ECEN 4638: Lab X

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Purpose

Lab X is designed to review the basics of feedback control. This will be accomplished through a series of procedures that analyze a cruise control system. The end goal of this project is to gain enough knowledge and background information to create a feasible control solution for use with the torsion disk system.

Background

Feedback is important in nature and technology. Without feedback life as we know it would not exist and the technological marvels that exits today would not be possible. There are numerous example of existing feedback systems but a few of the most important are listed below.

E1

1. Airplane: The autopilot can be set to maintain a level orientation at a given altitude. Disturbances in the air may cause the wings to dip and this movement is read by a gyroscope. The information from the gyroscope is read by the control system and adjustments are made to the ailerons to bring the plane back to a level orientation.

2. Thermostat: The user can select a desired temperature and the thermostat will compare this value to the measured temperature. If hot air from outside the house raises the temperature above the set value the ac will be activated.

3. Climate change: Greenhouse gases increase the Earth's temperature and this rise in temperature increases water vapor in the air. The increase in water vapor further increases the Earth's temperature. In addition, there is ice albedo feedback where the reduction of ice due to melting reduces the amount of reflected radiation and therefore the surface temperature rises which leads to further ice melting.

4. Metabolism: The metabolism of an animal is adjusted depending on food intake. As the food intake decreases the metabolism slows and vice versa.

Procedure

A simple example of a feedback system that has wide spread real world application is cruise control. The most common example of this appears in vehicles. The driver can set the desired speed and the feedback system will maintain that speed in the presence of disturbances such as wind and hills.

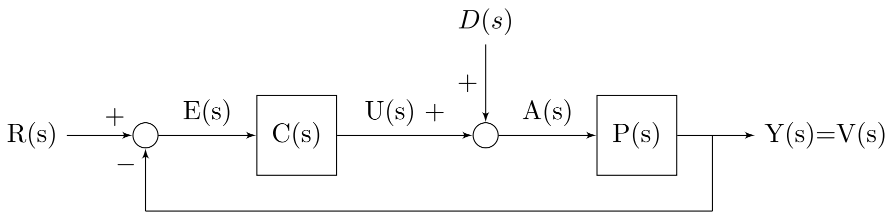

Figure 1: Cruise Control Feedback System

As seen in Figure 6 there are two inputs signal to the cruise control feedback system: the reference $R(s)$ and the disturbance $D(s)$. There is one output signal $Y(s)$ for this system but there are many other points of interest as labeled in Figure 6. With these signals many useful transfer functions can be derived that will help in the selection of an appropriate controller $C(s)$. The selection for $C(s)$ is dependent on many factors including desired rise time, settle time, overshoot, and steady state error.

Signal or Transfer Function	Description
$Y(s) = V(s)$	vehicle velocity
$R(s)$	reference velocity
$E(s) = R(s) - Y(s)$	velocity error
$U(s)$	controller output
$A(s)$	vehicle acceleration
$D(s)$	disturbance acceleration
$P(s) = \frac{k}{s+b}$	plant **transfer function**
$C(s)$	controller **transfer function**
$H_{yr}(s) = \frac{Y(s)}{R(s)}$	closed loop **transfer function** from $R(s)$ to $Y(s)$
$H_{yd}(s) = \frac{Y(s)}{D(s)}$	closed loop **transfer function** from $D(s)$ to $Y(s)$
$H_{ur}(s) = \frac{U(s)}{R(s)}$	closed loop **transfer function** from $R(s)$ to $U(s)$
$H_{ud}(s) = \frac{U(s)}{R(s)}$	closed-loop **transfer function** from $D(s)$ to $U(s)$

Table 1: Signals and Transfer Functions

Table 1 provides definitions for the signals and transfer functions that will be used for the remainder of Lab X. Common notation will be to exclude the (s) when writing equations i.e. $R(s)$ will be represented as R.

E2

The first transfer of function of interest is from the input R to the output Y. This transfer function can be found by setting the disturbance D to zero and chasing the signals from figure 6. The reason it is possible to set $D = 0$ and find an individual piece of the overall transfer function is due to the principle of superposition.

$$H_{yr} = \frac{Y}{R} = \frac{CP}{1 + CP} \tag{1}$$

$$\text{Substituting in for P yields: } H_{yr} = \frac{Ck}{(s+b) + Ck} \tag{2}$$

E3

A second transfer function of interest is from the disturbance D to the output Y. Once again, the principle of superposition can be utilized to find the desired transfer function H_{yd}. This time the input R is set to 0. In this case, it can be seen that "$b \geq 0$ indicates dissipation (friction, wind, etc...) and $k \geq 0$ indicates that a positive command should result in an increase in velocity"

$$H_{yd} = \frac{Y}{D} = \frac{P}{1+CP} \tag{3}$$

$$\text{Substituting in for P yields: } H_{yd} = \frac{k}{(s+b)+Ck} \tag{4}$$

E4

The purpose of a feedback control system is to design the controller C in such a manner that the output behaves as desired. A controller that commands an acceleration proportional to the velocity error is proportional feedback.

$$u(t) = k_p(r(t) - y(t))$$

This is an example of a negative feedback system. For sections E4-E8 a proportional controller will be used:

$$C = C_p = k_p$$

Substituting this proportional controller into equations 2 and 4 gives the following transfer functions.

$$H_{yr} = \frac{k_p k}{(s+b)+k_p k} \tag{5}$$

$$H_{yd} = \frac{k}{(s+b)+k_p k} \tag{6}$$

Examination of these two transfer functions shows that there are no zeros in either case. The poles of the two transfer functions are identical at $s = -b - k_p k$. In order for the system to be BIBO stable there can be no right-half-plane poles or zeros. For this to be true

$$k_p > \frac{-s-b}{k}$$

E5

Another useful piece of information is the impulse response of H_{yr}. The impulse response $h_{yr}(t)$ can be found by taking the inverse Laplace transform of H_{yr}.

$$H_{yr} = \frac{k_p k}{(s+b)+k_p k} = (k_p k)\frac{1}{s-(-b-k_p k)} \tag{7}$$

$$h(t) = \mathcal{L}^{-1}\{H_{yr}\} = k_p k \exp\{-t(b+k_p k)\} \tag{8}$$

The impulse response found in equation 8 can be used to find the time constant of the system. Equation 8 is of the form $e^{-t/\tau}$, where τ is the time constant of interest.

$$\text{time const. } \frac{1}{\tau} = (b+k_p k) \implies \tau = \frac{1}{(b+k_p k)} \tag{9}$$

For a desired time constant of 2 seconds a suitable value of k_p for the proportional controller is:

$$k_p = \frac{1-2b}{2k} \tag{10}$$

A suitable time constant for the torsion disc system is less than 0.5 seconds.

E6

We can model the cruise control system in Simulink, using proportional controller

$$C = C_p = k_p + k_I \frac{1}{s}$$

First we experiment with a step reference input at $t = 1 : r(t) = 1(t-1)$ and a disturbance input $d(t) = 0$. The results of this simulation are shown in the plot below. top left = r(t) and d(t), top right = u(t), bottom left = e(t), bottom right = y(t)

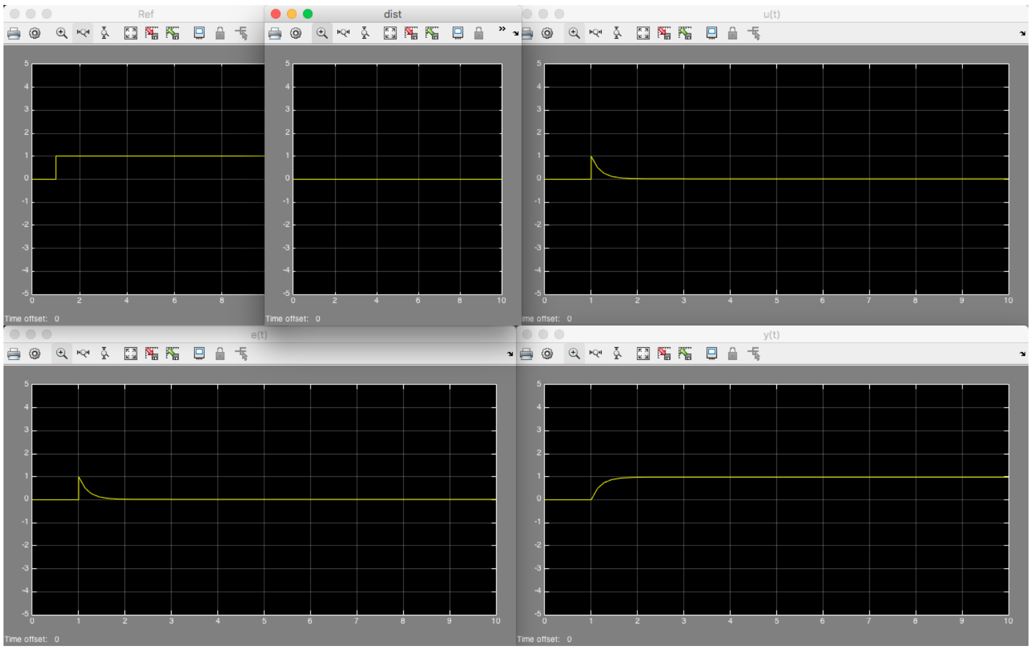

Figure 2: Step Reference 0 Disturbance

Next we introduce a step disturbance input at $t = 2 : d(t) = 1(t-2)$ and zero reference input. The results of the simulation are plotted below. top left = r(t) and d(t), top right = u(t), bottom left = e(t), bottom right = y(t)

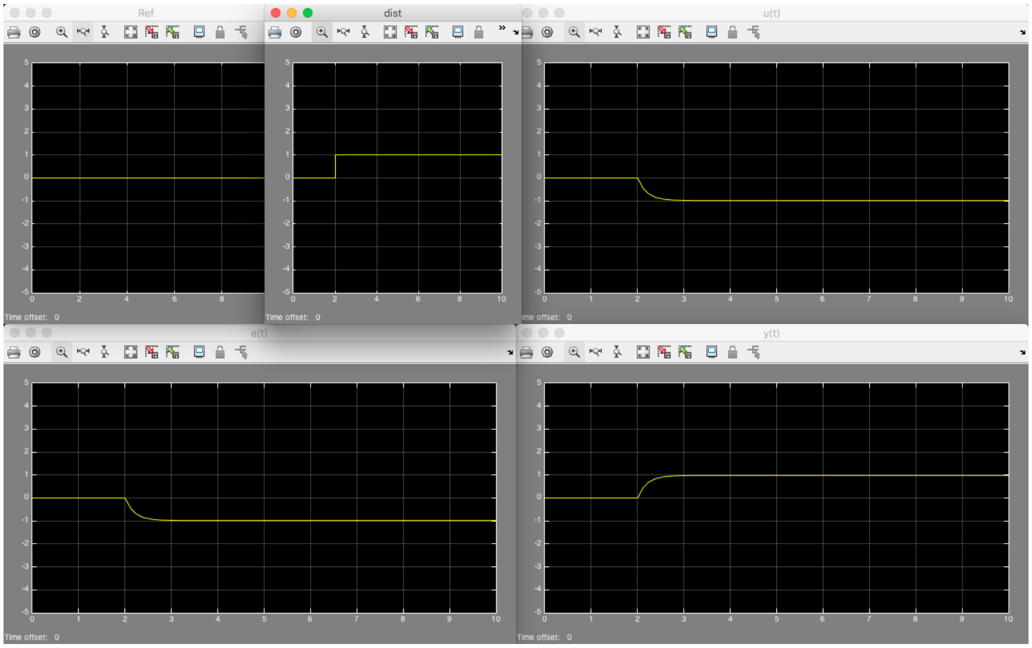

Figure 3: 0 Reference Step Disturbance

If we apply both a step reference input $r(t) = 1(t-1)$ and a step disturbance $d(t) = 1(t-2)$, we see that there is the initial system response and a second response to the disturbance. top left = r(t) and d(t), top right = u(t), bottom left = e(t), bottom right = y(t)

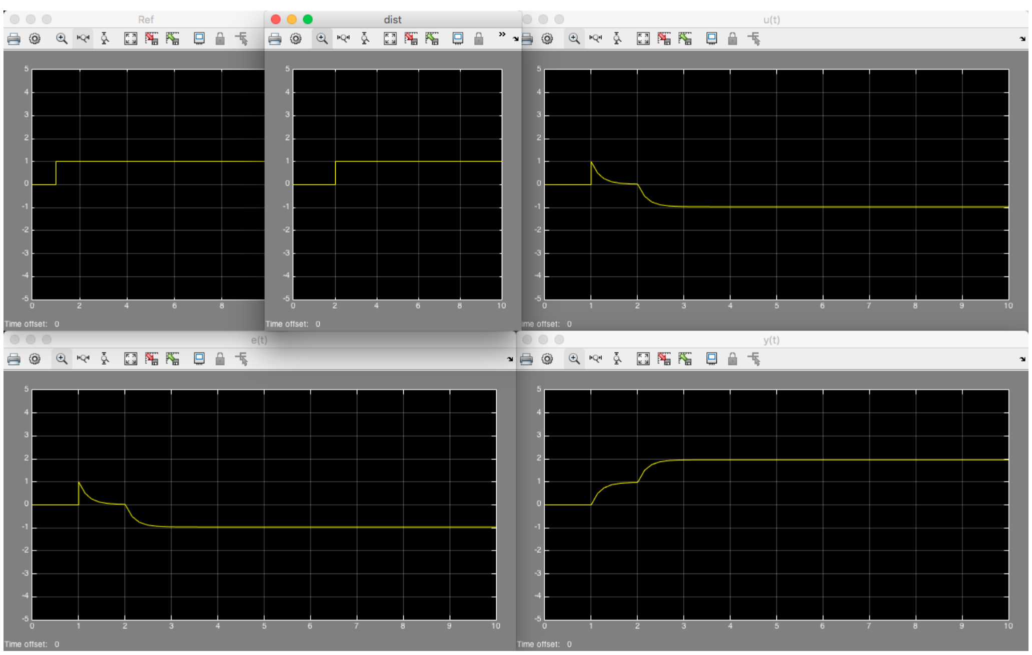

Figure 4: Step Reference Step Disturbance

E7

To determine the system's steady state error with respect to disturbance rejection, we use transfer function

$$H_{ed} = \frac{E}{D} = \frac{-P}{1+CP} \tag{11}$$

$$\text{Substituting in for C and P yields: } H_{ed} = \frac{-k}{(s+b)+k_p k} \tag{12}$$

The steady state error with respect to the disturbance can be found using the final value theorem:

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) \tag{13}$$

When $r(t) = 0$ and $d(t) = 1(t)$, the error is

$$e_{ss} = \lim_{s\to 0} s\frac{E}{D}D = \lim_{s\to 0} s\frac{-k}{s+b+k_p k}\frac{1}{s} = \frac{-k}{b+k_p k} \tag{14}$$

when we assume k_p is in the stability range, i.e. $k_p > \frac{-b}{k}$.

If we include a nonzero reference input, e.g. $r(t) = 1(t)$, the error signal is no longer $E = -Y$; we must now include the reference signal in calculations of transfer function $H'_{ed} = \frac{E}{D}$:

$$H'_{ed} = \frac{E}{D} = \frac{-(s+b+k)}{s+b+k_p k} \tag{15}$$

So with a nonzero reference input our steady state error with respect to disturbance becomes

$$e_{ss} = \lim_{s\to 0} s\frac{-(s+b+k)}{s+b+k_p k}\frac{1}{s} = \frac{-(b+k)}{b+k_p k} \tag{16}$$

while there is an additional term in the numerator, the error remains finite and the system type with respect to disturbance rejection does not change. By choosing k_p arbitrarily large this error can be made zero.

For a constant acceleration disturbance input the steady state error cannot be made zero. The steady state error in this case becomes

$$e_{ss} = \lim_{s\to 0} s\frac{-(s+b+k)}{s+b+k_p k}\frac{1}{s^2} = \infty$$

This result was verified via simulation in Simulink; plots for error output with large k_p are included below.

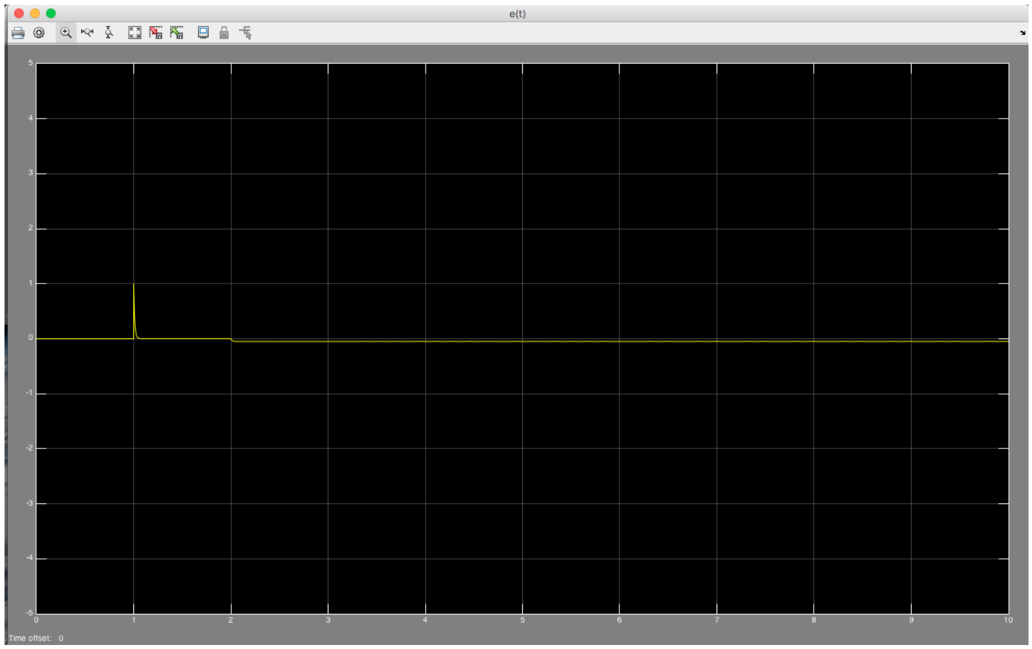

Figure 5: Error for large k_p

When there is a measurement and/or actuation bias a similar effect can be observed when looking at the error signal. An additional bias is in the feedback path can be analyzed using superposition and it can be shown the the steady state error changes in a similar manner.

E8

The result observed for a constant acceleration input is similar to that observed for actuator or measurement biases. An actuator bias does not allow accurate knowledge of the transfer function $H_y r$, thus additional nonzero error is introduced. Similarly, Measurement bias does not allow accurate knowledge of the error signal. $E \neq R - Y$; an additional error noise function is introduced but not accounted for by our model. To account for these biases in our system we could include additional blocks for actuator and measurement bias, $B(s)$ and $H(s)$, respectfully, as shown below.

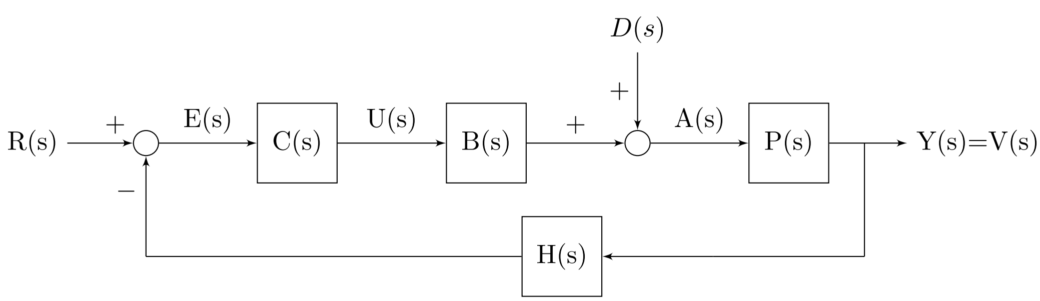

Figure 6: Cruise Control with Actuator and Measurement Bias

E9

In order to ensure a cruise control feedback system has zero steady state error in the face of a constant disturbance input, integral feedback can be introduced along with proportional control. The PI controller is now:

$$C_{PI} = k_p + k_I \frac{1}{s} \tag{17}$$

With this new controller, we can derive $H_{yr}(s)$ and $H_{yd}(s)$ as follows:

$$H_{yr}(s) = \frac{C_{PI}P}{1 + C_{PI}P} = \frac{(k_p + k_I)s}{s^2 + (b + k_p k)s + k_I k} \tag{18}$$

$$H_{yd}(s) = \frac{P}{1 + C_P IP} = \frac{ks}{s^2 + (b + k_p k)s + k_I k} \tag{19}$$

$H_{yr}(s)$ and $H_{yd}(s)$ have the same characteristic equation, and thus have common poles as well as a common zero at $s = 0$.

E10

The characteristic polynomial of the system with the new PI controller becomes

$$s^2 + (b + k_p k)s + k_I k = 0 \tag{20}$$

E11

We can determine the stability of the system from this characteristic equation using the Routh Stability Criterion:

s^2	1	$k_I k$
s^1	$b + k_p$	0
s^0	$k_I k$	0

For the system to be BIBO stable, we need

$$b + k_p > 0 \implies k_p > -b \tag{21}$$

$$k_I k > 0 \implies k_I > 0 \tag{22}$$

for $k > 0$

E12

Assuming both k_p and k_I are within the region for system stability, we can determine the steady state error of the system with a PI controller for a step disturbance input $d(t) = 1(t)$ and zero reference input $r(t) = 0$:

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) \tag{23}$$

$$= \lim_{s\to 0} \frac{-ks}{s^2 + (b + k_p k)s + k_i k} = 0 \tag{24}$$

The system can reject a constant disturbance input perfectly, with zero steady state error.

E13

A PI controller can be used in Simulink to view the effects for different values of k_I. As k_I is increased the rise time decreases but the overshoot and settle time both increase. The advantage of adding integral control is that steady state error can be eliminated for a step or ramp input. Below is a snapshot of the modified system in Simulink as well as plots for various values of k_I.

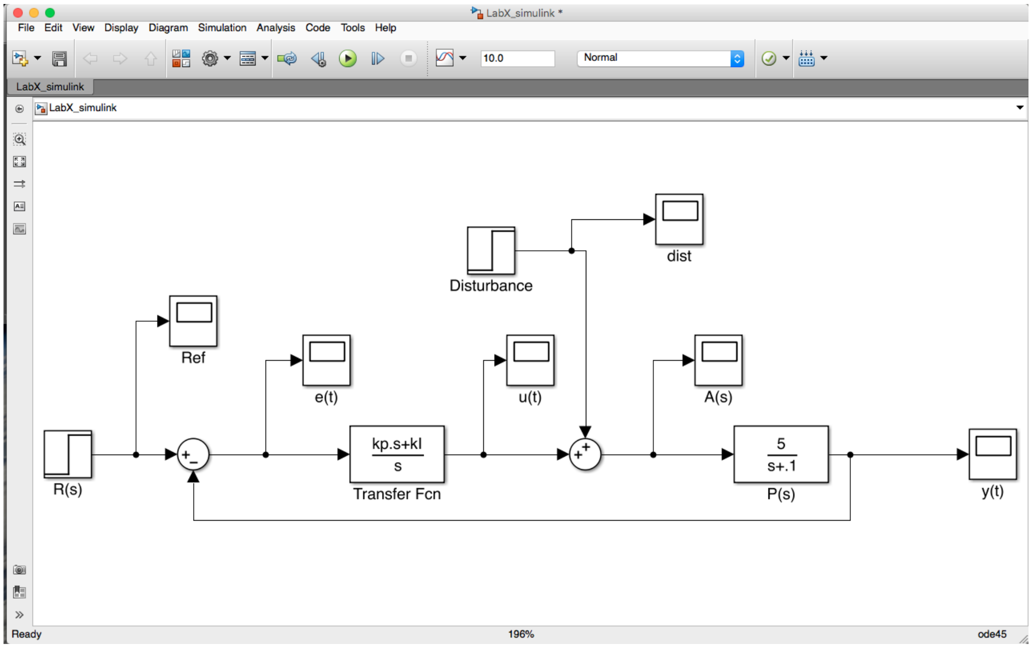

Figure 7: Simulink Model

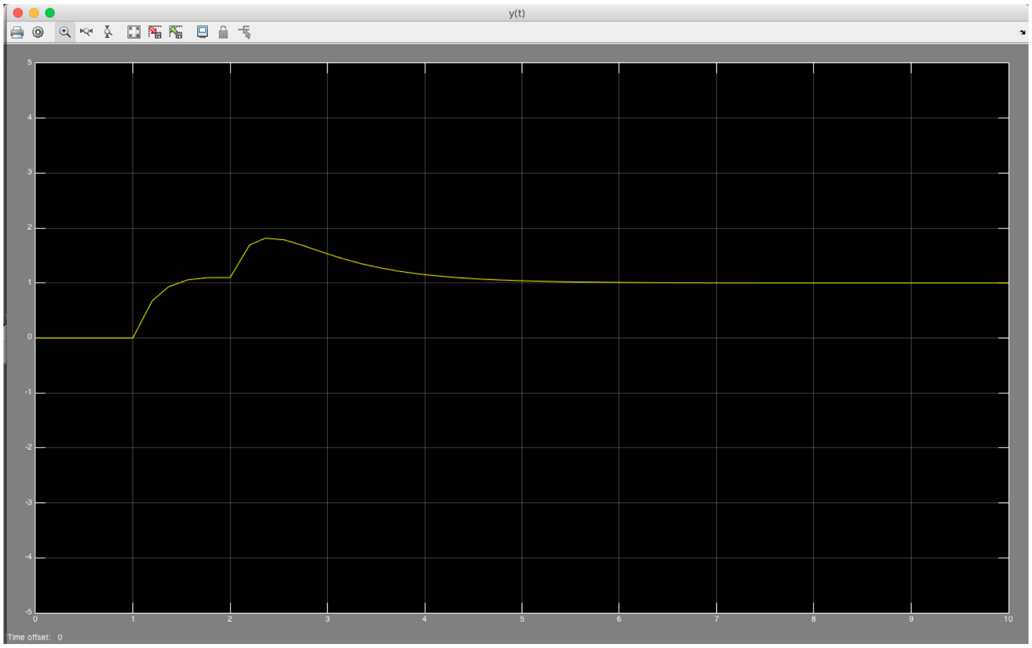

Figure 8: Small k_I

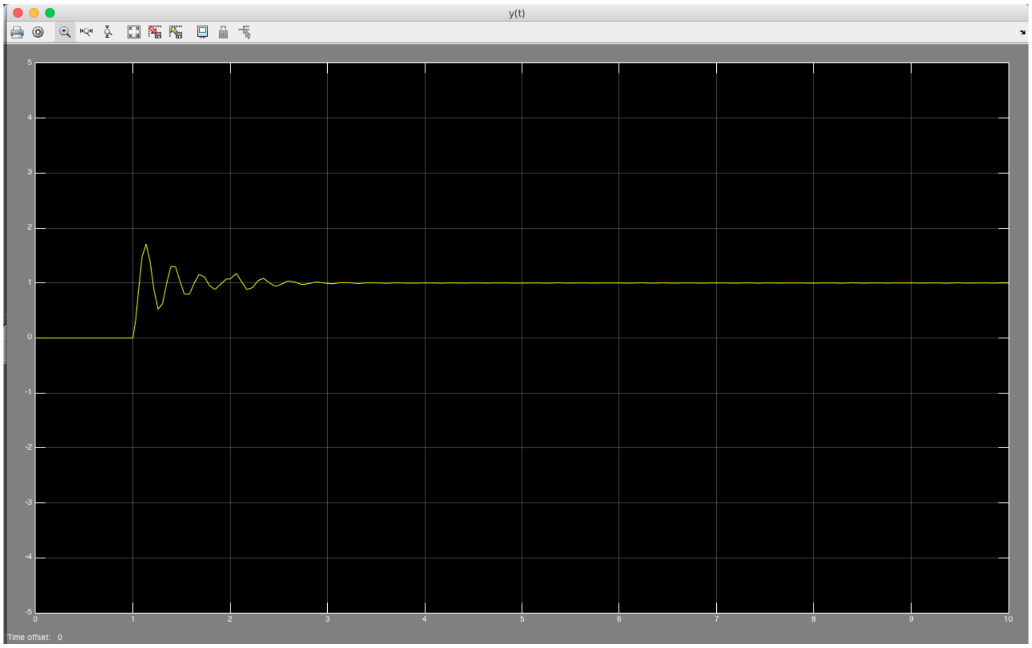

Figure 9: Large k_I

E14

The introduction of an integrator means we must now consider other aspects of the transient response in addition to steady state error. Overshoot and damping are must be considered as well for a well-functioning system. To more easily understand how our chosen parameters k_p and k_I, affect overshoot and damping, we consider the characteristic equation in the form

$$s^2 + 2\xi\omega_n s + \omega_n^2 \tag{25}$$

where ω_n is the natural frequency and ξ is the damping ratio. Both k_p and k_I can be determined in terms of ω_n and ξ:

$$b + k_p k = 2\xi\omega_n \implies k_p = \frac{2\xi\omega_n - b}{k} \tag{26}$$

$$k_I k = \omega_n^2 \implies k_i = \frac{\omega_n^2}{k} \tag{27}$$

For the system to be stable, we need $\xi > 0$ and $\omega_N > 0$. This agrees with the stability region we obtained from using Routh's Stability Criterion, we need $k_I k > 0$ and $k_p k + b > 0$ for stability.

E15

If we consider an experiment that began long before time $t = 0$ with $r(t) = 0$ and $d(t) = 1(-t)$ and assume all signals are constant at $t = 0$, we can verify that for $t = 0$, we have signals
$e(t) = 0$, $u(t) = -1$, $a(t) = 0$, and $y(t) = 0$. Since we assume we are using PI control for the system, there will be zero steady state error, so we know that $e(t) = 0$ at $t = 0$. Furthermore,

$$\begin{aligned}
y(t) &= \mathcal{L}^{-1}Y(s) = \mathcal{L}^{-1}E(s) - R(s) = 0 \\
u(t) &= \mathcal{L}^{-1}A(s) = \mathcal{L}^{-1}(Y(s) - D(s)) \\
&= 0 - -(-1) \\
&= -1 \\
a(t) &= \mathcal{L}^{-1}A(s) = \mathcal{L}^{-1}(U(s) + D(s)) = 0
\end{aligned}$$

Our disturbance input becomes 0 at $t = 0$, and again, because we have a PI controller, as $t \to \infty$, with $d(t) = 0$ for $t > 0$, $e(t)$ will be 0, and thus $u(t) \to 0$, $y(t) \to 0$ and $a(t) \to 0$:

$$\begin{aligned}
y(t) &= \mathcal{L}^{-1}Y(s) = \mathcal{L}^{-1}E(s) - R(s) = 0 \\
u(t) &= \mathcal{L}^{-1}A(s) = \mathcal{L}^{-1}(Y(s) - D(s)) = 0 \\
a(t) &= \mathcal{L}^{-1}A(s) = \mathcal{L}^{-1}(U(s) + D(s)) = 0
\end{aligned}$$

E16

A cruise control system should provide zero steady state error for both tracking a constant velocity reference and rejecting constant acceleration disturbances, hills and bumps for example. Additionally, the response should be quick, but not so fast that a the system does not ensure a smooth acceleration. Thus, if a step function is applied as the reference input, the system should not seek to match that input exactly. Other considerations include good damping - the speed should increase or decrease to meet the constant reference, then remain steady, with minimal oscillations, as well as little to no overshoot - we don't want the user to get a speeding ticket.

For such a system, a value of $k_p = 20$ and $k_I = 5$ will provide the $e_{ss} = 0$ for tracking a constant input and rejecting disturbances, a suitable rise time, good damping and little overshoot.

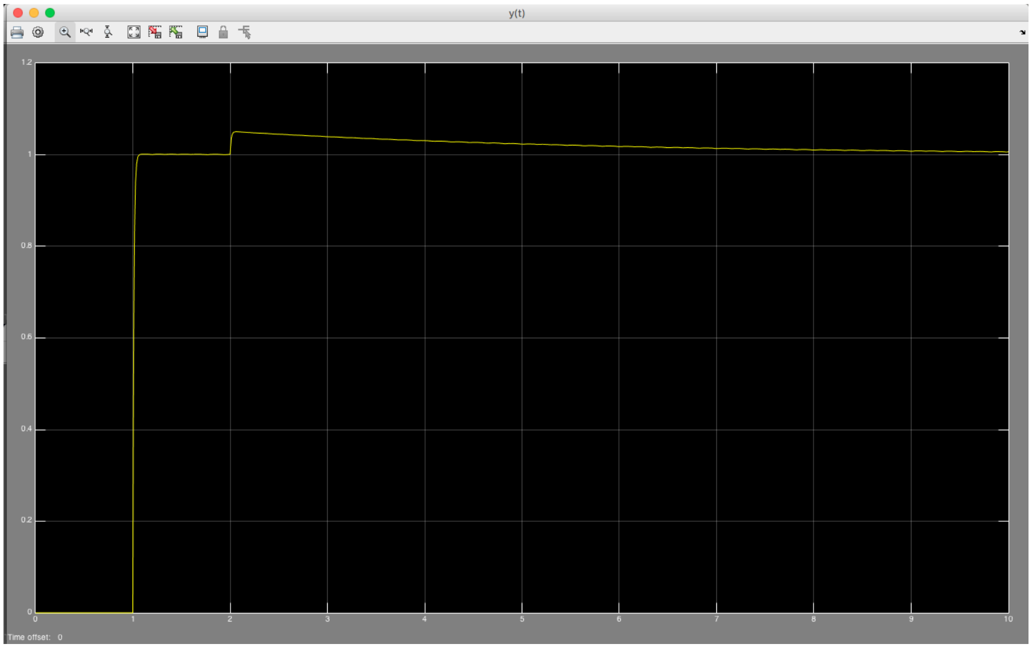

Figure 10: Good k_I and k_p values

E17+

The goal of this project was to explore different control systems for a basic cruise control system. The above equations and simulations show that a PI controller works well for obtaining a quick rise and limiting overshoot. These ideas can be incorporated into the torsion disc system where a motor will be used to drive the discs.