

Solve equations in Finite Fields

Notebook

Loading the package

Remember:

- The file `solvefinitefield.wl` must be in the same folder that your notebook file.
- You must change the directory where *Mathematica* searches packages (to the current one).
- Typically, save first the notebook and then load the package as follows:

```
SetDirectory[NotebookDirectory[]];
```

```
<< solvefinitefield.wl
```

Examples

`solveGF::usage`

`solveGF[p,n,variables,ideal]` finds the solutions of
the equations given by the ideal (without '`== 0`') in $GF(p^n)$.

```
solveGF[2, 3, {X, Y}, {X^2 - X}]
```

```
{ {X → 0, Y → 0}, {X → 0, Y → 1}, {X → 0, Y → x}, {X → 0, Y → 1 + x},  
  {X → 0, Y → x^2}, {X → 0, Y → 1 + x^2}, {X → 0, Y → x + x^2}, {X → 0, Y → 1 + x + x^2},  
  {X → 1, Y → 0}, {X → 1, Y → 1}, {X → 1, Y → x}, {X → 1, Y → 1 + x},  
  {X → 1, Y → x^2}, {X → 1, Y → 1 + x^2}, {X → 1, Y → x + x^2}, {X → 1, Y → 1 + x + x^2} }
```

```
solveGF[2, 3, {X, Y}, {X^2 - X, Y + 1}, False, True]
```

```

Simple solutions  $\{Y \rightarrow \{1, 0, 0\}_2\}$ 
Vars solved  $\{Y\}$ 
All vars  $\{X, Y\}$ 
Vars to solve  $\{X\}$ 
Ideal before  $\{1 + Y, X + X^2\}$ 
Ideal after  $\{X + X^2\}$ 

Solutions  $\{\{X \rightarrow 0, Y \rightarrow \{1, 0, 0\}_2\}, \{X \rightarrow \{1, 0, 0\}_2, Y \rightarrow \{1, 0, 0\}_2\}\}$ 
Solutions(tuple index)  $\{1, 2\}$ 
Assignations  $\{\{0\}, \{0\}, \{0, 1, 1\}_2\},$ 
 $\{0, 1, 1\}_2\}, \{1, 1, 0\}_2\}, \{1, 1, 0\}_2\}, \{1, 0, 1\}_2\}, \{1, 0, 1\}_2\}$ 
Relations  $\{\{X \rightarrow 0\}, \{X \rightarrow \{1, 0, 0\}_2\}, \{X \rightarrow \{0, 1, 0\}_2\}, \{X \rightarrow \{1, 1, 0\}_2\},$ 
 $\{X \rightarrow \{0, 0, 1\}_2\}, \{X \rightarrow \{1, 0, 1\}_2\}, \{X \rightarrow \{0, 1, 1\}_2\}, \{X \rightarrow \{1, 1, 1\}_2\}\}$ 
GroebnerBasis  $\{X + X^2\}$ 
Tuples  $\{\{0\}, \{1, 0, 0\}_2\}, \{0, 1, 0\}_2\},$ 
 $\{1, 1, 0\}_2\}, \{0, 0, 1\}_2\}, \{1, 0, 1\}_2\}, \{0, 1, 1\}_2\}, \{1, 1, 1\}_2\}$ 
Elements of GF  $\{0, \{1, 0, 0\}_2, \{0, 1, 0\}_2, \{1, 1, 0\}_2,$ 
 $\{0, 0, 1\}_2, \{1, 0, 1\}_2, \{0, 1, 1\}_2, \{1, 1, 1\}_2\}$ 
Ideal  $\{-X + X^2, 1 + Y\}$ 
Variables  $\{X\}$ 
 $\{\{X \rightarrow 0, Y \rightarrow \{1, 0, 0\}_2\}, \{X \rightarrow \{1, 0, 0\}_2, Y \rightarrow \{1, 0, 0\}_2\}\}$ 

```