TSLS

$$y_i = \beta_0 + \beta_1 x_i^{(1)} + \ldots + \beta_p x_i^{(P)} + \beta_{p+1} W_i^{(1)} + \ldots + \beta_{p+r} W_i^{(M)} + \varepsilon_i$$

X_i — endogenuous rgr.

W_i — exogenuous rg 2.

Z_i — instrument

$\begin{matrix} P \\ \sim \\ m \end{matrix}$

$m = p$ (IV)   exactly identified

$m > p$ (TSLS)   overidentification

$m < p$ (—)   underidentified

$\left. \right\} m \geq p$

1. $X_i^{(1)}$ | $Z_i^{(1)}, \ldots, Z_i^{(P)}, W_i^{(1)}, \ldots, W_i^{(2)}$

$\vdots$

$X_i^{(P)}$ | $Z_i^{(1)}, \ldots, Z_i^{(P)}, W_i^{(1)}, \ldots, W_i^{(2)}$

2. y_i | $\hat{X}_i^{(1)}, \ldots, \hat{X}_i^{(P)}, W_i^{(1)}, \ldots, W_i^{(2)}$

$\hat{\beta}^{TSLS}$ consistent

exogenous $Cov(Z_i^{(1)}, \varepsilon_i) = 0$

$\ldots$

$Cov(Z_i^{(P)}, \varepsilon_i) = 0$

Relevant

Tests for IV

Wu - Hausman Test

$$\left(\hat{\beta}^{TSLS} - \hat{\beta}^{OLS}\right)'\left(\hat{V}(\hat{\beta}^{TSLS}) - \hat{V}(\hat{\beta}^{OLS})\right)^{-1}\left(\hat{\beta}^{TSLS} - \hat{\beta}^{OLS}\right) \sim \chi^2_k$$

if H_0 is not rej $\Rightarrow$ $\hat{\beta}^{OLS}$

if H_0 is rej $\Rightarrow$ $\hat{\beta}^{TSLS}$

	inconsistent	consistent efficiency
	Consistent	consistent not efficient

endogeneity exogeneity

Exogeneity of instruments

Sargen test
(J - tat) $m > p$

H_0: all instrument are exog.

1. $y_i \mid \hat{X}_i^{(1)}, \dots, \hat{X}_i^{(p)}, W_i^{(1)}, \dots, W_i^{(2)} \Rightarrow \hat{\varepsilon}_i$

2. $\hat{\varepsilon}_i \mid Z_i^{(1)}, \dots, Z_i^{(n)}, W_i^{(1)}, \dots W_i^{(2)}$

$J = m \cdot F \sim \chi^2(m-p)$ m — # instruments

F — F-test stat.

$\gamma_i^{(1)} = \dots = \gamma_i^{(m)} = 0$

Relevance of instruments

(F-test for weak instruments)

$$\hat{X}_i \mid z_i^{(1)}, ..., z_i^{(N)}, w_i^{(1)} ... w_i^{(2)} \qquad : UR$$

$$\hat{X}_i \mid w_i^{(1)}, ..., w_i^{(2)} \qquad R$$

F-test $\qquad H_0: \gamma_1 = ... = \gamma_m = 0$

if H_0 rej $\Rightarrow$ relevant $\left(F > 10 \right)$