

“Control System Modelling for Reusable Rocket Landing”

PROJECT REPORT

Submitted for the course: Advanced Control Theory (EEE4018)

By

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Introduction

The aerospace industry has been experiencing tremendous growth in recent years. With exponential growth and increasing prices, new developments by way of reusable rockets are becoming exceedingly popular. Landing and reusing of first stage of rockets has been demonstrated by SpaceX and is in development by many space agencies across the world. This project proposes a control scheme for vertical landing the first stage of the rocket with a Matlab model.

Literature Review

1. The high cost of rocket technology has led to system reusability developments, particularly in the field of first stage rockets. With the motivation of decreasing production costs, successful vertical rocket landing attempts by SpaceX and Blue Origin have led the path for autonomous recovery and reusability of rocket engines. Such a feat can only be accomplished by complex control algorithms executing in real-time aboard the rocket. This project aims to develop a vertical rocket landing simulation environment where algorithms based on classical control and machine learning can be designed and evaluated.
2. The SpaceX reusable launch system development program is a privately funded program to develop a set of new technologies for an orbital launch system that may be reused many times in a manner similar to the reusability of aircraft. The company SpaceX is developing the technologies over a number of years to facilitate full and rapid reusability of space launch vehicles. The project's long-term objectives include returning a launch vehicle first stage to the launch site in minutes and to return a second stage to the launch pad following orbital realignment with the launch site and atmospheric reentry in up to 24 hours. SpaceX's long term goal is that both stages of their orbital launch vehicle will be designed to allow reuse a few hours after return.
3. The paper focuses on reusable launch space systems. It aims to describe the current state of reusability in space systems and to analyze the launch cost of current Falcon carrier rockets. The first chapter is dedicated to general information about reusable launch space systems. This includes definition of reusable and expendable launch systems or history of reusable launch systems. The second part is focused on the era after STS and new the concept of RLV. The following parts of this paper aim to analyze the launch price of current Falcon carrier rockets.
4. During the early stage of reusable launch vehicle (RLV) reentry flight, reaction control system (RCS) is the major attitude control device. RCS, which is much different from the atmospheric steer's control, requires a well designed control allocation system to fit the attitude control in high altitude. In this paper, an indexed control method was proposed for RCS preallocation, a 0-1 integer programming algorithm was designed for RCS allocation controller, and then this RCS scheme's effect was analyzed. Based on the specified flight mission simulation, the results show that the control system is satisfied. Moreover, several comparisons between the attitude control effect and RCS relevant parameters were studied.

5. The goals of this project are to significantly reduce the time and cost associated with guidance and control design for reusable launch vehicles, and to increase their safety and reliability. Success will lead to reduced cycle times during vehicle design and to reduced costs associated with flying to new orbits, with new payloads, and with modified vehicles. Success will also lead to more robustness to unforeseen circumstances in flight, thereby enhancing safety and reducing risk. There are many guidance and control methods available that hold some promise for improvement in the desired areas. Investigators are developing a representative set of independent guidance and control methods for this project. These methods are being incorporated into a high-fidelity reusable launch vehicle simulation. A simulation flyoff is being conducted across a broad range of flight requirements. The guidance and control methods that perform the best will have demonstrated the desired qualities.

This paper describes the design of attitude control for reusable launch vehicles during reentry phase using adaptive fuzzy control strategy with compensation controller that assures a stable and accurate attitude tracking despite having parameter uncertainties and external disturbances in the plant model. Firstly, the six-degrees-of-freedom dynamic model of the reusable launch vehicles is established, followed by a model transformation of rotational and kinematic dynamic equations, which results in a strict-feedback form attitude control system. Secondly, a fuzzy logic system combined with the adaptive technique is employed to model the system uncertainty term online. Then an attitude tracking controller based on adaptive fuzzy control approach is designed to assure tracking of guidance commands. Furthermore, to attenuate the adverse effects of fuzzy modeling errors on the control performance and system stability, a compensation controller is introduced to the control strategy. In addition, the stability of the closed-loop system is proved by using the Lyapunov theory, and the attitude tracking error converges to a small neighborhood of the origin. Finally, the simulation results and discussions are provided to verify the effectiveness of the proposed control strategy in tracking the guidance commands.

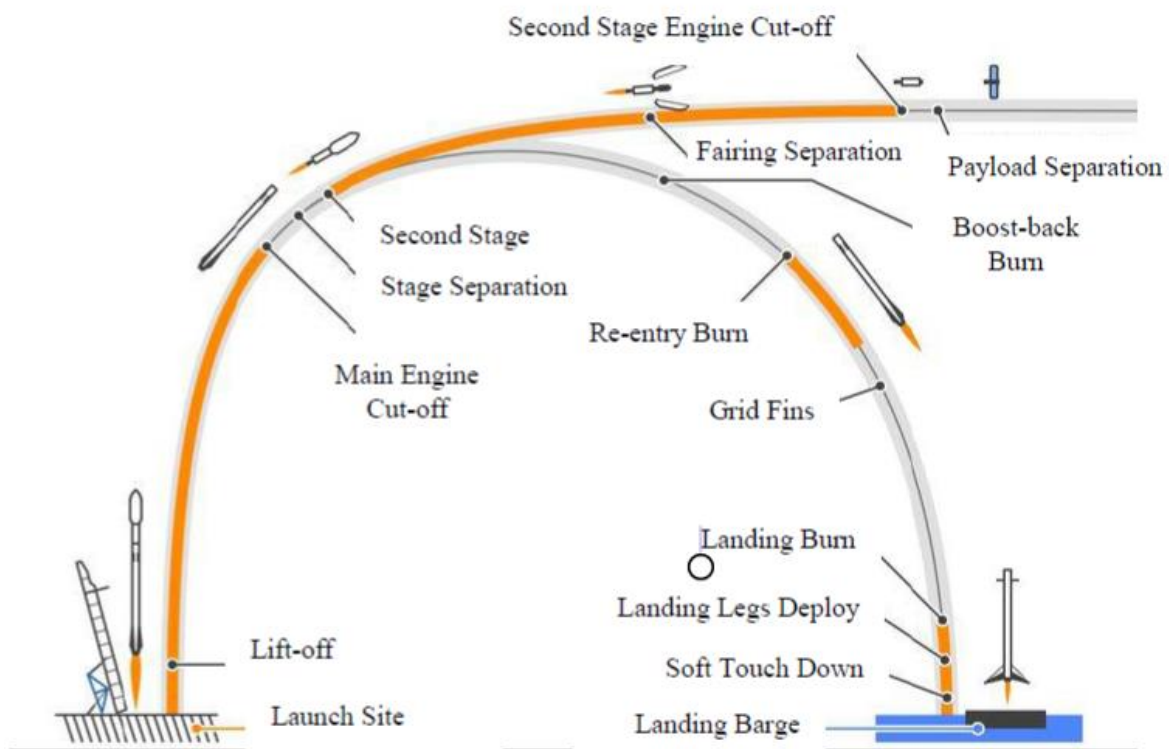
Aim of the Project

- To list the parameters involved in the control of Rocket
- To design the state space representation of the system
- To plot the control parameters in MATLAB/SIMULINK

Methodology

Problem definition

The following diagram represents the process undergone by the system when the first stage of a rocket needs to land.



Constraints

The control of the rocket are achieved by –

1. Main Engine Thrust, F_E
2. Side Thrusters, F_L, F_R – Equivalent to $F_S = F_L - F_R$
3. Nozzle Angle, φ

The constraints for the inputs based are assumed to be (As per Falcon 9 specifications) –

$$0 \text{ N} \leq F_E \leq 6486 \text{ N}$$

$$-130 \text{ N} \leq F_S \leq 130 \text{ N}$$

$$-15^\circ \leq \varphi \leq 15^\circ$$

Let the state of the rocket at any time be defined by $\mathbf{x}_i = [x_i, \dot{x}_i, z_i, \dot{z}_i, \theta_i, \dot{\theta}_i]$ and the final state by \mathbf{x}_τ .

The limiting states to allow the rocket to land safely are given as follows –

$$-Left \text{ Barge Edge} \leq x_\tau \leq Right \text{ Barge Edge}$$

$$-2 \text{ m/s} \leq \dot{x}_\tau \leq 2 \text{ m/s}$$

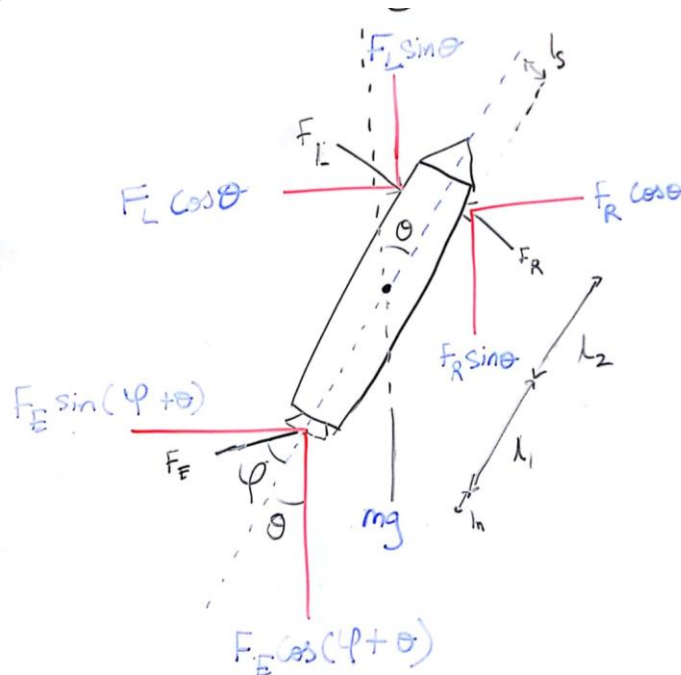
$$z_\tau = \text{Barge Height}$$

$$\dot{z}_\tau = 0 \text{ m/s}$$

$$-10^\circ \leq \theta_\tau \leq 10^\circ$$

$$-2^\circ/\text{s} \leq \dot{\theta}_\tau \leq 2^\circ/\text{s}$$

Free Body Diagram



F_E =Main Thruster Force

F_R =Right Thruster Force

F_L =Left Thruster Force

$F_S = F_L - F_R$

θ =Angle between the z-axis and the longitudinal axis of the rocket

φ =Angle between the Nozzle and the longitudinal axis of the rocket

l_1 =Longitudinal length between the Center of Gravity (COG) and F_E

l_2 =Longitudinal length between the COG and F_R, L

l_n =Nozzle length

m =Rocket Dry Mass+Fuel Mass

x =Horizontal Position of the Rocket

z =Vertical Position of the Rocket

α =Real Constant

Mathematical Modelling

The following equations define the rocket dynamics –

Balancing Forces in x direction

$$m\ddot{x} = F_E \sin(\theta + \varphi) + F_S \cos \theta$$

$$\ddot{x} = \frac{F_E \sin \theta + F_E \varphi + F_S}{m} \text{ (using small angle approximation)}$$

Balancing Forces in z direction

$$m\ddot{z} = F_E \cos(\theta + \varphi) - F_S \sin \theta - mg$$

$$\ddot{z} = \frac{F_E - F_E \varphi \ddot{\theta} - F_S \theta - mg}{m} \text{ (using small angle approximation)}$$

Torque

$$J\ddot{\theta} = -F_E \sin \varphi (l_1 + l_n \cos \varphi) + l_2 F_S$$

$$\ddot{\theta} = \frac{-F_E \varphi (l_1 + l_n) + l_2 F_S}{J} \text{ (using small angle approximation)}$$

Fuel Burn

$$\dot{m} = -\alpha(\beta F_E - F_S)$$

State Space Representation

From the system definition we can see that it's a Multiple Input Multiple Output (MIMO) system where F_E , F_S and φ represent the variable inputs that must be adjusted, while θ , x and z represent the output. We can see that the equations are Non-linear, so a Jacobian Linearization was performed to approximate the non-linear functions using Taylor series expansion as follows,

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du \\ A &= \nabla_x f \quad B = \nabla_u f \\ C &= I \quad D = 0\end{aligned}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{F_E}{m} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-F_E\varphi - F_S}{m} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 \\ \frac{\theta + \varphi}{m} & \frac{1}{m} & \frac{F_E}{m} \\ 0 & 0 & 0 \\ \frac{1 - \varphi\theta}{m} & -\frac{\theta}{m} & -\frac{F_E\theta}{m} \\ 0 & 0 & 0 \\ -\frac{\varphi(l_1 + l_n)}{J} & \frac{l_2}{J} & -\frac{F_E(l_1 + l_n)}{J} \end{bmatrix}$$

equilibrium point by setting the derivatives of the state variables to 0 and solving the simultaneous equations.

Initial conditions - $u = [mg, 0, 0]$

Simulation Code

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Declaration of variables

```
close all
clear all
clc

m=25.222;      %Mass of the rocket
g=9.81;        %Acceleration due to gravity
l1=3.8677;
l2=3.7;
ln=0.1892;
J=482.2956;
```

```

Fe=realp('Fe',m*g);
Fs=realp('Fs',0);
psi=realp('psi',0);
theta=realp('theta',deg2rad(0));

Fe.Minimum=0;
Fe.Maximum=6486;

Fs.Minimum=-130;
Fs.Maximum=130;

psi.Minimum=deg2rad(-15);
psi.Maximum=deg2rad(15);

theta.Minimum=deg2rad(-10);
theta.Maximum=deg2rad(10);

```

State Space Modelling

```

A=[0 1 0 0 0 0; 0 0 0 0 Fe/m 0; 0 0 0 1 0 0; ...
    0 0 0 0 (-Fe*psi-Fs)/m 0; 0 0 0 0 0 1; 0 0 0 0 0 0];
B=[0 0 0; (theta+psi)/m 1/m Fe/m; 0 0 0; ...
    (1-psi*theta)/m -theta/m -Fe*theta/m; 0 0 0;
    -psi*(1+ln)/J 12/J -Fe*(1+ln)/J];
C=eye(6);
D=0;

model=ss(A,B,C,D);
model=tunableSS('model',size(A,2),size(C,1),size(B,2),'full');
model.StateName={'X-Position','X-Velocity'...
    'Z-Position','Z-Velocity','Angle','Angular Velocity'};
model.InputName={'Main Thruster','Side Thruster','Thruster Angle'};
model.OutputName={'X-Position','X-Velocity'...
    'Z-Position','Z-Velocity','Angle','Angular Velocity'};

```

Analysis

```

i=pole(model)
ctrb_mat=ctrb(model);
if(rank(ctrb_mat)== size(A,1))
    disp("System is controllable");
end
figure;
step(model)

```

```

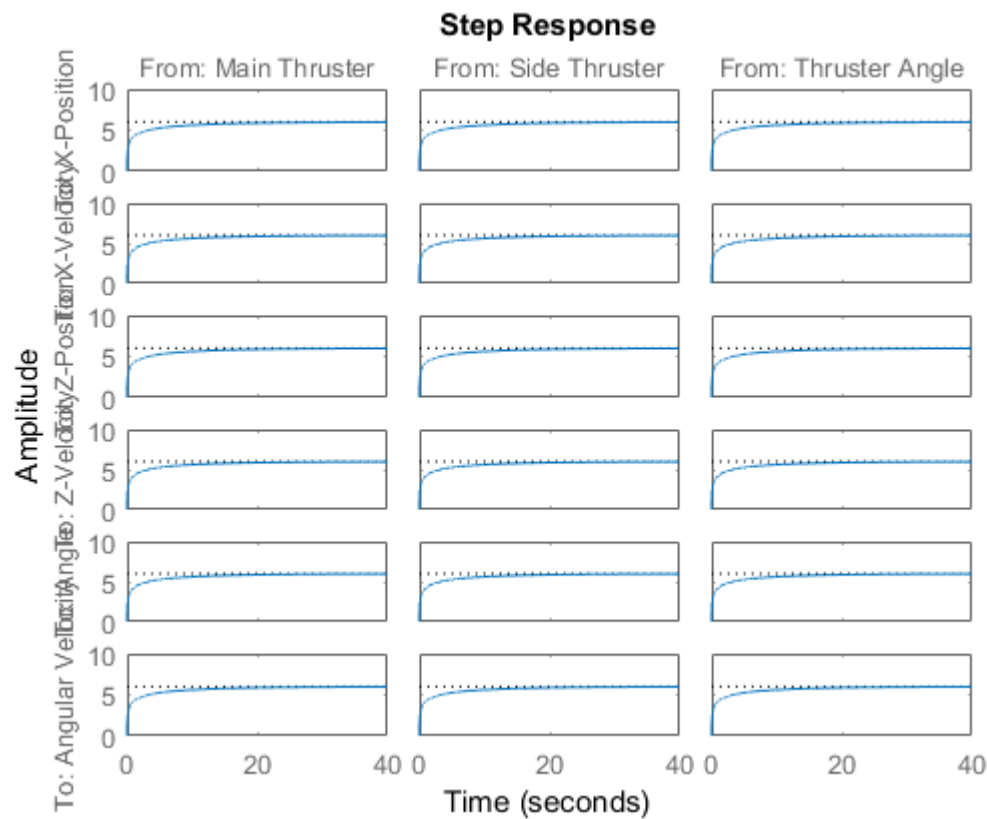
i =

-100.0000
-25.1189
-6.3096
-1.5849
-0.3981

```

-0.1000

System is controllable



Tuning of Parameters

```
responseTime=2;
steadyError=0.1;
RTrack=TuningGoal.Tracking(model.InputName,model.OutputName,responseTime,steadyError);
RTrack.InputScaling=[6000 100 1];
Rreject = TuningGoal.Gain(model.InputName,model.OutputName,0.1);
%[model,fsoft]=systune(model,[RTrack, Rreject]);
```

Controller Design

```
p=[-100,-25,-6,-2,-1,-0.5];
k=place(ss(model).a,ss(model).b,p)
controlmodel=feedback(ss(model),k);
```

k =

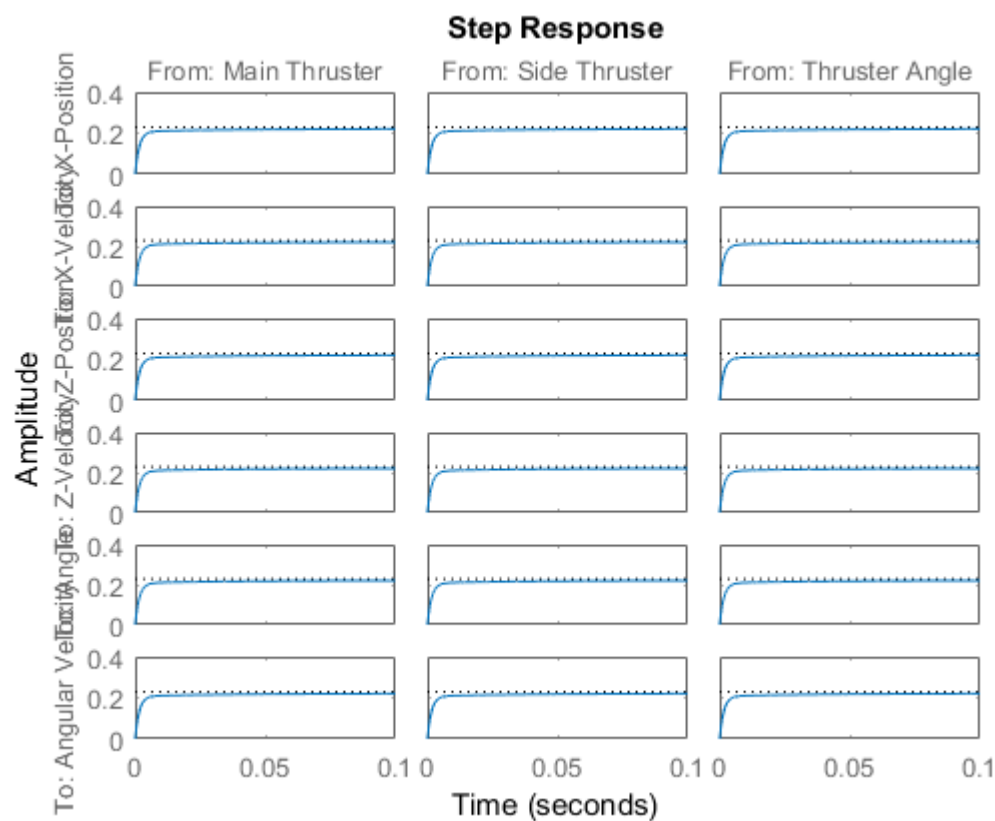
| | | | | | |
|--------|---------|--------|---------|---------|---|
| 1.5402 | -0.1383 | 0.0368 | -0.0313 | -0.0076 | 0 |
| 1.5402 | -0.1383 | 0.0368 | -0.0313 | -0.0076 | 0 |
| 1.5402 | -0.1383 | 0.0368 | -0.0313 | -0.0076 | 0 |

Results

```
c=pole(controlmodel)
figure;
step(controlmodel);
```

c =

```
-642.6378
-40.1652
-8.7478
-2.0342
-0.1174
-0.4860
```



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Conclusion

We can see the difference in rise time and settling time between the open loop and closed loop system due to the state feedback matrix K. Practical Limitations were used to determine the constraints and poles of the systems.

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