Stochastic regressor model

"*When an explanatory variable in a regression model has a random component, it is described as a stochastic regressor.* + *When a stochastic regressor is used in a regression model, the Gauss-Markov condition that the explanatory variables should be independent of the disturbance term is violated.* ⌐ *Consequently OLS regression estimates will be biased.* ⌐ *However, they will be consistent because the bias will disappear in large samples.*" ⌐

Endogenous $\quad$ $cov(x, u) \neq 0$

1. omitted variable
2. measurement errors
3. simultaneity

GM:
1. $y = X\beta + u$

2. w.p.1 . no perfect multicollinearity

3. $E(u|X) = 0$

4. $Var(u|X) = \sigma^2 I \quad \begin{bmatrix} \sigma^2 & & 0 \\ & \ddots & \\ 0 & & \sigma^2 \end{bmatrix}$

$\hat{\beta}$ — BLUE (linear by y, conditionally unbiased)

$E(\hat{\beta}) \neq \beta \qquad E(\hat{\beta}|X) = \beta$

$$Q_2. \quad y = \beta_2 Z + u$$

$$X = Z + \omega \qquad \omega \perp Z$$

$$y = \beta_2 X + \underbrace{u - \beta_2 \omega}_{V}$$

$$\hat{\beta}_2 = \frac{\sum X_i Y_i}{\sum X_i^2} = \frac{\sum X_i (\beta_2 X + u_i - \beta_2 \omega)}{\sum X_i^2} =$$

$$= \beta_2 + \frac{\sum X_i u_i}{\sum X_i^2} - \beta_2 \frac{\sum X_i \omega_i}{\sum X_i^2} =$$

$$= \beta_2 + \frac{\sum (X_i - \bar{X})(u_i - \bar{u}) + n \bar{X} \bar{u}}{\sum (X_i - \bar{X})^2 + n \bar{X}^2} -$$

$$\beta_2 \cdot \frac{\sum (X_i - \bar{X})(\omega_i - \bar{\omega}) + n \cdot \bar{X} \bar{v}}{\sum (X_i - \bar{X})^2 + n \bar{X}^2} \xrightarrow{p}$$

$$\beta_2 + \frac{Cov(X_i, u_i)^{=0} + \overset{=0}{\mu_x \mu_u}}{var(x_i) + \mu_x^2} -$$

$$- \beta_2 \cdot \frac{Cov(X_i, \omega_i) + \overset{=0}{\mu_x \mu_\omega}}{var(x_i) + \mu_x^2} =$$

$$= \beta_2 - \beta_2 \frac{cov(z_i + w_i, w_i)}{Var(x_i) + \mu_x^2} =$$

$$= \beta_2 - \beta_2 \frac{\sigma_w^2}{\sigma_r^2 + \mu_k^2}$$

Instrumental variable

(2SLS)

$$y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$$

endogenuous $cov(x_i, \varepsilon_i) \neq 0$

relevant $cov(z_i, x_i) \neq 0$

exogenuous $cov(z_i, \varepsilon_i) = 0$

∟ instrumental variable (valid)

1 step $\qquad \widehat{x}_i = \hat{\theta}_1 + \hat{\theta}_2 z_i \qquad \hat{\theta}_2 = \frac{\widehat{Cov}(x, z)}{\widehat{Var}(z)}$
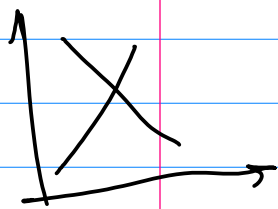
2 step $\qquad y_i = \hat{\beta}_1 + \hat{\beta}_2 \hat{x}_i + \varepsilon_i$

$$\beta_2^{TSLS(IV)} = \frac{\widehat{cov}(y, z)}{\widehat{cov}(x, z)} \qquad - \quad consistent$$

$$\triangleright \quad \hat{\beta}_2^{TSLS} = \frac{\widehat{Cov}(\hat{X}, y)}{\widehat{Var}(\hat{X})} = \frac{\widehat{Cov}(\hat{\theta}_1 + \hat{\theta}_2 z, y)}{\widehat{Var}(\hat{\theta}_1 + \hat{\theta}_2 z)} =$$

$$= \frac{\hat{\theta}_2 \, \widehat{Cov}(z, y)}{\hat{\theta}_2^2 \, \widehat{Var}(z)} = \frac{\widehat{Cov}(z, y)}{\hat{\theta}_2 \cdot \widehat{Var}(z)} =$$

$$= \frac{\widehat{Cov}(z, y)}{\frac{\widehat{Cov}(X, z)}{\widehat{Var}(z)} \cdot \widehat{Var}(z)} = \frac{\widehat{Cov}(z, y)}{\widehat{Cov}(x, z)}$$

$$\hat{\beta}_2^{TSLS} \xrightarrow{P} \frac{Cov(z_i, y_i)}{Cov(x_i, z_i)} = \frac{Cov(z_i, \beta_1 + \beta_2 x_i + \epsilon_i)}{Cov(x_i, z_i)} =$$

$$= \beta_2 \frac{\overbrace{Cov(z_i, x_i)}^{\neq 0} + \overbrace{Cov(z_i, \epsilon_i)}^{=0 \text{ \textcolor{red}{exogenous}}}}{\underset{\textcolor{red}{relevance}}{Cov(z_i, x_i)}} = \beta_2$$

Q_3 Demand: $\ln Q_i = \beta_1 + \beta_2^{<0} \ln P_i + c_i$

Supply: $\ln Q_i = \gamma_1 + \gamma_2 \cdot \ln P_i + \gamma_3 \ln T_i + u_i$ $\downarrow_0$

a) $\ln Q_i = \hat{\beta_1} + \hat{\beta_2} \ln P_i + c_i$ — α_S

$\hat{\beta_2}^{OLS}$ consistent?

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$\beta_1 + \beta_2 \ln P_i + c_i = \gamma_1 + \gamma_2 \cdot \ln P_i + \gamma_3 \ln T_i + u_i$

$$\ln P_i = \frac{\beta_1 - \gamma_1 - \gamma_3 \ln T_i - u_i + c_i}{\gamma_2 - \beta_2}$$

$$\text{cov}(\ln P_i, c_i) = \text{cov}\left(\frac{\beta_1 - \gamma_1 - \gamma_3 \ln T_i - u_i + c_i}{\gamma_2 - \beta_2}, c_i\right) =$$

$$= \frac{1}{\gamma_2 - \beta_2} \cdot \text{cov}(-\gamma_3 \cdot \ln T_i + c_i - u_i, c_i)$$

$$= \frac{1}{\gamma_2 - \beta_2} \cdot \left(-\gamma_3 \overset{\parallel^0}{\text{cov}(\ln T_i, c_i)} + \sigma_c^2 - \overset{\parallel^0}{\text{cov}(c_i, u_i)}\right)$$

$$= \frac{\sigma_c^2}{\gamma_2 - \beta_2} \neq 0$$

$$\hat{\beta}_2^{ols} = \frac{\widehat{cov}(\ln P, \ln Q)}{\widehat{Var}(\ln P)} = \beta_2 + \frac{\widehat{cov}(\ln P, \varepsilon)}{\widehat{Var}(\ln P)} \xrightarrow{P}$$

$$\beta_2 + \frac{cov(\ln P_i, \varepsilon_i)}{Var(\ln P_i)} = \beta_2 + \frac{\sigma_\varepsilon^2 / \gamma_2 - \beta_2}{\underbrace{\sigma^2}_{\ln P_i}} \ne \beta_2$$

$\searrow 0$

$\color{red}{\gamma_2 - \beta_2 > 0}$

$$\beta_2 < 0 \implies \text{will be closer to } 2$$

b) Demand: $\ln Q_i = \beta_1 + \beta_2 \ln P_i + \varepsilon_i$

Supply: $\ln Q_i = \gamma_1 + \gamma_2 \cdot \ln P_i + \gamma_3 \ln T_i + u_i$

$\ln T_i$ is valid instrument?

$cov(\ln T_i, \varepsilon_i) = 0$

$cov(\ln T_i, \ln P_i) > 0$

1 step: $\ln P_i \mid \ln T_i$

2 step: $\ln Q_i \mid \widehat{\ln P_i}$ $\hat{\beta}_2^{TSLS}$ — consistent