Design of Experiments Project / IE 6308 / Spring 2021

Ankit Ranjan - 1001832327

Rushikesh Rane - 1001836272

Project Title: Paper Towel Absorbency Experiment

Design a balanced complete factorial experiment with two factors (one with three or more levels)

Describe your problem:

A reviewing article (#1) by cleaning experts, evaluated paper various paper towel brands against their absorbency, strength, shedding tendency, softness and aesthetics. Two were selected at a similar price range, to ensure a fair comparison between different classes of paper towel brands. They were "Viva Signature Cloth" and "Bounty Essentials". Additionally, a third brand was chosen based on its regular usage among many households – "Great Value Ultra Strong". However, due to inventory absence of the "Viva Signature" brand, "Great Value Everyday Strong" was chosen as its replacement, with it taking the role of "Great Value Ultra Strong", and "Great Value Ultra Strong" taking the role of "Viva Signature Cloth" in this study.

This experiment (#2) will be conducted to observe the absorbency of paper towels of the different brands, and on 2 different types of solutions. At the end of this experiment, we will be able to conclude which brand of paper towel is better overall in absorbing the 2 types of solutions.

- **The response variable and its units.**

  The amount of solution absorbed by the paper towel in mL (milliliters).

- **The factors, their levels, and the resulting set of treatments.**

  Two factors:

  1. Factor A – Brands of paper towel (a = 3 levels)
     Brand X - Great Value Everyday Strong ($1.98) (i = 1)
     Brand Y - Great Value Ultra Strong ($3.67) (i = 2)
     Brand Z - Bounty Essentials ($4.64) (i = 3)

  2. Factor B – Types of solution (b = 2 levels)
     Cold tap water (j = 1)
     Vegetable oil (j = 2)

Treatments:

Treatment	Factor A	Factor B	Factor-level combination	Number of Replications
1	Brand X	Cold tap water	Brand X, Vegetable oil	2
2	Brand Y	Cold tap water	Brand Y, Vegetable oil	2
3	Brand Z	Cold tap water	Brand Z, Vegetable oil	2
4	Brand X	Vegetable oil	Brand X, Cold tap water	2
5	Brand Y	Vegetable oil	Brand Y, Cold tap water	2
6	Brand Z	Vegetable oil	Brand Z, Cold tap water	2

- **The experimental units and number of replications.**

  Experimental unit: 3 brands of paper towels with 2 types of solution each and 2 replications, resulting in a*b*r number of experimental units = 3*2*2 = 12 experimental units (12 distinct paper towels will be experimented on)

  Number of replications for each treatment = 2

- **The goals of the study.**

  To decide which brand of paper towel is the better one for the absorption of the two solutions – overall, in the same time period.

- **Describe how the data will be collected.**

  Materials required:
  3 above mentioned brands of paper towels (double-ply), vegetable oil, cold tap water, 2 Borosilicate glass beakers of 100 mL, ruler to measure immersion depth.

  The factors that affect absorbency of paper towels (#3) are material characteristics (paper towel brand), immersion time, sheet dimensions, solution type, solution temperature, environmental temperature and humidity. The factors are being studied from these are paper towel brand and liquid type.

  The sheet dimensions will be cut to a uniform dimension of 28.1 cm x 14.1 cm because the purchased sheets of double ply were of different dimensions. The immersion time for each replication will be 10 seconds. The solution temperature of cold tap water will be generally uniform because the same setting of the tap handle (extreme right for cold water) will be applied throughout the replications, and the vegetable oil will be at room temperature. The data will be collected in the same afternoon in a closed off room, to ensure constant temperature and humidity despite the possible HVAC interference.

  Procedure:

  1. Cut the paper towels for each replication into uniform dimension of 28.1 cm x 14.1 cm (Member 2 performed this task, while Member 1 set up the paper towels in the appropriate order the cutting is required and at the appropriate time to keep things moving smoothly).
  2. Pour 100 mL cold tap water and 100 mL vegetable oil in two separate beakers (Member 2 performed this task).
  3. Fold the cut paper towels lengthwise first, widthwise next, and again widthwise firmly (Member 1 performed this task).
  4. Immerse it lengthwise into the filled beaker at a length of 3 cm and hold it in place for 10 seconds. (Member 2 performed this task)

5. After 10 seconds, lift the wet paper towel above the solution surface for 5 seconds to allow the wet paper towel to drip into the solution. This is done as to ensure that the paper towels were absorbing the solution AND retaining the absorbed solution (Member 2 performed this task while Member 1 recorded the time on a stopwatch and instructed Member 2 to lift the wet paper towel above the beaker and then finally throw the wet paper towel at the 15 second mark).
6. Measure the remaining height of solution and subtract it from the original height (100 mL) (Member 1 performed this task).
7. Repeat steps 4-6 to record the replications for all the treatments.

- **Randomized assignment of the treatments to the experimental units**

Before sorting

Obs	TREATMNT	RANNO
1	1	0.62931
2	1	0.50537
3	2	0.43144
4	2	0.13276
5	3	0.46339
6	3	0.44380
7	4	0.69846
8	4	0.41678
9	5	0.88562
10	5	0.02275
11	6	0.99316
12	6	0.68249

After sorting

Obs	TREATMNT	RANNO
1	5	0.02275
2	2	0.13276
3	4	0.41678
4	2	0.43144
5	3	0.44380
6	3	0.46339
7	1	0.50537
8	1	0.62931
9	6	0.68249
10	4	0.69846
11	5	0.88562
12	6	0.99316

The randomized order obtained after sorting will be followed to collect the data.

References:

#1 https://www.goodhousekeeping.com/home-products/g29389533/best-paper-towel-brands-reviews/

#2 https://www.fundafundaacademy.com/diy-summer-camp-activities-paper-towel-absorbency-experiment/

#3 http://www.stat.ualberta.ca/statslabs/casestudies/files/towel2.pdf

2. Preliminary Model Assumptions.

a) Raw Data Plot & Discussion:

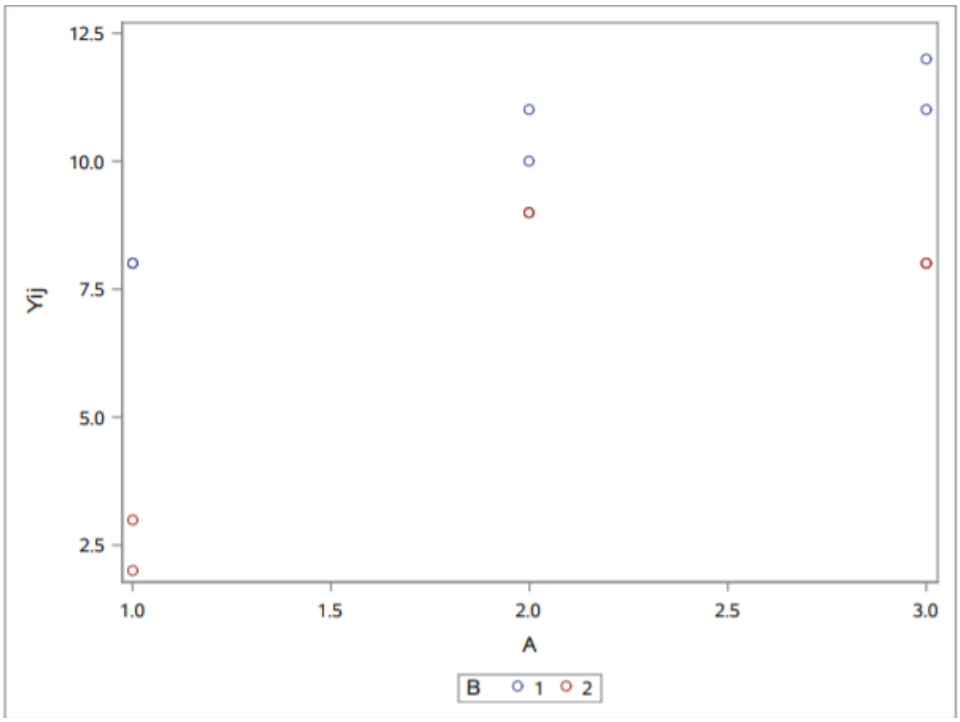


Figure 1.a

There are 12 observations on this dot plot (grouped by Factor B: Solution type), some overlapping with other observations. It seems that Great Value Everyday Strong seems to have much worse absorbency for vegetable oil than Great Value Ultra Strong and Bounty Essentials. Bounty Essentials seems to have better absorbency for water whereas Great Value Ultra Strong seems to have better absorbency for vegetable oil.

Model Form:

Full interaction model:

$$\mu_{ij} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij}$$

where:

$\mu_{..}$ is the overall mean of the volume of solution absorbed in mL (Response).

α_i is the main effect for the brand of paper towel (factor A).

β_j is the main effect for the type of solution (factor B),

$(\alpha\beta)_{ij}$ is the interaction effect between the brand of paper towel (factor A) and type of solution (factor B).

b) Checking Model Assumptions:

1- Constant Variance Assumption

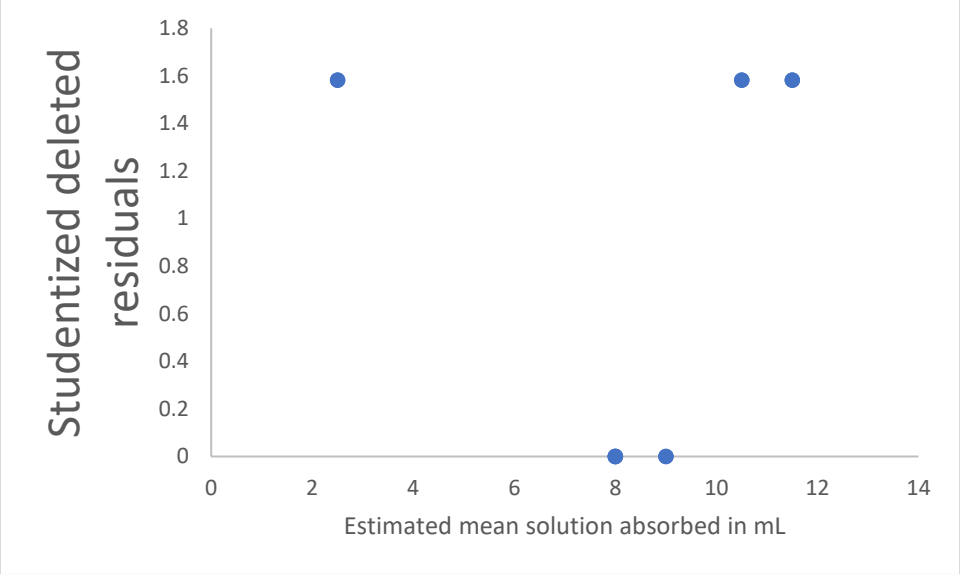


Figure 1.b.1

From the above plot, we can observe presence of constant variance across the different levels of Paper Towel brand and Solution type.

A random point cloud pattern can be observed in the plot of residuals vs Estimated mean solution absorbed in mL. This suggests that the variance of the response, amount of solution absorbed by the paper towel in mL is constant with the mean of response, amount of solution absorbed by the paper towel in mL.

2- Normality

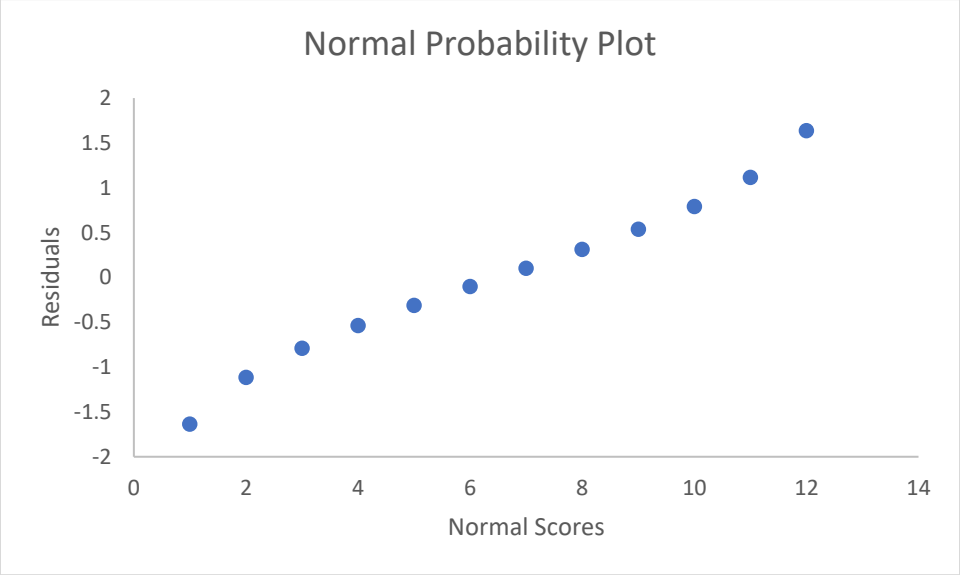


Figure 1.b.2

From the above normal probability plot, there is a slight S shape which might indicate a very small deviation from normality. Thus, it is overall close to normality.

Obs	A	B	avgY	varY	avgYnrm	expYnrm
1	1	2	2.5	0.5	-1.28155	4.2228
2	1	1	8	0.0	-0.42262	6.9219
3	3	2	8	0.0	-0.42262	6.9219
4	2	2	9	0.0	0.20189	8.8844
5	2	1	10.5	0.5	0.64335	10.2717
6	3	1	11.5	0.5	1.28155	12.2772

The scatter plot in blue is between normal scores of ordered treatments' means i.e., avgYnrm (X) vs avgY i.e., ordered treatments' means (Y).

3- Normal Probability Plot

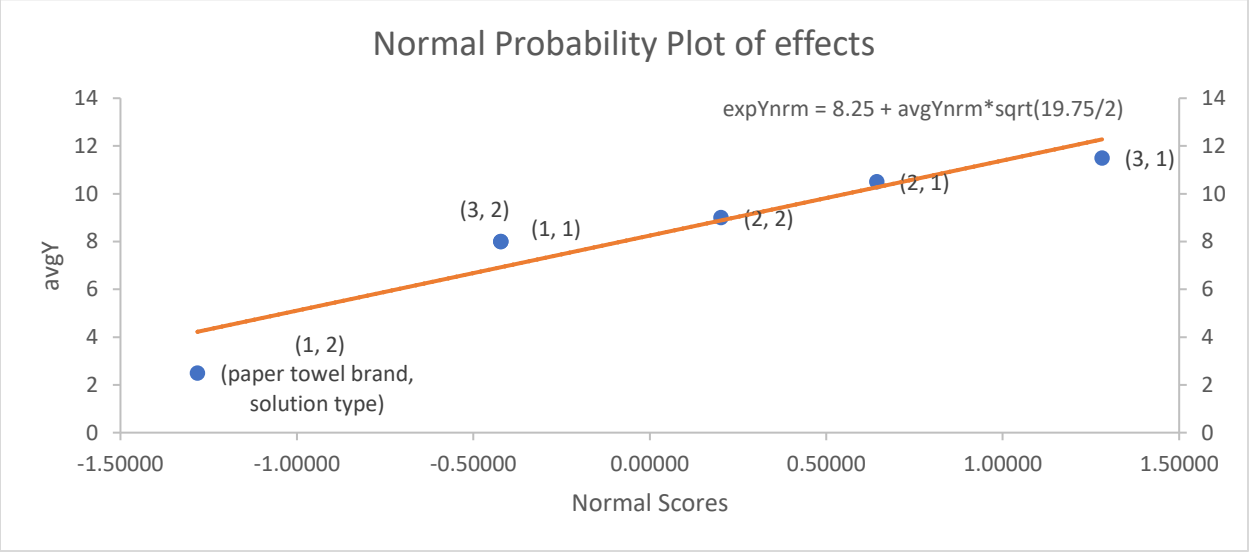


Figure 1.b.3

In the above orange trendline equation (between 'Expected value under H0 ('expYnrm') and ('Normal Scores'),

8.25 -> overall mean of response, amount of solution absorbed by the paper towel in mL

19.75 -> MSE, and 2 -> number of replications (r).

From the normal probability plot of effects, it appears that there is no interaction, as the points are in an organized manner.

However, we can verify this result from the normality test conducted below.

4- Serial Correlation

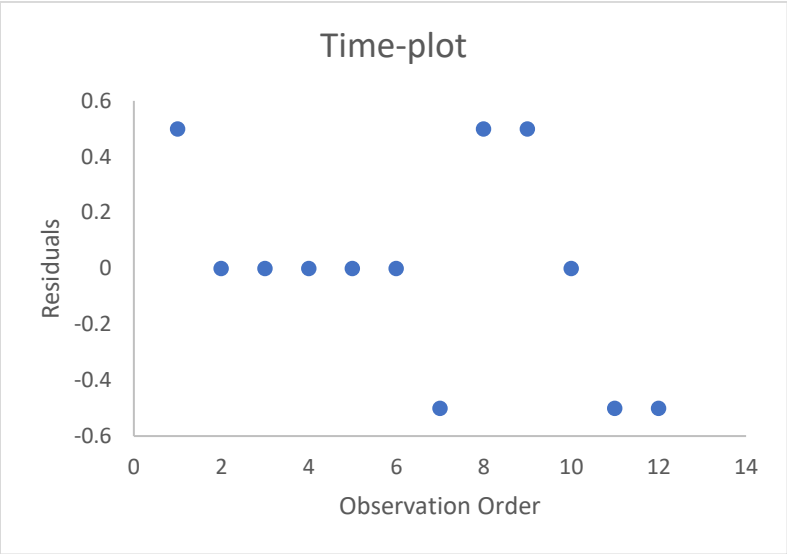


Figure 1.b.4

In the above time plot, we observe a randomly jagged pattern, which is an indication of uncorrelation of the observation order and the response, amount of solution absorbed by the paper towel in mL. Since the order of collection of observations was randomized and the conditions during data collection were generally constant throughout, this was the expected outcome.

5- Other Factors

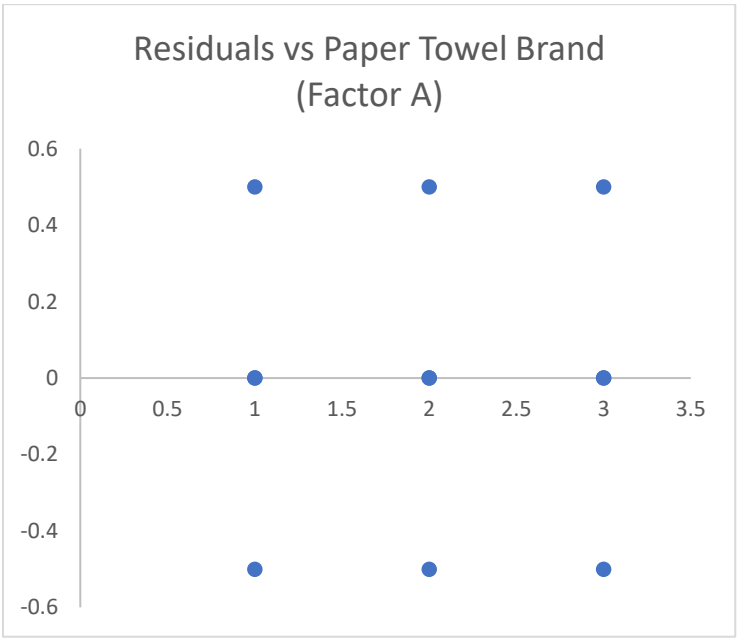


Figure 1.b.5a

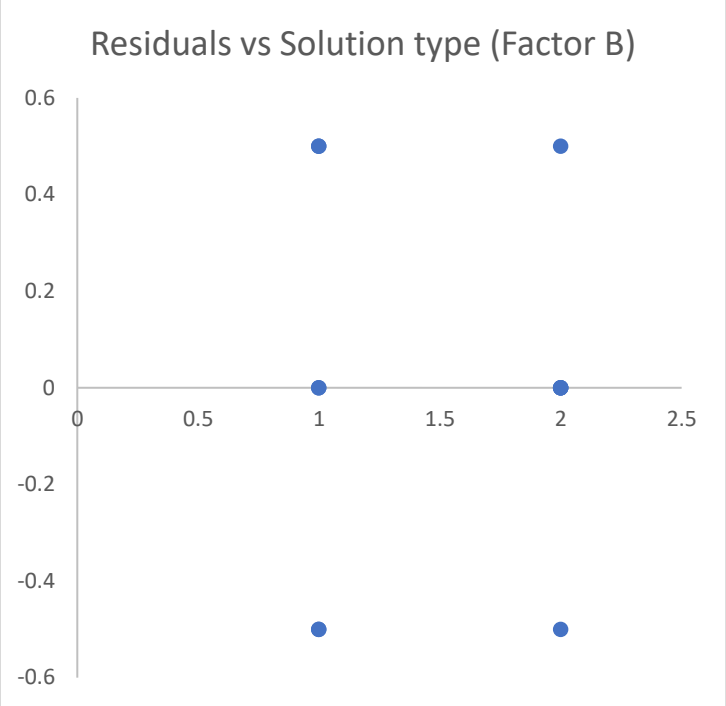


Figure 1.b.5b

The plots of residual vs Paper towel brand and residual vs Solution type, suggest that the variance of the residuals is constant across levels of both factors, Paper towel brand and Solution type. Hence the levels of the factors, Paper towel brand & Solution type do not show any discernable pattern with the different values of the response, amount of solution absorbed by the paper towel in mL.

6- Modified-Levene Test

Anova: Single Factor

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Treatment combination 1 (1,1)	2	0	0	0
Treatment combination 2 (1,2)	2	1	0.5	0
Treatment combination 3 (2,1)	2	1	0.5	0
Treatment combination 4 (2,2)	2	0	0	0
Treatment combination 5 (3,1)	2	1	0.5	0
Treatment combination 6 (3,2)	2	0	0	0

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	0.75	5	0.15	65535	#DIV/0!	4.387374
Within Groups	0	6	0			
Total	0.75	11				

Figure 1.b.6

H_0: Average variability are all equal.

H_1: Average variability is not equal for all.

From the above Excel output for One Way ANOVA F-test using the absolute differences (grouped by treatment), we observe that, F* = 4.38 ≯ F-statistic = 65535 and hence we reject the hypothesis that constant variance is OK.

7- Test for Normality

Pearson Correlation Coefficients, N = 12 Prob > |r| under H0: Rho=0		
	e	**enrm**
e	1.00000	0.95956 <.0001
enrm **Rank for Variable e**	0.95956 <.0001	1.00000

Figure 1.b.7

H_0: Normality is OK.

H_1: Normality is violated.

c (α = 0. 05, n = 12) = 0.928; $\hat{\rho}$= 0.95956 > 0.928 ⇒ non normality is not detected at the level of significance = 0.05.

8- Bonferroni Outlier Test

Trt combination (replication)	|Studentized deleted residuals|
1,1,1	0
1,2,1	1.58113883
2,1,1	1.58113883
2,2,1	0
3,1,1	1.58113883
3,2,1	0
1,1,2	0
1,2,2	1.58113883
2,1,2	1.58113883
2,2,2	0
3,1,2	1.58113883
3,2,2	0
t-cut-off	**5.839661196**

Figure 1.b.8

Since the $|t_{ijt}| \not> t_{n-v-1,\,(\alpha/2n)}$ = 5.839, hence we do not conclude any outliers from this dataset.

Discussion on validity of model assumptions, the presence of outliers, and the need for transformations or weighted least squares.

The constant variance assumption was not satisfied from the modified-Levene test. The normality assumption was satisfied from the test for linearity in the Normal Probability Plot. Thus, there is no need for a variance stabilizing transformation like Box-Cox, Yeo-Johnson etc. as the constant variance assumptions is not OK. The normally distributed errors assumption holds true.

From a graphical standpoint, we do not observe any outliers in the raw data dot plot, normal probability plot and the plots to check for nonconstant variance (studentized deleted residuals vs estimated mean solution absorbed in mL). Furthermore, from the Bonferroni outlier test, we do not conclude any outliers from the dataset at the level of significance $\alpha = 0.05$. This is an indication that the no outlier assumptions are valid.

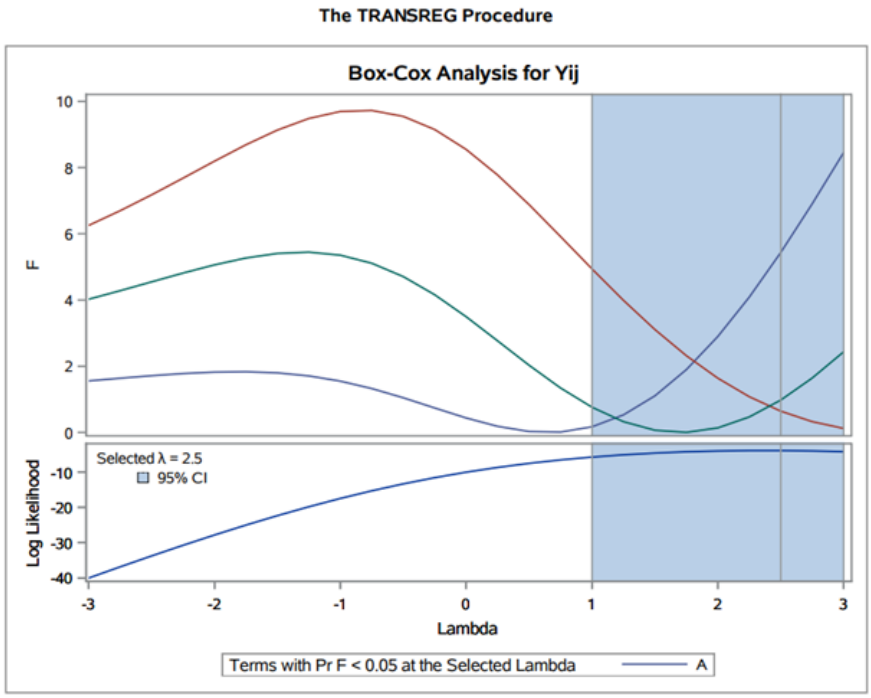
There was no serial correlation was not found from the time plot due to the random jagged pattern, and hence the model assumptions of uncorrelated errors hold true.

The other factors assumption was checked in the plots of residuals vs paper towel brand and residuals vs vegetable oil. The levels of the factors (Paper towel brands & Solution type) did not affect the distribution of the variance much. Hence, the other factors assumption holds true.

From the above conclusions reached from the modified-Levene test, we conclude that a necessary transformation would not be necessary as the normality is OK and constant variance is not OK. Thus, a Weighted Least Squares might be necessary, but is not performed because it will not allow us to proceed beyond that.

9- Transformation

A Box-Cox transformation was attempted as follows:



Model Statement Specification Details				
Type	**DF**	**Variable**	**Description**	**Value**
Dep	1	BoxCox(Yij)	Lambda Used	2.5
			Lambda	2.5
			Log Likelihood	-3.9463
			Conv. Lambda	1
			Conv. Lambda LL	-5.7905
			CI Limit	-5.8670
			Alpha	0.05
			Label	Yij
Ind	1	Identity(A)	Label	A
Ind	1	Identity(B)	Label	B
Ind	1	Identity(AB)	Label	A * B

The TRANSREG Procedure

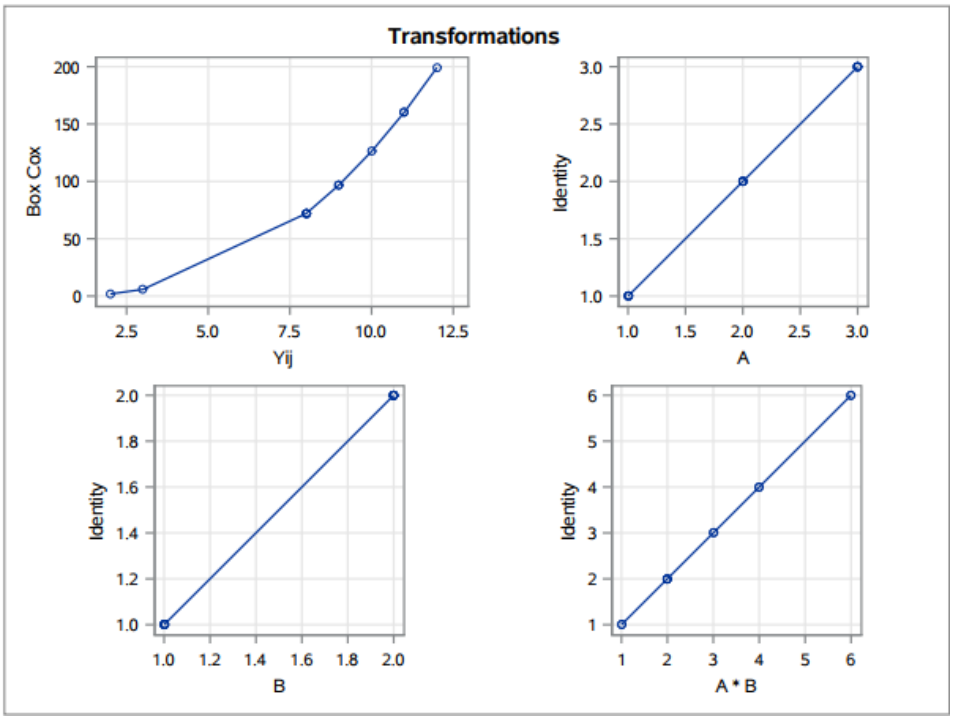


Figure 1.b.9.

From the Box-Cox output from SAS above, we can conclude that the appropriate lambda to be used for practical purpose for the response variable, amount of solution absorbed by the paper towel in mL is $\lambda = 2.0$.

This would make the ANOVA model full interaction as follows:

$\mu_{ij}^{2} = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij}$ => $\mu_{ij}' = \mu_{..} + \alpha_i + \beta_j + (\alpha\beta)_{ij}$; where $\mu_{ij}' = \mu_{ij}^{2}$ to fit the linear model assumption.

3. Analysis of Variance.

a) Interaction Plot & Discussion:

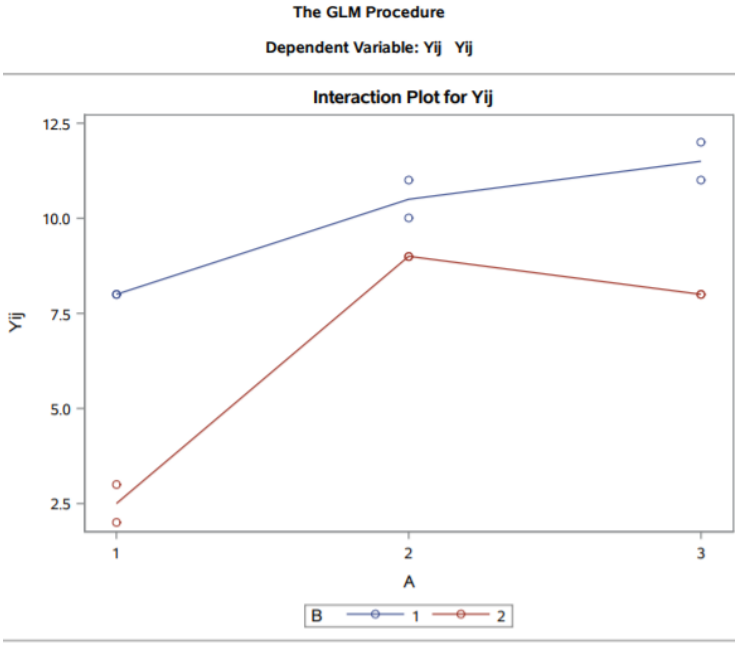


Figure 2.a

Main A effect present due to the presence of a slope from A1 to A2 (or A2 to A3).

Main B effect present due to the presence of more than 1 distinct lines among B1 and B2.

Interaction effects AB may be present due to presence of unequal slopes for B1 and B2.

b) Conducting Analysis of Variance & Discussion:

The GLM Procedure

Class Level Information		
Class	Levels	Values
A	3	1 2 3
B	2	1 2

Number of Observations Read	12
Number of Observations Used	12

The GLM Procedure

Dependent Variable: Yij Yij

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	98.7500000	19.7500000	79.00	<.0001
Error	6	1.5000000	0.2500000		
Corrected Total	11	100.2500000			

R-Square	Coeff Var	Root MSE	Yij Mean
0.985037	6.060606	0.500000	8.250000

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	2	54.00000000	27.00000000	108.00	<.0001
B	1	36.75000000	36.75000000	147.00	<.0001
A*B	2	8.00000000	4.00000000	16.00	0.0039

Source	DF	Type III SS	Mean Square	F Value	Pr > F
A	2	54.00000000	27.00000000	108.00	<.0001
B	1	36.75000000	36.75000000	147.00	<.0001
A*B	2	8.00000000	4.00000000	16.00	0.0039

H_0^{AB}: AB interaction is negligible. (Additive model)

H_1^{AB}: AB interaction is not negligible. (Full interaction model)

From the ANOVA output above, we see that the p-value of interaction AB = 0.0039 < significance level = 0.05, hence we reject H_0. Thus, for future work, the Full interaction model is chosen. Given this conclusion, the individual factors are significant independently as well, and thus hypothesis testing is not required for the same. Additionally, it is evident from their p-values (< significance level = 0.05) that they are significant.

4. Analysis of Effects.

a) Pairwise Comparisons of Paper Towel Brand & Solution Type with Discussion:

The GLM Procedure
Least Squares Means
Adjustment for Multiple Comparisons: Scheffe

A	B	Yij LSMEAN	LSMEAN Number
1	1	8.0000000	1
1	2	2.5000000	2
2	1	10.5000000	3
2	2	9.0000000	4
3	1	11.5000000	5
3	2	8.0000000	6

Least Squares Means for Effect A*B

i	j	Difference Between Means	Simultaneous 95% Confidence Limits for LSMean(i)-LSMean(j)	
1	2	5.500000	3.158159	7.841841
1	3	-2.500000	-4.841841	-0.158159
1	4	-1.000000	-3.341841	1.341841
1	5	-3.500000	-5.841841	-1.158159
1	6	0	-2.341841	2.341841
2	3	-8.000000	-10.341841	-5.658159
2	4	-6.500000	-8.489852	-4.510148
2	5	-9.000000	-10.989852	-7.010148
2	6	-5.500000	-7.489852	-3.510148
3	4	1.500000	-0.489852	3.489852
3	5	-1.000000	-2.989852	0.989852
3	6	2.500000	0.510148	4.489852
4	5	-2.500000	-4.489852	-0.510148
4	6	1.000000	-0.989852	2.989852
5	6	3.500000	1.510148	5.489852

The GLM Procedure
Least Squares Means
Adjustment for Multiple Comparisons: Tukey

A	B	Yij LSMEAN	LSMEAN Number
1	1	8.0000000	1
1	2	2.5000000	2
2	1	10.5000000	3
2	2	9.0000000	4
3	1	11.5000000	5
3	2	8.0000000	6

Least Squares Means for Effect A*B

i	j	Difference Between Means	Simultaneous 95% Confidence Limits for LSMean(i)-LSMean(j)	
1	2	5.500000	3.510148	7.489852
1	3	-2.500000	-4.489852	-0.510148
1	4	-1.000000	-2.989852	0.989852
1	5	-3.500000	-5.489852	-1.510148
1	6	0	-1.989852	1.989852
2	3	-8.000000	-9.989852	-6.010148
2	4	-6.500000	-8.841841	-4.158159
2	5	-9.000000	-11.341841	-6.658159
2	6	-5.500000	-7.841841	-3.158159
3	4	1.500000	-0.841841	3.841841
3	5	-1.000000	-3.341841	1.341841
3	6	2.500000	0.158159	4.841841
4	5	-2.500000	-4.841841	-0.158159
4	6	1.000000	-1.341841	3.341841
5	6	3.500000	1.158159	5.841841

From above table, we can see the $(MSD)_{Tukey}$ = 1.989852 – (- 1.989852) = 3.979704

$(MSD)_{Scheffe}$ = 2.341841 – (-2.341841) = 4.683682

Thus, $(MSD)_{Tukey} < (MSD)_{Scheffe}$ and so $W_T < W_S$ and so the Tukey procedure is more efficient than the Scheffe procedure.

b) Line Plots:

The GLM Procedure
Least Squares Means
Adjustment for Multiple Comparisons: Tukey

A	B	Yij LSMEAN	LSMEAN Number
1	1	8.0000000	1
1	2	2.5000000	2
2	1	10.5000000	3
2	2	9.0000000	4
3	1	11.5000000	5
3	2	8.0000000	6

We observe the above SAS output and calculate the mean response differences between the treatments from it below:

Since the smallest mean response i.e., mean response of Great Value Everyday Strong with vegetable oil's difference with next smallest mean response i.e., mean response of Great Value Everyday Strong with vegetable oil is 8.0 – 2.5 = 5.5 > $(MSD)_{Tukey}$ = 3.979704, we conclude that for treatment combination Great Value Everyday Strong with vegetable oil; we reject $H_{0,}$ and it is thus statistically significant/different at level of significance = 0.05. Thus, other remaining treatment's mean response differences with that of Great Value Everyday Strong with vegetable oil are statistically different as well, as these estimated differences > $(MSD)_{Tukey}$.

Furthermore, the other remaining treatment's mean response differences which are statistically same (as they are lesser than the $(MSD)_{Tukey}$) are:

Great Value Everyday Strong with water and Great Value Ultra Strong with water,

Great Value Everyday Strong with water and Bounty Essentials with water,

Great Value Ultra Strong with water and Bounty Essentials with water,

Bounty Essentials with vegetable oil and Great Value Ultra Strong with water,

Bounty Essentials with vegetable oil and Bounty Essentials with water,

Great Value Ultra Strong with vegetable oil and Bounty Essentials with water,

Great Value Ultra Strong with vegetable oil and Great Value Everyday Strong with water,

Great Value Ultra Strong with vegetable oil and Great Value Ultra Strong with water,

Great Value Everyday Strong with water and Bounty Essentials with vegetable oil,

Bounty Essentials with vegetable oil and Great Value Ultra Strong with vegetable oil.

In the line plot below, the treatments Great Value Everyday Strong with water and Bounty Essentials with vegetable oil share the same position as their mean responses are the same.

Finally, constructing the line plot (from the above information) below:

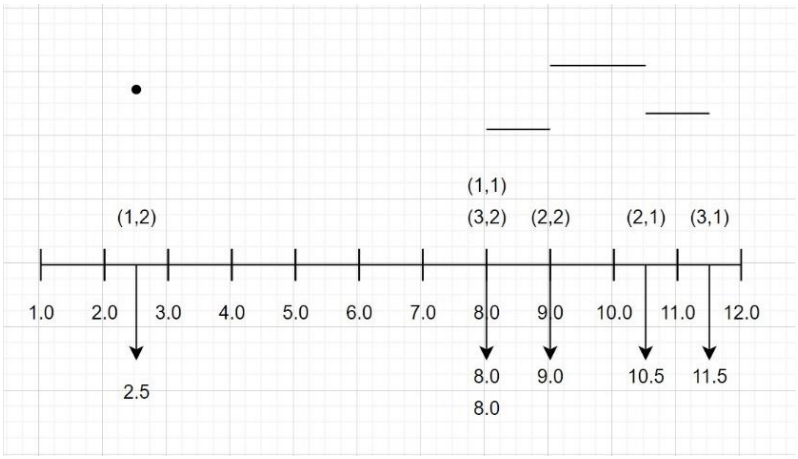


Figure 4.b

c) **Multiple Comparisons for Pre-Selected Set of Three Differences:**

H_0: D = 0

H_1: D ≠ 0

The value of m chosen prior to data collection is m = 4 for level of significance, alpha = 0.05. The first two comparisons (D) are between Great Value Everyday Strong with water and Great Value Ultra Strong with water, and Great Value Everyday Strong with water and Bounty Essentials with water. The first is chosen because both belong to the same company "Great Value" just differing in price with Great Value Ultra Strong being slightly more expensive, and the solution is the same (water) for fairness. The second difference was chosen to compare the most expensive paper towel (Bounty Essentials) and least expensive (Great Value Everyday Strong) with water solution. The third comparison is between Great Value Ultra Strong with vegetable oil and Bounty Essentials with vegetable oil. This is to compare the most expensive brands in this study with the same solution (vegetable oil) for the sake of fairness.

The GLM Procedure
Least Squares Means
Adjustment for Multiple Comparisons: Bonferroni

A	B	Yij LSMEAN	LSMEAN Number
1	1	8.0000000	1
1	2	2.5000000	2
2	1	10.5000000	3
2	2	9.0000000	4
3	1	11.5000000	5
3	2	8.0000000	6

Least Squares Means for Effect A*B				
i	j	Difference Between Means	Simultaneous 95% Confidence Limits for LSMean(i)-LSMean(j)	
1	2	5.500000	3.151039	7.848961
1	3	-2.500000	-4.848961	-0.151039
1	4	-1.000000	-3.348961	1.348961
1	5	-3.500000	-5.848961	-1.151039
1	6	0	-2.348961	2.348961
2	3	-8.000000	-10.348961	-5.651039
2	4	-6.500000	-8.848961	-4.151039
2	5	-9.000000	-11.348961	-6.651039
2	6	-5.500000	-7.848961	-3.151039
3	4	1.500000	-0.848961	3.848961
3	5	-1.000000	-3.348961	1.348961
3	6	2.500000	0.151039	4.848961
4	5	-2.500000	-4.848961	-0.151039
4	6	1.000000	-1.348961	3.348961
5	6	3.500000	1.151039	5.848961

The chosen differences are shown below, with the values obtained from the SAS output above:

$\widehat{D}_1 = \hat{\mu}_{11} - \hat{\mu}_{21}$ = -2.5, CI of $\widehat{D}_1$ = (-4.848961, -0.151039)

$\widehat{D}_2 = \hat{\mu}_{11} - \hat{\mu}_{31}$ = -3.5, CI of $\widehat{D}_2$ = (-5.848961, -1.151039)

$\widehat{D}_3 = \hat{\mu}_{22} - \hat{\mu}_{32}$ = 1.0, CI of $\widehat{D}_3$= (-1.348961, 3.348961)

α, level of significance = 0.05

Since CI of $\widehat{D}_1$ does not contain 0, and CI of $\widehat{D}_2$ does contain 0, we fail to reject H_0 at the level of significance 0.05.

Also, CI of $\widehat{D}_3$ contains 0, we reject H_0 at the level of significance 0.05.

Thus, we are 95% confident that true mean of the amount of solution absorbed by the paper towel (in mL) is different for Great Value Everyday Strong with water and Great Value Ultra Strong with water, Great Value Everyday Strong with water and Bounty Essentials with water and is same for Great Value Ultra Strong with vegetable oil and Bounty Essentials with vegetable oil, simultaneously.

5. Final Discussion.

Present a final discussion (summary, conclusions, other possible factors) in paragraph form.

The goal of this experiment was to the decide which brand of paper towel is the better one for the absorption of the two solutions – overall, in the same time period. A full interaction model was assumed to be the ANOVA model for these response and factor variables. From the preliminary plots, we observed that Great Value Everyday Strong seemed to have much worse absorbency for vegetable oil than Great Value Ultra Strong and Bounty Essentials. Also, Bounty Essentials seemed to have better absorbency for water whereas Great Value Ultra Strong seems to have better absorbency for vegetable oil. Next, several plots were constructed to perform a preliminary interpretation of the ANOVA model assumptions, specifically the constant variance and normality assumptions were of most importance. The linearity and Modified-Levene tests were used to confirm the normality and constant variance results from the interpretation of the two previously mentioned plots. A weighted least square (WLS) transformation was the required next step as the constant variance assumption was violated from the Modified-Levene test and normality was satisfied from the linearity test. However, given that further steps could not be performed with a WLS transformation, a mere Box-Cox transformation was attempted with the square transformation of the response, was chosen as the preferred transformation.

Analysis of variance was performed to check the significance of interaction terms in the model. The result was that combination of both factors were significant and thus, they were significant independently as well. Thus, the full interaction model was confirmed for future work as well.

We have observed from the constructed line plot that the Great Value Everyday Strong paper towel is not up to the mark to absorb a satisfactory amount of vegetable oil i.e., it was very poor compared to the other brands. However, it performed satisfactorily when it was tested against absorbing water. Bounty Essentials performed the bare minimum when it was tested against absorbing vegetable oil but was the outright best when it came to testing against absorbing water. Finally, Great Value Ultra Strong performed the second best when it came to testing it against absorbing water but was the best when it came to testing against absorbing vegetable oil.

Other possible factors that influence the absorbency of paper towel brands and might be examined in a future experiment include: Immersion Time, Sheet Size, Temperature and Humidity.

To sum up, we would not recommend buying this brand due to its appalling performance when tested against absorbing vegetable oil, even though it performed to the bare minimum levels when tested against absorbing water. Great Value Ultra Strong and Bounty Essentials performed better when tested against absorbing water and vegetable oil, respectively. It should be noted that they performed similarly against water and vegetable oil. However, Great Value Ultra Strong was cheaper than Bounty Essentials by about a dollar. Thus, for frequent purchasers of paper towels, unless one for some reason prioritizes a slightly higher absorbency of water (Bounty Essentials) over an overall great performance on the two common types of solutions, we would recommend going for the Great Value Ultra Strong paper towel.

Appendix:

Obs	A	B	Yij
1	2	1	11
2	2	2	9
3	1	1	8
4	2	2	9
5	3	2	8
6	3	2	8
7	1	2	2
8	1	2	3
9	3	1	12
10	1	1	8
11	2	1	10
12	3	1	11