العمليات في مجموعة الأعداد الحقيقية

التمرين 1:

$$A = x^2 + 2x + \frac{8}{9}$$

أ-

ب- حيث $x = 0$

$$A = \frac{8}{9}$$

حيث $x = -2$

$$A = (-2)^2 + 2 \times (-2) + \frac{8}{9}$$

$$= 4 - 4 + \frac{8}{9}$$

$$= \frac{8}{9}$$

ب- $A = (x+1)^2 - \frac{1}{9} = ?$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(x+1)^2 - \frac{1}{9} = x^2 + 2x + 1 - \frac{1}{9}$$

$$= x^2 + 2x + \frac{9}{9} - \frac{1}{9}$$

$$= x^2 + 2x + \frac{8}{9} = A$$

وبالتالي $A = (x+1)^2 - \frac{1}{9}$

ج-

$$A = (x+1)^2 - \frac{1}{9}$$

$$= \underbrace{(x+1)}_{a}^2 - \underbrace{\left(\frac{1}{3}\right)}_{b}^2$$

$$\boxed{a^2 - b^2 = (a-b) \times (a+b)}$$

$$A = \left((x+1) - \frac{1}{3}\right)\left((x+1) + \frac{1}{3}\right)$$ يعني

$$= \left(x+1-\frac{1}{3}\right)\left(x+1+\frac{1}{3}\right)$$

$$\boxed{A = \left(x+\frac{2}{3}\right)\left(x+\frac{4}{3}\right)}$$

2- $B = 3x^2 + 5x + \frac{4}{3}$; $x \in \mathbb{R}$

أ-

$$(3x+1)\left(x+\frac{4}{3}\right) = 3x^2 + 3x.\frac{4}{3} + x + \frac{4}{3}$$

$$= 3x^2 + 4x + x + \frac{4}{3}$$

$$= 3x^2 + 5x + \frac{4}{3} = B$$

$B = (3x+1)\left(x+\frac{4}{3}\right)$ يعني ..

ب-

$$\frac{A}{B} = \frac{\left(x+\frac{2}{3}\right)\left(x+\frac{4}{3}\right)}{(3x+1)\left(x+\frac{4}{3}\right)} = \frac{x+\frac{2}{3}}{3x+1}$$

التمرين 2 :

1- $A = (\sqrt{3} - x)(\sqrt{2} + x) - (2x - \sqrt{2})(x - \sqrt{3})$

أ-

$A = \sqrt{3} \times \sqrt{2} + \sqrt{3}.x - \sqrt{2}x - x^2 - (2x^2 - 2\sqrt{3}.x - \sqrt{2}x + \sqrt{6})$

$= \sqrt{6} + \sqrt{3}x - \sqrt{2}x - x^2 - 2x^2 + 2\sqrt{3}.x + \sqrt{2}x - \sqrt{6}$

$= 3.\sqrt{3}x - 3x^2$

$\boxed{A = 3x(\sqrt{3} - x)}$

ب- $x = -1$ يعني :

$A = 3 \times (-1) \times (\sqrt{3} - (-1))$

$= -3 \times (\sqrt{3} + 1)$

$A = -3\sqrt{3} - 3$

ج- $x = -\sqrt{3}$

$A = 3 \times (-\sqrt{3})(\sqrt{3} - (-\sqrt{3}))$

$= -3\sqrt{3} \times (\sqrt{3} + \sqrt{3})$

$= -3\sqrt{3} \times 2\sqrt{3}$

$= -6 \times 3$

$= -18$

د-

$A = 3x(\sqrt{3} - x) = 0$ يعني $A = 0$

يعني $3x = 0$ أو $\sqrt{3} - x = 0$

$x = \sqrt{3}$ يعني $\sqrt{3} - x = 0$ *

$x = 0$ يعني $3x = 0$ *

2- $B = \sqrt{27} - 3x$

أ- $B = \sqrt{3 \times 9} - 3x$

$= \sqrt{3} \times \frac{\sqrt{9}}{3} - 3x$

$= \boxed{3}\sqrt{3} - \boxed{3}x$

عامل مشترك

$B = 3(\sqrt{3} - x)$ يعني

ب- $A - B = 3x(\sqrt{3} - x) - 3(\sqrt{3} - x)$

عامل مشترك

$= 3(\sqrt{3} - x)(3x - 1)$

ج- $A - B = 0$ يعني $3(\sqrt{3} - x)(x - 1) = 0$

لحينئذ $x-1=0$ أو $\sqrt{3}-x=0$

- $x-1=0$ لحينئذ $x=1$
- $\sqrt{3}-x=0$ لحينئذ $x=\sqrt{3}$

التمرين 3:

$a=x\sqrt{\frac{242}{45}}$ حيث $x \in \mathbb{R}$

1-

$$a=x\sqrt{\frac{242}{45}}=\frac{x\sqrt{242}}{\sqrt{45}}$$

$$=x.\frac{\sqrt{121\times 2}}{\sqrt{9\times 5}}$$

$$=x.\frac{\sqrt{121}\times\sqrt{2}}{\sqrt{9}\times\sqrt{5}}$$

$$=x.\frac{11.\sqrt{2}}{3.\sqrt{5}}$$

$$\boxed{a=\frac{11\sqrt{2}}{3\sqrt{5}}x}$$

ب - $x = \sqrt{2}$ يعني

$$a = \frac{11.\sqrt{2}}{3.\sqrt{5}} \times \sqrt{2}$$

$$= \frac{11 \times 2}{3\sqrt{5}} = \frac{22}{3\sqrt{5}}$$

$x = \sqrt{10}$ يعني

$$a = \frac{11\sqrt{2} \times \sqrt{10}}{3.\sqrt{5}}$$

$$= \frac{11\sqrt{2} \times \sqrt{5} \times \sqrt{2}}{3.\sqrt{5}}$$

$$a = \frac{22}{3}$$

ج- تذكير:

$|x| = x \quad ; \quad x > 0 \quad ; x \in \mathbb{R}$

$|x| = -x \quad ; \quad x < 0$

مقابل

$|a| = \left| \frac{11\sqrt{2}}{3\sqrt{5}} . x \right| = - \frac{11\sqrt{2}}{3\sqrt{5}} . x$

سالب

2-

$$b = \frac{1}{x} \frac{\sqrt{180}}{\sqrt{968}} \quad ; x \in \mathbb{R}^*$$

أ-

$a \times b = ?$

$$b = \frac{1}{x} \times \frac{\sqrt{180}}{\sqrt{968}} = \frac{1}{x} \times \frac{\sqrt{36 \times 5}}{\sqrt{484 \times 2}}$$

$$= \frac{1}{x} \frac{\sqrt{36} \cdot \sqrt{5}}{\sqrt{484} \times \sqrt{2}}$$

$$= \frac{1}{x} \frac{6 \cdot \sqrt{5}}{22\sqrt{2}}$$

$$a \times b = \frac{11 \cdot \sqrt{2}}{3\sqrt{5}} \cdot x \times \frac{1}{x} \frac{6 \cdot \sqrt{5}}{22 \cdot \sqrt{2}} = \frac{66}{66} = 1$$

ب- بما أن $a \times b = 1$ يعني a هو مقلوب b و b هو مقلوب a.

التمرين 4:

1- $x \in \mathbb{R}_-$ يعني $A = -|x| + x$

$$= -x + x$$

$$= 0$$

$A = -|x| + x$ بحيث $x \in \mathbb{R}_-$.

$|x| = -x$

$= -(-x) + x$

$= x + x$

$= 2x$

$B = -x - |x+2|$ -2

$x + 2 \geq 0$ وبالتالي $x \geq -2$: بحيث .

$|x+2| = x+2$

بحيث :

$B = -x - (x+2)$

$= -x - x - 2$

$= -2x - 2$

$= -2(x+1)$

$x \leq -2$ بحيث $x+2 \leq 0$ وبالتالي

$|x+2| = -(x+2)$

بحيث :

$B = -x - (-(x+2))$

$= -x + (x+2)$

$= -x + x + 2$

$= 2$

$c = \sqrt{2} - |\sqrt{2} - x|$ -3

$x \geq \sqrt{2}$. يعني $\sqrt{2} - x \leq 0$ و بالتالي

$|\sqrt{2} - x| = -(\sqrt{2} - x)$

يعني $c = \sqrt{2} - (-(\sqrt{2} - x))$

$= \sqrt{2} + (\sqrt{2} - x)$

$c = 2\sqrt{2} - x$

$x \leq \sqrt{2}$. يعني $\sqrt{2} - x \geq 0$ و بالتالي

$|\sqrt{2} - x| = \sqrt{2} - x$

يعني $c = \sqrt{2} - (\sqrt{2} - x)$

$= \sqrt{2} - \sqrt{2} + x$

$c = x$