第四次機率與統計作業

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4.24

Y\X	0	1	2	3
0	(5a) 0	$\frac{3}{70}$	$\frac{9}{70}$	$\frac{3}{70}$
1	$\frac{2}{70}$	$\frac{18}{70}$	$\frac{18}{70}$	$\frac{2}{70}$
2	$\frac{3}{70}$	$\frac{9}{70}$	$\frac{3}{70}$	0 (74)

a.)

$$E(X^2Y-2XY)=\sum_{x=0}^{3}\sum_{y=0}^{2}(x^2y-2xy)f(x,y)$$
$$=\sum_{x=0}^{3}\sum_{y=0}^{2}xy(x-2)f(x,y)$$
$$=0+1\cdot1\overset{-1}{(1-2)}\cdot\frac{18}{70}+0+$$
$$3\cdot1\overset{1}{(3-2)}\cdot\frac{2}{70}+0+1\cdot2\underset{-1}{(1-2)}\frac{9}{70}+0$$
$$=-\frac{18}{70}+\frac{6}{70}-\frac{18}{70}=-\frac{3}{7}\ \#$$

b.)

$$\mu_X-\mu_Y=E(X)-E(Y)$$
$$=\sum_{x=0}^{3}xf(x)-\sum_{y=0}^{2}yf(y)$$
$$=\left(0+\frac{30}{70}+\frac{60}{70}+\frac{15}{70}\right)-\left(0+\frac{40}{70}+\frac{30}{70}\right)$$
$$=\frac{35}{70}=\frac{1}{2}\ \#$$

4.44

$$\sigma_{XY}=E(XY)-\mu_X\mu_Y$$
$$=\sum_{x=0}^{3}\sum_{y=0}^{2}xyf(x,y)-\mu_X\mu_Y$$
$$=1\cdot\frac{18}{70}+2\cdot\frac{18}{70}+3\cdot\frac{2}{70}+2\cdot\frac{9}{70}+4\cdot\frac{3}{70}-\frac{\overset{3}{\cancel{105}}}{\underset{2}{\cancel{70}}}\times\frac{\cancel{70}}{\cancel{70}}$$
$$=\frac{18+36+6+18+12}{70}-\frac{3}{2}$$
$$=\frac{9}{7}-\frac{3}{2}=\frac{18-21}{14}=-\frac{3}{14}\ \#$$

4.78

1° Get the covarient and mean of X :

$$\sigma_X^2=E(X^2)-\mu_X^2=E(X^2)-(E(X))^2$$
$$=\int_0^1 30x^2(1-x)^2dx\cdot x^2-\left[\int_0^1 30x^2(1-x)^2dx\cdot x\right]^2$$
$$=\int_0^1 30(x^6-2x^5+x^4)dx-\left[\int_0^1 30(x^5-2x^4+x^3)dx\right]^2$$
$$=\left(\frac{30}{7}x^7-10x^6+6x^5\right)\Big|_0^1-\left[\left(5x^6-12x^5+\frac{30}{4}x^4\right)\Big|_0^1\right]^2$$
$$=\frac{30}{7}\overset{-4}{\cancel{-10+6}}-\left(\overset{-7}{\cancel{5-12}}+\frac{\overset{15}{\cancel{30}}}{\underset{2}{\cancel{4}}}\right)^2=\frac{2}{7}-\frac{1}{4}$$
$$=\frac{8-7}{28}=\frac{1}{28}$$
$$\therefore\sigma_X=\sqrt{\frac{1}{28}}=\frac{1}{2\sqrt{7}}\ ;\ \mu_X=\frac{1}{2}$$

2°

①: By Integral :

$$P(\mu-2\sigma<X<\mu+2\sigma)=\int_{\frac{1}{2}-\frac{1}{\sqrt{7}}}^{\frac{1}{2}+\frac{1}{\sqrt{7}}}30x^2(1-x)^2dx$$
$$=\int_{\frac{1}{2}-\frac{1}{\sqrt{7}}}^{\frac{1}{2}+\frac{1}{\sqrt{7}}}30(x^4-2x^3+x^2)dx=(6x^5-15x^4+10x^3)\Big|_{\frac{1}{2}-\frac{1}{\sqrt{7}}}^{\frac{1}{2}+\frac{1}{\sqrt{7}}}$$
$$\doteq 0.985-0.015\doteq 0.970$$

②: By Chebyshev's theorem:

$$P(\mu-2\sigma<X<\mu+2\sigma)\geq 1-\frac{1}{2^2}=\frac{3}{4}=0.75,$$

where $k=2$.

3°

By ①, ②, we get

$$\begin{cases}\text{true value result: } 0.970\\ \text{Chebyshev's theorem result : } 0.75\end{cases}\ \#,$$

which true value is way bigger then Chebyshev's be. #

4.60

a.)

$\because$ X & Y are independent

$$\therefore E(2X-3Y)=2E(X)-3E(Y)$$
$$=2(\overset{1.1}{\cancel{2\cdot0.55}}+\overset{1.8}{\cancel{4\cdot0.45}})-3(\overset{0.25}{\cancel{1\cdot0.25}}+\overset{3.0}{\overset{1.5}{\cancel{3\cdot0.50}}}+\overset{1.5}{\overset{1.25}{\cancel{5\cdot0.25}}})$$
$$=5.8-9.0$$
$$=-3.2\ \#$$

b.)

$$E(XY)=E(X)E(Y)$$
$$=2.9\times3.0$$
$$=8.7\ \#$$

4.98

a.)

x	0	1	2
$g(x)$	0.20	0.32	0.48

#

y	0	1	2
$h(y)$	0.26	0.35	0.39

#

$P(X|Y=2) = \frac{f(x,2)}{h(2)} = \frac{1}{0.39} f(x,2)$, where $0 \le x \le 2$

$\therefore$

x	0	1	2
$P(X\|Y=2)$	$\frac{4}{39}$	$\frac{5}{39}$	$\frac{30}{39}$

#

b.)

$E(X) = xg(x) = 0 + 0.32 + \overset{0.96}{\cancel{2 \cdot 0.48}}$

$= \underline{1.28}$ #

$Var(X) = \sigma_X^2 = E(X^2) - (E(X))^2$

$= x^2 g(x) - (xg(x))^2$

$= 0 + 0.32 + \underset{1.92}{\cancel{4 \times 0.48}} - (1.28)^2$

$= \underline{0.6016}$ #

c.)

$E(X|Y=2) = x\,P(x|Y=2)$

$= 0 + 1 \cdot \frac{5}{39} + 2 \cdot \frac{30}{39}$

$= \frac{65}{39} = \underline{\frac{5}{3}}$ #

$Var(X|Y=2) = E(X^2|Y=2) - (E(X|Y=2))^2$

$= x^2 P(x|y=2) - (\frac{65}{39})^2$

$= \frac{5}{39} + \frac{4 \cdot 30}{39} - \frac{65^2}{39^2}$

$= \frac{125 \cdot 39 - 65^2}{39^2}$

$= \frac{650}{39^2}$

$= \underline{\frac{50}{117}}$ #