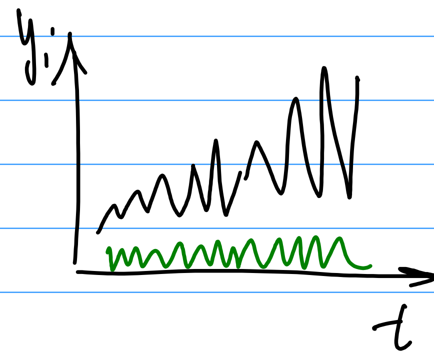
Logarithmic Model



Linear

$$y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$$

$$\frac{dy_i}{dx_i} = \beta_2 \qquad x \uparrow 1 \qquad y \uparrow \beta_2$$

Log-Log

$$\ln y_i = \beta_1 + \beta_2 \ln x_i + \varepsilon_i \qquad y_i = e^{\beta_1} x_i^{\beta_2} \cdot \varepsilon_i$$

$$Y = A K^{\beta} L^{1-\beta}$$

$$\frac{d \ln y_i}{d \ln x_i} = \frac{100 \cdot dy_i / y_i}{100 \cdot dx_i / x_i} = \beta_2 \qquad H_0: \beta_2 + \beta_3 = 1$$

$$x \uparrow 1\% \qquad y \uparrow \beta_2 \%$$

Log-linear

$$\ln y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$$

$$\frac{d \ln y_i}{dx_i} = \beta_2 \qquad \frac{100 \cdot dy_i / y_i}{dx_i} = \beta_2 \cdot 100$$

$$x \uparrow 1 \qquad y \uparrow \beta_2 \cdot 100 \%$$

Lin-log

$$y_i = \beta_1 + \beta_2 \ln x_i + \varepsilon_i$$

$$\frac{dy_i}{d \ln x_i} = \beta_2 \qquad \frac{dy_i}{100\, dx_i / x_i} = \beta_2 / 100$$

$$x \uparrow 1\% \qquad y \uparrow \beta_2 / 100$$

Problem 7. (UoL Exam) A regression of consumption (C) on income (Y) and unemployment (U) (all variables are index numbers) using annual data 1961-82 for the UK produced the following results:

log log log

$$\hat{C}_t = 1.7880 + 0.7327 Y_t + 0.930 U_t \quad R^2 = 0.992 \qquad (1)$$
$$(2817.0)\ (0.026)\qquad (0.798)$$

α β_1 β_2 0

(figures in brackets are standard errors) with a table of correlation coefficients between variables of:

	C	Y	U
C	1.00	0.996	0.783
Y	0.996	1.00	0.771
U	0.783	0.771	1.00

$\rho_{Y,U} > 0{,}7$

$n = 22$

1) interpretation, test

$t_{\hat{\alpha}} = 6{,}07$

$t_{0{,}975,\,19} = 2{,}09$

$t_{\hat{\beta}_1} = 28.8$

$t_{\hat{\beta}_2} = 1{,}16$

2) $C_i = \alpha + \hat{\beta}_1 \cdot Y_i + \hat{\varepsilon}_i$

assume $\beta_2 < 0 \Rightarrow \hat{\beta}_1 \downarrow$

3) assume : $\beta_1 \geq 0$ $\quad \beta_2 \leq 0$

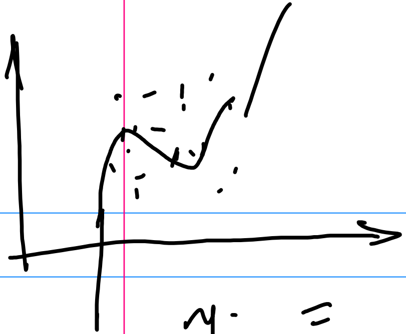
$H_0: \beta_1 = 0 \Leftrightarrow \beta_1 \geq 0$ | $H_0: \beta_2 \leq 0$

$H_a: \beta_1 < 0$ | $H_a: \beta_2 > 0$

F - test

$H_0: \beta_1 = \beta_2 = 0$

$$F = \frac{R^2 / k-1}{(1-R^2)/ n-k} = 1178$$



Model with quadratic terms

model with interactive terms

$$y_i = \beta_1 + \beta_2 \cdot x_i + \beta_3 \cdot x_i^2 + \epsilon_i$$

$$\frac{dy_i}{dx_i} = \beta_2 + 2\beta_3 \cdot x_i$$

β_2 rate of change of y_i

when $x_i = 0$

$$y_i = \beta_1 + (\beta_2 + \beta_3 x_i) x_i + \epsilon_i$$

β_3 rate of change of x_i

per unit change of x_i

- What happens with TSS, RSS, ESS, R^2

TSS: $\bar{y}' = \frac{(y_1 + \dots + y_n) + y_{n+1}}{n+1} =$

$\frac{n \cdot \bar{y} + y_{n+1}}{n+1} = \frac{n}{n+1}\bar{y} + \frac{1}{n+1} y_{n+1}$

$TSS' = \sum_{i=1}^{n+1} (y_i - \bar{y}')^2 = \sum_{i=1}^{n} (y_i - \bar{y} + \bar{y} - \bar{y}')^2$

$+ (y_{n+1} - \bar{y}')^2 = \sum_{i=1}^{n} (y_i - \bar{y})^2$

$+ \sum (\bar{y} - \bar{y}')^2 + 2 \underbrace{\sum (y_i - \bar{y})}(\bar{y} - \bar{y}')$

$+ (y_{n+1} - \bar{y}')^2 = TSS +$ $\sum (y_i - \bar{y}) = 0$

$\sum \left(\bar{y} - \frac{n}{n+1}\bar{y} - \frac{1}{n+1} y_{n+1} \right)^2$ $\sum y_i = \sum \bar{y}$

RSS: $\sum_{i=1}^{n+1} (y_i - \hat{y}_i')^2 =$

$\sum_{i=1}^{n} (y_i - \hat{y}_i')^2 + (y_{n+1} - \hat{y}_{n+1}')^2 \geq$

$\sum_{i=1}^{n} (y_i - \hat{y}_i')^2 \geq RSS_n^*$

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$$\underset{}{\hat{C}_t} = \underset{(2817.0)}{17880} + \underset{(0.026)}{0.7527} Y_t + \underset{(0.798)}{0.930} U_t, \quad R^2 = 0.992 \qquad (1)$$

(figures in brackets are standard errors) with a table of correlation coefficients between variables of:

	C	Y	U
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