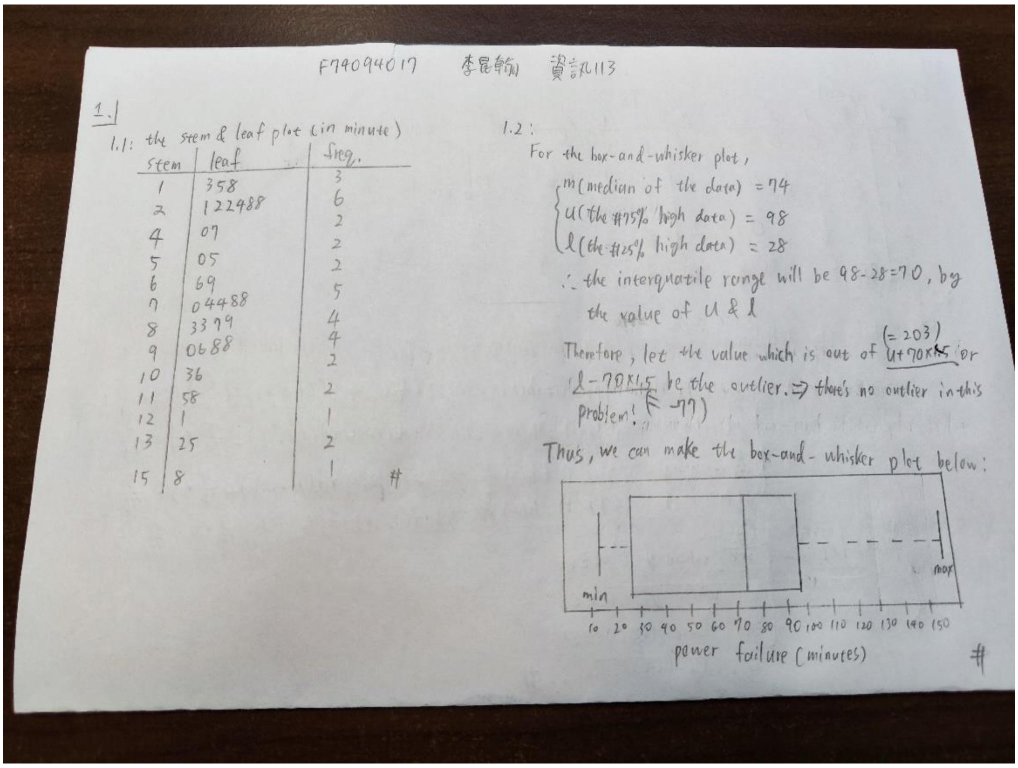
機率與統計期中考

F74094017 資訊113 李昆翰

一、手寫部分（第1～4題）：

1、



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1.

1.1: the stem & leaf plot (in minute)

stem	leaf	freq.
1	358	3
2	122488	6
4	07	2
5	05	2
6	69	2
7	04488	5
8	3379	4
9	0688	4
10	36	2
11	58	2
12	1	1
13	25	2
15	8	1

#

1.2:

For the box-and-whisker plot,

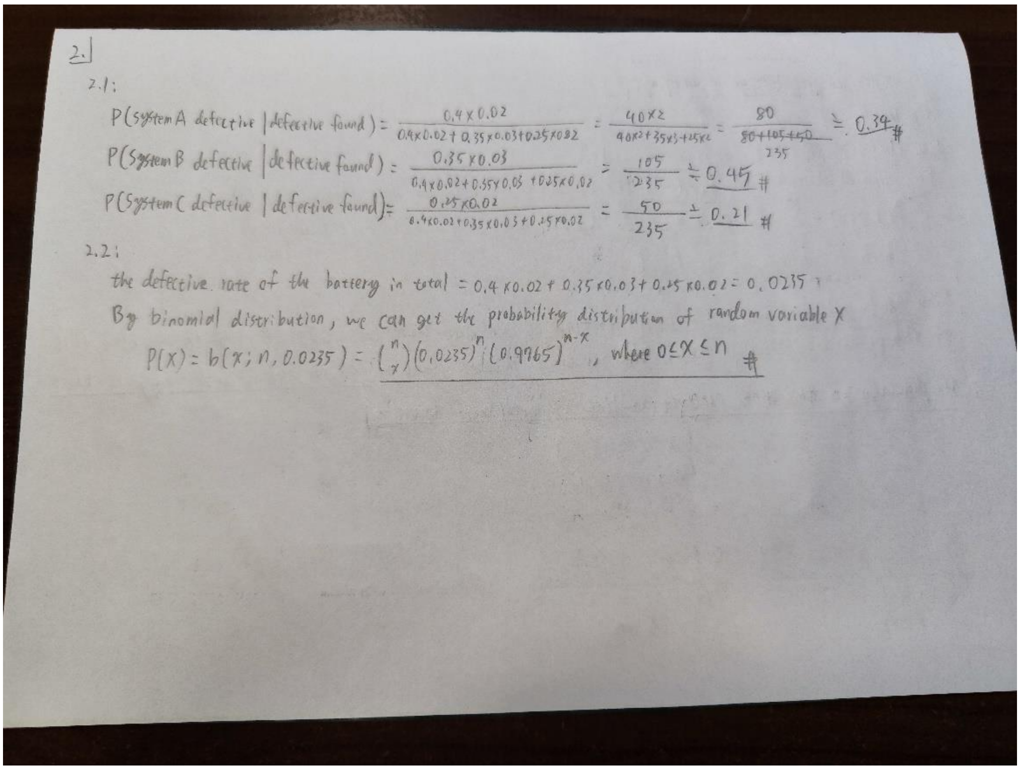
$$\begin{cases} m\text{(median of the data)} = 74 \\ u\text{(the \#75\% high data)} = 98 \\ l\text{(the \#25\% high data)} = 28 \end{cases}$$

∴ the interquatile range will be 98-28=70, by the value of u & l

Therefore, let the value which is out of $\underbrace{u+70\times1.5}_{(=203)}$ or $\underbrace{l-70\times1.5}_{(=-77)}$ be the outlier. ⇒ there's no outlier in this problem!

Thus, we can make the box-and-whisker plot below:

2、



2.

2.1:

$P(\text{system A defective} \mid \text{defective found}) = \frac{0.4\times0.02}{0.4\times0.02+0.35\times0.03+0.25\times0.02} = \frac{40\times2}{40\times2+35\times3+25\times2} = \frac{80}{\underbrace{80+105+50}_{235}} \doteq \underline{0.34}$ #

$P(\text{System B defective} \mid \text{defective found}) = \frac{0.35\times0.03}{0.4\times0.02+0.35\times0.03+0.25\times0.02} = \frac{105}{235} \doteq \underline{0.45}$ #

$P(\text{System C defective} \mid \text{defective found}) = \frac{0.25\times0.02}{0.4\times0.02+0.35\times0.03+0.25\times0.02} = \frac{50}{235} \doteq \underline{0.21}$ #

2.2:

the defective rate of the battery in total $= 0.4\times0.02+0.35\times0.03+0.25\times0.02 = 0.0235$

By binomial distribution, we can get the probability distribution of random variable X

$P(X) = b(x; n, 0.0235) = \underline{\binom{n}{x}(0.0235)^{n}(0.9765)^{n-x}, \text{ where } 0 \le X \le n}$ #

3、

3.

3.1:

(z=0)

y \ x	0	1	2	3
0	0	0	0	$\frac{1}{12}$
1	0	0	$\frac{1}{4}$	0
2	0	$\frac{1}{8}$	0	0
3	$\frac{1}{120}$	0	0	0

#

(z=1)

y \ x	0	1	2
0	0	0	$\frac{1}{6}$
1	0	$\frac{1}{4}$	0
2	$\frac{1}{20}$	0	0

#

(z=2)

y \ x	0	1	2
0	0	$\frac{1}{24}$	0
1	$\frac{1}{40}$	0	0

#

3.2:

The probability distribution of $g(x+y|x=1)$ in the table as in 3.1. will be the followings:

$$g(x+y|x=1) = \begin{cases} g(1+0|x=1) = g(1|x=1) = h(1)\times(f(1,0,0)+f(1,0,1)+f(1,0,2)) = \overset{\frac{5}{12}}{\left(\frac{1}{8}+\frac{1}{4}+\frac{1}{24}\right)}\times\frac{1}{24} = \frac{5}{288} \\ g(1+1|x=1) = g(2|x=1) = h(1)\times(f(1,1,0)+f(1,1,1)+f(1,1,2)) = \frac{5}{12}\times\frac{1}{4} = \frac{5}{48} \\ g(1+2|x=1) = g(3|x=1) = h(1)\times(f(1,2,0)+f(1,2,1)+f(1,2,2)) = \frac{5}{12}\times\frac{1}{8} = \frac{5}{96} \\ g(1+3|x=1) = g(4|x=1) = h(1)\times(f(1,3,0)+f(1,3,1)+f(1,3,2)) = \frac{5}{12}\times 0 = 0 \end{cases}$$

$$\therefore g(x+y|x=1) = \begin{cases} \frac{5}{288}, \text{ where } y=0, 0\leq z\leq 2 \\ \frac{5}{48}, \text{ where } y=1, 0\leq z\leq 2 \\ \frac{5}{96}, \text{ where } y=2, 0\leq z\leq 2 \\ 0, \text{ elsewhere} \end{cases}$$

#

4、

4.

the conjectured probability $p = 40\% = 0.4$;

the number of sample $n = 14$ 。

1° We assume that the conjesture is true.

By binomial distribution, find if it's possible to find 10 or more defective LCDs

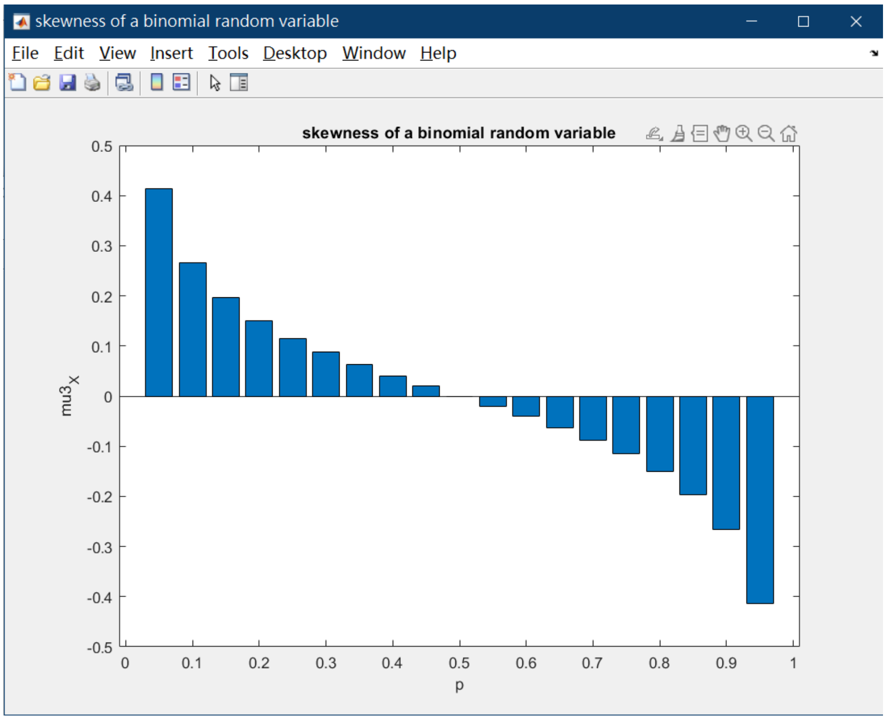
$$P(X\geq 10) = \sum_{x=0}^{14} b(x;14,0.4) - \sum_{x=0}^{9} b(x;14,0.4) = 1 - 0.9825 = 0.0175,$$

which is unlikely to be found.

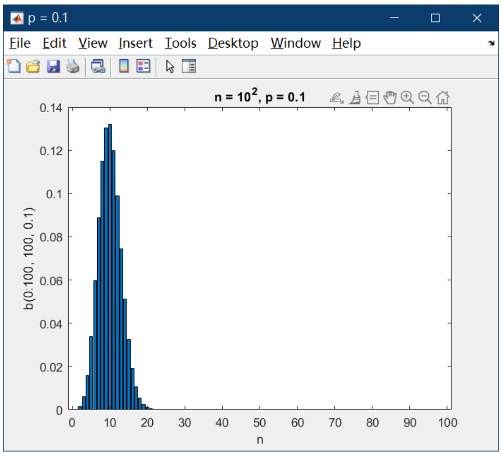
Since it is unlikely (with 1.75% chance) to find 10 or more defective LCDs if only the defective rate of this production line is 40%, this cases considerable doubt on the conjesture and suggests that the defective in this new production line is much more severe #

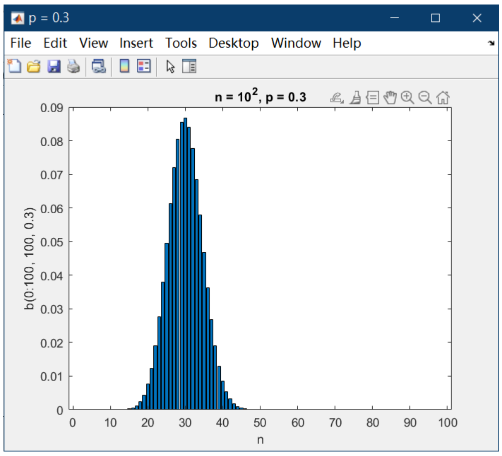
二、matlab 部分（第 5 題）：

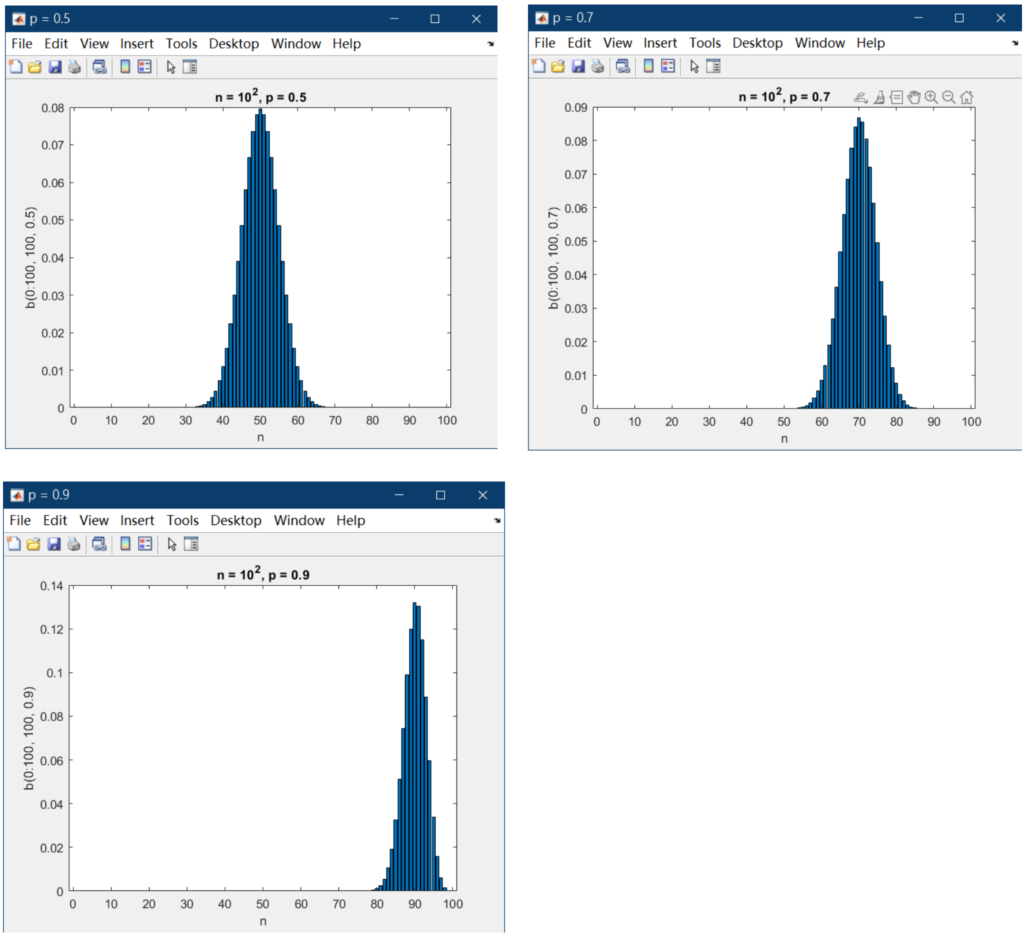
5_1：



圖(一)：skewness of a binomial random variable





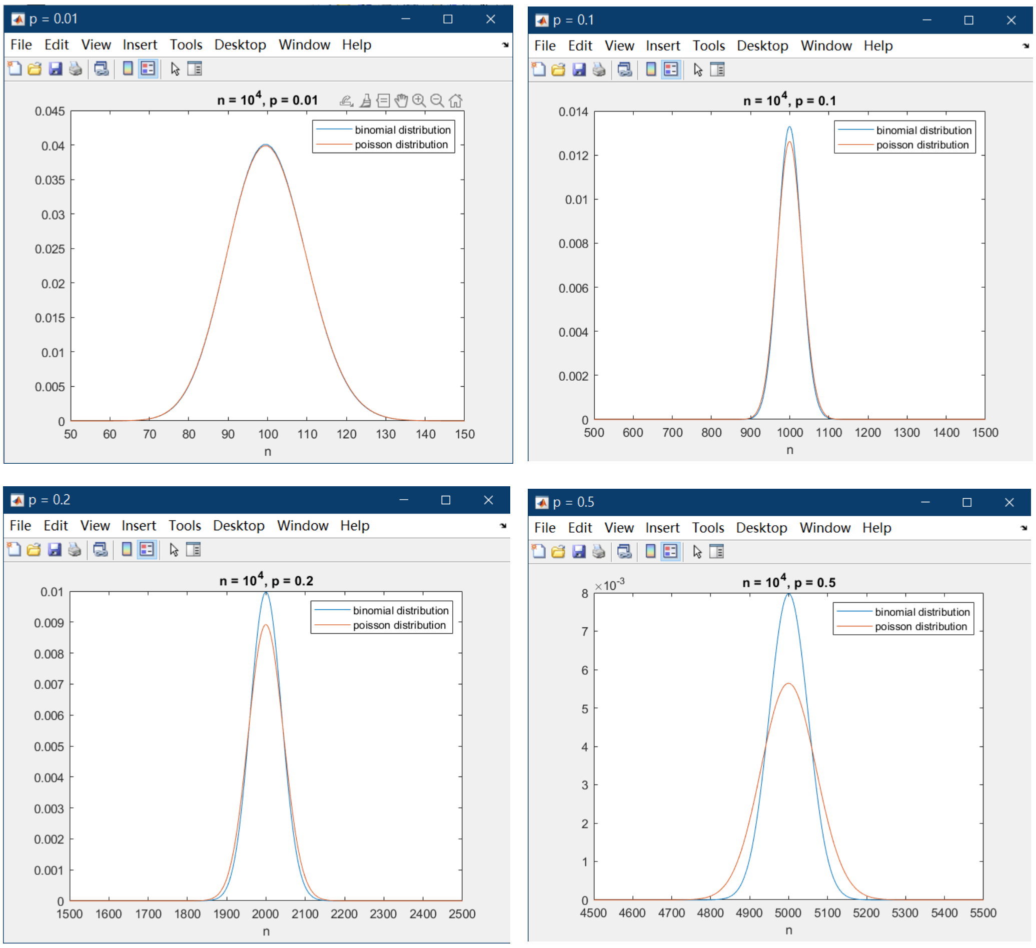


圖(二～六) binomial distribution in p = 0.1, 0.3, 0.5, 0.7, 0.9, respectively

在比對實驗得出來的 skewness 以及製作出來的 binomial distribution 中，我們可以觀察到，在 p = 0.1 和 0.3 的 binomial distribution 中，他們的 right skew 在 skewness 會是正值，且會依序遞減，如同兩個 binomial distribution 相較於 bell shape graph 的右傾程度；在 p = 0.5 時，binomial distribution 剛好是一個 bell shape graph，而他的 skewness 也剛好就是 0；在 p = 0.7 和 0.9 時，他們的 left skew 在 skewness 會是負值，且會依序遞減，如同兩個 binomial distribution 相較於 bell shape graph 的左傾程度。

所以，總結而言：當 skewness 的值越大，則該 binomial distribution bar plot 會是 right skewed（也就是 0 <= p < 0.5）；而相反的，skewness 的值越小，則該 binomial distribution bar plot 會是 left skewed（也就是 0.5 < p <= 1）。

5_2：



圖(七～十) binomial distribution and poisson distribution in p = 0.01, 0.1, 0.2, 0.5, respectively

在本次的實驗中，因為 n 是固定的，所以我們藉由 p 的大小來得出結論。在以上的 4 個圖表中，可以發現當 p 值越小時，binomial distribution 和 poisson distribution 的誤差很小，幾乎快要看不出來，如同上圖 p = 0.01 的結果；相反的，p 值越大時，binomial distribution 和 poisson distribution 的誤差會越來越大，如同上圖 p = 0.5 的結果。

而我們在此觀察到的情形和 approximation of binomial distribution by a poisson distribution 中的理論是吻合的，也就是：當 n 趨近餘無限大，且 p 趨近於 0 時，則可將 binomial distribution 的平均 mu = np 代換進 poisson distribution 的 p(x;mu)之中來求近似。因此，我們上一段所觀察到的情況是合理的。