$$y_i = \beta_1 + \beta_2 \cdot X_{2i} + \beta_3 X_{3i} + u_i \qquad i = \overline{1,N}$$

$$\hat{\beta} = (X'X)^{-1} X'y$$

$$\hat{y} = \hat{\beta}_1 + \hat{\beta}_2 \cdot X_2 + \hat{\beta}_3 \cdot X_3$$

$$(1) \quad \hat{\beta}_1 = \overline{Y} - \hat{\beta}_2 \cdot \overline{X}_2 - \hat{\beta}_3 \cdot \overline{X}_3$$

$Cov(X_2, Y)$; $Cov(X_3, Y)$!

$$(2) \quad \hat{\beta}_2 \cdot Var(X_2) + \hat{\beta}_3 \cdot Cov(X_2, X_3) = Cov(X_2, Y)$$

$$(3) \quad \hat{\beta}_2 \cdot Cov(X_2, X_3) + \hat{\beta}_3 \cdot Var(X_3) = Cov(X_3, Y)$$

$$\left| \begin{array}{cc|c} Var(X_2) & Cov(X_2, X_3) & Cov(X_2, Y) \\ Cov(X_2, X_3) & Var(X_3) & Cov(X_3, Y) \end{array} \right|$$

$$\Delta = Var(X_2) \cdot Var(X_3) - Cov^2(X_2, X_3)$$

$$\Delta_1 = Cov(X_2, Y) \cdot Var(X_3) - Cov(X_3, Y) \cdot Cov(X_2, X_3)$$

$$\Delta_2 = Var(X_2) \cdot Cov(X_3, Y) - Cov(X_2, X_3) \cdot Cov(X_2, Y)$$

$$\hat{\beta}_2 = \frac{\Delta_1}{\Delta} \qquad \hat{\beta}_3 = \frac{\Delta_2}{\Delta}$$

$$R^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS}$$

$$R^2_{adj} = 1 - \frac{RSS / n-k}{TSS / n-1}$$

$F = t^2$ for pair linear reg

$$y_i = \beta_0 + \beta_1 \cdot x_{i} \dots + \beta_k x_{1i} + u_i$$

$H_0: \beta_1 = \dots = \beta_k = 0$

$H_a: \exists i \quad \beta_i \neq 0$

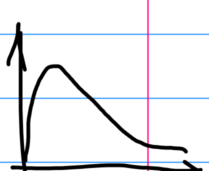
$$F = \frac{R^2/(k-1)}{(1-R^2)/(n-k)} = \frac{ESS/(k-1)}{RSS/(n-k)}$$

UR : $y_i = \beta_0 + \beta_1 x_{1i} + \dots + \beta_k x_{ki} + u_i$

R : $\begin{bmatrix} \beta_1 = 0 \\ \beta_2 = 0 \end{bmatrix}$ $\quad y_i = \beta_0 + \beta_3 x_{3i} + \dots + \beta_k x_{ki} + u_i$

$H_0: \beta_1 = 0 , \ \beta_2 = 0$

$H_a: \beta_1 \neq 0$ or $\beta_2 \neq 0$ $\qquad \overset{H_0}{\sim} F(q, n-k-1)$



$$F = \frac{(RSS_R - RSS_{UR})/q}{RSS_{UR}/(n-k-1)} = \frac{(R^2_{UR} - R^2_R)/q}{(1-R^2_{UR})/(n-k-1)}$$

$$RSS_R \geq RSS_{UR}$$

$$TSS_R = TSS_{UR} \quad \Sigma(y_i - \bar{y})^2$$

$$ESS_R \leq RSS_{UR}$$

$$R^2_R \leqslant R^2_{UR}$$

Problem 1 $\quad y_i = \beta_1 + \beta_2 \cdot X_{2i} + \beta_3 X_{3i} + u_i$

(a) $H_0: \beta_2 = 1$ $\quad n$ – sample size

$H_a: \beta_2 \neq 1$ $\quad k = 3$

$$t = \frac{\hat{\beta}_2 - 1}{se(\hat{\beta}_2)} \overset{H_0}{\sim} t_{n-3}.$$

$$|t| > t_{1-\frac{\alpha}{2}, n-3} \Rightarrow H_0 \text{ is rejected}$$

(b) $H_0: \beta_2 = \beta_3 = 0$ $\quad y_i = \beta_1 + \beta_2 \cdot X_{2i} + \beta_3 X_{3i} + u_i$

$H_a: \beta_2 \neq 0$ or

$\beta_3 \neq 0$

$$F = \frac{R^2/k-1}{1-R^2/n-k} = \frac{R^2/2}{1-R^2/n-3} \overset{H_0}{\sim} F(2, n-3)$$

$$F_{obs} > F_{1-\alpha; 2, n-3} \Rightarrow H_0 \text{ is rejected}$$

(c) $H_0: \beta_2 = \beta_3$

UR: $y_i = \beta_1 + \beta_2 \cdot X_{2i} + \beta_3 X_{3i} + u_i$

R: $y_i = \beta_1 + \beta_2 (\underbrace{X_{2i} + X_{3i}}_{Z_1}) + u_i$

$$F = \frac{(RSS_R - RSS_{UR}) / 1}{RSS_{UR} / n-3} = \frac{(R^2_{UR} - R^2_R) / 1}{(1 - R^2_{UR}) / n-3}$$

(d) $H_0: \beta_2 + \beta_3 = 1$ $\quad$ $H_a: \beta_2 + \beta_3 \neq 1$

UR: $y_i = \beta_1 + \beta_2 \cdot X_{2i} + \beta_3 X_{3i} + u_i$

R: $y_i = \beta_1 + \beta_2 X_{2i} + (1 - \beta_2) X_{3i} + u_i$

$y_i = \beta_1 + X_{3i} + \beta_2 (X_{2i} - X_{3i}) + u_i$

$\underbrace{y_i - X_{3i}}_{W_i} = \beta_1 + \beta_2 \underbrace{(X_{2i} - X_{3i})}_{Z_i} + u_i$

$$F = \frac{(RSS_R - RSS_{UR}) / 1}{RSS_{UR} / n-3}$$

(e) $H_0: (\beta_2 + \beta_3 - 1) = 0 \qquad H_a: \beta_2 + \beta_3 < 1$

$$y_i = \beta_1 + \beta_2 \cdot X_{2i} + \beta_3 X_{3i} + u_i$$

$$\underbrace{y_i - X_{2i}}_{\sigma} = \beta_1 + (\overbrace{\beta_2 + \beta_3 - 1}^{\gamma}) X_{2i} +$$

$$\beta_3 (X_{3i} - X_{2i}) + u_i$$

$H_0: \gamma = 0 \qquad H_a: \gamma < 0$

$\Rightarrow t$