Lecture 1: Three-Dimensional Coordinate Systems (§12.1)

Goals:

1. Identify and construct right-handed coordinate frames in three-dimensional space.
2. Compute the distance between two points in space.
3. Describe elementary spatial regions (including spheres) with equalities and inequalities.
4. Sketch and verbally describe an elementary spatial region defined in terms of equalities and inequalities.

Before we start:

Read syllabus carefully. A few "highlights":

- MyLab homeworks:
  - Monday lecture assignment due the following Thursday at 11:59 PM.
  - Wednesday lecture assignment due the following Sunday at 11:59 PM.
  - Friday lecture assignment due the following Tuesday at 11:59 PM.
  - ~~There will be a 50% penalty deduction applied to the score of any late submission.~~

- Written homeworks:
  - Typically harder than MyLab.
  - You may work together, write answers in your own words.
  - Due Monday at 11:59PM. **No late submission is allowed** (do not solve at the last minute)
- Grade distribution:
  - Final Exam 30%
  - Midterm 1 20%
  - Midterm 2 20%
  - Written Homework 15%
  - MyLab Math Homework 15%.
- ~~**Office hours (Zoom): Monday 14:00-16:00. Wednesday 14:00-15:00.**~~
- ~~**Do not come to class if you have Covid symptoms** (there will be annotated notes online and recorded lectures by Prof. Kored)~~.

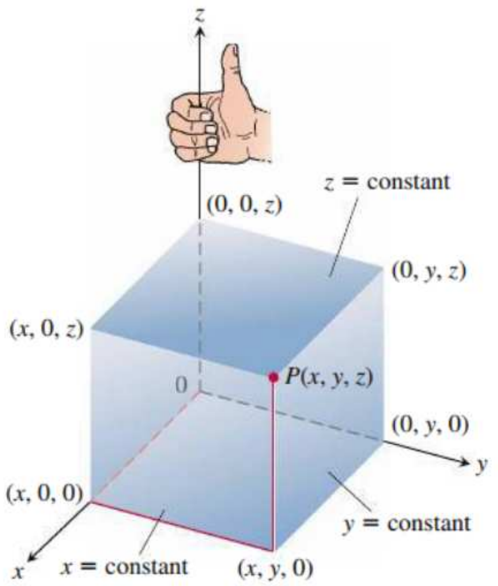
Office hours: Monday $13^{00} - 15^{00}$

Wednesday $13^{00} - 14^{00}$

At Lunt 220

Three-Dimensional Coordinate Systems (§12.1)

To describe a point p in space, we use a **three dimensional Cartesian coordinate system**, formed using three mutually perpendicular coordinate axes. The orientation is chosen according to the **right hand rule**.



There are three coordinate planes:

- xy-plane described by equation $z = 0$: the collection of all points $(x, y, 0)$
- xz-plane described by equation $y = 0$: the collection of all points $(x, 0, z)$
- yz-plane described by equation $x = 0$: the collection of all points $(0, y, z)$.

The three coordinate planes divide the space into eight cells called **octants**.

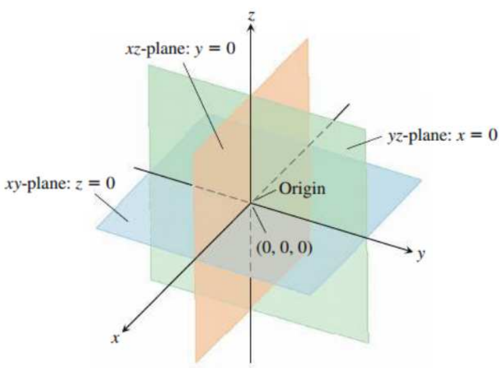
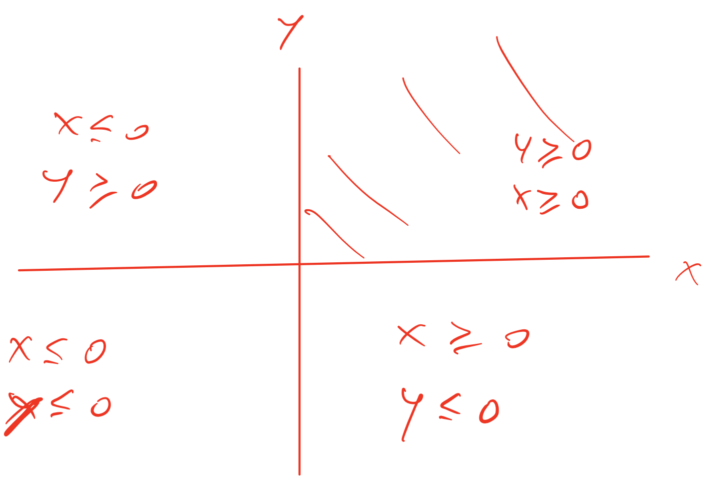
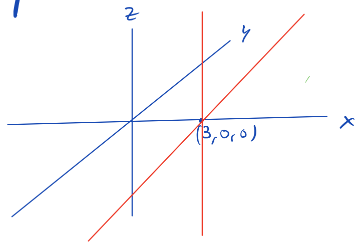


FIGURE 12.2 The planes $x = 0$, $y = 0$, and $z = 0$ divide space into eight octants.



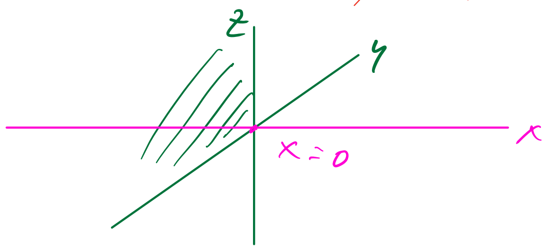
Example 0.1. Give a geometric description for the following equations and inequalities:

- $z \geq 0$? the upper half space
- $x = 3$? A plane parallel to the yz plane



- $x \geq 0, y \geq 0, z \geq 0$? The positive octant
- $x = 0$, $y \leq 0$, $z \geq 0$? A quarter of a plane

  A plane



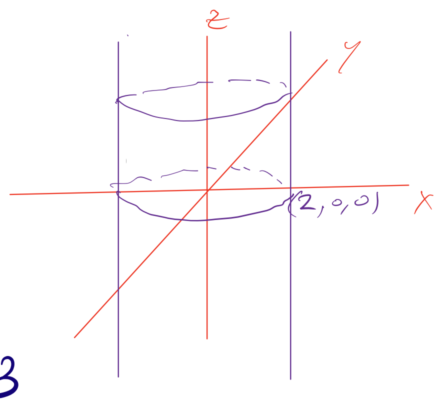
- $x = y = 1$?

  (1,1,z) describes a line parallel to the z axis

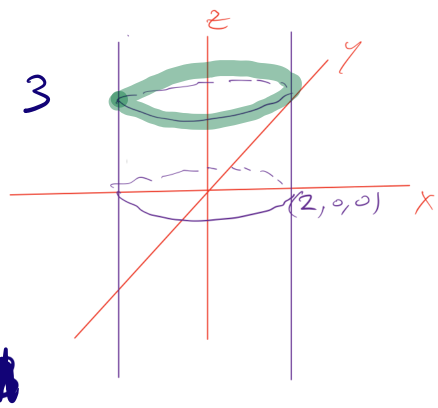
Example 0.2.

1. What points satisfy the equation $x^2 + y^2 = 4$?
2. What points satisfy the equations $x^2 + y^2 = 4$ and $z = 3$?

① A cylinder of radius 2 around the z-axis



② This is a circle at height 3 of radius 2 centered at (0,0,3)

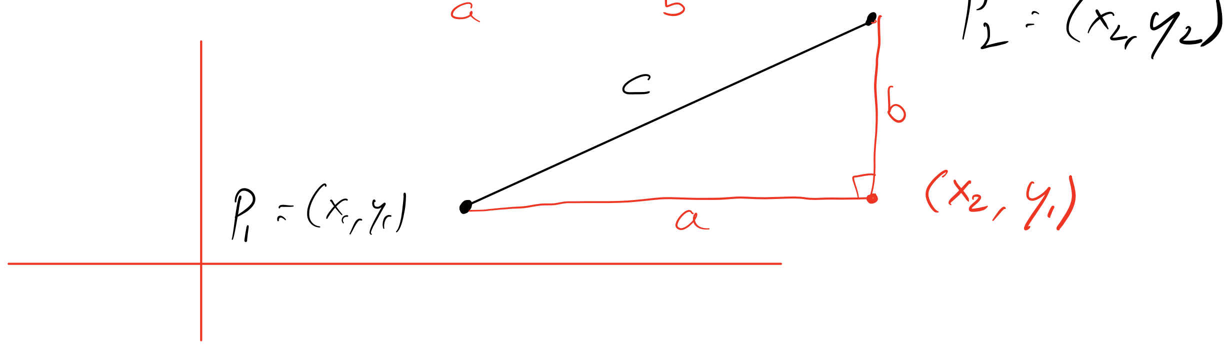


Distance and spheres in space

Recall that the distance between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on the plane is given by

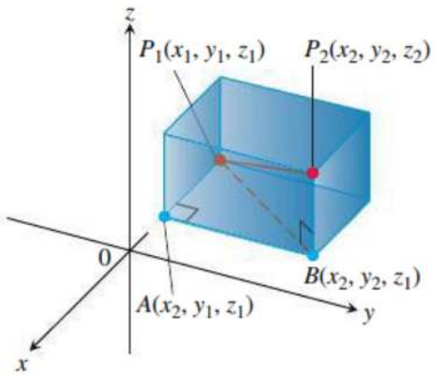
$$|P_1P_2| = \sqrt{\underbrace{(x_1 - x_2)^2}_{a} + \underbrace{(y_1 - y_2)^2}_{b}}.$$

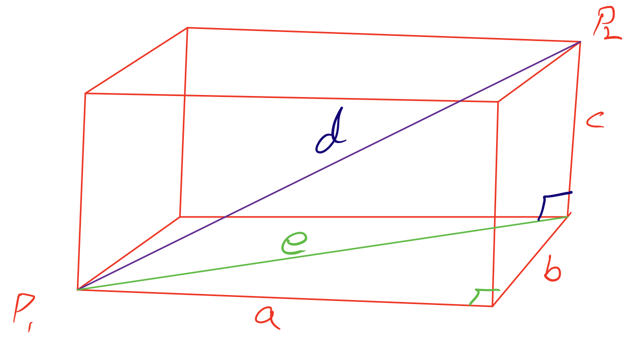
$a = |x_1 - x_2|$



This formula can be generalized to the case of points on the three dimensional space:

> The distance between two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is
>
> $$\underbrace{|P_1P_2|}_{d} = \sqrt{\underbrace{(x_1 - x_2)^2}_{a} + \underbrace{(y_1 - y_2)^2}_{b} + \underbrace{(z_1 - z_2)^2}_{c}}$$





$e^2 = a^2 + b^2$

$d^2 = e^2 + c^2 = a^2 + b^2 + c^2$

Example 0.3. What is the distance between $P_1(2, 1, 5)$ and $P_2(0, 3, 6)$?

$$|P_1P_2| = \sqrt{(2-0)^2 + (1-3)^2 + (5-6)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$$

We can use the distance formula to write equations for spheres in space:

The equation for the sphere of radius a and center (x_0, y_0, z_0) is

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$$

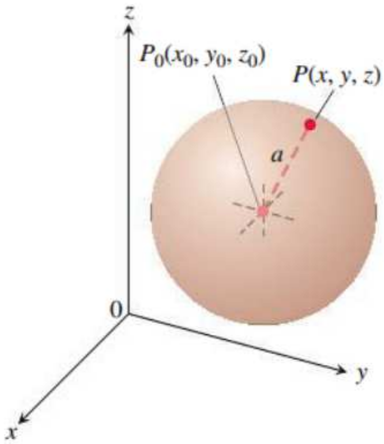


FIGURE 12.6 The sphere of radius a centered at the point (x_0, y_0, z_0).

Example 0.4.

1. Describe the following geometric object (is it a sphere?):

$$x^2 + y^2 + z^2 - 4x - 6y - 3 = 0.$$

yes

2. Describe the following geometric object:

$$x^2 + y^2 + z^2 - 4x - 6y - 3 \leq 0.$$

no

3. How about

$$x^2 + y^2 + z^2 - 4x - 6y - 3 < 0?$$

① $x^2 + y^2 + z^2 - 4x - 6y - 3 = 0$

$(x-2)^2 - 4 + (y-3)^2 - 9 + z^2 - 3 = 0$

$(x-2)^2 + (y-3)^2 + z^2 = 16$

This is a sphere of radius 4 centered at $(2,3,0)$

② $(x-2)^2 + (y-3)^2 + z^2 \leq 16$

This is a closed ball of radius 4 centered at $(2,3,0)$.

③ $(x-2)^2 + (y-3)^2 + z^2 < 16$ the open ball of all points at did all distance < 4 from $(2,3,0)$

Example 0.5. Describe the following geometric object

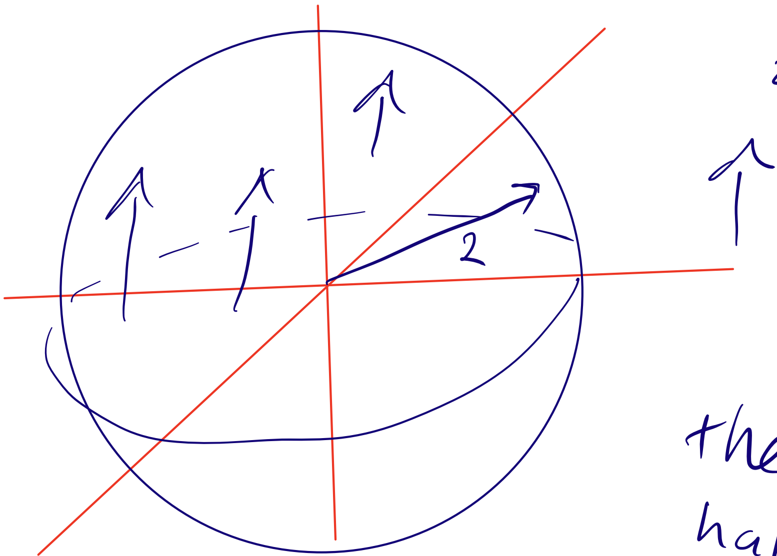
$$x^2 + y^2 + z^2 \leq 4,\ z \geq 0.$$

$x^2+y^2+z^2 \leq 4$

is a closed ball

$z \geq 0$

is the upper half space



the upper half ball of radius 2 around $(0,0,0)$.