

TSLS

$$y_i = \beta_0 + \beta_1 x_i^{(1)} + \dots + \beta_p x_i^{(p)} + \beta_{p+1} w_i^{(1)} + \dots + \beta_{p+2} w_i^{(2)} + \epsilon_i$$

x_i - endogenous variables p
 w_i - exogenous variables 2
 z_i - instruments m

$m = p$ (IV) exactly identified
 $m > p$ (TSLS) overidentified

$m < p$ (—) underidentified

1 step $x_i^{(1)} \mid z_i^{(1)}, \dots, z_i^{(m)}, w_i^{(1)}, \dots, w_i^{(2)}$

\vdots
 $x_i^{(p)} \mid z_i^{(1)}, \dots, z_i^{(m)}, w_i^{(1)}, \dots, w_i^{(2)}$

2 step $y_i \mid \hat{x}_i^{(1)}, \dots, \hat{x}_i^{(p)}, w_i^{(1)}, \dots, w_i^{(2)}$

relevance

exogenous $\text{cov}(z_i^{(1)}, \epsilon_i) = 0, \dots, \text{cov}(z_i^{(m)}, \epsilon_i) = 0$

Test for TSLS

Wu-Hausman Test

H_0 : $\hat{\beta}^{OLS}$ are consistent

$$(\hat{\beta}^{2SLS} - \hat{\beta}^{OLS})' \left(\hat{V}(\hat{\beta}^{2SLS}) - \hat{V}(\hat{\beta}^{OLS}) \right)^{-1} (\hat{\beta}^{2SLS} - \hat{\beta}^{OLS}) \sim \chi^2_k$$

k - # of vars at

		2 step	
H_0 is not rejected	$\hat{\beta}^{OLS}$	consistent efficient	inconsistent
H_0 is rejected	$\hat{\beta}^{TSLS}$	consistent exogen.	Consistent endog.

Test for exogeneity of instruments

Sargan test (y-test)

$$(2 \text{ step}) \quad y_i \mid \hat{x}_i^{(1)}, \dots, \hat{x}_i^{(p)}, w_i^{(1)}, \dots, w_i^{(2)}$$

$$\Downarrow$$
$$\hat{\epsilon}_i$$

$$\hat{\epsilon}_i \mid z_i^{(1)}, \dots, z_i^{(n)}, w_i^{(1)}, \dots, w_i^{(n)}$$

$$J = m \cdot F \sim \chi^2_{m-p}$$

F stat for

$$H_0: \gamma_1 = \dots = \gamma_n = 0$$

$$m > p$$

Test for relevance

F-test for weak instruments

$$(1 \text{ step}) \quad x_i \mid z_i^{(1)}, \dots, z_i^{(n)}, w_i^{(1)}, \dots, w_i^{(2)}$$

F test for $H_0: \gamma_1 = \dots = \gamma_n = 0$

$$F > 10$$

instruments are relevant

$$F \leq 10$$

instruments are weak