第三次機率與統計作業

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$\boxed{3.6}$

(a.)

$$P(X \ge 200) = \int_{200}^{\infty} \frac{20000}{(x+100)^3}\,dx = -\frac{1}{2} \times \frac{20000}{(x+100)^2}\Big|_{200}^{\infty} = \frac{-10000}{(x+100)^2}\Big|_{200}^{\infty} = 0 - \left(-\frac{10000}{\cancel{300^2}}\right) = \underline{\frac{1}{9}}\ \#$$

(b.)

$$P(80 \le X \le 120) = \int_{80}^{120} \frac{20000}{(x+100)^3}\,dx = \frac{-10000}{(x+100)^2}\Big|_{80}^{120} = \frac{-10000}{220^2} - \left(-\frac{10000}{180^2}\right) = \frac{-10000}{48400} + \frac{10000}{32400}$$

$$= 25\left(\frac{1}{81} - \frac{1}{121}\right) \doteq \underline{0.102}\ \#$$

$\boxed{3.15}$

(a.)

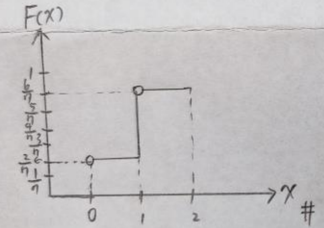
$$P(X=1) = F(1) = f(0) + f(1) = \frac{C^5_3}{C^7_3} + \frac{C^5_2 \times C^2_1}{C^7_3} = \frac{\frac{5\cdot4}{2\cdot1}}{\frac{7\cdot6\cdot5}{3\cdot2\cdot1}} + \frac{\frac{5\cdot4}{2\cdot1}\times2}{\frac{7\cdot6\cdot5}{3\cdot2\cdot1}} = \frac{2}{7} + \frac{4}{7} = \underline{\frac{6}{7}}\ \#$$

(b.)

$$P(0 < X \le 2) = F(2) - F(0) = f(1) + f(2) = \frac{C^5_2 \times C^2_1}{C^7_3} + \frac{C^5_1 \times C^2_2}{C^7_3} = \frac{4}{7} + \frac{5\cdot1}{\frac{7\cdot5\cdot2}{3\cdot2\cdot1}} = \underline{\frac{5}{7}}\ \#$$

$\boxed{3.16}$

x	0	1	2
$f(x)$	$\frac{2}{7}$	$\frac{4}{7}$	$\frac{1}{7}$



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$\boxed{3.24}$

The general solution:

$$P(X=x) = \frac{C^5_x C^5_{4-x}}{C^{10}_4} = \frac{\binom{5}{x}\binom{5}{4-x}}{\binom{10}{4}}\ \#$$

Therefore, we can get the results

x	0	1	2	3	4
$f(x)$	$\frac{\binom{5}{0}\binom{5}{4}}{\binom{10}{4}} = \frac{5}{210}$	$\frac{\binom{5}{1}\binom{5}{3}}{\binom{10}{4}} = \frac{50}{210}$	$\frac{\binom{5}{2}\binom{5}{2}}{\binom{10}{4}} = \frac{100}{210}$	$\frac{50}{210}$	$\frac{5}{210}$

\#

$\boxed{3.30}$

(a.)

$$P(X=x) = \int_{-\infty}^{\infty} k(3-x^2)\,dx = \int_{-1}^{1} k(3-x^2)\,dx$$

$$= \left(3kx - \frac{1}{3}kx^3\right)\Big|_{-1}^{1} = \left(3k - \frac{1}{3}k\right) - \left(-3k + \frac{1}{3}k\right)$$

$$= 6k - \frac{2}{3}k = \frac{16}{3}k = 1$$

$\therefore$ when $k = \frac{3}{16}$, then it renders $f(x)$ a valid density function \#

(b.)

$$P\left(X \le \frac{1}{2}\right) = \int_{-\infty}^{\frac{1}{2}} \frac{3}{16}(3-x^2) = \int_{-1}^{\frac{1}{2}} \frac{3}{16}(3-x^2) = \left(\frac{9}{16}x - \frac{1}{16}x^3\right)\Big|_{-1}^{\frac{1}{2}}$$

$$= \left(\frac{9}{32} - \frac{1}{128}\right) - \left(-\frac{9}{16} + \frac{1}{16}\right) = \frac{36-1+72-8}{128} = \frac{99}{128}$$

(c.)

$$P(X \ge 0.8\ \&\ X \le -0.8) = \int_{0.8}^{\infty} \frac{3}{16}(3-x^2) + \int_{-\infty}^{-0.8} \frac{3}{16}(3-x^2)$$

$$= \int_{0.8}^{1} \frac{3}{16}(3-x^2) + \int_{-1}^{-0.8} \frac{3}{16}(3-x^2)$$

$$= \left(\frac{9x}{16} - \frac{x^3}{16}\right)\Big|_{0.8}^{1} \times 2$$

$$= \left[\left(\frac{9}{16} - \frac{1}{16}\right) - \left(\frac{9}{16} \times \frac{4}{5} - \frac{1}{16}\left(\frac{4}{5}\right)^3\right)\right] \times 2$$

$$= \left[\frac{1}{2} - \frac{1}{16}\left(\frac{\overset{836}{\cancel{9\cdot4\cdot25-4^3}}}{125}\right)\right] \times 2$$

$$= (0.5 - 0.418) \times 2$$

$$= \underline{0.164}\ \#$$

$\boxed{3.40}$

(a.)

For marginal density of X

$$g(x) = \int_0^1 \frac{2}{3}(x+2y)\,dy$$

$$= \frac{2}{3}(xy + y^2)\Big|_0^1$$

$$= \frac{2}{3}(x+1) - 0$$

$$= \underline{\frac{2}{3}(x+1), \text{ for } 0 \le x \le 1} \ \#$$

(b.)

For marginal density of Y

$$h(y) = \int_0^1 \frac{2}{3}(x+2y)\,dx$$

$$= \frac{2}{3}\left(\frac{1}{2}x^2 + 2xy\right)\Big|_0^1$$

$$= \frac{2}{3}\left(\frac{1}{2} + 2y\right) - 0$$

$$= \underline{\frac{4}{3}y + \frac{1}{3}, \text{ for } 0 \le y \le 1}$$

(c.)

$$P(X < 0.5) = \int_0^{0.5}\int_0^1 \frac{2}{3}(x+2y)\,dy\,dx$$

$$= \int_0^{0.5} \frac{2}{3}(x+1)\,dx$$

$$= \frac{2}{3}\left(\frac{1}{2}x^2 + x\right)\Big|_0^{0.5}$$

$$= \frac{2}{3}\left(\frac{1}{8} + \frac{1}{2}\right) - 0$$

$$= \frac{2}{3} \times \frac{5}{8_4} = \underline{\frac{5}{12}} \ \#$$

$\boxed{3.50}$

(a.)

For X:

x	2	4
$g(x)$	0.40	0.60

\#

(b.)

For Y:

y	1	3	5
$h(y)$	0.25	0.50	0.25

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