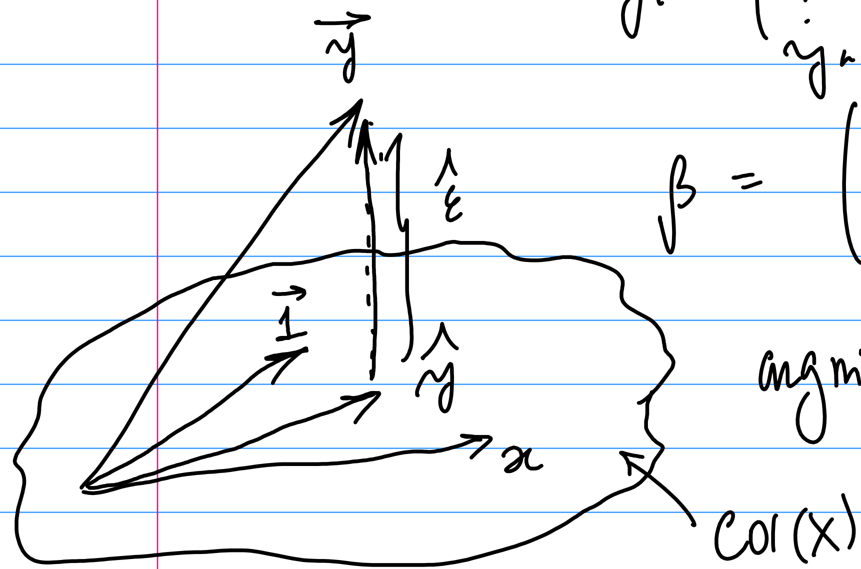
$$y_i = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} \qquad X = \begin{pmatrix} 1 & x_{11} & \cdots & x_{k1} \\ \vdots & \vdots & & \vdots \\ 1 & x_{1n} & \cdots & x_{kn} \end{pmatrix}$$

$$\beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{pmatrix}$$



$$\arg\min \|y - \hat{y}\|^2$$

$$X'\hat{\varepsilon} = 0$$

$$X'(y - \hat{y}) = 0$$

$$X'(y - X \cdot \hat{\beta}) = 0$$

$$\frac{X'y}{X'X}$$

$$X'y - X'X \cdot \hat{\beta} = 0$$

$$X'y = X'X\hat{\beta}$$

$$\hat{\alpha} = \bar{y} - \hat{\beta} \cdot \bar{x}$$

$$\hat{\beta} = \frac{Cov(x,y)}{Var(x)} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} \qquad \boxed{(X'X)^{-1} X'y = \hat{\beta}}$$

$$y_i = \hat{\alpha} + \hat{\beta} x_i + \hat{\varepsilon}_i$$

$$y_i = \alpha + \beta \cdot x_i + \varepsilon_i$$

$$\vec{1}'\hat{\varepsilon} = 0$$

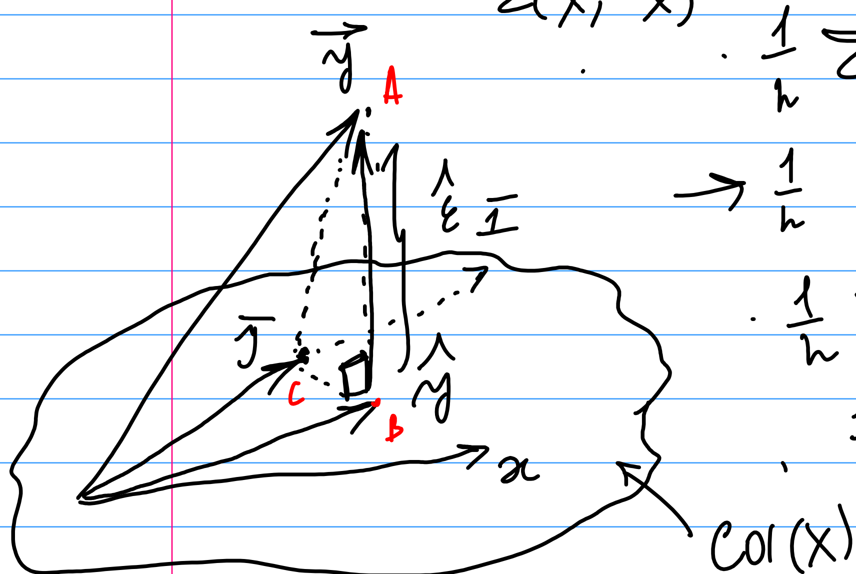
$$\sum \hat{\varepsilon}_i = 0$$

$$\frac{1}{n} \sum \hat{\varepsilon}_i = 0$$

$$\rightarrow \frac{1}{n} \sum \hat{Y}_i = \frac{1}{n} \sum y_i$$

$$\frac{1}{n} \sum X_i \hat{\varepsilon}_i = 0$$

$$\frac{1}{n} \sum \hat{Y}_i \hat{\varepsilon}_i = 0$$



$$\underline{TSS = ESS + RSS}$$

$$AC^2 = \sum (y_i - \bar{y})^2 = TSS$$

$$AB^2 = \sum (y_i - \hat{y}_i)^2 = \sum \hat{\varepsilon}_i^2 = RSS$$

by Pythagor.-Thm

$$BC^2 = \sum (\hat{y}_i - \bar{y})^2 = ESS$$

$$\frac{1}{n} \sum \hat{\varepsilon}_i = 0 \quad \longleftrightarrow \quad \hat{\varepsilon}_i^2 \xrightarrow[\beta_1, \beta_2]{} \min$$

$$-2 \sum \underbrace{(y_i - \hat{\beta}_1 - \hat{\beta}_2 \cdot x)}_{\hat{\varepsilon}_i} = 0$$

$$\frac{1}{n} \sum x_i \hat{\varepsilon}_i = 0 \quad \longleftrightarrow \quad -2 \sum (y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i) x_i = 0$$

$$y_i = \alpha + \beta \cdot x_i + \varepsilon_i \qquad \varepsilon_i \sim N(0, \sigma^2)$$

$$y_i = \hat{\alpha} + \hat{\beta} x_i + \hat{\varepsilon}_i$$

$\hat{\alpha}, \hat{\beta}$ - OLS estimators

1) $\boxed{\alpha = 0}$ $\quad y_i = \tilde{\beta} x_i + \tilde{\varepsilon}_i$

$$\hat{y}_i = \tilde{\beta} x_i$$

$$\hat{\varepsilon}_i^2 = \sum (y_i - \hat{y}_i)^2 = \sum (y_i - \tilde{\beta} x_i)^2$$

$$\frac{\partial RSS}{\partial \beta} = 0 \Rightarrow \tilde{\beta} = \frac{\sum x_i y_i}{\sum x_i^2}$$

$$Var(\tilde{\beta}) = \frac{\sigma^2}{\sum x_i^2}$$

$$Var(\tilde{\beta}) < Var(\hat{\beta})$$

$$Var(\hat{\beta}) = \frac{\sigma^2}{\sum x_i^2 - n\bar{x}^2}$$