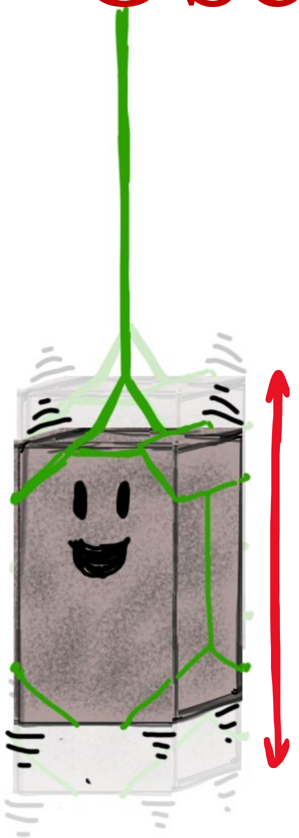
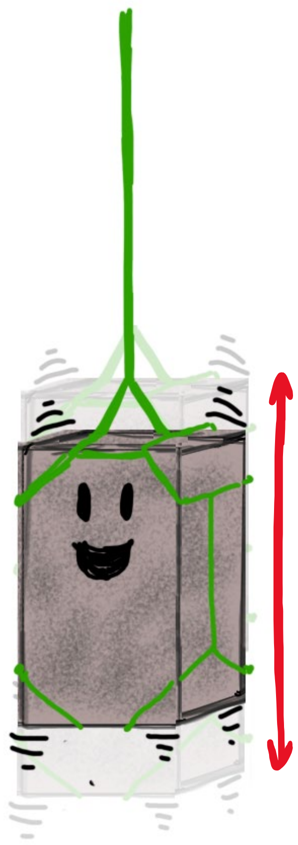
PHYSICS 157 PART II
Oscillations & Waves



Oscillations And the Tacoma Narrows Bridge Collapse

Details: https://www.youtube.com/watch?v=mXTSnZgrfxM



Lecture 26.

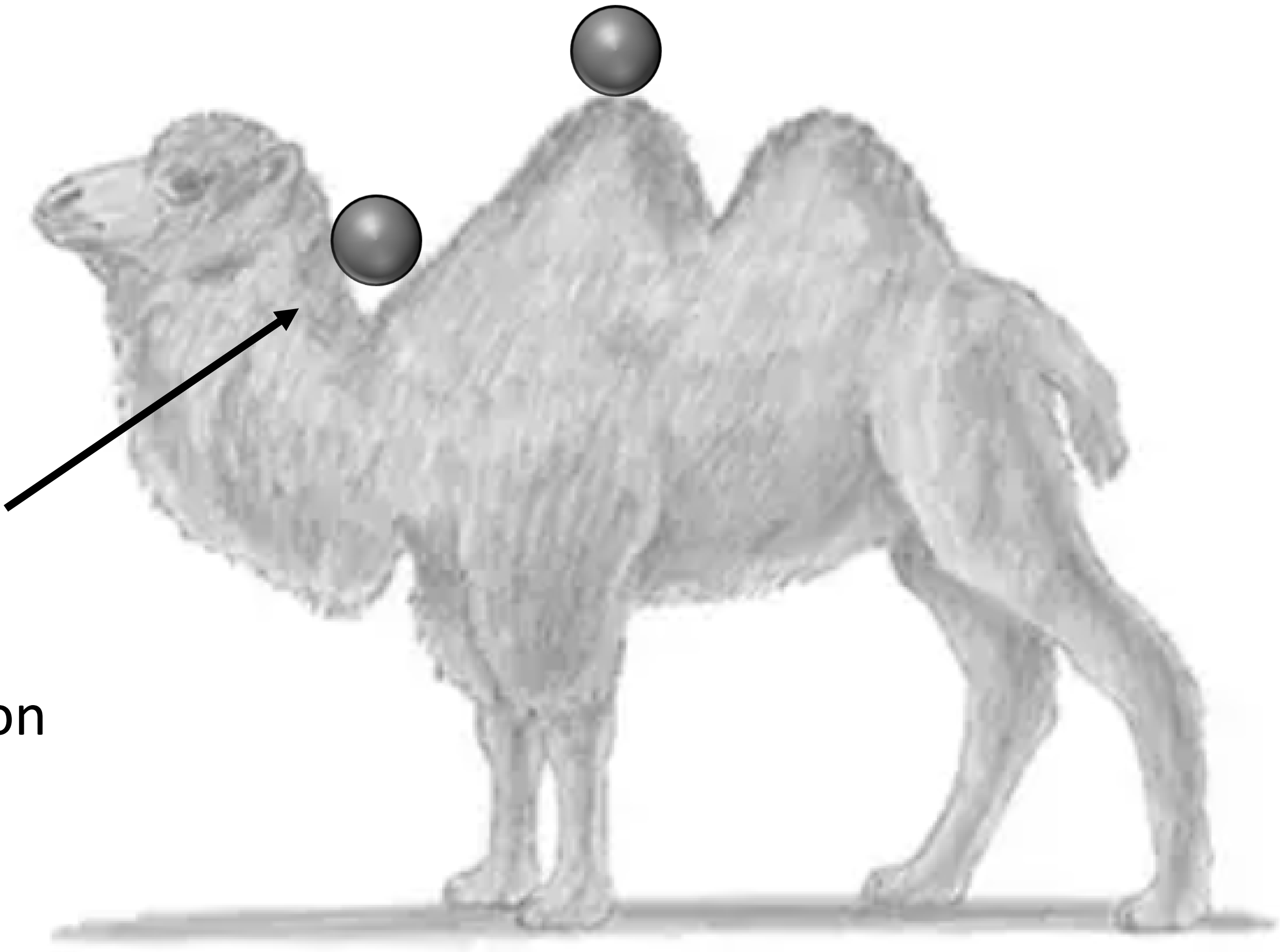
Restoring force.

Simple Harmonic Motion.

Last Time

- Unstable equilibrium
  - ➢ Small deflection => force carries the object away from equilibrium position

- In general, equilibrium occurs when forces (and torques) on each part of the system add to zero

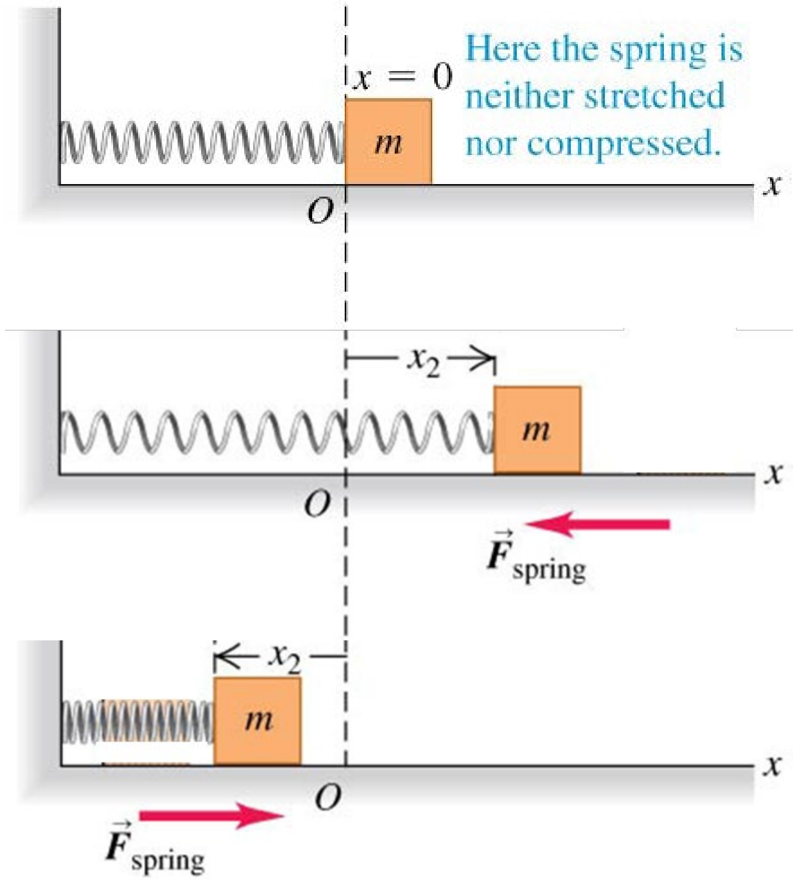


- Stable equilibrium
  - ➢ Small deflection => force carries the object back to its equilibrium position
  - ➢ Restoring force

Restoring Forces

- For a stable equilibrium configuration, a displacement in one direction leads to a net force in the OTHER direction

- Example:

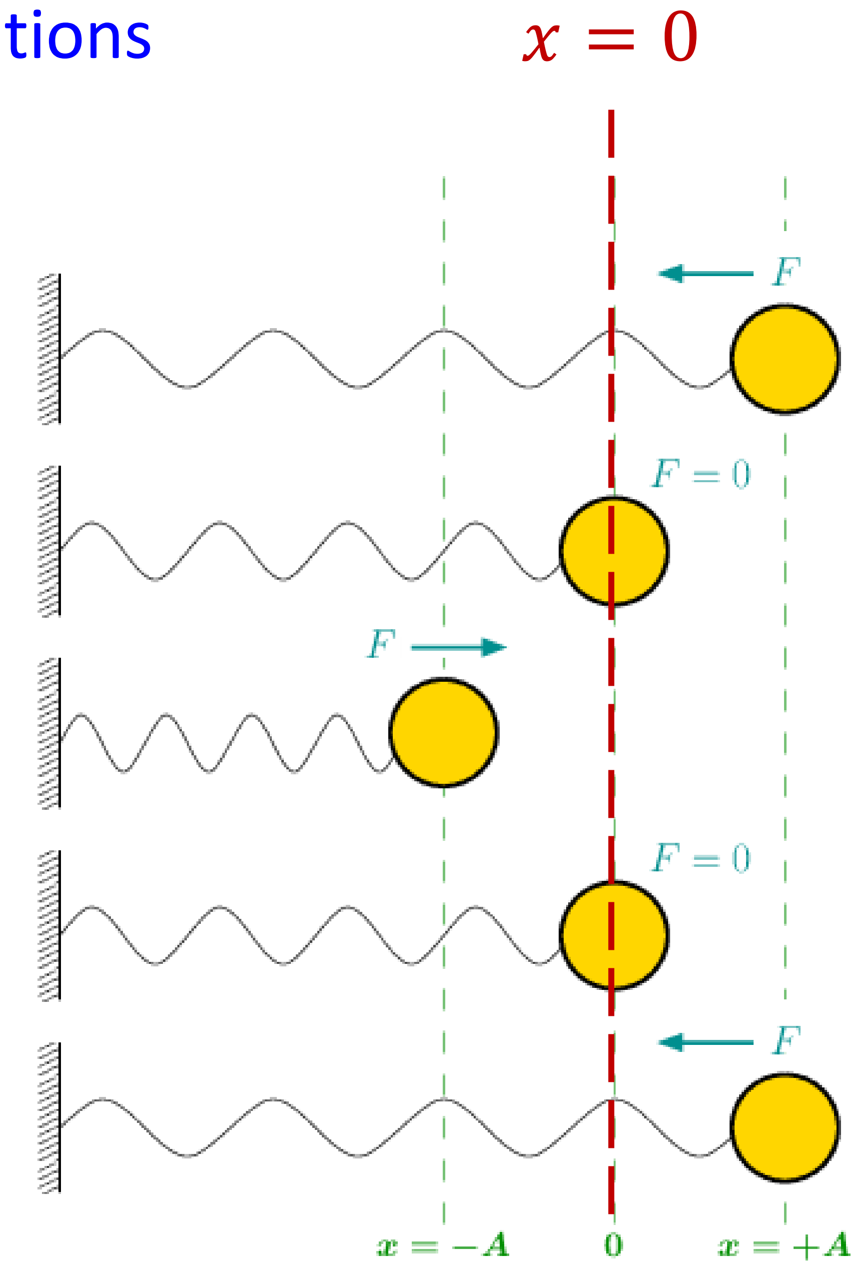


Equilibrium: the spring is neither stretched nor compressed

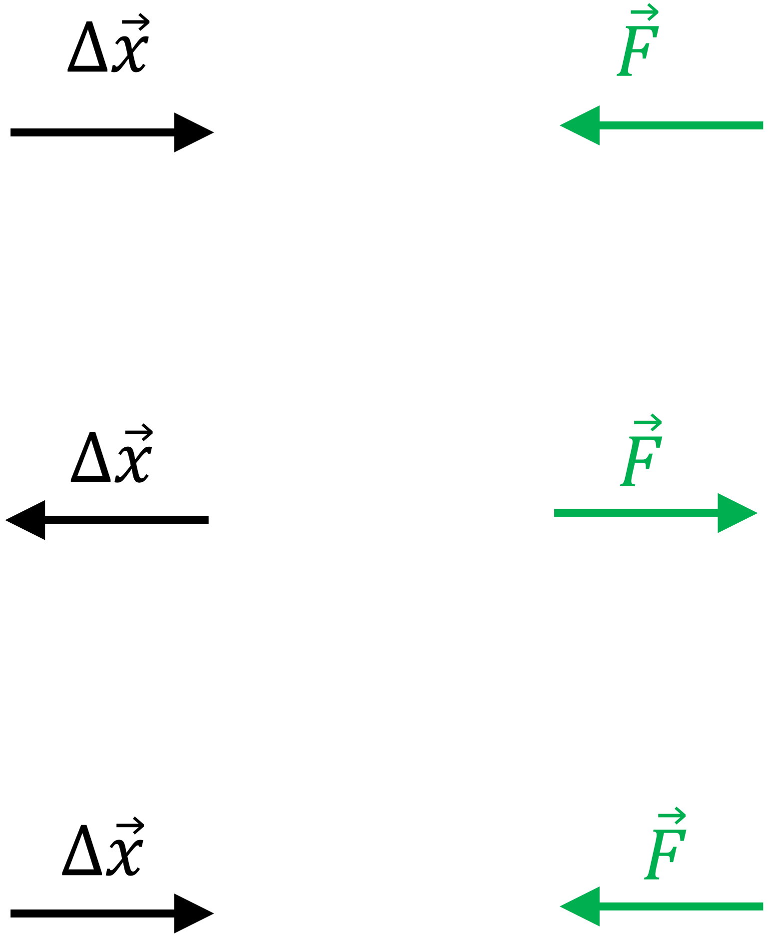
Stretched: $\Delta x > 0$, $F_{\text{spring}} < 0$

Compressed: $\Delta x < 0$, $F_{\text{spring}} > 0$

Oscillations



- Restoring forces result in periodic motion



…and the cycle repeats over and over again

Periodic Motion

- Periodic motion is any type of motion that repeats itself

- heartbeat
- violin string
- pendulum/swing
- mass on a spring
- rocking chair
- buoy bobbing in the water
- car stuck in muddy ditch
- e^- carrying current (AC)
- atoms trapped in a magneto-optical trap
- ions trapped in a Penning trap
- vibrations of molecules in a solid
- atomic clocks†

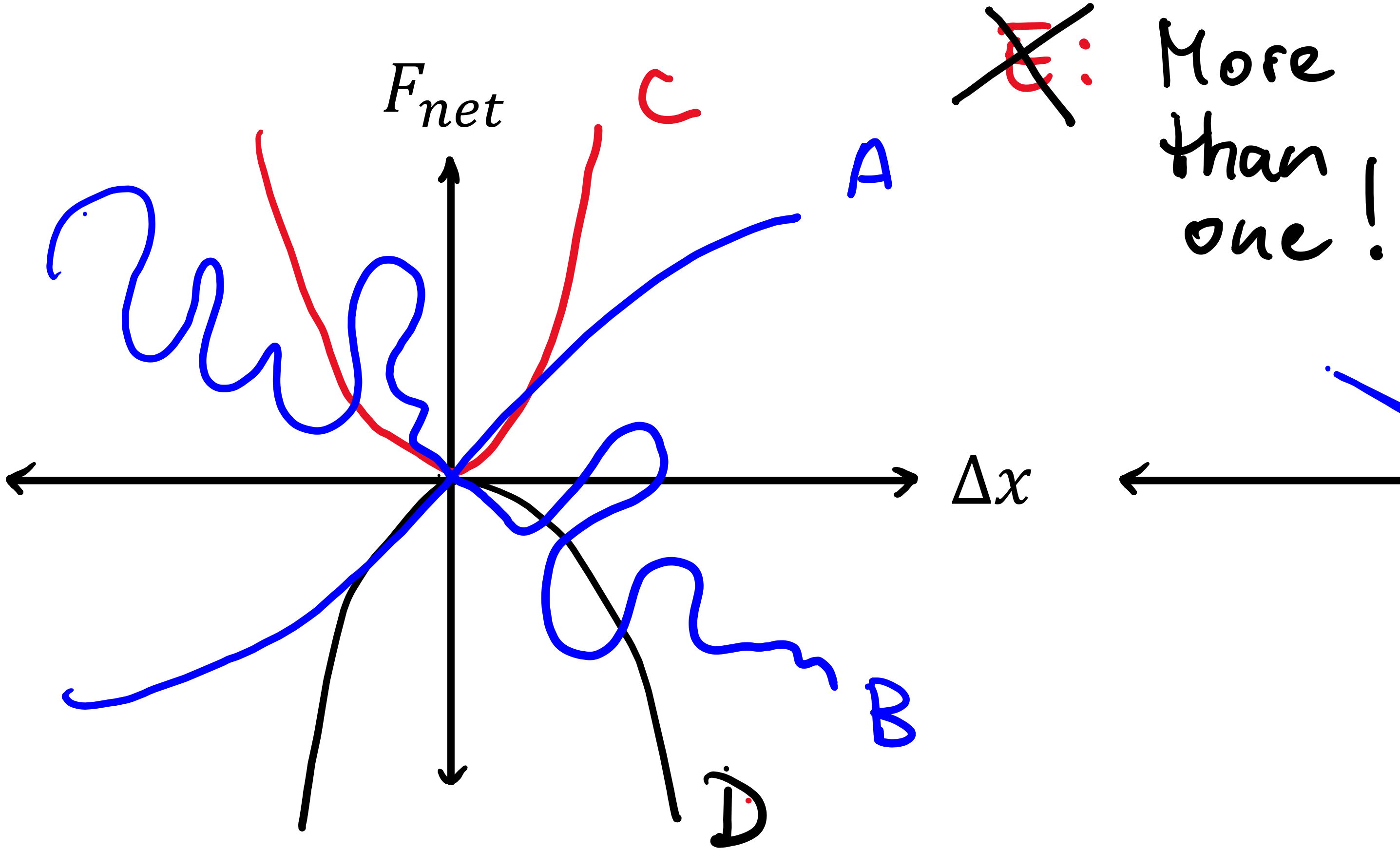
- We will concentrate on a particular subset of periodic motion, called simple harmonic motion (SHM)

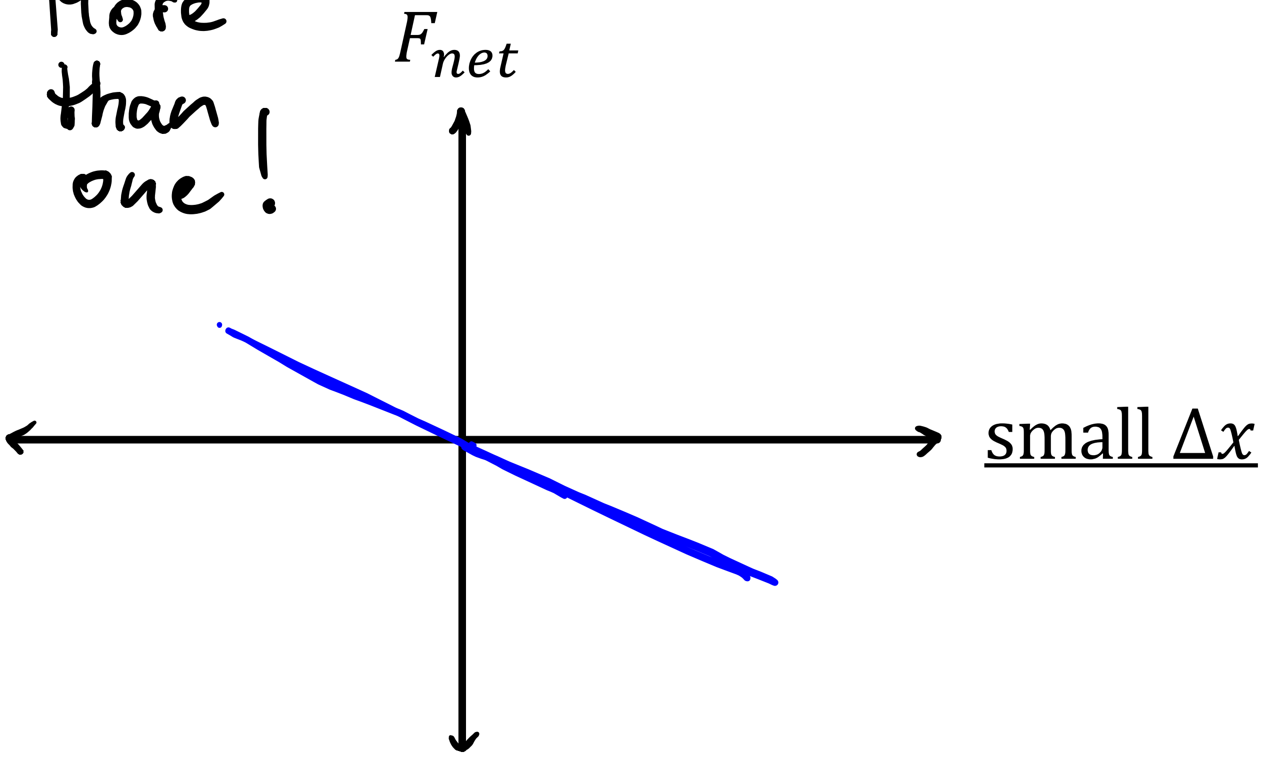
- SHM comes up again and again in Nature (1st yr undergrad, 2nd yr physics, 3rd and 4th yr; every grad school course; research, …)

†The frequency of the valence e^- of ^{133}Cs *defines* the second

Almost everything is a spring!

Q: For an object in a stable equilibrium configuration, draw some possible graphs of the net force on this object as a function of the displacement Δx.

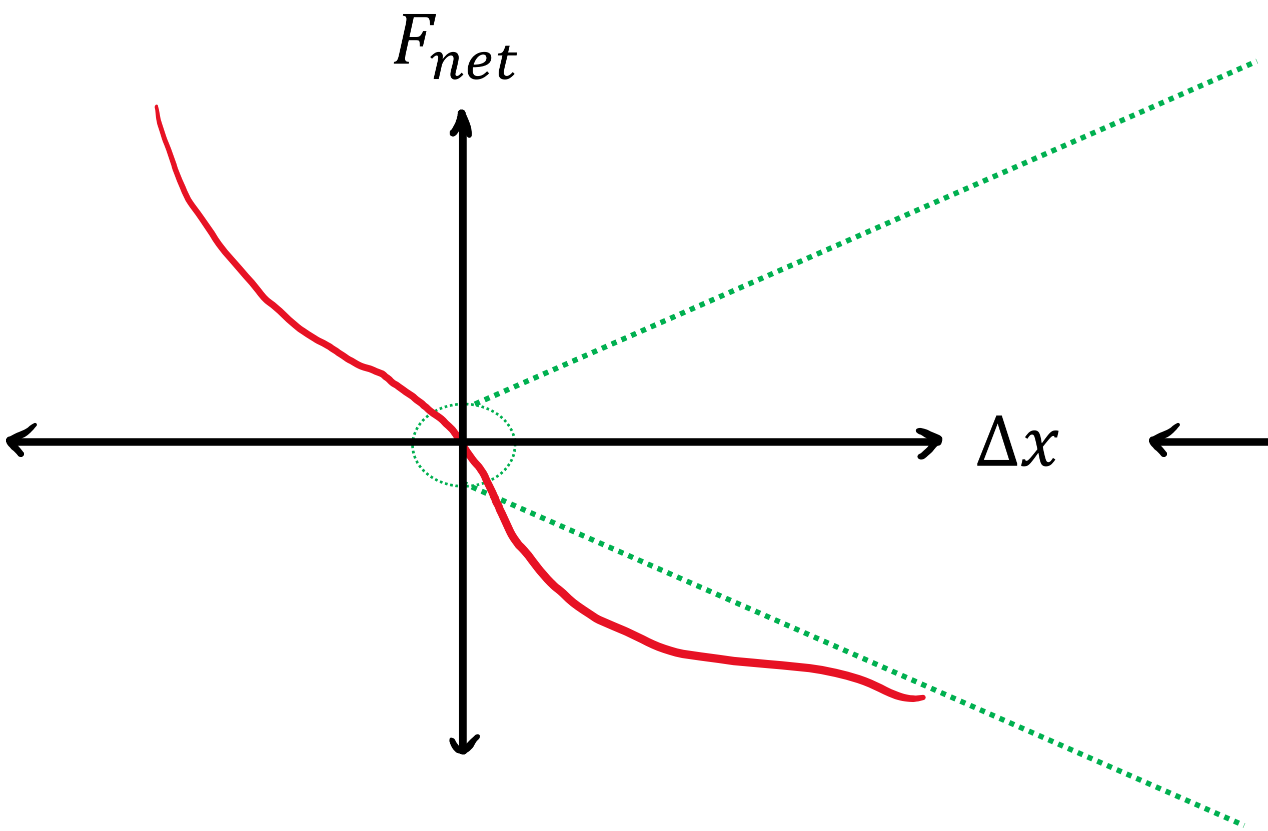


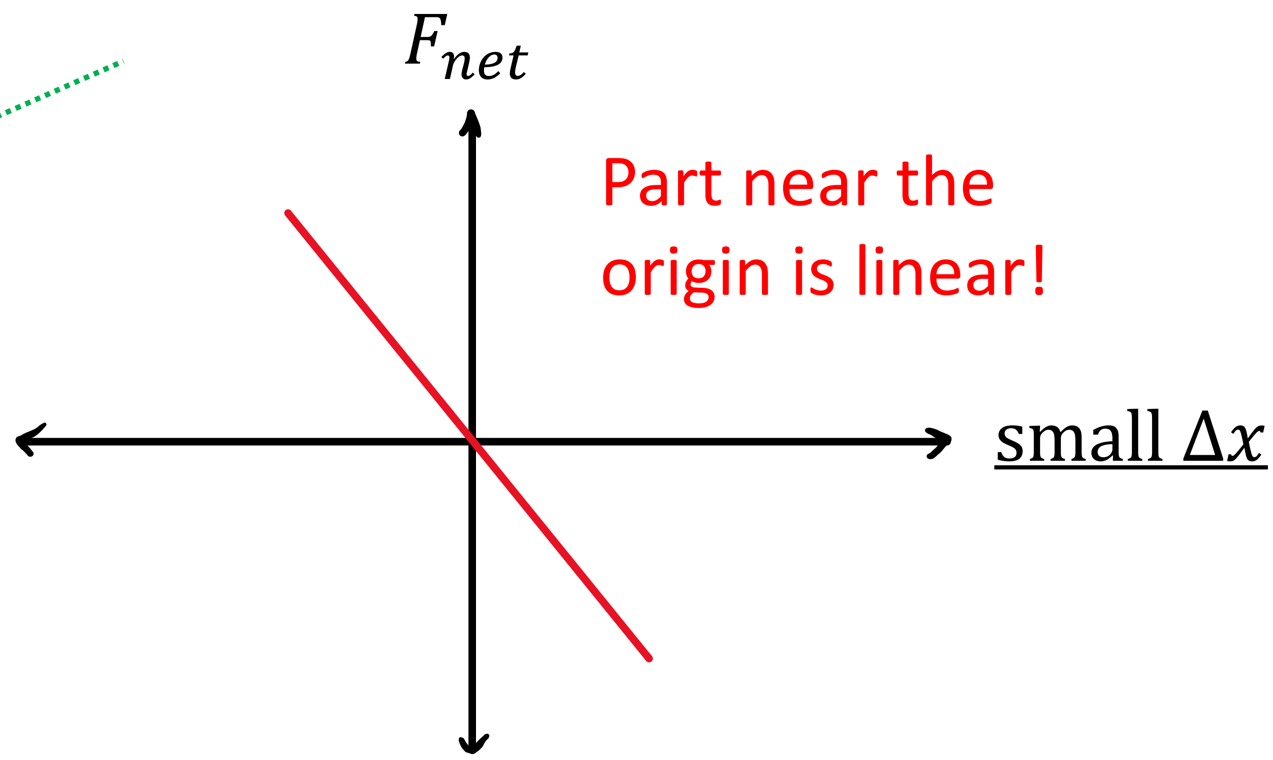


Q: What does your graph look like if you zoom into the region of small Δx? Can you write down an equation that describes F vs Δx in this region?

Almost everything is a spring!

Q: For an object in a stable equilibrium configuration, draw some possible graphs of the net force on this object as a function of the displacement Δx.

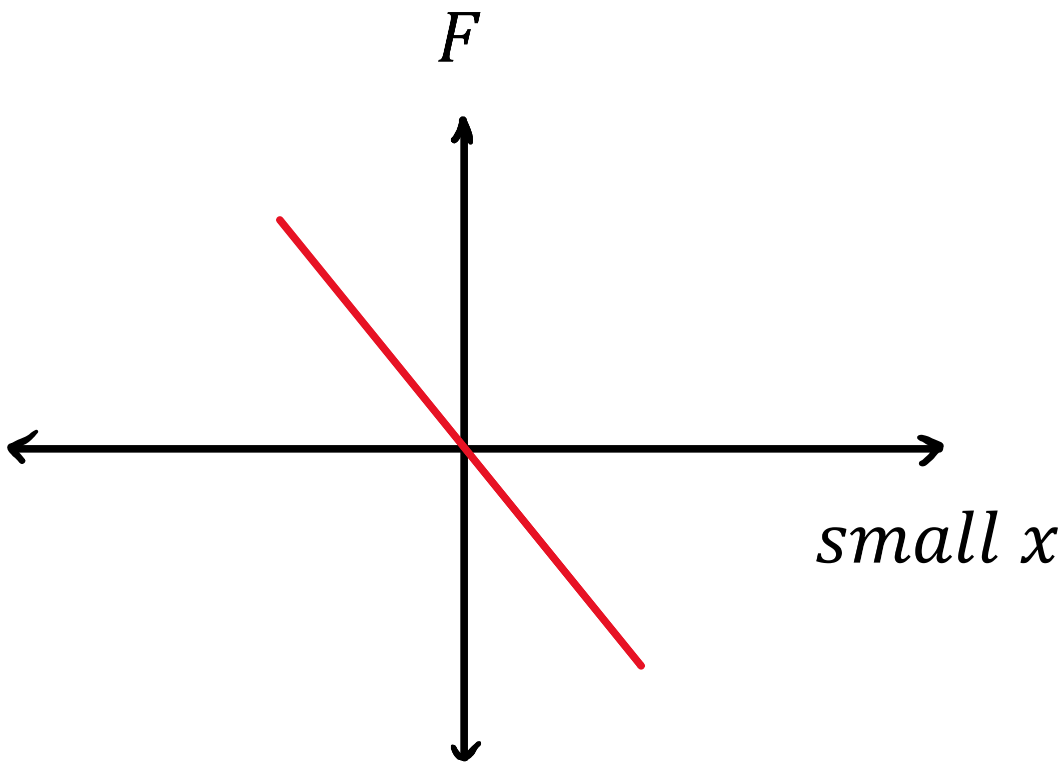




Q: What does your graph look like if you zoom into the region of small Δx? Can you write down an equation that describes F vs Δx in this region?

Hooke's Law

- Applies to almost any system perturbed a small amount from stable equilibrium



$$\boxed{F = -kx}$$

- ➢ F is the restoring force
- ➢ x is the displacement from the equilibrium position
- ➢ "$-$" captures the restoring character of the force

- Exact for an "ideal" spring

Oscillations with Hooke's Law

Hook's law   Newton's 2nd law

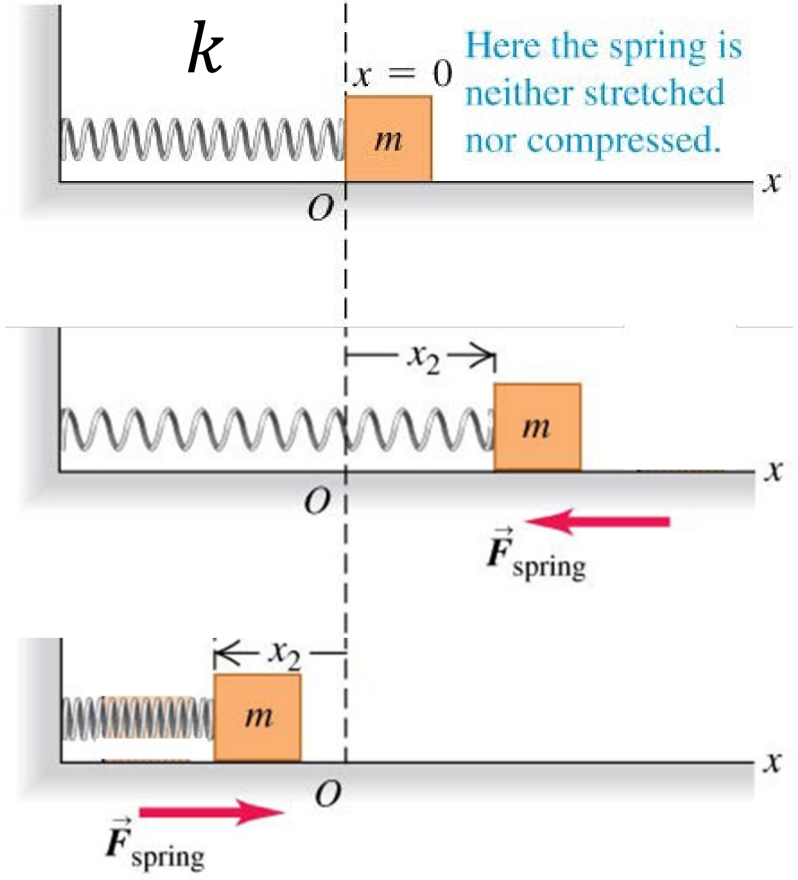
- $F = -kx$ and $F = ma$

gives $-kx = ma$.

- Now: $a = \frac{d^2x}{dt^2}$. Then:

$\frac{k}{m} = \omega^2$

$\omega = \sqrt{\frac{k}{m}}$



$a = -\omega^2 x$ (later) $\longrightarrow$

$$\frac{d^2x}{dt^2} = -\boxed{\frac{k}{m}}x \equiv -\omega^2 x$$

where $\omega = \sqrt{k/m}$.

$= \sqrt{\frac{k}{m}}$

Math: most general function with this property is:

const   const   const

$$\boxed{x(t) = A\cos(\omega t + \phi)}$$

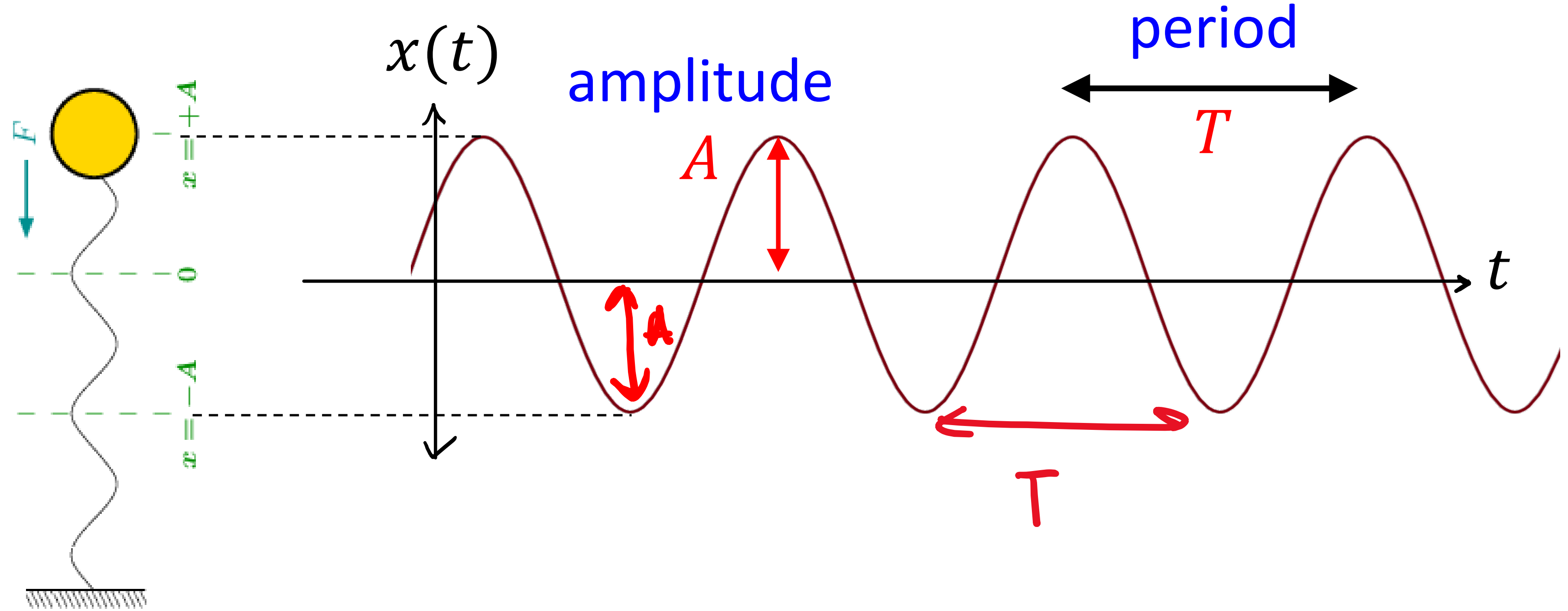
This function describes motion of a particle driven by Hook's restoring force

Simple Harmonic Motion

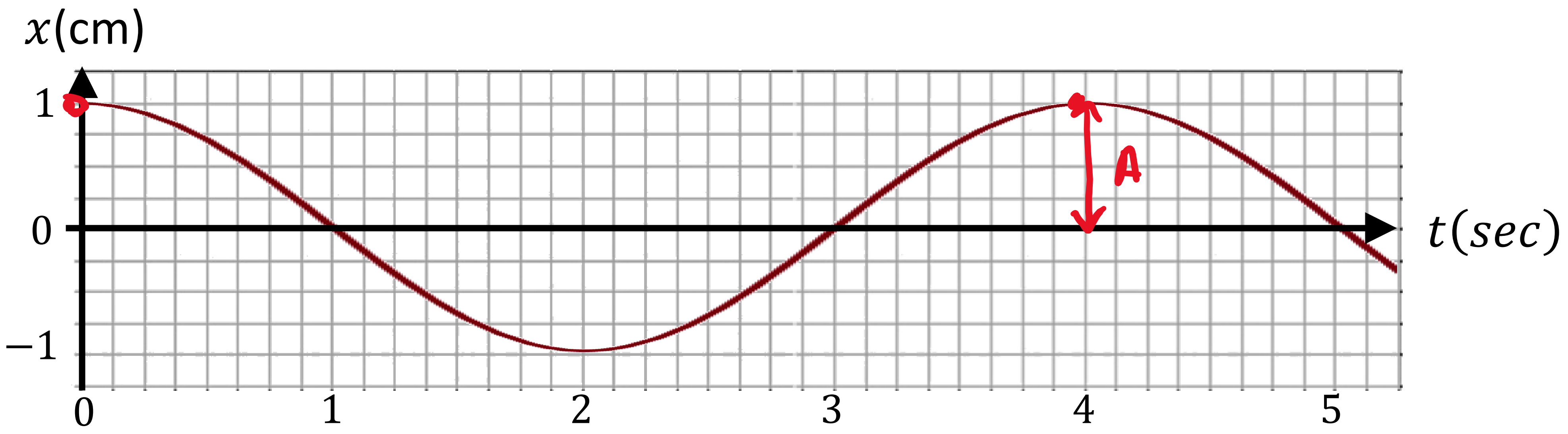
$$\boxed{x(t) = A\cos(\omega t + \phi)}$$

SHM: motion described by this harmonic function

- A = amplitude ✓
- ω = angular frequency → T?
- ϕ = phase ?



Q: A plot of displacement (in cm) as a function of time (in s) is shown below. What are the period and amplitude of this simple harmonic motion?



A. T = 1 s, A = 2 cm

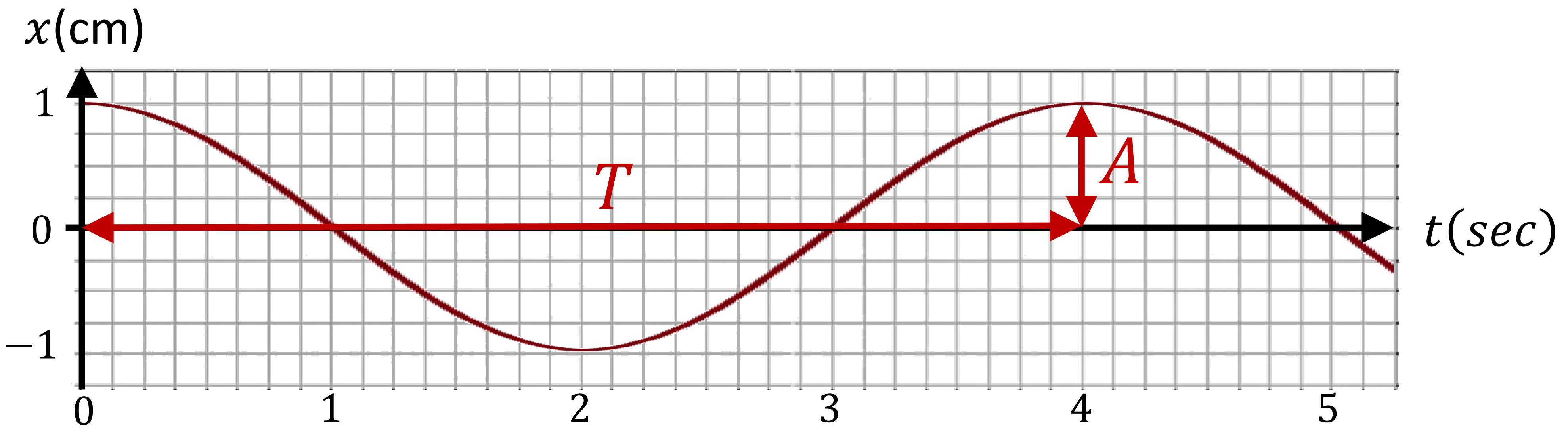
B. T = 2 s, A = 2 cm

C. T = 4 s, A = 2 cm

D. T = 2 s, A = 1 cm

E. T = 4 s, A = 1 cm

Q: A plot of displacement (in cm) as a function of time (in s) is shown below. What are the period and amplitude of this simple harmonic motion?



A. T = 1 s, A = 2 cm
B. T = 2 s, A = 2 cm
C. T = 4 s, A = 2 cm
D. T = 2 s, A = 1 cm
E. T = 4 s, A = 1 cm ✓

$$x(t) = A\cos(\omega t + \phi)$$

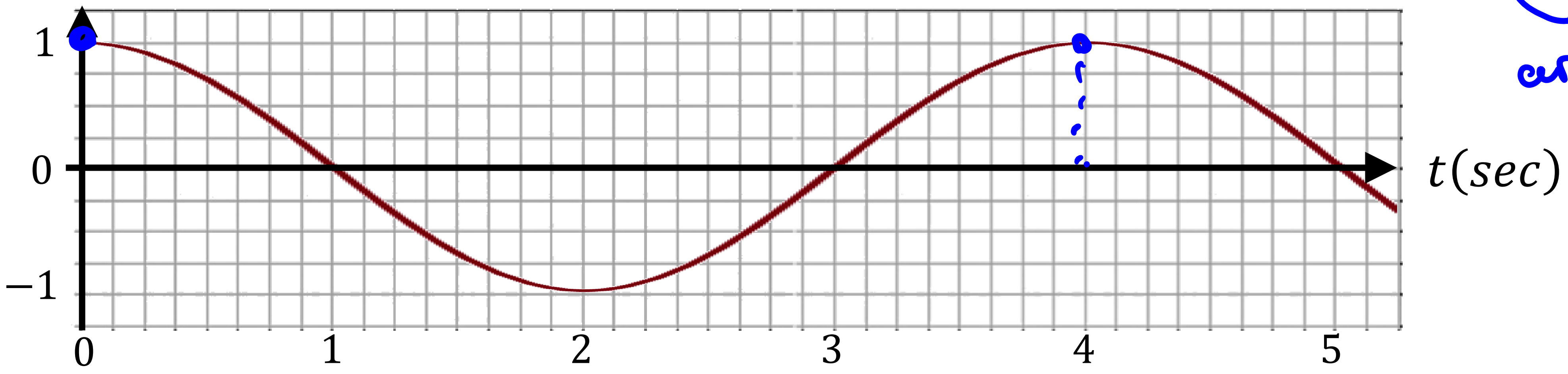
A = 1 cm

T = 4 s

Q: A plot of displacement (in cm) as a function of time (in s) is shown below. Which function below describes this motion?

$T = 4s$

$x(t) = A\cos(\underbrace{\omega t}_{\text{argument}})$



A. $x(t) = \cos(t)$

B. $x(t) = \cos(4t)$

C. $x(t) = \cos(2\pi t)$

D. $x(t) = \cos(\pi t)$

E. $x(t) = \cos(\pi/2\, t)$

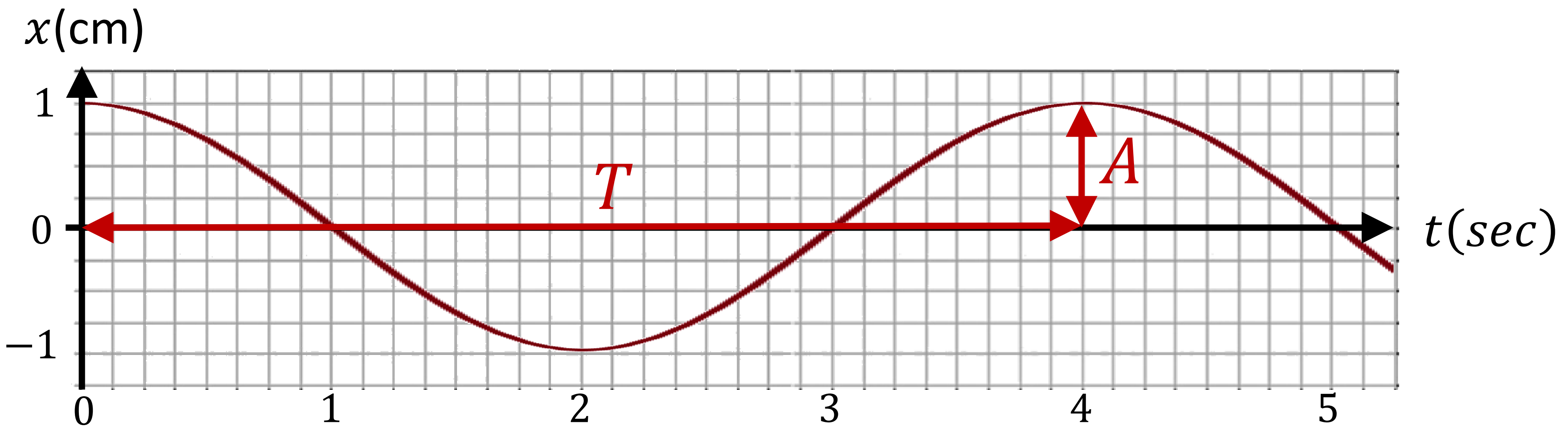
1) $t = 0$: $x(t) = A \cdot \cos(0) = A$

2) argument $= 2\pi$ @ $t = 4s$

$\omega t|_{t=4s} = 2\pi$

$\omega T = 2\pi$

Q: A plot of displacement (in cm) as a function of time (in s) is shown below. Which function below describes this motion?



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E. $x(t) = \cos(\pi/2\, t)$ ✓

- Period of cos is 2π
- Graph is $\cos(\omega t)$: when $t = 4$ s, graph goes back to 1 for the first time, so must have $\omega t = 2\pi$ at $t = 4$ s

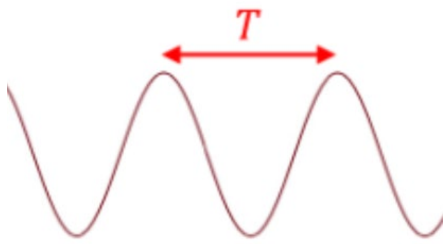
$$\omega = \frac{2\pi}{4\ s} = \frac{\pi}{2} s^{-1}$$

Period (T), frequency (f), angular frequency (ω)

$$\boxed{x(t) = A\cos(\omega t + \phi)}$$

- A = amplitude
- ? ω = angular frequency
- ϕ = phase

- Period T: time from max → max (full cycle)
  - $T = \frac{2\pi}{\omega}$, since cos repeats every 2π
  - Units: seconds



- Frequency, f: number of cycles per second
  - $f = \frac{1}{T}$
  - Units: Hertz ($1\ \mathrm{Hz} = 1\ \mathrm{s}^{-1}$)

$$\boxed{f = \frac{1}{T} \Leftrightarrow T = \frac{1}{f}}$$

- Angular frequency, ω:
  - $\omega = \sqrt{k/m}$, for a mass on a spring
  - Units: rad/s
  - Meaning: time rate change of the phase.

$$\boxed{\omega = 2\pi f = \frac{2\pi}{T}}$$

Q: Which of the following would increase the oscillation frequency of a mass on an ideal spring:

$$f$$

$$\hookrightarrow \frac{\omega}{2\pi}$$

$$\omega = \sqrt{\frac{k}{m}}$$

A. Increasing the mass
B. Increasing the spring constant
C. Increasing the initial displacement
D. Both A and C
E. Both B and C

Q: Which of the following would increase the oscillation frequency of a mass on an ideal spring:

$$f = \frac{\omega}{2\pi}$$

$$\omega = \sqrt{\frac{k}{m}}$$ doesn't depend on displacement or amplitude

$\omega \uparrow$ if $k \uparrow$

A. Increasing the mass
B. Increasing the spring constant ✓
C. Increasing the initial displacement
D. Both A and C
E. Both B and C

Q: You attach a 0.100 kg glider to a spring and start it oscillating. The elapsed time from when it first moves through the equilibrium point to the second time it moves through that point is 0.5 s. What is the spring constant?

$$\frac{2\pi}{T} = \omega = \sqrt{\frac{k}{m}}$$

SHM

system parameters

A. 1 N/m

B. 2 N/m

C. 4 N/m

D. 8 N/m

E. 16 N/m

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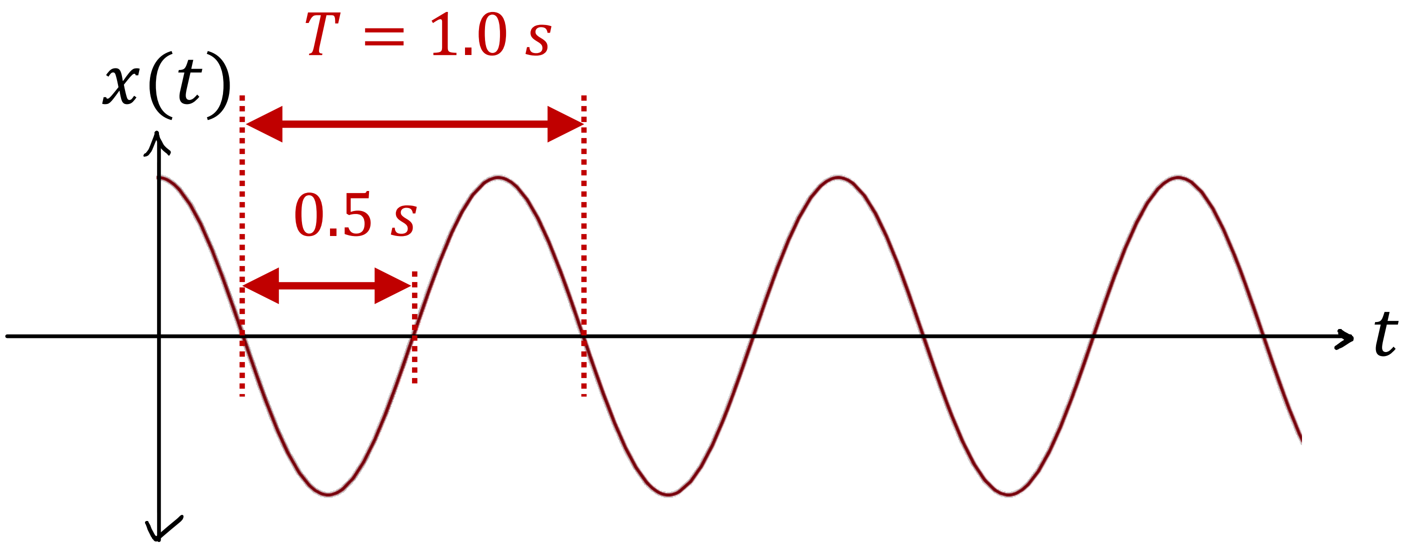

A. 1 N/m
B. 2 N/m
C. 4 N/m
D. 8 N/m
E. 16 N/m

- We have: $T = 1$ sec $\Longrightarrow\ \omega = \frac{2\pi}{T} = 6.28\ s^{-1}$
- Using $\omega = \sqrt{k/m}$, we have:

$$k = m\omega^2 = 0.1 \cdot (6.28)^2 = 4\,\frac{N}{m}$$

Q: What a bicycle wheel and a mass on a spring have in common?

Q: What a bicycle wheel and a mass on a spring have in common?

- If we project circular motion under constant speed onto one direction, we find that it follows SHM!