A GRASP based approach for the Bin Packing Problem with General Precedence Constraints

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1. Introduction

In modern software development environments, organizations face increasingly complex challenges in managing the delivery of software features across multiple development cycles and deployment windows.

Software teams must regularly decide how to group and schedule features for delivery, while respecting complex dependencies between components, modules, and functionalities. This real-world scenario mirrors a classic combinatorial optimization challenge, where limited resources (delivery windows, development capacity, testing environments) must be allocated efficiently while maintaining the correct execution order of interdependent tasks.

The complexity intensifies when considering that resources often have sizes, priorities, and precedence relationships that cannot be violated during the delivery process.

The Bin Packing Problem with General Precedence Constraints (BPP-GP) can be formally defined as follows: given a set of items $N = \{1, 2, ..., n\}$ with associated weights w_j, a set of identical bins with capacity C, subject to a set of precedence constraints between item pairs that define their partial order relationships, the objective is to pack all items into the minimum number of bins, ensuring that no bin exceeds its capacity and that all precedence relationships are respected within a bin and across bins.

This optimization problem extends the classical bin packing problem by incorporating ordering dependencies that are fundamental in software development contexts.

The Bin Packing Problem with General Precedence Constraints belongs to the class of NP-Hard optimization problems. This classification arises from the fact that even the basic bin packing problem without precedence constraints is NP-Hard, and the addition of these constraints only allows for the description of more expressive problems.

The decision version of this problem, which asks whether there exists a feasible packing using at most k bins while respecting all constraints, is NP-Complete, making it unlikely that polynomial-time exact algorithms exist to solve large instances of this problem.

Several related problems share structural similarities with BPP-GP, including the classical Bin Packing Problem (without precedence constraints), the Job Shop Scheduling Problem with precedence constraints, the Resource-Constrained Project Scheduling Problem (RCPSP), and the Knapsack Problem with Precedence Constraints.

Furthermore, variants such as the Bin Packing Problem with Conflicts, where certain items cannot be placed in the same bin, and the Multidimensional Bin Packing Problem provide insights into techniques for handling constraints.

The Strip Packing Problem with precedence constraints also offers relevant algorithmic approaches, particularly in the context of scheduling and resource allocation.

BPP-GP presents several unique features that distinguish it from standard combinatorial optimization problems. Precedence constraints create a partial ordering that can result in complex dependency graphs, potentially including parallel branches and multiple hierarchy levels. In software delivery contexts, these constraints may represent technical dependencies, business priorities, relationships between user stories, or testing requirements. The problem allows for intra-bin precedence (dependencies within the same delivery window) and inter-bin precedence (dependencies spanning multiple delivery cycles), significantly complicating the solution space and requiring sophisticated constraint propagation mechanisms.

This problem deserves investigation due to its direct applicability to critical challenges in modern software engineering and project management. As software systems become increasingly complex and development teams adopt agile methodologies with frequent delivery cycles, the efficient allocation of resources to delivery windows while respecting dependencies becomes fundamental to project success. Inadequate scheduling of resources can lead to development bottlenecks, release delays, inefficient resource utilization, and increased technical debt.

Furthermore, the investigation of a meta-heuristic based approach for such problem can advance the broader field of combinatorial optimization with constraints and provide insights applicable to other domains, such as supply chain management, manufacturing scheduling, and resource allocation problems.

This research builds upon the baseline methodology proposed by [Kramer et al. 2017] for the Bin Packing Problem with General Precedence Constraints (BPP-GP). The original approach provides a structured framework for addressing precedence-aware packing through iterative local search techniques. In this work, we take this baseline as a starting point and extend it by implementing a GRASP-based metaheuristic, aiming to enhance the efficiency of precedence propagation and packing strategies. The objective remains to minimize the number of bins while ensuring feasibility with respect to both intra-bin and inter-bin dependencies, leveraging the strengths of the baseline while exploring improvements through the GRASP methodology.

The justification for this work lies both in its practical relevance and in the theoretical contribution it may offer. In practice, the optimization of software resource delivery under precedence constraints addresses a real need faced by development teams, where the complexity of manual scheduling often leads to inefficiencies and delays. From a theoretical perspective, the formulation and analysis of heuristics for BPP-GP expand the body of knowledge in combinatorial optimization under complex constraints, providing new perspectives that may be applied in domains beyond software engineering, such as logistics, manufacturing, and project management. Thus, the research is justified by its ability to generate direct impact on critical engineering processes while simultaneously strengthening the academic understanding of NP-Hard problems with additional constraints.

The development of this work will follow a structured roadmap in progressive steps, as follows:

- **Literature review:** Investigation of the BPP-GP problem and related combinatorial optimization problems, with emphasis on metaheuristic strategies previously applied in similar contexts, including the baseline proposed by [Kramer et al. 2017];
- **Problem definition:** Formal and computational description of the BPP-GP, highlighting its constraints, objectives, and relevance to resource allocation and scheduling problems;
- **Development of the metaheuristic algorithm:** Design and implementation of a new metaheuristic approach to replace or extend the baseline method proposed in [Kramer et al. 2017], aiming to improve solution quality and robustness. Particular attention will be given to mechanisms that effectively manage precedence constraints;
- **Definition of the experimental methodology:** Establishment of the experimental protocol, including test instance selection or generation, parameter tuning strategies, performance metrics, and reference algorithms for comparison;
- **Execution of computational experiments:** Implementation, execution, and evaluation of the proposed metaheuristic on benchmark instances. The results will be analyzed in terms of computational efficiency, convergence behavior, and quality of the obtained solutions, with comparisons to baseline and state-of-the-art methods;
- **Conclusions and future research directions:** Summary of the main findings and assessment of the proposed metaheuristic's effectiveness in solving the BPP-GP. Limitations will be discussed, and perspectives for future work will be outlined, such as algorithmic refinements, hybridizations with other techniques, or applications to extended problem variants.

2. Related work

The Bin Packing Problem with Precedence Constraints (BPP-P) can be can be seen as an extension to the well known Bin Packing Problem in order to incorporate precedence constraints. A BPP-P instance is described by a set of items $N = 1, 2, \ldots, n$, with non-negative weights and a set of bins $M = 1, 2, \ldots, m$, each with capacity C. In addition to that, in the set of items, there is also a **precedence** relation. The goal is to pack all items in the bins respecting the capacity and the precedence: items with less precedence must be packed into lower numbered bins.

A solution to this problem was introduced in the work of [Dell'Amico et al. 2012]. They consider different, independent approaches: A branch-and-bound procedure, a greedy constructive heuristic (first fit), together with a local search procedure, and also a *Variable Neighborhood Search*.

Focusing on the classic problem in the same year, the work by [Layeb and Chenche 2012] addresses the one-dimensional Bin Packing Problem (1-BPP), the simplest version of the problem. The authors propose a new approach based on the GRASP (Greedy Randomized Adaptive Search Procedure) meta-heuristic. The methodology is implemented in two phases: the first is a construction phase that uses a new randomized greedy heuristic, notable for being a hybrid between the First Fit (FF) and Best Fit (BF) heuristics; the second phase consists of a local search based on the Tabu Search (TS) algorithm, used to refine and improve the solution found in the first phase. The experimental results, conducted on benchmark instances (easy, medium, and hard), demonstrate that the proposed GRASP approach significantly outperforms the simple greedy heuristic and achieving results very close to the best-known solutions.

The work by [Pereira 2016] proposes a heuristic based on dynamic programming and an exact branch-and-bound method, incorporating several new lower bounds and dominance rules to improve performance. The computational results show the notable effectiveness of the proposed methods, which were able to find the optimal solution for all previously unsolved ("open") instances from the classic benchmark set, significantly outperforming prior approaches in terms of solution quality and speed. The study concludes that the developed procedures represent the new state-of-the-art, and it identifies that instances with large item weights and a low number of precedence relations (low order strength) are the most challenging for the algorithm.

Later, the work of [Kramer et al. 2017] proposes an extension of this problem, the Bin Packing Problem with general precedence constraints, which is the topic of interest of our research project. This problem further generalizes the BPP-P by introducing weighted precedences: Each precedence relation between items a and b is described by a tuple (a, b) with an associated integer t, the goal is to have $i_a - i_b \geq t$, where i_a and i_b are the bins where a, b are inserted. This work also introduces a simple algorithm, based on the *Iterated Local Search* procedure. Using a set of constructive heuristics to build an initial solution that are then refined. The computational results show that the proposed method, is robust and provides high-quality solutions within a reasonable time, comparing well with state-of-the-art methods tailored for particular problem cases. The authors conclude that their simple and flexible approach is effective, although the introduction of heterogeneous precedences presents new challenges for both exact and heuristic methods, leaving many instances unsolved to proven optimality and suggesting avenues for future research.

Expanding on the application of heuristics for similar optimization problems, [Lalaoui and El Afia 2018] address the Simple Assembly Line Balancing Problem of type I (SALBP-I) by proposing an adaptive metaheuristic. Their approach employs a generalized simulated annealing algorithm whose parameters are dynamically adjusted by a fuzzy inference system, aiming to quickly find effective solutions for rebalancing production lines in response to factors like demand fluctuation. The paper's conclusion is based on the performance analysis of their fuzzy simulated annealing method on a well-known benchmark dataset for SALBP-I, demonstrating its applicability in minimizing the number of workstations for a given cycle time.

The existing literature presents robust methods for the Bin Packing Problem with Generalized Precedence Constraints (BPP-GP) and its variants. Notably, [Kramer et al. 2017] proposed an effective baseline method for the BPP-GP based on Iterated Local Search (ILS). Concurrently, other metaheuristics have demonstrated great success on similar problems, such as GRASP, which was effectively applied by [Layeb and Chenche 2012] to the classic 1-BPP. While the ILS approach from [Kramer et al. 2017] serves as an established benchmark, the success of GRASP in combinatorial optimization motivates its application to the BPP-GP.

This project aims to extend the baseline method proposed in [Kramer et al. 2017] by developing and integrating an adapted GRASP metaheuristic, which will replace the original ILS procedure to explore a new solution pathway for this complex problem.

3. Methodology

In this work, we will implement a GRASP approach which uses ideas from the work of [Kramer et al. 2017]. We will use this same study as our control method. The authors of this study propose an algorithm based on Iterated local search, as presented below:

Algorithm 1: Control Metaheuristic

```
function BM-ILS(Instance):
    ImprovedInstance ← PreProcessing(Instance)
    LB ← ComputeBounds(ImprovedInstance)
    /* Construct first solution                    */
    x_incumbent ← ConstructInitialSolution(ImprovedInstance)
    x_best ← x_incumbent
    repeat
        /* Intensification using batch of moves        */
        x_incumbent ← BM − LS(x_incumbent) if z(x_incumbent) < z(x_best) then
            x_best ← x_incumbent
        ρ ← random(1, K_MAX(ImprovedInstance))
        x_incumbent ← DestroyShrinkRepair(x_incumbent, ρ)
         // Diversification
    until z(x_best) = LB or Timeout()
    return x_best
```

This algorithm is comprised of three main phases:

1. Pre-Processing and lower-bound computation: This phase contributes to a speed-up in the convergence of the algorithm. They adapt the procedures presented in [Dell'Amico et al. 2012].
2. Initial Solution Computation: The procedure employ two independent **greedy constructive** heuristics and a linear integer programming approach. At the end, the best solution is selected.
3. Local Search: The goal of this phase is to refine the initial solution either until we reach a fixed timeout or reach the lower bound.

In our work, we plan to construct a GRASP based approach derived from this prior work. Each iteration of the GRASP metaheuristic contains two main steps:

- Construction of initial solutions: In this step, a feasible solution is built using a **randomized adaptative greedy** algorithm.
- Local-Search step: This initial solution is refined until a local optima is reached.

This randomized greedy algorithm is controlled by a parameter α which defines the compromise between search exploitation and exploration. In our work, this α will be adaptative, that is, its value will be tuned through the procedure.

In the constructive step, we plan to adapt the two greedy constructive heuristics employed by [Kramer et al. 2017] to be randomized in order to be incorporated into a GRASP step. We will also explore other possible heuristics that could bring benefits to our approach. In the local search step, we plan to use the local search (BM-LS) algorithm also proposed by [Kramer et al. 2017].

We hope that our work could produce at least comparable solutions to [Kramer et al. 2017] given the same execution times, and in some cases even better solutions to some benchmarks, due to the explorative/adaptative factor in GRASP.

3.1. Evaluation

For this phase, we will utilize already propose benchmarks sets for this problem. The work of [Kramer et al. 2017] also expands a standard benchmark set proposed by [Otto et al. 2013]. We plan to ask the authors of such work in order to use this extended set.

During our work, we will employ some intermediate evaluation steps, to decide how we should handle the use of the constructive heuristics, trying to answer the question: “Should we be inspired by [Kramer et al. 2017] and also run both, selecting the better result or should we also employ some randomness in this selection?”.

After completing this implementation phase, we will compare our approach with established techniques in the literature, such as the Iterated Local Search method proposed by [Kramer et al. 2017] and other state-of-the-art metaheuristics for the BPP-GP problem. The comparison will focus on both solution quality and computational efficiency.

To ensure a rigorous evaluation, we will adopt the following methodology:

- **Performance metrics:** We will consider multiple measures of solution quality, including the number of bins used, average gap to the known lower bounds, and the distribution of solution values across multiple runs. Computational efficiency will be assessed through CPU time, number of iterations, and convergence behavior.
- **Statistical analysis:** To account for the stochastic nature of GRASP, each experiment will be repeated multiple times (e.g., 10 independent runs), and results will be statistically analyzed. Metrics such as mean, standard deviation, and confidence intervals will be reported.
- **Benchmark instances:** Experiments will be conducted on standard benchmark sets, including the extended set from [Kramer et al. 2017] and the original set from [Otto et al. 2013]. Instances will cover a wide range of sizes and complexity levels to assess the scalability and robustness of the algorithm.
- **Parameter tuning:** Sensitivity analysis will be conducted for key algorithmic parameters, particularly the adaptive α controlling the randomized greedy construction, to identify configurations that maximize performance.
- **Ablation studies:** To understand the contribution of each component of the GRASP approach (e.g., randomized constructive heuristics, BM-LS local search, adaptive α), we will perform ablation experiments by selectively removing or modifying components and observing their impact on solution quality and convergence.

This evaluation framework aims to provide a thorough and fair comparison between our proposed GRASP metaheuristic and existing methods, highlighting potential improvements in solution quality, robustness, and computational efficiency. By combining detailed performance metrics with statistical analysis and controlled experiments, we aim to draw rigorous conclusions about the effectiveness of the proposed approach for the BPP-GP problem.

4. Schedule

To ensure a structured and timely execution of this research, the project has been divided into distinct stages, each with defined tasks, deadlines, and assigned responsibilities. Table 1 presents the detailed schedule, including the duration of each stage and the collaborators responsible for its completion. This structured timeline provides a clear roadmap for implementing the baseline, developing the proposed GRASP metaheuristic, conducting experiments, analyzing results, and preparing the final deliverables.

Table 1. Project Schedule and Responsibilities

Stage	Task	Deadline	Duration	Responsible
1	Review and Planning	27–29 Oct	3 days	Rangel
2	Baseline Implementation	30 Oct–2 Nov	4 days	Carlos
3	New Implementation	3–7 Nov	5 days	Carlos
4	Integration and Debugging	8–10 Nov	3 days	Carlos, Pedro
5	Execution of Experiments	11–14 Nov	4 days	Pedro
6	Results Analysis	15–17 Nov	3 days	Pedro, Rangel
7	Article Writing	18–22 Nov	5 days	Rangel
8	Presentation and Final Review	23–25 Nov	3 days	All
Total			**30 days**	

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