For office use only	Team Control Number	For office use only
T1 ________________	**65380**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**B**	F4 ________________

2017
MCM/ICM
Summary Sheet

Merge After Toll

Summary

Based on the existing toll plaza model, this paper discusses the merging pattern after toll, and selects the best model to analyze the performance of each model, discussing the variation of the traffic flow and the ratio of the three kinds of tollbooths. Finally we make relativity and sensitivity analysis.
Firstly, a two-dimensional NS optimization model is built based on matrix theory to simulate the existing toll plaza model. Determine the On-Ramp Rule, Moving Forward Rule, Lane-Switching Rule and Off-Ramp Rule so that we can simulate the hourly average throughput and total waiting these time two indicators, and design the form, size and merging pattern of the toll plaza in combined consideration of the construction cost.
On the basis of the above model, we summarize the other nine ways of toll plaza merging pattern by adjusting the position of the toll station, and establish the corresponding optimization model to simulate the model in order to determine the optimal design scheme. Further analysis shows that our model is very smooth under low traffic flow, and tends to be stable with the increase in traffic flow, which manifests great loading capacity!
With the increase of autonomous vehicles, we build the autonomous vehicles queue, adding the *Autonomous-Only Lane* and Emergency Lane to the original model in order to prevent the congestion brought by autonomous-vehicle accidents. Considering the types of tollbooths, the optimization model for proportion of tollbooths is established, and so that we can discuss the influence of the proportion of the three mixed tollbooths on the design scheme. We make correlation analysis on the throughput, congestion levels and the number of three kinds of tollbooths to eventually determine the optimal ratio of ETC: AUTO: HUMAN = 2: 3: 1. Then the sensitivity analysis of the waiting time of the automated tollbooth and throughput are carried out, and the conclusion that the model result is sensitive to the waiting time and insensitive to the traffic flow is obtained.
Realize the design towards GUI by programming and finally make assessment and promotion upon the model above, which has reference value to deal with relative reality problems to a certain degree!

Keywords: 2-D NS enhanced Model; Merging pattern; Autonomous-vehicle Queue

Dear New Jersey Turnpike Authority,

Hello, We are college students who are interested in Mathematical Modeling. We have learnt that the NJ is about to reform the tollbooths in order to meet the new traffic changes in coming future. As autonomous vehicles come to our life, the traffic situation turns to be more and more complicated and makes it urgent for the trunpikes to be changed. We are writing to provide you with my suggestions which is based on the delicately simulated model.

Figure 1: Pic. of NJ turnpike

This is the model we set, which manifests the best efficiency under simulation. On the basis of the above model, we summarize the other nine ways of toll plaza merging pattern by adjusting the position of the toll station, and establish the corresponding optimization model to simulate the model in order to determine the optimal design scheme. Further analysis shows that our model is very smooth under low traffic flow, and tends to be stable with the increase in traffic flow, which manifests great loading capacity!
As the figure shown clearly, this is the ultimate model we designed for the New

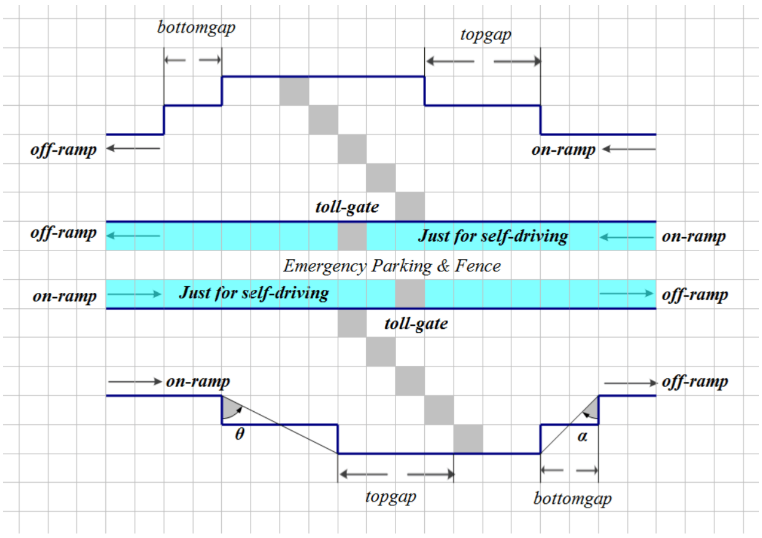

Figure 2: Model

Jersey highways. Our model are assessed by these factors:

- Accident Prevention
- Throughput
- Cost

After that, the solution to deal with the coming autonomous vehicles is to set *Self-driving Only* lanes which can separate the autonomous vehicles from the usual traffic. Through the separating method you can better manage the autonomous vehicles and eliminate their negative impacts on human-driving cars. What's more, when *Emergency Lanes* are added to the margin of lanes instead of the middle, the tollbooths in the middle are able to reverse their orientations to deal with the imbalance two-way traffic flow.
When it finally comes to the proportion of different types of toll collection booths, We set ETC:AUTO:HUMAN=2:3:1. And ETC is in the center of our toll plaza, for three lanes, we use six toll station, set top-gap = 5 and bottom-gap=2, such that the toll plaza enjoys the maximum efficiency.
The advantages of our ultimate model:

- Greater throughput
- Congestion is reduced
- Vehicles can change lanes when they close to the toll, so the drivers has full free time to choose toll station which is more close to them.

I would appreciate it if you could take my suggestions into serious consideration. I am looking forward to hearing from you or seeing the changes in the near future.
Yours, Sincerely

Contents

1 Introduction

1.1 Restatement of the Problem

In this problem, we need to figure out the most efficient model of barrier tolls which are on a two-way highway. A toll highway has L lanes of travel in each direction and a barrier toll contains B tollbooths ($B > L$) in each direction. The shape, size and merging pattern should be determined to design the final turnpikes that meet the requirements. What's more, accident prevention, throughput and cost are also need to be urgently considered. Meanwhile, as more autonomous vehicles added to the traffic mix, the traffic situation becomes more and more complicated. With types of toll collection methods utilized at the same time, there must be influence from the proportion. Faced with these furthermore complex circumstances, the performance of our model need to be assessed and thus be adjusted.
At last, a letter is required to written to the New Jersey Turnpike Authority providing them with our improvement suggestions which are based on our enhanced model.

1.2 Initial Assumption

- Two-way lanes
- Make the length of vehicles the same
- Model initially based on moderate traffic
- Model initially based only one certain type of tollbooth
- The maximum velocity in tolls and turning corners are artificially set
- The velocity of vehicles is average speed, the velocity passing the tolls is the same
- The velocity of vehicles keeps the same regardless of changing time

1.3 Additional Assumption

1. **On-ramp Rule** *(Or called Emerging Rule)* How the vehicles emerge at on-ramp is stipulated by this rule. Divide the traffic flow into $\boldsymbol{i}(i < L)$ levels. As there are L lanes, the *i th* level means that there are i vehicles emerge at the same time at on-ramp.
2. **Off-ramp Rule** *(Or called Vanishing Rule)* How the vehicles leave at off-ramp is stipulated by this rule. When it is time for a vehicle to leave our model at off-ramp, just set its value in the matrix (MatLab codes) into *0*.

1.4 Analysis of the Problem

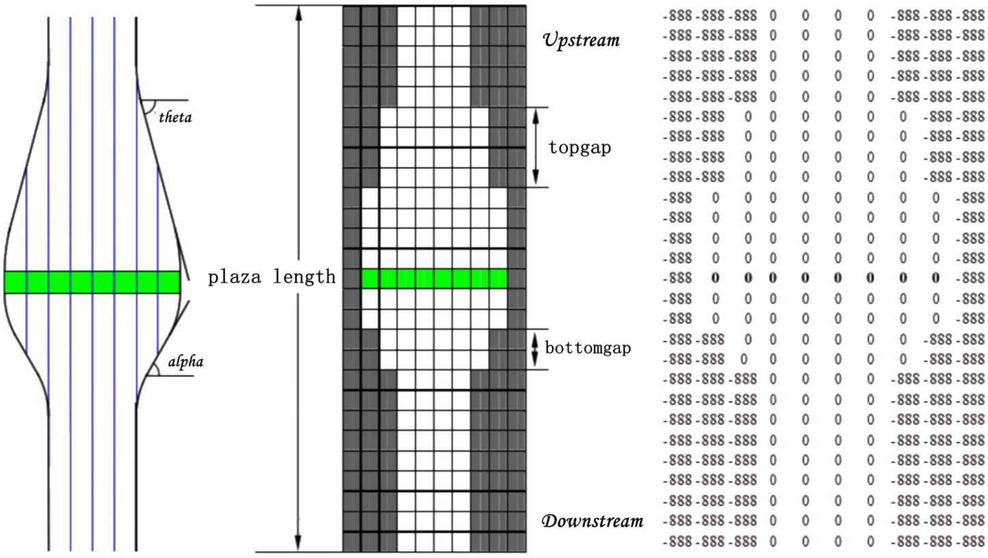

Figure 3: Analysed model

By analyzing the problem, we convert the actual figure of the turnpike into pixels. The pixels ultimately turn into simplified codes in *MatLab*, making it convenient for us to model the traffic situations. Eventually, the suitable strategy of arranging the length of *top-gap* and *bottom-gap* we concluded from our model paves the way for designing the paraments of the toll plaza (θ and α).
The position and line style of the certain tolls haven't been determined yet.

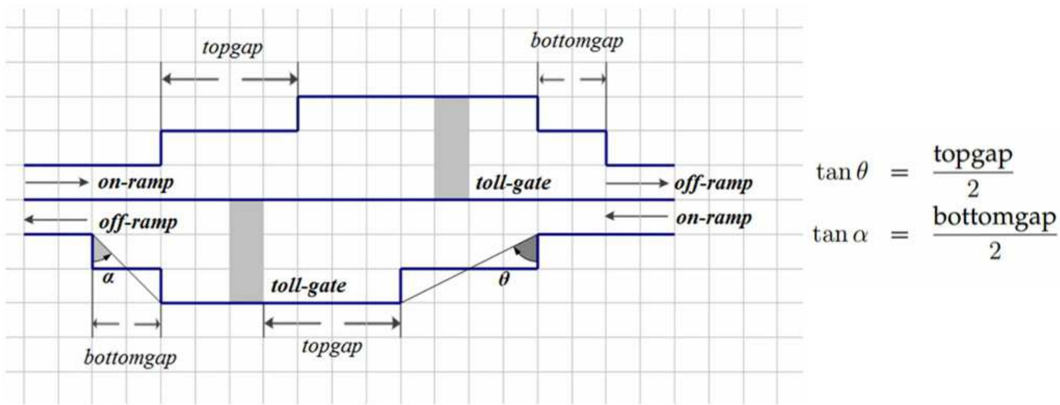

Figure 4: Angles

Considering the actual highway being two-way, we adjust our model and make the figure clear for calculation. The former model cost to much to fit two-way situation. We are convinced that this rotating symmetrical model can both low the cost and make the tolls artistic. The angles of the toll plaza are just dependent on the *top-gap* and *bottom-gap*, which is not relative with the numbers of tolls and lanes(B & L).

2 Modeling traffic with Cellular Automata

The aim of traffic-simulation-algorithms is to gain an understanding of road-traffic including its various phenomena. With the help of a proper simulation, we can make a prediction towards the development of real traffic situations and furthermore utilize the simulation to optimise traffic planning.

2.1 Overview

Looking back to the history, the first attempt to make important step forward was the Nagel-Schreckenberg model (simplified as 'NaSch model' or 'NS model') raised by Kai Nagel and Michael Schreckenberg. This model took into account the imperfect behaviour of human drivers and thus was the first model to explain the spontaneous formation of traffic jams. Cellular automata(CA) are models that are discrete in space, time and state variables. Due to the discreteness, CA are extremely efficient in implementations on a computer.[2] The NS model is a probabilistic cellular automata which contains a one-lane-road with discrete positions. And the state of each vehicle is determined by its velocity and position.

2.2 Introduction of NS model

The following rules are raised in NS Model.[1]

1. **Acceleration Rule.** The drivers always expect to drive at the maximum speed, if the n *th* car doesn't reach the maximum speed, its speed then increase 1: $\underline{v_n \to min(v_n + 1, v_{\max})}$
2. **Collision Prevention.** If the distance between the vehicle and the car ahead of it, d_n, is less than or equal to v_n, that is, the n *th* vehicle will collide if it doesn't slow down, so its velocity v_n is forced to be $d_n - 1$: $\underline{v_n \to min(v_n, d_n - 1)}$
3. **Random Deceleration.** Vehicles on the roads often decelerate for various kinds of reasons (cell phones, even drunk) and drivers occasionally make irrational choices. With these kind of existed probability , we draw $v_n \to v_n - 1$, presuming $v_n > 0$. Then we get: $\underline{v_n \to max(v_n - 1, 0)}$
4. **Position Updating.** We've emulated vehicle driving, knowing that if the velocity of the n *th* car is v_n, the position is x_n, then the position of next moment should be $\underline{x_n = x_n + v_n}$.

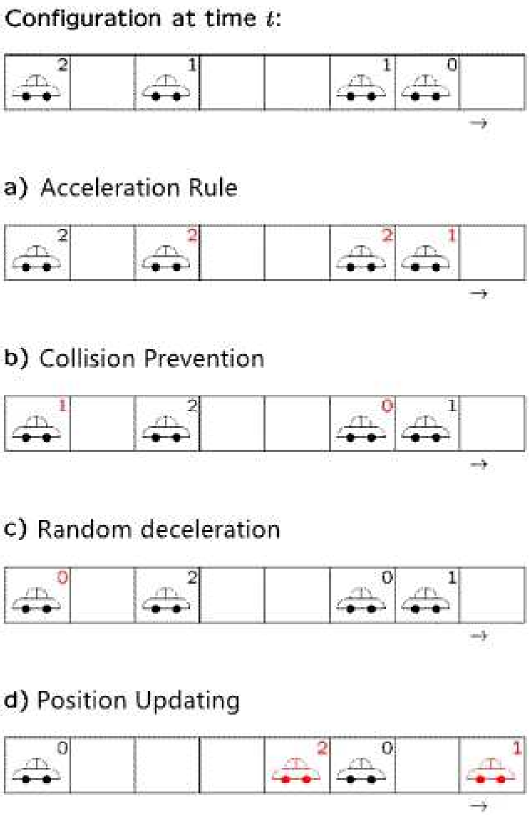

Figure 5: Rules of NS model

3 Model

In order to emulate the real traffic situations at the turnpikes, we structured the model basing on the classical *NS model*. Two rules have been proposed to perfect the model. Then comes the model we established according to the former assumption and basic *NS Cellular Automata*.

3.1 Behavior Principle

1. **Lane-Switching RULE**
   As for the driver, the probability to switch lane — p_s when there is a vehicle in front of him seems to be very high.
   At this time ,we set p_s to be 80%.

$$p_s = 80\%$$

   The probability of the driver being random first to the left or to the right first is equal.
   If he first switches to the left lane, then he must meet the ***lane-switching condition to the left*** – the left and the left rear must be no cars or obstacles.
   If the ***lane-switching condition to the left*** is not met but the ***lane-switching***

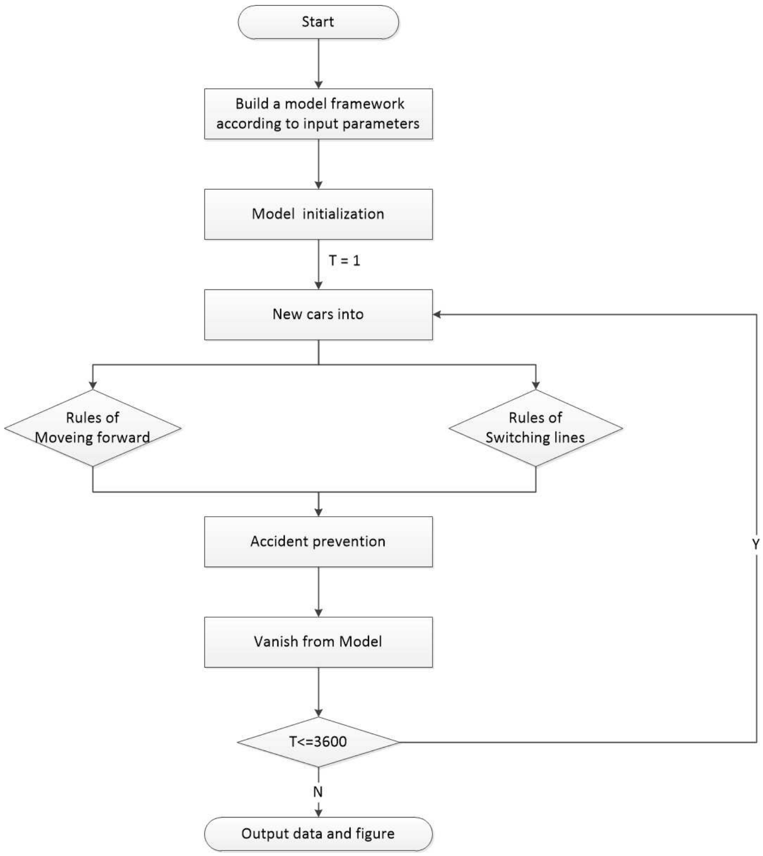

Figure 6: Model Initializing

condition to the right is satisfied - there must be no car or obstacle to the right and the right rear, he will switch lane to the right lane. More specifically, if the ***lane-switching condition to the left*** and the ***lane-switching condition to the right*** are not satisfied, the driver has to hold his velocity and waits for the next chance.
If he first switches to the right lane, the circumstance is the same as the above-mentioned first switching to the left method.

If the driver doesn't obey the ***Lane-Switching Rule*** strictly, he takes high risk to be involved into an **accident**.

2. **Moving Forward RULE**
We divide the implementation of this rule into three phases, *upstream of the tollbooth*, *in the tollbooth*, and *downstream of the tollbooth*.Then we introduce another parament — p_m which represent the probability of vehicles moving forward.
At this time we set p_m to be 70%.

$$p_m = 70\%$$

The probability p_m leads to the average velocity of vehicles v_n set to be 0.7grids/second

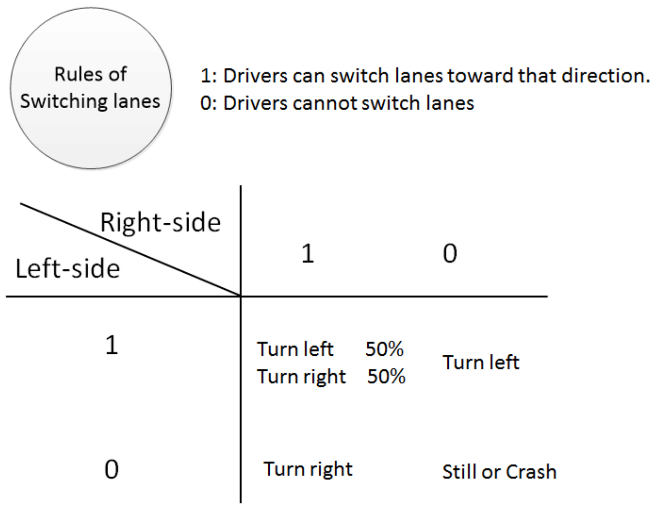

Figure 7: lane-switching

- If there is an obstacle (car or border) in front of the car at the *down-stream* or *upstream of the tollbooth*, there is a probability p_s to switch lane. In case that there is no obstacles, with 70% of the probability (p_m) the driver would move forward for a grid.
- When *in the toll station*, the definition of t_{delay} is the toll service time, when time accumulates to the t_{delay} the driver just drives out.
- When there is a toll on the left (or right) side of the vehicle, the vehicle cannot turn left (or right) and need to move forward or switch lane to its right (or left).

3.2 Further Study of Models

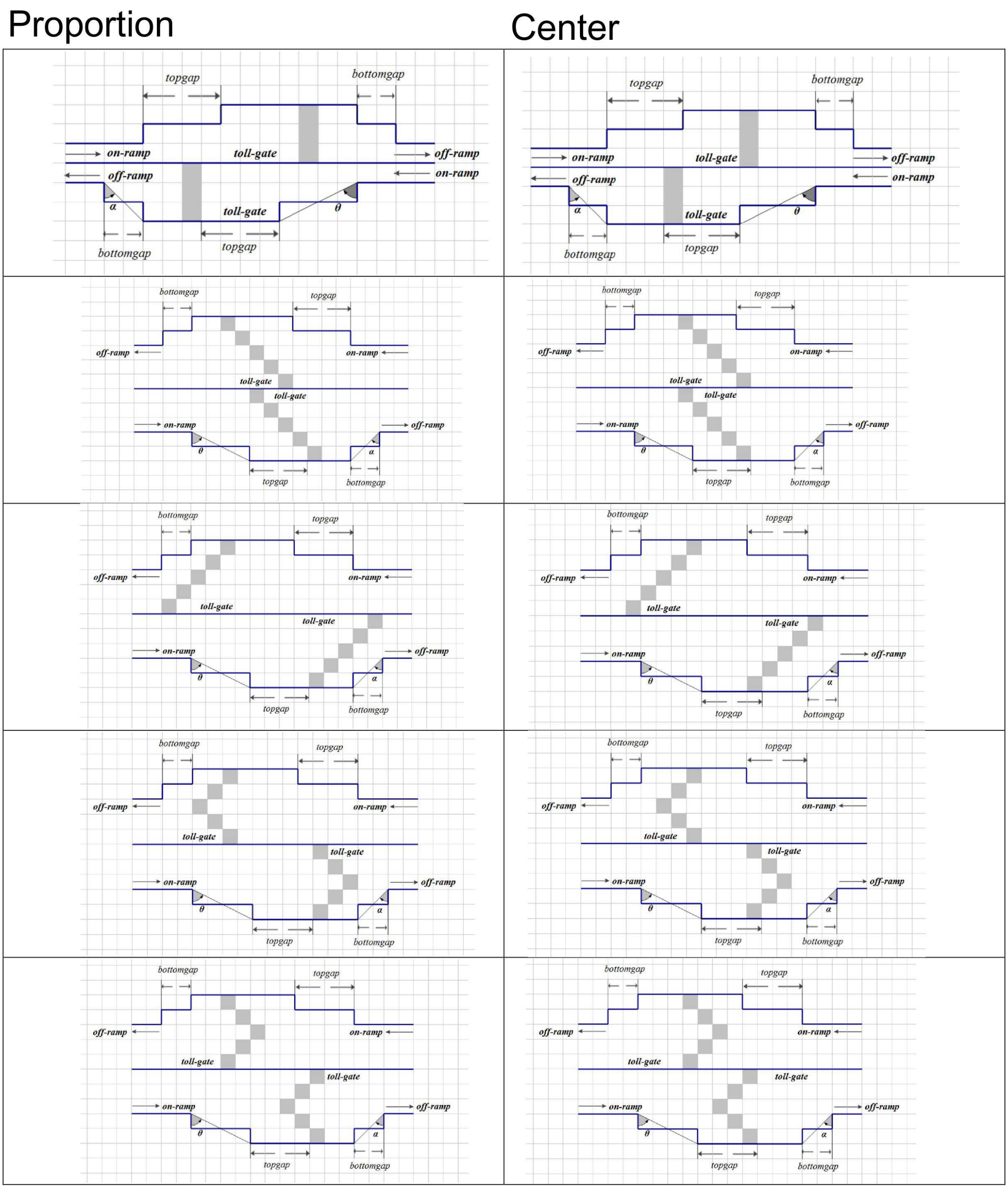

Figure 8: Prototypes

As can be seen from this table, we list a series of prototypes arranging from **Center 1 to 5** and **Proportion 1 to 5**. These two rows of models show kinds of toll position arranging. The proportion of empty plaza and shape of line style both influence the traffic efficiency of the turnpike design. There are steps to figure out which prototype hold the most efficient position.

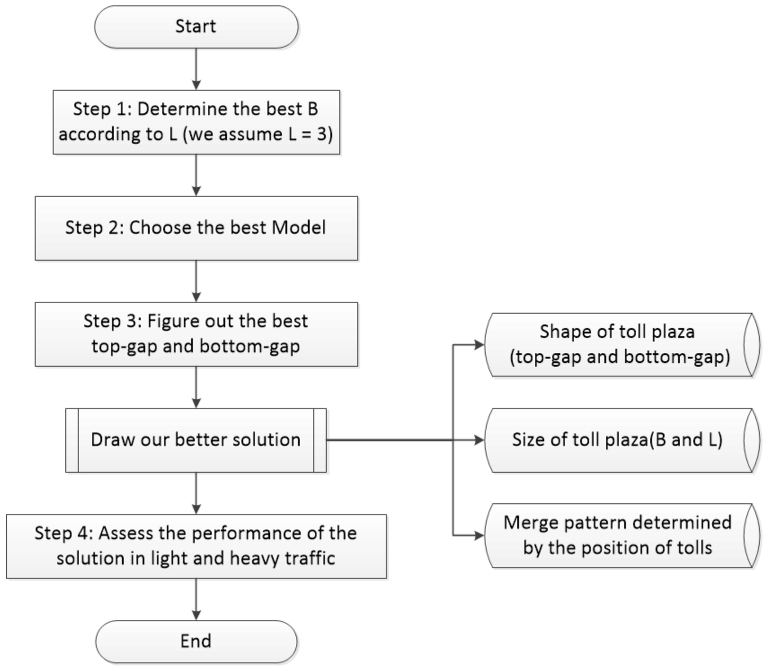

Figure 9: Flowchart

1. Firstly, we choose moderate throughput to emulate the reality traffic situations. Determine the number of tollbooths **B** according to the number of lanes **L** (We set **L** to be *3* temporarily.)

2. After **(B,L)** is fixed, take the position of tollbooths into consideration which can lead to the line style of tolls. By that we can determine the most efficient model.

3. After **(B,L)** is fixed, choose the best prototype we got in *Step 2* and try to determine the best values of **(topgap, bottomgap)**. This make it clear for us to calculate the actual paraments of the turnpike so that we can get the size of it.

4. Define the ultimate scheme and observe its performance under circumstances of throughput ranging from small to big.

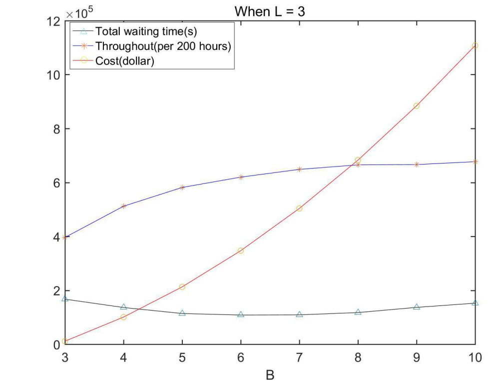

Figure 10: Performance of Different B

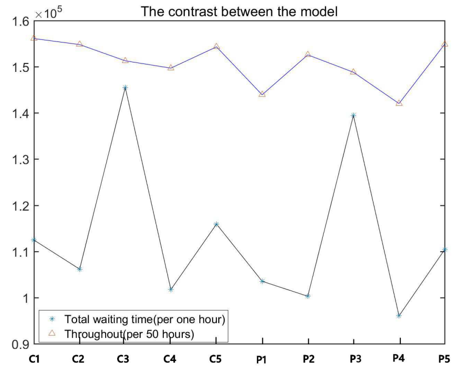

Figure 11: Contrast of Models

$$\begin{aligned} \textbf{C1} \sim \textbf{C5} &= \textbf{Center 1} \sim \textbf{5} \\ \textbf{P1} \sim \textbf{P5} &= \textbf{Proportion 1} \sim \textbf{5} \end{aligned}$$

Figure 12: *
(Description of Fig.7)

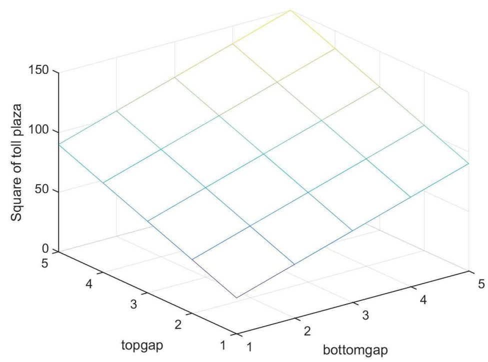

Figure 13: Difference in Size

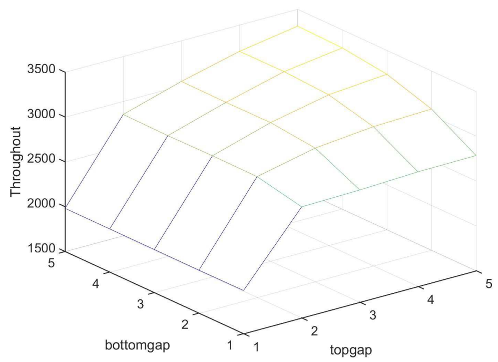

Figure 14: Difference in Throughput

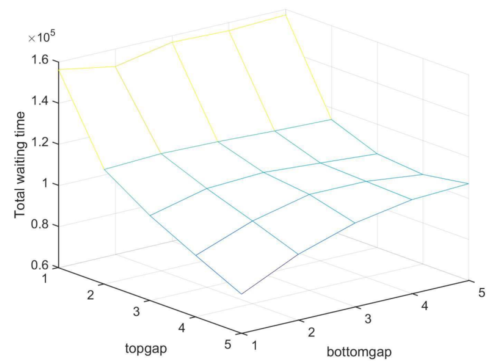

Figure 15: Difference in Waiting time

3.3 Established Model

- **Shape** The values of paraments — **(B,L) and** (θ, α)lead to the ultimate shape of the model.

$$\begin{aligned}\tan\theta &= \frac{\text{topgap}}{2} = \frac{5}{2}\\ \tan\alpha &= \frac{\text{bottomgap}}{2} = \frac{2}{2}\end{aligned}$$

  So we can get

$$\begin{aligned}(B, L) &= (6, 3)\\ (\theta, \alpha) &= (68.199°, 45°)\end{aligned}$$

- **Size** The calculation of size is according to the number of grids in our model.

$$Total\_Grid\_Number = (topgap + bottomgap)\sum_{i=L+1}^{B} i + B$$

  On the basis of the values we drew above, we can get

$$Total\_Grid\_Number = (5 + 2) \times 15 + 6 = 111$$

4 Important Considerations

4.1 Accident Prevention

Accidents only happen when the rules are not obeyed thoroughly, mostly because of violation against *Lane-Switching Rule*. The measure to low the accident rate is to artificially adjust the shape and size of the turnpikes which can low the rate of drivers' irrational behavior to a certain degree. We are convinced that risk of accident is directly dependent on the total waiting time, which means that the key to lower the accident rate is to decrease the total waiting time. Basing on the model we created, the change of total waiting time is a vivid result from changing paraments — **topgap** and **bottomgap**. Through this assumption, we stretch the figure showing the relationship between them.

4.2 Throughput

Through the emulation from *MatLab*, we can draw the change of throughput with the changing **topgap** and **bottomgap**.

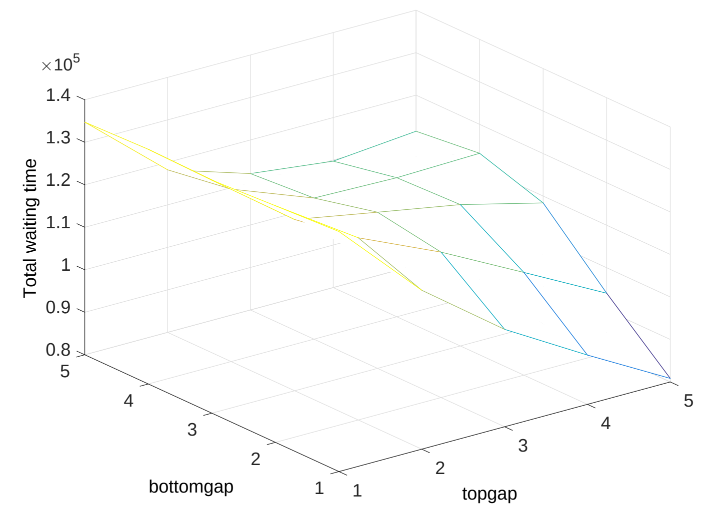

Figure 16: Total Waiting Time

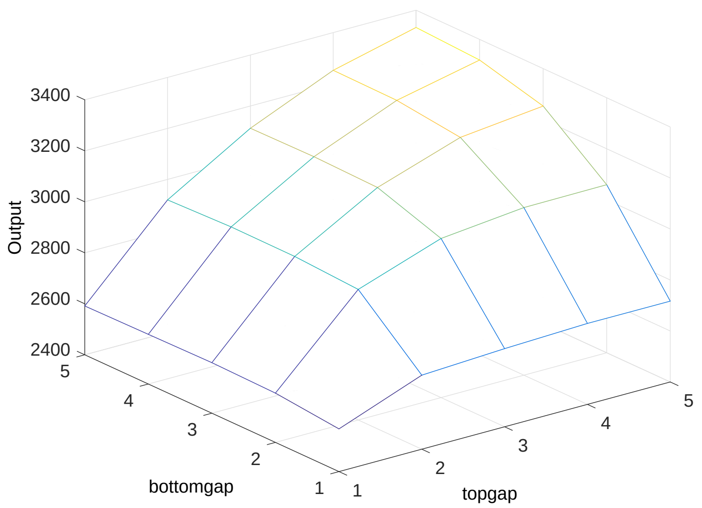

Figure 17: Throughput

4.3 Cost

When it comes to the cost, we find that it's hard to estimate the construction fee accurately. According to the information we know, we decide to take only land cost into account. That's to say, our cost estimation is just about the land cost, ignoring other forms of expenses. Due to that we have to minimize the area of our model. The relationship between the size of the plaza and **topgap** & **bottomgap** is shown in the figure.

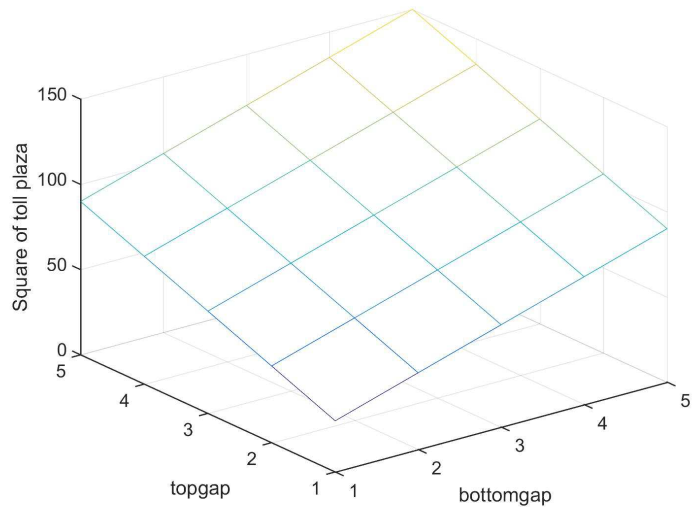

Figure 18: Size

4.4 Merging Pattern

As can be seen from the above three graphs, throughput increases with topgap and bottomgap, Total waiting time decreases as topgap and bottomgap decrease, but considering after the number of topgap and bottomgap all reach 3 there will be a rapid increase in cost. Integratedly considering, we choose the situation that topgap = 5, bottomgap = 2, at this time the cost is not too high; throughput is also close to the saturation value; Total waiting time is very small, that's to say, the road is not crowded.
Actually the vehicles' merging patterns basically obey the On-ramp & Off-ramp Rules, Moving Forward Rule and Lane-Switching Rule. The vehicle attempts to merge if its forward path is obstructed. The vehicle randomly chooses an intended direction, right or left. If that intended direction is blocked, the car moves in the other direction unless both directions are blocked (the car is surrounded).[3] This is what determine the merging pattern after tolls.

5 Performance Under Ranging Traffic Flow

In order to assess our model, we divide the traffic flow into three levels.

6 Introduction of Self-driving

As more autonomous vehicles added to the traffic mix, the traffic situation turns to be more and more complicated, which might increase the risk of traffic jams

Traffic flow level	Total_waiting_time	Throughput
1	24614	2662.1
2	91390	3068.2
3	94733	3074.2

Figure 19: Performance

and accidents. We tend to analyze the influence brought by self-driving car from these aspects.[4]

- **Imperfection of related laws**
- **Different moving pattern against human-driving vehicles**
- **Uncertainty of Acceleration & Deceleration RULES**
- **Lacking experience of handling accidents**

From these mentioned above, we know that the negative influence of self-driving car mostly lies in that they bring about uncertainty to the traffic when autonomous vehicles are mixed with normal traffic. We decide to make our focus on eliminating the uncertainty and thus ensuring the safety.

6.1 Our Solution

We try to lower the negative impact brought by self-driving as much as possible. It occurred to us that why not build a **Autonomous-Only Lane** which is similar to the idea of introducing *Bus-Only Lane* in big cities in Mainland China. The advantage is obvious that the authority would find it easy to manage the traffic of autonomous vehicles and eliminate its influence on the normal human-driving traffic. However, this kind of separating measure could bring about another problem — *Low Efficiency* sometimes when the autonomous vehicles are far less than estimated.
As the figure showed, our solution is to build an extra *Autonomous-Only Lane* at the middle or margin of the former established model.

Then we take two another factors into consideration.

- Extra addition of Emergency Lane
- Ability to reverse orientations of tollbooths for sake of imbalance efficiency in two-way lanes

Weighing these two kinds of models, we decide to choose ***Adjusted Model 2*** with the sacrifice of little more land cost. In which extra emergency lane can be added to the periphery of the whole model and the tollbooths in the middle can also be reversed to another orientation.

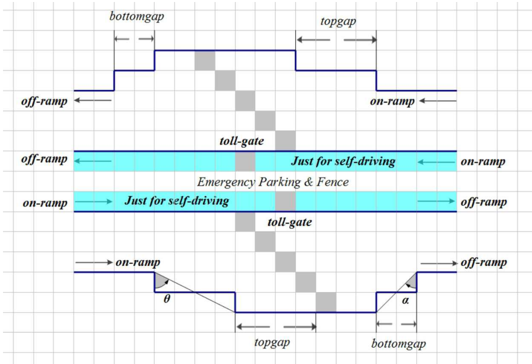

Figure 20: Adjusted Model 1

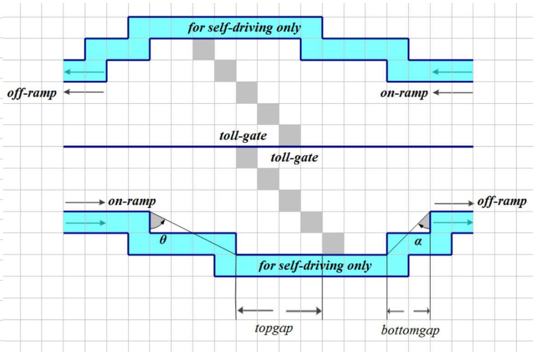

Figure 21: Adjusted Model 2

7 Proportion of Tollbooths

Nobody in New Jersey can avoid the traffic congestion on the Garden State Parkway, during the rush hours. As a matter of fact, it is common to experience long delays at a toll plaza even during non-rush hours.[5] The design of a toll plaza is clearly an important problem in any highway. Typically, the three types of toll collection methods mentioned in the problem are all existed in NJ system, they are

(a) Conventional (human-staffed) tollbooths
(b) Exact-change (automated) tollbooths
(c) Electronic toll collection booths

Our ultimate goal is to propose a scheme helping determine how would these three toll collection methods be mixed. The advantage of such a design should be that every vehicle has access and adjust to each type of toll booths. There are a number of important research issues in the toll plaza design problem. Some of them are

1. How many of each type of booth should we have
2. How should we layout and what should be the relative positions of the different types of toll payment methods
3. What is the traffic delay as a function of the numbers of different types of toll booths and layouts

7.1 The Best Proportion

According to the data in the news report, we set: Delay_human-staffed = 35s; Delay_ETC = 5s. Delay_Auto ranges from 15s $\sim$ 25s.
Adjust the proportion of the tollbooths under different levels of traffic flow, and finally we figure out the total waiting time and throughput in each situation.

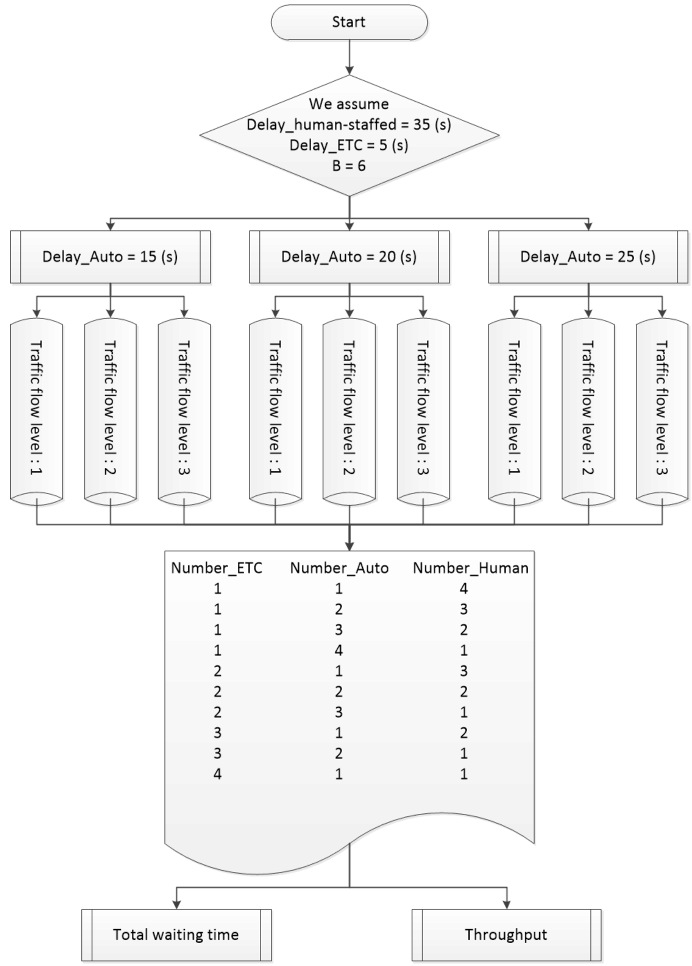

Figure 22: Flowchart

$$
\begin{aligned}
Delay\_human-staffed &= \text{The delay time of conventional tollbooths}\\
Delay\_ETC &= \text{The delay time of ellectronic toll collection booths}\\
Delay\_Auto &= \text{The delay time of exact-change tollbooths}
\end{aligned}
$$

7.2 Correlation Analysis

The charts above are got through correlations analysis. According to *Chart 1*, the total waiting time would gradually decrease with the increase of proportion of electronic tollbooths because the correlation is *-0.982*. However, in the relationship between proportion of conventional tollbooths and total waiting time the correlation is *0.628* while significance is *0.05*. It manifests that there is a significant correlation between them, while there is little correlation between exact-change tollbooths and total waiting time.
The same analyzing method is applied to *Chart 2*, we can draw the conclusion that there is a highly significant positive correlation with the throughput and the proportion of electronic tollbooths. While throughput and proportion of con-

Correlations

Control Variables			Electronic toll collection booths	Total waiting time	Conventional tollbooths	Exact-change tollbooths
-none-[a]	Electronic toll collection booths	Correlation	1.000	-.982	-.500	-.500
		Significance (2-tailed)	.	.000	.141	.141
		df	0	8	8	8
	Total waiting time	Correlation	-.982	1.000	.628	.354
		Significance (2-tailed)	.000	.	.052	.316
		df	8	0	8	8
	Conventional tollbooths	Correlation	-.500	.628	1.000	-.500
		Significance (2-tailed)	.141	.052	.	.141
		df	8	8	0	8
	Exact-change tollbooths	Correlation	-.500	.354	-.500	1.000
		Significance (2-tailed)	.141	.316	.141	.
		df	8	8	8	0
Conventional tollbooths & Exact-change tollbooths	Electronic toll collection booths	Correlation	1.000	.		
		Significance (2-tailed)	.	.		
		df	0	6		
	Total waiting time	Correlation	.	1.000		
		Significance (2-tailed)	.	.		
		df	6	0		

a. Cells contain zero-order (Pearson) correlations.

Figure 23: Chart 1

Correlations

Control Variables			Electronic toll collection booths	Throughput	Conventional tollbooths	Exact-change tollbooths
-none-[a]	Electronic toll collection booths	Correlation	1.000	.788	-.500	-.500
		Significance (2-tailed)	.	.007	.141	.141
		df	0	8	8	8
	Throughput	Correlation	.788	1.000	-.657	-.132
		Significance (2-tailed)	.007	.	.039	.717
		df	8	0	8	8
	Conventional tollbooths	Correlation	-.500	-.657	1.000	-.500
		Significance (2-tailed)	.141	.039	.	.141
		df	8	8	0	8
	Exact-change tollbooths	Correlation	-.500	-.132	-.500	1.000
		Significance (2-tailed)	.141	.717	.141	.
		df	8	8	8	0
Conventional tollbooths & Exact-change tollbooths	Electronic toll collection booths	Correlation	1.000	.		
		Significance (2-tailed)	.	.		
		df	0	6		
	Throughput	Correlation	.	1.000		
		Significance (2-tailed)	.	.		
		df	6	0		

a. Cells contain zero-order (Pearson) correlations.

Figure 24: Chart 2

ventional tollbooths are significantly negatively correlated, and the relationship between it and exact-change toll station is not obvious.

7.3 Sensitivity Analysis

Carry out the varying situations of total waiting time and throughput by changing traffic flow. Because the choose of value *(x,y,z)* doesn't influence the change of waiting time and throughput, make sensitivity analysis with *(x,y,z)* = (1,4,1)
According to Chart 1, it can be seen that the change of traffic flow by 50% makes

Traffic flow changes	Total waiting time	Throughput
-50%	17.84%	-1.65%
50%	0.94%	-0.29%

Figure 25: Chart 1

the waiting time significantly change by 17.84%, so it has higher sensitivity to waiting time and less sensitivity to throughput. Since the initial value of waiting time is 60304s, it can be estimated that the waiting time threshold is about 103315s, that is, the traffic flow will increase by about 71% and the waiting time will no longer increase.

And then control the traffic flow to be 2, and change the value of Delay to make sensitivity analysis:
According to Chart 2, range of delay in the range of 10% causes waiting time

Delay=20	Total waiting time	Throughput
-25%	39.41%	-1.34%
-10%	41.61%	3.15%
10%	45.57%	3.36%
25%	25.10%	0.50%

Figure 26: Chart 2

increases by about 40%, indicating its high sensitivity to the waiting time, that 1% change of delay will lead to 4% change of waiting time, while the change of throughput is too small, so the sensitivity is not high.

8 Conclusions

Assume that at the tollbooths electronic toll collection booths are in the middle with conventional tollbooths arranged at the edge of the turnpike. When it comes to the cost, exact-change tollbooths hold the cheapest position and the conventional tollbooths are the most expensive. At the time of *(x,y,z)* = (2,3,1), the throughput basically reaches the peak and the total waiting time is just normal. That's why we recommend to take *(x,y,z)* as (2,3,1).

GUI

9 Strengths and weaknesses

9.1 Strengths

- An overall consideration of aspects
- The introduction of Autonomous Vehicle Queue
- Correlation Analysis

9.2 Weaknesses

- Occasionality exists in Sensitivity Analysis
- The assumption of the same velocity is against reality

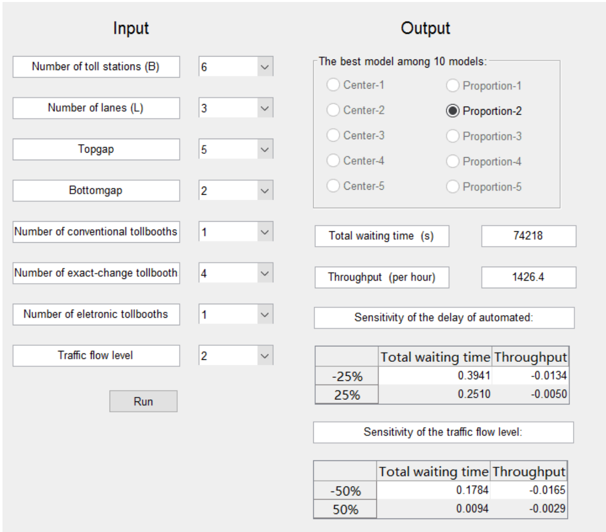

Figure 27: GUI

- The throughput and total waiting time set is average amount of 24hrs which don't reflect the ups and downs in a day

symbol	Significance
Ps	The probability of switching lane
Pm	The probability of verhicles moving forward
B	Number of tolls
L	Number of lanes
x	Electronic toll collection booths
y	Exact-change tollbooths
z	Conventional tollbooths

Figure 28: Symbols

References

[1] K. Nagel and M. Schreckenberg A cellular automaton model for freeway traffic, J. Phys. I France, 2221-2229 (1992)

[2] S. Wolfram, Theory and Applications of Cellular Automata, (World Scientific, 1986)

[3] Brian Camley, Bradley Klingenberg, Pascal Getreuer,For Whom the Booth Tolls, University of Colorado

[4] James M. Anderson, Nidhi Kalra, Karlyn Stanley, Paul Sorensen, Constantine Samaras, Oluwatobi Oluwatola ,Autonomous Vehicle Technology — A Guide for Policymakers

[5] Xiuli Chao, Design and Evaluation of Toll Plaza Systems, Department of Industrial and Manufacturing Engineering, New Jersey Institute of Technology