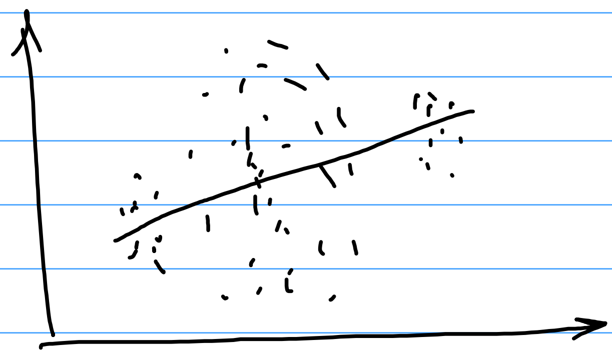
Heteroscadicity

- $\hat{\beta}$ consistent, unbiased, inefficient
- Test: GQ test, White test
- Solve: WLS, Robust s.e. (White s.e.)



Models with stochastic regressors

$\ln y_i = \beta_1 + \beta_2 \cdot t + \varepsilon_t$ $\qquad$ $x_t = \beta_1 + \beta_2 \cdot x_{t-1} + \beta_3 \cdot X_t + \varepsilon_t$

$\uparrow$
\# year

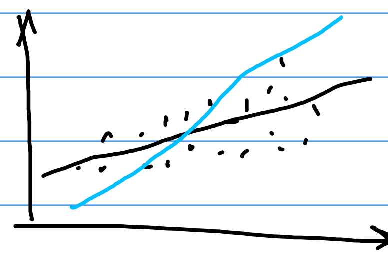
deterministic $\qquad\qquad$ stochastic

Assumptions

of model with stoch. regressors

1. Model is linear and correctly spefiied: $\quad y_i = \beta_1 + \beta_2 \cdot x_i + \varepsilon_i$.

2. $\{(x_i, y_i),\ i = \overline{1,n}\}$ i.i.d



3. $E(X_i^4) < \infty$, $E(y_i^4) < \infty$

4. $E(\varepsilon_i \mid X_i) = 0$ $\nearrow E\varepsilon = 0$ $\searrow \rho_{\varepsilon, x} = 0$

5*. no perf. multicollinearity w.p. 1

Under 1-4 $\quad \beta_{OLS}$ is consistent and asympt. normal

Col. 1 $\quad E(\varepsilon_i \mid x_i) = 0 \implies E(\varepsilon_i) = 0$

$$0 = E(E(\varepsilon_i \mid x_i)) = E(\varepsilon_i) = 0$$

Col. 2. $E(\varepsilon_i | X_i) = 0 \Rightarrow Cov(X_i, \varepsilon_i) = 0$

$$Cov(X_i, \varepsilon_i) = E(X_i \varepsilon_i) - E(X_i) \cdot \underbrace{E(\varepsilon_i)}_{=0}$$

$$E(X_i \varepsilon_i) = E(E(X_i \varepsilon_i | X_i)) =$$

$$= E(X_i \cdot \underbrace{E(\varepsilon_i | X_i)}_{=0}) = 0$$

1. X and ε are independent

$\Rightarrow \hat{\beta}$ – unbiased, consistent

2. $cov(X, \varepsilon) = 0 \Rightarrow \hat{\beta}$ – consistent

3. $cov(X, \varepsilon) \neq 0 \Rightarrow \hat{\beta}$ – inconsistent

X – endogenuous regressor $\Leftrightarrow Cov(X, \varepsilon) \neq 0$

1) Omitted variable

2) Measurement error

3) Simultaneity

$$\begin{cases} Q = \beta_1 + \beta_2 \cdot P + \varepsilon_i \\ P = \alpha_1 + \alpha_2 \cdot Q + v_i \end{cases}$$

Q1. Omitted variable

$$y_i = \beta_1 + \beta_2 \cdot x_i + \beta_3 \cdot w_i + \varepsilon_i \quad , \beta_3 \neq 0$$

$$\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i$$

$$\hat{\beta}_2 = \frac{\hat{\mathrm{Cov}}(x,y)}{\hat{\mathrm{Var}}(x)} \xrightarrow{P} \frac{\mathrm{Cov}(x_i,y_i)}{\mathrm{Var}(x_i)} =$$

$$= \frac{\mathrm{Cov}(x_i, \beta_1 + \beta_2 \cdot x_i + \beta_3 \cdot w_i + \varepsilon_i)}{\mathrm{Var}(x_i)} =$$

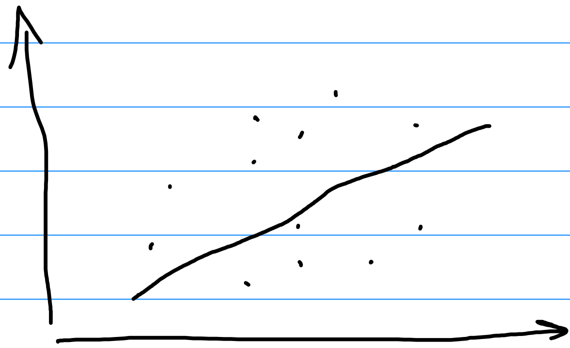
$$= 0 + \beta_2 + \beta_3 \frac{\mathrm{cov}(x_i, w_i)}{\mathrm{Var}(x_i)} + \frac{\overset{0}{\overset{\shortparallel}{\mathrm{Cov}(x_i, \varepsilon_i)}}}{\mathrm{Var}(x_i)}$$

$$\hat{\beta}_2 \xrightarrow{P} \beta_2 + \beta_3 \frac{\mathrm{cov}(x_i, w_i)}{\mathrm{Var}(x_i)}$$

$\beta_3 > 0$, $\mathrm{cov}(x_i, w_i) > 0$ $\Rightarrow$

$\hat{\beta}_2$ inconsistent, biased upwards

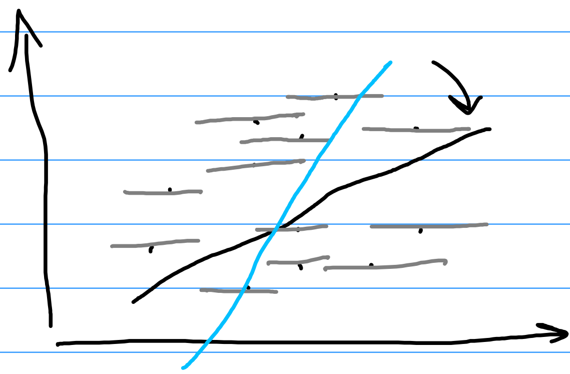
Q2. Measurement error Model



(1) $y_i = \beta_1 + \beta_2 \cdot x_i^*$

$x_i = x_i^* + \varepsilon_i$

$cov(x_i^*, \varepsilon_i) = 0$



$y_i = \beta_1 + \beta_2 x_i + u_i$

(1) $y_i = \beta_1 + \beta_2 (x_i - \varepsilon_i) =$

$= \beta_1 + \beta_2 x_i - \beta_2 \cdot \varepsilon_i$

$u_i = -\beta_2 \cdot \varepsilon_i$

$$\hat{\beta}_2 \xrightarrow{P} \beta_2 + \frac{Cov(x_i, u_i)}{Var(x)} = \beta_2 + \frac{Cov(x_i^* + \varepsilon_i, -\beta_2 \cdot \varepsilon_i)}{Var(x)} =$$

$$= \beta_2 - \beta_2 \frac{\overset{=0}{Cov(x_i^*, \varepsilon_i)} + Var(\varepsilon_i)}{Var(x)} =$$

$$= \beta_2 - \beta_2 \frac{Var(\varepsilon_i)}{Var(x_i^* + \varepsilon_i)} =$$

$$= \beta_2 - \underbrace{\beta_2 \frac{Var(\varepsilon_i)}{Var(x_i^*) + Var(\varepsilon_i)}}$$ $=$

$$= \beta_2 \cdot \frac{Var(x_i^*)}{Var(x_i^*) + Var(\varepsilon_i)}$$

$$\left| \frac{Var(x_i^*)}{Var(x_i^*) + Var(\varepsilon_i)} \right| < 1$$

$\hat{\beta}_2$ inconsistent and biased towards zero

$$\text{plim } \hat{\beta}_2 = \beta_2 + \frac{-\beta_2 \sigma_u^2}{\sigma_u^2 + \sigma_{x^*}^2}$$

$\beta > 0$