# TSLS

$$y_i = \beta_0 + \beta_1 \cdot x_i^{(1)} + \dots + \beta_p x_i^{(p)} + \beta_{p+1} w_i^{(1)} + \dots + \beta_{p+2} w_i^{(2)} + \varepsilon_i$$

| | | | |
|---|---|---|---|
| $x_i$ | – | endogenous variables | $p$ |
| $w_i$ | – | exogenous variables | $k$ |
| $z_i$ | – | instruments | $m$ |

$m = p$ (IV) exactly identified
$m > p$ (TSLS) overidentification
$m < p$ (—>) underidentification

$\Big\} \; m \geq p$

1 step

$$x_i^{(1)} \mid z_i^{(1)}, \dots, z_i^{(m)}, w_i^{(1)}, \dots, w_i^{(r)}$$
$$\vdots$$
$$x_i^{(p)} \mid z_i^{(1)}, \dots, z_i^{(m)}, w_i^{(1)}, \dots, w_i^{(r)}$$

2 step

$$y_i \mid \hat{x}_i^{(1)}, \dots, \hat{x}_i^{(p)}, w_i^{(1)}, \dots, w_i^{(m)}$$

$\hat{\beta}^{TSLS}$ consistent

exogenous $\quad cov(z_i^{(1)}, \varepsilon_i) = 0, \dots, cov(z_i^{(p)}, \varepsilon_i) = 0$

relevant

# Test for TSLS

## Wu-Hausman test

$H_0$: exogencity of regressors

$$(\hat{\beta}^{TSLS} - \hat{\beta}^{OLS})' \left(\hat{V}(\hat{\beta}^{TSLS}) - \hat{V}(\hat{\beta}^{OLS})\right)^{-1} (\hat{\beta}^{TSLS} - \hat{\beta}^{OLS}) \sim \chi^2_k$$

| | | | |
|---|---|---|---|
| $H_0$ is rejected | $\hat{\beta}^{TSLS}$ | consistent | consistent |
| $H_0$ is not rejected | $\hat{\beta}^{OLS}$ | cons., efficient | inconsistent |
| | | exog | end |

## Sargan's Test for exogeneity ($\gamma$-test)

$H_0$: instruments are exogenous $\quad m > p$

$$y_i \mid \hat{x}_i^{(1)}, \dots, \hat{x}_i^{(p)}, w_i^{(1)}, \dots, w_i^{(r)} \Rightarrow \hat{\varepsilon}_i$$

$$\hat{\varepsilon}_i \mid z_i^{(1)}, \dots, z_i^{(m)}, w_i^{(1)}, \dots, w_i^{(r)}$$

$\gamma = m \cdot F \sim \chi^2_{m-p}$ $\quad m$ - # instruments

$F$ - F-stats for

$H_0: \gamma_1 = \dots = \gamma_m = 0$

# F-test for weak instruments (for relevance)

$X_i \mid z_i^{(1)}, \ldots, z_i^{(m)}, w_i^{(1)}, \ldots, w_i^{(r)}$

F stats for $H_0: \gamma_1 = \ldots = \gamma_m$

$F > 10 \Rightarrow$ relevance

$F \leq 10 \Rightarrow$ weak instruments