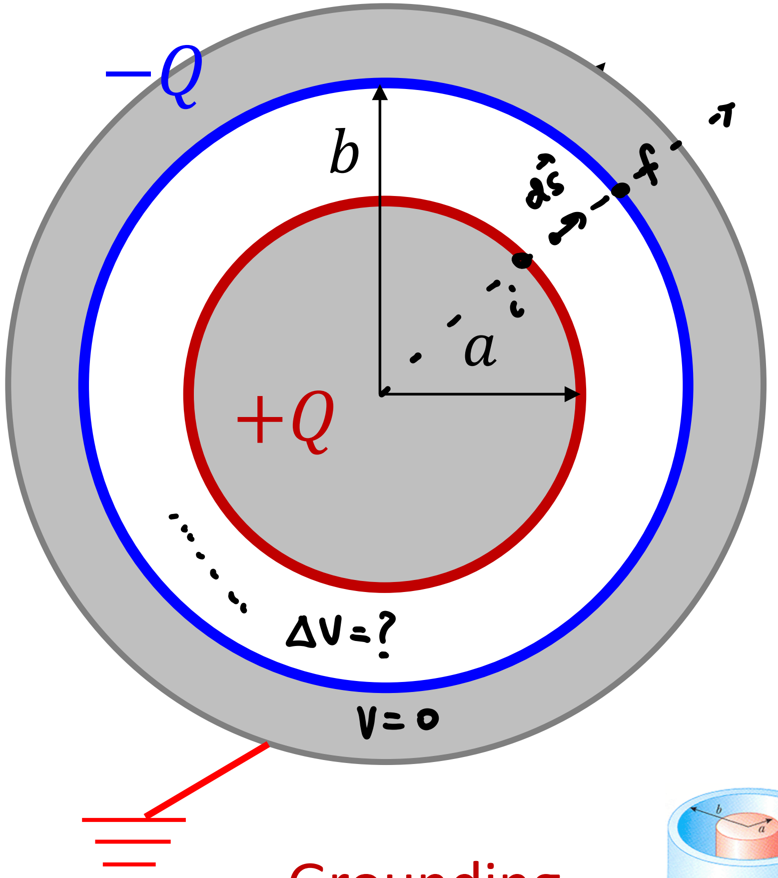
$$\boxed{E = -\frac{dV}{dr}}$$

$$\boxed{V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}}$$

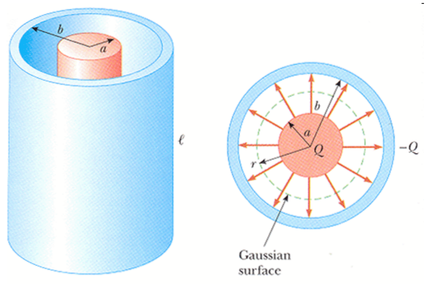
- Way 1: Integrate E-field for each part of space (remember to integrate outwards!) – on your own

- Way 2: Recall that $\frac{d\ln(r)}{dr} = \frac{1}{r}$
Can this help?

Charge distribution



Grounding.
Here $V = 0$!



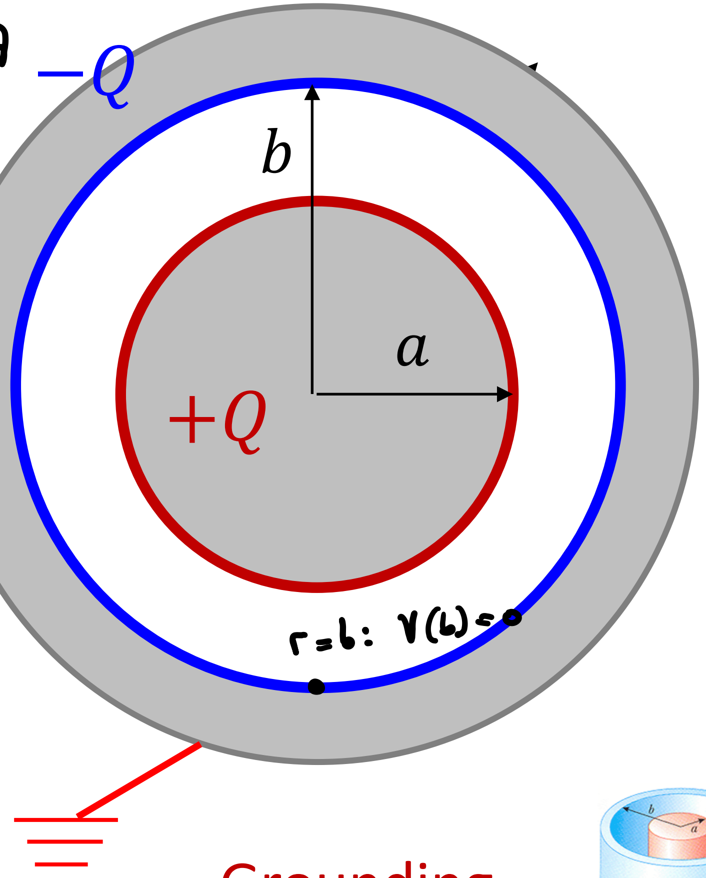
- From Gauss's law: $E = \frac{2k\lambda}{r}$ with $\lambda = \frac{Q_{in}}{L}$

Charge distribution

$$\boxed{E = -\frac{dV}{dr}} \qquad \frac{d\ln(r)}{dr} = \frac{1}{r}$$

$$-\frac{dV}{dr} = E = \frac{2k\lambda}{r}$$

$$V(r) = -2k\lambda \ln(r) + \underline{\text{const}}$$



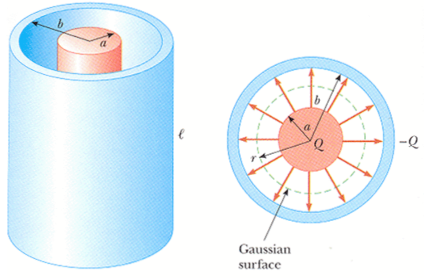
- Find the integration constant from boundary condition: $V(b) = 0$

$$V(b) = -2k\lambda \ln(b) + \text{const} = 0 \quad \Rightarrow$$

$$\text{const} = 2k\lambda \ln(b) \quad \Rightarrow$$

$$V(r) = -2k\lambda \ln(r) + 2k\lambda \ln(b)$$

Grounding.
Here $V = 0$!



Q: What is the potential difference, $V_a - V_b$, across this cylindrical capacitor?

$$V(r) = -2k\lambda \ln(r) + 2k\lambda \ln(b)$$

A. $V_a - V_b = 2k\left(\frac{Q}{L}\right)\ln\left(\frac{b}{a}\right)$

B. $V_a - V_b = 2k\left(\frac{Q}{L}\right)\ln\left(\frac{a}{b}\right)$

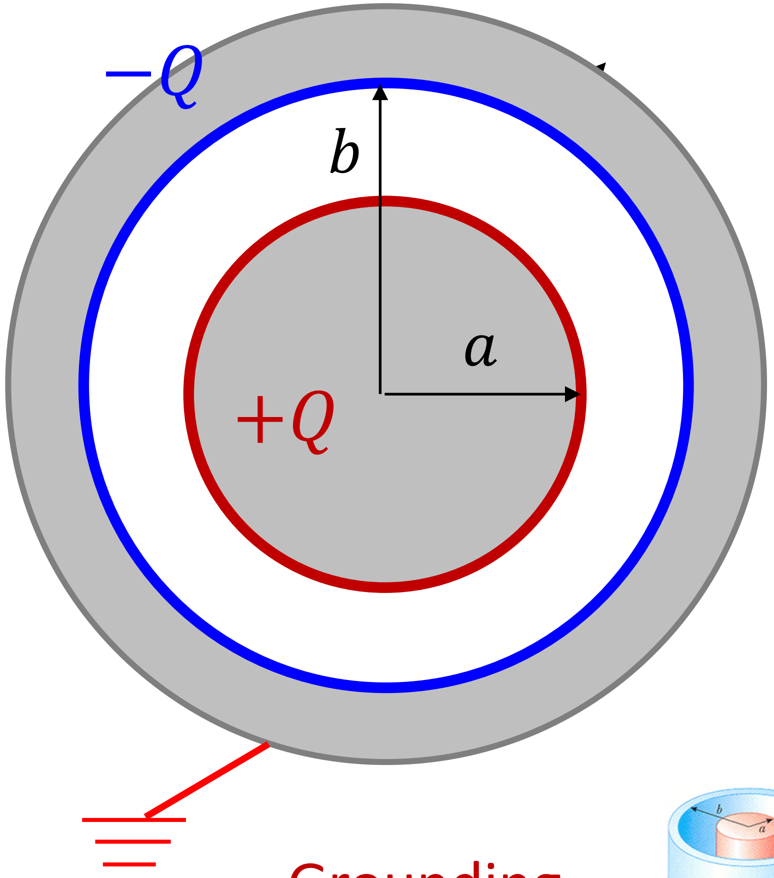
C. $V_a - V_b = 2k\left(\frac{Q}{L}\right)$

D. $V_a - V_b = 2kQ\ln\left(\frac{b}{a}\right)$

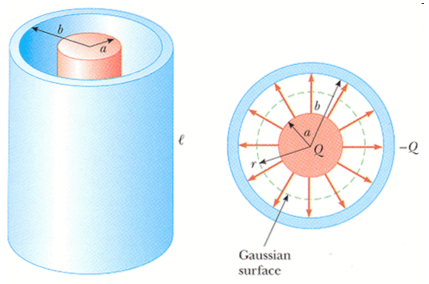
E. $V_a - V_b = 2kQ\ln\left(\frac{a}{b}\right)$

$\ln b - \ln a = \ln b/a$

$V(a) - \underset{\substack{\shortparallel \\ 0}}{V(b)}$



Grounding.
Here $V = 0$!



Q: What is the capacitance, C_{cyl}, of this cylindrical capacitor?

$$V(r) = -2k\lambda \ln(r) + 2k\lambda \ln(b)$$

A. $C_{\text{cyl}} = \frac{2k}{L} \ln\left(\frac{b}{a}\right)$

B. $C_{\text{cyl}} = \frac{2k}{L} \ln\left(\frac{a}{b}\right)$

C. $C_{\text{cyl}} = \frac{2k}{L}$

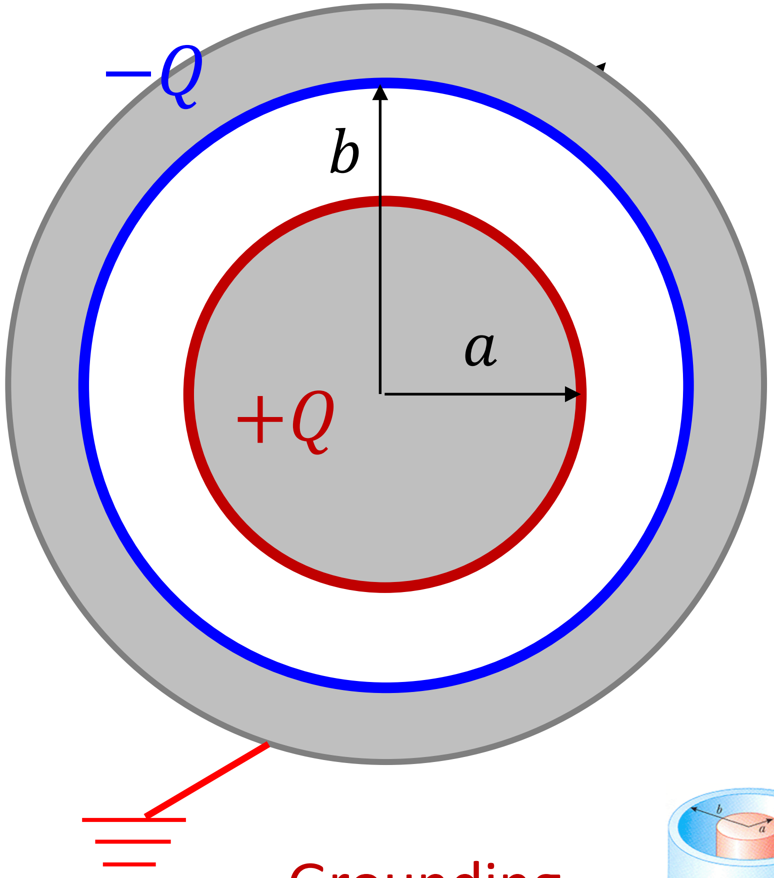
D. $C_{\text{cyl}} = \frac{L}{2k \ln\left(\frac{b}{a}\right)}$

E. $C_{\text{cyl}} = \frac{L}{2k \ln\left(\frac{a}{b}\right)}$

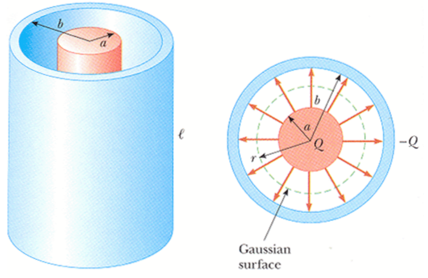
$$\Delta V = 2k\lambda \ln b/a$$

$$\lambda = \frac{Q}{L}$$

$$C = \frac{Q}{\Delta V}$$



Grounding.
Here $V = 0$!



$$V(r) = -2k\lambda \ln(r) + 2k\lambda \ln(b)$$

Charge distribution

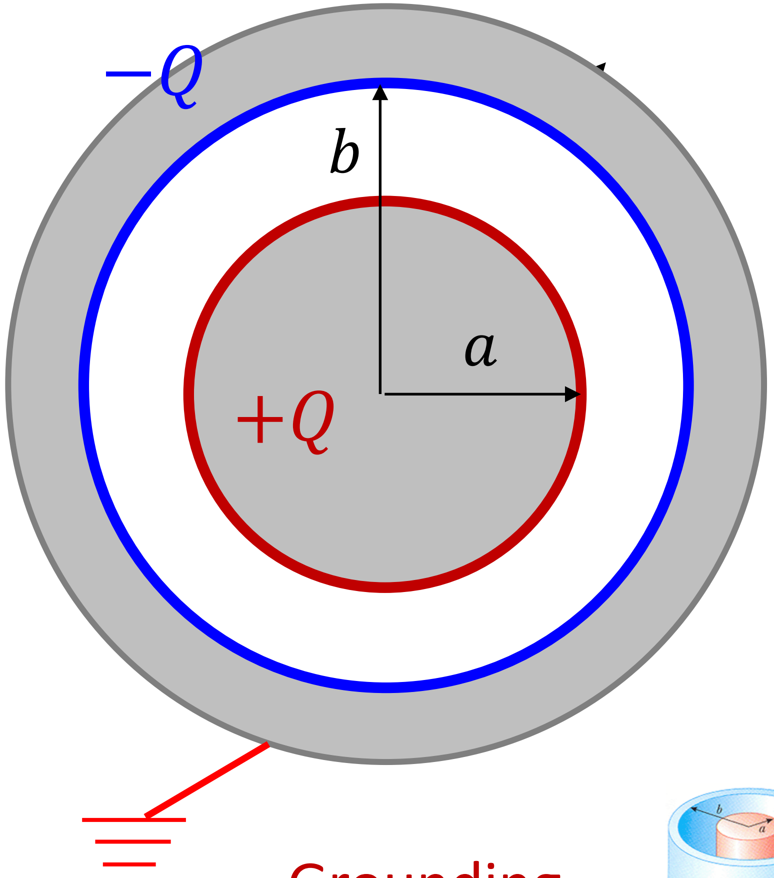
- Now we can find the potential difference across the two conductors:

$$V(a) = -2k\lambda \ln(a) + 2k\lambda \ln(b)$$

$$V(b) = 0$$

$$\Delta V = V_a - V_b = 2k\lambda(\ln b - \ln a) = 2k\frac{Q}{L}\ln\left(\frac{b}{a}\right)$$

$$C_{\text{cyl}} = \frac{Q}{\Delta V} = \frac{L}{2k\ln\left(\frac{b}{a}\right)}$$



Grounding.
Here $V = 0$!