OPTIMIZATION

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February 19, 2019

QUESTION 3.3

Given:

$f(\mathbf{x}) = (x_1 - 8)^2 + (x_2 - 6)^2 = r^2$

$g(\mathbf{x}) = x_1 + x_2 - 9 = 0$

$L(\mathbf{x},\lambda) = f(\mathbf{x}) - \lambda g(\mathbf{x})$

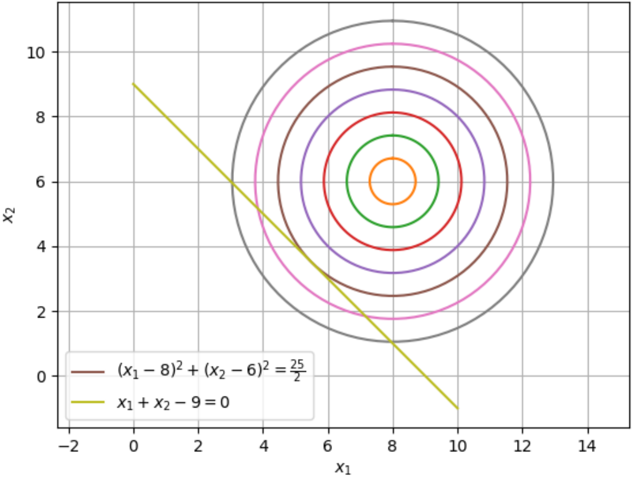
and

$$\nabla = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial \lambda} \end{bmatrix}$$

Solve the equation:

$\nabla L(\mathbf{x},\lambda) = 0$

What is the sign of λ?



$(x_1 - 8)^2 + (x_2 - 6)^2 = \frac{25}{2}$
$x_1 + x_2 - 9 = 0$
x_1
x_2
−2
0
2
4
6
8
10
12
14

Solution - 3.3

$$\nabla \mathrm{L}(\mathbf{x},\lambda) = (x_1 - 8)^2 + (x_2 - 6)^2 - \lambda(x_1 + x_2 - 9)$$

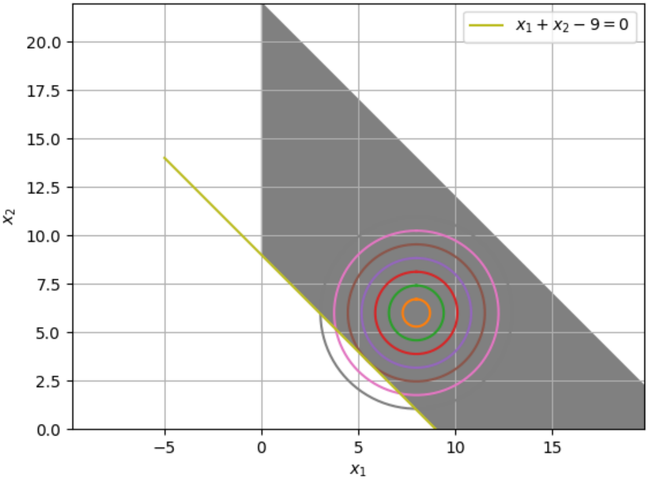
$$\nabla \mathrm{L}(\mathbf{x},\lambda) = \begin{bmatrix} 2x_1 - 16 - \lambda \\ 2x_2 - 12 - \lambda \\ x_1 + x_2 - 9 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \\ 9 \end{bmatrix} = 0$$

Hence the solution matrix is: $\begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} \frac{11}{2} \\ \frac{7}{2} \\ -5 \end{bmatrix}$

Therefore, λ is negative.

So,minimum value of f($\mathbf{x}$) over the constraint g($\mathbf{x}$)=0 is found.

Find the graphical solution for minimization of f($\mathbf{x}$) over the constraint g($\mathbf{x}$)$\geq$0



$x_1 + x_2 - 9 = 0$
20.0
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
x_2
−5
0
5
10
15
x_1

Solution 3.4

As we know that minimum possible radius for the circle centered at (6,8) in shaded region (constraint) is 0.
So, minimum value of $f(\mathbf{x})$ over the constraint $g(\mathbf{x}) \geq 0$ is **0**.

QUESTION 3.5

Use the method of Lagrange multiplier to solve problem 3.4 and compare with graphical solution.

$f(\mathbf{x}) = (x_1 - 8)^2 + (x_2 - 6)^2$
$g(\mathbf{x}) = x_1 + x_2 - 9 \geq 0$
Applying Lagrangian multipliers :
$\nabla L(\mathbf{x},\lambda) = (x_1 - 8)^2 + (x_2 - 6)^2 - \lambda(x_1 + x_2 - 9)$

$$\nabla L(\mathbf{x},\lambda) = \begin{bmatrix} 2x_1 - 16 - \lambda \\ 2x_2 - 12 - \lambda \\ x_1 + x_2 - 9 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \\ 9 \end{bmatrix} = 0$$

Hence the solution matrix is: $\begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} \frac{11}{2} \\ \frac{7}{2} \\ -5 \end{bmatrix}$

So, we can observe that solution obtained by Lagrange multiplier i.e (11/2,7/2) is different from the graphical solution.