第五次機率與統計作業

F74094017 資訊113 李昆翰

一、手寫部分：

F74094017 資訊系113 李昆翰

5.14

a.) By binomial distribution

$P_1(4\text{-}0) = b(4;4,0.9) = \sum_{x=0}^{4} b(x;4,0.9) - \sum_{x=0}^{3} b(x;4,0.9) = 1.0000 - 0.3439 = \underline{0.6561}$ #

b.) By binomial distribution

$P_2(\text{win}) = [b(3;4,0.9) + b(3;5,0.9) + b(3;6,0.9)] + b(4;4,0.9) = P(\text{3-lead}) + P(\text{4-0 win})$

$= (0.3439 - 0.0523) + (0.0815 - 0.0086) + (0.0159 - 0.0013) + 0.6561$

$= \underline{0.9972}$ #

c.)

By the problem, the very important assumption for answering a.) & b.) is made winning percentage $= 0.9$ #

5.26

a.) By binomial distribution

$P(X=6) = b(6;8,0.6) = \binom{8}{6}(0.6)^6(0.4)^2 \doteq \underline{0.2090}$ #

b.) By binomial distribution

$P(X=6) = b(6;8,0.6) = \sum_{x=0}^{6} b(x;8,0.6) - \sum_{x=0}^{5} b(x;8,0.6) = 0.8936 - 0.6846 = \underline{0.2090}$ #

5.50

a.) By negative binomial distribution

$b^*(7;3,0.5) = \binom{7-1}{3-1}\cdot(0.5)^3\cdot(0.5)^7 = \frac{\cancel{6}^{3}\cdot 5}{\cancel{2}\cdot 1}\cdot(0.5)^{10} \doteq \underline{0.01465}$ #

b.) By negative binomial distribution

$b^*(4;1,0.5) = \binom{4-1}{1-1}\cdot(0.5)^1\cdot(0.5)^4 = (0.5)^5 = \underline{0.03125}$ # $(= g(4,0.5))$

5.56

Since all the problem is based on "a month" ∴ let $t=1$, $\lambda=3 \Rightarrow t\lambda = 1\times 3 = 3.0$.

a.)

By poisson distribution,

$p(5;3) = \frac{e^{-3}\cdot 3^5}{5!} = \sum_{x=0}^{5} p(x;3) - \sum_{x=0}^{4} p(x;3) = 0.9161 - 0.8153 = \underline{0.1008}$ #

b.) By poisson distribution

$P(X<3) = \sum_{x=0}^{2} p(x;3) = \underline{0.4232}$ #

c.) By poisson distribution

$P(X\geq 2) = 1 - P(X<2) = 1 - \sum_{x=0}^{1} p(x;3) = 1 - 0.1991 = \underline{0.8009}$ #

5.80 $\lambda = 2.7$

a.) By poisson distribution

$$P(X \le 4) = \sum_{x=0}^{4} p(x; 2.7) = \frac{e^{-2.7} \cdot (2.7)^0}{0!} + \frac{e^{-2.7} \cdot (2.7)^1}{1!} + \frac{e^{-2.7} \cdot (2.7)^2}{2!} + \frac{e^{-2.7} \cdot (2.7)^3}{3!} + \frac{e^{-2.7} \cdot (2.7)^4}{4!}$$

$$\doteq \underline{0.8629} \ \#$$

b.) By poisson distribution

$$P(X<2) = P(X \le 1) = \sum_{x=0}^{1} p(x; 2.7) = e^{-2.7}(1+2.7) = \underline{0.2487} \ \#$$

c.) $t = 5 \Rightarrow \lambda t = 13.5 \Rightarrow$ By poisson distribution

$$\therefore P(X>10) = 1 - P(X \le 10) = 1 - \sum_{x=0}^{10} p(x; 13.5) \doteq 1 - 0.2112 = \underline{0.7888} \ \#$$

二、matlab 部分：

1(c)：

Binomial distribution table

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n	r	p0_10	p0_20	p0_25	p0_30	p0_40	p0_50	p0_60	p0_70	p0_80	p0_90
1	0	0.9	0.8	0.75	0.7	0.6	0.5	0.4	0.3	0.2	0.1
	1	1	1	1	1	1	1	1	1	1	1
2	0	0.81	0.64	0.5625	0.49	0.36	0.25	0.16	0.09	0.04	0.01
	1	0.99	0.96	0.9375	0.91	0.84	0.75	0.64	0.51	0.36	0.19
	2	1	1	1	1	1	1	1	1	1	1
3	0	0.729	0.512	0.4219	0.343	0.216	0.125	0.064	0.027	0.008	0.001
	1	0.972	0.896	0.8438	0.784	0.648	0.5	0.352	0.216	0.104	0.028
	2	0.999	0.992	0.9844	0.973	0.936	0.875	0.784	0.657	0.488	0.271
	3	1	1	1	1	1	1	1	1	1	1
4	0	0.6561	0.4096	0.3164	0.2401	0.1296	0.0625	0.0256	0.0081	0.0016	0.0001
	1	0.9477	0.8192	0.7383	0.6517	0.4752	0.3125	0.1792	0.0837	0.0272	0.0037
	2	0.9963	0.9728	0.9492	0.9163	0.8208	0.6875	0.5248	0.3483	0.1808	0.0523
	3	0.9999	0.9984	0.9961	0.9919	0.9744	0.9375	0.8704	0.7599	0.5904	0.3439
	4	1	1	1	1	1	1	1	1	1	1
5	0	0.5905	0.3277	0.2373	0.1681	0.0778	0.0313	0.0102	0.0024	0.0003	0
	1	0.9185	0.7373	0.6328	0.5282	0.337	0.1875	0.087	0.0308	0.0067	0.0005
	2	0.9914	0.9421	0.8965	0.8369	0.6826	0.5	0.3174	0.1631	0.0579	0.0086
	3	0.9995	0.9933	0.9844	0.9692	0.913	0.8125	0.663	0.4718	0.2627	0.0815
	4	1	0.9997	0.999	0.9976	0.9898	0.9688	0.9222	0.8319	0.6723	0.4095
	5	1	1	1	1	1	1	1	1	1	1
6	0	0.5314	0.2621	0.178	0.1176	0.0467	0.0156	0.0041	0.0007	0.0001	0
	1	0.8857	0.6554	0.5339	0.4202	0.2333	0.1094	0.041	0.0109	0.0016	0.0001
	2	0.9842	0.9011	0.8306	0.7443	0.5443	0.3438	0.1792	0.0705	0.017	0.0013
	3	0.9987	0.983	0.9624	0.9295	0.8208	0.6563	0.4557	0.2557	0.0989	0.0158
	4	0.9999	0.9984	0.9954	0.9891	0.959	0.8906	0.7667	0.5798	0.3446	0.1143
	5	1	0.9999	0.9998	0.9993	0.9959	0.9844	0.9533	0.8824	0.7379	0.4686
	6	1	1	1	1	1	1	1	1	1	1
7	0	0.4783	0.2097	0.1335	0.0824	0.028	0.0078	0.0016	0.0002	0	0
	1	0.8503	0.5767	0.4449	0.3294	0.1586	0.0625	0.0188	0.0038	0.0004	0
	2	0.9743	0.852	0.7564	0.6471	0.4199	0.2266	0.0963	0.0288	0.0047	0.0002
	3	0.9973	0.9667	0.9294	0.874	0.7102	0.5	0.2898	0.126	0.0333	0.0027
	4	0.9998	0.9953	0.9871	0.9712	0.9037	0.7734	0.5801	0.3529	0.148	0.0257
	5	1	0.9996	0.9987	0.9962	0.9812	0.9375	0.8414	0.6706	0.4233	0.1497
	6	1	1	0.9999	0.9998	0.9984	0.9922	0.972	0.9176	0.7903	0.5217
	7	1	1	1	1	1	1	1	1	1	1

1(d)：

Poisson distribution table

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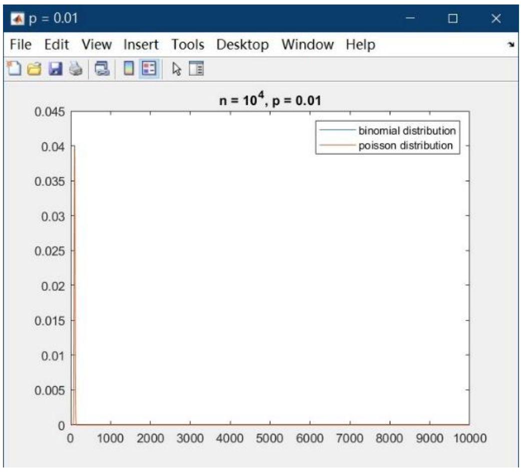
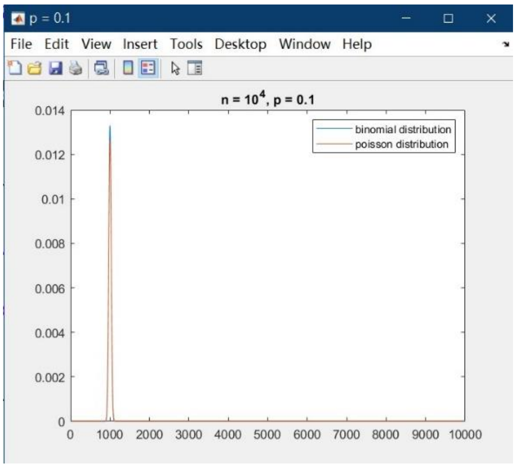
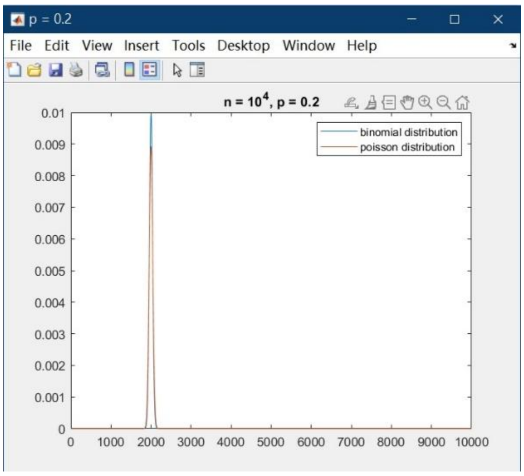
r	mu5_5	mu6_0	mu6_5	mu7_0	mu7_5	mu8_0	mu8_5	mu9_0	mu9_5
0	0.0041	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001
1	0.0266	0.0174	0.0113	0.0073	0.0047	0.003	0.0019	0.0012	0.0008
2	0.0884	0.062	0.043	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042
3	0.2017	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149
4	0.3575	0.2851	0.2237	0.173	0.1321	0.0996	0.0744	0.055	0.0403
5	0.5289	0.4457	0.369	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885
6	0.686	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649
7	0.8095	0.744	0.6728	0.5987	0.5246	0.453	0.3856	0.3239	0.2687
8	0.8944	0.8472	0.7916	0.7291	0.662	0.5925	0.5231	0.4557	0.3918
9	0.9462	0.9161	0.8774	0.8305	0.7764	0.7166	0.653	0.5874	0.5218
10	0.9747	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.706	0.6453
11	0.989	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.803	0.752
12	0.9955	0.9912	0.984	0.973	0.9573	0.9362	0.9091	0.8758	0.8364
13	0.9983	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981
14	0.9994	0.9986	0.997	0.9943	0.9897	0.9827	0.9726	0.9585	0.94
15	0.9998	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.978	0.9665
16	0.9999	0.9998	0.9996	0.999	0.998	0.9963	0.9934	0.9889	0.9823
17	1	0.9999	0.9998	0.9996	0.9992	0.9984	0.997	0.9947	0.9911
18	1	1	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957
19	1	1	1	1	0.9999	0.9997	0.9995	0.9989	0.998
20	1	1	1	1	1	0.9999	0.9998	0.9996	0.9991
21	1	1	1	1	1	1	0.9999	0.9998	0.9996
22	1	1	1	1	1	1	1	0.9999	0.9999
23	1	1	1	1	1	1	1	1	0.9999
24	1	1	1	1	1	1	1	1	1

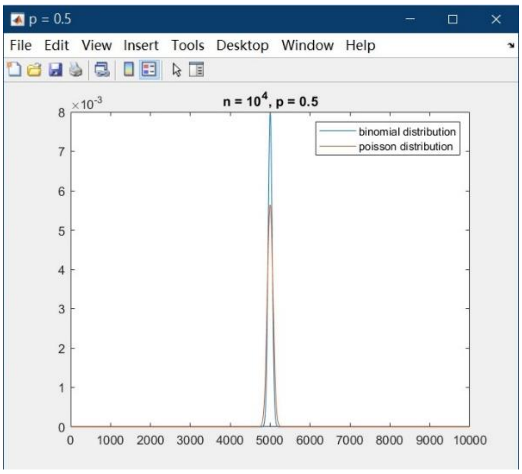
1(e)：

（表格一）Binomial 和 Poisson 的機率分布圖如下：

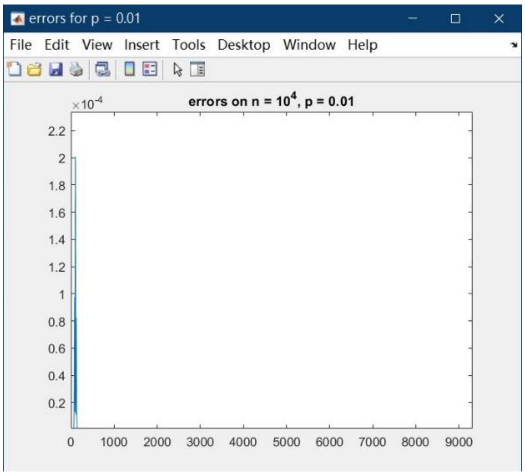


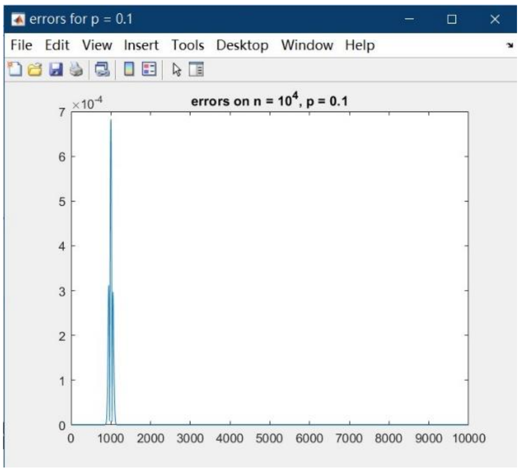


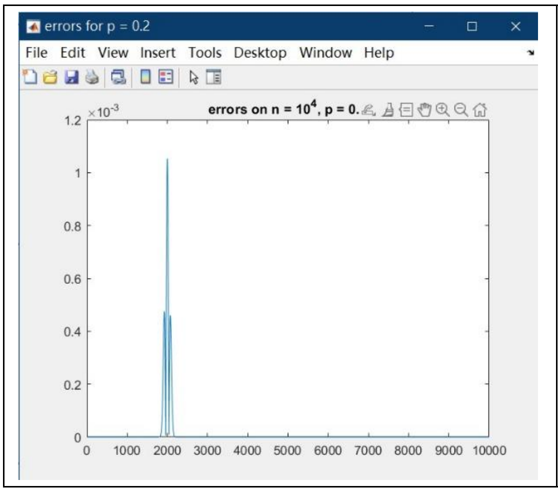


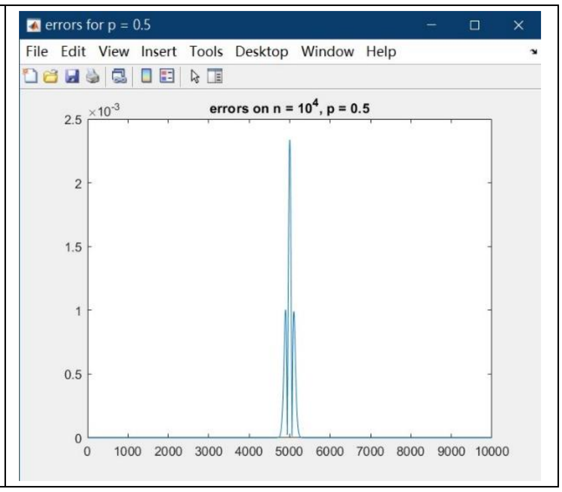


（表格二）Binomial 和 Poisson 機率分布的精準度（即兩者的差距）：



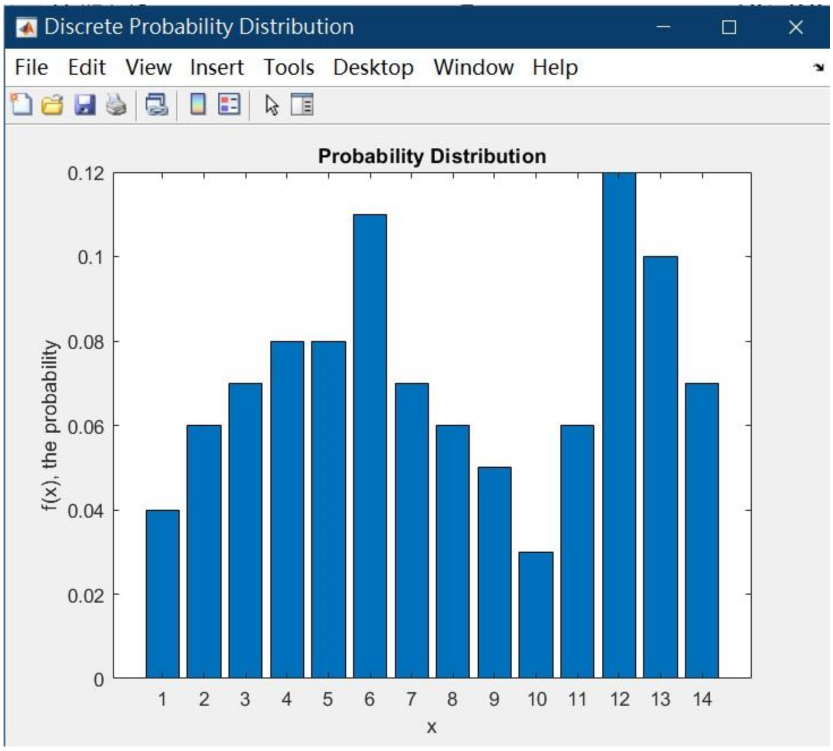




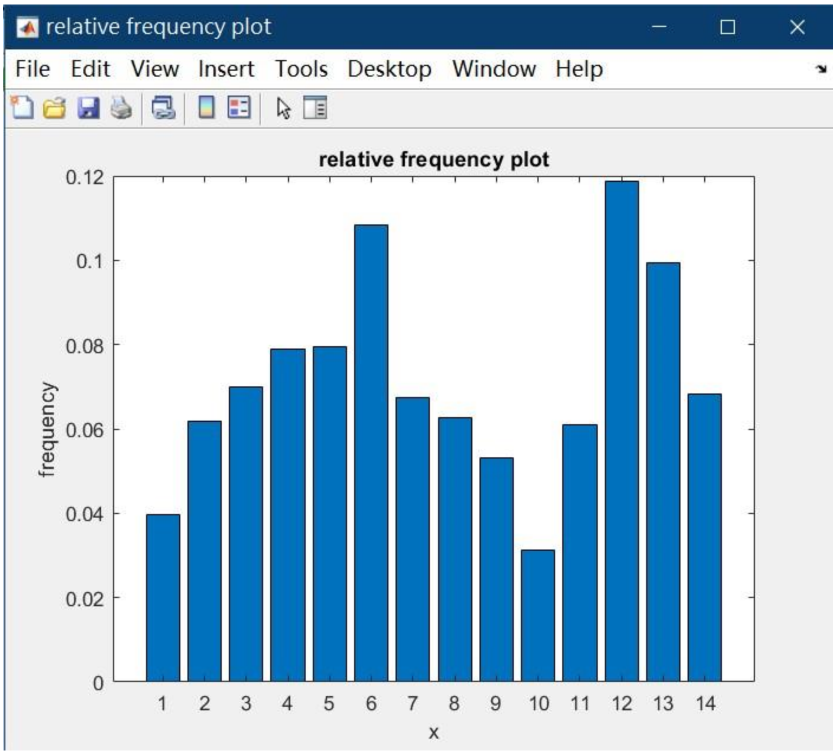


在 binomial distribution 中，若 n 值逼近無限大，且 p 值逼近 0，則可以將平均值 np 用 u 換進 poisson distribution 之 p(x;u)中。
而在此 matlab 實驗中，由於 n 是固定的，因此可以看 p 的變化來進行分析。由實驗的結果圖可以知道，當 p 越來越小時，binomial distribution 和 poisson distribution 的差距越來越小（p = 0.01 時之誤差高峰約 2 * 10^-4）；反之，則雙方的誤差逐漸增大（p = 0.5 時之誤差高峰約 2.4 * 10^-3）。以上的結果是合乎我於第一段寫的趨勢，因此實驗結果是合理的。

2(a)：



2(b)：



在本次的實驗中，由於有約 10^4 的樣本數，所以用機率出來 plot 出來的結果圖和 2(a)的圖非常的相像，顯現出 2(a)的機率分布趨勢。若數據的數量沒有到此實驗的量級的話（例如：100），則可能沒法顯示出 2(a)的機率分布趨勢。