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EN1060 SIGNALS AND SYSTEMS: TUTORIAL 03 *

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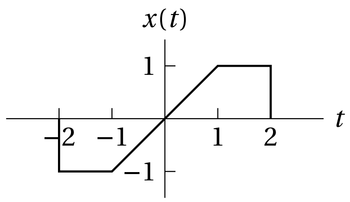
1. Use the Fourier transform analysis equation to calculate the Fourier transforms of
   (a) $e^{-2(t-1)}u(t-1)$
   (b) $e^{-2|t-1|}$
2. Use the Fourier transform analysis equation to calculate the Fourier transforms of
   (a) $\delta(t+1)+\delta(t-1)$
   (b) $\frac{d}{dt}[u(-2-t)+u(t-2)]$

   Sketch and label the magnitude of each Fourier transform.
3. Determine the Fourier transform of each of the following periodic signals:
   (a) $\sin(2\pi t+\pi/4)$
   (b) $1+\cos(6\pi t+\pi/8)$
4. Use the Fourier transform synthesis equation to determine the inverse Fourier transform of
   (a) $X_1(j\omega)=2\pi\delta(\omega)+\pi\delta(\omega-4\pi)+\pi\delta(\omega+4\pi)$
   (b) $X_2(j\omega)=\begin{cases}2, & 0\le\omega\le 2,\\ -2, & -2\le\omega<0,\\ 0, & |\omega|>2.\end{cases}$
5. Use the Fourier transform synthesis equation to determine the inverse Fourier transform of $X(j\omega)=|X(j\omega)|e^{j\angle X(j\omega)}$ where

$$|X(j\omega)|=2[u(\omega+3)-u(\omega-3)]$$
$$\angle X(j\omega)=-\frac{3}{2}\omega+\pi$$

6. Compute the Fourier transform of each of the following signals:

*All the questions are from Oppenheim *et al.* chapter 4.



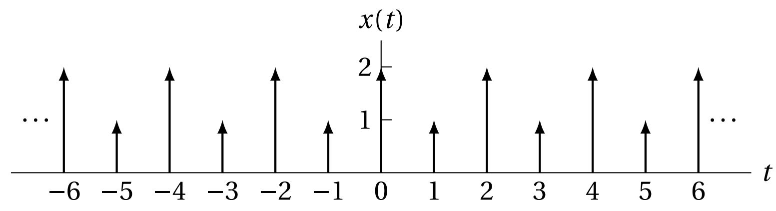


Figure 1: Figure for Q6

(a) $[e^{-\alpha t}\cos\omega_0 t]u(t), \quad \alpha > 0$

(b) $e^{-3|t|}\sin 2t\omega_0 t$

(c) $x(t) = \begin{cases} 1+\cos\pi t, & |t| < 1 \\ 0, & |t| > 1 \end{cases}$

(d) $\sum_{k=0}^{\infty} \alpha^k \delta(t-kT), \quad |\alpha| < 1$

(e) $[te^{-2t}\sin 4t]u(t)$

(f) $\left[\frac{\sin\pi t}{\pi t}\right]\left[\frac{\sin 2\pi(t-1)}{\pi(t-1)}\right]$

(g) $x(t)$ as shown in Figure 1a.

(h) $x(t)$ as shown in Figure 1b.

7. Determine the continuous-time signal corresponding to each of the following transfroms:

(a) $X(j\omega) = \dfrac{2\sin[3(\omega-2\pi)]}{\omega-2\pi}$

(b) $X(j\omega) = \cos(4\omega+\pi/3)$

(c) $X(j\omega)$ as given in the magnitude and phase plots of Figure 2a

(d) $X(j\omega) = 2[\delta(\omega-1)-\delta(\omega+1)]+3[\delta(\omega-2\pi)+\delta(\omega+2\pi)]$

(e) $X(j\omega)$ as in Figure 2b

8. Given that $x(t)$ has the Fourier transform $X(j\omega)$, express the Fourier transforms of the signals listed below in terms of $X(j\omega)$. You may find useful the Fourier transform properties listed in the table in the book.

(a) $x_1(t) = x(1-t)+x(-1-t)$

(b) $x_2(t) = x(3t-6)$

(c) $x_3(t) = \frac{d^2}{dt^2}x(t-1)$

9. For each of the following Fourier transforms, use Fourier transform properties to determine whether the corresponding time-domain signal is (i) real, imaginary, or neither and (ii) even, odd, or neither. Do this without evaluating the given transforms.

(a) $X_1(j\omega) = u(\omega)-u(\omega-2)$

(b) $X_2(j\omega) = \cos(2\omega)\sin(\omega/2)$

(c) $X_3(j\omega) = A(\omega)e^{jB(\omega)}$ where $A(\omega) = (\sin 2\omega)/\omega$ and $B(\omega) = 2\omega+\pi/2$

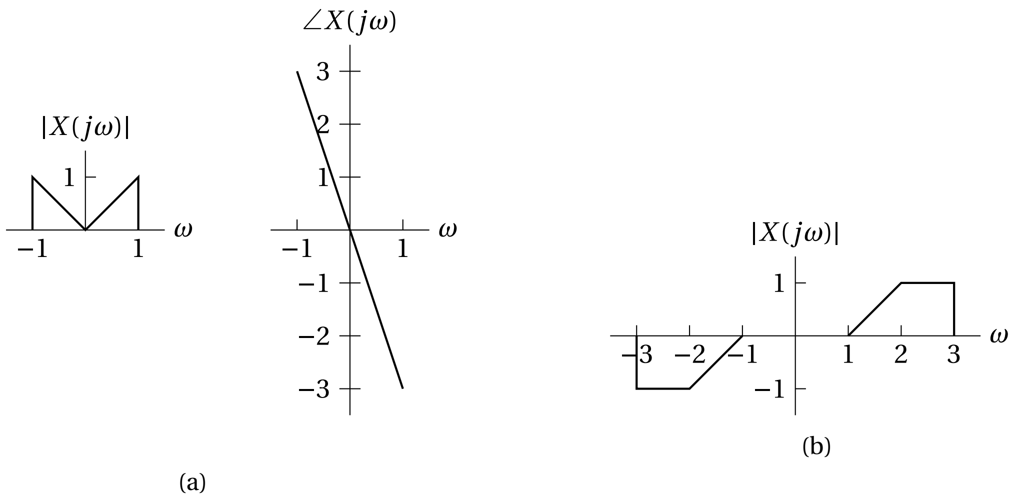


Figure 2: Figure for Q7

(d) $X_4(j\omega) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{|k|} \delta\left(\omega - \frac{k\pi}{4}\right)$

10. Consider the signal

$$x(t) = \begin{cases} 0, & t < \frac{1}{2}, \\ t + \frac{1}{2}, & -\frac{1}{2} \le t \le \frac{1}{2}, \\ 1, & t > \frac{1}{2}. \end{cases}$$

(a) Use differentiation and integration properties and the Fourier transform pair for the rectangular pulse to find a closed-form expression for $X(j\omega)$.

(b) What is the Fourier transform of $g(t) = x(t) - \frac{1}{2}$?

11. Consider the signal

$$x(t) = \begin{cases} 0, & |t| > 1, \\ (t+1)/2, & -1 \le t \le 1. \end{cases}$$

(a) With the help of tables, determine the closed-form expression for $X(j\omega)$.

(b) Take the real part of your answer above, and verify that it is the Fourier transform of the even part of $x(t)$.

(c) What is the Fourier transform of the odd part of $x(t)$?

12. (a) Use tables to help determine the Fourier transform of the following signal:

$$x(t) = t\left(\frac{\sin t}{\pi t}\right)^2.$$

(b) Use Pasrseval's relation and the result of the previous part to determine the numerical value of

$$A = \int_{-\infty}^{\infty} t^2 \left(\frac{\sin t}{\pi t}\right)^4.$$

13. Given the relationship

$$y(t) = x(t) * h(t),$$

and

$$g(t) = x(3t) * h(3t),$$

and given that $x(t)$ has Fourier transform $X(j\omega)$ and $h(t)$ has Fourier transform $H(j\omega)$, use Fourier transform properties to show that $g(t)$ has the form

$$g(t) = Ay(Bt).$$

Determine the values of A and B.

14. Consider the Fourier transform pair

$$e^{-|t|} \overset{\mathscr{F}}{\longleftrightarrow} \frac{2}{1+\omega^2}.$$

   (a) Use the appropriate Fourier transform properties to find the Fourier transform of $te^{-|t|}$.

   (b) Use the result from part (a), along with the duality property, to determine the Fourier transform of

$$\frac{4t}{(1+t^2)^2}.$$

Hint: See 15.

15. .Let $x(t)$ be a signal whose Fourier transform is

$$X(j\omega) = \delta(\omega) + \delta(\omega - \pi) + \delta(\omega - 5),$$

and let

$$h(t) = u(t) - u(t-2).$$

   (a) Is $x(t)$ periodic?

   (b) Is $x(t) * h(t)$ periodic?

   (c) Can the convolution of two aperiodic signals be periodic?

16. Consider a signal $x(t)$ with Fourier transform $X(j\omega)$. Suppose we are given the following facts:

   (a) $x(t)$ is real and non-negative.

   (b) $\mathscr{F}^{-1}(1+j\omega)X(j\omega) = Ae^{-2t}u(t)$, where A is independent of t.

   (c) $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2\pi$.

Determine a closed-form expression for $x(t)$.

17. Let $x(t)$ be a signal with Fourier transform $X(j\omega)$. Suppose we are given the following facts:

   (a) $x(t)$ is real.

   (b) $x(t) = 0$ for $t \leq 0$.

   (c) $\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathfrak{Re}\{X(j\omega)\} e^{j\omega t} d\omega = |t|e^{-|t|}$.

Determine a closed-form expression for $x(t)$.

18. Consider the signal

$$x(t) = \sum_{-\infty}^{\infty} \frac{\sin\left(k\frac{\pi}{4}\right)}{k\frac{\pi}{4}} \delta\left(t - k\frac{\pi}{4}\right).$$

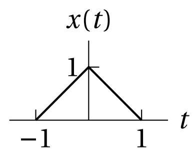


Figure 3: Figure for Q19

(a) Determine $g(t)$ such that

$$x(t) = \left(\frac{\sin t}{\pi t}\right) g(t).$$

(b) Use the multiplication property of the Fourier transform to argue that $X(j\omega)$ is periodic. Specify $X(j\omega)$ over one period.

19. Consider the signal $x(t)$ in Figure 3.

(a) Find the Fourier transform $X(j\omega)$ of $x(t)$.

(b) Sketch the signal

$$\tilde{x}(t) = x(t) * \sum_{k=-\infty}^{\infty} \delta(t - 4k).$$

(c) Find another signal $g(t)$ such that $g(t)$ is not the same as $x(t)$ and

$$\tilde{x}(t) = g(t) * \sum_{k=-\infty}^{\infty} \delta(t - 4k).$$

(d) Argue that, although $G(j\omega)$ is different from $X(j\omega)$, $G(j\frac{\pi k}{2}) = X(j\frac{\pi k}{2})$ for all integers k. You should explicitly evaluate $G(j\omega)$ to answer this question.

20. Let $x(t)$ be any signal with Fourier transform $X(j\omega)$. The frequency-shift property of the ft may be stated as

$$e^{j\omega_0 t} \overset{\mathscr{F}}{\longleftrightarrow} X(j(\omega - \omega_0)).$$

(a) Prove the frequency-shift property by applying the frequency shift to the analysis equation

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt.$$

(b) Prove the frequency-shift property by utilizing the Fourier transform of $e^{j\omega_0 t}$ in conjunction with the multiplication property of the Fourier transform.