Department of Electronic and Telecommunication Engineering
University of Moratuwa
Sri Lanka

EN1020 CIRCUITS, SIGNALS, AND SYSTEMS: TUTORIAL 01*

February 1, 2026

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1. A continuous time signal is given in Fig. 1. Sketch and label the following signals.

   (a) $x(t-2)$ (b) $x(t+1)$ (c) $x(-t+1)$ (d) $x(3t/2)$ (e) $x(t/3)$

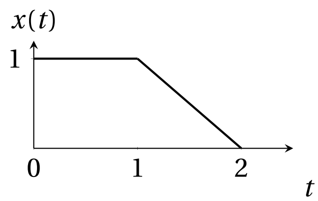

Figure 1:

2. In order to determine the effect of transforming the independent variable of a given signal $x(t)$ to obtain a signal of the form $x(\alpha t+\beta)$, where α and β are given constants, the systematic approach is to first delay or advance $x(t)$ in accordance with the value of β, and then to perform time scaling and/or time reversal on the resulting signal in accordance with the value of α. The delayed or advanced signal is linearly stretched if $|\alpha|<1$, linearly compressed if $|\alpha|>1$, and reversed in time if $\alpha<0$. Determine $x(\frac{3}{2}t+1)$ for $x(t)$ given in Fig. 1.

3. A discrete time signal is shown in Figure 2. Sketch and label each of the following signals.

   (a) $x[n+1]$ (b) $x[2n]$ (c) $x[-n]$ (d) $x[-n+2]$ (e) $x[-2n+1]$

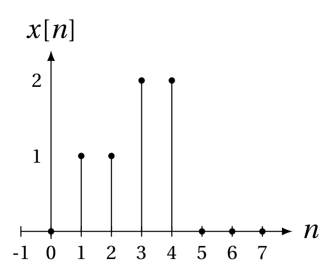

Figure 2:

4. Plot the magnitude of the signal

$$x(t)=e^{j2t}+e^{j3t}$$

   by expressing it as a single sinusoid.

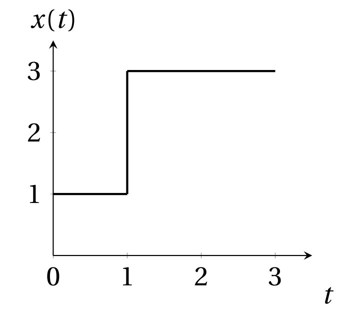

Figure 3:

5. Find the even and odd parts of the $x(t)$ signal given in Figure 3.

6. Using the discrete time signals $x_1[n]$ and $x_2[n]$ shown in Fig. 4, represent each of the following signals by a graph.

   (a) $y[n] = x_1[n] + x_2[n]$ (b) $y[n] = 2x_1[n]$ (c) $y[n] = x_1[n]x_2[n]$

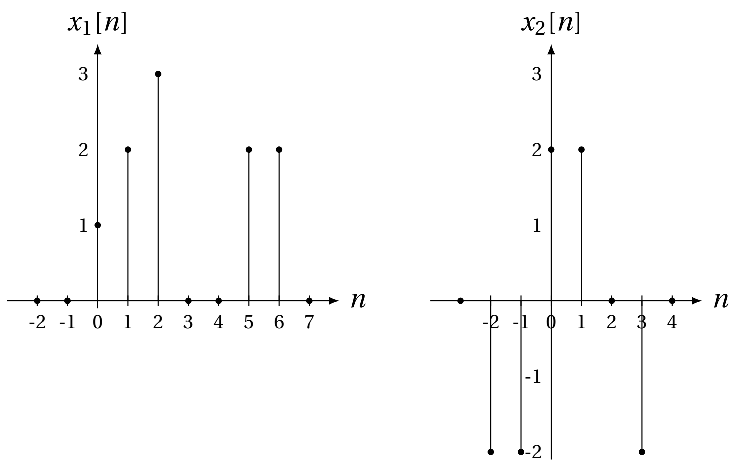

Figure 4:

7. Show that

$$\int_{-a}^{a} x(t)\,dt = \begin{cases} 2\int_0^a x(t)\,dt, & \text{if } x(t) \text{ is even,} \\ 0, & \text{if } x(t) \text{ is odd.} \end{cases}$$

8. Show that the complex exponential signal $x(t) = e^{j\omega t}$ is periodic and that its fundamental period is $2\pi/\omega$.

9. Show that the complex exponential signal $x[n] = e^{j\omega n}$ is periodic only if $\omega/2\pi$ is a rational number.

10. Consider the sinusoidal signal $x(t) = \cos(15t)$.

    (a) Find the value of sampling interval T such than $x[n]$ is a periodic sequence.

    (b) Find the fundamental period of $x[n]$ if $T = 0.1\pi$ seconds.

11. Determine whether or not each of the following signals are periodic. If periodic, find the fundamental period.

*Questions are from Oppenheim.

(a) $x(t) = 2e^{j(t+\pi/4)}$

(b) $x[n] = e^{j(\pi/4)n}$

(c) $x(t) = \cos(t + \pi/4)$

(d) $x(t) = \cos(t) + \sin(\sqrt{2}t)$

(e) $x[n] = \cos^2(\pi n/8)$

12. Determine whether the following signals are energy signals, power signals, or neither.

    (a) $x(t) = e^{-at}u(t), a > 0$ (b) $x(t) = A\cos(\omega t + \theta)$ (c) $x[n] = 3u[n]$ (d) $x[n] = 3e^{j3n}$

13. Determine the fundamental period of

$$x[n] = e^{j(2\pi/3)n} + e^{j(3\pi/4)n}.$$

14. In regard to general properties of systems, a system may or may not be:

    (a) Memoryless
    (b) Time invariant
    (c) Linear
    (d) Causal
    (e) Stable

    For each of the following continuous-time systems, determine which of these properties hold and which do not. Justify your answers in each case. In each example, $y(t)$ denotes the system output and $x(t)$ denotes the system input.

    (a) $y(t) = x(t-2) + x(2-t)$

    (b) $y(t) = [\cos(3t)]\, x(t)$

    (c)
$$y(t) = \int_0^{2t} x(\tau)\, d\tau$$

    (d)
$$y(t) = \begin{cases} 0, & t < 0 \\ x(t) + x(t-2), & t \geq 0 \end{cases}$$

    (e)
$$y(t) = \begin{cases} 0, & x(t) < 0 \\ x(t) + x(t-2), & x(t) \geq 0 \end{cases}$$

    (f) $y(t) = x(t/3)$

    (g) $y(t) = \dfrac{d}{dt}x(t)$

15. Determine which of the properties listed in Problem 14 hold and which do not hold for each of the following discrete-time systems. Justify your answers. In each example, $y[n]$ denotes the system output and $x[n]$ is the system input.

    (a) $y[n] = x[-n]$

    (b) $y[n] = x[n-2] - 2x[n-8]$

    (c) $y[n] = n\, x[n]$

    (d) $y[n] = \mathfrak{Ev}\{x[n-1]\}$

    (e)
$$y[n] = \begin{cases} x[n], & n \geq 1 \\ 0, & n = 0 \\ x[n+1], & n \leq -1 \end{cases}$$

    (f)
$$sy[n] = \begin{cases} x[n], & n \geq 1 \\ 0, & n = 0 \\ x[n], & n \leq -1 \end{cases}$$

    (g) $y[n] = x[4n+1]$

16. Determine if each of the following systems is invertible. If it is, construct the inverse system. If it is not, find two input signals to the system that have the same output.

(a) $y(t) = x(t-4)$

(b) $y(t) = \cos[x(t)]$

(c) $y[n] = n\,x[n]$

(d)
$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$

(e)
$$y[n] = \begin{cases} x[n-1], & n \ge 1 \\ 0, & n = 0 \\ x[n], & n \le -1 \end{cases}$$

(f) $y[n] = x[n]\,x[n-1]$

(g) $y[n] = x[1-n]$

(h)
$$y(t) = \int_{-\infty}^{t} e^{-(t-\tau)} x(\tau)\,d\tau$$

(i)
$$y[n] = \sum_{k=-\infty}^{n} \left(\tfrac{1}{2}\right)^{n-k} x[k]$$

(j)
$$y(t) = \frac{d}{dt} x(t)$$

(k)
$$y[n] = \begin{cases} x[n+1], & n \ge 0 \\ x[n], & n \le -1 \end{cases}$$

(l) $y(t) = x(2t)$

(m) $y[n] = x[2n]$

(n)
$$y[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

17. Determine the values of P_∞ and E_∞ for each of the following signals:

(a) $x_1(t) = e^{-2t} u(t)$

(b) $x_2(t) = e^{j(2t+\pi/4)}$

(c) $x_3(t) = \cos(t)$

(d) $x_1[n] = \left(\tfrac{1}{2}\right)^n u[n]$

(e) $x_2[n] = e^{j(\pi n/2+\pi/8)}$

(f) $x_3[n] = \cos\left(\tfrac{\pi}{4} n\right)$

18. Let $x(t)$ be the continuous-time complex exponential signal

$$x(t) = e^{j\omega_0 t}$$

with fundamental frequency ω_0 and fundamental period $T_0 = 2\pi/\omega_0$. Consider the discrete-time signal obtained by taking equally spaced samples of $x(t)$—that is,

$$x[n] = x(nT) = e^{j\omega_0 nT}.$$

(a) Show that $x[n]$ is periodic if and only if T/T_0 is a rational number—that is, if and only if some multiple of the sampling interval *exactly equals* a multiple of the period of $x(t)$.

(b) Suppose that $x[n]$ is periodic—that is, that

$$\frac{T}{T_0} = \frac{p}{q}, \tag{1}$$

where p and q are integers. What are the fundamental period and fundamental frequency of $x[n]$? Express the fundamental frequency as a fraction of $\omega_0 T$.

(c) Again assuming that T/T_0 satisfies Eq. 1, determine precisely how many periods of $x(t)$ are needed to obtain the samples that form a single period of $x[n]$.