θ $\qquad \hat{\theta}$

$$MSE(\hat{\theta}) = Var(\hat{\theta}) + Bias^2$$

1) Unbiasedness $\qquad E(\hat{\theta}) = \theta$

2) Consistency $\qquad \text{plim}_{n \to \infty} \hat{\theta} = \theta$

$$\lim_{n \to \infty} P\{|\theta - \hat{\theta}| < \varepsilon\} = 1$$

3) Efficiency $\qquad \hat{\theta}, \tilde{\theta} \in C_{LUE}$

$$Var(\hat{\theta}) \leq Var(\tilde{\theta}) \qquad \forall \vec{\theta}$$

$$y_i = \beta_0 + \beta_1 \cdot x_{1i} + \dots + \beta_k x_{ki} + \varepsilon_i$$

1) Model is correctly specified

2) $X = \begin{bmatrix} 1 & x_{11} & & x_{k1} \\ \vdots & \vdots & \dots & \vdots \\ 1 & x_{1n} & & x_{kn} \end{bmatrix}$ – deterministic

3) $Var(\varepsilon_i) = \sigma^2_\varepsilon$ – const

4) $Cov(\varepsilon_i, \varepsilon_j) = 0$

5) $\varepsilon_i \sim N(0, \sigma^2_\varepsilon)$

GMT: 1–5 hold $\Rightarrow \hat{\beta}_{OLS} = \begin{bmatrix} \hat{\beta}_0 \\ \vdots \\ \hat{\beta}_k \end{bmatrix}$ – BLUE

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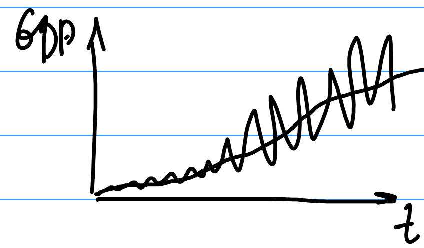
1) Log transf.; Quadr. Terms; Endogeneity → omitted variable; simultaneity; measurement error

2) Stochastic Regressors

3) $Var(\varepsilon_i) \neq \sigma^2_\varepsilon$ – Heteroscedasticity



4) Autocorrelation

5) Asymptotic theory

(N 1) $\nexists$ X, Y - independent $\Rightarrow$ $\rho_{x,y} = 0$

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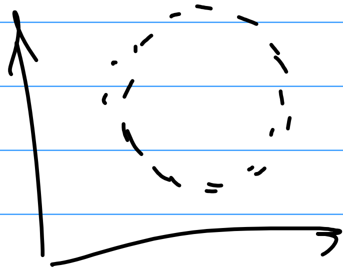

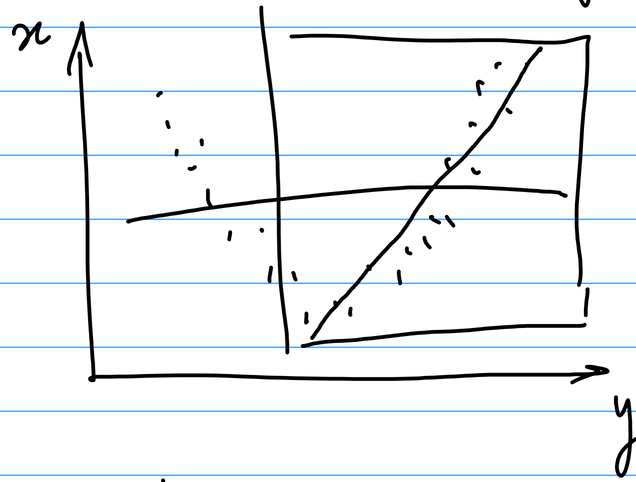

Non-parametric

Rank correlation

$$p(X, Y) = P(X)P(Y) \quad \text{independent}$$

$$E(Y|X) = E(Y) \qquad E(X|Y) = E(X) \quad \text{unpredictability}$$

$$E(XY) = E(X)E(Y) \quad \text{uncorrelated}$$

(N 2) X $\quad E(X) = \mu_x \quad Var(X) = \sigma_X^2$

$\{X_1, \ldots, X_n\}$

$$\hat{\mu}_x = \frac{X_1 + \ldots + X_{2K-1}}{k} \qquad \bar{X} = \frac{\Sigma x_i}{n}$$

1) Unbiased

2) $Var(\hat{\mu}_x) = \frac{\sigma_x^2}{k} > Var(\bar{X}) = \frac{\sigma_x^2}{n}$ $(n > k)$

N3 X $E(X) = \mu_x$ $Var(X) = \sigma_x^2$

$$Z = \frac{1}{2}X_1 + \frac{1}{4}X_2 + \dots + \frac{1}{2^n}X_n$$

1) n - finite biased

n - infinity $\lim_{n\to\infty} E(Z) = \lim_{n\to\infty}\left(\frac{1}{2} + \dots + \frac{1}{2^n}\right)\mu_x =$

$$= \frac{1/2}{1-1/2}\mu_x = \mu_x$$

2) $\lim_{n\to\infty} Var(Z) = \lim_{n\to\infty}\left(\frac{1}{4} + \dots + \frac{1}{2^{2n}}\right)\sigma_x^2 =$

$$\frac{1/4}{1-1/4}\sigma_x^2 = \frac{\sigma_x^2}{3}$$

N4 X $E(X) = \mu_x$ $Var(X) = \sigma_x^2$

$$\hat{\mu}_x = \frac{n+2}{n^2+3n+1}\sum_{i=1}^{n} X_i$$

1) $\hat{\mu}_x$ - biased, asymptotically unbiased

2) $\hat{\mu}_x$ - consistent, since $Var(\hat{\mu}_x) \to 0$