

Department of Electronic and Telecommunication Engineering  
University of Moratuwa  
Sri Lanka  
**EN1060 SIGNALS AND SYSTEMS: TUTORIAL 02 \***

February 24, 2023

- 
1. A continuous-time periodic signal  $x(t)$  is real valued and has a fundamental period  $T = 8$ . The non-zero Fourier series coefficients for  $x(t)$  are specified as

$$a_1 = a_{-1}^* = j, \quad a_5 = a_{-5}^* = 2.$$

Express  $x(t)$  in the form

$$x(t) = \sum_{k=0}^{\infty} \cos(\omega_k t + \phi_k).$$

2. Determine the Fourier series representation of the following signals:

- (a) Each  $x(t)$  illustrated in Figure ??.
- (b)  $x(t)$  periodic with period 2 and

$$x(t) = e^{-t} \quad \text{for} \quad -1 < t < 1.$$

- (c)  $x(t)$  periodic with period 4 and

$$x(t) = \begin{cases} \sin \pi t, & 0 \leq t \leq 2, \\ 0, & 2 < t \leq 4. \end{cases}$$

3. In each of the following, we specify the Fourier series coefficients of a continuous-time signal that is periodic with period 4. Determine the signal  $x(t)$  in each case.

(a)  $a_k = \begin{cases} 0, & k = 0, \\ (j)^k \frac{\sin k\pi/4}{k\pi}, & \text{otherwise.} \end{cases}$

(b)  $a_k = (-1)^k \frac{\sin k\pi/8}{2k\pi}$

(c)  $a_k = \begin{cases} jk, & |k| < 3, \\ 0, & \text{otherwise.} \end{cases}$

(d)  $a_k = \begin{cases} 1, & k \text{ even,} \\ 2, & k \text{ odd.} \end{cases}$

---

\*All the questions are from Oppenheim *et al.* chapter 3.

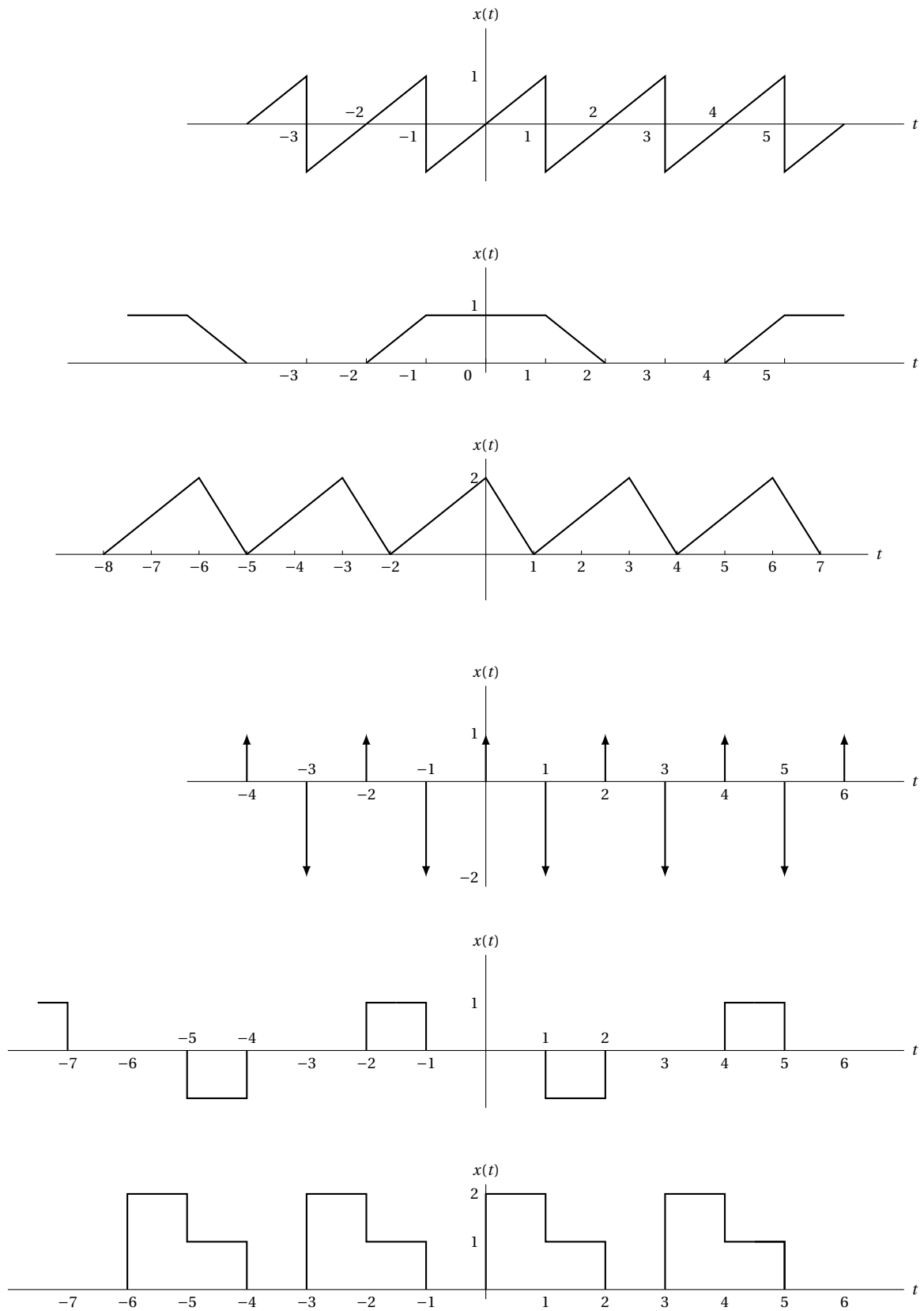


Figure 1: Figure Q02

4. Let

$$x(t) = \begin{cases} t, & 0 \leq t \leq 1, \\ 2 - t, & 1 \leq t \leq 2, \end{cases}$$

be a periodic signal with fundamental period  $T = 2$  and Fourier coefficients  $a_k$ .

- Determine the value of  $a_0$ .
- Determine the Fourier series representation of  $dx(t)/dt$ .
- Use this result and the differentiation property of the continuous-time Fourier series to help determine the Fourier series coefficients of  $x(t)$ .

5. Consider the following three continuous-time signals with a fundamental period of  $T = 1/2$ :

$$\begin{aligned} x(t) &= \cos(4\pi t), \\ y(t) &= \sin(4\pi t), \\ z(t) &= x(t)y(t). \end{aligned}$$

- Determine the Fourier series coefficients of  $x(t)$ .
- Determine the Fourier series coefficients of  $y(t)$ .
- Use these results along with the multiplication property of the continuous-time Fourier series to determine the Fourier series coefficients of  $z(t)$ .
- Determine the Fourier series coefficients of  $z(t)$  through direct expansion of  $z(t)$  in trigonometric form, and compare the result with that of part ??.

6. Let  $x(t)$  be a periodic signal whose Fourier series coefficients are

$$a_k = \begin{cases} 2, & k = 0, \\ j\left(\frac{1}{2}\right)^{|k|}, & \text{otherwise.} \end{cases}$$

Use Fourier series properties to answer the Following questions:

- Is  $x(t)$  real?
- Is  $x(t)$  even?
- Is  $dx(t)/dt$  even?

7. Let  $x(t)$  be a periodic signal with fundamental period  $T$  and Fourier series coefficients  $a_k$ . Derive the Fourier series coefficients of each of the following signals in terms of  $a_k$ :

- $x(t - t_0) + x(t + t_0)$
- $\Re\{x(t)\}$
- $\Im\{x(t)\}$
- $\frac{d^2 x(t)}{dt^2}$
- $x(3t - 1)$  [for this part, first determine the period of  $x(3t - 1)$ ]

8. Suppose we are given the following information about a continuous-time periodic signal with period 3 and Fourier coefficients  $a_k$ .

- (a)  $a_k = a_{k+2}$ .
- (b)  $a_k = a_{-k}$ .
- (c)  $\int_{-0.5}^{0.5} x(t) dt = 1$ .
- (d)  $\int_{0.5}^{1.5} x(t) dt = 2$ .

Determine  $x(t)$ .

9. Let  $x(t)$  be a real-valued signal with fundamental period  $T$  and Fourier series coefficients  $a_k$ .
  - (a) Show that  $a_k = a_{-k}^*$  and  $a_0$  must be real.
  - (b) Show that if  $x(t)$  is even, then its Fourier series coefficients must be real and even.
  - (c) Show that if  $x(t)$  is odd, then its Fourier series coefficients are imaginary and odd and  $a_0 = 0$ .
  - (d) Show that the Fourier series coefficients of the even part of  $x(t)$  are equal to  $\Re\{a_k\}$ .
  - (e) Show that the Fourier series coefficients of the odd part of  $x(t)$  are equal to  $j\Im\{a_k\}$ .
10. Let  $x(t)$  be a real periodic signal with Fourier series representation given in the sine-cosine form

$$x(t) = a_0 + 2 \sum_{k=1}^{\infty} [B_k \cos k\omega_0 t - C_k \sin k\omega_0 t]. \quad (1)$$

- (a) Find the exponential Fourier series representation of the even and odd parts of  $x(t)$ , that is, find the coefficients  $\alpha_k$  and  $\beta_k$  in terms of the coefficients in eq. ?? so that

$$\begin{aligned} \mathfrak{Ev}\{x(t)\} &= \sum_{k=-\infty}^{\infty} \alpha_k e^{jk\omega_0 t}, \\ \mathfrak{Ov}\{x(t)\} &= \sum_{k=-\infty}^{\infty} \beta_k e^{jk\omega_0 t}. \end{aligned}$$

- (b) What is the relationship between  $\alpha_k$  and  $\alpha_{-k}$ ? What is the relationship between  $\beta_k$  and  $\beta_{-k}$ ?