Heteroscedasity

- unbiased, consistent, inefficient est.

Test: GQ test, White test

Solve: WLS, Robust s.e. (White s.e.)

stochastic Regressors

$\ln y_t = \beta_1 + \beta_2 \cdot t + e_t$ $\pi_t = \beta_1 + \beta_2 \pi_{t-1} + \beta_2 x_t + \varepsilon_t$

deterministic stochastic

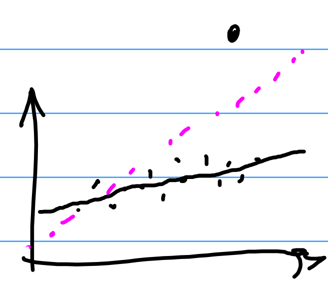
Model with

stoch. regressors

1. model is linear and correctly specified:

$$y_i = \beta_1 + \beta_2 \cdot x_i + e_i$$

2. Obs $\{(x_i, y_i), i = \overline{1,n}\}$ i.i.d.



3. $E(x_i^4) < \infty$, $E(y_i^4) < \infty$

4. $E(\varepsilon_i | x_i) = 0$

5*. no multicol. w.p. 1.

Under 1–4 $\hat{\beta}_{OLS}$ is consistent and as. normal if 4 th ass. violated =>

regr. are endogenious =>

$\hat{\beta}_{OLS}$ are inconsistent and biased

Col. 1 $\quad E(\varepsilon_i | x_i) = 0 \Rightarrow E(\varepsilon_i) = 0$

$$0 = E(E(\varepsilon_i | x_i)) = E(\varepsilon_i) = 0$$

Col. 2 $\quad E(\varepsilon_i | x_i) = 0 \Rightarrow cov(\varepsilon_i, X_i) = 0$

$$cov(\varepsilon_i, x_i) = E(\varepsilon_i X_i) - \underbrace{E(\varepsilon_i)}_{0} E(X_i)$$

$$E(\varepsilon_i X_i) = E(E(\varepsilon_i X_i | X_i)) =$$

$$E(X_i E(\varepsilon_i | X_i)) = 0 \Rightarrow cov(X_i, \varepsilon_i) = 0$$

if 4 th ass. violated =>

regr. are endogenious $\quad cov(X_i, \varepsilon_i) \neq 0$

$\hat{\beta}_{OLS}$ are inconsistent and biased

1) if x and ε are independent

$\Rightarrow$ $\hat{\beta}_{OLS}$ unbiased and consistent

2) if $cov(x, \varepsilon) = 0$ $\Rightarrow$

$\Rightarrow$ $\hat{\beta}_{OLS}$ consistent

3) if $cov(x, \varepsilon) \neq 0$ $\Rightarrow$

$\Rightarrow$ $\hat{\beta}_{OLS}$ inconsistent

Endogeneity:
1) Omitted variable bias
2) Measurement errors
3) Simultaneity
$y \sim x$ and $x \sim y$

Q1. OVB

True: $y_i = \beta_1 + \beta_2 \cdot x_i + \beta_3 \cdot W_i + \varepsilon_i, \ \beta_3 \neq 0$

Est: $\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 \cdot x_i$

$$\hat{\beta}_2 = \frac{\hat{Cov}(x,y)}{\hat{Var}(x)} \xrightarrow{p} \frac{Cov(x_i, y_i)}{Var(x_i)} =$$

$$\frac{Cov(x_i,\ \beta_1 + \beta_2 x_i + \beta_3 \cdot w_i + \varepsilon_i)}{Var(x_i)} =$$

$$0 + \beta_2 + \beta_3 \frac{Cov(x_i, w_i)}{Var(x_i)} + \frac{\overbrace{Cov(x_i, \varepsilon_i)}^{=0}}{Var(x_i)} =$$

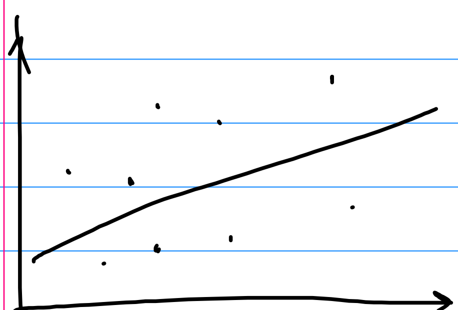
$$= \beta_2 + \beta_3 \frac{Cov(x_i, w_i)}{Var(x_i)} \xleftarrow{P} \hat{\beta}_2$$

$\beta_3 > 0$, $cov(x_i, w_i) > 0$ $\Rightarrow$

$\hat{\beta}_2$ is inconsistent

and biased upwards

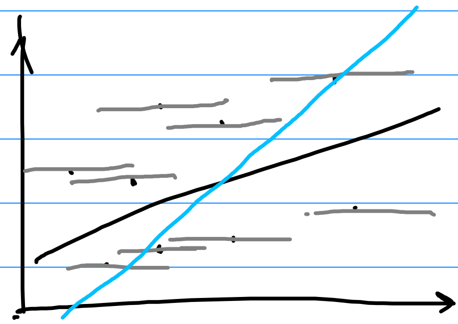
Q2. Measurement error Model



$$y = \beta_1 + \beta_2 \cdot x_i^*$$

$$x_i = x_i^* + \varepsilon_i$$

$$cov(x_i^*, \varepsilon_i) = 0$$



$$y_1 = \beta_1 + \beta_2 \cdot x_i + u_i$$

$$\|$$

$$y_1 = \beta_1 + \beta_2\ x_i - \beta_2 \varepsilon_i$$

$$\hat{\beta}_2 \xrightarrow{P} \beta_2 + \frac{cov(x_i, u_i)}{var(x_i)} = \beta_2 + \frac{cov(x_i^* + \varepsilon_i, -\beta_2 \cdot \varepsilon_i)}{Var(x_i)} =$$

$$= \beta_2 - \beta_2 \cdot \frac{cov(x_i^*, \varepsilon_i) + cov(\varepsilon_i, \varepsilon_i)}{var(x_i)} =$$

$$= \underbrace{\beta_2 - \beta_2 \frac{Var(\varepsilon_i)}{Var(\varepsilon_i) + Var(x_i^*)}}$$

$$= \beta_2 \frac{Var(x_i^*)}{Var(\varepsilon_i) + Var(x_i^*)}$$

$$\left| \frac{Var(x_i^*)}{Var(\varepsilon_i) + Var(x_i^*)} \right| < 1$$

regardless of sign of β_2

$\hat{\beta}_2$ is incons. and biased

towards zero.