

# ECEN 4638: Lab W2

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# 1 Description

The goal of this lab is to design a controller and investigate the response of a two disc system when using  $\omega_1$  for feedback and measurement versus using  $\omega_2$ . The key difference between these two methods is  $\omega_1$  is collocated control while  $\omega_2$  is non-collocated. We anticipate using  $\omega_1$  will produce a more robust controller and during the course of this lab we will confirm or refute this prediction.

## 2 Setup

For the following experiments we will arrange the TDS with the lower disc containing four weights at 6.5 cm and the middle disc with two weights at 6.5 cm. Using this setup the system parameters calculated from the methods described in Lab W are the following:

$$\begin{aligned} b &= 0.305 & k &= 2.55 \\ c_1 &= 0.004 & c_2 &= 0.0016 \\ J_1 &= 0.011475 & J_2 &= 0.0064375 \end{aligned}$$

## 3 System model

### 3.1 LTI model

The two disc torsional disc system can be modeled as an LTI system with the following equations:

$$J_1\ddot{\theta}_1 + c_1\dot{\theta}_1 + k(\theta_1 - \theta_2) = bu \quad (1)$$

$$J_2\ddot{\theta}_2 + c_2\dot{\theta}_2 + k(\theta_2 - \theta_1) = 0 \quad (2)$$

The difference in position of the discs,  $\beta$  is also of interest, where  $\beta = \theta_1 - \theta_2$ . If we make this substitution, as well as substituting in angular velocity  $\omega$  for  $\dot{\theta}$ , our system becomes:

$$J_1\dot{\omega}_1 + c_1\omega_1 + k\beta = bu \quad (3)$$

$$J_2\dot{\omega}_2 + c_2\omega_2 - k\beta = 0 \quad (4)$$

### 3.2 State Space Representation

Lab W2 will use the state space representation of the LTI model. State space methods are used because they are easier to manipulate when dealing with a system with multiple outputs such as the two disc TDS.

$$\begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -\frac{c_1}{J_1} & 0 & -\frac{k}{J_1} \\ 0 & -\frac{c_2}{J_2} & -\frac{k}{J_2} \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \beta \end{bmatrix} + \begin{bmatrix} \frac{b}{J_1} \\ 0 \\ 0 \end{bmatrix} \quad (5)$$

## 4 Matlab Controller Design $\omega_1$

### 4.1 Specifications $\omega_1$

- rise time The our controller should provide a relatively quick response; thus a rise time under 0.5 seconds is desired.
- overshoot We would like to reduce overshoot for the system, keeping it within 10% if possible (within 15% if that is more feasible)
- bandwidth System bandwidth should be idk!!!

To meet these specifications, we used Matlab's sisotool to design a lead-lag controller. The lead network increases the speed of system response, increases phase margin, and decreases overshoot, giving us a faster system with good stability. The lag network provides improvement in steady state tracking, and with the lead network, allows attenuation of low and high frequencies. The controller we used was

$$C = 1.7241 \frac{(1 + .68s)(1 + 0.1s)}{(1 + 0.097s)(1 + 3.9s)}$$

### 4.2 Step Response $\omega_1$

W

### 4.3 Frequency Response $\omega_1$

## 5 TDS Controller Implementation $\omega_1$

### 5.1 Step Response $\omega_1$

### 5.2 Frequency Response $\omega_1$

## 6 Matlab Controller Design $\omega_2$

### 6.1 Specifications $\omega_2$

- rise time
- overshoot
- bandwidth

## **6.2 Step Response $\omega_2$**

## **6.3 Frequency Response $\omega_2$**

# **7 TDS Controller Implementation $\omega_2$**

## **7.1 Step Response $\omega_2$**

## **7.2 Frequency Response $\omega_2$**

# **8 Controller Comparison**