Physics 158 HW-2 Hand-In Jan 21

Problem 1

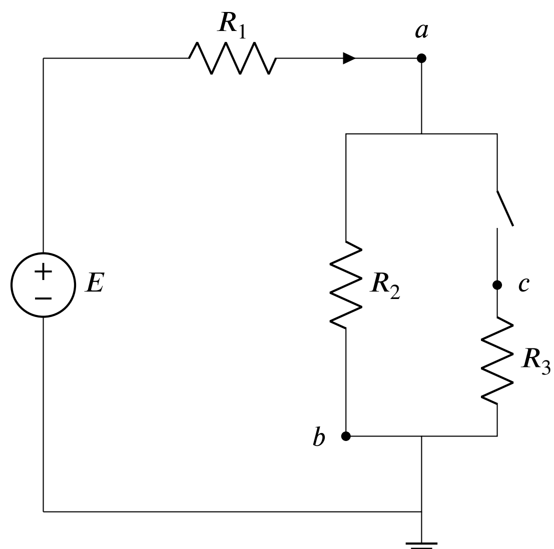
Difficulty: ★☆☆
If the switch is open,

a) Find all currents and potentials at the labelled points.

If the switch is then closed,

b) Find all currents and potentials at the labelled points.

$E = 12\,\text{V}$, $R_1 = 7\,\Omega$, $R_2 = 4\,\Omega$, $R_3 = 10\,\Omega$



Solution:

a) When the switch is open, there is a short circuit in the branch with R_3 so the circuit will act as a series circuit containing R_1, R_2, and E. The potential and voltage at point c will be 0.
The equivalent resistance will be $R_{\text{eq}} = R_1 + R_2$. The current can be computed from Ohm's law:

$$E = I_a R_{\text{eq}}$$
$$I_a = I_b = \frac{E}{R_{\text{eq}}} = \frac{E}{R_1 + R_2} = \frac{12\,\text{V}}{11\,\Omega} = \boxed{1.09\,\text{A}}$$

Point b is connected to ground so the potential of point b will be 0 V. The potential of point a can be computed as the voltage drop across R_2.

$$V_{R_2} = V_{ab} = V_a - V_b = V_a$$
$$V_{R_2} = I_a R_2 = \frac{12}{11}\,\mathrm{A} \cdot 4\,\Omega = \boxed{4.36\,\mathrm{V}}$$

b) When the switch is closed, R_2 and R_3 will be in parallel. We can compute the equivalent resistance for the circuit as

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{4} + \frac{1}{10} = \frac{7}{20} \Rightarrow R_{23} = \frac{20}{7}\,\Omega = 2.67\,\Omega$$
$$R_{\mathrm{eq}} = R_1 + R_{23} = 7 + \frac{20}{7} = \frac{69}{7}\,\Omega$$

Then we can compute the total current as

$$E = I_a R_{\mathrm{eq}} \Rightarrow I_a = \frac{E}{R_{\mathrm{eq}}} = \frac{12\,\mathrm{V}}{\frac{69}{7}\,\Omega} = \boxed{1.22\,\mathrm{A}}$$

We can use this to compute the volatge across R_1

$$V_{R_1} = I_a R_1 = 8.52\,\mathrm{V}$$

Then we can use Kirchoff's voltage law to find the remaining voltages

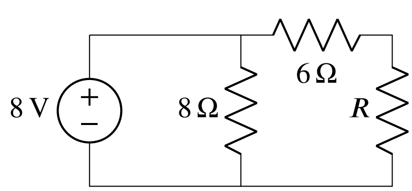
$$V_{R_2} = V_{R_3}$$
$$V_E = V_{R_1} + V_{R_2} \Rightarrow V_{R_2} = V_{R_3} = V_E - V_{R_1} = 3.48\,\mathrm{V}$$

From this we can get that the potential at c and the potential at a will be $V_a = V_c = \boxed{3.48\,\mathrm{V}}$ while the potential at b will be 0 V as in part a.
We can also use the voltages across the resistors to compute the currents in each branch.

$$I_b = \frac{V_{R_2}}{R_2} = \frac{3.48}{4} = \boxed{0.87\,\mathrm{A}}$$
$$I_c = \frac{V_{R_3}}{R_3} = \frac{3.48}{10} = \boxed{0.35\,\mathrm{A}}$$

Problem 2

Difficulty: ★☆☆
If the total power dissipated in the circuit is 15W, what is the value of R?



Solution:
The total power dissipated in the circuit will be the sum of the power dissipated in each resistor. We know that the power dissipated across a resistor is $P = IV$. We also know $V = IR$ and can rearrange to get $P = \frac{V^2}{R}$.
The voltage across the $8\,\Omega$ resistor is $8\,\mathrm{V}$ so power across that resistor will be

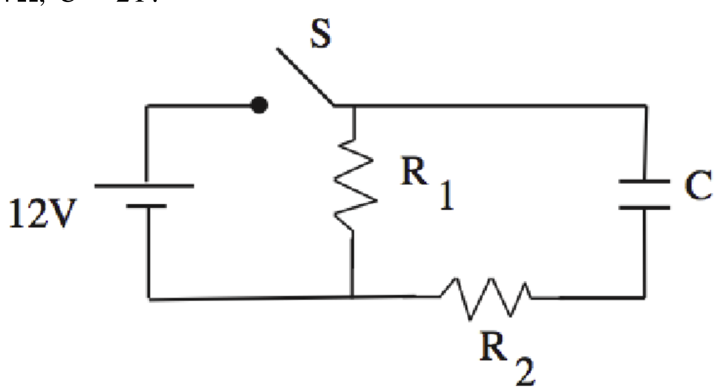
$$P_1 = \frac{(8\,\mathrm{V})^2}{8\,\Omega} = 8\,\mathrm{W}$$

The sum of the power across the remaining two resistors will be $P_{\text{total}} - P_1 = 15\,\mathrm{W} - 8\,\mathrm{W} = 7\,\mathrm{W}$.
We can combine resistor R and the $6\,\Omega$ to get an equivalent resistor with value $R + 6$. The volatge across this equivalent resistor will be $8\,\mathrm{V}$ and will have a power usage of $7\,\mathrm{W}$. We can then solve for R as

$$7\,\mathrm{W} = \frac{(8\,\mathrm{V})^2}{R + 6\,\Omega}$$
$$R = \frac{(8\,\mathrm{V})^2}{7\,\mathrm{W}} - 6\,\Omega = \boxed{3.14\,\Omega}$$

Problem 3

Difficulty: ★★☆
In the circuit shown below the switch has been open for a long time and there is no charge on the capacitor. Take $R_1 = 2\,\Omega$, $R_2 = 4\,\Omega$, $C = 2\,\mathrm{F}$.



a) The switch S is now closed. Find all currents just after the switch is closed.

b) Find all currents after the switch has been closed for a very long time.

c) After the switch has been closed for a very long time it is reopened. Calculate the current through R_2 as a function of time.

Solution:

a) The capacitor will want to initially act as a wire so we can analyze the circuit as two resistors in parallel. Due to Kirchoff's loop law, we can say that each resistor must have a voltage drop of $12\,\mathrm{V}$ and we can get the current of each from Ohm's law:

$$I_1 = \frac{\varepsilon}{R_1} = \frac{12}{2} = \boxed{6\,\mathrm{A}}$$

$$I_2 = \frac{\varepsilon}{R_2} = \frac{12}{4} = \boxed{3\,\text{A}}$$

b) After the switch has been closed for a long time, the capacitor will be fully charged and act as a short circuit. The circuit can then be analyzed as the loop going through the battery and R_1

$$I_1 = \frac{\varepsilon}{R_1} = \frac{12}{2} = \boxed{6\,\text{A}}$$
$$\boxed{I_2 = 0\,\text{A}}$$

c) After the switch is opened the current will flow through the loop containing R_1, R_2, and C. We can write the voltage loop equation as

$$0 = V_C + V_{R_1} + V_{R_2}$$

$$0 = \frac{q}{C} + iR_1 + iR_2$$

We know that $i = \frac{dq}{dt}$ and can take the derivative of both sides to get a 1st order differential equation and solve for $i(t)$

$$0 = \frac{i}{C} + \frac{di}{dt}(R_1 + R_2)$$
$$\frac{di}{dt} = -\frac{i}{(R_1 + R_2)C}$$
$$\frac{di}{i} = -\frac{dt}{(R_1 + R_2)C}$$
$$\int \frac{di}{i} = -\int \frac{dt}{(R_1 + R_2)C}$$
$$\ln|i| = -\frac{t}{(R_1 + R_2)C} + \text{Constant}$$
$$i = i_0 e^{-\frac{t}{(R_1+R_2)C}}$$

We can solve for the initial current by using our same voltage loop equation and knowing that the initial voltage across the capacitor is 12 V from the charge stored on it. The capacitor will be discharging so the potential in the equation can be thought of as negative.

$$0 = V_C + i_0(R_1 + R_2)$$
$$0 = -12 + 6i_0 \Rightarrow i_0 = 2\,\text{A}$$

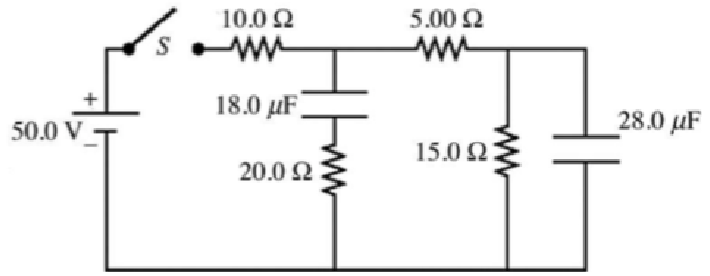
Plugging this all in we get,

$$\boxed{i(t) = 2e^{-\frac{t}{12}}\ \text{Amps}}$$

Problem 4

Difficulty: ★★☆

The circuit shown below initially has no charge on the capacitors and the switch S is originally open.



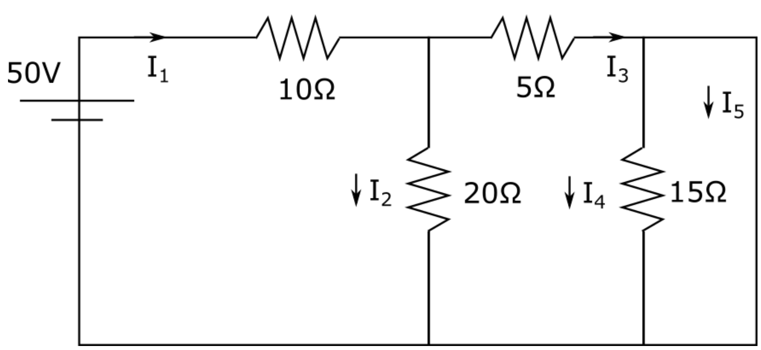
a) Just after closing the switch S, find all the currents.

b) After the switch has been closed for a very long time, find all the currents.

c) After the switch S has been closed for a very long time, find the potential difference across the 28.0 μF capacitor.

Solution:

a) just after we close the switch there is NO charge on the Capacitors. Hence we have the following circuit:



Note the Capacitors act like a wire when uncharged

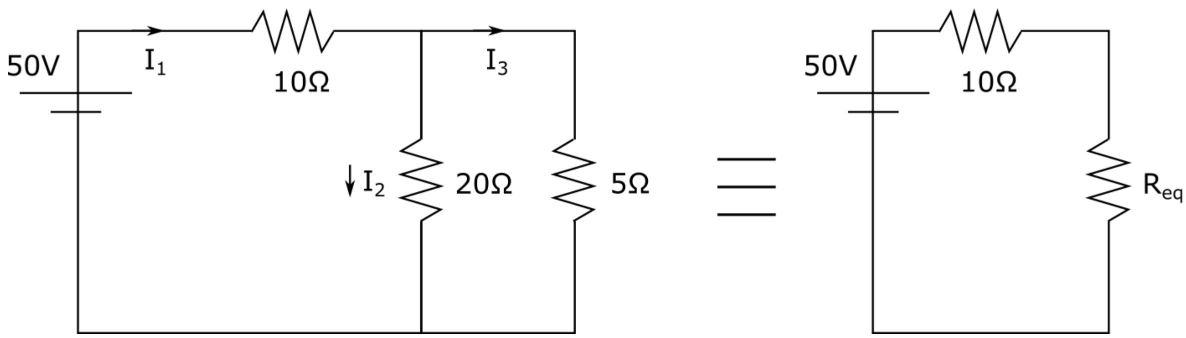
$K_1 \implies$ Hence $\underline{I_4 = 0}, I_1 = I_2 + I_3, I_3 = I_4 + I_5 = I_5$

$K_2 \implies 50 - 10I_1 - 20I_2 = 0$

$0 = -5I_3 + 20I_2$

You now have 3 equations and 3 unknowns. Solve

EQUIVALENT Circuit



$$\frac{1}{R_{eq}} = \frac{1}{20} + \frac{1}{5} = \frac{1}{20} + \frac{4}{20} = \frac{5}{20} = \frac{1}{4}$$
$$\rightarrow R_{eq} = 4\Omega$$

Hence $I_1 = \frac{50V}{14\Omega}$, Since $I_1 = I_2 + I_3$ and $20I_2 = 5I_3$ we have

$$\underline{I_1 = \frac{50}{14}\text{ Amps}} = 3.57\,A = I_2 + 4I_2 = 5I_2$$

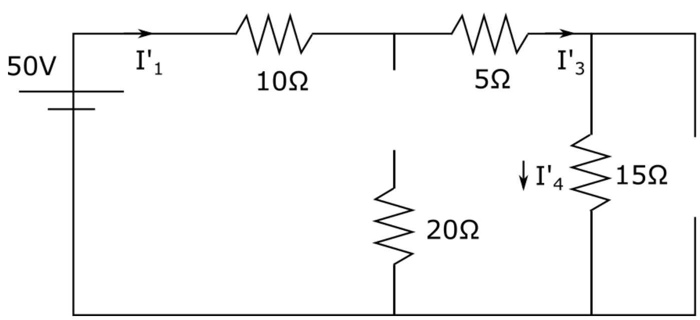
$$\therefore \underline{I_2 = \frac{10}{14}\text{ Amps}} = 0.714\,A$$

and finally

$$I_3 = 4I_2 = \underline{\frac{40}{14}\text{ Amps}} = 2.86\,A$$

b) after a long time the Capacitors are fully charged

$$\therefore I_2' = 0, I_5' = 0 \implies I_1' = I_3' = I_4'$$



$$\therefore \underline{\frac{50V}{30\Omega} = I_1' = \frac{5}{3}\text{Amp}}$$

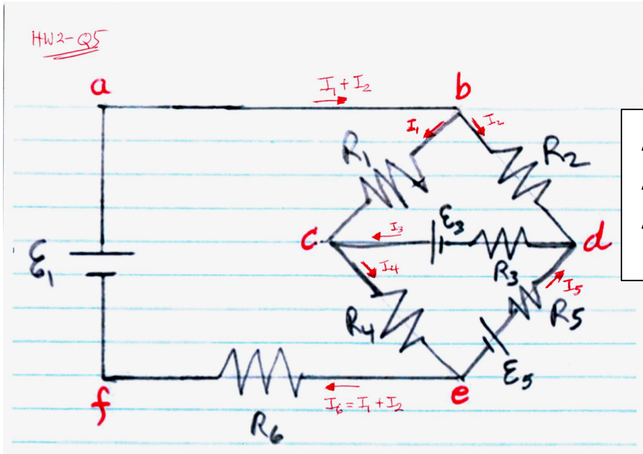
c) $V_{28\mu F} = V_{15\Omega} = I_4'(15\Omega) = \frac{5}{3}(15) = \underline{25V}$

5. (Difficulty: **)

Given the following circuit with resistors $R_1 = 1.0\Omega, R_2 = 2.0\Omega, R_3 = 3.0\Omega, R_4 = 4.0\Omega,\ R_5 = 5.0\Omega,$ $and\ R_6 = 6.0\Omega, with\ \mathcal{E}_1 = 10.0V, \mathcal{E}_3 = 10.0V, and\ \mathcal{E}_5 = 15.0V.$

(a) What is the current I_2 passing through resistor R_2 ??

(b) What is the current I_5 passing through resistor R_5 ?? (give both magnitude and direction)



Apply K1 Junction rule at c → $I_4 = I_1 + I_3$

Apply K1 Junction rule at d → $I_5 = I_3 - I_2$

Apply K1 Junction rule at e → $I_6 = I_4 - I_5 = I_1 + I_3 - (I_3 - I_2)$

$I_6 = I_1 + I_2$

Apply K2 Loop rule to abcefa, abdefa, bdcb (all CW)

$\mathcal{E}_1 - I_1R_1 - I_4R_4 - I_6R_6 = 0 = \mathcal{E}_1 - I_1R_1 - (I_1 + I_3)R_4 - (I_1 + I_2)R_6$

$\mathcal{E}_1 - I_2R_2 + I_5R_5 - \mathcal{E}_5 - I_6R_6 = 0 = \mathcal{E}_1 - I_2R_2 - (I_3 - I_2)R_5 - 15 - (I_1 + I_2)R_6$

$0 - I_2R_2 - I_3R_3 + \mathcal{E}_3 + I_1R_1 = \mathcal{E}_3 - I_2R_2 - I_3R_3 + I_1R_1$

Substitute the known values for R_i and $\mathcal{E}_i$ and write in standard form

$11\ I_1 + 6\ I_2 + 4\ I_3 = 10$

$-6\ I_1 - 13\ I_2 + 5\ I_3 = 5$

$I_1 - 2\ I_2 - 3\ I_3 = -10$

Solve using Cramer's Rule as ratio of Determinants

$$I_1 = \frac{\begin{vmatrix} 10 & 6 & 4 \\ 5 & -13 & 5 \\ -10 & -2 & -3 \end{vmatrix}}{\begin{vmatrix} 11 & 6 & 4 \\ -6 & -13 & 5 \\ 1 & -2 & -3 \end{vmatrix}} = \frac{10(49) - 6(35) + 4(140)}{11(49) - 6(13) + 4(25)} = \frac{-280}{561} = \boxed{-0.50A}$$

Solve for $I_2 = 0.847A$ ↘ , $I_3 = 2.60A$ ←

Hence $I_4 = 2.10A$ ↘ , $I_5 = 1.75A$ ↗ as shown