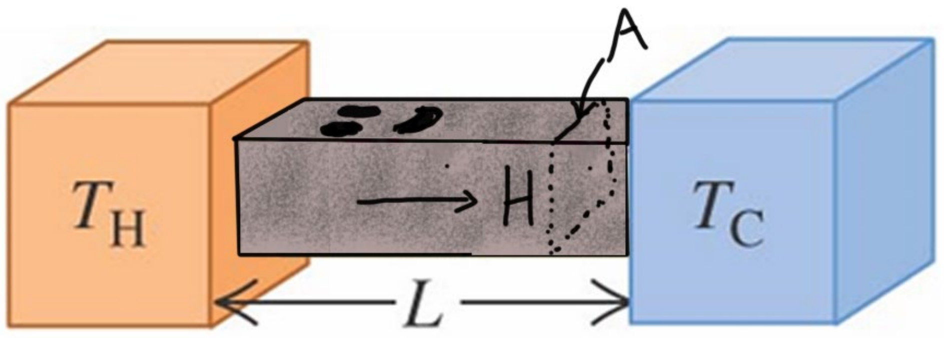
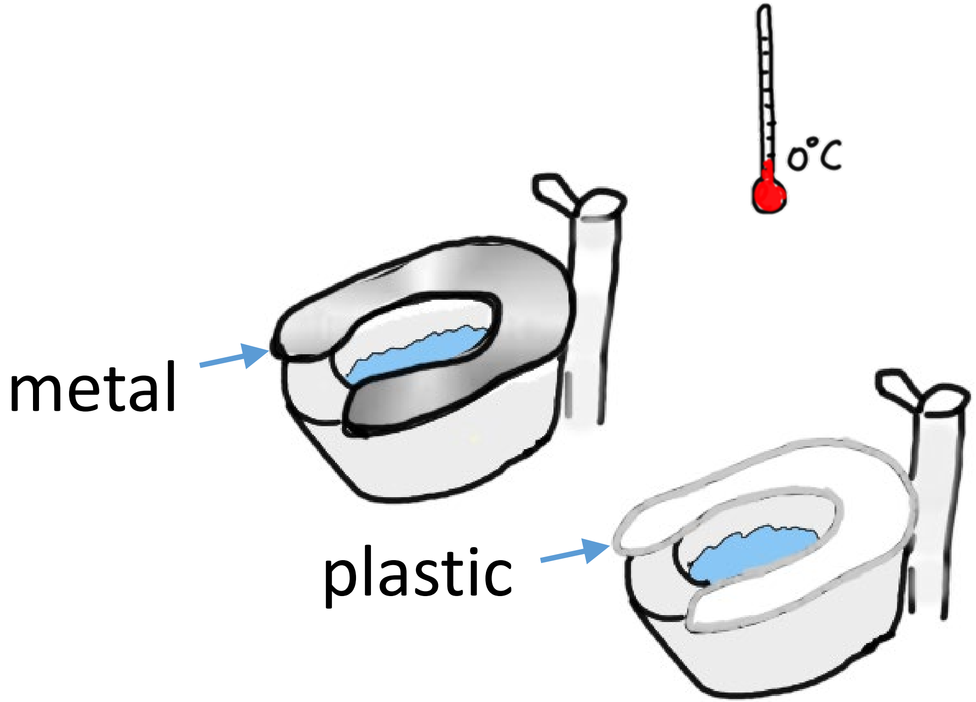


Lecture 10.
Thermal conductivity.
Heat current.

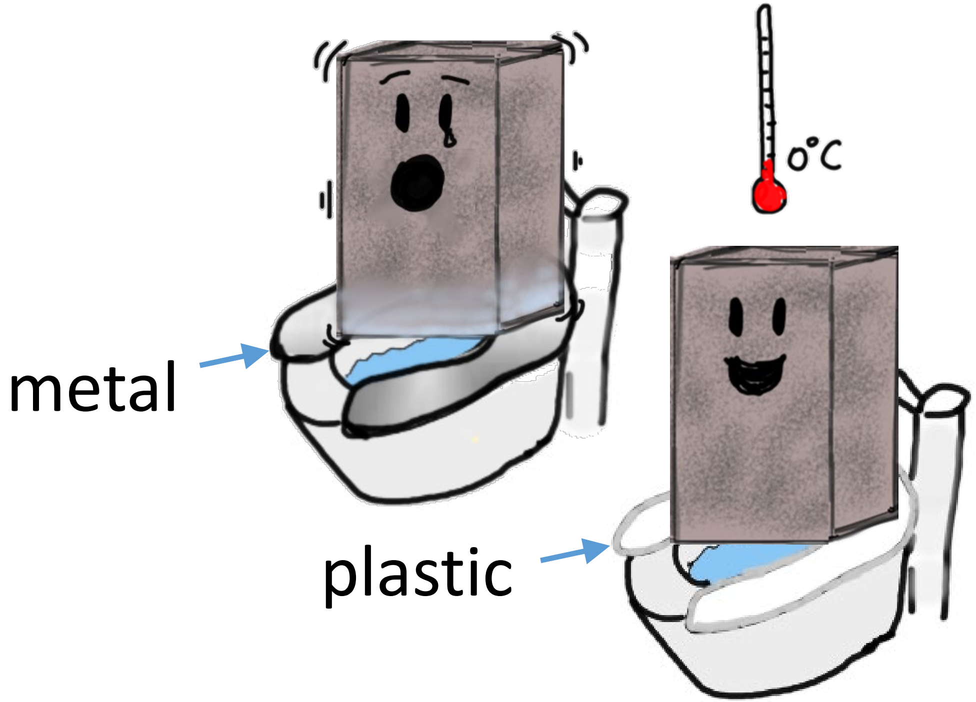
Q: During a break from skiing, you enter an unheated washroom building (0 ^{o}C). You notice there are two toilets, one with a metal seat (c ~ 200 J/kg·K) and one with a plastic seat (c ~ 1600 J/kg·K). Assuming that you need to sit down, and that both seats are clean, which do you choose?



A. The metal seat
B. The plastic seat
C. It doesn't matter: they are the same temperature
D. My head says A) but my heart says B)

Q: During a break from skiing, you enter an unheated washroom building (0 ^{o}C). You notice there are two toilets, one with a metal seat (c ~ 200 J/kg·K) and one with a plastic seat (c ~ 1600 J/kg·K). Assuming that you need to sit down, and that both seats are clean, which do you choose?

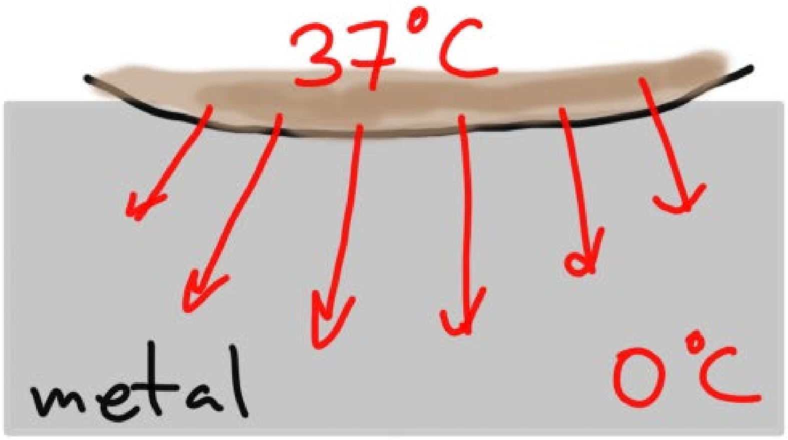
The choice is of course yours, but one decision is better than the other.



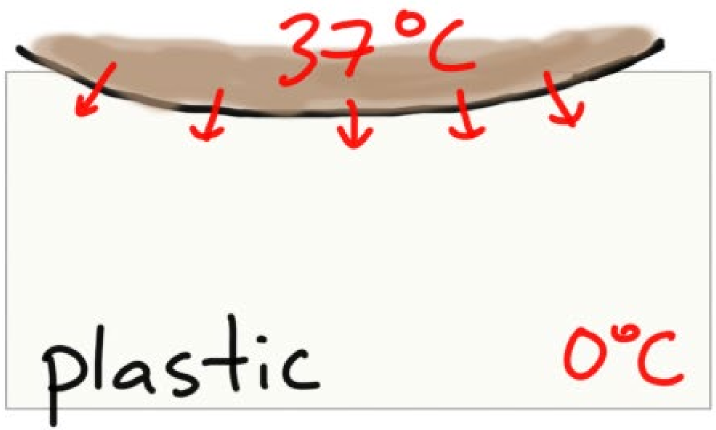
A. The metal seat
B. The plastic seat ✓
C. It doesn't matter: they are the same temperature
D. My head says A) but my heart says B)

Thermal conductivity

- Heat moves more quickly through some materials than others in response to a temperature gradient



Good thermal conductor

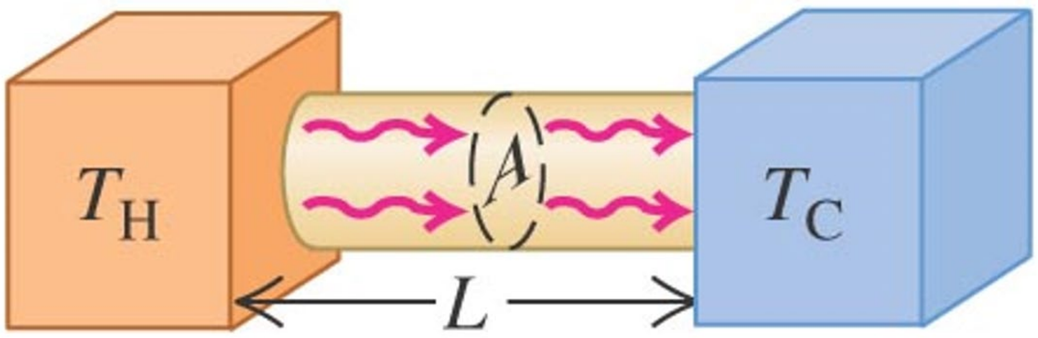


Poor thermal conductor (insulator)

The metal feels colder since it cools our skin quicker

Thermal conductivity

- Determines heat flow in response to a temperature gradient



$$H = \frac{Q}{\Delta t} = k\,A\,\frac{T_H - T_C}{L}$$

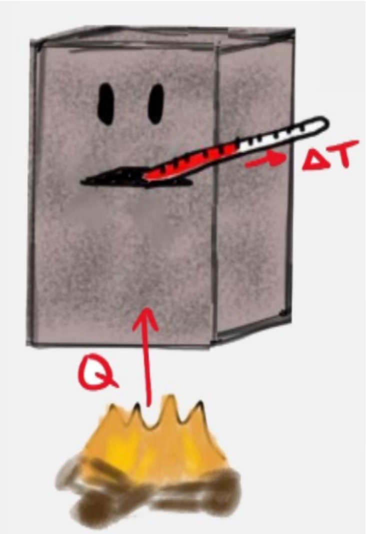
- H = Heat current, or heat flow (Joules/second)
- k = Thermal conductivity, a basic property of a material (material constant)
- A = cross sectional area through which heat flows
- $\frac{T_H - T_C}{L}$ = temperature gradient (calculus version: dT/dx)

Our toolkit

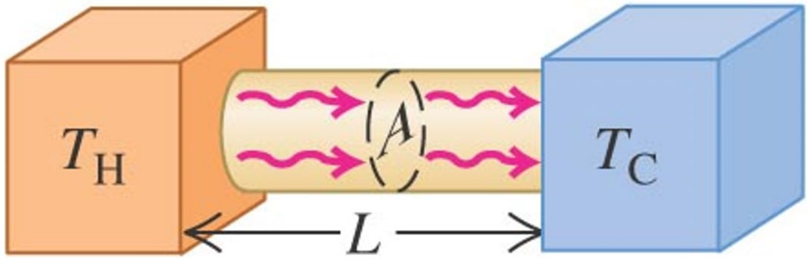
- What do these two equations mean?
- What exactly do they describe?
- When to use which?

$$Q = mc\,\Delta T$$

$$H = \frac{Q}{\Delta t} = k\,A\,\frac{T_H - T_C}{L}$$



- Heating / cooling an object by ΔT.



- Transfer of heat from something at $T = T_H$ to something with $T = T_C$.

Q: Objects A and B have the same size and same mass and both are at room temperature. We have specific heat $\boxed{c_A < c_B}$ and thermal conductivity $\boxed{k_A > k_B}$. If each is dropped into an equivalent volume of 80 °C water (insulated from the environment), we can say that:

$$H = \frac{Q}{\Delta t} = \frac{kA(T_H - T_C)}{L}$$

A : $k_A \uparrow \rightarrow$ faster reaches eq.

$$Q = mc\Delta T$$

A : $C_A \downarrow \rightarrow \Delta T \uparrow$

↳ measure of waisting heat for vibrations/rotatons

A. Object A will reach equilibrium faster and end up at a higher temperature.

B. Object A will reach equilibrium faster and end up at a lower temperature.

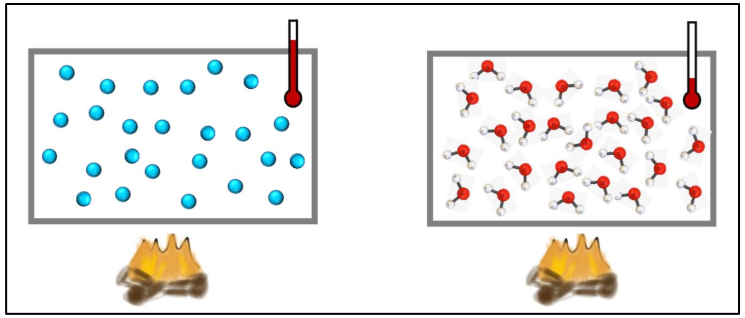
C. Object A will reach equilibrium slower and end up at a higher temperature.

D. Object A will reach equilibrium slower and end up at a lower temperature.

E. Both objects A and B will end up at the same temperature.

Q: Objects A and B have the same size and same mass and both are at room temperature. We have specific heat $c_A < c_B$ and thermal conductivity $k_A > k_B$. If each is dropped into an equivalent volume of 80 ^oC water (insulated from the environment), we can say that:

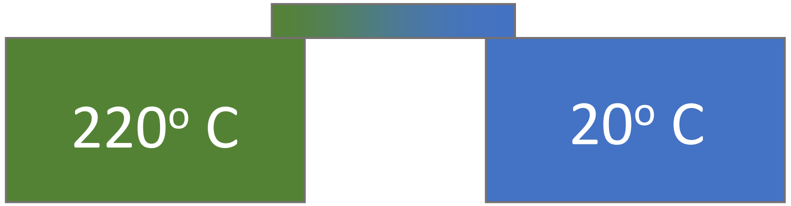
Larger k_A means heat flows in faster (better thermal conductor), and smaller c_A takes less energy for a given temperature change (less energy is "wasted" for rotations/vibrations). So heat from water will increase the temp of A more than that of B and it will reach equilibrium faster.



Week 3 Lecture 3

A. Object A will reach equilibrium faster and end up at a higher temperature. ✓
B. Object A will reach equilibrium faster and end up at a lower temperature.
C. Object A will reach equilibrium slower and end up at a higher temperature.
D. Object A will reach equilibrium slower and end up at a lower temperature.
E. Both objects A and B will end up at the same temperature.

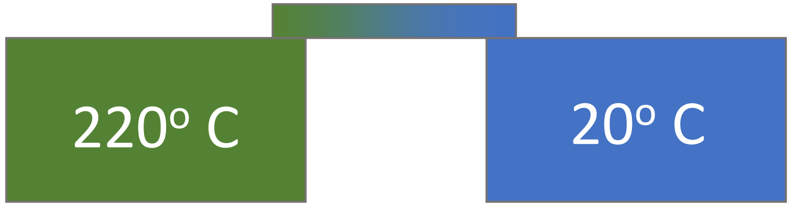
Thermal Conduction Problem



A block of aluminum is at room temperature ($T_1 = 20\ ^oC$) and another equivalent block of aluminum is at $T_2 = 220\ ^oC$. They are then connected together by another strip of aluminum.

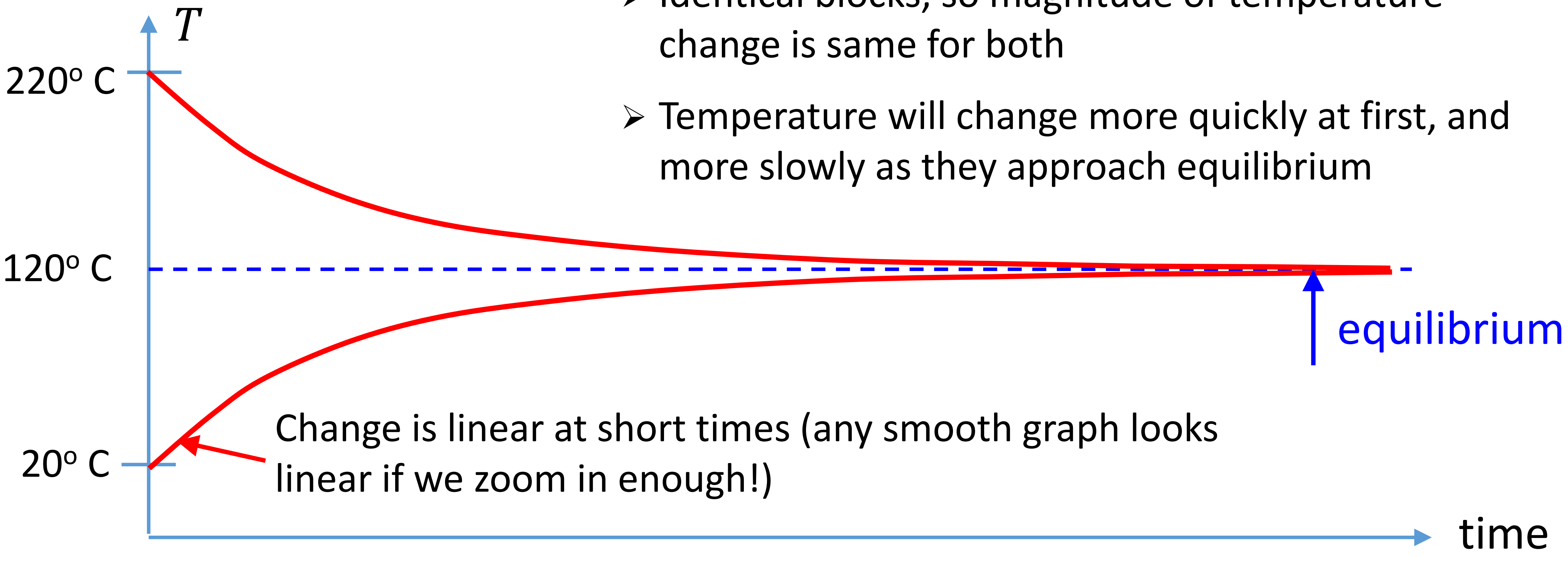
How would you expect the temperatures of the two blocks to behave as a function of time?

- Identical blocks, so the magnitude of temperature change is same for both (by symmetry) $T_f = 20°C + (220° - 20°)/2 = 120°C$
- Temperature will change more quickly at first, and more slowly as they approach equilibrium (since the closer they are to the equilibrium, the smaller is the temperature gradient, which drives the heat flow) $H = \frac{kA\Delta T}{L}$



Graph showing how the temperature of the two blocks changes as a function of time:

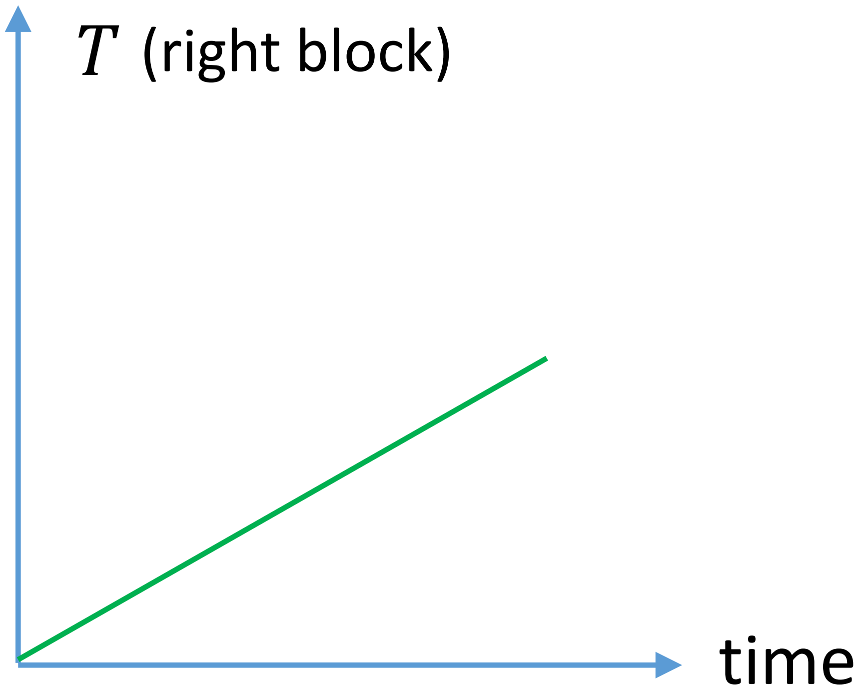
- Identical blocks, so magnitude of temperature change is same for both
- Temperature will change more quickly at first, and more slowly as they approach equilibrium



Q: What happens if we have double the number of connecting strips?

1

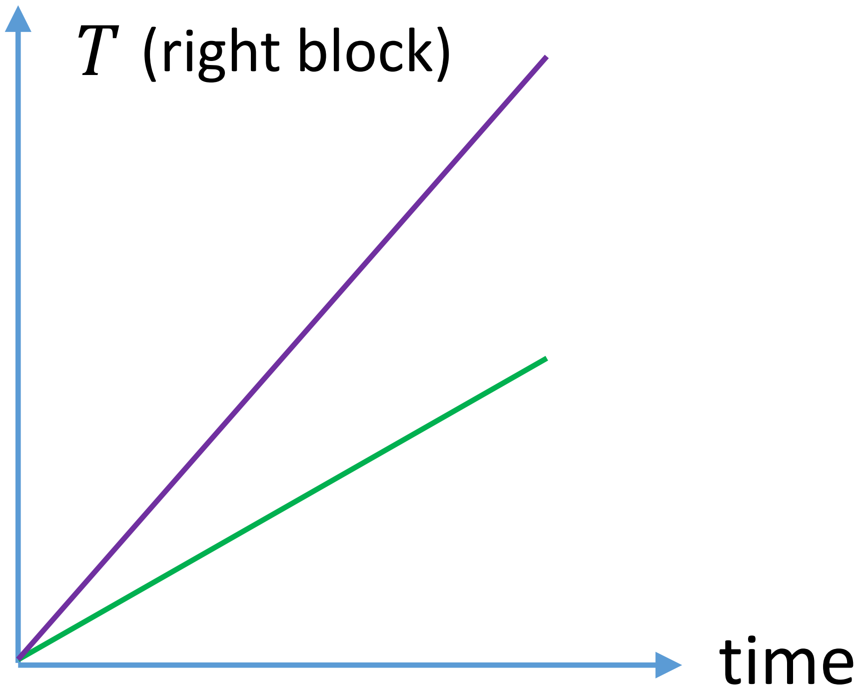
2



$$\boxed{H = k\,A\,\frac{T_H - T_C}{L}}$$

A graph of how the temperature behaves as a function of time at early times.

Q: What happens if we have double the number of connecting strips?

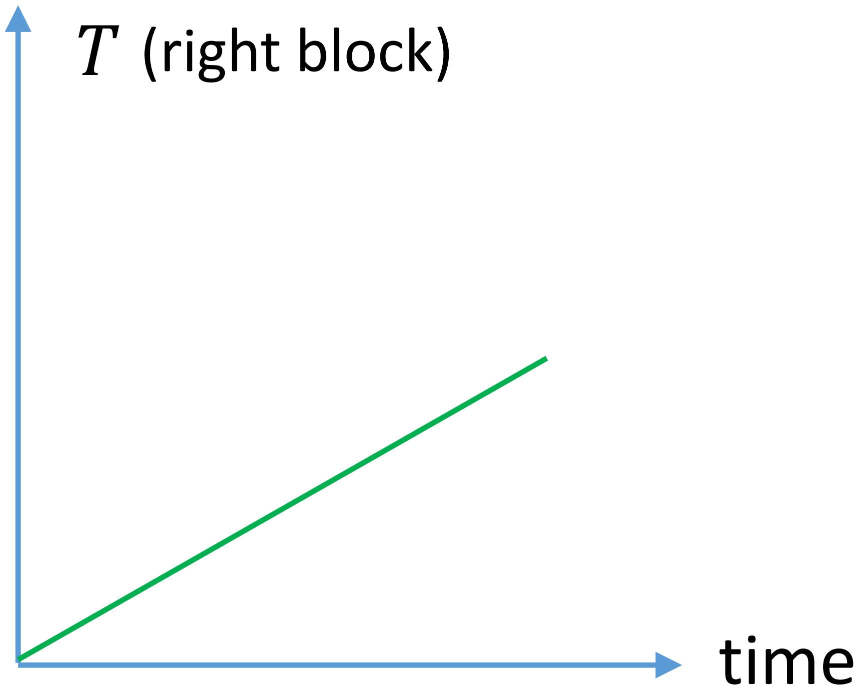


$$\boxed{H = k\,A\,\frac{T_H - T_C}{L}}$$

- A is doubled, so larger heat current and faster heating/cooling

A graph of how the temperature behaves as a function of time at early times.

Q: What happens if we have double the length of connecting strips?



$$\boxed{H = k\,A\,\frac{T_H - T_C}{L}}$$

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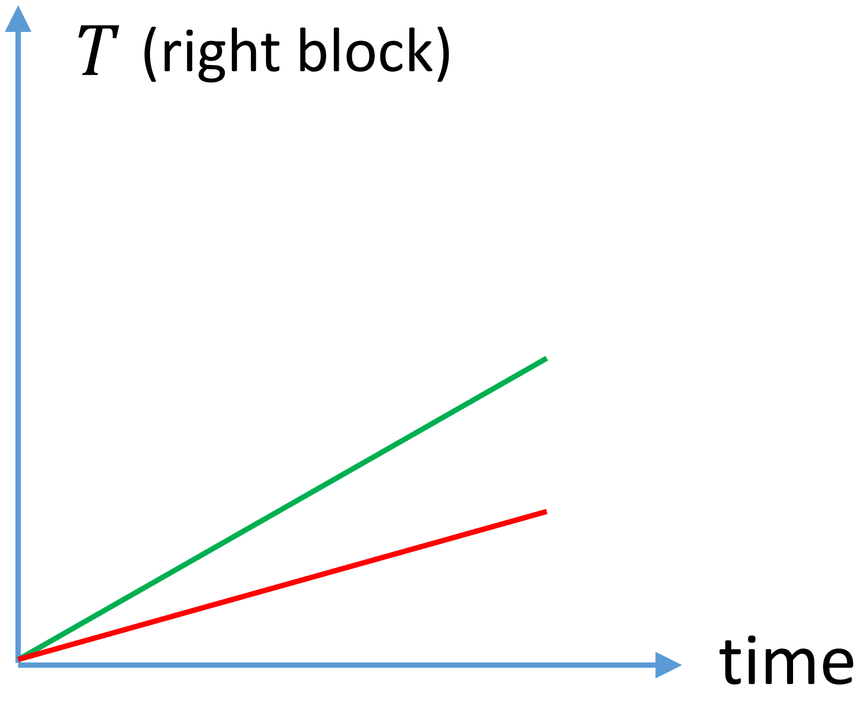


$$\boxed{H = k\,A\,\frac{T_H - T_C}{L}}$$

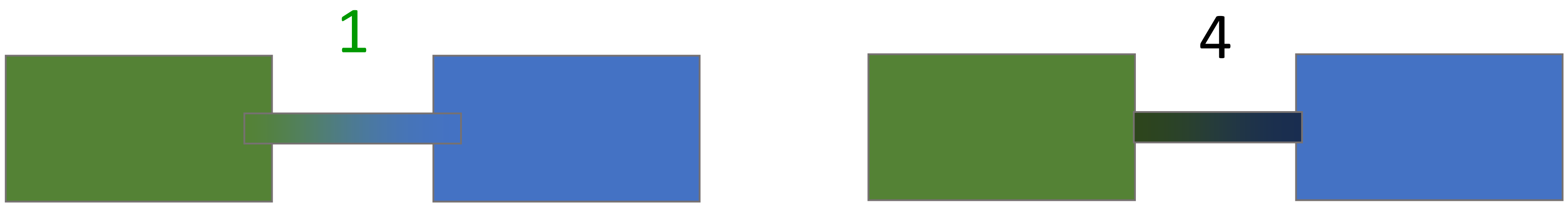
- L is doubled, so smaller heat current and smaller heating/cooling

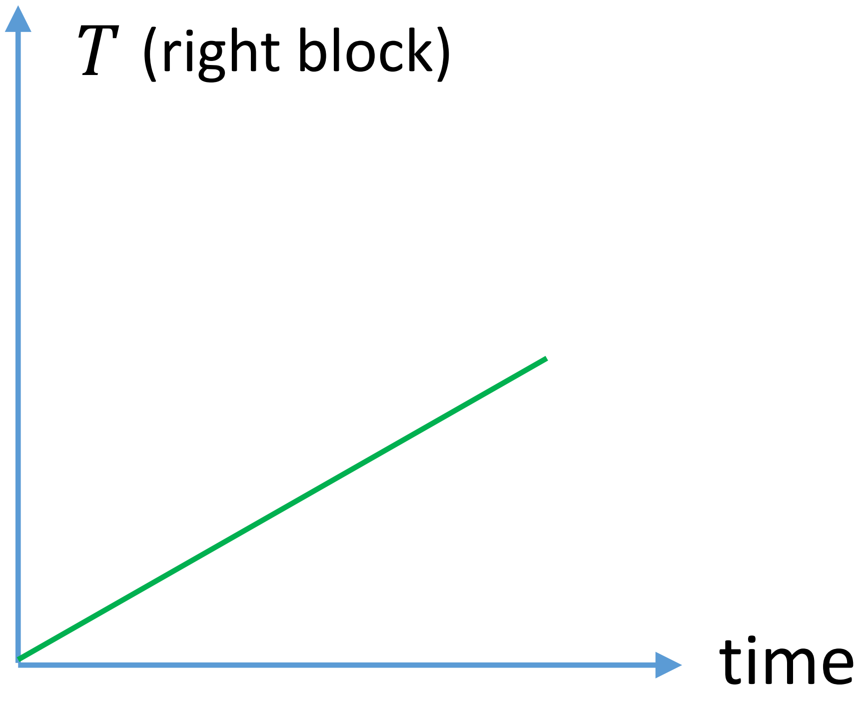
A graph of how the temperature behaves as a function of time at early times.

Q: What happens if the connecting strip is steel instead of aluminum?

$$k_{Al} \sim 200 \frac{W}{mK}$$

$$k_{steel} \sim 50 \frac{W}{mK}$$





$$\boxed{H = k\,A\,\frac{T_H - T_C}{L}}$$

A graph of how the temperature behaves as a function of time at early times.

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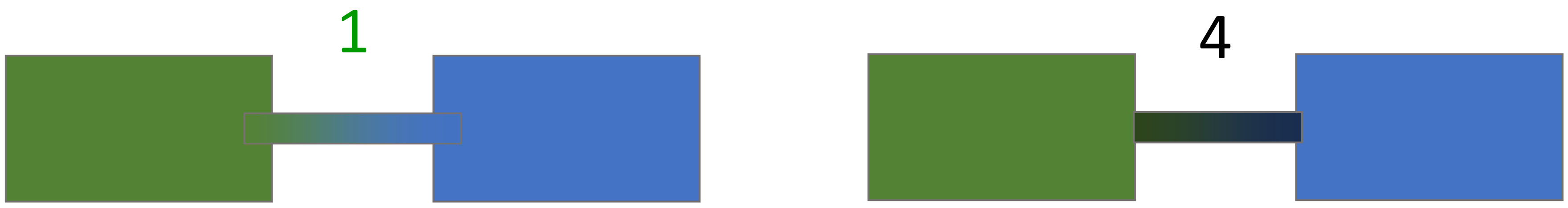
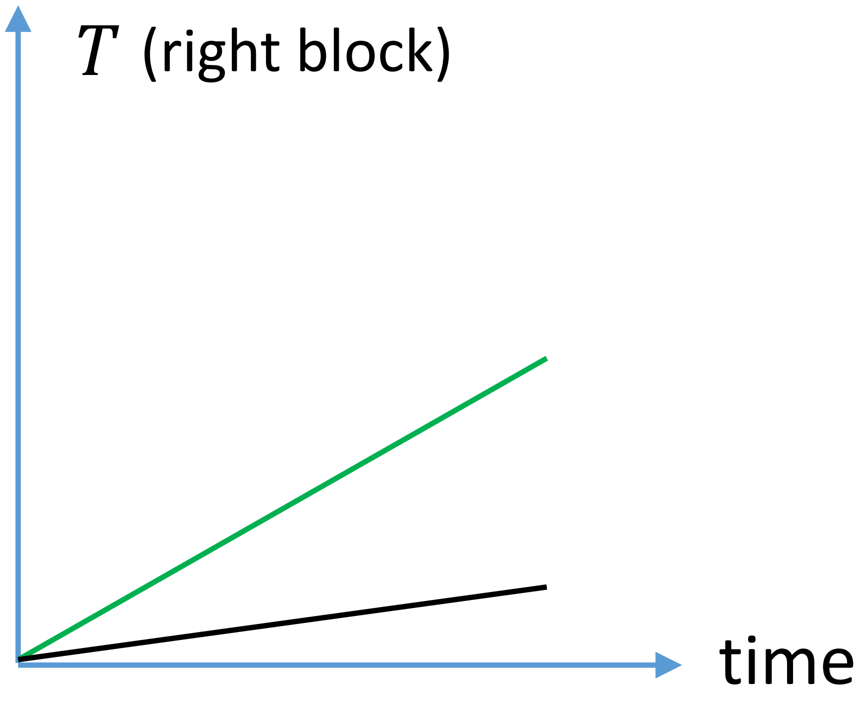


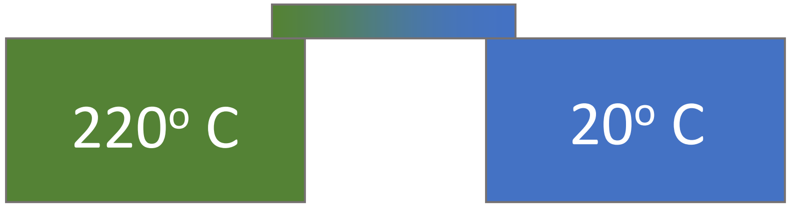


$$H = k\,A\,\frac{T_H - T_C}{L}$$

- k is 4x smaller for steel than for aluminum, so smaller heat current and slower heating/cooling

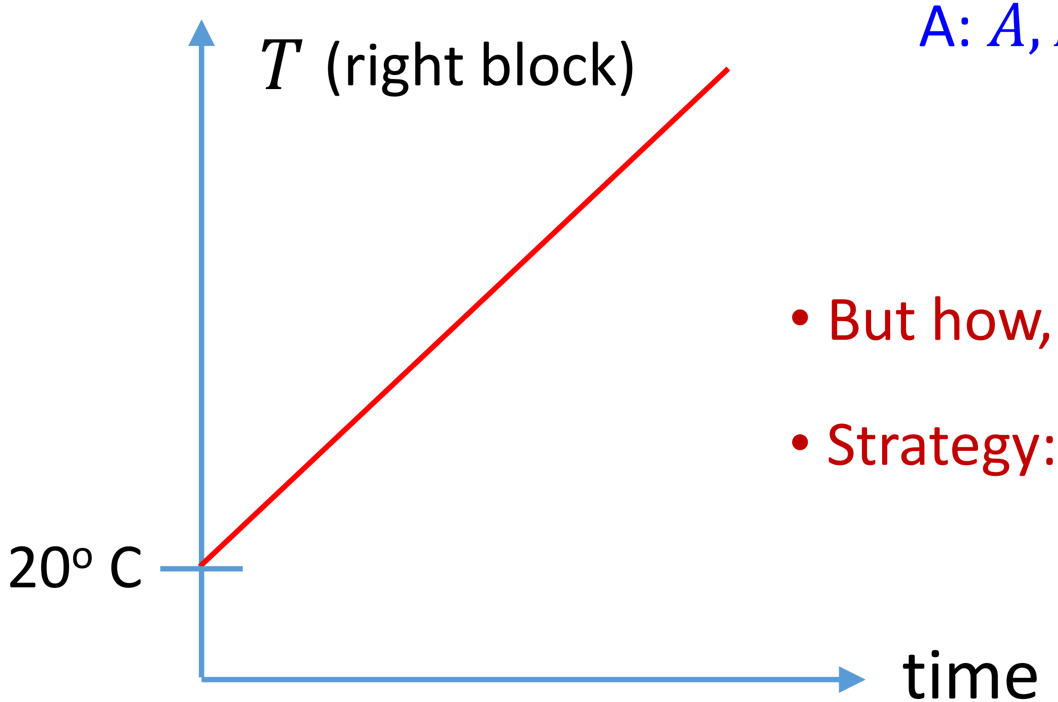
A graph of how the temperature behaves as a function of time at early times.

This is a graph of how the temperature of the right block behaves as a function of time at early times.



Q: What determines the initial slope of the temperature vs time?

A: A, L, k



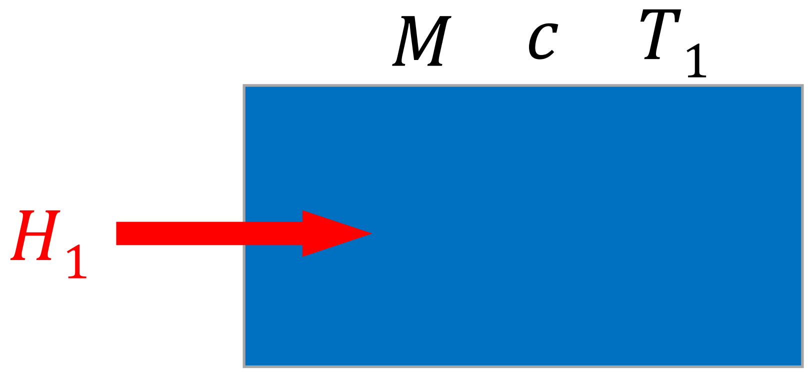
- But how, exactly, does it depend? Any other parameters?
- Strategy: First consider the parts separately…

$$\boxed{Q = m\,c\,\Delta T}$$

$$\boxed{H = k\,A\,\frac{T_H - T_C}{L}}$$

A heat current H_1 flows into the cooler block.

In a time dt, what is the change dT in the temperature of the cooler block (in terms of dt and the quantities shown)?



$$T_1 \stackrel{?}{=} T_1(t)$$

- *First: how much heat, dQ, enters the block during the time dt?*

$$H := \frac{dQ}{dt} \longrightarrow dQ = H_1 \cdot dt \qquad (*)$$

- *Second: How much does the temperature change, dT, from heat dQ?*

$$dQ = Mc\, dT \longrightarrow dT = \frac{dQ}{Mc}$$

$$\boxed{dT_1 = \frac{H_1}{Mc}\, dt}$$

- *Third: How fast does the temperature change?*

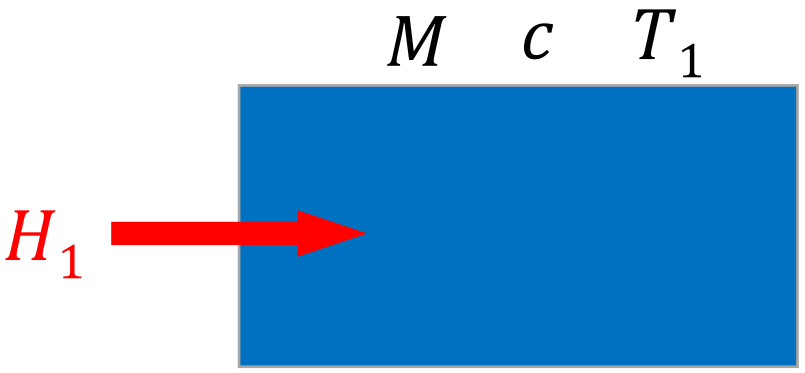
$$dT = \frac{dQ}{Mc} \stackrel{(*)}{=} \frac{H_1}{Mc}\, dt$$

$$\boxed{Q = m\, c\, \Delta T}$$

$$\boxed{H = k\, A\, \frac{T_H - T_C}{L}}$$

A heat current H_1 flows into the cooler block.

In a time dt, what is the change dT in the temperature of the cooler block (in terms of dt and the quantities shown)?



- *First: how much heat, dQ, enters the block during the time dt?*

  H_1 is heat per unit time, and dt is time, so heat added is $dQ = H_1\, dt$

- *Second: How much does the temperature change, dT, from heat dQ?*

  We have $dT_1 = \frac{dQ}{Mc}$ so $dT_1 = \frac{H_1}{Mc} dt$

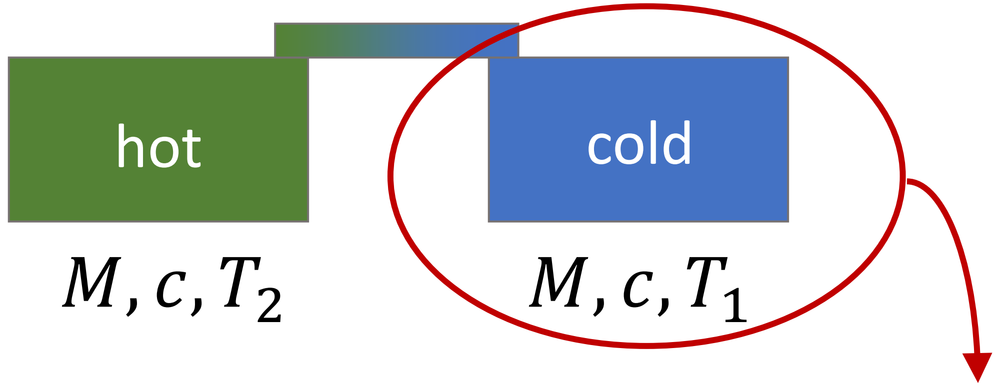
- *Third: How fast does the temperature change?*

  Divide by dt: $\frac{dT_1}{dt} = \frac{H_1}{Mc}$

$$\boxed{Q = m\, c\, \Delta T} \qquad \boxed{H = k\, A\, \frac{T_H - T_C}{L}}$$

A heat current H_1 flows into the cooler block.

In a time dt, what is the change dT in the temperature of the cooler block (in terms of dt and the quantities shown)?



$$dT_1 = \frac{H_1}{Mc}\, dt \quad (1)$$

➢ *Now, consider the strip:*

Q: Are all H's the same? If so, why?

A: Yes! All H's are the same by energy conservation:

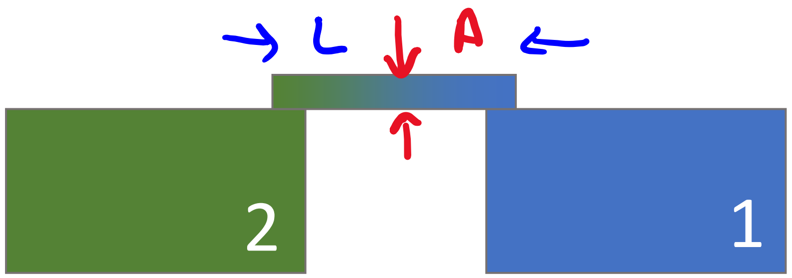
$$H_1 = H_2 = H = k\,A\,\frac{T_2 - T_1}{L} \quad (2)$$

➢ *Finally, combining (1) and (2) we get:*

$$\boxed{dT_1 = \underbrace{\frac{kA}{McL}(T_2 - T_1)}_{\text{Slope!}}\, dt}$$

$$\boxed{Q = m\,c\,\Delta T}$$

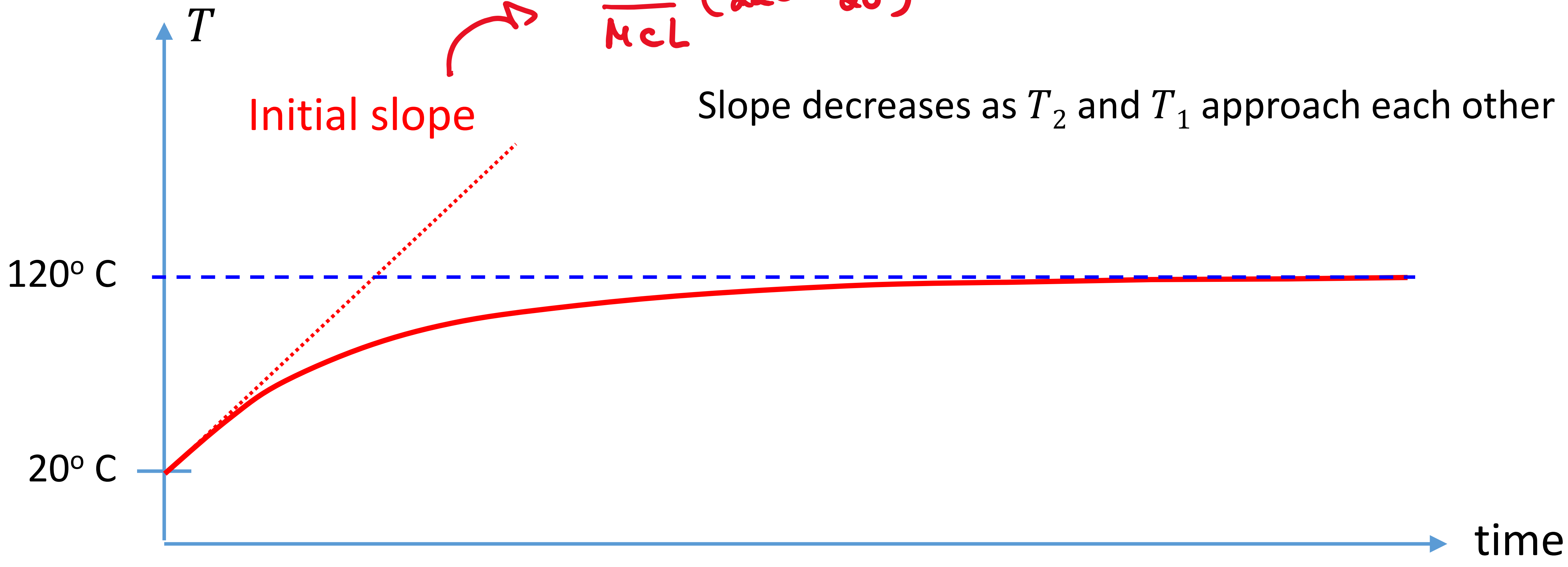
$$\boxed{H = k\,A\,\frac{T_H - T_C}{L}}$$



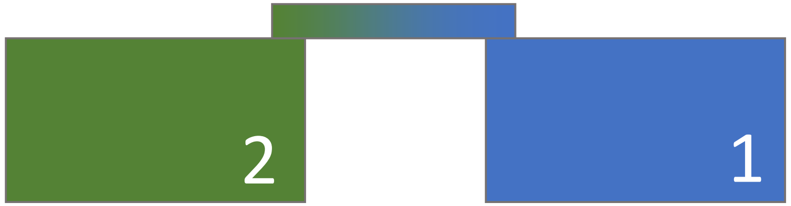
$$dT_1 = \frac{kA}{McL}(T_2 - T_1)\, dt$$

$T_{2,i}$ $T_{1,i}$

$$\frac{kA}{McL}(220^\circ - 20^\circ)$$



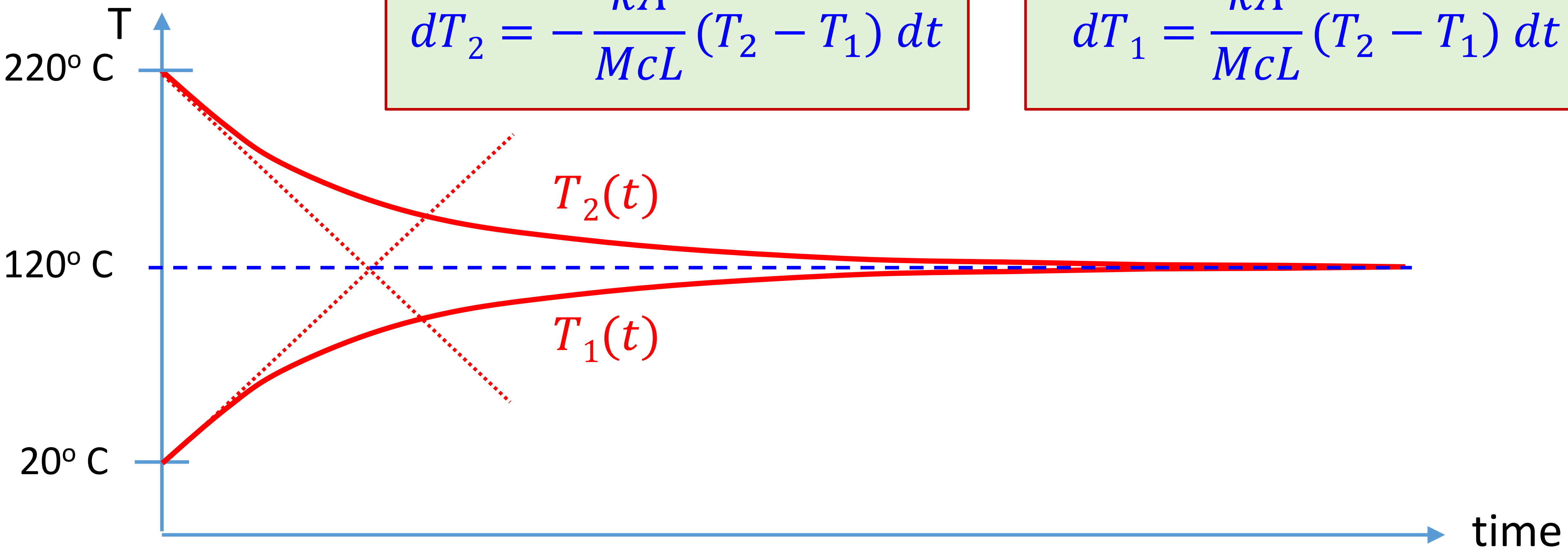
Q: How fast does the temperature of the hot block (T_2) change?



- M and c are the same for both blocks, but H is negative for block 2 (heat is leaving):

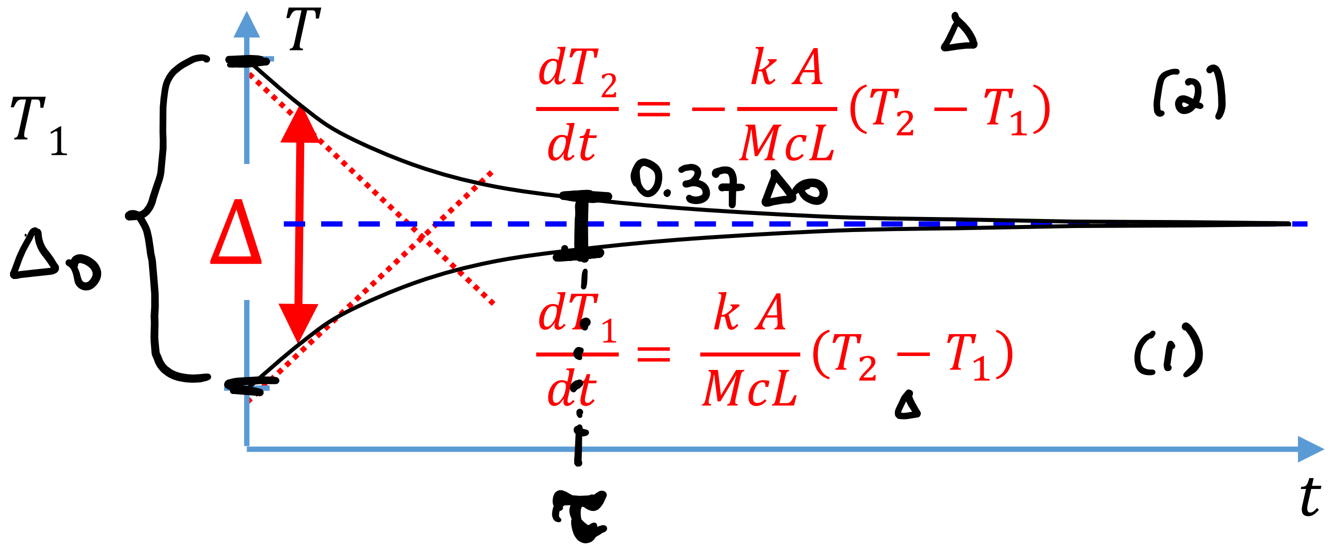
$$dT_2 = -\frac{kA}{McL}(T_2 - T_1)\, dt$$

$$dT_1 = \frac{kA}{McL}(T_2 - T_1)\, dt$$



Full time dependence

- At small times: (2) − (1)



- The gap between the two curves $\Delta = T_2 - T_1$ decreases twice as fast as T_1 increases:

$$\frac{d(T_2 - T_1)}{dt} = \frac{d\Delta}{dt} = -\frac{2k\,A}{McL}\Delta$$

- Rate of decrease of Δ is proportional to Δ
- Math: This means $\Delta(t)$ is an EXPONENTIAL
- The solution of this equation is: $\Delta(t) = \Delta_{t=0} \cdot e^{-\left(\frac{2kA}{McL}\right)t} = \Delta_{t=0} \cdot e^{-t/\tau}$ ($\frac{2kA}{McL} = 1/\tau$)

$$\Delta(t=\tau) = \Delta_0 e^{-\frac{\tau}{\tau}} = \Delta_0 e^{-1} = \frac{\Delta_0}{e} = \frac{\Delta_0}{2.71} = 0.37\Delta_0$$

- Here $\boxed{\tau = \frac{McL}{2kA}}$ is called the time constant
- Physical meaning: Δ drops to ~37% of its original value in time τ — origin of 37%
- At small times, $\Delta(t) \approx -\Delta_{t=0} \cdot \left(\frac{t}{\tau}\right)$ is linear in t – as we got in the beginning!