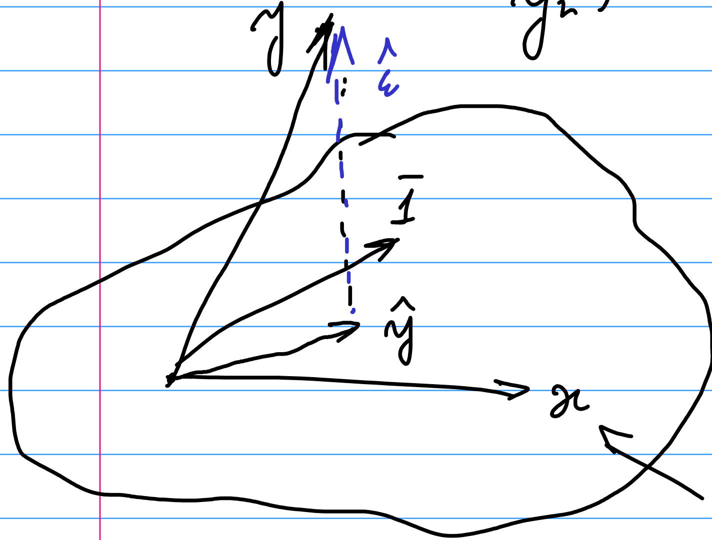
$$y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \qquad X = \begin{pmatrix} 1 & x_{11} & \cdots & x_{k1} \\ \vdots & \vdots & & \vdots \\ 1 & x_{1n} & & x_{kn} \end{pmatrix}$$



col(X)

$$\hat\beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{pmatrix} \qquad \hat y = \operatorname{argmin} \|\hat\varepsilon\|^2$$

$$X'\hat\varepsilon = 0$$

$$X'(y - \hat y) = 0$$

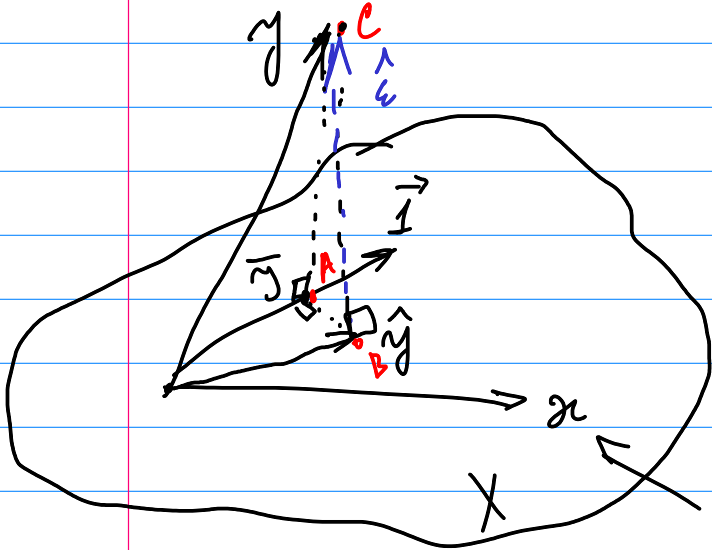
$$X'(y - X\hat\beta) = 0$$

$$X'y = X'X\hat\beta$$

$$\hat\beta = (X'X)^{-1}X'y = \begin{pmatrix} \hat\beta_0 \\ \hat\beta_1 \end{pmatrix}$$

$$\hat\alpha = \bar y - \hat\beta \cdot \bar x$$

$$\hat\beta = \frac{\widehat{\mathrm{Cov}}(X,Y)}{\widehat{\mathrm{Var}}(X)}$$



col(X)

$$\vec{1}'\hat\varepsilon = 0$$

(1). $\frac{1}{n}\sum \hat\varepsilon_i = 0 \iff \frac{\sum \hat\varepsilon_i}{n} = 0$

(4) $\frac{1}{n}\sum \hat Y_i = \frac{1}{n}\sum Y_i = \bar y$

(2). $\frac{1}{n}\sum X_i \hat\varepsilon_i = 0$

(4) $\frac{1}{n}\sum \hat Y_i \hat\varepsilon_i = 0$

$$AC^2 = \sum (y_i - \bar y)^2 = TSS$$

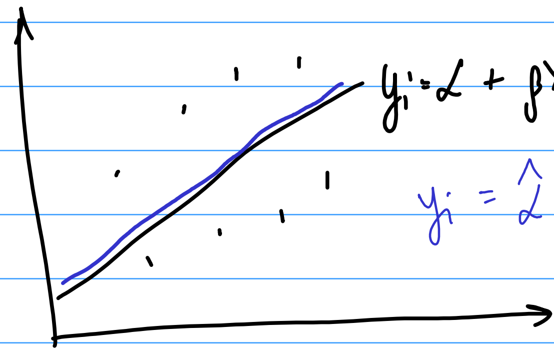
$$TSS = ESS + RSS$$

$$BC^2 = \sum (y_i - \hat{y})^2 = \sum \hat{\varepsilon}_i^2 = RSS$$

$$AB^2 = \sum (\hat{y}_i - \bar{y})^2 = ESS$$

$\hat{\alpha}$, $\hat{\beta}$, $\hat{y}_i = \hat{\alpha} + \hat{\beta} x_i$

$$\begin{cases} \frac{1}{n} \sum \hat{\varepsilon}_i = 0 \\ \frac{1}{n} \sum x_i \hat{\varepsilon}_i = 0 \end{cases}$$



$y_i = \alpha + \beta x_i + \varepsilon_i$

$y_i = \hat{\alpha} + \hat{\beta} x_i + \hat{\varepsilon}_i$

$$TSS = ESS + RSS$$

$$R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$$

$$y_i = \alpha + \beta x_i + \varepsilon_i$$

$\hat{\alpha}, \hat{\beta}$

$$\hat{\beta} = (X'X)^{-1}X'y$$

$\alpha = 0$; $\tilde{\beta} = \dfrac{\sum x_i y_i}{\sum x_i^2}$

$\alpha = 0 \Rightarrow \tilde{\beta}$ - unbiased

$\alpha \neq 0 \Rightarrow \tilde{\beta}$ - biased

$$Var(\tilde{\beta}) = \frac{\sigma^2}{\sum X_i^2}$$

$$Var(\hat{\beta}) = \frac{\sigma^2}{\sum X_i^2 - n\bar{X}}$$

$$Var(\hat{\beta}) > Var(\tilde{\beta})$$

W_i - wages

P_i - aggregate profits

Y_i - aggregate income

$$Y_i = W_i + P_i$$

$$\hat{W}_i = \hat{\alpha}_1 + \hat{\alpha}_2 \cdot Y_i$$

$$\hat{P}_i = \hat{\beta}_1 + \hat{\beta}_2 \cdot Y_i$$

$$\left| \begin{array}{l} \hat{\alpha}_2 + \hat{\beta}_2 = 1 \\ \hat{\alpha}_1 + \hat{\beta}_1 = 0 \end{array} \right|$$

$Y_i \quad , \quad X_i^* = \mu_1 + \mu_2 X_i$

$\hat{\beta}^* = \frac{\hat{\beta}}{\mu_2}$ $\qquad \hat{\beta} = \frac{Cov(X, Y)}{Var(X)}$

$\hat{\beta}^*$