$$y_i = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k + \varepsilon_i$$

1) model is correctly specified

2) $X = \begin{bmatrix} 1 & x_{11} & \dots & x_{k1} \\ \vdots & \vdots & & \\ 1 & x_{1n} & & x_{kn} \end{bmatrix}$ — deterministic

3) $Var(\varepsilon_i) = const$

4) $Cov(\varepsilon_i; \varepsilon_j) = 0$

5) $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$

GMT : 1–5 $\Rightarrow$ $\hat{\beta}_{OLS} = \begin{bmatrix} \hat{\beta}_0 \\ \vdots \\ \hat{\beta}_k \end{bmatrix}$ — BLUE

$\hat{\theta}$ $\qquad MSE(\hat{\theta}) = Var(\hat{\theta}) + bias^2$

1) Unbiasedness $\quad E(\hat{\theta}) = \theta$

2) Consistency $\quad \operatorname{plim}_{n \to \infty} \hat{\theta} = \theta$

$$P\{|\hat{\theta} - \theta| < \varepsilon\} = 1$$

3) Efficiency $\quad Var(\hat{\theta}) \le Var(\tilde{\theta})$

$$\hat{\theta}, \tilde{\theta} \in C$$

1) Wrong specification

Endogeneity

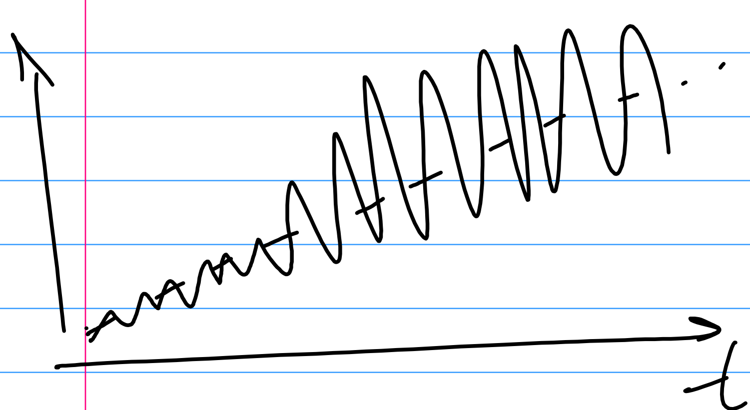
- ommitted variable
- simultaneity
- measurement errors

Log - transform

. . . .

2) $X \Rightarrow$ stochastic

3) Homoscedasticity $\Rightarrow$ heteroscedasticity



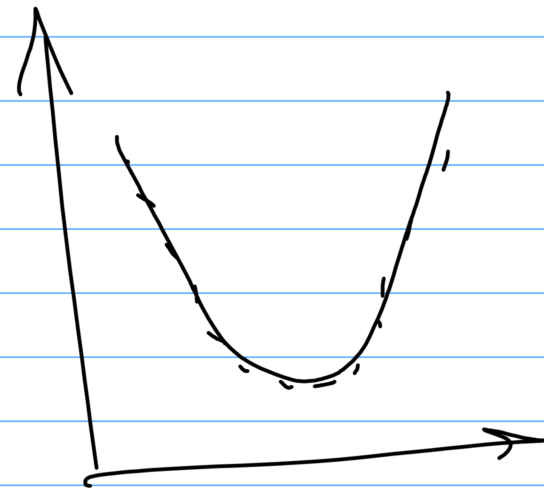
4) Autocorrelation

⑤ Binary Choice; Panel Data

№1 X, Y independent $\Rightarrow$

$$\rho_{X,Y} = 0$$

№2 $\Leftarrow$



$$Y = X\beta + e_i$$

№3

- $P(X,Y) = P(X) \cdot P(Y)$ — independency

$$\Downarrow$$

- $E(Y|X) = E(Y)$ — unpredictability

$$\Downarrow$$

- $E(XY) = E(X)E(Y)$ — uncorrelated

N4 $\quad X \qquad E(X) = \mu_x$

$$Var(X) = \sigma_x^2$$

$$\{X_1, \dots, X_n\}$$

$$\hat{\mu}_x = \frac{X_1 + \dots + X_{2k-1}}{k}$$

1) $\hat{\mu}_x$ – unbiased $\quad E(\hat{\mu}_x) = \frac{k E(X_i)}{k} = \mu_x$

2) is $\hat{\mu}_x$ – efficient?

$$Var(\hat{\mu}_x) = \frac{\sigma^2}{k} \qquad Var(\bar{X}) < Var(\hat{\mu}_x)$$

$$Var(\bar{X}) = \frac{\sigma^2}{n}$$

(№5) X $\quad E(X) = \mu_x \quad Var(X) = \sigma_x^2$

$\{X_1, \dots, X_n\}$

$$Z = \frac{1}{2}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \dots + \frac{1}{2^n}X_n$$

1) biased

2) $n \to \infty \qquad \frac{1}{2} + \dots + \frac{1}{2^n} \to 1$

$E(Z) \longrightarrow 1 \cdot \mu_x \implies$ asymptotic unbiasedness

3) $Var(Z) = \left(\frac{1}{4} + \dots + \frac{1}{2^{2n}}\right)\sigma_x^2 \xrightarrow{n\to\infty} \left(\frac{1/4}{1 - 1/4}\right)\sigma_x^2 = \frac{\sigma_x^2}{3}$

$\implies$ sufficiency condition for consistency doesn't hold

$\boxed{N6}$ $\cdot E(X) = \mu$ $\quad Var(X) = \sigma^2$

$$\hat{\mu} = \frac{n+2}{n^2+3n+1} \sum_{i=1}^{n} X_i$$

$\bar{X}$ - consistent

$\hat{\mu}$ - ?

plim $\hat{\mu} \to \mu$

$E(\hat{\mu}) \to \mu$

- $E(\hat{\mu}) \longrightarrow \mu$
- $Var(\hat{\mu}) \longrightarrow 0$

N7

$$\boxed{y_i = \alpha + \beta x_i + \varepsilon_i} \quad i = \overline{1, n}$$

$\hat{\alpha}, \hat{\beta}$ — OLS

$$\hat{\beta} = \frac{\widehat{\mathrm{cov}}(x, y)}{\widehat{\mathrm{Var}}(x)} = \frac{\cancel{1/n} \sum (x_i - \bar{x})(y_i - \bar{y})}{\cancel{1/n} \sum (x_i - \bar{x})^2}$$

$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$$

$\tilde{\beta}_{OLS}$. assuming $\alpha = 0$

$$\tilde{\beta} = \frac{\sum y_i x_i}{\sum x_i^2}$$