Math_226 Notes

Randy Truong

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Contents

Chapter 1

10.1 (Part One): Sequences (Part One) (03/28/23)

1.1 Reminders

- The first MyLab homework is due on **Wednesday, March 28, 2023**.
  - Series (Part 0)
- The first **written homework** is going to be due on **Friday, March 31, 2023**.

1.2 Objectives

- We want to be able to derive the concept of a series and a sequence.
- We want to be able to understand where the idea of a series and sequence come from, especially from seemingly-ordinary objects.
- We want to explore the idea of a limit in relation to a sequence.
- We want to be able to discretely express a sequence.

1.3 Motivation

In former calculus classes, we have observed the idea of limits, derivatives, and integrals at face value. We know how to evaluate these different calculations, but what exactly do they mean in the context of math? How can we better observe what exactly happens in these calculations, and understand them outside the context of visualizing graphs or projectile motion.

1.4 A third...

Chapter 2

10.1 (Part Two): Sequences (Part Two) (03/29/23)

2.1 Reminders

- MyLab Math Assignment 1 - **Sequences (Part 0)** is due **tonight, March 29, 2023**.
- Written Homework 1 is due **Friday, March 31, 2023** at the **beginning of class**.
- Friday's lesson is going to be over **sections 4.6 and 10.1** and we will be talking about **Newton's Method.**

2.1.1 Course Philosophy

Remember that the entire point of this class is to develop an intuition and a larger understanding and appreciation of calculus.

"Calculus is just algebra with a tiny drop of limits"

2.2 Motivation

In the last class, we got our first taste of sequences by exploring the idea of a third and eventually relating it to the **geometric sequence**, which results in the formula of

$$a_n = \frac{1}{1-x}$$

In this class, we were essentially taking our "preview" of sequences, actually defining different aspects of our sequence, doing operations on sequences, and finally, exploring one of the most important ideas of sequences, which are limit convergence and divergence, which led us to the famous $\varepsilon - N$ proof, which is also known as the **precise definition of convergence.**

2.3 Sequences

Definition 1. *Sequences*

A function with a domain of natural numbers and a co-domain of real numbers.

$$f : \mathbb{N} \to \mathbb{R}$$

Meaning.

Although our intuition would tell us that a sequence is just a list or a collection of numbers, a sequence is more precisely just a function in which we input an **index** (a natural number) and we output a **term** (a real number). We, of course, then, collect these outputs, and this is what we generally see.

Example.

$$\{a_n\}$$

$$\{a_n\}_{n=1}^{\infty}$$

$$\{a_n\}_{n=0}^{\infty}$$

$$1, 2, 3, 4, \ldots$$

$$1.1, 2.2, 3.3, 4.4, \ldots$$

2.4 Convergence

Definition 2. *Sequence Convergence*

Informal Definition.
A sequence $\{a_n\}$ converges to a limit L if the terms get arbitrarily close to L as n gets sufficiently large, which is also known as

$$\lim_{n \to \infty} a_n = L$$

Formal Definition.
A sequence $\{a_n\}$ converges to a limit L if, for every $\varepsilon > 0$, where ε is the distance from the range to the limit L, there exists such a number that

$$|a_n - L| < \varepsilon \text{ for } n \geq N$$

The preceding expression is also known as the $\varepsilon - N$ proof.

2.5 Divergence

Definition 3. *Sequence Divergence*

Informal Definition.
A sequence diverges when it doesn't converge. If the sequence $\{a_n\}$ does not get arbitrarily close to limit L as n gets sufficiently large. This is also known as when the limit L **does not exist.**

$$\lim_{n\to\infty} a_n \text{ does not exist}$$

Formal Definition.
A sequence a_n diverges to (positive) infinity if, for every $M > 0$, there exists an N such that

$$a_n > M \text{ whenever } n > M$$

Additionally, a sequence a_n diverges to negative infinity, if, for every $M < 0$, there exists an N such that

$$a_n < M \text{ whenever } n > M$$

2.6 $\varepsilon - N$ Proof

In this proof, we are proving the existence of a limit when given a sequence a_n

2.7 Properties of Sequence Limits

Chapter 3

10.1: Sequences (Part 3) (03/31/23)

3.1 Summary of Lesson

In this lesson, we further develop the idea of convergence and divergence by **expanding it to non-elementary sequences**. We apply several new theorems as a result of this, such as the Sandwich Theorem. The homework, as a result, is all about convergence and divergence, testing on how well we are able to determine convergence and divergence given sequences including factorials, logarithms, and exponential functions.

3.2 Reminders

- The **third MyLab Math assignment** (Sequences (Part 2)) is due on **Tuesday, April 3rd**.
- The second written assignment is due on **Friday, April 7, 2023**.

3.3 Motivation

In the previous lesson, we learned about the **precise definition of convergence and divergence**. We learned about what exactly convergence and divergence means, and we were able to apply these concepts to various elementary sequences. However, what if we were given a sequence like

$$\{a_n\} = \frac{\cos(n)}{n}$$

Additionally, what if we were given a sequence like

$$\{a_n\} = \left(1 + \frac{x}{n}\right)^n$$

These sequences are undoubtedly more complex than our former examples and are not nearly as intuitive whenever it comes to actually solving them.

Therefore, we need to learn a few more **theorems** as well as **techniques** in order to determine convergence and divergence in more complex sequence functions.

Theorem 1. *Sandwich Theorem*

Let $\{a_n\}$, $\{b_n\}$, $\{c_n\}$ be sequences of real numbers. If $a_n \leq b_n \leq c_n$ holds for all n beyond some index N, and if $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$, then $\lim_{n\to L} b_n = L$ also.

$$\text{let } \{a_n\}, \{b_n\}, \{c_n\} \text{ be sequences}$$

$$\text{if } a_n \leq b_n \leq c_n \text{ and}$$

$$\lim_{n\to\infty} a_n = L \text{ and } \lim_{n\to\infty} c_n = L, \text{ then}$$

$$\lim_{n\to\infty} b_n = L$$

Theorem 2. *The Continuous Function Theorem For Sequences*

Theorem 3. *Other Theorem*

Theorem 5

Chapter 4

4.6, 10.1: Finishing Sequences + Newton's Method (04/03/2023)

4.1 Reminders

- The **third MyLab Math assignment** (Sequences (Part 2)) is due on **Tuesday, April 3rd**.
- The second written assignment is due on **Friday, April 7, 2023**.

4.2 Motivation

Chapter 5

10.2 (Part One): Infinite Series (Part One)

5.1 Reminders

- The **third MyLab Math assignment** (Sequences (Part 2)) is due on **Tuesday, April 3rd**.
- The second written assignment is due on **Friday, April 7, 2023**.

5.2 Motivation

Chapter 6

10.2 (Part Two): Infinite Series (Part Two) (04/05/23)

6.1 Reminders

- The **third MyLab Math assignment** (Sequences (Part 2)) is due on **Tuesday, April 3rd**.
- The second written assignment is due on **Friday, April 7, 2023**.

6.2 Motivation

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