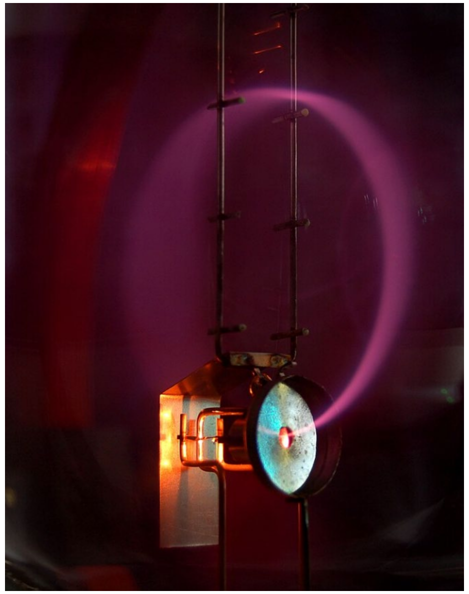
Lecture 25.

Magnetic field.
Magnetic force.

Magnetic field exists!

- Magnets fill the space around them with a "magnetic influence" (= "magnetic field")

- Attract objects made of some materials (act at a distance)



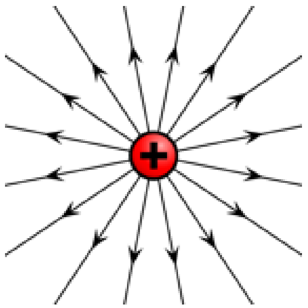
- Deflect charged particles (beam of electrons moving in a "magnetic field")

$$\vec{F} = q_{\pm}\, \vec{v} \times \vec{B}$$

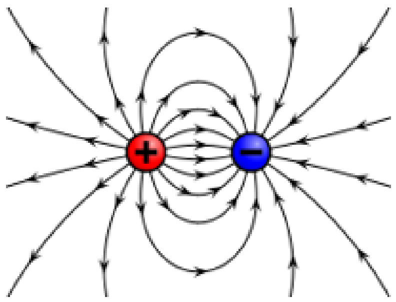
- Force acting on a charge $q_{\pm}$ moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$.

Electric vs Magnetic field sources

- There are no magnetic charges (monopoles). The most basic magnetic field source is a dipole. This is one reason why the magnetic force law is complicated.
- The magnetic field lines always exit from the North pole and they always enter at the South pole of a magnet.
- Magnetic field lines always form closed loops.



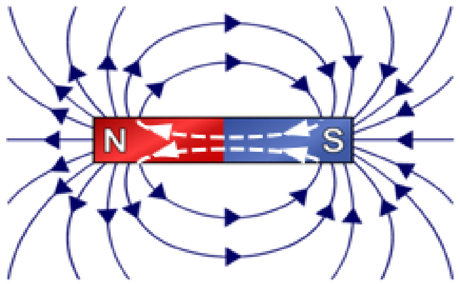
&



vs

N/A

&



Typical magnitudes for B field

- **Units:** The magnetic field is measured in units called Tesla (denoted by T).

  Since a 1 Tesla magnetic field is very large, B is sometimes measured in Gauss denoted by G.

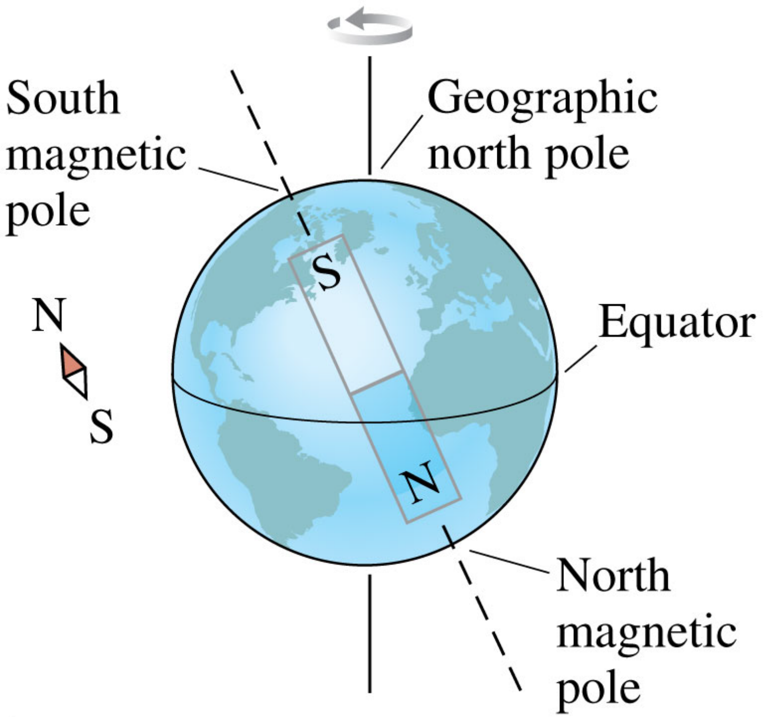
  1 Tesla = 10^4 G

Some examples of B-field values:

• Interstellar Space	10^{-10} Tesla
• Surface of Earth	10^{-5} T
• Refrigerator magnet	10^{-2} T
• Surface of Sun	10^{-1} T
• Surface of pulsar	10^{8} T
• Near atomic nucleus	10^{12} T

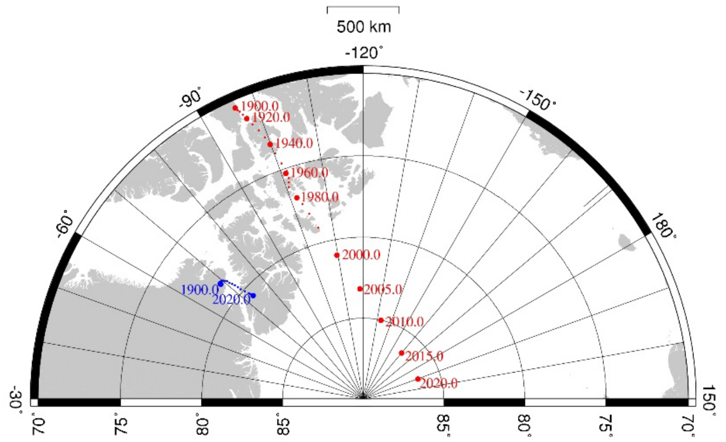
Magnetic field of the Earth

- Due to currents in the molten iron core, the earth itself acts as a large magnet.
- The poles are slightly offset from the poles of the rotation axis.
- The geographic north pole is actually a *south* magnetic pole!



From Wikipedia:

The North Magnetic Pole moves over time due to magnetic changes in [Earth's core] … In 2009, while still situated within the Canadian Arctic territorial claim at [84.9°N 131.0°W], it was moving toward [Russia] at between 55 and 60 kilometres per year. As of 2019, the pole is projected to have moved beyond the Canadian Arctic territorial claim to [86.448°N 175.346°E].

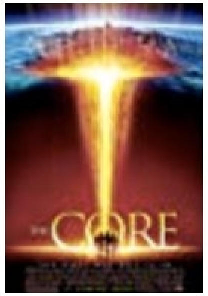


http://www.geomag.bgs.ac.uk/education/poles.html

Earth’s Magnetic Field

One of the most mysterious aspects of Earth's magnetic field is that once every 200,000 years, on average, it reverses. It also periodically grows weak, then strong again. This is not just theory. The magnetic field's meandering history is indelibly inscribed in rocks: Ferrous minerals, especially magnetite, in one stratum show strikingly different orientations from those in another.

Nuclear Planet -- geophysicist J. Marvin Herndon



The Core (2003)

Plot

Movie 2003 -- The core

https://www.discovermagazine.com/the-sciences/nuclear-planet

https://www.businessinsider.com/earth-north-south-poles-flip-magnetic-field-2018-4

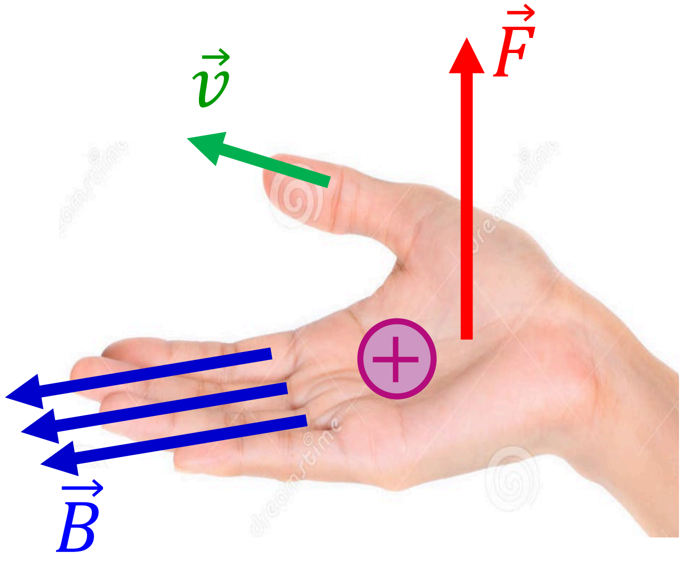
Magnetic force

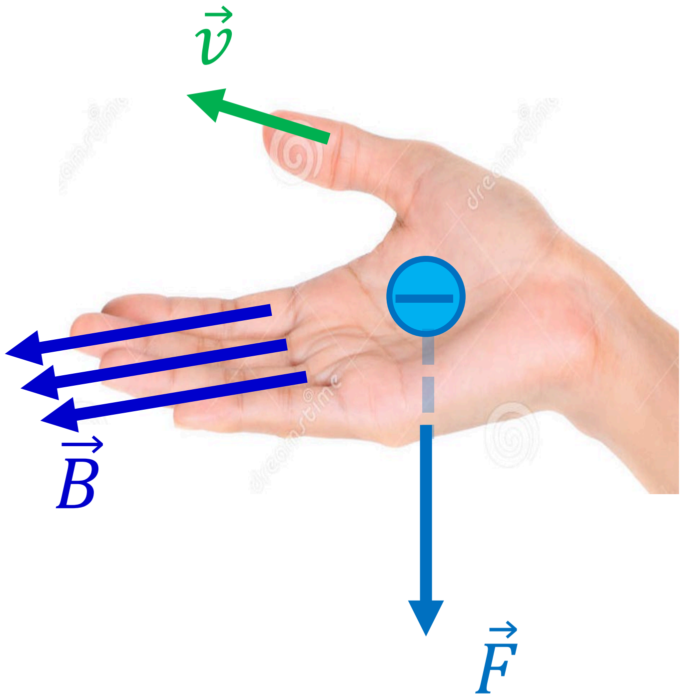
$$\boxed{\vec{F}_{B \text{ on } q} = q_{\pm}\, \vec{v} \times \vec{B}}$$

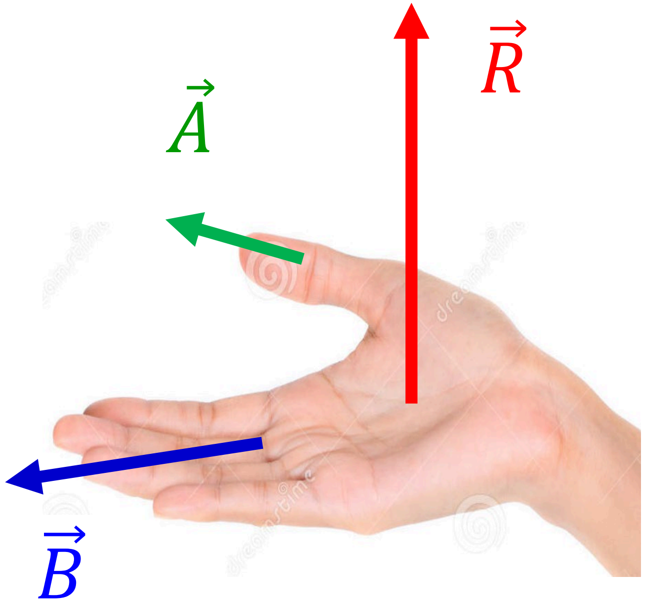
$$\boxed{\vec{A} \times \vec{B} = \vec{R}}$$

First Second Result

Note $q_{\pm}$: direction depends on the sign of the charge!

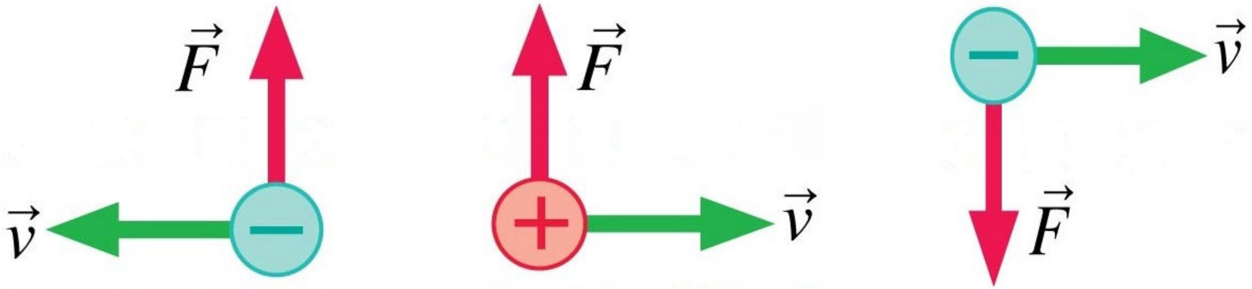






Open-palm version (my favorite)

Q: In which of the three cases shown below is the magnetic field direction pointing into the page?



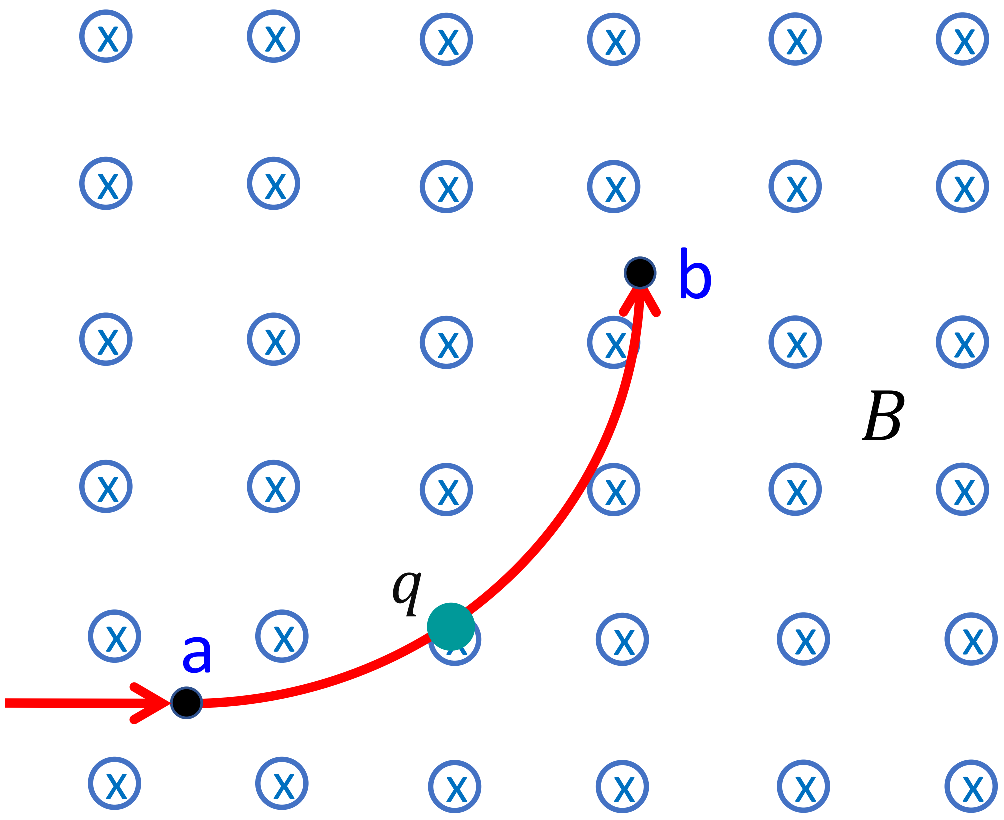
1 2 3

A. Only 1
B. Only 2
C. Only 3
D. 1 and 2
E. 1, 2, 3

$$\boxed{\vec{F}_{B\text{ on }q} = q_{\pm}\,\vec{v} \times \vec{B}}$$

Q: A charged particle with a charge $|q|$ enters a region with a constant B-field pointing into the page as shown. If the particle follows the path from a to b as shown, what is the sign of q?

$$\vec{F}_{B \text{ on } q} = q_{\pm}\, \vec{v} \times \vec{B}$$



A. Positive

B. Negative

C. Not enough info to answer

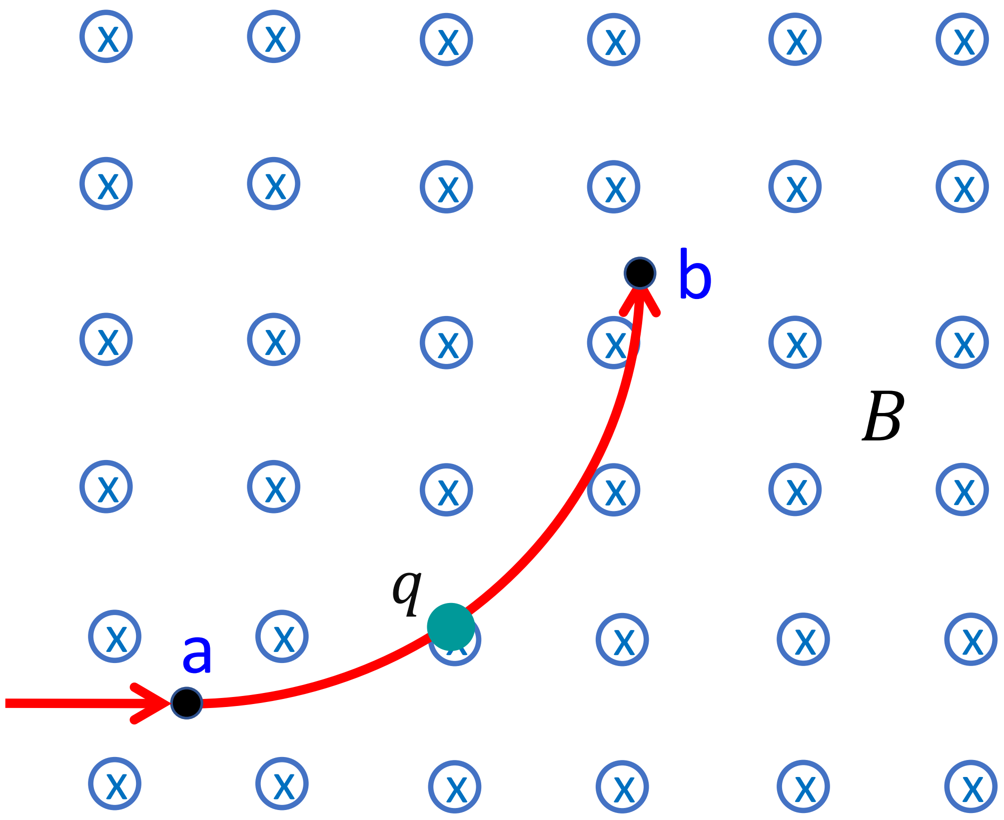
Q: A charged particle with a charge $|q|$ enters a region with a constant B-field pointing into the page as shown. If the particle follows the path from a to b as shown, its kinetic energy

$$\vec{F}_{B \text{ on } q} = q_{\pm}\, \vec{v} \times \vec{B}$$

Energy conservation

$$\Delta K + \Delta U = 0$$

$$\Delta U = -W_B$$



A. increases
B. decreases
C. stays the same
D. not enough info to answer

Force and Work done by B-field

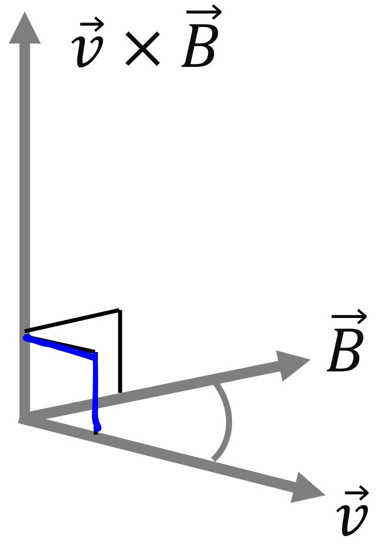
$$d\vec{r} = \vec{v}\,dt$$

- Force acting on the charge due to B-field: $\vec{F}_B = q_{\pm}\,\vec{v} \times \vec{B}$

- Work of that force: $W_B = \int \vec{F}_B \cdot d\vec{r} = \int \left(q_{\pm}\,\vec{v} \times \vec{B}\right) \cdot d\vec{r} = q_{\pm} \int \left(\vec{v} \times \vec{B}\right) \cdot \vec{v}\;\, dt$

- Now: $\left(\vec{v} \times \vec{B}\right) \cdot \vec{v} \equiv 0,$ always!

since the angle between $\vec{v} \times \vec{B}$ and $\vec{v}$ is 90°.



- We get that the work of magnetic force is always zero: $W_B = 0$

$W_B = 0 \;\Rightarrow\; \Delta U = 0 \;\Rightarrow\; \Delta K = 0$ (total energy – kinetic plus potential – conserves)

- Kinetic energy conserves => the particle moves with a constant speed.

Force and Work done by B-field

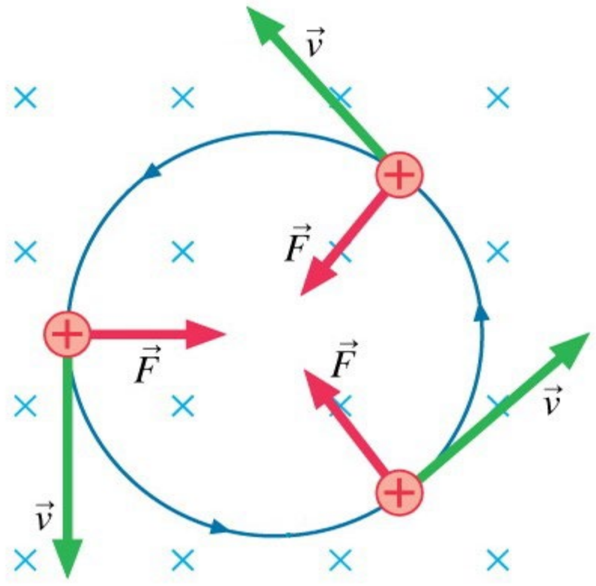
- Kinetic energy conserves => the particle moves with a constant speed.

- Wait… There is a force acting on the particle, $\vec{F}_B = q_\pm \, \vec{v} \times \vec{B}$ ⇨

there must be acceleration, $\vec{F}_B = m\vec{a}$!

- How then the particle can move with a constant speed ??

- The only way the speed can be constant is if the particle moves in a circle (uniform circular motion). The direction of the velocity changes, but the magnitude remains the same.



Cyclotron motion

$|\vec{v} \times \vec{B}| = vB\sin\theta$

$\downarrow$

$90°$

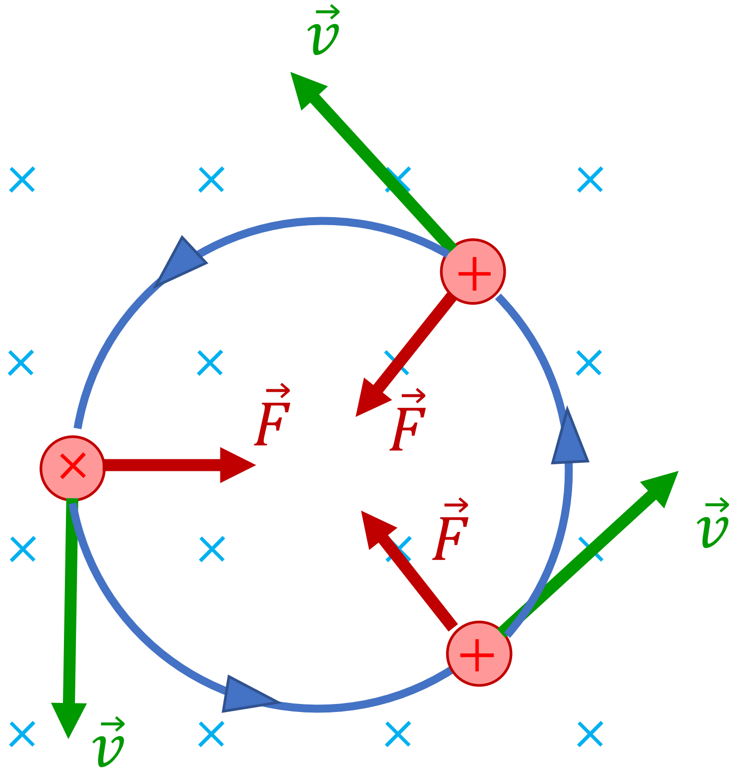
- $\vec{F}_B = q_\pm \, \vec{v} \times \vec{B}$
  always perpendicular to $\vec{v}$

$$F_B = ma_n \quad \Longrightarrow \quad q\cancel{v}B = m\frac{v^{\cancel{2}}}{r}$$

- Radius: $$\boxed{r_{cyc} = \frac{mv}{qB}}$$

- Frequency: $$f_{cyc} = \frac{1}{T} = \frac{v}{2\pi r_{cyc}} = \frac{\cancel{v}}{2\pi\,(m\cancel{v}/qB)}$$

$$\boxed{f_{cyc} = \frac{qB}{2\pi m}}$$

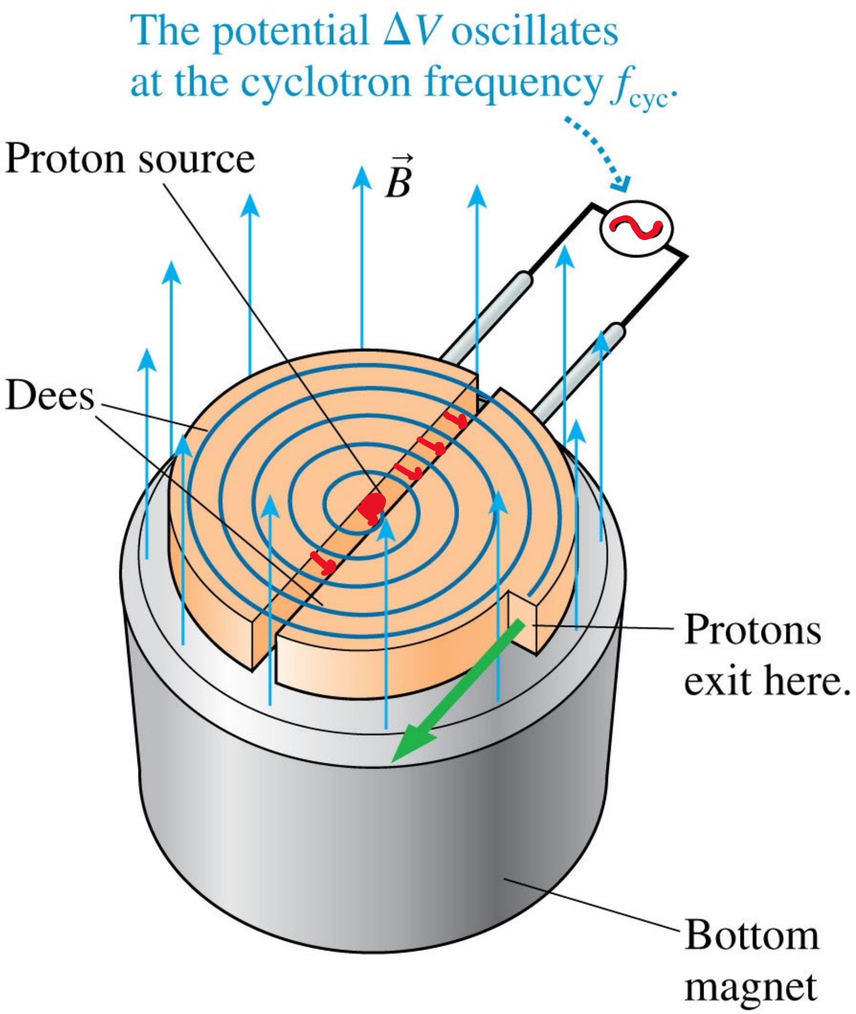


- Note: the frequency does not depend on particle's velocity, while the radius does

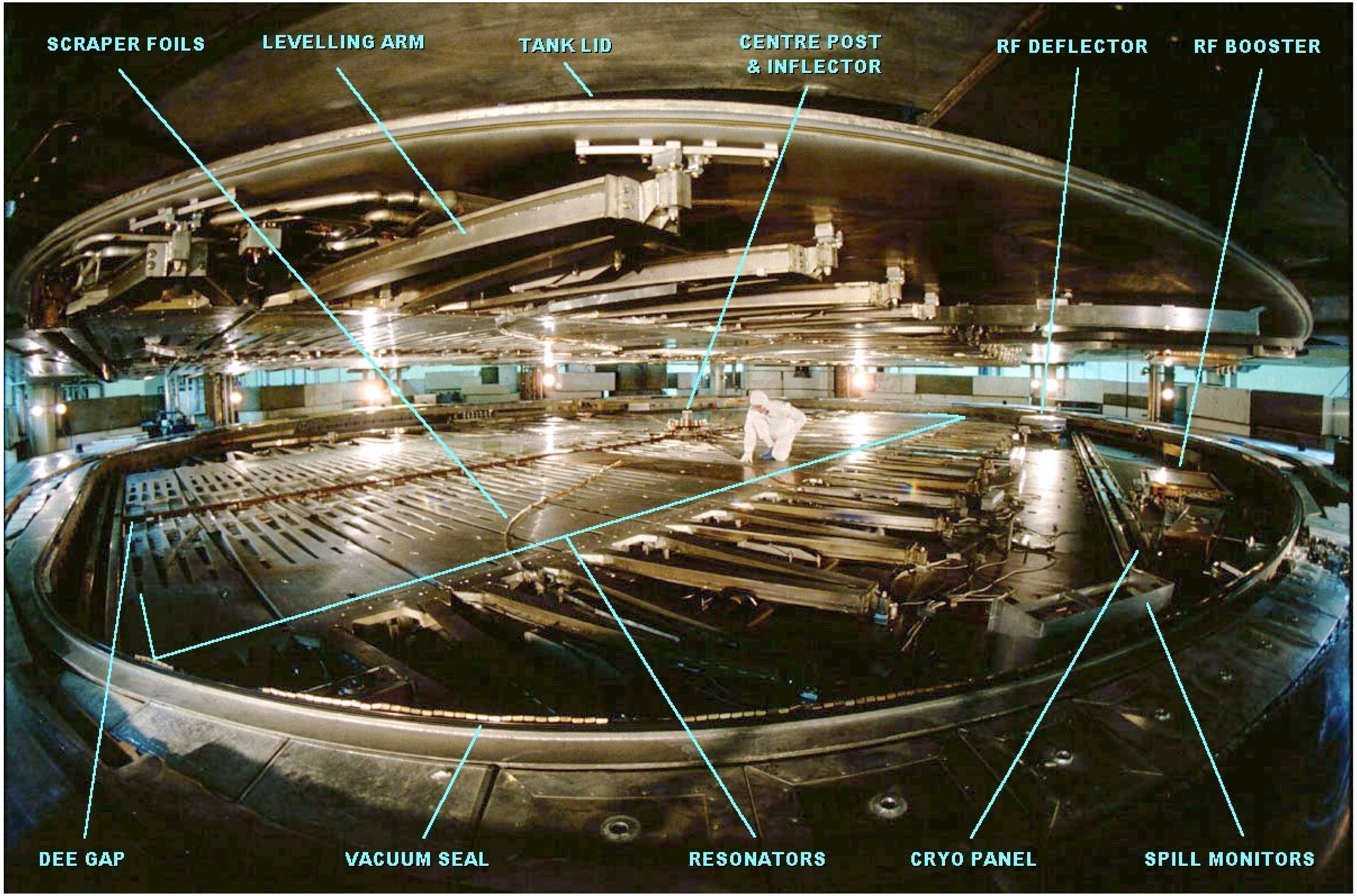
The Cyclotron: a Particle Accelerator

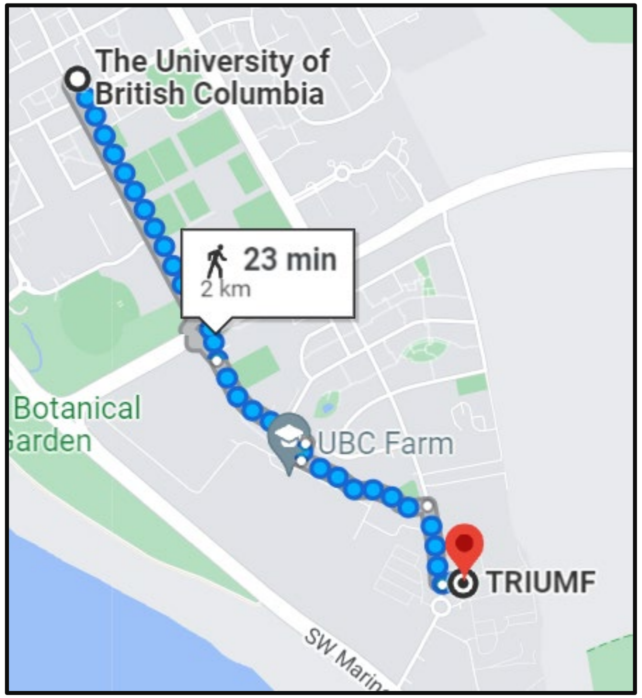
$$f_{cyc} = \frac{qB}{2\pi m} \qquad r_{cyc} = \frac{mv}{qB}$$

- Proton is injected at the center with some initial speed v => starts going around a circle (r_{cyc})
- There is a gap with applied E-field => the proton would accelerate at the segment when it goes in the direction of E, and decelerate when it goes against E (no net effect)
- But!! Proton's rotation period $T = 1/f_{cyc}$ is independent of its speed; what if we swap the polarity of ΔV exactly with the cyclotron frequency?
- The particle will gain energy $e\Delta V$ each time it crosses the gap => acceleration !!!
- The radius of the orbit increases as the particle accelerates => it will eventually leave the accelerator at the exit.



UBC TRIUMF cyclotron (2 km from here)





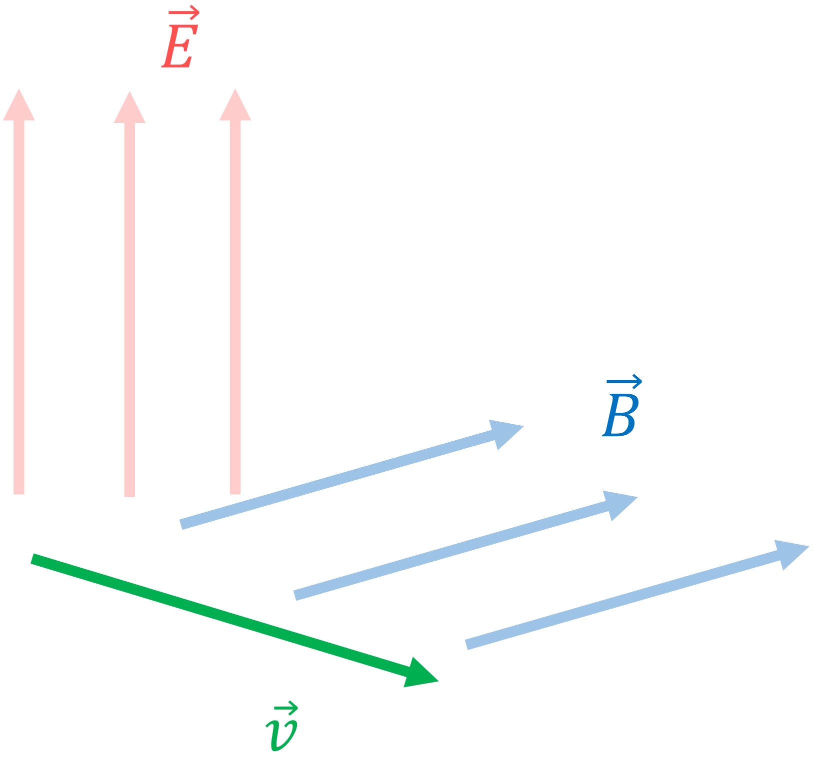
Combined effect of electric and magnetic fields

- If a charge is moving in a region where both electric and magnetic fields are present:

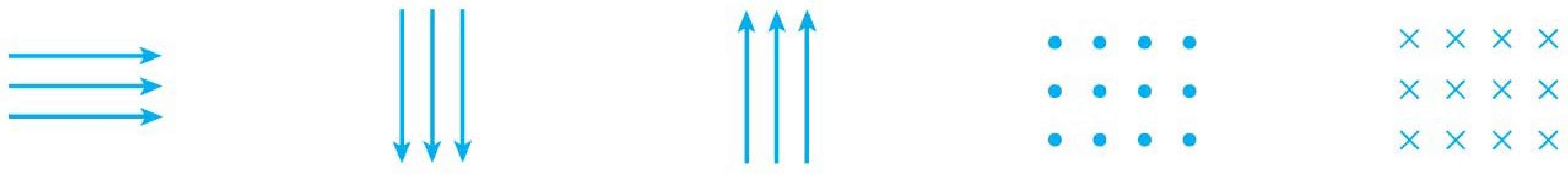
$$\vec{F} = \vec{F}_E + \vec{F}_B$$

$$\boxed{\vec{F} = q_{\pm}\vec{E} + q_{\pm}\vec{v} \times \vec{B}}$$

Lorentz force

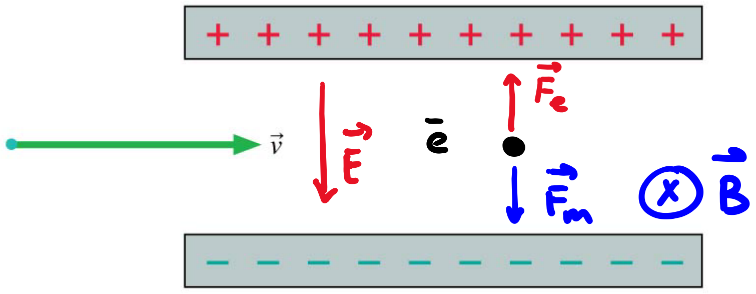


Q: Which direction of magnetic field (if it has the correct strength) allows the electron to pass through the charged plates without being deflected?



A. B. C. D. E.

Q: What should be the magnitude of he field B that will let the electron pass?



$$\underbrace{\vec E \perp \vec B \perp \vec v}$$

$$F_e = F_m$$

$$\cancel{e} E = \cancel{e} v B \qquad B = \frac{E}{v}$$

all other v's - won't

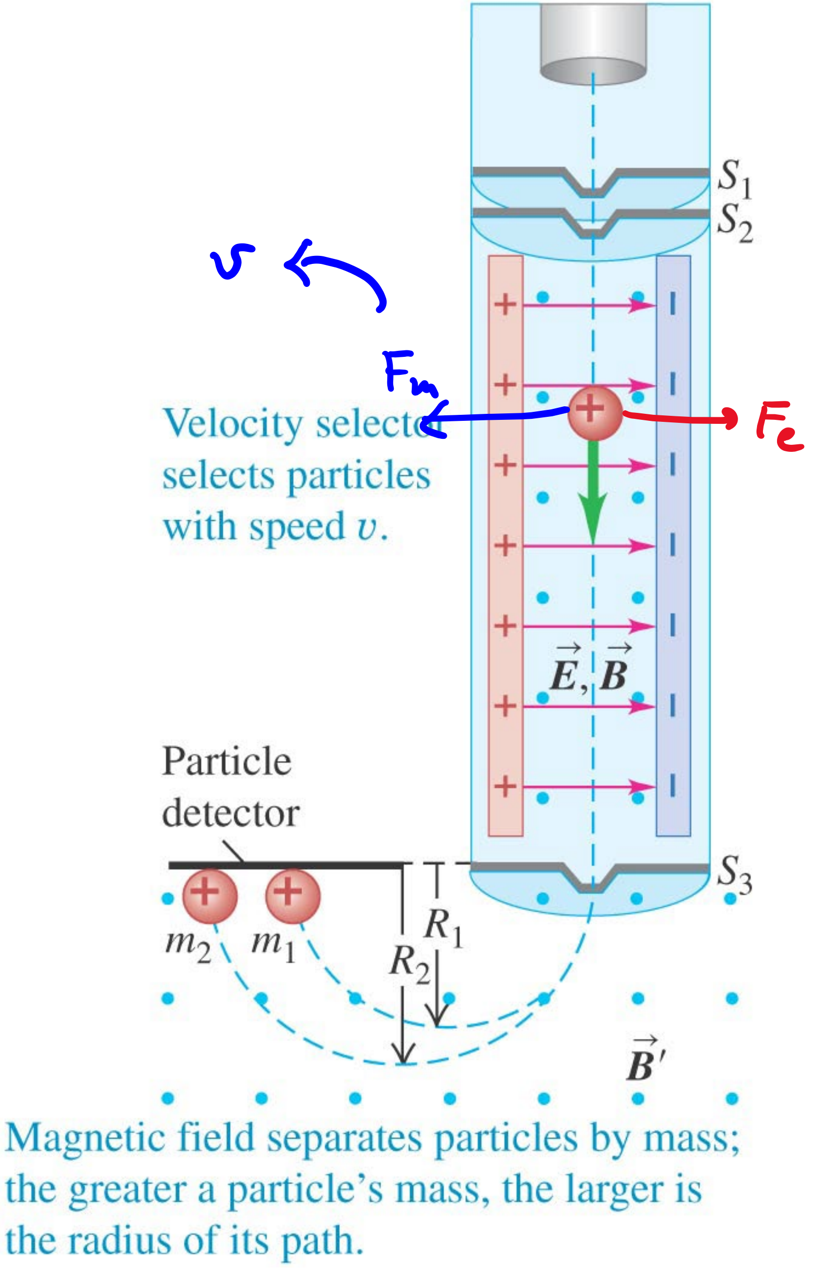
$$\boxed{E = vB} \qquad v = \frac{E}{B} \longrightarrow \text{pass}$$

$$\vec F = q_\pm \vec E + q_\pm \vec v \times \vec B$$

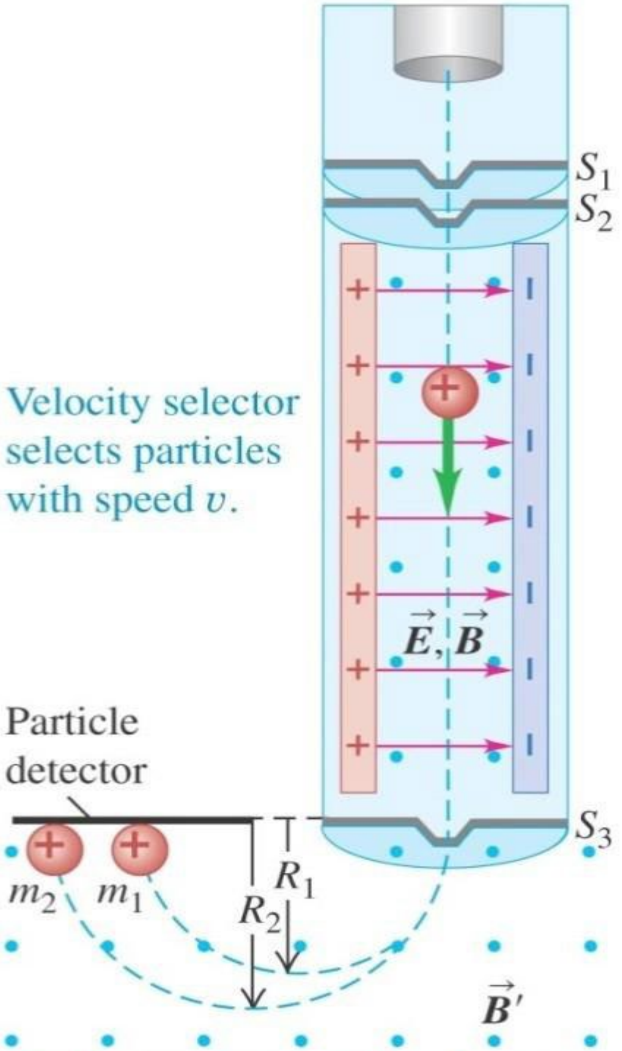
Mass Spectrometer

$$r_{cyc} = \frac{mv}{qB}$$

- A charged particle moves through a velocity selector at a constant speed in a straight line.
- Particle exits the velocity selector at S_3:
  $E = 0$, but same magnetic field.
- v stays the same, but the direction changes
  - ➢ **cyclotron motion**
- Selects particles according to m/q
  - ➢ good tool for chemical analysis



Q: Assume that the electric field strength E of the velocity selector of a mass spectrometer is set to 5.65×10^3 N/C, while the magnetic field strength B is set to 0.224 T. After exiting the velocity selector, the charged particle is exposed only to the magnetic field and moves in a circular orbit with radius 2.9 cm. Assuming that the particle is singly charged (i.e. carries charge e), find the mass of the particle.



Magnetic field separates particles by mass; the greater a particle's mass, the larger is the radius of its path.