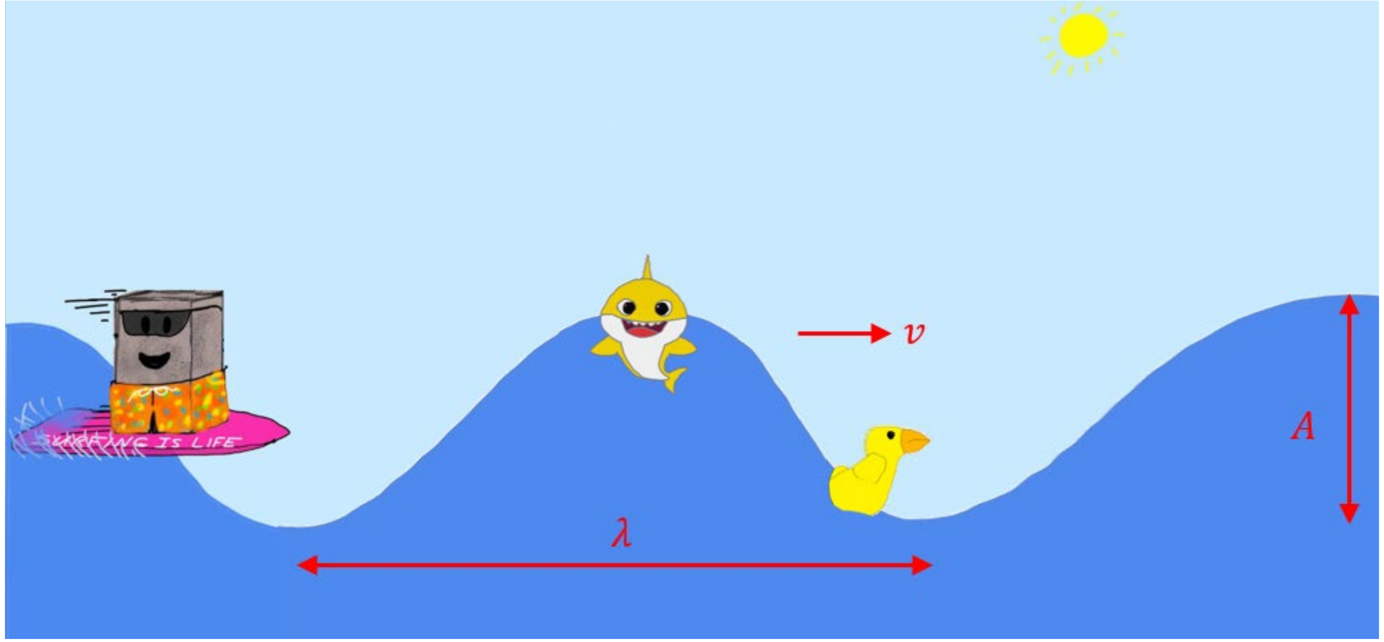
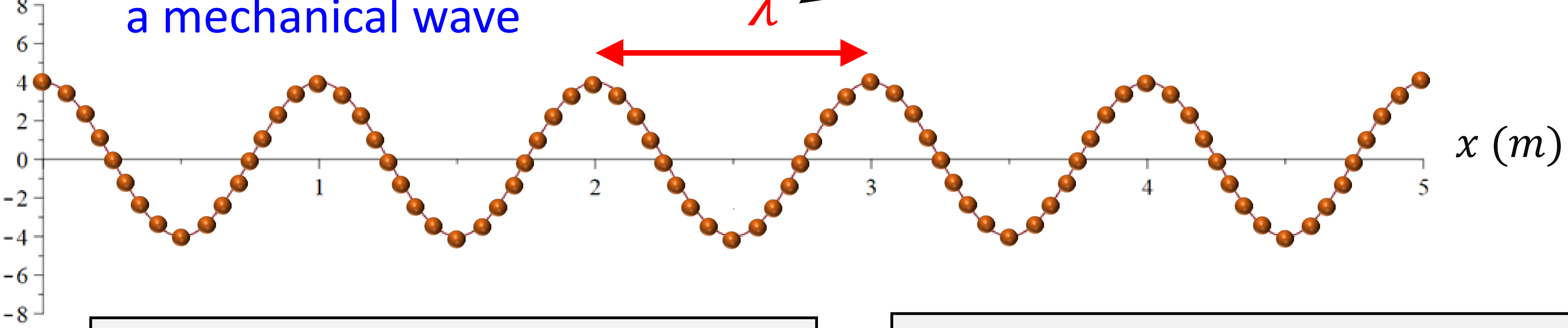
Lecture 32.

Travelling waves.



Last Time

“Snapshot” graph of
a mechanical wave



Snapshot of a wave:

$$\boxed{D(x) = A \cdot \cos(kx + \phi)}$$

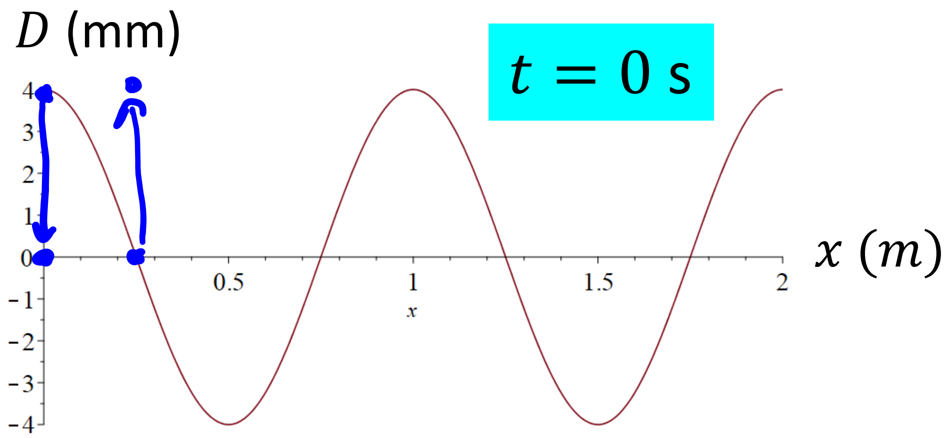
wave number: $k = \dfrac{2\pi}{\lambda}$

Simple Harmonic Oscillator:

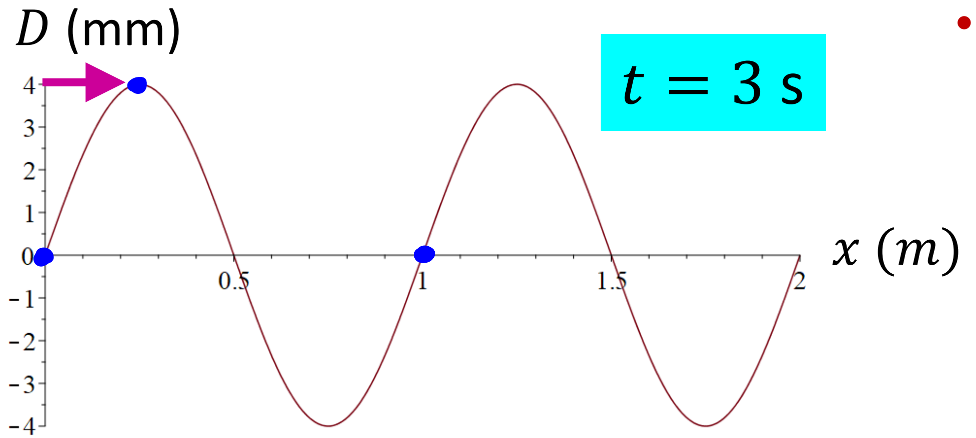
$$\boxed{x(t) = A \cdot \cos(\omega t + \phi)}$$

angular frequency: $\omega = \dfrac{2\pi}{T}$

Last Time



$$D(x) = 4\text{ mm}\cdot\cos\left(\frac{2\pi}{1\,m}\cdot x\right)$$



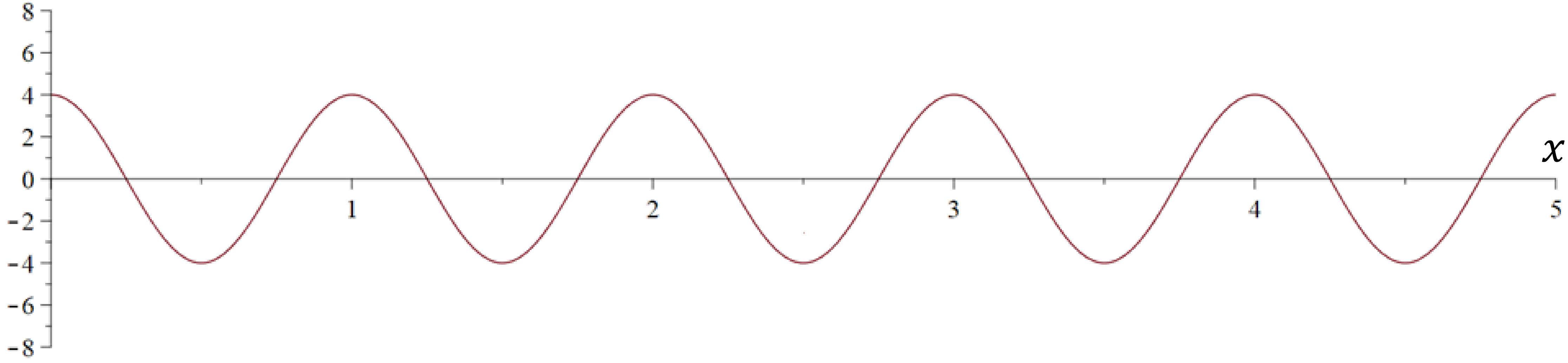
- Shifted $\frac{1}{4}$ period to the right, so phase is $-\frac{2\pi}{4} = -\frac{\pi}{2}$

$$D(x) = 4\text{ mm}\cdot\cos\left(\frac{2\pi}{1\,m}\cdot x - \frac{\pi}{2}\right)$$

Q: How will it further change as the time goes?

Travelling waves

Displacement (mm)



$x\ (m)$

$$D(x) = A \cdot \cos(kx + \Phi(t))$$

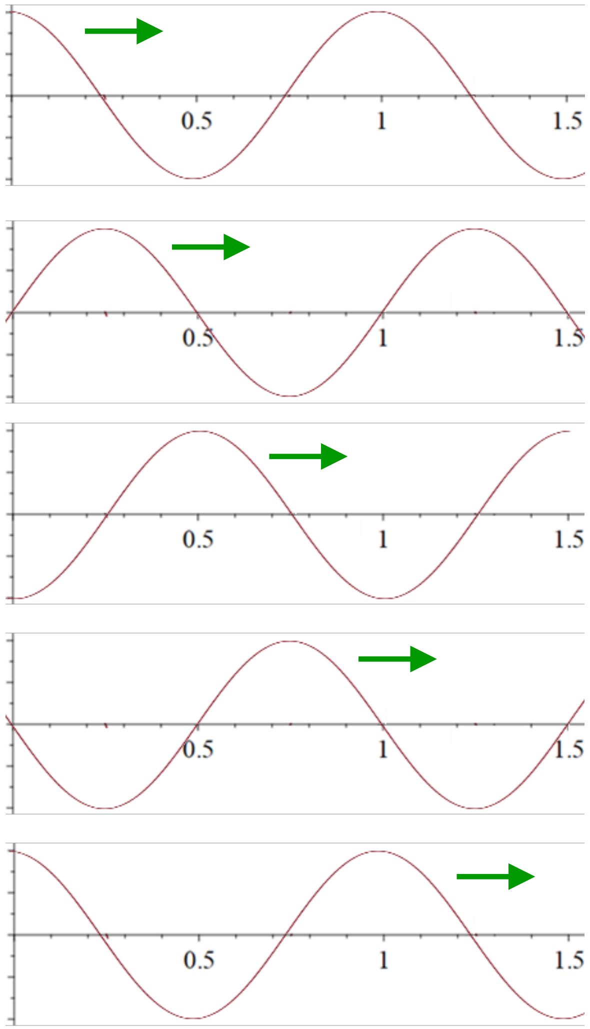
- Constant velocity:

phase is proportional to time!

- Right-moving wave: $\Phi(t) = -\text{constant} \cdot t$
- Left-moving wave: $\Phi(t) = +\text{constant} \cdot t$

- Let us figure out what $\Phi(t)$ is

Q: Which of the following represents the displacement of the wave shown as a function of position?



$x(m)$ $t = 0\ s$

$x(m)$ $t = 3\ s$

$x(m)$ $t = 6\ s$

$x(m)$ $t = 9\ s$

$x(m)$ $t = 12\ s$

A. $D(x) = A\cos\left(\frac{2\pi}{1\ m}\cdot x - \frac{t}{12\ s}\right)$

B. $D(x) = A\cos\left(\frac{2\pi}{1\ m}\cdot x - 12\ s\cdot t\right)$

C. $D(x) = A\cos\left(\frac{2\pi}{1\ m}\cdot x - \frac{2\pi}{12 s}t\right)$

D. $D(x) = A\cos\left(\frac{2\pi}{1\ m}\cdot x - \frac{12\ s}{2\pi}t\right)$

E. $D(x) = A\cos\left(\frac{2\pi}{1\ m}\cdot x - \frac{\pi}{2}t\right)$

$\frac{2\pi}{\cancel{12s}} \cdot \cancel{12s}$

2π

Q: Which of the following represents the displacement of the wave shown as a function of position?

- Moves to the right => $\Phi(t) = -$something
- Shift by full period in 12 s, so want $\Phi = -2\pi$ at $t = 12$ s

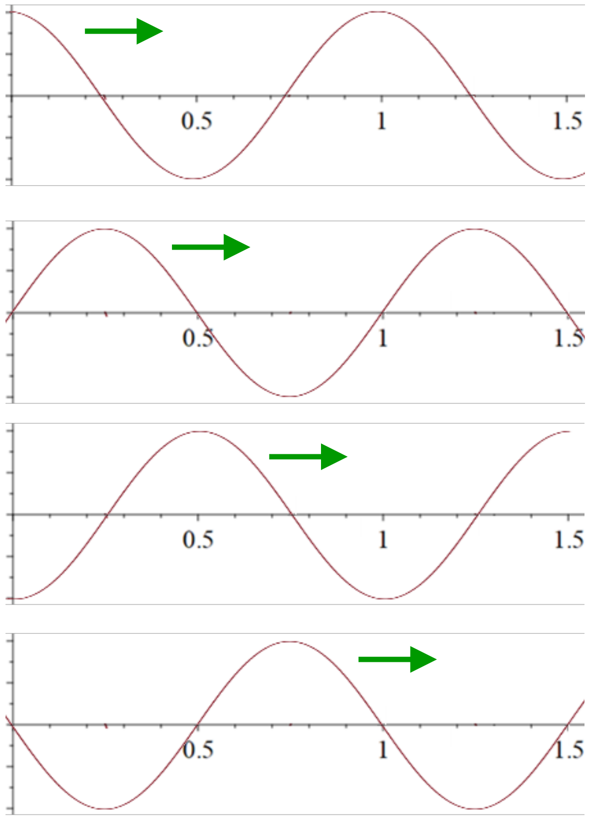
A. $D(x) = A\cos\left(\frac{2\pi}{1\,m} \cdot x - \frac{t}{12\,s}\right)$

B. $D(x) = A\cos\left(\frac{2\pi}{1\,m} \cdot x - 12\,s \cdot t\right)$

C. $D(x) = A\cos\left(\frac{2\pi}{1\,m} \cdot x - \frac{2\pi}{12s}t\right)$ ✓

D. $D(x) = A\cos\left(\frac{2\pi}{1\,m} \cdot x - \frac{12\,s}{2\pi}t\right)$

E. $D(x) = A\cos\left(\frac{2\pi}{1\,m} \cdot x - \frac{\pi}{2}t\right)$



$x(m)$ $t = 0\ s$

$x(m)$ $t = 3\ s$

$x(m)$ $t = 6\ s$

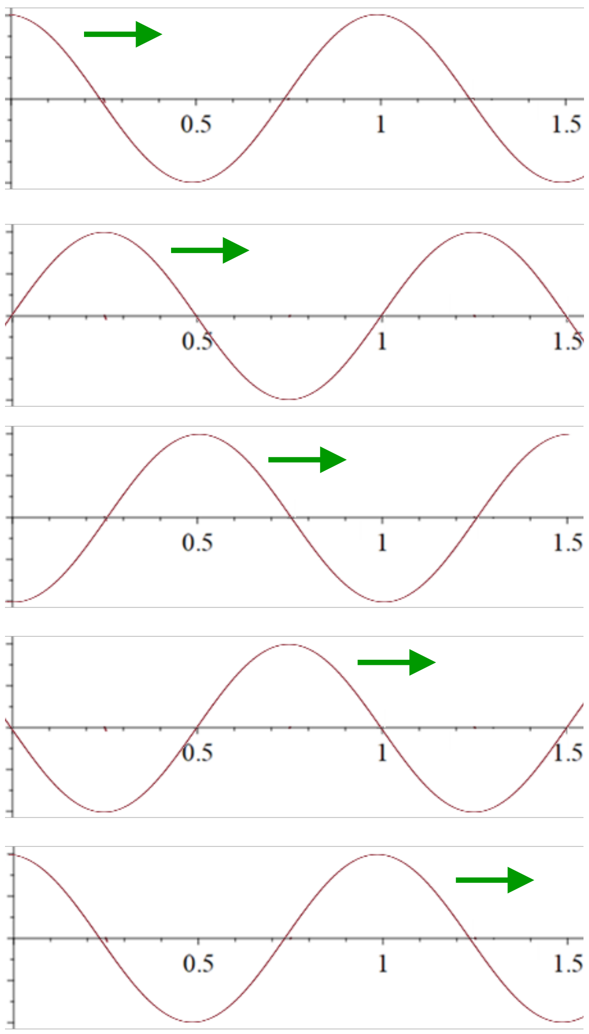
$x(m)$ $t = 9\ s$

$x(m)$ $t = 12\ s$

Moves to the right => $\Phi(t) = -$

Shift by full period in 12 s, so wa
at $t = 12$ s

Time-dependent phase Φ(t)



$x(m)$ $t = 0\ s$

$x(m)$ $t = 3\ s$

$x(m)$ $t = 6\ s$

$x(m)$ $t = 9\ s$

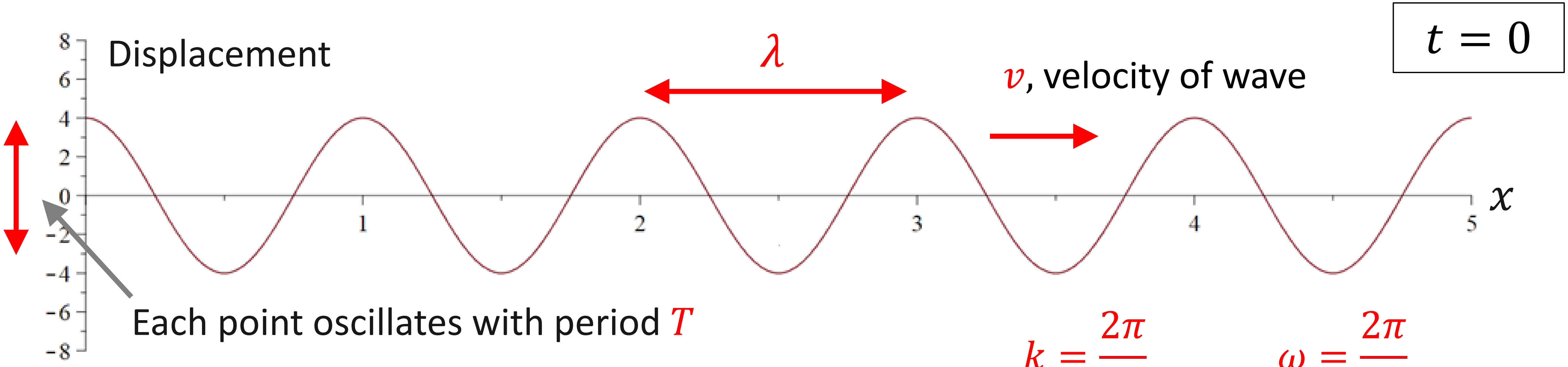
$x(m)$ $t = 12\ s$

- 12 s is the period T, since each point on the string has made a complete oscillation
- So $\omega = \dfrac{2\pi}{12\ s}$ is the angular frequency
- Phase for right-moving wave is $\Phi = -\omega t \quad \Rightarrow$
- $D(x) \;\Rightarrow\; D(x,t) = A \cdot \cos(kx - \omega t)$

$k = \dfrac{2\pi}{\lambda}$ $\qquad$ $\omega = \dfrac{2\pi}{T}$

Travelling harmonic waves

Q: How is v related to ω and k ?



$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

- Right-moving wave: $D(x,t) = A \cdot \cos(kx - \omega t + \phi_0)$
- Left-moving wave: $D(x,t) = A \cdot \cos(kx + \omega t + \phi_0)$

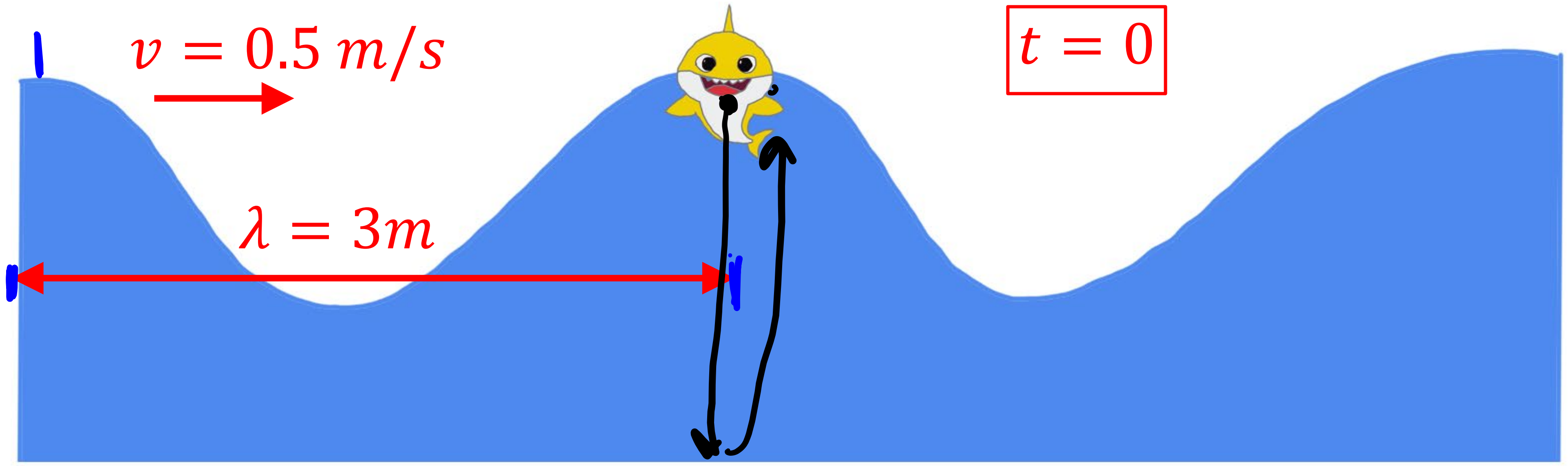
ϕ_0 is the initial condition:

$$D(x=0, t=0) = A\cos\phi_0$$

➢ Now we can tell what happens to each point in space at all moment of time!

- Note: Could also write displacement as $D(x,t) = A \cdot \sin(kx - \omega t + \phi_0)$
- This describe a wave that starts at $D = 0$ at $(kx - \omega t + \phi_0) = 0$ instead of $D = A$

Q: Baby Shark is floating at the surface of the water as waves pass by. At what time will Baby Shark next reach a maximum height?



λ

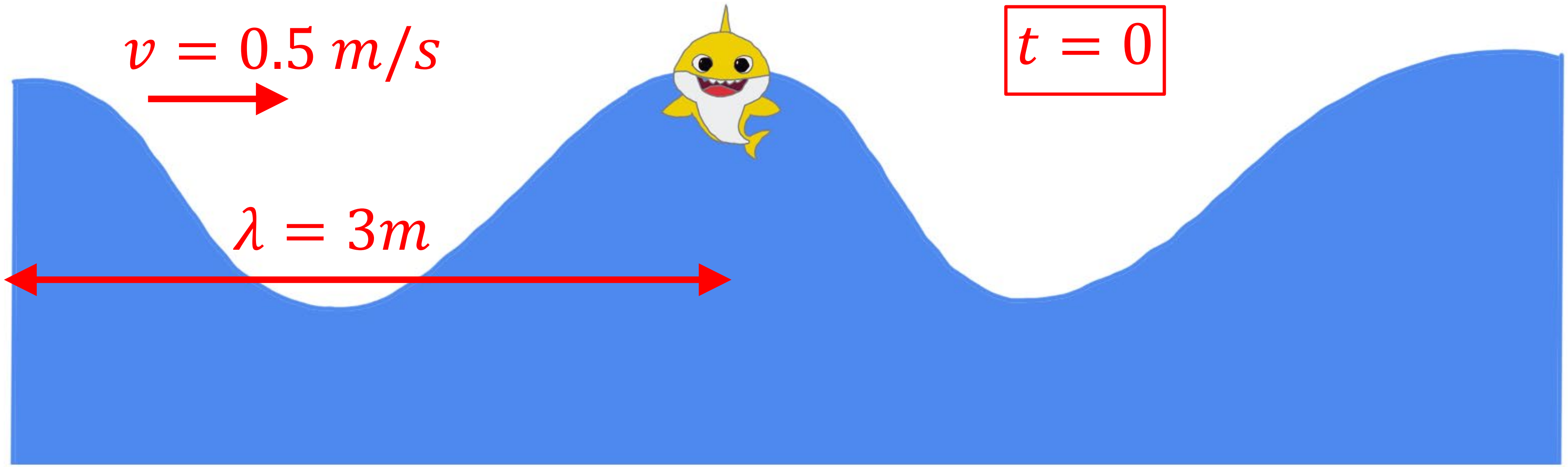
$$t = \frac{3m}{0.5\ m/s} = 6s$$

v

$$\frac{2\pi}{\omega} = T$$

A. 0.17 s
B. 1.5 s
C. 3 s
D. 6 s
E. 12 s

Q: Baby Shark is floating at the surface of the water as waves pass by. At what time will Baby Shark next reach a maximum height?

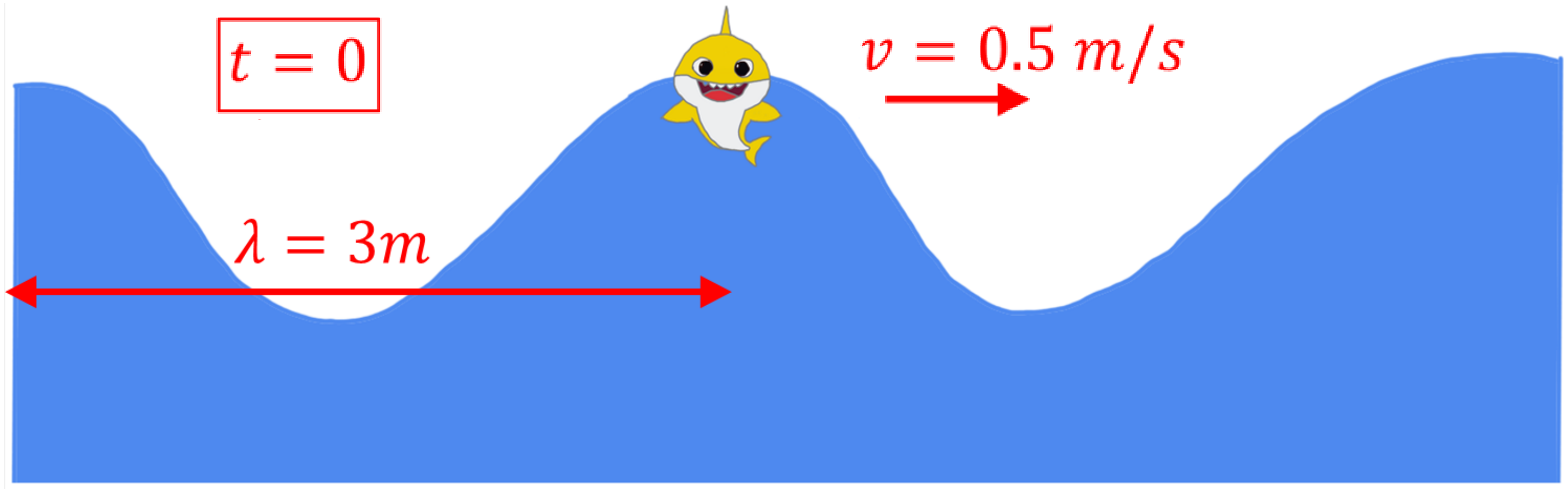


A. 0.17 s
B. 1.5 s
C. 3 s
D. 6 s
E. 12 s

- Baby Shark will be at max height again when wave moves distance $\lambda = 3\ m$.
- This takes time $T = \frac{\lambda}{v} = \frac{3\ m}{0.5\ m/s} = 6\ s$.

Period, wavelength, velocity of the wave

- wave velocity = velocity of the peaks



Common sense

$$T = \frac{1}{f}$$

$$f = \frac{\omega}{2\pi} \quad \text{and} \quad \lambda = \frac{2\pi}{k}$$

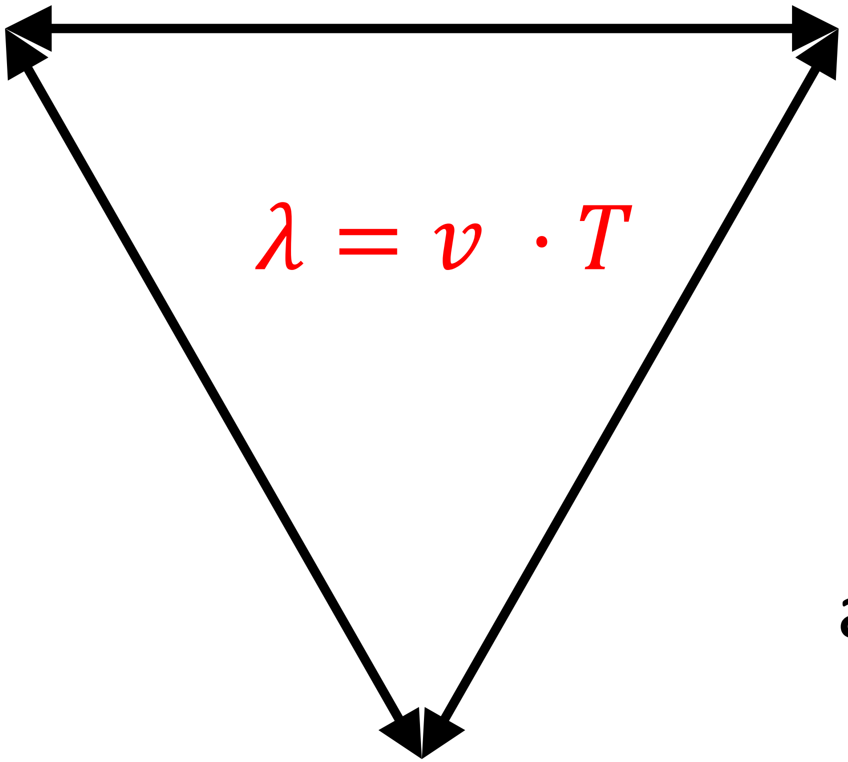
$$\boxed{\lambda = v \cdot T} \longleftrightarrow \boxed{v = \lambda f} \longleftrightarrow \boxed{v = \frac{\omega}{k}}$$

Properties of waves

$\lambda \leftrightarrow k$

wavelength

wave number

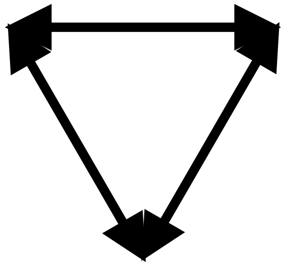


$\lambda = v \cdot T$

v

velocity

f T



ω

period

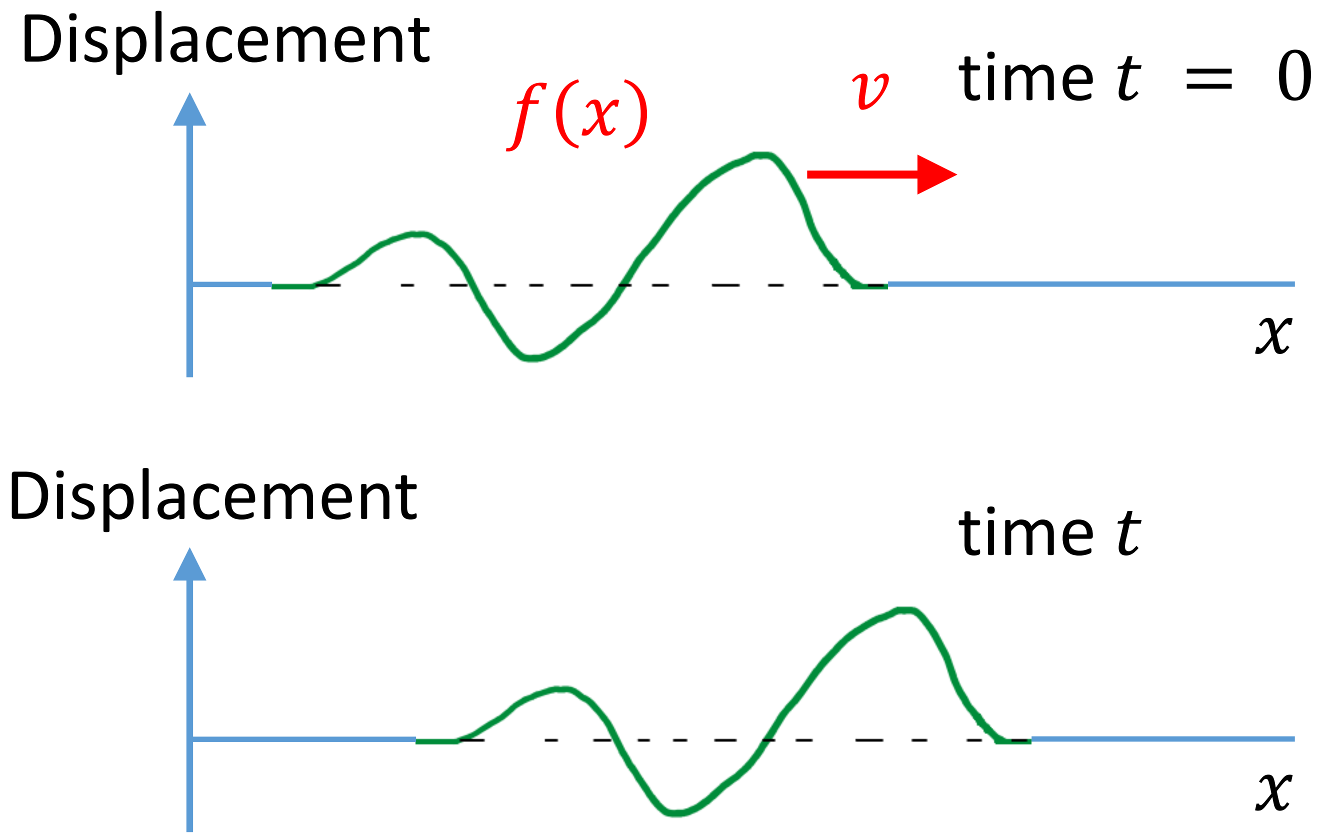
frequency

angular frequency

energy !

A amplitude

Q: At time $t = 0$, a right-moving wave pulse has displacement $D(x, t = 0) = f(x)$ shown in the top picture. At a later time t, the displacement will be described by:



A. $D(x, t) = f(x)$

B. $D(x, t) = f(x) + vt$

C. $D(x, t) = f(x) - vt$

D. $D(x, t) = f(x + vt)$

E. $D(x, t) = f(x - vt)$

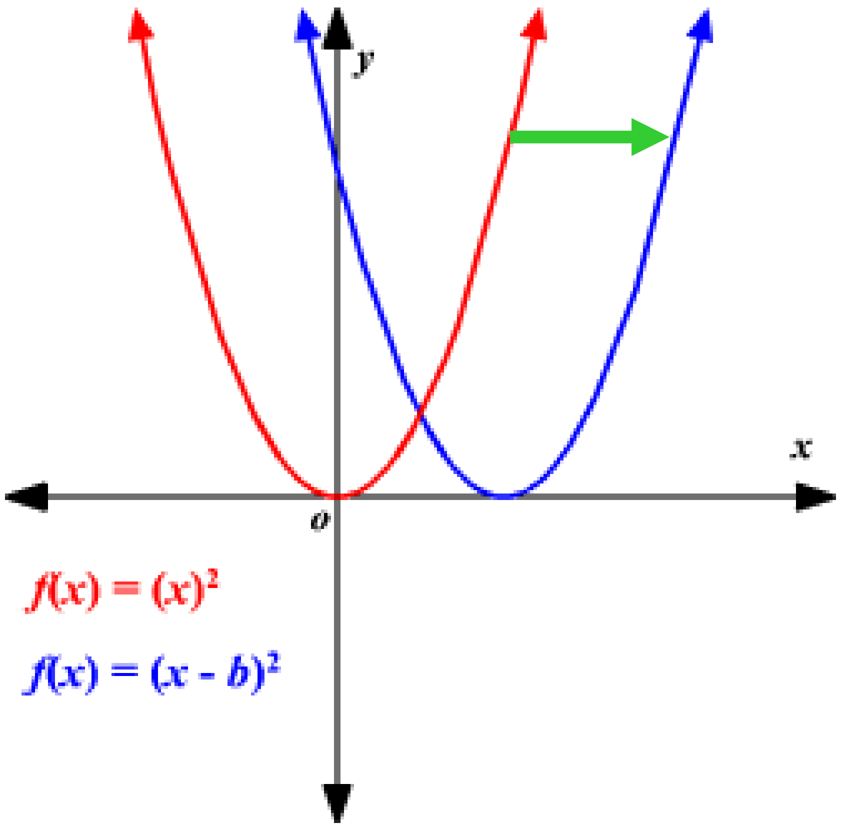
Reminder: Horizontal shift of a function

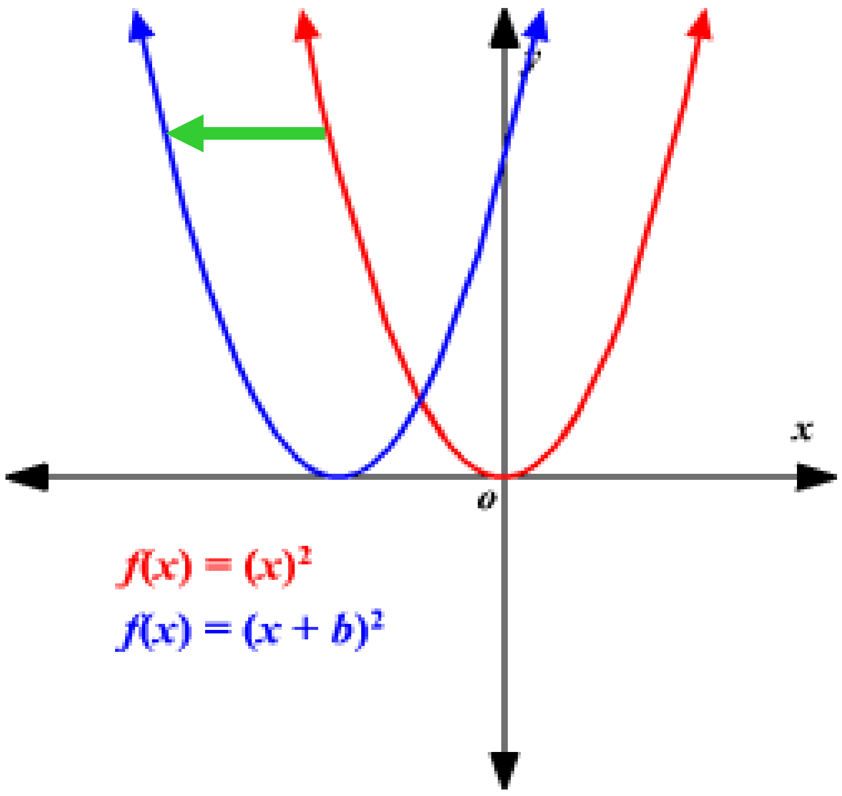
Shift function to the right by x_0:

$$x \to x - x_0$$

Shift function to the left by x_0:

$$x \to x + x_0$$



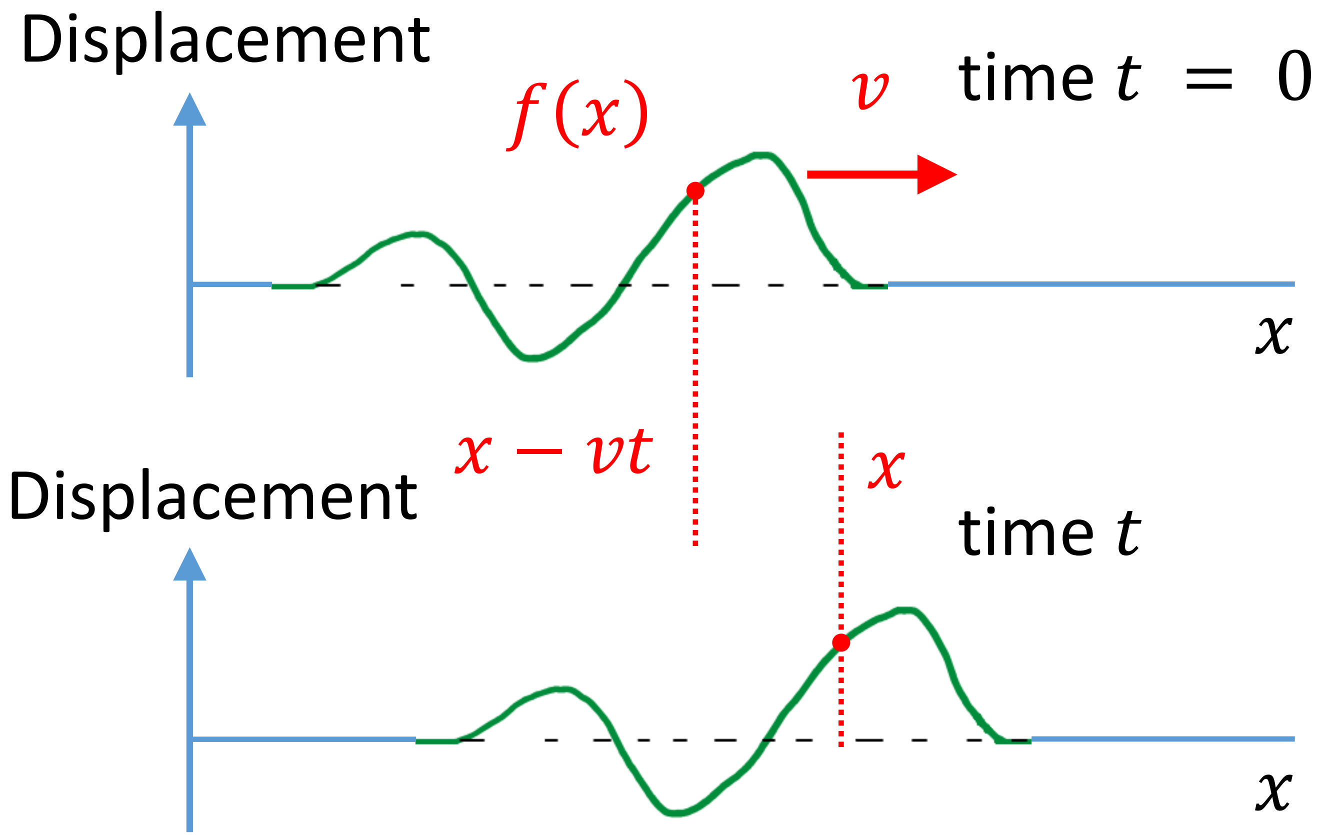


Q: At time $t = 0$, a right moving wave pulse has displacement $D(x, t = 0) = f(x)$ shown in the top picture. At a later time t, the displacement will be described by:

In time t: wave shifted by vt to the right

Displacement at position x at time t is displacement at position $x - vt$ in original graph

So new displacement is $f(x - vt)$



A. $D(x, t) = f(x)$

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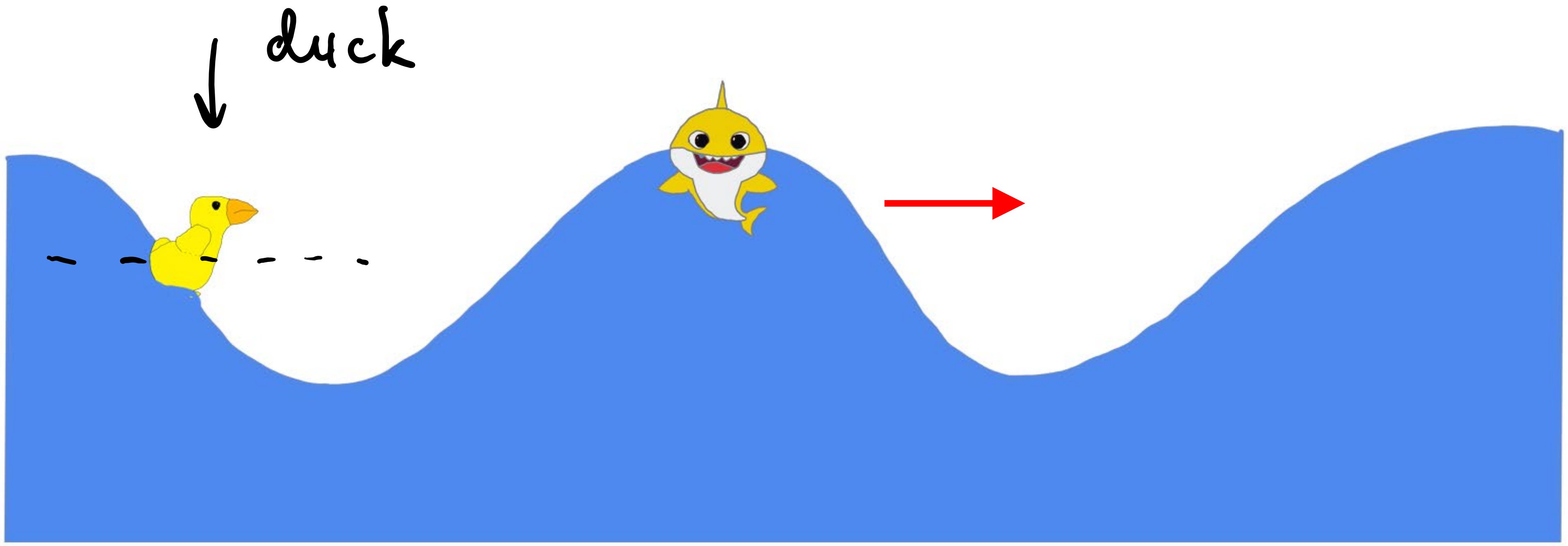
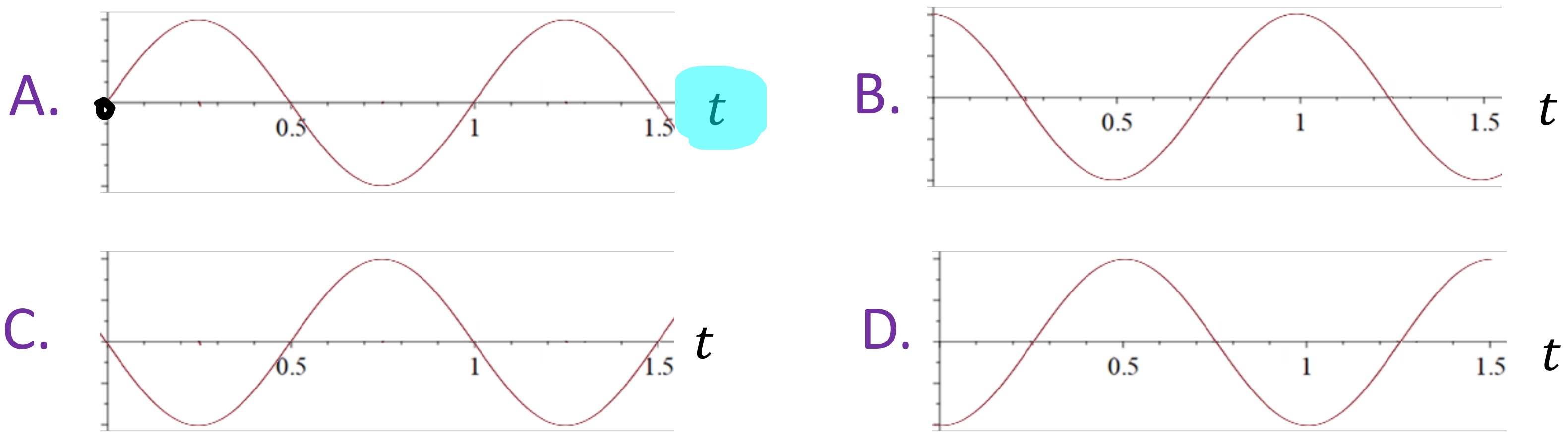
D. $D(x, t) = f(x + vt)$

E. $D(x, t) = f(x - vt)$ ✓

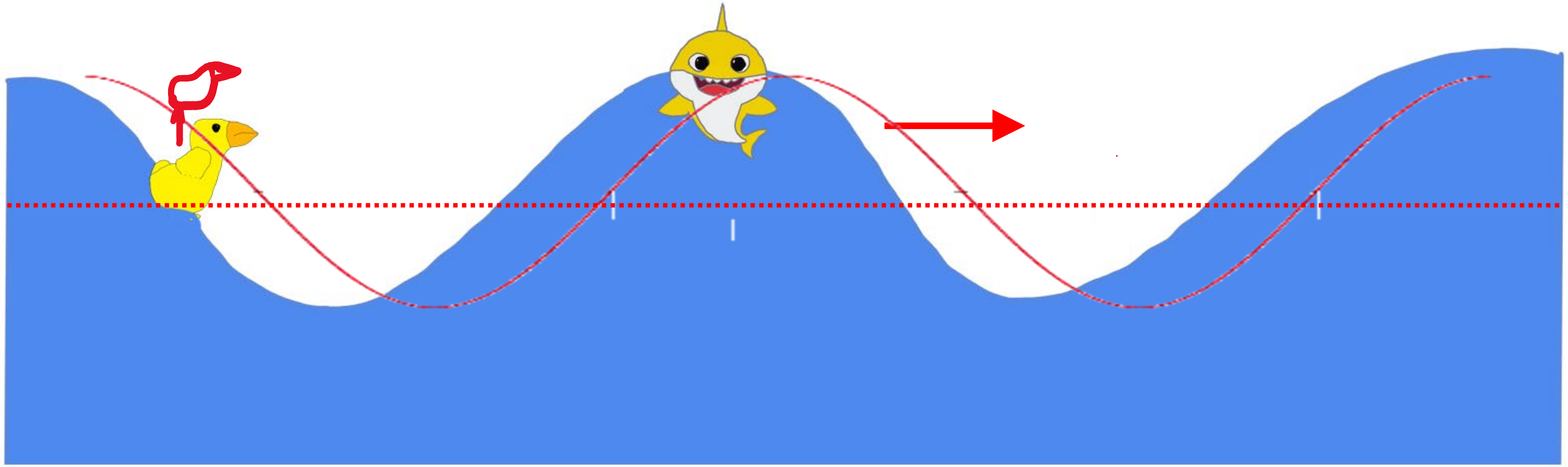
Q: Which graph represents the duck's vertical displacement as a function of time?





history graph of a wave

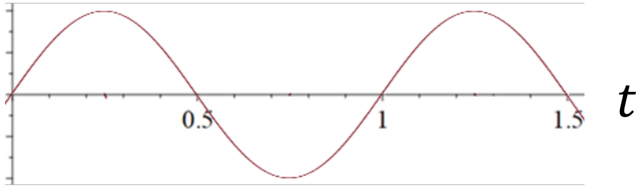
Q: Which graph represents the duck's vertical displacement as a function of <u>time</u>?



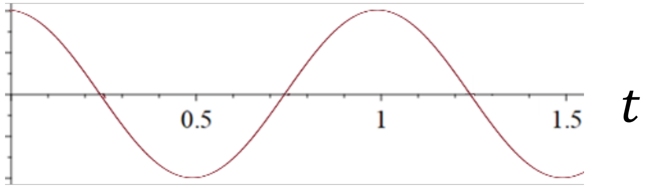
After short amount of time, duck moves up.

Eventually, will be lower than the original height.

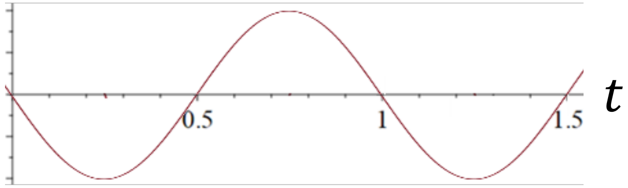
A.



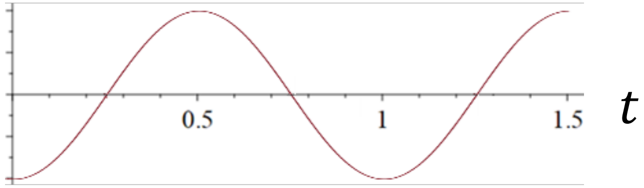
B.

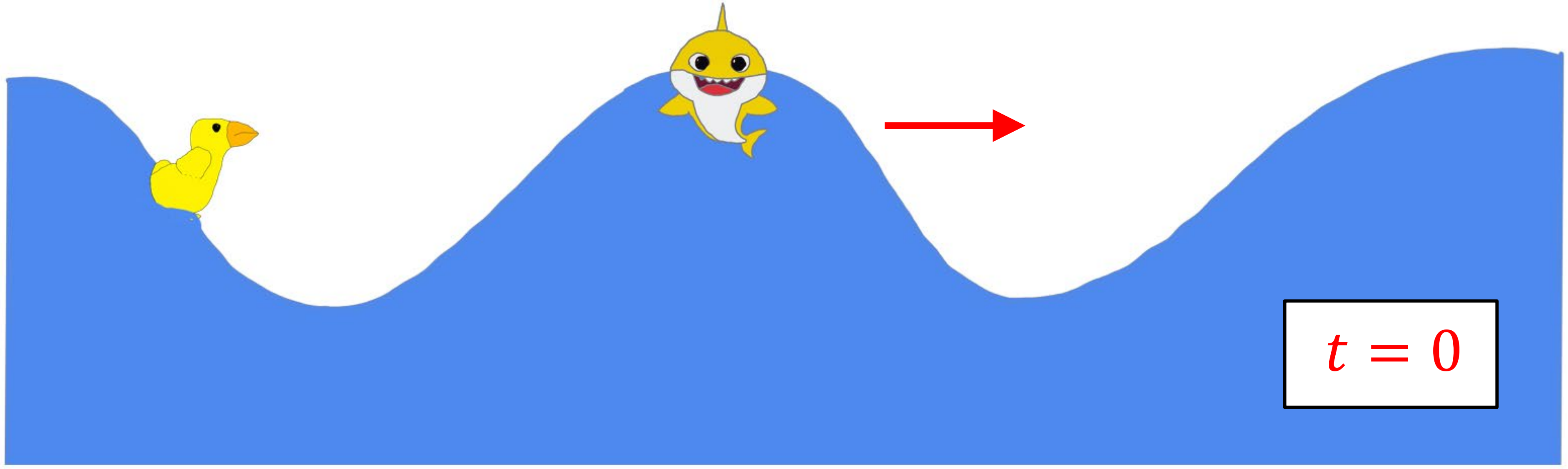


C.

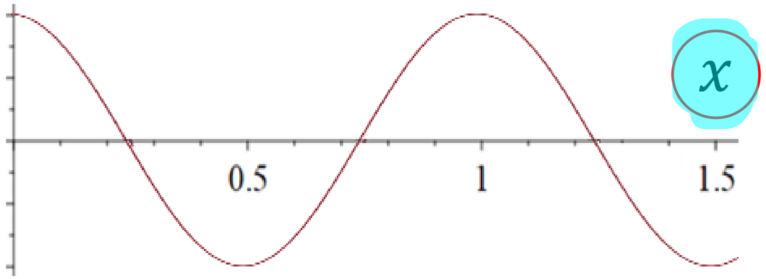


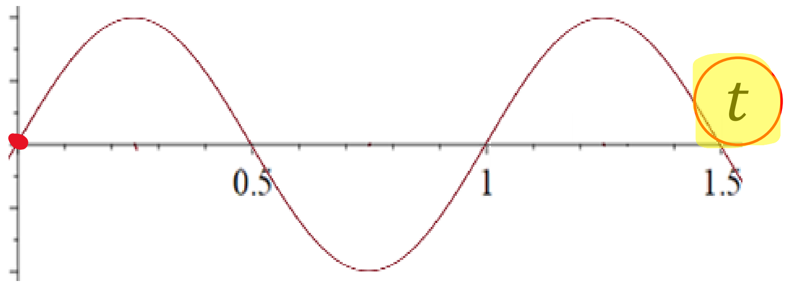
D.



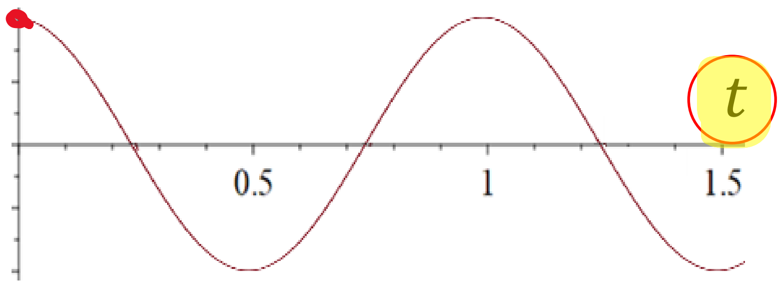


- Snapshot Graph (at $t = 0$)
- History Graphs



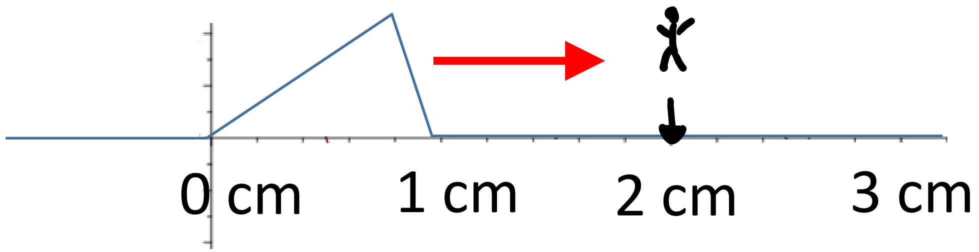


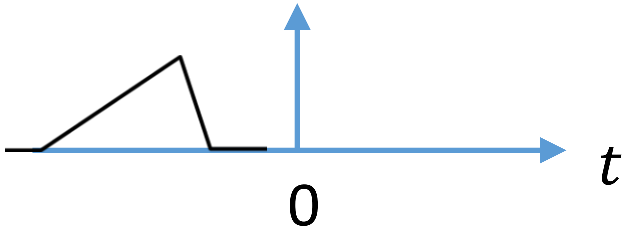
at x = Duck

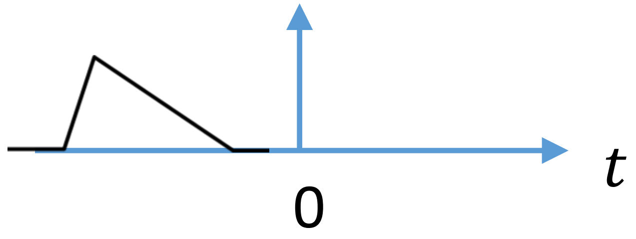


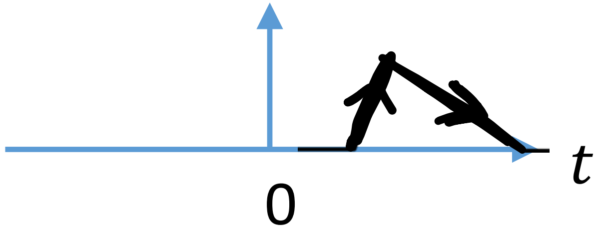
at x = Baby Shark

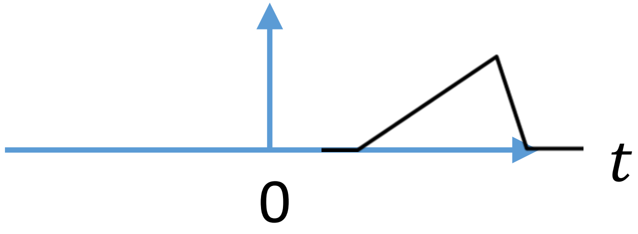
Q: The graph shows $D(x,0)$, the snapshot of a right-moving wave pulse at time $t = 0$. Which of the graphs below could represent $\underbrace{D(2\ cm, t)}_{\text{history}}$?



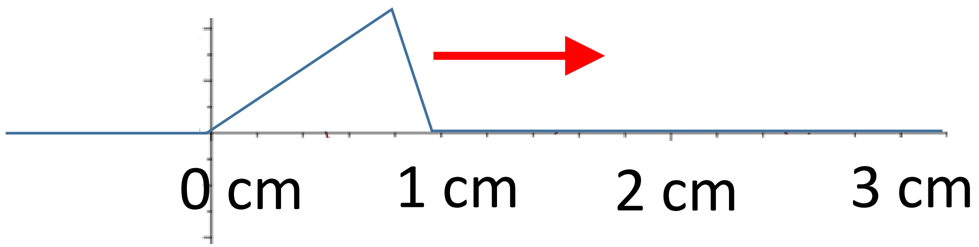
A. 

B. 

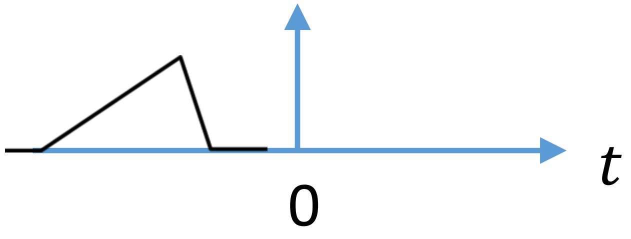
C. 

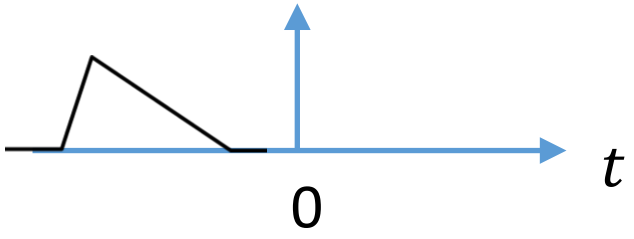
D. 

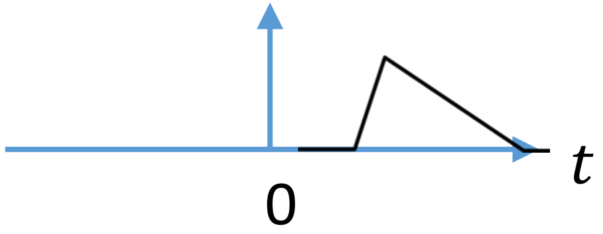
Q: The graph shows $D(x, 0)$, the snapshot of a right-moving wave pulse at time $t = 0$. Which of the graphs below could represent $D(2\ cm, t)$?

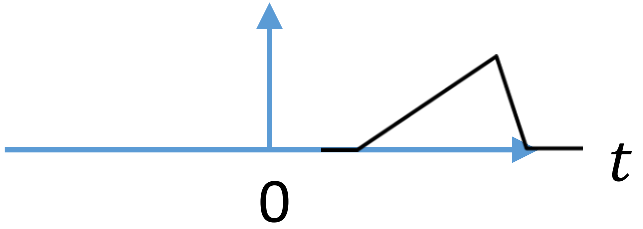


- No motion before $t = 0$.
- At some time after $t = 0$, pulse reaches $x = 2\ cm$.
- Front of pulse is steeper, so displacement increases quickly, then decreases slowly.

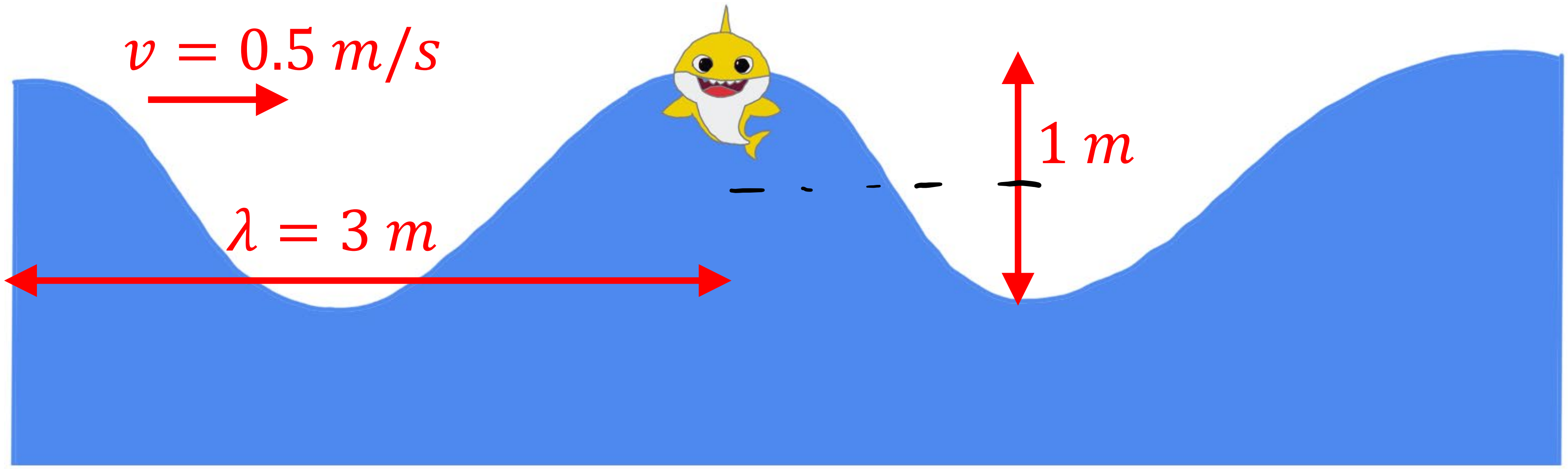
A.


B.


C.


D.


Q: What will be Baby Shark's maximum vertical velocity?



A. $\pi/6\ m/s$

B. $\pi/3\ m/s$

C. $\pi\ m/s$

D. $2\pi\ m/s$

E. $3\pi\ m/s$

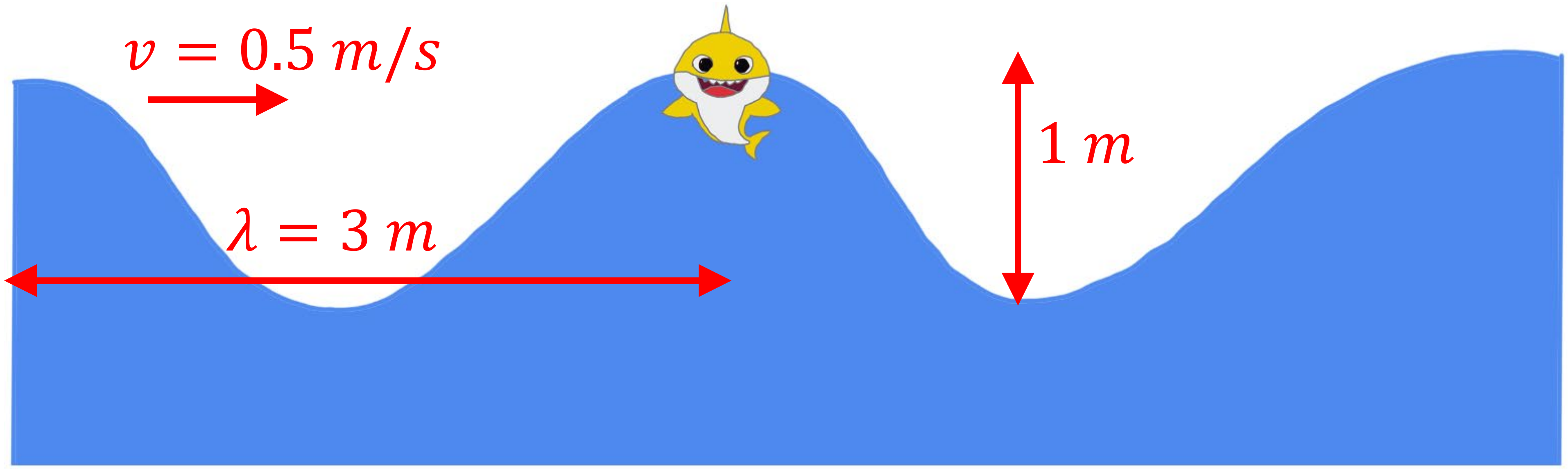
$$\boxed{v_{max} = A \cdot \omega}$$

$$\boxed{A = 0.5\,m}$$

λ, v

$$\boxed{\begin{gathered} \lambda f = v \\ f = \frac{v}{\lambda} = \frac{\omega}{2\pi} \end{gathered}}$$

Q: What will be Baby Shark's maximum vertical velocity?



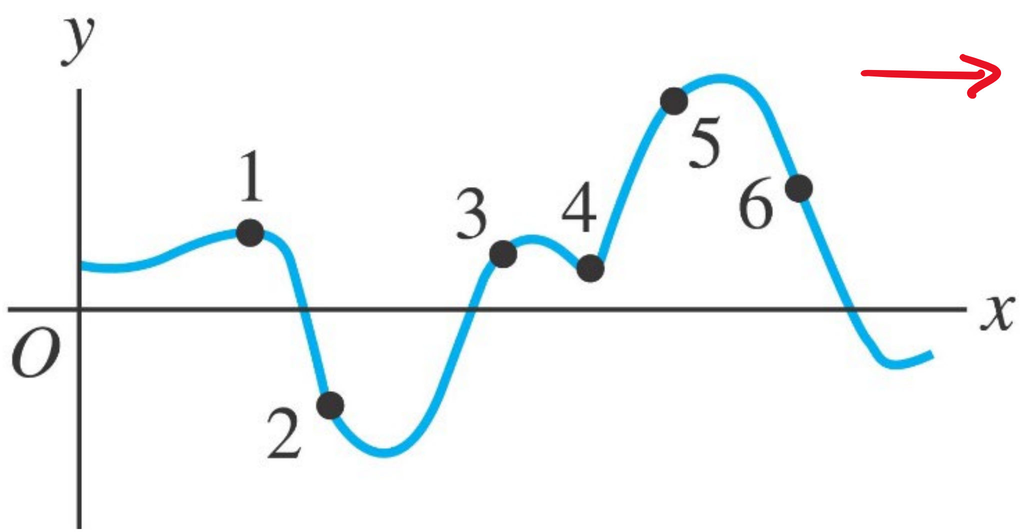
A. $\pi/6\ m/s$
B. $\pi/3\ m/s$
C. $\pi\ m/s$
D. $2\pi\ m/s$
E. $3\pi\ m/s$

- Baby Shark is in simple harmonic motion, so

$$D(t) = A\cos(\omega t + \phi)$$

- Velocity is $\frac{dD}{dt} = -A\omega\sin(\omega t + \phi)$.
- Max v is $A\omega = A\frac{2\pi}{T} = A\frac{2\pi}{\lambda/v} = 0.5\,m\frac{2\pi}{6s} = \frac{\pi}{6}m/s$

Q: A transverse wave on a string is moving to the right. Here is a snapshot graph of the shape of part of the string at time t. At this time, the velocity of a particle on the string is upward at:



A. only one of points 1, 2, 3, 4, 5, and 6

B. point 1 and point 4 only

C. point 2 and point 6 only

D. point 3 and point 5 only

E. three or more of points 1, 2, 3, 4, 5, and 6

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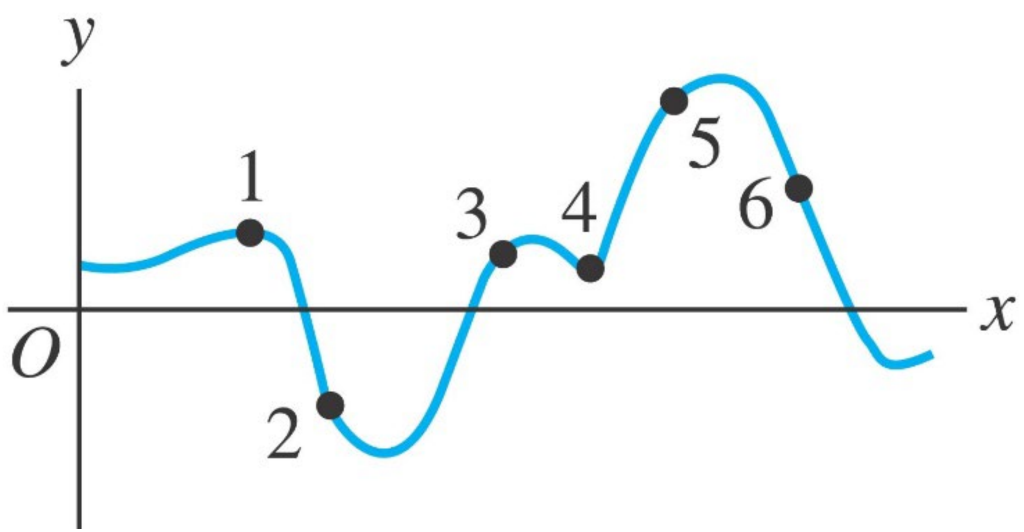


A. only one of points 1, 2, 3, 4, 5, and 6

B. point 1 and point 4 only

C. point 2 and point 6 only ✓

D. point 3 and point 5 only

E. three or more of points 1, 2, 3, 4, 5, and 6

Summary:

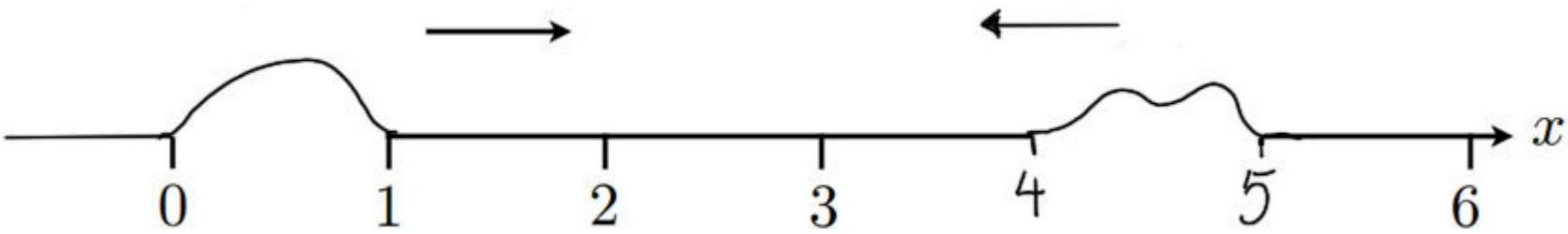
- Wave equation: $D(x,t) = A \cdot \cos(kx \pm \omega t + \phi_0)$

- Speed, wavelength, frequency: $v = \lambda f = \frac{\omega}{k}$

- Snapshot graphs vs History graphs of a wave

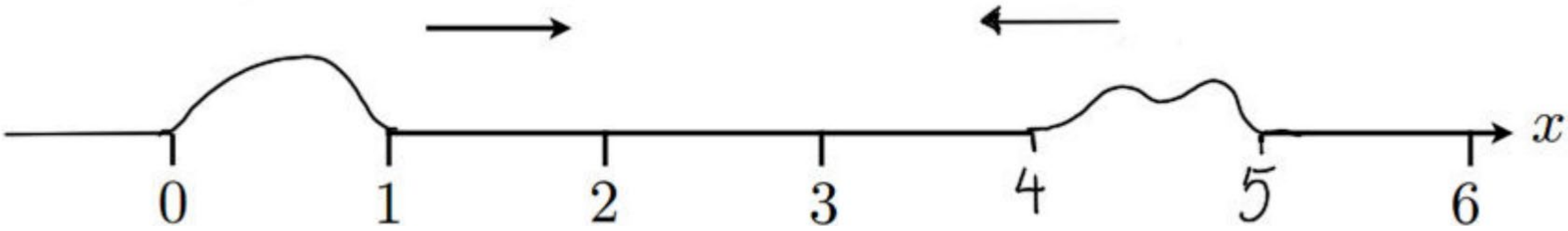
- Max vertical speed of a particle ($v_{max} = \omega A$) vs speed of the wave ($v = \lambda f$)

Q: Two waves are travelling towards each other as shown. When they meet, they will:



A. Bounce off each other and reflect backwards
B. Destroy each other, leaving a few random ripples going in either direction
C. Pass right through each other

Q: Two waves are travelling towards each other as shown. When they meet, they will:



The presence of one wave does not change the equation of motion for the other wave. They each behave like the other were not there, so they pass right through each other

A. Bounce off each other and reflect backwards
B. Destroy each other, leaving a few random ripples going in either direction
C. Pass right through each other ✓