Announcement

From: Engineering Undergraduate Society

To celebrate the end of midterms, we are putting on one more event called "PP semi" which is essentially a semi formal dance for first year engineering students and their plus ones. The event will take place on the 27th of March from 7:30-10:00pm.

PHYS 170

Week 11: Work and Energy

Section 201 (Mon Wed Fri 12:00 – 13:00)

What is energy?

Extra reading topic

...The law is called the *conservation of energy*. It states that there is a certain quantity, which we call energy, that does not change in the manifold changes which nature undergoes. ... It is not a description of a mechanism, or anything concrete; it is just a strange fact that we can calculate some number and when we finish watching nature go through her tricks and calculate the number again, it is the same.

A story of a child who has 28 identical blocks, and his mother who puts him with his 28 blocks into a room at the beginning of the day. At the end of the day, being curious, she counts the blocks very carefully, and discovers a phenomenal law—no matter what he does with the blocks, there are always 28 remaining!

- 26 blocks: the window was open.
- 30 blocks: a friend came by with his blocks.
- 25 blocks: locked toy box, and

  (number of blocks seen) + [(weight of box)–16 ounces]/(3 ounces) = constant.
- dirty water in the bathtub is changing its level, and

(number of blocks seen) + [(weight of box)–16 ounces]/(3 ounces)

+ [(height of water) – 6 inches] / (1/4 inch) = constant

- ... etc.

http://www.feynmanlectures.caltech.edu/I_04.html Richard Feynman, v.1, Lecture 4, “Conservation of energy”

ENERGY AT A GLANCE

PHYS 158

K

U

W

Notations (from text):

- Kinetic energy: T
- Potential energy: V
- Work: U

There are various kinds of energy:

- Kinetic energy is due to motion: $T = mv^2/2$

- Potential energy is due to interaction between objects. Related to the concept of work.
  - Gravitational energy: $V^{(g)} = mgh$

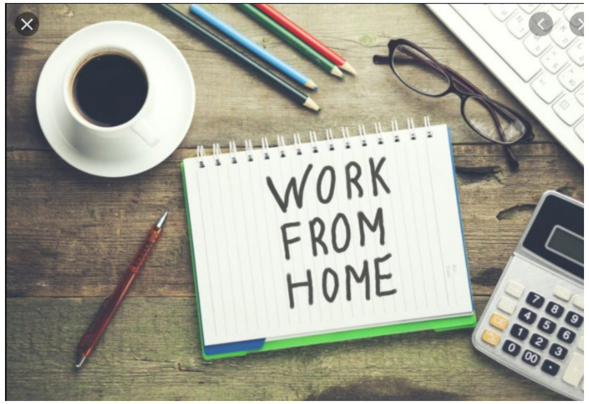
    (attraction between the Earth and the object)
  - Elastic energy: $V^{(s)} = k\frac{\Delta x^2}{2}$

    (interaction between atoms and molecules in a spring that tends to preserve its shape)
  - Electric energy of two charges: $V^{(el)} = K\frac{q_1 q_2}{r}$

    (attraction/repulsion between two charges)

- Total energy of a <u>closed</u> system is always conserved.

Work of a Force

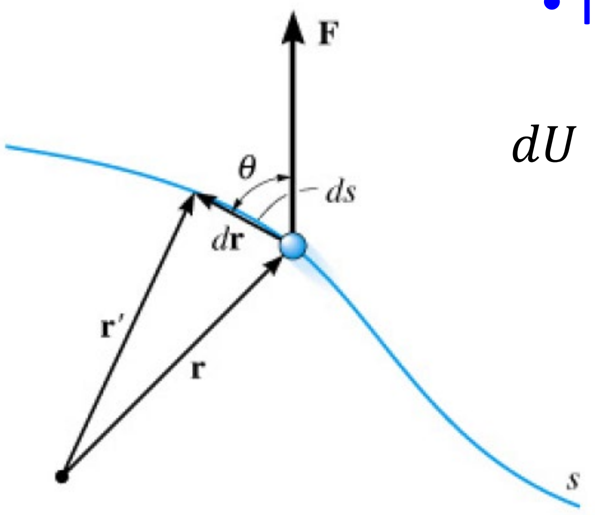


Text: 14.1

Content:

- Definition of the work of a force
- Sign of the work of a force
- Work of various forces (weight, elastic force)

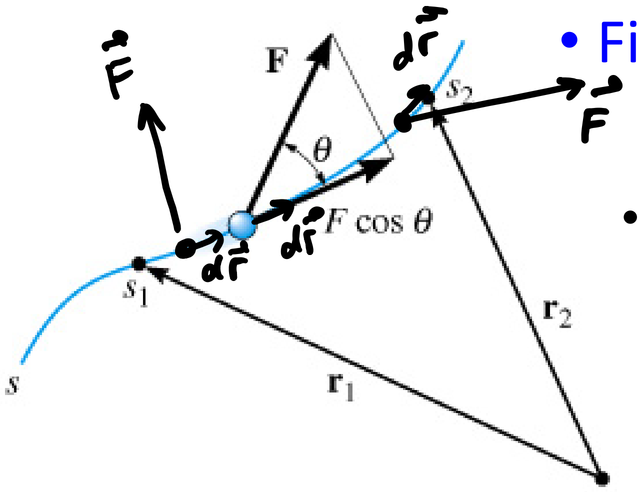
WORK OF A FORCE: definition



- Infinitesimal displacement:

dU = work of the force $\vec{F}$ acting on the object, which is displaced by $d\vec{r}$

$$\boxed{dU = \vec{F} \cdot d\vec{r} = F\, ds \cos\theta}$$



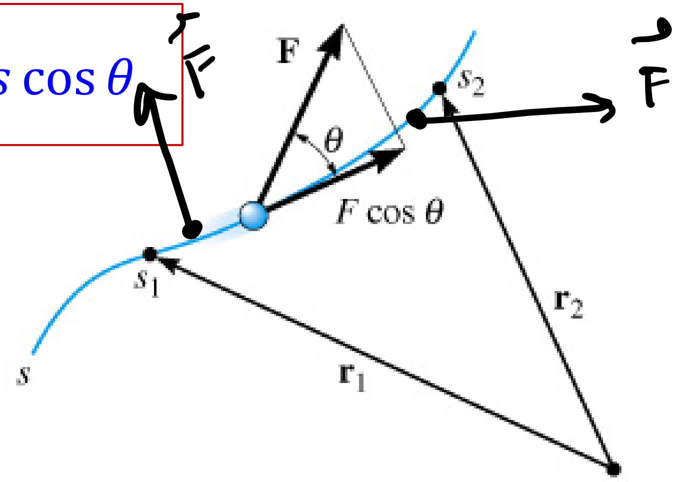
- Finite displacement: $U = \int dU$

- Work done by a force $\vec{F}$ acting on the object that travels from point 1 to point 2:

$$\boxed{U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = \int_{s_1}^{s_2} F\, ds \cos\theta}$$

WORK OF A FORCE: Properties

$$\boxed{U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = \int_{s_1}^{s_2} F\, ds \cos\theta}$$



1) Work is a scalar quantity

2) Units:

- SI: Joules (J) 1 J = 1 N · 1 m
- FPS: lb· ft

3) Sign depends on the mutual orientation of the force $\vec{F}$ and the caused displacement $d\vec{r}$:

- $0 \leq \theta < 90^o$: $\cos\theta > 0$ => $\vec{F}$ and $d\vec{r}$ have the same sense => Work is positive
- $90^o < \theta \leq 180^o$: $\cos\theta < 0$ => $\vec{F}$ and $d\vec{r}$ have opposite sense => Work is negative
- $\theta = 90^o$: $\cos\theta = 0$ => $\vec{F}$ and $d\vec{r}$ are orthogonal => Work is zero

4) Note that in general $\vec{F} = \vec{F}(s)$, so that $U_{1\to 2} = \int_{s_1}^{s_2} F(s) \cos\theta(s)\, ds$ ☹

… but it considerably simplifies when: $\vec{F} = const$ (uniform force):

$$U_{1\to 2} = \vec{F} \cdot \vec{r}_{12} \quad ☺$$

Q: In this picture, at the instant shown, the work on the barbell is done by:

A. Gravitational force
B. The sportsman
C. Both
D. Nobody
E. Not sure

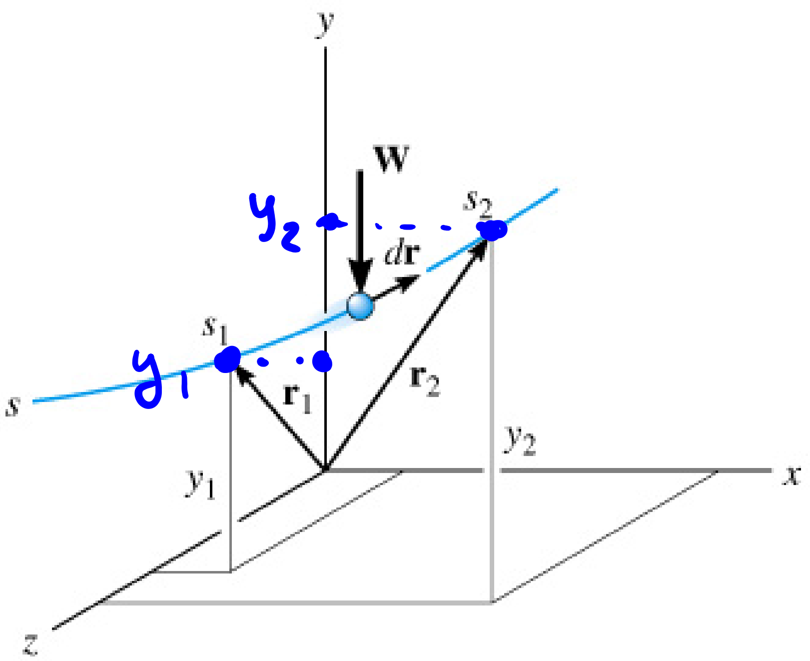
$$U = \vec{F} \cdot \underline{\underline{\Delta \vec{s}}}$$

Example: WORK OF THE GRAVITY FORCE (WEIGHT)

$$U_{1\to 2} = \int_{\vec{r}_1}^{\vec{r}_2} \underbrace{\vec{F}\cdot d\vec{r}}$$

$$\vec{j}\cdot\vec{i} = \vec{j}\cdot\vec{k} = 0$$

$$\vec{j}\cdot\vec{j} = 1$$



$$d\vec{r} = \vec{i}\,dx + \vec{j}\,dy + \vec{k}\,dz$$

$$\vec{W} = -mg\,\vec{j}$$

$$(-mg\,\vec{j})\cdot(\vec{i}\,dx + \vec{j}\,dy + \vec{k}\,dz)$$

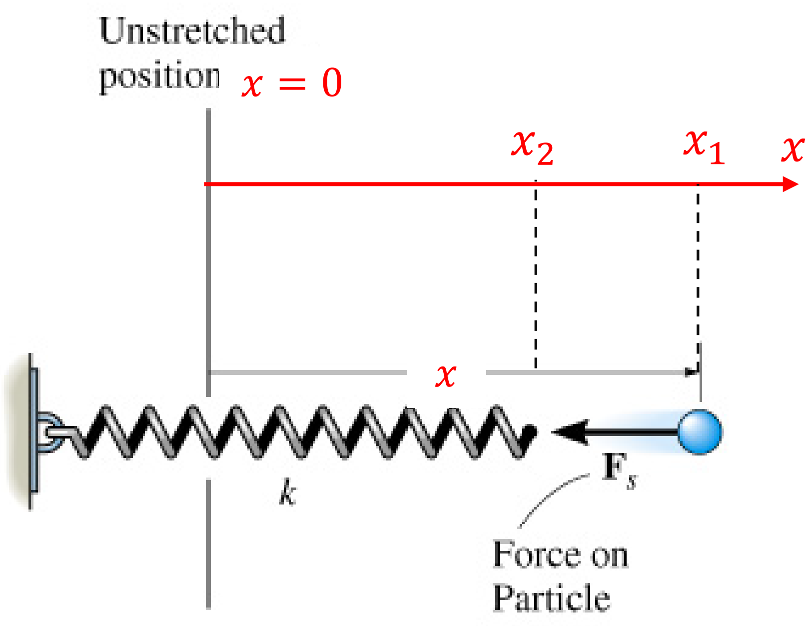
$$= -mg\,dy$$

$$U_{1\to 2}^{(g)} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{W}\cdot d\vec{r} = -mg\int_1^2 dy = -mg(y_2 - y_1) = mg(y_1 - y_2)$$

- Work of the gravity force is positive when $y_1 > y_2$, i.e. when motion is downwards
  - Indeed, then the gravity force is parallel to the object's displacement
- The work of the gravity force only depends on the heights of the two points (not on the path)

Example: WORK OF AN ELASTIC FORCE

$$U_{1\to2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$$



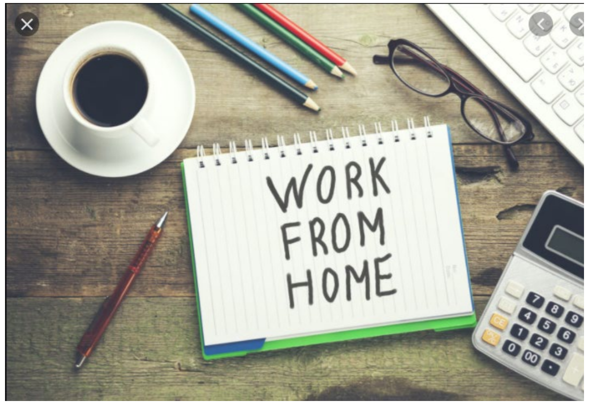
$$d\vec{r} = \vec{i}\,dx$$

$$\vec{F}_s(x) = -kx\,\vec{i}$$

$$\vec{F}_s \cdot d\vec{r} = -kx\,dx$$

$$U_{1\to2}^{(s)} = \int_{x_1}^{x_2} \vec{F}_s\cdot d\vec{r} = -k\int_{x_1}^{x_2} x\,dx = -k\left(\frac{x_2^2}{2} - \frac{x_1^2}{2}\right) = \frac{kx_1^2}{2} - \frac{kx_2^2}{2}$$

- Work of the elastic force is positive when $x_1 > x_2$, i.e. when the spring goes to its equilibrium position
    - Indeed, then the elastic force is parallel to the object’s displacement
- The work of elastic force only depends on initial and final positions of the particle



Work-Energy Principle

Text: 14.2-3

Content:

- Change of object's kinetic energy is equal to the work of all forces acting on it
- Applications: Finding velocity

Kinetic energy and Work

What if the speed of the object changes?

Changing speed means non-zero acceleration

$dv = a\,dt$

Non-zero acceleration means non-zero external net force (from somewhere)

$\vec{F}_R = m\vec{a}$

Non-zero external net force means that a work was done on the object

$U_{1\to 2} = \int_1^2 \vec{F}_R \cdot d\vec{r}$

Changing speed means changing kinetic energy of the object

$T = \dfrac{m\,v^2}{2}$

Change of kinetic energy of an object can be linked to the work done on it

Net work done on an object = change in its kinetic energy

$$a_t\, ds = v\, dv$$

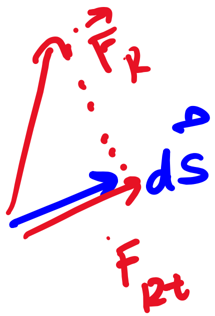
W7-1

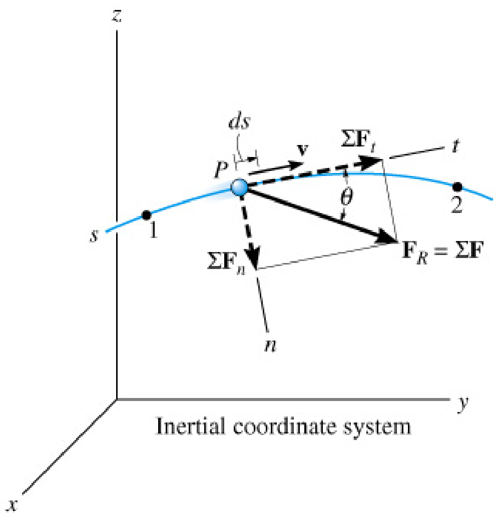
$$m a_t\, ds = m v\, dv$$

$$F_{R,t}\, ds = m v\, dv$$

$$\vec{F}_R \cdot d\vec{r} = m v\, dv$$

$$\underbrace{\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_R \cdot d\vec{r}}_{U_{1\rightarrow 2}} = \int_{v_1}^{v_2} m v\, dv = \underbrace{\frac{m v_2^2}{2} - \frac{m v_1^2}{2}}_{T_2 - T_1}$$





$$\boxed{\Delta T = \int_1^2 \vec{F}_R \cdot d\vec{r} = U_{1\rightarrow 2}}$$

Work of all the forces acting on the particle (= work of $\vec{F}_R$)

Change in the particle's kinetic energy between points 1 and 2: $\Delta T = T_2 - T_1$

WORK-ENERGY PRINCIPLE: Summary

$T_2 - T_1$

work!

$$\boxed{\Delta T = U_{1\rightarrow 2} = \int_1^2 \vec{F}_R \cdot d\vec{r}}$$

$$\boxed{T_2 = T_1 + U_{1\rightarrow 2}}$$

$$\frac{m v_2^2}{2} = \frac{m v_1^2}{2} + U_{1\rightarrow 2}$$

- In words: final kinetic energy of the particle is equal to the initial kinetic energy of the particle plus the work done by all the forces acting on the particle as it moves from its initial to final position.

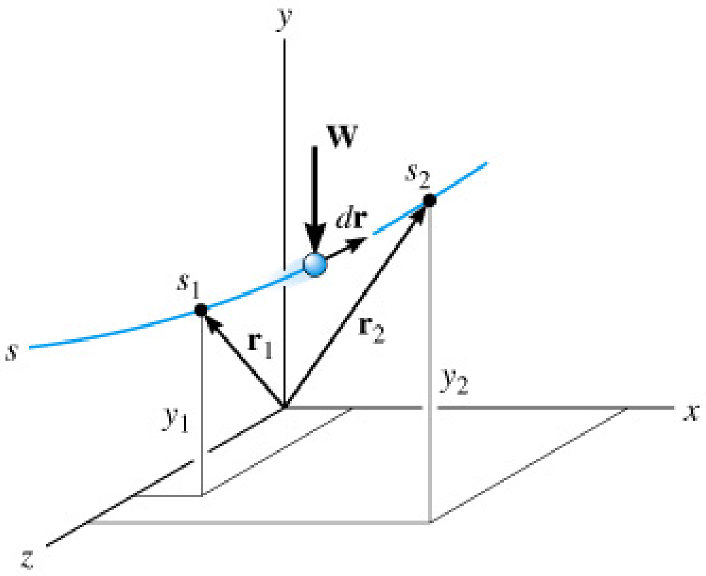
- Especially useful when final or initial speeds of particles are to be determined

- In the derivation of work-energy principle we made use of $\sum F_t = ma_t$ => work-energy principle can be used instead of $\sum F_t = ma_t$ (convenient when you need to find *velocity*, not acceleration a_t)

- Note that all the information about normal forces is NOT captured by this approach: equation $\sum F_n = ma_n$ remains valid and useful (can be used to find some unknowns)

- Section 14-3: Read about work and energy principle for a system of particles and about work of friction forces
- Section 14-4, Power and efficiency: you can skip it (or read if you have time)

WORK-ENERGY PRINCIPLE and GRAVITATIONAL POTENTIAL ENERGY



- Assume the only force that acts on the particle is the gravity force. Then:

$$T_2 = T_1 + U_{1\to2}^{(g)} = T_1 + mgy_1 - mgy_2$$

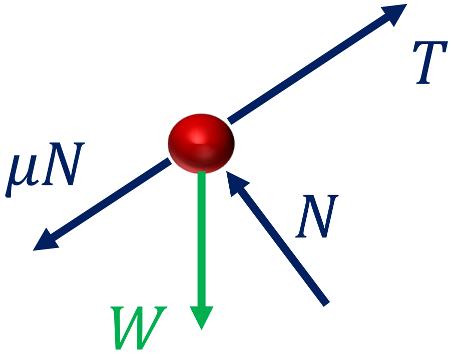
$$U_{1\to2}^{(g)} = -W\int_1^2 dy = mg(y_1 - y_2)$$ (see slide #8)

$$\boxed{T_2 + mgy_2 = T_1 + mgy_1}$$

- *Well-known formula: kinetic + potential (gravitational) energy conserve when there are no other forces acting on the particle*

- General case (when THERE ARE other forces aside from gravity):

  - $U_{1\to2} = U_{1\to2}^{(g)} + U_{1\to2}^{(\text{All\_but\_Gravity})}$
  - $\boxed{T_2 + mgy_2 = T_1 + mgy_1 + U_{1\to2}^{(\text{All\_but\_Gravity})}}$

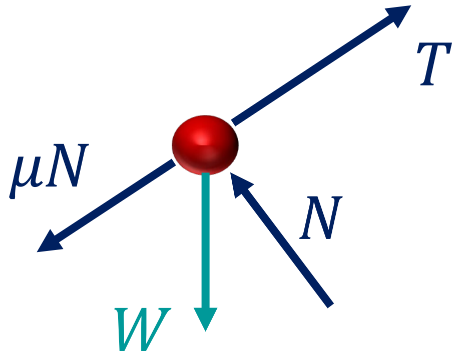


- *We will soon expand this idea of potential energy to other "conservative" forces*

WORK-ENERGY PRINCIPLE and GRAVITATIONAL POTENTIAL ENERGY

- Let us slow down and clearly state what we have done:

- Work-energy principle, general form:



$$\boxed{T_2 = T_1 + U_{1\to 2}^{(\text{All forces})}}$$

- Out of all the forces, we single out the gravity force (Q: why??) and calculate its work explicitly.

- We get:

$$\boxed{T_2 + \underline{mgy_2} = T_1 + \underline{mgy_1} + U_{1\to 2}^{(\text{All\_but\_gravity})}}$$

- *Since we account for the work of the gravity force by explicitly including the gravitational potential energy term, mgy, we don't double-count it in the work term, $U_{1\to 2}$.*

Q: A handle 20 inches long is used to turn the screw, which has 10 threads to the inch. We would like to know how much force would be needed at the handle to lift one ton (2000 pounds) by one inch.

(*Feynman's lectures*)

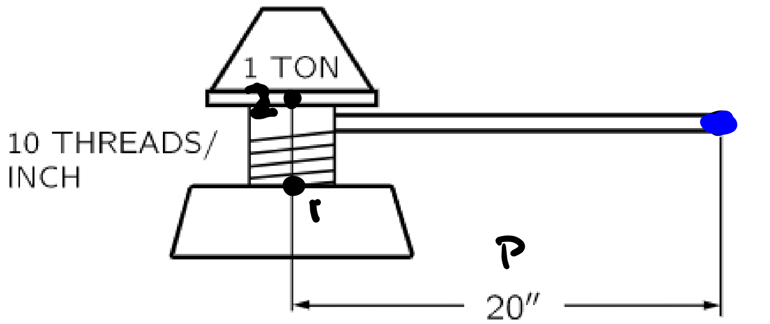


Fig. 4–5. A screw jack.

$$\underbrace{\cancel{T_2}^{\,0} + mgy_2} = \underbrace{\cancel{T_1}^{\,0} + mgy_1} + U_{1\to 2}^{(AbG)\to P}$$

$$mg(y_2 - y_1) = U_{1\to 2}^{(P)} = \int_1^2 \vec{P}\cdot d\vec{r} = \int_1^2 P\,dr = P\cdot 2\pi R\cdot 10$$

$$P = \frac{mg\,\Delta y}{10\cdot 2\pi R} = \frac{(2000\ \text{lb})(1'')}{10\cdot 2\cdot \pi\cdot (20'')} = 1.59\ \text{lb}$$

A. 0.5 lb
B. 1.6 lb
C. 2.7 lb
D. 5.7 lb
E. Not sure

$$\boxed{T_2 + mgy_2 = T_1 + mgy_1 + U_{1\to 2}^{(\text{All\_but\_gravity})}}$$

https://www.feynmanlectures.caltech.edu/I_04.html