Heteroscedasticity

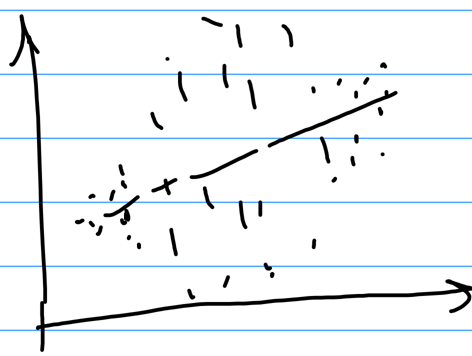
- unbiased, consistent, ineffective
- Test: GQ, White tests
- Remove: WLS, Robust s.e. (White s.e.)

White:

$$se(\hat{\beta}_2) = \sqrt{\frac{1}{n} \frac{\frac{1}{n} \sum (x_i - \bar{x})^2 \cdot \hat{\varepsilon}_i^2}{\widehat{var}(x)^2}}$$

$$\begin{bmatrix} \hat{\delta}_2 & & \\ & \ddots & \\ & & \ddots \end{bmatrix}$$

$$\begin{bmatrix} \hat{\varepsilon}_1^2 & & \\ & \ddots & \\ & & \hat{\varepsilon}_n^2 \end{bmatrix}$$



Models with stochastic regressors

$\ln y_i = \beta_1 + \beta_2 t + \varepsilon_t$ # Year

Deterministic

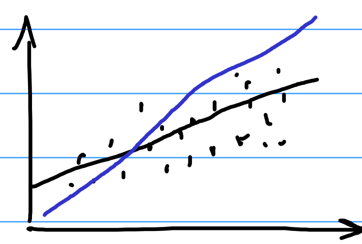
$\pi_t = \beta_1 + \beta_2 \pi_{t-1} + \beta_2 x_t + \varepsilon_t$

Stochastic

Assumptions of Model with stochastic regr.

1. Model is linear and correctly specified

$$y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$$



2. $\{(x_i, y_i),\ i = \overline{1,n}\}$ i.i.d

3. $E(x_i^4) < \infty,\ E(y_i^4) < \infty$

4. $E(\varepsilon_i \mid x_i) = 0 \to E(\varepsilon_i) = 0$

$\searrow \varepsilon_i, x_i$ uncorr.

5.* no multicol. w.p. 1

Under 1-4 $\hat{\beta}_{OLS}$ – consistent, asymptotically normal

Col. 1 $E(\varepsilon_i \mid x_i) = 0 \Rightarrow E(\varepsilon_i) = 0$

$$0 = E(E(\varepsilon_i \mid x_i)) = E(\varepsilon_i) = 0$$

Col. 2 $\quad E(\varepsilon_i | X_i) = 0 \quad \Rightarrow \quad cov(\varepsilon_i, X_i) = 0$

$$cov(\varepsilon_i, X_i) = E(\varepsilon_i X_i) - \overset{=0}{E(\varepsilon_i)} E(X_i)$$

$$E(\varepsilon_i X_i) = E(E(\varepsilon_i X_i | X_i)) =$$

$$= E(x_i \underset{=0}{E(\varepsilon_i | X_i)}) = 0$$

(1) X and ε are independent

$\Rightarrow \hat{\beta}$ - unbiased, consistent

(2) $cov(X, \varepsilon) = 0$

$\Rightarrow \hat{\beta}$ - consistent

(3) $cov(X, \varepsilon) \neq 0$

$\Rightarrow \hat{\beta}$ - inconsistent

$Cov(X, \varepsilon) \neq 0 \Rightarrow X$ - endogenous regressor

Endogeneity

1. Omitted variable
2. Measurement errors
3. Simultaneity

$$\begin{cases} Q^d = \beta_1 + \beta_2 P^d + \varepsilon_i \\ P^d = \alpha_1 + \alpha_2 Q^d + u_i \end{cases}$$

Q1. Omitted variable bias:

$$y_i = \beta_1 + \beta_2 \cdot x_i + \beta_3 w_i + \epsilon_i \,, \quad \beta_3 \neq 0$$

$$\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i$$

$$\hat{\beta}_2 = \frac{\widehat{cov}(x,y)}{\widehat{Var}(x)} \xrightarrow{P} \frac{cov(x,y)}{Var(x)} =$$

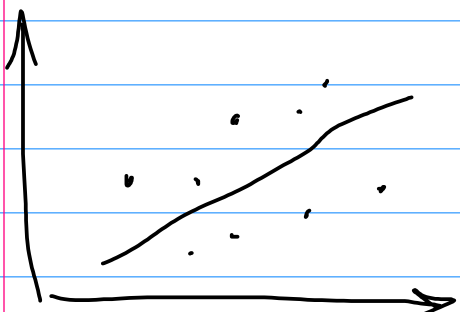
$$= \frac{Cov(x_i, \beta_1 + \beta_2 x_i + \beta_3 w_i + \epsilon_i)}{Var(x_i)} =$$

$$= 0 + \beta_2 + \beta_3 \frac{Cov(x_i, w_i)}{Var(x_i)} + 0$$

$$\hat{\beta}_2 \xrightarrow{P} \beta_2 + \beta_3 \frac{Cov(x_i, w_i)}{Var(x_i)}$$

$\beta_3 > 0$, $cov(x_i, w_i) > 0 \Rightarrow \hat{\beta}_2$ is inconsistent and biased upwards

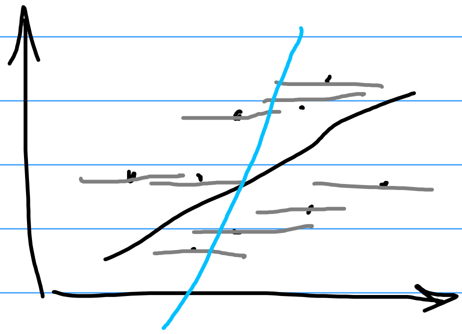
Q2. Model with measurement errors



(1) $y_i = \beta_1 + \beta_2 x_i^*$

$x_i = x_i^* + \varepsilon_i$

$\mathrm{cov}(x_i^*, \varepsilon_i) = 0$



$y_i = \beta_1 + \beta_2 \cdot x_i + u_i$

(1) $y_i = \beta_1 + \beta_2 (x_i - \varepsilon_i)$

$\to \beta_1 + \beta_2 x_i - \beta_2 \cdot \varepsilon_i$

$u_i = -\beta_2 \cdot \varepsilon_i$

$$\hat{\beta}_2 = \beta_2 + \frac{\widehat{\mathrm{Cov}}(x_i, u_i)}{\widehat{\mathrm{Var}}(x_i)} \xrightarrow{p} \beta_2 + \frac{\mathrm{Cov}(x_i, u_i)}{\mathrm{Var}(x_i)} =$$

$$= \beta_2 + \frac{\mathrm{Cov}(x_i^* + \varepsilon_i, -\beta_2 \varepsilon_i)}{\mathrm{Var}(x_i)} =$$

$$= \beta_2 - \beta_2 \frac{\mathrm{Var}(\varepsilon_i)}{\mathrm{Var}(x_i^*) + \mathrm{Var}(\varepsilon_i)} =$$

$$= \beta_2 \cdot \frac{\mathrm{Var}(x_i^*)}{\mathrm{Var}(x_i^*) + \mathrm{Var}(\varepsilon_i)}$$

$$\frac{Var(X_i^*)}{Var(X_i^*) + Var(\varepsilon_i)} < 1$$

regardless of sign of β_2

$\hat{\beta}_2$ is inconsistent and

biased towards zero