機率與統計期末考

F74094017 資訊113 李昆翰

一、手寫題：

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1.)

Since it's a binomial distribution in the problem

∴ we can get $\begin{cases} \text{the mean } \mu = np = 1000 \times 0.55 = 550 \\ \text{the variance } \sigma^2 = 1000 \times 0.55 \times (1-0.55) = 1000 \times 0.55 \times 0.45 = 247.5 \end{cases}$

⇒ the standard diviation $\sqrt{\sigma^2} = \sigma \doteq 15.732$

Since $n=1000$ is large and the probability $p=0.55$ isn't close to 0 or 1

∴ we apply the normal distribution approximation to binomial distribution

Therefore, we calculate the limiting form Z, on the boundary $X=530$ & $X=560$.

$$Z_{400} = \frac{530-550}{15.732} \doteq -1.27 \quad ; \quad Z_{700} = \frac{560-550}{15.732} \doteq 0.64$$

∴ we can write the binomial probability distribution to the normal probability distribution approximation as:

$$P(400 \le X \le 700) \doteq P(-1.27 < Z < 0.64) = P(Z < 0.64) - P(Z < -1.27)$$
$$= 0.7389 - 0.1020 = 0.6369$$

Thus, there's a probability of 0.6369# that 400 ~ 700 students (both included) own a mortorcycle of the 1000 randomly selected sample from NCKU students #

2.)

$\mu = 3$, $\sigma^2 = 1$; for $n=30$, $\bar{X} = 3.1$, $S^2 = 1.65$

Suppose that the null hypothesis $H_0: \sigma^2 = 1$ is true.

Since the probability distribution in the Gogoro battery is a normal probability distribution,

apply χ^2-values estimation between σ^2 & s^2 with $v = n-1 = 29$ degrees of freedom:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{29 \times 1.65}{1} = 47.85 \quad ①$$

By checking the chi-squared distribution table with $\begin{cases} \alpha = 0.95\ (95\%) \\ \alpha = 0.05\ (5\%) \end{cases}$ and 29 degrees of freedom, we find that

$17.708 <$ 95% of the χ^2-values with 29 degrees of freedom < 42.557,

which doesn't contain ①

∴ the null hypothesis $H_0: \sigma^2 = 1$ is false. (means that the alternate hypothesis $H_1: \sigma^2 \neq 1$ is true.)

⇒ people in Gogoro shouldn't # be convinced that the battery lifetime has a variance of 1 year #

3.)

By the probability distribution of the random variable $Y=X^2$

$\Rightarrow$ change to $X = \pm\sqrt{Y}$, for $0<y<2$

And by the range of the random value x, we can get

for $0<x_1<2$: $y=x_1^2 \Rightarrow x_1=\sqrt{y}$

for $-1<x_2\leq 0$: $y=x_2^2 \Rightarrow x_2=-\sqrt{y}$.

Then, we can get the Jacobian of x_1 & x_2 :

$J_{x_1} = x_1 \frac{dx_1}{dy} = \frac{1}{2\sqrt{y}}$

$J_{x_2} = x_2 \frac{dx_2}{dy} = -\frac{1}{2\sqrt{y}}$

Next, we do the transformation of variable from X to Y :

$$g(y) = f(x_1)|J_{x_1}| + f(x_2)|J_{x_2}| = \frac{2(1+x_1)}{9} \times \frac{1}{2\sqrt{y}} + \frac{2(1+x_2)}{9} \times \frac{1}{2\sqrt{y}} = \frac{1}{2\sqrt{y}}\left(\frac{2(1+\sqrt{y})+2(1-\sqrt{y})}{9}\right)$$

$$= \frac{1}{\cancel{2}\sqrt{y}} \times \frac{\cancel{4}\,2}{9} = \frac{2}{9\sqrt{y}} \quad \text{, for } 0<y<2$$

$$\therefore g(y) = \begin{cases} \frac{2}{9\sqrt{y}} , & 0<y<2 \\ 0 , & \text{elsewhere} \end{cases}$$

#

4.

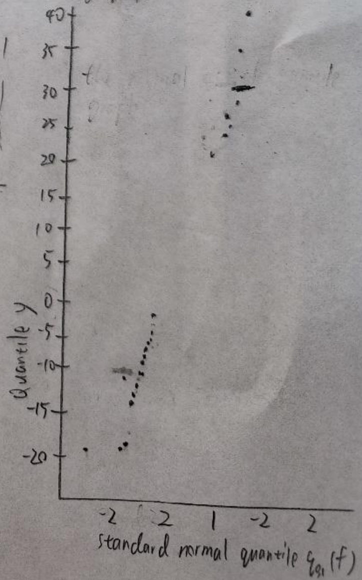
data (i)	fraction	normal quantile
(1) -19.6	≈ -0.032	≈ -2.185
-19.6	≈ -0.084	≈ -1.83
-19.3	≈ -0.138	≈ -1.63
-14.2	≈ -0.260	≈ -1.39
-13.6	≈ -0.346	≈ -1.294
-12.4	≈ -0.463	≈ -1.19
-12.2	≈ -0.554	≈ -1.32
-10.4	≈ -0.751	≈ -1.04
-10.3	≈ -0.858	≈ -1.01
-10.1	≈ -0.977	≈ -0.97
-8.5	≈ -1.288	≈ -0.91
-7.7	≈ -1.560	≈ -0.87
-7.6	≈ -1.718	≈ -0.86
-7.5	≈ -1.879	≈ -0.84
-6.1	≈ -2.5	≈ -0.81
-5.3	≈ -3.094	≈ -0.78
-2	≈ -9.5	≈ -0.74
-1.4	≈ -15.326	≈ -0.75
21.7	≈ 0.849	≈ 1.03
(20) 24.7	≈ 0.787	≈ 0.79

* $f_i = \frac{i-\frac{3}{8}}{n+\frac{1}{4}}$

$q_{0,1} = 4.91[f^{0.14} - (1-f)^{0.14}]$

data	fraction	normal quantile
(21) 25.4	≈ 0.804	≈ 0.85
25.8	≈ 0.830	≈ 0.95
26	≈ 0.862	≈ 1.09
28.2	≈ 0.830	≈ 0.95
28.7	≈ 0.851	≈ 1.04
30.4	≈ 0.836	≈ 0.98
30.4	≈ 0.869	≈ 1.12
33.2	≈ 0.826	≈ 0.94
35.6	≈ 0.798	≈ 0.83
(30) 43.2	≈ 0.682	≈ 0.47

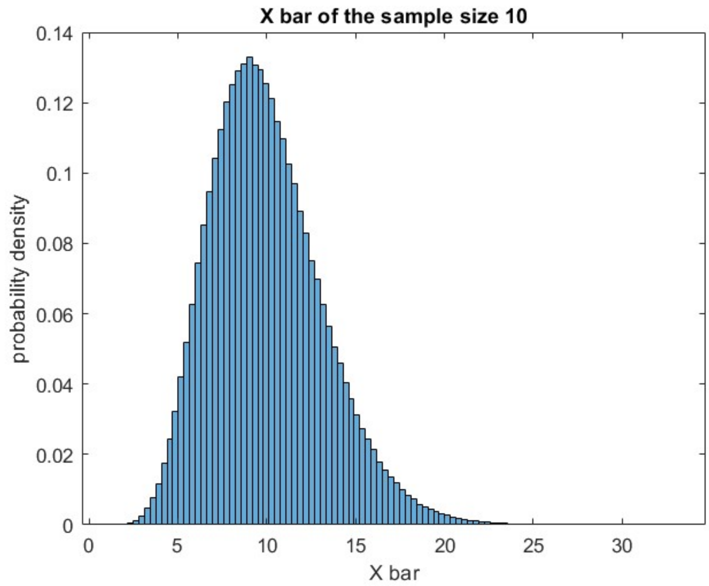
the normal quantile-quantile graph :

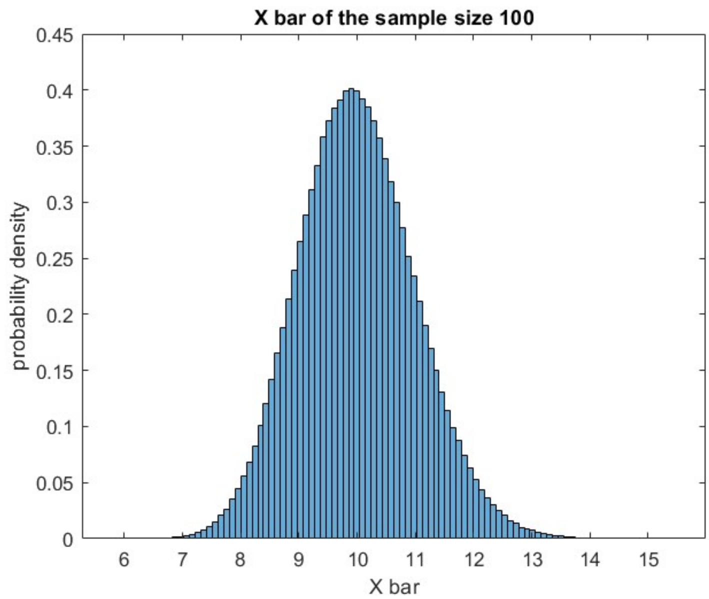


二、Matlab 題：

Problem 5:

以下分別是 n = 10, 100 的 X bar 的 histogram 結果圖

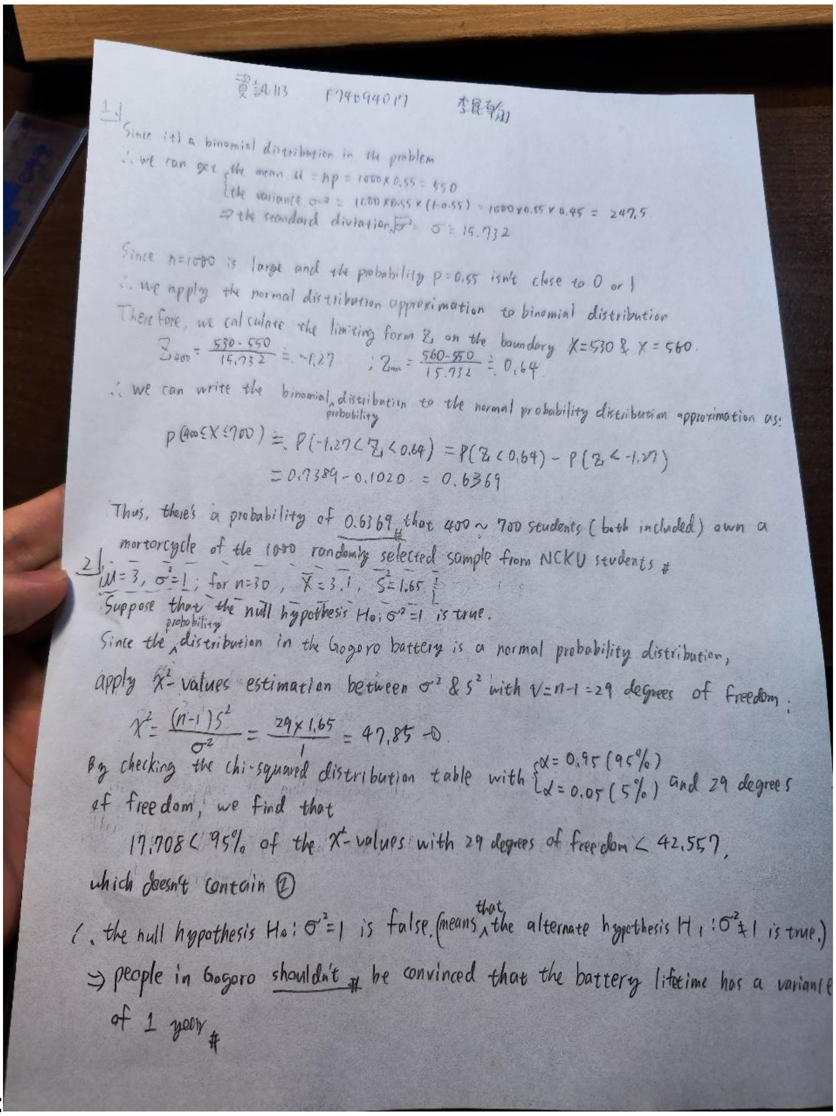




如果我們要使用隨機樣本的 x bar 來對自然對數分布的 mu 來做估計的話，會使用到中央極限定理，來使得取出來的隨機樣本可以近似為常態分佈，以便於估計原分布的 mu 在何處。

而從上面兩張結果圖中，以我們已知的 mu 來對比可以發現到，由 n = 10 的樣本數做出來的圖的最高點所在的 x bar 比起 mu 來說靠更左了些；而由 n = 100 的樣本數做出來的圖的最高點所在的 x bar 已經快要估出正確的 mu 了。從這可以發現到，當樣本量 n 越大時，我們求得的估似精度會更加的準確。

Problem 6:



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