

# ECEN 4638: Lab X.1P

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February 14, 2016

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# 1 Description

The purpose of this lab is to explore the torsion disc system and experimentally collect data that will help in the design of a proportional controller. The torsion disc system consists of a platform with three discs aligned above a driving motor. There are various configurations for the torsion disc system and this lab will use a basic setup described below.

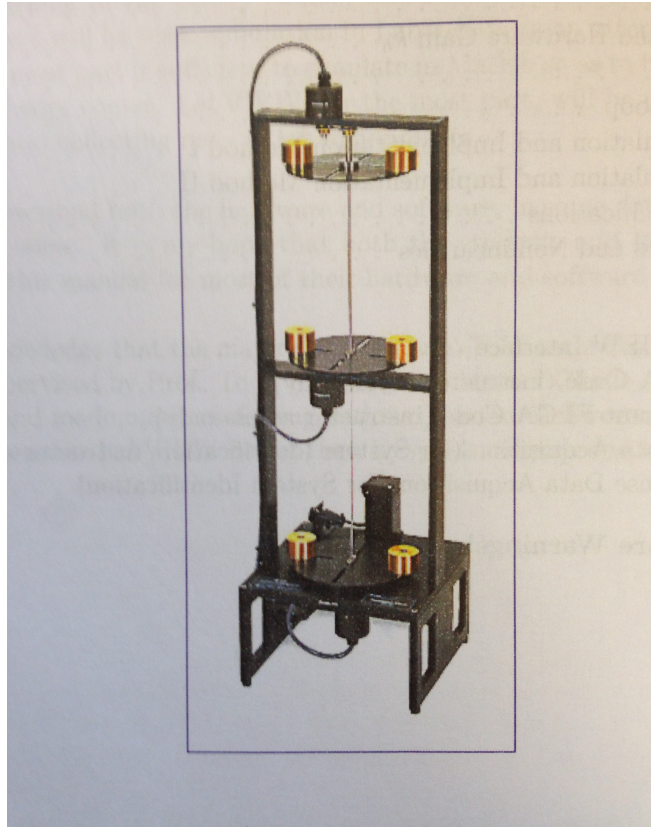


Figure 1: Torsion Disc System

## 2 Setup

There are three main components that will be necessary for this lab: LabView, Matlab, and the Torsion Disc System. LabView and Matlab do not require any setup as they are installed on all lab computers. On the other hand, the torsion disc system must be configured for proper use. For this experiment the top two discs and any attached weights should be removed. After the weights and discs are removed the system cables can be connected.

### Detailed Steps

1. loosen allen key screws on all weights attached to the top two discs and remove the weights.
2. loosen allen key screws on top two discs and detach the discs (each disc is two pieces)

3. attach all labeled connectors and power lines

### 3 LabView Intro

LabView is a high level program that will be used to control the torsion disc system. Data from the system will also be collected using LabView. The general idea is to build a block diagram of the system with appropriate input and output values as well as a configurable controller. In order to understand the operation of LabView it is useful to create a demo system before beginning work on the Torsion Disc system.

#### 3.1 Simulation Loop

Figure 2 shows a simulation loop that was created in LabView. Transfer function blocks were used for the vehicle and controller while the reference and disturbance inputs use lookup tables. The output of the system is written to an external file.

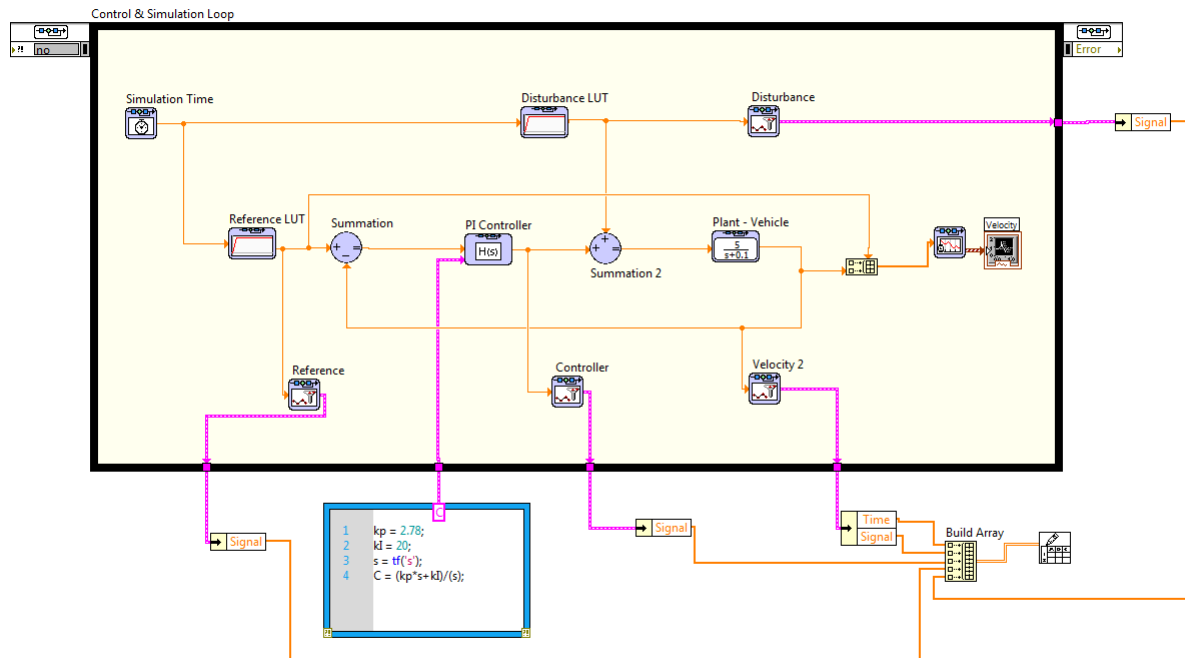


Figure 2: LabView Intro Model

#### 3.2 Controller Results

The created simulation was run with various values of  $K_p$  and  $K_I$  and the output was observed and recorded. The collected results from the LabView simulations were then taken and compared

to the Simulink model used in LabX. As seen in figure 3 and figure 4, the results from LabView and simulink are comparable.

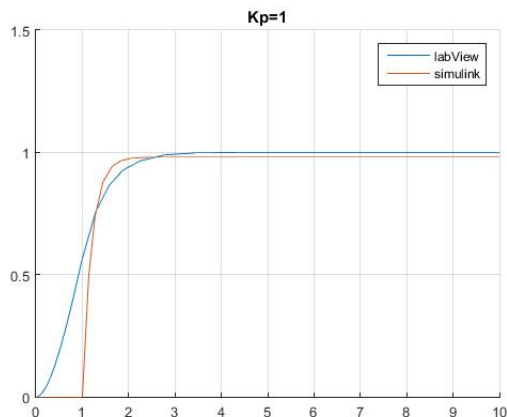


Figure 3: P Controller  $K_p = 1$

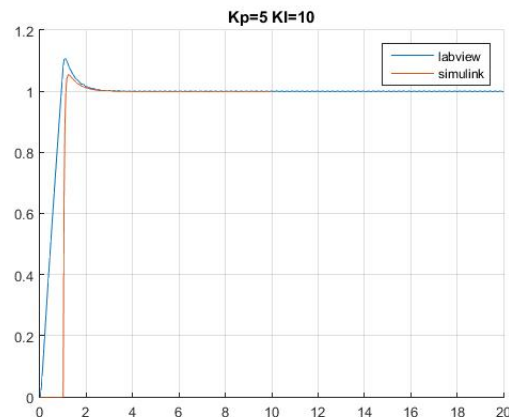


Figure 4: PI Controller  $K_p = 5$   $K_I = 10$

### 3.3 Disturbance

As a final experiment a disturbance was introduced to the LabView model that simulates a hill. The  $P$  and  $P_I$  values were chosen in order to produce a fast response with a small amount of overshoot and a reasonable settle time. In this case, values of  $K_I = 20$  and  $K_I = 60$  worked well. The results of this experiment are shown in figure 5.

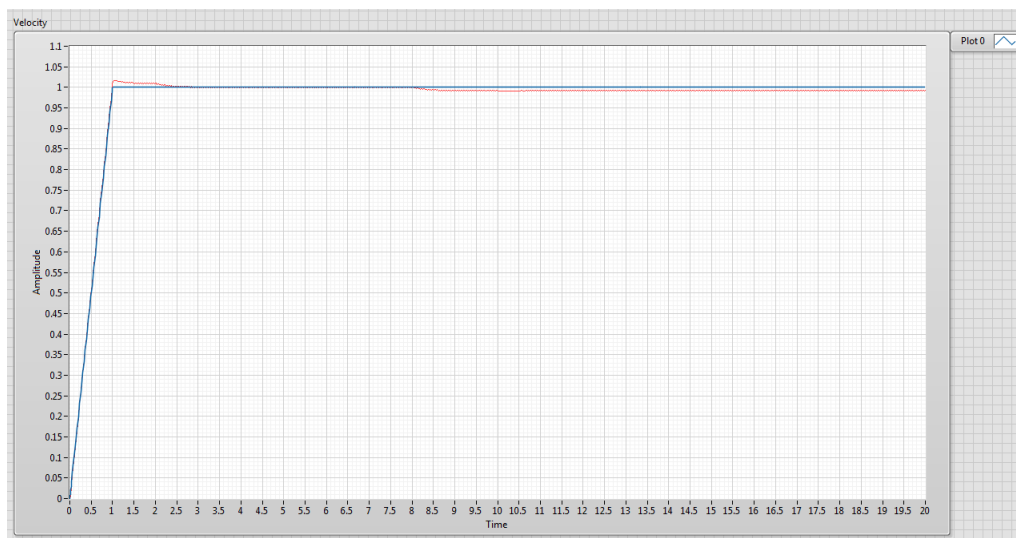


Figure 5: Response with disturbance

## 4 Data Collection

To create a LTI model for the Torsion Disc system it was necessary to collect data that could be used to estimate system parameters. A LabView model was used to control the system and collect necessary data. The LabView model shown in figure 6 was provided for this experiment and can be found on the ITLL share drive. Two experiments were conducted to collect the needed data.

*NOTE: equations are found in section 5.*

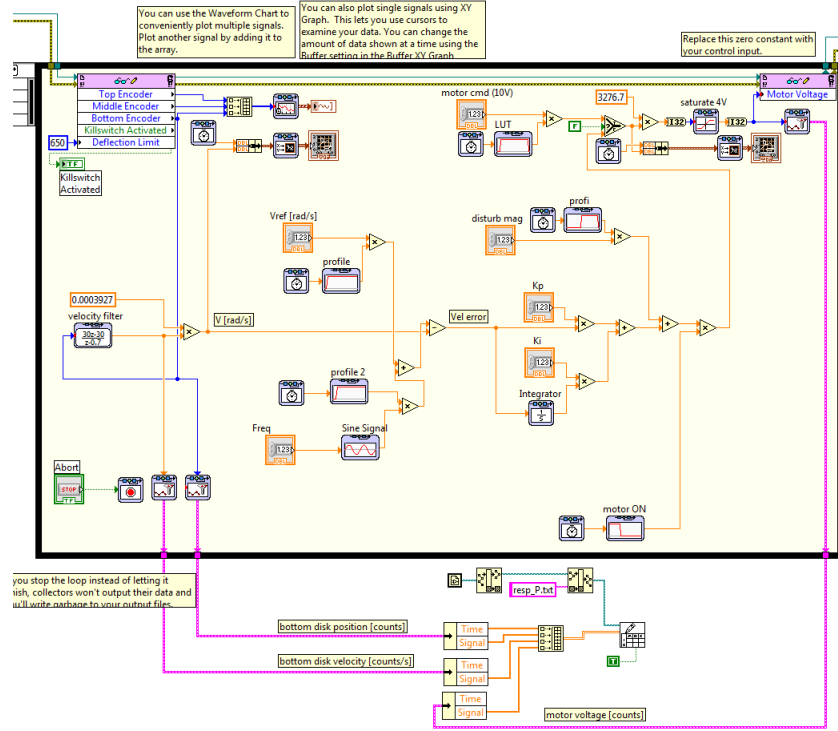


Figure 6: LabView Control System

### 4.1 No motor voltage

The first experiment was conducted with no power connected to the Torsion Disc motor. This was done in order to eliminate some of the parameters from the LTI model in order to estimate the value of  $c$ . Data was collected for three different weight positions as shown in table 1. Each weight position gives a different inertia value for the system. The data is collected by first selecting a weight position and then manually spinning the disc by hand. *If only 2 weights are used they must be located along the hub split line.*

#### Steps

1. set the weights to a desired radius on the lower disc
2. make sure the bolts holding the weights are firmly tightened

3. begin data collection in LabView
4. manually spin the disc and allow it to come to a stop
5. data is saved as a text file
6. change filename to reflect the weight position

Number of weights	Radius	Inertia $J$
4	9 cm	0.0192
4	7.5 cm	0.0143
2	4.5 cm	0.0043

Table 1: Weights and Position

## 4.2 Powered Torsion Disc System

The second data collection experiment was conducted with the power connected to the Torsion Disc system. In this case the values of various system parameters are adjusted in LabView while the weights remain in the same position. The collected data is then used to create a proportional controller.

### Steps

1. set reference speed to 3.14 rad/sec
2. set disturbance to 0
3. set  $K_p$  to 1
4. adjust experiment length to 12 seconds in order to collect data as the system comes to a stop
5. **Ensure there is someone with their finger on the power of button in case the system goes unstable**
6. turn the power on
7. press the run button in LabView, data is saved as a text file
8. rename file to reflect parameter values
9. repeat for the values listed in table 2

Reference rad/sec	$K_p$	Disturbance
3.14	1,5,10	0
3.14	1,5,10	1
6.28	1,5,10	0
6.28	1,5,10	1

Table 2: Data Collection Parameters

## 5 LTI Model

The Torsion Disc system can be modeled as an LTI system with the below equation.

$$J\dot{\omega} + c\omega = k_h u \quad (1)$$

Where  $J$  = total system inertia,  $\omega$  = velocity rad/sec,  $c$  = system drag,  $k_h$  = hardware gain,  $u$  = reference.

### 5.1 Calculating parameter $J$

The inertia of the system can be calculated from the system parameters (inertia of the torsion disk,  $J_{disk}$ , inertia of the motor,  $J_{motor}$ , and inertia of the weights,  $J_{weight}$ ) as follows:

$$\begin{aligned} J_{disk} &= 0.0019 \\ J_{motor} &= 0.0005 \\ J_{weight} &= \sum_{i=1}^N \frac{1}{2}mr^2 + mR^2 \end{aligned} \quad (2)$$

Thus the total inertia is:

$$\begin{aligned} J_{tot} &= J_{disk} + J_{motor} + J_{weight} \\ J_{tot} &= 0.0019 + 0.0005 + \sum_{i=1}^N \frac{1}{2}mr^2 + mR^2 \end{aligned} \quad (3)$$

### 5.2 Estimating parameter $c$

From the LTI model above, system drag  $c$  can be estimated as follows:

$$\begin{aligned} \omega(t) &= e^{\frac{-c}{J}t} \omega(t_0) \\ \ln \left( \frac{\omega(t_1)}{\omega(t_0)} \right) &= \frac{-c}{J} (t_1 - t_0) \\ c &= \frac{-J \ln \left( \frac{\omega(t_1)}{\omega(t_0)} \right)}{t_1 - t_0} \end{aligned} \quad (4)$$

To estimate  $c$  from collected data, the natural logarithm of angular velocity  $\omega$  is plotted against time. A reasonably straight part of the curve is chosen, from time  $t_0$  to time  $t_1$ . The total system inertia and angular velocities  $\omega(t_0)$  and  $\omega(t_1)$  are then used with equation 4 to estimate system drag  $c$ . Plots of filtered and unfiltered angular velocity data were analyzed, and filtering was found not to significantly affect the plot or  $c$  value obtained from the data. Thus, the unfiltered velocity data alone was used to estimate  $c$ . This was repeated for several experiments and  $c$  values were averaged, resulting in a  $c$  value of

$$c = 0.0079$$

A sample plot is shown below for  $k_p = 1$  and a reference input of  $\omega_{ref} = 6.28$  rad/s. The relatively linear section from  $t_0 = 7.5s$  to  $t_1 = 8.5s$  was chosen for estimating  $c$ . Choosing a time period when angular velocity was well above 0 reduced the influence of motor poles on the system and the estimation of  $c$ .



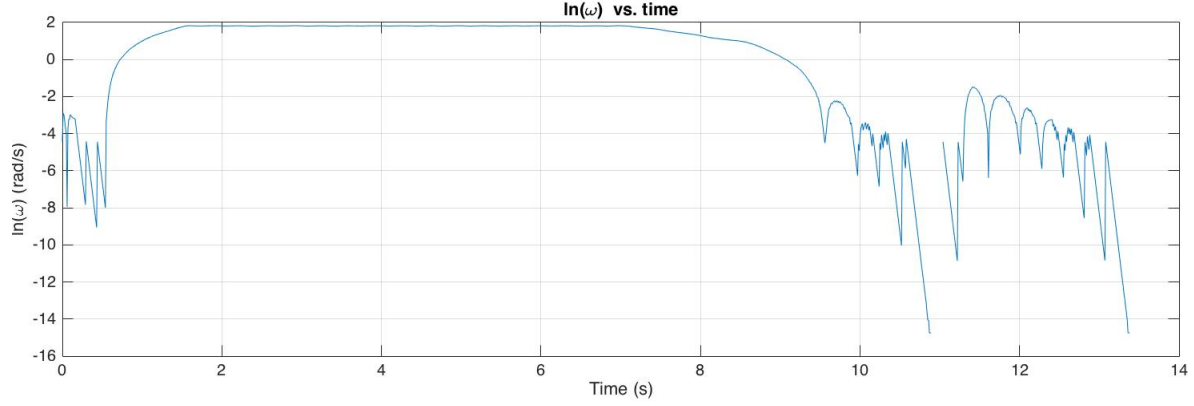


Figure 7:  $\ln(\omega)$  vs. time

### 5.3 Estimating parameter $k$

When angular acceleration  $\dot{\omega} = 0$ , the equation describing our LTI system becomes

$$\begin{aligned} c\omega &= k_h u \\ k_h &= \frac{\omega}{u} c \end{aligned} \quad (5)$$

Hardware gain  $k_h$  can be estimated from the above equation, where  $c$  is calculated as in the previous section,  $\omega$  is the average rotational velocity of the system, and  $u$  is the average motor voltage when the system is running at a constant speed. Parameters  $\omega$  and  $u$  were plotted against time to determine when the system was at a constant speed.  $\omega$  and  $u$  were averaged over the chosen time period and equation 5 was then used to estimate  $k_h$ . This was repeated for several experiments and  $\omega$  and  $u$  values were averaged. The resulting  $k_h$  value is

$$k_h = 0.3598$$

This value for  $k_h$  is very close to the nominal value calculated from system parameters

<i>Pulley Magnification</i>	$k_p = 3$	$\frac{Nm}{Nm}$
<i>Drive Motor Torque Constant</i>	$k_m = 0.086$	$\frac{Nm}{Nm}$
<i>Drive Motor Amplifier Current Gain</i>	$k_a = 1.5$	$\frac{A}{V}$

$$\begin{aligned} k_{hnominal} &= k_p k_m k_a \\ k_{hnominal} &= 0.387 \end{aligned}$$

Sample plots used to evaluate average rotational velocity  $\omega$  and average motor voltage  $u$  are shown below. The time period from  $t_0 = 2$  seconds to  $t_1 = 7$  seconds was used to calculate average  $\omega$  and  $u$ .

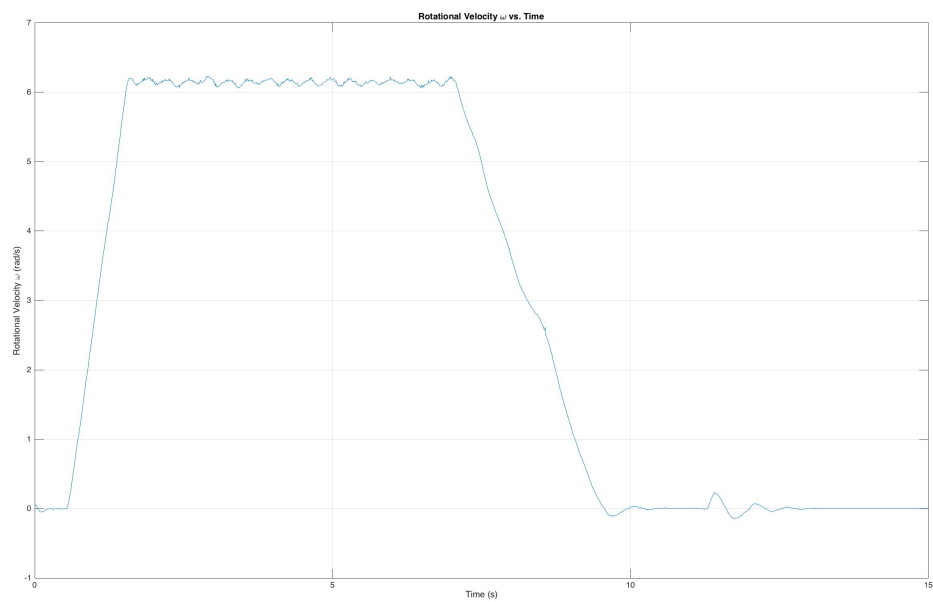


Figure 8: Rotational Velocity  $\omega$  vs. Time

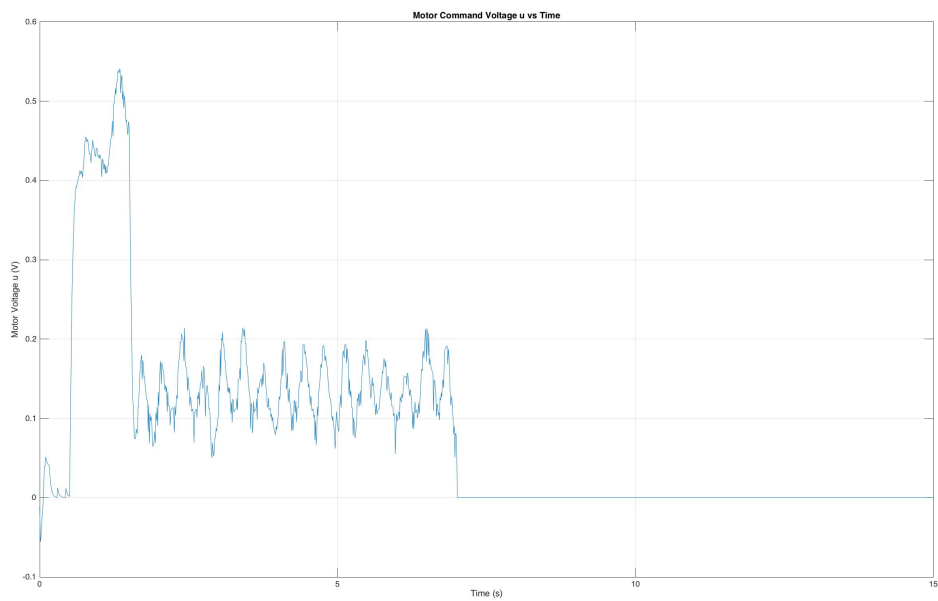


Figure 9: Motor Voltage  $u$  vs. Time

## 6 Introducing sinusoid to reference

With proportional control  $k_p$ , we see that the closed-loop transfer function of our system is

$$H(s) = \frac{\frac{k_p k}{J}}{s + \frac{c}{J} + \frac{k_p k}{J}} \quad (6)$$

Substituting our values for  $k_p$  and  $c$  in the system, for a gain of  $k = 1$ , and an arrangement of 4 masses at 9 cm from the center of the disk ( $J = 0.0192$ ), the transfer function becomes

$$H(s) = \frac{18.74}{s + 19.15} \quad (7)$$

A Bode plot of the system transfer function is shown in figure 10. The bandwidth of the system is determined from the Bode plot to be 19.015 Hz.

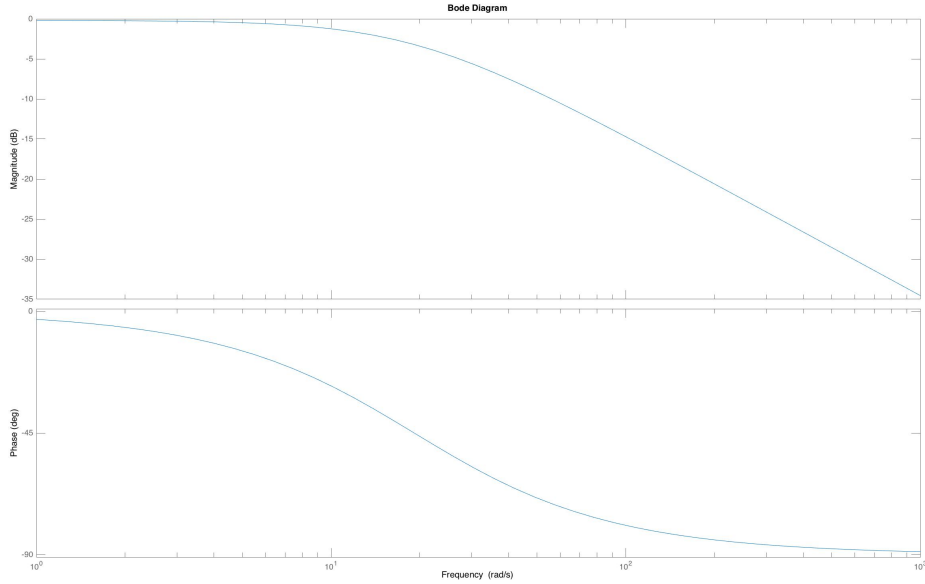


Figure 10: Bode Plot of System Transfer Function  $H(s)$

A sinusoidal input can be introduced to the system to further investigate the frequency response and time/phase delay of the system. For the LTI model, the introduction of a time delay results in the system equation

$$\begin{aligned} J(\omega b_\omega \cos(\omega t) - \omega a_\omega \sin(\omega t) + c(a_\omega \cos(\omega t) + b_{\omega} \sin(\omega t))) \\ = a_u \cos(w(t - T_D)) + b_u \sin(w(t - T_D)) \end{aligned} \quad (8)$$

This representation of the system is explored in more detail in the third data collection experiment, which was conducted with the power connected to the Torsion Disc system and a sinusoidal reference input  $\delta V_{ref} = 2 \sin \pi f t$  added to the constant reference input. In this case the values system parameters  $k_p$  and  $f$  are adjusted in LabView while the weights remain in the same position. The collected data is then used to investigate frequency response of the system.

## Steps

1. set reference speed to 6.28 rad/sec
2. set disturbance to 0
3. set  $K_p$  to 1
4. adjust experiment length to 15 seconds in order to collect data as the system comes to a stop
5. **Ensure there is someone with their finger on the power of button in case the system goes unstable**
6. turn the power on
7. press the run button in LabView, data is saved as a text file
8. rename file to reflect parameter values
9. repeat for the values listed in below table

Reference rad/sec	$K_p$	Frequency
6.28	0.5	4Hz
6.28	1	5Hz

Table 3: Frequency Response Parameters

The sinusoidal input for motor voltage  $u$  and output for rotational velocity  $\omega$  can be graphically fit using fourier series, and the fourier coefficients can be obtained from this nonlinear curve fitting. Below the data for  $u$  and  $\omega$  are plotted (data points) along with their best fit sinusoids (blue line) for  $k_p = 1$  and  $f = 5\text{Hz}$ .

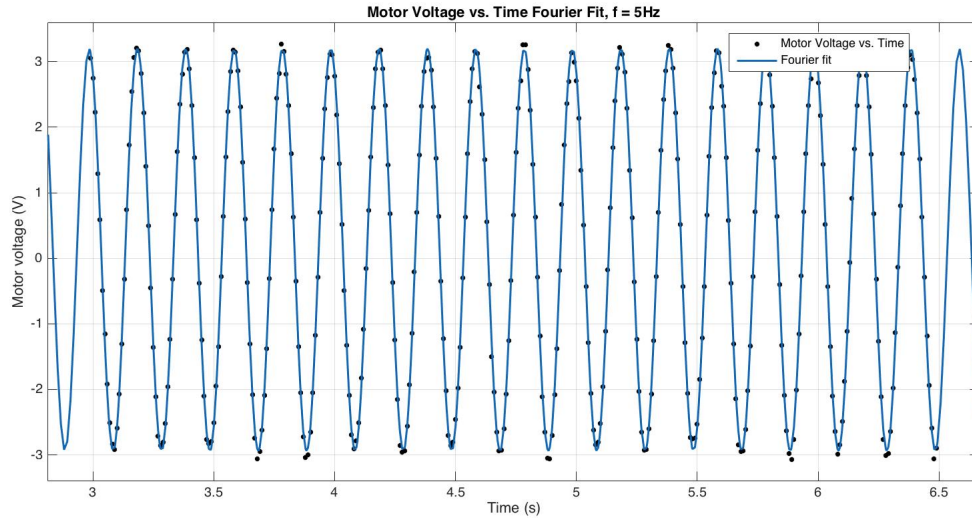


Figure 11: Fourier Fit of Motor Command Voltage  $u$

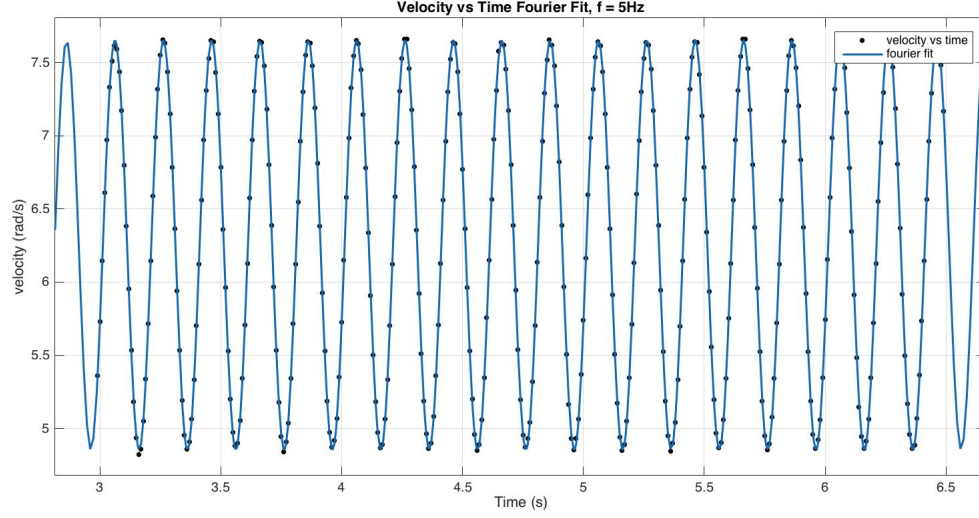


Figure 12: Fourier Fit of Rotational Velocity  $\omega$

The fourier coefficients obtained from the sinusoidal curve fitting are

$$a_{\omega} = -0.5444 \quad b_{\omega} = 1.289 \quad a_u = 2.712 \quad b_u = -1.435$$

Inserting the above parameters into the fourier form of our LTI system equation, and separating into sine and cosine parts, we get

$$J\omega b_{\omega}(\omega) + ca_{\omega}(\omega) = k(a_u(\omega)\cos(\omega T_D) - b_u(\omega)\sin(\omega T_d)) \quad (9)$$

$$J\omega 1.289 + (-0.544)c = k(2.712\cos(\omega T_D) + 1.435\sin(\omega T_d)) \quad (10)$$

**what to do with this??** The time delay  $T_D$  is estimated to be on the order of several hundredths of a second.

## 7 Proportional Controller Design