On double-obstacle problems:

An option has barriers at levels H_u and H_d, above and below the initial asset price, respectively. Formulate the following barrier option pricing problems as partial differential equation model or binomial tree model with suitable boundary and final conditions:

(a) If the underlying asset price touches both barrier before expiry then the option knocks out. Otherwise, at expiry the option has a payoff of $\max(S - X, 0)$.

(b) If the underlying asset price touches H_u but never touches H_d, the option has a terminal payoff of $\max(S - X, 0)$. Otherwise the option is worthless.