For office use only	Team Control Number	For office use only
T1 ________	**74076**	F1 ________
T2 ________		F2 ________
T3 ________	Problem Chosen	F3 ________
T4 ________	**D**	F4 ________

2018 MCM/ICM Summary Sheet

Out of Gas and Driving on Electric!

Summary

In order to achieve a complete electric vehicle (EV) transition, we present a comprehensive model to determine the amount and location of charging stations, and discuss task problems thoroughly.

First, we design the Quantity Determination Model to estimate the amount of charging stations to build in the given area and compute the total amounts of charging stations in the United States as well as their distributing proportion between urban, suburban, and rural areas. We also explore the network of Tesla charging stations and hold that Tesla is on track to switch to all-electric in the United States.

Second, we select Ireland as target nation. To solve Task 2a, we pick out 18 important city coordinates and select a set of 160 possible candidate points on the *Google Map*. Then, we use *CFLP*(Capacitive Facility Location Problem) Model to obtain the optimal number of chargers and placements on the roads between cities with sensitivity analysis, assuming that the transition to all-electric happens instantaneously. Similarly, we use *Two-level CFLP* to attain optimal number of chargers and placements in a simulated city with sensitivity analysis. To solve Task 2b, we design the Central Determination Algorithm to pick central cities and cluster nearby cities based on the coverage range to form city groups, in order to determine the order of constructions of charging stations in each city. To solve Task 2c, we introduce Electric Vehicle Growth Model to depict the relationship between the mounting population of EVs and charging stations. Three hypotheses are analyzed and we acquire the best function of hypothesis, ending up with a 12-year timeline for the full evolution of EVs in Ireland.

Last, we think that the previous Central Determined *CFLP* fails to apply to countries with very different geographies, population density and wealth distributions. Thus, we ameliorate the model by pre-sorting by population before choosing central cities, which adapts to all countries in Task 3. We believe a classification system that helps a nation to diesel vehicles to all EVs is possible. Furthermore, we consider that new technologies in Task 4 have positive impacts on the increasing use of EVs. For Task 5, we list key factors in a handout and give suggestions to national leaders.

Besides, the evaluation of our model system shows that strengths outweigh weaknesses. Our comprehensive model is adaptable to various conditions. *CFLP* can help solve similar locating problems. *Two-level CFLP* consists with the fact that both destination charging stations and supercharging stations are inside cities. Nonetheless, the model does not take consideration of the population change and economic development. We wish it could be improved in our further researches.

Keywords: Quantity Determination Model; *CFLP*; *Two-level CFLP*; Centra Determination Algorithm; Electric Vehicles Growth Model

Contents

1 Introduction

1.1 Background

Electric vehicles (EVs) are popular in many countries, and the adoption of EV gains active support from the governments. In order to support the development of EVs, government grants have been pledged with hundreds of bills and programs. In addition, the "Thirteen Five" emission reduction program authorized by the central government set a goal for renewable energy industry to achieve CNY 10 trillion market value in the year of 2020 with full support.

Unlike normal motor-vehicles, EVs are powered by electricity, a clean and renewable energy with nearly half the price of gasoline. As the company that have the biggest market share, Tesla provide two types of charging stations are in practice: destination charging and supercharging. The charging stations offer Tesla owners alternative choices to not only charge vehicles at home in a personal garage with power but also outside during a trip or at work. The destination charging stations are mostly distributed in areas with high population density, like west and east coasts and other inland big cities. The supercharging stations are mainly constructed along highways that connect cities and states with each other. It is easy to find out that only supercharging stations can meet the demand of longer road trip because supercharging does not need hours like destination charging.

1.2 Related Work

Charge station location planning problem[1] is similar to the gas station siting problem, substation site selecting problem and other siting problems. Most traditional methods use *integer programming*[Thompsongl,Walldl 1981] to solve substation planning problems. However, as the scale of problems increases, the computation time of integer programming methods increases exponentially and is only suitable for solving small-scale problems. In recent years, *expert systems, artificial intelligence* [YANG Lixi,WANG Jiayao 2003] methods have been used for charging station site, but the effect is not very satisfactory. Others have used computational Voronoi diagram to plan charging station locations. The method does not need to know the number of charging stations to be built, and can simultaneously calculate the number and location of charging stations of different kinds. However, this method also has drawbacks. Different levels of charging stations make the geometry into a weighted V map, the calculation is very large, and it requires high-precision data of the target area's demand distribution, which is quite difficult to measure. What's more, all of these methods failed to introduce some important factors affecting the EV charging station, especially the traffic factors, into the mathematical model of the charging station siting. In 2010, David P. Williamson proposed *CFLP method*, which was specifically designed to solve *capacitated facility location problem*. During the years, the method has shown a good adaptability.

1.3 Our work

The models mentioned above do provide several effective methods to deal with the locating problems. However, they can only solve the problem of siting one kind of item within a small area. We divide residents' demand of travel into two kinds: intercity travel and intracity travel.

Considering the features of each kind of demand, we expanded the basic CFLP to the Two-level CFLP, which allows to locate two different kinds of charging stations at the same time. We also calculate the optimal amount of charging stations intercity and intracity separately and determine the placement and distribution of the stations in Ireland to provide a complete electric vehicle promoting process for the Ireland government. We also built the growth model of the EVs and the charging stations to study the internal relation between them.

We find some useful points when we do research.

- The promotion of EV should start from affluent and densely populated areas.
- Imperfect EV infrastructure, such as the lack of charging stations, can limit the promotion of EV.
- The growth rate of EV can be described by normal distribution.

2 Fundamental Assumptions

- All the indicators for all vehicles are the same
- The charging device can be used normally at any time.
- One supercharge station's building cost is 20 times one destination charge station's.
- Charging stations can be built at any location without technical limitations
- The population of the target country or city remains unchanged, with no large-scale population movements among cities or countries.
- The density function of electric demand intercity subject to normal distribution.

3 Notation and Description

Notations	Description
k_l	The amount of l kind of charging stations
c_l	Building cost of each l kind of charging station
a_l	Charging efficiency of each l kind of charging station
t_l	Effective charging time of kind of charging stations per day
C	Collection of all urban center points
R	Collection of all candidate points along the highway
q_i	Electricity demand at point i
D_i	Electricity powered by destination charging stations at i
S_i	Electricity powered by supercharging stations at i
P_{income}	Average income affects the effective charging time of i charging stations per day
$P_{population}$	Income affects the effective charging time of i charging stations per day
p_i^I	Per capita income of candidate point i
p_i^P	Population of candidate point i
x_ij	Decision variable associated with availability of station j to point i
d_{ij}	Distance between point i and point j
z_j	decision variable associated with building station j
r	Coverage radius
A	The total acreage of the target area
B	The charging station building budget
E	Energy consumption per kilometer

4 Methods and Objectives

In order to help the government achieve a complete EV transition, we need to create a comprehensive model to help determine the amount and location of charging stations in a given area or even a country. Also, the model should give several complete planning processes for different regions and countries. Helping them finding the most suitable process to migrate all their personal passenger vehicles to all-electric vehicles[2].
The Basic model can be divided into the following sub-models:

- Quantity Determine Model: This model is designed to estimate the amount of charge stations need to be built in the given area.
- Site Selection Model Between Cities: In order to locate the SCS along the highway for long road trips use.
- Site Selection Model Inside City: This model designed to figure out the best amount and location of charging stations to be built .
- Evaluation Model: Model designed to evaluate each station's contribution to the whole charging station network and help arrange the construction order of the stations.
- Electric Vehicle Growth Model: This model is based on a biological population growth model. It provides a proposal for evolving the charging network from zero chargers to a full electric-vehicle system to the target area government.

4.1 Quantity Determination Model

In this section, we develop the basic model to calculate the amount of charging stations needed in a given area.
From the problem description, there are two main types of public charging stations: destination charging stations(DCS) and supercharging stations(SCS). Destination charging stations are mainly accumulating around urban area, while supercharging stations are normally located along the highway. As everyone switched to all-electric personal passenger vehicles, there are mainly three constraints for the amount of charging stations:

- 1) Total demand constraints: The total charging station network daily power supply should be able to cover the daily electricity demand.

  The amount of destination charging stations and supercharging stations are k_1 and k_2, respectively. The charging efficiency of destination charging stations and supercharging stations are a_1 and a_2.t_1 and t_2 are the effective charging time per day of the two kinds of stations. To estimate the effective charging time of DCS, take the peak of the demand as the supply capacity of the charging station. The proportion of effective charging time per day of DCS equals to the real output proportion of the supply capacity, with which the station charging the EVs. Also, the nearby area's per capita income, which affect the frequency of using personal vehicle, and the area's population density, which affect the vehicle density, are important factors. t_1 can be calculated by the following formula:

$$t_1 = \sum_{t=1}^{24} \frac{p_t}{max\{p_1, p_2, ..., p_{24}\}} P_{income} P_{population}$$

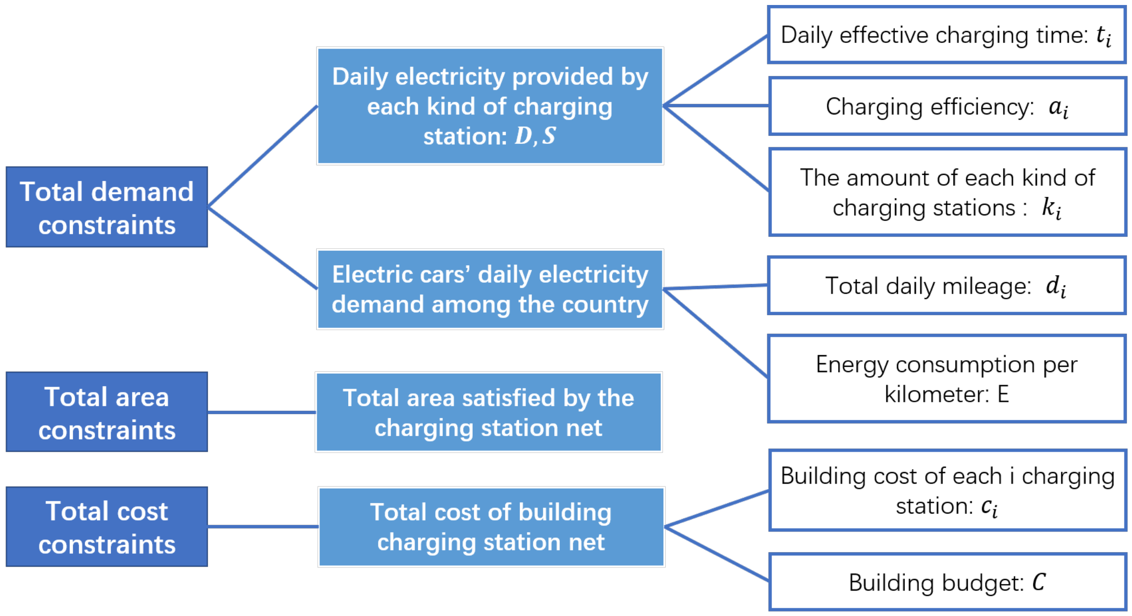


Figure 1: Quantity Determination Model

$$P_{income} = \sum_{i=1}^{m} \frac{p_i^I}{\sum_{i=1}^{m} p_i^I} \times \frac{p_i^I}{max\{p_1^I, p_2^I, ..., p_{24}^I\}}$$

$$P_{population} = \sum_{i=1}^{m} \frac{p_i^P}{\sum_{i=1}^{m} p_i^P} \times \frac{maxp_i^P}{max\{p_1^P, p_2^P, ..., p_{24}^P\}}$$

m is the amount of regions in the given area.

Then we can calculate the daily electricity provided by destination charging station D and the daily electricity provided by destination charging station S in the charging network:

$$D = k_1 t_1 a_1, \quad S = k_2 t_2 a_2$$

E is the energy consumption per kilometer and Q^* is the total demand of electricity. The complete constraint inequality is:

$$D + S = k_1 t_1 a_1 + k_2 t_2 a_2 \geq Q^*$$

$$D + S = k_1 \sum_{t=1}^{24} \frac{p_t}{max\{p_1, p_2, ..., p_24\}} a_1 + 24 k_2 a_2 \geq Q^*$$

- 2) Total area constraints: The coverage of all charging stations should be larger than the target land area.
  A circle with the station as center point and r as radius is the coverage area of the station. Take the smallest travel range that can be provided by one charge as the radius. The coverage of all charging stations should be larger than the U.S. land area. The formula is expressed as:

  $$\pi r^2 (k_1 + k_2) \geq A$$

  A is the total land area of the given area.

- 3) Total cost constraints: The total building cost of the charging station net should be both controllable and affordable. To build the charging station network,would cost the company and government a large amount of money. Also, the building cost of each kinds of station is quite different. With the given building budget of the stations, B, we can get the constraint formula:

$$\sum_{i=1}^{2} c_i k_i \leq B$$

The decision variables k_1 and k_2d will have different combinations as the budget changes.

The objective function is to minimize the amount of all the charging stations $k_1 + k_2$, subject to the three constraints mentioned above. The basic model shows as follow:

$$Min \qquad k_1 + k_2$$

$s.t.$

$$D + S \geq Q^*$$

$$\sum_{i=1}^{2} c_i k_i \leq B$$

$$(k_1 + k_2)\pi r^2 \geq A$$

Q^* is the total demand of electricity of the target area.

4.2 Intercity with *CFLP* Model

In this model, we seek the minimum cost of building stations and the cost of seeking for charging stations, while meeting the electric demand of each road.
In order to guarantee the long road trips between cities, we need to build adequate SCD at suitable locations to charge the EVs. Use the proportion of city i ($i \in C$,)population in the total population of the country as i's proportion of the area's electric demand. As stated in the assumptions, although there are inflow and outflow of population, city's population dynamically balance. In this case, we can take city i's highway traveling demand of electric as the total demand in the vicinity. The total charging capacity $S_j z_j$, of the charging stations available to city i should be adequate to cover i's demand $\sum_{i \in C} q_i x_{ij}$.

$$\sum_{i \in C} q_i x_{ij} \leq S_j z_j, \forall j \in R$$

The basic model shows as follow:

$$Min \qquad \sum_{j \in R} c_2 z_j + \sum_{i \in C} \sum_{j \in R} \theta d_{ij} x_{ij}$$

$s.t.$

$$\sum_{j \in R} x_{ij} = 1, \forall i \in C$$

$$\sum_{i \in C} q_i x_{ij} \leq S_j z_j, \forall j \in R$$

$$x_{ij}, z_j \in 0, 1$$

θ is the cost per kilometer.

4.3 Intracity with Two-level *CFLP* Model

Given the candidate points inside city, this model can output the location and amount of both DCS and SCS in city.

$$Min \quad \sum_{l=1,2}\sum_{j_l\in C_l} c_l z_{j_l} + \sum_{i\in C_0}\sum_{j_1\in C_1} c_{ij_1}x_{ij_1} + \sum_{i\in C_0}\sum_{j_2\in C_2} c_{ij_2}x_{ij_2}$$

$s.t.$

$$\sum_{j_1\in C_1} x_{ij_1} = 1, \forall i \in C_0$$

$$\sum_{j_2\in C_2} x_{ij_2} = 1, \forall i \in C_0$$

$$\sum_{i\in C_0} q_{1i}x_{ij_1} \le D_{j_1}z_{j_1}, \forall j_1 \in C_1$$

$$\sum_{i\in C_0} q_{2i}x_{ij_2} \le S_{j_2}z_{j_2}, \forall j_2 \in C_2$$

$$x_{ij_1}, x_{ij_2}, z_{j_1}, z_{j_2} \in \{0,1\}$$

C_0 is the collection of cities. C_1 is the collection of DCS's candidate points.C_2 is the collection of SCS's candidate points.

4.4 Central Determination Algorithm

This model is design to estimate the value of locating the $NO.k$ charging station at candidate point j . One charging station has two aspects of value: the demand covering value, which satisfy the demand in the covering range, and the network value, which work with the existing charging stations to enable EVs to travel long-distance.

According to the law of diminishing marginal utility, the value function is monotonically decreasing. Assume it matches the negative exponential distribution of e. The basic model shows as follow:

$$V_j^k = e^{-k}\sum_{i\in C} x_{ij}p_i^P + \alpha\sum_{t\in O} y_{tj}\sum_{i\in C} x_{it}p_i^P$$

4.5 Electric Vehicle Growth Model

In order to present a proposal of the whole charging station network evolving process, build Electric Vehicle Growth Model to describe the relationship between the mounting popularity of electric vehicles and charging stations. According to product life cycle theory, the quantitative growth model for electric vehicles is similar to the growth model for biological stocks. EVs growth model can be described as:

$$\frac{ds}{dt} = \alpha s(1 - \frac{s}{k})$$

$$s = \frac{ks_0e^{\alpha t}}{k + s_0(e^{\alpha t} - 1)}$$

s is the current amount of EVs, and s_0 is the initial value of it . α is the controlling variable. k is the maximum amount of EVs under the conditions of time t. β is the best ratio between the amount of EVs and the amount of charging stations, and y is the current amount of charging stations. Then k can be expressed as:

$$k = \beta y$$

5 Calculating and Simplifying the Model

5.1 Model Solution for Task 1

In February 7th 2018, SpaceX launched a rocket with a Tesla Roadster on it to the space, which is a milestone for both Tesla Motors and Elon Musk, the CEO of SpaceX and Tesla, Inc. Elon won attention by successful marketing. Moreover, he proved once again to the world that solar city, Tesla electric vehicle, Falcon Heavy, those anti-traditional innovative ideas and inventions are doable, and are, like it or not, changing the world. Though charging stations are still not enough for everyone to use if the all-electric era just came overnight, the charging network has already covered the whole country. With more stations to build in the future, the current electric vehicle network will be extending simultaneously as the number of electric vehicles increases. Therefore, it is safe to say that Tesla is on track to allow a complete switch to all-electric in the United States.

In order to figure out whether US can completely switch to all-electric personal passenger vehicles, firstly we need to find out how many charging stations would be needed and how much would they cost.
For t_1, as the DCS are mainly located near the urban area, use the statistics from a survey ($https : //sites.hks.harvard.edu/hepg/Papers/2010Braz_Aubrey_HEPG_Feb2010.pdf$), which was conducted in New York City about the distribution of the charging demand of the electric vehicles during a day. Charging demand distribution during a day is shown in Figure 1. We can get the sum of the charging

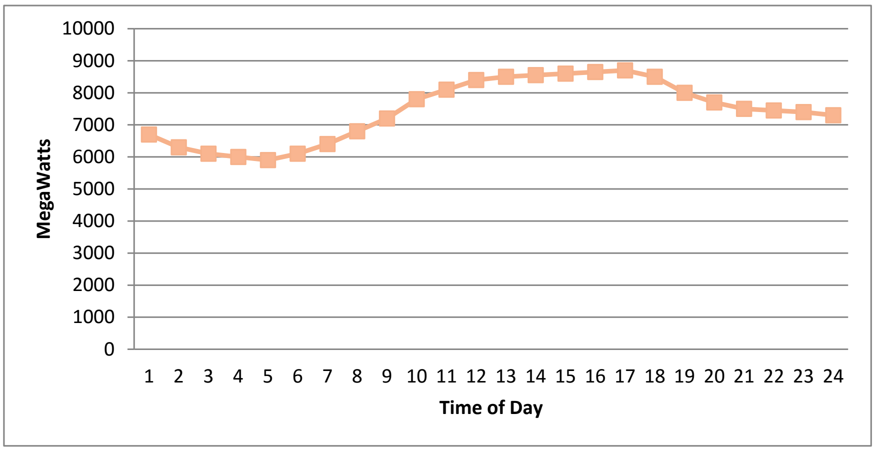


Figure 2: Average MegaWatts at each hour

demand of the electricity during a day:

$$t_1 = \sum_{t=1}^{24} \frac{p_t}{p_1 7} P_{income} P_{population} = 20.5344 P_{income} P_{population}$$

Referring to statistical data, we get the population density and per capita annual income of the 3133 regions in the United States. $m = 3133$, use the following formula to calculate P_{income} and $P_{population}$.

$$P_{income} = \sum_{i=1}^{m} \frac{p_i^I}{\sum_{i=1}^{m} p_i^I} \times \frac{p_i^I}{max\{p_1^I, p_2^I, ..., p_{24}^I\}} = 0.39925$$

$$P_{population} = \sum_{i=1}^{m} \frac{p_i^P}{\sum_{i=1}^{m} p_i^P} \times \frac{max p_i^P}{max\{p_1^P, p_2^P, ..., p_{24}^P\}} = 0.90145$$

$$t_1 = 20.5344 P_{income} P_{population} = 7.3904$$

For t_2, as the SCSs are located along the highway and the volume of traffic is relatively stable, the charging demand is easy to estimate. In this case, the amount of SCSs' effective charging time per day can be assumed as $t_2 = 24$. According to the problem description, the SCS is designed for longer road trips to provide up to 170 miles of range in as little as 30 minutes of charging, the distance between any two charging stations should be no more than 85 miles.$a_2 = 340E(W/h)$, $r = 85(mile)$. Tesla announces that the DCS can provided the vehicle with up to 80 ranges in 1 hour of charging, which indicates $a_1 = 80E(W/h)$

$$D + S = 7.3904 \times 80Ek_1 + 24 \times 340Ek_2$$

Referring to the U.S. Department of Transportation Statistic data base, the total vehicle travel is 2710556 million miles and the total land area of the United States $A_a = 9629091(km^2)$ (2014).

$$D + S = 591.232Ek_1 + 8160Ek_2 \geq Q^* = 2710556000000E$$

$$85^2(k_1 + k_2)\pi \geq 9629091$$

According to the information released by Tesla, the building cost of one SCD, c_2,is 17.5-20 times that of one DCS, c_1. In order to simplify the calculation, set $c_1 = 10000$ and $c_2 = 200000$.

$$10000k_1 + 200000k_2 \leq B$$

In theory, every point on the map of the United States can be the candidate locating points for charging stations. However, to build a simplified model, set the collection of intersections of highways as candidated points and concentrate the population of nearby cities, suburbs to the point.

$$Min \qquad k_1 + k_2$$

$$subject \quad to : 591.232Ek_1 + 8160Ek_2 \geq Q^* = 2710556000000E$$

$$85^2(k_1 + k_2)\pi \geq 9629091$$

$$10000k_1 + 200000k_2 \leq B$$

Finding the best combinations of k_1 and k_2 under different budget limitations: B. The results are shown in the following table:

Table 1: K1, K2

Cost/per Des-Charge	K1	K2	K1 + K2
14000000	13508871	24556	13533427
15000000	11282020	185897	11467917
16000000	9055259	347237	9402496
17000000	6828453	508577	7337030
18000000	4601646	669918	5271564
19000000	2374840	831258	3206098
19500000	1261438	911928	2173366
20000000	148035	992598	1140633
20500000	0	1003323	1003323

To describe the distribution of stations between urban, suburban and rural area, firstly we need to divide the 3133 regions into the three categories. Referring to the survey conducted by Chicago

Council Survey (2016), the current proportion of cities, suburbs and villages in the United States is: 32%,53%,15%. Take the per capita income of the region as the standard for regional classification. Then take each category's population share of the country as the category's personal vehicle share. There suppose to be 32.2% charge stations in urban areas, 57.4% in suburban area and 10.4% in the rural area.The result shows in the following table:

Table 2: Distribution among cities

	Population Distribution	Charges Distribution
urban	0.29874095	0.322347542
rural	0.152823266	0.103826342
suburban	0.548435785	0.573826116

5.2 Model Solution for Task 2

We chose Ireland as the target nation.

5.2.1 Solution and Sensitivity analysis for Task 2a

In task 2a, assuming there is no transition time required, we need to determine the optimal number, placement, and distribution of charging stations in Ireland when it has converted to all-electric vehicles.To accomplish this task, we need to collect the basic data of Ireland (city layout, road network structure, ect.) and put it into CFLP and Two-level CFLP to get the optimal number, placement, and distribution of charging stations. However, we can not find the exact demand of each city nor the demand distribution rules. Because of that, we provide intercity locating problem and intracity locating problem each with one effective model. We also test our models with several pair of data and present the results as references.
There are mainly two steps for this task:

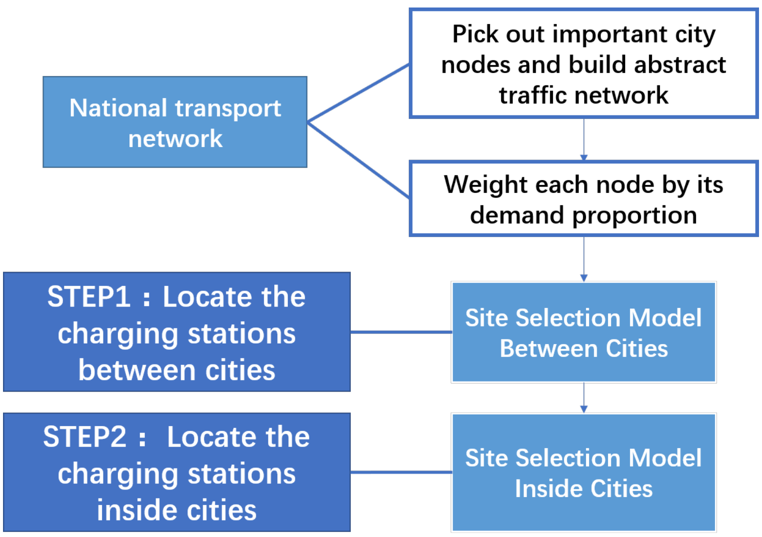


Figure 3: Total model includes CFLP and Two-level CFLP

Firstly, we need to determine the amount and locations of charging stations between cities, where only SCD should be used. Get the road network and the length of each section of the road from the public data on wikipedia. Then select a set of all possible candidate points for charging stations on the map.The selection of points follows the two rules below:

- All points are on the freeway.
- The distance between any two adjacent points does not exceed the maximum single-form mileage of the vehicle.

Thus, we have the coordinates of 18 big cities and 160 candidate points along the road. The initial network and distance matrixs are as follow. (Red points represent the cities and green points represent the candidate points.)

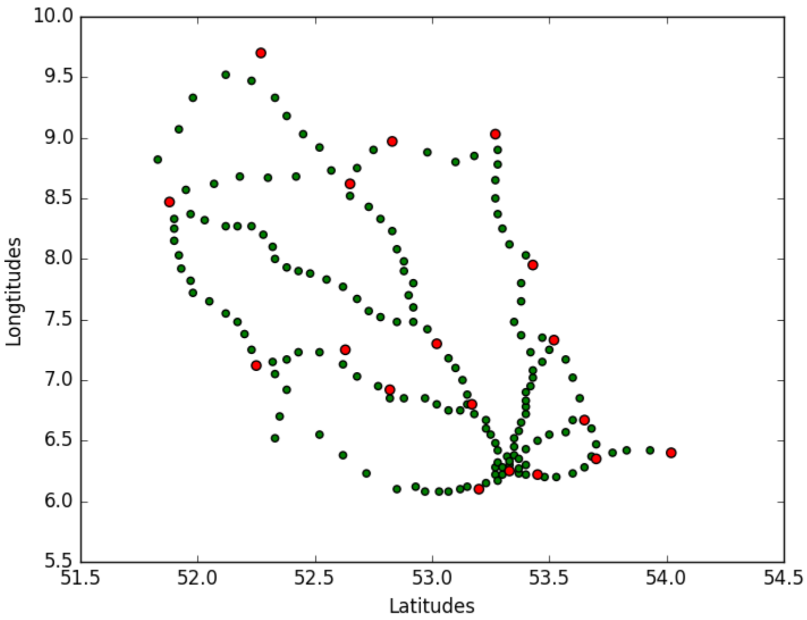


Figure 4: The initial network

Contune to take $c_2 = 200000$ and $q_i = \sum_{i \in C} \frac{p_i^P}{Min_{i \in C}\{p_i^P\}} n$. Control variable n is the proportion of distance traveled by electric vehicles.

$$Min \qquad \sum_{j \in R} 200000 z_j + \sum_{i \in C} \sum_{j \in R} \theta d_{ij} x_{ij}$$

$s.t.$

$$\sum_{j \in R} x_{ij} = 1, \forall i \in C$$

$$\sum_{i \in C} \frac{p_i^P}{Min_{i \in C}\{p_i^P\}} n x_{ij} \le S_j z_j, \forall j \in R$$

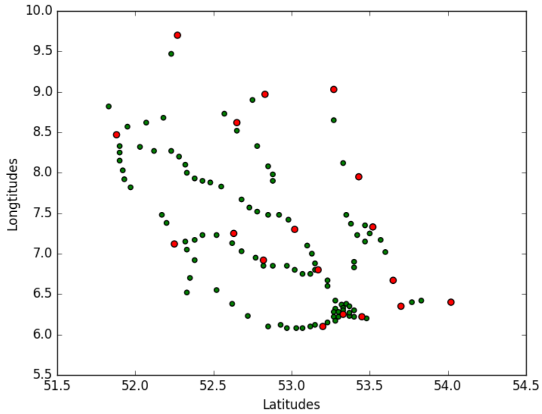
$$x_{ij}, z_j \in 0, 1$$

Complete the data in Site Selection Model Between Cities. When $n = 0.1 - 1$, the results are as follow:

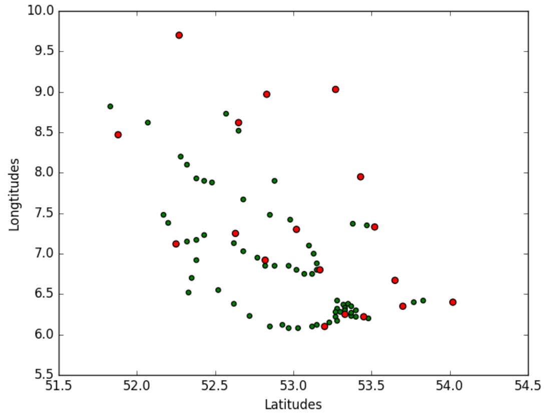
As the Ireland only use all-electric vehicles, $n = 100\%$.The result shows that 101 charging stations should be built along the freeway. In order to cover the demand,each charging station contains 12 SCDs. The distribution of the stations when $n = 60\%, 100\%$ are shown in Figure 6.

Table 3: Number of charges areas

n	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2
number of charges areas	23	30	38	46	54	64	73	81	91	101	109	118



(a) n=100%



(b) n=60%

Figure 5: Distribution of intracity charging stations

Total change figures are in ***First Appendix***

The key factor that lead to this distribution is the various demand of intracity traveling among cities. Due to the different demand, the cities have different weight in the network, which makes charging piles gathered around the cities with higher demand.

For the travel inside cities, take the city group near $(N52°, W8°)$ for an example. As it is unable to get each intercity point's specific electrical demand, we assume the density function of demand subject to normal distribution, which decrease from the city center to the edge. Abscissa of the point $x \sim N(\mu_1, \sigma_1)$,$\mu_1 = 52, \sigma_1 = 0.5$. Ordinate of the point $y \sim N(\mu_2, \sigma_2), \mu_2 = 8, \sigma_2 = 0.2$.Randomly select 200 points as the collection of demand points C_0, 80 points as C_1 and 40 points as C_2:

$$C_0 = \{(x, y)|x \sim N(52, 0.5), y \sim N(8, 0.2)\}, |C_0| = 200;$$

$$C_1 = \{(x, y)|x \sim N(52, 0.5), y \sim N(8, 0.2)\}, |C_1| = 80;$$

$$C_2 = \{(x, y)|x \sim N(52, 0.5), y \sim N(8, 0.2)\}, |C_2| = 40.$$

Figure 8(a) shows the initial distribution of C_0(red dots), C_1(green dots) and C_2(blue dots).

θd_{ij_1} is the cost of driving from point i to point $j_1(i \in C_0,j_1 \in C_1)$, and so is θd_{ij_2}. Continue to take $c_1 = 10000, \quad c_2 = 200000$, $D_{j_1} = 80E$, $S_{j_2} = 340E$. $q_1 i = \frac{p_i^P}{Min_{i \in C_0}\{p_i^P\}} n_1$, and $q_2 i = \frac{p_i^P}{Min_{i \in C_0}\{p_i^P\}} n_2$. Control variable n_1 is the average demand of DCS in the city and n_2 is that of SCS. Bring the data into the model:

$$Min \qquad \sum_{l=1,2} \sum_{j_l \in C_l} c_l z_{j_l} + \sum_{i \in C_0} \sum_{j_1 \in C_1} \theta d_{ij_1} x_{ij_1} + \sum_{i \in C_0} \sum_{j_2 \in C_2} \theta d_{ij_2} x_{ij_2}$$

$s.t.$

$$\sum_{j_1 \in C_1} x_{ij_1} = 1, \forall i \in C_0$$

$$\sum_{j_2 \in C_2} x_{ij_2} = 1, \forall i \in C_0$$

$$\sum_{i \in C_0} \frac{p_i^P}{Min_{i \in C_0}\{p_i^P\}} n_1 x_{ij_1} \leq 80 E z_{j_1}, \forall j_1 \in C_1$$

$$\sum_{i \in C_0} \frac{p_i^P}{Min_{i \in C_0}\{p_i^P\}} n_2 x_{ij_2} \leq 340 E z_{j_2}, \forall j_2 \in C_2$$

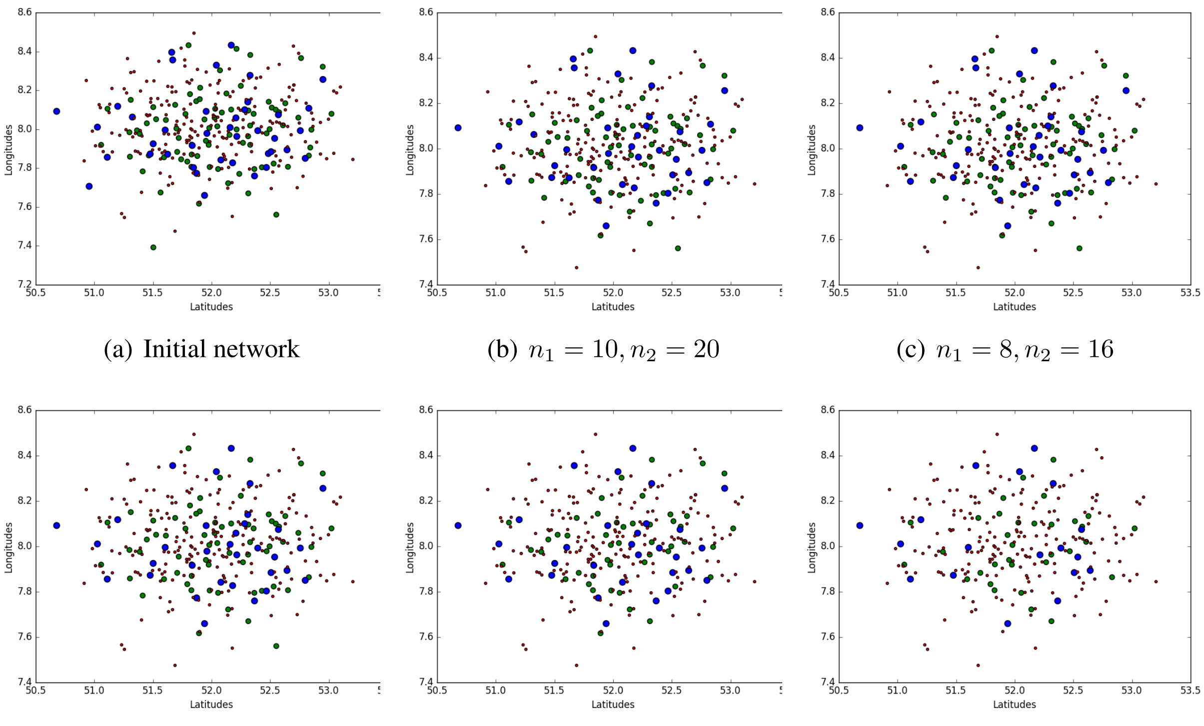
$$x_{ij_1}, x_{ij_2}, z_{j_1}, z_{j_2} \in \{0, 1\}$$

According to Irish official data, we can infer that the ratio of demand for DCS and SCS is about 1: 2, which means $n_1 : n_2 = 1 : 2$. Under different total demand limitations, the results are as follow:

Table 4: n1, n2

(n1,n2)	DCS	SCS
(10,20)	71	37
(8,16)	69	34
(6,12)	67	33
(4,8)	49	30
(2,4)	33	18

The placement of the charging stations is shown in Figure 8:



(a) Initial network (b) $n_1 = 10, n_2 = 20$ (c) $n_1 = 8, n_2 = 16$

(d) $n_1 = 6, n_2 = 12$ (e) $n_1 = 4, n_2 = 8$ (f) $n_1 = 2, n_2 = 4$

Figure 6: Distribution of the charging stations

As we cannot get the precise electricity demand of each city and the more people live in a city, the more electricity the city need, use the city's population[3] to represent its demand. Referring to the population data released on wikipedia, we can have the distribution of charging stations among

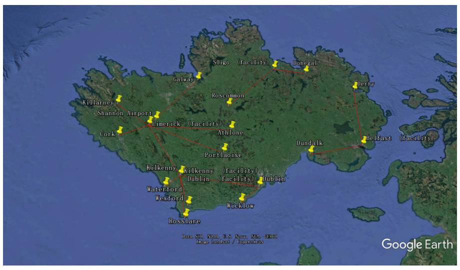
Table 5: Cities Distribution

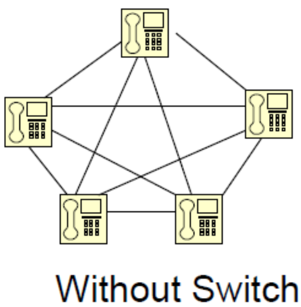
city	proportion	city	proportion	city	proportion
Dublin	0.593030103	Swords	0.019839467	Carlow	0.01226925
Cork	0.105480066	Dundalk	0.019716127	Tralee	0.011975561
Limerick	0.047613102	Bray	0.01647897	Newbridge	0.011495851
Galway	0.040405827	Navan	0.015252146	Portlaoise	0.011146052
Waterford	0.02704573	Kilkenny	0.013401548	Athlone	0.010791703
Drogheda	0.020702843	Ennis	0.012776762	Mullingar	0.010578892

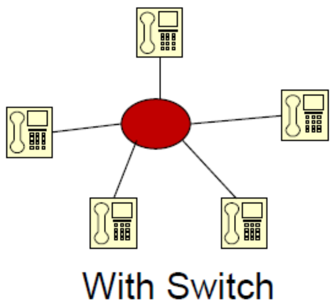
the 18 cities as follow:
In summary, given the precise data, the comprehensive model consist of CFLP and Two-level CFLP model can give the amount and distribution of two kinds of charging stations all around the country.

5.2.2 Solution and Sensitivity analysis for Task 2b

According to Task 2b, we are supposed to present a proposal for evolving the charging network from zero chargers to a full electric-vehicle system. In order to figure out the best charging station building order, we need to evaluate the each candidate point's value each time a new charging station is going to be built.
When developing a value evaluation model, we can compare the traffic network with *Computer Network*, which developed fast and thoroughly in latest years. In *Computer Network*, Internet-working is one of the most significant technologies for network expansion and utilization. Because of the huge quantity of computers, there are mainly two assignment criteria for evaluating the network architecture: *connectivity and* expansibility. Analogously, we will build a traffic network evaluating model.
To begin with the location of Center cities, we develop an coverage-based algorithm to divide different city groups among Ireland, finding the best charging location where has maximize neighbour nodes in its coverage region. Next, we can expand our charging station outward to form a star network, like this:







Take advantages of Star network can save unnecessary expenses for city group.
Then we consider the connectivity among these charging locations on the basis of highway distribution, rather than a huge network that include everything.

Since this coverage-based algorithm mainly focuses on city groups, it is natural to consider building all city-based chargers first. The advantage is that newly-built charging stations can serve more potential Tesla owners in the cities than in the rural area, for the population density and average income of the cities heavily outnumbered those of the rural area. As a result, the newly-built stations will also be more frequently used by Tesla owners in the cities than in the rural area. Moreover, the average distance between an EV and the newly-built station is less in the cities than in the rural area because of the difference of population density. Consequently, a potential Tesla owner will spend less

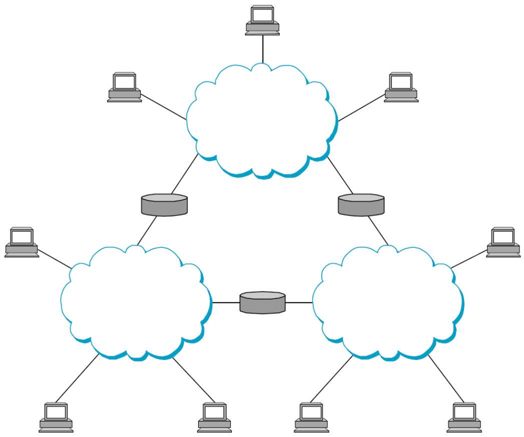


Figure 7: Network Architecture

time and waste less electricity on the way to get his vehicle recharged in the cities than in the rural area. To sum up, the pros of building stations in the city first outweigh the cons.
For the "EV first or station first" quandary, Ramteen Sioshansi, an associate professor from Ohio State University, has given the answers, quoted, "having a one-year view, you may end up making very poor decisions if you're that myopic". He backed up the decision to build charging infrastructure first because consumers can be reticent to jump on board with new technology. Only when the stations are built and everything is ready, people may have enough confidence in purchasing EVs. If Tesla has enough capital to build EV network and Ireland government determines to back up, it is absolutely wise to invest in infrastructures first. The number of EVs will increase as a response.

5.2.3 Solution and Analysis for Task 2c

For this task, we need to figure out the relationship between the amount of EVs and the amount of charging stations. Since Ireland has announced that only EVs can be sold in the country after 2030[4], we can assume that by that time the transition has been completed, which means $t \in [0, 12]$. As we can see from the EVs Growth Model[5], the amount of EVs (s) is limited by the number of charging stations(y). Therefore, we need to establish a suitable model to describe the charging station's growth process. Here are several possible hypothesis about y:

- **Hypothesis 1:** y is a linear function of t: $y = \omega t$
- **Hypothesis 2:** y is a exponential function of t: $y = e^t$
- **Hypothesis 3:** The density function of y follows a normal distribution:

$$y = \int_0^t f \mathrm{d}x$$

$$f(t) = \begin{cases} \frac{1}{\sigma_s \sqrt[2]{2\pi}} e^{\frac{(t-\mu_s)^2}{2\sigma_s^2}} & \mu_s - 6 < x \leq 12 \\ \frac{1}{\sigma_s \sqrt[2]{2\pi}} e^{\frac{(t-\mu_s)^2}{2\sigma_s^2}} & 0 < x \leq \mu_s - 6 \end{cases}$$

In order to figure out which hypothesis is the most feasible, use the Quantity Determine Model to calculate the final value of y, which is the amount of charging stations in a full electric-vehicle system. As we assumed the amount of the country's vehicle will not change, with 4773 thousand population, 9437 million km traveling distance, the total amount of charging stations is $s_{max} = 22038268$. Use the maximum value of S and the range of t to fit the function expressed in each hypothesis.

the results are shown in **Second Appendix**.

To evaluate the 3 functions, set the function with a constant $y = y_max$ as the control group. Set $\alpha = 1$.

$$s = \frac{ks_0e^t}{k + s_0(e^t - 1)}$$

The image of the four groups of functions and their promotion schedules are as follows:

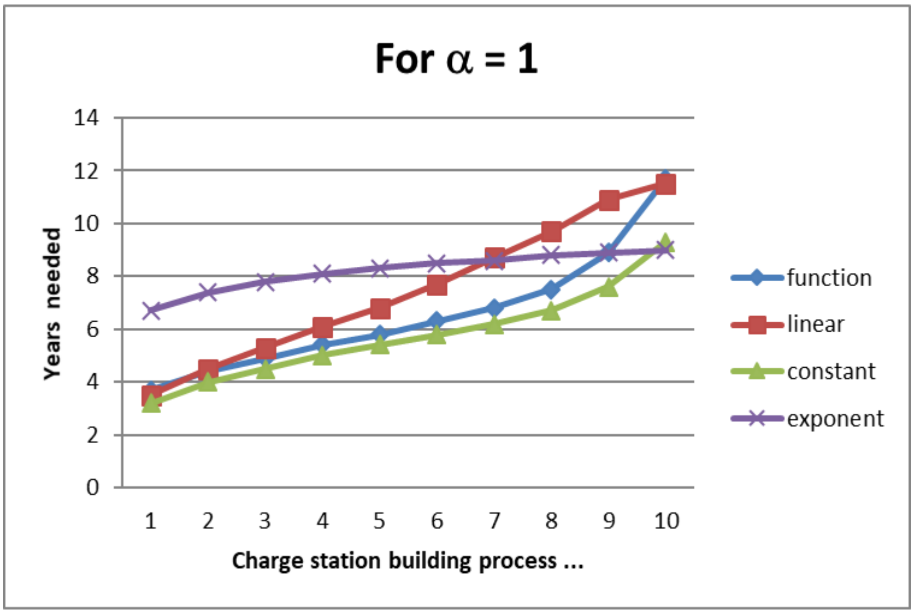


Figure 8: Function images comparison

Table 6: Evolving schedules

Charge Station Building ...	function	linear	Constant	Exponent
10%	3.7	3.5	3.2	6.7
20%	4.4	4.5	4	7.4
30%	4.9	5.3	4.5	7.8
40%	5.4	6.1	5	8.1
50%	5.8	6.8	5.4	8.3
60%	6.3	7.7	5.8	8.5
70%	6.8	8.7	6.2	8.6
80%	7.5	9.7	6.7	8.8
90%	8.9	10.9	7.6	8.9
98%	11.7	11.5	9.3	9

As the Figure 10-13 shows, the function of **Hypothesis 3** provide the best description of the growth

of EVs while the other functions deviate from the reference function in vary degrees.
Also, the model shows that it will take Ireland 3.7 years for there to be 10% electric vehicles, 4.9 years for 30% electric vehicles, 5.8 years for 50% electric vehicles and 11.7 years for 100% electric vehicles[3].

5.3 Model Solution for Task 3

The countries listed in the task have very different geographies, population density distributions and wealth distributions. For instance, in both Australia and China, most of the populations and wealth are distributed on the East Coast; moreover, Middle China also live a considerable number of people, while the West China happens to be a vast territory with a sparse population. In Indonesia, most people are living on the island of Java where the capital Jakarta locates. Besides Java, several other islands are also populated with quite a few people. However, some of the cities are divided by the sea, making the intercity land transportation impossible. Saudi Arabia, locating on a desert, has a few population centers. Beyond these centers, only few people live in the desert. Singapore is a highly developed city state with zero demand for intercity transportation within the country.

After analyzing each nation's case, we draw the conclusion that our proposed plan in the case of Ireland for growing and evolving the network of chargers cannot apply to most of the countries above. A counter example is that, in China, Eastern China is much richer and more highly populated than Western China. Applying the model in Task 2b would result in building charging stations simultaneously in Eastern China and Western China, which contradicts the initial idea that we should build stations in more populated and richer cities first. The reason is that Eastern China and Western China each has biggest cities in its own independent area that could become a central city in a group. Thus, a middle-sized central city in Western China might be prior to a larger-sized non-central city in Eastern China.

To apply our model to all nations, some changes to the previous CD-CFLP model in Task 2b are necessary. The procedure of the new algorithm is as follows:

- 1. Sort all cities by a rank A based on the multiplication of population and wealth.
- 2. Descend from the TOP 1 city to choose the central city and sort it in a new rank B. Cluster cities in the circle of this central city as the center of the circle to a group. Please remind to skip the city that is already in a group when picking the central city by the rank A.
- 3. Build charging stations inside city in the TOP 1 city of rank B. Then, build charging stations between cities on the highways (if no highway, then national road) connected to the city. Afterwards, build charging stations in the nearby cities which have been connected with the central city by highways in the group.
- 4. Build charging stations inside city in the TOP 2 city of rank B. Then copy the procedure 3 and for loop until there remains no cities unused in rank B.
- 5. Connect the groups with each other by building stations on the roads that enables communications between them. (like the concept of connected map in graph theory)

In this way, it is able to build stations in an order from the richest and most populated city as well as nearby cities in the same group to the less rich and less populated city as well as cities in the same group and so on. It prevents the situation that happens in the counter example without hurting other properties. Whatever uneven the population density and wealth distribution could be in a country, this new algorithm works.

Nonetheless, city state may not be well operated in our model, because it has no intercity transportation as well as groups and connectivity. To tackle this problem, an additional model (revised version) which is even simpler should be built. For Singapore, a typical city state, only charging stations inside the city and their placements need to be determined by the algorithm, whereas the intercity part of the model is totally superfluous and should be abandoned. This is a proper model for city state.

As for the feasibility of creating a classification system, we believe it is doable. The nations could be classified by their wealth, namely, developed country and developing country, which represents their purchase capability; by their population distribution, namely, unbalanced population distribution and balanced population distribution; by their geographic, namely, continental country and island country, which indicates their intercity land transportation ability. Other ways of classification such as education level could also be considered when creating such a system. We believe that each country should develop its own plan based on the condition of each country. Blind obedience is not encouraged because there remain countries that are still underdeveloped. All in all, a classification system is always feasible.

5.4 Comment for Task 4

The technological world continues to change and is impacting transportation options such as car-share and ride-share services, self-driving cars, rapid battery-swap stations for electric cars, and even flying cars and a Hyperloop. Comment on how these technologies might impact your analyses of the increasing use of electric vehicles.

A major benefit of car-share and ride-hailing services is that it is easier for those without cars to get around and for households to reduce the number of cars they own as well as their maintenance. Since apps such as Plugshare and ChargePoint provide locations about the public charging stations, ride-share driver can find an EV charging station (even the slower destination charging type) to charge vehicles during the time waiting for the next ride, saving hundreds of dollars on gas cost daily. Moreover, car-sharing companies like BlueLA and BlueIndy place 24/7 affordable shared electric vehicles in low-income neighborhoods, allowing everyone to use electric vehicles despite his/her affordability. Consequently, car-share and ride-share facilitate the adoption of electric vehicles by lowering the cost on fuel and allowing low-income community to afford. The increasing use of electric vehicles will even be more rapid than we thought.

Self-driving technology will also help boost the development of electric vehicles. Believe it or not, autonomous future is likely to be an electric one. First, due to regulatory reasons, gasoline vehicles have gas mileage requirement, whereas electric vehicles use unconstrained clean electricity power. Second, electric vehicles are easier for computers to drive. Third, it is cheaper to charge an unmanned car than to gas it up. Furthermore, self-driving vehicles will more likely populate urban areas first, so will the electric vehicles because of higher population density and affordability. Thus, self-driving technology and electric vehicles are probably complementary commodities, which means the development of either self-driving application and electric vehicles will accelerate one another, faster than we may previously thought.

Rapid battery-swap stations are alternatives for EV owners to get a brand new battery to power their vehicles and save the time on recharging. It is a great solution, however, has encountered bottlenecks. According to Tesla owners who have experienced Tesla's battery swap program, the battery exchange took seven minutes on average. It is indeed faster than recharge, which should have resulted in positive feedbacks. Nonetheless, the Tesla battery swap program looks quite closed for the time being. The program is just too costly to run. The good news, however, is that the battery-swap mode has been widely applied to electric buses in cities such as Beijing in China to avoid taking the buses off the road for hours to recharge during the working day. It is a proof in different countries that battery swapping are feasible provided that they are operated efficiently. It remains uncertain whether or not it will accelerate the increasing use of EVs, mainly depending on the operation status in the specific society.

The flying car is a type of vehicle that provides transportation by both ground and air, and may require enough space to take off. Some prototypes have already managed to fly and excited the market with flying colors, but it still takes time for widespread use. Although most of the prototypes are not electricity-powered, companies including Lilium are working on electric flying car that takes off vertically for saving the space. Backed by strong investors, it will not only be a dream in the near future. At present, Hyperloop is still under researched and unreachable for electric vehicles. We could only imagine that such an amazing invention will blazing shorter the travel time and change people's attitude toward as well as strengthen their confidence in neoteric high-tech products. In other word, the invention of flying cars and Hyperloop may overcome a big obstacle to the adoption of electric vehicles.

5.5 Handout for Task 5

The key factors for our all-electric future

Thank you for your concern about the energy crisis and environmental issues. Your presence in the summit show the remarkable responsibility of your country and your people. We will be honored if you may consider our counsel for the development of EV in your country. It would greatly help relieve the energy crisis.

As you know, gasoline vehicle causes air pollution, green-house effect, and overexploitation of oil and gas. In order to ameliorate the environment and embrace the new energy future, you may help the country adopt electric vehicles (EVs), which produce zero emission, to replace gasoline vehicles.

At present, companies such as Tesla Motors are working to build EV networks in the aim of providing more complete and convenient service to personal EV owners. The transition of gas vehicles to EVs needs time. To migrate personal transportation towards all-electric cars, the governments are responsible to guide and incent the people to support the changes. Moreover, a gas vehicle-ban date should be set in order to stimulate the whole country to adopt EV positively.

There are some key factors that impact on the development of EV network. We list these factors below and give suggestions for your consideration.

- First, we highly recommend you to allow the constructions of EV charging infrastructure before the promotion of EV. With charging stations, it is easier for EVs to find a place to charge in public. Moreover, it is vital to overcome the reticence of people to new and high-tech inventions. People's recognition and purchase intention to EV will grow as the number of infrastructures increases.

- Second, we suggest you to focus on the construction of EV infrastructure in the richest and most populated cities first. Cities have stronger economic strength to afford EVs and more people to use the charging stations than rural villages. Afterwards, you may further consider building infrastructures on the highway, allowing long travels and connectivity between these cities.

- Third, we advise you to balance the number of two types of charging stations. Destination charging and Supercharging are two ways to charge EVs. D is slower charging, while S is faster charging. For intercity travel, S is appropriate because travel takes time and S will not cost drivers much time. Inside cities, EV owners can charge vehicles while at work or sleeping at night, so we may build D and S as well.

- Fourth, we propose that you consider the particular geographic, wealth distribution, population density and other conditions of your country while growing the EV network. For isolated small island or city state countries, it may be superfluous to consider intercity transportation. For countries with unbalanced population density, the ultimate distribution of EVs and charging stations with in the country is mainly determined by population. For countries with unbalanced wealth distribution, it is wiser to consider building more charging stations in rich area, especially at the beginning of EV adoption. Other factors such as education level and economic power may also influence other factors such as the purchase intentions and capability.

Thank you again for your time and consideration. We have no doubt that your country will embrace an all-EV future with your leadership and foresight.

6 Conclusions

6.1 Strengths and weaknesses

6.1.1 Strengths

Our model system is adaptable to many conditions. Not only can it help determine the optimal number, placement and distribution of charging stations, but also it can present the suitable all-electric vehicle promoting process for a small community, a city or even a country.

- CFLP model can help solve many kinds of locating problems.
- The Two-level CFLP Model considers to build both destination charging stations and super-charging stations inside cities, which is consistent with the facts.
- The relational model between EVs and charging stations can be used to describe the relationship between complementary products in the market.

6.1.2 Weaknesses

- In Electric Vehicle Growth Model, we choose the optimal function among 3 hypotheses. There might be a function that can describe the growth of electric vehicles better.
- Our model take no consideration of the population change and economic development, which reduces the accuracy of the model and limits its application in a long period of time.

6.2 Further research

- In the future, we will include the population changes and economic development into consideration and build a more complete model system.
- The question whether or not can local economy benefit from the promotion of electric vehicles is also worth studying.

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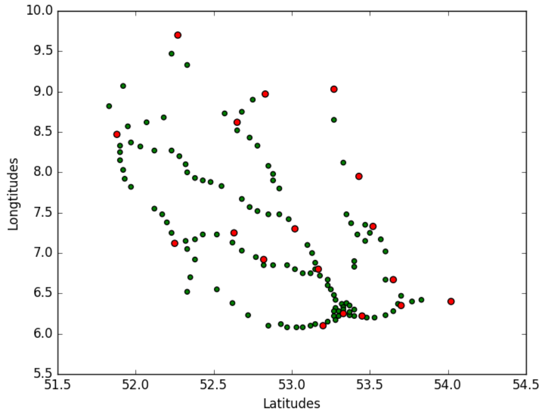
Appendices

Appendix A First appendix



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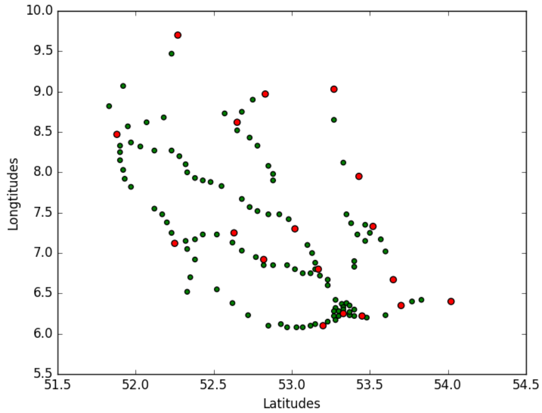

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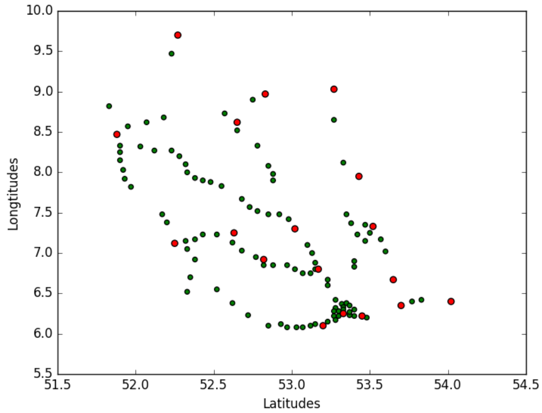

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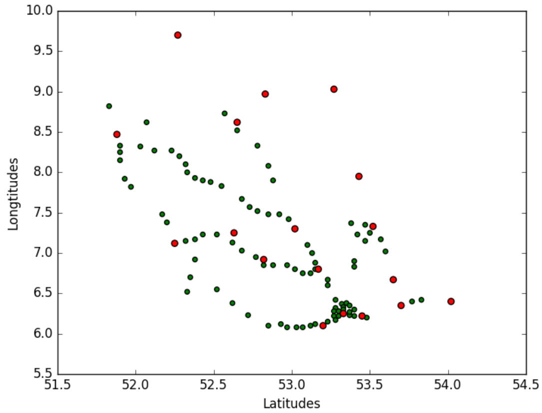

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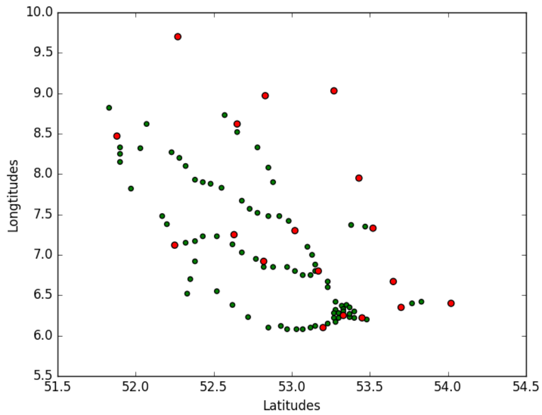

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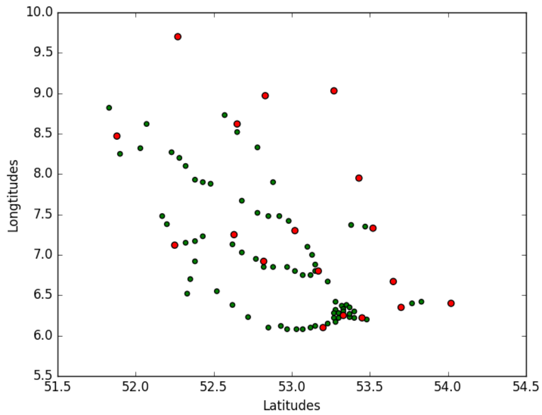

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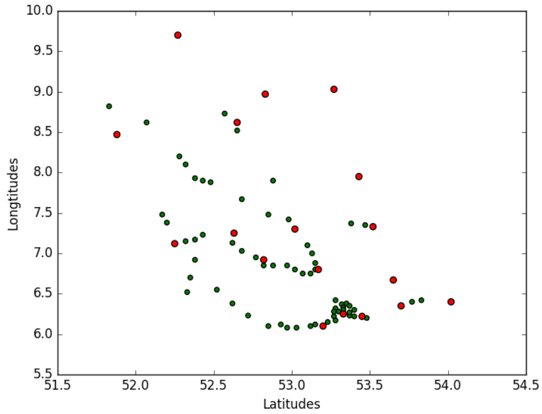

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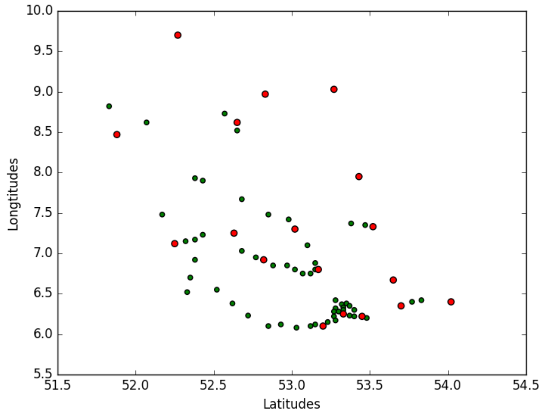

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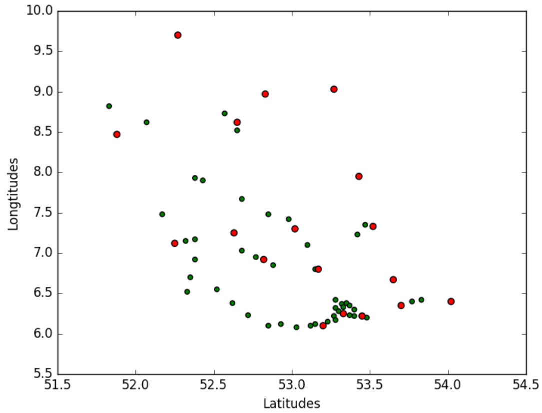

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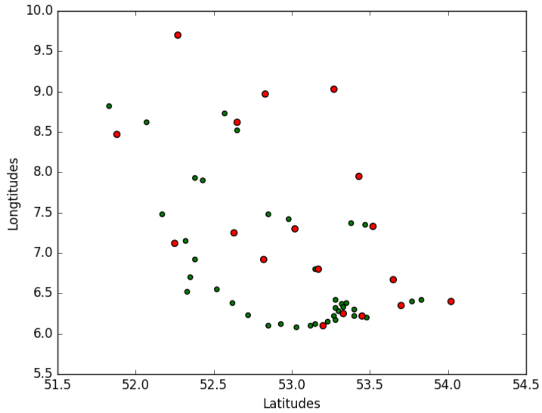

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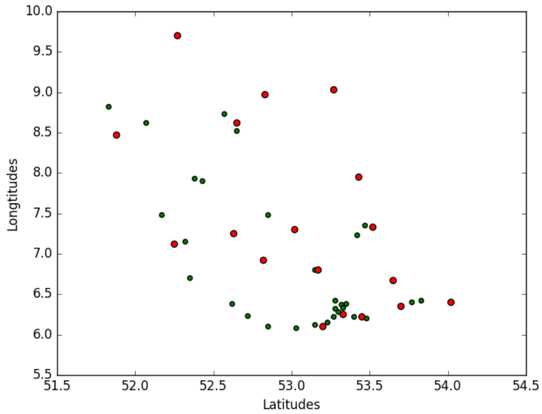
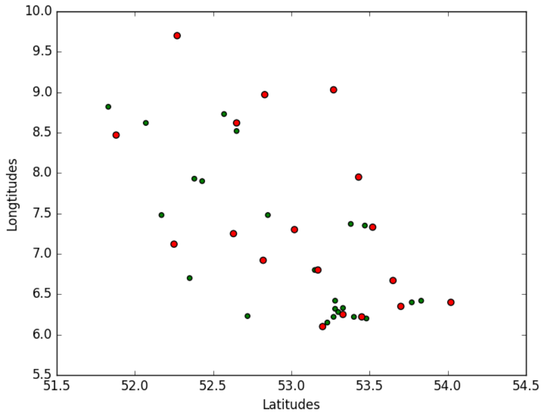


(k) n=20%



(l) n=10%

Figure 9: Distribution of intracity charging stations

Appendix B Second appendix

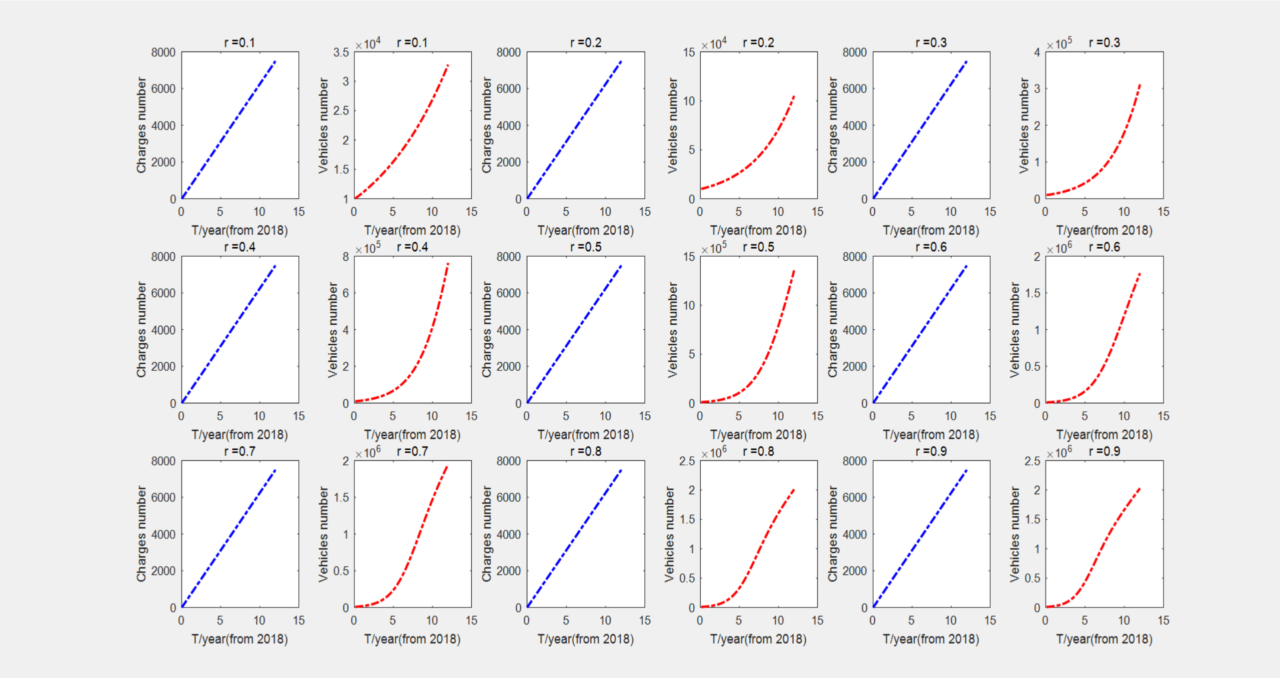


Figure 10: Hypothesis 1

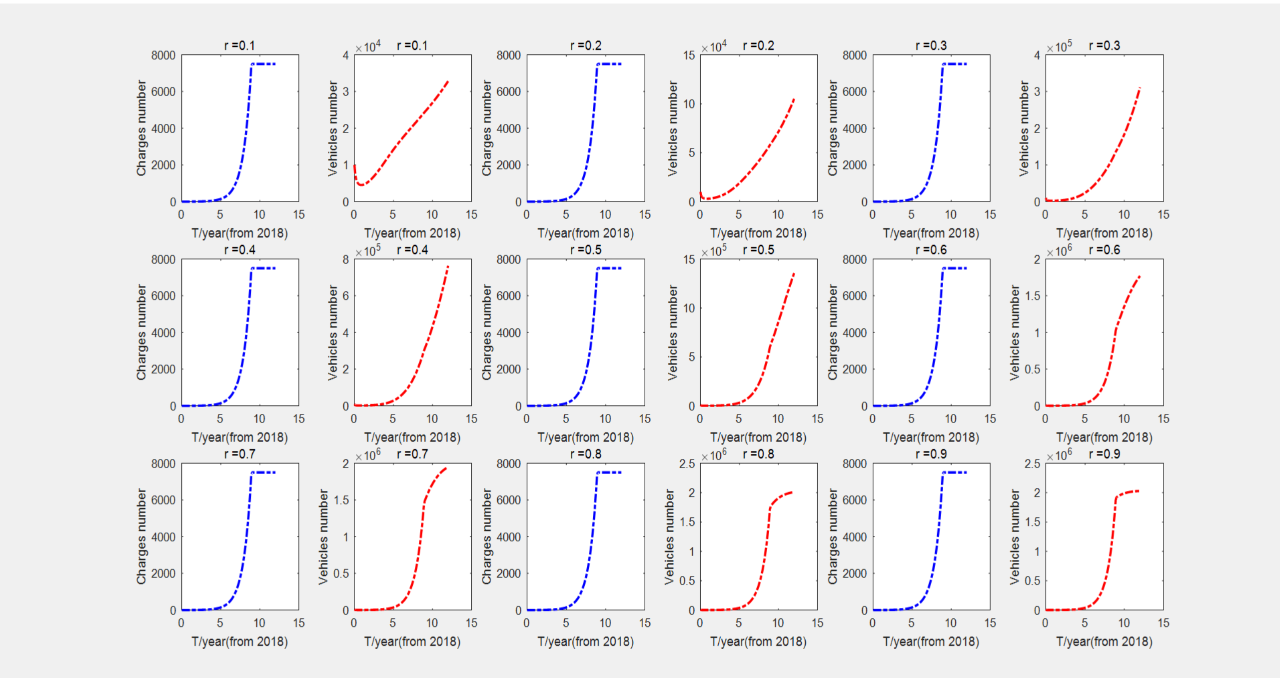


Figure 11: Hypothesis 2

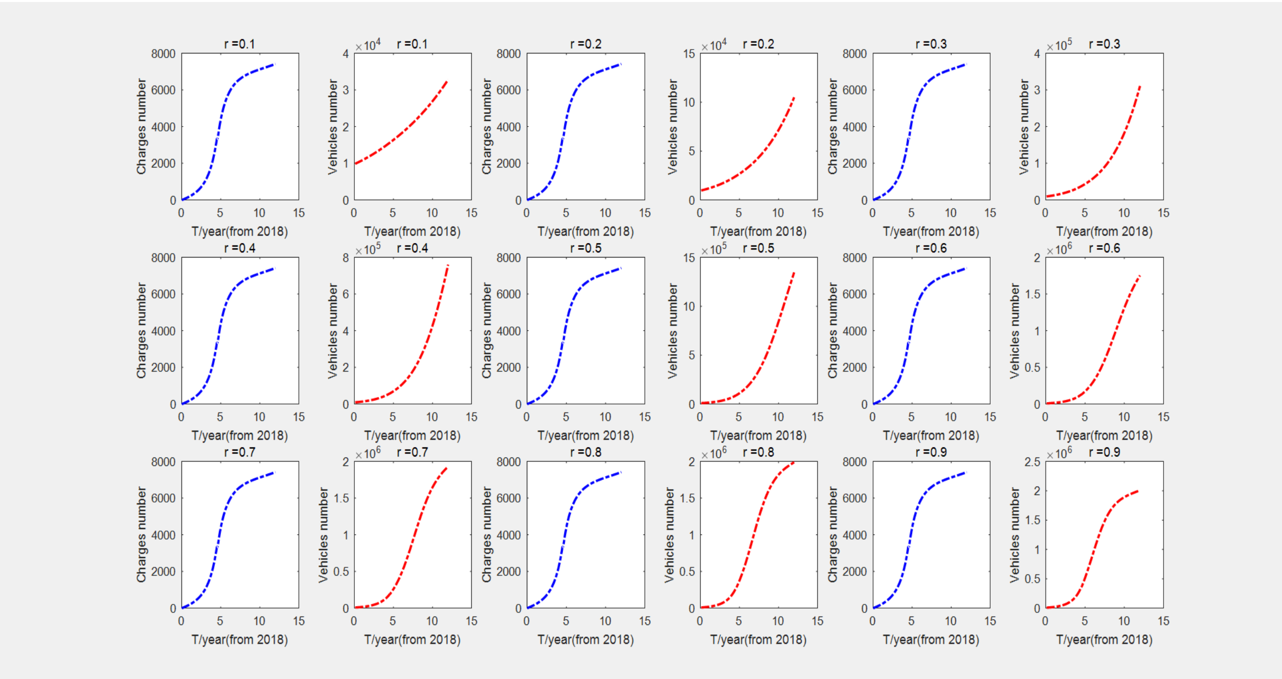


Figure 12: Hypothesis 3