NED University of Engineering and Technology Department of Electrical Engineering

Digital Signal Processing *B.E (EE)*
Lab Session 01

Effects of Sampling in Discrete Time Signals

OBJECTIVE:

1. Simulate and plot two CT signals of 10 Hz and 110 Hz for $0 < t < 0.2$ secs.
2. Sample at Fs = 100 Hz and plot them in discrete form.
3. Observe and note the *aliasing* effects.
4. Explore and learn.

PRE_LAB (Introduction)

- A *Signal* is defined as any physical quantity that varies with an independent variable.Signals are physical quantities that carry information in their pattern of variation.

- A ***Continuous Time Signal*** is defined for every value of time i.e. continuous function of time. Examples are sound wave form or a cosine wave.

  Continuous Time sinusoids

  $$x_1(t) = Cos 2\pi Ft$$

  where, 'F' is cycles/sec (Hz.)

  Range for F is $-\infty \le F \le \infty$

- A ***Discrete Time Signal*** is defined only at certain specific values of time or *Discrete Time Signals are sequence of numbers* . A *Discrete valued Signal* takes on values from a finite set of possible values

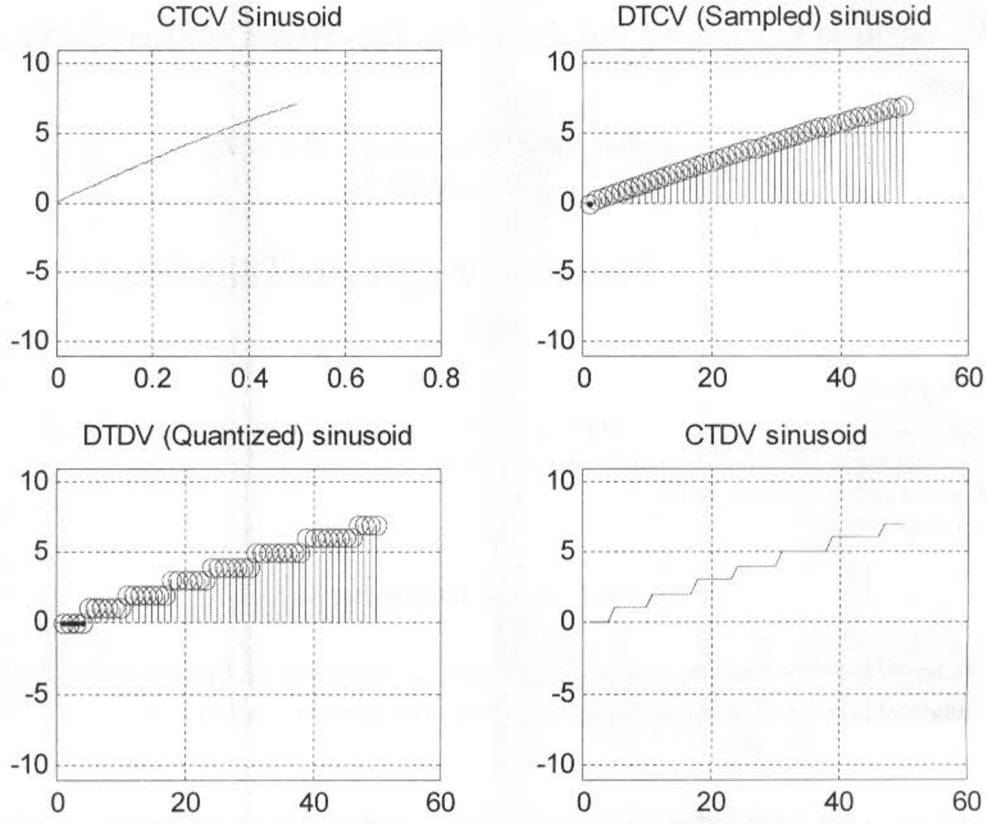
  If the values of a sequence are chosen from a finite set of numbers ,the sequence is known as a digital signal.

  Discrete Time sinusoids where, f_d is in cycles/sample.

  $$x_1(n) = Cos 2\pi f_d n$$

- The process of converting CTCV signal into a DTCV signal is called ***"sampling".***

- The process of converting DTCV signal into a DTDV signal is called **"Quantization"**.



Sampling of Analog Signals

- If $x_a(t)$ is an analog signal, 'n' is a sequence of integers, and 'T' is the *sampling period* by which we want to sample this analog signal then, the sampled signal is defined as

$$x(n) = x_a(nT) \qquad \text{where, } \quad n = 0, \pm1, \pm2, ...$$

From here we define the sampling rate or frequency.

$$F_s = \frac{1}{T}$$

Where **Sampling Rate** is the number of samples taken per unit time is called the sampling rate.

Relationship between F, f_d and F_S

We know that, $\quad x_a(t) = Cos 2\pi Ft$

Converting into discrete signal,

$$x(n) = x_a(nT) = Cos 2\pi nFT = Cos \frac{2\pi nF}{F_S}$$

PROCEDURE:

1. Make a folder at desktop and name it as your current directory within MATLAB®.
2. Open M-file editor and type the following code:

```
% Plotting the two CTCV
sinusoids

clear all;
close all;
clc;
F1 = 10;
F2 = 110;
Fs = 100;
Ts = 1/Fs;
t = [0 : 0.0005 : 0.2];
x1t = cos(2*pi*F1*t);
x2t = cos(2*pi*F2*t);
figure,
plot(t,x1t,t,x2t, 'LineWidth',2);
xlabel('cont time (sec)');
ylabel('Amp');
xlim([0 0.1]);
grid on;
legend('10Hz','110Hz');

title('Two CTCV sinusoids
plotted');
```

3. Save the file as P011.m in your current directory and 'run' it, either using F5 key or

writing the file name at the command window.

4. Check for the correctness of the time periods of both sinusoids.

5. Now add the following bit of code at the

bottom of your P011.m file and save.

```
%sampling the two CTCV sinusoids
nTs = [0 : Ts : 0.2];
n = [1 : length(nTs)-1 ];
x1n = cos(2*pi*F1*nTs);
x2n = cos(2*pi*F2*nTs);
figure,
subplot(2,1,1),
stem(nTs,x1n,'r','LineWidth',2);
grid on;
title('10Hz DTCV(sampled)SIGNAL');
xlabel('discrete time (sec)');
ylabel('Amp');
xlim([0 0.1]);
subplot(2,1,2)
stem(nTs,x2n,'LineWidth',2);
grid on;
title('110Hz DTCV(sampled)SIGNAL')
xlabel('discrete time (sec)');
ylabel('Amp');
```

But, discrete signal is also given by

$$x(n) = Cos2\pi f_d n$$

Comparing the last two equations, we get $f_d = \dfrac{F}{F_S}$

Units are, $\Rightarrow \dfrac{cycle}{sample}(f_d) = \dfrac{cycle/\sec(F)}{sample/\sec(F_S)}$

f_d is called normalized or relative frequency.

Frequency ranges…

We know from previous discussion,

$$-\infty \leq F \leq \infty \qquad \text{And} \qquad -\frac{1}{2} \leq f_d \leq \frac{1}{2}$$

We also know that

$$f_d = \frac{F}{F_S}$$

Therefore, the new range for F in discrete systems is

$$-\frac{F_S}{2} \leq F \leq \frac{F_S}{2}$$

$$-\frac{1}{2T} \leq F \leq \frac{1}{2T}$$

- Since the highest frequency in DT signal is $\boldsymbol{f_d = 1/2}$, it follows that with a sampling rate of Fs, corresponding highest frequency for F becomes,

$$F_{\max} = \frac{F_S}{2} = \frac{1}{2T}$$

Sampling Theorem

- So, this is suggested by this theorem that we should choose ***Fs*** to be at least <u>twice</u> that of the <u>maximum frequency component</u> present in our CT signal.

$$F_S \geq 2F_{\max}$$

- Now, we define **Nyquist Rate** as

$$F_N = 2B = 2F_{\max}$$

<u>**Aliasing:**</u> **A** common problem that arises when sampling a continuous signal is aliasing ,where a sampled signal has replications of its sinusoidal components which can interfere with other components .It is an effect that causes two discrete time signals to become indistinct due to improper sampling .

<u>In</u> general ,aliasing is a situation where one thing takes the form or identity of another.

IN LAB TASKS

Task-1: Sampling and Aliasing in Sinusoids

1. Simulate and plot two CT sinusoids of 10 Hz and 110 Hz for 0 < t < 0.2 sec.
2. Sample both sinusoids at Fs = 100 samples/sec and plot them in discrete form.

```
xlim([0 0.1]);
```

6. Before hitting the 'run', just try to understand what the code is doing and try to link it with what we have studied in classes regarding concepts of frequency for DT signals.
7. Now 'run' the file and observe both plots.
8. To see what is really happening, type the following code at the bottom of your existing P011.m file and run again.

```
figure,
plot(t,x1t,t,x2t);
hold
stem(nTs,x1n,'r','LineWidth',2);
grid on;
title('Analog Sinusoids and sampled signal');
xlabel('time (sec)');
ylabel('Amp');
xlim([0 0.1]);
legend('10Hz sinusoid','110Hz sinusoid','sampled sinusoid');
```

9. Observe the plots.
10. Now, Explain (write) in your own words the cause and effects of what you just saw.

Task-2: Sampling and Aliasing in Audio signals

You have to generate a tone in **MATLAB®** and listen to it with the `sound` command. The frequency of tone should be 1kHz at a sampling rate of 8kHz and the duration should be 3 sec. Run the following code which plays back tones of various frequencies. Fill the table given after this code.

```
clear all;close all;clc;
F=1000;
t=3;
Fs=8000;
nTs=[0:(1/Fs):t];
x=sin(2*pi*F*nTs);
sound(x,Fs)
```

POST LAB TASKS

Task-1:

Change the value of F which is 1000 Hz currently to these values, listen to the tones (make sure your headphones or speakers are on). In later scenarios, change Fs as well. *You might want to add a phase shift where F = Fs/2.* Can you tell why we need a phase shift at F=Fs/2 to produce *any* sound?

Sr. No.	F in Hz	Fs in Hz or samples/sec	Sounds like what frequency?	Reason. Specify whether Aliasing is happening or not.
1	1000	8000		
2	2000	8000		
3	3000	8000		
4	4000	8000		
5	5000	8000		
6	6000	8000		
7	7000	8000		
8	8000	8000		
9	9000	8000		

10	-1000	8000		
11	-2000	8000		
12	-3000	8000		
13	-4000	8000		
14	9000	19000		
15	25000	55000		

Task-2:

Consider the continuous time sinusoidal signal: $(t) = si(2\ \pi F_0 t)$.
The sampled version will be: $(n) = si(2\ \pi(F_0/Fs)n)$, where $'n'$ is a set of integers and sampling interval Ts $= 1/Fs$. Plot the signal (n) for Fs = 5 kHz and take

i. F_0 =0.5 kHz iii) F_0 =3 kHz

ii. F_0 =2 kHz iv) F_0 =4.5 kHz

Explain the similarities and differences among various plots. Also mention that whether aliasing occurs or not.

Task-3:

(a)Generate an analog signal of F= 5 Hz.
plot it for t=1 sec.
Take Fs =10
count no of samples using MATLAB and print it.
(b)Repeat above exercise for t=2 sec.
(c) Repeat above exercises for Fs=20.
(d) Repeat above exercises for F=10Hz
(e)) Repeat above exercises for Fs=40.
Observe the differences in above and explain.

Task-4:

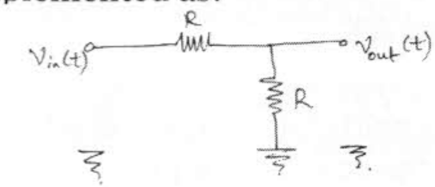
A cosine signal of 40 Hz is sampled at 30 samples per second .what will be f_d of the corresponding discrete signal?

Digital Signal Processing

Lab 01

Effects of Sampling in Discrete Time Signals

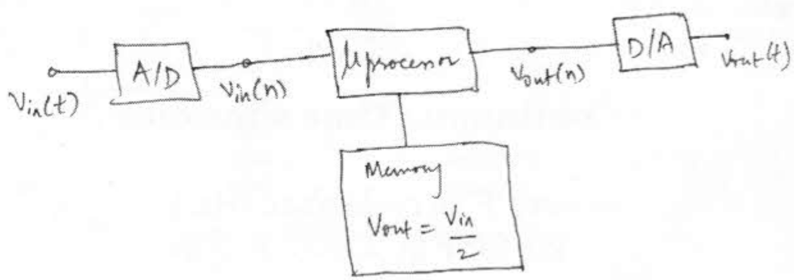
- A simple example to create a feeling of how DSP works.
- In analog, a model y = x/2 can be implemented as:



$$V_{out}(t) = \frac{R}{2R} v_{in}(t)$$

$$V_{out}(t) = \frac{1}{2} v_{in}(t).$$

- Where as, in digital domain the same thing would look like:



- DSP offers advantages such as:
  - ❑Programmable operations
  - ❑Flexibility
  - ❑Precision
  - ❑Recording capability, etc.
- DSP has developed rapidly in the last 40 years.
- This development is based on advances in:
  - ❑Digital computing technology
  - ❑Integrated circuit (IC) fabrication

- Both digital computing and IC fabrication has grown cheaper and faster over time.

- Enabling conversion/translation of more and more 'signal processing tasks' into 'digital domain'.

- No matter how complex a task – if it justifies
  a) the cost of IC and
  b) real time processing speed requirements – it can be performed digitally.

- DSP application areas:
  - Telecommunication systems
  - Robotics and Control Systems
  - Power Systems
  - Geophysical sciences (Mapping, seismic apps)
  - Industrial processes and controls
  - Speech Processing
  - Aviation and Navigation
  - Warfare
  - Biomedical / Health Sciences
  - Media and Entertainment
  - Image Processing

- A *Signal* is defined as any <u>physical quantity</u> that varies with an <u>independent variable</u>.
- This dependency can be single or multi-dimensional.
- Electric Current is a single dimension signal, while an image is three dimensional. *Continuous valued Signal* : If the values of a CT or DT signal are continuous then this is a CV signal.

- A *Discrete valued Signal* takes on values from a finite set of possible values.

- A *Continuous Time Signal* is defined for every value of time. Examples are sound wave form or a cosine wave.

- **Continuous Time sinusoids**

where, 'F' is cycles/sec (Hz.)
Range for F is $-\infty \leq F \geq \infty$

- A *Discrete Time Signal* is defined only at certain specific values of time.
- Time instants need not to be equidistant but usually they are.
- Example,

$$x_1(t_n) = e^{-|t_n|}$$

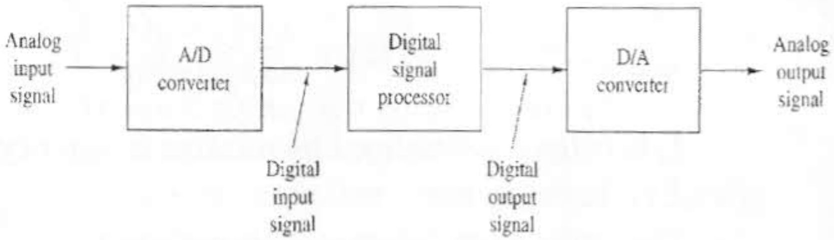
where, $n = 0, \pm 1, \pm 2, ...$

Discrete Time sinusoids where, f_d is in cycles/sample.

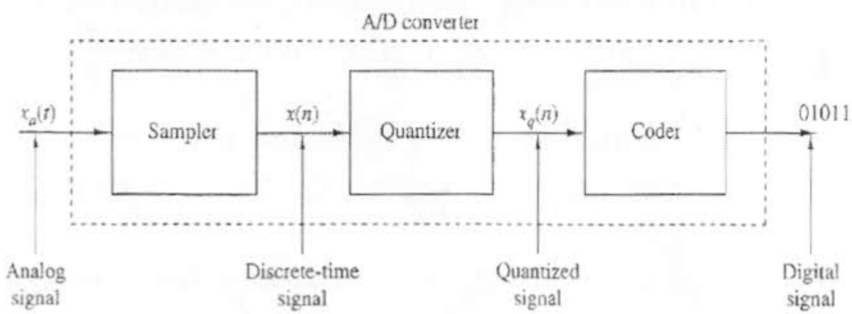
$$x_1(n) = Cos\, 2\pi f_d n$$

The process of converting CTCV signal into a DTCV signal is called "***sampling***".

The process of converting DTCV signal into a DTDV signal is called "**Quantization**".



Block diagram of a digital signal processing system



Basic parts of an analog-to-digital (A/D) converter

SAMPLING:

This is the conversion of a continuous-time signal into a discrete-time signal obtained by taking "samples" of the continuous-time signal at discrete-time instant. Thus, if $x_a(t)$ is the input to the sampler, the output is $x_a(nT)$ where T is called the sampling interval.

- Sampling of Analog Signals
- If $x_a(t)$ is an analog signal, 'n' is a sequence of integers, and 'T' is the *sampling period* by which we want to sample this analog signal then, the sampled signal is defined as
- $x(n) = x_a(nT)$ where, $n = 0, \pm 1, \pm 2, \ldots$
- From here we define the sampling rate or frequency.
- Where **Sampling Rate** is the number of samples taken per unit time is called the sampling rate.

- **Relationship between F, f_d and F_S**
- We know that, $x_a(t) = Cos\ 2\pi Ft$
- Converting into discrete signal,(put t=nT& T=1/F)

$$x(n) = x_a(nT) = Cos\, 2\pi nFT = Cos\, \frac{2\pi nF}{F_S}$$

- But, discrete signal is also given by

$$x(n) = Cos 2\pi f_d n$$

Comparing the last two equations, we get

$$f_d = \frac{F}{F_S}$$

- Units are,

$$\Rightarrow \frac{cycle}{sample}(f_d) = \frac{cycle/\sec\,(F)}{sample/\sec\,(F_S)}$$

f_d is called normalized or relative frequency.

Frequency ranges...

We know from previous discussion,

$$-\infty \le F \le \infty$$

And

$$-1/2 \le f_d \le 1/2$$

We also know that

$$f_d = \frac{F}{F_S}$$

Therefore, the new range for F in discrete systems is

$$-\frac{F_S}{2} \le F \le \frac{F_S}{2}$$

$$-\frac{1}{2T} \le F \le \frac{1}{2T}$$

- Since the highest frequency in DT signal is $f_d = 1/2$, it follows that with a sampling rate of Fs, corresponding highest frequency for F becomes,

$$F_{max} = \frac{F_S}{2} = \frac{1}{2T}$$

- **Sampling Theorem**

It sets out a guideline for sampling rates. Any real signal is a combination of numerous frequency components.
But as we know from previous slide, there is a maximum limit to F, below which all frequencies can be sampled correctly at a given Fs.
So, first we restrict our signal to a certain F, by filtering the upper frequencies

- So, this is suggested by this theorem that we should choose ***Fs*** to be at least <u>twice</u> that of the <u>maximum frequency component</u> present in our CT signal.

$$F_S \ge 2F_{max}$$

- Now, we define **Nyquist Rate** as

$$F_N = 2B = 2F_{max}$$

- Two important methods to sample an analog signal are:

1. Periodic sampling or uniform sampling

$$x(n) = x_a(nT) = Cos\, 2\pi nFT = Cos\, \frac{2\pi nF}{F_S}$$

2. Sample and hold : The process of extending each sample till the next sample arrives is called sample and hold.

<u>Aliasing</u>

- An infinite no.of continous-time sinusoids is represented by sampling the same discrete time signal(i.e. by the same set of samples).consequently if we are given the sequence x(n),an ambiguity exists as to which continous time signal x(t)these values represent.

<u>Aliasing</u>

- **A** common problem that arises when sampling a continuous signal is aliasing ,where a sampled signal has replications of its sinusoidal components which can interfere with other components .It is an effect that causes two discrete time signals to become indistinct due to improper sampling .
- **<u>In</u>** general ,aliasing is a situation where one thing takes the form or identity of another.

Quantization:

The process of converting DTCV signal into a DTDV signal is called "Quantization".

Coding:

In the coding process, each discrete value is represented by *b*-bit binary sequence.