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University of Moratuwa  
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**EN1060 SIGNALS AND SYSTEMS: TUTORIAL 03**\*

August 10, 2022

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1. Use the Fourier transform analysis equation to calculate the Fourier transforms of

(a)  $e^{-2(t-1)}u(t-1)$

(b)  $e^{-2|t-1|}$

2. Use the Fourier transform analysis equation to calculate the Fourier transforms of

(a)  $\delta(t+1) + \delta(t-1)$

(b)  $\frac{d}{dt}[u(-2-t) + u(t-2)]$

Sketch and label the magnitude of each Fourier transform.

3. Determine the Fourier transform of each of the following periodic signals:

(a)  $\sin(2\pi t + \pi/4)$

(b)  $1 + \cos(6\pi t + \pi/8)$

4. Use the Fourier transform synthesis equation to determine the inverse Fourier transform of

(a)  $X_1(j\omega) = 2\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)$

(b)  $X_2(j\omega) = \begin{cases} 2, & 0 \leq \omega \leq 2, \\ -2, & -2 \leq \omega < 0, \\ 0, & |\omega| > 2. \end{cases}$

5. Use the Fourier transform synthesis equation to determine the inverse Fourier transform of  $X(j\omega) = |X(j\omega)|e^{j\angle X(j\omega)}$  where

$$|X(j\omega)| = 2[u(\omega + 3) - u(\omega - 3)]$$

$$\angle X(j\omega) = -\frac{3}{2}\omega + \pi$$

6. Compute the Fourier transform of each of the following signals:

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\*All the questions are from Oppenheim *et al.* chapter 4.

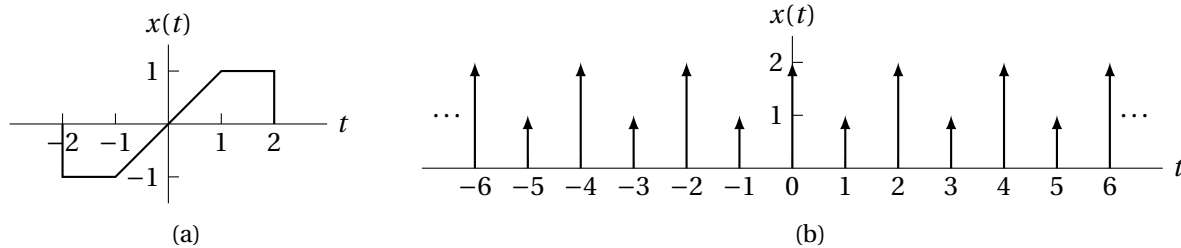


Figure 1: Figure for Q6

(a)  $[e^{-\alpha t} \cos \omega_0 t] u(t), \quad \alpha > 0$

(b)  $e^{-3|t|} \sin 2t \omega_0 t$

(c)  $x(t) = \begin{cases} 1 + \cos \pi t, & |t| < 1 \\ 0, & |t| > 1 \end{cases}$

(d)  $\sum_{k=0}^{\infty} \alpha^k \delta(t - kT), \quad |\alpha| < 1$

(e)  $[te^{-2t} \sin 4t] u(t)$

(f)  $\left[ \frac{\sin \pi t}{\pi t} \right] \left[ \frac{\sin 2\pi(t-1)}{\pi(t-1)} \right]$

(g)  $x(t)$  as shown in Figure 1a.

(h)  $x(t)$  as shown in Figure 1b.

7. Determine the continuous-time signal corresponding to each of the following transforms:

(a)  $X(j\omega) = \frac{2 \sin[3(\omega - 2\pi)]}{\omega - 2\pi}$

(b)  $X(j\omega) = \cos(4\omega + \pi/3)$

(c)  $X(j\omega)$  as given in the magnitude and phase plots of Figure 2a

(d)  $X(j\omega) = 2[\delta(\omega - 1) - \delta(\omega + 1)] + 3[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)]$

(e)  $X(j\omega)$  as in Figure 2b

8. Given that  $x(t)$  has the Fourier transform  $X(j\omega)$ , express the Fourier transforms of the signals listed below in terms of  $X(j\omega)$ . You may find useful the Fourier transform properties listed in the table in the book.

(a)  $x_1(t) = x(1 - t) + x(-1 - t)$

(b)  $x_2(t) = x(3t - 6)$

(c)  $x_3(t) = \frac{d^2}{dt^2} x(t - 1)$

9. For each of the following Fourier transforms, use Fourier transform properties to determine whether the corresponding time-domain signal is (i) real, imaginary, or neither and (ii) even, odd, or neither. Do this without evaluating the given transforms.

(a)  $X_1(j\omega) = u(\omega) - u(\omega - 2)$

(b)  $X_2(j\omega) = \cos(2\omega) \sin(\omega/2)$

(c)  $X_3(j\omega) = A(\omega) e^{jB(\omega)}$  where  $A(\omega) = (\sin 2\omega)/\omega$  and  $B(\omega) = 2\omega + \pi/2$

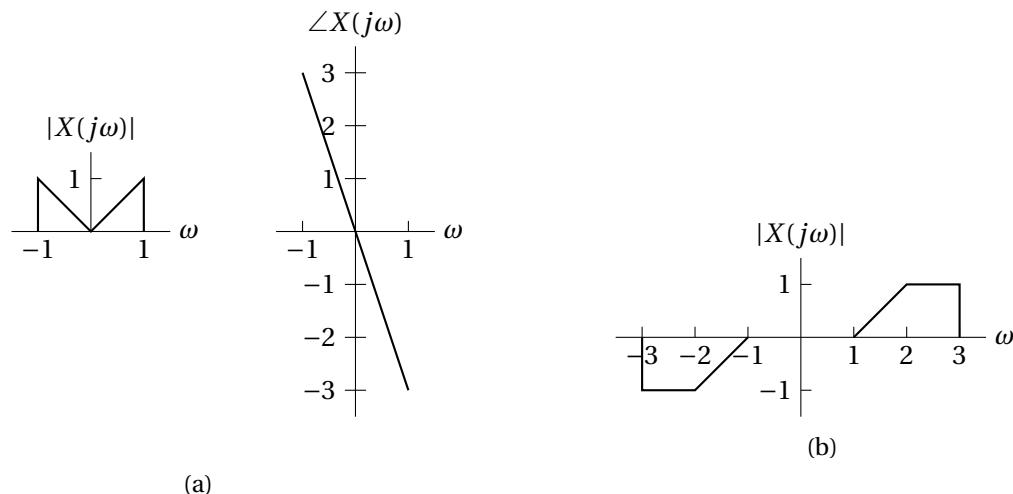


Figure 2: Figure for Q7

(d)  $X_4(j\omega) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{|k|} \delta\left(\omega - \frac{k\pi}{4}\right)$

10. Consider the signal

$$x(t) = \begin{cases} 0, & t < \frac{1}{2}, \\ t + \frac{1}{2}, & -\frac{1}{2} \leq t \leq \frac{1}{2}, \\ 1, & t > \frac{1}{2}. \end{cases}$$

(a) Use differentiation and integration properties and the Fourier transform pair for the rectangular pulse to find a closed-form expression for  $X(j\omega)$ .

(b) What is the Fourier transform of  $g(t) = x(t) - \frac{1}{2}$ ?

11. Consider the signal

$$x(t) = \begin{cases} 0, & |t| > 1, \\ (t+1)/2, & -1 \leq t \leq 1. \end{cases}$$

(a) With the help of tables, determine the closed-form expression for  $X(j\omega)$ .

(b) Take the real part of your answer above, and verify that it is the Fourier transform of the even part of  $x(t)$ .

(c) What is the Fourier transform of the odd part of  $x(t)$ ?

12. (a) Use tables to help determine the Fourier transform of the following signal:

$$x(t) = t \left( \frac{\sin t}{\pi t} \right)^2.$$

(b) Use Parseval's relation and the result of the previous part to determine the numerical value of

$$A = \int_{-\infty}^{\infty} t^2 \left( \frac{\sin t}{\pi t} \right)^4.$$

13. Given the relationship

$$y(t) = x(t) * h(t),$$

and

$$g(t) = x(3t) * h(3t),$$

and given that  $x(t)$  has Fourier transform  $X(j\omega)$  and  $h(t)$  has Fourier transform  $H(j\omega)$ , use Fourier transform properties to show that  $g(t)$  has the form

$$g(t) = Ay(Bt).$$

Determine the values of  $A$  and  $B$ .

14. Consider the Fourier transform pair

$$e^{-|t|} \xleftrightarrow{\mathcal{F}} \frac{2}{1+\omega^2}.$$

- (a) Use the appropriate Fourier transform properties to find the Fourier transform of  $te^{-|t|}$ .
- (b) Use the result from part (a), along with the duality property, to determine the Fourier transform of

$$\frac{4t}{(1+t^2)^2}.$$

*Hint:* See 15.

15. Let  $x(t)$  be a signal whose Fourier transform is

$$X(j\omega) = \delta(\omega) + \delta(\omega - \pi) + \delta(\omega - 5),$$

and let

$$h(t) = u(t) - u(t - 2).$$

- (a) Is  $x(t)$  periodic?
  - (b) Is  $x(t) * h(t)$  periodic?
  - (c) Can the convolution of two aperiodic signals be periodic?
16. Consider a signal  $x(t)$  with Fourier transform  $X(j\omega)$ . Suppose we are given the following facts:
- (a)  $x(t)$  is real and non-negative.
  - (b)  $\mathcal{F}^{-1}(1 + j\omega)X(j\omega) = Ae^{-2t}u(t)$ , where  $A$  is independent of  $t$ .
  - (c)  $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi$ .

Determine a closed-form expression for  $x(t)$ .