Math_226 Written Homework 2: Series

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Problem 1.1. Look at the following problems from 10.1: 22, 60, 108, 126

Problem 1.1.1 (Problem 22). Find a formula for the nth formula for the nth term of the sequence.

$$2, 6, 10, 14, 18, \ldots$$

$$\text{let}\{a_n\} = 2, 6, 10, 14, 18, ...$$

$$\{a_n\}_{n=1} = 4n - 2$$

Problem 1.1.2 (Problem 60). Determine whether the sequence diverges or converges. If the sequence converges, find its limit.

$$\{a_n\} = \sqrt[n]{n^2}$$

$$\text{let } \{a_n\} = (n^2)^{\frac{1}{n}}$$

$$a_n = e^{\ln(a_n)}$$

$$\Rightarrow a_n = e^{\ln((n^2)^{\frac{1}{n}})}$$

$$\Rightarrow a_n = e^{\frac{1}{n}\ln(n^2)}$$

$$\text{let } f(x) = e^x, \text{ let } x(n) = \frac{1}{n}\ln(n^2)$$

$$\lim_{n\to\infty} \frac{\ln(n^2)}{n} = \frac{\infty}{\infty}$$

Because this is in indeterminate form, use L'Hopital's Rule

$$\Rightarrow \frac{\frac{d}{dx}(\ln(n^2))}{\frac{d}{dx}(n)}$$

$$\Rightarrow \frac{\frac{1}{n^2} \cdot 2n}{1} = \frac{2}{n}$$

$$\Rightarrow \lim_{n\to\infty} \frac{2}{n} = 0$$

The sequence converges to 0.

Problem 1.1.3 (Problem 108). Suppose the following recursive sequence converges, find its limit.

$$\sqrt{1}, \sqrt{1+\sqrt{1}}, \sqrt{1+\sqrt{1+\sqrt{1}}}, \ldots$$

Intuition. Use the monotonic convergence theorem

$$\text{let } \{a_{n+1}\} \text{ be a recursive sequence}$$

$$\text{Since } 1 = \sqrt{1}, \text{ we can let } a_1 = 1$$

$$\{a_{n+1}\}_{a_1=1} = \sqrt{1+a_n}$$

Given that we know that a_{n+1} converges, we are able to assume that

$$\text{Suppose } \lim_{n\to\infty} a_n = L$$

$$\begin{aligned} \{a_{n+1}\}_{a_1=1} &= \sqrt{1+a_n} \\ \Rightarrow L &= \sqrt{1+L} \\ \Rightarrow L^2 &= 1+L \\ \Rightarrow L^2 - L - 1 &= 0 \end{aligned}$$

Use the quadratic formula

$$\Rightarrow \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{1 \pm \sqrt{1+4}}{2(1)} = \frac{1 \pm \sqrt{5}}{2}$$

Since we know that all terms a_{n+1} are positive, we know that the limit L is

$$L = \lim_{n\to\infty} a_{n+1} = \frac{1+\sqrt{5}}{2}$$

Problem 1.1.4 (Problem 126). Determine if the following sequence converges or diverges. If the sequence converges, find its limit.

$$\{a_n\} = n - \frac{1}{n}$$

$$\Rightarrow \lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(n - \frac{1}{n}\right)$$

$$\Rightarrow \lim_{n\to\infty} n - \lim_{n\to\infty} \frac{1}{n}$$

$$\Rightarrow \infty - 0$$

$$\Rightarrow \infty$$

The sequence does not converge and therefore diverges.

Problem 2.1. Determine whether the following series converges or diverges, and show that your answers are correct by citing (with full justification) the appropriate definitions or theorems.

Problem 2.1.1.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{2n} - \sin^2 n}$$

Problem 2.1.2.

$$1 + \frac{3}{4} + \frac{4}{6} + \frac{5}{8} + \frac{6}{10} + \frac{7}{12} + \dots$$

Problem 2.1.3.

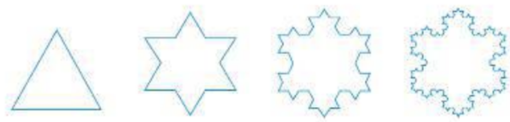
$$\sum_{n=2}^{\infty} \frac{1}{n(ln(n))^2}$$

Problem 3.1. Take the following as *fact*: a ball dropped from a height H meters takes $\sqrt{H/5}$ seconds to hit the ground. (Likewise, a ball launched from the ground to a maximum height H takes $\sqrt{H/5}$ seconds to reach tis peak.) Using this, determine the total time taken for the infinite processin Example 3 of 10.2.

104. Helga von Koch's snowflake curve Helga von Koch's snowflake is a curve of infinite length that encloses a region of finite area. To see why this is so, suppose the curve is generated by starting with an equilateral triangle whose sides have length 1.

a. Find the length L_n of the nth curve C_n and show that $\lim_{n \to \infty} L_n = \infty$.

b. Find the area A_n of the region enclosed by C_n and show that $\lim_{n \to \infty} A_n = (8/5) A_1$.



C_1 C_2 C_3 C_4

Problem 4.1.

Problem 5.1. Determine all values of p for which the following series converges using the Integral Test. Make sure you justify why the intergral test is applicable

$$\sum_{n=3}^{\infty} \frac{1}{n(ln(n))^{p+2}}$$

Problem 6.1. Give a value of n for which the expression

$$\sum_{k=1}^{\infty} \frac{1}{k^3}$$

is within 0.001 of its exact value. Justify your claim.