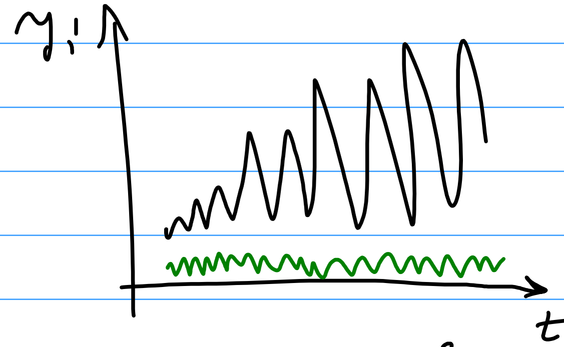
Logarithmic models



Linear: $y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$

$$\frac{dy}{dx} = \beta_2 \qquad x \uparrow 1 \qquad y \nearrow \beta_2$$

Log-log: $\ln y_i = \beta_1 + \beta_2 \ln x_i + \varepsilon_i$ $\qquad y_i = e^{\beta_1} \cdot x_i^{\beta_2} \cdot \varepsilon_i$

$$\frac{d \ln y_i}{d \ln x_i} = \frac{100\, dy_i / y_i}{100\, dx_i / x_i} = \beta_2$$

$$\underline{Y = A \cdot K^{\beta} \cdot L^{1-\beta}}$$

$H_0: \beta_2 + \beta_3 = 1$

$\rightarrow$ $x \uparrow 1\%$ $\quad y \uparrow \beta_2 \%$

$\ln Y = \beta_1 + \beta_2 \ln K + \beta_3 \ln L$

Log-lin: $\ln y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$

$x \uparrow 1 \quad y \uparrow 100\beta_2\%$

$$\frac{d \ln y_i}{d x_i} = \beta_2 \qquad \frac{100 \quad dy_i / y_i}{d x_i} = 100\, \beta_2$$

Lin-log: $y_i = \beta_1 + \beta_2 \ln x_i + \varepsilon_i$

$x \uparrow 1\% \quad y \nearrow \frac{\beta_2}{100}$

<...>

log log log

Problem 7. (UoL Exam) A regression of consumption (C) on income (Y) and unemployment (U) (all variables are index numbers) using annual data 1961-82 for the UK produced the following results:

$\alpha \quad \beta_1 \quad \beta_2$

$$\hat{C}_t = 17880 + 0.7527Y_t + 0.030U_t, \quad R^2 = 0.992 \qquad (1)$$
$$(2817.0) \quad (0.026) \qquad (0.798)$$

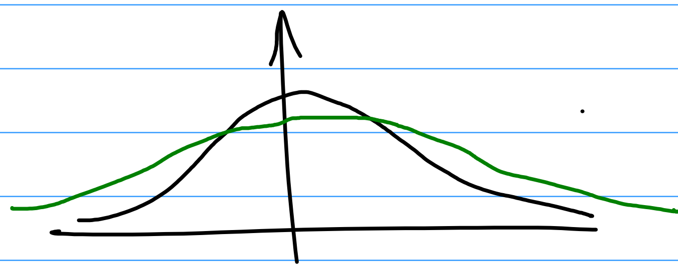
(figures in brackets are standard errors) with a table of correlation coefficients between variables of:

	C	Y	U
C	1.00	0.996	0.783
Y	0.996	1.00	0.771
U	0.783	0.771	1.00

$> 0,7$

$n = 22$

1) $t_{\hat{\alpha}} = \dfrac{1,7}{0,3} = 6,7 \sim t_{19}$



2) $C_i = \alpha + \beta Y_i + \varepsilon_i$

assume $\beta_2 < 0$

$\Rightarrow \hat{\beta}_1 \downarrow$

3) Assume : $\beta_1 \geq 0$; $\beta_2 \leq 0$

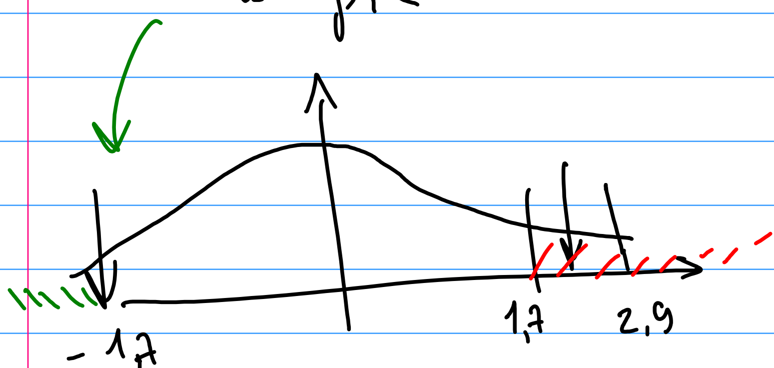
$H_0: \beta_1 \geq 0 \ (\beta_1 = 0)$

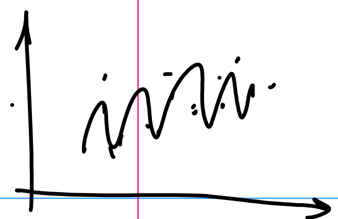
$H_a: \beta_1 < 0$

$t = 28$

$H_0: \beta_1 = 0$

$H_a: \beta_1 > 0$





Models with quadratic and interactive terms

$$Y_i = \beta_1 + \beta_2 X_i + \beta_3 X_i^2 + \varepsilon_i$$

$$\frac{dy_i}{dX_i} = \beta_2 + 2\beta_3 x_i$$

→ β_2 rate of change of y when $x_i = 0$

$$Y_i = \beta_1 + (\beta_2 + \beta_3 X_i) X_i + \varepsilon_i$$

→ β_3 rate of change of X_i per unit change of x_i