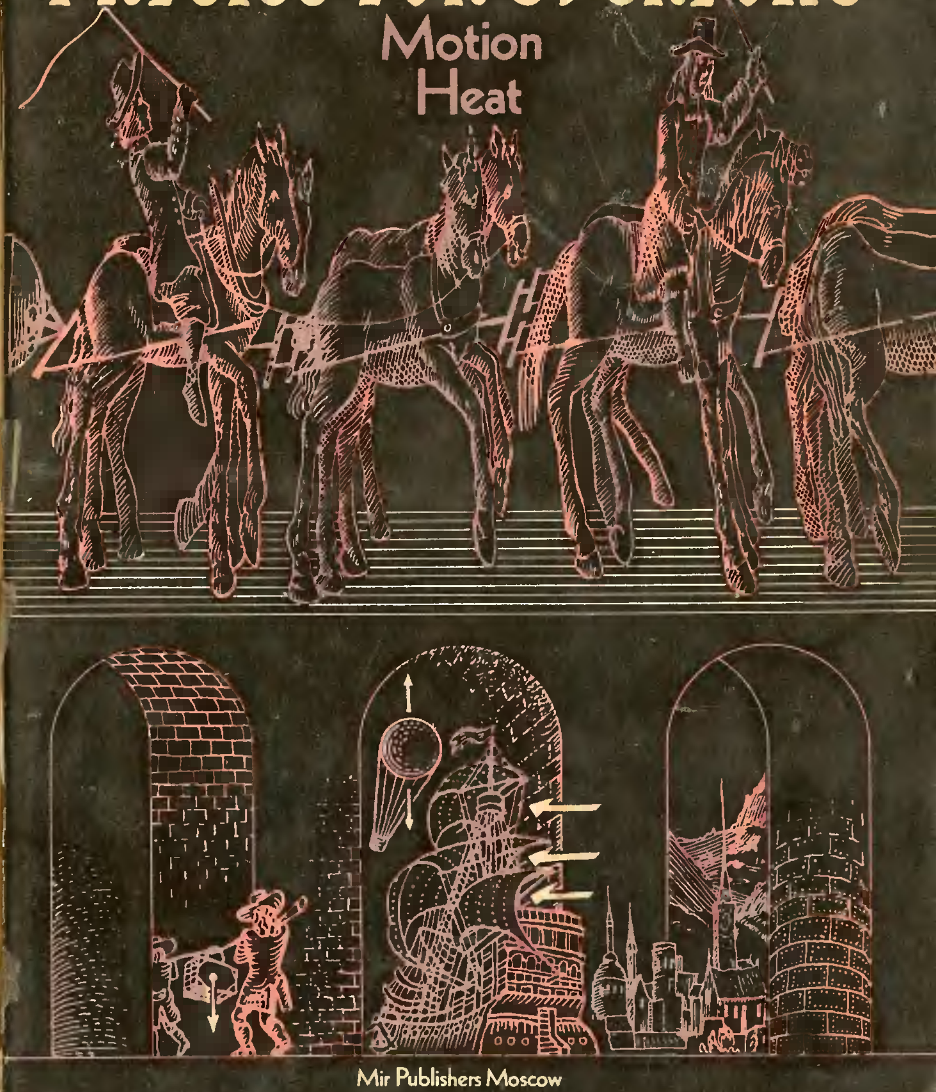
L. Landau and A. Kitaigorodsky

PHYSICS FOR EVERYONE

Motion
Heat



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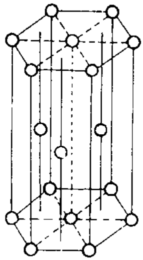

Heat

Translated from the Russian
by
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PREFACE

The first question which the reader, taking this book in his hands, asks himself is: for whom is this book "for everyone"?

Of course, there is some exaggeration in the title. An acquaintance with the fundamentals of high school algebra is sufficient for the reader of this book. No knowledge of physics is needed: it can be your first book on physics. It is possible, however, that the book will also turn out to be interesting for those who have chosen physics as their specialty.

We have tried to write this book in a light and simple style, not denying ourselves the pleasure of an occasional joke with the reader. But this does not in any way mean that our *Physics for Everyone* is an easy book. Many of its pages must be read attentively for a long time; in order to understand physics, one must very often think hard and tensely.

The book's main concern is the fundamental laws and concepts of physics. However, we have tried not to forget about illustrations from life and technology, true, not having the aim of dealing in any way with the inexhaustible field of applied physics.

A few historical digressions are devoted exclusively to the foundations of physics, but not to its applications.

So far *Physics for Everyone* covers only the part of physics pertaining to mechanical and molecular motion. We hope that under this same title the reader will in the future find a succession of books devoted to electricity, optics and atomic structure.

L. Landau

A. Kitaigorodsky

One

BASIC CONCEPTS

About Centimeters and Seconds

Everyone has to measure lengths, reckon time and weigh various bodies. Therefore, everyone knows just what a centimeter, a second and a gram are. But these measures are especially important for a physicist—they are necessary for making judgements about most physical phenomena. People try to measure distances, intervals of time and weights, which are called the basic concepts of physics, as accurately as possible.

Modern physical apparatuses permit us to determine a difference in length between two-meter long rods, even if it is less than one billionth of a meter. It is possible to distinguish intervals of time differing by one millionth of a second. Good scales can determine the weight of a poppy seed with a very high degree of accuracy.

Measurement techniques started developing only a few hundred years ago, and agreement on what segment of length and what body's weight to take as units has been reached relatively recently.

But why were the centimeter and the second chosen to be such as we know them? As a matter of fact, it is clear that there is no special significance to whether the centimeter or the second be longer.

A unit of measurement should be convenient—we require nothing further of it. It is very good for a unit of measurement to be at hand, and simplest of all to take the hand itself for such a unit. This is precisely what was done in ancient times; the very names of the units testify to this: for example, an "ell" or "cubit" is the distance between the elbow and the fingertips of a stretched-out hand, an "inch" is the width of a thumb at its base. The foot is also used for measurement—hence the name of the length "foot".

Although these units of measurement are very convenient in that they are always part of oneself, their disadvantages are obvious: there are just too many differences between individuals for a hand or a foot to serve as a unit of measurement which does not give rise to controversy.

With the development of trade, the need for agreeing on units of measurement arose. Standards of length and weight were at first established within a separate market, then for a city, later for an entire country and, finally, for the whole world. A standard is a model measure: a ruler, a weight. Governments carefully preserve these standards, and other rulers and weights must be made to correspond exactly to them.

The basic measures of weight and length in tsarist Russia—they were called the pound and the arshin—were first made in 1747. Demands on the accuracy of measurements increased in the 19th century, and these standards turned out to be imperfect. The complicated and responsible task of creating exact standards was carried out from 1893 to 1898 under the guidance of Dmitri Ivanovich Mendeleev. The great chemist considered the establishment of exact standards to be very important. The Central Bureau of Weights and Measures, where the standards are kept and their copies made, was founded at the end of the 19th century on his initiative.

Some distances are expressed in large units, others in smaller ones. As a matter of fact, we wouldn't think of expressing the distance from Moscow to Leningrad in centimeters, or the weight of a railroad train in grams. People therefore agreed on definite relationships between large and small units. As everyone knows, in the system of units which we use, large units differ from smaller ones by a factor of 10, 100, 1000 or, in general, a power of ten. Such a condition is very convenient and simplifies all computations. However, this convenient system has not been adopted in all countries. Meters, centimeters and kilometers, as well as grams and kilograms, are still used infrequently in England and the USA, in spite of the obviousness of the metric system's conveniences.*

In the 17th century the idea arose of choosing a standard which exists in nature and does not change in the course of years and even centuries. In 1664 Christiaan Huygens proposed that the length of a pendulum making one oscillation a second be taken as the unit of length. About a hundred years later, in 1771, it was suggested that the length of a freely falling body's path during the first second be regarded as the standard. However, both variants proved to be inconvenient and were not accepted. A revolution was necessary for the emergence of the modern units of measurement—the Great French Revolution gave birth to the kilogram and the meter.

* The following measures of length were officially adopted in England: the nautical mile (equals 1852 m), the ordinary mile (1609 m), the foot (30.48 cm), a foot is equal to 12 inches, an inch is 2.54 cm; a yard, 0.9144 m, is the "tailors' measure" used to mark off the amount of material needed for a suit.

In Anglo-Saxon countries, weight is measured in pounds (454 g). Small fractions of a pound are an ounce (1/16 pound) and a grain (1/7000 pound); these measures are used by druggists in weighing out medicine.

In 1790 the French Assembly created a special commission, containing the best physicists and mathematicians, for the establishment of a unified system of measurements. From all the suggested variants of a unit of length, the commission chose one ten-millionth of the Earth's meridian quadrant, calling this unit a *meter*. Its standard was made in 1799 and given to the Archives of the Republic for safe keeping.

Soon, however, it became clear that the theoretically correct idea about the advisability of choosing models for our measures by borrowing them from nature cannot be fully carried out in practice. More exact measurements, performed in the 19th century, showed that the standard made for the meter is approximately 0.08 of a millimeter shorter than one forty-millionth of the Earth's meridian. It became obvious that new corrections would be introduced as measurement techniques developed. If the definition of the meter as a fraction of the Earth's meridian were to be retained, it would be necessary to make a new standard and recalculate all lengths anew after each new measurement of the meridian. It was therefore decided after discussions at the International Congresses of 1870, 1872 and 1875 to regard the standard of the meter, made in 1799 and now kept at the Bureau of Weights and Measures at Sèvres, near Paris, rather than one forty-millionth of a meridian, as the unit of length.

The history of the meter does not end here. At the present time, new physical ideas constitute the foundations of our definition of this fundamental quantity. The unit of length has again been borrowed from nature, but in a far more cunning manner.

Together with the meter, there arose its fractions: one thousandth, called a *millimeter*, one millionth, called a *micron*, and the one which is used most frequently, one hundredth—a *centimeter*.

Let us now say a few words about the *second*. It is much older than the centimeter. There were no disagreements in establishing a unit for measuring time. This is understandable: the alternation of day and night and the eternal revolution of the Sun suggest a natural means of choosing a unit of time. The expression "determine time by means of the Sun" is well known to everyone. When the Sun is high up in the sky, it is noon, and, by measuring the length of the shadow cast by a pole, it is not difficult to determine the moment when it is at its summit. The same instant of the next day can be marked off in the same way. The interval of time which elapses constitutes a day. And the further division of a day into hours, minutes and seconds is all that remains to be done.

The large units of measurement—the year and the day—were given to us by nature itself. But the hour, the minute and the second were devised by man.

The modern division of the day goes far back to antiquity. The sexagesimal, rather than the decimal, number system was prevalent in Babylon. Since 60 is divisible by 12 without any remainder, the Babylonians divided the day into 12 equal parts.

The division of the day into 24 hours was introduced in Ancient Egypt. Minutes and seconds appeared later. The fact that 60 minutes make an hour and 60 seconds make a minute is also a legacy of Babylon's sexagesimal system.

In Ancient Times and the Middle Ages, time was measured with the aid of sun dials, water clocks (by the amount of time required for water to drip out of large vessels) and a series of subtle, but rather imprecise devices.

With the aid of modern clocks, it is easy to convince oneself that the duration of a day is not exactly the same at all times of the year. It was therefore stipulated that the average solar day for an entire year would be taken as the unit of measurement. One twenty-fourth of this yearly average interval of time is what we call an hour.

But in establishing units of time—the hour, minute and second—by dividing the day into equal parts, we assume that the Earth rotates uniformly. However, lunar-solar ocean tides slow down, although to an insignificant degree, the rotation of the Earth. Thus, our unit of time—the day—is incessantly becoming longer.

This slowing down of the Earth's rotation is so insignificant that only recently, with the invention of atomic clocks measuring intervals of time with great accuracy—up to a millionth of a second—has it become possible to measure it directly. The change in the length of a day amounts to 1-2 milliseconds in 100 years.

But a standard should exclude, when possible, even such an insignificant error. According to the latest determination, a second is 1/315 569 25.9747 of a quite definite year, but by no means a fraction of an average solar day.

Weight and Mass

Weight is the force with which a body is attracted by the Earth. This force can be measured with a spring balance. The more the body weighs, the more the spring on which it is suspended will be stretched. With the aid of a weight, taken as the unit, it is possible to calibrate the spring—make marks which will indicate how much the spring has been stretched by a weight of one, two, three, etc., kilograms. If, after this, a body is suspended on such a scale, we shall be able to find the force (gravity) of its attraction by the Earth, expressed in kilograms, by observing the stretching of the spring (Figure 1*a*). For measuring weights, one uses not only stretching, but also contracting springs (Figure 1*b*). Using springs of various thicknesses, one can

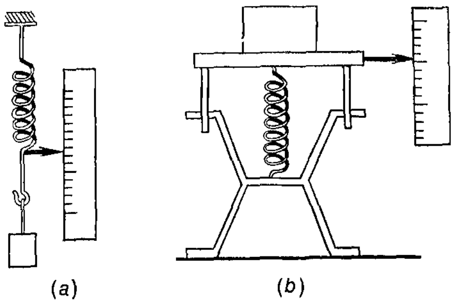


Fig. 1

make scales for measuring very large and also very small weights. Not only coarse commercial scales are constructed on the basis of this principle, but also precise instruments used for physical measurements.

A calibrated spring can serve for measuring not only the force of the Earth's attraction, i.e. weight, but also other forces. Such an instrument is called a dynamometer, which means a measurer of forces. You may have seen how a dynamometer is used for measuring a person's muscular force. It is also convenient to measure a motor's tractive force by means of a stretching spring (Figure 2).

A body's weight is one of its very important properties. However, the weight depends not only on the body itself. As a matter of fact, the Earth attracts it. And what if we were on the Moon? It is obvious that its weight would be different—about six times less, as shown by computations. In fact, even on the Earth, weight is different at various latitudes. At a pole, for example, a body weighs 0.5% more than at the equator.

However, for all its changeability, weight possesses a remarkable peculiarity—the ratio of the weights of two bodies remains unchanged under any conditions, as experiments have shown. If two different loads stretch a spring

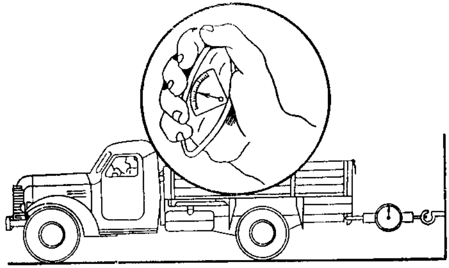


Fig. 2

identically at a pole, this identity is completely preserved even at the equator.

In measuring weight by comparing it with the weight of a standard, we find a new property of bodies, which is called *mass*.

The physical meaning of this new concept—mass—is related in the most intimate way to the identity in comparing weights which we have just noted.

Unlike weight, mass is an invariant property of a body, depending on nothing except the given body.

A comparison of weights, i.e. a measurement of mass, is most conveniently carried out with the aid of ordinary balance scales (Figure 3). We say that the masses of two bodies are equal if the balance scale on whose pans these bodies are placed is in perfect equilibrium. If a load is in equilibrium on a balance scale at the equator, and then the load and the weights are transported to a pole, the load and the weights change their weight identically. A weighing at the pole will therefore yield the same result: the scale will remain balanced.

We can even verify this state of affairs on the Moon. Since the ratio of bodies' weights will not change there either, a load placed on a scale will be balanced by the

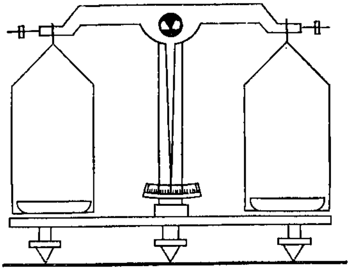


Fig. 3

same weights there. A body's mass remains the same, no matter where it is.

Units of mass and weight are related to the choice of a standard weight. Just as in the case of the meter and the second, people tried to find a natural standard of mass. The same commission used a definite alloy to make a weight which balanced one cubic decimeter of water at four degrees Centigrade*. This standard received the name *kilogram*.

Later, however, it became clear that it isn't so easy to "take" one cubic decimeter of water. Firstly, the decimeter, as a fraction of a meter, changed along with the refinement of the meter's standard. Secondly, what kind of water should we take? Chemically pure water? Twice distilled? Without any trace of air? And what should be done about admixtures

* This temperature was not chosen by chance. Its significance lies in the fact that the volume of water changes with heating in a very peculiar manner, unlike most bodies. A body ordinarily expands when heated, but water contracts as its temperature rises from 0 to 4 °C, and only starts expanding after it gets above 4 °C. Thus, 4 °C is the temperature at which water stops to contract and begins to expand.

of "heavy water"? To top off all our misfortunes, accuracy in measuring a volume is noticeably less than in weighing.

It again became necessary to reject a natural unit and accept a specially made weight as the unit of mass. This weight is also kept in Paris together with the standard for the meter.

One thousandth and one millionth of a kilogram—the *gram* and the *milligram*—are widely used for measuring mass. The weight of a standard weight at the Earth's 45th parallel is called a kilogram and denoted by kgf; the mass of this weight is also called a kilogram, and is denoted by kg. This weight's mass will still be 1 kg on the Moon, but its weight will become approximately 0.17 kgf. Thus, force and mass have units of measurement which were named identically. This state of affairs introduces a serious confusion into our understanding of the "interrelationship" between weight and mass.

In order to bring clarity to these questions, the Tenth and Eleventh (1960) General Conferences of Weights and Measures developed a new International System of Units (SI), which was then ratified by most countries as national standards. The name "kilogram" (kg) is retained by mass in the new system. Every force, including of course weight, is measured in newtons (N) in the new system. We shall find out a bit later why this unit was given such a name and how it is defined.

The new system will undoubtedly not be immediately and universally applied, and so it is still helpful to recall that a kilogram of mass (kg) and a kilogram of force (kgf) are distinct units and it is necessary to perform arithmetical operations on them as on different concrete numbers. Writing 5 kg + 2 kgf = 7 is just as meaningless as adding meters to seconds.

Density

What do we mean when we say: as heavy as lead and as light as a feather? It is clear that a grain of lead will be light, while a mountain of feathers has considerable weight. Those who use such comparisons have in mind not a body's mass, but the density of the material of which it consists.

The mass of a unit volume of a body is called its *density*. It is evident that a grain of lead and a massive block of lead have the same density.

In denoting density, we usually indicate how many grams (g) a cubic centimeter (cm^3) of the body weighs—after this number we place the symbol g/cm^3. In order to determine the density, the number of grams must be divided by the number of cubic centimeters; the fractional line in the symbol reminds us of this.

Certain metals are among the heaviest materials—osmium, whose density is equal to 22.5 g/cm^3, iridium (22.4), platinum (21.5), tungsten and gold (19.3). The density of iron is 7.88, that of copper, 8.93.

The lightest metals are magnesium (1.74), beryllium (1.83) and aluminium (2.70). Still lighter bodies should be sought among organic materials: various sorts of wood and plastic may have a density as low as 0.4.

It should be stipulated that we are dealing with continuous bodies. If there are pores in a solid, it will of course be lighter. Porous bodies—cork, foam glass—are frequently used in technology. The density of foam glass may be less than 0.5, although the solid matter from which it is made has a density greater than one. As all other bodies whose density is less than one, foam glass floats superbly on water.

The lightest liquid is liquid hydrogen; it can only be obtained at extremely low temperatures. One cubic centimeter of liquid hydrogen has a mass of 0.07 g. Organic liquids—alcohol, benzine, kerosene—do not differ significantly from water in density. Mercury is very heavy—it has a density of 13.6 g/cm^3.

And how can the density of gases be characterized? For gases, as is well known, occupy whatever volumes we let them. If we empty gas-bags with the same mass of gas into vessels of different volumes, the gas will always fill them up uniformly. How then can we speak of density?

We define the density of gases under so-called normal conditions—a temperature of 0 °C and a pressure of one atmosphere. The density of air under normal conditions is equal to 0.001 29 g/cm^3, of chlorine, 0.003 22 g/cm^3. Gaseous hydrogen, just as the liquid, holds the record: the density of this lightest gas is equal to 0.000 09 g/cm^3.

The next lightest gas is helium; it is twice as heavy as hydrogen. Carbon dioxide is 1.5 times as heavy as air. In Italy, near Naples, there is a famous "canine cave"; carbon dioxide continually exudes from its lower part, hangs low and slowly escapes from the cave. A person can enter this cave without difficulty, but such a stroll will end badly for a dog. Hence the cave's name.

A gas' density is extremely sensitive to external conditions—pressure and temperature. Without an indication of the external conditions, the value of a gas' density has no meaning. The densities of liquids and solids also depend on temperature and pressure, but to a considerably less degree.

The Law of Conservation of Mass

If we dissolve some sugar in water, the mass of the solution will be precisely equal to the sum of the masses of the sugar and the water.

This and an infinite number of similar experiments show that the mass of a body is an unchangeable property. No matter how the body is crushed or dissolved, its mass remains fixed.

The same also holds for arbitrary chemical transformations. Suppose that coal burns up. It is possible to establish by means of careful weighings that the mass of the coal and the oxygen from the air which was used up during the burning will be exactly equal to the mass of the end products of the combustion.

The law of conservation of mass was verified for the last time at the end of the 19th century, when the technique of fine weighing had already been very highly developed. It turned out that mass does not even change by an insignificant fraction of its value during the course of any chemical transformation.

Mass was considered to be unchangeable as far back as Ancient Times. This law first underwent an actual experimental verification in 1756. This was done by Mikhail Lomonosov, who proved the conservation of mass during the kilning of metals by means of experiments in 1756, and demonstrated the scientific significance of the law.

Mass is the most important unchanging characteristic of a body. The majority of a body's properties are, so to say, in the hands of human beings. A soft iron bar, which is easily bent by hand, can be made hard and brittle by hardening it. With the aid of ultrasonic waves, one can make a turbid solution transparent. Mechanical, electrical and thermal properties can be changed by means of external actions. If no matter is added to a body and not a single particle is separated from it, then it is impossible* to change its mass, regardless of what external actions we resort to.

* The reader will later discover that there are certain limitations to this assertion.

MIKHAIL LOMONOSOV (1711-1765)—an outstanding Russian scientist, the initiator of science in Russia, a great educator.
In the field of physics, Lomonosov struggled resolutely against the notion, widespread in the 18th century, of electrical and thermal "fluids", upholding the molecular-kinetic theory of matter. Lomonosov was the first to experimentally prove the constancy of the mass of matter participating in chemical transformations. Lomonosov carried out extensive research in the field of atmospheric electricity and meteorology. He constructed a series of remarkable optical instruments and discovered the atmosphere on Venus. Lomonosov created the basis of scientific Russian, he succeeded in translating the basic physical and chemical terms from the Latin exceptionally well.

Action and Reaction

We frequently fail to notice that every action of a force is accompanied by a reaction. If a valise is placed on a bed with a spring mattress, the bed will sag. The fact that the valise's weight acts on the bed is obvious to everyone. Sometimes, however, we forget that the bed also exerts a force on the valise. As a matter of fact, the valise lying on the bed does not fall; this means that from the region of the bed there is a force acting on it, which is equal to the weight of the valise and directed upwards.

Forces which are opposite in direction to gravity are often called reactions of the support. The word "reaction" means "counteraction". The action of a table on a book which is lying on it and the action of a bed on a valise which has been placed upon it are reactions of the support.

As we have just said, the weight of a body can be determined with the aid of a spring balance. The pressure of the body on the spring which has been placed under it, or the force stretching the spring on which the load has been suspended, is equal to the weight of the body. It is obvious, however, that the spring's compression or tension can just as well be used to obtain the value of the reaction of the support.

Thus, measuring the magnitude of some force by means of a spring balance, we measure the value of not one, but of two forces, opposite in direction. Spring balances measure the pressure exerted by the load on the pan, and also the reaction of the support—the action of the pan on the load. Fastening a spring to a wall and pulling it by hand, we can measure the force with which our hand pulls the spring and, simultaneously, the force with which the spring pulls our hand.

Therefore, forces possess a remarkable property: they are always found in pairs and are, moreover, equal in magnitude and opposite in direction. It is these two forces which are usually called an action and a reaction.

"Single" forces do not exist in nature—only mutual reactions between bodies have a real existence; moreover, the strengths of an action and a reaction are invariably equal—they are related to each other as an object is related to its mirror image.

One should not confuse balancing forces with forces of action and reaction.

We say that forces are balanced if they are applied to a single body; thus, the weight of a book lying on a table (the action of the Earth on the book) is balanced by the reaction of the table (the action of the table on the book).

In contrast to the forces which arise in balancing two interactions, the forces of action and reaction characterize one interaction, for example, of a table with a book. The action is "table-book" and the reaction is "book-table". These forces, of course, are applied to different bodies.

Let us try to clear up the following traditional misunderstanding: "The horse is pulling the waggon, but the waggon is also pulling the horse; why then do they move?" First of all, we must recall that the horse will not move the waggon if the road is slippery. Hence, in order to explain the motion, we must take into account not one but two interactions—not only "waggon-horse", but also "horse-road". The motion will begin when the force of the interaction between the horse and the road (the force with which the horse pushes off from the road) exceeds that of the interaction "horse-waggon" (the force with which the waggon pulls the horse). As for the forces "waggon pulls horse" and "horse pulls waggon", they characterize one and the same interaction, and will therefore be identical in magnitude when at rest and at any instant during the course of the motion.

How Velocities Are Added

If I waited half an hour and then another hour, I would lose one and a half hours of time all told. If I were given a rouble and then two more, I would receive three roubles in all. If I bought 200 g of grapes and then another 400 g, I would have 600 g of grapes. We say that time, mass and other similar magnitudes are added algebraically.

However, not every quantity can be added and subtracted so simply. If I say that it is 100 km from Moscow to Kolomna and 40 km from Kolomna to Kashira, then it does not follow from this that Kashira is located at a distance of 140 km from Moscow. Distances are not added algebraically.

How else can quantities be added? We shall easily find the required rule on the basis of our example. Let us draw three points on a piece of paper, indicating the relative locations of the three places of interest to us (Figure 4). We can construct a triangle with these three points as vertices. If two of its sides are known, it is possible to find the third. For this, however, we must know the angle between the two given segments.

One finds the unknown distance in the following way: mark off the first segment, and then construct the second in the given direction, beginning at the end point of the first. Now join the initial point of the first segment to the end point of the second. The required path is represented by the last drawn segment.

The kind of addition just described is called geometrical, and quantities which are added in this manner are called *vectors*.

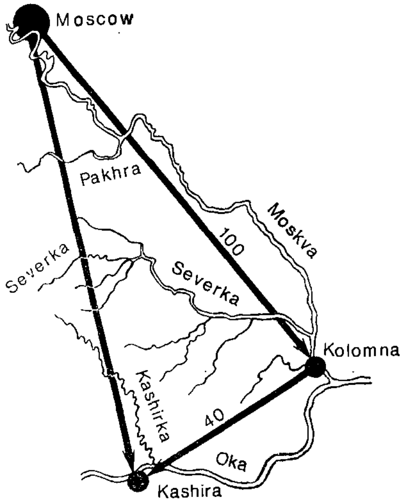


Fig. 4

In order to distinguish a segment's initial point from its end point, we add an arrow to it. Such a segment—a vector—indicates a length and a direction.

This rule is also applied in adding several vectors. Passing from the first point to the second, from the second to the third, etc., we cover a path which can be represented by a broken line. But it is possible to go directly from the starting point to the terminal point. This segment, closing up the polygon, will be precisely the vector sum.

A vector triangle also shows, of course, how to subtract one vector from another. For this we draw them from one point. The vector drawn from the end point of the second vector to the end point of the first will be the difference between the two vectors.

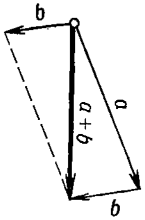


Fig. 5

Besides the triangle rule, one may make use of the equivalent parallelogram rule (Figure 5). This rule requires that we construct a parallelogram on the vectors we are adding, and draw the diagonal from the point of their intersection. It is clear from the figure that the diagonal of the parallelogram is precisely the segment which closes up the triangle. Hence, both rules are equally suitable.

Vectors are used for describing not only displacements. Vector quantities are frequently found in physics.

Consider, for example, a velocity of motion. *Velocity* is the displacement during a unit of time. Since the displacement is a vector, the velocity is also a vector, and it has the same direction. In the course of motion along a curve, the direction of displacement is changing all the time. How then can we answer the question about the velocity's direction? A small segment of a curve has the same direction as a tangent. Therefore, a body's displacement and velocity are directed along the tangent to the path of motion at each given instant.

In many cases one must add and subtract velocities according to the rule for vectors. The need to add velocities arises when a body participates simultaneously in two motions. Such cases are not uncommon: a person walks inside a train and, in addition, moves together with the train; a drop of water trickling down a train's window pane

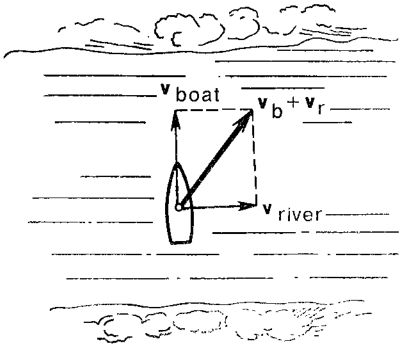


Fig. 6

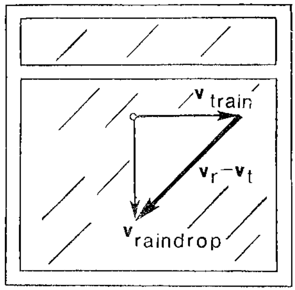


Fig. 7

moves downwards under the action of its weight and travels along with the train; the Earth moves around the Sun and, together with the Sun, moves with respect to the other stars. In all these and other similar cases, velocities are added in accordance with the rule for adding vectors.

If both motions take place along a single line, then vector addition reduces to ordinary addition when both motions have the same direction, and to subtraction when they have opposite directions.

But what if the motions take place at an angle? Then we turn to geometrical addition.

If, in crossing a swiftly flowing river, you steer perpendicular to the current, you will be carried downstream. The boat participates in two motions: across the river and along the river. The boat's total velocity is shown in Figure 6.

Another example. What does the motion of a stream of raindrops look like from the window of a train? You have no doubt observed rain from train windows. Even in windless weather it moves slantwise, as if a wind, blowing towards the train from ahead, were deflecting it (Figure 7).

If the weather is windless, a raindrop falls vertically downwards. But during the time the drop is falling near the window, the train has travelled a fair distance, leaving the vertical line of fall behind; this is why the rain seems to be slanting.

If the velocity of the train is $\mathbf{v}_t$ and the velocity of the raindrop's fall is $\mathbf{v}_r$, then the velocity of its fall relative to a passenger of the train is obtained by the vector subtraction of $\mathbf{v}_t$ from $\mathbf{v}_r$.* The velocity triangle is shown in Figure 7. The direction of the slanting vector indicates the rain's direction; now it is clear why we see the rain slanting. The length of the slantwise arrow yields the magnitude of this velocity in the chosen scale. The faster the train goes and the slower the raindrop falls, the more the stream of raindrops seems to slant.

Force Is a Vector

Force, just as velocity, is a vector quantity. For it always acts in a definite direction. Therefore, forces should also be added according to the rules which we have just discussed.

We often observe examples in real life which illustrate the vector addition of forces. A rope on which a package is hanging is shown in Figure 8. A person is pulling the package to one side with a string. The rope is being stretched by the action of two forces: the force of the package's weight and the person's force.

The rule of vector addition of forces allows us to determine the direction of the rope and compute the force of its tension. The package is at rest; hence, the sum of the forces

* Here and in what follows we shall use bold-face letters to denote vectors, i.e. characteristics for which not only magnitude, but also direction is of significance.

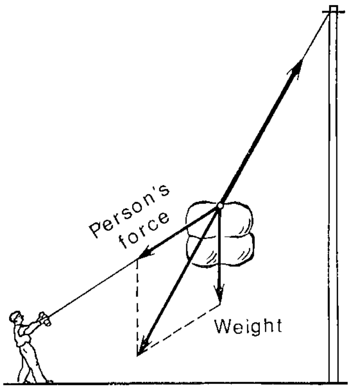


Fig. 8

acting on it must be equal to zero. And we can also put it this way—the tension in the rope must be equal to the sum of the force of the package's weight and the force pulling it to one side with the aid of the string. The sum of these forces yields the diagonal of a parallelogram, which will be directed along the rope (for otherwise it could not be "annihilated" by the force of the rope's tension). The length of this arrow will represent the force of the rope's tension. The two forces acting on the package could be replaced by such a force. The vector sum of forces is therefore sometimes called the resultant.

There very often arises a problem which is inverse to the addition of forces. A lamp is suspended on two ropes. In order to determine the forces of tension in the ropes, we must decompose the weight of the lamp along these two directions.

From the end point of the resultant vector (Figure 9) we draw lines parallel to the ropes, up to the points of intersection. The parallelogram of forces is constructed. Measuring the lengths of the parallelogram's sides, we find

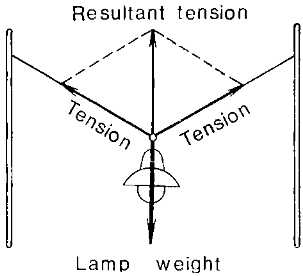


Fig. 9

(in the same scale in which the weight is represented) the magnitude of the tension in the rope.

Such a construction is called a decomposition of force. Every number can be represented in an infinite number of ways as the sum of two or several numbers; the same thing can also be done with a force vector: any force can be decomposed into two forces—sides of a parallelogram—one of which can always be chosen arbitrarily. It is also clear that to each vector there can be attached an arbitrary polygon.

It is often convenient to decompose a force into two mutually perpendicular forces—one along a direction of interest to us and the other perpendicular to this direction. They are called the tangential and normal (perpendicular) components of force.

The component of force in a particular direction, constructed by a decomposition along the sides of a rectangle, is also called the projection of the force in this direction.

It is clear that in Figure 10

$$F^2 = F_t^2 + F_n^2$$

where F_t and F_n are the projections of the force in the chosen direction and normal to it.

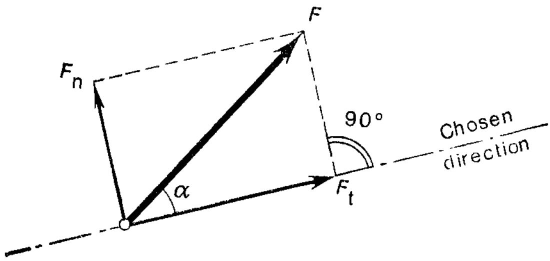


Fig. 10

Those who know some trigonometry will establish without difficulty that

$$F_t = F \cos \alpha$$

where α is the angle between the force vector and the direction onto which it is projected.

A very curious example of the decomposition of forces is given by the motion of a sailboat. How does it manage to sail against the wind? If you ever watched a sailboat doing this, you might have noticed that it zigzagged. Sailors call such a motion tacking.

Of course, it is impossible to sail directly against the wind, but why is it possible to sail against the wind at all, if only at an angle?

The possibility of beating against the wind is based on two circumstances. Firstly, the wind always pushes the sail at a right angle to the latter's plane. Look at Figure 11*a*: the wind's force is decomposed into two components—one of them makes the air slip past the sail, while the other—the normal component—exerts pressure on the sail. Secondly, the boat does not move to where the wind's force pushes it, but to where its force is facing.

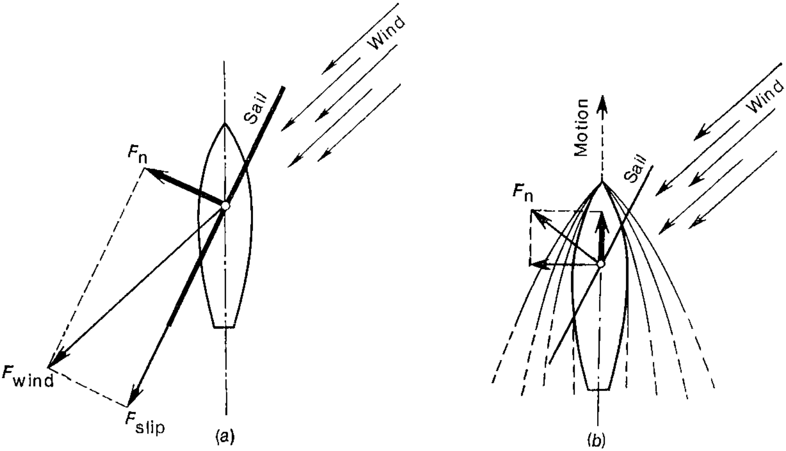


Fig. 11

This is explained by the fact that a movement of a boat across its keel line would meet with a very strong resistance on the part of the water. Therefore, in order for a boat to move forward, it is necessary that the force pressing on the sail should have a forward component along the keel line.

Figure 11*b*, which depicts a boat sailing against the wind, should now become clear to you. The sail is set in such a way that its plane bisects the angle between the direction of the boat's path and that of the wind.

In order to find the force which drives the boat forward, we must decompose the force of the wind twice: first along and perpendicular to the sail—only the normal component is significant—and then we have to decompose this normal component along and perpendicular to the keel line. It is just the tangential component that drives the boat at an angle towards the wind.

Inclined Plane

It is more difficult to overcome a steep rise than a gradual one. It is easier to roll a body up an inclined plane than to lift it vertically. Why is this so, and how much easier is it? The law of the addition of forces permits us to gain an understanding of these matters.

Figure 12 depicts a waggon on wheels, which is held on an inclined plane by the tension in a string. Besides this pull, two other forces are acting on the waggon—its weight and the force of the reaction of the support, which always acts along the normal to a surface, regardless of whether the surface of the support is horizontal or inclined.

As has already been said, if a body weighs on the support, the latter counteracts the pressure or, as we say, creates the *reaction force.*

We want to know to what degree it is easier to pull a waggon up along an inclined plane than to lift it vertically.

We decompose the forces in such a way that one component is directed along, and the other perpendicular to, the surface on which the body is moving. In order for the body to be at rest on the inclined plane, the force of the tension in the string must balance only the tangential component. As for the second component, it is balanced by the reaction of the support.

We can find the force we are interested in, i.e. the tension T in the rope, either by means of a geometrical construction or with the aid of trigonometry. The geometrical construction consists in dropping a perpendicular from the end point of the weight vector P to the plane.

One can find two similar triangles in the figure. The ratio of the length l of the inclined plane to its height h is equal to the ratio of the corresponding sides of the force triangle.

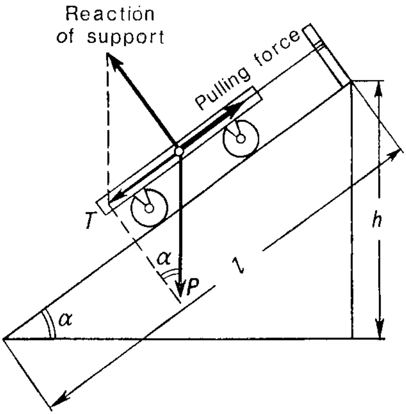


Fig. 12

Thus,

$$\frac{T}{P}=\frac{h}{l}$$

The less the plane is inclined (the smaller the value of h/l), the easier it will be, of course, to pull the body upwards.

And now, for those who are acquainted with trigonometry: since the angle between the normal component of the weight and the weight vector is equal to the angle of inclination α of the plane (these are angles with mutually perpendicular sides), we have

$$\frac{T}{P}=\sin\alpha \quad \text{and} \quad T=P\sin\alpha$$

Therefore, it is $\sin\alpha$ times easier to wheel a waggon up a plane with the angle of inclination α than to lift it vertically.

It is helpful to memorize the values of the trigonometric functions for angles of 30, 45 and 60°. Knowing these numbers for the sine ($\sin 30° = 1/2$; $\sin 45° = \sqrt{2}/2$; $\sin 60° = \sqrt{3}/2$), we get a good idea of the amount of force saved by moving up an inclined plane.

It is evident from our formulas that for a 30° angle of inclination, the force we exert will be half the body's weight: $T = P/2$. For angles of 45 and 60°, we have to pull the rope with forces equal to about 0.7 and 0.9 of the waggon's weight. As we see, such steeply inclined planes do not make our task much easier.

Two

LAWS OF MOTION

Various Points of View About Motion

The valise is standing in the baggage rack. At the same time, it is moving together with the train. The house is standing on the Earth, but it is also moving together with it. It is possible to say about one and the same body: it is moving in a straight line, it is at rest, it is rotating. And all these statements will be true, but from different points of view.

Not only the graph of the motion, but also its properties, can be entirely different if regarded from different points of view.

Recall what happens to objects on a ship which is being rocked by the sea. How they misbehave! The ash-tray on the table overturned and dove headlong under the bed. The water splashes in the bottle, and the lamp vibrates like a pendulum. Without any visible cause, some objects begin moving and others stop. An observer on such a ship might say that the basic law of motion is that at any moment an unfastened object can start travelling in any direction with an arbitrary speed.

This example shows that among the various points of view on motion there are those which are obviously awkward.

But what point of view is the most "reasonable"?

If suddenly, for no reason whatsoever, the lamp on the table were to bend over, or the paper-weight were to jump, then at first you would think that it was only your imagination. If these miracles were repeated, you would urgently start looking for the cause which drove these bodies out of the state of rest.

It is therefore perfectly natural to regard the point of view on motion, according to which bodies at rest do not budge without the action of a force, as a rational one.

Such a point of view seems quite natural: a body is at rest—hence, the sum of the forces acting on it is equal to zero; it moved—this happened under the action of a force.

This point of view presupposes the presence of an observer. However, it is not the observer himself who is of interest to us, but his location. Therefore, instead of "point of view on motion", we shall say "frame of reference in which the motion is regarded", or simply "frame of reference".

For us, inhabitants of the Earth, an important frame of reference is the Earth. However, bodies moving on the Earth, say, a ship or a train, can also frequently serve as frames of reference.

Let us now return to the "point of view" on motion which we called rational. This frame of reference has a name—it is called *inertial*.

We shall see a bit later where this term comes from.

Consequently, the properties of an inertial frame of reference are as follows: bodies in a state of rest with respect to such a frame do not feel the action of forces. Therefore, not a single motion in such a frame is begun without the action of a force. The simplicity and convenience of such a frame are obvious. It is clear that it would pay to study motion in them.

The fact that the Earth does not differ greatly from an inertial frame of reference is extremely important. We can therefore begin our investigation of the basic regularities of motion, considering them from the point of view of the Earth. Nevertheless, we must bear in mind that, strictly speaking, everything that will be said in the next section deals with an inertial frame of reference.

The Law of Inertia

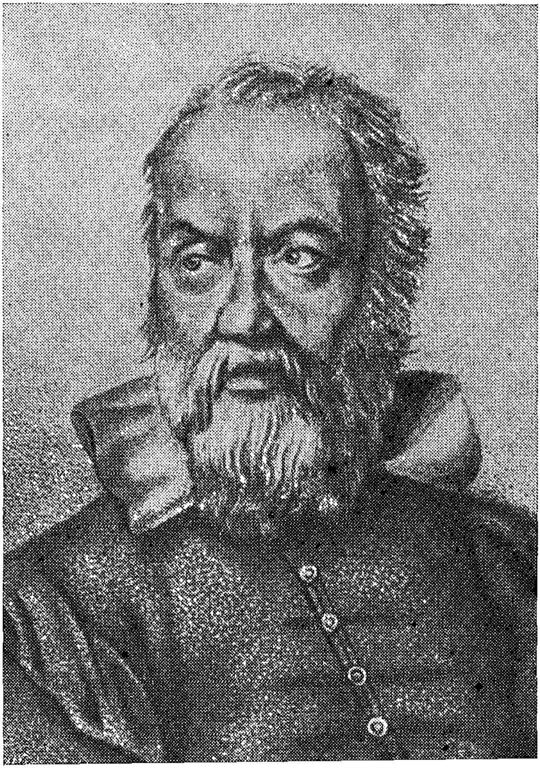
There can be no quarrel—an inertial frame of reference is convenient and has invaluable advantages.

But is such a frame unique or do there, perhaps, exist many inertial frames? The Ancient Greeks, for example, took the former point of view. In their writings we find many naive reflections on the causes of motion. These ideas find their completion in Aristotle. In the opinion of this philosopher, the natural state of a body is rest—of course, with respect to the Earth. Every displacement of a body with respect to the Earth must have a cause—a force. But if there is nothing causing a body to keep moving, it must halt, return to its natural state. And this is what rest with respect to the Earth is. From this point of view, the Earth is the unique inertial frame.

We are indebted to a great Italian, Galileo Galilei (1564-1642), for discovering the truth and disproving this false, but congenial to naive psychology, opinion.

Let us think over the Aristotelian explanation of motion and search familiar phenomena for confirmation or refutation of the idea that rest is the natural state of bodies on the Earth.

Imagine that we are in an airplane taking off from an airport at dawn. The Sun has not yet warmed up the air, so there are no "air-pockets", which cause many passengers unpleasantness. The airplane is moving smoothly, imperceptibly. If you don't look out of the window, you won't



GALILEO GALILEI (1564-1642)—a great Italian physicist and astronomer, the first to apply the experimental method of investigation in science. Galileo introduced the concept of inertia, established the relativity of motion, investigated the laws of free fall, of a body's motion on an inclined plane, and of the motion of an object thrown at an angle to the horizontal, used a pendulum for the measurement of time. For the first time in the history of mankind, he looked at the sky through a telescope, discovered many new stars, proved that the Milky Way consists of an enormous number of stars, discovered Jupiter's satellites, sunspots and the rotation of the Sun, investigated the structure of the Moon's surface. Galileo actively supported Copernicus' heliocentric system, banned in those days by the Catholic church. Persecution by the Inquisition darkened the last ten years of the great scientist's life.

even notice that you're flying. A book is lying on an empty seat; an apple is at rest on a table. All objects inside the airplane are motionless. Is this how things should be if Aristotle were right? Of course not. As a matter of fact, according to Aristotle, a body's natural state is rest on the Earth. But then why are all the objects not piled up at the rear wall of the airplane, trying to lag behind its motion, "wanting" to return to the state of "true" rest? What makes the apple lying on the table, hardly touching the surface of the table, move with an enormous speed of several hundred kilometers an hour?

What is the correct answer to the question of the cause of motion? Let us first take up the question of why moving bodies come to a stop. For example, why does a ball, rolling along the Earth's surface, come to a stop? In order to give a correct answer, we should consider in which cases a ball comes to a stop quickly, and in which cases slowly. We don't need any special experiments for this. We know perfectly well from our practical experience that the smoother the surface on which a ball is moving, the farther it will roll. From these and similar experiences, there arises the natural idea of the force of friction as a hindrance to motion, as the cause for the slowing down of an object which is rolling or slipping along the Earth. Friction can be decreased in various ways. The more we work on the destruction of every kind of resistance to motion (for example, the smoother we construct our roads, the better we lubricate our engines and the more we perfect our ball-bearings), the greater the distance a moving body will cover freely, without being acted on by any force.

The following question arises: and what would happen if there were no resistance, if the force of friction were absent? Obviously, in such a case a motion would continue infinitely, with unchanging speed and along one and the same straight line.

We have formulated the law of inertia in about the same form as it was first given by Galileo. Inertia is a brief designation for this ability of a body to move rectilinearly and uniformly ... without any cause, contrary to Aristotle. Inertia is an inalienable property of each particle in the Universe.

In what way can we check the validity of this remarkable law? As a matter of fact, it is impossible to create conditions under which no forces would be acting on a moving body. Even though this is true, we can, on the other hand, observe the opposite. In every case when a body changes the speed or direction of its motion, it is always possible to find a cause—a force responsible for this change. A body acquires speed in falling to the Earth; the cause is the Earth's gravitation. A stone twirles on a string, circumscribing a circle; the cause deflecting the stone from a rectilinear path is the tension in the string. If the string breaks, the stone will fly off in the same direction in which it was moving at the moment the string broke. The motion of an automobile running with the motor turned off slows down; the causes are air resistance, friction of the tires on the road and imperfections in the ball-bearings.

The law of inertia is the foundation on which the entire study of the motion of bodies rests.

Motion Is Relative

The law of inertia leads to a derivation of the multiplicity of inertial frames.

Not one, but many frames of reference exclude "causeless" motions.

If one such frame is found, we can immediately find another, moving forward (without rotation), uniformly and rectilinearly with respect to the first. Moreover, one inertial frame is not the least bit better than the others, does not in any way differ from the others. It is in no way possible to find a best frame among the multitude of inertial frames of reference. The laws of motion of bodies are identical in all inertial frames: a body is brought into motion only under the action of forces, is slowed down by forces, and in the absence of any forces acting on it, either remains at rest or moves uniformly and rectilinearly.

The impossibility of distinguishing some particular inertial frame with respect to the others by means of any experiments whatsoever constitutes the essence of the *Galilean principle of relativity*—one of the most important laws of physics.

But even though the points of view of observers studying phenomena in two inertial frames are fully equivalent, their judgements about one and the same fact will differ. For example, one of the observers will say that the seat on which he is sitting in a moving train is located at the same place in space all the time, but another observer, standing on the platform, will assert that this seat is moving from one place to another. Or one observer, firing a rifle, will say that the bullet flew out with a speed of 500 m/sec, while another observer, if he is in a frame which is moving in the same direction with a speed of 200 m/sec, will say that the bullet is flying considerably slower, with a speed of 300 m/sec.

Who of the two is right? Both. For the principle of the relativity of motion does not allow a preference to be given to any single inertial frame.

It turns out that no unconditionally true, as is said, absolute statements can be made about a location in space or the velocity of motion. The concepts of a location in space and the velocity of motion are relative. In speaking about such relative concepts, it is necessary to indicate which inertial frame of reference one has in mind.

Therefore, the absence of a single unique "correct" point of view on motion leads us to recognize the relativity of space. Space could have been called absolute only if we were able to find a body at rest in it—at rest from the point of view of all observers. But this is precisely what is impossible to do.

The relativity of space means that space may not be pictured as something into which bodies have been sprinkled.

The relativity of space was not recognized immediately by science. Even such a brilliant scientist as Newton regarded space as absolute, although he also understood that it would be impossible to prove this. This false point of view was widespread among a considerable number of physicists up to the end of the 19th century. The reasons for this are apparently of a psychological nature: we are simply very much accustomed to see the immovable "same places in space" around us.

We must now figure out what absolute judgements can be made about the nature of motion.

If bodies move with respect to one frame of reference with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$, then the difference (vector, of course) $\mathbf{v}_1 - \mathbf{v}_2$ will be identical for all inertial observers, since both of the velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ undergo the same change when the frame of reference is changed.

Thus, the vector difference between the velocities of two bodies is absolute. If so, then the vector increment in the velocity of one and the same body for a definite interval of time is also absolute, i.e. its value is identical for all inertial observers.

Just as the change in velocity, the rotation of a body has an absolute character. The direction of a rotation and the number of revolutions per minute will be identical from the point of view of all inertial frames.

The Point of View of a Celestial Observer

We decided to study motion from the point of view of an inertial frame. Won't we then have to reject the services of an Earth-bound observer? As a matter of fact, the Earth rotates about its axis and revolves around the Sun, as was proved by Nicolaus Copernicus. It may be difficult for the reader to feel now how revolutionary Copernicus' discovery was to realize that Giordano Bruno was burned at the stake, while Galileo suffered humiliation and exile, for championing the truth of Copernicus' ideas.

What was it that Copernicus' genius accomplished? Why may we place the discovery of the Earth's rotation and revolution on one plane with the ideas of human justice, for which progressive-minded people have been willing to give up their lives?

In his *Dialogue on the Two Chief Systems of the World* (the Ptolemaic and the Copernican), for whose writing he was persecuted by the Church, Galileo gave the opponent of the Copernican system the name Simplicio, which means "simpleton".

In fact, from the point of view of a simple direct observer of the world, that which is not very aptly called "common sense", the Copernican system seems mad. How can the Earth rotate? As a matter of fact, I see it and it is stationary, but the Sun and the stars are really moving.

The attitude of theologians to Copernicus' discovery is shown by the following conclusion of the Assembly of Theologians (1616):

"The doctrine that the Sun is located at the center of the world and immovable is false and absurd, formally heretical and contrary to the Sacred Writings, while the doctrine to the effect that the Earth does not lie at the center of the world and moves, possessing in addition a daily rotation, is false and absurd from the philosophical point of view, and at least erroneous from the theological."

This conclusion, in which a lack of understanding of the laws of nature and a belief in the infallibility of religious dogmas are mixed up with a false "common sense", testifies better than anything else to the strength of Copernicus' spirit and mind, and those of his disciples, having so resolutely broken with the "truths" of the 17th century.

But let us return to the question posed above.

If the velocity of an observer's motion changes or if he rotates, then he must be deleted from the list of "correct" observers. But it is precisely under these conditions that an observer on the Earth is found. However, if the change in velocity or the observer's rotation during the time he is investigating a motion is small, then such an observer may be conditionally regarded as "correct". Will this pertain to an observer on the Earth?

During a second the Earth will turn 1/240 of a degree, i.e. about 0.000 07 radian. This isn't so very much. The Earth is therefore quite inertial with respect to a great many phenomena.

Nevertheless, one can no longer forget about the Earth's rotation when dealing with prolonged phenomena.

Under the dome of St. Isaac Cathedral an enormous pendulum was hung. If we start oscillating this pendulum, within a short time it will be possible to notice that the plane of its oscillation is slowly turning. After several hours, the plane of oscillation will turn through a noticeable angle. Such an experiment with this kind of pendulum was first performed by the French scientist Foucault, and has born his name ever since. Foucault's experiment yields a visual demonstration of the Earth's rotation (Figure 13).

Thus, if the observed motion continues for a long time, we shall be forced to reject the services of an Earth-bound

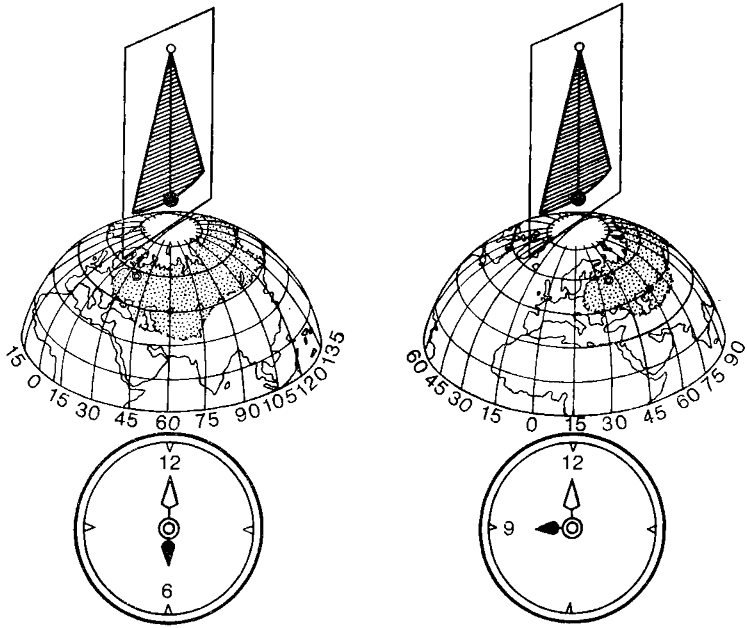


Fig. 13

observer and take a frame of reference connected to the Sun and the stars as our basis. Such a frame was used by Copernicus, assuming the Sun and the surrounding stars to be fixed.

However, in reality Copernicus' frame is not completely inertial.

The Universe consists of a great number of star-clusters—islands of the Universe, which are called galaxies. In the galaxy to which our solar system belongs, there are approximately one hundred billion stars. The Sun is revolving around the center of this galaxy with a period of about 180 million years and a speed of 250 km/sec.

What error will be made by assuming a solar observer to be inertial?

For a comparison of the merits of terrestrial and solar observers, let us compute the angle through which a solar frame of reference turns during a second. If a complete revolution takes place every 180×10^6 years (6×10^{15} sec), then in one second a solar frame of reference will turn through an angle of 6×10^{-14} degree or 10^{-15} radian. We may say that a solar observer is 100 billion times "better" than a terrestrial one.

Desiring an even closer approximation to an inertial frame, astronomers take a frame of reference connected to several galaxies as a basis. Such a frame of reference is the most inertial of all possible kinds. It is impossible to find a better frame.

Astronomers may be called star gazers in two senses: they observe stars and describe the motions of heavenly bodies from the point of view of the stars.

Acceleration

In order to characterize non-constant velocities, physicists use the concept of acceleration.

The change in velocity during a unit of time is called *acceleration*. Instead of saying "the body's velocity changed by a in 1 second," we say more briefly "the body's acceleration is equal to a."

If we denote by v_1 the speed of a rectilinear motion at the first instant, and by v_2 at the next, then our rule for calculating the acceleration a is expressed by the formula

$$a = \frac{v_2 - v_1}{t}$$

where t is the time during which the speed changed.

Speed is measured in cm/sec (or m/sec, etc.), time, in seconds. Hence, acceleration is measured in cm/sec per second. A number of centimeters per second is divided by seconds. Thus, the unit of acceleration will be cm/sec^2 (or m/sec^2, etc.).

Of course, the acceleration can change during the course of a motion. However, we shall not complicate our treatment with this inessential circumstance. We shall implicitly assume that the velocity changes uniformly during the course of a motion. Such a motion is called *uniformly accelerated*.

What is acceleration of curvilinear motion?

Since velocity is a vector, a change (difference) in velocity is a vector, and so acceleration is also a vector. In order to find the acceleration vector, one must divide the vector difference between the velocities by the time. But we have already described how to construct a vector change in velocity.

The highway takes a turn. Let us note two nearby positions of a car and represent its velocities by vectors (Figure 14). Subtracting these vectors, we obtain a quantity which is by no means equal to zero; dividing it by the elapsed time, we find the acceleration vector. An acceleration took place even when the speed around the turn did not change. Curvilinear motion is always accelerated. Only uniform rectilinear motion is unaccelerated.

In speaking about the velocity of a body's motion, we always stipulated what our point of view was with regard to the motion. A body's velocity is relative. From the point

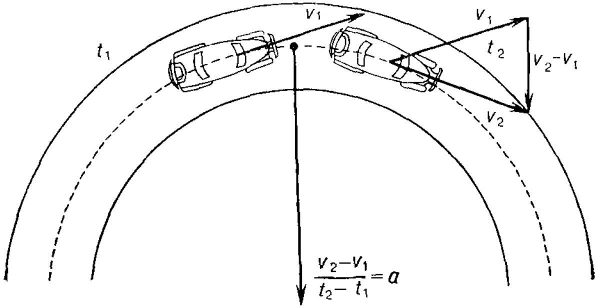


Fig. 14

of view of one inertial frame it can be large, and from the point of view of another inertial frame it can be small. Don't we have to make the same kind of stipulations when speaking about accelerations? Of course not. Unlike a velocity, an acceleration is absolute. From the point of view of all imaginable inertial frames, an acceleration will be identical. As a matter of fact, an acceleration depends on the difference in a body's velocity between the first and second instants, and this difference, as we already know, will be identical from all points of view, i.e. is absolute.

Acceleration and Force

If no force is acting on a body, it can only move without acceleration. Conversely, the action of a force on a body accelerates it; moreover, the greater the force, the greater will be the acceleration. The faster we want to move a loaded waggon, the more we have to strain our muscles. As a rule, two forces act on a moving body: accelerating—the pulling force, and decelerating—the force of friction or air resistance.

The difference between these two forces, the so-called resultant force, may be directed along or against the motion.

In the first case, the body speeds up its motion; in the second case, it slows it down. If these oppositely acting forces are equal to each other (balance), the body will move uniformly, just as though there were no forces acting on it.

But how is a force related to the acceleration it creates? The answer turns out to be very simple. The acceleration is proportional to the force:

$$a \propto F$$

(The symbol $\propto$ denotes "is proportional to".)

But another question still remains to be answered: how do a body's properties influence its ability to accelerate its motion under the action of one or another force? For it is clear that one and the same force acting on different bodies will give them different accelerations.

We shall find the answer to the question we have posed in the remarkable circumstance that all bodies fall to the Earth with the same acceleration. This acceleration is denoted by the letter g. In the vicinity of Moscow $g = 981$ cm/sec^2.

Direct observation will not, at first sight, confirm the identity of acceleration for all bodies. The fact of the matter is that when a body is falling under ordinary conditions, besides gravity there is another, "hindering" force acting on it—air resistance. Philosophers of antiquity were quite confused by the difference in the way light and heavy bodies fall. A piece of iron falls quickly, but a feather glides through the air. A sheet of paper falls slowly to the ground, but if we roll it up, this same sheet will fall considerably faster. The fact that the atmosphere distorts the "true" picture of the motion of a body under the action of the Earth was already understood by the Ancient Greeks. However, Democritus thought that even if the air were deleted, heavy bodies would always fall faster than light ones. But air resistance can have the opposite effect—for example, a sheet of aluminium foil (all unrolled) will fall more slowly than a small ball made by crumpling a piece of the same foil.

Incidentally, metallic wire of such a thinness (several microns) is now manufactured that it glides through the air like a feather.

Aristotle thought that all bodies should fall identically in a vacuum. However, he used this theoretical conclusion to make the following paradoxical deduction: "The falling of different bodies with the same speed is so absurd that the impossibility of a vacuum's existence is clear."

None of the scientists of antiquity or the Middle Ages guessed that it could be experimentally verified whether bodies fall to the Earth with different or the same accelerations. Only Galileo demonstrated by means of his remarkable experiments (he investigated the motion of balls down an inclined plane and the fall of bodies thrown from the top of the leaning tower of Pisa) that at any given point on the Earth, all bodies fall with the same acceleration, regardless of their mass. At the present time such experiments are quite easily performed with the aid of a long tube, out of which the air has been pumped. A feather and a stone fall identically in such a tube: only one force acts on the bodies—weight—air resistance has been reduced to zero. In the absence of air resistance, the fall of any body is a uniformly accelerated motion.

Let us now return to the question posed above. How does the ability of a body to accelerate its motion under the action of a given force depend on its properties?

Galileo's law states that all bodies, regardless of their masses, fall with one and the same acceleration; hence, a mass of m kg under the action of a force of m kgf moves with an acceleration g.

Now suppose we are no longer talking about falling bodies, and a force of 1 kgf is acting on a mass of m kg. Since acceleration is proportional to force, it will be m times less than g.

We have arrived at the conclusion that the acceleration a of a body for a given force (1 kgf in our example) is inversely proportional to its mass.

Uniting both conclusions, we may write:

$$a \propto \frac{F}{m}$$

i.e. the acceleration is directly proportional to the force for a constant mass and inversely proportional to the mass for a constant force.

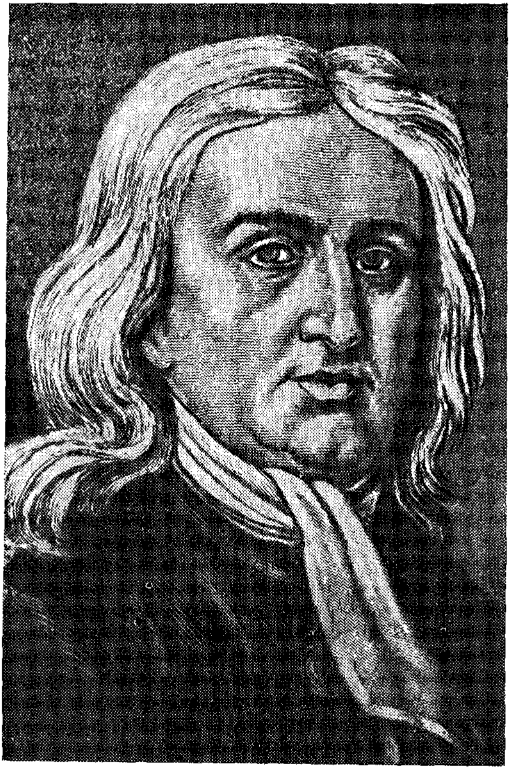
This law, relating acceleration to a body's mass and the force acting on it, was discovered by the great English scientist Sir Isaac Newton (1643-1727), and bears his name.*

Acceleration is directly proportional to the acting force and inversely proportional to the body's mass, and does not depend on any other properties of the body. It follows from Newton's law that it is precisely the mass which is the measure of a body's "inertness". For identical forces, it is more difficult to accelerate a body of larger mass. We see that the concept of mass, which we first knew as a "modest" quantity, determined by weighing a body on a balance scale, has acquired a new deep meaning: the mass characterizes a body's dynamical properties.

Newton's law may be written as follows:

$$kF = ma$$

* Newton himself showed that motion is subject to three laws. The law which we are now discussing appears on Newton's list as the second. He called the law of inertia the first law, and the law of action and reaction, the third.



SIR ISAAC NEWTON (1643-1727)—a brilliant English physicist and mathematician, one of the greatest scientists in the history of mankind. Newton formulated the basic concepts and laws of mechanics, discovered the law of universal gravitation, creating by the same token a physical picture of the world which remained inviolable until the beginning of the 20th century. He developed a theory of the motion of celestial bodies, explained the most important special features of the Moon's motion and gave an explanation for the tides. In optics, some remarkable discoveries, facilitating the rapid growth of this branch of physics, are due to Newton. Newton devised a powerful method of the mathematical investigation of nature; the honour of creating the differential and integral calculus belongs to him. This exerted an enormous influence on the entire subsequent development of physics and facilitated the introduction of mathematical methods of research.

where k is a constant coefficient. This coefficient depends on what units we choose.

Instead of making use of the unit of force (kgf) we already have available, we shall act in a different manner. As physicists often try to do, we shall choose our unit of force in such a way that the coefficient of proportionality in Newton's law becomes equal to one. Then Newton's law takes the following form:

$$F = ma$$

As we have already said, in physics it is customary to measure mass in grams, distance in centimeters and time in seconds. The system of units based on these three fundamental quantities is called the cgs system.

Let us now choose, using the principle formulated above, the unit of force. A force will then obviously be equal to one in case it imparts an acceleration of 1 cm/sec^2 to a mass of 1 g. Such a force received the name *dyne* in this system.

According to Newton's law, $F = ma$, the force will be expressed in dynes if we multiply m grams by a cm/sec^2. One therefore makes use of the following notation:

$$1 \text{ dyne} = 1 \text{ g-cm/sec}^2$$

The weight of a body is usually denoted by the letter P. The force P gives the body an acceleration g, and in dynes we obviously have

$$P = mg$$

But we already had a unit of force—the kilogram-force (kgf). We immediately find the relation between our new and old units from the last formula:

$$1 \text{ kgf} = 981\,000 \text{ dynes}$$

A dyne is a very small force. It is equal to about one milligram of weight.

We have already mentioned the new system of units (SI) which has been worked out recently. The name for the new unit of force, *newton* (N), is fully deserved. For such a choice of units, Newton's law will look as simple as possible; this new unit is defined as follows:

$$1\ \mathrm{N} = 1\ \mathrm{kg\text{-}m/sec^2}$$

i.e. 1 N is the force necessary to impart an acceleration of 1 m/sec^2 to a mass of 1 kg.

It is not difficult to relate this new unit to the dyne and the kilogram:

$$1\ \mathrm{N} = 100\,000\ \mathrm{dynes} = \frac{1}{9.8}\ \mathrm{kgf}$$

Rectilinear Motion with Constant Acceleration

Such a motion arises, according to Newton's law, when the resultant force acting on a body, speeding it up or slowing it down, is constant.

Such conditions arise rather frequently, even though only approximately: a car moving with its motor cut off slows down under the action of the more or less constant force of friction; a weighty object falls from a height under the action of a constant force.

Knowing the magnitude of the resultant force, and also the body's mass, we can find the magnitude of the acceleration according to the formula $a = F/m$. Since

$$a = \frac{v - v_0}{t}$$

where t is the duration of the motion, v is the final speed, and v_0 is the initial speed, with the aid of this formula it is possible to answer a series of questions of, say, the following type: how long will it take a train to come to a halt (where the decelerating force, the train's mass and the initial speed are known)? Or how much speed will a car gather (where the motor's power, the resistance, the car's mass and the acceleration's duration are known)?

We are often interested in knowing the distance covered by a body in a uniformly accelerated motion. If the motion is uniform, then the distance covered is found by multiplying the speed of the motion by its duration. If the motion is uniformly accelerated, then the calculation of the distance covered is carried out as though the body were moving uniformly for the same time t with a speed equal to half the sum of the initial and final speeds:

$$s = \frac{1}{2}(v_0 + v)t$$

Thus, for uniformly accelerated (or decelerated) motion, the distance covered by a body is equal to the product of half the sum of the initial and final speeds by the time of the motion. The same distance would be covered during the same time in a uniform motion with speed $(v_0 + v)/2$. In this sense, one can say that $(v_0 + v)/2$ is the average speed of the uniformly accelerated motion.

It is helpful to compose a formula which would show the dependence of the distance covered on the acceleration. Substituting $v = v_0 + at$ in the last formula, we find:

$$s = v_0 t + \frac{1}{2}at^2$$

or, if the motion occurs without any initial speed,

$$s = \frac{1}{2}at^2$$

If a body travels 5 m in one second, then in two seconds it will travel (4×5) m, in three seconds (9×5) m, etc.

The distance travelled grows in proportion to the square of the time.

A heavy body falls from a height in accordance with this law. The acceleration in free fall is equal to g, and our formula acquires the following form:

$$s = \frac{981}{2} t^2 \ [\mathrm{cm}]$$

if t is expressed in seconds.

If a body could fall without hindrance for some 100 seconds, it would cover an enormous distance from the beginning of its fall—about 50 km. Moreover, only a mere 0.5 km would be covered in the first 10 seconds—this is what accelerated motion means.

But what speed will a body develop in falling from a given height? For the answer to this question we shall need formulas relating the covered distance to the acceleration and the speed. Substituting $t = (v - v_0)/a$ in $s = (1/2)(v_0 + v) t$, we obtain:

$$s = \frac{1}{2a}(v^2 - v_0^2)$$

or, if the initial speed is equal to zero,

$$s = \frac{v^2}{2a}, \quad v = \sqrt{2as}$$

Ten meters is the height of a small two- or three-storey house. Why is it dangerous to jump to the ground from the roof of such a house? A simple calculation shows that the speed of such a free fall would reach the value $v = \sqrt{2 \times 9.8 \times 10}$ m/sec $= 14$ m/sec ≈ 50 km/hr, and this is, after all, the speed of a car within city limits.

Air resistance will not reduce this speed much.

The formulas we have singled out are employed for the most varied computations. Let us apply them in order to see how motions take place on the Moon.

In H. G. Wells' novel *The First Men in the Moon* we read about the surprises experienced by travellers in their fantastic trips. On the Moon, the acceleration of gravity is approximately 6 times less than terrestrial. If a falling body on the Earth covers 5 m in the first second, it will "float" down only 80 cm in all on the Moon (the acceleration there is about 1.6 m/sec^2).

The formulas we have written out permit us to rapidly calculate the lunar "miracles".

A jump from a height of h meters takes $t = \sqrt{2h/g}$ seconds. Since lunar acceleration is 6 times less than terrestrial, the jump will require $\sqrt{6} \approx 2.45$ times more time on the Moon. By how many times will the final speed of the jump be decreased ($v = \sqrt{2gh}$)?

One can jump safely from the roof of a three-storey house on the Moon. The height of a jump with the same initial speed will be increased by a factor of 6 ($h = v^2/2g$). A child will be able to jump higher than the record set on the Earth.

Path of a Bullet

People have been solving the problem of throwing an object as far as possible from time immemorial. A stone thrown by hand or shot from a sling, an arrow flown from a bow, a rifle bullet, an artillery shell, a ballistic missile—here is a brief list of successes in this field.

The thrown object will move in a curved line, called a parabola. It can be constructed without difficulty if we regard the thrown body's motion as the sum of two motions—horizontal and vertical—taking place simultaneously

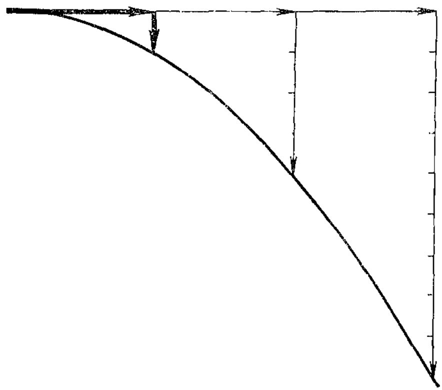


Fig. 15

and independently. The acceleration of gravity is vertical, and so a flying bullet moves horizontally by inertia with a constant velocity and simultaneously falls to the Earth vertically with a constant acceleration. But how can we add these two motions?

Let us begin with a simple case—when the initial velocity is horizontal (say, we are dealing with a shot from a rifle whose barrel is horizontal).

Take a sheet of graph paper and draw a vertical and a horizontal lines (Figure 15). Since the two motions are taking place independently, after t seconds the body is displaced by an interval of v_0t to the right and an interval of $gt^2/2$ downwards. Mark off the segment v_0t along the horizontal line, and from its end point, the vertical segment $gt^2/2$. The end point of the vertical segment represents the point where the body will be after t seconds.

This construction must be carried out for several points, i.e. for several instants. A smooth curve—the parabola representing the body's trajectory—will pass through these

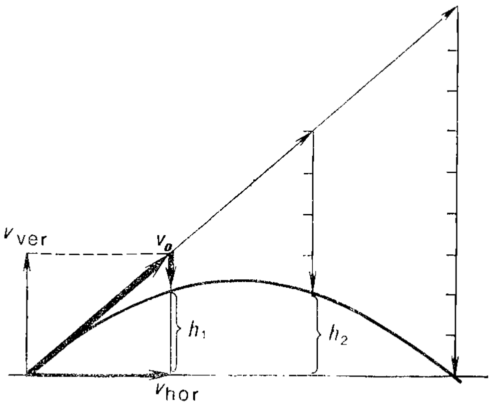


Fig. 16

points. The more frequently one lays off these points, the more accurately will the trajectory of the bullet's flight be constructed.

A trajectory has been constructed in Figure 16 for the case when the initial velocity v_0 is directed at an angle.

The vector v_0 should first of all be decomposed into its vertical and horizontal components. On the horizontal line we mark off $v_{hor}t$—the distance through which the bullet will move horizontally in t seconds.

But the bullet simultaneously performs an upward motion. After t seconds the body will rise to a height of $h = v_{ver}t - gt^2/2$. By means of this formula, substituting in it the times of interest to us, we can compute the vertical displacements and mark them off on the vertical axis. The value of h will first increase (rise) and then decrease.

It now remains to mark the trajectory's points on the graph, just as we did in the preceding example, and draw a smooth curve through them.

If the rifle barrel is held horizontally, the bullet will soon burrow into the ground; if the barrel is vertical, it will fall at the place where the shot was fired. Therefore, in order to shoot as far as possible, one must fix the barrel of the rifle at some angle to the horizontal. But at what angle?

Let us again employ the same device—decompose the initial velocity vector into its two components: a vertical vector equal to v_1 and a horizontal vector, to v_2. The time between the moment the shot is fired until the moment the bullet reaches its summit is equal to v_1/g. Note that the bullet will be falling downwards for the same length of time, i.e. the complete time of the bullet's flight until it lands on the ground is $2v_1/g$.

Since the horizontal motion is uniform, the range of the flight is equal to

$$s = \frac{2v_1v_2}{g}$$

(we have ignored the height of the rifle above ground level in our calculation).

We have obtained a formula which shows that the range of the flight is proportional to the product of the velocity's components. But for what firing direction will this product be greatest? This question can be expressed in the language of geometry. The velocities v_1 and v_2 form the sides of the velocity rectangle; a diagonal in it is the combined velocity v. The product v_1v_2 is equal to the area of this rectangle.

Our question reduces to the following: given the length of a diagonal, what sides must be taken for the rectangle's area to be maximal? It is proved in geometry that this condition is satisfied by a square. Therefore, the range of the bullet's flight will be greatest when $v_1 = v_2$, i.e. when the velocity rectangle reduces to a square. A diagonal of the velocity square forms an angle of 45° with the horizontal—this is precisely the angle at which the rifle must be held for the bullet to fly as far as possible.

If v is the bullet's combined velocity, then in the case of a square we have $v_1 = v_2 = v/\sqrt{2}$. The range-of-flight formula for this optimal case looks as follows: $s = v^2/g$, i.e. the range will be twice as great as the maximal height of a bullet fired upwards with the same initial speed.

The maximal height of a bullet fired at an angle of 45° will be $h = v_1^2/2g = v^2/4g$, i.e. four times less than the range of flight.

It should be admitted that the formulas we have been applying yield exact results only in the case, quite remote from practice, when air is absent. In many cases air resistance plays a decisive role and radically changes the entire picture.

Circular Motion

If a point moves around a circle, the motion is accelerated, if only because the velocity is changing its direction all the time. The speed may remain unchanged, and we shall confine our attention to precisely such a case.

We shall draw the velocity vectors at successive time intervals, transferring their initial points to a single point. (We have the right to do this.) If a velocity vector is rotated through a small angle, the change in velocity, as we know, will be represented by the base of an isosceles triangle. Let us construct the changes in velocity during the course of a complete revolution of the body (Figure 17). The sum of the magnitudes of the changes in velocity during a complete revolution will be equal to the sum of the sides of the depicted polygon. In constructing each small triangle, we have implicitly assumed that the velocity vector changed by jumps, but its direction is actually changing continuously.

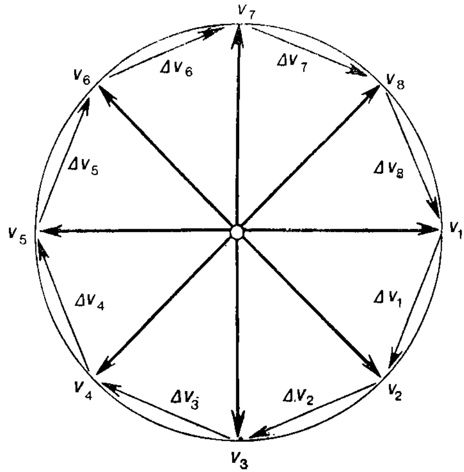


Fig. 17

It is perfectly clear that the smaller we take the vertex angles of the small triangles, the less will be our error The smaller the sides of our polygon, the closer will they cling to the circle of radius v. Consequently, the exact value of the sum of the magnitudes of the changes in velocity during the course of the point's revolution will be the circumference $2\pi v$ of the circle. The magnitude of the acceleration is found by dividing it by the time T of a complete revolution.

Thus, the magnitude of the acceleration in a uniform motion around a circle is given by the formula $a = 2\pi v/T$.

But the time of a complete revolution in motion around a circle of radius R can be expressed in the form $T = 2\pi R/v$. Substituting this expression in the preceding formula, we obtain the following for the acceleration: $a = v^2/R$.

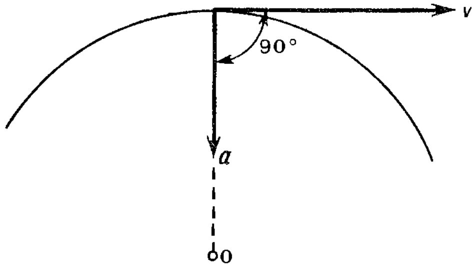


Fig. 18

For a fixed radius of rotation, the acceleration is proportional to the square of the speed. For a given speed, the acceleration is inversely proportional to the radius.

This same reasoning shows us how the acceleration of a circular motion is directed at each given instant. The smaller the vertex angle of the isosceles triangles which we used for our proof, the nearer the angle between the increase in velocity and the velocity will be to 90°.

Therefore, the acceleration of a uniform circular motion is directed perpendicular to the velocity; but how are the velocity and acceleration directed relative to the trajectory? Since the velocity is tangent to the path, the acceleration is directed along the radius towards the center of the circle. These relationships are clearly seen in Figure 18.

Try to twirl a stone on a string. You will clearly feel the need for muscular exertion in order to do this. But why is force necessary? After all, isn't the body moving uniformly? The whole point here is that it isn't! The body is moving with a constant speed, but the continuous change in the direction of the velocity makes this motion accelerated. Force is necessary in order to deflect the body from an inertial straight path. Force is needed in order to create the acceleration v^2/R, which we have just computed.

According to Newton's law, where the acceleration is directed, there the force is pointed. Consequently, a body revolving around a circle with a constant speed should be subject to the action of a force directed along a radius towards the center. The force acting on the stone, exerted by the string, is just what supplies the acceleration v^2/R. Hence, the magnitude of this force is mv^2/R.

The string pulls the stone; the stone pulls the string. In these two forces we recognize "an object and its mirror image"—forces of action and reaction. The force with which the stone acts on the string is frequently called *centrifugal*. The centrifugal force is, of course, equal to mv^2/R and directed along the radius out from the center of the circle. The centrifugal force acts on the body which is counteracting the inertial tendency of the revolving body to move rectilinearly.

What we have said applies also to the case when the role of the "string" is played by gravity. The Moon revolves around the Earth. What is it that retains our satellite? Why doesn't it go off, following the law of inertia, in an interplanetary trip? The Earth is holding on to the Moon with an "invisible string"—a gravitational force. This force is equal to mv^2/R, where v is the speed of the motion along the lunar orbit, and R is the distance to the Moon. The centrifugal force in this case acts on the Earth, but, because of the Earth's great mass, it only slightly influences the character of our planet's motion.

Suppose that it is required to send an artificial Earth satellite into a circular orbit at a distance of 300 km from the Earth's surface. What should be the speed of such a satellite? At a distance of 300 km, the acceleration of gravity is somewhat less than on the surface of the Earth, and is equal to 8.9 m/sec^2. The acceleration of a satellite moving in a circle is equal to v^2/R, where R is the distance from the center of the circle (i.e. from the center of the Earth)—about 6600 km = 6.6×10^6 m. On the other hand, this acceleration is equal to the acceleration of gravity, g. Consequently $g = v^2/R$, from which we find the speed of the satellite's orbital motion:

$$v = \sqrt{gR} = \sqrt{8.9 \times 6.6 \times 10^6} = 7700 \text{ m/sec} = 7.7 \text{ km/sec}$$

The minimal speed necessary for a horizontally propelled body to become an Earth satellite is called the orbital velocity. It is clear from the example we have given that this speed is close to 8 km/sec.

Three

MOTION FROM AN "UNREASONABLE" POINT OF VIEW

Principle of Equivalence

In the preceding chapter we found a "reasonable point of view" on motion. True, the "reasonable" points of view, which we called inertial frames, turned out to be infinite in number.

Now, armed with a knowledge of the laws of motion, we can become interested in what motion looks like from an "unreasonable" point of view. Our interest in how inhabitants of non-inertial frames live is by no means idle, if only because we ourselves are dwelling in such a system.

Let us imagine that, having grabbed our measuring instruments, we settled down in an interplanetary spaceship and went travelling in the stellar universe.

Time flies quickly. The Sun already resembles a little star. The engine has been cut off and the ship is far away from gravitating bodies.

Let us now see what's going on in our flying laboratory. Why does the thermometer that slid off its nail float in the air and not fall to the floor? In what a strange position, deviating from the "vertical", has the pendulum hanging on the wall gotten stuck! We immediately find the solution: after all, the ship is not on the Earth, but in interplanetary space. The objects have lost their weight.

Having feasted our eyes on this extraordinary scene, we decide to change our course. We turn on the jet engine by pressing a button, and suddenly ... the objects surrounding us seemed to come to life. All bodies which hadn't been made fast were brought into motion. The thermometer fell down, the pendulum began oscillating and, gradually coming to rest, assumed a vertical position, the pillow obediently sagged under the weight of the valise lying on it. Let us take a look at the instruments which indicate the direction in which our ship started accelerating. Upwards, of course! The instruments show that we chose a motion with an acceleration of 9.8 m/sec^2, not very great, considering the possibilities of our ship. Our sensations are quite ordinary; we feel the way we did on Earth. But why so? As before, we are unimaginably far from gravitational masses, there is no gravity, but objects have acquired weight.

Let us drop a marble and measure the acceleration with which it falls to the floor of the spaceship. It turns out that the acceleration is equal to 9.8 m/sec^2. This is the number we have just read on the instruments measuring the rocket's acceleration. The ship is moving upwards with the same acceleration with which the bodies in our flying laboratory are falling downwards.

But what is "up" or "down" in a flying ship? How simple things were when we lived on the Earth. There the sky was up and the Earth was down. And here? Our up has one unquestionable property—it is the direction of the rocket's acceleration.

It isn't hard to understand the meaning of our observations: no forces were acting on the marble we dropped. It moves as a result of inertia. It is the rocket that moves with an acceleration relative to the marble, and it seems to us, who are inside the rocket, that the marble is "falling" in the direction opposite to that of the rocket's acceleration.

Naturally, the acceleration of this "fall" is equal in magnitude to the true acceleration of the rocket. It is also clear that all bodies in the rocket will "fall" with the same acceleration.

We may draw an interesting conclusion from all that has been said. Bodies start "weighing" when the rocket's motion accelerates. Moreover, the "gravitational force" has a direction opposite to that of the rocket's acceleration, and the acceleration of free "fall" is equal in magnitude to that of the jet ship's motion. And what is most remarkable is the fact that in practice we are unable to distinguish a frame's accelerated motion from the corresponding gravitational force.* If we were inside a spaceship with closed windows, we could not tell whether we were at rest on the Earth or moving with an acceleration of 9.8 m/sec^2. This indistinguishability of an acceleration from the action of a gravitational force is called in physics the *equivalence principle*.

This principle, as we shall now see illustrated with a series of examples, permits one to quickly solve many problems by adding to real forces the fictitious gravitational force existing in an accelerating frame of reference.

The elevator can serve as our first example. Let us take along a spring balance with weights and go up in an elevator. We shall follow the behaviour of the scale's pointer after placing a kilogram weight on it (Figure 19). The ascent has begun; we see that the scale reading has increased, as though the weight weighed more than a kilogram. The

* Only in practice. There is a difference in principle. Gravitational forces on the Earth are directed along radii towards the Earth's center. This means that the directions of acceleration at two different points form an angle. In a rocket moving with an acceleration, the directions of weight are strictly parallel at all points. Acceleration also changes with height on the Earth; this effect is absent in an accelerating rocket.

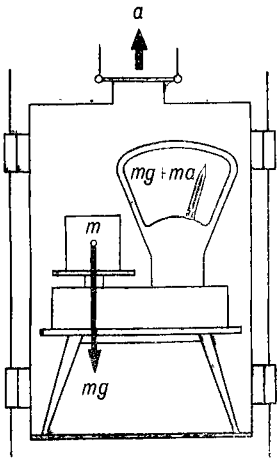


Fig. 19

equivalence principle will easily explain this fact. During the elevator's upward motion with an acceleration a, there arises an additional gravitational force, directed downwards. Since the acceleration of this force is equal to a, the additional weight is equal to ma. Hence, the scale shows a weight of $mg + ma$. The acceleration has ended, and the elevator is moving uniformly—the scale has returned to its initial position and shows a weight of 1 kgf. We are getting close to the top floor, and the elevator's motion is slowing down. What will now happen to the spring balance? Well, of course, the load now weighs less than one kilogram. When the motion is slowing down, the acceleration vector points downwards. Therefore, an additional fictitious gravitational force is directed upwards, opposite to the direction of the Earth's gravitation. Now a is negative, and so the scale shows a quantity less than mg. After the elevator comes to a halt, the scale returns to its initial position. Let us begin the descent. The elevator's motion speeds up; the acceleration vector is directed downwards; hence, an additional gravitational force is directed upwards. The load now weighs less than a kilogram. When the motion becomes uniform, the additional weight disappears, and towards the end of our trip on the elevator—when the downward motion is decelerating—the load will weigh more than a kilogram.

The unpleasant sensations experienced in rapidly accelerating and decelerating elevators are related to the change in weight under consideration.

If an elevator is falling with an acceleration, the bodies inside it seem to become lighter. The greater this acceleration, the greater will be the loss of weight. But what will happen when a frame of reference falls freely? The answer is clear: in this case, bodies stop pressing down on the scale—cease weighing: the Earth's gravitation will be balanced by the additional gravitational force existing in such a freely falling frame. Being in such an "elevator", one can calmly place a ton on one's shoulders.

At the beginning of this section, we described life "without weight" in an interplanetary spaceship which has left the sphere of gravitation. There is no weight in such a spaceship during uniform rectilinear motion, but the same thing also takes place during a frame's free fall. Hence, there is no need to leave the sphere of gravitation. Weight is absent in every interplanetary ship which is moving with its engine cut off. A free fall leads to the loss of weight in such systems. The equivalence principle brought us to the conclusion that a frame of reference moving rectilinearly and uniformly far from the action of gravitational forces is almost (see the footnote on p. 70) completely equivalent to a frame of reference falling freely under the action of its weight. In the first system there is no weight, while in the second the "downward weight" is balanced out by the "upward weight". We will not detect any difference between these systems.

Life "without weight" begins in an artificial Earth satellite at the moment when the ship is orbited and begins moving without the aid of a rocket.

The first space traveller was the dog Laika, and soon afterwards a human being adapted to life "without weight" in the spaceship's cabin. The Soviet cosmonaut, Yuri Gagarin, was the first to do so.

Life in the cabin of a spaceship cannot be called ordinary. A great deal of inventiveness and ingenuity were needed in order to make objects, so easily subordinated by gravity, obedient. Is it possible, for example, to pour water from a bottle into a glass? For water pours "downwards" under the action of gravity. Is it possible to cook food if water cannot be heated on a stove? (Warm water will not mix with cold one.) How can one write with a pencil on paper if a slight push of the former against a table is enough to drive him aside? Neither a match, nor a candle, nor a gas burner will burn, since burned-up gases will not rise upwards (after all, there is no up!) to make room for oxygen. It was even necessary to think about how to guarantee a normal course for the natural processes occurring in the human organism, for these processes are "accustomed" to the Earth's gravitation.

Let us now take up the question of physical observations in an accelerating bus or streetcar. A peculiarity of this example, distinguishing it from the preceding one, consists in the following. In the example with the elevator, the additional weight and the Earth's gravitation were directed along a single line. In a decelerating or accelerating streetcar, the additional weight is directed at right angles to the Earth's gravitation. This induces distinctive, although customary, sensation in the passenger. If the streetcar increases its speed, there arises an additional force, opposite in direction to that of motion. Let us add this force to that of the Earth's gravitation. The resultant force acting on a person in the car will be directed at an obtuse angle to the direction of the motion. Standing, as usual, face forward in the car, we sense that our "upwards" has moved. In order not to fall, we shall want to become "vertical"—as shown in Figure 20*a*. Our "vertical" is slanting. It is inclined at an acute angle to the direction of the motion. If a person

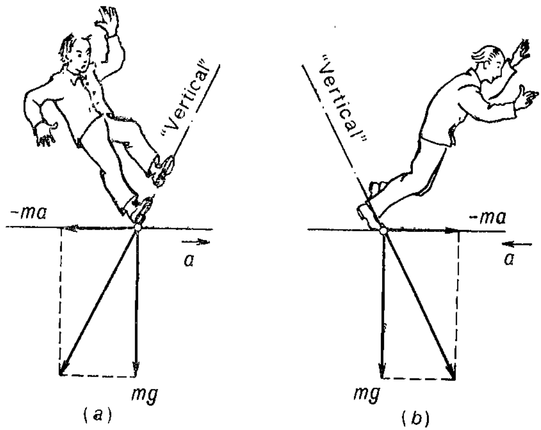


Fig. 20

stands without holding on to anything, he will be sure to fall backwards.

Finally, the streetcar's motion becomes uniform, and we can stand calmly. However, we are drawing close to the next stop. The driver applies the brakes and ... our "vertical" is deviating. It is now directed, as can be seen from the drawing in Figure 20*b*, at an acute angle to the motion. In order not to fall, the passenger leans backwards. However, he won't remain long in such a position. The car comes to a halt, the deceleration disappears and the "vertical" assumes its previous position. The position of one's body must again be changed. Check your sensations. Isn't it true that when the deceleration began you seemed to be pushed from behind (the "vertical" was in back of you)? You "straightened up", but the car has now come to a halt—the "vertical" is in front of you, and so you experience the sensation of being pushed in your chest.

Similar phenomena also occur when a streetcar moves around a curve. We know that motion around a circle, even with a fixed speed, is accelerated. The faster the streetcar moves and the smaller the radius of curvature R, the greater will be the acceleration v^2/R. The acceleration of such a motion is directed along a radius towards the center. But this is equivalent to the appearance of a new force, directed outwards from the center. Therefore, an additional force of mv^2/R will be acting on a streetcar passenger during a turn, throwing him out towards the external side of the curve. The radial force mv^2/R is called centrifugal. We have already met this force before, on p. 66, true, considered from a somewhat different point of view.

The action of a centrifugal force during the turning of a streetcar or bus can only lead to a slight unpleasantness. The force mv^2/R is not large in these cases. However, during a speedy motion around a curve, the centrifugal force can become great enough to pose a threat to one's life. Pilots come across large values of mv^2/R when their airplanes "loop-the-loop". While the airplane is describing a circle, the centrifugal force acts on the pilot, pinning him to his seat. The smaller the loop's circumference, the greater the additional force with which the pilot is pinned to the seat. If this weight becomes large enough, a person can be "torn"—for human tissue possesses limited strength and cannot withstand an arbitrary weight.

But how much weight can a person "put on" without seriously endangering his life? That depends on the duration of the overload. If it lasts a fraction of a second, a person is capable of withstanding an overload of 7-9g. A pilot can withstand an overload of 3-5g continuing for ten seconds. Cosmonauts are interested in the kind of overload a person is able to bear for tens of minutes and even, perhaps, hours.

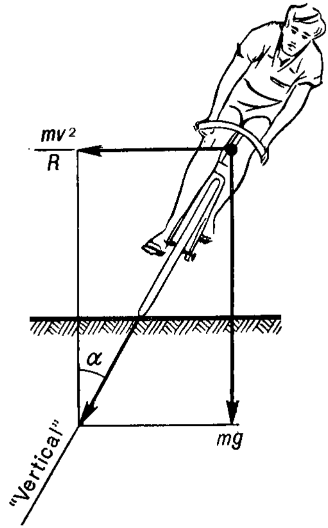


Fig. 21

In such cases, it is likely that the overload should be considerably lighter.

Let us compute the radii of a loop which an airplane flying at various speeds can describe without any danger to the pilot. Let us take the average figure of $4g$. This is the magnitude of the acceleration, i.e. $v^2/R = 4g$ and $R = v^2/4g$. For a speed of 360 km/hr = 100 m/sec, the loop's radius will be 250 m; but if the speed is 4 times greater, i.e. 1440 km/hr (and such speeds have already been surpassed by modern jet airplanes), the radius of the loop should be increased by a factor of 16. The loop's minimal radius becomes equal to 4 km.

Nor shall we leave a more modest form of transportation—the bicycle—without attention. Everyone has seen how

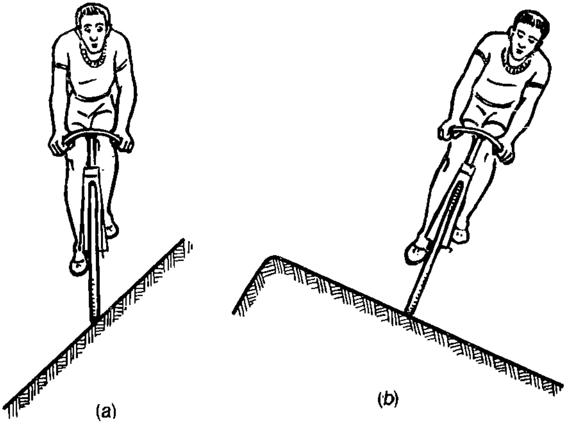


Fig. 22

a cyclist inclines while rounding a turn. Let us suggest to a cyclist that he should ride around a circle of radius R with speed v, i.e. move with acceleration v^2/R, directed towards the center. Then, besides the Earth's gravitation, an additional centrifugal force, directed horizontally outwards from the center of the circle, will act on the cyclist. These forces and their sum are shown in Figure 21. It is clear that the cyclist should hold himself "vertically", or else he will fall down. But ... his vertical does not coincide with that of the Earth. It can be seen from the drawing that the vectors mv^2/R and mg are the legs of a right triangle. The ratio of the leg opposite angle α to the adjacent one is called the tangent of angle α in trigonometry. We have $\tan \alpha = v^2/Rg$; the mass has been cancelled in full agreement with the equivalence principle. Hence, the cyclist's angle of inclination does not depend on his mass—both a stout and a thin riders must incline identically. The formula and the triangle drawn in the figure show the dependence of the incline on the motion's speed (it grows as the latter increases) and the circle's radius (it increases as the latter decreases).

We have explained why the vertical of the cyclist does not coincide with that of the Earth. What then will he feel? We must rotate Figure 21 in order to find it out. The road now looks like the slope of a mountain (Figure 22*a*), and it becomes clear that if the force of friction between the tires and the asphalt is insufficient (for example, when the road is wet), the bicycle may slip and a sharp turn may end with a fall into a ditch.

In order to forestall this, highways are built with sharp turns inclined, i.e. horizontal for a cyclist—as shown in Figure 22*b*. In this way, the tendency to slip can be greatly diminished, or even entirely eliminated. This is precisely how turns are constructed in bicycle tracks and superhighways.

Rotation

Let us now deal with rotating systems. The motion of such a system is determined by the number of revolutions per second which it makes about an axis. It is also necessary, of course, to know the direction of the axis of rotation.

In order to better understand the peculiarities of life in rotating systems, let us consider the "wheel of laughs"—a well-known ride (Figure 23). Its construction is rather simple. A smooth disc, several meters in diameter, rotates rapidly. Those who so desire are invited to get on it and to try to keep their balance. Even people who know no physics quickly acquire the secret of success: one must go to the center of the disc, since the farther one is from the center, the more difficult it is to keep one's balance.

Such a disc is a non-inertial system with several special features. Every object attached to the disc at a distance R

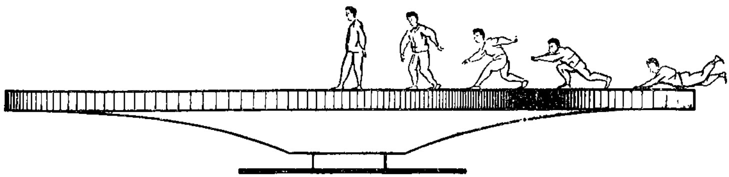


Fig. 23

from its center moves around a circle of radius R with speed v, i.e. with acceleration v^2/R. As we already know, from the point of view of a non-inertial observer this implies the presence of an additional force mv^2/R, directed along the radius outwards from the center. This radial force will act at each point of the "devilish wheel", creating there a radial acceleration of v^2/R. The magnitude of this acceleration will be identical for points lying on the same circle. But what about points on different circles? Don't rush to answer that according to the formula v^2/R, the smaller the distance from the center, the greater will be the acceleration. This isn't true; for the speed of points farther from the wheel's center will be greater. In fact, if we denote the number of revolutions made by the wheel in a second by n, then the path traced by a point of the wheel located at a distance R from the center in the course of one second, i.e. this point's speed, may be expressed as $2\pi Rn$.

The speed of a point is directly proportional to its distance from the center. We may now rewrite our formula for the acceleration:

$$a = 4\pi^2 n^2 R$$

But since the number of revolutions made in a second is the same for all points of the wheel, we arrive at the following result: the acceleration due to the force exerted by the "radial gravity" acting on a rotating wheel grows in proportion to the distance of a point from the wheel's center.

In this interesting non-inertial system the force of gravity is different on different circles. Therefore, the directions of the "verticals" will also be different for bodies located at different distances from the center. The Earth's gravitational force is, of course, the same at all points of the wheel. But the vector characterizing the additional radial force becomes longer as the distance from the center increases. Therefore, the diagonals of the rectangles deviate more and more from the vertical (normal to the Earth's surface).

If we imagine the successive sensations of a person slipping off the "wheel of laughs", from his point of view it can be said that the farther one gets from the center, the more the disc "inclines", making it impossible to stay on it.

However, might it not be possible to invent a contraption, analogous to an inclined highway, for this inertial system? Of course it is, but the disc would have to be replaced by a surface such that the resultant gravitational force is perpendicular to it at each of its points. Such a surface's form can be computed. It is called a paraboloid. This name isn't accidental: every vertical cut of a paraboloid is a parabola—the curve along which bodies fall. A paraboloid is obtained by rotating a parabola around its axis.

It is very easy to create such a surface by making a vessel containing water rotate rapidly. The rotating liquid's surface is precisely a paraboloid. The water particles will stop moving just when the force pressing each particle to the surface is perpendicular to it. To every rotational velocity there corresponds a distinct paraboloid (Figure 24).

It is possible to visually demonstrate this property by making a hard paraboloid. A small ball placed at any point of a paraboloid rotating with the appropriate velocity will remain at rest. This means that the force acting on it

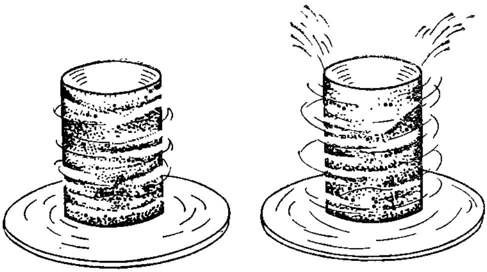


Fig. 24

will be perpendicular to the surface. In other words, a rotating paraboloid behaves as though it were a surface possessing properties of a horizontal surface. One can walk along such a surface and feel stable, just as on the Earth. However, the direction of the vertical will change during the walk.

Centrifugal phenomena are widely employed in technology. For example, the construction of a centrifuge is based on the use of these phenomena.

A centrifuge is a drum which rotates rapidly around its axis. What will happen if various objects are thrown into such a drum, filled to the brim with water?

Let us drop a metal ball into the water—it will go to the bottom, but not along our vertical; moving away from the axis of rotation all the time, it will come to a halt at the side. Now let us throw a cork ball into the drum—it, on the contrary, will immediately begin moving towards the axis of rotation and settle there.

If the drum of this model of a centrifuge has a large diameter, we shall notice that the acceleration increases sharply as the ball moves away from the center.

The phenomena which take place do not puzzle us at all. There is an additional radial force within the centrifuge. If the centrifuge is rotating rapidly enough, its "bottom" is the lateral surface of the drum. The metal ball "sinks" in the water, but the cork ball "floats". The farther a body "falling" in the water is from the axis of rotation, the "heavier" it becomes.

In sufficiently perfected centrifuges, the rotational velocity can be raised to 60 000 revolutions per minute, i.e. 10^3 revolutions per second. At a distance of 10 cm from the axis of rotation, the acceleration due to the radial gravitational force will be approximately equal to

$$40 \times 10^6 \times 0.1 = 4 \times 10^6 \text{ m/sec}^2$$

i.e. 400 000 times greater than terrestrial acceleration.

It is clear that the Earth's gravitation may be neglected for such machines; we really have the right to regard the drum's lateral surface as the "bottom" in a centrifuge.

A centrifuge's fields of application become clear from what we have said. If we want to separate the heavy particles in a mixture from the light ones, it is always advisable to apply a centrifuge. Everybody knows the expression: "The muddy liquid has settled." If dirty water stands long enough, the dirt (usually heavier than the water) will accumulate on the bottom. However, the process of accumulation may take months, but with the aid of a good centrifuge it is possible to clean up the water instantly.

Centrifuges rotating with velocities of tens of thousands of revolutions per minute are capable of separating the finest particles of dirt not only from water, but also from viscous fluids.

Centrifuges are applied in the chemical industry for separating crystals from the solution out of which they grew, for dehydrating salts and for cleaning varnishes; they are used in the food industry for separating syrup from sugar.

The centrifuges which are applied in separating solid or liquid components from a large number of fluids are called separators. Their main application is the processing of milk. Milk separators whirl with velocities of 2-6 thousand revolutions per minute; the diameters of their drums are as large as 5 m.

Centrifugal casting is widely applied in metallurgy. Even at velocities of 300-500 revolutions per minute the liquid metal flowing into the rotating cast is pressed against its outer surface with a considerable force. This is how metal pipes are cast, making them denser, more uniform and without blisters or cracks.

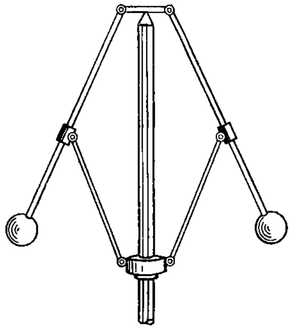


Fig. 25

Here is another application of centrifugal force. A simple instrument, which serves as a governor of the number of revolutions of a machine's rotating parts, is depicted in Figure 25. This instrument is called a centrifugal governor. As the velocity of rotation increases, the centrifugal force grows and the small balls move farther away from the governor's axis. The rods attached to the balls are deflected, and when the deflection reaches a definite level, computed by an engineer, some electrical contacts may be broken, while in the case of a steam engine, for example, valves may be opened, letting out excess steam. This will decrease the velocity of rotation and return the rods to their normal position.

The following experiment is interesting. Place a small cardboard disc on the axis of an electric motor. Turn on the electricity and bring a piece of wood in contact with the whirling disc. A fairly thick beam can be sawed in half as easily as by a steel saw.

An attempt to saw wood by means of cardboard can only evince a smile if one employs it as a hand saw. Why then does the rotating cardboard cut wood? An enormous centrifugal force is exerted on the cardboard particles situated on the boundary of the disc. The lateral forces which might alter the cardboard's plane are insignificant in comparison with the centrifugal. By keeping its plane fixed, the cardboard disc acquires the ability of gnawing into the wood.

The centrifugal force arising as a result of the Earth's rotation leads to the differences in a body's weight at various latitudes that we spoke of above.

A body weighs less at the equator than at a pole for two reasons. Bodies lying on the Earth's surface are at different distances from the Earth's axis, depending on the latitude of their locations. Of course, this distance grows in passing from a pole to the equator. Moreover, a body located at a pole is on the axis of rotation, so the centrifugal acceleration $a = 4\pi^2 n^2 R$ is equal to zero (the distance from the axis of rotation $R = 0$). At the equator, on the contrary, this acceleration is maximal. The centrifugal force reduces the gravitational force. Therefore, the pressure exerted by a body on a scale (the body's weight) is least at the equator.

If the Earth had a precisely spherical form, then a kilogram weight, carried from a pole to the equator, would lose 3.5 grams in weight. You can easily find this number with the aid of the formula

$$4\pi^2 n^2 Rm$$

by substituting $n = 1$ revolution per day, $R = 6300$ km, and $m = 1000$ g. Only don't forget to convert the units of measurements to seconds and centimeters.

However, a kilogram weight will actually lose 5.3 grams, and not 3.5 grams. This is the case because the Earth is an oblate sphere, called an ellipsoid in geometry. The distance from a pole to the center of the Earth is about 1/300 less than a terrestrial radius extended to the equator.

This contraction of the Earth was caused by the very same centrifugal force. In fact, it is exerted on all the particles of the Earth. In remote times, the centrifugal force "moulded" our planet—gave it an oblate form.

Coriolis Force

The peculiarities of a world of rotating systems are not exhausted by the existence of radial gravitational forces. We shall become acquainted with still another interesting effect, whose theory was presented in 1835 by the Frenchman Gaspard Gustave de Coriolis.

Let us pose the following question: what does rectilinear motion look like from the point of view of a rotating laboratory? A design of such a laboratory is depicted in Figure 26. The rectilinear trajectory of some body is shown by means of a ray passing through the center. We are considering the case when the body's path passes through our laboratory's center of rotation. The disc on which the laboratory is standing rotates uniformly; five positions of the laboratory with respect to the rectilinear trajectory are shown in the figure. This is how the relative positions of the laboratory and the body's trajectory look after one, two, three, etc., seconds. The laboratory, as you see, is rotating counterclockwise, if looked upon from above.

Arrows corresponding to the segments through which the body passed during one, two, three, etc., seconds have been drawn on the line of its path. The body covers the same distance during each second, since we are dealing with uniform and rectilinear motion (from the point of view of a fixed observer).

Imagine that the moving body is a freshly painted ball rolling along the disc. What kind of trace will remain on the disc? Our construction yields the answer to this question.

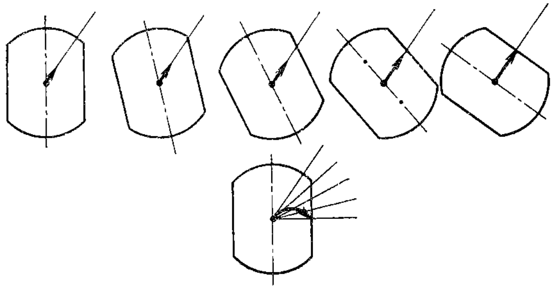


Fig. 26

The points which mark the ends of the arrows have been transferred from our five drawings to a single diagram. It remains to connect these points with a smooth curve. The result of our construction will not surprise us: rectilinear and uniform motion looks like curvilinear motion from the point of view of a rotating observer. The following rule attracts our attention: a moving body is deflected to the right of its path during the entire course of the motion. We now assume that the disc is rotating in the clockwise direction, and leave the repetition of our construction to the reader. It will show that, in this case, a moving body is deflected to the left of its path from the point of view of a rotating observer.

We know that centrifugal force arises in rotating systems. However, its action cannot serve as the cause of the path's deformation—for it is directed along the radius. Hence, besides the centrifugal force, another additional force arises in rotating systems. It is called the *Coriolis force*.

Why is it that in the previous examples we did not come across the Coriolis force and managed superbly with only centrifugal? The reason is that until now we have not regarded a body's motion from the point of view of a rotating observer, but a Coriolis force arises only in such a case. Only a centrifugal force is exerted on bodies which are at rest in rotating systems. A table in a rotating laboratory is screwed on to the floor—only a centrifugal force is exerted on it. But on a ball which has fallen from the table and rolled along the floor of the rotating laboratory, besides a centrifugal force, a Coriolis force is also exerted.

On what quantities does the magnitude of a Coriolis force depend? It can be calculated, but the computations are too complicated to be given here. We shall therefore present only the result of these computations.

Unlike a centrifugal force, whose magnitude depends on the distance from the axis of rotation, a Coriolis force is independent of the body's position. Its magnitude is determined by the moving body's velocity and, moreover, not only by its speed, but also by its velocity's direction with respect to the axis of rotation. If the body moves along the axis of rotation, the Coriolis force is equal to zero. The greater the angle between the velocity vector and the axis of rotation, the greater will be the Coriolis force; this force assumes its maximal value when the body's motion is at right angles to the axis.

As we know, it is always possible to decompose a velocity vector into any pair of its components and consider separately the two resulting motions in which the body is simultaneously involved.

If the body's velocity is decomposed into components $v_{\parallel}$ and $v_{\perp}$ —parallel and perpendicular to the axis of rotation, then the first motion will not be subject to the action of a Coriolis force. The magnitude F_C of the Coriolis force is determined by the component $v_{\perp}$ of the velocity. Computations lead to the formula

$$F_C = 4\pi n v_{\perp} m$$

Here m is the body's mass, while n is the number of revolutions made by the rotating system in a unit of time. As can be seen from the formula, the faster the system rotates and the faster the body moves, the greater will be the Coriolis force.

Calculations have also established the direction of a Coriolis force. This force is always perpendicular to the axis of rotation and the direction of the motion. Moreover, as has already been said above, the force is directed to the right of its path in a system rotating counterclockwise.

Many interesting phenomena occurring on the Earth are explained by the action of Coriolis forces. The Earth is a sphere, and not a disc. This makes the effects of Coriolis forces more complicated. These forces will not only influence motion along the Earth's surface, but also the falling of bodies to the Earth.

Does a body fall exactly along a vertical? Not quite. Only at a pole does a body fall exactly along a vertical. Here the direction of the motion and the Earth's axis of rotation coincide, so there is no Coriolis force. The situation is different at the equator; here the direction of the motion forms right angles with the Earth's axis. If looked upon from the North Pole, the Earth's rotation will appear to be counterclockwise. Hence, a freely falling body should be deflected to the right of its path, i.e. to the East. The magnitude of this eastward deflection, the greatest at the equator, decreases to zero as the poles are approached.

Let us compute the magnitude of the deflection at the equator. Since a freely falling body moves with a uniform acceleration, the Coriolis force increases as the Earth is approached. We shall therefore restrict ourselves to an approximate computation. If the body falls from a height, say, of 80 m, then its fall will last about 4 sec, according to the formula $t = \sqrt{2h/g}$. The average speed for the fall will be equal to 20 m/sec.

This is the speed that we shall substitute in our formula, $4\pi nv$, for the Coriolis acceleration. Let us convert the value $n = 1$ revolution in 24 hours to the number of revolutions per second; 24×3600 seconds are contained in 24 hours, so n is equal to 1/86 400 rev/sec; consequently, the acceleration created by the Coriolis force is equal to $\pi/1080$ m/sec^2. The distance covered during 4 sec with such an acceleration is equal to $(1/2)(\pi/1080) \times 4^2 = 2.3$ cm. This is precisely the magnitude of the eastward deflection in our example. An exact computation, taking into account the non-uniformity of the fall, yields a somewhat different number—3.1 cm.

While the deflection of a freely falling body is maximal at the equator and equal to zero at the poles, we shall see the opposite picture in the case of the deflection of a body, moving in a horizontal plane, under the action of a Coriolis force.

A horizontal site on the North or South Pole does not differ at all from the rotating disc with which we began our study of Coriolis forces. A body moving along such a site will be deflected to the right of its motion's direction by the Coriolis force at the North Pole, and to the left at the South. Using the same formula for the Coriolis acceleration, the reader can calculate without difficulty that a bullet fired from a rifle with an initial speed of 500 m/sec will be deflected from the target by 3.5 cm in a horizontal plane during one second (i.e. while it travels 500 m).

But why should the deflection in a horizontal plane at the equator be equal to zero? Without rigorous proofs, it is clear that this should be the case. At the North Pole a body is deflected to the right of the motion's path, at the South, to the left; hence, half-way between the poles, i.e. at the equator, the deflection will be equal to zero.

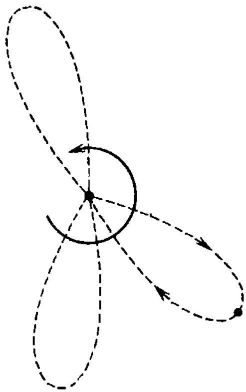


Fig. 27

Let us recall the experiment with Foucault's pendulum. A pendulum oscillating at a pole preserves the plane of its oscillations. The Earth, rotating, moves away from under the pendulum. This is how a stellar observer explains Foucault's experiment. But an observer rotating together with the Earth explains this experiment by means of a Coriolis force. As a matter of fact, a Coriolis force is directed perpendicularly to the Earth's axis and perpendicularly to the direction of the pendulum's motion; in other words, the force is perpendicular to the plane of the pendulum's oscillation and will continually turn this plane. It can be arranged so that the end of the pendulum traces the motion's trajectory. This trajectory is represented by the "rosette" shown in Figure 27. It can be seen from this figure that the "Earth" completes one quarter of a rotation during one and a half periods of the pendulum's oscillation. The Foucault pendulum turns much more slowly. At a pole, the pendulum's plane of oscillation will turn through 1/4 of a degree during one minute. At the North Pole the plane will be turned to the right of the pendulum's path, and at the South, to the left.

The Coriolis effect will be somewhat less at Central European latitudes than at the equator. A bullet in the example we have just given will be deflected not by 3.5 cm, but by 2.5 cm. The Foucault pendulum will be turned by about 1/6 of a degree during one minute.

Must a gunner take the Coriolis force into account? Big

Bertha, used by the Germans to shell Paris during World War I, was situated 110 km from the target. The Coriolis deflection is as much as 1600 m in such a case. This is no longer a small quantity.

If a flying projectile is sent very far without taking the Coriolis force into account, it will be deflected significantly from its course. This effect is large, not because the force is large (for a ten-ton projectile having a speed of 1000 km/hr, the Coriolis force will be about 25 kgf), but because it is exerted continually for a long period of time.

Of course, the wind's influence on a rocket projectile may be no less significant. Flight corrections made by a pilot depend on the action of the wind, the Coriolis effect and imperfections in the airplane or flying bomb.

What specialists, besides aviators and gunners, should be aware of the Coriolis effect? Strange as it may seem, among such specialists are railroaders. Under the action of Coriolis forces, one of a railroad's rails wears out on the inside noticeably more than the other. We know just which one: in the Northern Hemisphere it will be the right rail (relative to the train's motion), in the Southern, the left. Only the railroaders in equatorial countries are saved from trouble in connection with this.

The washing away of right banks in the Northern Hemisphere is explained in exactly the same way as the wearing out of rails. The deviation of a river bed is to a large extent related to the action of Coriolis forces. It turns out that rivers in the Northern Hemisphere pass obstacles on the right.

It is well known that streams of air flow into a low-pressure area. But why is such a wind called a cyclone? After all, this word's root suggests a circular (cyclic) motion.

This is precisely the case—a circular motion of air masses arises in a low-pressure area (Figure 28). The cause lies

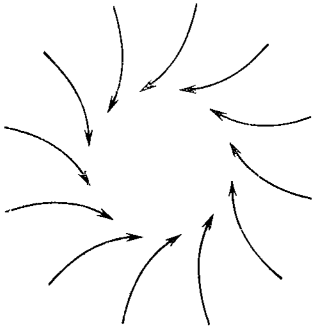


Fig. 28

Fig. 29

in the action of Coriolis forces. In the Northern Hemisphere, all air streams directed towards the place with lowered pressure are deflected to the right of their motion. Take a look at Figure 29—you see that this leads to a westward deflection of the winds blowing in both hemispheres from the tropics to the equator (trade-winds).

But why does such a small force play such a big role in the motion of air masses?

This is explained by the insignificance of the frictional forces. Air is extremely mobile, and a small, but constantly acting force can lead to important consequences.

Four

CONSERVATION LAWS

Recoil

Even those who have not been at war know that, when a gun is fired, it jumps back abruptly. When a rifle is fired, a recoil in the shoulder occurs. But it is possible to become acquainted with the phenomena of recoil without having recourse to firearms. Pour some water into a test-tube, cork it up and suspend it horizontally by two threads (Figure 30). Now turn on a burner under the test-tube—the water will begin boiling, and in a couple of minutes the cork will fly out in one direction, while the test-tube will be deflected in the opposite direction.

The force which drove the cork out of the test-tube is steam pressure. And the force deflecting the test-tube is

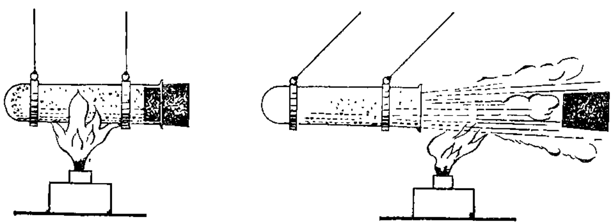


Fig. 30

also steam pressure. Both motions arose under the action of one and the same force. The same thing also happens in shooting, only there the action is not that of steam, but of gunpowder gas.

The phenomenon of recoil is an inevitable consequence of the principle of equality between an action and its reaction. If the steam acts on the cork, then the cork also acts on the steam in the opposite direction, while the steam transmits this reaction to the test-tube.

But perhaps the following objection occurs to you: can one and the same force really lead to such dissimilar effects? The rifle moves backwards only slightly, but the bullet flies far away. We hope, however, that such an objection has not occurred to the reader. Identical forces certainly can lead to different effects: for the acceleration which a body receives (and this is precisely the effect of the force's action) is inversely proportional to its mass. We should write out the acceleration of one of the bodies (shell, bullet, cork) in the form $a_1 = F/m_1$, but the acceleration of the body experiencing the recoil (gun, rifle, test-tube) will be $a_2 = F/m_2$. Since the force is one and the same, we arrive at an important conclusion: the accelerations imparted by the interaction of two bodies participating in a "shot" will be inversely proportional to their masses:

$$\frac{a_1}{a_2} = \frac{m_2}{m_1}$$

This means that the acceleration imparted to the gun when it recoils will be as many times less than the shell's acceleration as the gun weighs more than the shell.

The acceleration of the bullet, and also of the rifle during the recoil, lasts as long as the bullet is moving through the muzzle. Let us denote this time by t. When this time has elapsed, the accelerated motion will become uniform.

For the sake of simplicity, we shall assume the acceleration to be constant. Then the speed with which the bullet flies out of the rifle's muzzle is $v_1 = a_1 t$, while the speed of the recoil is $v_2 = a_2 t$. Since the time during which the accelerations act is one and the same, $v_1/v_2 = a_1/a_2$, and so

$$\frac{v_1}{v_2} = \frac{m_2}{m_1}$$

The speeds with which the bodies fly apart after the interaction will be inversely proportional to their masses.

If we recall the vector nature of velocity, we can rewrite the last relation as follows: $m_1\mathbf{v}_1 = -m_2\mathbf{v}_2$; the minus sign indicates that the velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ are oppositely directed.

Finally, let us rewrite our equation once again, bringing the products of mass by velocity to one side:

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = 0$$

The Law of Conservation of Momentum

The product of the mass of a body by its velocity is called the *momentum* of the body (another name for it is *linear momentum*). Since velocity is a vector, momentum is also a vector quantity. Of course, the momentum's direction coincides with that of the moving body's velocity.

With the aid of our new concept, Newton's law, $F = ma$, can be expressed differently. Since $a = (v_2 - v_1)/t$, we have $F = (mv_2 - mv_1)/t$, or $Ft = mv_2 - mv_1$. The product of the force by the duration of its action is equal to the change in the body's momentum.

Let us return to the phenomenon of recoil.

The result of our investigation of a gun's recoil can now be formulated more concisely: the sum of the momenta of the gun and the shell will remain equal to zero after the firing. It is obvious that this was also the case before the firing, when the gun and the shell were in a state of rest.

The velocities occurring in the equation $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = 0$ are the velocities immediately after the firing. During the subsequent motion of the shell and the gun, the forces of gravity and air resistance will begin acting on them, and the Earth will exert an additional friction force on the gun. But if the shot were fired in a vacuum from a gun hanging in the void, then the motion with the velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ would continue arbitrarily long. The gun would move in one direction, and the shell, in the opposite direction.

Guns mounted on a platform and firing while in motion are widely applied in current artillery practice. How should the equation we derived be changed in order that it be applicable to a shot fired from such a gun? We may write:

$$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = 0$$

where $\mathbf{u}_1$ and $\mathbf{u}_2$ are the velocities of the shell and the gun relative to the moving platform. If the platform's velocity is $\mathbf{V}$, then the velocities of the gun and the shell relative to an observer who is at rest will be $\mathbf{v}_1 = \mathbf{u}_1 + \mathbf{V}$ and $\mathbf{v}_2 = \mathbf{u}_2 + \mathbf{V}$.

Substituting for $\mathbf{u}_1$ and $\mathbf{u}_2$ in our previous equation, we obtain:

$$(m_1 + m_2)\,\mathbf{V} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$$

In the right-hand side of the equation we have the sum of the momenta of the shell and the gun after the firing. And in the left-hand side? Before the firing, the gun and the shell, with a total mass of $m_1 + m_2$, move together with the velocity $\mathbf{V}$. Therefore, in the left-hand side of the equation there is also the total momentum of the shell and the gun, but before the firing.

We have proved a very important law of nature, which is called the *law of conservation of momentum*. We proved it for two bodies, but it can easily be proved that the same result also holds for any number of bodies. What is the content of this law? The law of conservation of momentum asserts that the sum of the momenta of a number of interacting bodies does not change as a result of this interaction.

It is clear that the law of conservation of momentum will only be valid when no outside forces are exerted on the group of bodies under consideration. Such a group of bodies is called closed in physics.

A rifle and a bullet behave like a closed group of two bodies during a shooting, in spite of the fact that they are subject to the Earth's gravitation. The bullet's weight is small in comparison with the force exerted by gunpowder gases, and the phenomenon of recoil occurs in accordance with one and the same laws, regardless of where the shot will be fired—on the Earth or in a rocket flying through interplanetary space.

The law of conservation of momentum allows one to easily solve various problems dealing with colliding bodies. Let us try to strike one clay ball with another—they will stick together and continue the motion together; if we shoot from a rifle at a wooden ball, it will roll together with the bullet stuck in it; a standing cart will roll if a person takes a running jump into it. All the examples we have given are very similar from the point of view of physics. The rule relating the velocities of the bodies involved in such kinds of collisions can be immediately obtained from the law of conservation of momentum.

The momenta of the bodies prior to their collision were $m_1\mathrm{v}_1$ and $m_2\mathrm{v}_2$, they united after the collision, and their total mass is equal to $m_1 + m_2$. Denoting the velocity of

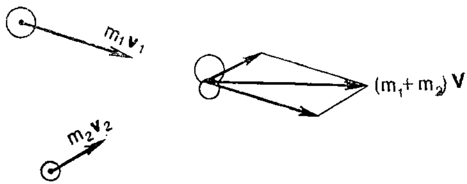


Fig. 31

the united body by **V**, we obtain:

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\,\mathbf{V}$$

or

$$\mathbf{V} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}$$

Let us recall the vector nature of the law of conservation of momentum. The momenta $m\mathbf{v}$ in the numerator of the formula must be added like vectors.

The "uniting hit" when bodies moving at an angle to each other meet is shown in Figure 31. In order to find the value of the speed, we must divide the length of a diagonal of the parallelogram formed by the momentum vectors of the colliding bodies by the sum of their masses.

Jet Propulsion

A person moves by pushing off from the Earth; a boat sails because the rowers push against the water with their oars; a ship also pushes against the water, only not with oars, but with propellers. A train moving on rails and an automobile also push off from the Earth—remember how hard it is for an automobile to get started on an icy road.

Thus, pushing off from a support seems to be a necessary condition for motion; even an airplane moves by pushing the air with its propeller.

But is it really? Might there not be some intricate means of moving without pushing off from anything? If you ice-skate, you can easily convince yourself on the basis of your experience that such motion is quite possible. Pick up a heavy stick and get on the ice. Throw the stick forwards—what will happen? You will glide backwards, although the thought of pushing against the ice with your foot didn't even cross your mind.

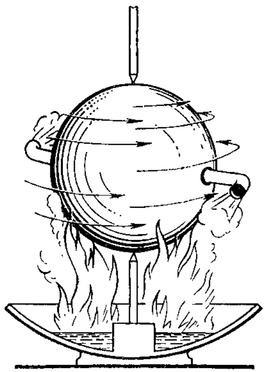


Fig. 32

The phenomenon of recoil, which we have just studied, yields us the clue to carrying out motion without support, without pushing off. Recoil presents a possibility of accelerating motion even in a vacuum, where there really is absolutely nothing to push off from.

The recoil caused by a steam jet being driven out of a vessel (the reaction of the jet) was used back in Ancient Times for creating curious toys. An ancient steam turbine, invented in the second century B.C., is pictured in Figure 32. A spherical cauldron was supported by a vertical axis. Escaping from the cauldron through elbow-shaped pipes, the steam pushed these pipes in the opposite direction, and the sphere rotated.

These days, the use of jet propulsion has already gone far beyond the realm of the creation of toys and the collection of interesting observations. The twentieth century is sometimes called the century of atomic energy, but with no less reason one could call it the century of jet propulsion, since the far-reaching consequences of the use of powerful jet engines can scarcely be exaggerated. This is not only a revolution in aircraft construction, but the beginning of mankind's contact with the Universe.

The principle of jet propulsion permits the creation of airplanes moving with a speed of several thousand kilometers per hour, flying missiles rising hundreds of kilometers above the Earth, artificial Earth satellites and cosmic rockets carrying out interplanetary flights.

A jet engine is a machine from which gases formed by the combustion of fuel are ejected with great force. The rocket moves in the direction opposite to that of the gas stream.

How strong is the thrust carrying the rocket off into space? We know that the force is equal to the change in momentum during a unit of time. According to our conservation law, the rocket's momentum changes by the total momentum mv of the ejected gas.

This law of nature allows us to compute, for example, the relation between the force of the jet propulsion and the expenditure of fuel necessary for this. In doing so, one must assume a value for the speed of discharge of the products of combustion. Let us take, say, the following values: the gases are ejected with a speed of 2000 m/sec at the rate of 10 tons per second. Then the force of the jet propulsion will be about 2×10^{12} dynes, i.e. approximately 2000 tonf.

Let us determine the change in speed of a rocket moving in interplanetary space.

The momentum of the mass ΔM of gas ejected with speed u is equal to $u\Delta M$. The momentum of a rocket of mass M will increase by the amount $M\Delta v$. According to our conservation law, these two quantities must be equal to each other:

$$u\Delta M = M\Delta v, \quad \text{i.e.} \quad \Delta v = u\frac{\Delta M}{M}$$

However, if we wish to compute a rocket's speed when the ejected mass is comparable to the rocket's mass, then the formula we have derived turns out to be useless. In fact, it assumes that the rocket's mass is constant. However, the following important result remains valid: identical relative changes in mass lead to one and the same change in speed. A calculation with the exact formula shows that when the rocket's mass is cut in half, its speed will reach $0.7u$.

In order to raise the rocket's speed to $3u$, it is necessary to burn up a mass $m = (19/20)\ M$. This means that only 1/20 of the rocket's mass can be preserved if we wish to raise its speed to $3u$, i.e. to 6-8 km/sec.

In order to attain a speed of $7u$, the rocket's mass must decrease by 1000 times during the speed-up.

These calculations warn us against striving to increase the mass of the fuel which can be put in the rocket. The more fuel we take, the more we must burn. For a given speed of gas ejection, it is very difficult to achieve an increase in the rocket's speed.

What is basic in the problem of attaining high rocket speeds is the increasing of the speed of gas ejection. In this respect, a significant role must be played by the application to rockets of engines running on a new, atomic fuel.

For a constant speed of gas ejection, a gain in speed with the same mass of fuel is obtained by using multi-stage rockets. In a single-stage rocket, the mass of the fuel decreases, but the empty tanks keep moving with the rocket. Additional energy is required to accelerate the mass of the unnecessary fuel tanks. It would be expedient to throw away the fuel tanks whose fuel has been consumed. In modern multi-stage rockets, not only are the fuel tanks and piping thrown away, but also the engines of the used stages.

Of course, it would be best to continuously throw away the rocket's unnecessary mass. Such a construction does not yet exist. The take-off weight of a three-stage rocket can be made 6 times less than that of a single-stage rocket with the same "ceiling". A "continuous" rocket would be more profitable in this sense than a three-stage rocket by an additional 15%.

Motion Under the Action of Gravity

We shall roll a small cart down two very smooth inclined planes. Let us take two boards, one much shorter than the other, and place them on one and the same support. Then one inclined plane will be steep, while the other will be gently sloping. The tops of both boards—the cart's starting places—will be at the same height. In which case do you suppose will the cart acquire the greater speed by rolling down its inclined plane? Many people will decide that it will be the one which rolls down the steeper board.

An experiment will show that they are wrong—in both cases the cart will acquire the same speed. While a body is moving along an inclined plane, it is subject to the action of a constant force, namely (Figure 33), the component of gravity directed along the line of its motion. The speed v which a body acquires, moving with acceleration a along a path of length s, is equal, as we know, to $\sqrt{2as}$.

But what makes it evident that this magnitude does not depend on the plane's angle of inclination? We see two triangles in Figure 33. One of them depicts the inclined plane. The small leg of this triangle, denoted by h, is the height from which the motion begins; the hypotenuse s is the path through which the body passes in its accelerated motion. The small force triangle with leg ma and hypotenuse mg is similar to the large one, since they are right triangles and their angles, as angles with mutually perpendicular sides, are equal. Hence, the ratio of the legs should be equal

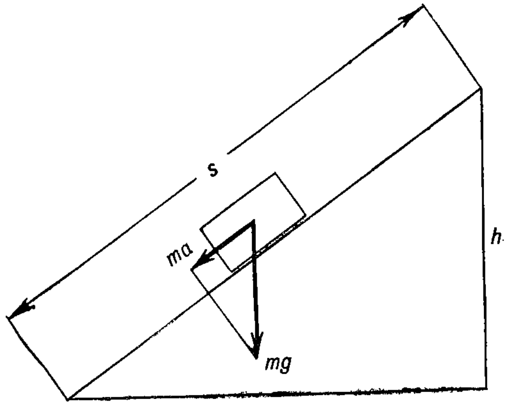


Fig. 33

to the ratio of the hypotenuses, i.e.

$$\frac{h}{ma}=\frac{s}{mg}, \quad \text{or } as=gh$$

We have proved that the product as, and hence also the final speed of a body rolling down an inclined plane, is independent of the angle of inclination, but depends only on the height from which the downward motion began. The speed $v=\sqrt{2gh}$ for all inclined planes subject to the sole condition that the motion began from one and the same height h. This speed turned out to be equal to the speed of free fall from height h.

Let us measure the speed of a body at two places on the inclined plane—at heights h_1 and h_2. Denote the body's speed when it passes through the first point by v_1, and its speed when it passes through the second point, by v_2.

If the initial height from which the motion began is h, then the square of the body's speed at the first point will be $v_1^2 = 2g(h - h_1)$, and at the second point, $v_2^2 = 2g(h - h_2)$. Subtracting the former from the latter, we shall find out how a body's speeds at the initial and end points of an arbitrary piece of an inclined plane are related to the heights of these points:

$$v_2^2 - v_1^2 = 2g(h_1 - h_2)$$

The difference between the squares of the speeds depends only on the difference in height. Note that the equation we have obtained is equally suitable for upward motion and downward motion. If the first height is less than the second (ascent), then the second speed is less than the first.

This formula can be rewritten in the following way:

$$\frac{v_1^2}{2} + gh_1 = \frac{v_2^2}{2} + gh_2$$

We wish to emphasize by means of this formulation that the sum of half the square of the speed and g times the height is identical for all points on the inclined plane. One may say that the quantity $(v^2/2) + gh$ is conserved during the motion.

What is most remarkable in the law we have found is that it is valid for frictionless motion on any hill and, in general, along any path consisting of alternating ascents and descents of various slopes. This follows from the fact that any path can be broken up into rectilinear portions. The smaller we take the segments, the closer will the broken line approximate the curve. Each straight line segment into which the curvilinear path has been broken up may be regarded as part of an inclined plane, and the rule we have found may be applied to it.

Therefore, the sum $(v^2/2) + gh$ is identical for all points of the trajectory. Consequently, a change in the square of the speed does not depend on the form or length of the path along which a body moved, but is determined solely by the difference in height of the initial and end points of the motion.

It may seem to the reader that our conclusion does not coincide with his daily experience: on a long, gently sloping path a body does not gather any speed at all, and eventually comes to a halt. This is the way things are, but we haven't taken the force of friction into account in our reasoning. The above formula is valid for motion within the Earth's gravitational field under the action of only the single force of gravity. If the frictional force is small, then the derived law will be satisfied rather well. A sled with metal runners slides down smooth icy mountains with very little friction. It is possible to build long icy paths, beginning with a steep descent, on which a great speed is gathered, but then twisting up and down fantastically. The finish to a trip on such a hill (when the sled stops by itself) would occur at a height equal to that of the start, provided that friction were entirely absent. But since it is impossible to avoid friction, the point at which the sled's motion started will be higher than the place where it stops.

The law, which asserts that the final speed of a motion subject to the force of gravity is independent of the path's form, can be applied to the solution of various interesting problems.

"Looping-the-loop" in a vertical circle has been frequently presented at a circus as an exciting number. A cyclist or a cart with an acrobat is placed on a high platform. An accelerated descent, then an ascent. There he is already in an upside-down position, again a descent—and the loop has been looped. Let us consider a problem which a circus engineer must solve. At what height should the platform from which the descent begins be made, so that the acrobat might loop-the-loop without falling? We know a necessary condition: the centrifugal force pressing the acrobat against the cart must balance the oppositely directed gravitational force.[*] Hence $mg \leqslant mv^2/r$, where r is the radius of the loop, and v is the speed of the motion at the summit of the loop. In order that this speed be attained, it is necessary to begin the motion from a place which is a certain quantity h higher than the loop's summit. Since the initial speed of the acrobat is equal to zero, we have $v^2 = 2gh$ at the loop's summit. But, on the other hand, $v^2 \geqslant gr$. Hence, between the height h and the loop's radius there is the relation $h \geqslant r/2$. The platform must be raised by at least half the loop's radius above the loop's summit. Taking into account the inevitable frictional force, we shall, of course, have to choose our height with a margin of safety.

And here is another problem. Let us take a large, very smooth dome, so that friction is minimal. Let us place a small object at the summit and give it the opportunity of sliding down the dome by means of a hardly noticeable push. Sooner or later the sliding body will get detached from the dome and start falling. We can easily answer the question as to just when the body breaks away from the surface of the dome: at the moment of the break the centrifugal force must equal the radial component of the weight (at this instant the body will cease pressing the dome, but this is precisely the moment of the break). Two similar triangles can be seen in Figure 34; the moment of the break is depicted. Let us form the ratio of a leg to the hypotenuse for the force triangle and set it equal to the corresponding ratio for the other triangle:

$$\frac{mv^2/r}{mg} = \frac{r-h}{r}$$

Here r is the radius of the spherical dome, and h is the difference in height between the start and finish of the

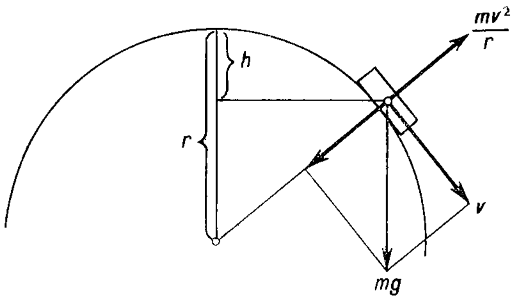


Fig. 34

sliding. Let us now make use of the final speed's independence of the path's form. Since the body's initial speed is assumed equal to zero, we have $v^2 = 2gh$. Substituting this value in the above proportion and performing arithmetical transformations, we find $h = r/3$. Hence, the body will break away from the dome at a height which is 1/3 of a radius lower than the dome's summit.

The Law of Conservation of Mechanical Energy

We have convinced ourselves in the examples just considered how helpful it is to know a quantity not changing its numerical value (conserving it) throughout a motion.

So far we know such a quantity for one body only. But if several associated bodies are moving within a gravitational field? It is evident that we may not assume that the expression $(v^2/2) + gh$ remains constant for each of them, since each of the bodies is subject to the action of not only the force of gravity, but also of the neighbouring bodies. Perhaps the sum of such expressions, taken over the group of bodies under consideration, is conserved?

We shall now show that this assumption is false. There exists a quantity conserved throughout the motion of many bodies; however, it is not equal to the sum

$$\left(\frac{v^2}{2}+gh\right)_{\text{body 1}}+\left(\frac{v^2}{2}+gh\right)_{\text{body 2}}+\dots$$

but rather to the sum of such expressions multiplied by the masses of the corresponding bodies; in other words, the sum

$$m_1\left(\frac{v^2}{2}+gh\right)_1+m_2\left(\frac{v^2}{2}+gh\right)_2+\dots$$

is conserved.

For the proof of this important law of mechanics, we turn to the following example.

Two loads are connected by a cord passing over a pulley, the large one of mass M and the small one of mass m. The large load pulls the small one, and this group of two bodies will move with increasing speed.

The driving force is the difference in weight of these bodies, $Mg - mg$. Since the masses of both bodies participate in the accelerated motion, Newton's law for this case will be written out as follows:

$$(M-m)\,g=(M+m)\,a$$

Let us consider two instants during the motion and show that the sum of the expressions $(v^2/2)+gh$, multiplied by the corresponding masses, really remains unchanged. Thus, it is required to prove the equality

$$m\left(\frac{v_2^2}{2}+gh_2\right)+M\left(\frac{V_2^2}{2}+gH_2\right)=$$

$$=m\left(\frac{v_1^2}{2}+gh_1\right)+M\left(\frac{V_1^2}{2}+gH_1\right)$$

Capital letters denote physical quantities characterizing the large load. The subscripts 1 and 2 refer here to the two instants which we are considering.

Since the loads are connected by a cord, $v_1 = V_1$ and $v_2 = V_2$. Using these simplifications and transferring all summands containing heights to the right-hand side, and summands with speeds to the left-hand side, we obtain:

$$\frac{m+M}{2}(v_2^2 - v_1^2) = mgh_1 + MgH_1 - mgh_2 - MgH_2 =$$
$$= mg(h_1 - h_2) + Mg(H_1 - H_2)$$

The differences in height of the loads are, of course, equal (but opposite in sign, since one load rises, while the other falls). Therefore,

$$\frac{m+M}{2}(v_2^2 - v_1^2) = g(M - m)s$$

where s is the distance covered.

We learned on p. 58 that the difference between the squares of the speeds at the initial and end points of a segment of length s of a path traversed with acceleration a is equal to $2as$:

$$v_1^2 - v_2^2 = 2as$$

Substituting this expression in the preceding formula, we find:

$$(m + M)\, a = (M - m)\, g$$

But this is Newton's law, which we have written out above for our example. With this we have proved what was required: for two bodies the sum of the expressions $(v^2/2) + gh$, multiplied by the corresponding masses*, remains constant

* Of course, the expression $(v^2/2) + gh$ could equally well be multiplied by $2m$, or $m/2$ and, more generally, by any coefficient in addition. We agreed to act in the simplest manner, i.e. to multiply it simply by m.

during the motion, or, as one says, is conserved, i.e.

$$\left(\frac{mv^2}{2}+mgh\right)+\left(\frac{MV^2}{2}+MgH\right)=\text{const}$$

For the case of a single body, this formula reduces to an earlier proved one:

$$\frac{v^2}{2}+gh=\text{const}$$

Half the product of the mass by the square of the speed is called the *kinetic energy* K:

$$K=\frac{mv^2}{2}$$

The product of a body's weight by its height is called the *potential energy* U of the body's gravitational attraction to the Earth:

$$U = mgh$$

We have proved that during the motion of a two-body system (and it is possible to prove the same thing for a system consisting of many bodies), the sum of the kinetic and potential energies of the bodies remains constant.

In other words, an increase in the kinetic energy of a group of bodies can only occur at the expense of a decrease in this system's potential energy (and, of course, conversely).

The law just proved is called the *law of conservation of mechanical energy*.

The law of conservation of mechanical energy is a very important law of nature. We have not yet shown its significance in full measure. Later, when we have become acquainted with the motion of molecules, its universality and its applicability to all natural phenomena will be evident.

Work

If we push or pull a body, meeting no hindrance to what we are doing, then the result will be an acceleration of the body. The change in kinetic energy taking place in this connection is called the *work* A performed by the force:

$$A = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}$$

According to Newton's law, a body's acceleration, and hence also the increase in its kinetic energy, is determined by the vector sum of all the forces applied to it. Therefore, in the case of many forces, the formula $A = (mv_2^2/2) - (mv_1^2/2)$ expresses the work performed by the resultant force. Let us express the work A in terms of the magnitude of the force.

For the sake of simplicity, we shall restrict ourselves to the case when motion is possible only in one direction—we shall push (or pull) a cart of mass m, standing on rails (Figure 35).

According to our general formula for uniformly accelerated motion, $v_2^2 - v_1^2 = 2as$. Therefore, the work performed by all the forces over a distance s is

$$A = \frac{mv_2^2}{2} - \frac{mv_1^2}{2} = mas$$

The product ma is equal to the component of the total force in the direction of the motion. Consequently, $A = f_t s$.

The work done by a force is measured by the product of the distance by the component of the force along the direction of the displacement.

This formula for the work is valid for forces of any origin and for motions along any trajectory.

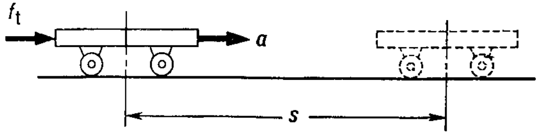


Fig. 35

Note that the work may be equal to zero even when forces act on a moving body.

For example, the work done by a Coriolis force is equal to zero, because such a force is perpendicular to the motion's direction. It has no tangential component, so the work is equal to zero.

Any twist in the trajectory, which is not accompanied by a change in speed, requires no work—for the kinetic energy does not change under such conditions.

Can work be negative? Of course, if the force is directed at an obtuse angle to the motion, then it does not help, but hinders the motion. The tangential component of the force in the direction of the motion will be negative. In this case we do say that the force performs negative work. The force of friction always slows down a motion, i.e. does negative work.

On the basis of the change in kinetic energy, one can only judge the work done by the resultant force.

As for the work done by the individual forces, we should compute them as products $f_t s$. An automobile is moving uniformly along a highway. There is no increase in kinetic energy, so the work done by the resultant force is equal to zero. But the work done by the motor is, of course, not equal to zero—it is equal to the product of the force of the motor's thrust by the distance covered, and is fully compensated by the negative work done by the forces of resistance and friction.

Using the concept of "work", we can describe more briefly and clearly the interesting peculiarities of the gravitational force with which we have just become acquainted. If a body goes from one place to another under the action of gravity, its kinetic energy will change. This change in kinetic energy is equal to the work A. But we know from the law of conservation of energy that an increase in kinetic energy takes place at the expense of a decrease in potential one.

Therefore, the work done by gravity is equal to the loss of potential energy:

$$A = U_1 - U_2$$

It is obvious that a loss (or gain) of potential energy, and hence an increase (or decrease) in kinetic energy, will be the same regardless of the path along which a body moved. This implies that the work performed by gravity does not depend on the form of the path. If a body went from the first point to the second with an increase in kinetic energy, it will go from the second point to the first with a decrease in kinetic energy of exactly the same amount. Moreover, it makes no difference whether or not the form of the path "there" coincides with the form of the path "back". Hence, the work "there" and "back" will also be identical. And if the body takes a long trip with the initial and end points of its path coinciding, then the work will be equal to zero.

Imagine a canal whose form is as fantastic as possible, through which a body slides without friction. Let us send it off on a trip from the highest point. The body rushes downwards, gathering speed. At the expense of the kinetic energy so obtained, the body will surmount ascents and return finally to the station where it departed. With what speed? With the same, of course, with which it left the station. Its potential energy will return to its previous value. But if so, then its kinetic energy could neither have decreased nor increased. Hence, the work is equal to zero.

Not for all forces is the work done along a circular (physicists say: a closed) path equal to zero. There is no need to prove that the longer the path, the greater will be the work performed by friction, for example.

In What Units Work and Energy Are Measured

Since work is equal to the change in energy, then work and energy—potential as well as kinetic, of course—are measured in one and the same units. Work is equal to the product of a force by a distance. The work done by a force of one dyne over a distance of one centimeter is called an *erg*:

$$1 \text{ erg} = 1 \text{ dyne} \cdot 1 \text{ cm}$$

This is a very small work. Such a work is performed against gravity by a mosquito in order to fly from the thumb to the forefinger of someone's hand. A larger unit of work and energy, used in physics, is the *joule*. It is 10 million times as great as an erg:

$$1 \text{ joule} = 10 \text{ million ergs}$$

A unit of work which is quite often used is 1 *kilogram-meter* (1 kgf-m)—this is the work which a force of 1 kgf performs in a displacement of 1 m. About this much work is done by a kilogram weight falling off a table to the floor.

As we know, a force of 1 kgf is equal to 981 000 dynes, and 1 m is equal to 100 cm. Hence, 1 kgf-m of work is equal to 98 100 000 ergs, or 9.81 joules. Conversely, 1 joule is equal to 0.102 kgf-m.

The new system of units (SI), which we have already mentioned and will mention again, suggests that the joule be used as the unit of work and energy, and defines it as the work done by a force of 1 newton (see p. 56) over a distance of 1 meter. Knowing how easily force is defined in this new system of units, one has no difficulty in understanding the reason for its advantages.

Loss of Energy

The reader has probably noticed that in our illustrations of the law of conservation of mechanical energy we persistently repeat: "in the absence of friction, if there were no friction ... ." But friction inevitably accompanies any motion. What significance does a law have which doesn't take into account such an important practical circumstance? We shall put off answering this question and consider now some consequences of friction.

Friction forces are directed against motion, and so perform negative work. This causes an unavoidable loss of mechanical energy.

Will this inevitable loss of mechanical energy lead to a cessation of the motion? It is not difficult to convince oneself that not every motion can be stopped by friction.

Imagine a closed system consisting of several interacting bodies. The law of conservation of momentum is valid, as we know, in relation to such a closed system. A closed system cannot change its momentum, so it moves rectilinearly and uniformly. Friction within such a system can destroy relative motions of parts of the system, but cannot affect the speed and direction of the motion of the entire system as a whole.

There exists still another law of nature, called the *law of conservation of angular momentum* (we shall make its acquaintance later), which does not permit friction to destroy the uniform rotation of an entire closed system.

Therefore, the presence of friction leads to the cessation of all movement within a closed system of bodies, not obstructing only the uniform rectilinear and the uniform rotational motion of this system as a whole.

If the Earth does slightly change the speed of its rotation, the cause of this is not the friction exerted by terrestrial bodies against one another, but the fact that the Earth is not an isolated system.

But as for the motions of bodies on the Earth, they are all subject to friction and lose their mechanical energy. Therefore, such motion will always cease, if not supported from without.

This is a law of nature. But if one succeeded in tricking nature? Then ... then one might be able to bring about perpetuum mobile, which is Latin for "perpetual motion".

Perpetuum Mobile

Bertold, a hero of Pushkin's *Scenes from the Days of Knighthood*, dreamed of bringing about perpetuum mobile. "What is perpetuum mobile?" asks his interlocutor. "It is perpetual motion," answers Bertold. "If I find perpetual motion, I see no bounds to human creativity. To make gold is a tempting problem, a discovery can be curious and profitable, but to find a solution to the problem of perpetuum mobile ... ."

Perpetuum mobile, or a perpetual motion machine, is a machine working not only contrary to the law of loss of mechanical energy, but also in violation of the law of conservation of mechanical energy, which, as we now know, holds only under ideal, unattainable conditions—in the absence of friction. A perpetual motion machine must, as soon as it is constructed, begin working "by itself"—for example, turning a wheel or lifting up a load. This work should take place perpetually and continually, and the machine should require neither fuel nor human hands nor the energy of falling water—in short, nothing gotten from without.

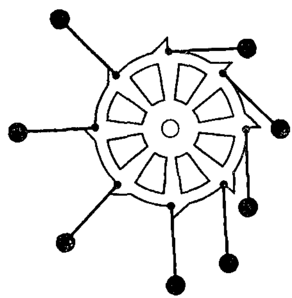


Fig. 36

The earliest reliable document known so far dealing with the "realization" of a perpetual motion machine goes back to the 13th century. It is a curious fact that after six centuries, in 1910, exactly the same "project" was presented for "consideration" in one of Moscow's scientific institutions.

The project for this perpetual motion machine is depicted in Figure 36. As the wheel rotates, the loads are thrown back and, according to the inventor, support the motion, since these loads, acting at a greater distance from the axis, press down much harder than the others. Having constructed this by no means complicated "machine", the inventor convinces himself that after turning once or twice by inertia, the wheel comes to a halt. But this does not make him lose heart. An error has been committed: the levers should have been made longer, the protuberances must be changed in form. And the fruitless labour, to which many self-made inventors have devoted their lives, continues, but of course with the same success.

On the whole, there have not been many variants of proposed perpetual motion machines: various self-moving wheels, not differing in principle from the one described; hydraulic machines—for example, the machine, shown in Figure 37, which was invented in 1634; machines using siphons or capillary tubes (Figure 38), the loss of weight in water (Figure 39) or the attraction of iron bodies to mag-

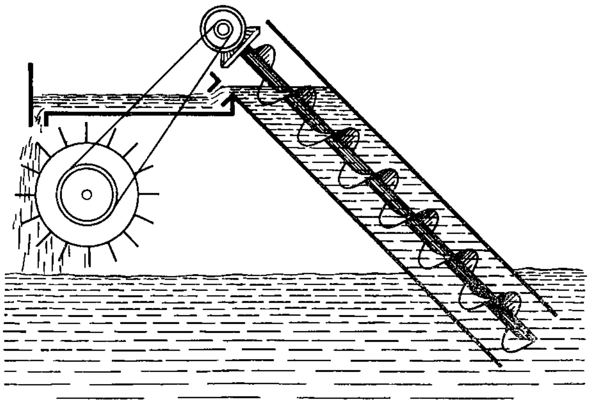


Fig. 37

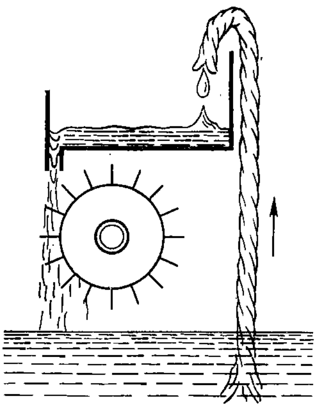


Fig. 38

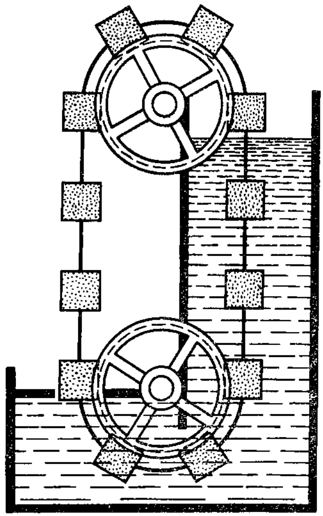


Fig. 39

nets. It is by no means always possible to guess at the expense of what, according to the inventor, the perpetual motion should have occurred.

Even before the law of conservation of energy was established, we find the assertion of the impossibility of perpetuum mobile in an official declaration of the French Academy, made in 1775, when it decided not to accept any more projects for perpetual motion machines to be examined and tested.

Many 17th and 18th century physicists had already assumed the axiom of the impossibility of perpetuum mobile as a basis of their proofs, in spite of the fact that the concept of energy and the law of conservation of energy entered science much later.

At the present time it is clear that inventors who try to create a perpetual motion machine not only come into contradiction with experiment, but also commit an error in elementary logic. For the impossibility of perpetuum mobile is a direct consequence of the laws of mechanics, which is what they proceed from in justifying their "inventions".

In spite of their complete fruitlessness, searches for perpetual motion machines probably played, nevertheless, some sort of useful role, since they led in the final analysis to the discovery of the law of conservation of energy.

Collisions

Momentum is conserved in every collision between two bodies. But as for energy, it will necessarily decrease, as we have just explained, because of various kinds of friction.

However, if the colliding bodies are made of elastic material, say of ivory or steel, then the loss of energy will be insignificant.

Such collisions, for which the sums of kinetic energy before and after the collision are identical, are called *ideally elastic*.

A small loss of kinetic energy takes place even in collisions of the most elastic materials—it reaches, for example, 3-4% with ivory billiard balls.

The conservation of kinetic energy in elastic collisions permits us to solve a number of problems.

Consider, for example, a head-on collision between balls of different mass. The momentum equation has the form (we assume that ball 2 has been at rest prior to the collision)

$$m_1 v_1 = m_1 u_1 + m_2 u_2$$

and the energy equation,

$$\frac{m_1 v_1^2}{2} = \frac{m_1 u_1^2}{2} + \frac{m_2 u_2^2}{2}$$

where v_1 is the speed of the first ball before the collision, and u_1 and u_2 are the speeds of the balls after the collision.

Since the motion takes place along a straight line (the one passing through the centers of the balls—this is just what is meant by a head-on collision), the bold-face type denoting vectors has been replaced by italics.

From the first equation we have:

$$u_2 = \frac{m_1}{m_2}(v_1 - u_1)$$

Substituting this expression for u_2 in the energy equation, we obtain:

$$\frac{m_1}{2}(v_1^2 - u_1^2) = \frac{m_2}{2}\left[\frac{m_1}{m_2}(v_1 - u_1)\right]^2$$

One of the solutions of this equation is $u_1 = v_1$, which yields $u_2 = 0$. But this answer doesn't interest us, since the equalities $u_1 = v_1$, $u_2 = 0$ imply that the balls did not collide at all. We therefore look for another solution of the equation. Dividing by $m_1(v_1 - u_1)$, we obtain:

$$\frac{1}{2}(v_1 + u_1) = \frac{1}{2}\frac{m_1}{m_2}(v_1 - u_1)$$

i.e.

$$m_2 v_1 + m_2 u_1 = m_1 v_1 - m_1 u_1$$

or

$$(m_1 - m_2)\, v_1 = (m_1 + m_2)\, u_1$$

which yields the following value for the first ball's speed after the collision:

$$u_1 = \frac{m_1 - m_2}{m_1 + m_2}\, v_1$$

In a head-on collision with a ball at rest, the moving ball rebounds (negative u_1) if its mass is less. If m_1 is greater than m_2, both balls continue the motion in the direction of the collision.

In case of an exact head-on collision during a game of billiards, one often observes the following scene: the driving ball comes to a sudden stop, and the target ball heads for a pocket. This is explained by the equation we have just found. The balls' masses are equal, the equation yields $u_1 = 0$, and so $u_2 = v_1$. The colliding ball halts, while the second ball begins its motion with the former's previous speed. It is as though the balls have exchanged speeds.

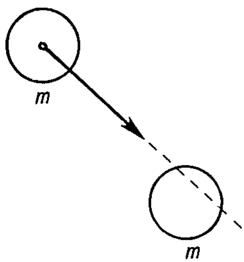


Fig. 40

Let us consider another example of a collision between bodies in accordance with the law of elastic collisions, namely an oblique collision between bodies of equal mass (Figure 40). The second body was at rest prior to the collision, so the laws of conservation of momentum and energy have the form:

$$m\mathbf{v}_1 = m\mathbf{u}_1 + m\mathbf{u}_2$$

$$\frac{mv_1^2}{2} = \frac{mu_1^2}{2} + \frac{mu_2^2}{2}$$

Cancelling the mass, we obtain:

$$\mathbf{v}_1 = \mathbf{u}_1 + \mathbf{u}_2$$

$$v_1^2 = u_1^2 + u_2^2$$

Vector $\mathbf{v}_1$ is the vector sum of $\mathbf{u}_1$ and $\mathbf{u}_2$. But this means that the lengths of the velocity vectors form a triangle.

What kind of triangle is this? Recall the Pythagorean theorem. Our second equation is an expression of it. This means that the velocity triangle must be a right triangle with hypotenuse v_1 and legs $\mathbf{u}_1$ and $\mathbf{u}_2$. Hence, $\mathbf{u}_1$ and $\mathbf{u}_2$ form right angles with each other. This interesting result shows that in any oblique elastic collision, bodies of equal mass fly apart at right angles.

Five

OSCILLATIONS

Equilibrium

In certain cases it is very difficult to maintain an equilibrium—try to walk across a tightrope. At the same time, nobody rewards a person sitting in a rocking-chair with applause. But he is also maintaining his equilibrium.

What is the difference between these two examples? In which case is equilibrium maintained "by itself"?

The condition for equilibrium seems to be obvious. For a body not to be displaced from its position, the forces exerted on it must balance; in other words, the sum of these forces must be equal to zero. This condition really is necessary for a body's equilibrium, but is it sufficient?

A side-view of a hill, which can be easily built out of cardboard paper, is depicted in Figure 41. A ball will behave in different ways, depending on the part of the hill where it is placed. A force which makes it roll down will be exerted on the ball at any point on the hill's slope. This active force is gravity, or rather its projection on the tangent to the hill's section, passing through the point in which we are interested. It is therefore clear that the more gentle the slope, the smaller will be the force acting on the ball.

We are interested above all in the points at which the force of gravity is completely balanced by the reaction of

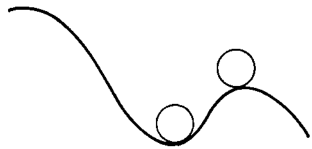


Fig. 41

the support, and hence the resultant force acting on the ball is equal to zero. This condition will be fulfilled at the summits of the hill and at its lowest points—the hollows. The tangents are horizontal at these points, and the resultant forces acting on the ball are equal to zero.

However, in spite of the fact that the resultant force is equal to zero at a summit, we won't be able to put a ball there, but even if we could, we would immediately detect the accessory cause of our success—friction. A small push or a light puff will overcome the friction force, and the ball will leave its place and roll down.

For a smooth ball on a smooth hill, only the low points of the hollows will be positions of equilibrium. If a push or an air stream displaces the ball from such a position, it will return there by itself.

A body is undoubtedly in equilibrium in a hollow, a hole or a depression. Having been deflected from such a position, a body will come under the action of a force returning it back. The picture is different at the summits of the hill: if a body has left such a position, not a returning, but a "sending-away" force is exerted on it. Consequently, a resultant force equal to zero is a necessary, but not a sufficient condition for stable equilibrium.

The equilibrium of a ball on a hill can also be regarded from another point of view. The hollows correspond to minima, and the summits to maxima of potential energy. The law of conservation of energy prevents a change in positions for which the potential energy is minimal. Such a change would make the kinetic energy negative, but this is impossible. The situation is entirely different at the summits. A departure from these points entails a decrease in potential energy, and hence not a decrease, but an increase in kinetic energy.

Thus, in a position of equilibrium, the potential energy must assume a minimal value with respect to its values at neighbouring points.

The deeper the hole, the greater will be the stability. Since we know the law of conservation of energy, we can immediately say under what conditions a body will roll out of a depression. For this it is necessary to impart to the body a kinetic energy which would be enough for raising it to the edge of the hole. The deeper the hole, the greater will be the kinetic energy needed for disturbing the stable equilibrium.

Simple Oscillations

If a ball lying in a depression is pushed, it will begin moving up the hill, gradually losing kinetic energy. When it is completely lost, an instantaneous halt will occur and a downward motion will begin. Potential energy will now be transformed into kinetic. The ball will gain speed, rush past the equilibrium position by inertia and begin ascending again, only in the opposite direction. If the friction is insignificant, such an "upward-downward" motion can continue very long, while in the ideal case—in the absence of friction—it will continue indefinitely.

Therefore, motions near a position of stable equilibrium always have an oscillatory nature.

For studying oscillation, a pendulum is perhaps more suitable than a ball rolling back and forth in a hole, at least to the extent that it is easier to reduce the friction exerted on a pendulum to a minimum.

When a pendulum bob is deflected to its highest position, its velocity and kinetic energy are equal to zero. Its potential energy is greatest at this moment. The bob goes down—the potential energy decreases and is transformed into kinetic. Hence, the motion's speed increases too. When the bob passes through its lowest position, its potential energy is least and the corresponding kinetic energy and speed are maximal. As the motion continues, the bob again rises. The speed now diminishes and the potential energy increases.

If we abstract from the losses due to friction, the bob will be deflected by the same distance to the right as it was originally deflected to the left. Its potential energy was transformed into kinetic, and then the same quantity of "new" potential energy was created. We have described the first half of a single oscillation. The second half takes place in the same way, only the bob moves in the opposite direction.

Oscillatory motion is a repeating or, as one says, periodic motion. Returning to its starting point, the bob repeats its motion each time (if the changes resulting from friction are not taken into account) both with respect to its path and to its velocity and acceleration. The time spent on a single oscillation, i.e. in returning to the starting point, is identical for the first, second and all subsequent oscillations. This time—one of the oscillation's most important characteristics—is called the *period*; we shall denote it by T. After this time, the motion is repeated, i.e. after the time T, we shall always find a vibrating body at the same point in space and moving in the same direction. After a half-period, the body's displacement and also the motion's direction change sign. Since the period T is the time for one oscillation, the number n of oscillations in a unit of time will be equal to $1/T$.

But what does the period of vibration of a body, moving not far from a position of stable equilibrium, depend on? In particular, what does a pendulum's period of oscillation depend on? Galileo was the first to pose and solve this problem. We shall now derive Galileo's formula.

However, it is difficult to apply the laws of mechanics to non-uniformly accelerated motion in an elementary manner. Therefore, in order to by-pass this difficulty, we shall not make the pendulum bob oscillate in a vertical plane, but have it describe a circle, remaining at a fixed height all the time. It isn't difficult to create such a motion—one merely has to remove the pendulum from its equilibrium position and give it an initial push with a properly chosen force in a direction exactly perpendicular to the radius of deflection.

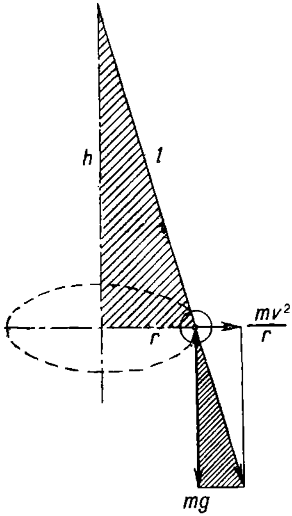


Fig. 42

Such a "conical pendulum" is depicted in Figure 42.

The bob of mass m moves around a circle. Hence, besides the force of gravity mg, a centrifugal force of mv^2/r, which we may also represent in the form $4\pi^2n^2rm$, is exerted on it. Here n is the number of revolutions per second. We may therefore also write out our expression for the centrifugal force as follows: $4\pi^2rm/T^2$. The resultant of these two forces pulls on the pendulum's cord.

Two similar triangles—the force triangle and the distance triangle—are shaded in our figure. The ratios of the corresponding legs are equal; hence

$$\frac{mgT^2}{m4\pi^2 r} = \frac{h}{r}, \quad \text{or} \quad T = 2\pi\sqrt{\frac{h}{g}}$$

But what factors does a pendulum's period of oscillation depend on? If we perform experiments at one and the same location on the Earth (g doesn't change), then the period of oscillation depends only on the difference in height between the point of suspension and the point where the bob is. The bob's mass, as always in a gravitational field, does not affect the period of oscillation.

The following circumstance is of interest. We are investigating motion near a position of stable equilibrium. But for small deflections, we may replace the difference in height h by the pendulum's length l. It is easy to verify this. If the length of the pendulum is 1 m and the radius of deflection is 1 cm, then

$$h = \sqrt{10\,000 - 1} = 99.995 \text{ cm}$$

The difference between h and l will reach 1% only when the deflection is 14 cm. Consequently, the period of a pendulum's free oscillations for not too large a deflection from the equilibrium position is

$$T = 2\pi\sqrt{\frac{l}{g}}$$

i.e. depends only on the pendulum's length and the value of the acceleration of gravity at the place where the experiment is performed, but does not depend on the magnitude of the pendulum's deflection from its equilibrium position.

The formula $T = 2\pi\sqrt{l/g}$ has been proved for a conical pendulum; but what will it look like for a simple "plane" pendulum? It turns out that this formula retains its form. We shall not prove this rigorously, but call attention to the fact that the shadow cast onto a wall by a conical pendulum's bob will oscillate almost like a plane pendulum: the shadow completes one oscillation during just the same time in which the bob describes a circle.

The use of small oscillations about an equilibrium position permits the measurement of time with very great accuracy.

According to legend, Galileo established the independence of a pendulum's period of oscillation from its amplitude and mass while observing during services in a cathedral how two enormous chandeliers were swinging.

Therefore, a pendulum's period of oscillation is proportional to the square root of its length. Thus, the period of oscillation of a meter-long pendulum is twice that of a 25-cm pendulum. It also follows from our formula for a pendulum's period of oscillation that one and the same pendulum will not oscillate equally fast at different latitudes on the Earth. As we move closer to the equator, the acceleration of gravity decreases and the period of oscillation grows.

It is possible to measure a period of oscillation with a very great degree of accuracy. Therefore, experiments with pendulums enable us to measure the acceleration of gravity very accurately.

Displaying Oscillations

Let us attach a piece of soft lead to the bob of a pendulum and hang the pendulum over a sheet of paper in such a way that the lead touches the paper (Figure 43). Now we deflect the pendulum slightly. The oscillating lead will trace a small line segment on the paper. At the midpoint of the oscillation, when the pendulum is passing through its equilibrium position, the pencil line will be thicker, since in this position

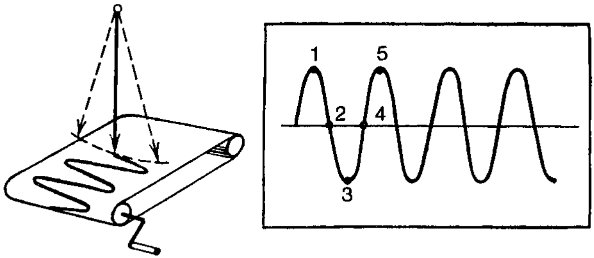


Fig. 43

the lead presses down harder on the paper. If we pull the sheet of paper in a direction perpendicular to the plane of the oscillation, then the curve depicted in Figure 43 will be traced. It is not difficult to see that the wavelets so obtained will be dense if the paper is pulled slowly, and sparse if the sheet of paper moves with a considerable speed. In order for the curve to turn out accurate, it is necessary that the sheet of paper move uniformly.

In this manner we have in a sense "displayed" the oscillations.

The display is needed in order to say where the pendulum bob was located and where it was moving at one or another instant. Imagine that the paper moves with a speed of 1 cm/sec from the time when the pendulum was as far as possible from, say to the left of, the midpoint. This initial position corresponds to the point on our graph, which has been marked with the number *1*. After 1/4 of a period, the pendulum will pass through the midpoint. During this time the paper has moved $T/4$ centimeters (point *2* in the figure). The pendulum now moves to the right and the paper simultaneously crawls along. When the pendulum comes to its extreme right position, the paper will have moved $T/2$ centimeters (point *3* in the figure). The pendulum again moves towards the midpoint and arrives at its equilibrium position in $3T/4$ (point *4* in the diagram). Point *5* finishes a complete oscillation, after which the motion is repeated every T seconds or every T centimeters on our graph.

Thus, a vertical line on the graph is the scale of a point's displacement from the equilibrium position, while the central horizontal line is the time scale.

The two magnitudes which characterize an oscillation in an exhaustive manner are easily found from such a graph. The period can be determined by the distance between two equivalent points, for example, between two neighbouring summits. The maximal displacement of a point from the equilibrium position can also be measured at once. This displacement is called the *amplitude* of the oscillation.

Displaying an oscillation permits us, moreover, to answer the question posed above: where is an oscillating point at one or another instant? For example, where will a vibrating point be after 11 seconds if the period of oscillation is equal to 3 seconds and the motion began at the extreme left position? The oscillation begins from the very same point after every 3 seconds. Therefore, after 9 seconds the body will also be at the extreme left position.

Consequently, there is no need of a graph in which the curve is extended over several periods; a figure depicting the curve corresponding to one oscillation is quite enough. After 11 seconds the state of an oscillating point will be the same as after 2 seconds if the period is 3 seconds. Laying off 2 centimeters on our diagram (for we stipulated that the paper be pulled with a speed of 1 cm/sec or, in other words, that the scale of our diagram be 1 cm equals 1 sec), we see that after 11 seconds the point will be on its way from the extreme right position to that of equilibrium. The magnitude of the displacement at this instant can be found from the figure.

It isn't necessary to turn to a graph in order to find the magnitude of the displacement of a point making small oscillations about its equilibrium position. Theory shows that in this case the curve depicting the dependence of the displacement on the time is a sinusoid. If we denote the displacement of a point by y, the amplitude by a, and the period of the oscillation by T, then we can find the magnitude of the displacement at a time t after the beginning of the oscillation by means of the formula

$$y = a \sin 2\pi \frac{t}{T}$$

An oscillation taking place in accordance with this law is called *harmonic*. The argument of the sine is equal to the product of 2π by t/T. The quantity $2\pi t/T$ is called the *phase*.

Having trigonometric tables at hand and knowing the period and amplitude, we can easily compute the magnitude of a point's displacement and figure out, on the basis of the value of the phase, in which direction it is moving.

It is not difficult to derive the formula for vibratory motion by considering the motion of the shadow cast on a wall by a bob moving around a circle.

We shall mark off the displacements of the shadow from its central position. At the extreme positions, the displacement y is equal to the radius a of the circle. This is the amplitude of the shadow's vibration.

If the bob has moved along the circle through an angle φ from the central position, its shadow (Figure 44) will deviate from the midpoint by $a \sin \varphi$.

Let the period of the bob's motion (which, of course, is also the period of oscillation of the shadow) be T; this means that the bob passes through 2π radians during the time T. We may form the proportion $\varphi/t = 2\pi/T$, where t is the time required for a revolution through an angle φ.

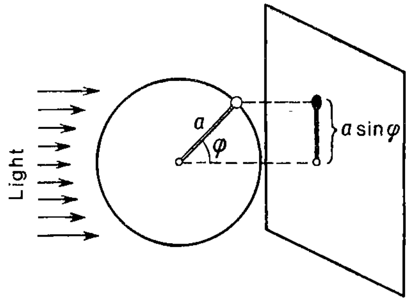


Fig. 44

Consequently, $\varphi = 2\pi t/T$ and $y = a \sin 2\pi t/T$. This is precisely what we wished to prove.

The velocity of a vibrating point also changes according to a sinusoidal law. The same kind of reasoning about the movement of a bob describing a circle will lead us to this conclusion. This bob's velocity is a vector of constant length v_0. The velocity vector revolves together with the bob. Let us think of the velocity vector as a physical arrow capable of casting a shadow. At the extreme positions of the bob, the vector will lie along a ray of light and will not create a shadow. When the bob has moved along the circle from an extreme position through an angle of θ, the vector velocity will have turned through the same angle and its projection will be equal to $v_0 \sin\theta$. But on the same basis as before, $\theta/t = 2\pi/T$, and so the vibrating body's instantaneous speed

$$v = v_0 \sin \frac{2\pi}{T} t$$

Note that in the formula for determining the magnitude of the displacement, the time is equal to zero at the central position, but in the formula for the speed, at an extreme position. The displacement of a pendulum equals zero when the bob is at the central position, but the speed of oscillation is zero at the extreme positions.

There is a simple relation between the maximal speed v_0 of an oscillation and the maximal displacement (or amplitude): the bob describes a circle with circumference $2\pi a$ during a time equal to the period T of the oscillation. Therefore, $v_0 = 2\pi a/T$ and $v = 2\pi a/T \sin 2\pi t/T$.

Force and Potential Energy During an Oscillation

During every oscillation about an equilibrium position, there is a force acting on the vibrating body, "desiring" to return it to the equilibrium position. When a point is receding from its equilibrium position, the force decelerates its motion; when it is nearing this position, the force accelerates its motion.

Let us examine this force in the case of a pendulum. The pendulum bob is acted upon by the force of gravity and the force of tension in the string. Let us decompose the force of gravity into two components—one directed along the string and the other perpendicular to it, along the tangent to the path. Only the tangential component of the gravitational force is of significance for the motion. It is precisely the restoring force in this case. As for the force directed along the string, it is balanced by the reaction on the part of the nail on which the pendulum is hanging, and it is only necessary to take it into account when we are interested in whether the string will withstand the vibrating body's weight.

Denote the magnitude of the bob's displacement by x. The motion takes place along an arc, but we have agreed to investigate oscillations near an equilibrium position. We therefore make no distinction between the magnitude of a displacement along the arc and the bob's deviation from the vertical. Let us consider two similar triangles (Figure 45). The ratio of the corresponding legs is equal to the ratio of the hypotenuses, i.e.

$$\frac{F}{x}=\frac{mg}{l}, \quad \text{or} \quad F=\frac{mg}{l}x$$

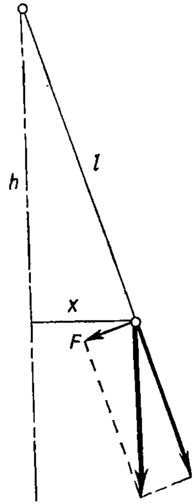


Fig. 45

The quantity mg/l does not change during the oscillation. If we denote this constant by k, then the restoring force F is given by the formula $F = kx$. We arrive at the following important conclusion: the magnitude of the restoring force is directly proportional to the magnitude of the oscillating point's displacement from its equilibrium position. The restoring force is maximal at the vibrating body's extreme positions. When the body passes through the midpoint, the force vanishes and changes sign or, in other words, direction. While the body is displaced to the right, the force is directed to the left, and conversely.

The pendulum serves as the simplest example of an oscillating body. However, we are interested in the possibility of extending the formulas and laws which we find to arbitrary vibrations.

A pendulum's period of oscillation was expressed in terms of its length. Such a formula applies only to a pendulum. But we can express the period of free oscillations in terms of the restoring force constant k. Since $k = mg/l$

we have $l/g = m/k$, and so

$$T = 2\pi\sqrt{\frac{m}{k}}$$

This formula extends to all cases of oscillations, since any free oscillation takes place under the action of a restoring force.

Let us now express a pendulum's potential energy in terms of its displacement x from the equilibrium position. We may take the bob's potential energy to be zero when it passes through the lowest point, and then the height of its ascent should be measured from this point. Denoting the difference in height between the point of suspension and the level of the deflected bob by h, we express the potential energy as follows: $U = mg\,(l - h)$ or, using the formula for the difference of squares,

$$U = mg\frac{l^2 - h^2}{l + h}$$

But, as can be seen from the figure, $l^2 - h^2 = x^2$, l and h differ very slightly and, therefore, $2l$ may be substituted for $l + h$. Thus, $U = mgx^2/2l$, or

$$U = \frac{kx^2}{2}$$

The potential energy of an oscillating body is proportional to the square of its displacement from the equilibrium position.

Let us check the correctness of the formula we have just derived. The loss of potential energy must be equal to the work performed by the restoring force. Consider two of the body's positions—x_2 and x_1. The difference in potential energy

$$U_2 - U_1 = \frac{kx_2^2}{2} - \frac{kx_1^2}{2} = \frac{k}{2}(x_2^2 - x_1^2)$$

But a difference of squares may be written as the product of the sum by the difference. Hence,

$$U_2 - U_1 = \frac{k}{2}(x_2 + x_1)(x_2 - x_1) = \frac{kx_2 + kx_1}{2}(x_2 - x_1)$$

But $x_2 - x_1$ is the length of the path covered by the body, kx_1 and kx_2 are the magnitudes of the restoring force at the beginning and end of the motion, and $(kx_1 + kx_2)/2$ is equal to the average force.

Our formula led us to the correct result: the loss in potential energy is equal to the work performed.

Oscillation of Springs

It is easy to make a ball oscillate by hanging it on a spring. Let us fasten one end of the spring and pull the ball (Figure 46). The spring will be in a stretched position as long as we pull the ball with our hand. If we let go, the spring will unstretch and the ball will begin moving towards its equilibrium position. Just as the pendulum, the spring will not come to a state of rest immediately. The equilibrium position will be passed by inertia, and the spring will begin contracting. The ball's motion slows down, and at a certain instant it comes to a halt in order to start moving at once in the opposite direction. There arises an oscillation with the same typical features with which our study of the pendulum acquainted us.

In the absence of friction, the oscillation would continue indefinitely. In the presence of friction, the oscillations are damped; moreover, the greater the friction, the faster they are damped.

The roles of a spring and a pendulum are frequently analogous. Both one and the other serve to maintain constancy in the period of clocks. The precise movement of

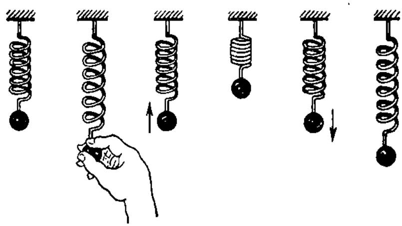


Fig. 46

modern spring watches is ensured by the oscillatory motion of a small flywheel balance. It is set oscillating by a spring which winds and unwinds tens of thousands of times a day.

For the ball on a string, the role of the restoring force was played by the tangential component of gravity. For the ball on a spring, the restoring force is the elastic force of a contracted or stretched spring. Therefore, the magnitude of the elastic force is directly proportional to the displacement: $F = kx$.

The coefficient k has another meaning in this case. It is now the stiffness of the spring. A stiff spring is one which is difficult to stretch or contract. The coefficient k has precisely such a meaning. The following is clear from the formula: k is equal to the force necessary for a stretching or contraction of the spring by a unit of length.

Knowing a spring's stiffness and the mass of the load hung on it, we find the period of free oscillation with the aid of the formula $T = 2\pi\sqrt{m/k}$. For example, a load of mass 10 g on a spring with a stiffness coefficient 10^5 dyne/cm (this is a rather stiff spring—a hundred-gram weight will stretch it by 1 cm) will oscillate with a period $T = 6.28 \times \times 10^{-2}$ sec. During one second, 16 oscillations will take place.

The more flexible the spring, the slower will be the oscillation. An increase in the mass of the load has the same effect.

Let us apply the law of conservation of energy to a ball on a spring.

We know that the sum $K + U$ of kinetic and potential energy for a pendulum does not vary:

$$K + U \text{ is conserved}$$

We know the values of K and U for a pendulum. The law of conservation of energy states that

$$\frac{mv^2}{2} + \frac{kx^2}{2} \text{ is conserved}$$

But the same thing is also true for a ball on a spring.

The deduction which we must inevitably make is quite interesting.

Aside from the potential energy with which we became acquainted earlier, there also exists, therefore, a potential energy of a different kind. The former is called *gravitational potential energy*. If the spring were hanging horizontally, then the gravitational potential energy would, of course, not change during the oscillation. The new potential energy we discovered is called *elastic potential energy*. In our case it is equal to $kx^2/2$, i.e. it depends on the stiffness of the spring and is directly proportional to the square of the amount of contraction or stretching.

The total energy of the oscillation, remaining constant, may be expressed in the following form: $E = ka^2/2$, or $E = mv_0^2/2$.

The quantities a and v_0 occurring in the last formulas are the maximal values which the displacement and speed take on during the oscillation. (They are sometimes called the amplitude values of the displacement and speed.) The origin of these formulas is quite clear. In an extreme position, when $x = a$, the kinetic energy of oscillation is equal to zero, and the total energy is equal to the potential energy.

In the central position, the displacement of the point from the equilibrium position, and hence the potential energy, is equal to zero, the speed at this instant is maximal, $v = v_0$, and the total energy is equal to the potential energy.

The study of oscillations is an extensive branch of physics. One often has to deal with pendulums and springs. But this, of course, does not exhaust the list of bodies whose oscillations must be investigated. Engine seatings vibrate; bridges, parts of buildings, beams and high-voltage lines can begin vibrating. Sound is a vibration of the air.

We have listed several examples of mechanical oscillations. However, the concept of oscillation may refer not only to mechanical displacements of bodies or particles from an equilibrium position. We also come across oscillations in many electrical phenomena; moreover, these oscillations occur in accordance with laws closely resembling those which we have considered above. The study of oscillations permeates all branches of physics.

More Complex Oscillations

What has been said so far refers to oscillations near an equilibrium position, taking place under the action of a restoring force whose magnitude is directly proportional to the displacement of a point from its equilibrium position. Such motions occur in accordance with a sinusoidal law. They are called harmonic. The period of harmonic oscillations is independent of the amplitude.

Oscillations with a large amplitude are much more complex. Such oscillations do not occur in accordance with a sinusoidal law, but their display yields more complicated curves, different for various oscillating systems. The period is no longer a characteristic property of the oscillation, but depends on the amplitude.

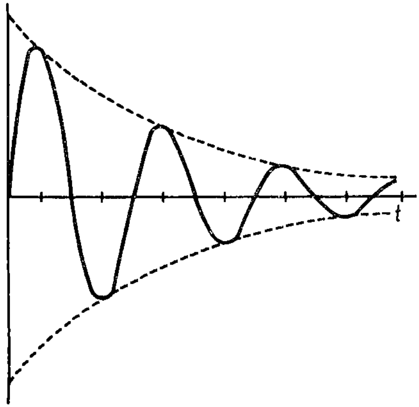


Fig. 47

Friction will significantly change any oscillations. In the presence of friction, oscillations gradually die out. The greater the friction, the faster the damping occurs. Try making a pendulum, immersed in water, oscillate. It is unlikely that you will succeed in getting the pendulum to complete more than one or two oscillations. If a pendulum is immersed in a very viscous medium, there may fail to be any oscillation at all. The deflected pendulum will simply return to its equilibrium position. A typical graph for a damped oscillation is shown in Figure 47. The deviation from the equilibrium position has been plotted along the vertical axis, and the time, along the horizontal. The amplitude of a damped oscillation diminishes with each oscillation.

Resonance

A child is seated on a swing. His feet do not reach the ground. In order to swing him, one can, of course, raise the swing high up and then let it go. But this would be rather

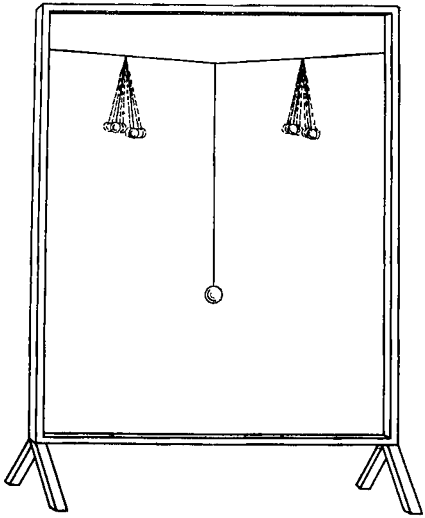


Fig. 48

difficult and also quite unnecessary; it is enough to gently push the swing in time with the oscillations, and in a short time the child will be swinging vigorously.

In order to swing a body, it is necessary to act in time with the oscillations. In other words, it is necessary to make one's pushes occur with the same period as that of the body's own oscillations. In such cases one speaks of *resonance.*

The phenomenon of resonance, widespread in nature and technology, merits careful consideration.

You can observe a very amusing and peculiar occurrence of resonance if you construct the following apparatus. Extend a string horizontally and hang three pendulums on it (Figure 48)—two short ones of identical length and a longer one. Now deflect and release one of the short pendulums.

After a few seconds you will see how the other pendulum of the same length gradually begins oscillating too. A few more seconds—and the second short pendulum will swing, so that it will no longer be possible to tell which of the two pendulums first began moving.

What is the reason for this? Pendulums of the same length have identical periods of free oscillation. The first pendulum swings the second. The oscillations are transmitted from one to the other through the string connecting them. True, but there is yet another pendulum, of different length, hanging on the string. And what will happen to it? Nothing will happen to it. This pendulum's period is different, and a short pendulum will not be able to swing it. The third pendulum will be present at this interesting energy "transfusion" from one pendulum to another, taking no part in it.

Each of us often comes across mechanical resonance phenomena. Perhaps you simply did not pay any attention to them, even though resonance is sometimes very bothersome.

A streetcar passed by your window, and the dishes in the sideboard began jingling. What is the matter? Oscillations of the ground were transmitted to the building and simultaneously to the floor of your room, so your sideboard and the dishes in it started to vibrate. The oscillation was propagated so far and through so many objects. This happened as a result of resonance. The external oscillations were in resonance with the natural oscillations of the bodies. Almost any rattling which we hear in a room, a factory or a car occurs because of resonance.

The phenomenon of resonance, as, incidentally, many phenomena, can be both useful and harmful.

A machine is standing on an engine seating. Its moving parts move rhythmically, with a definite period. Imagine that this period coincides with the engine seating's period of free oscillation. What will happen? The engine seating will very soon be swinging, and the matter can come to a bad end.

The following fact is known. A company of soldiers was marching in step across a bridge in St. Petersburg. The bridge collapsed. An investigation into this matter was begun. It seemed that there were no grounds for anxiety over the fate of the bridge or the people: how many times had crowds gathered on this bridge, had heavy vehicles, weighing much more than a company of soldiers, slowly crossed it!

But a bridge sags by an insignificant amount under the action of a heavy weight. An incomparably greater sagging occurs when a bridge swings. The resonance amplitude of an oscillation can be thousands of times greater than the displacement caused by a stationary load of the same weight.

This is precisely what the investigation showed—the bridge's period of free oscillation coincided with the period of an ordinary marching step.

Therefore, when a military subunit crosses a bridge, a command is given to break step. If people's movements are not coordinated, the phenomenon of resonance will not set in and bridges will not swing. Incidentally, this tragedy is well remembered by engineers. In designing bridges, they try to make its period of free oscillation far from the period of a marching step.

Constructors of engine seatings have similar problems. They try to make the engine seating in such a way that its period of oscillation be as far as possible from that of the machine's moving parts.

Six
MOTION OF SOLID BODIES

Torque

Try to get a heavy flywheel rotating by hand. Pull one of the spokes. You will find it difficult if you grasp it too near to the axle. Move your hand towards the rim, and things will become easier.

But what has changed? After all, the force is the same in both cases. The point of application of the force has changed.

In all that preceded, the question of where a force is applied did not arise, since a body's form and size played no role in the problems under consideration. What we essentially did was to conceptually replace a body by a point.

The example with the rotation of a wheel shows that the question of the point of application of a force is far from idle when we are dealing with the rotation or revolution of a body.

In order to understand the role of the point of application of a force, let us compute the work which must be performed to turn a body through a certain angle. In this calculation, of course, it is assumed that all the body's particles are rigidly bound to one another (we are ignoring at present a body's ability to bend, contract and, in general, to change its form). Therefore, a force applied to one point of a body imparts kinetic energy to all its parts.

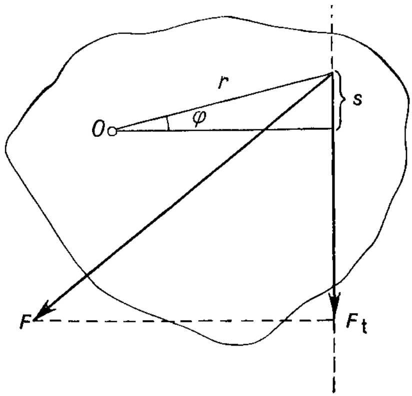


Fig. 49

In computing this work, the role of the point of application of a force is clearly seen.

A body fastened to an axis is shown in Figure 49. When the body turns through a small angle φ, the point of application of a force moves along an arc—it is displaced by a distance s.

Projecting the force onto the direction of the motion, i.e. onto the tangent to the circle around which the point of application moves, we find a familiar expression for the work A:

$$A = F_t s$$

But the arc s may be represented as follows:

$$s = r\varphi$$

where r is the distance from the axis of rotation to the point of application of the force. Thus,

$$A = F_t r\varphi$$

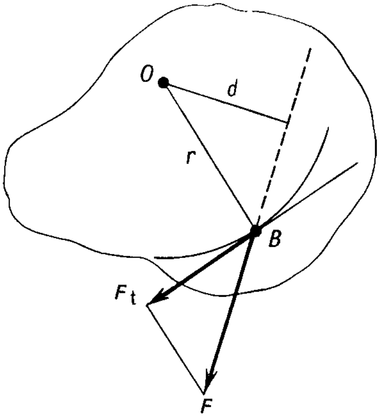


Fig. 50

Turning the body through one and tbe same angle in various ways, we may expend different amounts of work, depending on where the force is applied.

If the angle is given, the work is determined by the product $F_t r$. This product is called the *moment of force*, or the *torque* of the force:

$$M = F_t r$$

Our formula for the torque can be given another form. Let O be the axis of rotation, and B the point of application of the force (Figure 50). The length of the perpendicular dropped from O to the direction of the force is denoted by d. The two triangles constructed in the figure are similar. Therefore,

$$\frac{F}{F_t} = \frac{r}{d}, \quad \text{or} \quad F_t r = Fd$$

The quantity d is called the *arm*, or the *lever arm* of the force.

Our new formula $M = Fd$ reads as follows: the torque is equal to the product of the force by its lever arm.

If we displace the point of application of the force along its direction, then the lever arm d, and with it the torque, will not be changed. Hence, it makes no difference just where the point of application lies on the line of action of the force.

With the aid of the new concept, the formula for the work can be written out more concisely:

$$A = M\varphi$$

i.e. the work is equal to the product of the torque by the angle of rotation.

Let two forces act on a body with moments M_1 and M_2. When the body is rotated through an angle φ, the work done will be $M_1\varphi + M_2\varphi = (M_1 + M_2)\varphi$. This equality shows that two forces with moments M_1 and M_2 rotate a body just as a single force with moment $M = M_1 + M_2$ would. Moments of force can help, as well as hinder, each other. If torques M_1 and M_2 tend to rotate a body in one and the same direction, then we should regard them as magnitudes having the same sign. On the contrary, torques rotating a body in opposite directions have different signs.

As we know, the work done by all the forces acting on a body effects a change in its kinetic energy.

The rotation of a body slowed down or speeded up—hence, its kinetic energy changed. This can only take place in case the resultant torque is not equal to zero.

But what if the resultant torque is equal to zero? The answer is obvious—the kinetic energy does not change; consequently, the body either rotates uniformly by inertia or remains at rest.

Thus, equilibrium of a body capable of rotating requires the balancing of all torques acting on it. If there are two such torques, equilibrium requires that

$$M_1 + M_2 = 0$$

While we were interested in problems in which a body could be regarded as a point, the conditions for equilibrium were simple: in order for a body to remain at rest or move uniformly, stated Newton's law for such problems, it is necessary that the resultant force be equal to zero; the forces acting upwards must balance those directed downwards; the rightward force must compensate for the leftward force.

This law is also valid for our case. If a flywheel is at rest, the forces acting on it are balanced by the reaction of the axle, around which the wheel can turn.

But these necessary conditions become insufficient. Besides the balancing of forces, the balancing of torques is also required. The balancing of moments of force is the second necessary condition for rest or uniform rotation of a solid body.

Torques, if there are several of them, can be easily separated into two groups: some tend to rotate a body in a clockwise direction, others, counterclockwise. These are precisely the groups of forces which must compensate for each other.

Lever

Can a person keep 100 tons from falling? Can one crush a piece of iron with one's hand? Can a child resist a strong man? Yes, they can.

Ask a strong man to turn a flywheel in the clockwise direction while holding a spoke near the axle. The torque will be small in this case: the force is large, but the lever arm is short. If a child pulls the wheel in the opposite direction, holding a spoke near the rim, the torque may turn out to be large: the force is small, but the lever arm is long. The condition for equilibrium will be

$$M_1 = M_2, \quad \text{or} \quad F_1 d_1 = F_2 d_2$$

Using the law of moments, a person can acquire fabulous strength.

The action of levers serves as the most striking example of this.

You want to lift an enormous stone with the aid of a crow-bar. This task will turn out to be possible for you to accomplish, even though the stone weighs several tons. The crow-bar is placed on a pivot and is the solid body of our problem. The pivot is the center of rotation. Two torques act on the body: a hindering one, from the weight of the stone, and a helping one, from your hand. If the subscript 1 refers to the muscular force, and the subscript 2 to the weight of the stone, then the possibility of lifting the stone can be expressed concisely: M_1 must be greater than M_2.

The stone can be supported above the ground provided that

$$M_1 = M_2, \quad \text{i.e.} \quad F_1 d_1 = F_2 d_2$$

If the short lever arm—from the pivot to the stone—is 15 times smaller than the long one—from the pivot to the hand, then a person acting with his entire weight on the long end of the lever will support a 1-ton stone in a raised position.

A crow-bar placed on a pivot is a rather widespread and the simplest example of a lever. A ten- to twenty-fold gain in force is usually achieved with the aid of a crow-bar. The length of a crow-bar is about 1.5 m, but it is usually difficult to place the pivot nearer than 10 cm from its bottom. Therefore, one of the lever arms will be 15-20 times as long as the other, and so this will also be the gain in force.

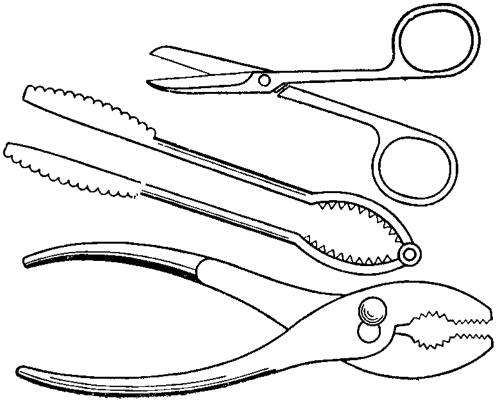


Fig. 51

A chauffeur will easily raise an automobile weighing several tons with the aid of a jack. A jack is a lever, of the same type as a crow-bar, placed on a pivot. The points of application of the forces (the hand, the car's weight) lie on opposite sides of the jack's pivot. Here the gain in force is about forty- to fifty-fold, which makes it possible to easily lift an enormous weight.

A pair of scissors, a nutcracker, pliers, pincers, nippers and many other instruments are all levers. You can easily find the centers of rotation (pivots) of the solid bodies depicted in Figure 51, as well as the points of application of the two forces—active and hindering.

In cutting tin-plate with a pair of scissors, one tries to open them as wide as possible. What is accomplished by this? One succeeds in slipping the piece of metal closer to the center of rotation. The lever arm of the torque one is overcoming becomes shorter, and so the gain in force is greater. When moving a pair of scissors, an adult ordinarily acts with a force of 40-50 kgf. One of the lever arms can be

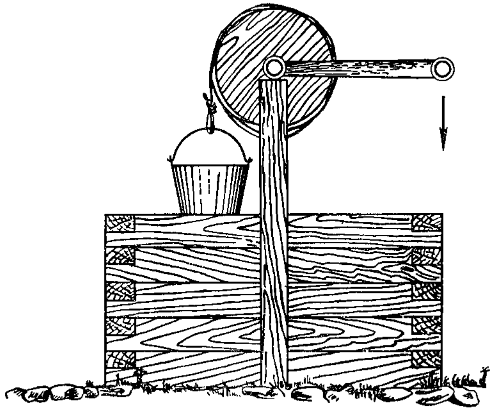


Fig. 52

20 times longer than the other. It turns out that we are able to cut into metal with a force of 1 ton. And this with the aid of such simple instruments.

The windlass is a variety of lever. With the aid of a windlass (Figure 52), water is taken out of a well in many villages.

Loss in Path

Instruments make a person strong, but it by no means follows from this that instruments permit one to expend a little work and obtain a lot. The law of conservation of energy convinces us that a gain in work, i.e. the creation of work out of "nothing", is impossible.

The work obtained cannot be greater than the work performed. On the contrary, the inevitable loss of energy due to friction leads to the fact that the work obtained with the aid of an instrument will always be less than that performed. In the ideal case, these works can be equal.

Properly speaking, we are wasting time by explaining this obvious truth: for the rule of torques was derived from the condition of equality of the work performed by the active and overcome forces.

If the points of application of the forces moved distances s_1 and s_2, the condition of equality of the work assumes the following form:

$$F_1^t s_1 = F_2^t s_2$$

In overcoming some force F_2 along a path of length s_2 with the aid of a lever, we can make this by means of force F_1 much less than F_2. But the displacement s_1 of our hand must be as many times greater than s_2 as the muscular force F_1 is less than F_2.

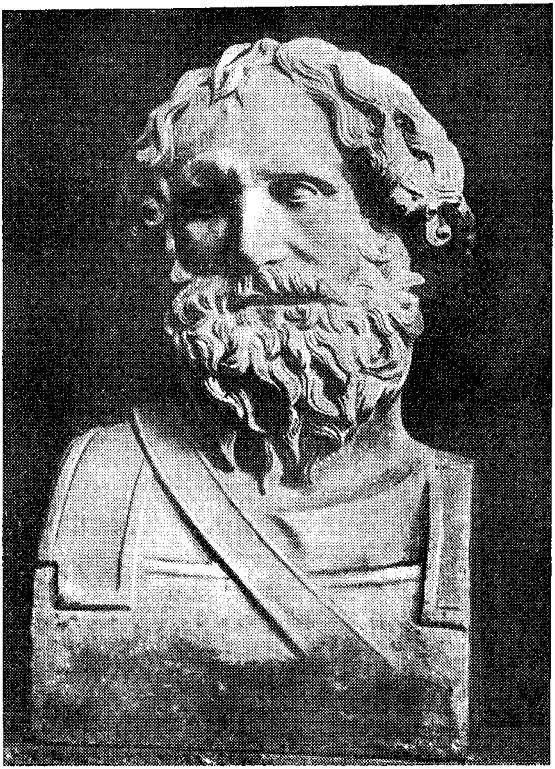
This law is often expressed by the following brief sentence: the gain in force is equal to the loss in path.

The law of the lever was discovered by the greatest scientist of antiquity—Archimedes. Amazed at the strength of his proof, this remarkable scientist of antiquity wrote to King Hiero II of Syracuse: "If there were another world and I could go to it, I would move this one." A very long lever, whose pivot is near the Earth, would make it possible to solve such a problem.

We shall not grieve with Archimedes over the absence of a fulcrum, which, as he thought, was all that he lacked to move the Earth.

Let us dream: take the strongest possible lever, place it on a pivot and "suspend a small sphere" of weight ... 6×10^{24} kgf on its short end. This modest number shows how much the Earth, "pressed into a small sphere", weighs. Now apply muscular force to the long end of the lever.

If the force exerted by Archimedes can be taken as 60 kgf, then in order to displace the "Earth nut" by 1 cm, Archimedes' hand would have to cover a distance $(6 \times 10^{24})/60 =$



ARCHIMEDES (*circa* 287-212 B.C.)—the greatest mathematician, physicist and engineer of antiquity. Archimedes computed the volume and the surface area of a sphere and its parts, of a cylinder and of bodies formed by rotating an ellipse, hyperbola or parabola. He was the first to compute the ratio of a circle's circumference to its diameter with a high degree of accuracy, showing that it satisfies the inequalities $3.1408 < \pi < 3.1429$. In mechanics he established the laws of lever, the conditions governing floating bodies (Archimedes' principle), the laws of composition of parallel forces. Archimedes invented a machine for pumping water (Archimedean screw, used in our times for transporting free-flowing or viscous cargo), systems of levers and blocks for raising heavy weights and military engines, successfully employed during the siege of his native city, Syracuse, by the Romans.

$= 10^{23}$ times greater. But 10^{23} cm are 10^{18} km, which is three billion times greater than the diameter of the Earth's orbit!

This fantastic example clearly demonstrates the scale of the "loss in path" involved in the work of a lever.

Any of the examples considered by us above can be used to illustrate not only a gain in force, but also a loss in path. The hand of the chauffeur jacking up his car covers a path which will be as many times longer than the height to which he raises it as his muscular force is less than the automobile's weight. Moving a pair of scissors in order to cut a sheet of tin-plate, we perform work along a path which is as many times longer than the depth of the cut as our muscular force is less than the resistance of the tin-plate. The stone lifted by the crow-bar will rise to a height as many times less than that by which the hand is lowered as the muscular force is less than the stone's weight. This rule makes the principle of a screw's action clear. Imagine that we are screwing in a bolt, whose threading has a 1-mm screw pitch, with the aid of a wrench of length 30 cm. The bolt will advance 1 mm along its axis during a single turn, while our hand will cover a 2-m path during this same time. Our gain in force is two thousand-fold, and we either safely fasten the components together or move a heavy weight with a slight effort of our hand.

Other Very Simple Machines

A loss in path as payment for a gain in force is a general law not only for levers, but also for all other devices and mechanisms used by man.

A tackle is widely used for lifting loads. This is what we call a system consisting of several movable pulleys joined to one or several fixed pulleys. The load in Figure 53 is

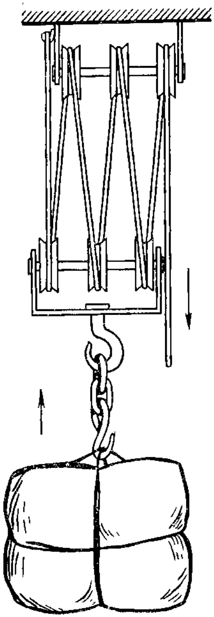


Fig. 53

suspended by six strings. It is clear that the weight is distributed among the strings, and so the tension in a string will be six times less than the weight. The lifting of a one-ton load will require an application of 1000/6 = 167 kgf. However, it is not difficult to see that in order to raise the load by 1 m, one must haul in 6 m of string. For raising the load by 1 m, 1000 kgf-m of work are needed. We must supply this work in "some form"—a force of 1000/6 kgf must act along a 6-m path, a force of 10 kgf along a 100-m path, and a force of 1 kgf along a 1-km path.

An inclined plane, which we discussed on p. 34, is also a device permitting a gain in force at the expense of a loss in path.

A blow is a distinctive means of multiplying forces. A blow with a hammer, an axe, a ram and even a blow with a fist can create an enormous force. The secret of a strong blow isn't complicated. Driving a nail into an unyielding wall with a hammer, one must take a good wind-up. A big swing, i.e. a big path along which the force acts, generates a significant kinetic energy in the hammer. This energy is released along a small path. If the swing covers 1/2 m and the nail enters 1/2 cm into the wall, then the force is intensified by a factor of 100. But if the wall is harder and the nail, after the same swing of the hand, enters 1/2 mm into the wall, then the blow will be 10 times as strong as in the former case. The nail does not enter a hard wall as deeply, and the same work is expended on a shorter path. It turns out that a hammer works like an automaton: it strikes harder where there are greater difficulties.

If a hammer is "speeded up" to a force of one kilogram, then it will strike a nail with a force of 100 kgf. But in splitting logs with a heavy wood-chopper, we break the wood with a force of several tons. Heavy forging hammers fall from moderate heights—of the order of a meter. Flattening a forged piece by 1-2 mm, a hammer weighing a ton comes down on it with an enormous force—thousands of tons.

How to Add Parallel Forces Acting on a Solid Body

On the preceding pages, when we solved mechanical problems in which a body was conceptually replaced by a point, the question of how to add forces was answered simply. The parallelogram law of forces yielded an answer to this question, while if the forces were parallel, we added their magnitudes like numbers.

Now matters are more complicated. For the effect of a force on an object is characterized not only by its magnitude and direction, but also by the point of its application, or—we have explained above that this is the same thing—its line of action.

To add forces means to replace them by a single force. This is by no means always possible.

The replacement of parallel forces by a single resultant is a problem which can always be solved (except in a special case, which will be discussed at the end of this section). Let us consider the addition of parallel forces. Of course, the sum of forces of 3 kgf and 5 kgf is equal to 8 kgf, provided that they have the same direction. The problem con-

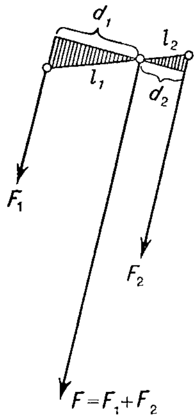


Fig. 54

sists in finding the resultant's point of application (line of action).

Two forces acting on a body are depicted in Figure 54. The resultant force F replaces the forces F_1 and F_2, but this means not only that $F = F_1 + F_2$; the action of F will be equivalent to that of F_1 and F_2 in case the torque produced by F is equal to the sum of the torques produced by F_1 and F_2.

We are looking for the line of action of the resultant force F. Of course, it is parallel to the forces F_1 and F_2, but how far is this line from F_1 and F_2?

A point lying on the segment joining the points of application of F_1 and F_2 is depicted in the figure as F's point of application. With respect to the chosen point, the moment of F is, of course, equal to zero. But then the sum of the moments of F_1 and F_2 should also be equal to zero with respect to this point, i.e. the torques produced by F_1 and F_2, opposite in sign, will be equal in magnitude.

Denoting the lever arms of F_1 and F_2 by d_1 and d_2, we may write out this condition as follows:

$$F_1 d_1 = F_2 d_2, \quad \text{i.e.} \quad \frac{F_1}{F_2} = \frac{d_2}{d_1}$$

It follows from the similarity of the shaded triangles that $d_2/d_1 = l_2/l_1$, i.e. the point of application of the resultant force divides the distance on the uniting segment between the added forces into parts, l_1 and l_2, which are inversely proportional to the forces.

Denote the distance between the points of application of F_1 and F_2 by l. It is obvious that $l = l_1 + l_2$.

Let us solve the following system of two equations in two variables

$$F_1 l_1 - F_2 l_2 = 0$$
$$l_1 + l_2 = l$$

We obtain

$$l_1 = \frac{F_2 l}{F_1 + F_2}, \quad l_2 = \frac{F_1 l}{F_1 + F_2}$$

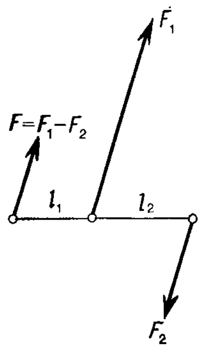


Fig. 55

By means of these formulas, we can find the point of application of the resultant force not only in the case when the forces have the same direction, but also in the case of forces with opposite directions (antiparallel forces, as we say). If the forces have different directions, then they have opposite signs, and the resultant is equal to the difference $F_1 - F_2$ of the forces, and not to their sum. Taking the smaller of the two forces, F_2, to be negative, we see by our formulas that l_1 becomes negative. This means that the point of application of F_1 lies not to the left (as before), but to the right of the resultant's point of application (Figure 55); moreover, as in the previous case,

$$\frac{F_1}{F_2} = \frac{l_2}{l_1}$$

An interesting result is obtained for equal antiparallel forces. Then $F_1 + F_2 = 0$. The formulas show that l_1 and l_2 will then become infinitely large. But what physical meaning does this assertion have? Since it is meaningless to put the resultant at infinity, it is therefore impossible to replace equal antiparallel forces by a single force. Such a combination of forces is called a *couple*.

The action of a couple cannot be reduced to the action of one force. Any other pair of parallel or antiparallel

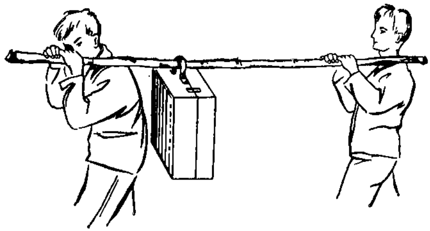


Fig. 56

forces can be balanced by a single force, but a couple cannot.

Of course, it would be false to say that the forces constituting a couple cancel each other. A couple has quite a significant effect—it rotates a body; a peculiarity of a couple's action consists in the fact that it does not produce a translational motion.

In certain cases, the question may arise not of adding parallel forces, but of decomposing a given force into two parallel ones.

Two persons, carrying a heavy valise together on a pole, are depicted in Figure 56. The weight of the valise is distributed between the two of them. If the load presses down on the center of the pole, they both feel the same weight. If the distances from the load's point of application to the hands which carry it are d_1 and d_2, then the force F is decomposed into forces F_1 and F_2 according to the rule

$$\frac{F_1}{F_2}=\frac{d_2}{d_1}$$

The stronger person should take hold of the pole nearer to the load.

Center of Gravity

All particles of a body possess weight. Therefore, a solid body is subject to the action of an infinite number of gravitational forces. Moreover, all these forces are parallel. If so, it is possible to add them according to the rule which we have just considered, and replace them by a single force. The point of application of the resultant force is called the *center of gravity*. It is as if the body's weight were concentrated at this point.

Let us suspend a body by one of its points. How will it then be situated? Since we may conceptually replace the body by one load concentrated at the center of gravity, it is clear that in equilibrium this load will lie on the vertical passing through the pivot. In other words, in equilibrium the center of gravity lies on the vertical passing through the pivot, and is at its lowest possible position.

One can place the center of gravity on the vertical passing through the axis and above the pivot. It will be possible to do this with a great deal of difficulty and only because of the presence of friction. Such an equilibrium is *unstable*.

We have already discussed the condition for stable equilibrium—the potential energy must be minimal. This is precisely the case when the center of gravity lies below the pivot. Any deflection raises the center of gravity and, therefore, increases the potential energy. On the contrary, when the center of gravity lies above the pivot, then any puff, removing the body from this position, leads to a decrease in potential energy. Such a position is unstable.

Cut a figure out of cardboard. In order to find its center of gravity, hang it up twice, attaching the suspending thread first to one and then to another point of the body. Attach the figure to an axis passing through its center of gravity. Turn the figure to one position, a second, a third, ...

We observe the complete neutrality of the body towards our operations. A special case of equilibrium is attained in any position. This is just what we call it—*neutral.*

The reason for this is clear—in any of the figure's positions, the material point replacing it is located at one and the same place.

In a number of cases, the center of gravity can be found without any experiments or computations. It is clear, for example, that the centers of gravity of a sphere, circle, square and rectangle are located at the centers of these figures, since they are symmetrical. If we conceptually break up a symmetrical body into small parts, then each of them will correspond to another, symmetrically located on the other side of the center. But for each pair of such particles, the center of the figure will be the center of gravity.

A triangle's center of gravity lies at the intersection of its medians. In fact, let us break up a triangle into narrow strips, parallel to one of the sides. A median divides each of the strips in half. But a strip's center of gravity lies, of course, half-way along it, i.e. on the median. The centers of gravity of all the strips occur on the median, and when we add their weights, we arrive at the conclusion that the triangle's center of gravity lies somewhere on the median. But this argument is valid with respect to any of the medians. Therefore, the center of gravity must lie at their intersection.

But perhaps you are not convinced that the three medians intersect in a point. This is proved in geometry; but our argument also proves this interesting theorem. For a body cannot have several centers of gravity; but since the center of gravity is one and lies on a median, no matter from which vertex we draw it, all three medians therefore intersect in a single point. The formulation of a physical problem helped us prove a geometric theorem.

It is more difficult to find the center of gravity of a homogeneous cone. It is only clear from considerations of symmetry that the center of gravity lies on the axis. Computations show that it is located at the distance of one-fourth of the height from the apex.

The center of gravity is not necessarily located inside a body. For example, the center of gravity of a ring is located at its center, i.e. outside the ring.

Can a pin be stably placed in a vertical position on a glass pedestal?

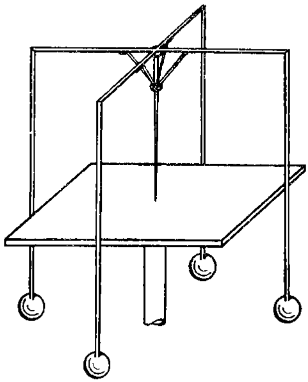


Fig. 57

It is shown in Figure 57 how to do this. A small apparatus, consisting of wires in the form of a double yoke with four small loads, should be rigidly fastened to the pin. Since the loads are hanging lower down than the pivot, while the weight of the pin is small, the center of gravity lies below the pivot. The position is stable.

So far we have been dealing with bodies possessing a point of support. What is the situation in those cases when a body is supported over an entire area element?

It is clear that in this case the location of the center of gravity above the support does not at all imply that the equilibrium is unstable. How else could glasses stand on a table? It is necessary for stability that the line of action of the gravitational force, drawn from the center of gravity, pass through the area of support. On the contrary, if the force's line of action passes outside the area of support, then the body will fall.

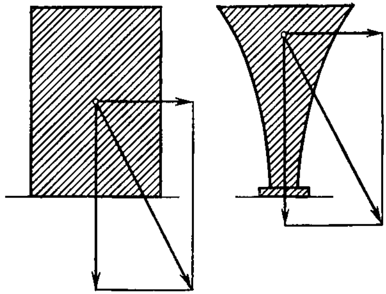


Fig. 58

Degrees of stability may be very different, depending on how high above the support the center of gravity is. Only a very clumsy person will overturn a glass of tea, but a flower vase with a small base can be overturned by a careless touch. What is the point here?

Take a look at Figure 58. One and the same overturning force, added to the force of gravity, yields a resultant force which presses the body to the support if the center of gravity is low, but is directed to one side, instead of passing through the area of support, for a high location of the center of gravity.

We have said that for a body to be stable, the force applied to it must pass through the area of support. But the area of support needed for equilibrium does not always correspond to the actual area of support. A body whose area of support has the form of a crescent is depicted in Figure 59. It is easy to see that the body's stability will not change if the crescent is completed to a solid half-disc. Thus, the area of support determining the condition for equilibrium may be greater than the actual one.

In order to find the area of support for the tripod depicted in Figure 60, one must join its tips with straight-line segments.

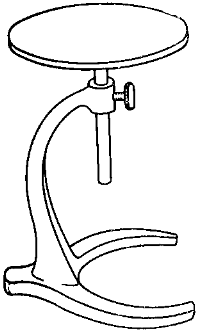


Fig. 59

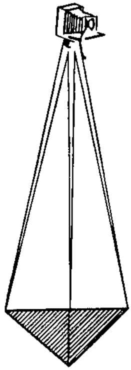


Fig. 60

Why is it so hard to walk a tightrope? Because the area of support has sharply decreased. It isn't easy to walk a tightrope, and skilful tightrope walkers aren't rewarded with applause for nothing. However, sometimes viewers make the mistake of acclaiming clever tricks simplifying the task as the epitome of artistry. The performer takes a heavily bent yoke with two pails of water; the pails turn out to be on the level of the tightrope. With a straight face, while the orchestra has ceased playing, the performer takes his walk along the tightrope. How complicated has the trick become, thinks the inexperienced viewer. As a matter of fact, the performer has simplified his task by lowering the center of gravity.

Center of Mass

It is entirely legitimate to ask the following question: where is the center of gravity of a group of bodies? If many people are on a raft, then its stability will depend on the location of their (together with the raft's) center of gravity.

The meaning of this concept remains the same. The center of gravity is the point of application of the sum of the gravitational forces of all the bodies in the group under consideration.

We know the result of the computation for two bodies. If two bodies of weights F_1 and F_2 are located at a distance x from each other, then their center of gravity is situated at distances x_1 from the first and x_2 from the second body, where

$$x_1 + x_2 = x \quad \text{and} \quad \frac{F_1}{F_2} = \frac{x_2}{x_1}$$

Since weight may be represented as a product mg, the center of gravity of the pair of bodies satisfies the condition

$$m_1 x_1 = m_2 x_2$$

i.e. lies at the point which divides the distance between the masses into segments inversely proportional to the masses.

Let us now recall the firing of a gun attached to a platform. The momenta of the gun and the shell are equal in magnitude and opposite in direction. The following equalities hold:

$$m_1 v_1 = m_2 v_2, \quad \text{or} \quad \frac{v_2}{v_1} = \frac{m_1}{m_2}$$

where the ratio of the speeds retains this value during the entire interaction. In the course of the motion arising as a result of the recoil, the gun and the shell are displaced with respect to their initial positions by distances x_1 and x_2 in opposite directions. The distances x_1 and x_2—the paths covered by the two bodies—increase, but for a constant ratio of speeds, they will also be in the same ratio to each other all the time:

$$\frac{x_2}{x_1} = \frac{m_1}{m_2}, \quad \text{or} \quad x_1 m_1 = x_2 m_2$$

Here x_1 and x_2 are the distances of the gun and the shell from their original positions. Comparing this formula with the formula determining the position of the center of gravity, we observe their complete identity. It immediately follows from this that the center of gravity of the gun and the shell remains at its original position all the time after the firing.

In other words, we have arrived at a very interesting result—the center of gravity of the gun and the shell remains at rest after the firing.

Such a conclusion is always true: if the center of gravity of two bodies was initially at rest, then their interaction—regardless of its nature—cannot change the position of the center of gravity. This is precisely why it is impossible to pick oneself up by the hair or pull oneself up to the Moon by the method of the French writer Cyrano de Bergerac, who proposed (jokingly, of course) to this end that one threw a magnet upwards while holding a piece of iron, which would be attracted by the magnet.

A resting center of gravity is moving uniformly from the point of view of a different inertial system. Hence, a center of gravity is either at rest or moving uniformly and rectilinearly.

What we have said about the center of gravity of two bodies is also true for a group of many bodies. Of course, for an isolated group of bodies—this is always stipulated when we are applying the law of conservation of momentum.

Consequently, every group of interacting bodies has a point which is at rest or is moving uniformly, and this point is their center of gravity.

Wanting to emphasize the new property of this point, we give it an additional name: the *center of mass*. As a matter of fact, the question of, say, the solar system's weight (and hence its center of gravity) can have only a hypothetical meaning.

No matter how the bodies forming a closed group move, the center of mass (gravity) will be at rest or, in another frame of reference, will move by inertia.

Angular Momentum

We shall now become acquainted with another mechanical concept, which permits us to formulate a new (for us), important law of motion.

This concept is called *angular momentum*, or *moment of momentum*. The very names suggest that we are dealing with a quantity which somehow resembles a moment of force.

A moment of momentum, just as a moment of force, requires an indication of the point with respect to which the moment is defined. In order to define the angular momentum relative to some point, one must construct the momentum vector and drop a perpendicular from the point to its direction (Figure 61). The product of the momentum mv by the lever arm d is precisely the angular momentum, which we shall denote by the letter N:

$$N = mvd$$

If a body is moving freely, its velocity does not change; the lever arm with respect to any point also remains constant, since the motion takes place along a straight line. Consequently, the angular momentum also remains constant during such a motion.

Just as for the moment of force, we can also obtain a different formula for the moment of momentum. Draw a radius between the body's position and the point with respect to which we are interested in the angular momentum (see Figure 61). Construct also the projection of the velocity on the direction perpendicular to the radius. It follows from the similar triangles constructed in the figure that $v/v_{\perp} =$

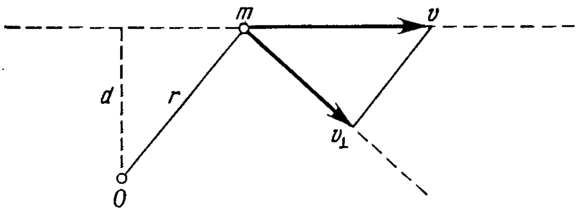


Fig. 61

$= r/d$. Therefore, $vd = v_{\perp}r$, and the formula for angular momentum may be also written in the following form: $N = mv_{\perp}r$.

During free motion, as we have just said, angular momentum remains constant. Well, but if a force is acting on the body? Computations show that the change in angular momentum during a second is equal to the torque.

This law can be extended without difficulty to systems of bodies. If we add the changes in the angular momenta of all the bodies belonging to the system, then their sum turns out to be equal to the sum of the torques acting on the bodies. Consequently, the following statement is valid for a group of bodies: the change in the total moment of momentum during a unit of time is equal to the sum of the moments of all the forces.

Law of Conservation of Angular Momentum

If two stones are connected with a string and one of them is hurled, then the other stone will fly after the first one at the end of the stretched string. Each stone will pass the other, and this forward motion will be accompanied by a rotation. Let us forget about the gravitational field—assume that the throw was made in interstellar space.

The forces acting on the stones are equal in magnitude and directed towards each other along the string (for these are forces of action and reaction). But then the lever arms of both forces with respect to an arbitrary point will also be the same. Equal lever arms and equal, but oppositely directed forces yield torques which are equal in magnitude and opposite in sign.

The resultant torque will be equal to zero. But it follows from this that the change in angular momentum will also equal zero, i.e. that the angular momentum of such a system remains constant.

We only needed the string connecting the stones for visualization. The law of conservation of angular momentum is valid for any pair of interacting bodies, no matter what the nature of this interaction.

Yes, and not only for a pair. If a closed system of bodies is being investigated, then the forces acting between the bodies can always be divided up into an equal number of forces of action and reaction, whose moments will cancel each other in pairs.

The law of conservation of total angular momentum is universal; it is valid for any closed system of bodies.

If a body is rotating about an axis, then its angular momentum is

$$N = mvr$$

where m is the mass, v is the speed, and r is the distance from the axis. Expressing the speed in terms of the number n of revolutions per second, we have:

$$v = 2\pi nr \quad \text{and} \quad N = 2\pi mnr^2$$

i.e. the angular momentum is proportional to the square of the distance from the axis.

Sit down on a swivel stool. Pick up heavy weights, spread your arms wide apart and ask somebody to get you rotating slowly. Now press your arms to your chest by means of a rapid motion—you will suddenly begin rotating faster. Arms out—the motion slows down, arms in—the motion speeds up. Until the stool stops turning because of friction, you will have time to change your rotational velocity several times.

Why does this happen?

For a constant number of revolutions per second, the angular momentum would decrease in case the weights approached the axis. In order to "compensate" for this decrease, the rotational velocity increases.

Acrobats make good use of the law of conservation of angular momentum. How does an acrobat turn a somersault in mid-air? First of all, by pushing off from an elastic floor or his partner's hand. When pushing off, his body bends forward and his weight, together with the force of the push, creates an instantaneous torque. The force of the push creates a forward motion, but the torque causes a rotation. However, this rotation is slow, incapable of impressing the audience. The acrobat bends his knees. "Gathering his body" closer to the axis of rotation, the acrobat greatly increases the rotational velocity and quickly turns over. This is the mechanics of the somersault.

The movements of a ballerina, performing a succession of rapid turns, are based on this same principle. Ordinarily the initial angular momentum is imparted to the ballerina by her partner. At this instant the dancer's body is bent; a slow rotation begins, then a graceful and rapid movement—the ballerina straightens up. Now all points of her body are closer to the axis of rotation, and conservation of angular momentum leads to a sharp increase in speed.

Angular Momentum as a Vector

So far we have been dealing with the magnitude of angular momenta. But a moment of momentum possesses the properties of a vector quantity.

Consider the rotation of a point with respect to some "center". Two nearby positions of the point are depicted in Figure 62. The motion in which we are interested is characterized by the magnitude of its angular momentum and the plane in which it takes place. The plane of the motion is shaded in the figure—it is the area swept out by the radius drawn from the "center" to the moving point.

Information about the direction of the plane of the motion and about the magnitude of the angular momentum can be combined. The angular momentum vector, directed along the normal to the motion's plane and equal in magnitude to the absolute value of the angular momentum, serves for this purpose. However, this is still not all—one must take into account the direction of the motion in the plane: for a body can rotate about a center in the clockwise, as well as in the counterclockwise, direction.

It is customary to draw an angular momentum vector in such a manner that we see the point rotating in the counterclockwise direction when we look at it facing the vector. This can also be said otherwise: the direction of the angular momentum vector is related to the direction of the rotation in the same way as the direction of a turning corkscrew is related to the direction of its handle's motion.

Thus, if we know the angular momentum vector, we can determine the magnitude of the angular momentum, how the motion's plane is situated in space, and the direction of the rotation with respect to the "center".

If the motion takes place in one and the same plane, but the lever arm and speed change, then the angular momentum

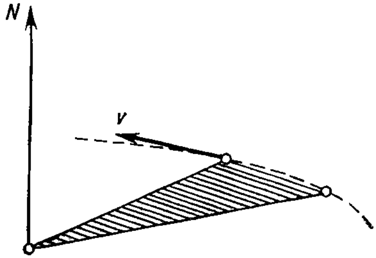


Fig. 62

vector preserves its direction in space, but changes in length. But in the case of an arbitrary motion, the angular momentum vector changes in direction as well as in magnitude.

It may seem that such a fusion into one concept of the direction of a motion's plane and the magnitude of an angular momentum serves only the purpose of saving words. In reality, however, when we are dealing with a system of bodies which are moving in more than one plane, we obtain the law of conservation of angular momentum only when we add moments of momentum as vectors.

This circumstance shows that the attribution of a vector nature to angular momentum has a profound content.

Angular momentum is always defined with respect to some conditionally chosen "center". It is only natural that this quantity depends, generally speaking, on the choice of this point. Nevertheless, it can be shown that if the system of bodies under consideration is at rest on the whole (its total momentum is equal to zero), then its angular momentum vector is independent of our choice of "center". This angular momentum may be called the internal angular momentum of the system of bodies.

The law of conservation of angular momentum vector is the third and last conservation law in mechanics. However, we are not being entirely precise when speaking of three conservation laws. In fact, momentum and angular momentum are vector quantities, and a law of conservation of a vector quantity implies that not only its numerical value remains constant, but also its direction. To put it otherwise, the three components of a vector in three mutually perpendicular directions in space remain constant. Energy is a scalar quantity, momentum is a vector quantity, and angular momentum is also a vector quantity. It would therefore be more precise to say that seven conservation laws hold in mechanics.

Tops

Try to place a plate topside up on a thin stick and keep it in a position of equilibrium. Nothing will come of your efforts. However, such a trick is the favourite number of Chinese jugglers. They succeed in performing it with several sticks simultaneously. A juggler doesn't even attempt to maintain his thin sticks in a vertical position. It appears to be a miracle that the plates, slightly supported by the ends of the horizontally inclined sticks, do not fall, but practically hang in the air.

If you have the opportunity of observing jugglers at work at close range, note the following significant detail: the juggler twists the plates in such a fashion that they rotate rapidly in their planes.

Juggling maces, rings or hats, the performer will in all cases impart a spin to them. Only then will the objects return to his hand in the same state in which they were put at the beginning.

What is the cause of such stability, imparted by a rotation? It is related to the law of conservation of angular momentum. For when there is a change in the direction of the axis of rotation, the direction of the angular momentum vector also changes. Just as a force is needed to change a velocity's direction, so a torque is needed to change a rotation's direction; the faster the body rotates, the greater the torque required.

The tendency of a rapidly rotating body to preserve the direction of its axis of rotation can be observed in many cases similar to those mentioned. Thus, a spinning top does not tip over, even though its axis is inclined.

Try to overturn a spinning top with your hand. It proves to be not so easy to do this.

The stability of a rotating body is utilized in the artillery. You have probably heard that gun barrels are rifled. An outgoing projectile spins about its axis and, because of this, does not "tumble" through the air. A rifled gun gives incomparably better aiming and greater range than an unrifled one.

It is necessary for a pilot or a sea navigator to always be aware of the location of the true terrestrial vertical relative to the position of the airplane or the ship at the given instant. The use of a plumb-line is unsuitable for this purpose, since it is deflected during an accelerated motion. A rapidly spinning top of special construction is therefore employed—it is called a gyrovertical. If we set its axis of rotation along a terrestrial vertical, it will then remain in this position regardless of how the airplane changes its position in space.

But what does the top stand on? If it is located on a support which is turning together with the airplane, then how can its axis of rotation preserve its direction?

An apparatus like the Cardan suspension (Figure 63) serves as the support. In this apparatus, with a minimum of friction at the pivots, a top can behave as though it were suspended in air.

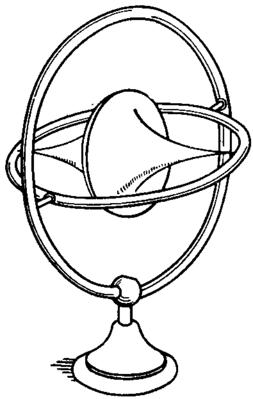


Fig. 63

With the aid of spinning tops, it is possible to automatically keep torpedoes and airplanes on a given course. This is done by means of mechanisms "watching" the deviation of the direction of the torpedo's axis from that of the top's axis.

Such an important instrument as the gyrocompass is based on an application of the spinning top. It can be proved that under the action of the Coriolis force and friction, a top's axis eventually settles down parallel to the Earth's axis, and so points to the North.

Gyrocompasses are widely applied in navigation. Their main part, an engine with a heavy flywheel, does up to 25 000 rpm. In spite of a number of difficulties involved in the elimination of various hindrances, in particular those due to the pitching of a ship, gyrocompasses have an advantage over magnetic compasses. The drawback of the latter is the distortion of the readings because of the influence of iron objects and electrical appliances aboard the ship.

Flexible Shaft

Shafts of modern steam turbines are important parts of these mighty machines. The manufacture of such shafts, attaining 10 m in length and 0.5 m in diameter, is a complex technological problem. A powerful turbine's shaft can withstand a load of about 200 t and rotate with a speed of 3000 rpm.

At first glance, it might seem that such a shaft should be exceptionally hard and durable. This, however, is not so. At tens of thousands of revolutions per minute, a rigidly fastened and unbendable shaft will inevitably break, no matter how strong it may be.

It isn't difficult to see why rigid shafts are unsuitable. No matter how precisely engineers work, they cannot avoid at least a slight asymmetry in a turbine's wheel. Enormous centrifugal forces arise when such a wheel rotates—recall that their magnitudes are proportional to the square of the rotational speed. If they are not exactly balanced, then the shaft will start "beating" against the ball-bearings (for the unbalanced centrifugal forces "rotate" together with the machine), break them and smash the turbine.

At one time, this phenomenon created an unsurmountable obstacle to the increase of a turbine's rotational speed. A way out of the situation was found at the last turn of the century. The flexible shaft was introduced into the technology of turbine construction.

In order to understand the idea behind this remarkable invention, we must compute the total effect of the centrifugal forces. But how can these forces be added? It turns out that the resultant of these centrifugal forces acts at the center of gravity of the shaft and has the same magnitude as if the entire mass of the turbine's wheel were concentrated at the center of gravity.

Let us denote the distance from the center of gravity of the turbine's wheel to its axis, distinct from zero because of a slight asymmetry in the wheel, by a. During rotation, centrifugal forces will act on the shaft, which will bend. Denote the displacement of the shaft by l. Let us compute this magnitude. We know the formula for centrifugal force (see p. 75)—this force is proportional to the distance from the center of gravity to the axis, which is now $a + l$, and is equal to $4\pi^2 n^2 M\ (a + l)$, where n is the number of revolutions per minute, and M is the mass of the rotating parts. The centrifugal force is balanced by an elastic force, which is proportional to the magnitude of the shaft's displacement and will be equal to kl, where the coefficient k characterizes the shaft's rigidity. Thus:

$$kl = 4\pi^2 n^2 M\ (a + l)$$

whence

$$l = a\,\frac{1}{k/(4\pi^2 n^2 M) - 1}$$

Judging by this formula, fast rotations are no problem for a flexible shaft. For very large (even infinitely large) values of n, the deflection l of the shaft does not grow without bound. The quantity $k/(4\pi^2 n^2 M)$, figuring in our last formula, tends to zero, and the deflection l of the shaft becomes equal in magnitude to the asymmetry, but opposite in sign.

This computational result implies that, for fast rotations, the wheel's asymmetry, instead of smashing the shaft, bends it in such a way as to cancel the asymmetry's effect. The bending shaft centers the rotating parts, transfers the center of gravity to the axis of rotation by means of its deformation, and thus nullifies the centrifugal force's action.

The shaft's flexibility is by no means a drawback; on the contrary, it is a necessary condition for stability. As a matter of fact, it is necessary for stability that the shaft bend by a distance of a without breaking.

An attentive reader may have noticed an error in the reasoning employed. If we displace a shaft, "centering" during fast rotations, from the position of equilibrium we have found and consider only centrifugal and elastic forces, then it is easy to see that this equilibrium is unstable. It turns out, however, that Coriolis forces save the situation and make this equilibrium quite stable.

A turbine starts turning slowly. At first, when n is very small, the fraction $k/(4\pi^2n^2M)$ will have a large value. As long as it is greater than one and n is increasing, the shaft's deflection will have the same sign as that of the original displacement of the wheel's center of gravity. Therefore, during the beginning of the motion the bending shaft does not center the wheel, but, on the contrary, increases the total displacement of the center of gravity by means of its deformation, and hence also the centrifugal force. To the degree that n increases [but while the condition $k/(4\pi^2n^2M) > 1$ is preserved], the displacement grows and, finally, the critical moment is reached. The denominator of our formula for the displacement l vanishes when $k/(4\pi^2n^2M) = 1$, and so the deflection of the shaft formally becomes infinitely large. The shaft will break at such a speed of rotation. In starting a turbine, this moment must be passed very quickly; it is necessary to slip by the critical number of revolutions per minute and pass over to a much faster motion of the turbine, for which the phenomenon of self-centering, described above, will begin.

But what is this critical moment? We can rewrite its condition in the following form:

$$4\pi^2 \frac{M}{k} = \frac{1}{n^2}$$

Or, expressing the number of revolutions per minute in terms of the period of rotation by means of the relation $n = 1/T$ and extracting square roots, we can rewrite it as follows:

$$T = 2\pi \sqrt{\frac{M}{k}}$$

But what kind of quantity have we obtained in the right-hand side of the equality? Our formula looks rather familiar. Turning to page 136, we see that the period of free vibration of the wheel on the shaft figures in our right-hand side. The period $2\pi\sqrt{M/k}$ is that with which a turbine's wheel of mass M would vibrate on a shaft of rigidity k if we were to deflect the wheel to one side, so that it might vibrate by itself.

Thus, the dangerous instant is when the rotational period of the turbine's wheel coincides with the period of free vibration of the system turbine-shaft. The phenomenon of resonance is responsible for the existence of a critical number of revolutions per minute.

Seven
GRAVITATION

What Holds the Earth Up?

In the distant past, people gave a simple answer to this question: the three whales. True, it remained unclear what was holding the whales up. However, this did not disturb our naive forefathers.

Correct ideas about the nature of the Earth's motion, the Earth's form and many regularities in the motion of the planets around the Sun had arisen long before an answer was given to the question of the causes for the planets' movements.

And really, what does "hold up" the Earth and the planets? Why do they move around the Sun along definite paths, instead of flying away from it?

There was no answer to these questions for a long time, and the Church, struggling against the Copernican system of the Universe, used this to negate the fact of the Earth's motion.

We are obliged to the great English scientist Isaac Newton for his discovery of the true answers.

A well-known historical anecdote asserts that while sitting in an orchard under an apple-tree, thoughtfully observing how one apple after another fell to the ground because of gusts of wind, Newton arrived at the idea of the existence of gravitational forces between all bodies in the Universe.

As a result of Newton's discovery, it became clear that many apparently miscellaneous phenomena—the free fall of bodies to the Earth, the apparent motions of the Moon and the Sun, the ocean tides, etc.—are manifestations of one and the same law of nature: the *law of universal gravitation.*

Between all bodies of the Universe, asserts this law, be they grains of sand, peas, stones or planets, there are exerted forces of mutual attraction.

At first sight, this law seems false: we somehow haven t noticed that the objects surrounding us were attracted to each other. The Earth attracts all bodies to itself; no one will have any doubt about this. But perhaps this is a special property of the Earth? No, that isn't so. The attraction of two arbitrary objects is slight, and this is the only reason why it doesn't arrest our attention. Nevertheless, it can be detected by means of special experiments. But more about that later.

The presence of universal gravitation, and nothing else, explains the stability of the solar system and the motion of the planets and other celestial bodies.

The Moon is kept in orbit by terrestrial gravitational forces, the Earth on its trajectory, by solar gravitational forces.

The circular motion of celestial bodies occurs in the same way as the circular motion of a stone twirled on a string.

The forces of universal gravitation are invisible ropes compelling celestial bodies to move along definite paths.

The assertion of the existence of universal gravitational forces didn't really mean much. Newton discovered the law of gravitation and showed what these forces depend on.

Law of Universal Gravitation

The first question which Newton asked himself was the following: how does the Moon's acceleration differ from an apple's? To put it otherwise, what is the difference between the acceleration g which the Earth creates on its surface, i.e. at the distance r from its center, and the acceleration created by the Earth at the distance R at which the Moon is located from the Earth?

In order to calculate this acceleration, v^2/R, it is necessary to know the speed of the Moon's motion and its distance from the Earth. Both these figures were known by Newton. The Moon's acceleration turned out to be approximately equal to 0.27 cm/sec^2. This is about 3600 times less than the value of g, 980 cm/sec^2.

Hence, the acceleration created by the Earth decreases as one recedes from the center of the Earth. But how rapidly? The distance to the Moon equals sixty terrestrial radii. But 3600 is the square of 60. Increasing the distance by a factor of 60, we decrease the acceleration by a factor of 60^2.

Newton concluded that the acceleration, and therefore also the gravitational force, is inversely proportional to the square of the distance. Further, we already know that the force exerted on a body in a gravitational field is proportional to its mass. Therefore, the first body attracts the second with a force proportional to the second body's mass; the second body attracts the first with a force proportional to the first body's mass.

We are dealing with identically equal forces—forces of action and reaction. Consequently, the mutual gravitational force must be proportional to the mass of the first, as well as to the mass of the second body or, to put it otherwise, to the product of the masses.

Thus,

$$F = G\frac{Mm}{r^2},$$

This is precisely the law of universal gravitation. Newton assumed that this law will be valid for any pair of bodies.

This bold hypothesis is now completely proved. Therefore, the attractive force of two bodies is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

But what is this G that entered the formula? This is the coefficient of proportionality. May we assume it to be equal to one, as we have already repeatedly done? No, we may not: we have agreed to measure mass in grams, distance in centimeters, and force in dynes. The value of G is equal to the force of attraction between two masses of 1 g, located at a distance of 1 cm from each other. We may not assume that the force is equal to anything, in particular, to one dyne: the coefficient G must be measured.

In order to find G, we don't, of course, have to measure the forces of attraction between gram weights. We are interested in carrying out measurements on massive bodies—then the force will be greater.

If we determine the mass of two bodies, know the distance between them and measure the force of attraction, then G will be found by a simple calculation.

Such experiments were performed many times. They showed that the value of G is always one and the same, independent of the material of the attracting bodies and also of the properties of the medium in which they are situated. The quantity G is called the *gravitational constant*. It is equal to 6.67×10^{-8} cm^3/g$\cdot$sec^2.

The diagram of one of the experiments on measuring G is shown in Figure 64. Two balls of identical mass are hung

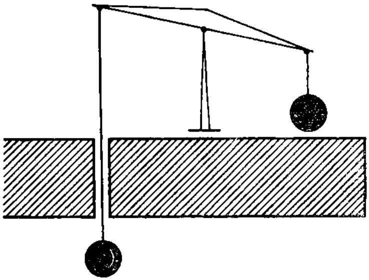


Fig. 64

on the ends of a scale's balance. One of them is situated above a lead plate, the other, beneath it. By means of its attraction, the lead (100 tons of it are taken for the experiment) increases the weight of the ball on the right and decreases that of the ball on the left. The former outweighs the latter. The value of G is computed on the basis of the magnitude of the deflection of the scale's balance.

The difficulty in detecting gravitational forces between two objects is explained by the negligible value of G.

Two heavy 1000-kilogram loads pull each other with an insignificant force, equal in all to only 6.7 dynes, i.e. 0.007 gf, if these objects are situated, say, at a distance of 1 m from each other.

But how great are the attractive forces between celestial bodies! Between the Moon and the Earth

$$F = 6.7 \times 10^{-8} \frac{6 \times 10^{27} \times 0.74 \times 10^{26}}{(38 \times 10^{9})^{2}} = 2 \times 10^{25} \text{ dynes} \approx$$
$$\approx 2 \times 10^{19} \text{ kgf}$$

between the Earth and the Sun

$$F = 6.7 \times 10^{-8} \frac{2 \times 10^{33} \times 6 \times 10^{27}}{(15 \times 10^{12})^{2}} = 3.6 \times 10^{27} \text{ dynes} \approx$$
$$\approx 3.6 \times 10^{21} \text{ kgf}$$

Weighing the Earth

Before beginning to make use of the law of universal gravitation, we must turn our attention to an important detail.

We have just calculated the force of attraction between two loads located at a distance of 1 m from each other. But if this distance were 1 cm? What would we then substitute in the formula—the distance between the bodies' surfaces or the distance between their centers of gravity or some other value?

The law of universal gravitation, $F = Gm_1m_2/r^2$ can be applied with complete rigour when such doubts do not arise. The distance between the bodies should be much greater than their dimensions; we should have the right to regard the bodies as points. But how should we apply the law to two nearby bodies? This is simple in principle: we must conceptually break up the bodies into small pieces, calculate the force F for each pair and then add (vectorially) all the forces.

In principle this is simple, but it is rather complicated in practice.

However, Nature has helped us. Computations show that if the particles of a body act on each other with a force proportional to $1/r^2$, then spherical bodies possess the property of attracting like points located at the centers of the spheres. For two nearby spheres, the formula $F = Gm_1m_2/r^2$ is exactly valid, just as for distant spheres, if r is the distance between their centers. We have already used this rule above in computing the acceleration on the Earth's surface.

We now have the right to apply the gravitational formula for computing the forces with which the Earth attracts bodies. We should take the distance from the center of the Earth to the body as r.

Let M be the mass and R, the radius of the Earth. Then the attractive force acting on a body of mass m at the surface of the Earth

$$F = Gm\frac{M}{R^2}$$

But this is in fact the body's weight, which we always express as mg. Hence, the acceleration due to gravity,

$$g = G\frac{M}{R^2}$$

Now at last we can say how the Earth was weighed. The quantities g, G and R are known, so the Earth's mass can be computed from this formula. The Sun can also be weighed in the same manner.

But can we really call such a computation a weighing? Of course we can; indirect measurements play at least as great a role in physics as direct measurements.

Let us now solve a curious problem.

An essential role in the plans for creating world-wide television is played by the creation of a "suspended" sputnik, i.e. one which will always be situated at one and the same point above the Earth's surface. Will such a sputnik experience a significant frictional force? This depends on how far from the Earth it will have to perform its rotation.

A suspended sputnik should revolve with a period T equal to 24 hours. If r is the distance from the sputnik to the center of the Earth, then its speed $v = 2\pi r/T$ and its acceleration $v^2/r = 4\pi^2 r/T^2$. On the other hand, this acceleration, whose source is the Earth's attraction, is equal to $GM/r^2 = gR^2/r^2$. Equating our two expressions for the acceleration, we obtain:

$$g\frac{R^2}{r^2} = \frac{4\pi^2 r}{T^2}, \quad \text{i.e. } r^3 = \frac{gR^2T^2}{4\pi^2}$$

Substituting the rounded-off values of $g = 10$ m/sec^2, $R = 6 \times 10^6$ m and $T = 9 \times 10^4$ sec, we obtain: $r^3 = 7 \times 10^{22}$ m^3, i.e. $r \approx 4 \times 10^7$ m $= 40\,000$ km. There is no air friction at such a height, and a suspended sputnik, if we succeed in creating it, will not slow down its "motionless running".

Measurements of g in the Service of Prospecting

The topic is geological prospecting, whose aim is to find deposits of useful minerals under the Earth without digging a pit or sinking a shaft.

There exist several methods of determining the acceleration due to gravity very accurately. It is possible to find g by simply weighing a standard weight on a spring balance. Geological balances should be extremely sensitive—their spring changes its expansion when a load of less than a millionth of a gram is added. Quartz torsion balances yield excellent results. Their construction isn't complicated in principle. To a horizontally stretched quartz thread, a lever is welded, whose weight slightly twists the thread (Figure 65).

A pendulum is used for the same purposes. Not very long ago, pendulum methods of measuring g were the only ones, and only in the last 10-20 years have the more convenient and precise balance methods begun to supplant them. In any case, measuring a pendulum's period of oscillation, one can find the value of g accurately enough from the formula $T = 2\pi \sqrt{l/g}$.

Measuring values of g at different places with the same apparatus, we can detect relative changes in the force of gravity up to one millionth.

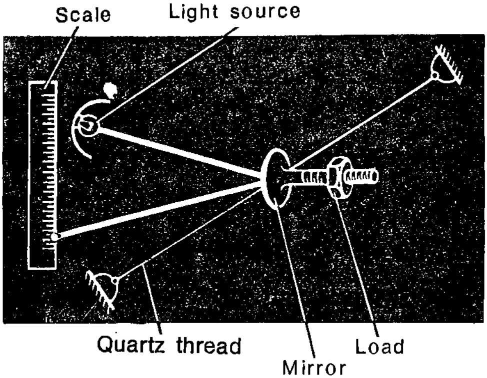


Fig. 65

Measuring the value of g at some place on the Earth's surface, the experimenter ascertains: here the value is anomalous; it is so much less than the norm or such an amount greater than the norm.

But what is the norm for the value of g?

There are two law-governed changes, which have long been observed and are well known to researchers, in the value of the acceleration due to gravity on the Earth's surface.

First of all, g changes as a natural result of passing from a pole to the equator. This has been spoken of above. Let us only recall that such a change occurs as a result of two causes: firstly, the Earth isn't a sphere, and a body at a pole will be nearer to the center of the Earth; secondly, the more a body advances towards the equator, the more will the force of gravity be weakened by the centrifugal force.

The second law-governed change in g is the decrease due to elevation. The greater the distance from the Earth's center, the smaller will be the value of g, in accordance with the formula $g = GM/(R + h)^2$, where R is the radius of the Earth, and h is the height above sea level.

Therefore, at one and the same latitude and at one and the same height above sea level, the acceleration due to gravity should be identical.

Accurate measurements show that deviations from this norm—gravitational anomalies—are found quite often. The cause of an anomaly consists in the heterogeneity of the mass distribution near the place of measurement.

As we explained, the gravitational force due to a large body can be conceptually represented as the sum of forces emanating from the individual particles of the large body. The attraction of a pendulum to the Earth is the result of the action of all the particles of the Earth on it. But it is clear that the nearby particles make the greatest contribution to the resultant force—for the attraction is inversely proportional to the square of the distance.

If heavy masses are concentrated near the place of measurement, g will be greater than the norm; in the opposite case, g will be smaller than the norm. If, for example, we measure g on a mountain and in an airplane flying over a sea at an altitude equal to the mountain's height, then a greater value will be obtained in the former case. For example, the value of g is 0.292 cm/sec^2 greater than the norm on Mount Etna in Italy. The value of g is also higher than the norm on isolated ocean islands. It is clear that in both cases the growth of g is explained by the concentration of additional masses at the place of measurement.

Not only the value of g, but also the direction of the gravitational force, can deviate from the norm. If a load is suspended on a thread, the stretched thread will indicate the vertical for the given place. This vertical may deviate from the norm. The "normal" direction of a vertical is known to geologists from special maps, on which the "ideal" contour of the Earth is constructed on the basis of data giving values of g.

Imagine that you are performing experiments with a plumb-line at the foot of a large mountain. The load of the plumb-line is attracted by the Earth towards its center, and by the mountain, to one side. Under such conditions, the plumb-line must be deflected from a normal vertical's direction (Figure 66). Since the Earth's mass is much greater than the mountain's, such a deflection will not exceed several seconds of arc.

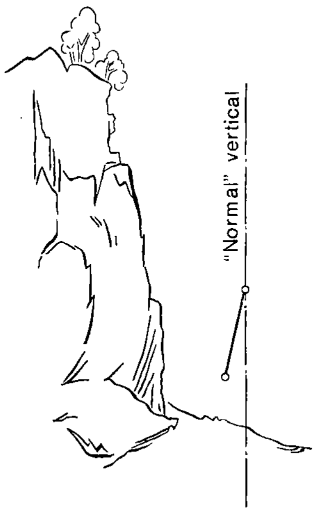


Fig. 66

A normal vertical can be determined by the stars, since it has been calculated for any geographical point at what place in the sky the vertical to the "ideal" contour of the Earth is "set" at a given instant of a day and year.

Plumb-line deflections sometimes yield strange results. For example, in Florence the influence of the Appenines leads not to an attraction, but to a repulsion of a plumb-line. The explanation can only be as follows: there are enormous empty spaces in the mountains.

Measurements of the acceleration due to gravity on the scale of continents and oceans yield remarkable results. Continents are considerably heavier than oceans; therefore, it would seem that values of g over continents should be greater than over oceans. But in reality, values of g measured along a single latitude are identical, on the average, over oceans and continents.

Again there is only one explanation: continents lie on lighter bed-rocks, and oceans, on heavier ones. And as a matter of fact, where direct prospecting is possible, geologists ascertain that oceans lie on heavy basaltic bedrocks, and continents, on light granite ones.

But the following question immediately arises: why do heavy and light bed-rocks compensate so exactly for the difference in weights between continents and oceans? Such a compensation cannot be a matter of chance; its cause must be rooted in the construction of the Earth's shell.

Geologists assume that it is as though the upper part of the Earth's shell were floating on an underlying plastic (i.e. easily deformed, like wet clay) mass. The pressure at depths of about 100 km should be identical everywhere, just as the pressure on the bottom of a vessel filled with water, in which pieces of wood of various weights are floating, is identical everywhere. Consequently, a column of matter with an area of a square meter, from the surface to a depth of 100 km, should have the same weight under an ocean and under a continent.

This equalizing of pressures (it is called isostasy) is just what leads to the situation where along a single latitude the values of the acceleration due to gravity g do not differ significantly over oceans and continents.

Local gravitational anomalies serve us just as the magic wand, which banged on the ground where there was gold or silver, served little Mook in Hauf's fairy-tale.

One must look for heavy ore in those places where g is greatest. On the contrary, light salt deposits are discovered by finding localities with lowered values of g. It is possible to measure g with an accuracy up to a hundred-thousandth of 1 cm/sec^2.

Methods of prospecting with the aid of pendulums and superexact scales are called gravitational. They are of great practical value, in particular when looking for oil. The fact is that with gravitational methods of prospecting, it is easy to discover underground salt domes, but it very often turns out that where there is salt, there is oil. Moreover, the oil lies at some depth, while the salt is nearer to the Earth's surface. Oil was discovered in Kazakhstan and in other places by the method of gravitational prospecting.

Weight Underground

It remains for us to throw light on another interesting question. How will the force of gravity change if we go deep underground?

An object's weight is a result of the tension in the invisible threads reaching out to this object from every piece of matter in the Earth. Weight is the resultant force, the result of the addition of the elementary forces exerted on the object by the Earth's particles. All these forces, even though directed at different angles, pull a body "down"—towards the center of the Earth.

But what will be the weight of an object situated in an underground laboratory? Forces of attraction will be exerted on it by the internal and external layers of the Earth.

Consider the gravitational forces exerted at a point, lying inside the Earth, by an external layer. If we break up this layer into thin shells, cut out in one of them a small square with side a_1 and draw lines from the square's perimeter through the point O, the weight at which we are interested in, then in another part of the shell we obtain a square of a different size with side a_2 (Figure 67). The attractive forces exerted at O by the two squares are oppositely directed and proportional, according to the law of gravitation, to m_1/r_1^2 and m_2/r_2^2. But the masses of the squares, m_1 and m_2, are proportional to their areas. Therefore, the gravitational

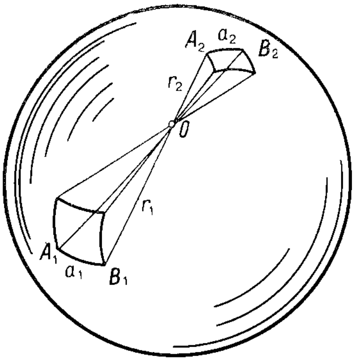


Fig. 67

forces are proportional to the expressions a_1^2/r_1^2 and a_2^2/r_2^2.

However, these ratios are equal. One can see from Figure 67 that a_1/r_1 and a_2/r_2 are the ratios of the corresponding sides of triangles OA_1B_1 and OA_2B_2, which will be similar if the squares' sides, A_1B_1 and A_2B_2, are taken to be very small. But we can always do this.

In fact, if the squares are small, the straight-line segments A_1B_1 and A_2B_2 do not differ much from the corresponding arcs. But $\angle B_1A_1O = \angle OA_2B_2$, since these angles are measured by one and the same arc. Furthermore, the vertical angles $\angle A_1OB_1$ and $\angle B_2OA_2$ are also equal. Consequently, the triangles are similar.

It follows from this geometrical proof that $a_1/r_1 = a_2/r_2$, and so the forces of attraction exerted at the point O by the two triangles balance each other.

Having broken up a thin shell into pairs of "opposite" similar triangles, we established a remarkable fact: a thin homogeneous spherical shell does not act on a point situated within it. But this is true for all the thin shells into which we broke up the spherical layer lying above the underground point we are interested in.

Hence, the layer of the Earth which lies over the body might just as well be absent. The action of its individual parts on the body is neutralized, and the resultant force of attraction exerted by the external layer is equal to zero.

Of course, throughout this reasoning we have assumed the Earth's density to be constant within each shell.

The result of our reasoning permits us to easily obtain a formula for the gravitational force exerted at any depth H under the Earth. A point situated at a depth H only experiences an attraction exerted by the internal layers of the Earth. The formula for the acceleration due to gravity, $g = GM/R^2$, also applies to this case, but M and R are the mass and radius not of the entire Earth, but of its "internal" parts with respect to this point.

If the Earth had the same density in all its shells, the formula for g would assume the following form:

$$g = G\frac{\rho\frac{4}{3}\pi(R_E - H)^3}{(R_E - H)^2} = \frac{4}{3}\pi G\rho(R_E - H)$$

where ρ is the density, and R_E is the radius of the Earth.

This implies that g would be directly proportional to $(R_E - H)$: the greater the depth H, the smaller would be g.

But as a matter of fact, the behaviour of g near the Earth's surface—we are able to observe it up to a depth of 5 km (below sea level)—does not obey this law at all. Experiments show that g, on the contrary, increases with depth within these layers. The lack of agreement between the experiments and our formula is explained by the fact that the difference in density at various depths was not taken into account.

The average density of the Earth is easily found by dividing its mass by its volume. This yields a value of 5.52. At the same time, the density of the surface bed-rocks is much smaller—it is equal to 2.75. The density of the Earth's shells increases with depth. Within the surface layers of the Earth, this effect dominates the ideal decrease which follows from the formula just derived, and so the value of g increases.

Gravitational Energy

We have already become acquainted with gravitational energy through a simple example. A body raised to a height h above the Earth possesses a potential energy of mgh. However, this formula may be used only when the height h is much smaller than the Earth's radius.

Gravitational energy is an important quantity, and it would be interesting to obtain a formula for it which would apply to a body raised to an arbitrary height above the Earth and also, more generally, two masses attracting each other in accordance with the universal law:

$$F = G\frac{m_1 m_2}{r^2}$$

Let us assume that the bodies approached each other somewhat under the action of their mutual attraction. The distance between them was r_1, but it became r_2. Moreover, the work $A = F(r_1 - r_2)$ is performed. The value of the force must be taken at some intermediate point. Thus,

$$A = G\frac{m_1 m_2}{r_{\text{int}}^2}(r_1 - r_2)$$

If r_1 and r_2 do not differ much from each other, we may replace r_{int}^2 by the product $r_1 r_2$. We obtain:

$$A = G\frac{m_1 m_2}{r_2} - G\frac{m_1 m_2}{r_1}$$

This work is performed at the expense of the gravitational energy:

$$A = U_1 - U_2$$

where U_1 is the initial and U_2 the final value of the gravitational potential energy.

Comparing these two formulas, we find the following expression for the potential energy:

$$U = -G\frac{m_1 m_2}{r}$$

It resembles the formula for the gravitational force, but r is raised to the first power in the denominator.

According to this formula, the potential energy $U = 0$ for very large r's. This is reasonable, since the attraction will no longer be felt at such distances. But when the bodies approach each other, the potential energy should decrease. After all, the work takes place at its expense.

But in what direction can it decrease from zero? In the negative direction. Hence there is a minus sign in the formula. After all, —5 is less than zero, while —10 is less than —5.

If we are dealing with motion near the Earth's surface, we may replace the general expression for the gravitational force by mg. Then with greater accuracy, we have $U_1 - U_2 = mgh$.

But on the surface of the Earth, a body has a potential energy $-GMm/R$, where R is the Earth's radius. Therefore, at a height h above the Earth's surface,

$$U = -G\frac{Mm}{R} + mgh$$

When we first introduced the formula for potential energy, $U = mgh$, we agreed to measure height and energy from the surface of the Earth. Using the formula $U = mgh$, we discard the constant term $-GMm/R$, regarding it as conditionally equal to zero. Since we are interested only in differences of energy—for it is work, which is an energy difference, that is ordinarily measured—the presence of the constant term $-GMm/R$ in the potential energy formula does not play any role.

Gravitational energy determines the strength of the chains "binding" a body to the Earth. How can we break these chains? How can we ensure that a body thrown from the Earth will not return to the Earth? It is clear that to do this we must impart a large initial velocity to the body. But what is the minimal velocity that is required?

As a body (missile, rocket) thrown from the Earth increases its distance from the Earth, its potential energy will rise (the absolute value of U will fall); its kinetic energy will fall. If its kinetic energy becomes equal to zero prematurely, before we break the Earth's gravitational chains, the missile that was thrown will fall back to the Earth.

It is necessary that the body conserves its kinetic energy as long as its potential energy has not actually vanished. Before its departure, a missile possesses a potential energy of $-GMm/R$ (M and R are the mass and radius of the Earth). Therefore, a missile must be given a velocity which would make its total energy positive. A body with a negative total energy (a potential energy of greater absolute value than kinetic energy) will not get beyond the bounds of the sphere of gravity.

Hence, we arrive at a simple condition. In order for a body of mass m to break away from the Earth, it must, as has been already said, overcome a gravitational potential energy of

$$G\frac{Mm}{R}$$

For this, the missile's speed should be increased to the value of the escape velocity from the Earth, v_2, which is easily computed by equating kinetic and potential energies:

$$\frac{mv_2^2}{2}=G\frac{mM}{R},\quad \text{i.e.}\quad v_2^2=2G\frac{M}{R}$$

or, since $g = GM/R^2$,

$$v_2^2 = 2gR$$

The value of v_2 computed by means of this formula is 11 km/sec—of course, without taking air resistance into account. This speed is $\sqrt{2} = 1.41$ times as great as the orbital velocity $v_1 = \sqrt{gR}$ of an artificial satellite revolving near the surface of the Earth, i.e. $v_2 = \sqrt{2}v_1$.

The Moon's mass is 81 times as small as the Earth's; its radius is four times as small as the Earth's. Consequently, gravitational energy on the Moon is 20 times less than on the Earth, and a speed of 2.5 km/sec is sufficient to break away from the Moon.

A kinetic energy of $mv_2^2/2$ is spent in order to break the gravitational chains to the planet—the take-off station. If we want the rocket, having overcome gravity, to move with speed v, then an additional energy of $mv^2/2$ is needed for this. In such a case, when launching the rocket, it is necessary to give it an energy of $mv_0^2/2 = mv_2^2/2 + mv^2/2$. Therefore, the three speeds in question are connected by a simple relation:

$$v_0^2 = v_2^2 + v^2$$

What should be the speed v_3, necessary for overcoming the gravitation of the Earth and the Sun—the minimal speed of a missile sent to distant stars? We denoted this speed by v_3 because it is called the escape velocity from the solar system.

First of all, let us determine the speed necessary for overcoming only the single attraction of the Sun.

As we have just shown, the speed needed for a departure from the sphere of the Earth's attraction by a missile sent on a flight is $\sqrt{2}$ times as great as the speed with which an Earth satellite is sent into orbit. Our reasoning is equally valid for the Sun, i.e. the speed needed to escape from the

Sun is $\sqrt{2}$ times as great as the speed of a satellite of the Sun (i.e. the Earth). Since the speed of the Earth's motion around the Sun is about 30 km/sec, the speed necessary for an escape from the sphere of the Sun's attraction is 42 km/sec. This is a very great speed, but for sending a missile to distant stars, we must, of course, use the Earth's motion and launch the body in the direction in which the Earth is moving. We then need to add only $42 - 30 = 12$ km/sec.

Now we can finally compute the escape velocity from the solar system. This is the speed with which a rocket must be launched in order that, leaving the sphere of the Earth's attraction, it have a speed of 12 km/sec. Using the formula just adduced, we obtain:

$$v_3^2 = 11^2 + 12^2$$

from which $v_3 = 16$ km/sec.

Thus, having a speed of about 11 km/sec, a body will leave the Earth, but such a missile will not go "far" away; the Earth let it go, but the Sun will not free it. It will turn into a satellite of the Sun.

It turns out that the speed necessary for interstellar travel is only one and a half times as great as the speed needed for travelling through the solar system within the Earth's orbit. True, as has been already said, every appreciable increase in the initial speed of a missile is accompanied by many technical difficulties (see p. 101).

How Planets Move

The question as to how planets move can be answered briefly: obeying the law of gravitation. For the forces of gravitation are the only forces applied to planets.

Since the mass of the planets is much less than the mass of the Sun, the forces of interaction between the planets do not play a large role. Each of the planets moves almost the way the gravitational force of the Sun alone dictates, as though the other planets did not even exist.

The laws of planetary motion around the Sun follow from the law of universal gravitation.

Incidentally, this isn't the way things developed historically. The laws of planetary motion were discovered by the outstanding German astronomer Johannes Kepler, before Newton and without the aid of the law of gravitation, on the basis of an almost twenty-year processing of astronomical observations.

The paths, or, as astronomers say, the orbits, which planets describe around the Sun, are very close to circles.

How is a planet's period of revolution related to the radius of its orbit?

The gravitational force exerted on a planet by the Sun is equal to

$$F = G\frac{mM}{r^2}$$

where M is the mass of the Sun, m is the mass of the planet, and r is the distance between them.

But F/m is, according to the basic law of mechanics, none other than the acceleration; moreover, it is centripetal:

$$\frac{F}{m} = \frac{v^2}{r}$$

The planet's speed can be represented as the length $2\pi r$ of the circumference divided by the period of revolution T. Substituting $v = 2\pi r/T$ and the value of the force F in the acceleration formula, we obtain:

$$\frac{4\pi^2 r}{T^2} = \frac{GM}{r^2}, \quad \text{i.e. } T^2 = \frac{4\pi^2}{GM} r^3$$

The coefficient of proportionality preceding r^3 is a quantity depending only on the mass of the Sun; it is identical for all planets. Consequently, the following relation holds for two planets:

$$\frac{T_1^2}{T_2^2}=\frac{r_1^3}{r_2^3}$$

The ratio of the squares of the periods of revolution of the planets turns out to be equal to the ratio of the cubes of their orbital radii. This interesting law was derived empirically by Kepler. The law of universal gravitation explained Kepler's observations.

A circular motion of one celestial body around another is only one of the possibilities.

The trajectories of one body, revolving around another due to gravitational forces, can be very different. However, as shown by calculations and as Kepler had observed before any calculations were made, they all belong to one and the same class of curves, called ellipses.

If we tie a thread to two pins stuck in a sheet of drawing paper, stretch the thread with the point of a pencil and move the pencil in such a way that the thread remains stretched, then a closed curve will eventually be drawn on the paper—this is an ellipse (Figure 68). The points where the pins are stuck will be the foci of the ellipse.

Ellipses can have various forms. If the thread is taken much longer than the distance between the pins, then the ellipse will be very similar to a circle. If, on the contrary, the length of the thread barely exceeds the distance between the pins, then an elongated ellipse—almost a stick—will be obtained.

A planet describes an ellipse, at one of whose foci is the Sun.

But what kind of ellipses do the planets describe? It turns out that they are very close to circles.

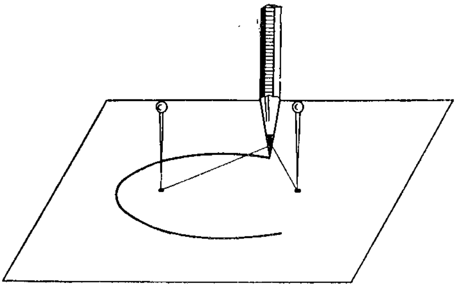


Fig. 68

The path of the planet nearest to the Sun—Mercury—differs most from a circle. But even in this case, the longest diameter of the ellipse is only 2% greater than the shortest one. The situation is different with the orbits of artificial planets. Take a look at Figure 69. You can't distinguish the orbit of Mars from a circle.

However, since the Sun is located at one of the foci of the ellipse, and not at its center, the planet's distance from the Sun changes more noticeably. Let us draw a line through the two foci of an ellipse. This line intersects the ellipse at two places. The point nearest to the Sun is called the perihelion, the farthest from the Sun, the aphelion. Mercury, when located at the perihelion, is 1.5 times closer to the Sun than at the aphelion.

The major planets describe ellipses around the Sun which are close to circles. However, there are celestial bodies which move around the Sun in highly drawn-out ellipses. Among them are comets. Their orbits are not at all comparable, with respect to elongation, to those of the planets. With regard to the celestial bodies moving in ellipses, it can be said that they belong to the solar family. However, casual newcomers also drop in at our system.

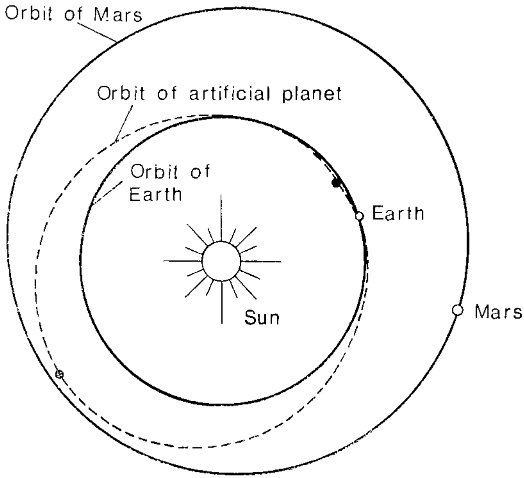


Fig. 69

There have been observed comets describing curves around the Sun, whose forms suggest the following conclusion: the comet will not return; it does not belong to the family of the solar system. The "open" curves described by comets are called hyperbolas.

Such comets move especially fast when passing near the Sun. This is understandable—a comet's total energy is constant, but passing by the Sun, it has the least potential energy. Hence, its kinetic energy will be greatest at this time. Of course, such an effect takes place for all the planets and for our Earth. However, this effect is slight, since the difference in potential energy at the aphelion and perihelion is small.

An interesting law of planetary motion follows from the law of conservation of angular momentum.

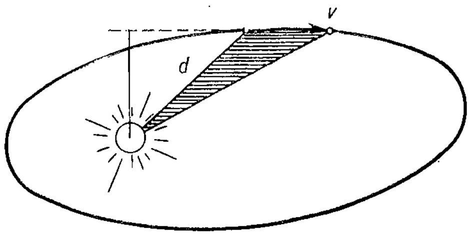


Fig. 70

Two positions of a planet are depicted in Figure 70. From the Sun, i.e. from a focus of the ellipse, the two radii have been drawn to the two positions of the planet, and the sector so formed has been shaded. We are to determine the area swept out by a radius in a unit of time. For a small angle, the sector swept out by a radius during a second may be replaced by a triangle. The base of the triangle is the speed v (the distance covered during a second), while the altitude of the triangle is equal to the lever arm d of the velocity. Therefore, the triangle's area is $vd/2$.

The constancy of the quantity mvd during the motion follows from the law of conservation of angular momentum. But if mvd is constant, so is the triangle's area $vd/2$. We can draw sectors for any one-second intervals—they will turn out identical in area. A planet's speed changes, but the so-called areal velocity remains fixed.

Not all stars are surrounded by planetary systems. There are quite a few double stars in the sky. Two enormous celestial bodies revolve around each other.

The Sun's enormous mass makes it the center of the family. In double stars, both celestial bodies have masses of the same order of magnitude. In this case, we may not assume that one of the two stars is at rest. But how does the motion proceed in this case? We know that each closed system has

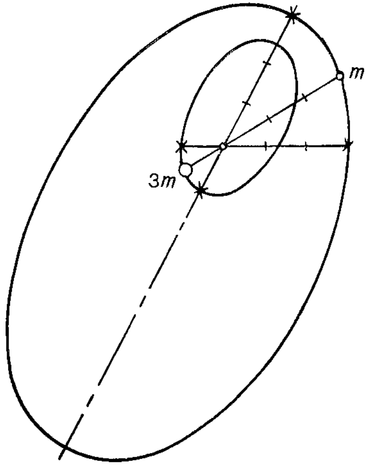


Fig 71

one point which is at rest (or moving uniformly)—its center of mass. Both stars revolve around this very point. Moreover, they describe similar ellipses, which follows from the condition written on p. 166, $m_1/m_2 = r_2/r_1$. The ellipse of one star is as many times greater than the ellipse of the other as the mass of one star is greater than the mass of the other (Figure 71). In the case of equal masses, both stars will describe identical trajectories around the center of mass.

The planets of the solar system are in ideal conditions: they are not subject to friction. The small, artificial celestial bodies created by people—sputniks—are not in such an ideal situation: frictional forces, however insignificant they may be at first, but none the less perceptible, interfere decisively in their motion.

The total energy of a planet remains constant. The total energy of a sputnik falls slightly with every revolution. At first sight, it would seem that friction will slow down the motion of a sputnik. In reality, the opposite occurs.

First of all, recall that a sputnik's speed is equal to $\sqrt{gR}$ or $\sqrt{GM/R}$, where R is its distance from the center of the Earth and M, its mass.

The total energy of a sputnik is equal to

$$E = -G\frac{Mm}{R} + \frac{mv^2}{2}$$

Substituting the value of the sputnik's speed, we find the expression $GmM/2R$ for the kinetic energy. We find that the kinetic energy is half as great in absolute value as the potential one, while the total energy is equal to

$$E = -\frac{G}{2}\frac{mM}{R}$$

In the presence of friction, the total energy will fall, i.e. (since it is negative) grow in absolute value; the distance R will start decreasing: the sputnik descends. What will happen to the energy summands in this connection? The potential energy decreases (grows in absolute value), the kinetic energy grows.

Nevertheless, the net change is negative, since the potential energy decreases twice as fast as the kinetic energy increases.

Friction leads to a growth in speed of a sputnik's motion, and not to a deceleration.

It is now clear why a large launch vehicle outflies a small sputnik. The friction acting on a large rocket is greater.

If There Were No Moon

We shall not discuss the sad consequences of the absence of the Moon for poets and lovers. The title of this section should be understood much more prosaically: how the Moon's presence affects terrestrial mechanics.

In our previous discussion of what forces act on a book lying on a table, we confidently stated: the Earth's gravity and the reaction force. But, strictly speaking, a book lying

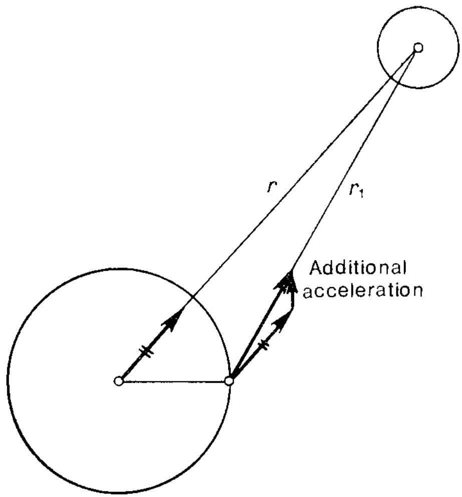


Fig. 72

on a table is also attracted by the Moon, the Sun and even the stars.

The Moon is our nearest neighbour. Let us forget about the Sun and the stars and consider how much the weight of a body on the Earth will change under the influence of the Moon.

The Earth and the Moon are in relative motion. With respect to the Moon, the Earth as a whole (i.e. all points of the Earth) is moving with an acceleration of Gm/r^2, where m is the mass of the Moon, and r is the distance from the center of the Moon to the center of the Earth.

Consider a body lying on the surface of the Earth. We are interested in how much its weight will change owing to the Moon's action. Terrestrial weight is determined by acceleration with respect to the Earth. In other words, we are therefore interested in how much the acceleration with respect to the Earth of a body lying on the Earth's surface will be changed by the Moon's action.

The acceleration of the Earth with respect to the Moon is Gm/r^2; the acceleration with respect to the Moon of a body lying on the surface of the Earth is Gm/r_1^2, where r_1 is the distance from the body to the Moon (Figure 72).

But we need the additional acceleration of the body with respect to the Earth: it will be equal to the geometrical difference between the appropriate accelerations.

The value of Gm/r^2 is a constant number for the Earth, while the value of Gm/r_1^2 is different at various points of the Earth's surface. Hence, the geometrical difference of interest to us will differ at various places on the Earth.

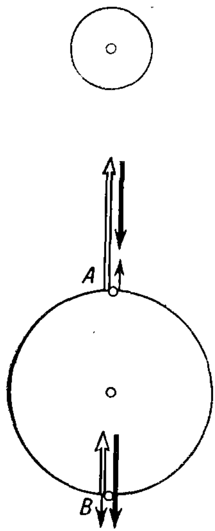


Fig. 73

What will the terrestrial weight be at the place nearest to the Moon, farthest from it and half-way along the Earth's surface?

For finding the acceleration with respect to the center of the Earth, induced by the Moon on a body, i.e. the correction to the terrestrial g, it is necessary to subtract the constant Gm/r^2 from the value of Gm/r_1^2 at the indicated places on the Earth (light arrows in Figure 73). Moreover, it should be remembered that the acceleration Gm/r^2—of the Earth towards the Moon—is directed parallel to the line joining their centers. The subtraction of a vector is equivalent to the addition of the inverse vector. The vectors $-Gm/r^2$ are shown by means of bold-face arrows in the figure.

Adding the vectors depicted in the figure, we find what we are interested in: the change in the acceleration of free fall on the surface of the Earth, arising as a result of the influence of the Moon.

At the place nearest to the Moon, the resulting additional acceleration will be equal to

$$G\frac{m}{(r-R)^2}-G\frac{m}{r^2}$$

and directed towards the Moon. Earth's gravity diminishes; a body at point A becomes lighter than in the absence of the Moon.

Bearing in mind that R is much less than r, we are able to simplify the formula written above. Reducing to a common denominator, we obtain:

$$\frac{GmR\,(2r-R)}{r^2\,(r-R)^2}$$

Discarding from the parentheses the relatively small magnitude R, subtracted from the much larger magnitudes r or $2r$, we obtain

$$\frac{2GmR}{r^3}$$

Let us now transfer to the antipode. At point B the acceleration of a body due to the Moon isn't greater, but less than the acceleration of the Earth. But we are now at the far side of the Earth from the Moon. A decrease in the Moon's attraction at this side of the Earth leads to the same result as an increase in attraction at point A—to a decrease in the acceleration due to gravity. An unexpected result, isn't it? Here too a body becomes lighter owing to the action of the Moon. The difference

$$G\frac{m}{(r+R)^2}-G\frac{m}{r^2}\approx-\frac{2GmR}{r^3}$$

turns out to be the same in absolute value as at point A.

Things are different at the median line. Here the accelerations are directed at an angle to each other, and so the subtraction of the acceleration Gm/r^2 of the Earth as a whole by the Moon and the acceleration Gm/r_1^2 of a body lying on the Earth by the Moon must be carried out geometrically (Figure 74). We shall depart insignificantly from the median line if we place the body on the Earth in a way such that r_1 and r are equal in magnitude. The vector difference between the accelerations is the base of an isosceles triangle. From the similarity of the triangles depicted in Figure 74, it is obvious that the required acceleration is as many times less than Gm/r^2 as R is less than r. Consequently, the required addition to g at the median line on the Earth's surface equals

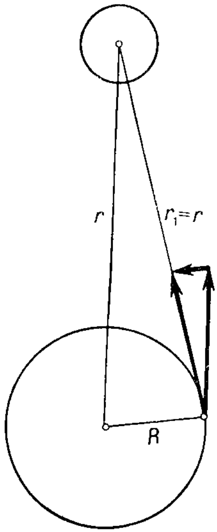


Fig. 74

$$\frac{GmR}{r^3}$$

in magnitude this is one-half of the weakening of the Earth's attractive force at the extreme points. As for the direction of this additional acceleration, it again practically coincides, as can be seen from the figure, with the vertical at the given point of the Earth's surface. It is directed downwards, i.e. leads to an increase in weight.

Thus, the influence of the Moon on terrestrial mechanics consists in a change in weight of bodies located on the surface of the Earth. Moreover, weight diminishes at the nearest and farthest points from the Moon, but grows on the median line, this change in weight in the latter case being half as great as in the former.

Of course, the reasoning carried out is valid for any planet, for the Sun or for a star.

It is not difficult to calculate that neither a planet nor a star give even an insignificant fraction of the lunar acceleration.

It is very easy to compare the action of any celestial body with that of the Moon: we must divide the additional acceleration due to this body by the "lunar addition":

$$\frac{GmR}{r^3} : \frac{Gm_{\text{Moon}}R}{r^3_{\text{Moon}}}$$

We obtain

$$\frac{m}{m_{\text{Moon}}} \frac{r^3_{\text{Moon}}}{r^3}$$

This product will only fail to be much less than unity for the Sun. The Sun is much farther from us than the Moon, but the mass of the Moon is tens of millions of times less than the mass of the Sun.

Substituting numerical values, we find that under the influence of the Moon, terrestrial weight is changed 2.17 times as much as under the influence of the Sun.

Let us now estimate by how much the weight of terrestrial bodies would be changed if the Moon were to leave its orbit around the Earth. Substituting numerical values in the expression $2GmR/r^3$, we find that lunar acceleration is of the order of magnitude of 0.0001 cm/sec^2, i.e. of one ten-millionth of g.

Almost nothing, it would seem. Was it worthwhile to follow with strained attention the solution to a rather complicated mechanical problem for the sake of such an insignificant effect? Don't hurry with such a conclusion. This "insignificant" effect is the cause of powerful tidal waves. It creates 10^{16} kgf-m of kinetic energy daily, moving enormous masses of water. This energy equals that borne by all the Earth's rivers.

In fact, the percentagewise change in the quantity we computed is very small. A body which becomes lighter by such an "insignificant" amount will move a bit farther away from the center of the Earth. But the radius of the Earth is 6 000 000 m, and an insignificant deviation will be measured in tens of centimeters.

Imagine that the Moon stopped its motion relative to the Earth and is shining somewhere over an ocean. Calculations show that the level of the water at this place would rise by 54 cm. Such a jump in the water level would also occur at the antipode. On the median line between these extreme points, the level of the ocean water would drop by 27 cm.

Thanks to the Earth's rotation about its axis, the "places" of rises and falls in the ocean are moving all the time. These are tides. During about six hours, a rise in the water level takes place and the water moves up the shore—this is high tide. Then low tide sets in; it also lasts six hours. Two high tides and two low tides occur in every lunar day. The picture of tidal phenomena is greatly complicated by the friction of water particles, the form of the sea bottom and the contour of the shores.

For example, tides are impossible in the Caspian Sea simply because the entire surface of the sea is subject to the same conditions.

Tides are also absent from internal seas, connected to an ocean by long and narrow straits—for example, the Black and Baltic seas.

Especially big tides occur in narrow bays, where a tidal wave coming in from the ocean rises steeply. For example, in the Gizhiginskaya Inlet on the Sea of Okhotsk, the height of waves attains several meters.

If the ocean shore is sufficiently flat (for example, in France), the water's rise during high tide can change the location of the boundary between land and sea by many kilometers.

Tidal phenomena hinder the Earth's rotation. For the motion of tidal waves is related to friction. Work must be expended to overcome this friction—it is called tidal. Therefore, the rotational energy, and with it the Earth's rotational speed about its axis, falls.

This phenomenon leads to the lengthening of the day, which was discussed on p. 14.

Tidal friction enables us to understand why one and the same side of the Moon always faces the Earth.

At one time, the Moon was probably in a liquid state. The rotation of this liquid sphere with respect to the Earth was accompanied by strong tidal friction, which gradually slowed down the motion of the Moon. Finally, the Moon stopped rotating relative to the Earth, the tides ceased and the Moon hid half of its surface from our sight.

Eight

PRESSURE

Hydraulic Press

The hydraulic press is an ancient machine, but it has retained its significance to the present day.

Take a look at Figure 75, depicting a hydraulic press. Two pistons—small and large—can move in a vessel with water. If we press one piston with our hand, the pressure is transmitted to the other piston—it rises. Just as much water will rise above the initial position of the second piston as the first piston presses down into the vessel.

If the areas of the pistons are S_1 and S_2, and their displacements are l_1 and l_2, then the equality of the volumes yields: $S_1 l_1 = S_2 l_2$, or

$$\frac{l_1}{l_2} = \frac{S_2}{S_1}$$

We must discover an equilibrium condition for the pistons.

We shall find such a condition without difficulty, starting out from the fact that the work performed by the balancing forces should be equal to zero. If so, then during the displacement of the pistons, the work done by the forces exerted on them should be equal (with opposite signs). Therefore,

$$F_1 l_1 = F_2 l_2, \quad \text{or} \quad \frac{F_2}{F_1} = \frac{l_1}{l_2}$$

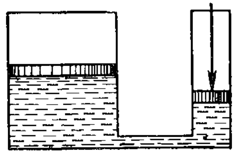


Fig. 75

Comparing this with the preceding equality, we see that

$$\frac{F_2}{F_1}=\frac{S_2}{S_1}$$

This modest equation implies the possibility of an enormous multiplication of force. The piston transmitting pressure can have an area which is hundreds or thousands of times smaller. The force acting on the large piston will be just as many times greater than the muscular force.

With the aid of a hydraulic press, one can forge and punch metals, press the juice out of grapes and raise weights.

Of course, the gain in force will be accompanied by a loss in path. In order to compress a body by 1 cm with a press, one's hand would have to cover a path as many times greater as the forces F_2 and F_1 differ.

Physicists call the ratio of the force to the area, F/S, the *pressure*. Instead of saying, "One kilogram-force acts on an area of one square centimeter," we shall say more concisely, "The pressure, denoted by p, is 1 kgf/cm^2."

Instead of the relation $F_2/F_1 = S_2/S_1$, one can now write:

$$\frac{F_2}{S_2}=\frac{F_1}{S_1}, \qquad \text{i.e.} \quad p_1=p_2$$

Thus, the pressure on both pistons is the same.

Our reasoning does not depend on where the pistons are located or whether their surfaces are horizontal, vertical or inclined. And in general, it is not a matter of pistons. One may conceptually choose any two portions of a surface enclosing a liquid, and assert that the pressures on them are identical.

It turns out, therefore, that the pressure within a liquid is the same at all its points and in all directions. In other words, an identical force is exerted on area elements of a definite size, wherever and however they be situated. This fact is called *Pascal's law*.

Hydrostatic Pressure

Pascal's law is valid for liquids and gases. However, it fails to take into account an important circumstance—the existence of weight.

Under the conditions prevailing on the Earth, this should not be forgotten. Even water has weight. It is therefore obvious that two area elements situated at different depths under water will experience different pressures. But what will this difference be equal to? Let us conceptually single out within a liquid a right cylinder with horizontal bases. The water inside it presses on the surrounding water. The resultant force of this pressure is equal to the weight mg of the liquid in the cylinder (Figure 76). This resultant force is made up of forces acting on the bases of the cylinder and on its lateral surface. But the forces acting on opposite sides of the lateral surface are equal in magnitude and opposite in direction. Therefore, the sum of all the forces acting on the lateral surface is equal to zero. Hence, the weight mg will be equal to the difference in force, $F_2 - F_1$. If the height of the cylinder equals h, the area of its base equals S, and the density of the liquid equals ρ, then we may write ρghS instead of mg. The difference in force is equal to this quantity. In order to obtain the difference in pressure, we must divide the weight by the area S. The difference in pressure turns out equal to ρgh.

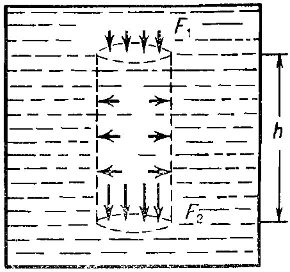


Fig. 76

In accordance with Pascal's law, the pressure on differently oriented area elements located at the same depth will be identical. Hence, at two points of a liquid, situated one above the other at a height h, the difference in pressure will equal the weight of a column of the liquid whose cross-sectional area is equal to one and whose height is h:

$$p_2 - p_1 = \rho g h$$

A pressure exerted by water, caused by its weight, is called *hydrostatic*.

Under terrestrial conditions, air most often presses down on the free surface of a liquid. The pressure exerted by air is called *atmospheric*. The pressure at a depth is composed of atmospheric and hydrostatic pressures.

In order to compute the force due to water pressure, it is only necessary to know the size of the area element on which it is exerted and the height of the column of liquid above it. By virtue of Pascal's law, nothing else plays any role.

This may seem surprising. Is it possible for the forces acting on the identical bottoms of the two vessels depicted in Figure 77 to be the same? Indeed, there is much more water in the vessel on the left. In spite of this, the forces acting on the bottoms are equal to $\rho g h S$ in both cases. This is

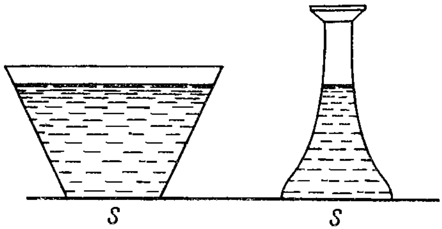


Fig. 77

greater than the weight of the water in the vessel on the right and less than the weight of the water in the vessel on the left. The sloping walls of the vessel on the left support the weight of the "extra" water, but on the right, on the contrary, they add reaction forces to the weight of the water. This interesting phenomenon is sometimes called the hydrostatic paradox.

If two vessels of different form, but with water at the same level, are connected by means of a tube, then water will not flow from one vessel to another. Such a flow could take place in case the pressures in the vessels were different. But this is not the case, and so the liquid in the communicating vessels will always stand at one and the same level.

On the contrary, if the water levels in communicating vessels are different, then water will begin moving and the levels will equalize.

Water pressure is much greater than air pressure. At a depth of 10 m, water presses on one square centimeter with a force of 1 kgf, in addition to the atmospheric pressure. At a depth of a kilometer, with a force of 100 kgf on one square centimeter.

Oceans have depths greater than 10 km at certain places. The forces due to water pressure at such depths are exceptionally great. Pieces of wood which are lowered to a depth of 5 km are so compressed by this enormous pressure that after such a "baptism", they sink like bricks in a barrel of water.

This enormous pressure puts big obstacles in the path of investigators of marine life. Deep-sea descents are carried out in steel globes—the so-called bathyspheres or bathyscaphes —which have to withstand pressures greater than 1 ton on a square centimeter.

But submarines can dive to a depth of only 100-200 m.

Atmospheric Pressure

We live on the bottom of an ocean of air—the atmosphere. Each body, every grain of sand, any object situated on the Earth is subject to air pressure.

Atmospheric pressure isn't so small. A force of about 1 kgf acts on each square centimeter of a body's surface.

The cause of atmospheric pressure is obvious. Just as water, air possesses weight and, therefore, exerts a pressure equal (just as for water) to the weight of the column of air situated above a body. The higher we climb up a mountain, the less air there will be above us and, therefore, the lower will the air pressure become.

One must know how to measure pressure for scientific and everyday purposes. There exist special instruments—*barometers*—for this.

It isn't difficult to make a barometer. Mercury is poured into a tube with one end sealed off. Stopping up the open end with a finger, one turns the tube upside-down and submerges its open end in a cup of mercury. When this is done, the mercury in the tube will fall, but will not all pour out. The space above the mercury in the tube is undoubtedly airless. The mercury is supported in the tube by the pressure of the external air (Figure 78).

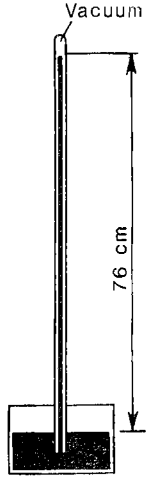


Fig. 78

Whatever be the dimensions of the cup with mercury we take, whatever be the diameter of the tube, the mercury will stand at about one and the same height—76 cm.

If we take a tube shorter than 76 cm, it will be completely filled by mercury and we will not see any empty space. A 76-cm column of mercury presses down on the support with the same force as the atmosphere.

A mercury column of 76 cm with a cross-sectional area of 1 cm^2 weighs about one kilogram, more precisely 1.033 kgf. This number is the volume of the mercury, 1×76 cm^3, multiplied by its density, 13.6 g/cm^3. One kilogram to one square centimeter—this is just the value of normal atmospheric pressure.

The number 76 cm means that such a column of mercury balances the column of air passing through the entire atmosphere which is located above the same area element.

Computing the area of the Earth's surface with the aid of formula $4\pi R^2$, we find that the weight of the entire atmosphere is expressed by the enormous figure of 5×10^{18} kgf.

Barometer tubes can have the most varied forms; only one thing is important: one of the tube's ends must be sealed off in such a way that there be no air above the surface of the mercury. Atmospheric pressure acts on the other level of the mercury.

Atmospheric pressure can be measured by mercury barometer with very great accuracy. Of course, it isn't necessary to use mercury; any other liquid is suitable. But mercury is the heaviest liquid, and so the height of a mercury column under normal pressure will be least.

Various units are used in measuring pressures. One often simply indicates the height of a mercury column in millimeters. For example, we say that the pressure is above normal today, it is equal to 768 mm Hg (i.e. of mercury).

Knowing the density of mercury, we can always translate a pressure into kgf/cm^2. Each millimeter of a mercury column is equal to 1.36 gf/cm^2.

A pressure of 760 mm Hg is sometimes called a *standard atmosphere*. A pressure of 1 kgf/cm^2 is called a *technical atmosphere*.

Physicists also make frequent use of another unit of pressure, the *bar*. One bar equals 10^6 dyne/cm^2. Since 1 gf = = 981 dynes, one bar is approximately equal to one atmosphere. More precisely, normal atmospheric pressure roughly equals 1013 millibars.

The mercury barometer is not a particularly convenient instrument. It is not good to leave a surface of mercury open (mercury vapour is poisonous); furthermore, this instrument is not portable.

These drawbacks are not shared by aneroid barometers — aneroids (i.e. airless).

Everyone has seen such a barometer. It is a small round metal box with a scale and a pointer. Values of pressure are marked on the scale, usually in centimeters of a mercury column.

The air has been pumped out of the metal box. The cover of the box is kept in place by a strong spring, since it would otherwise be crushed by atmospheric pressure. With a change in atmospheric pressure, the cover either bends or straightens. The pointer is connected to the cover in such a manner that the pointer moves to the right when the cover is bent.

Such a barometer is graduated by comparing its readings with those of a mercury barometer.

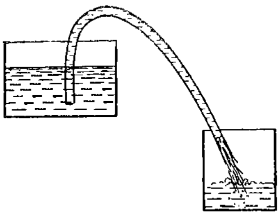


Fig. 79

If you want to know the pressure, don't forget to knock on the barometer with your finger. The dial's pointer experiences considerable friction and usually gets stuck at "yesterday's weather".

A simple mechanism—the siphon—is based on atmospheric pressure.

A driver wants to help his friend, who is out of gas. But how can gasoline be poured from the tank of his car? It can't be inclined like a tea-kettle.

A rubber tube comes to his aid. He lowers one of its ends into his gas tank and orally sucks the air out of the other end. Then a rapid motion—the open end is stopped up with a finger and placed at a height below the gas tank. Now the finger can be removed—the gasoline will pour out of the hose (Figure 79).

A bent rubber tube is just what a siphon is. The liquid moves in this case for the same reason as through a straight inclined tube. In the final analysis, the liquid flows downwards in both cases.

Atmospheric pressure is necessary for the action of a siphon: it "props up" the liquid and doesn't let the column of liquid in the tube break. If there were no atmospheric pressure, the column would break at the transfer point and the liquid would slip into both vessels.

The siphon starts functioning when the liquid in the right-hand (i.e. the "pouring") part of the tube drops below the level of the liquid being siphoned off, into which the left end of the tube has been lowered. The liquid would otherwise flow back.

How Atmospheric Pressure Was Discovered

Suction pumps were already known to ancient civilizations. Water could be raised to a considerable height with their aid. Water very obediently follows the piston of such a pump.

Ancient philosophers thought about the causes for this and arrived at the following profound conclusion: water follows the piston because nature fears a vacuum and so does not leave any free space between the piston and the water.

It is told that an artisan constructed a suction pump for the Duke of Tuscany in Florence, whose piston was supposed to draw water to a height of more than 10 m. But no matter how they tried to begin sucking up water with this pump, nothing came of it. The water rose 10 m with the piston, but after that the piston left the water behind, and so the very same vacuum which nature fears was formed.

When Galileo was asked to explain the cause of this failure, he answered that nature really dislikes a vacuum, but only up to a certain point. A disciple of Galileo, Torricelli, evidently used this case as an excuse to perform his famous experiment in 1643 with a tube filled with mercury. We have just described this experiment—the constructing of a mercury barometer is precisely Torricelli's experiment.

Taking a tube of height more than 76 cm, Torricelli created a vacuum over the mercury (it is often called a *Torri-*

cellian vacuum in his honour) and therefore proved the existence of atmospheric pressure.

By means of this experiment, Torricelli cleared up the misunderstanding of the Duke of Tuscany's artisan. In fact, it is easy to see how many meters water will humbly follow the piston of a suction pump. This motion will continue until the column of water with an area of a square centimeter acquires a weight of 1 kgf. Such a column of water will have a height of 10 m. This is why nature fears a vacuum..., but only up to 10 m.

In 1654, 11 years after Torricelli's discovery, the action of atmospheric pressure was graphically demonstrated by the Burgomaster of Magdeburg, Otto von Guericke. It wasn't so much the physical essence of the experiment, as the theatricality of its performance, that brought the author renown.

Two copper hemispheres were connected by an annular washer. The air was pumped out of the sphere so obtained through a pipe attached to one of the hemispheres, after which it was impossible to separate the hemispheres. A detailed description of Guericke's experiment has been preserved. The atmospheric pressure on the hemispheres can now be calculated: for a diameter of 37 cm, the force was approximately equal to four tons. In order to separate the hemispheres, Guericke ordered that two teams of eight horses each be harnessed. Ropes passing through the rings attached to the hemispheres were tied to the harnesses. The horses proved unable to separate the Magdeburg hemispheres.

The forces supplied by eight horses (exactly eight, and not sixteen, since the second team, harnessed for greater effect could have been replaced by a hook nailed to the wall, with no change in the force acting on the hemispheres) were not enough to break the Magdeburg hemispheres.

If there is an empty cavity between two bodies in contact, then these bodies will not come apart, because of atmospheric pressure.

Atmospheric Pressure and Weather

Pressure fluctuations caused by the weather are very irregular in character. At one time people thought that pressure alone determines the weather. Therefore, the following inscriptions have been placed on barometers up to the present day: clear, dry, rain, storm. One even finds the inscription "earthquake".

Changes in pressure really do play a big role in changing the weather. But this role is not decisive. Average or normal pressure at sea level is equal to 1013 millibars. Pressure fluctuations are comparatively small. The pressure rarely falls below 935-940 millibars or rises to 1055-1060.

The lowest pressure was registered on August 18, 1927, in the South China Sea—885 millibars. The highest—about 1080 millibars—was registered on January 23, 1900, at the Barnaul station in Siberia (all figures are taken with respect to sea level).

A map used by meteorologists analyzing changes in the weather is depicted in Figure 80. The lines drawn on the map are called isobars. The pressure is the same along each such line (its value is indicated). Note the regions of the lowest and highest pressures—the pressure "peaks" and "pockets".

The directions and strengths of winds are related to the distribution of atmospheric pressure.

Pressures are not identical at different places on the Earth's surface, and a higher pressure "squeezes" air into places with a lower pressure. It would seem that a wind should blow in a direction perpendicular to the isobars, i.e.

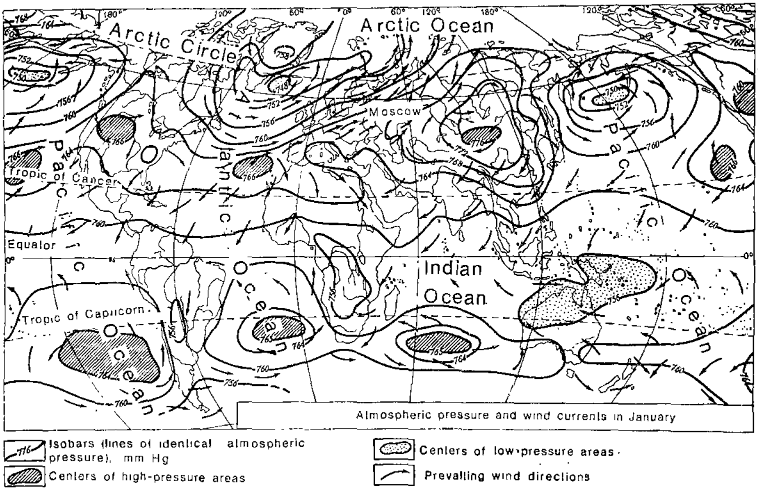


Fig. 80

where the pressure is falling most rapidly. However, wind maps show otherwise. Coriolis forces interfere in the matter of air pressure and contribute their corrections, which are very significant.

As we know, a Coriolis force, directed to the right of the motion, acts on any body moving in the Northern Hemisphere. This also pertains to air particles. Squeezed out of places of higher pressure and into places where the pressure is lower, the particle should move across the isobar, but the Coriolis force deflects it to the right, and so the direction of the wind forms an angle of about 45° with the direction of the isobar.

A strikingly large effect for such a small force! This is explained by the fact that the obstacles to the action of the

Coriolis force—the friction between layers of air—are also very insignificant.

The influence of Coriolis forces on the direction of winds at pressure "peaks" and "pockets" is even more interesting. Owing to the action of Coriolis forces, the air leaving a pressure "peak" does not flow in all directions along radii, but moves along curved lines—spirals. These spiral air streams twist in one and the same direction and create a circular whirlwind, displacing air masses clockwise, in a high-pressure area. Figure 28 (see p. 92) clearly shows how a radial motion is converted into a spiral motion under the action of a constant deflecting force.

The same thing also happens in a low-pressure area. In the absence of Coriolis forces, the air would flow towards this area uniformly along all radii. However, along the way air masses are deflected to the right. In this case, as is clear from the figure, a circular whirlwind is formed, moving the air counterclockwise.

Winds in low-pressure areas are called *cyclones*; winds in high-pressure areas are called *anticyclones*.

You shouldn't think that every cyclone implies a hurricane or a storm. The passing of cyclones or anticyclones through the city where we live is an ordinary phenomenon, related, it is true, more often than not to a change in weather. In many cases, the approach of a cyclone means the coming of bad weather, while the approach of an anticyclone, the coming of good weather.

Incidentally, we shall not embark on the path of a weather forecaster.

Archimedes' Principle

Let us hang a weight on a spring balance. The spring will stretch and show how much the weight weighs. Without taking the weight off the spring balance, let us submerge it in water. Will the reading of the spring balance change? Yes, the body's weight seems to decrease. If the experiment is done with an iron kilogram weight, then the "loss" in weight will constitute approximately 140 grams.

But what is the matter? For it is clear that neither the mass of the weight nor its attraction by the Earth could have changed. There can be only one cause of the loss in weight: an upward force of 140 gf acts on the weight submerged in water. But where does this buoyant force, discovered by the great scientist of antiquity, Archimedes, come from? Before considering a solid body in water, let us consider "water in water". We conceptually single out an arbitrary volume of water. This volume possesses weight, but does not fall to the bottom. Why? The answer is obvious—the hydrostatic pressure of the surrounding water prevents this. This implies that the resultant of this pressure on the volume under consideration is equal to the weight of the water and directed vertically upwards.

If this same volume is now occupied by a solid body, it is clear that the hydrostatic pressure will remain the same.

Thus, as a result of hydrostatic pressure, a force acts on a body immersed in a fluid, which is directed vertically upwards and is equal in magnitude to the weight of the fluid displaced by the body. This is *Archimedes' principle.*

It is said that Archimedes lay in a bath-tub and thought about how to determine whether or not there is any silver in a gold crown. A person taking a bath distinctly feels a buoyant force. Suddenly the principle came to light, presented itself to Archimedes in its remarkable simplicity. With a cry of "Eureka!" (which means "I found it!"), Archimedes jumped out of the bath-tub and ran into the room containing the precious crown in order to immediately determine its loss of weight in water.

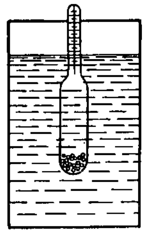


Fig. 81

The loss of weight of a body in water, expressed in grams, will be equal to the weight of the water displaced by the body. Knowing the weight of the water, we shall immediately determine its volume, which is equal to the volume of the crown. Knowing the weight of the crown, we can immediately find the density of the material out of which it was made and, knowing the density of gold and silver, find the fraction of silver in the crown.

Archimedes' principle is valid, of course, for any fluid. If a body of volume V is immersed in a fluid of density ρ, then the weight of the displaced fluid—and this is just the buoyant force—will be equal to ρgV.

The behaviour of simple instruments controlling properties of fluid products is based on Archimedes' principle. If alcohol or milk is diluted with water, then its density will change; but it is possible to judge its composition on the basis of its density. Such a measurement is simply and easily performed with the aid of an areometer (Figure 81). An areometer lowered into a liquid will be immersed to a greater or lesser depth depending on its density.

An areometer will be in a state of equilibrium when the buoyant force becomes equal to the weight of the areometer.

Divisions are marked off on an areometer, and a liquid's density is read from the marking which appears at its surface. Areometers applied for the control of alcohol are called alcoholometers, for the control of milk, lactometers.

The average density of a person's body is somewhat greater than one. Anyone unable to swim will drown in fresh water. Salt water has a density greater than one. The salinity of the water in most seas is insignificant, and its density, although greater than one, is less than the average density of the human body. The density of the water in the Bay of Kara-Bogaz-Gol in the Caspian Sea is 1.18. This is greater than the average density of the human body. It is impossible to drown in this bay. One can lie on the water and read a book.

Ice floats on water. The preposition "on", incidentally, is somewhat out of place here. The density of ice is about 10% less than that of water, so it follows from Archimedes' principle that approximately 0.9 of a piece of ice is submerged in water. It is precisely this circumstance that makes so dangerous for ocean liners to come across icebergs.

If a balance scale is in equilibrium in the atmosphere, this does not imply that it will be in equilibrium in a vacuum. Archimedes' principle refers to air to the same degree as to water. A buoyant force, equal to the weight of the displaced air, acts on a body situated in the atmosphere. A body "weighs" less in the atmosphere than in a vacuum. The greater the volume of a body, the greater will be its loss of weight. A ton of wood loses more weight than a ton of lead. To the humorous question of which is lighter, there is the same kind of answer: a ton of lead is heavier than a ton of wood, if they are weighed in the atmosphere.

The loss of weight in the atmosphere is slight as long as we are considering small bodies. However, in weighing a piece the size of a room, we would "lose" several tens of kilograms. For an exact weighing, the correction due to the loss of weight in air should be taken into account.

The buoyant force in air permits us to construct balloons, aerostats and dirigibles of various types. For this one must have a gas lighter than air.

If a balloon of volume 1 m^3 is filled with hydrogen, 1 m^3 of which has a weight equal to 0.09 kgf, then the lift—the difference between the buoyant force and the weight of the gas—will equal

$$1.29 \text{ kgf} - 0.09 \text{ kgf} = 1.20 \text{ kgf}$$

1.29 kg/m^3 is the density of air.

Hence, a load of about a kilogram can be attached to such a balloon, and this will not prevent it from flying above the clouds.

It is clear that with relatively small volumes—several hundred cubic meters—hydrogen balloons are capable of raising considerable loads into the air.

A serious defect of hydrogen balloons is the inflammability of hydrogen. Together with air, hydrogen forms an explosive mixture. Tragic accidents have marked the history of the creation of aerostats.

Therefore, when helium was discovered, people started filling balloons with it. Helium is twice as heavy as hydrogen and the lift of a balloon filled with it is smaller. But will this difference be significant? The lift of a 1-m^3 balloon filled with helium is found as the difference 1.29 kgf — — 0.18 kgf = 1.11 kgf. The lift has decreased by only 8%. At the same time, the advantages of helium are obvious.

The balloon was the first apparatus with whose aid people rose in the air. Balloons with a hermetically sealed car have been used up to the present day for investigating the upper layers of the atmosphere. They are called stratospheric balloons. They rise to a height of more than 20 km.

Balloons equipped with various measuring devices and transmitting the results of their measurements by radio

(Figure 82) are widely used at the present time. Such radiosondes contain miniature radio transmitters with batteries, which report on the humidity, temperature and air pressure at various heights by means of prearranged signals.

One can send an unguided aerostat on a long journey and determine where it will land rather accurately. For this it is necessary that the aerostat climb to a great height, of the order of 20-30 km. Air currents are extremely stable at such heights, and the aerostat's path can be calculated quite well beforehand. When necessary, one can automatically change an aerostat's lift by letting out gas or throwing off ballast.

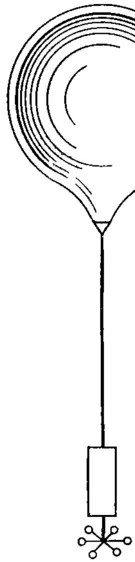


Fig. 82

Airships on which a motor with a propeller was installed had been used previously for flights. Such airships—they are called dirigibles (which means "guided")—were streamlined.* Dirigibles lost the competition with airplanes; even in comparison with planes of 30 years ago, they are clumsy, awkward to control, move slowly and have a "low ceiling". It is believed that dirigibles would be advantageous for carrying cargo.

Pressures of Millions of Atmospheres

We daily come across large pressures exerted on small areas. Let us estimate, for example, what the pressure will be at the point of a needle. Assume that the tip of a needle or nail has a linear dimension of 0.1 mm. This implies that the area of the point will be about 0.0001 cm^2. If a rather

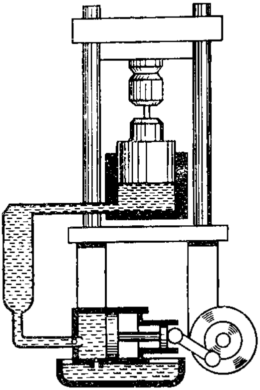


Fig. 83

modest force of 10 kgf acts on such a nail, then the tip of the nail will exert a pressure of 100 000 atmospheres. It's no wonder that the points of objects so easily penetrate deeply into dense bodies.

It follows from this example that the creation of large pressures on small areas is quite a common thing. The situation is completely different if the question is the creation of high pressures on large surfaces.

The creation of high pressures under laboratory conditions is accomplished with the aid of powerful presses, for example, hydraulic (Figure 83). The force of the press is transmitted to a piston of small area, which forces its way into the vessel within which we wish to create a high pressure.

Pressures of several thousand atmospheres can be created in this manner without any particular difficulty. But in order to obtain ultrahigh pressures, we must complicate the experiment, since the material composing the vessel cannot withstand such pressures.

Here nature has met us half-way. It turns out that metals become considerably stronger under pressures of the order of 20 000 atmospheres. Therefore, an apparatus for obtaining ultrahigh pressures is submerged in a liquid which is under a pressure of the order of 30 000 atmospheres. In this case, one is able to create pressures of several hundred thousands of atmospheres (but again with a piston). The highest pressure—400 000 atmospheres—was obtained by the American physicist Percy Williams Bridgman.

Our interest in obtaining ultrahigh pressures is far from idle. Phenomena which are impossible to induce by other methods can occur at such pressures. Artificial diamonds were obtained in 1955. A pressure of 100 000 atmospheres and, in addition, a temperature of 2300 °C were required for this.

Ultrahigh pressures of the order of 300 000 atmospheres on large surfaces are formed during explosions of solid or liquid explosive materials—nitroglycerine, trotyl, etc.

Incomparably higher pressures, attaining 10^{13} atmospheres, arise within an atomic bomb during its explosion.

Pressures during an explosion exist for a very short time. There are constant high pressures deep inside celestial bodies, including the Earth, of course. The pressure at the center of the Earth is equal to approximately 3 million atmospheres.

Surface Tension

Is it possible to emerge dry from a swimming pool? Of course it is, if one smears oneself with a non-wettable substance.

Rub your finger with paraffin and put it under water. When you take it out, you will find that except for two or three drops, there is no water on your finger. A slight motion—and the drops are shaken off.

In this case we say that water does not wet paraffin. Mercury behaves in such a manner towards almost all solid bodies: mercury does not wet leather, glass, wood, ...

Water is more capricious. It adheres closely to some bodies and tries not to touch others. Water does not wet oily surfaces, but thoroughly wets clean glass. Water wets wood, paper and wool.

If a drop of water is placed on a clean plate of glass, it will spread out and form a very shallow, small puddle. If such a drop is put on a piece of paraffin, it will just remain a drop, almost spherical in form and slightly flattened by gravity.

Among the substances which "stick" to almost all bodies is kerosene. Striving to flow along glass or metal, kerosene is capable of creeping out of a loosely closed vessel. A puddle of spilled kerosene can spoil one's existence for a long time: kerosene will seize a large surface, creep into cracks and penetrate one's clothes. That is why it is so difficult to get rid of its not very pleasant odour.

The failure to wet bodies can lead to curious phenomena. Take a needle, grease it and carefully place it flat on water. The needle will not sink. Looking attentively, you can notice that the needle depresses the water and calmly lies in the small hollow so formed. However, a slight pressure is enough to make the needle go to the bottom. For this it is necessary that a considerable part of it turns out to be in the water.

This interesting property is made use of by water-bugs running swiftly along the surface of the water without wetting their feet.

Wetting is used to dress ores by means of floatation. The word "floatation" means "surfacing". The essence of the phenomenon is as follows. Finely crushed ore is loaded into a vat containing water. A small amount of special oil is added, which should possess the property of wetting particles of the useful mineral and not wetting particles of the "dirt" (this is what the superfluous portion of the ore is called). When mixed, the particles of the useful mineral are coated with an oily film.

Air is blown into the black mixture of ore, water and oil. There is formed a mass of little bubbles of air—foam. The air bubbles come to the surface. The process of floatation is based on the fact that the particles covered with oil cling to the air bubbles. A large bubble carries up a small particle like a balloon.

The useful mineral passes in a foam to the surface. The dirt remains on the bottom. The foam is removed and sent for further processing in order to obtain a so-called "concentrate", which contains tens of times as little dirt.

Forces of adhesion between surfaces are capable of violating the levelling of a liquid in communicating vessels. It is very easy to verify the truth of this.

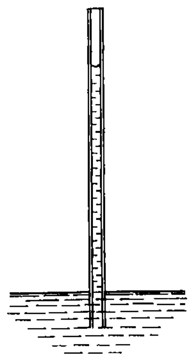


Fig. 84

If a thin glass tube (with diameter of a fraction of a millimeter) is lowered into water, then in violation of the law of communicating vessels, the water in it will quickly begin rising, and its level will become considerably higher than in the large vessel (Figure 84).

But what took place? What forces are supporting the weight of the column of liquid that has risen up? The rise is accomplished by the forces of adhesion between the water and the glass.

Forces of adhesion between surfaces clearly manifest themselves only when a liquid rises in a sufficiently thin tube. The narrower the tube, the higher will the liquid rise and the more distinct will the phenomenon be. The name of these surface phenomena is related to the name of the tubes. The inside diameter of such a tube is measured in fractions of a millimeter; such a tube is called capillary (meaning "thin as a hair"). The phenomenon of the rise of liquids in thin tubes is called capillarity.

But to what height are capillary tubes capable of raising a liquid? It turns out that water rises to a height of 1.5 mm in a tube of 1-mm diameter. For a diameter of 0.01 mm, the

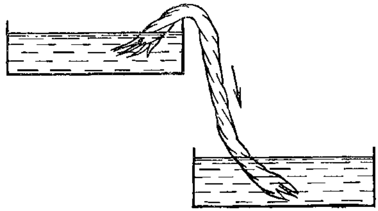


Fig. 85

height of the rise will increase as many times as the tube's diameter decreases, i.e. to 15 cm.

Of course, the elevation of a liquid is possible only in the case of wetting. It is not hard to guess that mercury will not rise in glass tubes. On the contrary, mercury falls in glass tubes. Mercury is so "intolerant" of contact with glass that it strives to reduce the total surface to the minimum allowed by gravity.

There exist many bodies which are something like systems of very thin tubes. Capillary phenomena can always be observed in such bodies.

Plants and trees have an entire system of long ducts and pores. The diameters of these ducts are less than hundredths of a millimeter. Because of this, capillary forces raise soil moisture to a considerable height and distribute water throughout the plant.

Blotting paper is a very convenient thing. You spilled some ink on a page and want to turn it over. But you're not going to wait until the blot dries up! You take a sheet of blotting paper, dip one of its edges in the drop, and the ink swiftly runs upwards against gravity.

A typical capillary phenomenon has occurred. If you look at the blotting paper through a microscope, you can see its structure. Such a paper consists of a sparse network of paper fibers, forming thin and long ducts with each other. These ducts play the role of capillary tubes.

The same kind of system of long pores or ducts, formed by fibers, exists in wicks. Kerosene rises through the wick of a lamp. A siphon can also be created with the aid of a wick by placing one of its ends in a glass partially filled with water, in such a way that the other end, hanging over the edge, is lower than the first (Figure 85).

In the technology of the dyeing industry, frequent use is also made of a fabric's ability to draw in a liquid through the thin pores formed by the fabric's threads.

Nine
BRICKS OF THE UNIVERSE

Elements

What is the world surrounding us made of? The first answers to this question which have reached us originated in Ancient Greece more than 25 centuries ago.

At first glance, the answers seem as strange as can be, and we would have to waste a lot of paper in order to explain to the reader the logic of the ancient sages—Thales, having asserted that everything consists of water, Anaximander, having said that the world is made of air, or Heraclitus, in whose opinion everything consists of fire.

The incongruity of such explanations forced later Greek "lovers of wisdom" (that's how the word "philosopher" is translated) to increase the number of fundamental qualities or, as they were called in antiquity, *elements*. Empedocles asserted that there are four elements: earth, water, air and fire. Aristotle contributed the final (for a very long time) amendment to this investigation.

According to Aristotle, all bodies consist of one and the same substance, but this substance can assume different qualities. There are four immaterial element-qualities: cold, hot, wet and dry. Combining in pairs and being imparted to a substance, Aristotle's element-qualities form the elements of Empedocles. Thus, a dry and cold substance yields earth, dry and hot, fire, wet and cold, water and, finally, wet and hot, air.

However, in view of the difficulty involved in answering many questions, ancient philosophers added a "divine quintessence" to the four element-qualities. This was a kind of god-cook, cooking the various element-qualities together. Of course, it isn't hard to explain away any perplexity by reference to a god.

But for a very long time—almost up to the 18th century—few dared be perplexed and ask questions. Aristotle's teachings were avowed by the Church, and any doubt in their validity was a heresy.

But these doubts arose anyway. They were engendered by alchemy.

In the distant past, into the heart of which we can look by reading ancient manuscripts, people knew that all bodies surrounding us were capable of being transformed into others. Combustion, the roasting of ores, the melting of metals—all these phenomena were well known.

This, it would seem, did not contradict Aristotle's teaching. The so-called "dosage" of the elements changed during any transformation. If the whole world consists of only four elements, then the possibilities of transforming bodies should be very great. It is merely necessary to find the secret of what to do in order that any body might be obtained from any other one.

How tempting is the problem of making gold or finding a special, extraordinary "philosophers' stone", giving its possessor wealth, power and eternal youth! The science of manufacturing gold and a philosophers' stone, of transforming any body into any other one, was called alchemy by the ancient Arabs.

The labour of people devoting themselves to the solution of this problem continued for centuries. Alchemists did not learn how to make gold, did not find a philosophers' stone, but made up for this by collecting many valuable facts about the transformation of bodies. In the final analysis, these facts served as the death sentence for alchemy. In the 17th century, it became obvious to many people that the number of basic substances—elements—is incomparably greater than four. Mercury, lead, sulphur, gold and antimony turned out to be indecomposable substances; one could no longer say that these substances were made out of elements. On the contrary, one had to rank them among the elements of the world.

Robert Boyle's book *The Sceptical Chemist, or Doubts and Paradoxes Concerning the Elements of the Alchemists* was published in England in 1661. Here we find a completely new definition of an element. This is no longer the elusive, mysterious immaterial element of the alchemists. An element is now a substance, a component part of a body.

This is consistent with the modern definition of the concept of an element.

Boyle's list of elements was not very large. He added fire to a correct list. Incidentally, the idea of element-qualities lived on even after Boyle. Even in a list of the great Frenchman, Antoine Laurent Lavoisier (1743-1794), who is regarded as the founder of chemistry, side by side with real elements there also appear imponderable elements: a heat-producing and a light-producing substances.

In the first half of the 18th century, there were 15 known elements, and their number rose to 35 by the end of the century. True, only 23 of them were real elements, but the rest were either non-existent elements or else substances like caustic soda and caustic potash, which turned out to be compounds. By the middle of the 19th century, more than 50 indecomposable substances had already been described in chemical handbooks.

The periodic law of the great Russian chemist Mendeleev provided the stimulus for a conscious search for undiscovered elements. It is still too early to speak about this law here. Let us merely say that by means of his law, Mendeleev showed how one must look for the elements which had not yet been discovered.

Almost all the elements occurring in nature had been discovered by the beginning of the 20th century. There turned out to be 88 of them.

Atoms

About 2000 years ago, an original poem was written in Ancient Rome. Its author was the Roman poet Lucretius. His poem was called *On the Nature of Things.*

With sonorous lines, Lucretius told of the ancient Greek philosopher Democritus' views on the world in his poetic work.

What views were these? These were teachings about the minutest, invisible particles which our whole world is made of. Having observed various phenomena, Democritus tried to give them an explanation.

Take water, for example. When sufficiently heated, it evaporates and disappears. How can this be explained? It is clear that such a property of water is related to its internal structure.

Or why, for example, do we perceive the scent of flowers at a distance?

Meditating on similar questions, Democritus became convinced that bodies only seem to be solid, but in fact consist of the minutest particles. These particles are different in form for different bodies, but they are so small that they cannot be seen. That is why all bodies seem to us to be solid.

Democritus called such very tiny particles, which cannot be further divided and of which water and all other bodies consist, *atoms* (derived from the Greek *atomos*, meaning "indivisible").

This remarkable guess of ancient Greek thinkers, born 24 centuries ago, was later long forgotten. Aristotle's erroneous teaching exercised complete sway over the scientific world for more than a thousand years.

Asserting that all substances are mutually transmutable, Aristotle categorically denied the existence of atoms. Any body can be infinitely divided, taught Aristotle.

In 1647, a Frenchman, Pierre Gassendi, published a book in which he courageously denied Aristotle's teachings and asserted that all substances in the world consist of small indivisible particles—atoms. Atoms differ from each other in form, size and weight.

Agreeing with the teachings of the ancient atomists, Gassendi developed these teachings further. He explained exactly how the millions of diverse bodies of nature can and do arise in the world. For this, he asserted, a large number of different atoms is not necessary. For an atom is the same thing as building materials for houses. It is possible to construct an enormous number of the most diverse houses from three different kinds of building material—bricks, boards and logs. In precisely the same way, nature can create thousands of the most diverse bodies from several tens of different atoms. Moreover, in each body various atoms are united in small groups; Gassendi called these groups *molecules*, i.e. "small masses" (derived from the Latin *moles*, meaning "mass").

Molecules of various bodies differ from each other in the number and kind ("sort") of atoms belonging to them. It is not difficult to understand that an immense quantity of different combinations of atoms—molecules—can be created from several tens of different atoms. This is why there is such a great variety in the bodies surrounding us.

However, Gassendi's views still contained much that was incorrect. Thus, he believed that there were special atoms for heat, cold, taste and smell. As other scientists of that time, he too could not completely free himself from Aristotle's influence, and recognized his immaterial elements.

The following ideas, experimentally verified much later, are contained in the writings of M. V. Lomonosov—the great enlightener and founder of science in Russia.

Lomonosov writes that molecules can be homogeneous or heterogeneous. In the former case, similar atoms are grouped in a molecule. In the latter, a molecule consists of atoms differing from each other. If some body is composed of homogeneous molecules, then it must be regarded as simple. If, on the contrary, a body consists of molecules built up from various atoms, Lomonosov calls it compound.

We now well know that nature's various bodies have precisely such a structure. In fact, take the gas oxygen, for example; two identical atoms of oxygen are contained in each of its molecules. This is a molecule of simple matter. But if the atoms composing a molecule are different, it is a "compound", or a complex chemical union. Its molecules consist of atoms of those chemical elements which occur in the composition of this union.

This can also be said otherwise—each simple substance is constructed from atoms of one chemical element; a compound substance contains atoms of two or more elements.

A number of thinkers spoke about atoms, adducing logical arguments in favour of their existence. The English scientist Dalton introduced atoms into science in the right way and made them an object of research. Dalton showed that there exist chemical regularities which can be explained in a natural manner only by making use of the idea of an atom.

After Dalton, atoms firmly entered science. However, for a very long time there still were scientists who did not believe in atoms. Even at the very end of the last century, one of them wrote that after several decades "it will be possible to find [atoms] only in the dust of libraries".

Such reasoning seems funny now. We now know so many details about the "life" of an atom that to doubt its existence is the same thing as to doubt the reality of the Black Sea.

The relative weights of the atoms were determined by chemists. At first the weight of an atom of hydrogen was taken as the unit of atomic weight. The atomic weight of nitrogen turned out to be approximately equal to 14, oxygen, approximately 16, chlorine, approximately 35.5. Since oxygen compounds are most widespread, a somewhat different choice of the relative units of atomic weight was later made, for which the number 16.0000 was assigned to oxygen. The atomic weight of hydrogen turned out equal to 1.008 in this scale. At the present, these units are determined from the atomic weight of carbon, 12.

As a result of a series of interesting experiments, physicists were able to measure the absolute weight of atoms. Since their relative weights were known, it was sufficient to measure in grams the weight of an atom of any one sort, say of hydrogen.

Of course, physicists did not construct scales on which one could place a single atom and balance it with a weight. For the determination of the weight of atoms, physicists used other measurements, which, however, are not a bit less reliable than a direct weighing.

The unit of atomic weight turned out equal to

$$m = 1.66 \times 10^{-24}\,\text{g}$$

In order to grasp the minuteness of this figure, imagine yourself demanding a billion molecules from each person on the Earth (whose population is more than four billion). How much matter will you collect in this way? Several millionths of a gram.

Or another such comparison: the Earth is as many times heavier than an apple as an apple is heavier than a hydrogen atom.

The inverse of m is called *Avogadro's number*:

$$N_A = \frac{1}{m} = 6.023 \times 10^{23}$$

This enormous number has the following significance. Take such a large quantity of a substance that its weight in grams is equal to the relative weight M of one of its atoms or molecules. Such a quantity is called 1 *gram-atom* or 1 *gram-molecule* (instead of a "gram-molecule", one often says a "*mole*" for the sake of conciseness). The weight of such a molecule in grams is equal to Mm. Therefore, the number of molecules in a gram-molecule of any substance is

$$\frac{M}{Mm} = N_A$$

i.e. is equal to Avogadro's number.

What Heat Is

How does a hot body differ from a cold one? Up until the 19th century, this question was answered as follows: a hot body contains more heat-producing matter (or "caloric") than a cold one, in exactly the same sense as soup is saltier if it contains more salt. But what is caloric? The following answer was given to this question: "Caloric is the matter of heat, it is the elementary fire." Mysterious and incomprehensible. And this answer is in essence the same as the following explanation of what a rope is: "A rope is simple 'ropeness'."

Along with the caloric theory, a different view on the nature of heat had long been in existence. It was brilliantly advocated by many outstanding scientists of the 16-18th centuries.

Francis Bacon wrote in his book *Novum Organum*: "Heat itself in its essence is nothing but motion ... . Heat consists in a variable motion of a body's minutest particles."

Robert Hooke asserted in his book *Micrographia*: "Heat is a continuous motion of a body's parts ... . There is no such body whose particles would be at rest."

We find particularly clear statements of this kind in Lomonosov's work (1745) *Reflections on the Cause of Heat and Cold*. The existence of caloric is denied in this work, where it is said that "heat consists of the internal motion of particles of matter".

Count Rumford put it very graphically at the end of the 18th century: "The more intensively the particles composing a body move, the hotter the body will be, analogous to how the more vigorously a bell vibrates, the louder it rings."

In these remarkable guesses, far ahead of their time, are concealed the bases of our modern views on the nature of heat.

There are sometimes quiet, calm, clear days. The leaves lie still on the trees, not even a slight ripple disturbs the water's glassy surface. The entire surroundings have frozen in strict, triumphant immobility. The visible world is at rest. But what is taking place in the world of atoms and molecules?

Contemporary physicists can say much about this. Never, not under any circumstances is there a cessation to the invisible motion of the particles that the world is made of.

But why don't we see all these motions? Particles move, but the body is at rest. How is this possible?

Have you ever watched a swarm of midges? When there is no wind, the swarm appears to be suspended in air. But an intensive life is going on inside the swarm. Hundreds of insects flew off to the right, but just as many flew off to the left at the same instant. The swarm as a whole remained at the same place and did not change its form.

The invisible motions of atoms and molecules are of the same chaotic, disorderly nature. If some molecules leave a volume, their place is occupied by others. But since the newcomers do not in the least differ from the departed molecules, the body remains entirely as it was. A disorderly, chaotic motion of particles does not change the properties of the visible world.

"However, isn't this idle talk?" the reader might ask us. In what sense are these arguments, however beautiful, more convincing than the caloric theory? Has anyone actually seen the eternal thermal motion of particles of matter?

It is possible to see the thermal motion of particles and, moreover, with the aid of the simplest microscope. This phenomenon was first observed more than a hundred years ago by the English botanist Brown.

Looking at the internal structure of a plant through a microscope, he noticed that tiny particles of matter, floating in the plant's sap, were continually moving in all directions. The botanist became interested: what forces made the particles move? Perhaps they were living beings of some kind? The scientist decided to examine under a microscope small particles of clay making some water turbid. But neither were these, undoubtedly lifeless, particles at rest; they were engaged in a continual chaotic motion. The smaller the particles were, the faster they moved. The botanist examined this drop of water for a long time, but still he couldn't see any end to the particles' motion. Some invisible forces seemed to be constantly pushing them.

The Brownian motion of particles is just a thermal motion. Thermal motion is inherent in large and small particles, clots of molecules, individual molecules and atoms.

Energy Is Always Conserved

Thus, the world is composed of moving atoms. Atoms possess mass, moving atoms possess kinetic energy. Of course, the mass of an atom is unimaginably small, and so its energy will also be minute, but there are billions of billions of atoms.

We now remind the reader that although we spoke of the law of conservation of energy, this was not a sufficiently universal conservation law. Linear and angular momenta were conserved experimentally, but energy was only conserved ideally—in the absence of friction. But as a matter of fact, energy always decreased.

But we did not say anything previously about the energy of atoms. A natural idea arises: where at first sight we noticed a decrease in energy, some energy was transmitted to the atoms of a body in a manner which is imperceptible to the naked eye.

Atoms are subject to the laws of mechanics. True (you will have to learn this from another book), their mechanics is somewhat peculiar, but this does not change matters—with respect to the law of conservation of mechanical energy, atoms do not differ at all from large bodies.

Hence, the complete conservation of energy will be detected only when along with a body's mechanical energy, the internal energy of this body and the environment is taken into account. Only in this case will the law be universal.

What does a body's total energy consist of? We have, in essence, already named its first component—it is the sum of the kinetic energies of all its atoms. But it must not be forgotten that atoms interact with each other. Therefore, the potential energy of this interaction is added. Thus, the total energy of a body is equal to the sum of the kinetic energies of its particles and the potential energy of their interaction.

It is not difficult to comprehend that the mechanical energy of a body as a whole is only part of its total energy. For when a body is at rest, its molecules do not stop moving and do not cease interacting with each other. The energy of the particles' thermal motion, which remains in a body at rest, and the energy of the interaction between the particles constitute the internal energy of the body. A body's total energy is therefore equal to the sum of its mechanical and internal energies.

In the mechanical energy of a body as a whole, its gravitational energy is also included, i.e. the potential energy of its particles' interaction with the Earth.

Considering internal energy, we no longer detect vanishing of energy. When we consider nature through glasses magnifying the world millions of times, the picture seems to us to be of rare harmoniousness. There are no losses of mechanical energy, but there is only a transformation of it into internal energy of a body or its surroundings. Has any work disappeared? No! The energy went into an acceleration of the relative motion of the molecules or a change in their mutual distribution.

Molecules obey the law of conservation of mechanical energy. There are no frictional forces in the world of molecules; the world of molecules is controlled by transformations of potential energy into kinetic one and vice versa. Only in the coarse world of large objects, which does not notice molecules, does "energy vanish".

If mechanical energy totally or partially vanishes during some occurrence, then the internal energy of the bodies and media participating in this occurrence will grow by the same amount. Putting it otherwise, mechanical energy is transformed without any loss whatsoever into energy of molecules or atoms.

The law of conservation of energy is a most strict bookkeeper of physics. The income and outgo of energy should exactly balance during any occurrence. If this did not take place in some experiment, then the implication is that something important escaped our attention. In such a case, the law of conservation of energy gives us a signal: researcher, repeat the experiment, increase the accuracy of your measurements, look for the cause of the loss! Physicists have repeatedly made new, important discoveries along these lines, convincing themselves time and time again of the perfectly strict validity of this remarkable law.

Calorie

We already have two units of energy—the erg and the kilogram-meter. It would seem that this is enough. However, it is traditional to employ yet a third unit—the *calorie*—in the study of thermal phenomena.

We shall see later that, even with the calorie, the list of units adopted for designating energies is not exhausted.

It is possible that in each individual case, the use of its "own" unit of energy is convenient and expedient. But in any example dealing with the transformation of energy from one form to another, which is the least bit complicated, an inconceivable mix-up with units arises.

In order to simplify computations, the new system of units (SI) provides for a single unit for work, energy and a quantity of heat—the joule (see p. 114). However, considering the strength of tradition and the length of time which will be required for this system to become the only system of units in general use, it is helpful to acquaint ourselves more closely with the "departing" unit of heat—the calorie.

A *small calorie* (cal) is the quantity of energy which must be supplied to 1 g of water in order to heat it by 1°.

The word "small" must be mentioned because one sometimes uses a "large" calorie, which is a thousand times as great as the chosen unit (a *large calorie* is often denoted by kcal, which means "kilocalorie").

The relationship between a calorie and a mechanical unit of work, such as an erg or a kilogram-meter, is found by heating water mechanically. Such experiments have been performed repeatedly. It is possible, for example, to raise the temperature of water by stirring it energetically. The mechanical work expended for the heating can be evaluated with sufficient accuracy. It was found from such measurements that

$$1 \text{ cal} = 0.427 \text{ kgf-m} = 4.18 \text{ J}$$

Since energy and work have units in common, it is also possible to measure work in calories. One must expend 2.35 calories in order to raise a kilogram weight by one meter. This sounds unusual, and it really is inconvenient to compare the raising of a load with the heating of water. Therefore, calories are not employed in mechanics.

Some History

The law of conservation of energy could only be formulated when the idea of the mechanical nature of heat had become sufficiently clear and when technology had posed in practice the important question of the equivalence between heat and work.

The first experiment establishing a quantitative relationship between heat and work was carried out by the well-known physicist Rumford (1753-1814). He worked in a factory where cannon were manufactured. When the muzzle of a gun is bored, heat is given off. How could it be estimated? What should be taken as the measure of heat? It occurred to Rumford to relate the work performed in boring with the heating of one or another quantity of water by one or another number of degrees. This investigation was perhaps the first precise expression of the idea that heat and work should have a measure in common.

The next step towards the discovery of the law of conservation of energy was the establishment of an important fact: a disappearance of work is accompanied by an appearance of a proportional amount of heat; with this a common measure for heat and work was found.

The original definition of the so-called mechanical equivalent of heat was given by the French physicist Sadi Carnot. This outstanding person died at the age of 36 in 1832 and left behind a manuscript, which was published only after 50 years. The discovery made by Carnot remained unknown and did not influence the development of science. Carnot calculated in this work that the raising of a cubic meter of water to a height of 1 m requires just as much energy as is needed for heating 1 kg of water by 2.7 degrees (the correct figure is 2.3 degrees).

The Heilbronn doctor Julius Robert von Mayer published his first work in 1842. Although Mayer called physical concepts familiar to us by entirely different names, a careful reading of his work leads nevertheless to the conclusion that the essential features of the law of conservation of energy are presented in it. Mayer distinguished between the internal energy ("thermal"), gravitational potential energy and energy of motion of a body. He tried to infer the necessity of conservation of energy under various transformations from purely theoretical considerations. In order to check this assertion experimentally, one must have a common measure for measuring these energies. Mayer calculated that the heating of a kilogram of water by one degree is equivalent to the raising of one kilogram by 365 m.

In his second work, published three years later, Mayer noted the universality of the law of conservation of energy—the possibility of applying it to questions of chemistry, biology and cosmic phenomena. To the various known forms of energy, Mayer added magnetic, electric and chemical.

A lot of credit for the discovery of the law of conservation of energy goes to the remarkable English physicist (a brewer from Salford in England) James Prescott Joule, working independently of Mayer.

While a certain inclination to an indeterminate philosophy is characteristic of Mayer, Joule's basic trait is a strict experimental approach towards the phenomena under consideration. Joule posed a question before nature and obtained an answer to it by means of special experiments set up in an exceptionally painstaking manner. There is no doubt that in the entire series of experiments performed by Joule, he was guided by a single idea—to find a common measure for evaluating thermal, chemical, electrical and mechanical actions, to demonstrate that energy is conserved in all these phenomena. Joule formulated his idea as follows: "The destruction of forces performing work does not occur in nature without a corresponding action."

Joule reported on his first work on January 24, 1843, and on August 21 of the same year, he communicated his results on the establishment of a common measure for heat and work.

Heating a kilogram of water by one degree proved equivalent to raising one kilogram by 460 m.

In the following years, Joule and a number of other researchers spent a great deal of effort in order to find a more precise value for the thermal equivalent, and also attempted to prove the complete universality of the equivalent. During the late forties, it became clear that regardless of how work is transformed into heat, the quantity of heat arising will always be proportional to the quantity of work expended. In spite of the fact that Joule laid the experimental basis for the law of conservation of energy, he did not give a clear formulation of this law in his works.

The credit for this belongs to the German physicist Helmholtz. On July 23, 1847, at a meeting of the Berlin Physical Society, Hermann Helmholtz gave a lecture on the principle of conservation of energy. The mechanical basis of the law of conservation of energy was clearly presented for the first time in this talk. The world consists of atoms; atoms possess potential and kinetic energy. The sum of the potential and kinetic energies of the particles which a body or system is made of cannot change, if the body or system is not subjected to external influences. The law of conservation of energy, as we outlined it several pages above, was first formulated by Helmholtz.

Helmholtz' long lecture contained not only statements of general ideas. Helmholtz examined in detail all physical phenomena (thermal, chemical, electromagnetic), demonstrated the universality of the principle of equivalence and presented rules for computing energies.

After the work of Helmholtz, it remained for other physicists to merely verify and apply the law of conservation of energy. The success of these investigations led to the fact that by the end of the fifties, the law of conservation of

HERMANN HELMHOLTZ (1821-1894)—a famous German scientist. Helmholtz worked in the fields of physics, mathematics and physiology with great success. He was the first (1847) to give a mathematical interpretation of the law of conservation of energy, emphasizing the universal character of this law. Helmholtz obtained outstanding results in thermodynamics; he was the first to apply it to the study of chemical processes. By means of his work on the vortex motion of liquids, Helmholtz laid the foundations of hydrodynamics and aerodynamics. He carried out a number of valuable investigations in the fields of acoustics and electromagnetism. Helmholtz developed a physical theory of music. He applied powerful and original mathematical methods in his physical research.

energy had been already universally recognized as a fundamental law of natural science.

Phenomena casting doubt on the law of conservation of energy have already been observed in the 20th century. However, explanations were later found for the apparent discrepancies. The law of conservation of energy has so far always stood the test with credit.

Ten
STRUCTURE OF MATTER

Molecules

Molecules consist of atoms. Atoms are bound in molecules by forces which are called chemical forces.

There exist molecules consisting of two, three, four atoms. The largest molecules—protein molecules—consist of tens and even hundreds of thousands of atoms.

The molecule kingdom is exceptionally varied. By now, millions of substances, built up out of various molecules, have already been isolated by chemists from natural materials and created in their laboratories.

Properties of molecules are determined not only by how many atoms of one or another sort participate in their construction, but also by the order and configuration in which they are bound. A molecule is not a heap of bricks, but a complicated architectural structure, where each brick has its place and its completely determined neighbours. The atomic structure forming a molecule can be rigid to a greater or lesser degree. In any case, each of the atoms carries out an oscillation about its equilibrium position. In certain cases, some parts of a molecule can even revolve around other parts, giving different and the most fantastic configurations to a free molecule in the process of its thermal motion.

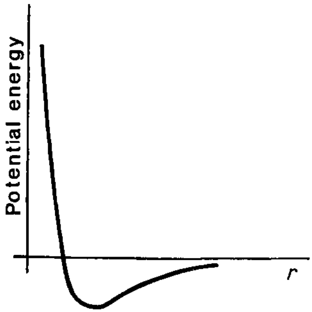


Fig. 86

Let us analyze the interaction of atoms in greater detail. The potential energy curve of a diatomic molecule is depicted in Figure 86. It has a characteristic form—it first goes down, then turns up, forming a "well", and afterwards rises more slowly towards the horizontal axis, on which the distance between the atoms is marked.

We know that the state in which the potential energy has the least value is stable. When an atom forms a part of a molecule, it "sits" in a potential well, carrying out small thermal oscillations about its equilibrium position.

The distance from the vertical axis to the bottom of the well can be called the equilibrium distance. The atoms would be situated at this distance if the thermal motion were to cease.

The potential energy curve tells about all the details of the interaction between the atoms. Whether the particles attract or repel each other at one or another distance, whether the strength of the interaction increases or decreases when the particles separate or approach—all this information can be obtained from an analysis of the potential energy curve. Points to the left of the "bottom" correspond to a repulsion. On the contrary, portions of the curve to the right of the bottom of the well characterize attraction. The steepness of the curve also yields important information: the steeper the curve, the greater the force.

When atoms are at great distances from each other, they are attracted; this force decreases rather rapidly with an increase in the distance between them. As they approach each other, the attractive force grows and reaches its greatest value when the atoms come very close to each other. As they come even closer, the attraction weakens and, finally, at the equilibrium distance the force of the interaction vanishes. When the atoms become closer than the equilibrium distance, repulsive forces arise, which sharply increase and quickly make a further decrease in distance practically impossible.

Equilibrium distances (below we shall say distances for the sake of brevity) between atoms are different for various sorts of atoms.

For various pairs of atoms, not only are the distances between the vertical axis and the bottom of the well different, but so are the depths of the wells.

The depth of a well has a simple meaning—in order to roll out of the well, an energy just equal to the depth is needed. Therefore, the depth of a well can be called the binding energy of the particles.

The distances between the atoms of a molecule are so small that it is necessary to choose appropriate units for their measurement; otherwise, their values would have to be expressed, for example, in the following form: 0.000 000 012 cm. This figure is for an oxygen molecule.

Units especially convenient for describing the atomic world are called *angstroms* (true, the name of the Swedish scientist in whose honour these units were named is properly spelt Angström; in order to remember this, a small circle is placed over the letter A);

$$1\ \text{Å} = 10^{-8}\ \text{cm}$$

i.e. one-hundred millionth of a centimeter.

Distances between the atoms of a molecule lie within the limits of 1 to 4 Å. The equilibrium distance for oxygen, which has been written out above, is equal to 1.2 Å.

Interatomic distances, as you see, are very small. If we gird the Earth with a string at the equator, then the length of the "belt" will be as many times greater than the width of your palm as the latter is greater than the distance between the atoms of a molecule.

Ordinary calories are used for measuring binding energies, not in a single molecule, which would, of course, yield a negligible number, but in a gram-molecule, i.e. in the number of grams equal to the relative molecular weight.

It is clear that the binding energy in a gram-molecule divided by Avogadro's number, $N_A = 6.023 \times 10^{23}$, yields the binding energy of a single molecule.

The binding energy of the atoms in a molecule, just as interatomic distances, varies within narrow limits.

For the same oxygen, the binding energy is equal to 116 000 calories per gram-molecule, for hydrogen, 103 000 calories, etc.

We have already said that the atoms in a molecule are distributed in an entirely definite manner with respect to each other, forming in complicated cases rather intricate structures.

Let us present several simple examples. In a molecule of CO_2 (carbon dioxide), all three atoms are lined up in a row, with the carbon atom in the middle. A molecule of H_2O (water) has an angular form, with the oxygen atom at the vertex of the angle (it is equal to 105°).

In a molecule of NH_3 (ammonia), the nitrogen atom is situated at the vertex of a three-faced pyramid; in a molecule of CH_4 (methane), the carbon atom is located in the center of a four-faced figure with equal sides, which is called a tetrahedron.

The carbon atoms of C_6H_6 (benzene) form a regular hexagon. The bonds of the carbon atoms with the hydrogen atoms go from all the vertices of the hexagon. All the atoms are situated in one plane.

Diagrams of the distribution of the centers of the atoms in these molecules are shown in Figures 87 and 88. The lines symbolize the bonds.

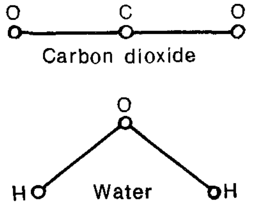


Fig. 87

A chemical reaction has occurred; there were molecules of one sort, and then others were formed. Some bonds have been broken, while others have been newly created. In order to break the bonds between atoms—recall Figure 86—it is necessary to perform work, just as in rolling a ball out of a well. On the contrary, energy is given off when new bonds are formed, just as when a ball rolls into a well.

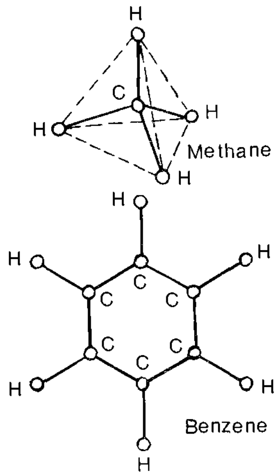


Fig. 88

Which is greater, the work involved in breaking or in creating bonds? We come across reactions of both types in nature.

The excess energy is called the *thermal effect*, or more concisely, the *heat of transformation* (*reaction*). A heat of reaction is usually a quantity of the order of tens of thousands of calories per mole. The heat of reaction is often included as a summand in the formula for a reaction.

For example, the reaction whereby carbon in the form of graphite burns, i.e. unites with oxygen, is written out as follows:

$$C + O_2 = CO_2 + 94\,250 \text{ cal}$$

This means that when C combines with O_2, an energy of 94 250 calories is given off.

The sum of the internal energies of a gram-atom of carbon in the form of graphite and a gram-molecule of oxygen is equal to the internal energy of a gram-molecule of carbon dioxide plus 94 250 calories.

Thus, such formulas have the transparent meaning of algebraic equalities written in terms of the values of the internal energies.

With the aid of such equations, one can find the heats of reaction for which direct methods of measurement, as a result of one or another cause, are unsuitable. Here is an example: if carbon (graphite) were to combine with hydrogen, the gas acetylene would be formed:

$$2C + H_2 = C_2H_2$$

The reaction does not proceed in this manner. Nevertheless, it is possible to find its thermal effect. We write down three known reactions

the oxidation of carbon:

$$2C + 2O_2 = 2CO_2 + 188\,000$$

the oxidation of hydrogen:

$$H_2 + \frac{1}{2}O_2 = H_2O + 68\,000$$

the oxidation of acetylene:

$$C_2H_2 + \frac{5}{2}O_2 = 2CO_2 + H_2O + 312\,000$$

All these equalities may be regarded as equations for the binding energies of molecules. If so, then we may operate on them as on algebraic equalities. Subtracting the first two equalities from the third, we obtain:

$$2C + H_2 = C_2H_2 - 56\,000$$

Therefore, the reaction we are interested in is accompanied by the consumption of 56 000 calories per gram-molecule.

Interaction of Molecules

There can be no doubt of the fact that molecules attract each other. If they stopped doing so for an instant, all liquid and solid bodies would decompose into molecules.

Molecules repel each other, and neither can this be doubted, because liquids and solids would otherwise contract with extraordinary ease.

Forces are exerted between molecules, which resemble in many respects the forces between atoms spoken of above. The potential energy curve, which we have just drawn for atoms, gives a true picture of the basic features of a molecular interaction. However, there are also essential differences between these interactions.

Let us compare, for example, the equilibrium distances between atoms of oxygen forming a molecule and atoms of oxygen in two neighbouring molecules, drawn to their equilibrium positions in solidified oxygen. The difference will be very noticeable: oxygen atoms forming a molecule settle down at a distance of 1.2Å, while oxygen atoms of different molecules approach each other to within 2.9 Å.

Analogous results have also been obtained for other atoms. Atoms of different molecules settle down farther from each other than atoms of the same molecule. It is therefore easier to tear molecules apart from each other than atoms from a molecule; moreover, the difference in energy is much greater than the difference in distance. While the energy necessary for breaking the bonds between oxygen atoms forming a molecule is about 100 kcal/mol, the energy needed to pull oxygen molecules asunder is less than 2 kcal/mol.

Hence, on a potential energy curve for molecules, the "well" lies farther away from the vertical axis and, furthermore, the "well" is much shallower.

However, this does not exhaust the difference between the interaction of atoms forming a molecule and the interaction of molecules.

Chemists have shown that an atom is bound in a molecule with a fully determined number of other atoms. If two hydrogen atoms have formed a molecule, then no third atom will join with them to this end. An oxygen atom in water is bound to two hydrogen atoms, and it is impossible to bind another atom to them.

We do not find anything similar in intermolecular interactions. Having attracted one neighbour to itself, a molecule does not to any degree lose its "attractive force". The approach of neighbours will continue as long as there is enough room.

What does "there is enough room" mean? Are molecules really something like apples or eggs? Of course, in a certain sense such a comparison is justified: molecules are physical bodies possessing definite "sizes" and "forms". The equilibrium distance between molecules is nothing but their "sizes".

What Thermal Motion Looks Like

The interaction between molecules can have greater or smaller values during the "lives" of the molecules.

The three states of matter—gaseous, liquid and solid—

differ from one another in the role which molecular interaction plays in them.

The word "gas" was thought up by scientists (derived from the Greek *chaos*, meaning "disorder").

And as a matter of fact, the gaseous state of matter is an example of the existence in nature of complete, perfect disorder in the mutual distribution and motion of particles. There is no microscope which would permit one to see the motion of gaseous molecules, but in spite of this, physicists can describe the life of this invisible world in sufficient detail.

There is an enormous number of molecules, approximately 2.5×10^{19} (i.e. 25 billion billions of molecules), in a cubic centimeter of air under normal conditions (room temperature and atmospheric pressure). Each molecule's share is a volume of 4×10^{-20} cm^3, that of a small cube whose sides are approximately 3.5×10^{-7} cm = 35 Å. However, the molecules are much smaller. For example, molecules of oxygen and nitrogen—the basic components of air—have an average size of about 4 Å.

Therefore, the average distance between the molecules is 10 times as great as the size of the molecules. And this in turn implies that the average molecule's share of atmospheric volume is approximately 1000 times as great as the volume of the molecule itself.

Imagine a plane surface on which coins have been thrown in a random manner, where there is an average of a hundred coins to each square meter. This means one or two coins on a page of the book which you are reading. This is roughly how sparsely gas molecules are distributed.

Every molecule of a gas is in a state of continual thermal motion.

Let us follow a single molecule. Here it is swiftly moving somewhere to the right. If it met no obstacles in its path, the molecule would continue its motion with the same velocity along a straight line. But the molecule's path is crossed by its innumerable neighbours. Collisions are inevitable, and the molecules fly apart like two colliding billiard balls. In which direction will our molecule gallop? Will it acquire or lose its speed? Anything is possible: for its collisions can be of the most various kinds. Blows are possible from the front or from behind, from the right or the left, which are strong or weak. It is clear that being subject to such irregular impacts during these random collisions, the molecule which we are observing will rush about through all parts of the vessel in which gas is confined.

How far are gas molecules able to go without a collision?

It depends on the sizes of the molecules and the density of the gas. The larger the molecules and the more there are of them in a vessel, the more often will they collide. The average distance travelled by a molecule without any impact—it is called the mean free path—is equal to 11×10^{-6} cm = 1100 Å for hydrogen molecules and 5×10^{-6} cm = 500 Å for oxygen molecules under ordinary conditions. The distance of 5×10^{-6} cm (one twenty-thousandth of a millimeter) is very small, but it is far from small in comparison with molecular sizes. A distance of 10 m for a billiard ball corresponds in scale to a path of 5×10^{-6} cm for an oxygen molecule.

The structure of a liquid differs essentially from that of a gas, whose molecules are far from each other and only rarely collide. In a liquid, a molecule is constantly found in the immediate vicinity of others. The molecules of a liquid are distributed like potatoes in a sack. True, with one distinction—the molecules of a liquid are in a state of continual chaotic thermal motion. Because they are so crowded, they cannot move around as freely as the molecules of a gas. Each of them is always "marking time" in practically one and the same place, surrounded by the same neighbours, and only gradually moves through the volume occupied by the liquid. The more viscous the liquid, the slower will be this displacement. But even in such a "mobile" liquid as water, a molecule moves 3 Å during the time required by a gas molecule to cover 700 Å.

The forces of interaction between molecules deal very resolutely with their thermal motion in solid bodies. In solid matter, the molecules are almost always in a fixed position. The only effect of the thermal motion is that the molecules are continually vibrating about their equilibrium positions. The lack of systematic displacements by the molecules is precisely the cause of what we call solidity. In fact, if molecules do not change neighbours, then all the more will the separate parts of the body remain in a fixed bond with one another.

Compressibility of Bodies

As raindrops drum on a roof, so do gas molecules beat against the walls of a vessel. The number of these blows is immense, and it is their united action that creates the pressure which can move the piston of an engine, explode a shell or blow up a balloon. A hail of molecular blows—this is atmospheric pressure, this is the pressure that makes the lid of a boiling tea-kettle jump, this is the force driving a bullet out of a rifle.

What is gas pressure related to? It is clear that the stronger the blow inflicted by a single molecule, the greater will be the pressure. It is no less obvious that the pressure will depend on the number of blows inflicted in a second. The more molecules in a vessel, the more frequent the blows and the greater the pressure. Hence, the pressure p of a given gas is proportional, first of all, to its density.

If the mass of a gas is constant, then decreasing its volume, we increase its density by the corresponding factor. Therefore, the pressure of a gas in a closed vessel will be inversely proportional to its volume. Or, in other words, the product of the pressure by the volume should be constant:

$$pV = \text{const}$$

This simple law was discovered by the English physicist Boyle and the French scientist Mariotte. Boyle's law (also known as Mariotte's law) is one of the first quantitative laws in the history of physical science. Of course, it holds when the temperature is constant.

As a gas is compressed, the Boyle equation is satisfied worse and worse. The molecules approach each other and the interactions between them begin to influence the behaviour of the gas.

Boyle's law is valid in those cases when the interference of the forces of interaction in the lives of the gas molecules is completely insignificant. One therefore speaks of Boyle's law as a law of ideal gases.

The adjective "ideal" sounds rather funny when modifying the word "gas". Ideal means perfect, so that it is impossible to be better.

The simpler a model or diagram, the more ideal it is for the physicist. Computations are simplified, explanations of physical phenomena become easy and clear. The term "ideal gas" pertains to the simplest model of a gas. The behaviour of sufficiently rarefied gases is practically indistinguishable from the behaviour of ideal gases.

Liquids are much less compressible than gases. In a liquid, the molecules already are in "contact". Compression consists only in improving the "packing" of the molecules, and for very high pressures, in pressing the molecules themselves.

The degree to which the forces of repulsion hinder the compression of a liquid can be seen from the following figures. A rise in pressure from one to two atmospheres entails a decrease in the volume of a gas by a factor of two, while the volume of water changes by 1/20 000 and that of mercury, by a total of 1/250 000.

Even the enormous pressure in the depths of an ocean is incapable of compressing water to any noticeable extent. In fact, a pressure of one atmosphere is created by a ten-meter column of water. The pressure under a 10-km layer of water is equal to 1000 atmospheres. The water's volume decreases by 1000/20 000, i.e. by 1/20.

The compressibility of solid bodies differs little from that of liquids. This is understandable—in both cases the molecules already are in contact, and so a compression can only be achieved at the expense of a further drawing together of molecules which are already strongly repelling each other. By means of ultrahigh pressures of 50-100 thousand atmospheres, we are able to compress steel by 1/1000, and lead by 1/7, of its volume.

It is clear from these examples that under terrestrial conditions, we cannot succeed in compressing solid matter to any significant extent.

But in the Universe, there are bodies where matter is compressed with incomparably greater strength. Astronomers discovered the existence of stars in which the density of matter reaches 10^6 g/cm^3. Inside these stars—they are called white dwarfs ("white"—for the nature of their luminosity, "dwarfs"—because of their relatively small sizes)—there should therefore be enormous pressure.

Change of Pressure with Altitude

Pressure falls with an increase in altitude. This was first clarified by the Frenchman Périer in 1648 on the instructions of Blaise Pascal. Mt. Puy de Dôme, near where Florin Périer lived, was 975 m high. Measurements showed that the mercury in a Torricellian tube falls by 8 mm when this mountain is climbed.

A fall in air pressure with an increase in altitude is quite natural. For a smaller column of air then presses down on the instrument.

If you have ever flown in an airplane, then you should know that there is an instrument on the front wall of the cabin, indicating the airplane's altitude with an accuracy to within tens of meters. This instrument is called an altimeter. This is an ordinary barometer, but it has been calibrated to show heights above sea level.

Pressure falls with an increase in altitude; let us find a formula giving this dependence. We single out a small layer of air with an area of a square centimeter, located between altitudes h_1 and h_2. The change of density with altitude is hardly noticeable within a layer which is not too large. Therefore, the weight of the volume of air we have singled out (it is a small cylinder of height $h_2 - h_1$ and base area of a square centimeter) will be $mg = \rho (h_2 - h_1) g$. This weight is just what yields the fall in pressure caused by rising from altitude h_1 to altitude h_2. That is

$$\frac{p_1 - p_2}{\rho} = g(h_2 - h_1)$$

But according to Boyle's law, the density of a gas is proportional to its pressure. Consequently,

$$\frac{p_1 - p_2}{p} \propto (h_2 - h_1)$$

On the left is the fraction by which the pressure grew when the altitude was lowered from h_2 to h_1. Hence, a growth in pressure by one and the same per cent will correspond to identical drops of $h_2 - h_1$.

Measurements and calculations, in complete agreement with each other, show that the pressure will fall by 0.1 for each kilometer rise above sea level. The same also holds for descents into deep shafts under sea level—the pressure will increase by 0.1 of its value when we descend by one kilometer.

We are talking about a change of 0.1 from the value at the previous altitude. This means that during an ascent of one kilometer, the pressure decreases to 0.9 of the pressure at sea level; during an ascent through the next kilometer, it will become equal to 0.9 of 0.9 of the pressure at sea level; at an altitude of 3 kilometers, the pressure will be equal to 0.9 of 0.9 of 0.9, i.e. $(0.9)^3$, of the pressure at sea level. It is not difficult to continue this reasoning further.

Denoting the pressure at sea level by p_0, we can write out the pressure at altitude h (expressed in kilometers):

$$p = p_0 (0.87)^h = p_0 \times 10^{-0.06h}$$

A more precise number is written in parentheses: 0.9 is the rounded-off value. The formula presupposes the identical temperature at all altitudes. But as a matter of fact, the temperature of the atmosphere changes with altitude and does so, moreover, in accordance with a rather complicated law. Nevertheless, the formula yields fairly good results and may be used for altitudes up to hundreds of kilometers.

It is not hard to determine with the aid of this formula that on the top of the Elbrus—about 5.6 km—the pressure will fall by a factor of approximately two, while at an altitude of 22 km (the record height of a stratospheric balloon's ascent with people), the pressure will fall to 50 mm Hg.

When we say that a pressure of 760 mm Hg is normal, we must not forget to add, "at sea level". At an altitude of 5.6 km, the normal pressure will not be 760, but 380 mm Hg.

Along with pressure, air density also falls with an increase in altitude, according to the same law. At an altitude of 160 km, not much air will remain.

In fact,

$$(0.87)^{160} = 10^{-10}$$

The air density at the Earth's surface is equal to about 1000 g/m^3, which means that according to our formula, there should be 10^{-7} g of air in a cubic meter at an altitude of 160 km. But in reality, as measurements performed with the aid of rockets show, the air density at this height is ten times as great.

Our formula gives us an even greater underestimation for heights of several hundreds of kilometers. The change in temperature with altitude and also a particular phenomenon—the decay of air molecules under the action of solar radiation—are responsible for the fact that the formula becomes useless at great heights. Here we shall not go into this.

Vacuum

A vessel which is technically empty still contains an enormous number of molecules.

Molecules of gas constitute a considerable hindrance in many physical instruments. Radio tubes, X-ray tubes, accelerators of elementary particles—all these instruments require a vacuum (from the Latin word *vacuus*—empty), i.e. space free of gas molecules. There should also be a vacuum in an ordinary electric lamp. If air enters a lamp, it will oxidize and immediately burn out.

In the best vacuum instruments vacuum of the order of 10^{-8} mm Hg is produced. A completely negligible pressure, it would seem: the level of mercury in a manometer would move by a hundred-millionth of a millimeter if the pressure changed by such an amount.

However, there are still several hundred million molecules in a cubic centimeter at this meagre pressure.

It is interesting to compare the void of interstellar space with such a vacuum—there one finds an average of one elementary particle of matter in several cubic centimeters.

Special pumps are employed in order to obtain vacuum. An ordinary pump, removing gas by means of the motion of a piston, can create a vacuum of at best 0.01 mm Hg. A good or, as one says, high vacuum can be obtained with the aid of a so-called diffusion pump, mercury or oil, in which gas molecules are caught up in a stream of mercury or oil vapour.

Mercury pumps, bearing the name of their inventor, Langmuir, start working only after a preliminary exhaustion to a pressure of about 0.1 mm Hg; such a preliminary rarefaction is called creating a forevacuum.

The principle behind its action consists of the following. A small glass container is connected to a vessel containing mercury, the space being exhausted and a fore-pump. The mercury is heated and the fore-pump carries away its vapour. The mercury vapours capture molecules of the gas along the way and bring them to the fore-pump. The atoms of mercury condense (cooling by means of running water is provided for), and the liquid trickles down into the vessel from which the mercury began its trip.

A vacuum obtained under laboratory conditions, as we have just said, is still far from empty in the absolute sense of the word. A vacuum is greatly rarefied gas. The properties of such a gas may differ essentially from those of an ordinary gas.

The motion of the molecules "forming a vacuum" changes its character when the mean free path of a molecule becomes greater than the dimensions of the vessel containing the gas. The molecules then rarely collide with each other and carry out their trips in straight zigzags, striking against first one and then another wall of the vessel.

Let us compute a pressure at which this will occur. It was said above that a molecule's mean free path in air at atmospheric pressure is equal to 5×10^{-6} cm. If we increase it by a factor of 10^7, it will be 50 cm, i.e. will be noticeably greater than an average sized vessel. Since the mean free path is inversely proportional to the density, and hence also to the pressure, the pressure should be 10^{-7} of atmospheric pressure for this, or approximately 10^{-4} mm Hg.

Even interplanetary space is not entirely empty. But the density of the matter in it is about 5×10^{-24} g/cm^3. The main component of interplanetary matter is atomic hydrogen. At the present time, it is considered that cosmic space contains several hydrogen atoms per cubic centimeter. If a hydrogen molecule were enlarged to the size of a pea and placed in Moscow, then its nearest "cosmic neighbour" would prove to be in Tula.

Crystals

Many people think that crystals are beautiful, rarely found stones. They occur in various colours, are usually transparent and, what is most remarkable, possess a beautiful regular form. Crystals are most often polyhedra with ideally plane sides (faces) and strictly straight edges. They please the eye with a marvellous play of colours at the faces and an amazingly regular structure.

Among them are the unassuming crystals of rock salt—natural sodium chloride, i.e. common salt. They are found in nature in the form of rectangular parallelepipeds or cubes. Calcite crystals also have a simple form—transparent oblique-angled parallelepipeds. Quartz crystals are much more complicated. Each little crystal has a great many faces of different forms, intersecting in edges of different lengths.

However, a crystal is anything but a museum-piece. Crystals surround us everywhere. The solid bodies with which we build homes and make machines, the substances which we use in our daily lives—almost all of them are crystals. But why do we not see this? The reason is that in nature we rarely come across bodies in the form of single individual crystals (or, as is said, monocrystals). Matter is most often found in the form of firmly linked crystalline grains of very small size—less than a thousandth of a millimeter. Such a structure can only be seen through a microscope.

Bodies consisting of tiny crystalline grains are called polycrystalline (from the Greek *polys*, meaning "many").

Of course, polycrystalline bodies must also be included among the crystals. It will then turn out that almost all the solid bodies surrounding us are crystals. Sand and granite, copper and iron, the salol sold in a drug store and paint—all these are crystals.

There are exceptions; glass and plastics do not consist of crystals. Such solid bodies are called amorphous.

Thus, studying crystals means studying almost all the bodies surrounding us. It is obvious that this is important.

Single crystals are recognized at once by the regularity of their forms. Plane faces and straight edges are characteristic properties of a crystal; the regularity of form is undoubtedly related to the regularity of the internal structure of a crystal. If a crystal has been especially stretched in a certain direction, then it means that the structure of the crystal in this direction is also special in some way.

But imagine that a ball has been made by machine out of a large crystal. Will we succeed in figuring out that we have a crystal in our hands, in distinguishing this ball from a glass ball? The natural form of a crystal shows that a crystal is different in different directions. If this difference becomes apparent with respect to form, it should also exist with respect to other properties. The strength of a crystal, its electrical properties, its conduction of heat—all these properties may be different in different directions. This peculiarity of a crystal is called the anisotropy of its properties. Anisotropic means different properties in different directions.

Crystals are anisotropic. On the contrary, amorphous bodies, liquids and gases are isotropic, i.e. possess identical (from the Greek *isos*, meaning "equal") properties in different directions (from the Greek *tropos*, meaning "turning").

The anisotropy of crystals' properties is precisely what permits us to find out whether or not a transparent, formless piece of matter is a crystal.

Structure of Crystals

Why is the regular form of a crystal so beautiful? Its faces, smooth and shining, look as though a skilful polisher has worked on them. Separate parts of a crystal repeat each other, forming a beautiful symmetric figure.

There can be only one answer to the question we posed—to the external beauty there must correspond an internal regularity. This regularity consists in a multiple repetition of one and the same basic parts.

Imagine a park fence made out of rods of different lengths and distributed helter-skelter. An ugly scene! A good fence is constructed from identical rods distributed in a regular sequence at identical distances from each other.

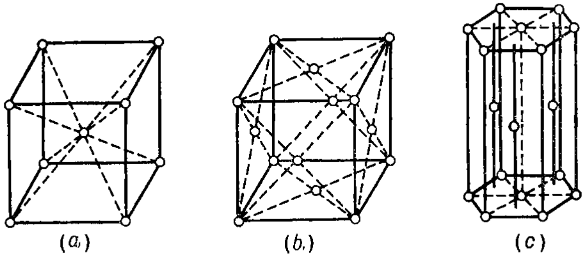


Fig. 89

We find such a self-repeating picture on wall-paper. Here an element of a drawing—say, a girl playing with a ball—is repeated not only in one direction, as in a park fence, but so that it fills a plane.

But what relationship do a park fence and wall-paper have to a crystal? A most direct one. A park fence consists of links repeating along a line, wall-paper, of pictures repeating in a plane, and a crystal, of groups of atoms repeating in space. One therefore says that the atoms of a crystal form a space (or crystal) lattice.

The structure of many hundreds of crystals is known at the present time. We shall tell the reader about the structure of the simplest crystals, and first of all, about those which are built up out of atoms of a single sort.

Three types of lattices are most common. They are shown in Figure 89. The centers of the atoms are represented by points; the lines joining the points do not have any actual meaning. They have been drawn merely to make the nature of the atoms' spatial distribution clearer to the reader.

Figures 89*a* and *b* depict cubic lattices. In order to visualize these lattices more clearly, imagine that you have arranged building blocks in the simplest manner—edge to edge and face to face.

If you now conceptually place points at the vertices and centers of the cubes, then the cubic lattice depicted in Figure 89*a* appears. Such a structure is called body-centered cubic. If points are placed at the vertices of the cubes and in the centers of their faces, then the cubic lattice depicted in Figure 86*b* appears. It is called face-centered cubic.

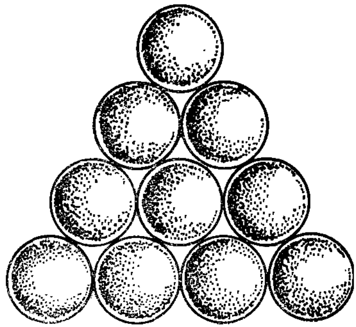


Fig. 90

The third lattice (Figure 89*c*) is called close-packed hexagonal (i.e. having six angles). In order to understand the origin of this term and more clearly visualize the distribution of the atoms in this lattice, let us take some billiard balls and start packing them as closely as possible. First of all, let us form a close layer—it looks like billiard balls which have been gathered in by a "triangle" before the beginning of a game (Figure 90). Note that the ball in the middle of the triangle is in contact with six neighbours, and these six neighbours form a hexagon. We continue the packing by laying one layer upon another. If we place the balls of the second layer directly above the balls of the first layer, then such a packing would not be close. Trying to distribute the greatest number of balls in a definite volume, we should place the balls of the second layer in the holes formed by the first layer, the balls of the third in the holes of the second, etc. In a close-packed hexagonal lattice, the balls of the third layer are placed in such a way that their centers lie directly above the centers of the balls of the first layer.

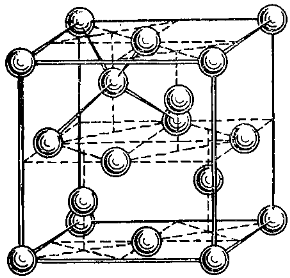


Fig. 91

The centers of the atoms in a close-packed hexagonal lattice are distributed just like those of the balls which are closely packed in the manner described.

A great many elements crystallize into lattices of the three types described above:

Close-packed hexagonal lattice . .	Be, Co, Hf, Ti, Zn, Zr
Face-centered cubic	Al, Cu, Co, Fe, Au, Ge, Ni, Ti
Body-centered cubic	Cr, Fe, Li, Mo, Ta, Ti, U, V

We shall mention only a few of the other structures. The structure of diamond is depicted in Figure 91. What is characteristic for this structure is that a carbon atom in diamond has four immediate neighbours. Let us compare this number with the corresponding numbers for the three most common structures just described. As is evident from the figures, each atom has 12 immediate neighbours in a close-packed hexagonal lattice, the atoms forming a face-centered cubic lattice have just as many neighbours, and each atom has eight neighbours in a body-centered lattice.

We shall say a few words about graphite, whose structure is shown in Figure 92. This structure has a striking peculiarity. Graphite consists of layers of atoms, where atoms in

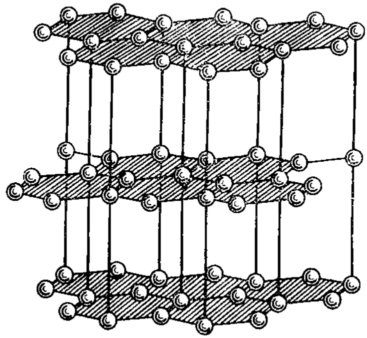


Fig. 92

a single layer are more firmly bound to each other than atoms of neighbouring layers. This is related to the values of the interatomic distances: the distance between neighbours in a single layer is 2.5 times as small as the shortest distance between layers.

The presence of weakly bound atomic layers makes it easy for graphite crystals to break up along these layers. This is why solid graphite can serve as a lubricant in those cases when it is impossible to apply lubricating oil—for example, at very low or very high temperatures. Graphite is a solid lubricant.

Friction between two bodies reduces, roughly speaking, to the fact that microscopic protuberances of one body sink into hollows of the other. The force required for breaking up a microscopic graphite crystal is much smaller than the frictional forces; therefore, if there has been a graphite lubrication, the sliding of one body along another will be considerably eased.

There is an endless variety in the structures of crystals of chemical compounds. The structures of rock salt and carbon dioxide, depicted in Figures 93 and 94, can serve as extreme examples.

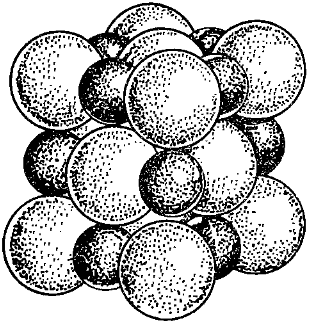


Fig. 93

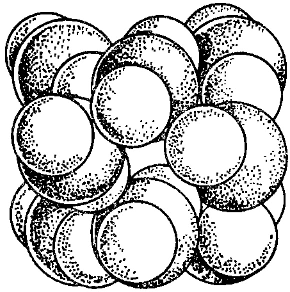


Fig. 94

Crystals of rock salt (Figure 93) consist of atoms of sodium (small dark balls) and chlorine (large light balls) alternating along the axes of a cube.

Each sodium atom has six equidistant neighbours of the other sort. The same is also true of chlorine. But where are the molecules of sodium chloride? There are none; not only are groups of single atoms of sodium and single atoms of chlorine absent from salt crystals, but in general, no group of atoms whatsoever can be distinguished from the others by their proximity.

The chemical formula of NaCl does not give us any grounds for saying that "this substance is built up out of molecules of NaCl". The chemical formula merely indicates that the substance is constructed from the same number of atoms of sodium and chlorine.

The question of the existence of molecules in a substance is decided by its structure. If no groups of close atoms can be distinguished in it, then there are no molecules. Crystals without molecules are called atomic.

A crystal of carbon dioxide, CO_2 (the dry ice which lies in cartons of ice cream), is an example of a molecular crystal (Figure 94).

The centers of the oxygen and carbon atoms of a molecule of CO_2 are situated along a straight line. The distance C—O is equal to 1.3 Å. But the distance between oxygen atoms of neighbouring molecules is about 3 Å. It is obvious that under such conditions, we immediately "recognize" a molecule in a crystal.

Molecular crystals are close packings of molecules. In order to see this, we must outline the contours of the molecules. This is precisely what has been done in Figure 94.

Eleven
TEMPERATURE

Thermometer

If two differently heated bodies are brought into contact, the warmer one will cool off and the colder one will warm up. It is said that two such bodies exchange heat; of course, a case when one person gives another a hundred roubles, and the other person takes them, is not called an exchange in ordinary life, but this is the accepted terminology in physics.

As we have already said, heat exchange is a kind of energy transfer; the body which gives off energy is called hotter. We feel that a body is hot if it warms our hand, i.e. transfers energy to it. On the contrary, if a body is felt to be cold, then this means that it is taking energy away from our body.

Concerning a body which is giving off heat (i.e. giving off energy by means of heat exchange), we say: its temperature is higher than the temperature of the body which is taking in this heat.

Observing whether an object of interest to us is cooling off or warming up in the presence of one or another body, we find "its place" in a row of heated bodies. Temperature is a kind of mark, indicating for which bodies the object of interest to us will be a giver, and for which a receiver, of heat.

Temperature is measured by thermometers.

The structure of thermometers can be based on the utilization of various properties of bodies, sensitive to temperature. The most frequently utilized property is the expansion of bodies during a rise in temperature.

If the body of a thermometer changes its volume when in contact with various bodies, this implies that these bodies have different temperatures. When the volume of the thermometer's body is greater, the temperature is higher, and when the volume is smaller, the temperature is lower.

There are various substances that can serve as thermometers: liquids, such as mercury or alcohol, solids, such as metals, and gases. But different substances expand differently, and so mercurial, alcoholic, gaseous and other "degrees" will not coincide. Of course, two basic points—the melting point of ice and the boiling point of water—can always be marked on all thermometers. Therefore, all thermometers will indicate 0 and 100 degrees centigrade identically. But bodies will not expand identically between 0 and 100 degrees. One body expands rapidly between 0 and 50 degrees on a mercury thermometer, and slowly in the second part of this interval, but another, vice versa.

Having made thermometers with differently expanding bodies, we will discover noticeable discrepancies in their readings, in spite of the fact that their readings will coincide for the basic points. Moreover, a water thermometer would lead us to the following discovery: if a body cooled to zero is placed on an electric stove, its "water temperature" would first fall and then rise. This happens because water at first decreases its volume when heated, and only later behaves "normally", i.e. increases its volume when heated.

We see that a rash choice of material for a thermometer can bring us to an impasse.

But then what should we be guided by in choosing a "true" thermometer? Which body would be ideal for this purpose?

We have already spoken about such ideal bodies. They are the ideal gases. There are no interactions between the particles of an ideal gas and, studying the expansion of an ideal gas, we study how the motion of its molecules changes. This is precisely the reason why an ideal gas is an ideal body for a thermometer.

And it really is a striking fact that while water does not expand like alcohol (nor alcohol like glass, nor glass like iron), hydrogen, oxygen, nitrogen and any other gas in a sufficiently rarefied state to deserve being called ideal expand in exactly the same fashion when heated.

Therefore, the changes in volume undergone by a definite quantity of ideal gas serve as the basis for defining temperature in physics. Of course, in view of the fact that gases are highly compressible, one must be especially careful in seeing to it that the gas is under constant pressure.

In order to graduate a gas thermometer, we should accurately measure the volume of the gas we have chosen at 0° and at 100 °C. We shall divide the difference between the volumes V_{100} and V_0 into 100 equal parts. In other words, the change in the volume of the gas by $(V_{100} - V_0)/100$ corresponds to one degree centigrade (1 °C).

Let us now suppose that our thermometer shows a volume V. What temperature t °C corresponds to this volume? It is not difficult to comprehend that

$$t\,^{\circ}\mathrm{C} = \frac{V - V_0}{V_{100} - V_0} 100$$

i.e.

$$\frac{t\,^{\circ}\mathrm{C}}{100} = \frac{V - V_0}{V_{100} - V_0}$$

By means of this equality, we assign each volume V to a temperature t and obtain the temperature scale* which physicists use.

As the temperature increases, the volume of the gas increases without bound—there is no theoretical limit to the growth in temperature. On the contrary, low (negative on the centigrade scale) temperatures have a limit.

For what will happen when the temperature is lowered? A real gas will eventually turn into a liquid, and with an even greater fall in temperature, will solidify. The gas molecules will gather in a small volume. But what will this volume be equal to for a thermometer filled with an ideal gas? Its molecules do not interact with each other and do not have any volume of their own. Hence, a decrease in temperature brings an ideal gas to a zero volume. It is quite possible to come as close as we wish in practice to a behaviour that is characteristic of an ideal gas, for example, to a zero vol-

* The *centigrade scale*, at which 0 °C is taken as the melting point of ice, and 100 °C, as the boiling point of water (both at the normal pressure of 760 mm Hg), is very convenient. In spite of this, the British and the Americans have so far been using a temperature scale which seems very strange to us. How, for example, will you react to the following sentence, taken from an English novel: "The summer wasn't hot, the temperature was 60-70 degrees." A misprint? No, the Fahrenheit scale (°F).

The temperature in England rarely falls below —20 °C. Fahrenheit selected a mixture of ice and salt, having approximately such a temperature, and took this temperature for his zero. In the words of the inventor, the normal temperature of a human body was taken for 100° on this scale. However, in order to determine this point, Fahrenheit probably made use of the services of a slightly feverish person. On the *Fahrenheit scale*, the average normal temperature of a human body is 98 °F. On this scale, water freezes at +32 °F and boils at 212 °F. The conversion formula will be:

$$t\ ^\circ\mathrm{C} = \frac{5}{9}(t-32)\ ^\circ\mathrm{F}$$

ume. For this it is necessary to fill up the thermometer with more and more rarefied gas. Therefore, we won't go wrong by assuming the minimum volume equal to zero.

According to our formula, the lowest possible temperature corresponds to a zero volume. This temperature is called the *absolute zero* of temperature.

In order to determine the position of absolute zero on the centigrade scale, we must substitute zero for the volume ($V = 0$) in the temperature formula just derived. Consequently, the temperature of absolute zero is equal to $-100V_0/(V_{100} - V_0)$.

It turns out that this remarkable point corresponds to a temperature of about -273 °C (more precisely, -273.15 °C).

Thus, there are no temperatures below absolute zero; for they would correspond to negative volumes of a gas. It doesn't make sense to speak of lower temperatures. It is just as impossible to obtain temperatures below absolute zero as to make a wire with diameter less than zero.

It is impossible to cool a body at absolute zero, i.e. one cannot take energy away from it. In other words, bodies and the particles they are made of have the least possible energy at absolute zero. This implies that the kinetic energy equals zero and the potential energy assumes its least possible value at absolute zero.

Since absolute zero is the lowest temperature, it is only natural that an absolute scale, in which readings begin at absolute zero, be used in physics, especially in those of its branches where low temperatures play an important role. It is clear that $T_{abs} = (t + 273)$ °C. Room temperature will be about 300° on the *absolute scale*. The absolute scale is also called the *Kelvin scale*, in honour of a well-known 19th century English scientist, and the notation T K is employed in place of T_{abs}.

A formula for a gas thermometer determining the absolute temperature T can be written down in the form

$$T = 100\,\frac{V - V_0}{V_{100} - V_0} + 273$$

Using the equality $100V_0/(V_{100} - V_0) = 273$, we arrive at the following simple result:

$$\frac{T}{273} = \frac{V}{V_0}$$

Therefore, the absolute temperature is simply proportional to the volume of an ideal gas.

Exact measurements of temperature require all kinds of contrivances on the part of the physicist. Mercury, alcohol (for Arctic regions) and other thermometers are graduated by comparison with a gas thermometer over a rather wide temperature interval. However, it too is also unsuitable for temperatures very close to absolute zero (below 0.7 K), when all gases liquefy, and also for temperatures above 600 °C, when gases penetrate glass. Other principles of temperature measurement are used for high and very low temperatures.

As for practical methods of measuring temperature, they are manifold. Instruments based on electrical phenomena are of great significance. It is now important to remember only one thing—during all measurements of temperature, we should be convinced that the reading obtained completely coincides with what a measurement of a rarefied gas' expansion would give.

High temperatures arise in ovens, furnaces and burners. Temperatures of 220-280 °C are attained in baking ovens. Higher temperatures are applied in metallurgy—hardening furnaces yield 900-1000 °C, forges yield 1400-1500 °C. Temperatures of 2000 °C are attained in steel furnaces.

The records for high temperature in a furnace are obtained with the aid of electric arcs (about 5000 °C). The flames of an arc make it possible "to deal" with the most refractory metals.

And what is the temperature of a gas burner's flame? The temperature in the inner bluish cone of flame is only 300 °C. The temperature in the outer cone attains 1800 °C.

Incomparably higher temperatures arise during the explosion of an atomic bomb. Judging by indirect estimates, the temperature at the center of the explosion attains several million degrees.

Attempts have recently been made to obtain such ultrahigh temperatures in special laboratory installations (Ogra, Zeta) built in the Soviet Union and in other countries. It has proved possible to attain temperatures up to two million degrees for very brief moments.

Ultrahigh temperatures also exist in nature, not on the Earth, but on other bodies in the Universe. In the centers of stars, the Sun in particular, the temperature attains tens of millions of degrees.

But the surfaces of stars have a considerably lower temperature, not exceeding 20 000 °C. The surface of the Sun gets heated up to 6000 °C.

Ideal Gas Theory

The properties of an ideal gas, giving us the definition of temperature, are very simple. The Boyle law is valid for constant temperatures: during changes in volume or pressure, the product pV remains fixed. For a constant pressure, the quotient V/T is conserved, no matter how the volume or temperature changes. It is easy to unite these two laws. It is clear that the expression pV/T remains the same as for a constant temperature, but with changing V and p, just

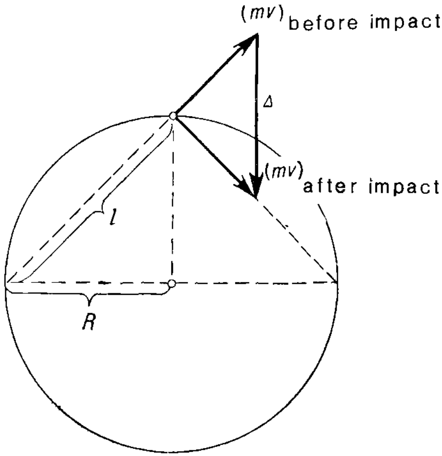


Fig. 95

as for a constant pressure, but with changing V and T. The expression pV/T remains constant during a change not only in any pair, but also simultaneously in all three, of the quantities p, V and T. The law $pV/T = \text{const}$ is known as the *equation of state of an ideal gas*.

An ideal gas is chosen for a thermometer because its properties depend only on the motion (but not on the interaction) of its molecules.

What is the nature of the relationship between the motion of molecules and temperature? For an answer to this question, it is necessary to find the relationship between the pressure of a gas and the motion of the molecules in it. In a spherical vessel of radius R N molecules are contained (Figure 95). Let us follow an arbitrary molecule, for example, one which is moving at a given moment from left to right along a chord of length l. We shall not pay attention to molecular collisions: such impacts do not affect the pressure. Having flown to the boundary of the vessel, the molecule will strike against the wall and fly off in some other direction with the same speed (the collision is elastic). Ideally, such a trip through the vessel might continue eternally. If v is the molecular velocity, each impact will occur after l/v seconds, i.e. each molecule will strike the wall v/l times a second. The continual hail of impacts by the N molecules unites into a single force of pressure.

According to Newton's law, the force is equal to the change in momentum during a unit of time. Let us denote the change in momentum at each impact by Δ. This change occurs v/l times a second. Consequently, the contribution to the force on the part of a single molecule will be $\Delta v/l$.

The momentum vectors before and after an impact, and also the momentum transfer Δ, have been constructed in Figure 95. It follows from the similarity of the triangles arising in the construction that $\Delta/l = mv/R$. The contribution to the force on the part of a single molecule will take the following form:

$$\frac{mv^2}{R}$$

Since the length of the chord does not occur in the formula, it is clear that molecules moving along arbitrary chords make an identical contribution to the force. Of course, the change in momentum will be smaller for an oblique impact, but then the impacts in this case will be more frequent. Calculations show that these two effects exactly compensate each other.

Since there are N molecules in the sphere, the resultant force will be equal to

$$\frac{Nmv_{\text{av}}^2}{R}$$

where v_{av} is the average molecular velocity.

The pressure p of the gas, equal to the force divided by the surface area of the sphere, $4\pi R^2$, is equal to

$$p=\frac{Nmv_{av}^2}{4\pi R^2 R}=\frac{\frac{1}{3}Nmv_{av}^2}{\frac{4}{3}\pi R^3}=\frac{Nmv_{av}^2}{3V}$$

where V is the volume of the sphere.

Therefore,

$$pV=\frac{1}{3}Nmv_{av}^2$$

This equation was first derived by Daniel Bernoulli in 1738*.

It follows from the equation of state of an ideal gas that $pV = \text{const}\cdot T$; we see from the equation just derived that pV is proportional to v_{av}^2. Hence,

$$T \propto v_{av}^2, \quad \text{or } v_{av} \propto \sqrt{T}$$

i.e. the average velocity of an ideal gas molecule is proportional to the square root of the absolute temperature.

Avogadro's Law

Assume that a substance is a mixture of different molecules. Isn't there a physical quantity, characterizing a motion, which would be identical for all these molecules, say for hydrogen and oxygen, provided their temperatures are identical?

* Of Swiss origin, D. Bernoulli worked and lived in Russia; he was a member of the St. Petersburg Academy of Sciences. No less well known is the activity of Johann (Jean) Bernoulli and Jakob (Jacques) Bernoulli. All three were brothers, not namesakes.

Mechanics yields an answer to this question. It can be proved that the average kinetic energy $mv_{av}^2/2$ of the translational motion will be identical for all molecules.

This implies that for a given temperature, the average square of the molecular velocities is inversely proportional to the mass of the particles, or

$$v_{av} \propto \frac{1}{\sqrt{m}}$$

Let us return to the equation $pV = (1/3)\ Nmv_{av}^2$. Since the quantities mv_{av}^2 are identical for all gases at a given temperature, the number N of molecules contained in a given volume at a definite pressure p and temperature T is identical for all gases. This remarkable law was first formulated by Avogadro.

But how many molecules are there in a cubic centimeter? It turns out that there are 2.7×10^{19} molecules in a cubic centimeter at 0 °C and 760 mm Hg. This is an enormous number. So that you can feel just how great it is, let us give an example. Suppose that gas is flowing out of a 1-cm^3 vessel with such a speed that a million molecules leave each second. It isn't hard to calculate that it will take the vessel a million years to get rid of the gas.

Avogadro's law shows that under a definite pressure and temperature, the ratio of the number of molecules to the volume in which they are contained, N/V, is a quantity that is identical for all gases.

Since the density of a gas $\rho = Nm/V$, the ratio of the densities of gases is equal to the ratio of their molecular weights:

$$\frac{\rho_1}{\rho_2} = \frac{m_1}{m_2}$$

The relative weights of molecules can therefore be established by means of a simple weighing of gaseous substances.

Such measurements once played a great role in the development of chemistry and have a significance today, too, when one has to find the molecular weight of a newly synthesized substance: it is only necessary to transform the substance (without spoiling it) into a gaseous state. Air is a mixture of gases, and in order to compare its density with that of other gases, it is convenient to introduce the average molecular weight of air. It turns out to be equal to 28.8. Knowing this figure, it is easy to find the density of various gases with respect to air. For example, water vapour with a molecular weight of 18 has a density of 18/28.8 = 0.62 with respect to air.

Molecular Velocities

Theory shows that for a constant temperature, the average kinetic energy of molecules, $mv_{\mathrm{av}}^2/2$, is identical. According to our definition of temperature, this average kinetic energy of the translational motion of the molecules of a gas is proportional to the absolute temperature. In the form of an equality, this important law can be written out as follows:

$$\left(\frac{mv^2}{2}\right)_{\mathrm{av}} = 2.1 \times 10^{-16} T$$

where the energy is measured in ergs.

We have already understood that temperature is some sort of measure of the intensity of thermal motion. But now we see that temperature measurements with a thermometer filled with an ideal gas add a meaning of rare simplicity to this measure. The temperature is proportional to the average value of the energy of the translational motion of the molecules.

Let us determine the average speed of oxygen molecules at room temperature, which we take to be 27 °C = 300 K in round numbers. The molecular weight of oxygen is 32, so the weight of one molecule equals $32/6 \times 10^{23}$. A simple computation yields $v_{av} = 4.8 \times 10^4$ cm/sec, i.e. about 500 m/sec. Molecules of hydrogen move considerably faster. Their masses are 16 times as small, and their speeds are $\sqrt{16} = 4$ times as great, i.e. are about 2 km/sec at room temperature. Let us estimate the thermal speed of a small particle which is visible through a microscope. An ordinary microscope permits us to see a dust particle of 1 micron (10^{-4} cm) in diameter. The mass of such a particle, with density close to one, will be in the neighbourhood of 5×10^{-13} g. We obtain about 0.5 cm/sec for its speed. It is not surprising that such a motion is quite noticeable.

The speed of the Brownian movement of a particle with a mass of 0.1 g will be only 10^{-6} cm/sec in all. It is no wonder that we do not see the Brownian movement of such particles.

We have spoken of the average spee of a molecule. But not all molecules move with the same speed; a certain fraction of the molecules move faster, but others move slower. It turns out that this can all be calculated. We shall only present the results.

At a temperature of about 15 °C, for example, the average speed of nitrogen molecules is equal to 500 m/sec; 59% of the molecules move with speeds between 300 and 700 m/sec. Only 0.6% of the molecules move with small speeds—from 0 to 100 m/sec. There are only 5.4% of fast molecules with speeds greater than 1000 m/sec (Figure 96).

It is also possible to calculate the distribution of molecules due to the energy of their translational motion.

The number of molecules whose energy is more than double the average is less than 10%. The fraction of still more "energetic" molecules falls off faster and faster as the energy increases. Thus, the number of molecules whose energy is at least 4 times as large as the average is only 0.7%, 8 times

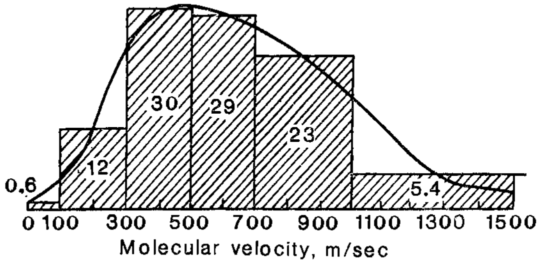


Fig. 96

as large as the average, $0.06 \times 10^{-4}\%$, 16 times as large as the average, $2 \times 10^{-8}\%$.

The energy of an oxygen molecule moving with a speed of 11 km/sec is equal to 32×10^{-12} erg. The average energy of a molecule at room temperature is equal to only 6×10^{-14} erg. Therefore, the energy of an "eleven-kilometer molecule" is at least 500 times as great as the energy of a molecule with average speed. It is not surprising that the fraction of the molecules with speeds higher than 11 km/sec is equal to an unimaginably small number—of the order of 10^{-300}.

But why are we intrigued with the speed of 11 km/sec? On p. 199 we spoke of the fact that only bodies having this speed can escape from the Earth. Hence, molecules which have risen to a great height can lose their ties to the Earth and take off in a distant interplanetary trip, but for this it is necessary to have a speed of 11 km/sec. The fraction of such fast molecules, as we have seen, is so negligible that there is no danger of the Earth's losing its atmosphere even in the course of a billion years.

The rate of leaving the atmosphere depends to an extraordinarily great degree on the gravitational energy GMm/r. If the average kinetic energy of a molecule is many times less than the gravitational energy, the escape of the molecules from the Earth is practically impossible. The gravitational energy on the surface of the Moon is 20 times as small, which gives an oxygen molecule a "runaway" energy of 1.5×10^{-12} erg. This value exceeds the value of a molecule's average kinetic energy by a factor of only 20-25. The fraction of the molecules capable of breaking away from the Moon is equal to 10^{-17}. This is already entirely different than 10^{-300}, and computations show that the air would leave the Moon quickly enough for interplanetary space. It is not surprising that there is no atmosphere on the Moon.

Thermal Expansion

If a body is heated, the motion of its atoms (molecules) will be more intensive. They will start pushing each other away and will occupy more space. The following well-known fact is explained by this: solids, liquids and gases expand when heated.

We don't have to say much about the thermal expansion of gases: in fact, the proportionality of temperature to volume was made the basis of our temperature scale.

We see from the formula $V = V_0T/273$ that the volume of a gas under a constant pressure grows by 1/273 (i.e. by 0.0037) of its value at 0 °C for each 1 °C increase in temperature (this situation is sometimes called Gay-Lussac's law).

Under ordinary conditions, i.e. at room temperature and normal atmospheric pressure, most liquids expand one-third to one-half as much as gases.

We have already spoken more than once of the anomalous expansion of water. The volume of water decreases as it is heated up from 0 to 4 °C. This anomaly in the expansion of water plays a colossal role in organic life on the Earth.

In autumn, the upper layers of water become denser and sink to the bottom as they cool off. Warmer water, rising from below, takes their place. But such a mixing takes place only until the temperature of the water falls to 4 °C. With a further lowering of temperature, the upper layers will no longer contract, will therefore not become heavier and will not sink to the bottom. Starting with this temperature, the upper layer, gradually cooling off, reaches zero degrees and freezes.

It is only this peculiarity of water that prevents rivers from freezing down to their beds. If water were to suddenly lose its remarkable anomaly, the disastrous consequences of this can be easily pictured even by a person without a rich imagination.

The thermal expansion of solids is considerably less than that of liquids. It is hundreds and thousands of times less than the expansion of gases.

Thermal expansion is an annoying hindrance in many cases. Thus, a change in the sizes of the moving parts of a clock-work with a change in temperature would lead to a change in the clock's speed, if a special alloy, invar (invariant means unchanging, whence the name "invar"), were not used for these delicate components. Invar, steel with a large nickel content, is widely used in the instrument manufacture. An invar rod is lengthened by only one millionth when its temperature increases by 1 °C.

An apparently negligible thermal expansion of a solid body can lead to serious consequences. The reason for this is that the low compressibility of solids makes it hard to hinder their thermal expansion.

When heated by 1 °C, a steel rod will increase in length by only one-hundred thousandth, i.e. by an amount unnoticeable to the unaided eye. However, in order to prevent the expansion and compress the rod one-hundred thousandth, a force of 20 kgf on a square centimeter is needed. And this is merely for cancelling the effect of a rise in temperature by only 1 °C!

The forces arising from thermal expansion can lead to breakages and catastrophes if they are not reckoned with. Thus, in order to avoid the action of these forces, the rails of a railroad-bed are laid with clearances. One has to remember these forces when handling glassware, which is easily cracked by non-uniform heating. It is therefore the practice in laboratories to use vessels made of quartz glass (fused quartz—silicon dioxide—exists in an amorphous state), which lack this drawback. For one and the same rise in temperature, a copper bar will be lengthened by a millimeter, while the same sized bar of quartz glass will change its length by the unnoticeable amount of 30-40 microns. The expansion of quartz is so insignificant that a quartz vessel can be heated by several hundred degrees and then thrown into water without any fear.

Heat Capacity

The internal energy of a body depends, of course, on its temperature. The more a body must be heated, the greater is the energy required. In order to raise a body's temperature from T_1 to T_2, it is required to supply an energy

$$Q = C\,(T_2 - T_1)$$

to it in the form of heat. Here C is the proportionality factor, which is called the *heat capacity* of the body. The definition of the concept of heat capacity follows from the formula: C is the amount of heat necessary for raising the temperature by 1 °C. The heat capacity, itself, also depends on the temperature: rises in temperature from 0 to 1 °C and from 100 to 101 °C require somewhat different amounts of heat.

The quantity C is usually defined for one gram and called the *specific heat*. It is then denoted by the small letter c.

The amount of heat which goes to heat up a body of mass m is given by the following formula:

$$Q = mc\,(T_2 - T_1)$$

In what follows we shall make use of the concept of specific heat capacity, but shall speak of a body's heat capacity for the sake of conciseness. An additional guide will always be the dimension of the quantity.

The value of heat capacity varies within a rather wide range. Of course, the heat capacity of water in calories per degree is equal to one by definition.

Most bodies have a heat capacity less than that of water. Thus, most oils, alcohols and other liquids have heat capacities close to 0.5 cal/g·deg. Quartz, glass and sand have a heat capacity of the order of 0.2 cal/g·deg. The heat capacity of iron and copper is about 0.1 cal/g·deg. And here are examples of the heat capacities of some gases: hydrogen—3.4 cal/g·deg, air—0.24 cal/g·deg.

The heat capacities of all bodies decrease, as a rule, with a fall in temperature, and assume negligible values for most bodies at temperatures close to absolute zero. Thus, the heat capacity of copper is equal to only 0.0035 at 20 K; this is twenty-four times less than at room temperature.

A knowledge of heat capacities may prove useful for solving various problems on the distribution of heat among bodies.

The difference between the heat capacities of water and soil is one of the causes determining the distinction between maritime and continental climates. Possessing approximately five times as great a heat capacity as soil, water warms up slowly and cools off just as slowly.

During the summertime in maritime regions, the water, having warmed up more slowly than the land, cools the air, but in the wintertime, the warm sea gradually cools off, yielding heat to the air and making the frost less severe. It is not difficult to calculate that a cubic meter of sea water, cooling off by 1 °C, warms up 3000 m^3 of air by 1 °C. Consequently, in maritime regions the variations in temperature and the difference between winter and summer temperatures are less substantial than in continental regions.

Thermal Conductivity

Each object can serve as a "bridge" along which heat passes from a warmer body to a cooler body. For example, a tea spoon placed in a glass of hot tea is such a bridge. Metallic objects conduct heat very well. The top of the spoon placed in the glass will become warm in the course of a second.

If it is necessary to stir a hot mixture, then the handle of the stirrer must be made of wood or plastic. These solids conduct heat 1000 times worse than metals. We say "conduct heat", but could just as well have said "conduct cold". Of course, a body's properties do not change as a result of the direction in which a heat flow is passing through it. In freezing weather we are careful not to touch metals with our bare hands outdoors, but grasp wooden handles without fear.

Among the poor heat conductors—they are also called heat insulators—are wood, brick, glass and plastic. The walls of houses, ovens and refrigerators are made of these materials.

Among the good conductors are all the metals. The best conductors are copper and silver—they conduct heat twice as well as iron.

Of course, not only solids can serve as "bridges" for the passage of heat. Liquids also conduct heat, but much worse than metals. The thermal conductivity of metals is hundreds of times greater than that of solid and liquid non-metallic bodies.

In order to demonstrate the poor thermal conductivity of water, the following experiment is performed. A piece of ice is fastened to the bottom of a test-tube filled with water, while the top of the test-tube is heated over a gas burner; the water begins boiling, but the ice still has no intention of melting. If the test-tube were without water and made of metal, then the piece of ice would begin melting almost immediately. Water conducts heat about two hundred times worse than copper.

Gases conduct heat tens of times worse than condensed non-metallic bodies. The thermal conductivity of air is 20 000 times smaller than that of copper.

The poor thermal conductivity of gases permits us to hold in our hands a piece of dry ice whose temperature is —78 °C, and to even hold on our palms a drop of liquid nitrogen having a temperature of —196 °C. If we do not squeeze these cold objects with our fingers, then there will be no "burn". The reason consists in the fact that when the drop of liquid or the piece of solid is boiling very energetically, it is covered by a "vapour jacket", and the layer of gas so formed serves as a heat insulator.

The spheroidal state of a liquid—this is what one calls the state in which drops are covered by vapour—is formed whenever water gets into a very hot frying-pan. Drops of boiling water, having fallen on one's palm, severely burn one's hand, although the difference in temperature between boiling water and a human body is less than that between a hand and liquid air. Since one's hand is colder than the drops of boiling water, heat leaves the drops, the boiling ceases and no vapour jacket is formed.

It isn't hard to understand that the best heat insulator is a vacuum—emptiness. There are no carriers of heat in a vacuum, and so the thermal conductivity will be at a minimum.

Therefore, if we want to create a thermal barrier, hide something warm from something cold or vice versa, then the best thing to do is to erect a casing with double walls and pump the air out of the space between them. When doing this, we come across the following curious phenomenon. If we keep track of the change in thermal conductivity of the gas as it is being rarefied, then we shall observe that right until the moment when the pressure reaches several millimeters of mercury column the thermal conductivity remains practically constant. Our expectations are only justified when, with the passage to a higher vacuum, the thermal conductivity sharply falls off.

But what is the cause of this?

In order to understand this phenomenon, we must try to visualize the process of heat transfer in a gas.

The transmission of heat from a warm place to cold ones takes place by means of the transmission of energy from one molecule to a neighbouring one. It is clear that collisions between fast and slow molecules usually lead to an acceleration of the slow molecules and a deceleration of the fast ones. And this means that the hot place will become colder, while the cold place will warm up.

But how will a decrease in pressure affect the transfer of heat? Since a decrease in pressure lowers the density, the number of collisions between fast and slow molecules, during which a transmission of energy occurs, will also decrease. This would decrease the thermal conductivity. However, a decrease in pressure leads, on the other hand, to an increase in the mean free path of the molecules, which therefore transfer heat by greater distances, and this tends to increase the thermal conductivity. Computations show that these effects compensate each other, and so the ability to transmit heat does not change for some time as the air is being pumped out.

This will be the case until the vacuum becomes so considerable that the mean free path is comparable to the distance between the walls of the vessel. Now a further lowering of the pressure can no longer change the mean free path of the molecules, which are "running around" from wall to wall: the fall in density is not "cancelled out", and so the thermal conductivity rapidly falls in proportion to the pressure, reaching negligible values as a high vacuum is attained. The construction of a thermos (a vacuum bottle) is based on the use of the properties of a vacuum. Thermoses are very widespread: they are applied not only for the conservation of hot and cold foods, but also in science and technology. In such a case they are called Dewar flasks (vessels), in honour of their inventor. Liquid air, nitrogen and oxygen are transported in such vessels. Later we shall tell how these gases are obtained in a liquid state*.

Convection

But if water is such a poor heat conductor, then how does it warm up in a tea-kettle? Air conducts heat even worse; then it isn't clear why the same temperature is established in all parts of a room.

* Everyone who has seen cylinders of vacuum bottles noticed that their walls are always silver-plated. But why? The fact is that thermal conductivity is not the only means of transferring heat. There exists yet another way to transfer heat, which we shall speak of in another book—so-called radiation. Under ordinary conditions, it is much weaker than thermal conductivity, but is nevertheless quite noticeable. The walls of thermos bottles are covered with a coating of silver precisely in order to weaken the radiation.

Water in a tea-kettle quickly boils because of gravity. The lower layers of the water, having warmed up, expand, become lighter and rise to the top, with cold water taking their place. A rapid heating occurs thanks only to convection (from the Latin *convectus*, meaning "bring together"). It wouldn't be so easy to heat up water in a tea-kettle located in an interplanetary rocket.

Somewhat earlier explaining why rivers do not freeze down to the bottom, we spoke about another case of convection currents of water, without using this word.

Why are the radiators of a central heating placed near the floor? Why is the ventilation window made in the upper part of a window? It might be more convenient to open a ventilation window if it were lower down, and it might not be a bad idea to place radiators under the ceiling, so that they did not get in the way.

If we were to follow such advice, we would soon discover that the room is not being warmed up by the radiator and not being aired when the ventilation window is open.

The same thing takes place with the air in a room as with the water in a tea-kettle. When the radiator is turned on, the air in the lower layers of the room begins warming up. It expands, becomes lighter and rises towards the ceiling. Heavier layers of cold air arrive in its place. And they, having warmed up, leave for the ceiling. A continuous air current thus arises in the room, with warm air moving up from below and cold air moving down from above. Opening a ventilation window in winter, we admit a stream of cold air into the room. It is heavier than the air in the room, and so goes down, forcing out the warm air, which rises towards the top of the room and leaves through the ventilation window.

A kerosene lamp flames up well only when it is covered with a tall piece of glass. One should not think that the glass is needed only in order to shield the flame from the wind. Even in the calmest weather, the brightness of the flame immediately increases when the glass is put on the lamp. The role of the glass consists in intensifying the stream of air approaching the flame—in creating a draught. This occurs for the reason that the air inside the glass, deprived of the oxygen that was used for the burning, quickly warms up and rises, while pure cold air moves into its place through the holes made in the burner of the lamp.

The taller the glass, the better will the lamp burn. In fact, the speed with which cold air rushes into the lamp's burner depends on the difference in weight between the heated column of air in the lamp and the cold air outside the lamp. The higher the column of air, the greater will this difference in weight be, and so the faster will the movement of air be.

Factory chimneys are also made high for this reason. An especially rapid influx of air, a good draught, is needed for a factory furnace. It is achieved as a result of a high chimney.

The lack of convection in a rocket devoid of weight makes it impossible to use matches, lamps or gas burners: the products of combustion would smother the flame.

Air is a poor conductor; we can conserve heat with its aid, but only under one condition: if we avoid convection—the mixing of warm and cold air—which will bring to naught the thermal-insulation properties of air.

The elimination of convection is achieved by applying various kinds of porous and fibrous bodies. It is difficult for air to move inside such bodies. All bodies of this kind are good heat insulators, thanks only to their ability to retain a layer of air. But the thermal conductivity of the substance itself, of which the fibers or the walls of the pores consist, can be not very small.

A good fur coat is made of a dense fur containing as many fibers as possible; eider-down can be used to make warm sleeping-bags weighing less than half a kilogram, due to the exceptional thinness of its fibers. Half a kilogram of this down can "detain" as much air as tens of kilograms of sheet wadding.

Storm windows are made in order to reduce the convection. The air between the panes does not participate in the mixing of layers of air which takes place within the room.

On the contrary, every movement of the air intensifies the mixing and increases the transfer of heat. This is precisely why we fan ourselves or turn on the ventilator when we want the heat to go away faster. This is also what makes it colder in the wind. But if the air temperature is higher than one's body temperature, then a mixing has the opposite effect, and a wind feels like a hot breath of air.

The problem involved in a steam boiler consists in obtaining steam, heated to the required temperature, as quickly as possible. The natural convection in a gravitational field is quite insufficient for this. Therefore, the creation of an intensive circulation of water and steam, leading to the mixing of warm and cold layers, is one of the basic problems in the construction of steam boilers.

Twelve

STATES OF MATTER

Iron Vapour and Solid Air

A strange combination of words, isn't it? However, this is by no means nonsense: iron vapour and solid air exist in nature, only not under ordinary conditions.

But what conditions are we talking about? The state of a substance is determined by two circumstances: temperature and pressure.

Our lives proceed under conditions which change relatively little. The air pressure varies by several per cent about a value of one atmosphere (1 kgf/cm^2); the temperature of the air, say near Moscow, lies in the interval from -30 to $+30°$; in the absolute scale, in which the lowest possible temperature ($-273°$) is taken as zero, this interval will look less impressive: 240-300 K, which is also only $\pm 10\%$ of the average value.

It is quite natural that we have become accustomed to these ordinary conditions, and so when speaking simple truths such as, "iron is a solid, air is a gas", etc., we forget to add, "under normal conditions".

If iron is heated, it will first melt and then vaporize. If air is cooled, it will first liquefy and then solidify.

Even if the reader has never come across iron vapour or solid air, he or she will probably believe without difficulty that by means of a change in temperature, it is possible to obtain any substance in a solid and in a liquid and in a gaseous state or, as is also said, in a solid, liquid or gaseous phase.

It is easy to believe this because everyone has observed one substance, without which life on the Earth would be impossible, in the form of a gas, and as a liquid, and in the form of a solid. We are speaking, of course, about water.

But under what conditions does a transformation of a substance from one state to another occur?

Boiling

If we lower a thermometer into water which has been poured into a tea-kettle, turn on the electric stove and watch the mercury in the thermometer, then we shall see the following: the level of the mercury will inch upwards almost immediately. Now it is already 90, 95 and finally 100°. The water begins boiling and simultaneously the mercury stops rising. The water has already been boiling for many minutes, but the level of the mercury does not change. The temperature will not change until all the water has boiled away (Figure 97).

But what is the heat used for if the temperature of the water does not change? The answer is obvious. The process of transforming water into steam requires energy.

Let us compare the energy of a gram of water and a gram of the steam created out of it. The molecules of the steam are distributed farther from each other than the water molecules. It is obvious that because of this, the potential energy of the water will differ from the potential energy of the steam.

The potential energy of attracting particles decreases as they approach. The energy of the steam is therefore greater than the energy of the water, and so the transformation of

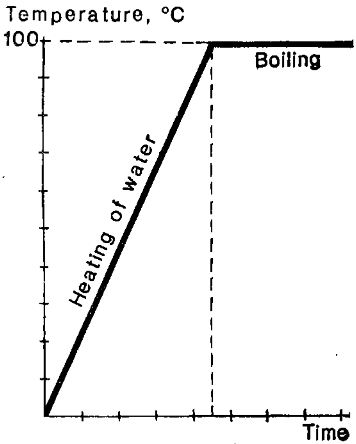


Fig. 97

water to steam requires energy. This excess energy is imparted by the electric stove to the water boiling in the teakettle.

The energy needed for transforming water to steam is called its *heat of vaporization*. In order to transform 1 g of water into steam, 539 calories are required (this figure is for a temperature of 100 °C).

If 539 calories are used for 1 g, then $18 \times 539 \approx 9700$ calories will be supplied to 1 gram-molecule of water. This quantity of heat must be consumed in breaking the intermolecular bonds.

One can compare this figure with the amount of work necessary for breaking the intramolecular bonds. In order to split one gram-molecule of steam into atoms, about 220 000 calories are required, i.e. 25 times as much energy. This directly proves the weakness of the forces binding molecules to each other, as compared to the forces tying atoms together in a molecule.

Dependence of Boiling Point on Pressure

The boiling point of water is equal to 100 °C; one might think that this is an inherent property of water, that water will always boil at 100 °C, no matter where and under what conditions it may be.

But this is not so, and people who live high up in the mountains are perfectly well aware of this.

There is a tourist cabin and a scientific station near the top of Mt. Elbrus. Novices are sometimes amazed at "how hard it is to boil an egg in boiling water" or wonder "why boiling water doesn't scald". In such cases, it is pointed out to them that water is already boiling at 82 °C on the top of Mt. Elbrus.

But what causes this? What physical factor interferes with the phenomenon of boiling? What is the significance of the height above sea level?

This physical factor is the pressure acting on the surface of the liquid. It isn't necessary to climb to the top of a mountain in order to check the validity of what we have said.

If we place a bell-glass over water that is being heated and pump air into or out of it, we can convince ourselves that the boiling point is raised by an increase in pressure and lowered by a decrease in pressure.

Water boils at 100 °C only at a definite pressure—760 mm Hg.

The curve showing the dependence of the boiling point on the pressure is depicted in Figure 98. The pressure is equal to 0.5 atm on the top of Mt. Elbrus, and a boiling point of 82 °C corresponds to this pressure.

But it is even possible to refresh oneself in hot weather with water boiling at 10-15 mm Hg. At such pressures, the boiling point will fall to 10-15 °C.

One can even obtain "boiling water" having a temperature of freezing water. One has to lower the pressure to 4.6 mm Hg for this.

It is possible to observe an interesting scene by placing an uncovered vessel with water under a bell-glass and pumping the air out. This will make the water boil, but boiling requires heat. There is nowhere it could be gotten from, and

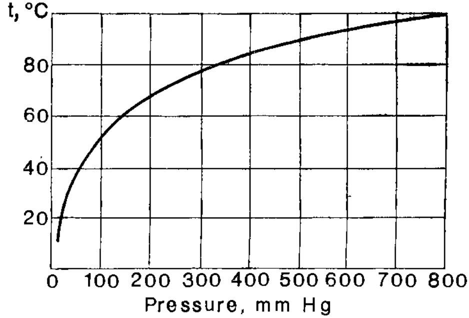


Fig. 98

so the water has to give up its own energy. The temperature of the boiling water starts falling, but since the pumping continues, the pressure also falls. Therefore, the boiling will not cease, the water will continue cooling off and will finally freeze.

Such a boiling of cold water occurs not only as a result of air being pumped out. For example, when a marine screw propeller rotates, the pressure in the layer of water moving rapidly about the metallic surface will fall sharply, and so the water in this layer will boil, i.e. many bubbles filled with steam will appear in it. This phenomenon is called *cavitation* (from the Latin *cavus*, meaning "hollow").

Decreasing the pressure, we lower the boiling point. And raising it? A graph analogous to ours answers this question. A pressure of 15 atm can so delay boiling that it will begin only at 200 °C, while a pressure of 80 atm will make water boil only at 300 °C.

Thus, a definite boiling point corresponds to a definite external pressure. But we may also turn this assertion "around" by saying that "a definite pressure corresponds to each boiling point of water". This pressure is called the *vapour pressure*.

The curve depicting the dependence of the boiling point on the pressure is simultaneously the curve of the vapour pressure as a function of the temperature.

The numbers plotted on the boiling point graph (or on the vapour pressure graph) show that the vapour pressure changes very sharply with a change in the temperature. At 0 °C (i.e. 273 K) the vapour pressure is equal to 4.6 mm Hg, at 100 °C (373 K) it equals 760 mm Hg, i.e. has increased by a factor of 165. With a doubling of the temperature from 0 °C (i.e. 273 K) to 273 °C (i.e. 546 K), the vapour pressure grows from 4.6 mm Hg to almost 60 atm, i.e. by a factor of about 10 000.

Therefore, the boiling point, on the contrary, changes rather slowly with a change in the pressure. When the pressure is doubled—from 0.5 to 1 atm, the boiling point grows from 82 °C (i.e. 355 K) to 100 °C (i.e. 373 K), and with a doubling from 1 to 2 atm, from 100 °C (i.e. 373 K) to 120 °C (i.e. 393 K).

The same curve that we are now considering also controls the condensation of steam to water.

Steam can be transformed into water by either compressing or cooling it.

During the course of condensation, just as for boiling, the point will not move off the curve until the transformation of steam into water or water into steam is completely finished. This can also be formulated as follows: the coexistence of the liquid and the vapour phase is possible under the conditions of our curve and only under these conditions. If, moreover, no heat is supplied or removed, then the amount of vapour and liquid in a closed vessel will remain constant. We say that such a vapour and liquid are in equilibrium, and a vapour in equilibrium with its liquid is called saturated.

The boiling and condensation curve has, as we see, yet another meaning—it is the curve of the equilibrium of liquid and vapour. The equilibrium curve divides the plane of the diagram into two parts. To the left and above the curve (towards higher temperatures and lower pressures) is located the region of stable vapour state. To the right and below the curve is the region of stable liquid state.

The vapour-liquid equilibrium curve, i.e. the curve of the dependence of the boiling point on the pressure, or, what is the same thing, of the vapour pressure on the temperature, is approximately identical for all liquids. In some cases, the change may be somewhat sharper, in others, somewhat slower, but the vapour pressure always grows rapidly with an increase in temperature.

We have already used the words "gas" and "vapour" many times. These two words are more or less synonyms. One may say: water gas is the vapour of water, oxygen gas is the vapour of liquid oxygen. Nevertheless, a certain habit has been formed regarding the usage of these two words. Since we are accustomed to a definite, rather small range of temperatures, we usually apply the word "gas" to those substances whose vapour pressure is higher than atmospheric pressure at ordinary temperatures. On the contrary, we speak of a vapour when a substance is more stable in the form of a liquid at room temperature and atmospheric pressure.

Evaporation

Boiling is a rapid process, and not even a trace of boiling water remains after a short time—it is transformed into steam.

But there is also another phenomenon whereby water or some other liquid is transformed into a vapour: *evaporation*. Evaporation takes place at any temperature and regardless of the pressure, which is always close to 760 mm Hg under ordinary conditions. Evaporation, unlike boiling, is a very slow process. A bottle of eau-de-cologne which we forgot to close will turn out to be empty after several days; water will remain in a saucer for a longer time, but sooner or later it too will turn out to be dry.

Air plays a big role in the process of evaporation. It does not, by itself, prevent water from evaporating. As soon as we uncover the surface of a liquid, water molecules will begin moving into the nearest layer of air. The density of the vapour in this layer will quickly increase; after a short time, the pressure of the vapour will become equal to the vapour pressure of water at the temperature of the surroundings. Moreover, the vapour pressure will be exactly the same as in the absence of air.

The passage of vapour into the air does not, of course, mean an increase in pressure. The total pressure in the space on top of the water surface does not increase; it is only the fraction of this pressure which is borne by the vapour that increases, and the air's fraction correspondingly decreases as it is displaced by the vapour.

There is vapour mixed with air over the water; higher up are layers of air without vapour. They will inevitably mix. Water vapour will continually move into higher layers, and its place in the lower layer will be taken by air which does not contain any water molecules. Therefore, room will always be made for new water molecules in the layer closest to the water. Water will continually evaporate, maintaining the pressure of the water vapour at the surface equal to the vapour pressure, and the process will continue until the water has completely evaporated.

We began with examples involving eau-de-cologne and water. It is well known that they evaporate with different speeds. Ether flies away with exceptional rapidity, alcohol is rather quick and water is much slower. We shall immediately understand why this is so if we find the values of the vapour pressure for these liquids in a handbook, say, at room temperature. Here are the figures: ether—437 mm, alcohol—44.5 mm and water—17.5 mm Hg.

The greater the vapour pressure, the more vapour there will be in the adjacent layer of air and the faster the liquid will evaporate. We know that vapour pressure increases with an increase in temperature. It is clear why the rate of evaporation increases with heating.

It is also possible to influence the rate of evaporation by other means. If we want to aid the evaporation, we must take the vapour away from the liquid more rapidly, i.e. speed up the mixing with air. This is precisely why evaporation is greatly speeded up by blowing on the liquid. Water, although it has a relatively low vapour pressure, will disappear rather quickly if the saucer is placed in the wind.

It is therefore clear why a swimmer, having come out of the water, feels cold in the wind. The wind speeds up the mixing of air with vapour and so increases the rate of evaporation, but the swimmer's body is forced to give up heat for the evaporation.

The way a person feels depends on how much water vapour there is in the air. Both dry and moist air are unpleasant. The humidity is regarded as normal when it is equal to 60%. This means that the density of the water vapour is 60% of the density of saturated water vapour at the same temperature.

If moist air is cooled, the pressure of the water vapour in it will eventually equal the vapour pressure at this temperature. The vapour will become saturated and begin condensing into water when the temperature is lowered further. The morning dew moistening the grass and the leaves appears precisely as a result of this phenomenon.

At 20 °C the density of saturated water vapour is about 0.000 02 g/cm^3. We shall feel fine if the amount of water vapour in the air is 60% of this figure—hence, only a bit more than one-hundred thousandth of a gram per cubic centimeter.

Although this is a small number, it leads to an impressive quantity of water in a room. It is not difficult to calculate that in an average sized room of 12-m^2 area and 3-m height, "there will be room" for about a kilogram of water in the form of saturated vapour.

Consequently, if we place an open barrel of water in a room sealed up tight, then regardless of the barrel's volume a liter of water will evaporate.

It is interesting to compare this result for water with the corresponding figures for mercury. At the same temperature of 20 °C, the density of saturated mercury vapour is 10^{-8} g/cm^3. There will be room for at most 1 g of mercury in a room of the size we have just considered.

Incidentally, mercury vapour is very poisonous, and one gram of it can seriously injure any person's health. When working with mercury, it is necessary to see to it that not even the smallest drop is spilt.

Critical Temperature

How can we turn a gas into a liquid? The boiling point graph answers this question. A gas can be turned into a liquid by either lowering the temperature or raising the pressure.

In the 19th century, the problem of raising pressures seemed to be easier than that of lowering temperatures. At the beginning of that century, the great English physicist Michael Faraday succeeded in compressing gases to the value of their vapour pressures and in this manner transforming many gases into liquids (chlorine, carbon dioxide, etc.).

However, certain gases, such as hydrogen, nitrogen, oxygen, simply could not be liquefied. No matter how much

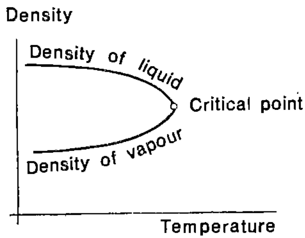


Fig. 99

the pressure was increased, they did not turn into liquids. One might have thought that oxygen and other gases cannot be liquid. They were regarded as true, or constant, gases.

But as a matter of fact, the failures were caused by a lack of understanding of one important circumstance.

Let us consider a liquid and a vapour which are in equilibrium, and think of what happens to them with an increase in the boiling point and, of course, a corresponding increase in the pressure. In other words, let us imagine that a point on the boiling point graph is moving upwards along the curve. It is clear that as the temperature rises, the liquid expands and its density falls. But as for the vapour, an increase in the boiling point is, of course, conducive to its expansion, but, as we have already said, the pressure of the saturated vapour grows considerably faster than the boiling point. Therefore, the density of the vapour does not fall, but on the contrary, rapidly rises with an increase in the boiling point.

Since the density of a liquid falls, but the density of a vapour rises, moving "upwards" along the boiling point curve, we shall inevitably arrive at a point for which the densities of the liquid and the vapour are equal (Figure 99).

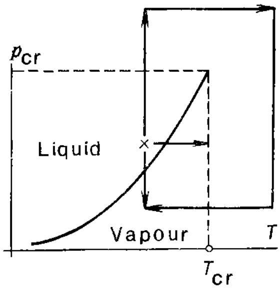


Fig. 100

At this remarkable point, called *critical*, the boiling point curve breaks off. Since all distinctions between a gas and a liquid are related to a difference in density, the properties of a liquid and a gas become identical at the critical point. Each substance has its own critical temperature and critical pressure. Thus, the critical point for water corresponds to a temperature of 374 °C and a pressure of 218.5 atm.

If we compress a gas whose temperature is below critical, then the process of its compression can be depicted by an arrow intersecting the boiling point curve (Figure 100). This implies that at the moment of attaining a pressure equal to the vapour pressure (the point of intersection of the arrow with the boiling point curve), the gas starts condensing to a liquid. If our vessel were transparent, then at this moment we would see the beginning of the formation of a layer of liquid at the bottom of the vessel. If the pressure does not change, the layer of liquid will grow until all of the gas has finally been transformed to a liquid. A further compression will now require an increase in pressure.

The situation is entirely different for the compression of a gas whose temperature is above critical. The process of compression can again be still depicted in the form of an arrow going upwards. But now this arrow does not intersect the boiling point curve. Hence, during the compression the vapour will not condense, but will only continually become denser.

The existence of a gas-liquid interface is impossible at a temperature above critical. When compressed to arbitrary densities, a uniform substance will be found under the piston, and it is hard to say when one may call it a gas and when a liquid.

The presence of a critical point shows that there is no difference in principle between the liquid and gaseous states. At first sight, it might seem that there is no such difference in principle only in case we are dealing with temperature above critical. This, however, is not so. The existence of a critical point indicates the possibility of transforming a liquid—a most genuine liquid, which can be poured into a glass—to the gaseous state without any sign of boiling.

The path of such a transformation is shown in Figure 100. An unquestionable liquid is marked by a cross. If we lower the pressure somewhat (arrow pointing downwards), it will boil; it will also boil in case we raise the temperature somewhat (arrow pointing to the right). But we shall act completely otherwise. Let us compress the liquid so hard that its pressure exceeds critical. The point representing the state of the liquid will rise vertically. Then let us heat the liquid—this process will be depicted by a horizontal line. Now, after we have found ourselves to the right of the critical temperature, let us lower the pressure to its initial value. If we now decrease the temperature, we can obtain a most genuine vapour, which could have been obtained from this liquid along a simpler and shorter path.

Therefore, by changing the pressure and temperature so that the critical point is rounded, it is always possible to obtain a vapour by means of a continuous transition from the liquid phase, or a liquid from a vapour. Such a continuous transition does not require boiling or condensation.

Early attempts to liquefy such gases as oxygen, nitrogen and hydrogen were unsuccessful because the existence of a critical temperature was unknown. These gases have very low critical temperatures: —147 °C for nitrogen, —119 °C for oxygen, —240 °C or 33 K for hydrogen. The record holder is helium, whose critical temperature is equal to 4.3 K. It is possible to transform these gases to the liquid phase in only one way: we must lower their temperatures below the indicated values.

Obtaining Low Temperatures

A significant decrease in temperature can be attained by various means. But the idea involved in all these methods is one and the same: we must force the body we want to cool to expend its internal energy.

But how can this be done? One of the ways is to make the liquid boil, not bringing in any heat from without. For this, as we know, it is necessary to decrease the pressure—to reduce it to the value of the vapour pressure. The heat expended on the boiling will be taken from the liquid, and hence the temperature of the liquid and vapour will fall; therefore, so will the vapour pressure. Consequently, in order that the boiling not cease, but proceed more rapidly, it is necessary to continually pump air out of the vessel with the liquid.

However, a limit is reached to the fall in temperature during this process: the vapour pressure will eventually become completely negligible, and so even the most powerful pumps will be unable to create the required pressure.

In order to continue the lowering of temperature, we can, by cooling a gas with the aid of the liquid already obtained, convert it into a liquid with a lower boiling point. It is now possible to repeat the pumping process with the second substance and in this way to obtain lower temperatures. In case of necessity, such a "cascade" method of obtaining low temperatures can be extended.

Precisely in such a manner was this problem dealt with at the end of the past century, the liquefaction of gases was carried out in stages: ethylene, oxygen, nitrogen and hydrogen—substances with boiling points of —103, —183, —196 and —253 °C—were successively converted into liquids. Having liquid hydrogen available, one can also obtain the lowest boiling liquid—helium (—269 °C). The neighbour "to the left" helped obtain the neighbour "to the right".

The cascade method of cooling is about a hundred years old. Liquid air was obtained by this method in 1877. Liquid hydrogen was first obtained in 1884-1885. Finally, the last stronghold was taken after another twenty years: helium, the substance with the lowest critical temperature, was converted into a liquid in 1908 by Heike Kamerlingh Onnes in Leiden, Holland. The 50th anniversary of this important scientific achievement was widely celebrated.

For many years, the Leiden laboratory was the only "low-temperature" laboratory. But now there exist tens of such laboratories in many countries, not to mention the factories producing liquid air for technical purposes.

The cascade method of obtaining low temperatures is now rarely applied. In technical installations for lowering temperatures, a different means of decreasing a gas' internal energy is applied: the gas is forced to expand rapidly and perform work at the expense of its internal energy.

If, for example, air is compressed up to several atmospheres and let into an expander, then when it performs the work involved in displacing a piston or rotating a turbine, it will cool off so abruptly that it liquefies. If carbon dioxide is let out of a cylinder with great speed, it will cool off so abruptly that it is converted into "ice" in the air.

Liquid gases have found wide application in technology. Liquid oxygen is used for explosives and as a component of the fuel mixture in jet engines.

The liquefaction of air is used in technology for separating the gases constituting air, as will be discussed below.

The temperature of liquid air is widely used in various branches of technology. But this temperature isn't low enough for many physical investigations. In fact, if we convert the relevant temperatures expressed on the centigrade scale to their values on the Kelvin scale, then we shall see that the temperature of liquid air is approximately 1/3 of room temperature. Much more interesting for physics are "hydrogen" temperatures, i.e. temperatures of the order of 14-20 K, and especially "helium" temperatures. The lowest temperature obtained by pumping out liquid helium is 0.7 K.

Physicists have succeeded in coming much closer to absolute zero. At the present time, temperatures have been obtained which are only several thousandths of a degree above absolute zero. However, these extremely low temperatures are obtained by methods which do not resemble those described above.

Supercooled Vapours and Superheated Liquids

In passing through the boiling point, a vapour ought to condense, be transformed to a liquid. However, it turns out that if a vapour is very pure and does not come in contact with a liquid, then we are able to obtain it in the form of a supercooled or supersaturated vapour—a vapour which should have already become a liquid a long time ago.

Supersaturated vapour is very unstable. Sometimes it is sufficient to shake the vessel containing the vapour or throw a couple of grains into it for the delayed condensation to immediately begin.

Experience shows that the condensation of steam molecules is greatly eased by the introduction of small alien particles. The supersaturation of water vapour does not occur in dusty air. It is possible to bring about condensation with puffs of smoke, since smoke consists of tiny solid particles. When the particles enter the steam, they gather molecules around themselves and become centers of condensation.

Thus, even though unstable, a vapour can exist in the region of temperatures fit for liquid "life".

But can a liquid "live" in the region of a vapour under those same conditions? In other words, is it possible to superheat a liquid?

It turns out to be possible. For this it is necessary to prevent the molecules of the liquid from breaking away from its surface. A radical means of achieving this is liquidating the free surface, i.e. placing the liquid in a vessel where it would be compressed on all sides by solid walls. Liquids have been successfully superheated in this manner by several degrees, i.e. one is able to displace a point depicting a liquid's state to the right of its boiling point curve (see Figure 100).

Superheating is the displacement of a liquid into the region of a vapour; therefore, the superheating of a liquid can be achieved by lowering the pressure, as well as by supplying heat. The former method can be used to obtain a surprising result.

Water or some other liquid, thoroughly freed of dissolved gases, which is not easy to do, is placed in a vessel with a piston reaching the surface of the liquid. The vessel and the piston should be wet by the liquid. If we now draw the piston towards ourselves, then the water, cohering to the bottom of the piston, will move along with it. But the layer of water adhering to the piston will pull the next layer of water after it, this layer will pull the one lying below it—as a result, the liquid will stretch.

The column of water will finally break (it is precisely the column that will break, but the water will not break away from the piston), but this will occur when the force on a unit of area attains tens of kilograms. In other words, a negative pressure of tens of atmospheres is created in the liquid.

The vapour phase of a substance is stable for even small positive pressures. And a liquid can be made to have a negative pressure. You couldn't think of a more striking example of "superheating".

Melting

There is no solid body which would withstand a continual rise in temperature. Sooner or later a solid piece is transformed to a liquid; true, in certain cases we will not succeed in reaching the melting point—a chemical decomposition may take place.

The molecules will move more and more intensively as the temperature increases. Finally, the moment arrives when the preservation of order among the wildly "swinging" molecules becomes impossible. The solid body melts. Tungsten has the highest melting point: 3380 °C. Gold melts at 1063 °C, iron, at 1539 °C. Incidentally, there are also easily melted metals. Mercury, as is well known, will even melt at a temperature of —39 °C. Organic substances do not have high melting points. Naphthalene melts at 80 °C, toluene, at —94.5 °C.

It is not at all difficult to measure a body's melting point, especially if it melts within the interval of temperatures which can be measured by an ordinary thermometer. It is

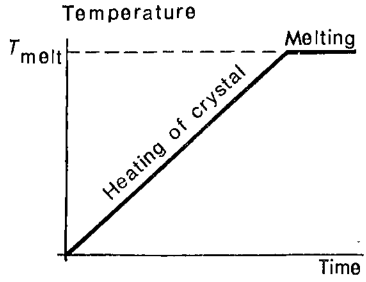


Fig. 101

completely unnecessary to keep one's eye on the melting body. It is sufficient to look at the mercury column of the thermometer (Figure 101). The temperature of the body increases until the melting begins. As soon as the melting begins, the rise in temperature ceases, and the temperature remains constant until the process of melting is completely finished.

Just as in the conversion of a liquid to a vapour, the conversion of a solid body to a liquid requires heat. The quantity of heat needed for this is called the *latent heat of fusion*. For example, the melting of one kilogram of ice requires 80 kilocalories.

Ice is one of the bodies possessing a large heat of fusion. The melting of ice requires, for example, more than 10 times as much energy as the melting of the same mass of lead. Of course, we are talking about the melting proper; here we are not dealing with the fact that before lead begins to melt, it must be heated up to +327 °C. The thawing of snow is delayed as a result of the large heat of fusion possessed by ice. Imagine that its heat of fusion were 10 times smaller. Then the spring thaws would lead each year to inconceivable disasters.

Thus, the heat of fusion of ice is great, but it is also small if we compare it with water's heat of vaporization—540 kilocalories per kilogram (about seven times as small). This difference, by the way, is completely natural. Converting a liquid to a vapour, we must tear its molecules away from each other, but in melting a solid we only have to destroy the order in the distribution of the molecules, leaving them almost at the same distances from each other. It is clear that less work is required in the latter case.

The possession of a definite melting point is an important feature of crystalline substances. It is on the basis of precisely this property that they are easily distinguished from other solids, called amorphous bodies or glasses. Glasses are found among organic substances, as well as among inorganic. Window glass is usually made out of silicates of sodium and calcium; an organic glass (it is also called Plexiglas) is often placed on a desk.

In contrast to crystals, amorphous substances do not have a definite melting point. Glass does not melt, but softens. When heated, a piece of hard glass becomes soft, so that it can be easily bent or stretched; it begins to change its form at a higher temperature under the influence of its own weight. As it is further heated, the thick viscous mass of glass assumes the form of the vessel in which it is lying. This mass is at first as thick as honey, then as sour cream and, finally, becomes almost like a liquid with a small viscosity, such as water. With the best will in the world, here we are unable to single out a definite temperature at which the solid entered the liquid phase. The reasons for this lie in the radical difference between the structure of glass and that of crystalline bodies. As has been said above, the atoms in amorphous bodies are distributed disorderly. The structure of glass resembles that of a liquid. The molecules in hard glass are already distributed disorderly. Hence, a rise in the temperature of glass merely increases the amplitude of its molecules' oscillations, gradually giving them more and more freedom of movement. Therefore, glass softens gradually and does not display the sharp transition from "solid" to "liquid", which is characteristic of a transition from the distribution of molecules in a strict order to their disorderly distribution.

When discussing a boiling point curve, we said that a liquid and a vapour can exist, although in an unstable state, in alien regions—a vapour can be supercooled and moved to the left of the boiling point curve, while a liquid can be superheated and drawn off to the right of the boiling point curve.

Are the analogous phenomena with respect to a crystal and a liquid possible? It turns out that the analogy here is incomplete.

If a crystal is heated, it will begin melting at its melting point. We shall not succeed in superheating a crystal. On the contrary, in cooling a liquid, we can, if we take certain steps, "slip past" the melting point with comparative ease. We are able to achieve considerable supercoolings of certain liquids. There are even such liquids which are easy to supercool, but hard to make them crystallize. As such a liquid is cooled, it becomes more and more viscous and finally hardens, without having crystallized. Glass is like that.

It is also possible to supercool water. Drops of mist can fail to freeze even during the most severe frosts. If a crystal of a substance (priming) is thrown into a supercooled liquid, then crystallization will immediately begin.

Finally, in many cases a delayed crystallization can begin as a result of shaking or other random events. It is known, for example, that crystalline glycerine was first obtained while being transported by train. After lying around for a long time, glass can begin to crystallize (devitrify).

How to Grow a Crystal

We said that most solids consist of the minutest crystals, visible in general only through a microscope. But as for single crystals, large enough to be seen by the naked eye and having such external features of a crystal as plane faces, straight edges and a regular symmetric form, they are found quite infrequently in nature. And this is no accident. The fact is that unless special measures are taken, a fine-crystalline substance, and not a single crystal, is always formed when a melt is cooled. This is explained by the fact that the growth of crystals begins simultaneously at a great many places in the melt, and the entire melt gradually sprouts into an enormous number of tiny crystals.

If we wish to grow a single crystal, we should take steps to have a crystal grow in one place. But if several crystals have already started growing, then in any case we must take steps to make the conditions for growth favourable for only one of them.

Here, for example, is what one does in growing crystals of fusible metals. The metal is melted in a glass test-tube with a drawn-out bottom. The test-tube, suspended by a thread inside a vertical cylindrical oven, is slowly lowered. The drawn-out bottom gradually leaves the oven and cools off. Crystallization begins. At first, several crystals are formed, but those which grow sideways come up against the test-tube wall and their growth is slowed down. Only the crystal which grows along the axis of the test-tube, i.e. into the heart of the melt, proves to be in favourable conditions. As the test-tube is lowered, new portions of the melt, coming into a low-temperature region, will "feed" this unique crystal. Therefore, of all the tiny crystals, it alone will survive; as the test-tube is lowered, it continues to grow along its axis.

At last, all of the melted metal hardens in the form of a single crystal.

The same idea underlies the growing of refractory crystals of ruby. Fine ruby powder is poured in a stream through a flame. As a result, the particles of powder melt; tiny droplets fall on a refractory support of very small area, forming a mass of crystals. During the further fall of droplets on the support, all the tiny crystals will grow, but once again only the one which is in the most advantageous position for "receiving" the falling drops will develop. Crystals are quite often grown from solutions. We shall speak about this crystallization a bit later.

But what are large crystals needed for?

Industry and science are often in need of large single crystals. Of great significance for technology are crystals of Seignette salt and quartz possessing the remarkable property of transforming mechanical action (for example, pressure) into voltage.

The optical industry needs large crystals of calcite, rock salt, fluorite, etc.

Crystals of ruby, sapphire and certain other precious stones are needed for the watchmaking. The reason for this is that individual movable parts of ordinary watches perform up to 20 000 oscillations per hour. Such a great speed makes unusually heavy demands on the quality of the tips of the axes and the bearings. The wear will be least when ruby or sapphire serves as the bearing for the tip of an axis of 0.07-0.15 mm in diameter. Artificial crystals of these substances are very durable and are worn out very little by steel. It is remarkable that artificial stones prove to be better for this purpose than the same natural stones.

It is important to have single large crystals of iron, copper, etc. in order to investigate properties of metals.

Influence of Pressure on Melting Point

If the pressure is changed, then the melting point will also change. We came across such a regularity when dealing with boiling. The greater the pressure, the higher will be the boiling point. As a rule, this is also true for melting. However, there are a small number of substances which behave anomalously: their melting points decrease with an increase in pressure.

The fact of the matter is that the vast majority of solids are denser than their liquids. The exceptions to this rule are precisely those substances whose melting points change somewhat unusually with a change in pressure, for example, water. Ice is lighter than water, and the melting point of ice is lowered by an increase in pressure.

Compression facilitates the formation of the denser state. If the solid is denser than the liquid, then compression aids solidification and hinders melting. But if melting is obstructed by compression, then this means that the substance remains a solid, whereas previously it would have melted at this temperature, i.e. the melting point rises with an increase in pressure. In an anomalous case, the liquid is denser than the solid, and so the pressure helps to form the liquid, i.e. lowers the melting point.

The influence of the pressure on the melting point is much less than the analogous effect for boiling. An increase in pressure of more than 100 kgf/cm^2 lowers the melting point of ice by 1 °C.

Incidentally, it can be seen from this how naive is the frequently met explanation that skates slide on ice because its melting point has been lowered by pressure. The pressure exerted by the blade of a skate does not in any case exceed 100 kgf/cm^2, and so the lowering of the melting point caused by this cannot play any role for an ice-skater.

Evaporation of Solids

When we say that "a substance is evaporating", it is usually implied that a liquid is evaporating. But solids can also evaporate. The evaporation of solids is sometimes called *sublimation*.

Naphthalene, for example, is an evaporating solid. Naphthalene melts at 80 °C, but evaporates at room temperature. It is precisely this property of naphthalene which enables it to be used for the extermination of moths. A fur coat powdered with naphthalene will become saturated with naphthalene vapours, creating an atmosphere which moths cannot bear. Every solid with an odour sublimes to a significant degree. In fact, the odour is created by the molecules which have broken away from the substance and reached our nose. However, the cases where a substance sublimes to an insignificant degree, sometimes to a degree that cannot be detected by even the most careful investigation, are more frequent.

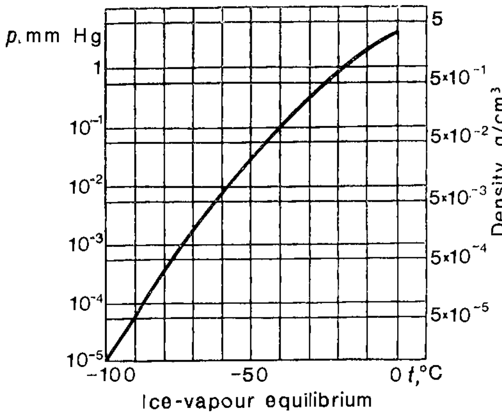


Fig. 102

In principle, any solid (yes, any, even iron or copper) evaporates. If we do not detect sublimation, then this merely means that the saturated vapour density is very negligible.

The saturated vapour density above the solid grows rapidly with an increase in temperature (Figure 102). It is possible to convince oneself that a number of substances, having sharp odours at room temperature, loss them at low temperatures.

In most cases it is impossible to considerably increase the vapour density of a solid for a simple reason—the substance will melt beforehand.

Ice also evaporates. This is well known to housewives who hang up their wash to dry during a frost. At first the water will freeze, but then the ice evaporates and the wash turns out to be dry.

Triple Point

Thus, there are conditions under which a vapour, a liquid and a crystal can exist pairwise in equilibrium.

Can all three states be in equilibrium? Such a point exists on the pressure-temperature diagram; it is called the *triple point*. Where is it located?

If ice floating on water is placed in a closed vessel at a temperature of zero degrees, then water (and "ice") vapours will start entering the free space. At a pressure of 4.6 mm Hg, evaporation will cease and saturation will set in. Now the three phases—ice, water and vapour—will be in a state of equilibrium. This is precisely the triple point.

The relationships between the various states are graphically and clearly shown by the diagram for water depicted in Figure 103.

Such a diagram can be constructed for any substance.

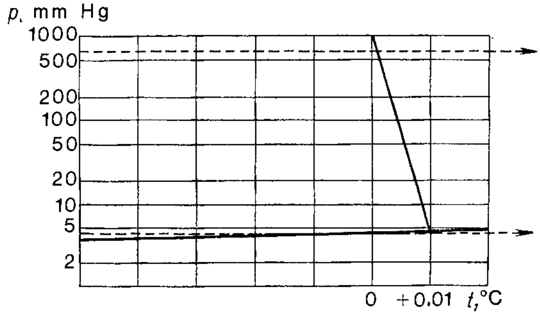


Fig. 103

We are acquainted with the curves in the diagram—they are the equilibrium curves for ice and water vapour, ice and water, and water and water vapour. As is customary, the pressure is plotted along the vertical axis, and the temperature, along the horizontal.

The three curves intersect in the triple point and divide the diagram into three regions—the living spaces for ice, water and water vapour.

A phase diagram is a concise handbook. Its aim is to answer questions as to what state of a substance is stable at a given pressure and a given temperature.

If water or water vapour is placed under the conditions of the "left-hand region", it will turn into ice. If water or ice is introduced into the "lower region", water vapour will be obtained. In the "right-hand region", water vapour will condense and ice will melt.

The phase diagram permits one to immediately say what will happen to a substance when it is heated or compressed. Heating at a constant pressure will be depicted on the diagram by a horizontal line. The point depicting the phase of the substance will move from left to right along this line.

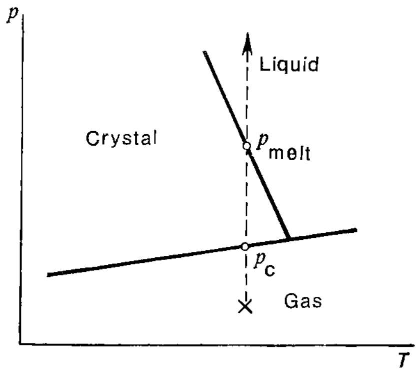


Fig. 104

Two such lines are depicted in the figure, one of which is heating under normal pressure. This line lies above the triple point. Therefore, it will first intersect the melting point curve, and then, beyond the bounds of the diagram, the evaporation curve too. Under normal pressure, ice melts at a temperature of 0 °C, while the water so formed will boil at 100 °C.

The situation will be different for ice heated under a very low pressure, say a bit less than 5 mm Hg.

The heating process is depicted by a line passing beneath the triple point. The melting and boiling point curves do not intersect this line. Under such a negligible pressure, heating leads to a direct transition of ice to water vapour.

In Figure 104, this same diagram shows what an interesting phenomenon will occur when water vapour in the state marked on the figure by a cross is compressed. The vapour will first be converted into ice, which will then melt. The diagram enables us to say at once at what pressure the growth of crystals will begin and when the melting takes place.

The phase diagrams for all substances resemble each other. Significant, from the everyday point of view, differences arise because the location of the triple point on the diagram can be most diverse for various substances.

After all, we exist under conditions which are close to "normal", i.e. in the first place, under a pressure close to one atmosphere. The way in which the triple point of a substance is located with respect to the normal pressure line is very important for us.

If the pressure at the triple point is less than atmospheric, then for us, living under "normal" conditions, the substance will be one of those that melt. As the temperature rises, it will first be transformed into a liquid, which will then boil. In the opposite case, when the pressure at the triple point is higher than atmospheric, we will not see any liquid when the substance is heated, since the solid will be converted directly into a vapour. This is how "dry ice" behaves, which is very convenient for ice cream vendors. They can distribute pieces of "dry ice" among the portions of ice cream without worrying that the ice cream will become wet as a result. "Dry ice" is solid carbon dioxide, CO_2. Its triple point lies at 73 atm. Therefore, when solid CO_2 is heated, the point depicting its state will move along a horizontal line intersecting only the sublimation curve (just as for ordinary ice at a pressure of about 5 mm Hg).

The Same Atoms, but Different Crystals

The dull, soft graphite with which we write and the bright, transparent, hard, cutting glass, diamond, are made of one and the same atoms—atoms of carbon. Why then are there such differences between the properties of these two substances, identical in composition?

Recall the lattice of flaky graphite, each of whose atoms has three nearest neighbours, and the lattice of diamond, whose atoms have four nearest neighbours. It is clearly evident from this example how the properties of crystals are determined by the mutual distribution of their atoms. Fireproof crucibles, withstanding temperatures up to two or three thousand degrees, are made of graphite, but diamond burns at temperatures above 700 °C; the specific gravity of diamond is 3.5, of graphite, 2.3; graphite conducts electricity, but diamond does not, etc.

This property of forming various crystals is possessed not only by carbon. Almost every chemical element, and not only element, but also any chemical substance, can exist in several varieties. Six varieties of ice, nine varieties of sulphur and four varieties of iron are known.

In discussing a phase diagram, we did not speak of the various types of crystals, but drew a single region for the solid. But for a great many substances, this region is divided up into sections, each of which corresponds to a definite "sort" of solid body or, as one says, a definite solid phase (a definite crystal modification).

Each crystal phase has its own region of stable state, bounded by definite intervals of pressure and temperature. The laws of transformation of one crystal variety to another are the same as the laws of melting and evaporation.

Given any pressure, one can find a temperature at which both types of crystals will peacefully coexist. If we raise the temperature, a crystal of one form will be converted into a crystal of the second form. If we lower the temperature, the reverse transformation will take place.

In order to convert red sulphur into yellow under normal pressure, a temperature below 110 °C is needed. Above this temperature, right up to the melting point, the order of the distribution of atoms, characteristic of red sulphur, is stable.

When the temperature falls, the oscillations of the atoms decrease, and, beginning with 110 °C, nature finds a more convenient order for the distribution of the atoms. A transformation of one crystal to another occurs.

Nobody has thought of names for the six different ices. This is what one says: ice one, ice two, ..., ice seven. How come seven if there are only six varieties? The reason is that repeated experiments have failed to detect ice four.

If water is compressed at a temperature of about zero, then ice five will be formed at a pressure of about 2000 atm, and at a pressure of about 6000 atm, ice six.

Ice two and ice three are stable at temperatures below zero degrees centigrade.

Ice seven is a hot ice; it appears when hot water is subjected to a pressure of about 20 000 atm.

All ices, except the ordinary one, are heavier than water. Ice obtained under normal external conditions behaves anomalously; on the contrary, ice obtained under conditions differing from the norm behaves normally.

We say that each crystal modification is characterized by a definite region of existence. But if so, then how can graphite and diamond possibly exist under identical conditions?

Such "lawlessness" is found very often in the world of crystals. The ability to live under "alient" conditions is almost a rule for crystals. While one must turn to various tricks in order to transfer a vapour or a liquid to alien regions of existence, a crystal, on the contrary, can almost never be forced to stay within the frontiers marked off for it by nature.

The superheating and supercooling of crystals are explained by the difficulty in transforming one order into another under conditions of extreme overcrowding. Yellow sulphur should be transformed into red at 95.5 °C. During a more or less rapid heating, we "slip past" this transformation point and drive it up to 113 °C.

The true transformation point is most easily detected when different crystals are in contact. If we put one up close against the other and maintain a temperature of 96 °C, then the yellow sulphur will be eaten up by the red, but the yellow will swallow the red at 95 °C. Unlike a "crystal-liquid" transition, a "crystal-crystal" transformation is ordinarily delayed during superheating, just as during supercooling.

In certain cases, we come across states of a substance which are supposed to exist at entirely different temperatures.

White tin should turn into grey when the temperature falls to +13 °C. We ordinarily use things made of white tin and know that nothing will happen to them in winter. It withstands supercoolings of 20-30 degrees perfectly well. However, under conditions of a severe winter, white tin is transformed into grey. The lack of knowledge of this fact was one of the circumstances destroying Scott's expedition to the South Pole (1912). The liquid fuel taken along by the expedition was kept in vessels soldered with tin. During severe frosts, the white tin was transformed into a grey powder, the vessels came unsoldered and the fuel was spilt. Not without reason is the appearance of grey spots on white tin called tin plague.

Just as in the case of sulphur, white tin can be converted into grey at a temperature a bit lower than 13 °C, provided that a tiny grain of the grey variety falls on a tin object.

The existence of several varieties of one and the same substance and the delays in their mutual transformations have great significance for technology.

At room temperature, iron atoms form a body-centered cubic lattice, occupying the vertices and center of each cube. Every atom has 8 neighbours. At a high temperature, iron atoms form a denser "packing"—each atom has 12 neighbours. Iron with 8 neighbours per atom is soft; iron with 12 neighbours per atom is hard. It turns out that it is possible to obtain iron of the latter type at room temperature. The method whereby this is done—tempering—is widely applied in metallurgy.

Tempering is accomplished quite simply—the metallic object is made red-hot and then thrown into water or oil. Cooling occurs so rapidly that there is no time for the transformation of the structure stable at a high temperature to take place. Consequently, the high-temperature structure will exist indefinitely under conditions unnatural to it: recrystallization into a stable structure occurs so slowly that it is practically unnoticeable.

In speaking of the tempering of iron, we were not quite exact. Steel, i.e. iron containing a fraction of a per cent of carbon, is tempered. The presence of very small admixtures of carbon delays the transformation of hard iron into soft and permits the tempering to be carried out. As for completely pure iron, one cannot succeed in tempering it—there is time for the conversion of its structure to take place even during the most rapid cooling.

Depending on the form of the phase diagram, one or another transformation is achieved by changing the pressure or the temperature.

Many transformations of a crystal into a crystal are observed during a change of only the pressure. Black phosphorus was obtained in this way.

Graphite was able to be converted into diamond only by simultaneously using high temperature and pressure. The phase diagram for carbon is depicted in Figure 105. At pressures below ten thousand atmospheres and at temperatures less than 4000 K, graphite is the stable modification. Therefore, diamond exists under "alien" conditions, and so it can be transformed into graphite without particular difficulty. But the inverse problem is of practical interest. We cannot succeed in carrying out a transformation of graphite into

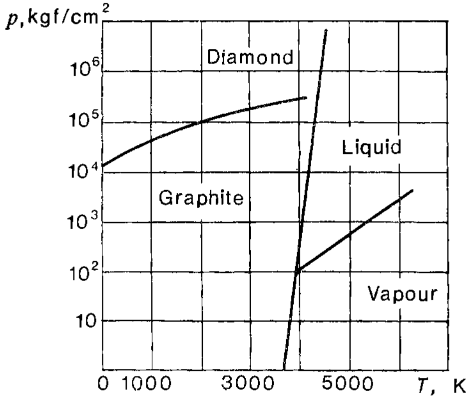


Fig. 105

diamond by only raising the pressure. A phase transformation in the solid state apparently goes too slowly. The form of the phase diagram suggests the correct solution: increase the pressure and heat the graphite simultaneously. We then obtain (upper right-hand corner of the diagram) melted carbon. Cooling it at a high pressure, we should enter the region of diamond.

The practical possibility of such a process was proved in 1955, and the problem is considered at the present time to be solved from technological point of view.

An Amazing Liquid

If we lower the temperature of a body, then sooner or later it will solidify and acquire a crystal structure. Moreover, it makes no difference at what pressure the cooling takes place. This circumstance seems perfectly natural and understandable from the point of view of the physical laws with which we have already become acquainted. In fact, by lowering the temperature, we decrease the intensity of the thermal motion. When the motion of the molecules becomes so weak that it has already ceased to interfere with the forces of interaction between them, the molecules will line up in an accurate order—will form crystals. A further cooling will take away from the molecules all the energy of their motion, and at absolute zero, a substance should exist in the form of molecules at rest, distributed in a regular lattice.

Experiments show that all substances behave in this manner. All but one unique substance: this "freak" is helium.

We have already informed the reader of certain facts concerning helium. It holds the record for the value of its critical temperature. Not a single substance has a critical temperature lower than 4.3 K. However, this record by itself does not imply anything amazing. Something else is startling: cooling helium below the critical temperature and practically reaching absolute zero, we will not obtain solid helium. Helium remains a liquid even at absolute zero.

The behaviour of helium is completely unexplainable from the point of view of the laws of motion presented by us, and is one of the signs of the limited validity of the laws of nature, which seemed universal.

If a substance is a liquid, then its atoms are in motion. But in cooling it down to absolute zero, we have taken all energy of motion away from it. We have to admit that helium has an energy of motion which cannot be taken away. This conclusion is incompatible with the mechanics which we have been studying so far. According to the mechanics we have learned, the motion of a body can always be slowed down to a complete halt by taking away all its kinetic energy; the motion of molecules can also be stopped in exactly the same way by taking energy away from them during collisions with the walls of the vessel being cooled. Such a mechanics will obviously not do for helium.

The "strange" behaviour of helium is an indication of a fact of enormous importance. This is the first time that we have come up against the impossibility of applying in the world of atoms the basic laws of mechanics, established by means of a direct investigation of the motion of visible bodies—laws which seemed to constitute a firm foundation for physics.

The fact that helium "refuses" to crystallize at absolute zero is by no means possible to reconcile with the mechanics which we have been studying until now. The contradiction which we have come across for the first time—the insubordination of the world of atoms to the laws of mechanics—is merely the first link in a chain of sharper and more glaring contradictions in physics.

These contradictions have led to the necessity of revising the foundations of the mechanics of the atoms. This revision is very profound and has led to a change in our entire understanding of nature.

The necessity for a radical revision of the mechanics of the atomic motion does not imply that we must give up the laws of mechanics we have studied as a bad job. It would have been unfair to make the reader study unnecessary things. The old mechanics is completely valid in the world of large bodies. This is already enough for us to regard the corresponding chapters of physics with complete respect. However, it is also important that a number of laws of the "old" mechanics pass over without change into the "new" mechanics. Among them, in particular, is the law of conservation of energy.

The presence of "inalienable" energy at absolute zero is not a special property of helium. It turns out that all substances have "zero" energy. Only for helium does this energy

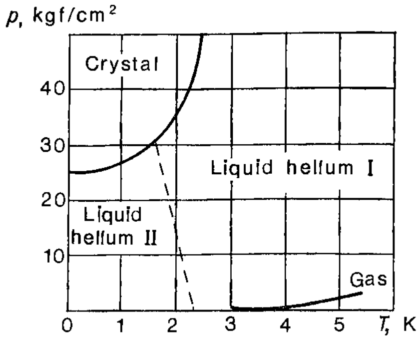


Fig. 106

prove sufficient to prevent the atoms from forming a regular crystal lattice.

One must not think that helium cannot occur in a crystalline state. It is merely necessary to raise the pressure to approximately 25 atm in order to crystallize helium. A cooling carried out above this pressure will lead to the formation of solid crystalline helium with perfectly ordinary properties. Helium forms a face-centered cubic lattice.

The phase diagram for helium is shown in Figure 106. It differs greatly from the diagrams of all other substances in the absence of a triple point. The melting and boiling point curves do not intersect.

Thirteen
SOLUTIONS

What a Solution Is

If broth is salted and stirred with a spoon, no traces of salt will remain. It should not be thought that the grains of salt are simply not visible to the unaided eye. One will not succeed in detecting the tiny salt crystals in any way, because they have dissolved. If pepper is added to the broth, no solution will be obtained. One can even stir the broth for days on end, but the tiny black grains will not disappear.

But what do we mean when we say, "a substance has dissolved"? For the atoms or molecules composing it cannot vanish without a trace, can they? Of course not, and they do not vanish. When dissolving, only the grains of a substance, crystals, accumulations of molecules of a single sort, disappear. A *dissolution* consists in stirring the particles of a mixture in such a manner that the molecules of one substance are distributed between the molecules of another. A *solution* is the mixture of molecules or atoms of different substances.

A solution can contain various amounts of a solute. The composition of a solution is characterized by its concentration, for example, the ratio of the number of grams of a solute to the number of liters of solution.

As we add a solute, the concentration of the solution grows, but not without bound. Sooner or later the solution will become saturated and will cease "taking in" the solute. The concentration of a saturated solution, i.e. the "limiting" concentration of the solution, is called the *solubility*.

It is possible to dissolve a surprising amount of sugar in hot water. At a temperature of 80 °C, a full glass of water will accept 720 g of sugar with no residue. This saturated solution will be thick and viscous, and is called a sugar syrup by cooks. The figure we just gave for sugar is for a cup, which holds 0.2 l. Hence, the concentration of sugar in water at 80 °C is equal to 3600 g/l. (which is read, "grams per liter").

The solubility of certain substances is highly dependent on the temperature. At room temperature (20 °C), the solubility of sugar in water falls to 2000 g/l. On the contrary, the solubility of salt changes quite insignificantly with a change in temperature.

Sugar and salt dissolve well in water. But naphthalene is practically insoluble in water. Different substances dissolve quite differently in different solvents.

Solutions are used for growing monocrystals. If a small crystal of a solute is suspended in a saturated solution, then as the solvent evaporates, the solute will settle out on the surface of this crystal. Moreover, the molecules will preserve a strict order, and as a result, the small crystal will be transformed into a large one, remaining a monocrystal.

Solutions of Liquids and Gases

Is it possible to dissolve a liquid in a liquid? Of course, it is. For example, vodka is a solution of alcohol in water (or, if you wish, of water in alcohol—depending on what there is more of). Vodka is a real solution; the molecules of water and alcohol are completely mixed up in it.

However, such a result is not always obtained when two liquids are mixed.

Try adding kerosene to water. No matter how you stir, you will not succeed in obtaining a uniform solution; this is just as hopeless as trying to dissolve pepper in soup. As soon as you stop stirring, the liquids arrange themselves in layers: the heavier water—on the bottom, the lighter kerosene—on the top. Kerosene with water and alcohol with water are systems having opposite properties with respect to their solubility.

However, there are also intermediate cases. If we mix ether with water, we shall clearly see two layers in the vessel. At first glance, it might appear that on the top is ether and on the bottom is water. But as a matter of fact, the lower and upper layers are solutions: the former is water in which part of the ether is dissolved (the concentration being 25 grams of ether to a liter of water), while the latter is ether in which there is a noticeable amount of water (60 g/l.).

Let us now turn our attention to solutions of gases. It is clear that an unlimited amount of an arbitrary gas will dissolve in any other gas. Two gases always intermingle in such a way that the molecules of one penetrate between the molecules of the other. For gas molecules interact but slightly with each other, and so each gas behaves in the presence of another gas by, in a certain sense, not paying any "attention" to its cohabitant.

Gases can also dissolve in liquids. However, not in arbitrary quantities, but in limited ones, not differing in this respect from solids. Moreover, different gases dissolve differently, and these differences can be very great. Immense amounts of ammonia can dissolve in water (about 100 grams to half a glass of cold water), and large quantities of hydrogen sulphide and carbon dioxide. Oxygen and nitrogen are soluble in water in only negligible quantities (0.07 and

0.03 per liter of cold water). Therefore, there is a total of only about one hundredth of a gram of air in a liter of cold water. However, even this small amount plays a large role in life on the Earth—for fish breathe the oxygen of the air dissolved in water.

The greater the pressure of a gas, the more of it will be dissolved in a liquid. If the amount of a dissolved gas is not very great, then there is a direct proportionality between it and the pressure of the gas above the surface of the liquid.

Who has not enjoyed a glass of cold soda, so good for quenching one's thirst. It is possible to obtain soda because of the dependence of the amount of a dissolved gas on the pressure. Carbon dioxide is driven into water under pressure (from the cylinders which are in every store where soda is sold). When soda is poured into a glass, the pressure falls to atmospheric and the water gives off the "superfluous" gas in the form of bubbles.

Taking such effects into account, deep-sea divers should not rise rapidly from under the water to the surface. Additional amounts of air dissolve in a diver's blood under the high pressures of deep water. When rising, the pressure falls and air begins separating out in the form of bubbles, which can stop up the blood vessels.

Solid Solutions

The word "solution" is applied to liquids in everyday life. However, there also exist solid mixtures whose atoms and molecules are uniformly distributed. But how can one obtain solid solutions? You won't get them with the aid of mortar and pestle. Therefore, we must first turn the substances to be mixed into liquids, that is melt them, then mix the liquids and allow the mixture to solidify. One can also act otherwise, dissolving the two substances, which we want to mix, in some liquid and then evaporating the solvent. Solid solutions might be obtained by such means. They might be, but they cannot ordinarily be so obtained. Solid solutions are rarities. If a piece of sugar is thrown into salty water, it will dissolve excellently. Evaporate the water; the minutest crystals of salt and sugar will be found at the bottom of the cup. Salt and sugar do not yield solid solutions.

It is possible to melt cadmium and bismuth in a single crucible. After cooling, we can see a mixture of cadmium and bismuth crystals through a microscope. Bismuth and cadmium do not form solid solutions either.

A necessary, although not a sufficient, condition for the emergence of solid solutions is the affinity of the molecules or atoms of the substances being mixed in form and dimensions. In this case, crystals of one sort are formed when the mixture freezes. The lattice points of each crystal are usually occupied randomly by atoms (molecules) of different sorts.

Alloys of metals of great technological value are frequently solid solutions. The dissolution of a small amount of an admixture can radically change the properties of a metal. A striking illustration of this is the obtaining of one of the most widespread materials in technology—steel, which is a solid solution of small quantities of carbon (of the order of 0.5 of weight per cent, that is one atom of carbon to 40 atoms of iron) in iron, where the atoms of carbon are randomly distributed between the atoms of iron.

Only a small number of carbon atoms dissolve in iron. However, some solid solutions are formed by mixing substances in arbitrary proportions. Alloys of gold and copper can serve as an example. Crystals of gold and copper have lattices of the same type—face-centered cubic. An alloy of copper with gold has the same lattice. An idea of the structure of alloys with an increasing share of copper can be obtained by conceptually deleting atoms of gold from the lattice and replacing them by atoms of copper. Moreover, the replacement occurs disorderly, with the copper atoms generally being distributed randomly among the lattice points.

Alloys of copper with gold may be called solutions of replacement, whereas steel is a solution of a different type, a solution of introduction.

In the vast majority of cases, solid solutions do not arise, and, as has been said above, after cooling we can see in the microscope that the substance consists of a mixture of tiny crystals of both substances.

How Solutions Freeze

If a solution of any salt in water is cooled, it will be discovered that the freezing point has been lowered. Zero mark is past, but solidification does not occur. Only at a temperature of several degrees below zero do crystals appear in the liquid. They are crystals of pure ice: salt does not dissolve in solid ice.

The freezing point depends on the concentration of a solution. Increasing the concentration of a solution, we shall lower its freezing point. A saturated solution has the lowest freezing point. The decrease in the freezing point of a solution is not at all small: thus, a saturated solution of sodium chloride in water freezes at —21 °C. With the aid of other salts, we can achieve an even greater decrease; calcium chloride, for example, enables us to bring the freezing point of a solution down to —55 °C.

Let us now consider how the process of freezing takes place. After the first ice crystals have fallen out of a solution, the concentration of the solution increases. Now the relative number of alien molecules grows, the obstacles to the process of the crystallization of water also increase and the freezing point falls. If we do not further lower the temperature, then crystallization will cease. With a further lowering of temperature, crystals of water (the solvent) continue separating out. Finally, the solution becomes saturated. A further enrichment of the solution by the solute becomes impossible, and the solution solidifies at once; moreover, if we look at the frozen mixture through a microscope, we can see that it consists of crystals of ice and crystals of salt.

Consequently, the freezing of a solution is unlike that of a simple liquid. The process of freezing is spread out over a wide temperature interval.

What will happen if we pour salt on some frozen surface? The answer to this question is well known to yard men—as soon as salt comes in contact with ice, the ice begins melting. In order that this phenomenon take place, it is necessary, of course, that the freezing point of a saturated salt solution be lower than the temperature of the air. If this condition is met, then the mixture of ice and salt is in an alien phase region, namely, in the region of a solution's stable existence. Therefore, the mixture of ice with salt will turn into a solution, i.e. the ice will melt and the salt will dissolve in the water so formed. Finally, either all the ice will melt or else a solution of such a concentration will be formed that its freezing point will be equal to the temperature of the surroundings.

Suppose that a yard whose area is 100 square meters is covered by a 1-cm layer of ice—this is quite a bit of ice, about one ton. Let us calculate how much salt is needed to melt this ice if the temperature is —3 °C, the freezing (melting) point of a salt solution with a concentration 45 g/l. Approximately one liter of water corresponds to one kilogram of ice. Hence, in order to melt one ton of ice at —3 °C, 45 kg of salt are needed. A much smaller quantity is used in practice since one does not seek the complete melting of all the ice.

Ice melts when mixed with salt, and salt dissolves in water. But heat is required for melting, and the ice takes it from its surroundings. Therefore, the addition of salt to ice leads to a decrease in temperature.

We are now in the habit of buying ice cream, but it was previously made at home. In this connection, a solution of ice with salt played the role of a refrigerator.

Boiling of Solutions

The phenomenon of the boiling of solutions has a lot in common with that of their freezing.

The presence of a dissolved substance hinders crystallization. For the very same reasons, a dissolved substance also hinders boiling. In both cases it is as though the alien molecules were fighting to retain a solution which is as diluted as possible. In other words, the alien molecules stabilize the state of the basic substance (i.e. facilitate its existence), which can dissolve them.

Therefore, alien molecules impede the crystallization of a liquid, and so lower the freezing point. In exactly the same way, alien molecules impede the boiling of a liquid, and so raise its boiling point.

It is curious that within certain limits of concentration (for not very concentrated solutions), neither the lowering of a solution's freezing point nor the raising of its boiling point depends in the least on the properties of the solute, but is determined only by the number of its molecules. This interesting circumstance is used for the determination of the molecular weight of soluble substances. This is done with the aid of a remarkable formula (we cannot give it here), which relates the change in the freezing or boiling point to the number of molecules in a unit volume of solution (and to the heat of fusion or vaporization).

The boiling point of water is raised one-third as much as its freezing point is lowered. Thus, sea water containing approximately 3.5% of salt has a boiling point of 100.6 °C, while its freezing point is lowered by 2°.

If one liquid boils at a higher temperature than another, then (at the same temperature) its vapour pressure is lower. Hence, the vapour pressure of a solution is less than that of the pure solvent. One can judge the difference on the basis of the following values: the vapour pressure at 20 °C equals 17.5 mm Hg and the vapour pressure of a saturated solution of sodium chloride at the same temperature is 13.2 mm Hg.

Vapour with a pressure of 15 mm Hg, unsaturated for water, will be supersaturated for a saturated salt solution. In the presence of such a solution, the vapour starts condensing and uniting with the solution. Of course, not only a salt solution, but also powdered salt, will take water vapour out of the air. In fact, the very first drop of water falling on the salt will dissolve it and create a saturated solution.

The absorption of water vapour from the air by salt leads to the salt's becoming moist. This is well known to hosts and affords them grief. But this phenomenon of the decrease in vapour pressure over a solution can also be of benefit: it is used for drying air in laboratory practice. Air is passed through calcium chloride, which is the record holder in taking moisture out of air. While a saturated solution of sodium chloride has a vapour pressure of 13.2 mm Hg, that of calcium chloride is 5.6 mm Hg. The pressure of the water vapour will fall to this value when it is passed through a sufficient amount of calcium chloride (1 kg of which "has room for" approximately 1 kg of water). This is a negligible humidity, and such air may be regarded as dry.

How Liquids Are Freed of Admixtures

Distillation is one of the most important means of freeing liquids of admixtures. The liquid is boiled and the vapour is sent into a refrigerator. When cooled, the vapour turns into a liquid once more, but this liquid will be purer than the initial one.

It is easy to get rid of solids dissolved in a liquid with the aid of distillation. Molecules of such substances are practically absent in water vapour. Distilled water is obtained in this way—completely tasteless pure water devoid of mineral admixtures.

However, using evaporation, we can also get rid of liquid admixtures and separate a mixture consisting of two or more liquids. In this connection, we make use of the fact that two liquids forming a mixture do not boil with the same "ease".

Let us see how a mixture of two liquids, for example, of water and ethyl alcohol, taken in equal quantities (100 proof vodka), will behave when boiled.

Under normal pressure, water boils at 100 °C and alcohol at 78 °C. The mixture we are dealing with will boil at the intermediate temperature of 81.2 °C. Alcohol boils more easily, so its vapour pressure is greater, and for an initial fifty-per cent composition of the mixture, the first portion of vapour will contain 80% alcohol.

We can draw off the portion of vapour so obtained into a refrigerator and obtain a liquid enriched with alcohol. This process can be further repeated. However, it is clear that such a method is not practical, for with each successive distillation, less and less substance will be obtained. In order that there be no such loss, so-called rectifying (i.e. purifying) columns are applied for the purpose of cleaning a liquid.

The idea behind the structure of this interesting apparatus consists in the following. Imagine a vertical column whose lower part contains a liquid mixture. Heat is supplied below the column and cooling is carried out above it. The vapour formed by boiling rises to the top and condenses; the resulting liquid flows down. For a fixed addition of heat from below and withdrawal from above, countercurrents of vapour, going up, and liquid, flowing down, are established in the closed column.

Let us fix our attention on some horizontal cross-section of the column. Liquid passes downwards, and vapour upwards, through this cross-section; moreover, none of the substances composing the liquid mixture stays behind. If we are dealing with a column containing a mixture of alcohol and water, then the amount of alcohol passing upwards and downwards, just as the amount of water passing upwards and downwards, will be equal. Since liquid is going down and vapour is coming up, this means that the composition of the liquid and the composition of the vapour are identical at any height in the column.

As has been just explained, the equilibrium of the liquid and gaseous phases of a mixture of two substances requires, on the contrary, a difference between the compositions of these phases. Therefore, a conversion of liquid into vapour, and of vapour into liquid, takes place at every height in the column. Moreover, the part of the mixture with high boiling point condenses and the component with low boiling point passes from a liquid to a vapour.

Consequently, it is as though the rising current of vapour were taking away the component with low boiling point at all heights while the down-flowing current of liquid were continually being enriched with the part with high boiling point. The composition of the mixture will become different at each height: the higher the mixture, the greater the percent-

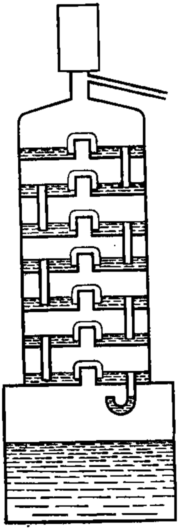


Fig. 107

age of the component with low boiling point. In the ideal case, a pure layer of the component with low boiling point will be on the top, while on the bottom, a pure layer of that with high boiling point. The substances should now be drawn off, only as slowly as possible, in order that the ideal picture just outlined not be disturbed, the one with low boiling point, from the top, and with high boiling point, from the bottom.

In order to carry out the separation, or rectification, in practice, we must present the countercurrents of vapour and liquid, with the possibility of thoroughly intermingling. To this end, the currents of liquid and vapour are delayed with the aid of plates distributed one above the other and connected by means of overflow pipes. Liquid can flow down to a lower level from an overfilled plate. Vapour, going upwards in a rapid current (0.3-1 m/sec), forces its way through a thin layer of liquid. The diagram of a column is shown in Figure 107.

One does not always succeed in purifying a liquid completely. Certain mixtures possess an "unpleasant" property: for a definite composition, the ratio of the components of the evaporating molecules is the same as the ratio of the components in the liquid mixture. In this case, of course, a further purification by means of the method described becomes impossible. Such is the mixture containing 96% of alcohol and 4% of water: it yields a vapour of the same composition. Consequently, 96% alcohol is the best that can be obtained by an evaporation method.

Rectification (or distillation) of liquids is an important process in chemical technology. For example, gasoline is obtained from oil with the aid of rectification.

It is a curious fact that rectification is the cheapest method of obtaining oxygen. For this, of course, one must change air to the liquid phase beforehand, after which one can separate it into almost pure nitrogen and oxygen by means of rectification.

Purification of Solids

On a phial containing a chemical substance, alongside of the chemical name one can see, as a rule, the following notation: "pure", "pure for analysis" or "spectroscopically pure": They are used to mark the degree of purity of a substance: "pure" means a rather slight degree of purity—there may possibly be an admixture of the order of 1% in the substance; "pure for analysis" means that admixtures constitute not more than several tenths of a per cent of the substance; "spectroscopically pure" is not easy to obtain, since spectral analysis detects an admixture of several thousandths of a per cent. The inscription "spectroscopically pure" permits us to be confident that the purity of the substance is characterized by at least "four nines", i.e. that the content of the basic substance is not less than 99.99%.

The need for pure solids is quite great. Admixtures of several thousandths of a per cent can change many physical properties, and in one special problem of exceptional interest to modern technology, namely the problem of obtaining semiconducting materials, technologists require a purity of seven nines. This means that one unnecessary atom among ten million necessary ones prevents us from solving engineering problems! We take recourse in special methods for obtaining such ultrapure materials.

Ultrapure germanium and silicon (these are the main representatives of semiconducting materials) can be obtained by slowly drawing out a growing crystal from a melt. A rod, on whose tip a seed crystal is attached, is brought to the surface of melted silicon (or germanium). We then begin slowly raising the rod; the crystal coming out of the melt is made up of atoms of the basic substance, while the atoms of the admixture remain in the melt.

The method of so-called zone refining has been applied more widely. A rod of arbitrary length and several millimeters in diameter is made out of the material being purified. A small cylindrical oven is placed alongside the rod, enveloping it. The temperature in the oven is high enough for melting, and the part of the metal which is inside the oven will melt. Therefore, a small zone of melted metal moves along the rod.

Atoms of an admixture usually dissolve much more easily in a liquid than in a solid. Consequently, at the boundary of the melted zone, atoms of the admixture pass from solid parts to the melted zone and do not pass back. It is as though the moving melted zone were dragging off atoms of the admixture together with the melt. The oven is turned off during the reverse motion, and the operation of pulling the melted zone past the rod of metal is repeated many times. After a sufficient number of cycles, it only remains to saw off the polluted end of the rod. Ultrapure materials are obtained in a vacuum or in an atmosphere of inert gas.

When there is a large fraction of alien atoms, purification is carried out by other methods, zone melting and pulling of a crystal out of a melt being applied only for the final purification of the material.

Adsorption

Gases rarely dissolve in solids, i.e. rarely penetrate crystals. But there exists a different kind of absorption of gases by solids. Molecules of a gas accumulate on the surface of a solid body—this peculiar adherence is called *adsorption**. Thus, adsorption occurs when molecules cannot penetrate within a body, but successfully stick to its surface.

To be adsorbed means to be absorbed by a surface. But can such a phenomenon really play any kind of significant role? In fact, a layer one molecule thick, even on a very large object, will weigh a negligible fraction of a gram.

Let us perform the appropriate calculations. The area of a small molecule is in the neighbourhood of 10 square angstroms, i.e. 10^{-15} cm^2. Hence, 10^{15} molecules will fit into 1 cm^2. That many molecules, say, of water do not weigh much: 3×10^{-8} g. Even in a square meter, there will be room for only 0.0003 g of water.

More noticeable quantities of a substance will form on surface areas of hundreds of square meters. There is as much as 0.03 g of water (10^{21} molecules) per 100 m^2.

But do we come across such sizable surface areas in our laboratory practice? However, it is not hard to grasp that sometimes very small bodies, fitting onto the tip of a teaspoon, have enormous areas of hundreds of square meters.

A cube whose edges are 1 cm long has a surface area of 6 cm^2. Let us cut up such a cube into 8 equal cubes with 0.5-cm edges. The small cubes have faces with an area of 0.25 cm^2. There are $6 \times 8 = 48$ such faces in all. Their total area is equal to 12 cm^2. The surface area has been doubled.

Thus, every splitting of a body increases its surface. Let us now break up a 1-cm cube into small parts of 1-micron

* Adsorption should not be confused with absorption, which simply means taking in.

length: 1 $\mu m = 10^{-4}$ cm, so the large cube will be divided into 10^{12} pieces. Each piece (let us assume for the sake of simplicity that it, too, is cubic) has a surface area of 6 μm^2, i.e. 6×10^{-8} cm^2. The total surface area of the pieces is equal to 6×10^4 cm^2, i.e. 6 m^2. And such a splitting is by no means the limit.

It is quite understandable that the specific surface area (i.e. the surface area of one gram of the substance) can be immense. It grows rapidly with the crushing of a substance—for the surface area of a grain decreases in proportion to the square of its linear dimension, but the number of grains in a unit of volume increases in proportion to the cube of this dimension. A gram of water poured onto the bottom of a glass has a surface area of several square centimeters. The same gram of water in the form of raindrops will now have a surface whose area measures tens of square centimeters. But one gram of droplets of fog has a surface area of several hundred square meters.

If you break up a piece of coal (the finer the better), it will be capable of adsorbing ammonia, carbon dioxide and many toxic gases. This last property has assured for coal an application in gas-masks. Coal breaks up particularly well, and the linear dimensions of its particles can be reduced to ten angstroms. Therefore, one gram of special coal has a surface area of several hundred square meters. A gas-mask with coal is capable of absorbing tens of liters of gas.

Adsorption is widely employed in the chemical industry. Molecules of different gases adsorbed on a surface come in close contact with each other and participate in chemical reactions more easily. In order to speed up chemical processes, one frequently makes use of finely split-up metals (nickel, copper and others) as well as of coal.

Substances increasing the rate of a chemical reaction are called catalysts.

Osmosis

Among the living tissues, there are peculiar membranes which have the ability to let water molecules pass through them, remaining impermeable to molecules of substances dissolved in water.

The properties of these membranes are the causes of physical phenomena bearing the name "osmotic" (or simply "osmosis").

Imagine that such a semipermeable partition divides a pipe, made in the form of the letter U, into two parts. A solution is poured into one of the pipe's elbows, and water or some other solvent, into the other elbow. Having poured the same amount of liquid into both elbows, we shall discover with surprise that there is no equilibrium when the levels are equal. After a short time, the liquids settle down at different levels. Moreover, the level is raised in the elbow containing the solution. The water separated from the solution by the semipermeable partition tends to dilute the solution. It is just this phenomenon that bears the name *osmosis*, and the difference in height is called the *osmotic pressure*.

But what is the cause giving rise to osmotic pressure?

In the right-hand elbow of the vessel (Figure 108), pressure is exerted only by water. In the left-hand elbow, the overall pressure is the sum of the pressure of the water and the pressure of the solute. But the door is open only for the water, and equilibrium in the presence of the semipermeable partition is established not when the pressure from the left equals the overall pressure from the right, but when the pressure of the pure water is equal to the "water" portion of the pressure of the solution. The arising difference between the overall pressures is equal to the pressure of the solute.

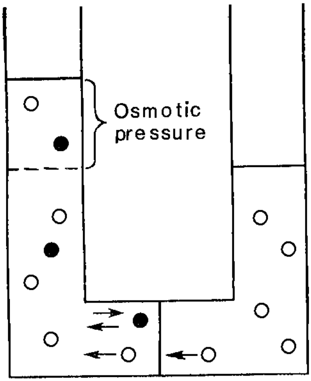


Fig. 108

This excess of pressure is precisely the osmotic pressure. As experiments and computations show, the osmotic pressure is equal to the pressure exerted by a gas composed of the solute and occupying the same volume. It is therefore not surprising that osmotic pressure is measured in impressive numbers.

Let us calculate the osmotic pressure arising in one liter of water when it dissolves 20 grams of sugar (the concentration of sugar in a glass of tea is probably higher). The molecular weight of sugar, whose chemical formula is $C_{12}H_{22}O_{11}$, equals 342. According to the conditions of the problem, there is 20/342 of a mole of sugar in one liter of water. Therefore, to one mole of sugar there is a volume of $342/20 = 17.1$ liters. But under the "normal" conditions of 0 °C and one atmosphere of pressure, one mole of gas occupies 22.4 liters. According to the laws of ideal gases at 0 °C, the pressure of sugar, regarded as a gas, would be equal to 22.4/17.1 atm, and at 20 °C, $(22.4/17.1) \times (293/273) = 1.4$ atm. This is the osmotic pressure of sugar. In the experiment with a semipermeable membrane, this osmotic pressure would balance a column of water 14 m in height.

Running the risk of arousing unpleasant memories in the reader, we shall now examine how the laxative action of certain salt solutions is related to osmotic pressure. The walls of the intestines are semipermeable to a number of solutions. If a salt does not pass through the walls of the intestines (this is true of Glauber's salt), then in the intestines there arises an osmotic pressure, which sucks water through the tissues from the organism into the intestines.

Why does very salty water fail to quench one's thirst? It turns out that osmotic pressure is guilty of this, too. The kidneys cannot eliminate urine with osmotic pressure greater than the pressure in the tissues of the organism. Consequently, an organism which has acquired salty sea water does not only fail to give it to the tissue liquids, but, on the contrary, eliminates water taken away from the tissues along with urine.

Fourteen
FRICTION

Frictional Forces

This isn't the first time that we are speaking of friction. And as a matter of fact, in telling about motion, how could we have managed without mentioning friction? Almost any motion of the bodies surrounding us is accompanied by friction. A car whose driver cut the motor comes to a halt; a pendulum comes to rest after many oscillations, a small metal ball, thrown into a jar of sunflower oil, slowly sinks. What makes bodies moving along a surface come to a halt? What is the cause of the slow falling of a ball in oil? We answer: the frictional forces arising during the motion of some bodies along the surfaces of others.

But frictional forces arise not only during motion.

You probably had to move furniture in a room. You know how hard it is to begin moving a heavy bookcase. The force counteracting this effort is called *static friction.*

Frictional forces also arise when we slide and roll objects. These are two somewhat different physical phenomena. We therefore distinguish between *sliding friction* and *rolling friction.* Rolling friction is tens of times less than sliding friction.

Of course, in certain cases sliding also proceeds with great ease. A sled slides easily along snow, and the sliding of skates along ice is even easier.

But what causes do frictional forces depend on?

A frictional force between solid bodies depends little on the speed of the motion and is proportional to a body's weight. If the weight of a body doubles, then it will be twice as hard to set it in motion and to keep pulling it. We haven't expressed ourselves with complete precision: what is important is not so much the weight as the force pressing the body to the surface. If a body is light, but we press down hard on it with our hand, then this will of course affect the force of friction. If we denote the force pressing a body to a surface (this is its weight in most cases) by P, then the following simple formula will be valid for the frictional force F_{fr}:

$$F_{fr} = kP$$

But how are properties of the surfaces taken into account? For it is well known that one and the same sled will slide completely differently on the very same runners, depending on whether or not the runners are bound with iron. These properties are taken into account by the proportionality factor k, which is called the *friction coefficient*.

The friction coefficient of metal on wood is approximately equal to 1/2. Only with a force of 1 kgf will one succeed in setting in motion a metallic slab weighing 2 kgf which is lying on a smooth wooden table. But the friction coefficient of steel on ice is equal to only 0.027. That same slab can be moved on ice with a force of only 54 gf.

The surface area does not occur in the above formula: the force of friction does not depend on the area of the contact surface between the bodies rubbing against each other. The same force is needed in order to set in motion, or to keep moving with a constant velocity, a wide sheet of steel weighing a kilogram and a kilogram weight which is supported by the surface of a small area.

And one more remark about forces of sliding friction. It is somewhat more difficult to set a body in motion than to keep it moving: the force of friction overcome at the first instant of motion (static friction) exceeds the subsequent values by 20-30%.

What can be said about the force of rolling friction, say for a wheel? Just as for sliding friction, the greater the force pressing a wheel to a surface, the greater will the rolling friction be. Furthermore, the force of rolling friction is inversely proportional to the radius of the wheel. This is understandable: the larger the wheel, the less perceptible will be the roughness of the surface along which it is rolling.

If we compare the forces which must be overcome in making a body slide and roll, then the difference we obtain is very impressive. For example, in order to pull a steel bar weighing 1 tonf along an asphalt pavement, a force of 200 kgf must be applied—only an athlete is capable of doing this. But even a child can roll this very same bar on a cart—a force not greater than 10 kgf is needed for this.

It's no wonder that rolling friction has "defeated" sliding friction. It was not without reason that humanity switched to wheel transport a very long time ago.

The replacement of runners by wheels is not yet a complete victory over sliding friction. For a wheel must be attached to an axle. At first glance it seems impossible to avoid the friction of the axle on the bearings. So people thought during the course of centuries, and tried to decrease the sliding friction in bearings only by means of various lubricants. The services rendered by lubricants have not been small—sliding friction has been reduced by a factor of 8-10. But in a great many cases, the sliding friction, even with lubrication, is so considerable that it costs too much. This circumstance greatly impeded the development of technology at the end of the past century. It was then that the remarkable

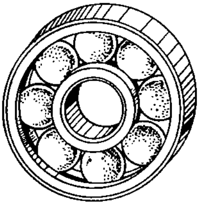


Fig. 109

idea arose of replacing the sliding friction in bearings by rolling friction. Ball-bearings have accomplished this replacement. Small balls were fitted in between the axle and the bush. The balls roll along the bush, and the axle on the balls, when the wheel turns. The construction of this mechanism is shown in Figure 109. Sliding friction was replaced by rolling friction in this manner, thus reducing frictional forces tens of times over.

The role played by ball- and roller-bearings in modern technology can scarcely be exaggerated. They are made with balls, with cylindrical rollers, with conical rollers. All machines, large and small, are equipped with such bearings. There exist ball-bearings a millimeter in diameter; some bearings for large machines weigh over a ton. Balls for bearings (you have seen them, of course, in certain store windows) are manufactured with the most varying diameters—from fractions of a millimeter to several centimeters.

Viscous Friction in Liquids and Gases

Until now we have been speaking of "dry" friction, i.e. of the friction arising from the contact of two solid objects. But floating and flying bodies are also subject to the action of frictional forces. The source of friction has changed—dry friction is replaced by "wet" friction.

The resistance which a body moving in water or air experiences is subject to other laws, essentially different from the laws of dry friction, which we have spoken of above.

The rules for the behaviour of a liquid and a gas with respect to friction cannot be distinguished. Therefore, everything said below pertains to liquids and gases to the same degree. If we speak of a "liquid" in what follows for the sake of brevity, what we say will equally well pertain to gases.

One of the distinctions between "wet" and dry friction consists in the absence of "wet static friction"; an object suspended in water or air can, generally speaking, be started in motion by an arbitrarily small force. But as for the frictional force experienced by a moving body, it will depend on the speed of the motion, on the form and dimensions of the body and on the properties of the liquid (gas). The study of the motion of bodies in liquids and gases has shown that there is no single law for "wet" friction, but there are two different laws: one is valid for motions with low speeds, and the other for high. The existence of two laws implies that the flow of a medium around a body moving in it takes place differently for motions of solids with high and low speeds.

For motions with low speeds, the resistance force is directly proportional to the speed and a linear dimension of the body:

$$F \propto vL$$

How should we understand the proportionality to dimension if we aren't told what form the body under consideration has? What is meant is that for two bodies completely similar in form (i.e. all of whose corresponding dimensions are in the same ratio), the ratio of the resistance forces is the same as that of the linear dimensions of the bodies.

The magnitude of the resistance depends to an enormous degree on the properties of the liquid. Comparing the frictional forces experienced by the same objects moving with identical speeds in various media, we see that the thicker or, as we say, the more viscous the medium, the greater will be the resistance force experienced by a body. It is therefore appropriate to call the friction under discussion *viscous friction*. It is quite understandable that air creates a negligible viscous friction, approximately 60 times less than water. A liquid can be "thin", like water, or very viscous, like sour cream and honey.

We can judge the degree of viscosity of a liquid either by the speed with which solid bodies fall in it or by the speed with which it pours through an opening.

It will take water several seconds to pour out of a half-liter funnel. A very viscous liquid will trickle out of it in hours, or even days. It is possible to give examples of even more viscous liquids. Geologists have called attention to the fact that in the craters of certain volcanoes, spherical pieces are found in accumulations of lava on the inner sides. At first sight, it was completely impossible to understand how such a sphere might be formed out of lava inside a crater. This would be incomprehensible if lava were regarded as a solid. But if lava behaves like a liquid, then it will drop down the crater just like any other liquid. But only a drop will be formed not in a fraction of a second, but in the course of decades. When the drop becomes too heavy, it will break away and fall on the bottom of the crater.

It is clear from this example that real solid bodies and amorphous bodies, which, as we know, resemble liquids much more than crystals, should not be put on the same level. Lava is precisely such an amorphous body. It seems to be a solid, but is actually a very viscous liquid.

What do you think, is sealing-wax a solid? Take two corks and place them on the bottom of two cups. Pour any melted salt (for example, saltpeter—it is easily obtained) into one, and pour sealing-wax into the other cup with a cork. Both liquids will harden and bury the corks. Put these cups away in the cupboard and forget about them for a long time. After several months, you will see the difference between sealing-wax and salt. The cork drowned in salt will be at rest, as before, on the bottom of its cup. But the cork drowned in sealing-wax will turn out to be on the top. But how did this occur? Very simply: the cork came to the surface in quite the same way as it would do in water. The only difference being is that of time: when the force of viscous friction is small, a cork comes to the surface instantly, but in very viscous liquids, floating up takes months.

Resistance Forces at High Speeds

But let us return to the laws of "wet" friction. As we explained, at low speeds the resistance depends on the viscosity of the liquid, the speed of the motion and the linear dimensions of the body. Let us now consider the laws of friction for high speeds. But first of all, we must say what speeds are to be regarded as low, and what speeds as high. We are interested not in the numerical value of a speed, but in whether a speed is sufficiently low for the law of viscous friction considered above to hold.

It turns out that it is impossible to name a number of meters per second, such that the laws of viscous friction be applicable in all cases with lower speeds. The bounds of applicability of the law we have studied depends on the dimensions of the body and on the degree of viscosity and density of the liquid.

For air, "low" is a speed less than

$$\frac{0.75}{L\,(\text{cm})}\,\frac{\text{cm}}{\text{sec}}$$

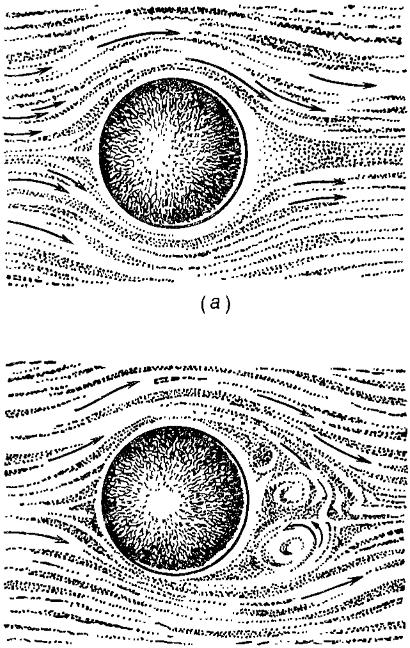
for water, less than

$$\frac{0.05}{L\,(\text{cm})}\,\frac{\text{cm}}{\text{sec}}$$

and for viscous liquids like thick honey, less than

$$\frac{100}{L\,(\text{cm})}\,\frac{\text{cm}}{\text{sec}}$$

Consequently, the laws of viscous friction are scarcely applicable to air and especially to water: even for low speeds, of the order of 1 cm/sec, they will apply only for tiny bodies with millimeter dimensions. The resistance experienced by a person diving into water is to no extent subject to the law of viscous friction.



(a) (b)

Fig. 110

But how can we explain the fact that the law governing a medium's resistance changes with a change in speed? The causes must be sought in the change in the character of the flow of a liquid around a body moving in it. Two circular cylinders moving in a liquid are depicted in Figure 110 (their axes are perpendicular to the drawing). For a slow motion, the liquid flows smoothly around the moving object—the resistance force which it has to overcome is the force of viscous friction (Figure 110*a*). For a high speed, behind the moving body there arises a complicated irregular motion of the liquid (Figure 110*b*). Various streams appear and disappear, forming fantastic figures, rings and eddies. The picture made by the streams is changing all the time. The appearance of this motion, called turbulent, radically changes the law of resistance.

Turbulent resistance depends on the speed and dimensions of an object in an entirely different way than viscous resistance: it is proportional to the square of the speed and the square of the linear dimensions. The viscosity of a liquid during this motion ceases to play an essential role; the determining property becomes its density, with the resisting force becoming proportional to the first power of the density of the liquid (gas). Therefore, the following formula holds for the force F of turbulent resistance:

$$F \propto \rho v^2 L^2$$

where v is the speed of the motion, L is a linear dimension of the object, and ρ is the density of the medium. The numerical proportionality factor, which we haven't written down, has various values, depending on the form of the body.

Streamline

Motion in the air, as has been said above, is almost always "speedy", i.e. the basic role is played by turbulent, and not viscous, resistance. Airplanes, birds and parachutists experience turbulent resistance. If a person falls through the air without a parachute, then after a certain time he begins falling uniformly (the resisting force balances the weight), but with quite a considerable speed, of the order of 50 m/sec. The opening of a parachute leads to a sharp deceleration of the fall—the same weight is now balanced by the resistance of the canopy. Since the resisting force is proportional to the speed of the motion, as well as to a linear dimension of the falling object, the speed will decrease as many times as the linear dimensions of the falling body increase. The diameter of a parachute is about 7 m, whereas a person's "diameter" is about one meter. The speed of the fall will be decreased to 7 m/sec. One can land safely with such a speed.

We must say that the problem of increasing the resistance is far more easily solved than the converse problem. Reducing the air resistance on a car or an airplane, or the water resistance on a submarine, is a most important and difficult technological problem.

It proves possible to decrease turbulent resistance by a large factor by changing the form of a body. For this one must reduce to a minimum the turbulent motion, which is the source of the resistance. This is achieved by, as they say, streamlining the object.

But what form is best in this sense? It would appear at first sight that the body should be shaped so that the forward side comes to a point. Such a point, it seems, should "cleave" the air most successfully. But it proves important not to cleave the air, but to disturb it as little as possible so that it flows smoothly around the object. The best profile for a body moving in a liquid or a gas is a form which is obtuse in front and pointed behind*. With such a form, the liquid would flow smoothly off the point and the turbulence would be reduced to a minimum. Sharp edges should not in any case be put in front, since there they would create turbulence.

* The pointed prows of boats and sea-going vessels are needed for "breaking" waves, i.e. only when the motion is taking place along the surface.

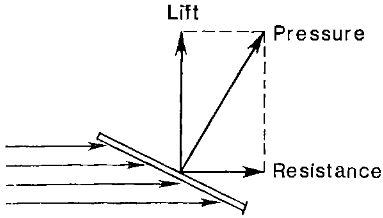


Fig. 111

The streamline of an airplane wing creates not only the least resistance to the motion, but also the greatest lift, when the fairing is inclined upwards from the direction of the motion. Flowing around a wing, the air pushes against it mainly in a direction perpendicular to its plane (Figure 111). It is clear that for an inclined wing this force is directed upwards.

The lift grows with an increase in the angle of attack. But reasoning based solely on geometrical considerations would lead us to the false conclusion that the greater the angle of attack, the better. But as a matter of fact, as the angle of attack is increased, it becomes more and more difficult for the air to flow smoothly around the wings, and at a certain value for this angle, as illustrated in Figure 112, violent turbulence arises; the resistance sharply increases and the lift decreases.

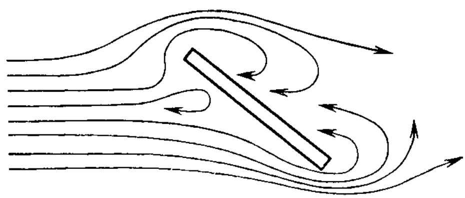


Fig. 112

Disappearance of Viscosity

Very often, in explaining some phenomenon or describing the behaviour of one or another group of bodies, we refer to well-known examples. It is quite obvious that this object should be moving in such a manner, we say, for other bodies also move according to the very same rules. An explanation reducing the new to that which we have already come across in the course of our lives always satisfies the majority. We therefore did not have any particular difficulty in explaining to the reader the laws in accordance with which liquids move—for everyone has seen how water flows, and the laws governing this motion seem quite natural.

However, there is on perfectly amazing liquid which does not resemble any other liquid and moves in accordance with special laws, characteristic of it, alone. It is liquid helium.

We have already said that liquid helium remains in the liquid phase right down to a temperature of absolute zero. However, helium above 2 K (more precisely, 2.19 K) and helium below this temperature are two completely different liquids. Above two degrees, the properties of helium in no way distinguish it from other liquids. Below this temperature, helium becomes a miraculous liquid. This miraculous helium is called helium II.

The most striking property of helium II is its *superfluidity*, i.e. complete lack of viscosity, discovered by P. L. Kapitza in 1938.

In order to observe superfluidity, a vessel is made with a very fine slit at the bottom—of only half a micron in width. An ordinary liquid hardly seeps through such a slit; helium also behaves this way at temperatures above 2.19 K. But as soon as the temperature becomes barely less than 2.19 K, the speed with which helium flows out of such a vessel grows by leaps and bounds—by a factor of at least several thousand.

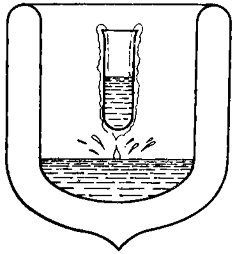


Fig. 113

Helium II almost momentarily flows through the narrowest clearance, i.e. it completely loses its viscosity. The superfluidity of helium leads to an even stranger phenomenon. Helium II is capable of "climbing" by itself out of the glass or test-tube into which it was poured.

A diagram for carrying out this experiment is shown in Figure 113. A test-tube with helium II is placed in a Dewar vessel over a helium bath. "Without rhyme or reason", helium rises along the wall of the test-tube in the form of a very fine, completely unnoticeable film and overflows; drops fall from the bottom of the test-tube.

It should be recalled that thanks to capillary forces, which were discussed on p. 237, the molecules of every liquid wetting the wall of a vessel climb up this wall and form the finest film on it, whose width is of the order of one millionth of a centimeter. This film cannot be seen by an eye, and in general does not manifest itself in any way when the liquid is of ordinary viscosity.

The picture changes completely if we are dealing with helium devoid of viscosity. In fact, a narrow slit does not hinder the movement of superfluid helium, and a thin film on a surface is just the same as a narrow slit. A liquid devoid of viscosity flows in a very fine layer. The film covering the surface forms a siphon over the edge of the glass or test-tube through which the helium overflows the vessel.

It is obvious that we do not observe anything of the kind in the behaviour of an ordinary liquid. A liquid of normal viscosity is practically unable to "make its way" through a siphon of negligible thickness. Such a motion is so slow that the outflow would last millions of years.

Thus, helium II is devoid of any viscosity. The conclusion that a solid should move without friction in such a liquid would appear to follow from this with iron logic. Let us take a disc on a string, place it in liquid helium II and twist the string. Leaving this uncomplicated device alone, we create something like a pendulum—the string with the disc will oscillate and periodically twist first in one and then in another direction. If there were no friction, we should expect the disc to oscillate perpetually. But nothing of the kind! After a comparatively short time, approximately the same as for ordinary normal helium I (i.e. helium at a temperature above 2.19 K), the disc comes to a halt. What kind of strangeness is this? When flowing through a slit, helium behaves like a liquid without viscosity, but behaves like an ordinary viscous liquid in relation to bodies moving in it. It is this, to be sure, that is in fact completely extraordinary and incomprehensible.

It now remains for us to recall what has been said as regards the very fact that helium does not solidify right up to absolute zero, for the point is one of the unsuitability of the ideas about motion which we are accustomed to. If helium has remained a liquid "illegally", then should we be surprised by the lawless behaviour of this liquid?

It is only possible to understand the behaviour of liquid helium from the point of view of the new conceptions of motion which have received the name of *quantum mechanics*. Let us try to give a general idea of how quantum mechanics explains the behaviour of liquid helium.

Quantum mechanics is an extremely intricate theory, which is very hard to understand, and so the reader should not be surprised that the explanation looks even stranger than the phenomena, themselves. It turns out that every particle of liquid helium participates simultaneously in two motions: one motion is superfluid, unrelated to viscosity, and the other is ordinary.

Helium II behaves as if it consisted of a mixture of two liquids moving completely independently, "one through the other". One liquid behaves normally, i.e. possesses an ordinary viscosity, while the other component part is superfluid.

When helium flows through a slit or overflows a glass, we observe the effect of superfluidity. But during the oscillation of a disc submerged in helium, the friction stopping the disc is created because friction is inevitable in the normal part of helium.

The ability to participate in two distinct motions also gives rise to the completely unusual heat-conducting properties of helium. As has been said already, liquids in general conduct heat rather poorly. Helium I also behaves like an ordinary liquid. But when a transformation into helium II takes place, its thermal conductivity grows about a billion-fold. Therefore, helium II conducts heat better than the best ordinary heat conductors, such as copper and silver.

The fact is that the superfluid motion of helium does not participate in the heat transfer. Consequently, when there is a difference in temperature within helium II, there arise two currents, going in opposite directions, and one of them—the normal one—carries heat with it. This does not at all resemble ordinary heat conduction. In an ordinary liquid, heat is transferred by means of molecular collisions. In helium II, heat flows together with the ordinary part of helium—flows like a liquid. Here at last the term "heat flow" is fully justified. It is precisely such a method of transferring heat that leads to an immense thermal conductivity.

This explanation of the thermal conductivity of helium may seem so strange that you will refuse to believe it. But it is possible to convince oneself first-hand of the validity of what has been said, by means of the following conceptually simple experiment.

In a tub with liquid helium there is a Dewar vessel, filled to the brim with helium. The vessel is connected to the tub by means of a capillary branch piece. The helium inside the vessel is heated by an electric spiral, but heat does not pass to the surrounding helium through the walls of the vessel, since they do not transmit heat.

There is a vane, suspended on a fine thread, near the end of the capillary tube. If the heat is flowing like a liquid, then it should turn the vane. This is precisely what happens. Moreover, the amount of helium in the vessel does not change. How can this miraculous phenomenon be explained? In only one way: during heating there arises a current of the normal part of the liquid from the heated place to the cold place, and a current of the superfluid part in the opposite direction. The amount of helium at each point does not change, but since the normal part of the liquid moves together with the heat transfer, the vane turns as a result of this part's viscous friction and remains deflected as long as heating continues.

Another conclusion also follows from the fact that superfluid motion does not transfer heat. We have spoken above about the "creeping" of helium over the brim of a glass. But the superfluid part "climbs out" of the glass, while the normal part remains there. Heat is connected to only the normal part of helium and does not accompany the superfluid part which is "climbing out" of the glass. Hence, as helium "climbs out" of the vessel, one and the same amount of heat will be shared by a smaller and smaller quantity of helium—the helium remaining in the vessel should warm up. This is actually observed during experiments.

The masses of the helium partaking in superfluid and normal motion are not identical. Their ratio depends on the temperature. The lower the temperature, the greater the superfluid part. All of the helium becomes superfluid at absolute zero. As the temperature rises, a larger and larger part of the helium begins to behave normally, and at a temperature of 2.19 K, all of the helium becomes normal, acquiring the properties of an ordinary liquid.

But the reader already has some questions on the tip of his tongue: what is this thing called superfluid helium, how can a particle of liquid participate simultaneously in two motions and how can the very fact of two motions of a single particle be explained?... Unfortunately, we are obliged to leave all these questions unanswered. The theory of helium II is too complicated, and it is necessary to know a great deal in order to understand it.

Plasticity

Elasticity is the ability of a body to recover its form after a force has stopped acting on it. If a kilogram weight is hung on a 1-m steel wire with cross-sectional area of 1 mm^2, then the wire will be stretched. The stretching is negligible, only 0.5 mm in all, but it is not difficult to observe. If the load is removed, then the wire will contract by the same 0.5 mm, and so its length will return to its former value. Such a deformation is called *elastic*.

Let us note that a wire of 1-mm^2 cross-section under the action of a force of 1 kgf and a wire of 1-cm^2 cross-section under the action of a force of 100 kgf are, as one says, under the same conditions of mechanical stress. Therefore, the behaviour of a material must always be described by indicating not the force (which would be pointless if the cross-section of a body is unknown), but the stress, i.e. the force per unit of area. Ordinary bodies—metals, glass, stones—can be stretched elastically by only several per cent at best. Rubber possesses outstanding elastic properties. Rubber can be stretched elastically by several hundred per cent (i.e. made two or three times longer than it was originally), and when we let such a rubber band go, we see that it returns to its initial state.

All bodies without exception behave elastically under the action of small forces. However, a limit of elasticity appears earlier for some bodies and considerably later for others. For example, the elastic limit for such soft metals like lead has already been reached when a load of 0.2-0.3 kgf is hung on a wire of 1-mm^2 cross-section. This limit is approximately 100 times as great, i.e. about 25 kgf, for such hard materials as steel.

With respect to large forces, exceeding the elastic limit, the various bodies can be roughly divided into two classes—those like glass, i.e. fragile, and those like clay, i.e. plastic.

If you press a piece of clay with your finger, it will leave its imprint, containing even the intricate whirls drawn on its skin. If you strike a piece of soft iron or lead with a hammer, a clear trace will be left. There is no longer any force, but the deformation remains—it is called *plastic* or *residual*. You will not succeed in obtaining such residual traces on glass: if you persist in this endeavour, the glass will break. Certain metals and alloys, say cast iron, are just as fragile. An iron pail will flatten, but a cast iron pot will crack, under the blows of a hammer.

One can judge the strength of fragile bodies by the following figures. In order to convert a piece of cast iron into powder, one must act with a force of about 50-80 kgf per square millimeter of surface area. This figure falls to 1.5-3 kgf for a brick.

As for every classification, the division of bodies into fragile and plastic is, to a fair degree, relative. First of all, a body which is fragile at a low temperature can become plastic at higher temperatures. Glass can be superbly processed as a plastic material if it is heated to a temperature of several hundred degrees.

Soft metals, such as lead, can be forged cold, but hard metals yield to forging only when burning hot. A rise in temperature sharply magnifies the plastic properties of a material.

One of the essential features of metals, which has made them irreplaceable building materials, is their hardness at room temperatures and plasticity at high temperatures: a burning hot metal can be easily given the required form, but it is only possible to change this form at room temperature by means of very substantial forces.

The internal structure of a material influences its mechanical properties in an essential way. It is obvious that cracks and holes weaken the apparent strength of a body and make it more fragile.

The ability of plastically deformable bodies to strengthen is remarkable. A single crystal of a metal is very soft when it has just grown up out of a melt. Crystals of many metals are so soft that they are easily bent with one's fingers, but... one will not succeed in straightening them out. Strengthening has taken place. This same specimen can now be plastically deformed only by means of a considerably greater force. It turns out that plasticity is not only a property of a material, but also a property of its treatment.

Why are instruments made not by casting metals, but by forging them? The reason is obvious—a metal subjected to forging (or rolling, or drawing) is much stronger than cast metal. No matter how much we forge a metal, we cannot increase its strength beyond a certain limit, which is called the yield stress. For steel it is between 30 and 50 kgf/mm^2.

This figure has the following meaning. If a one-pood weight (below the yield stress) is hung on a wire of 1-mm^2 cross-section, then the wire will begin stretching and strengthening simultaneously. The stretching will therefore quickly cease—the weight will hang calmly on the wire. But if a three- or four-pood weight (above the yield stress) is hung on such a wire, then the picture will be different. The wire will keep stretching until it breaks. Let us emphasize once again that the mechanical behaviour of a body is determined not by the force, but by the stress. A wire of a cross-section of 100 square microns will yield under the action of a load of (30-50) $\times$ 10^{-4} kgf, i.e. 3-5 gf.

Hardness

Strength and hardness do not go hand in hand with each other. A rope, a scrap of cloth and a silk thread can possess a great deal of strength—a considerable stress is needed to tear them. Of course, nobody will say that rope and cloth are hard materials. And conversely, the strength of glass is not great, but glass is a hard material.

The concept of hardness used in technology is taken from everyday practice. *Hardness* is the resistance to penetration. A body is hard if it is difficult to scratch it, difficult to leave an imprint on it. These definitions may seem somewhat vague to the reader. We are accustomed to physical concepts being expressed in terms of numbers. But how can this be done in relation to hardness?

One rather primitive method, which is nevertheless useful in practice, has already been employed for a long time by mineralogists. Ten definite minerals are arranged in a series. The first is diamond, followed by corundum, then topaz, quartz, feldspar, apatite, fluor-spar, calcite, gypsum and talc. The series is sorted out in the following manner: diamond leaves its mark on all minerals, but none of these minerals can scratch diamond. This means that diamond is the hardest mineral. The hardness of diamond is evaluated by the number 10. Corundum, which follows diamond in the series, is harder than all the minerals below it—corundum can scratch them. Corundum is assigned the hardness number 9. The numbers 8, 7 and 6 are assigned to topaz, quartz and feldspar, respectively, on the same grounds. Each of them is harder than (i.e. can scratch) all the minerals below it, and softer than (can itself be scratched by) the minerals having greater hardness numbers. The softest mineral—talc—has hardness number 1.

A "measurement" (this word must be taken in quotation marks) of hardness with the aid of this scale consists in finding the place for the mineral of interest to us in a series containing the ten chosen standards. If the unknown mineral can be scratched by quartz, but leaves its mark on feldspar, then its hardness is equal to 6.5.

Metallurgists use a different method for determining hardness. A dent is made on the material being tested by pressing a steel ball 1 cm in diameter against it with a standard force (ordinarily 3000 kgf). The radius of the small pit so formed is taken as the hardness number.

Hardness with respect to scratching and hardness with respect to pressing do not necessarily correspond, and one material may prove to be harder than another when tested by scratching, but softer when testing by pressing.

Consequently, there is no universal concept of hardness, independent of the method of measurement. The concept of hardness is a technological, but not a physical, concept.

Fifteen
SOUND

Sound Vibrations

We have already given the reader a lot of information about oscillations. How a pendulum and a ball on a spring oscillate, what regularities there are in the oscillation of a string—Chapter V of our book was devoted to these questions. We haven't spoken about what takes place within the air or some other medium when a body located in it performs oscillations. There is no doubt that the medium cannot remain indifferent to vibrations. An oscillating object pushes the air, displaces the air particles from the positions in which they were previously located. It is also obvious that the matter cannot be limited to an influence on only the adjacent layer of air. The body will push the nearest layer, this layer presses against the next one, and so layer after layer, particle after particle, all of the surrounding air is brought into motion. We say that the air has come to a vibrating state, or that *sound vibrations* are taking place in the air.

We call the vibrations of the medium sound vibrations, but this does not mean that we hear all of them. Physics uses the concept of sound vibrations in a broader sense. The question as to which sound vibrations are heard will be discussed below.

We are dealing with air only because sound is most often transmitted through air. But there are, of course, no special properties of air which would give it a monopoly on the right to perform sound vibrations. Sound vibrations arise in any medium capable of being compressed, but since there are no incompressible bodies in nature, the particles of any material can therefore find themselves in this state. The study of such vibrations is usually called *acoustics*.

Each particle of air remains at one place, on the average, during sound vibrations—it only performs oscillations about its equilibrium position. In the simplest case, a particle of air can perform a harmonic oscillation, which, as we recall, takes place in accordance with a sinusoidal law. Such an oscillation is characterized by the maximal displacement of a particle from its equilibrium position (amplitude) and the period of the oscillation, i.e. the time required to perform a complete oscillation.

The concept of the *frequency of vibration* is used more often than that of the period for describing the properties of sound vibrations. The frequency $\nu = 1/T$ is the reciprocal of the period. The unit of frequency is the inverse second (sec^{-1}). If the frequency of vibration is equal to 100 sec^{-1}, then this means that during one second a particle of air performs 100 complete vibrations. Instead of saying, "100 inverse seconds", one may say, "100 hertz" (Hz) or "100 cycles per second". Since in physics we must deal rather often with frequencies which are many times greater than a hertz, the units *kilohertz* (kilocycles) and *megahertz* (megacycles) are widely applied; 1 kHz = 10^3 Hz, 1 MHz = 10^6 Hz.

The speed of a vibrating particle is maximal when it is passing through its equilibrium position. On the contrary, in positions of maximal displacement, the speed of a particle is, naturally, equal to zero. We have already said that if the displacement of a particle is subject to a law of harmonic

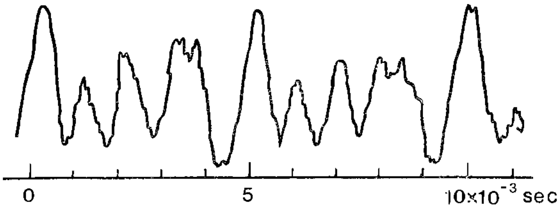


Fig. 114

oscillation, then the change in its speed of vibration obeys the same law. If we denote the amplitude value of the displacement by s_0, and of the speed by v_0, then $v_0 = 2\pi s_0/T$, or $v_0 = s_0 2\pi\nu$. Loud conversation brings air particles into vibration with an amplitude of only several millionth of a centimeter. The amplitude value of the speed will be of the order of 0.02 cm/sec.

Another important physical quantity, varying together with the displacement and speed of a particle, is the *excess pressure*, also called the *sound pressure*. A sound vibration of air consists in a periodic alternation of compression and rarefaction at each point in the medium. The air pressure at any place is now higher, now lower than the pressure which would be there in the absence of sound. This excess (or insufficiency) of pressure is just what is called the sound pressure. Sound pressure is a small fraction of normal air pressure. For our example—loud conversation—the sound pressure amplitude will be equal to approximately one millionth of the atmospheric pressure. Sound pressure is directly proportional to the speed of a particle's vibration, where the ratio of these physical quantities depends only on the properties of the medium. For example, a speed of vibration of 0.025 cm/sec corresponds to a sound pressure in air of 1 dyne/cm^2.

A string vibrating in accordance with a sinusoidal law brings air particles into harmonic oscillation. Noises and

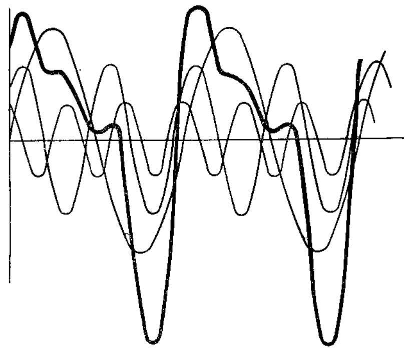


Fig. 115

musical sounds lead to a considerably more complicated picture. A graph of sound vibrations, namely of the sound pressure as a function of time, is shown in Figure 114. This curve bears little resemblance to a sine wave. It turns out, however, that any arbitrarily complicated vibration can be represented as the result of superimposing a large number of sine waves with different amplitudes and frequencies. These simple vibrations make up, as is said, the spectrum of the complex vibration. Such a superposition of vibrations is shown for a simple example in Figure 115.

Speed of Sound

One shouldn't be afraid of thunder after the lightning has flashed. No doubt you have heard about this. But why is this so? The fact is that light is propagated incomparably faster than sound—practically instantaneously. Thunder and lightning occur at one and the same moment, but we see the lightning when it comes into existence, whereas the sound of thunder reaches us with a speed of approximately one kilometer in three seconds (the speed of sound in air is 330 m/sec). Hence, when the thunder is audible, the danger of being struck by lightning has already been over.

Knowing the speed of propagation of sound, we can usually determine how far away a thunderstorm is raging. If 12 seconds have passed from the moment of the flash of lightning to that of the peal of thunder, the storm is therefore 4 kilometers away.

The speed of sound in gases is approximately equal to the average speed of the motion of their molecules. It is also independent of the density of a gas and proportional to the square root of its absolute temperature. Liquids propagate sound faster than gases. Sound is propagated in water with a speed of 1450 m/sec, i.e. 4.5 times as fast as in air. The speed of sound in solids is even greater; for example, it is about 6000 m/sec in iron.

When sound passes from one medium to another, the speed of its propagation changes. But another interesting phenomenon also occurs simultaneously—the partial reflection of sound from the boundary between the two media. The fraction of sound reflected depends mainly on the ratio of the densities. In the case when sound passing through air is incident upon a solid or liquid surface, or vice versa, the sound is almost completely reflected. When sound arrives from air to water or, conversely, from water to air, only 1/1000 of the sound passes into the latter medium. If both media are dense, then the ratio between the transmitted and the reflected sound can be small. For example, 13% of the sound will pass from water into steel or from steel into water, and 87% will be reflected.

The phenomenon of the reflection of sound is widely applied in navigation. The construction of an instrument for measuring depths—the sonic depth finder (Figure 116)—

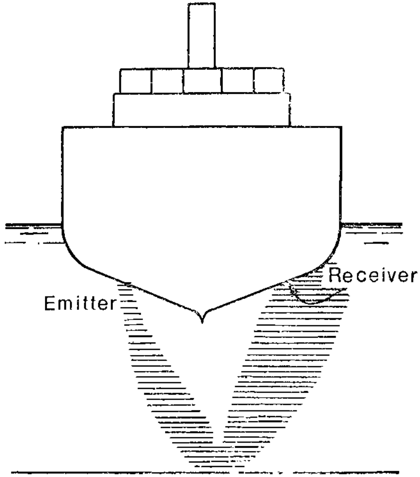


Fig. 116

is based on it. A source of sound is placed under water on one side of a ship. A discontinuous sound creates sound rays, which will make their way through the watery thickness to the bottom of the sea or river, be reflected from the bottom and return in part to the ship, where sensitive instruments will catch them. Accurate clocks will show how much time the sound needs for this trip. The speed of sound in water is known, and it is possible to obtain precise information about the depth by means of a simple calculation.

By aiming the sound forwards or sideways instead of downwards, it is possible to determine, with its aid, whether or not there are dangerous reefs or deeply submerged icebergs near the ship.

Sound Wave

If sound were propagated instantaneously, then all the air particles would vibrate in unison. But sound is not propagated instantaneously, and the masses of air lying on the lines of propagation are brought into motion in turn, as if caught up by a wave coming from some source. In exactly the same way, a chip lies calmly on the water until the circular waves from a pebble thrown into the water catch it up and make it vibrate.

Let us confine our attention to a single vibrating particle and compare its behaviour with the motion of other particles lying on the same line of sound propagation. An djacent particle will start vibrating somewhat later, the next particle, still later. The delay will keep increasing until we meet a particle which is lagging behind by a whole period and is therefore vibrating in time with the initial particle. So an unsuccessful runner, who has fallen behind the leader by an entire lap, can cross the finish line simultaneously with the leader. But at what distance will we meet a point which is vibrating in time with the initial particle? It is not hard to see that this distance λ is equal to the product of the speed of propagation of sound c by the period of vibration T. The distance λ is called the *wavelength*:

$$\lambda = cT$$

We will meet points vibrating in time after intervals of length λ. Points separated by a distance of $\lambda/2$ will move relative to each other like an object vibrating perpendicularly to a mirror moves with respect to its image.

If we depict the displacement (or speed, or sound pressure) of all the points lying on a line of propagation of a harmonic sound, then a sine wave is again obtained.

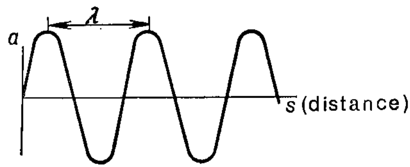


Fig. 117

The graphs of the wave motion and the vibration should not be confused. There is a strong resemblance between Figures 117 and 118, but the distance is plotted along the horizontal axis in the former, while the time, in the latter. One figure represents the development of the vibration in time, but the other, an instantaneous "photograph" of the wave. It is evident from a comparison of these two graphs that the wavelength may also be called its spatial period: the quantity λ plays the same role in space as T plays in time.

In a graph of a sound wave, the displacements of a particle are plotted along the vertical axis, and the horizontal axis is the direction of the wave's propagation along which the distance is marked off. This might suggest the false idea that the particles are displaced perpendicularly to the direction in which the wave is propagated. In reality, air particles always vibrate along the direction of propagation of the sound. Such a wave is called *longitudinal*.

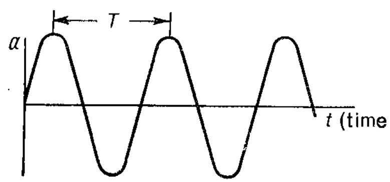


Fig. 118

Audible Sound

But what sounds can be perceived by the human ear? It turns out that the ear is capable of perceiving only the vibrations lying within the interval from approximately 20 to 20 000 Hz.

Sounds with a large frequency are called high-pitched, and with a small frequency, low-pitched.

But what wavelengths correspond to the limiting audible frequencies? Since the speed of sound is approximately equal to 300 m/sec, using the formula $\lambda = cT = c/\nu$, we find that the lengths of audible sound waves lie within the limits of 15 m for the lowest tones to 3 cm for the highest.

And how do we "hear" these vibrations?

The way the organ of hearing functions has not as yet been fully clarified. The crux of the matter is that the internal ear (the cochlea, a canal of few centimeters long and filled with fluid) contains several thousand sensory nerves capable of perceiving sound oscillations transmitted to the cochlea from the air through the tympanic membrane. This or that part of the cochlea vibrates more strongly depending on the frequency of sound. Although the sensory nerves are situated along the cochlea so closely that a large number of them are stimulated simultaneously, man (and animals) is capable, particularly in childhood, of distinguishing minute changes in frequency (thousands of a fraction). It is still not known precisely just how this occurs. The only obvious fact is that the analysis by the brain of the stimuli arriving from many different nerves is of utmost importance here. A mechanical model of the same design as the human ear that could be capable of discerning sound frequencies just as well has yet to be invented.

Some people possess the ability to distinguish tones perfectly: if you play a complex chord on a piano, such a listener will tell you which keys you struck. Hence, his or her ear is capable of decomposing a complex sound into its harmonic components.

Music

The difference between musical sound and noise has already been illustrated by means of sound pressure curves. A simple musical tone is created by periodic vibrations of a definite frequency. Complex sounds are combinations of pure tones.

The musicians in an orchestra produce almost all the audible frequencies. The range of a grand piano encompasses tones with frequencies from about 25 to 4000 Hz.

Not all combinations of sound give the listener pleasure. It turns out that pleasant sensations are created by sounds whose vibrational frequencies form simple ratios with each other. If sound frequencies are related to one another as $2 \div 1$, then one speaks of an octave, if $5 \div 4$, of a major third, if $4 \div 3$, of a fourth, and $3 \div 2$, of a quint. The sensation of harmony is lost if the frequencies of the sound vibrations cannot be expressed by such simple ratios. Then musicians speak of dissonance. The ear senses combinations of different tones quite well. Therefore, even people with a mediocre ear for music are sensitive to dissonance.

With the aid of instruments without keys, such as the violin, a musician can hit any tone and sound any combination of tones.

Things are different for such instruments as the grand piano. The strings of a grand piano are tuned to definite frequencies, and the tonalities of their sounds cannot be changed by striking the keys. The keyboard of a grand piano contains seven complete octaves. Lower C has a frequency of 32.64 Hz, and upper C, $32.64 \times 2^7 \approx 4178$ Hz. The problem consists in how to divide an octave, i.e. what intermediate tones should be introduced, so that the following two conditions are satisfied. Firstly, the frequencies should bear the simplest possible ratios to one another. Secondly, an octave must be divided into equal intervals (i.e. the ratios between adjacent frequencies must be equal), since only in this case is it possible to play one and the same melody, starting with any note of an octave (the same melody in a different tonality). Strictly speaking, these two requirements are contradictory. They are approximately realized with the use of the so-called equally tempered scale.

Let us consider what is obtained if an octave is divided into 12 equal intervals. Each of these intervals will be equal to $2^{1/12} = 1.059$. This means that the ratio of two adjacent tones will be equal to this number.

Let us now write out the following numbers:

(1) $2^{1/12} = 1.059$	(5) $2^{5/12} = 1.335$	(9) $2^{9/12} = 1.682$
(2) $2^{2/12} = 1.122$	(6) $2^{6/12} = 1.414$	(10) $2^{10/12} = 1.782$
(3) $2^{3/12} = 1.189$	(7) $2^{7/12} = 1.498$	(11) $2^{11/12} = 1.888$
(4) $2^{4/12} = 1.260$	(8) $2^{8/12} = 1.587$	(12) $2^{12/12} = 2$

To his or her complete satisfaction, a musician will note that the problem has been solved by arithmetic: an octave has been divided into equal intervals, and at the same time, the ratios of many tones are quite close to the ratios of prime numbers. Here we find a quint (7), a fourth (5) and a major third (4), since the following approximations are valid: $1.498 \approx 3/2$, $1.260 \approx 5/4$ and $1.335 \approx 4/3$. The situation is excellent in other cases, too, where the difference does not exceed 1%: $1.414 \approx 7/5$, $1.122 \approx 9/8$, $1.587 \approx 8/5$, $1.682 \approx 5/3$, $1.888 \approx 17/9$, and only the first interval ($1.059 \approx 18/17$) yields an obvious dissonance.

Small deviations from the pure scale (i.e. the one in which the ratios of the adjacent frequencies are exactly equal to the whole-number ratios) are slightly noticeable to the ear, and so grand pianos with equally tempered scales have become widespread.

Quality of Sound

You have seen how a guitar is tuned—a string is stretched on a pin. If a string's length and degree of stretching are properly selected, it will produce a quite definite sound when disturbed.

If, however, you listen to the sounds of a string which is disturbed in various places—in the middle, one-quarter of the distance from where it is fastened, at any other place—then you will hear sounds which are not quite identical. The tone will be one and the same, but its shade, or, as musicians say, the quality of the sound, will be different. But what is the essence of this difference and what gives a different shade to sounds of one and the same tonality?

The difference consists in the fact that one and the same string can vibrate not only in one, but in a great many ways. Several types of possible vibrations of a string are shown in Figure 119. The vibration with the least frequency (it is also called the *fundamental frequency*) is shown in the diagram on the left. The end points are fastened, and the midpoint performs vibrations with the greatest amplitude. In order that the reader might clearly visualize a vibration of the entire string as a whole, several of its successive positions have been depicted in the figure. Among them is the position for which the whole string is stretched out in a straight line—all points of the string are simultaneously passing through their equilibrium positions. The diagram in the center shows the vibration which takes place with approximately double the fundamental frequency. Now, besides the fastened end points, the midpoint of the string is

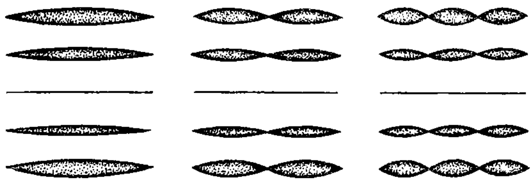


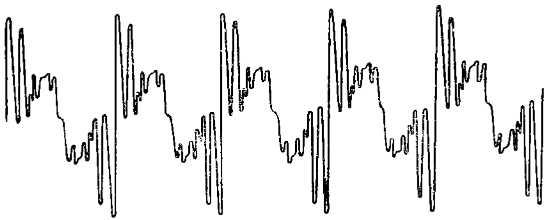
Fig. 119

also at rest. Such a point at rest is called a node of the vibration. The points located at a distance of 1/4 from an end of the string have the maximum amplitude of vibration. The loops, or antinodes, of the vibration are said to lie at these points. Several positions of the string are depicted for the sake of clarity. In this case, too, just as in all other cases, all the points of the string simultaneously pass through the zero position.

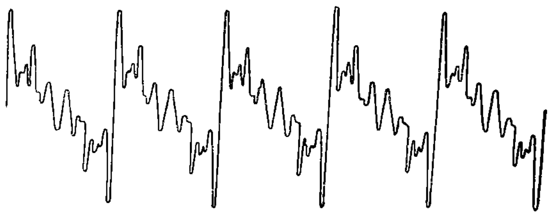
It is no longer necessary to comment on the diagram on the right, where the vibration with approximately triple the fundamental frequency is shown—two nodes and three loops characterize this vibration.

The string can also vibrate with greater frequencies, depending on what excites it. All these frequencies pertain, as one says, to the normal modes of vibration or eigenvibrations of the string.

The eigenvibrations of a string produce sounds which, except for the fundamental one, are called *overtones*. The sound of a string is composed of the sounds of the fundamental tone and the overtones. Disturbing a string at various points, we create various spectra of vibration. Thus, plucking at the midpoint leads to the fundamental tone's being very strong. Plucking at a distance of 1/4 leads to a substantial sounding of the overtone with doubled frequency. In an arbitrary case, the spectrum of the vibration will contain



(a)



(b)

Fig. 120

many overtones of different strengths. It is these overtones which create the shading (quality or timbre) of a sound.

We are now beginning to understand how one and the same tone can sound differently when sung by different voices or played on a piano or a violin. These are all sounds of a single tone, but with a different structure of overtones. It is this that gives sounds their specific shadings. Compare, for example, the two curves in Figure 120*a* and *b*. They are recordings of one and the same tone, played on a clarinet and a grand piano. We see that neither of these sounds is a simple sinusoidal vibration. The fundamental frequency of the vibration is identical in both cases—this is what creates the identity in tone. But the curves are different, which shows what is meant by the quality of a sound.

The ability of the ear to distinguish the note C on a grand piano from the same note on a clarinet is also based on the decomposition of a sound into its harmonic components, i.e. into the fundamental tone and the overtones.

The clarinet belongs to a large class of wind instruments. What are the vibrations in these instruments that create sounds of a definite tonality and various qualities? They are vibrations of air columns.

A musician playing a wind instrument is acting with his breathing not like a singer, but like a guitarist acts with his hand. The musician merely brings the air column of his trumpet into vibration. But as for tonality and quality, they are produced by the musician by varying the length of the air column. Just like a string, the air in a trumpet vibrates with definite frequencies, depending on the length of the air column.

Moving Orchestra

You are resting under a tree near a highway, and a truck with an orchestra in the process of playing drives past. Or the opposite case—you are driving through a village where a country celebration is in full swing. Several musical phrases rush past your ear in both these cases. Doesn't a sound change when we listen to it "on the run"?

Let us first deal with the musical impressions of a driver approaching an orchestra. If a car is moving towards a sound wave, then the number of concentrations of air reaching the driver's ear in a unit of time will, of course, be greater than if the car were parked.

The situation is exactly the same as if a string of running athletes, and not a sound wave, were moving towards a driver. In order that the analogy be complete, it must be assumed that the runners are maintaining the same distance between each other (this is the wavelength) and are running with a constant speed.

Of course, the number of runners passing by a car which is moving towards them will be greater per second because of the car's motion. The relative speed of the car and the runners is equal to $c + u$. The number of athletes running past the car in a unit of time will increase by just as many times as the relative speed has increased.

Therefore, the ratio of the frequency ν_{mov}, measured by a moving observer, to the frequency ν, measured by an observer at rest, is equal to the ratio of the speeds:

$$\frac{\nu_{mov}}{\nu} = \frac{c+u}{c}$$

or in another form,

$$\nu_{mov} = \nu\left(1 + \frac{u}{c}\right)$$

As the formula we have obtained shows, the frequency of the sound is raised when a car and an orchestra approach each other. If the car is travelling with a speed of 70 km/hr, the frequency of the sound will be raised by 6%.

If the car is moving away from the orchestra, the sign of the speed u must be changed to minus. The frequency of the sound will be lowered by such a relative motion. Consequently, when the car is rushing past the orchestra, the frequency of the sound will be changed by $2 \times 6 = 12\%$. A frequency of 100 Hz will be perceived as a frequency of 106 or 94 Hz, but this is a change in frequency by about a half tone. Even a listener without much musical training will sense such a change.

If $u = -c$, i.e. if the listener is flying away from the source of the sound with the speed of sound, then $\nu_{mov} = 0$; to put it simply, the sound will not be heard. If the speed of the flight exceeds the speed of the sound, then audibility will appear and the speed of the sound will grow with an increase in the speed of the flight. A minus sign will appear in the formula. It does not have a direct significance, since frequency is a positive quantity. However, the very phenomenon acquires, in a certain sense, the opposite character with the appearance of a minus sign. When flying away with a speed greater than the speed of sound, the listener is continually catching up with the sound, at first with the sound that left the source, say, a second ago, then with that which left two seconds ago, afterwards the traveller reaches the sound that departed for space three, four, etc. seconds ago. Thus, all the sounds will be heard in the opposite order.

Let us return to our general formula for the change in the frequency. Can this same formula be used for the case of the moving orchestra? It undoubtedly can, but it is only necessary to use it properly.

Two frequencies appear in the formula which we derived for the case of a moving observer—the frequency of sound in the medium, which coincides, naturally, with the frequency of a sound perceived by a listener at rest or emitted by a stationary instrument, and the frequency ν_{mov}, equal to the number of vibrations per second presented by the moving body to the air or coming to the moving body from the air.

Therefore, while in the first example the emitting and receiving frequencies are the frequency ν of the medium and the frequency ν_{mov} due to the motion, in the second example, on the contrary, the receiving frequency is ν and the emitting frequency is ν_{mov}.

For a moving observer, $\nu_{obs} = \nu_{tr} (1 + u/c)$.

For a moving source of sound, $\nu_{obs} = \nu_{tr}/(1 + u/c)$.

In addition, it must be borne in mind that a positive speed in the first case corresponds to the source and the observer getting closer to each other, but in the second case, to their separation.

It is not difficult to see that both formulas yield similar regularities for the shift of frequency with the speed. If, for example, $u/c = 0.2$, then the frequency increases by 20% when the observer is approaching the source, and will be raised by 25% when the source is moving towards the observer.

We have so far been tacitly assuming that the orchestra and the listener are moving along the line coinciding with the direction of propagation of the sound. What will be changed if the listener is moving not towards the orchestra, but past it? It is clear that only the component of a car's velocity along the line of propagation of the sound has any significance. The motion of an observer along the wave front, i.e. perpendicularly to the direction of propagation of the sound, plays no role.

The same considerations apply to the motion of an orchestra. Applying the formulas for this case, we ought to bear in mind that the speed of motion occurring in the formula should be taken not at the moment of reception, but at the moment of emission of the sound wave.

If the source of the sound, as well as the observer, are in motion with respect to the air, then the formulas are combined. The frequency of the sound that is received proves equal to

$$\nu_{obs} = \frac{1 + u/c}{1 - v/c} \nu_{tr}$$

where u is the speed of the observer, and v is the speed of the source of the sound.

The change in frequency caused by a motion of an observer or a source of sound is called the *Doppler effect.*

Sound Energy

All the particles of air surrounding an emitter of sound are in a state of vibration. As we explained in Chapter V, a mass point oscillating in accordance with a sinusoidal law possesses a definite and constant total energy.

When an oscillating point passes through its equilibrium position, its speed is maximal. Since the displacement of the point is equal to zero at this instant, the entire energy is kinetic:

$$E=\frac{mv_{\max}^2}{2}$$

Consequently, as we explained back on p. 139, the total energy is proportional to the square of the amplitude of the speed of vibration.

This is also true for particles of air vibrating in a sound wave. However, a particle of air is something indefinite. Therefore, the energy of a sound is given per unit of volume. This magnitude can be called the density of the sound energy.

Since the mass of a unit of volume is the density ρ, the density of the sound energy

$$w=\frac{\rho v_{\max}^2}{2}$$

We have spoken above about another important physical magnitude which vibrates according to a sinusoidal law with the same frequency as the speed. This is the sound, or excess, pressure. Since these magnitudes are proportional, we may say that the energy density is proportional to the square of the amplitude of the sound pressure.

The values of amplitudes of a sound vibration for loud conversation were given above. The speed amplitude was equal to 0.02 cm/sec. One cubic centimeter of air weighs about 0.001 g. Therefore, the energy density is equal to

$$\frac{1}{2} \times 10^{-3} \times 0.02^2 \,\mathrm{erg/cm^3} = 2 \times 10^{-7} \,\mathrm{erg/cm^3}$$

Suppose that a source is emitting sound. It is radiating sound energy in the surrounding air. It is as though the energy were "flowing" from the emitting body. During a second, a definite amount of energy flows through each area element, which is perpendicular to the line of propagation of the sound. This quantity is called the *energy flux* streaming through the area element. Furthermore, if we take an area element of one square centimeter, then the amount of energy which has streamed through it is called the *intensity of the sound wave*.

It is not difficult to see that the intensity I of sound is equal to the product of the energy density w by the speed of sound c. Imagine a small cylinder of 1-cm height and 1-cm^2 base area, whose generatrices are parallel to the direction of propagation of a sound. The energy w contained in such a cylinder will completely leave it during a time of $1/c$. Therefore, an energy of $w/(1/c)$, i.e. wc, will pass through a unit of area during a unit of time. It is as though the energy, itself, were moving with the speed of sound.

During a loud conversation, the intensity of the sound near the talkers will be approximately equal to (we are making use of the number obtained above) $2 \times 10^{-7} \times 3 \times \times 10^4 = 0.006$ erg/cm$^2\cdot$sec.

Weakening of Sound with Distance

The wave from an instrument emitting sound is propagated, of course, in all directions.

Let us conceptually construct two spheres with different radii and with the source of sound as center. Of course, the sound energy passing through the first sphere will also pass through the second spherical surface. If we denote the intensity of the sound by I, then the energy of the wave passing through a sphere can be written out as follows: $4\pi r^2 I$, since $4\pi r^2$ is the area of the surface of a sphere of radius r. If no energy was lost along a path from the first sphere to the second, then $4\pi r_1^2 I_1 = 4\pi r_2^2 I_2$.

Hence, the intensities I_1 and I_2 of a wave at distances r_1 and r_2 from the source of a sound are inversely proportional to the squares of these distances. Since the intensity of a sound is proportional to the energy density, the intensity, just like the energy density, is proportional to the square of the amplitude of the vibration. It follows from this that the amplitudes of the wave at distances r_1 and r_2 from the source of a sound are inversely proportional to these distances. The intensity of a sound is inversely proportional to the square of the distance from the source, and the amplitude is inversely proportional to the first power of the distance. As a matter of fact, sounds decrease somewhat faster since part of the energy is dissipated along the way. This occurs because when the particles of a medium are vibrating, a certain portion of the energy will be expended in overcoming viscous friction. However, these losses are relatively small, and the main reason for our hearing worse far away from, than near to, the source of the sound is the inverse square law.

Loudly and Quietly

Human sense organs are in many respects more perfect than the best instruments. This is true, in particular, of the ear. We are capable of perceiving waves with intensities from 10^{-9} to 10^{4} erg/cm$^2\cdot$sec in the form of sound. Therefore, the strongest sound differs from the weakest by a factor of 10^{13}.

But what kind of sound is the faintest sound that a human being is capable of perceiving? A hardly audible rustle creates a pressure on the ear drum, which is equal to 2×10^{-4} dyne/cm^2, i.e. approximately two ten millionths of a gram. The best microbalances are not as sensitive as the human ear.

If a sound is carrying more energy than 10^4 erg/cm$^2 \cdot$sec, a person no longer hears it, but experiences a painful sensation. The pressure on the ear drum reaches 0.2 gf/cm^2 in such a case. It is precisely the pressure wave, i.e. the rapidly alternating tremors of contraction and rarefaction, that is painfully perceived by the ear. But if the constant air pressure increases by such an amount, i.e. by 0.2 gf/cm^2, then of course, the ear "will not notice" this. Normal atmospheric pressure, equal to approximately 1 kgf/cm^2, will increase by more than 0.2 gf/cm^2 when you descend from the second floor to the street.

The energy of a wave carrying a strong sound is an enormous number of times greater than the energy of a wave bringing us a whisper or a rustle. It is therefore very inconvenient in practice to evaluate the loudness of a sound by the amount of its energy. Imagine that an official, trying to find means of noise abatement in the streets, has to give a talk at a session of the City Council and tell how much less noise there would be if streetcars were replaced by trolleybuses or buses, if drivers were forbidden to use their horns in the street, etc. In order that the picture be visual, it is necessary to resort to posters. As is customary in the construction of various types of diagrams, one can draw columns on a poster whose heights will represent degrees of noise. But if we define the loudness of a sound in terms of energy, then an insurmountable difficulty arises: quiet and noise differ from each other so much that it is a lot harder to represent them in a diagram in a single scale than to draw an elephant and a fly life-size on one poster.

When such cases arise in physics, refuge is taken in a so-called logarithmic scale.

If some magnitude increases 10-, 100-, 1000-, etc. fold, then its logarithm increases by a factor of 1, 2, 3, etc. Hence, if instead of the energy of a sound wave, we use the logarithm of this magnitude, we can always "fit" the noise of an airplane motor and the hum of a mosquito onto a single poster.

A scale of loudness is created in the following manner. A certain zero level of loudness, equal to 10^{-9} erg/cm$^2\cdot$sec, is chosen by convention. Sounds of such strength are not heard by a person with even the most sensitive ear. We can then determine how many times the energy E of the sound we are interested in is as great as this initial level E_0, i.e. the ratio E/E_0 is found.

The common logarithm of this ratio is taken as the measure of the sound's loudness. The unit of loudness bears the name *bel*; incidentally, one tenth of a bel, called a *decibel* (dB), is ordinarily used. Loudness in decibels is equal to $10 \log (E/E_0)$.

We can judge what a decibel is on the basis of the following table, indicating the loudness values of various sounds (in dB) at a distance of several meters from the source:

Rustle of leaves	10
Quiet street	30
Passing automobile	50
Loud conversation	70
Noisy street	90
Airplane	100

A table of logarithms will enable us to clearly visualize the decibel. Thus, an increase in the strength of a sound by

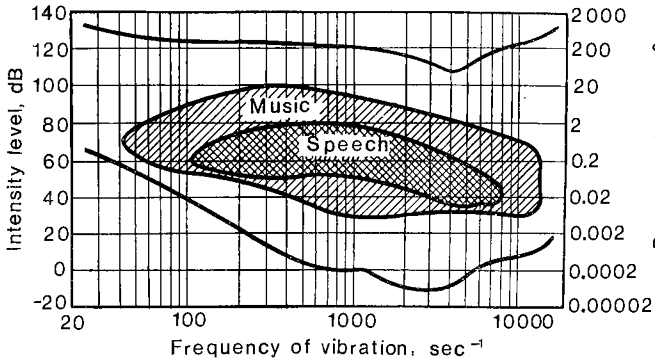


Fig. 121

1 dB corresponds to a growth in its intensity by a factor of $10^{0.1} = 1.26$, i.e. by 26%. A two-fold increase in the intensity of sound corresponds to a change in its loudness by 3 dB five-fold, 7 dB, ten-fold, 10 dB.

If the distance from the source of a sound is doubled, then its intensity will fall off by a factor of four and its strength will decrease by 6 dB. Suppose we were at a distance of a meter from a vibrating string and moved away to a distance of 10 m. The intensity of the wave reaching our ears has become 100 times as small, and the strength of the sound has decreased by 20 dB.

We have spoken earlier about the range of audible fre quencies being limited. Supplementing this information with our knowledge of the ear's sensitivity to quiet and loud sounds, we can depict them by means of a diagram of the auditory area typical of a person of normal hearing (Figure 121). Sound frequencies are marked off along the horizontal axis of this graph, the energy of a sound, along the vertical axis. The threshold of hearing and the pain threshold are shown in the figure. The auditory area lies between these two curves.

Inaudible Sounds

A sound frequency of 20 000 Hz is the value beyond which the human ear does not perceive the mechanical vibrations of a medium. It is possible to create vibrations of a higher frequency in various ways; a person will not hear them, but instruments can record them. Incidentally, not only instruments record such vibrations. Many animals—bats, bees, whales and dolphins (as can be seen, it isn't a question of a living being's dimensions)—are capable of perceiving mechanical vibrations with frequencies up to 100 000 Hz.

We are now able to obtain vibrations with frequencies up to a billion hertz. Such vibrations, although inaudible, are called *ultrasonic* or *supersonic* in order to affirm their affinity to sound.

Supersounds of the highest frequencies are obtained with the aid of quartz plates. Such plates are cut out of monocrystals of quartz. They possess the following interesting property: if voltage is applied to such a plate, it will contract or expand. But if an alternating voltage is applied to such a plate, it will alternately contract and expand, i.e. will start vibrating.

Powerful streams of ultrasound with an intensity of several thousand joules per second per square centimeter have been successfully created in this manner. It is interesting to compare the intensity of an audible sound with this figure. It attains only 0.005 joule per second per square centimeter in the immediate vicinity of a gun that is being fired.

Ultrasonic energy is so great that it can be felt. If you dip your hand in a liquid performing ultrasonic vibrations, you will feel a sharp pain.

Since supersounds are capable of carrying out interesting transformations on the substances, they have found a wide application in the most varied fields. One such transformation is the crushing of a substance. If a piece of lead or copper is placed in a liquid and subjected to an ultrasound, then the metal will crush and form the finest suspension. Crushing occurs in those cases when the dimensions of a particle are greater than the wavelength.

If the particles of a substance are small, then the influence of ultrasound will be the opposite. Acting with a supersound in a smoke-filled room, one can quickly and completely clean the air. It turns out that under the action of supersound the particles of smoke stick together (this phenomenon is called coagulation), become tens and hundreds of times heavier and settle on the floor.

The influence of ultrasound on biological objects is particularly interesting. Many cells, especially those which are thread-like in form, disintegrate under the action of supersound. Bacteria die or undergo significant changes. Ultrasound can sterilize milk.

An interesting area for the application of supersound is the search for cracks or other defects in metallic casts of great thickness (up to tens of meters). If a crack or a cavity is found in the path of an ultrasonic ray, then the ray will not pass through it, but will be reflected in the opposite direction. This reflection is caught by an instrument, and on the basis of the time spent by the supersound in travelling to and from the defect, the depth of the defect's location is determined.

Bats make use of ultrasound in an interesting manner. In order that the bat might exist in complete darkness, nature equipped it with an echo sounder of exceptional perfection. It works on ultrasonic frequencies. When in flight, a bat emits signals with a frequency of 25 000-50 000 Hz, which are inaudible to the human ear. Each signal lasts approximately 10-15 thousandths of a second. The ultrasonic signal, sent by a bat in a definite direction relative to that of its body, strikes an obstacle, is reflected from it and returns. A bat's auditory organs are also extraordinarily well developed—a bat is capable of hearing the reflected signal even if it is two thousand times weaker than the initial signal. Moreover, a bat is capable of distinguishing its own reflected signal from extraneous noise, even if this noise is thousands of times stronger than the echo of the signal it sent. On the basis of the time which has elapsed from the moment of the signal's emission to that of its return, the bat determines (instinctively, of course) how far away the obstacle is located.

How Sound Bends Around an Obstacle

You are in the interior of a room on the second floor and are talking. The window is open. Your friend is in the street beneath your window. Will he hear you? Yes, if you speak loud enough, but he will nevertheless hear much worse than in the case when he climbs up a ladder and stands facing the window. It is as though the sound waves coming out of the window were spreading out in all directions, but kind of unwillingly. This shows that sound waves are best of all propagated forwards along straight lines, but are to a certain degree deflected sideways. Is this true for any sound wave? No, it turns out not to be so.

An essential role is played by the relationship between the wavelength and the dimensions of the opening. If the wavelength is long in comparison with these dimensions,

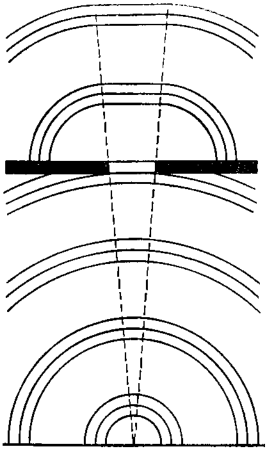


Fig. 122

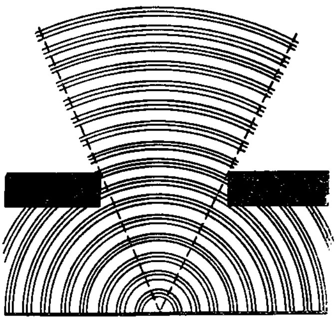


Fig. 123

then, leaving through the opening, the wave "spreads out" in all directions as if the opening itself were the source of sound. On the contrary, if the wavelength is much smaller than the dimensions of the opening, the sound is propagated along rays, and where a straight line drawn from the source of the sound to an observer strikes an obstacle (in our example—the wall), a "shadow" arises: the sound is almost inaudible.

In our example, a wavelength of 30 cm corresponds to 1000 Hz, the average frequency of the human voice. Therefore, such waves are most apt to be propagated forwards through a meter window opening, but are also noticeably deflected obliquely.

It is very hard to depict the bending of sound waves around an obstacle in a drawing.

It is much simpler to show how surface waves on water behave in an analogous situation. We shall speak about these waves a bit later. The properties of such waves are somewhat distinctive. However, as for the rules governing the bending of waves around obstacles, they are identical for water waves and for sound waves in the air.

Figures 122 and 123 depict the passage of water waves of different lengths through one and the same opening. In Figure 122, the wavelength is considerably greater than the dimension of the opening. In this case, the wave almost completely fills the region behind the screen. A wave of very small length is depicted in Figure 123. Now the wave propagation takes place along rays. The wave will hardly enter the region of geometrical shadow.

Therefore, it turns out that when the length of sound waves is considerably less than the dimensions of the objects with which they collide, the sound behaves entirely as though it were not vibrations in the air, but a stream of particles moving through the air. The difference from ordinary particles consists mainly in the fact that ordinary particles can move with arbitrary velocities, but sound is always propagated with one and the same speed.

The wave nature of sound manifests itself in the fact that it is nevertheless always deflected from a straight-line propagation. As we have already said, the smaller the wavelength, the smaller the deflection, but it always exists and can be measured in principle. This deflection is called *diffraction of sound*. The existence of diffraction could have served as a proof of the fact that sound is a wave motion, if we hadn't discovered this directly (by the way in which sound is obtained). Studying diffraction, we could have measured the length of sound waves, if again we hadn't already known it on the basis of the vibrational frequency of the source of the sound.

Reflection of Sound

We shall assume in this section that the length of a sound wave is sufficiently small and that sound is therefore propagated along rays. What happens when such a sound ray falls from the air onto a rigid surface? It is clear that a reflection of the sound will occur in this case. But where will it be reflected to?

An analogy between the propagation of sound and the motion of material particles shows that such a reflection should take place in the same way as a ball rebounds from a wall, with the only difference being that as a result of frictional processes, the speed of the ball will decrease during the collision, while the speed of propagation of the sound, depending only on properties of the air medium, will certainly not change. Here friction will manifest itself not in a change in the speed of the sound, but in the fact that during the reflection, part of the energy of the sound waves will be transformed into heat.

Since the reflection of a sound does not differ in principle from an elastic collision, the law of reflection of sound can be formulated in the following manner: the angle of incidence of a sound ray, i.e. the angle made by the ray and the normal (i.e. perpendicular) to the part of the surface on which it falls, is equal to the angle of reflection, where the reflected ray lies in the plane passing through the incident ray and the normal to the surface. This surface is called the ray's plane of incidence.

Thus, if we want to know where a reflected ray will go, we must do the following. Draw the normal where the ray strikes, measure the angle of incidence and construct the plane of incidence. Then, in this plane, mark off the angle equal to the angle of incidence on the other side of the

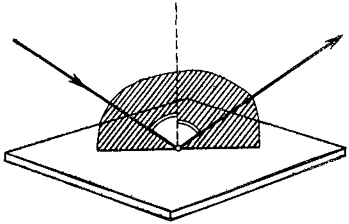


Fig. 124

normal; the straight line so obtained is the reflected ray (Figure 124).

Let us now solve an interesting problem.

As we know, sound is propagated from a source in all directions, and only a small fraction of the sound energy arrives at a distant point. What kind of reflecting surface is required in order to gather the sound from a source at a single point once more? The form of the reflecting surface should be such that the rays from a single point (the source of the sound), incident to it at various angles, would be reflected to a single point again. What sort of surface is this?

We already know what an ellipse is. On p. 202 we discussed this remarkable curve possessing the peculiarity that the distance from one of its foci to any of its points plus the distance from the other focus to this same point is identical for all points on the ellipse. Imagine that an ellipse is revolving around its major axis. The revolving curve describes a surface, which is called ellipsoidal, or simply an ellipsoid. The form of an ellipsoid resembles that of an egg.

An ellipse possesses the following geometrical peculiarity (Figure 125). The bisector of the angle whose vertex lies on an ellipse and whose sides pass through its foci will be a normal to the ellipse (i.e. the perpendicular to the tangent to the ellipse at this point). Hence, if a sound ray leaves

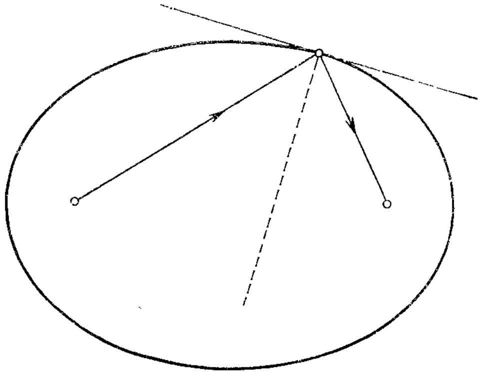


Fig. 125

one of the foci of an ellipsoid, then, being reflected from its surface, it will arrive at the other focus. All the rays will behave in this fashion, and so the entire sound flux which left one focus will gather at the other.

This property of such curved surfaces was already known in antiquity. In the Middle Ages, during the Inquisition, when controlling the thoughts of each person had become one of the most important aspects of governmental activity, arched surfaces were used for overhearing conversations. Two people revealing their thoughts to each other in a quiet voice did not even suspect that because of the arched ceiling the monk dosing in another corner of the café was hearing each word of their conversation almost as well as they themselves.

It is difficult to construct an ellipsoidal surface. But small portions of a spherical surface differ little in form from the portions of an ellipsoid.

If a vibrating object is placed in front of such a spherical "mirror", then the sound rays coming from it will gather once more in another region (not at a single point, as in the case of a real ellipsoid, but in a small region of space).

The following experiment can be performed even with an ordinary deep plate. If a watch, whose ticking is practically inaudible at a distance of the order of a meter, is placed near such a plate, then one can find a point rather far from the plate, at which the watch's ticking is heard with the same loudness as though it were brought up to one's ear. This same phenomenon is employed when constructing the prompt-box in a theater. The position of the prompter and the form of the box are the most appropriate for reflecting sound in the direction of the stage.

The reflection of sound from the walls of a room is of great interest to builders of theaters, concert halls and meeting rooms. This branch of construction engineering, dealing with the problem of optimal audibility in closed rooms, is called architectural acoustics.

Waves Moving Along a Surface

Submariners do not experience sea storms. During the roughest storms, calm prevails at a depth of several meters below sea level. A sea wave is an example of wave motion affecting only the surface of a body.

It may sometimes seem that a sea wave is a stream of a moving mass of water. But this is not so, and it is not at all difficult to convince oneself of the vibratory character of the motion of the water particles, if one watches how a boat rolls on the waves when the rowers are taking a rest. Up, down, a bit forwards, slightly backwards, but practically no progress. More precise observations will show that the water particles are performing a circular motion. Each particle of water is describing a trajectory which is close to a circle. The planes of the circles lie in the direction in which the wave is propagated, i.e. perpendicular to the wave front.

The picture of a rough sea can be exceedingly varied—tiny ripples, big waves, waves following each other frequently or infrequently. Speaking the language of a physicist, waves can be of different amplitudes and lengths.

As has been already said above, waves are rapidly damped with an increase in depth. The water particles lying under the surface perform vibrations with an ever smaller amplitude, and this amplitude is the smaller the greater the depth. At a depth of only half a wavelength, the amplitude of the vibration falls off by a factor of 20, and practically no motion remains at the depth of a wavelength.

Until now we have been speaking of waves whose speed of propagation depended only on properties of the medium. Surface waves are another matter: vibrations of different frequencies are propagated with different speeds. The speed of propagation and the period of vibration are related by a simple dependence: $c = gT/2\pi$, where g is the acceleration of gravity. The appearance of the acceleration of gravity g in this formula is quite natural, for it is precisely gravity that makes the surface of the water plane. According to this formula, when the vibrational frequency is 1 Hz, the waves will run with a speed of about 1.5 m/sec. This formula is valid for waves in an open sea; this simple dependence becomes more complicated near the shore and, in general, in shallow water. Since $\lambda = cT$, we have $c = \sqrt{g\lambda/2\pi}$. Hence, when a big storm arises in some region of a sea, the longest waves, which have the greatest speed of propagation, are the first to reach distant places.

How Solids Transmit Sound

There exists a difference of no small importance between the transmission of sound through liquids and gases, on the one hand, and through solid objects, on the other. This difference consists in the fact that besides longitudinal waves, transverse waves can also arise in solids.

This term speaks for itself—a *transverse wave* possesses the peculiarity that the particles participating in the wave process perform vibrations not in the direction of the wave's propagation, but in a transverse direction—perpendicular to the direction of propagation.

A sound wave in a gas or a liquid is a wave of alternating contractions and rarefactions. Such a wave can only be longitudinal—transverse vibrations of particles cannot produce local changes in volume, i.e. cannot lead to contractions and rarefactions. A transverse wave in a liquid or a gas is impossible, since these media resist a contraction or a rarefaction, but not a shear. A solid resists not only a change in its volume, but also a change in form, so aside from longitudinal waves, transverse waves can also arise in a solid.

When a transverse wave is propagated through a solid medium, a share wave is formed—particles of the body move in a wave by turns in various directions from the line of its propagation. But longitudinal waves in a solid medium are accompanied by contractions and rarefactions, just like the waves in liquids and gases.

Transverse and longitudinal waves transmit sound equally well, but not equally fast. Longitudinal waves are always propagated faster than transverse waves.

Here are some characteristic figures. In steel, the speed of transverse waves is about 3000 m/sec, and of longitudinal, 6000 m/sec. Sound in soft lead has a smaller speed of propagation—700 m/sec for transverse waves, and 2200 m/sec for longitudinal.

The ratio between the speeds of longitudinal and transverse waves in rubber is particularly great. Rubber offers very little resistance to a change in form, but it is not at all easy to change its volume. Transverse waves are propagated in rubber with a speed of only 30 m/sec—one tenth of the speed of sound in air.

Besides these two types of waves, *surface waves* are also propagated through a solid. However, they do not at all resemble sea waves, for which gravity is the force restoring the deflected particles. The waves on the surface of a solid are supported by the elastic forces binding its particles. It is therefore only natural that the speed of surface waves depends on the elastic properties. The speed of surface waves is approximately 0.9 the speed of propagation of transverse waves. Just as in the case of a liquid, the trajectories of the vibrating particles lie in planes perpendicular to the wave front. The points move in closed curves resembling ellipses. As the distance from the surface grows, the form of the ellipse changes, the amplitude of the vibration becomes smaller and the wave is damped.

Heralds of Earthquakes

The Earth transmits sound very well. In almost every novel written during the Middle Ages, you will find a scene where the hero, galloping on his horse, is being chased. "The rider suddenly pulled up his steed, dismounted and placed his ear to the ground. 'We are being pursued! We must hurry!'" In fact, the striking of a horse's hooves against the ground is transmitted farther than a kilometer. The Earth, like every other elastic body, serves as a conductor of sound waves.

Sound waves propagated through the Earth bring us information about earthquakes and acquaint us with processes occurring within the Earth's core. The sound waves arising during an earthquake are called *seismic*. The presence of a seismic wave, its amplitude, speed, wavelength

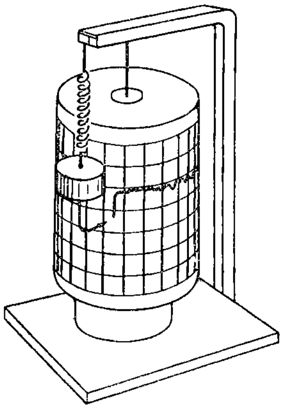


Fig. 126

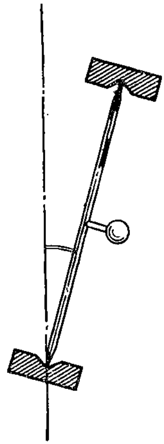


Fig. 127

and frequency of vibration can all be determined with the aid of very sensitive instruments—seismographs.

A seismograph is a complicated instrument. But it is easy to understand how it works. The basic part of a seismograph is a heavy weight suspended on a spring. During a vertical displacement of the ground, the point of suspension of the spring with the weight will shift as shown in Figure 126. Owing to the large inertia of the weight, it will at first remain at rest. A pen is attached to the weight, and a sheet of paper is rigidly fastened to the support. When the support is displaced, the pen will draw a vertical line on the paper. In order to record a seismic wave, it is necessary to turn the paper.

Besides such seismographs, recording vertical displacements of the ground, horizontal seismographs are also employed. The principle behind the operation of a horizontal seismograph is shown in Figure 127. The main part of the instrument is an almost vertical rod. An off-center weight converts this rod into a pendulum which is able to swing about its axis. If the ground is calm, the weight will be at rest in the lowest position. A tremor in a horizontal direction gives rise to a displacement of the pendulum's axis, while the heavy weight is at first kept in its place by inertia. The turning of the pendulum is registered by an automatic recorder.

If we install one vertical and two horizontal seismographs, vibrating in mutually perpendicular planes, then we can record the magnitude and direction of any displacement.

With the word "earthquake", one usually associates a picture of houses and trees being destroyed and falling through the fissures which are being formed, of people losing their lives. Such big earthquakes rarely occur, but seismologists apply the term "earthquake" to all underground incidents capable of bringing into motion the pen of a seismograph recording vibrations of the Earth's core. Nothing but a seismograph will even notice such earthquakes. About a hundred thousand of them occur on the Earth each year. It turns out that the "underground kingdom" is living a rather active life!

A seismic wave is propagated in all directions from the seismic center, and will be received by many seismographs, which have been installed in various cities and countries. Evidence of each underground tremor will be furnished three times, since all three types of waves which we have just considered will set out from the seismic center. The longitudinal wave will be the first to come to the observer, the transverse wave will follow and the surface wave will be the last to arrive.

Nevertheless, surface waves are the most significant for a seismologist, since (owing to easily understood reasons) they are the most intense.

On p. 407 we said that the intensity of a sound wave is inversely proportional to the square of the distance from the source of the sound. But this does not apply to surface waves. With a source of sound as center, let us construct not two spheres, but two circles. The energy of a wave passing through a circle is proportional to $2\pi rI$, where I is the intensity. Consequently, in the absence of a loss in energy, the intensity of a surface wave will be proportional to $1/r$, and not to $1/r^2$. Therefore, these waves will come to an observer considerably less weakened than longitudinal and transverse spatial waves.

Investigations of seismic waves do not only establish the seismic center, but enable us to carry out important large-scale work in the study of the Earth's structure. Signals coming from the heart of the Earth allow us to judge its structure, too. The reason for this is that the speeds of seismic waves are different at various depths. Near the Earth's surface, longitudinal waves have speeds of the order of 5.5 km/sec and transverse, 3.3 km/sec. However, the speed of propagation of seismic waves at the center of the Earth attains 11-12 km/sec.

Knowing what peculiarities of structure can influence the speed of propagation of waves, investigators draw conclusions about the structure of the Earth's core. It has been established, for example, that transverse waves do not penetrate the heart of the Earth's core, whence the conclusion is drawn that the core of the Earth is liquid, since transverse waves do not pass through liquids,

Shock Wave

With the word "wave" in everyday practice, there is associated a picture of a periodic process, a graphic example of which is a rough sea. Riding the "waves" is a favourite pastime of bathers.

In physics the word "wave" is used in a wider sense, and we speak of the propagation of a wave even in the case when the local increase or decrease in pressure is caused by a single blow, explosion or sucking in of air.

The air wave created by an explosion looks very distinctive. (We have already said that it is possible to photograph an air wave, so the word "looks" is quite appropriate for a wave of pressure.)

The instantaneous profile of such a blast wave is depicted in Figure 128: the curve depicts the pressure distribution along an arbitrary direction of propagation of the wave. The profile of the wave is composed of a gradual rise, culminating in a sheer descent. The direction of the wave's motion is shown from left to right in the diagram. The portions of air situated to the right of the wave front are at rest at the instant under consideration—the wave will get to them yet.

The basic feature of the blast or, as it is called, *shock wave* just described is the sharp jump in pressure at the

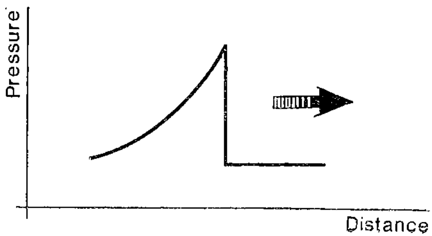


Fig. 128

"front"; points at rest are caught up by a maximum of pressure practically instantaneously: a particle of air was just under atmospheric pressure, but at the next instant, the pressure at this place is maximal. Then, as the shock wave advances further, the pressure at the point on which we have fixed our attention will gradually fall in accordance with the profile of the left slope of a gently sloping hill.

The pressure distribution along any line of propagation of the wave is depicted in Figure 128. The wave is propagated in space, and the wave front is a surface.

The front of a shock wave brings with it a jump not only in pressure, but also in density and temperature.

Aside from changes in pressure and temperature, a shock wave also brings motion with it. In a sound wave, too, the air is brought into motion along the line of propagation of the wave, but this phenomenon is hardly noticeable there. In a shock wave, the air is carried away so strongly that "carried away" becomes too mild an expression. A shock wave creates the strongest wind, hurricane ... . It is perhaps impossible to find an appropriate expression for the motion in powerful shock waves.

The jump in properties that we have spoken of is exceptionally sharp—the transition from complete rest to maximal speed takes place within an interval of length equal to several mean free paths of a gas molecule. For air this is a submicroscopic magnitude of the order of some hundred thousandths of a centimeter. The time required for such a jump is measured in ten billionths (10^{-10}) of a second. Such a truly instantaneous change in the state of the pressure, density, temperature and speed of motion is precisely an indication of a shock wave.

Depending on the force of the explosion, the jump in pressure which the shock wave brings with it or, in other words, the height of the wave front, can be quite different:

at the moment of the shock wave's arrival, the pressure can increase from several per cent to tens of times.

The values of the jumps of all the quantities along the front of the shock wave are related to each other. Knowing the jump in pressure, we can also calculate the jump in the density, the temperature, or the speed of motion. The height of the wave front also determines the speed of propagation of the shock wave. The speed of a weak shock wave does not differ from that of an ordinary sound wave. As the height of the wave front grows, so does the speed of propagation of the shock wave.

Let us give the figures for a "modest" shock wave, raising the pressure by a factor of one and a half. It turns out that such an increase in pressure entails an increase in air density by 30% and a 35° rise in temperature. The speed of the front of such a shock wave is about 400 m/sec. Even with a relatively small jump in pressure to 1.5 times the initial value, the shock wave will carry air away with it with a speed of about 100 m/sec, i.e. 360 km/hr. Not a single hurricane has such a wind velocity.

However, explosions capable of creating incomparably more powerful shock waves are possible. If a wave brings a ten-fold increase in pressure with it, then a spasmodic four-fold increase in density and a 500° rise in temperature take place at the wave front. Moreover, the wind velocity attains 725 m/sec. The speed of propagation of such a shock wave is equal to 1 km/sec.

The shock waves generated by powerful explosions are propagated tens of kilometers. The jump in properties which a shock wave brings with it acts like a severe shock to the obstacles found in the path of the wave. Weak shock waves knock out window panes, destroy walls of houses and pull up trees by the roots. The destructive action of a mortar is to a great extent based on the action of shock waves.

The destructive action of shock waves is highly dependent on many factors, especially on the duration of the wave's action. But in order to give some idea of the relation between the destructive action of a wave and its basic parameter, the rise in pressure, let us note that a shock wave with a height of only 2% is capable of knocking out a window, while a wave bringing a two-fold increase in pressure can break a thick wall.

Motion with a Supersonic Speed

Shock waves, as has been just said, are propagated with supersonic speeds. It turns out that when a solid body moves through the air with a supersonic speed, this also leads to the formation of a shock wave. Shock waves are therefore of great significance for modern aviation.

Motion with speeds considerably exceeding 330 m/sec, i.e. 1200 km/hr, has recently become a reality in aviation. The motion of airplanes and guided missiles, cutting through the atmosphere with speeds above the sound barrier—this is what the border-line value of 1200 km/hr is called—differs rather sharply from the motions lying on the other side of the sound barrier. This difference consists in the formation of a shock wave in front of a body flying with supersonic speed.

A diagram of a shock wave created by a round missile is shown in Figure 129. The wave front is a curved surface passing slightly in front of the moving body. As the distance from the line of motion increases, the wave front lags farther and farther behind the missile.

The picture looks somewhat different for a shock wave of a pointed body having the well-known form of a missile. Figure 130 shows that the shock wave "sat down on the

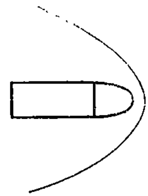


Fig. 129

nose" of the missile; the wave front has acquired a conical form.

It is possible to photograph a missile flying with supersonic speed. The sharp difference in air density around the missile clearly delineates the front of the shock wave generated by it. The faster the missile moves, the more pointed is the cone.

The shock wave is the main source of the resistance experienced by a body moving with supersonic speed. But for motions with speeds which are less than the speed of sound, the resistance is created, as we have said, mainly by the resulting turbulent motion. Therefore, the most advantageous forms of a body for motions of these two types are different. What is advantageous for fast motions is disadvantageous for slower ones, and conversely.

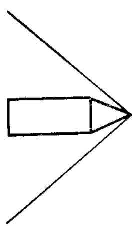


Fig. 130

A body which is pointed in front contributes to turbulence and, therefore, increases the resistance to motion with subsonic speeds. On the contrary, the pointed form of a missile decreases the resistance of its shock wave.

A body which is blunt in front decreases turbulence and is therefore more advantageous for subsonic speeds than pointed bodies. When passing through the sound barrier, this form becomes less advantageous, since the shock wave becomes the main source of resistance. This is the reason why missiles shot from guns are pointed in front—for they move with supersonic speeds.

It is, unfortunately, impossible to liquidate the shock wave and, along with it, the main source of resistance to a body cutting through the air with supersonic speed. The problem faced by constructors of missiles and airplanes consists in weakening the resistance created by a shock wave.

For missiles and airplane fuselages, a decrease in resistance is achieved by means of a pointed form. But what idea can be offered for the wings? Airplanes with very high operating velocities have acquired new outlines in the course of the past decades: the wings have been pressed to the fuselage and the airplane has acquired an arrow-shaped form. This has been done precisely in order to struggle against the resistance of shock waves (Figure 131).

Instead of discussing the motion of an airplane cutting through the air, we can speak of a stream of air rushing past the plane. For this is one and the same thing.

An airplane whose wings are inclined obliquely to an air stream is depicted in Figure 131. The velocity of the air at a wing can be decomposed into two vectors, one of them directed along, and the other perpendicular to, the wing. The air slides freely along the length of the wing, and this tangential sliding motion cannot be a significant source of resistance. The main resistance which will be

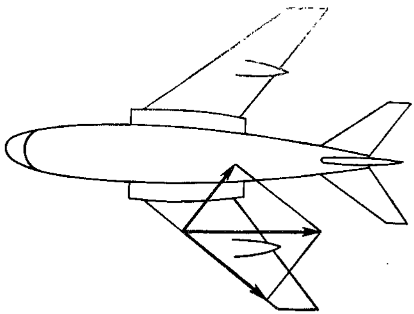


Fig. 131

experienced by the wing will be due to the motion of the air perpendicular to it. But the transverse component of the velocity with which the air is moving towards the wing may be considerably less than the head-on speed. It may even happen that during the motion of an airplane with supersonic speed, the transverse speed of the air with respect to its wings will be below the sound barrier. Such a decrease in the transverse speed will lead to a weakening of the shock waves and a diminishing of resistance. This is why airplanes with very high operating velocities are given an arrow-shaped form.

Incidentally, airplane constructors are faced with a difficult problem—to find a compromise between forms convenient for supersonic and ordinary speeds. Such a compromise is necessary for a simple reason—an airplane takes off and lands with relatively low speeds.

At the present time, there are jet planes flying with speeds of many thousands of kilometers per hour, and constructors are continuing their work in order to conquer even higher speeds. New difficulties arise. Having overcome the sound barrier, engineers came up against the heat barrier.

A swiftly moving airplane or missile compresses the air in front of it. The compression leads to a rise in temperature. The air through which the moving body cuts is heated, and so the surface of the airplane will also become warmer.

The rise in temperature proves proportional to the square of the speed of the air. The greater the speed, the more the air warms up. At the moment when the sound barrier is reached, the temperature of the air in front of the airplane is raised by only 60 °C. This does not yet have any great significance in practice. But when the speed of the airplane's motion is twice as great as the speed of sound, the air will already be heated up to 240 °C, and when the speed of sound is tripled, the air will acquire a temperature of the order of 820 °C, etc. It is not difficult to understand that this heating leads to considerable technological complications.

It is clear from the figures just given how rapidly the temperature increases with a growth in the speed of the motion. For motions with speeds of the order of 10 km/sec, the temperatures become so great that any body will melt and turn into a gas. Meteorites—stones and pebbles of various sizes—are continually falling into the atmosphere of the Earth from outer space. They move with speeds of several tens of kilometers per second. At a height of 150-200 km above the surface of the Earth, when the atmosphere is becoming less rarefied, these alien bodies become noticeably warmer, and at heights of the order of 130-600 km, their temperature increases so much that they evaporate. With an unaided eye, we notice a glowing pebble in the night sky. At the moment when we see it, it seems to us that a star has fallen out of the sky. The "star's fall" does not last long: a fraction of a second—and the pebble has evaporated.

Burning and Explosion

For burning to begin, it is necessary, as is well known, to bring a burning match to an inflammable object. But neither does a match light up by itself; it must be struck against a match-box. Therefore, in order that a chemical reaction start, a preliminary heating is necessary.

The reason for this is evident. A chemical reaction is a rearrangement of molecules. An energetic thermal motion of the atoms is absolutely necessary in order that such a rearrangement might take place. Therefore, the speeds of chemical reactions depend to a very great degree on the temperature. As a rule, a rise in temperature by 10° increases the rate of a reaction by a factor of 2-4.

While the rate of a reaction increases, say, by a factor of 3 with a 10° rise in temperature, a 100° rise in temperature yields a $3^{10} \approx 60\,000$-fold increase, a 200° rise, $3^{20} \approx \approx 4 \times 10^{9}$, and a 500° rise, 3^{50}, i.e. approximately 10^{24}.

It is no wonder that a reaction which proceeds at a normal rate at a temperature of 500 °C does not take place at all at room temperature. The ignition creates the required temperature for the reaction at the initial moment. A high temperature is further maintained by the heat which is liberated in the course of the reaction.

The initial local heating should be sufficient for the heat liberated by the reaction to exceed the heat emitted to the cold surrounding medium. Therefore, each reaction has its own, as one says, *ignition point*. Burning only begins if the initial temperature is higher than the ignition point. For example, the ignition point of wood is 610 °C, of benzine, about 200 °C, of white phosphorus, 50 °C.

The burning of wood, coal or oil is a chemical reaction uniting the substance with the oxygen of the air. Therefore, such a reaction proceeds at the surface: until the outside layer burns up, the next layer cannot participate in the burning. This is what explains the relative slowness of such burning.

It is not difficult to convince oneself in practice of the validity of what we have said. If fuel is broken up into small pieces, the rate of combustion may be considerably increased. This is why the crushing of coal is carried out in the fireboxes of many oven mechanisms.

The situation is entirely different in the case where the atmosphere is not needed, where everything that is necessary for the reaction is contained within the substance. An example of such a substance is a mixture of hydrogen and oxygen (it is called detonating gas). The reaction occurs not at the surface, but within the substance. Unlike the case of burning, all the energy forming in the course of the reaction is liberated almost instantaneously, as a result of which the pressure rises sharply and an explosion takes place. A detonating gas doesn't burn—it explodes.

Thus, an explosive should contain within it the atoms or molecules needed for a reaction. It is evident that we can prepare explosive gas mixtures. There also exist solid explosives. They are explosive precisely because in their compositions all the atoms occur which are necessary for chemical reactions giving off heat and light.

The chemical reaction taking place during an explosion is a decomposition reaction of the splitting up of molecules into parts. An example of an explosive reaction is shown in Figure 132—the splitting up of a nitroglycerine molecule into its parts. As is evident from the right-hand part of the diagram, molecules of carbon dioxide, water and nitrogen are formed out of the original molecule. We find the ordinary by-products of a combustion among the end products, but burning has occurred without the participation of oxygen

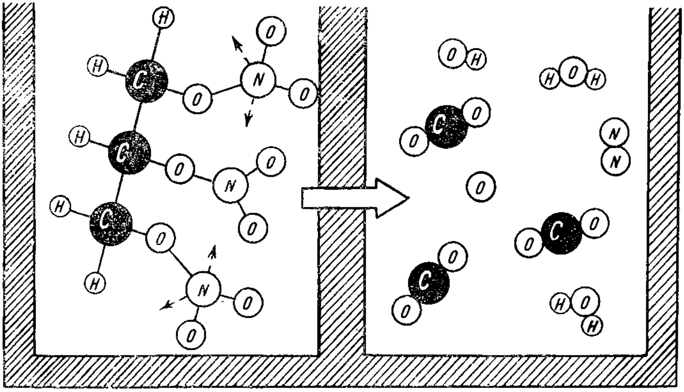


Fig. 132

molecules from the air; all atoms necessary for the combustion are contained within the nitroglycerine molecule.

How is an explosion propagated through an explosive, for example, a detonating gas? When we set fire to an explosive, a local heating arises. A reaction occurs in the heated region. But heat is liberated in the course of the reaction, and passes into the adjacent layers of the mixture by means of heat transfer. This heat is sufficient for the reaction to take place in the neighbouring layers, too. New amounts of the heat being liberated enter the next layers of fire-damp, and so with a rate related to that of the heat transfer, the reaction spreads throughout the entire substance. The rate of such a transfer is of the order of 20-30 m/sec. This is, of course, very rapid. A meter-long tube of gas explodes within one tenth of a second, i.e. almost instantaneously, while the rate of the burning of wood or pieces of coal that takes place at the surface, and not within the volume, is measured in centimeters per minute, i.e. is several thousand times less.

Nevertheless, such an explosion may also be called slow, since a different kind of explosion, hundreds of times faster than the one described, is possible.

A shock wave gives rise to a fast explosion. If the pressure rises sharply in some layer of a substance, then a shock wave will start being propagated from this place. As we already know, a shock wave leads to a considerable jump in temperature. Coming to a neighbouring layer, the shock wave will raise its temperature. The rise in temperature gives an impetus to an explosive reaction, and the explosion leads to a rise in pressure and maintains the shock wave, whose intensity would otherwise fall off rapidly with its propagation. Therefore, a shock wave gives rise to an explosion, while the explosion in turn maintains the shock wave.

The type of explosion described by us is called a detonation. Since a detonation is propagated through a substance with the speed of a shock wave (of the order of 1 km/sec), it is actually hundreds of times faster than a "slow" explosion.

But which substances explode "slowly" and which explode "rapidly"? The question should not be posed in this fashion: one and the same substance can explode "slowly" and also detonate under different conditions, while a "slow" explosion turns into a detonation in certain cases.

Some substances, for example, nitrogen iodide, explode on contact with a straw, on being slightly heated or as a result of a flash of light. An explosive substance like trotyl does not explode when it is dropped or even shot out of a rifle. A powerful shock wave is required in order to explode it.

There exist substances even less sensitive to external influences. The fertilizing mixture of ammonium nitrate and ammonium sulfate had not been considered to be explosive until a tragic accident occurred in 1921 at a chemical factory in Oppau, Germany. An explosive method was used there for pulverizing mixtures which had become caked. As a result, a warehouse and the entire factory were blown up. The engineers at the factory could not be blamed for the catastrophe: approximately twenty thousand blasts had proceeded normally, and only once had the conditions favourable for detonation been created.

Substances which explode only when subjected to a shock wave, but exist in a stable state and are not even afraid of fire under ordinary conditions, are quite convenient for specialists in creating explosions. Such substances can be manufactured and stored in large quantities. However, in order to bring these inert explosives into action, pioneers or, as is said, initiators of the explosion are needed. Such initiating explosives are absolutely necessary as sources of shock waves.

Mercury fulminate can serve as an example of an initiating substance. If a grain of it is placed on a sheet of tin and lighted, then an explosion takes place, making a hole in the tin. An explosion of such a substance under any conditions is a detonation.

If a bit of mercury fulminate is placed on the charge of a secondary explosive and lighted, then the explosion of the initiator creates a shock wave which is enough to detonate the secondary explosive. Explosions are produced in practice with the aid of a detonating capsule (1-2 g of an initiating substance). The capsule can be ignited at a distance, for example, with the aid of a long cord (Bickford fuse); the shock wave coming from the capsule will blow up the secondary explosive.

In a number of cases arising in technology, we must contend with detonational phenomena. "Slow explosions" of a mixture of benzine and air occur in the motor of a car under ordinary conditions. However, sometimes a detonation also arises. Shock waves in a motor are absolutely unacceptable as a systematic phenomenon, since they will soon put the walls of the motor's cylinders out of commission.

In order to cope with detonations in engines, it is necessary to either use a special benzine with high octane number or else mix the benzine with special substances—antiknocks—which do not let a shock wave develop. One of the most widespread antiknocks is tetraethyl lead (TEL). This substance is highly toxic, and so drivers are warned of the need to be careful when using such benzine.

Artillery cannons must be constructed so as to avoid detonations. Shock waves should not be formed inside the barrel when the gun is being fired, otherwise, the gun will be put out of commission.

Sixteen
ENERGY AROUND US

How to Transform Energy into Work

People need machines, and to get them they must be able to create motion—to move a piston, turn a wheel, pull the cars of a train. The motion of a machine requires work. How can it be obtained?

It would seem that we have already discussed this question: work originates at the expense of energy. We must take energy away from a body or a system of bodies—work will then be obtained.

This prescription is quite correct, but we haven't yet touched upon the question of how to achieve such a transformation. Is it always possible to take energy away from a body? What conditions are needed for this? We shall now see that almost all the energy around us is absolutely useless: it cannot be transformed into work. Such energy cannot in any sense be ranked among our energy reserves. Let us look into this matter.

A pendulum which has been deflected from its equilibrium position will sooner or later come to rest, a wheel of an upside-down bicycle, which has been spun by hand, will make a lot of turns, but it, too, will eventually stop moving. There is no exception to the following important law: all the spontaneously moving bodies surrounding us will eventually come to rest*.

If there are two bodies, one heated and the other cold, heat will be transferred from the former to the latter until their temperatures are equalized. Then the heat transfer will cease and the states of the bodies will stop changing. Thermal equilibrium will set in.

There is no phenomenon whereby a body spontaneously leaves a state of equilibrium. There cannot be a case in which a wheel at rest on an axle starts turning by itself. Neither does it happen that an ink-well on a table warms up by itself.

The tendency towards equilibrium implies that events take a natural course: heat passes from a hot body to a cold one, but cannot spontaneously pass from a cold body to a hot one.

As a result of air resistance and friction at the pivot, the mechanical energy of an oscillating pendulum will be converted into heat. However, not under any conditions will a pendulum begin swinging at the expense of the heat of the surrounding medium. Bodies come to a state of equilibrium, but cannot leave it spontaneously.

This law of nature shows at once what part of the energy surrounding us is absolutely useless. This is the energy of the thermal motion of the molecules of those bodies which are in a state of equilibrium. Such bodies are incapable of converting their energy into mechanical motion.

This part of the energy is immense. Let is calculate the value of this "died" energy. If the temperature is lowered by one degree, then a kilogram of earth, having a heat capacity of 0.2 kcal/kg, loses 0.2 kcal. A relatively small

* Here, of course, we do not have in mind a uniform translational motion or a uniform rotation of a system of bodies as a whole.

figure. However, let us estimate how much energy we would obtain if we were able to cool by only one degree a substance with the mass of the Earth, i.e. 6×10^{24} kg. Multiplying, we obtain an immense figure: 1.2×10^{24} kcal. In order that you might picture this value, let us state at once that at the present time, the yearly energy output of all the power stations in the world is equal to 10^{15}-10^{16} kcal, i.e. about a billion times less.

We needn't be astonished by the fact that such calculations act hypnotically on poorly informed inventors. We have spoken above of attempts to construct a perpetual motion machine ("perpetuum mobile") creating work out of nothing. Operating with the principal propositions of physics following from the law of conservation of energy, one cannot possibly refute this law with the creation of a perpetual motion machine (we shall now call it a *perpetual motion machine of the first kind*). The same sort of error is also committed by certain cleverer inventors, who create models of machines performing mechanical motion at the expense of nothing but by cooling the medium. Such, alas, unrealizable machines are called *perpetual motion machines of the second kind*. Here, too, a logical error is committed, since the inventor bases himself on laws of physics, which are consequences of the law of the tending of all bodies towards a state of equilibrium, and with the aid of these laws, tries to refute the foundations on which they are based.

Thus, it is impossible to produce work by merely taking heat from a medium. In other words, a system of bodies in equilibrium with each other is energetically barren.

Hence, in order to obtain work, it is first of all necessary to find bodies which are not in equilibrium with their neighbours. Only then will one succeed in realizing a process of transferring heat from one body to another or converting heat into mechanical energy.

The creation of an energy flux is a necessary condition for obtaining work. In the "path" of such a flux a conversion into work of some of the energy of bodies is possible.

Therefore, the energy of only those bodies which are not in equilibrium with the surrounding medium are ranked among the energy reserves which are of use to people.

Tendency Towards Disorder

Bodies left to themselves tend towards a state of equilibrium. Mechanical and thermal equilibrium is the natural state of all bodies. We have become acquainted with the practical consequences of this most important law in sufficiently great detail.

But what is the internal significance of this law? Why is the whole Universe a road to an equilibrium state? Why do bodies left to themselves inevitably approach a state in which mechanical motion has ceased and the temperatures of the bodies have been equalized?

These questions are very important and interesting. In order to answer them, we will have to begin from afar.

Everyday, frequently encountered events occur at every turn, they are *probable*. On the contrary, events which have occurred thanks to a rare coincidence are regarded as *improbable*.

An improbable event does not require a display of any supernatural forces whatsoever. There is nothing impossible about it, nothing contradicting the laws of nature. And nevertheless, in many cases we are perfectly sure that the improbable is identical with the impossible.

Consider a lottery prize-list. Count the number of winning tickets whose numbers end with a 4, 5 or 6. You will not be the least bit surprised when you find that approximately one tenth of the winning tickets correspond to each digit.

Well, but perhaps tickets with numbers ending with a 5 were to make up one fifth of the winners, instead of one tenth? Unlikely, you say. Well, and if half of the winning tickets were to have such numbers? No, that would be absolutely improbable ..., and therefore, also impossible.

Reflecting on what conditions are necessary for an event to be probable, we arrive at the following conclusion: the probability of an event depends on the number of ways in which it can be realized. The greater this number, the more frequently will such an event occur.

More precisely, the *probability* is the ratio of the number of ways of realizing a given event to the number of ways of realizing all possible events.

Write down the numbers from 0 to 9 on ten cardboard discs and place them in a sack. Now pull out a disc, note its number and put it back in the sack. This is very much like a lottery drawing. It can be confidently said that you will not draw one and the same number, say, 7 times in a row, even if you devote an entire evening to this boring occupation. Why? The drawing of seven identical numbers is an event which is realizable in only ten ways (7 zeros, 7 ones, 7 twos, etc.). But there are a total of 10^7 possible ways of drawing seven discs. Therefore, the probability of drawing seven discs in a row with identical numbers is equal to $10/10^7 = 10^{-6}$, i.e. only one millionth.

If black and white grains are poured into a box and mixed with a shovel, the grains will very soon be distributed uniformly throughout the entire box. Scooping up a handful of grain at random, we shall find approximately the same number of white and black grains in it. No matter how much we mix them, the result will always be the same—uniformity is preserved. But why doesn't a separation of the grains take place? Why won't we succeed in driving the black grains to the top and the white grains to the bottom by means of a prolonged mixing? Here, too, it is entirely a matter of probability. A state in which the grains are distributed disorderly, i.e. black and white grains are uniformly intermingled, can be realized in an enormous number of ways and, consequently, possesses the greatest probability. On the contrary, a state in which all the white grains are on the top and all the black grains, on the bottom, is unique. Therefore, the probability of its realization is negligibly small.

We can easily pass from grains in a sack to the molecules that bodies are made of. The behaviour of molecules is subject to chance. This can be seen particularly clearly in the case of gases. As we know, gas molecules collide randomly and move in all possible directions, first with one speed and then with another. This eternal thermal motion continually reshuffles the molecules, mixes them like a shovel mixes the grains in a box.

The room in which you are is filled with air. Why can't it happen at some moment that the molecules in the lower half of the room pass into the upper half—under the ceiling? Such a process is not impossible—it is very improbable. But what does very improbable mean? If such a phenomenon were even a billion times less probable than a disorderly distribution of molecules, then someone might nevertheless observe it. Will we really observe such a phenomenon?

A computation shows that such an event for a 1-cm^3 vessel in volume takes place once in $10^{3\times10^{19}}$ cases. It hardly pays to make a distinction between the words "extremely improbable" and "impossible". For the number written above is unimaginably great; if we divide it by the number of atoms not only on the Earth, but also in the entire solar system, then it will still remain enormous.

But what will be the state of the gas molecules? The most probable one. But the most probable state will be that which is realizable in the greatest number of ways, i.e. disorderly distribution of molecules, for which there are approximately the same number of molecules moving to the right as to the left, upwards as downwards, for which one finds identical numbers of molecules in all equal volumes, the same proportion of fast and slow molecules in the upper and lower halves of the vessel. Any deviation from such a disorder, i.e. from a uniform and disorderly intermingling of molecules with respect to position and velocity, is linked with a decrease in probability, or, more concisely, is an improbable event.

On the contrary, phenomena linked with an intermingling, with the creation of disorder out of order, increase the probability of a state.

Hence, it is these phenomena that will determine the natural course of events. The law of the impossibility of a perpetual motion machine of the second kind and the law of the tending of all bodies towards an equilibrium state receive their explanation. Why is mechanical energy transformed into thermal? Simply because mechanical motion is orderly, but thermal is disorderly. The transition from order to disorder increases the probability of a state.

Physicists often use an auxiliary magnitude, called entropy. *Entropy* characterizes the degree of order and is related by a simple formula to the number of ways of creating states. We shall not give the formula, but shall only say that the greater the probability, the greater the entropy.

The law of nature which we are now discussing asserts: all natural processes proceed in such a way that the probability of a state increases. In other words, this same law of nature can be formulated as the law of increasing entropy.

The law of increasing entropy is a most important law of nature. From it follows, in particular, the impossibility of constructing a perpetual motion machine of the second kind, or, which is the same, the assertion that bodies left to themselves tend to a state of equilibrium.

The law of increasing entropy is sometimes called the *second law of thermodynamics* (*thermodynamics* is the study of heat). And the first law? It is the law of conservation of energy.

The name "laws of thermodynamics" for these laws of nature took shape historically. It cannot be said that such a union "under a single banner" was fortunate. For the law of conservation of energy is a mechanical law, obeyed implicitly by individual atoms and molecules, as well as by large bodies. But as for the law of increasing entropy, as follows from what has been said above, it is applicable only to sufficiently large collections of particles, and is simply impossible to formulate for individual molecules.

The statistical (this means pertaining to a large collection of particles) character of the second law of thermodynamics does not in the least diminish its significance. The law of increasing entropy predetermines the direction of processes. In this sense, entropy may be called the managing director of natural resources, while energy serves as its book-keeper.

But who has the honour of discovering this important law of nature? Here it is impossible to limit ourselves to a single name. The second law of thermodynamics has its own history.

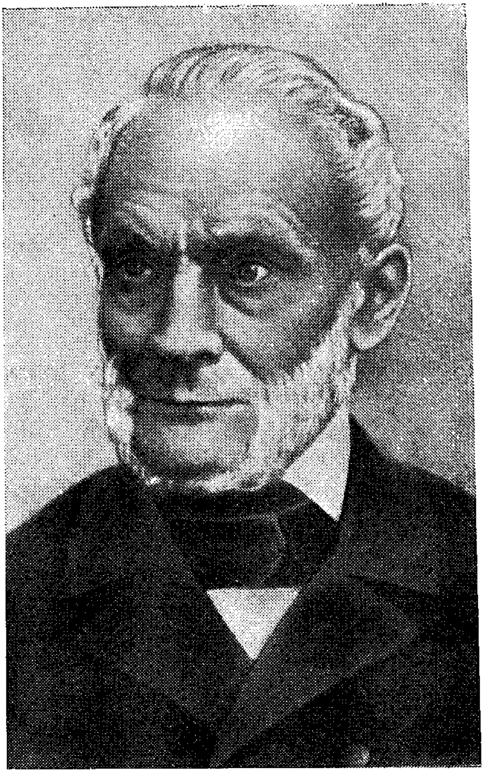
Here, too, just as in the history of the first law of thermodynamics, the name of the Frenchman Sadi Carnot should be mentioned first of all. In 1824, he published a work entitled *Reflections on the Motive Power of Fire* at his own expense. It was first demonstrated in this work that heat cannot pass from a cold body to a warm one without the consumption of work. Carnot also showed that the maximum efficiency of a heat engine (see below) is determined only by the difference in the temperatures of the heater and the cooling medium.

Only after Carnot's death in 1832 did other physicists pay attention to this work. However, it had little influence on the further development of science because all of Carnot's writings were based on the recognition of an indestructible and uncreatable "substance"—caloric.

Only after the work of Mayer, Joule and Helmholtz, who established the law of equivalence of heat and work, did the great German physicist Rudolf Clausius (1822-1888) arrive at the second law of thermodynamics and formulate it mathematically. Clausius introduced the concept of entropy and showed that the essence of the second law of thermodynamics can be reduced to the inevitable growth of entropy in all real processes.

The second law of thermodynamics permits one to formulate a number of general laws which all bodies should obey, regardless of their structures. However, there still remains the question of how to find a relationship between a body's structure and its properties. The branch of physics, which is called *statistical physics*, gives an answer to this question.

It is clear that in calculating physical quantities describing a system consisting of billions of billions of particles, a new approach is absolutely necessary. In fact, it would be pointless, not to say completely impossible, to follow the motions of all the particles and describe this motion with the aid of formulas of mechanics. However, it is precisely this enormous number of particles which enables us to apply new "statistical" methods to the study of bodies. These methods widely use the concept of the probability of events. The foundations of statistical physics were laid down by the outstanding Austrian physicist Ludwig Boltzmann (1844-1906). In a series of papers, Boltz-



RUROLF CLAUSIUS (1822-1888)—an outstanding German theoretical physicist. Clausius was the first to accurately formulate the second law of thermodynamics: in 1850, in the form of the thesis of the impossibility of heat being spontaneously transmitted from a colder body to a hotter one, and in 1865, with the aid of the concept of entropy, which he, himself, introduced. Clausius was one of the first to consider the questions of the heat capacity of polyatomic gases and the thermal conductivity of gases. Clausius' work in the kinetic theory of gases contributed to the development of statistical concepts of physical processes. A series of interesting investigations into electrical and magnetic phenomena are due to Clausius.

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mann showed how the indicated program can be carried out for gases.

The statistical interpretation of the second law of thermodynamics given by Boltzmann in 1877 was the logical culmination of these investigations. The formula relating the entropy and probability of a state of a system was carved out on Boltzmann's tombstone.

It would be difficult to overestimate the scientific achievement of Boltzmann, who discovered completely new paths in theoretical physics. Boltzmann's investigations were subjected to ridicule during his lifetime by conservative German professors: at that time, atomic and molecular conceptions were regarded by many as naive and unsuccessful. Boltzmann committed suicide, and the role played by the situation just described was undoubtedly far from the least important.

The edifice of statistical physics was perfected to a considerable degree by the work of the outstanding American physicist Josiah Willard Gibbs (1839-1903). Gibbs generalized Boltzmann's methods and showed how one might extend the statistical approach to all bodies.

Gibbs' last paper appeared at the beginning of the 20th century. A very modest researcher, Gibbs published his papers in the proceedings of a small provincial university. A considerable number of years had passed until his remarkable investigations were made known to all physicists.

Statistical physics has shown the way in which one can calculate properties of bodies consisting of a given quantity of particles. Of course, it should not be thought that these computational methods are all-powerful. If the nature of the motion of the atoms in a body is very complicated, as is the case for liquids, then the actual computation becomes unfeasible in practice.

Power

In order to judge a machine's ability to perform work, and also a consumption of work, the concept of power is used. *Power* is the work done in a unit of time.

There exist many different units for measuring power. The power unit erg/sec corresponds to the cgs system. But 1 erg/sec is a negligibly small power, and so this unit is inconvenient in practice. The unit of power obtained by dividing a joule by a second is much more widespread. This unit is called the *watt* (W); 1 W = 1 J/sec = 10^7 erg/sec.

When this unit is also too small, it is multiplied by a thousand and the *kilowatt* (kW) is used.

A power unit called the *horsepower* (hp) was inherited by us from old times. Some time during the dawn of the development of technology, this name had a profound meaning. A 10-hp machine can replace 10 horses — thus concluded the buyers, even if they had no idea of power units.

Of course, no two horses are alike. The inventor of the first unit of power apparently supposed that the "average" horse is capable of performing 75 kgf-m of work during one second. This is the unit that was adopted: 1 hp = = 75 kgf-m/sec.

A working horse is capable of doing a great deal of work, especially when beginning to move. However, the power of the average horse is rather close to half a horsepower.

Converting horsepowers to kilowatts, we obtain: 1 hp = = 0.735 kW.

We come across engines of the most different powers in our everyday practice and in technology. The power of a phonograph motor is 10 W, the power of a "Volga" engine is 75 hp = 55 kW and the power of the engines of an IL-18 passenger airplane is 16 000 hp. A small power station on a collective farm has a power of 100 kW. The record in this respect is held by the Krasnoyarsk hydroelectric power station, with a power of 5 million kW.

The power units with which we have become acquainted suggest another energy unit, well known wherever electric energy meters have been installed, namely the *kilowatt-hour* (kW-hr). One kilowatt-hour is the work done in the course of one hour by a one-kilowatt power. It is easy to convert this new unit to those with which we have been already acquainted: 1 kW-hr $= 3.6 \times 10^6$ J = 861 kcal = = 367 000 kgf-m. The reader may ask, "Was another energy unit really needed?" As a matter, there have already been quite a few of them! But the concept of energy permeates various branches of physics, and, thinking of the conveniences for a given branch, physicists kept introducing newer and newer units of energy. This finally led to the inference that it is necessary to introduce a common energy unit for all branches of physics, which is precisely what was done by the new system of units SI (see p. 18). However, a lot more time will go by until the "old" units make way for the lucky chosen one, and so meanwhile, the kilowatt-hour is not the last energy unit with which you will have to become acquainted in the process of studying physics.

Efficiency

With the aid of various machines, one can make sources of energy perform different kinds of work—raise loads, move machine tools, transport freight and people.

It is possible to calculate the amount of energy put into a machine and the value of the work obtained from it. In all cases, the figure for the output will prove to be less than the figure for the input—part of the energy is lost in the machine.

The fraction of the energy which is completely used in the machine for purposes necessary to us is called the *efficiency* (E) of the machine. Values of E are usually given in percentages.

If E equals 90%, this means that the machine loses only 10% of the energy. $E = 10\%$ means that the machine utilizes only 10% of the energy supplied to it.

If a machine converts mechanical energy into work, then its efficiency can in principle be made very large. An increase in efficiency is achieved in this case by a struggle with the inevitable friction. Improving the lubricant, introducing more perfect bearings and decreasing the resistance on the part of the medium in which the motion is taking place are means of bringing the efficiency nearer to one (to 100%).

When mechanical energy is being converted into work, electrotransmission is usually used as the intermediate stage (for example, in hydroelectric power stations). Of course, this also entails additional losses. However, they are small, and the losses involved in converting mechanical energy into work can also be reduced to several per cent in the case when electrotransmission is used.

Matters are entirely different in those cases when the machine makes use of the chemical energy of a substance.

Up to the present day, there have not existed machines, working on a large scale, which would convert the energy of a fuel directly into mechanical or electrical energy. Therefore, the intermediate stage of a transformation of chemical energy into thermal energy is inevitable. In order to obtain work from a fuel, it is necessary to burn it and create a high temperature in some region (furnace). A heat engine works on the temperature difference between the furnace and the surrounding medium. It takes away part of the thermal energy flux and converts it into work. But only part of the flux—under no conditions will the entire flux be so converted.

If the temperature difference is not great, then it is possible to take only a small streamlet of energy aside; when the furnace is at the same temperature as the medium, it is completely impossible to take heat from the source. If the temperature difference is large, then it is possible to convert a much greater part of the thermal flux into work.

The greater the temperature difference between the source of the heat flux and the surrounding medium, the greater the success with which the useful utilization of thermal energy can proceed.

This temperature difference sets a limit to the possibilities of perfecting heat engines. If we liquidate all losses in the machine, create ideal bearings and make use of such ideal heat-insulating and heat-conducting materials that do not even exist in nature, then the efficiency will still be unequal to one, but will merely attain a certain maximum. This limiting value of E for the conversion into work of a heat flux going from a heated body with temperature T_1 to a medium, which is at temperature T_0, equals

$$1-\frac{T_0}{T_1}$$

Thus, if the source of the heat flux has a temperature of 100 °C and the medium is at 20 °C, then the maximal value of E is equal to $1 - 293/373$, i.e. about 20%. When the temperature of the source is 1000 °C, we shall obtain 76%.

It is clear that we must strive to burn fuel in such a way that the highest possible temperature is attained.

From what has been said, it is obvious how unprofitable the utilization of a heat flux is for doing mechanical work. In the best modern gas turbines (see p. 466) we have only succeeded in attaining efficiencies of about 45%. It would be best of all to learn how to convert chemical energy directly into mechanical work, by-passing thermal energy. We know that in principle it would be possible to avoid the energy loss during such a direct transformation. However, as was already said, so far technology has not yet solved this problem.

Sources of Energy on the Earth

Not all sources of energy are of equal value. Some are of interest only theoretically; the existence of civilization is related to others. Some sources are practically inexhaustible; others will be depleted within the next few centuries or even decades.

The main guardian of our planetary system, the Sun, has been sending its life-giving rays to the Earth for several billion years. This source of energy may be safely called inexhaustible. The energy received by each square meter of the Earth's surface has an average power of about 1.5 kW; this will come to about 10 million kilocalories a year—such an amount of heat is released by burning hundreds of kilograms of coal. But how much heat does the whole Earth receive from the Sun? Computing the area of the Earth's surface and taking into account the fact that its illumination by the Sun's rays is non-uniform, we get about 10^{14} kW. This is 100 thousand times more than the energy which all the factories, plants, power stations and automobile and airplane engines yearly get from all the sources of energy on the Earth or, more concisely, 100 thousand times more than the power consumed by the entire population of the Earth (of the order of 10^9 kW).

However, in spite of a mass of projects, the use of solar energy has been quite negligible. And it is true that although our computation yielded an enormous figure, this amount of energy falls all over the Earth's surface: on the sides of inaccessible mountains, on the surface of the oceans, which occupies the greater part of the Earth's surface, and on the sands of uninhabited deserts.

Furthermore, the amount of energy falling on a small area isn't all that great. But it would hardly be expedient to create receivers of energy extending over several square kilometers. Finally, it is obvious that it only makes sense to engage in converting solar energy into heat in those localities where there are many sunny days.

Interest in the direct utilization of the Sun's energy has been somewhat heightened recently in connection with the emerging possibilities of converting solar energy directly into electrical energy. Such a possibility is naturally quite attractive. However, it has been realized to a very negligible extent so far.

Accumulators of solar energy have been discovered above us comparatively recently, in the upper layers of the atmosphere. It turned out that oxygen at a height of 150-200 km above the Earth's surface is in a dissociated state owing to the action of solar radiation: its molecules have been split up into atoms. If these atoms were to unite to form molecules of oxygen, the energy of 118 kcal/mol could be given off. But how great are the total reserves of this energy? In a 50-km thick layer at the indicated height 10^{13} kcal are stocked—as much as are released when several million tons of coal burn up completely. Such an amount of coal is mined in the USSR during several days. Although the energy of dissociated oxygen at great heights is continually renewed, here we again come across the problem of a small concentration: it isn't so easy to devise a mechanism for the practical utilization of this energy.

Let us return to our discussion of sources of energy. Masses of air in the Earth's atmosphere are continually in motion. Cyclones, storms, constantly blowing trade-winds and breezes are the manifold manifestations of energy in air streams.

The energy of winds was used for moving sailing vessels and windmills back in ancient times. The average annual total power of the air currents on the entire Earth is equal to no less than 10^{11} kW.

However, we shall not place great hopes in the wind as a source of energy. In addition to the fact that this source is unsteady (how many misfortunes and disappointments did calms lead to in the age of sailing vessels!), it possesses the same drawback as the solar energy: the amount of energy released per unit of area is relatively small; the blades of a wind turbine, if such be created for the production of energy on a large scale, would have to attain dimensions which are not realizable in practice. A no less essential drawback is the unconstancy of the wind intensity. Therefore, the energy of the wind, or, as it is called poetically, "blue coal", is used only in small engines, wind motors, or windmills. When it is windy, they supply agricultural machines with electrical energy and light up homes. If a surplus of energy is formed, it is stored in accumulators (this is what depositories of electrical energy are called). These surpluses can be utilized during a calm. Of course, you can't depend on a wind motor; it can only play the role of an auxiliary engine.

Moving water—the ocean waves, continually driven to the shores by the tides, and the streams of river waters flowing to the seas and oceans—is also a free source of energy.

The power of all the rivers on the Earth is measured in billions of kilowatts, but only about 40 million kW are being used at the present time, i.e. of the order of 1%. The potential power of the rivers of the USSR is about 400 million kW, but so far only about 20 million kW of them are being used.

If we are deprived of coal, oil and other sources of energy and were to convert exclusively to "white coal" (the energy of rivers), then even if this energy were to be fully utilized (assuming that all possible hydroelectric power stations were built on all the rivers on the Earth), we would have to decrease the consumption of energy on the Earth. The expenditure of energy on the Earth at the present time exceeds a billion kilowatts—water power alone would even now just barely suffice for the needs of humanity.

Well, but what about the waves due to tides? Their energy is quite considerable, although approximately ten times less than the energy of rivers. Alas, this energy has so far been utilized only to the most negligible degree: the pulsating character of the tides makes its use difficult. However, Soviet and French engineers discovered a practical means of overcoming this difficulty. A tidal power station can now ensure the distribution of a guaranteed power during the hours of maximal consumption. The St. Malo tidal power station was built in France and has been already in operation, while in the USSR, a tidal power station is being built across the Kislaya Guba near Murmansk. The latter will serve as an experiment for the construction of planned powerful tidal power stations across the Lumbovskii and Mezenskii Bays in the White Sea. A tidal station with a power of 240 thousand kW was put in operation in France.

The water in the oceans at great depths has a temperature which differs from the temperature of the surface layers by 10-20°. Hence, it is possible to construct a heat engine whose heater at middle latitudes would be the upper layer of water, and whose cooler, an abyssal layer. The efficiency of such an engine would be 1-2%. But this, of course, would be a very unconcentrated source of energy.

The Sun, air and water are free sources of energy.* Gift

* Of course, we must not put the Sun on the same level with other sources of energy. In the final analysis, all our energy is taken from the Sun.

in the sense that the utilization of their energy does not entail a decrease in any of the Earth's valuables whatsoever. The operation of windmills does not diminish the amount of air on the Earth; the operation of hydroelectric power stations does not decrease the depths of rivers; neither are the reserves of any substances on the Earth used during the operation of solar engines.

In this sense, the sources of energy described so far have a big advantage over fuel. A fuel burns up. The utilization of the energy of coal, oil and wood is an irreversible destruction of the Earth's valuables. It would be very tempting to try to invent a photochemical engine, i.e. to obtain energy with the aid of the mechanism of photosynthesis, which provides an accumulation of fuel energy. A green leaf of any plant is a factory which manufactures organic substances with a large supply of energy in their molecules out of molecules of water and carbon dioxide, thanks to the energy of sun rays. This process in plants has a low efficiency (about 1%), but in spite of this, the amount of energy annually stored by plants is about 2×10^{15} kW-hr, i.e hundreds of times greater than the yearly production of energy of all the power stations in the world. The mechanism of photosynthesis has not yet been completely unraveled, but there is no doubt that in the future we shall succeed in not only realizing photosynthesis under artificial conditions, but also in raising its efficiency. However, human beings are still unable to compete with nature in this field and are forced to make use of its gifts, burning wood, oil and coal.

But what reserves of fuel are there on the Earth? Coal and oil are classified as ordinary fuels, i.e. those which burn when treated with fire. Their reserves on the Earth are extremely small. Given the current rate of oil consumption, its known reserves will be exhausted already by the beginning of the next millennium. The reserves of coal are somewhat greater. The amount of coal on the Earth is about 10^{13} tons. A kilogram of coal yields 7000 kcal of heat when burned. Therefore, the total energy reserves of coal are of the order of 10^{20} kcal. This is about a thousand times greater than the yearly consumption of energy.

A thousand-year supply of energy must be recognized as very little. A thousand years are a lot only by comparison with the life-span of a human being, but a human life is a fleeting moment in comparison with the life of the Earth or the duration of the civilized world. Furthermore, the per capita consumption of energy is continually growing. Therefore, if the reserves of fuel were reduced to oil and coal, then the situation with respect to the energy reserves on the Earth would have to be regarded as catastrophic.

In the early forties of our century, the practical possibility of utilizing a completely new kind of fuel, called nuclear, was proved. We have available considerable reserves of nuclear fuel.

This is not the place to dwell on the structure of the atom and its core, the atomic nucleus, or on how one can extract the internal energy from atomic nuclei. The extraction of nuclear energy can only be accomplished on a large scale at so-called atomic power stations. Nuclear energy is released in the form of heat, which is used just as in a power station running on coal.

At the present time, we can extract energy in amounts suitable for industrial needs from two elements, uranium and thorium. A special feature of nuclear fuel, its main virtue, is the exceptionally high concentration of energy in it. A kilogram of nuclear fuel yields 2.5 million times more energy than a kilogram of coal.

Therefore, in spite of the relatively small prevalence of these elements, their reserves on the Earth are quite considerable when expressed in terms of energy. Approximate calculations show that the reserves of nuclear fuel are essentially greater than the reserves of coal. However, gaining access to such fuels as uranium and thorium does not solve the fundamental problem of liberating humanity from energy hunger, since the supply of minerals in the Earth's core is limited.

But now it is already possible to point to a truly unlimited source of energy. We are talking about so-called thermonuclear reactions. They are possible only at ultrahigh temperatures of the order of twenty million degrees. This temperature has so far been attained only by means of nuclear explosions.

The problem of obtaining high temperatures by non-explosive means is now before researchers, and the first attempts to attain a temperature of a million degrees have been crowned with success.

If physicists are able to work with the required high temperatures of tens of millions of degrees, obtained by a non-explosive method, then a controlled reaction of fusion of hydrogen nuclei (it bears the name thermonuclear) will become possible.

An enormous amount of energy per kilogram of fuel will be released during such a reaction. In order to supply humanity with energy for one year, it is enough to release thermonuclear energy by processing ten million tons of water.

So much thermonuclear energy is stored in the world ocean that it suffices for covering all the energy requirements of humanity for a period of time exceeding the age of the solar system. Here at last is a really unlimited source of energy.

Engines

A person living in the 20th century is accustomed to make use of a variety of engines, which do a great deal of work for him, facilitating his labour and increasing his power ten-fold.

Windmills have been used in agriculture in many countries up to the present day. This most simple engine, utilizing the energy of the wind, has already served people for many centuries. The blades of such an engine are plane. They are placed at certain angles to the direction of the wind. The stream of air striking the blades, which are distributed in a circle, will turn a wheel.

It is evident that a wind engine can be reversed: if any motor rotates it, then the blades will send forth a powerful stream of air along the axis of rotation. When such a system is installed on a glider, airplane or helicopter, we speak of a propeller. The jet reaction thrust by a propeller pulls a glider or an airplane and creates the lift for a helicopter.

To all appearances, the first engine used by human beings for their needs was a water (hydraulic) turbine in its most primitive form, that of a waterwheel.

Figure 133 depicts a waterwheel. Striking against one of the wheel's submerged blades, a stream of water gives it part of its kinetic energy. The blade is set into motion. Since it is rigidly connected to the wheel, the wheel begins to rotate. But it can be seen at once that only one blade can be perpendicular to the stream at a given instant. The others form acute angles with the rushing stream, taking less energy from it than the perpendicular blade. The efficiency of such a wheel is not high. The way to raise it is obvious: it must be arranged so that all the blades of the wheel are perpendicular to a rushing stream. We are able to realize this idea with the aid of a regulating ring. It is

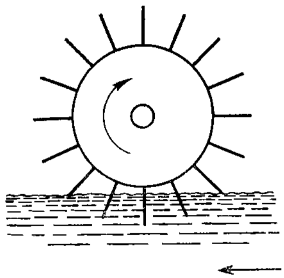

Fig. 133

clear from Figure 134 that in addition, the presence of a difference in water levels is necessary for the successful operation of such a turbine. We arrive at the diagram of a modern hydroelectric power station, whose powerful dam thrusts masses of water onto the turbine's blades with a huge force. Made with a high degree of modern engineering

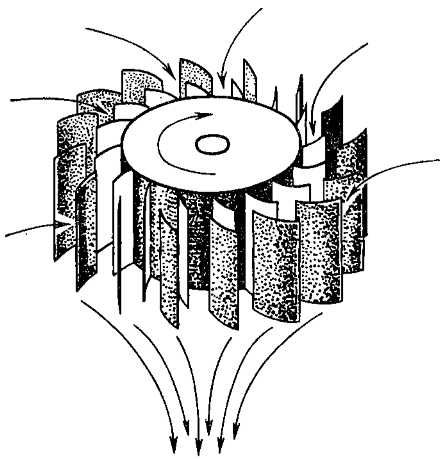

Fig. 134

skill, hydraulic turbines are designed for powers exceeding 100 000 kW and have, moreover, an efficiency of 95%. Since these powers are created for a rather slow rate of rotation (of the order of 100 rpm), the hydraulic turbines now being constructed stagger one by their dimensions and weight. Thus, the height of the driving wheel of the Lenin Volzhskaya hydroelectric power station is about 10 m, and its weight is approximately 420 tons.

An important advantage of the turbine is the extraordinary simplicity of the transformation of the translational motion of water into rotary motion. For this reason, this principle is widely used in engines bearing absolutely no external resemblance to the waterwheel whatsoever. When steam pushes the blades, we have a steam turbine. We already know that in order to increase the efficiency, it is necessary to raise the temperature of the working body. Steam having a temperature of 580 °C and a pressure of 240 atm is let into the turbines of modern thermal power stations. The theoretical limit to the efficiency of such a turbine is equal to 66%, if we assume that the cooler has a temperature of 20 °C. An efficiency of 42% is attained in practice. Therefore, steam turbines are good modern engines. They have a power of up to 300 000 kW in a single unit. Such a turbine consumes more than 900 tons of steam at high pressure every hour. But it is perfectly clear that obtaining such amounts of steam is a complicated technological problem. High-pressure steam boilers and the system preparing and feeding the fuel occupy the major part of the volume of a modern thermal power station. Steam turbines are therefore used for transportation purposes only on large ships, turboships.

The word "turboelectric motor ship" began appearing in print in recent years. The meaning of this name is easily explained: on such a ship, steam sets turbines in motion, the turbines in turn set powerful direct-current generators in motion and the propellers are placed on the shafts of the electromotors. Isn't this complication unnecessary? Why not place a propeller directly on the shaft of a turbine? Here we come across a new question, the tractive characteristic of an engine.

The fact is that a steam turbine develops its maximum power only at a very definite rate of rotation. Thus, the powerful turbines of Soviet power stations perform 3000 rotations per minute. The power falls off when the rotation slows down. It is clear that if the propellers were put directly on the shafts of the turbines, then a ship equipped with such a power unit would perform poorly. But a direct-current electric motor has an ideal tractive characteristic: the greater the force of resistance, the greater the tractive force which it develops; moreover, such a motor can develop a great power with a low rate of rotation at the moment when the ship casts off.

Consequently, the direct-current generator and the motor, which are in between a turbine and a propeller of a turbo-electric motor ship, play the role of an automatic variable-speed gear possessing a high degree of perfection. It may seem that such a system is somewhat unwieldy, but given the large powers of modern turboelectric motor ships, any other would be just as bulky, but less reliable.

The power unit of a turboelectric motor ship can be considerably perfected in another respect: it is quite advantageous to replace the unwieldy steam boilers by an atomic reactor. This yields an enormous economy in the volume of the fuel which has to be taken on a voyage.

The first Soviet nuclear-propelled ice-breaker *Lenin* has achieved world-wide renown. The power of its engines is equal to 44 000 hp and its displacement, 16 000 tons. The nuclear power unit of this turboelectric ship assures it a self-contained voyage of over a year.

Thus, a powerful outside source of heat is needed for a steam turbine. Whether the furnace of a steam boiler or a uranium nuclear reactor, at the present level of development of technology, these sources have such great dimensions and weight that the installation of a steam turbine in an automobile or an airplane would be completely inexpedient: the total weight of the engine and the heater per horsepower would be too great. Can t we get rid of the outside heater by transferring it to within the turbine?

Such a unit has been constructed and is already widely used. This is the gas turbine. Its immediate working fluid is the product of combustion of a high-calorific fuel. This is what determines the important advantages of a gas turbine over a steam turbine and the great technological difficulties involved in insuring its reliable functioning.

The advantages are obvious: the combustion chamber, where the fuel is burned, has small size and can be placed under the casing of the turbine, and the combustion gases consisting, for example, of atomized kerosene and oxygen have a temperature unattainable for steam. The heat flow formed in the combustion chamber of a gas turbine is very intense, which makes it possible to obtain a high efficiency.

But these advantages turn into shortcomings. The steel blades of the turbine function in streams of gas having a temperature up to 1200 °C and inevitably saturated with microscopic ash particles. It is easy to imagine what great demands have to be made of the materials out of which gas turbines are manufactured. During an attempt to construct a gas turbine with a power of about 200 hp for a car, it became necessary to deal with quite a peculiar difficulty: the turbine was of such small dimensions that the usual engineering solutions and the customary materials simply could not be used. However, the technological difficulties are already being overcome. The first experimental cars with gas turbines are being tested.

It turned out easier to utilize the gas turbine in railway transport. Locomotives with gas turbines, gas-turbine locomotives, have already received universal recognition.

But entirely different engines, in which the gas engine is a subordinate, although necessary, component part, paved the way for the widespread use of the gas turbine. We are speaking of the turbojet engine, the basic type of engine at the present time in jet aviation.

The principle on which the jet engine is based is extremely simple. A gas mixture is burned in a durable combustion chamber; the combustion gases, having an extraordinarily great speed (3000 m/sec when hydrogen is burned in oxygen, somewhat less for other types of fuel), are thrust through a smoothly expanding nozzle in the direction opposite to that of motion. Even relatively small amounts of products of combustion will carry a large momentum away from the engine.

With the creation of jet engines, people received the realistic opportunity of carrying out interplanetary flights.

Liquid propellant rocket engines have become very widespread. Definite portions of a fuel (for example, ethyl alcohol) and an oxidizer (usually liquid oxygen) are injected into the combustion chamber of such an engine. The mixture burns, creating traction. In high-altitude rockets, such as the V-2, the traction is of the order of 15 tons. The figures are sufficiently eloquent: 8.5 tons of fuel and oxidizer, which take 1.5 minutes to burn up, are poured into the rocket. The use of liquid propellant rocket engines is only expedient for flights at great heights or beyond the Earth's atmosphere. It makes no sense to pour large quantities of a special oxidizer into an airplane destined for flights in the lower layers of the atmosphere (up to 20 km), where

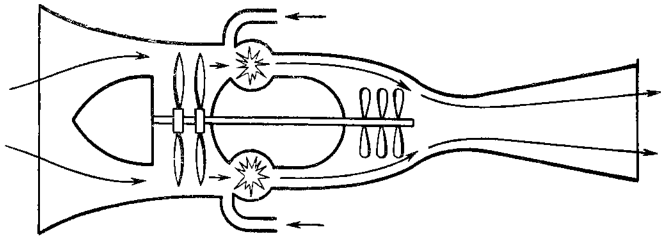

Fig. 135

there is enough oxygen. But then the problem arises of forcing huge amounts of air, necessary for intensive burning, into the combustion chamber. This problem is solved in a natural fashion: part of the energy of the gas stream created in the combustion chamber is taken away to rotate the powerful compressor forcing air into the chamber.

We have already said what engine can do work at the expense of the energy of a stream of scorching gases; of course, this is the gas turbine. The whole system is called a turbojet engine (Figure 135). A turbojet engine has no competitor for flights with speeds from 800 to 1200 km/hr.

For long-distance flights with speeds of 600-800 km/hr, an ordinary aircraft propeller is added to the shaft of a turbojet engine. This is a turboprop engine.

At flying speeds of about 2000 km/hr or more, the air pressure developed by the airplane is so great that the need for a compressor no longer arises. It is then only natural that neither is a gas turbine needed. The engine is converted into a pipe of variable cross-section, in which fuel is burned at a very definite place. This is a ramjet engine. It is clear that a ramjet engine cannot lift an airplane off the ground: it becomes capable of functioning only at very high flying speeds.

Jet engines are completely inexpedient for flights at small speeds owing to the large expenditures of fuel.

For motion on land, on water or in the air with speeds from 0 to 500 km/hr, people are reliably served by gasoline or diesel internal combustion piston engines. As indicated by the name, the main part of such an engine is the cylinder inside which the piston can move. The back-and-forth motion of the piston is transformed into a rotary motion of the shaft with the aid of the connecting rod and crank system (Figure 136).

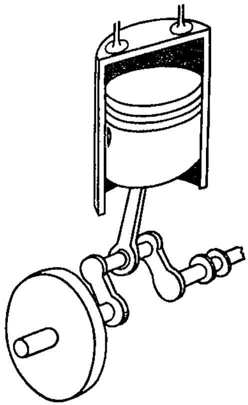

Fig. 136

The motion of the piston is transmitted through the connecting rod to the crank, which is part of the crankshaft. The motion of the crank sets the shaft into rotation. Conversely, if the crankshaft is turned, then this will give rise to an oscillation of the connecting rods and a displacement of the pistons inside the cylinders.

The cylinder of a gasoline engine is equipped with two valves, one of which is meant for the inlet of the gas mixture, and the other, for the exhaust of waste gases. In order to start the engine, we must turn it over, using the energy of some outside source. Assume that at a certain moment the piston is moving down and the inlet valve is open. A mixture of atomized gasoline and air is sucked into the cylinder. The inlet valve is connected to the shaft of the engine in such a way that it closes at the moment when the piston reaches its lowest position. As the shaft continues to turn, the piston starts coming up. The valves' automatic drive keeps them closed during this stroke, and the gas mixture is therefore compressed. When the piston reaches its highest position, the compressed mixture is ignited by an electric spark, which jumps between the electrodes of the spark plug. The mixture ignites and the expanding products of combustion act, pushing the piston down. The engine shaft receives a powerful push, and the flywheel on the shaft stores considerable kinetic energy. All of the next three preliminary strokes proceed at the expense of this energy: first the exhaust, when the exhaust valve is open and the piston is moving up, driving the exhaust gas out of the cylinder, next the intake and compression that we already know about, then a new ignition. The engine is started.

Gasoline engines have powers from fractions of a horsepower to 4000 hp, an efficiency of up to 40% and weigh up to 300 g per horsepower. Their widespread utilization in cars and airplanes is explained by such a good showing.

How might we increase the efficiency of a gasoline engine? The major means is by raising the degree of compression. For the surrounding air is the cooler for all heat engines used in transportation. Therefore, the efficiency can be increased only by raising the temperature of the working fluid; but for this it is necessary to compress the mixture as much as possible before the ignition. However, a serious complication arises in this connection: a highly compressed mixture detonates (see p. 437). The power stroke would acquire the character of a strong explosion, which could damage the engine. One has to take special measures to decrease the detonation properties of the gasoline, and this would greatly raise the cost of a fuel that is already quite expensive (see p. 439).

The problems of raising the temperature during the power stroke, eliminating detonation and reducing the cost of the fuel, have been successfully solved in the diesel engine.

A diesel engine resembles a gasoline engine to a great extent in its construction, but is designed for products of oil distillation, which are cheaper than and inferior in quality to gasoline. The cycle begins with the admission of pure air into the cylinder. The air is then compressed by the piston to approximately 20 atm. It would be very difficult to achieve such a high compression by turning over the engine by hand. Therefore, diesel engines are started with a special starting motor, usually a gasoline, or by compressed air.

When highly compressed, the temperature of the air in a cylinder rises so much that it becomes sufficient for the gas mixture to ignite. But how can it be admitted into the cylinder, where a high pressure has been attained? An inlet valve would not be suitable here. It is replaced by a sprayer, which forces the fuel into the cylinder through a tiny opening. It ignites as it enters, eliminating the danger of a detonation, considerable for a gasoline engine. Eliminating the danger of a detonation permits us to construct diesels with many thousands of horsepowers for slow ships. Naturally, they acquire quite considerable dimensions, but remain more compact than an aggregate consisting of a steam boiler and a turbine.

A ship in which there are direct-current generator and motor between a diesel engine and a blade is called a diesel-electric motor ship. Diesel locomotives, now being widely introduced on railroads, are built along the same lines; they may in a sense be called diesel-electric locomotives.

Internal combustion piston engines, considered last by us, have borrowed their basic constructive elements, the cylinder, the piston, the connecting rod and crank mechanism which helps obtain rotary motion, from the steam engine, now gradually leaving the scene. The steam engine could have been called an "external combustion piston engine". It is precisely this combination of the unwieldy steam boiler with the no less unwieldy system of transforming translational motion into rotary one that deprives the steam engine of the possibility of successfully competing with more modern engines. In order to convince ourselves of this, let us follow the operation of a double-acting steam engine.

Steam from the boiler enters the steam chest, within which there is a slide valve, a valve of a special form. The slide valve is connected to the piston by means of a system of levers in such a way that it moves in jerks, giving the steam access to different parts of the cylinder in turn. Therefore, the cylinder contains steam under high pressure at any moment. It would seem that a steam engine is better than a gasoline engine: in fact, it does not take any preliminary strokes; each of its strokes is a power one. But this superficial reasoning is absolutely incorrect.

It should be recalled that the satisfactory efficiency of a gasoline engine is determined by the high temperature of the gases pushing the piston. We already know that in order to raise the efficiency of a steam turbine, one uses steam under high pressure at a temperature which makes the steam pipelines and the blades red-hot. But the blades of a turbine rotate freely, without any friction against a metallic surface ... . Imagine what difficulties a dreamer would have to overcome if he or she intended to "improve" the steam engine by making a red-hot piston slide within an equally hot cylinder, where the piston should fit so closely to the cylinder that it can maintain a temperature difference of the order of 600 atm. Even if one were to display a miraculous inventiveness and construct such a machine, its efficiency would still be lower than that of a turbine with the same steam parameters, since the rotation is effected much more simply in the latter, while its dimensions and weight would be greater than those of an analogous internal combustion engine.

Modern steam engines have an efficiency of about 10%. The locomotives which are now being taken out of production released up to 95% of the fuel they burned through the smoke-stack to no advantage.

This "record-breaking" low efficiency is explained by the inevitable deterioration in the properties of a steam boiler designed for installation on locomotive as compared to a stationary steam boiler.

But why were steam engines employed so widely in transportation for such a long time? Besides adherence to customary solutions, the circumstance that a steam engine has a very good tractive characteristic also played a role: the greater the force with which the load resists a displacement of the piston, the greater the force exerted on it by the steam, i.e. the torque developed by a steam engine grows under difficult conditions, which is important in transportation. But, of course, the fact that the steam engine does not need a complicated system of variable transmission to the driving axles cannot in the least compensate for its basic defect, a low efficiency.

This is what explains the supplanting of the steam engine by other engines.

Fluctuations

Let us return to the second law of thermodynamics, that mighty law of nature governing the flow of natural phenomena. We have seen that spontaneous processes bring a system to its most probable state, i.e. to the growth in entropy. After the entropy of a system has become maximal, further changes in the system cease—equilibrium has been reached.

But a state of equilibrium does not by any means imply internal rest. An intensive thermal motion takes place within the system. Therefore, strictly speaking, any physical body "stops being itself" at each instant: the mutual distribution of its molecules at every successive moment is not the same as it was at the preceding one. Consequently, the values of all physical quantities are conserved only "on the average"; they are not exactly equal to their most probable values, but vary around them. Deviations from the most probable values at equilibrium are called *fluctuations*. The values of the various fluctuations are extremely negligible. The greater the value of a fluctuation, the less probable it is.

The average value of a relative fluctuation, i.e. the fraction of the magnitude of the physical quantity of interest by which it can change as a result of the chaotic thermal motion of molecules, can be approximately represented by the expression $1/\sqrt{N}$, where N is the number of molecules in the body, or in that part of it, which we are investigating. Therefore, fluctuations are appreciable for systems consisting of a small number of molecules, and completely insignificant for large bodies containing billions of billions of molecules.

The formula $1/\sqrt{N}$ shows that in one cubic centimeter of a gas, the density, pressure, temperature and also any other properties can change by $1/\sqrt{3 \times 10^{19}}$, i.e. by not more than $10^{-8}\%$. Such fluctuations are too small to be detected experimentally.

However, things are entirely different for a volume of a cubic micron. Here $N = 3 \times 10^{7}$ and fluctuations will attain measurable values of the order of hundredths of a per cent.

A fluctuation is an "abnormal" phenomenon in the sense that it implies a transition from a more probable state to a less probable one. In the course of a fluctuation, heat transfers from a cold body to a hot one, the uniform distribution of the molecules is violated and an orderly motion arises.

Will someone perhaps succeed in constructing a perpetual motion machine of the second kind on the basis of these violations?

Let us imagine, for example, a tiny turbine situated in a rarefied gas. Can't we arrange things in such a way that this small machine would react to all fluctuations in an arbitrary, but fixed, direction? For example, so that it would rotate if the number of molecules flying to the right became greater than the number of molecules moving to the left? Such small tremors might be accumulated, eventually giving rise to work. The law asserting the impossibility of a perpetual motion machine of the second kind would be refuted.

But, alas, such a mechanism is impossible in principle. A detailed examination, taking into account the fact that the turbine would have its own fluctuations (the smaller its dimensions, the greater they will be), shows that fluctuations can never perform any work whatsoever. Although violations of the tendency towards equilibrium continually arise around us, they cannot change the inexorable course of physical processes in the direction increasing the probability of a state, i.e. entropy.

Entropy and the Development of the Universe

Rivers flow down, stones roll down a mountain, movement ceases as a result of friction—all relative motions come to a halt. Hot bodies cool off and cold bodies warm up—the temperatures of all the bodies in the world are equalizing. Such is the irreversible course of events in the world surrounding us from the point of view of the law of increasing entropy.

It would seem that everything is clear. However, if we think it over, we shall find an obscure aspect to this question. If nature is tending towards an equilibrium, then the question arises as to why no equilibrium has yet been established.

In fact, even if a system is extremely non-equilibrium, the time needed for its transition to a state of equilibrium (physicists call it the relaxation time) cannot be infinitely long. The transition of our Universe to a state of equilibrium might last a long time, say many billions of years, but in any event, the transition from an arbitrary non-equilibrium state to a state of equilibrium would take a definite amount of time, and would not last indefinitely.

Why, then, didn't this equilibrium set in billions, or even billions of billions, of years ago?

This contradiction is very serious. It turns out that the very existence of our world, behaving in accordance with our observations, is in irreconcilable contradiction with known laws of physics.

Can't we get out of the difficulty by assuming that our entire Universe is a gigantic fluctuation? The world is infinite in time and space. A fluctuation arises first here, then there: molecules unite, their motion becomes orderly and there is created, for example, a planetary system similar to ours. After this, the fluctuation resolves, disappears, but in its place there arises another fluctuation in another part of the world.

However, regardless of how tempting such a hypothesis may be, it does not withstand a simple criticism. Such a fluctuation is too improbable. We saw that a spontaneous condensation of molecules into one-half of a cubic-centimeter vessel is one event in a colossal number. What can we then say about a fluctuation creating the visible Universe?

Such an explanation is obviously invalid. It would be far more naive to believe in its validity than to believe the sworn statements of a thief that it wasn't he who stole your wallet from your pocket, but a fluctuation of molecules led to a transfer of the wallet from your pocket to his hand. Incidentally, such a fluctuation is an unimaginably large number of times more probable than the fluctuation on a scale of the Universe under consideration.

You might try to object in the following manner. Let the probability of a gigantic fluctuation of the size of the Universe be negligibly small, but this shouldn't surprise us. After all, I, a human being discussing this question, am also the consequence of a fluctuation. Surely my existence is a completely improbable occurrence, and I ought to make judgements about what is probable or improbable with respect to myself.

This objection also has to be discarded.

The solar system is more than enough for our existence, but we see an unbalanced world on a scale in comparison with which our solar system is the most minute particle.

At the present time, astronomers have already, with the aid of telescopes, penetrated into the depths of the Universe by distances which are 10^{12}-10^{13} times as great as the size of the solar system. If the Universe is a fluctuation, this means that we are observing non-equilibrium states whose scale exceeds that needed for our lives by at least 10^{12} times. Therefore, our existence does not in the least justify the unimaginably small probability of a fluctuation leading to the formation of the Universe in its present form.

Consequently, the contradiction remains in full force. This indicates that there is something wrong with the fundamental conceptions of space and time, and also the basic laws, which we have so far regarded as beyond doubt.

We have come across a fundamental defect in our mechanics for the second time. However, now we have found a new defect in it, unrelated to a revision of concepts whose necessity we indicated when we became acquainted with the extraordinary properties of liquid helium. There we discussed the inapplicability of laws of the old mechanics to microparticles. Now we have discovered shortcomings in the foundations of our knowledge, trying to apply it to the Universe.

Our old mechanics proved to be unsuitable for the very large, just as for the very small.

We hope to speak to the reader in the future about what changes must be introduced into our previous formulation of the laws of nature in order that they might be applied in certain necessary cases to the microworld, and in others, to the entire Universe.

TO THE READER

Mir Publishers would be grateful for your comments on the content, translation and design of this book. We would also be pleased to receive any other suggestions you may wish to make.

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Molecular Physics

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In this second part of a general course in physics the authors acquaint first-year students specializing in the subject with the fundamentals of the molecular structure of matter on the basis of the kinetic theory. They give the essential elements of thermodynamics, statistical physics and the quantum theory needed to understand the subject. Great attention is devoted in the book to the experimental methods used in physics.

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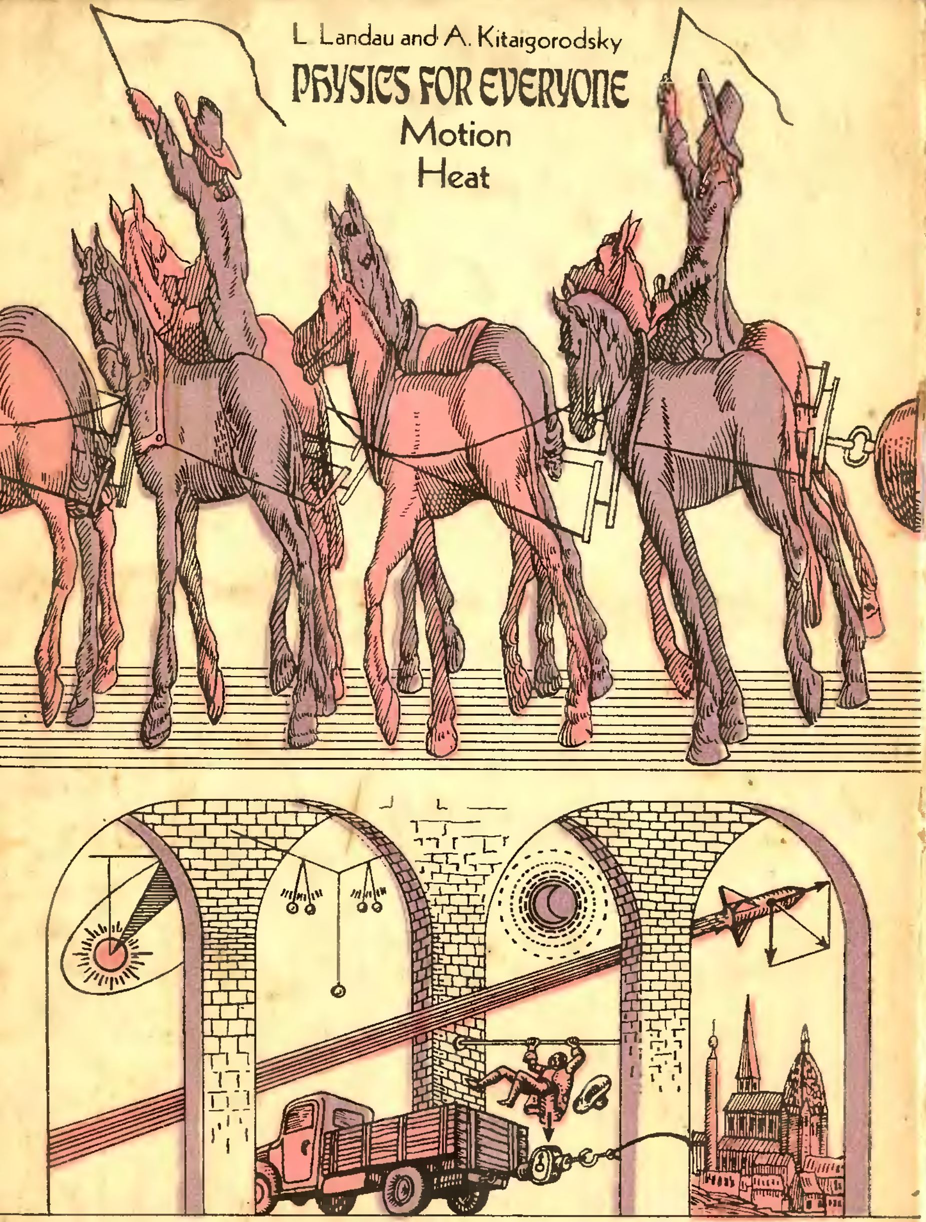
L. Landau and A. Kitaigorodsky

PHYSICS FOR EVERYONE

Motion

Heat



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