ECEN 4638: Lab W2

Rane Brown
Kate Schneider

April 17, 2016

Contents

List of Figures

List of Tables

1 Description

The goal of this lab is to design a controller and investigate the response of a two disc system when using ω_1 for feedback and measurement versus using ω_2. The key difference between these two methods is ω_1 is collocated control while ω_2 is non-collocated. We anticipate using ω_1 will produce a more robust controller and during the course of this lab we will confirm or refute this prediction.

2 Setup

For the following experiments we will arrange the TDS with the lower disc containing four weights at 6.5 cm and the middle disc with two weights at 6.5 cm. Using this setup the system parameters calculated from the methods described in Lab W are the following:

$$\begin{aligned} b &= 0.305 & k &= 2.55 \\ c_1 &= 0.004 & c_2 &= 0.0016 \\ J_1 &= 0.011475 & J_2 &= 0.0064375 \end{aligned}$$

3 System model

3.1 LTI model

The two disc torsional disc system can be modeled as an LTI system with the following equations:

$$J_1\ddot{\theta}_1 + c_1\dot{\theta}_1 + k(\theta_1 - \theta_2) = bu \tag{1}$$

$$J_2\ddot{\theta}_2 + c_2\dot{\theta}_2 + k(\theta_2 - \theta_1) = 0 \tag{2}$$

The difference in position of the discs, β is also of interest, where $\beta = \theta_1 - \theta_2$. If we make this substitution, as well as substituting in angular velocity ω for $\dot{\theta}$, our system becomes:

$$J_1\dot{\omega}_1 + c_1\omega_1 + k\beta = bu \tag{3}$$

$$J_2\dot{\omega}_2 + c_2\omega_2 - k\beta = 0 \tag{4}$$

3.2 State Space Representation

Lab W2 will use the state space representation of the LTI model. State space methods are used because they are easier to manipulate when dealing with a system with multiple outputs such as the two disc TDS.

$$\begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -\frac{c_1}{J_1} & 0 & -\frac{k}{J_1} \\ 0 & -\frac{c_2}{J_2} & -\frac{k}{J_2} \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \beta \end{bmatrix} + \begin{bmatrix} \frac{b}{J_1} \\ 0 \\ 0 \end{bmatrix} \tag{5}$$

4 Matlab Controller Design ω_1

4.1 Specifications ω_1

- Rise time:
  The controller should provide a relatively quick response; thus a rise time under 0.5 seconds is desired.
- Overshoot:
  We would like to reduce overshoot for the system, keeping it within 10% if possible, and within 15% if that is more feasible.
- Bandwidth:
  System bandwidth should be as high as is possible to achieve without compromising other areas of system performance.

To meet these specifications, we used several design iterations in Matlab's sisotool to design a lead-lag controller. The lead network increases the speed of system response, increases phase margin, and decreases overshoot, giving us a faster system with good stability. The lag network provides improvement in steady state tracking, and with the lead network, allows attenuation of low and high frequencies. The controller we used was

$$C = 1.7241\frac{(1+.68s)(1+0.1s)}{(1+0.097s)(1+3.9s)}$$

resulting in an overall system transfer function of

$$H_{\omega_1 u_{compensated}} = \frac{8.219s^4 + 96.34s^3 + 3419s^2 + 3.7600s + 48160}{s^5 + 19.38s^4 + 725.6s^3 + 10170s^2 + 41280s + 48670}$$

4.2 Step Response ω_1

Our controller provided a relatively good step response, with a rise time of 0.27 seconds, overshoot of 11.6%, and a settling time of 1.9 seconds. The step response is plotted below.

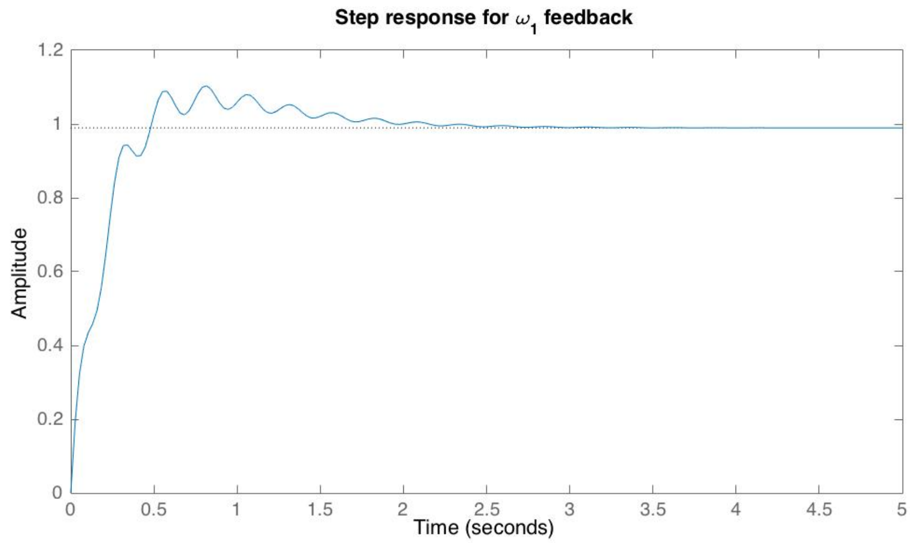

Figure 1: System step response with ω_1 feedback

4.3 Frequency Response ω_1

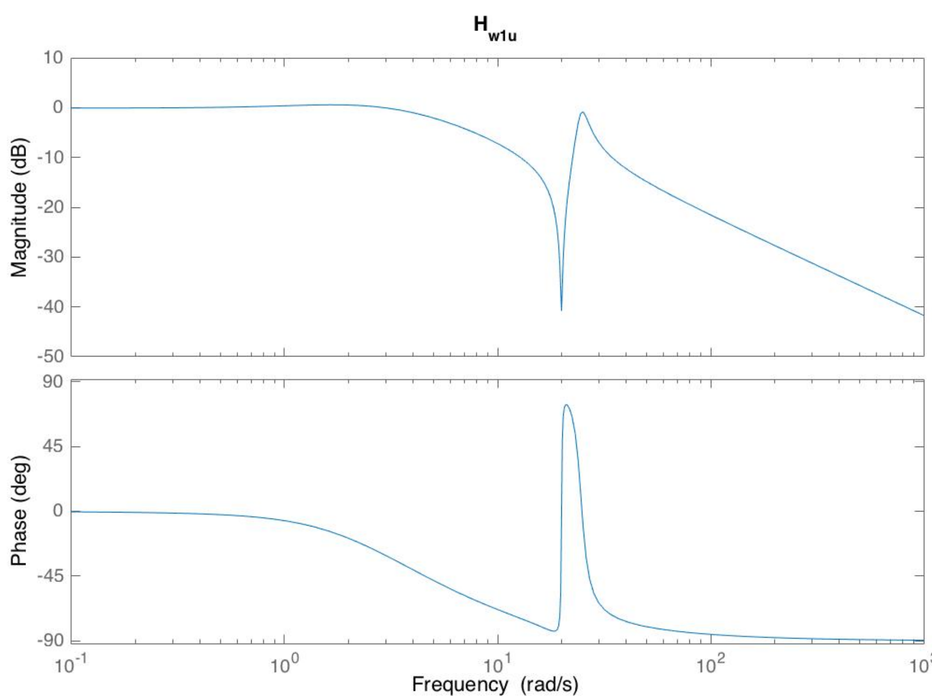

Figure 2: System frequency response with ω_1 feedback

With the addition of our controller, the closed loop system has infinite gain margin, a phase margin of 150°, and a bandwidth of 5.94 rad/s, which can be seen from the above frequency response plots. Thus, our theoretical system was shown to be stable with this controller, and it was tested on the physical torsional disc system to further evaluate its performance.

5 TDS Controller Implementation ω_1

5.1 Step Response ω_1

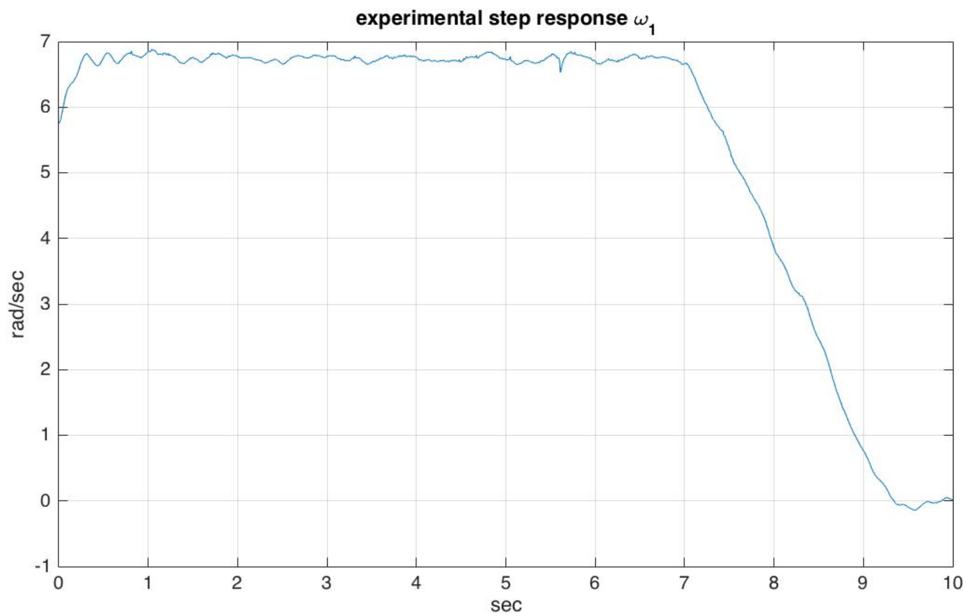

Figure 3: Experimental step response with ω_1 feedback

From the plot of the experimental step response above, we can see that this controller performed well on the physical system, producing fast rise time, minimal overshoot and little noise. For a collocated system, it appears that a simple lead-lag compensator is sufficient to provide decent control.

5.2 Frequency Response ω_1

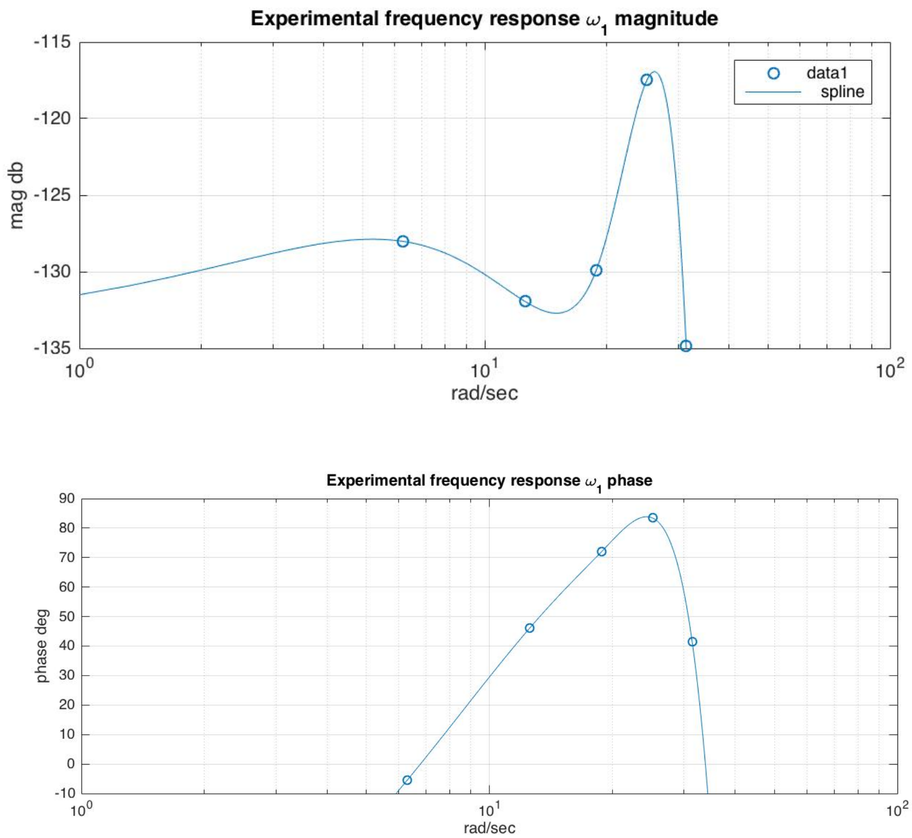

Figure 4: Experimental frequency response with ω_1 feedback

The experimental frequency response shown above follows the predicted response, albeit with fewer data points, (and over a much reduced data range) so the fit is not exact. In general, our controller performed similarly on the physical system as was predicted in Matlab analysis. The physical bandwidth appears to be lower than the 5.9 rad/s predicted by Matlab analysis, and the gain is less, with the magnitude plot shifted down.

6 Matlab Controller Design ω_2

6.1 Specifications ω_2

- rise time
- overshoot
- bandwidth

6.2 Step Response ω_2

6.3 Frequency Response ω_2

7 TDS Controller Implementation ω_2

7.1 Step Response ω_2

7.2 Frequency Response ω_2

8 Controller Comparison