$\theta \qquad \hat{\theta}$ - estimator

1) Unbiasedness $\quad E(\hat{\theta}) = \theta \qquad MSE(\hat{\theta}) = Var(\hat{\theta})$
$+ bias^2$

2) Consistency $\quad \underset{n \to \infty}{\text{plim}} \; \hat{\theta} = \theta$

$$\lim_{n \to \infty} P\{|\hat{\theta} - \theta| < \varepsilon\} = 1$$

3) Efficiency $\quad \hat{\theta}, \tilde{\theta} \in C_{LUE}$

$$Var(\hat{\theta}) \leq Var(\tilde{\theta}) \quad \forall \tilde{\theta} \in C_{LUE}$$

$$y_i = \beta_0 + \beta_1 x_{1i} + \dots + \beta_k x_{ki} + \varepsilon_i$$

1) Model is specified correctly

2) $X = \begin{bmatrix} 1 & x_{11} & & x_{k1} \\ \vdots & \vdots & \dots & \vdots \\ 1 & x_{1n} & & x_{kn} \end{bmatrix}$ - deterministic

3) $Var(\varepsilon_i) = \sigma^2_\varepsilon$ - const

4) $Cov(\varepsilon_i, \varepsilon_j) = 0, \; i \neq j$

5) $\varepsilon_i \sim N(0, \sigma^2_\varepsilon)$

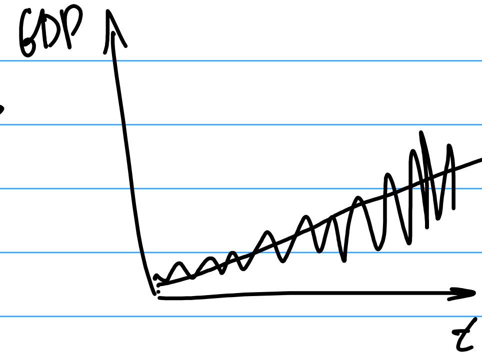
GMT: 1-5 $\Rightarrow \hat{\beta}_{OLS} = \begin{bmatrix} \hat{\beta}_0 \\ \vdots \\ \hat{\beta}_k \end{bmatrix}$ - BLUE

1) log transform, quadratic,

Endogeneity
- omitted variable
- simultaneity
- measurement errors

2) X - stochastic regressors

3) Homoscedastistic errors →



Heteroscedastistic

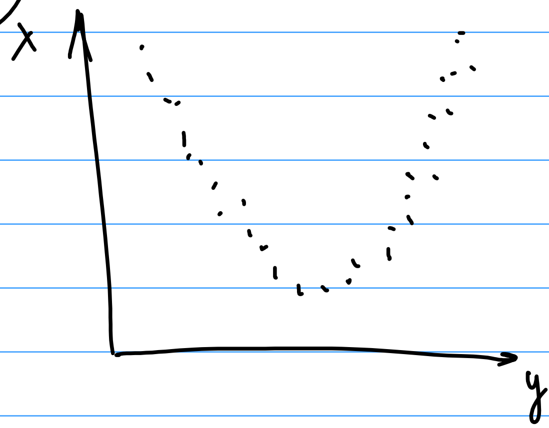
$Var(\varepsilon_i) \neq$ const

4) Autocorrelation

5) Asymptotic theory

Binary Choice Models

N1 X, Y independent $\Rightarrow$ $\rho_{x,y}=0$



$\Leftarrow$?

$\rho_{x,y}=0$

Non-parametric Statistics

$p(X,Y) = P(X)\,P(Y)$ independence

$E(Y|X) = E(Y)$ $\quad$ $E(X|Y) = E(Y)$ unpredictability

$E(XY) = E(X) \cdot E(Y)$ uncorrelatedness

N2 X $\quad$ $E(X) = \mu_x$ $\quad$ $Var(X) = \sigma_x^2$

$\{X_1, \ldots, X_n\}$

$$\hat{\mu}_x = \frac{X_1 + \ldots + X_{2k-1}}{k}$$

1) unbiased

2) $Var(\hat{\mu}_x) > Var(\bar{x})$

$$\frac{\sigma_x^2}{k} > \frac{\sigma_x^2}{n} \qquad k < n$$

N3 $\quad X \quad E(X_i) = \mu_x, \quad Var(X_i) = \sigma_x^2$

$$\{X_1, \dots, X_n\}$$

$$Z = \frac{1}{2} X_1 + \frac{1}{4} \cdot X_2 + \dots \frac{1}{2^n} \cdot X_n$$

1) n - finite $\Rightarrow$ Z - biased

n - infinite $\lim\limits_{n \to \infty} E(Z) = \lim\limits_{n \to \infty} \left(\frac{1}{2} + \dots + \frac{1}{2^n} \right) \mu_x =$

$$= \frac{1/2}{1 - 1/2} \mu_x = \mu_x$$

asymptotically unbiased

2) $\lim\limits_{n \to \infty} Var(Z) = \lim\limits_{n \to \infty} \left(\frac{1}{4} + \dots + \frac{1}{2^{2n}} \right) \sigma_x^2 = \frac{1/4}{3/4} \sigma_x^2 = \frac{\sigma_x^2}{3}$

N4 $\quad X \quad E(X) = \mu_x \quad Var(X) = \sigma_x^2$

$$\hat{\mu}_x = \frac{n+2}{n^2+3n+1} \sum_{i=1}^{n} X_i$$

$\hat{\mu}_x$ - biased

as. unbiased

$$y_i = \alpha + \beta x_i + \varepsilon_i \qquad i = \overline{1,n}$$

$$RSS = \sum (y_i - b x_i)^2 \to \min_b$$

$$\hat{\beta} = \frac{\widehat{cov}(x,y)}{\widehat{Var}(x)} = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{n} (X_i - \bar{X})^2}$$

$$\hat{\alpha} = \bar{y} - \hat{\beta} \cdot \bar{x}$$

assume $\alpha = 0$ $\qquad \tilde{\beta} = \dfrac{\sum x_i y_i}{\sum x_i^2}$