Aiden Lab Predoctoral Fellows Application 2017

Round II (Challenge 1A)

Mathematical Explanation

Denote the target value as t and the bucket sizes as an array of numbers $a_1, a_2, \ldots, a_n$. The goal is to determine if it is possible to find integers $x_1, x_2, \ldots, x_n$ such that,

$$a_1x_1 + \ldots + a_nx_n = t \qquad \text{(I)}$$

Notice that it is possible to reduce this problem to a much simpler problem by rearranging the terms as follows,

$$a_1x_1 + \ldots + a_{n-1}x_{n-1} = t - a_nx_n, \text{ for all } a_n = 0, 1, 2, \ldots, \text{floor}(t/x_n) \qquad \text{(II)}$$

Following in a similar fashion, we reach,

$$a_1x_1 = t - a_nx_n - \ldots - a_2x_2, \text{ for all feasible}^{(1)}\ a_k,\ k = 2, \ldots, n \qquad \text{(III)}$$

At this point, if the right-hand side is divisible by a_1, the answer to the problem is True. Otherwise it is False.

Hence, the algorithm is quite clear now. We first check if any of the individual bucket sizes can divide the target value. If the target value is divisible by any one of them, the algorithm outputs **True**. Otherwise the algorithm iterates through all feasible (non-negative right-hand side, in this case) values of a_n and checks if the values on the right-hand side of (II) are divisible by any of the a_k, $k = 1, 2, \ldots, n-1$. If the value $t - a_nx_n$ is divisible by any one of them, the algorithm outputs **True**. Otherwise, we keep descending into deeper levels of recursion until we reach (III), where the problem has been reduced to simply determining whether a_1 divides all non-negative values of $t - a_nx_n - \ldots - a_2x_2$. If it is divisible, the answer to both the simplified sub-problem and the original problem is **True**. Otherwise, if all these cases have failed (which is the worst case-scenario, in terms of speed optimality), the answer is **False**.

(1) Feasible values are the values of a_k, $k = 2, \ldots, n$ for which the right-hand side is non-negative.