Dummy Variables

$$c_i = \begin{cases} 1, & \text{if bad} \\ 2, & \text{if med.} \\ 3, & \text{if good} \end{cases}$$

$$y_i = \beta_0 + \beta_1 \cdot c_i + \beta_2 \cdot P_i + \varepsilon_i \quad \oplus \beta_4 \cdot d_m$$

$$y_i = \beta_0 + \beta_1 d_b + \beta_2 \cdot d_g + \beta_3 P_i + \varepsilon_i$$

Reference model : if $c_i = 1$ $\quad y_i = \beta_0 + \beta_3 P_i + \varepsilon_i$

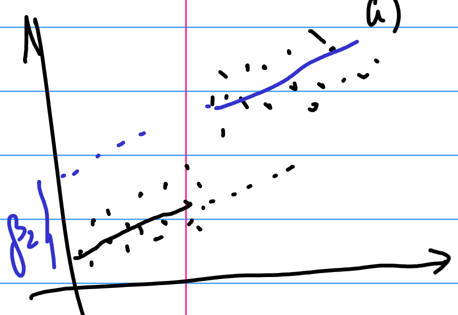
$$X = \begin{bmatrix} 1 & 1 & & & p_1 \\ \vdots & \vdots & & & \vdots \\ \vdots & & \vdots & & \vdots \\ \vdots & & & \vdots & \vdots \\ 1 & & & 1 & p_n \end{bmatrix}$$

dummy variable trap

$$\hat{\beta} = (X'X)^{-1} X'y$$

Question 1: $\quad Y_t = \beta_0 + \beta_1 \cdot P_t + \varepsilon_t$

a) $D_t = \begin{cases} 0, & \text{before crisis} \\ 1, & \text{after crisis} \end{cases}$

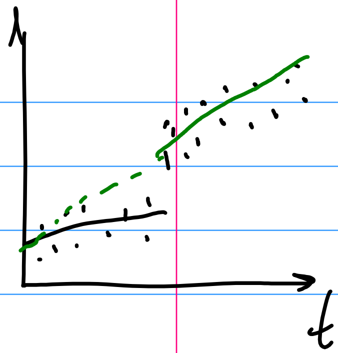


Change in the intercept

$$Y_t = \beta_0 + \beta_1 \cdot P_t + \beta_2 \cdot D_t + \varepsilon_t$$

Ref. mod. B.C. : $\quad y_t = \beta_0 + \beta_1 \cdot P_t + \varepsilon_t$

A.C. : $\quad y_t = (\beta_0 + \beta_2) + \beta_1 \cdot P_t + \varepsilon_t$



b) change in slope:

$$Y_t = \beta_0 + \beta_1 \cdot P_i + \beta_2 \cdot D_i \cdot P_i + \varepsilon_i$$

Ref. mod. B.C.: $Y_i = \beta_0 + \beta_1 P_i + \varepsilon_i$

A.C.: $y_i = \beta_0 + (\beta_1 + \beta_2) \cdot P_i + \varepsilon_i$

c) change in both const. and slope:

$$Y_t = \beta_0 + \beta_1 \cdot P_i + \beta_2 \cdot D_i + \beta_3 \cdot D_i \cdot P_i + \varepsilon_i$$

Ref. mod. BC: $Y_i = \beta_0 + \beta_1 \cdot P_i + \varepsilon_i$

AC: $Y_i = (\beta_0 + \beta_2) + (\beta_1 + \beta_3) P_i + \varepsilon_i$

F test: $H_0: \beta_2 = \beta_3 = 0$

Chow test (for structural breaks in t.s.)

Pooled reg. $y_i = \beta_1 + \beta_2 x_{1i} + \dots \beta_k x_{k-1,i} + \varepsilon_i$ $\quad N = n_A + n_B$

Subsample A $(D_i = 0)$ $\quad y_i = \beta_1^A + \beta_2^A x_{1,i} + \dots + \beta_k^A x_{k-1,i} + \varepsilon_i$ $\quad n_A$

Subsample B $(D_i = 1)$ $\quad y_i = \beta_1^B + \beta_2^B x_{1i} + \dots + \beta_k^B x_{k-1,i} + \varepsilon_i$ $\quad n_B$

$H_0: \beta_1^A = \beta_1^B, \dots, \beta_k^A = \beta_k^B$

H_a: at least one l.r. doesn't hold

$$F = \frac{(RSS_P - RSS_A - RSS_B)/k}{(RSS_A + RSS_B)/(n-2\cdot k)} \sim F(k, n-2k)$$

$$F = \frac{(\text{improvement in fit})/(\text{extra d.o.f.})}{(\text{remaining RSS})/(\text{remaining d.o.f})}$$

Problem 2. (ICEF exam) A student decided to investigate the market of private mathematics teachers in Moscow, with particular interest to those who can teach in English. He took a random sample of 30 profiles of teachers who provide private teaching in math (taken from population of 300 profiles registered in certain internet site) and run some regressions trying to find factors influencing the prices of teaching ($PRICE_i$ - price of a standard two-hour lesson in thousands of roubles, $DIST_i$ - distance in the number of metro stations from the center of Moscow to the teacher's place, $HOME_i$ - dummy variable indicating visit of the tutor to the client, ENG_i - dummy variable indicating ability to teach the subject in English):

$$\widehat{PRICE}_i = \underset{(0.49)}{6.59} - \underset{(0.06)}{0.16}DIST_i \qquad R^2 = 0.185 \qquad (1)$$

R: $$\widehat{PRICE}_i = \underset{(0.40)}{4.51} + \underset{(0.58)}{2.54}HOME_i \qquad R^2 = 0.40 \qquad (2)$$

UR: $$\widehat{PRICE}_i = \underset{(0.64)}{\overset{\beta_0}{5.13}} - \underset{(0.06)}{0.08}DIST_i + \underset{(0.95)}{1.95}HOME_i + \underset{(0.07)}{0.07}DIST*HOME_i \qquad R^2 = 0.437$$

(3)

$$\widehat{PRICE}_i = \underset{(0.61)}{4.52} - \underset{(0.06)}{0.08}DIST_i + \underset{(0.75)}{2.18}HOME_i + \underset{(0.76)}{1.58}ENG_i - \underset{(1.09)}{0.39}\underbrace{HOME}_{1}*\underbrace{ENG_i}_{1} \qquad R^2 = 0.553$$

(4)

a) $4,51 + 2,54 = 7,05$

b) home = 1 $\qquad \hat{P}_i = 7,08 - 0,01\, D_i$

c) is mod. (3) sig.:

$$F = \frac{0,437/3}{(1-0,437)/(30-4)} = 6,73$$

$$F_{crit} = 4,64$$

is factor "dist" sig.:

$$H_0: \beta_1 = \beta_3 = 0$$

$$F = \frac{(R^2_{UR} - R^2_R)/q}{(1-R^2_{UR})/n-k} = \frac{0,037/2}{(1-0,437)/(30-4)} = 0,85$$

$F_{crit, 5\%} = 3{,}37$

is factor "home" sig.:

$H_0: \quad \beta_2 = \beta_3 = 0$

$$F = \frac{(0{,}437 - 0{,}185)/2}{(1-0{,}437)/(30-4)} = 5{,}82$$

d)

$$F = \frac{(RSS_P - RSS_A - RSS_B)/2}{(RSS_A + RSS_B)/(30 - 2\cdot 2)} \sim F(k, n-2k)$$

Home < h/0, H/1

Eng < e/0, E/1

$$F = \frac{(RSS_P - RSS_{he} - RSS_{He} - RSS_{hE} - RSS_{HE})/2\cdot 3}{(RSS_{he} + RSS_{He} + RSS_{hE} + RSS_{HE})/(30 - 4\cdot 2)}$$

Alternative F-test:

$$UR: P_i = \beta_0 + \beta_1 \cdot D_i + \beta_2 \cdot H_i + \beta_3 \cdot E_i +$$
$$+ \beta_4 \cdot H_i D_i + \beta_5 \cdot E_i \cdot D_i +$$
$$+ \beta_6 \cdot H_i E_i D_i + \beta_7 \cdot H_i \cdot E_i + u_i$$

F-test: $H_0: \beta_2 = \ldots = \beta_7 = 0$

$$F = \frac{(R^2_{UR} - R^2_{(1)})/6}{(1-R^2_{UR})/(30-8)}$$