HW-8 — solns — March 21/24

HW8 soln' — Mar 11/24

1(a) **Model:** The potential at any point is the superposition of the potentials due to all charges. Outside a uniformly charged sphere, the electric potential is identical to that of a point charge Q at the center.
Visualize: Sphere A is the sphere on the left and sphere B is the one on the right.
Solve: The potential at point a is the sum of the potentials due to the spheres A and B:

$$V_a = V_{\text{A at a}} + V_{\text{B at a}} = \frac{1}{4\pi\varepsilon_0}\frac{Q_A}{R_A} + \frac{1}{4\pi\varepsilon_0}\frac{Q_B}{0.70\text{ m}}$$

$$= (9.0\times10^9\text{ N m}^2/\text{C}^2)\frac{100\times10^{-9}\text{ C}}{0.30\text{ m}} + (9.0\times10^9\text{ N m}^2/\text{C}^2)\frac{25\times10^{-9}\text{ C}}{0.70\text{ m}}$$

$$V_a = 3000\text{ V} + 321\text{ V} = 3321\text{ V}$$

Similarly, the potential at point b is the sum of the potentials due to the spheres A and B:

$$V_b = V_{\text{B at b}} + V_{\text{A at b}} = \frac{1}{4\pi\varepsilon_0}\frac{Q_B}{R_B} + \frac{1}{4\pi\varepsilon_0}\frac{Q_A}{0.95\text{ m}}$$

$$= (9.0\times10^9\text{ N m}^2/\text{C}^2)\left(\frac{25\times10^{-9}\text{ C}}{0.05\text{ m}} + \frac{100\times10^{-9}\text{ C}}{0.95\text{ m}}\right)$$

$$V_b = 4500\text{ V} + 947\text{ V} = 5447\text{ V}$$

Thus, the potential at point b is higher than the potential at a. The difference in potential is $V_b - V_a = 5447\text{ V} - 3321\text{ V} = 2126\text{ V} = 2.1\text{ kV}$.

Assess: $V_{\text{A at a}} = 3000$ V and the sphere B has a potential of 225 V at point a. The spherical symmetry dictates that the potential on a sphere's surface be the same everywhere. So, in calculating the potential at point a due to the sphere B we used the center-to-center separation of 1.0 m rather than a separation of 100 cm − 30 cm = 70 cm from the center of sphere B to the point a. The former choice leads to the same potential everywhere on the surface whereas the latter choice will lead to a distribution of potentials depending upon the location of the point a. Similar reasoning also applies to the potential at point b.

1(b) $V_{centre} = V_1 + V_2\ (d_2 = 1\text{ m})$

$$\boxed{V_2 = \frac{kQ_2}{d_2} = \frac{(9\times10^9)(25\times10^{-9})}{1} = 225\text{ V}}$$

$V_1(0.3\text{ m}) = \frac{kQ}{R_1} = \frac{9\times10^9(10^{-7})}{0.3} = 3000\text{ V}$ as given above.

Since $E_1(r) = \frac{kQ_1}{R_1^3}r \Rightarrow V_1(R_1) - V_1(0) = -\int_0^{R_1}\vec{E}_1\cdot d\vec{l} = -\int_0^{R_1}\frac{kQ_1}{R_1^3}r\,dr = -\frac{kQ_1}{R_1^3}\frac{R_1^2}{2}$

$$\boxed{V_1(0) = V_1(R_1) + \tfrac{1}{2}\tfrac{kQ_1}{R_1} = 3000 + 1500 = 4500\text{ V}}$$

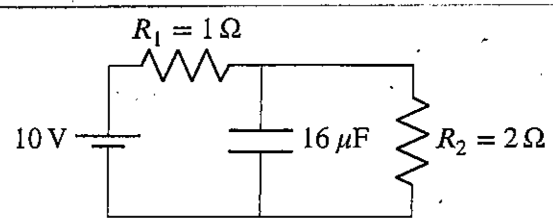
$$\boxed{\text{V1+V2 = 4,725 volts}}$$

①

3

Difficulty: ★★☆

A resistor ($R_2 = 2\,\Omega$) and a capacitor ($C = 16\,\mu$F) are connected in parallel to a 10 V battery with another resistor ($R_1 = 1\,\Omega$) as shown.



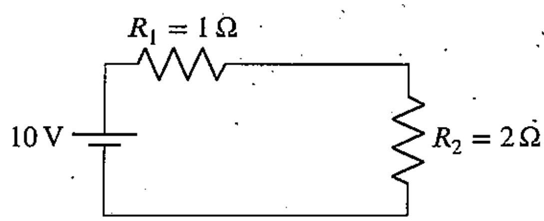
a) What is the current flowing through R_2 after a long period of time?

At time $t = t_0$ a dielectric of $\kappa = 4$ is inserted into the capacitor.

b) What is the new current flowing through R_2 at the instant just after the dielectric is inserted (at $t = t_0^+$)?

Solution:

a) The capacitor will act as a short so we are left with a series circuit



$$R_{eq} = R_1 + R_2 = 3\,\Omega$$
$$I = \frac{V}{R_{eq}} = \frac{10\text{ V}}{3\,\Omega} = \boxed{3.33\text{ A}}$$

b) At the instant just before the dielectric is inserted the voltage across R_1 is

$$V_{R_1} = IR_1 = 3.33\text{ A} \cdot 1\,\Omega = 3.33\text{ V}$$

This means that there must be $10\text{ V} - 3.33\text{ V} = 6.67\text{ V}$ across the capacitor just before the dielectric is inserted.

The charge on the capacitor just before the dielectric is inserted is

$$Q = CV = 16\,\mu\text{F} \cdot 6.67\text{ V} = 106.67\,\mu\text{C}$$

The new capacitance value with the dielectric will be

$$C = \kappa C_0 = 4 \cdot 16\,\mu\text{C} = 64\,\mu\text{C}$$

The charge that was accumulated on the capacitor cannot instantaneously jump in value so we know that the charge on the capacitor just after the dielectric is inserted is the same as the charge just before the dielectric is inserted ($q(t_0^-) = q(t_0^+)$). This means that the voltage across the capactor must have changed when the dielectric was inserted.

$$V = \frac{Q}{C} = \frac{106.67\,\mu\text{C}}{64\,\mu\text{C}} = 1.67\text{ V}$$

Because R_2 is in parallel with the capacitor, it must have the same voltage. We can use this to compute the current.

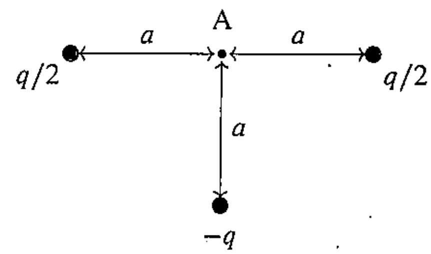
$$V_{R_2} = 1.67\text{ V}$$
$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{1.67\text{ V}}{2\,\Omega} = \boxed{0.834\text{ A}}$$

3

3 Difficulty: ★☆☆

A very rough approximation for a water molecule can be represented by the following diagram: (a negative oxygen atom (blue) with two positive hydrogen atoms (red) attached to it)

Assume $V(\infty) = 0$



a) The total electrical energy stored in this molecule is the sum of the potential energy of each pair of charges. The general expression for the potential energy between two point charges is

$$U = \frac{kq_1q_2}{r}$$

For the energy between either hydrogen atom and the oxygen atom we get

$$r = \sqrt{a^2 + a^2} = a\sqrt{2}$$
$$U = \frac{k(-q)(\frac{q}{2})}{a\sqrt{2}} = -\frac{kq^2}{2a\sqrt{2}}$$

For the energy between the two hydrogen atoms we get

$$r = 2a$$
$$U = \frac{k(\frac{q}{2})(\frac{q}{2})}{2a} = \frac{kq^2}{8a}$$

Summing all of the energies together we get

$$U = -\frac{kq^2}{2a\sqrt{2}} - \frac{kq^2}{2a\sqrt{2}} + \frac{kq^2}{8a}$$
$$U = -\frac{kq^2}{a\sqrt{2}} + \frac{kq^2}{8a}$$
$$\boxed{U = \frac{kq^2}{a}\left(-\frac{1}{\sqrt{2}} + \frac{1}{8}\right)}$$

b) $U < 0$ which means that the 2 positive hydrogen atoms are bound to the negative oxygen atom and energy is released when the water molecule is formed.

c) We know that the test charge starts at infinity and ends at point A. The work done on the point charge can be computed as

$$W = q'V$$

Q3 con't

This means to solve the problem we just need to compute the potential difference. To compute the potential, we can compute the potential difference between infinity and point A.
We already know that the potential at infinity is zero. We can compute the potential at point A by summing the potential due to each charge.

$$V = \frac{kq}{r}$$
$$V(A) = \frac{k(\frac{q}{2})}{a} + \frac{k(\frac{q}{2})}{a} + \frac{k(-q)}{a}$$
$$V(A) = 0$$
$$V(A) - V(\infty) = 0$$

We find that the potential at point A is the same as that at infinity. Therefore, no work is required to bring the charge from infinity to point A.

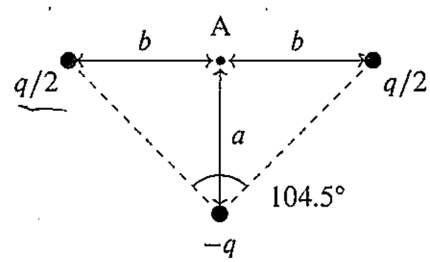
d) This is a bit of a trick question. Because there was no work done in bringing the charge into the system, the total energy stored in the system is the same as it was before.

$$\boxed{U = \frac{kq^2}{a}\left(-\frac{1}{\sqrt{2}} + \frac{1}{8}\right)}$$

e) The angle between the hydrogen atoms is 104.5° which means that the distance from the hydrogen atoms to point A is now no longer the same (now represented by b). We can also note that the angle in the original schematic was 90 degrees which is less than the new angle. This implies that $b > a$ and so the hydrogen atoms are now further away from point A. So we get

$$V(A) = \frac{k\left(\frac{q}{2}\right)}{b} + \frac{k\left(\frac{q}{2}\right)}{b} + \frac{k(-q)}{a} = kq\left(\frac{1}{b} - \frac{1}{a}\right)$$
$$b > a \Rightarrow V(A) < 0$$

Seeing that the potential is negative, we can conclude that the work done on the charge is negative ($W = q'V$). This means that energy is released when the charge q' is brought into the system.



4

IDENTIFY: This situation is analogous to having two capacitors C_1 in series, each with separation $\frac{1}{2}(d-a)$.

SET UP: For capacitors in series, $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$.

EXECUTE: **(a)** $C = \left(\dfrac{1}{C_1} + \dfrac{1}{C_1}\right)^{-1} = \frac{1}{2}C_1 = \frac{1}{2}\dfrac{\epsilon_0 A}{(d-a)/2} = \dfrac{\epsilon_0 A}{d-a}$.

(b) $C = \dfrac{\epsilon_0 A}{d-a} = \dfrac{\epsilon_0 A}{d}\dfrac{d}{d-a} = C_0 \dfrac{d}{d-a}$.

EVALUATE: **(c)** As $a \to 0$, $C \to C_0$. The metal slab has no effect if it is very thin. And as $a \to d$, $C \to \infty$. $V = Q/C$. $V = Ey$ is the potential difference between two points separated by a distance y parallel to a uniform electric field. When the distance is very small, it takes a very large field and hence a large Q on the plates for a given potential difference. Since $Q = CV$ this corresponds to a very large C.

5

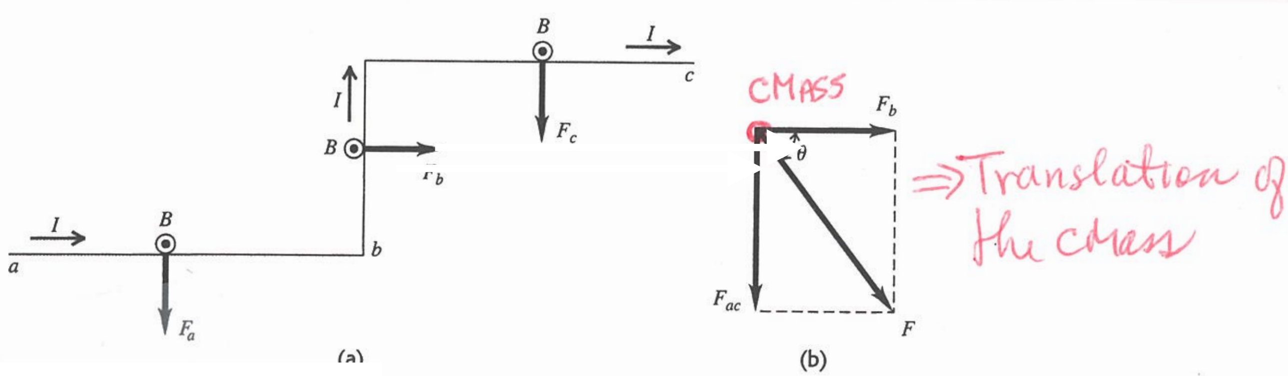
IDENTIFY: Apply $F = IlB\sin\phi$.

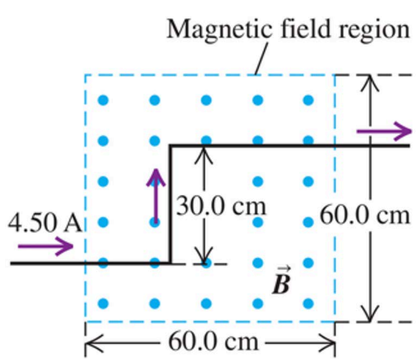
SET UP: Label the three segments in the field as a, b, and c. Let x be the length of segment a. Segment b has length 0.300 m and segment c has length $0.600\text{ m} - x$. Figure 27.29a shows the direction of the force on each segment. For each segment, $\phi = 90°$. The total force on the wire is the vector sum of the forces on each segment.

EXECUTE: $F_a = IlB = (4.50\text{ A})x(0.240\text{ T})$. $F_c = (4.50\text{ A})(0.600\text{ m} - x)(0.240\text{ T})$. Since $\vec{F}_a$ and $\vec{F}_c$ are in the same direction their vector sum has magnitude $F_{ac} = F_a + F_c = (4.50\text{ A})(0.600\text{ m})(0.240\text{ T}) = 0.648\text{ N}$ and is directed toward the bottom of the page in Figure 27.29a. $F_b = (4.50\text{ A})(0.300\text{ m})(0.240\text{ T}) = 0.324\text{ N}$ and is directed to the right. The vector addition diagram for $\vec{F}_{ac}$ and $\vec{F}_b$ is given in Figure 27.29b.

$F = \sqrt{F_{ac}^2 + F_b^2} = \sqrt{(0.648\text{ N})^2 + (0.324\text{ N})^2} = 0.724\text{ N}$. $\tan\theta = \dfrac{F_{ac}}{F_b} = \dfrac{0.648\text{ N}}{0.324\text{ N}}$ and $\theta = 63.4°$. The net force has magnitude 0.724 N and its direction is specified by $\theta = 63.4°$ in Figure 27.29b.

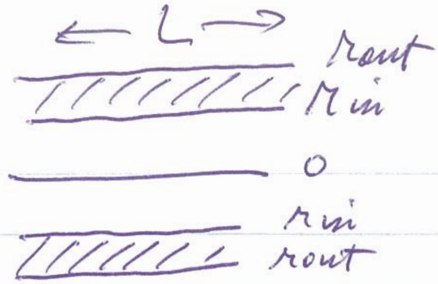
EVALUATE: All three current segments are perpendicular to the magnetic field, so $\phi = 90°$ for each in the force equation. The direction of the force on a segment depends on the direction of the current for that segment.



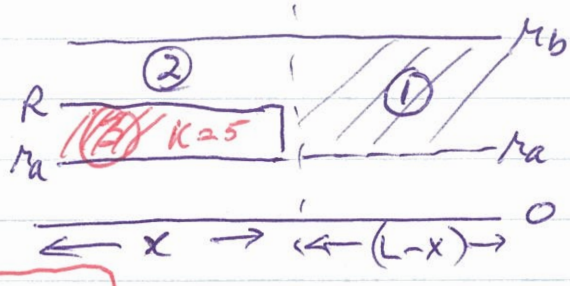


The wire will rotate CW since F_c ~ 0.432N and F_a ~ 0.216N. The moment arm of F_C is also 2x larger as well. We place the 2 forces at 50% of the distance of the wire section in the B field.

General Cylindrical capacitor



$$\Rightarrow C = \frac{2\pi\epsilon_0 L}{\ln\left(\frac{r_{out}}{r_{in}}\right)}$$



#6

Region 1 $\boxed{C_1 = \frac{2\pi\epsilon_0 (L-x)}{\ln\left(\frac{r_b}{r_a}\right)}}$ $L = 10\text{ cm} = 10^{-1}\text{ m}$

Region 2 has C_{2K}; C_{2air} in series.

$$C_{2K} = \frac{2\pi\epsilon_0 x K}{\ln\left(\frac{R}{r_a}\right)} \qquad C_{2air} = \frac{2\pi\epsilon_0 x}{\ln\frac{r_b}{R}}$$

add in Series $\Rightarrow$

$$\Rightarrow \frac{1}{C_{2T}} = \frac{1}{C_{2A}} + \frac{1}{C_{2K}}$$

$$C_{2\,TOTAL} = \frac{C_{2K}\, C_{2air}}{C_{2K} + C_{2air}} = \frac{(2\pi\epsilon_0 x)^2 \left[\frac{K}{\ln\frac{R}{r_a}}\right]\left[\frac{1}{\ln\frac{r_b}{R}}\right]}{2\pi\epsilon_0 x\left[\frac{K}{\ln\frac{R}{r_a}} + \frac{1}{\ln\frac{r_b}{R}}\right]}$$

$$\boxed{C_{2\,TOTAL} = 2\pi\epsilon_0 x\left[\frac{K}{K\ln\frac{r_b}{R} + \ln\frac{R}{r_a}}\right]}$$

C_1 & C_2 are parallel caps - same voltage $= V$

$$\Rightarrow C(x) = C_1 + C_{2\,TOTAL} = 2\pi\epsilon_0\left[\frac{L-x}{\ln\frac{r_b}{r_a}} + \frac{Kx}{K\ln\frac{r_b}{R} + \ln\frac{R}{r_a}}\right]$$

$$C(0) = \frac{2\pi\epsilon_0 L}{\ln \frac{r_b}{r_a}} = \frac{2\pi(8.85\times10^{-12})(10^{-1})}{\ln\left(\frac{4}{1}\right)} = 4.009\ pF$$

$$C(L) = 2\pi\epsilon_0 L k\left(\frac{1}{K\ln\frac{4}{3} + \ln 3}\right) = \frac{2\pi(8.85\times10^{-12})(5)(10^{-1})}{5\ln 1.33 + \ln 3}$$

$$C(L) = \frac{27.79\times10^{-12}}{2.525} = 11.00\times10^{-12} F = \underline{\underline{11.0\ pF}}$$

(c) $V = 500$ volts, $q = 3$ nC – find x

use $Q = C(x)\,V$

$$\frac{3\times10^{-9}}{500} = C(x) = 0.6\times10^{-11} = 2\pi\epsilon_0\left(\frac{L-x}{\ln 4} + \frac{Kx}{K\ln\frac{4}{3} + \ln 3}\right)$$

$$\Rightarrow \frac{0.6\times10^{-11}}{2\pi(8.85\times10^{-12})} = 0.108 = \left(\frac{0.10-x}{1.386} + \frac{5x}{2.525}\right)$$

$$\Rightarrow \boxed{x = 2.78\ cm}$$

if V = 500 volts and q = 6 nC then x = 11.4 cm (outside the capacitor (L=10 cm)
if V = 1000 volts and q = 6 nC then x = 2.78 cm (same as 500 V and 3 nC)

Note that the book problem, uses V = 1,000 volts, q = 6.0 C, and K = 3.21 so then x = 4.12 cm — see page 6-4 below

6-3

#6

IDENTIFY: Two coaxial conducting shells with dielectric in the space between them form a cylindrical capacitor. We assume that there are no appreciable effects due to the part of the dielectric that is not within the region between the cylinders.

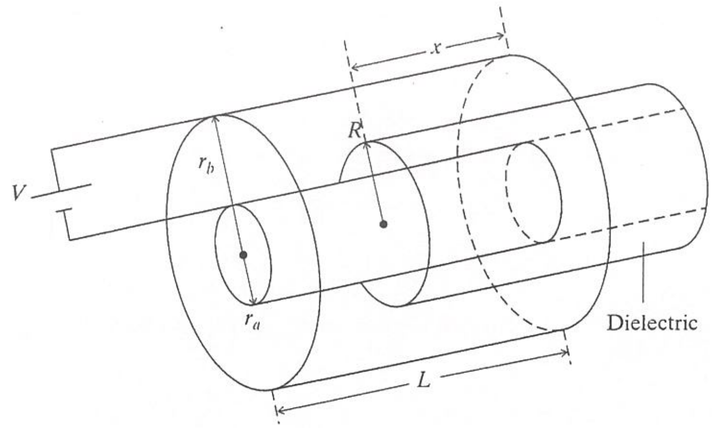


Figure 24.70a

SET UP: Refer to Fig. 24.70a. We can think of this combination as two capacitors in parallel. One capacitor is the section without dielectric and the other is the section with dielectric. They are in parallel because they share the same positive side and the same negative side. The section containing dielectric can be viewed as two cylindrical capacitors of length x in series, as shown in Fig. 24.70b. If R is the radius of the dielectric section, that part has an inner radius r_a and outer radius R. The part without dielectric has inner radius R and outer radius r_b. For a cylindrical capacitor of length ℓ with inner radius r_a and outer radius r_b, $C = \dfrac{2\pi\epsilon_0 \ell}{\ln(r_b/r_a)}$. We want the capacitance of this device.

EXECUTE: (a) The section containing no dielectric: Use $C = \dfrac{2\pi\epsilon_0 \ell}{\ln(r_b/r_a)}$ with $\ell = L - x$. Call this capacitance C_1 so $C_1 = \dfrac{2\pi\epsilon_0 (L-x)}{\ln(r_b/r_a)}$.

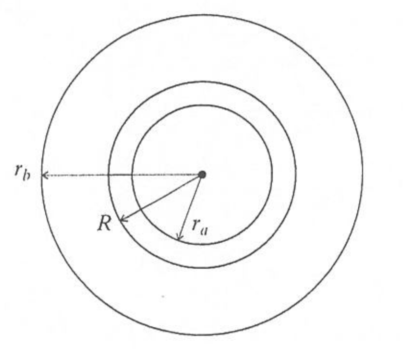


Figure 24.70b

The section partially filled with dielectric: Refer to Fig. 24.70b. Call the inner part (radii r_a and R) C_{aR}. Applying $C = \dfrac{2\pi\epsilon_0 \ell}{\ln(r_b/r_a)}$ with $\ell = x$ and allowing for the dielectric, we have $C_{aR} = \dfrac{K2\pi\epsilon_0 x}{\ln(R/r_a)}$. Call the outer part (radii R and r_b) C_{Rb}. Applying the same formula gives $C_{Rb} = \dfrac{2\pi\epsilon_0 x}{\ln(r_b/R)}$. Call C_2 the equivalent capacitance of C_{aR} and C_{Rb}. Since they are in series, their equivalent capacitance is given by $\dfrac{1}{C_2} = \dfrac{1}{C_{aR}} + \dfrac{1}{C_{Rb}} = \dfrac{\ln(R/r_a)}{K2\pi\epsilon_0 x} + \dfrac{\ln(r_b/R)}{2\pi\epsilon_0 x}$. This gives $C_2 = \dfrac{2\pi\epsilon_0 x}{\dfrac{\ln(R/r_a)}{K} + \ln(r_b/R)}$. Since C_1 and C_2 are in

#6

series, the equivalent capacitance of this device is $C = C_1 + C_2$, which gives

$$C = \frac{2\pi\epsilon_0 (L-x)}{\ln(r_b/r_a)} + \frac{2\pi\epsilon_0 x}{\frac{\ln(R/r_a)}{K} + \ln(r_b/R)}.$$

(b) We want the equivalent capacitance when $x = 0$. Using our result from part (a) gives $C = \frac{2\pi\epsilon_0 (L-x)}{\ln(r_b/r_a)}$. We recognize this as an ordinary air-filled cylindrical capacitor of length $L-x$. Using $L = 10.0$ cm, $r_a = 1.00$ cm, $r_b = 4.00$ cm, and $K = 3.21$, we get $C = 4.01$ pF.

(c) We want C when $x = L$. Using the result from (a) with $K = 3.21$ gives $C = 8.83$ pF.

(d) We want x. First find C. $C = Q/V = (6.00\text{ nC})/(1.00\text{ kV}) = 6.00$ pF. Use our result from (a) and solve for x: $C = \frac{2\pi\epsilon_0 (L-x)}{\ln(r_b/r_a)} + \frac{2\pi\epsilon_0 x}{\frac{\ln(R/r_a)}{K} + \ln(r_b/R)} = 6.00$ pF. The result is $x = 4.12$ cm.

EVALUATE: A device like this one could be used to make a variable capacitor that one could easily vary as desired simply by sliding the dielectric in and out.

here $q = 6$nC, $V = 1000$ volts and K = 3.21